Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.4-d+e-x-^m-f+g-x-^n-a+b-x+c-x^2-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [632]. This is test number [22].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (632)	0.00 (0)
Mathematica	100.00 (632)	0.00 (0)
Maple	99.84 (631)	0.16 (1)
Fricas	93.51 (591)	6.49 (41)
IntegrateAlgebraic	76.90 (486)	23.10 (146)
Maxima	51.90 (328)	48.10 (304)
Giac	43.67 (276)	56.33 (356)
Mupad	42.88 (271)	57.12 (361)
Sympy	27.53 (174)	% 72.47 (458)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

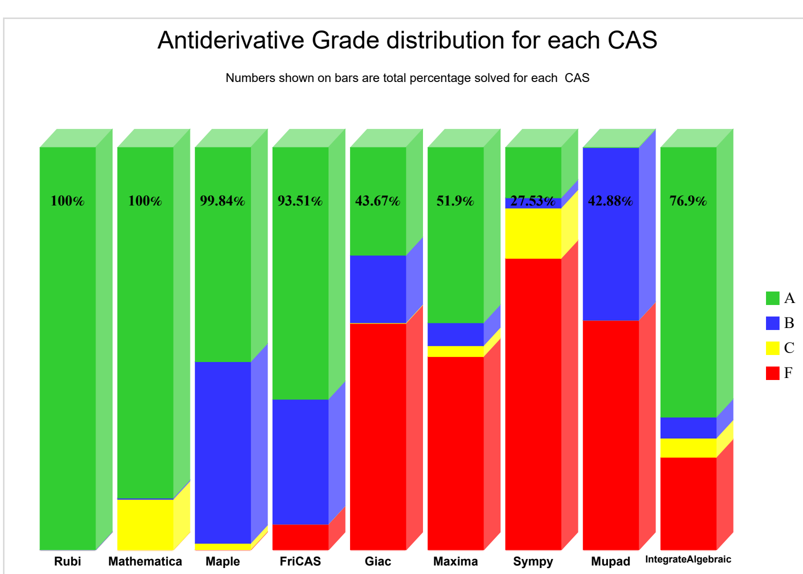
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

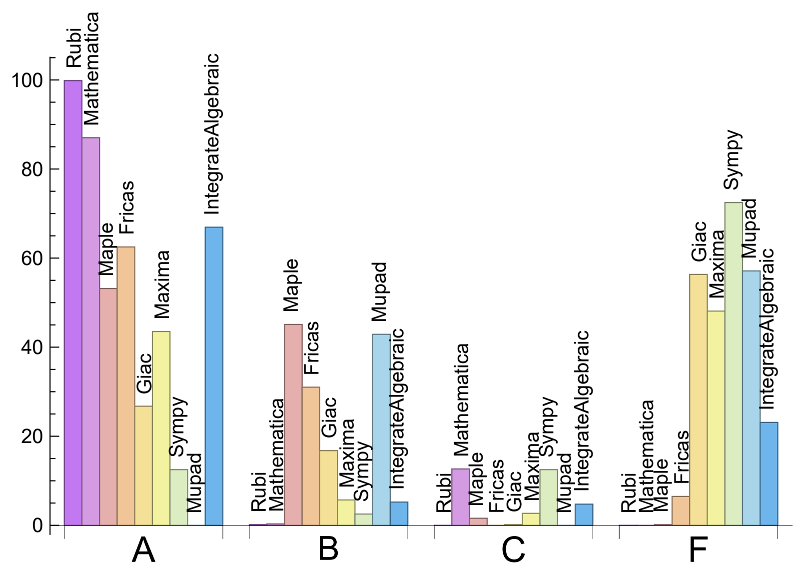
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.84	0.16	0.00	0.00
Mathematica	87.03	0.32	12.66	0.00
IntegrateAlgebraic	66.93	5.22	4.75	23.10
Fricas	62.50	31.01	0.00	6.49
Maple	53.16	45.09	1.58	0.16
Maxima	43.51	5.70	2.69	48.10
Giac	26.74	16.77	0.16	56.33
Sympy	12.50	2.53	12.50	72.47
Mupad	N/A	42.88	0.00	57.12

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Fricas	41	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	146	47.95 %	52.05 %	0.00 %
Giac	356	14.89 %	27.53 %	57.58 %
Maxima	304	72.04 %	0.99 %	26.97 %
Sympy	458	65.72 %	33.84 %	0.44 %
Mupad	361	98.06 %	1.94 %	0.00 %

Table 1.4: Failure statistics for each CAS

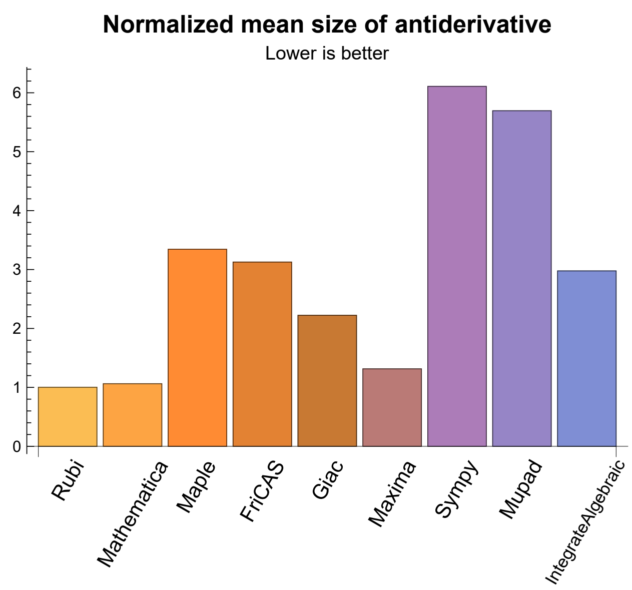
1.3 Performance

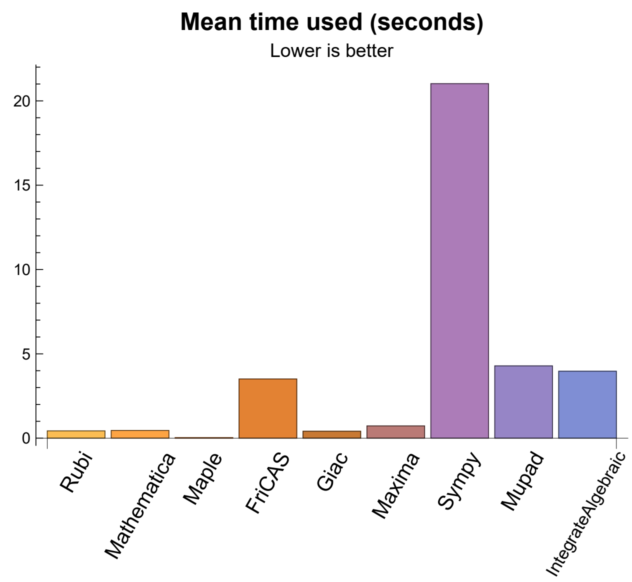
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.43	201.39	1.00	163.00	1.00
Mathematica	0.46	183.93	1.06	124.00	0.87
Maple	0.03	1172.33	3.34	260.00	1.69
Maxima	0.73	200.40	1.31	151.50	1.19
Fricas	3.51	724.40	3.13	258.00	1.92
Sympy	21.02	967.48	6.11	386.00	3.31
Giac	0.41	517.79	2.22	188.50	1.49
Mupad	4.29	2066.54	5.70	162.00	1.29
IntegrateAlgebraic	3.97	562.49	2.98	148.00	1.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
5
10
15
20
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {313, 328, 330, 332, 406, 576}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

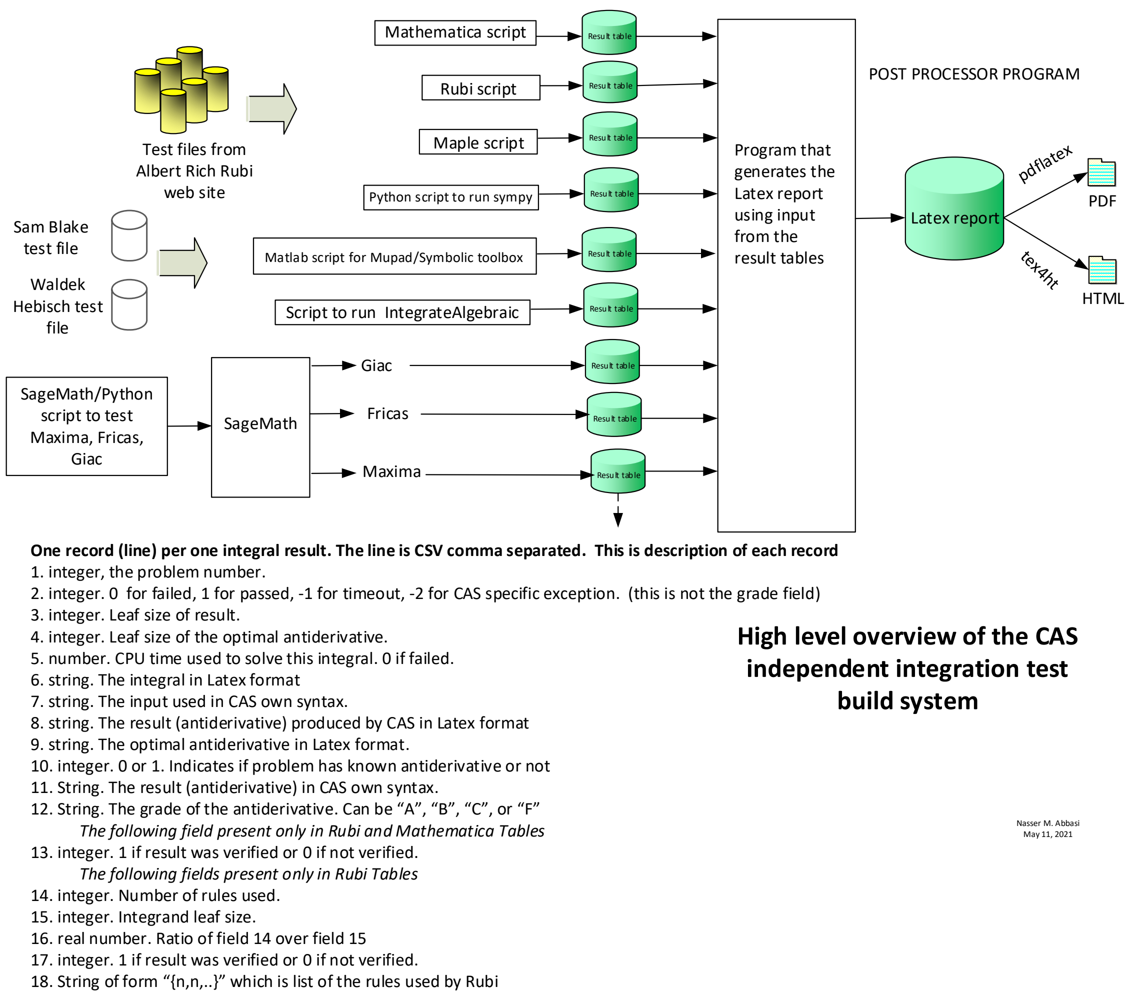
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632 }

B grade: { 576 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 327, 329, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 391, 392, 393, 394, 395, 396, 398, 399, 400, 401, 405, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 423, 424, 425, 426, 427, 428, 429, 432, 433, 434, 435, 439, 440, 441, 442, 446, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 460, 461, 463, 467, 468, 469, 470, 471, 472, 475, 479, 480, 481, 482, 483, 484, 485, 486, 489, 490, 491, 492, 493, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 513, 514, 515, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631 }

B grade: { 269, 632 }

C grade: { 8, 9, 10, 11, 13, 14, 15, 42, 43, 50, 51, 52, 53, 62, 63, 64, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 91, 226, 236, 250, 251, 252, 313, 324, 326, 328, 330, 332, 389, 390, 397, 402, 403, 404, 406, 421, 422, 430, 431, 436, 437, 438, 443, 444, 445, 453, 454, 455, 462, 464, 465, 466, 473, 474, 476, 477, 478, 487, 488, 494, 495, 512, 521, 522, 542, 581, 582 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 96, 103, 107, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 161, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 198, 200, 201, 211, 212, 213, 214, 215, 216, 218, 219, 224, 226, 237, 238, 239, 243, 244, 252, 261, 263, 264, 274, 304, 305, 306, 307, 308, 309, 310, 314, 315, 316, 321, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 350, 351, 352, 353, 354, 355, 359, 360, 361, 362, 363, 364, 365, 368, 371, 372, 373, 374, 375, 379, 380, 384, 385, 386, 390, 391, 392, 393, 394, 395, 398, 399, 400, 401, 402, 406, 421, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 467, 468, 469, 470, 471, 475, 479, 480, 481, 482, 483, 484, 485, 486, 489, 490, 491, 492, 493, 495, 496, 497, 498, 499, 502, 503, 504, 505, 506, 507, 508, 511, 513, 514, 515, 516, 517, 520, 524, 525, 526, 527, 528, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 556, 557, 563, 564, 565, 566, 570, 571, 572, 583, 584, 591, 592, 618 }

B grade: { 9, 10, 11, 45, 78, 79, 84, 85, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 136, 137, 138, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 191, 194, 195, 196, 197, 199, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 220, 221, 222, 223, 225, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 262, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 311, 312, 313, 317, 318, 319, 320, 322, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 366, 367, 369, 370, 376, 377, 378, 381, 382, 383, 387, 388, 389, 396, 397, 403, 404, 405, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 445, 455, 465, 466, 472, 473, 474, 476, 477, 478, 487, 488, 494, 500, 501, 509, 510, 512, 518, 519, 521, 522, 523, 540, 541, 542, 553, 554, 555, 558, 559, 560, 561, 562, 567, 568, 569, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 585, 586, 587, 588, 589, 590, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632 }

C grade: { 543, 544, 545, 546, 547, 548, 549, 550, 551, 552 }

F grade: { 533 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 137, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 182, 183, 184, 202, 203, 220, 224, 227, 228, 229, 230, 231, 232, 237, 238, 239, 240, 241, 245, 246, 247, 248, 249, 253, 254, 255, 256, 257, 258, 262, 263, 264, 265, 266, 267, 323, 325, 327, 329, 331, 333, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 386, 391, 392, 393, 394, 398, 399, 400, 401, 424, 425, 426, 427, 432, 433, 434, 435, 439, 440, 441, 442, 446, 447, 448, 449, 450, 456, 457, 458, 459, 460, 467, 468, 469, 470, 471, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 543, 544, 545, 548, 549, 550, 555, 556, 557, 558, 559, 562, 563, 564, 565, 569, 570, 571, 572 }

B grade: { 11, 19, 20, 21, 22, 44, 45, 83, 84, 85, 136, 138, 192, 193, 211, 212, 213, 214, 215, 222, 268, 269, 270, 283, 294, 381, 382, 383, 384, 385, 553, 554, 629, 630, 631, 632 }

C grade: { 102, 103, 104, 105, 106, 107, 157, 158, 159, 160, 161, 162, 198, 199, 200, 201, 406 }

F grade: { 98, 99, 100, 101, 124, 125, 126, 133, 134, 135, 144, 145, 146, 147, 154, 155, 156, 176, 177, 178, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 204, 205, 206, 207, 208, 209, 210, 216, 217, 218, 219, 221, 223, 225, 226, 233, 234, 235, 236, 242, 243, 244, 250, 251, 252, 259, 260, 261, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 326, 328, 330, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 387, 388, 389, 390, 395, 396, 397, 402, 403, 404, 405, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 428, 429, 430, 431, 436, 437, 438, 443, 444, 445, 451, 452, 453, 454, 455, 461, 462, 463, 464, 465, 466, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 539, 540, 541, 542, 546, 547, 551, 552, 560, 561, 566, 567, 568, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 28, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 222, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 252, 259, 260, 261, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 350, 351, 352, 353, 354, 359, 360, 361, 362, 364, 371, 372, 374, 375, 385, 386, 391, 392, 393, 394, 395, 398, 399, 400, 401, 405, 406, 422, 423, 424, 425, 426, 427, 428, 432, 433, 434, 435, 439, 440, 441, 446, 447, 448, 449, 450, 451, 456, 457, 458, 459, 460, 461, 462, 467, 468, 469, 470, 472, 473, 474, 479, 480, 481, 482, 487, 488, 489, 494, 495, 500, 501, 502, 503, 504, 509, 510, 511, 512, 513, 518, 519, 520, 521, 522, 523, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 543, 544, 545, 548, 549, 557, 558, 559, 562, 563, 564, 565, 566, 569, 570, 571, 572, 577, 578, 579, 580, 583, 585, 586, 587, 588, 600 }

B grade: { 18, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 131, 132, 138, 139, 140, 141, 142, 143, 144, 148, 149, 216, 221, 223, 225, 241, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 262, 263, 264, 265, 266, 267, 268, 269, 270, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 355, 356, 357, 358, 363, 365, 366, 367, 368, 369, 370, 373, 376, 377, 378, 379, 380, 381, 382, 383, 384, 387, 388, 389, 390, 396, 397, 402, 403, 404, 409, 410, 413, 414, 415, 417, 418, 420, 421, 429, 430, 431, 436, 437, 438, 442, 443, 444, 445, 452, 453, 454, 455, 463, 464, 465, 466, 471, 475, 476, 477, 478, 483, 484, 485, 486, 490, 491, 492, 493, 496, 497, 498, 499, 505, 506, 507, 508, 514, 515, 516, 517, 524, 525, 526, 527, 528, 540, 541, 542, 546, 550, 553, 554, 555, 556, 567, 568, 573, 574, 575, 576, 581, 582, 584, 589, 590, 591, 592, 594, 616, 617, 618, 623, 624, 625, 629, 630, 631, 632 }

C grade: { }

F grade: { 303, 349, 407, 408, 411, 412, 416, 419, 547, 551, 552, 560, 561, 593, 595, 596, 597, 598, 599, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 619, 620, 621, 622, 626, 627, 628 }

2.1.6 Sympy

A grade: { 3, 5, 6, 25, 32, 34, 36, 37, 54, 55, 56, 57, 58, 62, 65, 67, 69, 104, 106, 117, 157, 159, 161, 220, 222, 224, 226, 262, 263, 264, 265, 266, 267, 333, 338, 350, 351, 352, 353, 359, 360, 361, 362, 363, 364, 368, 371, 372, 373, 374, 375, 378, 379, 380, 390, 392, 393, 394, 395, 398, 399, 400, 401, 402, 554, 555, 556, 562, 563, 564, 565, 566, 569, 570, 571, 572, 573, 629, 630 }

B grade: { 19, 21, 23, 354, 355, 356, 357, 358, 365, 366, 367, 369, 370, 376, 377, 557 }

C grade: { 1, 2, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 26, 27, 28, 29, 30, 31, 33, 35, 38, 39, 40, 41, 42, 43, 59, 60, 61, 63, 64, 66, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 102, 103, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 158, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 406, 545 }

F grade: { 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 225, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 381, 382, 383, 384, 385, 386, 387, 388, 389, 391, 396, 397, 403, 404, 405, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 546, 547, 548, 549, 550, 551, 552, 553, 558, 559, 560, 561, 567, 568, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 631, 632 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 117, 130, 134, 151, 152, 153, 154, 186, 187, 195, 196, 197, 219, 226, 227, 228, 229, 230, 231, 233, 234, 235, 237, 238, 239, 240, 241, 243, 244, 246, 247, 248, 249, 251, 252, 259, 327, 333, 338, 348, 350, 354, 359, 360, 365, 366, 377, 378, 383, 384, 385, 386, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 423, 531, 532, 543, 544, 545, 557, 558, 559, 560, 563, 564, 565, 566, 567, 571, 572, 573, 574, 576, 577, 578, 579, 580, 581, 585, 588, 617 }

B grade: { 9, 10, 11, 12, 13, 14, 15, 40, 41, 42, 43, 60, 61, 62, 63, 64, 75, 76, 77, 78, 79, 80, 81, 82, 98, 118, 156, 236, 245, 262, 263, 264, 265, 266, 267, 268, 269, 270, 323, 325, 334, 335, 336, 337, 339, 341, 342, 343, 344, 345, 346, 347, 349, 351, 352, 353, 361, 362, 363, 364, 371, 372, 373, 374, 375, 376, 381, 382, 387, 389, 398, 421, 422, 528, 529, 530, 546, 548, 549, 550, 553, 554, 555, 556, 561, 562, 568, 569, 570, 575, 582, 583, 584, 586, 587, 589, 603, 620, 623, 624, 625, 628, 629, 630, 631, 632 }

C grade: { 388 }

F grade: { 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 232, 242, 250, 253, 254, 255, 256, 257, 258, 260, 261, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 326, 328, 329, 330, 331, 332, 340, 355, 356, 357, 358, 367, 368, 369, 370, 379, 380, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 547, 551, 552, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 618, 619, 621, 622, 626, 627 }

2.1.8 Mupad

A grade: { }

B grade: { 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 86, 87, 88, 117, 118, 123, 130, 131, 132, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 172, 173, 174, 175, 182, 183, 184, 192, 193, 210, 211, 212, 213, 214, 215, 220, 222, 224, 226, 262, 263, 264, 265, 266, 267, 268, 269, 270, 307, 314, 315, 316, 321, 322, 323, 325, 327, 329, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 384, 385, 386, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 420, 421, 422, 423, 424, 425, 426, 427, 432, 433, 434, 435, 439, 440, 441, 442, 446, 447, 448, 449, 450, 456, 457, 458, 459, 460, 467, 468, 469, 470, 471, 483, 484, 485, 486, 490, 491, 492, 493, 496, 497, 498, 499, 505, 506, 507, 508, 514, 515, 516, 517, 524, 525, 526, 527, 528, 529, 530, 531, 532, 534, 535, 536, 537, 538, 543, 544, 545, 546, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 579, 580, 583, 584, 587, 588, 591, 592, 629, 630, 631, 632 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 10, 11, 19, 20, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 50, 51, 52, 53, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 133, 134, 135, 136, 137, 138, 144, 145, 146, 147, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 176, 177, 178, 179, 180, 181, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 216, 217, 218, 219, 221, 223, 225, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 317, 318, 319, 320, 324, 326, 328, 330, 332, 381, 382, 383, 387, 388, 389, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 428, 429, 430, 431, 436, 437, 438, 443, 444, 445, 451, 452, 453, 454, 455, 461, 462, 463, 464, 465, 466, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 487, 488, 489, 494, 495, 500, 501, 502, 503, 504, 509, 510, 511, 512, 513, 518, 519, 520, 521, 522, 523, 533, 539, 540, 541, 542, 547, 548, 549, 550, 551, 552, 578, 581, 582, 585, 586, 589, 590, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628 }

2.1.9 IntegrateAlgebraic

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 222, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 273, 275, 276, 277, 278, 279, 284, 285, 286, 287, 288, 299, 305, 307, 308, 309, 310, 320, 328, 335, 336, 339, 340, 341, 344, 381, 382, 383, 384, 385, 386, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 421, 424, 425, 426, 427, 432, 433, 434, 435, 439, 440, 441, 442, 448, 449, 450, 458, 459, 460, 461, 462, 470, 471, 472, 473, 479, 480, 481, 482, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 515, 519, 520, 521, 522, 523, 533, 535, 536, 537, 538, 545, 546, 547, 551, 563, 564, 565, 566, 567, 568, 570, 571, 572, 573, 574, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 595, 596, 597, 598, 599, 600, 601, 602, 607, 608, 613, 614, 615, 616, 617, 618, 622, 623, 624, 625, 626 }

B grade: { 98, 241, 272, 274, 283, 306, 313, 314, 315, 316, 406, 422, 423, 446, 447, 483, 505, 514, 524, 534, 543, 544, 548, 549, 550, 562, 569, 575, 606, 609, 610, 619, 621 }

C grade: { 218, 226, 334, 337, 338, 342, 343, 345, 346, 347, 348, 349, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 428, 451, 452, 539 }

F grade: { 221, 223, 225, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 311, 312, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 387, 388, 389, 429, 430, 431, 436, 437, 438, 443, 444, 445, 453, 454, 455, 456, 457, 463, 464, 465, 466, 467, 468, 469, 474, 475, 476, 477, 478, 499, 516, 517, 518, 525, 526, 527, 528, 529, 530, 531, 532, 540, 541, 542, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 594, 603, 604, 605, 611, 612, 620, 627, 628, 629, 630, 631, 632 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	112	125	104	95	279	74	-1	114
N.S.	1	1.00	0.85	0.95	0.79	0.72	2.11	0.56	-0.01	0.86
time (sec)	N/A	0.076	0.124	0.059	0.983	0.397	5.478	0.258	0.000	0.889

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	157	198	177	138	830	117	-1	158
N.S.	1	1.00	0.78	0.99	0.88	0.69	4.13	0.58	-0.00	0.79
time (sec)	N/A	0.149	0.214	0.045	0.989	0.399	17.835	0.220	0.000	0.502

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	146	173	152	127	775	106	-1	147
N.S.	1	1.00	0.85	1.01	0.88	0.74	4.51	0.62	-0.01	0.85
time (sec)	N/A	0.101	0.191	0.023	0.977	0.413	17.170	0.226	0.000	0.458

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	135	148	127	116	653	96	-1	136
N.S.	1	1.00	0.85	0.93	0.80	0.73	4.11	0.60	-0.01	0.86
time (sec)	N/A	0.108	0.175	0.045	0.949	0.397	12.267	0.227	0.000	0.448

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	124	123	102	105	580	84	-1	125
N.S.	1	1.00	1.07	1.06	0.88	0.91	5.00	0.72	-0.01	1.08
time (sec)	N/A	0.034	0.142	0.015	0.984	0.403	12.059	0.212	0.000	0.407

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	124	123	102	105	580	84	-1	125
N.S.	1	1.00	1.07	1.06	0.88	0.91	5.00	0.72	-0.01	1.08
time (sec)	N/A	0.033	0.042	0.000	0.980	0.415	12.180	0.340	0.000	0.001

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	124	151	124	107	469	99	107	142
N.S.	1	1.00	1.10	1.34	1.10	0.95	4.15	0.88	0.95	1.26
time (sec)	N/A	0.098	0.184	0.019	0.985	0.414	22.411	0.273	2.904	0.510

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	124	182	129	124	386	157	114	143
N.S.	1	1.00	1.06	1.56	1.10	1.06	3.30	1.34	0.97	1.22
time (sec)	N/A	0.092	0.174	0.033	0.990	0.408	8.182	0.254	3.513	0.480

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	110	212	160	133	461	217	120	146
N.S.	1	1.00	0.91	1.75	1.32	1.10	3.81	1.79	0.99	1.21
time (sec)	N/A	0.094	0.077	0.022	0.988	0.415	9.533	0.276	3.735	0.685

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	111	235	184	129	457	261	-1	141
N.S.	1	1.00	0.92	1.96	1.53	1.08	3.81	2.18	-0.01	1.18
time (sec)	N/A	0.092	0.060	0.022	0.982	0.417	8.932	0.234	0.000	0.591

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	B	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	133	260	210	119	541	297	-1	141
N.S.	1	1.00	1.13	2.20	1.78	1.01	4.58	2.52	-0.01	1.19
time (sec)	N/A	0.092	0.089	0.025	0.983	0.408	11.056	0.232	0.000	0.595

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	133	158	155	98	774	368	93	155
N.S.	1	1.00	1.23	1.46	1.44	0.91	7.17	3.41	0.86	1.44
time (sec)	N/A	0.063	0.061	0.033	0.994	0.395	11.365	0.246	4.264	0.621

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	59	186	180	109	918	431	118	126
N.S.	1	1.00	0.41	1.30	1.26	0.76	6.42	3.01	0.83	0.88
time (sec)	N/A	0.095	0.021	0.031	0.988	0.427	15.254	0.276	4.660	0.616

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	72	211	205	120	1037	494	192	137
N.S.	1	1.00	0.42	1.23	1.19	0.70	6.03	2.87	1.12	0.80
time (sec)	N/A	0.125	0.020	0.042	0.990	0.421	16.620	0.256	5.334	0.684

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	73	236	230	131	1159	431	212	148
N.S.	1	1.00	0.36	1.17	1.14	0.65	5.77	2.14	1.05	0.74
time (sec)	N/A	0.156	0.022	0.073	0.995	0.446	22.545	0.256	6.044	0.741

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	70	102	81	72	177	54	112	92
N.S.	1	1.00	0.68	0.99	0.79	0.70	1.72	0.52	1.09	0.89
time (sec)	N/A	0.053	0.038	0.019	0.985	0.405	5.254	0.245	3.138	0.242

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	77	99	78	87	184	66	87	84
N.S.	1	1.00	1.05	1.36	1.07	1.19	2.52	0.90	1.19	1.15
time (sec)	N/A	0.042	0.031	0.019	0.979	0.403	9.708	0.252	2.956	0.466

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	52	55	88	104	231	51	55	59
N.S.	1	1.00	0.90	0.95	1.52	1.79	3.98	0.88	0.95	1.02
time (sec)	N/A	0.026	0.021	0.009	0.444	0.391	9.883	0.295	2.590	0.427

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	155	227	312	278	2004	120	-1	152
N.S.	1	1.00	0.96	1.41	1.94	1.73	12.45	0.75	-0.01	0.94
time (sec)	N/A	0.139	0.097	0.074	1.029	0.467	66.418	0.279	0.000	0.777

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	142	195	278	263	1821	109	-1	137
N.S.	1	1.00	0.97	1.33	1.89	1.79	12.39	0.74	-0.01	0.93
time (sec)	N/A	0.120	0.087	0.025	1.032	0.420	62.081	0.272	0.000	0.732

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	130	166	250	247	1739	97	-1	125
N.S.	1	1.00	1.07	1.36	2.05	2.02	14.25	0.80	-0.01	1.02
time (sec)	N/A	0.081	0.081	0.021	1.011	0.408	73.140	0.308	0.000	0.679

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	82	77	159	171	418	64	78	82
N.S.	1	1.00	0.98	0.92	1.89	2.04	4.98	0.76	0.93	0.98
time (sec)	N/A	0.052	0.025	0.010	0.453	0.391	63.869	0.272	2.701	0.521

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	82	77	134	172	337	58	78	82
N.S.	1	1.00	0.91	0.86	1.49	1.91	3.74	0.64	0.87	0.91
time (sec)	N/A	0.042	0.023	0.010	0.448	0.407	20.548	0.278	2.659	0.482

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	82	77	112	173	513	64	78	82
N.S.	1	1.00	0.87	0.82	1.19	1.84	5.46	0.68	0.83	0.87
time (sec)	N/A	0.046	0.025	0.009	0.438	0.416	21.308	0.269	2.615	0.517

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	82	77	87	172	432	57	78	82
N.S.	1	1.00	0.99	0.93	1.05	2.07	5.20	0.69	0.94	0.99
time (sec)	N/A	0.023	0.035	0.009	0.436	0.415	22.679	0.267	2.621	0.507

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	82	77	80	171	604	65	78	82
N.S.	1	1.00	1.02	0.96	1.00	2.14	7.55	0.81	0.98	1.02
time (sec)	N/A	0.021	0.029	0.009	0.435	0.420	24.406	0.260	2.584	0.002

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	131	163	157	244	2378	122	127	122
N.S.	1	1.00	1.12	1.39	1.34	2.09	20.32	1.04	1.09	1.04
time (sec)	N/A	0.103	0.063	0.013	0.445	0.423	41.141	0.274	3.079	0.724

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	147	195	189	270	2404	189	141	137
N.S.	1	1.00	0.96	1.27	1.24	1.76	15.71	1.24	0.92	0.90
time (sec)	N/A	0.127	0.073	0.016	0.463	0.442	31.225	0.286	3.305	0.610

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	183	227	221	291	2691	260	181	150
N.S.	1	1.00	0.99	1.23	1.20	1.58	14.62	1.41	0.98	0.82
time (sec)	N/A	0.160	0.135	0.016	0.468	0.511	35.060	0.359	3.428	0.813

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	104	99	135	239	903	77	164	104
N.S.	1	1.00	0.86	0.82	1.12	1.98	7.46	0.64	1.36	0.86
time (sec)	N/A	0.053	0.040	0.012	0.440	0.469	22.733	0.274	2.690	0.554

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	126	121	158	305	1401	90	202	126
N.S.	1	1.00	0.85	0.82	1.07	2.06	9.47	0.61	1.36	0.85
time (sec)	N/A	0.062	0.049	0.014	0.448	0.762	48.458	0.298	2.737	0.623

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	50	85	63	66	102	70	84	74
N.S.	1	1.00	0.93	1.57	1.17	1.22	1.89	1.30	1.56	1.37
time (sec)	N/A	0.034	0.029	0.016	0.964	0.401	8.328	0.211	0.088	0.409

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	103	174	153	105	558	84	-1	125
N.S.	1	1.00	0.60	1.01	0.88	0.61	3.23	0.49	-0.01	0.72
time (sec)	N/A	0.227	0.099	0.028	0.978	0.410	13.484	0.269	0.000	0.416

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	92	149	128	94	357	73	-1	114
N.S.	1	1.00	0.64	1.03	0.89	0.65	2.48	0.51	-0.01	0.79
time (sec)	N/A	0.186	0.090	0.010	0.973	0.413	7.871	0.263	0.000	0.420

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	81	124	103	83	386	63	-1	103
N.S.	1	1.00	0.70	1.08	0.90	0.72	3.36	0.55	-0.01	0.90
time (sec)	N/A	0.145	0.070	0.011	0.968	0.398	9.327	0.252	0.000	0.388

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	69	98	77	71	218	49	-1	89
N.S.	1	1.00	0.83	1.18	0.93	0.86	2.63	0.59	-0.01	1.07
time (sec)	N/A	0.085	0.054	0.009	0.970	0.401	5.617	0.253	0.000	0.389

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	58	71	53	60	269	40	-1	81
N.S.	1	1.00	0.70	0.86	0.64	0.72	3.24	0.48	-0.01	0.98
time (sec)	N/A	0.028	0.036	0.008	0.970	0.414	5.044	0.251	0.000	0.014

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	66	91	62	73	184	65	-1	104
N.S.	1	1.00	1.00	1.38	0.94	1.11	2.79	0.98	-0.02	1.58
time (sec)	N/A	0.111	0.027	0.010	0.971	0.397	6.958	0.257	0.000	0.382

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	93	64	79	207	107	-1	102
N.S.	1	1.00	1.00	1.37	0.94	1.16	3.04	1.57	-0.01	1.50
time (sec)	N/A	0.115	0.030	0.011	0.962	0.403	4.296	0.256	0.000	0.404

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	122	86	83	63	214	170	-1	122
N.S.	1	1.00	1.52	1.08	1.04	0.79	2.68	2.12	-0.01	1.52
time (sec)	N/A	0.109	0.243	0.012	0.951	0.392	6.721	0.269	0.000	0.462

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	87	114	108	74	303	239	-1	91
N.S.	1	1.00	0.81	1.07	1.01	0.69	2.83	2.23	-0.01	0.85
time (sec)	N/A	0.137	0.150	0.013	0.970	0.402	6.090	0.305	0.000	0.474

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	155	139	133	87	449	305	-1	104
N.S.	1	1.00	1.11	0.99	0.95	0.62	3.21	2.18	-0.01	0.74
time (sec)	N/A	0.172	0.148	0.015	0.970	0.402	10.316	0.290	0.000	0.546

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	79	164	158	98	510	365	-1	115
N.S.	1	1.00	0.47	0.97	0.93	0.58	3.02	2.16	-0.01	0.68
time (sec)	N/A	0.195	0.040	0.016	0.984	0.396	8.957	0.272	0.000	0.641

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	111	193	276	188	0	106	-1	126
N.S.	1	1.00	0.78	1.35	1.93	1.31	0.00	0.74	-0.01	0.88
time (sec)	N/A	0.271	0.212	0.010	1.016	0.402	0.000	0.298	0.000	0.652

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	96	236	298	172	0	95	-1	114
N.S.	1	1.00	0.79	1.95	2.46	1.42	0.00	0.79	-0.01	0.94
time (sec)	N/A	0.208	0.202	0.018	1.004	0.420	0.000	0.280	0.000	0.617

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	63	65	155	116	0	63	66	70
N.S.	1	1.00	0.65	0.67	1.60	1.20	0.00	0.65	0.68	0.72
time (sec)	N/A	0.173	0.060	0.008	0.445	0.400	0.000	0.287	2.893	0.493

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	63	66	131	117	0	61	67	70
N.S.	1	1.00	0.72	0.76	1.51	1.34	0.00	0.70	0.77	0.80
time (sec)	N/A	0.120	0.055	0.010	0.445	0.400	0.000	0.284	2.865	0.481

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	62	64	109	117	0	64	65	69
N.S.	1	1.00	0.70	0.72	1.22	1.31	0.00	0.72	0.73	0.78
time (sec)	N/A	0.033	0.051	0.008	0.438	0.411	0.000	0.276	2.862	0.546

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	63	65	78	116	0	61	66	70
N.S.	1	1.00	0.82	0.84	1.01	1.51	0.00	0.79	0.86	0.91
time (sec)	N/A	0.020	0.042	0.008	0.438	0.404	0.000	0.315	2.814	0.047

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	81	160	154	169	0	118	-1	111
N.S.	1	1.00	0.69	1.37	1.32	1.44	0.00	1.01	-0.01	0.95
time (sec)	N/A	0.159	0.042	0.010	0.456	0.393	0.000	0.288	0.000	0.652

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	90	193	187	195	0	188	-1	126
N.S.	1	1.00	0.62	1.33	1.29	1.34	0.00	1.30	-0.01	0.87
time (sec)	N/A	0.275	0.050	0.011	0.466	0.417	0.000	0.288	0.000	0.735

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	117	224	218	216	0	260	-1	139
N.S.	1	1.00	0.64	1.23	1.20	1.19	0.00	1.43	-0.01	0.76
time (sec)	N/A	0.358	0.056	0.016	0.469	0.428	0.000	0.298	0.000	0.930

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	105	249	243	227	0	325	-1	150
N.S.	1	1.00	0.50	1.19	1.16	1.09	0.00	1.56	-0.00	0.72
time (sec)	N/A	0.474	0.054	0.020	0.478	0.473	0.000	0.351	0.000	1.053

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	42	71	70	50	73	34	36	58
N.S.	1	1.00	0.52	0.88	0.86	0.62	0.90	0.42	0.44	0.72
time (sec)	N/A	0.093	0.036	0.012	0.968	0.400	1.409	0.182	2.501	0.232

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	37	57	56	45	60	30	31	53
N.S.	1	1.00	0.59	0.90	0.89	0.71	0.95	0.48	0.49	0.84
time (sec)	N/A	0.081	0.028	0.007	0.969	0.384	0.810	0.192	0.029	0.224

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	26	41	40	38	37	21	22	46
N.S.	1	1.00	0.63	1.00	0.98	0.93	0.90	0.51	0.54	1.12
time (sec)	N/A	0.048	0.017	0.005	0.971	0.396	0.408	0.220	0.029	0.186

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	25	29	28	33	27	19	21	41
N.S.	1	1.00	0.62	0.72	0.70	0.82	0.68	0.48	0.52	1.02
time (sec)	N/A	0.012	0.015	0.003	0.961	0.396	0.242	0.195	0.030	0.186

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	29	41	46	31	34	32	52
N.S.	1	1.00	1.00	0.91	1.28	1.44	0.97	1.06	1.00	1.62
time (sec)	N/A	0.059	0.010	0.005	0.979	0.390	6.295	0.180	0.047	0.183

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	30	42	53	51	55	35	57
N.S.	1	1.00	1.00	0.91	1.27	1.61	1.55	1.67	1.06	1.73
time (sec)	N/A	0.063	0.016	0.008	0.971	0.404	4.684	0.184	0.079	0.169

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	40	42	54	43	116	91	47	48
N.S.	1	1.00	0.78	0.82	1.06	0.84	2.27	1.78	0.92	0.94
time (sec)	N/A	0.061	0.020	0.007	0.969	0.403	7.033	0.179	2.487	0.165

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	43	56	68	48	128	125	67	47
N.S.	1	1.00	0.64	0.84	1.01	0.72	1.91	1.87	1.00	0.70
time (sec)	N/A	0.074	0.024	0.008	0.964	0.396	8.348	0.186	0.032	0.170

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	73	70	82	53	223	163	77	58
N.S.	1	1.00	0.82	0.79	0.92	0.60	2.51	1.83	0.87	0.65
time (sec)	N/A	0.090	0.038	0.009	0.972	0.403	11.059	0.184	0.032	0.165

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	50	84	96	58	201	199	90	63
N.S.	1	1.00	0.47	0.79	0.90	0.54	1.88	1.86	0.84	0.59
time (sec)	N/A	0.103	0.017	0.008	0.969	0.403	12.693	0.198	0.036	0.175

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	196	212	159	111	544	295	-1	134
N.S.	1	1.00	1.46	1.58	1.19	0.83	4.06	2.20	-0.01	1.00
time (sec)	N/A	0.215	0.243	0.022	0.982	0.420	10.027	0.273	0.000	0.559

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	212	291	270	194	2273	170	-1	213
N.S.	1	1.00	0.68	0.94	0.87	0.63	7.33	0.55	-0.00	0.69
time (sec)	N/A	0.487	0.358	0.107	1.002	0.433	101.431	0.258	0.000	0.879

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	200	266	245	183	2028	160	-1	202
N.S.	1	1.00	0.71	0.95	0.87	0.65	7.22	0.57	-0.00	0.72
time (sec)	N/A	0.406	0.330	0.018	0.987	0.415	64.638	0.256	0.000	0.626

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	189	241	220	172	1919	149	-1	191
N.S.	1	1.00	0.75	0.96	0.87	0.68	7.62	0.59	-0.00	0.76
time (sec)	N/A	0.364	0.297	0.015	0.992	0.414	59.744	0.242	0.000	0.676

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	178	216	195	161	1681	139	-1	180
N.S.	1	1.00	0.80	0.97	0.87	0.72	7.54	0.62	-0.00	0.81
time (sec)	N/A	0.306	0.267	0.014	0.993	0.419	40.610	0.292	0.000	0.693

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	167	191	170	150	1554	128	-1	169
N.S.	1	1.00	0.73	0.83	0.74	0.65	6.76	0.56	-0.00	0.73
time (sec)	N/A	0.121	0.379	0.010	0.987	0.404	40.183	0.240	0.000	0.589

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	156	154	136	139	1284	117	-1	158
N.S.	1	1.00	0.83	0.82	0.72	0.74	6.83	0.62	-0.01	0.84
time (sec)	N/A	0.082	0.326	0.010	0.985	0.406	25.884	0.249	0.000	0.485

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	168	231	204	151	1263	143	-1	186
N.S.	1	1.00	0.88	1.22	1.07	0.79	6.65	0.75	-0.01	0.98
time (sec)	N/A	0.307	0.355	0.011	1.004	0.426	47.722	0.260	0.000	0.580

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	221	243	217	167	1057	199	-1	187
N.S.	1	1.00	1.15	1.26	1.12	0.87	5.48	1.03	-0.01	0.97
time (sec)	N/A	0.305	0.525	0.013	0.998	0.427	19.884	0.245	0.000	0.575

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	207	259	252	229	179	1059	262	-1	189
N.S.	1	1.00	1.25	1.22	1.11	0.86	5.12	1.27	-0.00	0.91
time (sec)	N/A	0.313	0.640	0.017	1.004	0.427	22.217	0.252	0.000	0.716

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	251	277	226	179	911	318	-1	192
N.S.	1	1.00	1.20	1.32	1.08	0.85	4.34	1.51	-0.00	0.91
time (sec)	N/A	0.314	0.272	0.017	1.008	0.422	15.742	0.289	0.000	0.616

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	195	302	250	180	1028	374	-1	194
N.S.	1	1.00	0.93	1.44	1.20	0.86	4.92	1.79	-0.00	0.93
time (sec)	N/A	0.319	0.100	0.021	0.993	0.424	20.103	0.267	0.000	0.677

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	216	216	199	327	278	180	1178	430	-1	194
N.S.	1	1.00	0.92	1.51	1.29	0.83	5.45	1.99	-0.00	0.90
time (sec)	N/A	0.313	0.094	0.027	0.994	0.434	20.702	0.276	0.000	0.750

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	286	352	303	179	1397	485	-1	194
N.S.	1	1.00	1.34	1.64	1.42	0.84	6.53	2.27	-0.00	0.91
time (sec)	N/A	0.312	0.223	0.034	1.001	0.440	21.706	0.311	0.000	0.800

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	206	247	377	326	173	1513	510	-1	188
N.S.	1	1.00	1.20	1.83	1.58	0.84	7.34	2.48	-0.00	0.91
time (sec)	N/A	0.311	0.152	0.046	1.010	0.412	22.293	0.311	0.000	0.799

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	245	402	352	163	1719	538	-1	186
N.S.	1	1.00	1.20	1.97	1.73	0.80	8.43	2.64	-0.00	0.91
time (sec)	N/A	0.304	0.149	0.065	1.009	0.483	31.403	0.313	0.000	0.829

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	218	250	247	142	1889	620	-1	159
N.S.	1	1.00	1.17	1.34	1.32	0.76	10.10	3.32	-0.01	0.85
time (sec)	N/A	0.260	0.165	0.098	0.997	0.463	36.712	0.466	0.000	0.879

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	102	278	272	153	2159	683	-1	170
N.S.	1	1.00	0.45	1.24	1.21	0.68	9.60	3.04	-0.00	0.76
time (sec)	N/A	0.298	0.064	0.152	1.013	0.518	49.867	0.361	0.000	0.986

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	112	303	297	164	2397	746	-1	181
N.S.	1	1.00	0.44	1.19	1.17	0.65	9.44	2.94	-0.00	0.71
time (sec)	N/A	0.329	0.061	0.241	1.005	0.576	74.516	0.363	0.000	1.053

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	131	222	305	192	0	118	-1	123
N.S.	1	1.00	0.75	1.28	1.75	1.10	0.00	0.68	-0.01	0.71
time (sec)	N/A	0.405	0.237	0.013	1.028	0.434	0.000	0.301	0.000	0.580

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	119	262	324	177	0	107	-1	108
N.S.	1	1.00	0.84	1.85	2.28	1.25	0.00	0.75	-0.01	0.76
time (sec)	N/A	0.325	0.199	0.010	1.019	0.417	0.000	0.293	0.000	0.543

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	112	234	296	161	0	95	-1	96
N.S.	1	1.00	0.95	1.98	2.51	1.36	0.00	0.81	-0.01	0.81
time (sec)	N/A	0.216	0.151	0.013	1.017	0.404	0.000	0.292	0.000	0.619

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	58	55	154	106	0	72	49	53
N.S.	1	1.00	0.62	0.59	1.66	1.14	0.00	0.77	0.53	0.57
time (sec)	N/A	0.126	0.079	0.006	0.453	0.398	0.000	0.284	2.690	0.444

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	55	52	128	104	0	60	46	53
N.S.	1	1.00	0.64	0.60	1.49	1.21	0.00	0.70	0.53	0.62
time (sec)	N/A	0.036	0.176	0.009	0.443	0.390	0.000	0.308	2.656	0.425

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	58	55	101	106	0	70	49	53
N.S.	1	1.00	0.56	0.53	0.98	1.03	0.00	0.68	0.48	0.51
time (sec)	N/A	0.048	0.059	0.008	0.439	0.402	0.000	0.291	2.657	0.002

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	81	158	152	158	0	117	-1	93
N.S.	1	1.00	0.71	1.39	1.33	1.39	0.00	1.03	-0.01	0.82
time (sec)	N/A	0.159	0.059	0.014	0.458	0.406	0.000	0.291	0.000	0.669

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	96	190	184	184	0	185	-1	108
N.S.	1	1.00	0.66	1.31	1.27	1.27	0.00	1.28	-0.01	0.74
time (sec)	N/A	0.287	0.056	0.012	0.472	0.423	0.000	0.290	0.000	0.591

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	119	222	216	205	0	259	-1	121
N.S.	1	1.00	0.65	1.22	1.19	1.13	0.00	1.42	-0.01	0.66
time (sec)	N/A	0.362	0.069	0.014	0.475	0.418	0.000	0.331	0.000	0.721

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	91	208	125	95	0	77	-1	114
N.S.	1	1.00	0.62	1.41	0.85	0.65	0.00	0.52	-0.01	0.78
time (sec)	N/A	0.142	0.140	0.018	0.990	0.405	0.000	0.205	0.000	0.263

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	80	185	101	83	0	66	-1	103
N.S.	1	1.00	0.68	1.57	0.86	0.70	0.00	0.56	-0.01	0.87
time (sec)	N/A	0.099	0.101	0.013	0.990	0.404	0.000	0.211	0.000	0.265

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	69	160	77	73	0	54	-1	92
N.S.	1	1.00	0.80	1.86	0.90	0.85	0.00	0.63	-0.01	1.07
time (sec)	N/A	0.110	0.073	0.012	0.988	0.404	0.000	0.203	0.000	0.246

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	57	140	56	60	0	43	-1	80
N.S.	1	1.00	0.92	2.26	0.90	0.97	0.00	0.69	-0.02	1.29
time (sec)	N/A	0.041	0.059	0.010	0.971	0.401	0.000	0.220	0.000	0.224

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	43	77	31	52	0	31	-1	65
N.S.	1	1.00	0.93	1.67	0.67	1.13	0.00	0.67	-0.02	1.41
time (sec)	N/A	0.016	0.023	0.004	0.972	0.390	0.000	0.214	0.000	0.197

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	137	56	54	0	48	-1	84
N.S.	1	1.00	1.00	2.98	1.22	1.17	0.00	1.04	-0.02	1.83
time (sec)	N/A	0.059	0.038	0.013	0.988	0.425	0.000	0.212	0.000	0.209

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	53	222	0	50	0	102	-1	103
N.S.	1	1.00	1.04	4.35	0.00	0.98	0.00	2.00	-0.02	2.02
time (sec)	N/A	0.057	0.064	0.012	0.000	0.395	0.000	0.211	0.000	0.261

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	70	254	0	63	0	0	-1	79
N.S.	1	1.00	0.85	3.10	0.00	0.77	0.00	0.00	-0.01	0.96
time (sec)	N/A	0.078	0.104	0.011	0.000	0.385	0.000	0.000	0.000	0.287

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	84	280	0	75	0	0	-1	137
N.S.	1	1.00	0.74	2.46	0.00	0.66	0.00	0.00	-0.01	1.20
time (sec)	N/A	0.110	0.108	0.012	0.000	0.396	0.000	0.000	0.000	0.384

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	95	304	0	86	0	0	-1	104
N.S.	1	1.00	0.66	2.13	0.00	0.60	0.00	0.00	-0.01	0.73
time (sec)	N/A	0.135	0.127	0.013	0.000	0.392	0.000	0.000	0.000	0.410

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	C	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	112	222	174	94	279	0	-1	114
N.S.	1	1.00	0.99	1.96	1.54	0.83	2.47	0.00	-0.01	1.01
time (sec)	N/A	0.131	0.118	0.015	1.011	0.394	7.848	0.000	0.000	0.244

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	135	330	246	139	830	0	-1	158
N.S.	1	1.00	0.67	1.64	1.22	0.69	4.13	0.00	-0.00	0.79
time (sec)	N/A	0.160	0.174	0.016	1.042	0.410	25.151	0.000	0.000	0.376

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	C	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	124	305	221	127	775	0	-1	147
N.S.	1	1.00	0.72	1.77	1.28	0.74	4.51	0.00	-0.01	0.85
time (sec)	N/A	0.123	0.131	0.012	1.033	0.402	23.006	0.000	0.000	0.387

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	C	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	113	282	198	117	653	0	-1	136
N.S.	1	1.00	0.81	2.01	1.41	0.84	4.66	0.00	-0.01	0.97
time (sec)	N/A	0.150	0.102	0.014	1.045	0.406	16.657	0.000	0.000	0.392

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	C	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	102	260	176	105	580	0	-1	125
N.S.	1	1.00	0.88	2.24	1.52	0.91	5.00	0.00	-0.01	1.08
time (sec)	N/A	0.059	0.087	0.010	1.016	0.403	16.117	0.000	0.000	0.378

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	91	147	109	95	435	0	-1	114
N.S.	1	1.00	0.91	1.47	1.09	0.95	4.35	0.00	-0.01	1.14
time (sec)	N/A	0.031	0.055	0.006	0.992	0.397	10.461	0.000	0.000	0.384

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	108	245	124	107	469	0	-1	142
N.S.	1	1.00	0.96	2.17	1.10	0.95	4.15	0.00	-0.01	1.26
time (sec)	N/A	0.116	0.088	0.011	0.990	0.415	25.649	0.000	0.000	0.437

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	114	380	131	123	386	0	-1	143
N.S.	1	1.00	0.99	3.30	1.14	1.07	3.36	0.00	-0.01	1.24
time (sec)	N/A	0.118	0.125	0.011	0.989	0.394	10.205	0.000	0.000	0.452

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	119	411	138	135	461	0	-1	145
N.S.	1	1.00	0.98	3.40	1.14	1.12	3.81	0.00	-0.01	1.20
time (sec)	N/A	0.113	0.155	0.012	0.975	0.423	13.347	0.000	0.000	0.656

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	116	439	132	130	457	0	-1	141
N.S.	1	1.00	0.97	3.66	1.10	1.08	3.81	0.00	-0.01	1.18
time (sec)	N/A	0.114	0.154	0.013	0.986	0.412	11.684	0.000	0.000	0.501

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	111	463	159	119	541	0	-1	142
N.S.	1	1.00	0.93	3.89	1.34	1.00	4.55	0.00	-0.01	1.19
time (sec)	N/A	0.115	0.185	0.014	1.009	0.414	14.371	0.000	0.000	0.579

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	106	493	153	97	774	0	-1	155
N.S.	1	1.00	0.98	4.56	1.42	0.90	7.17	0.00	-0.01	1.44
time (sec)	N/A	0.089	0.147	0.016	1.002	0.405	13.939	0.000	0.000	0.587

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	117	521	178	108	918	0	-1	126
N.S.	1	1.00	0.82	3.64	1.24	0.76	6.42	0.00	-0.01	0.88
time (sec)	N/A	0.120	0.177	0.014	0.996	0.419	18.691	0.000	0.000	0.612

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	128	546	203	119	1037	0	-1	137
N.S.	1	1.00	0.74	3.17	1.18	0.69	6.03	0.00	-0.01	0.80
time (sec)	N/A	0.154	0.190	0.015	0.994	0.425	18.790	0.000	0.000	0.679

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	139	571	228	130	1159	0	-1	148
N.S.	1	1.00	0.69	2.84	1.13	0.65	5.77	0.00	-0.00	0.74
time (sec)	N/A	0.190	0.212	0.017	0.998	0.460	27.712	0.000	0.000	0.741

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	26	34	28	31	29	19	20	37
N.S.	1	1.00	0.96	1.26	1.04	1.15	1.07	0.70	0.74	1.37
time (sec)	N/A	0.017	0.038	0.007	0.976	0.410	3.350	0.164	0.039	0.180

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	49	238	68	74	170	125	74	95
N.S.	1	1.00	0.96	4.67	1.33	1.45	3.33	2.45	1.45	1.86
time (sec)	N/A	0.072	0.037	0.018	0.989	0.410	6.533	0.195	0.051	0.437

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	91	147	113	112	0	0	-1	109
N.S.	1	1.00	0.77	1.25	0.96	0.95	0.00	0.00	-0.01	0.92
time (sec)	N/A	0.096	0.088	0.014	1.007	0.407	0.000	0.000	0.000	0.456

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	80	120	86	101	0	0	-1	98
N.S.	1	1.00	0.88	1.32	0.95	1.11	0.00	0.00	-0.01	1.08
time (sec)	N/A	0.064	0.059	0.011	0.997	0.405	0.000	0.000	0.000	0.364

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	59	97	63	85	0	0	-1	81
N.S.	1	1.00	0.77	1.26	0.82	1.10	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.097	0.070	0.010	0.976	0.409	0.000	0.000	0.000	0.338

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	49	74	40	67	0	0	-1	71
N.S.	1	1.00	0.94	1.42	0.77	1.29	0.00	0.00	-0.02	1.37
time (sec)	N/A	0.022	0.029	0.010	0.974	0.397	0.000	0.000	0.000	0.289

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	32	29	30	35	0	0	29	31
N.S.	1	1.00	1.03	0.94	0.97	1.13	0.00	0.00	0.94	1.00
time (sec)	N/A	0.011	0.006	0.006	0.976	0.384	0.000	0.000	2.643	0.002

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	52	88	0	62	0	0	-1	70
N.S.	1	1.00	0.96	1.63	0.00	1.15	0.00	0.00	-0.02	1.30
time (sec)	N/A	0.044	0.037	0.012	0.000	0.385	0.000	0.000	0.000	0.397

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	62	108	0	88	0	0	-1	82
N.S.	1	1.00	0.77	1.33	0.00	1.09	0.00	0.00	-0.01	1.01
time (sec)	N/A	0.064	0.053	0.012	0.000	0.408	0.000	0.000	0.000	0.364

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	127	133	0	113	0	0	-1	97
N.S.	1	1.00	1.12	1.18	0.00	1.00	0.00	0.00	-0.01	0.86
time (sec)	N/A	0.091	0.317	0.014	0.000	0.407	0.000	0.000	0.000	0.440

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	106	208	151	190	0	0	-1	130
N.S.	1	1.00	0.83	1.62	1.18	1.48	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.105	0.170	0.021	1.015	0.403	0.000	0.000	0.000	0.520

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	93	179	124	175	0	0	-1	114
N.S.	1	1.00	0.82	1.58	1.10	1.55	0.00	0.00	-0.01	1.01
time (sec)	N/A	0.094	0.135	0.010	1.010	0.423	0.000	0.000	0.000	0.457

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	80	153	99	157	0	0	-1	102
N.S.	1	1.00	0.90	1.72	1.11	1.76	0.00	0.00	-0.01	1.15
time (sec)	N/A	0.070	0.122	0.011	1.003	0.419	0.000	0.000	0.000	0.438

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	48	86	103	0	1	56	60
N.S.	1	1.00	1.00	0.80	1.43	1.72	0.00	0.02	0.93	1.00
time (sec)	N/A	0.035	0.052	0.008	0.464	0.403	0.000	0.245	2.712	0.393

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	56	44	67	101	0	0	52	56
N.S.	1	1.00	0.97	0.76	1.16	1.74	0.00	0.00	0.90	0.97
time (sec)	N/A	0.020	0.045	0.008	0.458	0.411	0.000	0.000	2.712	0.368

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	58	46	65	102	0	0	56	60
N.S.	1	1.00	1.00	0.79	1.12	1.76	0.00	0.00	0.97	1.03
time (sec)	N/A	0.015	0.033	0.007	0.453	0.395	0.000	0.000	2.714	0.377

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	83	142	0	155	0	0	-1	99
N.S.	1	1.00	0.94	1.61	0.00	1.76	0.00	0.00	-0.01	1.12
time (sec)	N/A	0.075	0.103	0.012	0.000	0.401	0.000	0.000	0.000	0.530

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	101	188	0	181	0	1	-1	115
N.S.	1	1.00	0.84	1.57	0.00	1.51	0.00	0.01	-0.01	0.96
time (sec)	N/A	0.103	0.119	0.017	0.000	0.391	0.000	0.252	0.000	0.492

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	115	216	0	201	0	0	-1	128
N.S.	1	1.00	0.76	1.42	0.00	1.32	0.00	0.00	-0.01	0.84
time (sec)	N/A	0.129	0.104	0.016	0.000	0.418	0.000	0.000	0.000	0.572

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	128	318	289	274	0	0	-1	152
N.S.	1	1.00	0.79	1.96	1.78	1.69	0.00	0.00	-0.01	0.94
time (sec)	N/A	0.160	0.240	0.049	1.091	0.463	0.000	0.000	0.000	0.644

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	115	288	259	258	0	0	-1	137
N.S.	1	1.00	0.78	1.95	1.75	1.74	0.00	0.00	-0.01	0.93
time (sec)	N/A	0.137	0.192	0.011	1.091	0.437	0.000	0.000	0.000	0.574

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	103	259	234	241	0	0	-1	124
N.S.	1	1.00	0.84	2.12	1.92	1.98	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.102	0.145	0.010	1.060	0.427	0.000	0.000	0.000	0.535

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	82	70	134	168	0	0	78	82
N.S.	1	1.00	0.96	0.82	1.58	1.98	0.00	0.00	0.92	0.96
time (sec)	N/A	0.074	0.086	0.008	0.499	0.406	0.000	0.000	2.952	0.449

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	82	70	110	171	0	0	78	82
N.S.	1	1.00	0.90	0.77	1.21	1.88	0.00	0.00	0.86	0.90
time (sec)	N/A	0.069	0.071	0.008	0.470	0.406	0.000	0.000	2.835	0.445

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	82	70	110	170	0	0	78	82
N.S.	1	1.00	0.86	0.74	1.16	1.79	0.00	0.00	0.82	0.86
time (sec)	N/A	0.053	0.060	0.009	0.478	0.410	0.000	0.000	2.788	0.435

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	82	70	90	171	0	0	78	82
N.S.	1	1.00	0.96	0.82	1.06	2.01	0.00	0.00	0.92	0.96
time (sec)	N/A	0.029	0.055	0.008	0.490	0.418	0.000	0.000	2.784	0.418

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	70	85	168	0	0	78	82
N.S.	1	1.00	1.00	0.85	1.04	2.05	0.00	0.00	0.95	1.00
time (sec)	N/A	0.022	0.039	0.008	0.452	0.414	0.000	0.000	2.759	0.005

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	106	196	0	237	0	0	-1	122
N.S.	1	1.00	0.89	1.65	0.00	1.99	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.107	0.090	0.015	0.000	0.422	0.000	0.000	0.000	0.661

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	122	268	0	265	0	0	-1	137
N.S.	1	1.00	0.79	1.74	0.00	1.72	0.00	0.00	-0.01	0.89
time (sec)	N/A	0.136	0.124	0.016	0.000	0.448	0.000	0.000	0.000	0.760

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	137	298	0	286	0	0	-1	150
N.S.	1	1.00	0.74	1.60	0.00	1.54	0.00	0.00	-0.01	0.81
time (sec)	N/A	0.168	0.130	0.017	0.000	0.508	0.000	0.000	0.000	1.041

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	148	326	0	297	0	0	-1	161
N.S.	1	1.00	0.69	1.52	0.00	1.38	0.00	0.00	-0.00	0.75
time (sec)	N/A	0.210	0.162	0.021	0.000	0.520	0.000	0.000	0.000	1.425

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	104	92	133	239	0	0	161	104
N.S.	1	1.00	0.88	0.78	1.13	2.03	0.00	0.00	1.36	0.88
time (sec)	N/A	0.077	0.120	0.010	0.508	0.488	0.000	0.000	2.948	0.779

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	104	92	133	238	0	0	161	104
N.S.	1	1.00	0.85	0.75	1.08	1.93	0.00	0.00	1.31	0.85
time (sec)	N/A	0.058	0.080	0.010	0.485	0.491	0.000	0.000	2.883	0.681

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	54	100	68	75	0	0	116	86
N.S.	1	1.00	0.82	1.52	1.03	1.14	0.00	0.00	1.76	1.30
time (sec)	N/A	0.054	0.051	0.014	0.975	0.415	0.000	0.000	0.071	0.432

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	37	84	52	66	0	70	84	74
N.S.	1	1.00	0.67	1.53	0.95	1.20	0.00	1.27	1.53	1.35
time (sec)	N/A	0.086	0.053	0.008	0.978	0.400	0.000	0.198	0.067	0.318

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	31	65	33	58	0	52	57	67
N.S.	1	1.00	0.91	1.91	0.97	1.71	0.00	1.53	1.68	1.97
time (sec)	N/A	0.017	0.024	0.009	0.986	0.405	0.000	0.215	2.605	0.335

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	25	22	23	28	0	34	23	26
N.S.	1	1.00	0.96	0.85	0.88	1.08	0.00	1.31	0.88	1.00
time (sec)	N/A	0.010	0.006	0.006	0.975	0.387	0.000	0.200	2.588	0.316

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	58	0	52	0	74	58	55
N.S.	1	1.00	1.00	1.41	0.00	1.27	0.00	1.80	1.41	1.34
time (sec)	N/A	0.040	0.028	0.014	0.000	0.400	0.000	0.208	2.652	0.453

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	50	73	0	76	0	0	81	64
N.S.	1	1.00	0.78	1.14	0.00	1.19	0.00	0.00	1.27	1.00
time (sec)	N/A	0.053	0.041	0.013	0.000	0.398	0.000	0.000	2.592	0.444

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	63	94	0	97	0	213	105	76
N.S.	1	1.00	0.70	1.04	0.00	1.08	0.00	2.37	1.17	0.84
time (sec)	N/A	0.079	0.055	0.013	0.000	0.402	0.000	0.208	2.611	0.551

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	229	229	135	375	299	138	571	0	-1	158
N.S.	1	1.00	0.59	1.64	1.31	0.60	2.49	0.00	-0.00	0.69
time (sec)	N/A	0.315	0.213	0.020	1.072	0.402	17.476	0.000	0.000	0.725

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	C	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	124	350	275	128	690	0	-1	147
N.S.	1	1.00	0.62	1.75	1.38	0.64	3.45	0.00	-0.00	0.74
time (sec)	N/A	0.270	0.155	0.015	1.039	0.404	21.396	0.000	0.000	0.648

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	C	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	113	327	251	116	450	0	-1	136
N.S.	1	1.00	0.66	1.91	1.47	0.68	2.63	0.00	-0.01	0.80
time (sec)	N/A	0.215	0.127	0.016	1.038	0.411	12.029	0.000	0.000	0.646

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	C	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	102	303	230	106	541	0	-1	125
N.S.	1	1.00	0.72	2.13	1.62	0.75	3.81	0.00	-0.01	0.88
time (sec)	N/A	0.177	0.114	0.014	1.030	0.406	14.453	0.000	0.000	0.574

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	91	198	167	94	321	0	-1	114
N.S.	1	1.00	0.67	1.46	1.23	0.69	2.36	0.00	-0.01	0.84
time (sec)	N/A	0.057	0.076	0.013	1.002	0.393	8.566	0.000	0.000	0.463

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	C	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	80	194	119	84	350	0	-1	103
N.S.	1	1.00	0.74	1.80	1.10	0.78	3.24	0.00	-0.01	0.95
time (sec)	N/A	0.042	0.048	0.006	0.991	0.398	9.349	0.000	0.000	0.435

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	96	290	103	95	267	0	-1	128
N.S.	1	1.00	1.00	3.02	1.07	0.99	2.78	0.00	-0.01	1.33
time (sec)	N/A	0.162	0.096	0.012	0.987	0.414	14.833	0.000	0.000	0.478

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	100	425	112	111	347	0	-1	131
N.S.	1	1.00	0.95	4.05	1.07	1.06	3.30	0.00	-0.01	1.25
time (sec)	N/A	0.161	0.127	0.013	1.118	0.404	9.854	0.000	0.000	0.598

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	102	456	111	119	347	0	-1	128
N.S.	1	1.00	0.93	4.15	1.01	1.08	3.15	0.00	-0.01	1.16
time (sec)	N/A	0.163	0.140	0.014	0.986	0.406	10.175	0.000	0.000	0.504

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	96	479	134	106	338	0	-1	129
N.S.	1	1.00	0.94	4.70	1.31	1.04	3.31	0.00	-0.01	1.26
time (sec)	N/A	0.163	0.178	0.016	0.991	0.417	9.735	0.000	0.000	0.543

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	95	513	130	86	422	0	-1	144
N.S.	1	1.00	0.88	4.75	1.20	0.80	3.91	0.00	-0.01	1.33
time (sec)	N/A	0.147	0.175	0.016	1.008	0.392	12.355	0.000	0.000	0.719

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	106	541	155	97	660	0	-1	115
N.S.	1	1.00	0.76	3.86	1.11	0.69	4.71	0.00	-0.01	0.82
time (sec)	N/A	0.177	0.168	0.016	0.991	0.408	13.422	0.000	0.000	0.707

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	117	566	180	108	808	0	-1	126
N.S.	1	1.00	0.69	3.35	1.07	0.64	4.78	0.00	-0.01	0.75
time (sec)	N/A	0.208	0.234	0.017	1.010	0.416	19.721	0.000	0.000	0.752

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	128	591	205	119	835	0	-1	137
N.S.	1	1.00	0.65	2.98	1.04	0.60	4.22	0.00	-0.01	0.69
time (sec)	N/A	0.236	0.234	0.018	0.988	0.404	18.183	0.000	0.000	0.854

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	106	198	170	171	0	0	-1	114
N.S.	1	1.00	0.86	1.61	1.38	1.39	0.00	0.00	-0.01	0.93
time (sec)	N/A	0.238	0.158	0.017	1.015	0.410	0.000	0.000	0.000	0.670

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	70	65	157	116	0	0	66	70
N.S.	1	1.00	0.71	0.66	1.59	1.17	0.00	0.00	0.67	0.71
time (sec)	N/A	0.204	0.077	0.007	0.468	0.398	0.000	0.000	2.970	0.543

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	70	65	136	118	0	0	66	70
N.S.	1	1.00	0.79	0.73	1.53	1.33	0.00	0.00	0.74	0.79
time (sec)	N/A	0.142	0.064	0.009	0.468	0.390	0.000	0.000	2.896	0.542

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	69	64	138	116	0	0	65	69
N.S.	1	1.00	0.76	0.70	1.52	1.27	0.00	0.00	0.71	0.76
time (sec)	N/A	0.036	0.054	0.010	0.464	0.398	0.000	0.000	2.879	0.483

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	70	66	136	115	0	0	66	70
N.S.	1	1.00	0.77	0.73	1.49	1.26	0.00	0.00	0.73	0.77
time (sec)	N/A	0.031	0.037	0.008	0.445	0.385	0.000	0.000	2.847	0.516

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	95	187	0	168	0	0	-1	111
N.S.	1	1.00	0.81	1.58	0.00	1.42	0.00	0.00	-0.01	0.94
time (sec)	N/A	0.178	0.087	0.012	0.000	0.404	0.000	0.000	0.000	0.764

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	112	234	0	194	0	0	-1	126
N.S.	1	1.00	0.77	1.60	0.00	1.33	0.00	0.00	-0.01	0.86
time (sec)	N/A	0.300	0.108	0.012	0.000	0.420	0.000	0.000	0.000	0.779

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	127	259	0	215	0	0	-1	139
N.S.	1	1.00	0.69	1.42	0.00	1.17	0.00	0.00	-0.01	0.76
time (sec)	N/A	0.375	0.131	0.017	0.000	0.432	0.000	0.000	0.000	0.949

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	98	212	185	190	0	0	-1	121
N.S.	1	1.00	0.55	1.20	1.05	1.07	0.00	0.00	-0.01	0.68
time (sec)	N/A	0.439	0.193	0.019	1.000	0.434	0.000	0.000	0.000	0.689

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	85	187	160	174	0	0	-1	106
N.S.	1	1.00	0.58	1.28	1.10	1.19	0.00	0.00	-0.01	0.73
time (sec)	N/A	0.365	0.150	0.016	0.984	0.413	0.000	0.000	0.000	0.725

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	73	163	136	157	0	0	-1	93
N.S.	1	1.00	0.61	1.36	1.13	1.31	0.00	0.00	-0.01	0.78
time (sec)	N/A	0.261	0.113	0.013	0.985	0.407	0.000	0.000	0.000	0.604

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	52	55	125	104	0	0	48	52
N.S.	1	1.00	0.55	0.58	1.32	1.09	0.00	0.00	0.51	0.55
time (sec)	N/A	0.128	0.061	0.008	0.983	0.397	0.000	0.000	2.762	0.526

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	49	52	129	100	0	0	45	52
N.S.	1	1.00	0.51	0.54	1.33	1.03	0.00	0.00	0.46	0.54
time (sec)	N/A	0.045	0.049	0.008	0.984	0.397	0.000	0.000	2.590	0.483

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	52	55	128	104	0	0	48	52
N.S.	1	1.00	0.52	0.55	1.28	1.04	0.00	0.00	0.48	0.52
time (sec)	N/A	0.037	0.029	0.007	0.984	0.406	0.000	0.000	2.619	0.003

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	76	179	0	153	0	0	-1	92
N.S.	1	1.00	0.66	1.56	0.00	1.33	0.00	0.00	-0.01	0.80
time (sec)	N/A	0.179	0.120	0.012	0.000	0.401	0.000	0.000	0.000	0.905

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	92	199	0	181	0	1	-1	107
N.S.	1	1.00	0.63	1.36	0.00	1.24	0.00	0.01	-0.01	0.73
time (sec)	N/A	0.305	0.177	0.013	0.000	0.407	0.000	0.302	0.000	0.698

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	107	222	0	202	0	1	-1	120
N.S.	1	1.00	0.58	1.21	0.00	1.10	0.00	0.01	-0.01	0.66
time (sec)	N/A	0.380	0.173	0.014	0.000	0.421	0.000	0.293	0.000	0.955

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-1)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	109	297	0	200	0	0	-1	130
N.S.	1	1.00	0.53	1.46	0.00	0.98	0.00	0.00	-0.00	0.64
time (sec)	N/A	0.593	0.184	0.025	0.000	0.440	0.000	0.000	0.000	0.758

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-1)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	98	273	0	190	0	0	-1	121
N.S.	1	1.00	0.61	1.71	0.00	1.19	0.00	0.00	-0.01	0.76
time (sec)	N/A	0.415	0.179	0.014	0.000	0.428	0.000	0.000	0.000	0.823

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-1)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	85	212	0	174	0	0	-1	106
N.S.	1	1.00	0.57	1.43	0.00	1.18	0.00	0.00	-0.01	0.72
time (sec)	N/A	0.246	0.130	0.011	0.000	0.418	0.000	0.000	0.000	0.648

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	73	214	0	157	0	0	-1	94
N.S.	1	1.00	0.63	1.86	0.00	1.37	0.00	0.00	-0.01	0.82
time (sec)	N/A	0.147	0.122	0.011	0.000	0.413	0.000	0.000	0.000	0.686

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	50	42	125	102	0	0	46	52
N.S.	1	1.00	0.78	0.66	1.95	1.59	0.00	0.00	0.72	0.81
time (sec)	N/A	0.027	0.049	0.007	0.447	0.395	0.000	0.000	2.904	0.591

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	51	43	123	104	0	0	47	51
N.S.	1	1.00	0.76	0.64	1.84	1.55	0.00	0.00	0.70	0.76
time (sec)	N/A	0.024	0.028	0.006	0.445	0.408	0.000	0.000	2.776	0.619

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	76	196	0	153	0	0	-1	92
N.S.	1	1.00	0.69	1.78	0.00	1.39	0.00	0.00	-0.01	0.84
time (sec)	N/A	0.220	0.137	0.013	0.000	0.408	0.000	0.000	0.000	0.914

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	92	361	0	181	0	1	-1	107
N.S.	1	1.00	0.64	2.52	0.00	1.27	0.00	0.01	-0.01	0.75
time (sec)	N/A	0.306	0.209	0.012	0.000	0.415	0.000	0.283	0.000	0.588

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	107	389	0	202	0	1	-1	120
N.S.	1	1.00	0.58	2.13	0.00	1.10	0.00	0.01	-0.01	0.66
time (sec)	N/A	0.392	0.229	0.013	0.000	0.423	0.000	0.303	0.000	0.794

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	118	412	0	213	0	1	-1	131
N.S.	1	1.00	0.56	1.96	0.00	1.01	0.00	0.00	-0.00	0.62
time (sec)	N/A	0.494	0.266	0.019	0.000	0.456	0.000	0.368	0.000	1.148

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	131	416	478	156	0	0	-1	154
N.S.	1	1.00	0.52	1.65	1.90	0.62	0.00	0.00	-0.00	0.61
time (sec)	N/A	0.663	0.232	0.026	1.059	0.426	0.000	0.000	0.000	0.605

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	C	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	125	393	456	146	0	0	-1	143
N.S.	1	1.00	0.56	1.75	2.04	0.65	0.00	0.00	-0.00	0.64
time (sec)	N/A	0.534	0.163	0.015	1.040	0.413	0.000	0.000	0.000	0.649

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	109	285	407	134	0	0	-1	132
N.S.	1	1.00	0.57	1.48	2.12	0.70	0.00	0.00	-0.01	0.69
time (sec)	N/A	0.435	0.136	0.017	1.029	0.402	0.000	0.000	0.000	0.538

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	103	288	363	124	0	0	-1	121
N.S.	1	1.00	0.57	1.58	1.99	0.68	0.00	0.00	-0.01	0.66
time (sec)	N/A	0.219	0.119	0.012	1.023	0.407	0.000	0.000	0.000	0.539

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	83	290	235	111	0	0	-1	107
N.S.	1	1.00	0.64	2.23	1.81	0.85	0.00	0.00	-0.01	0.82
time (sec)	N/A	0.064	0.101	0.011	0.985	0.401	0.000	0.000	0.000	0.484

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	75	284	134	99	0	0	-1	98
N.S.	1	1.00	0.66	2.51	1.19	0.88	0.00	0.00	-0.01	0.87
time (sec)	N/A	0.048	0.065	0.007	0.978	0.404	0.000	0.000	0.000	0.451

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	79	378	0	111	0	0	-1	117
N.S.	1	1.00	0.89	4.25	0.00	1.25	0.00	0.00	-0.01	1.31
time (sec)	N/A	0.211	0.147	0.012	0.000	0.414	0.000	0.000	0.000	0.645

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	84	515	0	127	0	0	-1	117
N.S.	1	1.00	0.89	5.48	0.00	1.35	0.00	0.00	-0.01	1.24
time (sec)	N/A	0.216	0.187	0.013	0.000	0.421	0.000	0.000	0.000	0.539

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	85	504	0	112	0	0	-1	140
N.S.	1	1.00	0.77	4.58	0.00	1.02	0.00	0.00	-0.01	1.27
time (sec)	N/A	0.215	0.221	0.013	0.000	0.411	0.000	0.000	0.000	0.622

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	94	575	0	123	0	0	-1	109
N.S.	1	1.00	0.69	4.20	0.00	0.90	0.00	0.00	-0.01	0.80
time (sec)	N/A	0.297	0.247	0.018	0.000	0.397	0.000	0.000	0.000	0.762

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	107	600	0	136	0	0	-1	122
N.S.	1	1.00	0.63	3.53	0.00	0.80	0.00	0.00	-0.01	0.72
time (sec)	N/A	0.394	0.268	0.017	0.000	0.398	0.000	0.000	0.000	0.936

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	118	628	0	147	0	0	-1	131
N.S.	1	1.00	0.60	3.20	0.00	0.75	0.00	0.00	-0.01	0.67
time (sec)	N/A	0.517	0.331	0.018	0.000	0.406	0.000	0.000	0.000	1.063

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	50	200	0	126	0	0	220	85
N.S.	1	1.00	0.53	2.11	0.00	1.33	0.00	0.00	2.32	0.89
time (sec)	N/A	0.132	0.114	0.018	0.000	0.404	0.000	0.000	2.703	0.694

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	50	44	153	102	0	0	287	50
N.S.	1	1.00	0.57	0.50	1.74	1.16	0.00	0.00	3.26	0.57
time (sec)	N/A	0.123	0.072	0.007	0.444	0.402	0.000	0.000	0.061	0.723

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	137	132	399	316	0	0	252	137
N.S.	1	1.00	0.66	0.63	1.91	1.51	0.00	0.00	1.21	0.66
time (sec)	N/A	0.315	0.176	0.011	0.496	0.922	0.000	0.000	3.217	0.975

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	137	132	401	317	0	0	252	137
N.S.	1	1.00	0.66	0.63	1.92	1.52	0.00	0.00	1.21	0.66
time (sec)	N/A	0.207	0.100	0.013	0.493	0.965	0.000	0.000	3.189	0.772

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	137	132	405	316	0	0	252	137
N.S.	1	1.00	0.65	0.63	1.92	1.50	0.00	0.00	1.19	0.65
time (sec)	N/A	0.105	0.088	0.011	0.490	1.042	0.000	0.000	3.194	0.896

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	137	132	393	314	0	0	242	137
N.S.	1	1.00	0.67	0.64	1.92	1.53	0.00	0.00	1.18	0.67
time (sec)	N/A	0.091	0.066	0.010	0.500	1.139	0.000	0.000	3.118	0.047

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	161	385	0	432	0	0	-1	177
N.S.	1	1.00	0.69	1.65	0.00	1.85	0.00	0.00	-0.00	0.76
time (sec)	N/A	0.385	0.172	0.028	0.000	1.181	0.000	0.000	0.000	1.321

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	183	484	0	458	0	0	-1	192
N.S.	1	1.00	0.68	1.79	0.00	1.69	0.00	0.00	-0.00	0.71
time (sec)	N/A	0.680	0.210	0.018	0.000	1.779	0.000	0.000	0.000	1.309

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	93	95	0	217	0	0	-1	123
N.S.	1	1.00	0.91	0.93	0.00	2.13	0.00	0.00	-0.01	1.21
time (sec)	N/A	0.112	0.095	0.049	0.000	0.419	0.000	0.000	0.000	0.478

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	67	58	0	110	0	57	-1	54
N.S.	1	1.00	1.72	1.49	0.00	2.82	0.00	1.46	-0.03	1.38
time (sec)	N/A	0.040	0.035	0.018	0.000	0.406	0.000	0.172	0.000	0.337

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	62	48	92	83	0	38	54
N.S.	1	1.00	1.00	1.77	1.37	2.63	2.37	0.00	1.09	1.54
time (sec)	N/A	0.010	0.015	0.008	0.957	0.400	1.904	0.000	2.985	0.081

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	76	0	199	0	0	-1	0
N.S.	1	1.00	1.00	2.17	0.00	5.69	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.029	0.009	0.016	0.000	0.440	0.000	0.000	0.000	2.086

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	57	68	90	29	0	36	46
N.S.	1	1.00	1.00	1.68	2.00	2.65	0.85	0.00	1.06	1.35
time (sec)	N/A	0.009	0.012	0.006	0.955	0.418	1.966	0.000	3.000	0.064

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	86	0	208	0	0	-1	0
N.S.	1	1.00	1.00	2.53	0.00	6.12	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.028	0.012	0.014	0.000	0.451	0.000	0.000	0.000	2.206

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	49	79	82	111	148	0	54	74
N.S.	1	1.00	0.78	1.25	1.30	1.76	2.35	0.00	0.86	1.17
time (sec)	N/A	0.015	0.020	0.005	0.959	0.414	3.391	0.000	2.597	0.088

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	49	92	0	221	0	0	-1	0
N.S.	1	1.00	0.78	1.46	0.00	3.51	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.033	0.007	0.014	0.000	0.445	0.000	0.000	0.000	2.023

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	60	240	0	307	68	167	201	68
N.S.	1	1.00	0.28	1.12	0.00	1.43	0.32	0.78	0.94	0.32
time (sec)	N/A	0.247	0.043	0.117	0.000	0.420	11.157	0.896	0.111	0.383

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	259	560	249	1104	0	252	-1	283
N.S.	1	1.00	1.02	2.20	0.98	4.33	0.00	0.99	-0.00	1.11
time (sec)	N/A	0.629	0.613	0.021	0.682	6.900	0.000	0.210	0.000	0.912

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	225	515	207	963	0	201	-1	236
N.S.	1	1.00	1.07	2.44	0.98	4.56	0.00	0.95	-0.00	1.12
time (sec)	N/A	0.390	0.400	0.011	0.584	7.039	0.000	0.219	0.000	0.671

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	193	448	144	776	0	157	-1	203
N.S.	1	1.00	1.26	2.93	0.94	5.07	0.00	1.03	-0.01	1.33
time (sec)	N/A	0.211	0.319	0.013	0.524	0.689	0.000	0.210	0.000	0.591

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	175	423	122	684	0	135	-1	177
N.S.	1	1.00	1.38	3.33	0.96	5.39	0.00	1.06	-0.01	1.39
time (sec)	N/A	0.105	0.275	0.008	0.505	0.676	0.000	0.199	0.000	0.466

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	99	381	84	574	0	109	-1	151
N.S.	1	1.00	0.96	3.70	0.82	5.57	0.00	1.06	-0.01	1.47
time (sec)	N/A	0.071	0.024	0.006	0.483	0.510	0.000	0.191	0.000	0.006

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	113	420	103	1316	0	0	-1	181
N.S.	1	1.00	0.97	3.62	0.89	11.34	0.00	0.00	-0.01	1.56
time (sec)	N/A	0.100	0.048	0.010	0.499	1.150	0.000	0.000	0.000	0.395

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	178	486	0	599	0	145	-1	167
N.S.	1	1.00	1.70	4.63	0.00	5.70	0.00	1.38	-0.01	1.59
time (sec)	N/A	0.167	0.238	0.012	0.000	0.481	0.000	0.220	0.000	0.450

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	283	567	0	726	0	230	-1	184
N.S.	1	1.00	1.77	3.54	0.00	4.54	0.00	1.44	-0.01	1.15
time (sec)	N/A	0.211	0.397	0.013	0.000	0.489	0.000	0.217	0.000	0.635

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	191	191	301	600	0	824	0	309	-1	214
N.S.	1	1.00	1.58	3.14	0.00	4.31	0.00	1.62	-0.01	1.12
time (sec)	N/A	0.232	1.020	0.015	0.000	0.506	0.000	0.212	0.000	0.861

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	274	274	344	703	0	1007	0	596	-1	252
N.S.	1	1.00	1.26	2.57	0.00	3.68	0.00	2.18	-0.00	0.92
time (sec)	N/A	0.297	1.097	0.016	0.000	0.575	0.000	0.260	0.000	1.130

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	149	260	171	1060	0	163	-1	217
N.S.	1	1.00	0.76	1.33	0.88	5.44	0.00	0.84	-0.01	1.11
time (sec)	N/A	0.482	0.227	0.016	0.547	2.879	0.000	0.213	0.000	0.632

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	131	217	130	924	0	129	-1	194
N.S.	1	1.00	0.86	1.43	0.86	6.08	0.00	0.85	-0.01	1.28
time (sec)	N/A	0.274	0.222	0.010	0.518	2.802	0.000	0.224	0.000	0.466

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	105	172	90	745	0	105	-1	170
N.S.	1	1.00	0.96	1.58	0.83	6.83	0.00	0.96	-0.01	1.56
time (sec)	N/A	0.128	0.074	0.009	0.491	0.529	0.000	0.226	0.000	0.454

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	86	151	71	631	0	88	-1	150
N.S.	1	1.00	1.00	1.76	0.83	7.34	0.00	1.02	-0.01	1.74
time (sec)	N/A	0.044	0.026	0.010	0.485	0.532	0.000	0.201	0.000	0.371

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	127	52	211	0	59	-1	114
N.S.	1	1.00	1.00	2.35	0.96	3.91	0.00	1.09	-0.02	2.11
time (sec)	N/A	0.017	0.007	0.005	0.463	0.425	0.000	0.189	0.000	0.005

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	86	158	0	634	0	0	-1	161
N.S.	1	1.00	1.00	1.84	0.00	7.37	0.00	0.00	-0.01	1.87
time (sec)	N/A	0.083	0.045	0.012	0.000	0.461	0.000	0.000	0.000	0.348

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	107	180	0	767	0	142	-1	186
N.S.	1	1.00	0.96	1.62	0.00	6.91	0.00	1.28	-0.01	1.68
time (sec)	N/A	0.095	0.083	0.012	0.000	0.484	0.000	0.221	0.000	0.404

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	163	236	0	956	0	239	-1	203
N.S.	1	1.00	0.97	1.40	0.00	5.69	0.00	1.42	-0.01	1.21
time (sec)	N/A	0.140	0.642	0.014	0.000	0.504	0.000	0.222	0.000	0.690

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	179	396	251	1525	0	299	-1	223
N.S.	1	1.00	1.23	2.71	1.72	10.45	0.00	2.05	-0.01	1.53
time (sec)	N/A	0.312	0.444	0.020	0.621	4.653	0.000	0.265	0.000	0.846

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	153	354	211	1323	0	219	-1	189
N.S.	1	1.00	1.24	2.88	1.72	10.76	0.00	1.78	-0.01	1.54
time (sec)	N/A	0.165	0.280	0.010	0.587	4.690	0.000	0.235	0.000	0.691

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	95	311	171	455	0	174	-1	156
N.S.	1	1.00	1.00	3.27	1.80	4.79	0.00	1.83	-0.01	1.64
time (sec)	N/A	0.111	0.084	0.012	0.546	0.474	0.000	0.217	0.000	0.512

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	88	283	148	425	0	162	-1	149
N.S.	1	1.00	1.00	3.22	1.68	4.83	0.00	1.84	-0.01	1.69
time (sec)	N/A	0.052	0.053	0.007	0.540	0.479	0.000	0.205	0.000	0.531

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	94	260	123	456	0	172	-1	154
N.S.	1	1.00	1.00	2.77	1.31	4.85	0.00	1.83	-0.01	1.64
time (sec)	N/A	0.046	0.045	0.006	0.502	0.486	0.000	0.232	0.000	0.012

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	132	318	0	1325	0	0	-1	200
N.S.	1	1.00	0.90	2.16	0.00	9.01	0.00	0.00	-0.01	1.36
time (sec)	N/A	0.135	0.168	0.013	0.000	0.692	0.000	0.000	0.000	1.141

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	194	163	363	0	1556	0	266	-1	242
N.S.	1	1.00	0.84	1.87	0.00	8.02	0.00	1.37	-0.01	1.25
time (sec)	N/A	0.167	0.405	0.012	0.000	0.717	0.000	0.251	0.000	0.734

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	275	203	439	0	1943	0	358	-1	287
N.S.	1	1.00	0.74	1.59	0.00	7.04	0.00	1.30	-0.00	1.04
time (sec)	N/A	0.236	0.342	0.014	0.000	1.044	0.000	0.289	0.000	1.019

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	230	474	274	2025	0	0	-1	291
N.S.	1	1.00	0.94	1.94	1.12	8.30	0.00	0.00	-0.00	1.19
time (sec)	N/A	0.890	0.513	0.019	0.604	41.858	0.000	0.000	0.000	1.587

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	208	435	233	1786	0	0	-1	238
N.S.	1	1.00	1.02	2.13	1.14	8.75	0.00	0.00	-0.00	1.17
time (sec)	N/A	0.523	0.366	0.013	0.571	66.283	0.000	0.000	0.000	1.351

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	184	386	193	1449	0	0	-1	197
N.S.	1	1.00	1.15	2.41	1.21	9.06	0.00	0.00	-0.01	1.23
time (sec)	N/A	0.327	0.274	0.013	0.543	6.774	0.000	0.000	0.000	1.117

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	172	368	171	1260	0	0	-1	168
N.S.	1	1.00	1.26	2.69	1.25	9.20	0.00	0.00	-0.01	1.23
time (sec)	N/A	0.168	0.323	0.011	0.533	6.367	0.000	0.000	0.000	0.806

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	90	340	148	382	0	0	-1	150
N.S.	1	1.00	1.00	3.78	1.64	4.24	0.00	0.00	-0.01	1.67
time (sec)	N/A	0.038	0.046	0.009	0.515	0.458	0.000	0.000	0.000	0.609

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	115	210	93	381	0	0	-1	151
N.S.	1	1.00	1.26	2.31	1.02	4.19	0.00	0.00	-0.01	1.66
time (sec)	N/A	0.034	0.074	0.006	0.490	0.458	0.000	0.000	0.000	0.007

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	178	364	0	1261	0	126	-1	178
N.S.	1	1.00	0.99	2.03	0.00	7.04	0.00	0.70	-0.01	0.99
time (sec)	N/A	0.137	0.222	0.011	0.000	0.808	0.000	0.566	0.000	0.804

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	197	395	0	1512	0	0	-1	214
N.S.	1	1.00	0.93	1.86	0.00	7.13	0.00	0.00	-0.00	1.01
time (sec)	N/A	0.168	0.328	0.012	0.000	0.761	0.000	0.000	0.000	1.317

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	229	452	0	1867	0	0	-1	248
N.S.	1	1.00	0.85	1.69	0.00	6.97	0.00	0.00	-0.00	0.93
time (sec)	N/A	0.222	0.448	0.012	0.000	1.291	0.000	0.000	0.000	1.784

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	114	328	210	368	4134	624	363	0
N.S.	1	1.00	0.84	2.43	1.56	2.73	30.62	4.62	2.69	0.00
time (sec)	N/A	0.082	0.092	0.007	0.472	0.415	6.697	0.194	2.816	0.062

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	109	195	146	250	2181	410	255	0
N.S.	1	1.00	1.07	1.91	1.43	2.45	21.38	4.02	2.50	0.00
time (sec)	N/A	0.052	0.104	0.005	0.467	0.414	3.535	0.189	2.701	0.045

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	65	100	89	148	952	237	163	0
N.S.	1	1.00	0.93	1.43	1.27	2.11	13.60	3.39	2.33	0.00
time (sec)	N/A	0.031	0.041	0.004	0.457	0.411	2.073	0.164	2.629	0.041

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	232	232	199	1000	447	1027	14317	1750	932	0
N.S.	1	1.00	0.86	4.31	1.93	4.43	61.71	7.54	4.02	0.00
time (sec)	N/A	0.139	0.181	0.016	0.505	0.435	21.587	0.215	3.119	0.068

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	323	677	335	757	8940	1266	723	0
N.S.	1	1.00	1.75	3.66	1.81	4.09	48.32	6.84	3.91	0.00
time (sec)	N/A	0.100	0.498	0.012	0.500	0.424	13.801	0.215	3.051	0.060

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	160	420	235	519	5097	851	496	0
N.S.	1	1.00	1.14	3.00	1.68	3.71	36.41	6.08	3.54	0.00
time (sec)	N/A	0.068	0.178	0.010	0.477	0.418	7.128	0.200	2.837	0.055

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	302	2232	795	2165	0	3713	1796	0
N.S.	1	1.00	0.88	6.51	2.32	6.31	0.00	10.83	5.24	0.00
time (sec)	N/A	0.211	0.248	0.024	0.550	0.449	0.000	0.304	3.809	0.085

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	282	282	709	1639	625	1675	0	2851	1459	0
N.S.	1	1.00	2.51	5.81	2.22	5.94	0.00	10.11	5.17	0.00
time (sec)	N/A	0.168	1.468	0.018	0.532	0.446	0.000	0.253	3.478	0.073

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	347	1140	472	1244	0	2085	1144	0
N.S.	1	1.00	1.56	5.11	2.12	5.58	0.00	9.35	5.13	0.00
time (sec)	N/A	0.126	0.498	0.014	0.507	0.414	0.000	0.235	3.164	0.070

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	345	345	304	946	0	678	0	0	-1	0
N.S.	1	1.00	0.88	2.74	0.00	1.97	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.506	1.593	0.028	0.000	0.574	0.000	0.000	0.000	180.359

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	245	713	0	536	0	0	-1	13727
N.S.	1	1.00	0.98	2.84	0.00	2.14	0.00	0.00	-0.00	54.69
time (sec)	N/A	0.260	0.825	0.014	0.000	0.472	0.000	0.000	0.000	14.050

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	207	197	516	0	418	0	0	-1	330
N.S.	1	1.00	0.95	2.49	0.00	2.02	0.00	0.00	-0.00	1.59
time (sec)	N/A	0.192	0.661	0.010	0.000	0.434	0.000	0.000	0.000	2.221

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	155	205	0	337	0	0	-1	272
N.S.	1	1.00	1.18	1.56	0.00	2.57	0.00	0.00	-0.01	2.08
time (sec)	N/A	0.070	0.752	0.006	0.000	0.436	0.000	0.000	0.000	0.017

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	210	439	0	947	0	0	-1	325
N.S.	1	1.00	1.25	2.61	0.00	5.64	0.00	0.00	-0.01	1.93
time (sec)	N/A	0.167	0.182	0.015	0.000	0.655	0.000	0.000	0.000	0.683

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	117	594	0	355	0	0	-1	124
N.S.	1	1.00	0.85	4.34	0.00	2.59	0.00	0.00	-0.01	0.91
time (sec)	N/A	0.142	0.139	0.015	0.000	0.519	0.000	0.000	0.000	0.569

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	162	882	0	442	0	0	-1	170
N.S.	1	1.00	0.80	4.37	0.00	2.19	0.00	0.00	-0.00	0.84
time (sec)	N/A	0.276	0.160	0.017	0.000	0.914	0.000	0.000	0.000	0.895

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	286	286	210	1165	0	558	0	0	-1	228
N.S.	1	1.00	0.73	4.07	0.00	1.95	0.00	0.00	-0.00	0.80
time (sec)	N/A	0.404	0.239	0.019	0.000	2.116	0.000	0.000	0.000	1.225

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	389	389	273	1494	0	702	0	0	-1	307
N.S.	1	1.00	0.70	3.84	0.00	1.80	0.00	0.00	-0.00	0.79
time (sec)	N/A	0.594	0.362	0.023	0.000	9.699	0.000	0.000	0.000	1.745

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	449	449	425	1883	0	1044	0	0	-1	0
N.S.	1	1.00	0.95	4.19	0.00	2.33	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.570	2.288	0.026	0.000	0.508	0.000	0.000	0.000	180.434

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	352	352	497	1560	0	846	0	0	-1	0
N.S.	1	1.00	1.41	4.43	0.00	2.40	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.328	2.771	0.016	0.000	0.459	0.000	0.000	0.000	181.183

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	295	295	276	1279	0	676	0	0	-1	0
N.S.	1	1.00	0.94	4.34	0.00	2.29	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.282	1.240	0.012	0.000	0.459	0.000	0.000	0.000	180.774

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	264	566	462	532	0	0	-1	13228
N.S.	1	1.00	1.31	2.82	2.30	2.65	0.00	0.00	-0.00	65.81
time (sec)	N/A	0.117	0.675	0.010	0.503	0.428	0.000	0.000	0.000	0.228

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	275	1130	0	1327	0	0	-1	428
N.S.	1	1.00	1.10	4.50	0.00	5.29	0.00	0.00	-0.00	1.71
time (sec)	N/A	0.276	0.853	0.015	0.000	4.404	0.000	0.000	0.000	4.805

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	263	1310	0	1221	0	0	-1	383
N.S.	1	1.00	1.10	5.46	0.00	5.09	0.00	0.00	-0.00	1.60
time (sec)	N/A	0.273	1.201	0.017	0.000	1.801	0.000	0.000	0.000	2.289

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	256	256	285	1604	0	1375	0	0	-1	395
N.S.	1	1.00	1.11	6.27	0.00	5.37	0.00	0.00	-0.00	1.54
time (sec)	N/A	0.283	2.446	0.019	0.000	2.314	0.000	0.000	0.000	1.891

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	188	1945	0	558	0	0	-1	229
N.S.	1	1.00	0.89	9.22	0.00	2.64	0.00	0.00	-0.00	1.09
time (sec)	N/A	0.236	0.277	0.022	0.000	1.554	0.000	0.000	0.000	1.832

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	295	295	253	2427	0	704	0	0	-1	307
N.S.	1	1.00	0.86	8.23	0.00	2.39	0.00	0.00	-0.00	1.04
time (sec)	N/A	0.386	0.303	0.027	0.000	8.659	0.000	0.000	0.000	2.191

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	395	395	310	2888	0	872	0	0	-1	0
N.S.	1	1.00	0.78	7.31	0.00	2.21	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.514	0.487	0.033	0.000	20.245	0.000	0.000	0.000	180.107

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	498	498	380	3387	0	1072	0	0	-1	0
N.S.	1	1.00	0.76	6.80	0.00	2.15	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.725	0.775	0.045	0.000	55.728	0.000	0.000	0.000	180.876

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	574	574	681	3178	0	1524	0	0	-1	0
N.S.	1	1.00	1.19	5.54	0.00	2.66	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.695	3.608	0.028	0.000	0.574	0.000	0.000	0.000	180.030

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	452	452	562	2731	0	1272	0	0	-1	0
N.S.	1	1.00	1.24	6.04	0.00	2.81	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.410	5.706	0.018	0.000	0.507	0.000	0.000	0.000	180.074

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	381	381	506	2411	0	1046	0	0	-1	0
N.S.	1	1.00	1.33	6.33	0.00	2.75	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.386	2.735	0.013	0.000	0.491	0.000	0.000	0.000	180.018

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	274	274	384	1123	915	844	0	0	-1	0
N.S.	1	1.00	1.40	4.10	3.34	3.08	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.185	1.217	0.010	0.574	0.461	0.000	0.000	0.000	180.036

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	394	394	390	2180	0	1873	0	0	-1	0
N.S.	1	1.00	0.99	5.53	0.00	4.75	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.450	2.007	0.017	0.000	45.741	0.000	0.000	0.000	180.014

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	352	352	350	2364	0	1717	0	0	-1	0
N.S.	1	1.00	0.99	6.72	0.00	4.88	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.432	2.073	0.018	0.000	15.202	0.000	0.000	0.000	180.158

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	339	339	334	2688	0	1569	0	0	-1	0
N.S.	1	1.00	0.99	7.93	0.00	4.63	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.388	2.245	0.021	0.000	7.419	0.000	0.000	0.000	180.012

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	371	371	357	3144	0	1741	0	0	-1	0
N.S.	1	1.00	0.96	8.47	0.00	4.69	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.470	3.038	0.027	0.000	8.715	0.000	0.000	0.000	180.006

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	404	404	404	3646	0	1917	0	0	-1	544
N.S.	1	1.00	1.00	9.02	0.00	4.75	0.00	0.00	-0.00	1.35
time (sec)	N/A	0.456	3.482	0.033	0.000	24.351	0.000	0.000	0.000	2.567

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	295	3991	0	872	0	0	-1	0
N.S.	1	1.00	1.02	13.81	0.00	3.02	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.328	0.939	0.045	0.000	19.496	0.000	0.000	0.000	180.096

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	386	386	344	4735	0	1072	0	0	-1	0
N.S.	1	1.00	0.89	12.27	0.00	2.78	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.495	0.994	0.064	0.000	60.752	0.000	0.000	0.000	180.313

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	500	500	408	5353	0	1300	0	0	-1	0
N.S.	1	1.00	0.82	10.71	0.00	2.60	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.639	0.747	0.086	0.000	124.864	0.000	0.000	0.000	184.301

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	628	628	512	6030	0	0	0	0	-1	0
N.S.	1	1.00	0.82	9.60	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.885	1.456	0.128	0.000	0.000	0.000	0.000	0.000	180.027

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	298	331	391	0	758	0	0	-1	0
N.S.	1	1.10	1.22	1.44	0.00	2.80	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.341	0.514	0.021	0.000	0.903	0.000	0.000	0.000	180.011

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	255	241	0	586	0	0	-1	328
N.S.	1	1.00	1.31	1.24	0.00	3.01	0.00	0.00	-0.01	1.68
time (sec)	N/A	0.346	0.364	0.013	0.000	0.568	0.000	0.000	0.000	1.803

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	189	131	0	443	0	0	-1	296
N.S.	1	1.00	1.36	0.94	0.00	3.19	0.00	0.00	-0.01	2.13
time (sec)	N/A	0.109	0.416	0.010	0.000	0.551	0.000	0.000	0.000	0.740

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	42	51	0	59	0	0	50	52
N.S.	1	1.00	0.81	0.98	0.00	1.13	0.00	0.00	0.96	1.00
time (sec)	N/A	0.024	0.016	0.008	0.000	0.475	0.000	0.000	2.645	0.003

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	131	136	0	454	0	0	-1	146
N.S.	1	1.00	0.92	0.95	0.00	3.17	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.175	0.126	0.016	0.000	0.773	0.000	0.000	0.000	0.545

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	229	229	201	253	0	610	0	0	-1	178
N.S.	1	1.00	0.88	1.10	0.00	2.66	0.00	0.00	-0.00	0.78
time (sec)	N/A	0.292	0.123	0.016	0.000	1.538	0.000	0.000	0.000	0.973

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	329	329	283	414	0	792	0	0	-1	256
N.S.	1	1.00	0.86	1.26	0.00	2.41	0.00	0.00	-0.00	0.78
time (sec)	N/A	0.508	0.189	0.017	0.000	3.716	0.000	0.000	0.000	1.451

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	515	515	296	1680	0	2120	0	0	-1	0
N.S.	1	1.00	0.57	3.26	0.00	4.12	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.619	5.632	0.030	0.000	6.648	0.000	0.000	0.000	180.027

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	438	438	387	1266	0	1782	0	0	-1	0
N.S.	1	1.00	0.88	2.89	0.00	4.07	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.541	1.342	0.013	0.000	2.663	0.000	0.000	0.000	180.170

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	297	297	1443	977	0	1466	0	0	-1	10635
N.S.	1	1.00	4.86	3.29	0.00	4.94	0.00	0.00	-0.00	35.81
time (sec)	N/A	0.293	4.640	0.014	0.000	3.244	0.000	0.000	0.000	10.109

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	99	145	0	308	0	0	1071	20752
N.S.	1	1.00	0.79	1.15	0.00	2.44	0.00	0.00	8.50	164.70
time (sec)	N/A	0.108	0.065	0.011	0.000	2.825	0.000	0.000	3.603	152.020

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	100	149	0	314	0	0	499	25359
N.S.	1	1.00	0.72	1.08	0.00	2.28	0.00	0.00	3.62	183.76
time (sec)	N/A	0.094	0.035	0.010	0.000	2.719	0.000	0.000	3.319	153.712

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	95	138	0	306	0	0	120	27688
N.S.	1	1.00	0.79	1.14	0.00	2.53	0.00	0.00	0.99	228.83
time (sec)	N/A	0.042	0.032	0.010	0.000	2.297	0.000	0.000	2.885	0.024

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	262	682	0	1476	0	0	-1	0
N.S.	1	1.00	0.97	2.52	0.00	5.45	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.338	0.415	0.016	0.000	6.516	0.000	0.000	0.000	180.566

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	394	394	370	912	0	1812	0	0	-1	0
N.S.	1	1.00	0.94	2.31	0.00	4.60	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.590	0.569	0.016	0.000	19.593	0.000	0.000	0.000	180.035

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	522	522	467	1319	0	2162	0	0	-1	0
N.S.	1	1.00	0.89	2.53	0.00	4.14	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.800	0.935	0.020	0.000	42.287	0.000	0.000	0.000	180.165

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	664	664	593	1705	0	2526	0	0	-1	711
N.S.	1	1.00	0.89	2.57	0.00	3.80	0.00	0.00	-0.00	1.07
time (sec)	N/A	1.170	1.395	0.021	0.000	93.161	0.000	0.000	0.000	9.132

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	259	259	235	366	0	820	0	0	3099	0
N.S.	1	1.00	0.91	1.41	0.00	3.17	0.00	0.00	11.97	0.00
time (sec)	N/A	0.236	0.123	0.014	0.000	27.129	0.000	0.000	4.327	180.065

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	341	341	433	663	0	1540	0	0	11469	0
N.S.	1	1.00	1.27	1.94	0.00	4.52	0.00	0.00	33.63	0.00
time (sec)	N/A	0.289	0.195	0.020	0.000	167.967	0.000	0.000	7.725	180.032

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	22	22	0	67	22	0
N.S.	1	1.00	1.00	0.78	0.96	0.96	0.00	2.91	0.96	0.00
time (sec)	N/A	0.018	0.032	0.003	0.955	0.383	0.000	0.241	2.620	33.849

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	197	43	0	60	0	0	-1	0
N.S.	1	1.00	2.98	0.65	0.00	0.91	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.033	0.407	0.026	0.000	0.400	0.000	0.000	0.000	22.478

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	27	27	0	173	25	0
N.S.	1	1.00	1.00	0.78	1.17	1.17	0.00	7.52	1.09	0.00
time (sec)	N/A	0.021	0.042	0.005	0.974	0.386	0.000	0.610	0.117	71.299

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	201	57	0	65	0	0	-1	0
N.S.	1	1.00	2.14	0.61	0.00	0.69	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.041	0.296	0.011	0.000	0.398	0.000	0.000	0.000	48.003

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	22	17	0	18	9	0
N.S.	1	1.00	1.00	0.78	0.96	0.74	0.00	0.78	0.39	0.00
time (sec)	N/A	0.019	0.024	0.005	0.970	0.383	0.000	0.179	0.150	150.827

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	42	42	2463	33	0	43	0	0	-1	61
N.S.	1	1.00	58.64	0.79	0.00	1.02	0.00	0.00	-0.02	1.45
time (sec)	N/A	0.026	12.801	0.028	0.000	0.394	0.000	0.000	0.000	8.247

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	17	24	0	0	17	0
N.S.	1	1.00	1.00	0.78	0.74	1.04	0.00	0.00	0.74	0.00
time (sec)	N/A	0.021	0.030	0.006	0.978	0.377	0.000	0.000	2.692	173.280

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	66	66	2511	43	0	78	0	0	-1	0
N.S.	1	1.00	38.05	0.65	0.00	1.18	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.034	6.077	0.041	0.000	0.388	0.000	0.000	0.000	122.667

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	24	29	0	0	82	0
N.S.	1	1.00	1.00	0.78	1.04	1.26	0.00	0.00	3.57	0.00
time (sec)	N/A	0.021	0.040	0.005	0.971	0.396	0.000	0.000	2.875	180.004

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	96	96	2539	69	0	101	0	0	-1	0
N.S.	1	1.00	26.45	0.72	0.00	1.05	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.037	6.089	0.063	0.000	0.385	0.000	0.000	0.000	169.093

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	78	68	75	134	88	71	84	0
N.S.	1	1.00	0.80	0.70	0.77	1.38	0.91	0.73	0.87	0.00
time (sec)	N/A	0.071	0.052	0.013	0.960	0.391	0.243	0.183	0.129	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	490	490	568	2218	0	5507	0	1171	13879	784
N.S.	1	1.00	1.16	4.53	0.00	11.24	0.00	2.39	28.32	1.60
time (sec)	N/A	14.847	0.746	0.110	0.000	1.003	0.000	0.827	4.857	2.224

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	326	397	466	1764	0	4245	0	1045	11143	584
N.S.	1	1.22	1.43	5.41	0.00	13.02	0.00	3.21	34.18	1.79
time (sec)	N/A	7.466	0.531	0.058	0.000	0.718	0.000	0.564	4.366	1.505

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	375	1329	0	2966	0	868	8171	445
N.S.	1	1.00	1.19	4.21	0.00	9.39	0.00	2.75	25.86	1.41
time (sec)	N/A	3.158	0.442	0.045	0.000	0.524	0.000	0.447	3.911	1.163

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	287	287	301	926	0	1721	0	753	5664	387
N.S.	1	1.00	1.05	3.23	0.00	6.00	0.00	2.62	19.74	1.35
time (sec)	N/A	3.200	0.406	0.038	0.000	0.455	0.000	0.412	3.820	1.184

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	175	545	0	715	155	223	709	275
N.S.	1	1.00	0.88	2.75	0.00	3.61	0.78	1.13	3.58	1.39
time (sec)	N/A	0.271	0.397	0.027	0.000	0.422	50.096	0.268	2.992	0.002

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	275	275	267	581	0	2446	0	712	10894	274
N.S.	1	1.00	0.97	2.11	0.00	8.89	0.00	2.59	39.61	1.00
time (sec)	N/A	1.129	0.938	0.058	0.000	0.786	0.000	0.392	7.410	0.994

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	368	356	364	999	0	4860	0	0	19887	424
N.S.	1	0.97	0.99	2.71	0.00	13.21	0.00	0.00	54.04	1.15
time (sec)	N/A	3.664	1.591	0.050	0.000	9.995	0.000	0.000	6.814	1.708

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	531	531	516	1486	0	7425	0	1041	33838	577
N.S.	1	1.00	0.97	2.80	0.00	13.98	0.00	1.96	63.73	1.09
time (sec)	N/A	3.569	2.335	0.051	0.000	149.303	0.000	0.585	8.089	2.462

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	650	650	808	3685	0	14340	0	1577	31485	1229
N.S.	1	1.00	1.24	5.67	0.00	22.06	0.00	2.43	48.44	1.89
time (sec)	N/A	2.679	1.116	0.081	0.000	11.223	0.000	0.686	7.969	3.483

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	581	581	680	2988	0	11459	0	1362	25497	996
N.S.	1	1.00	1.17	5.14	0.00	19.72	0.00	2.34	43.88	1.71
time (sec)	N/A	15.247	0.904	0.063	0.000	6.061	0.000	0.584	7.139	2.968

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	441	441	538	2358	0	8530	0	1160	19465	702
N.S.	1	1.00	1.22	5.35	0.00	19.34	0.00	2.63	44.14	1.59
time (sec)	N/A	2.150	0.675	0.059	0.000	2.404	0.000	0.526	5.724	1.934

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	453	453	779	1714	0	5572	0	978	13841	599
N.S.	1	1.00	1.72	3.78	0.00	12.30	0.00	2.16	30.55	1.32
time (sec)	N/A	4.529	1.489	0.052	0.000	1.126	0.000	0.460	4.723	2.023

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	317	1138	0	2770	0	783	8334	436
N.S.	1	1.00	0.98	3.53	0.00	8.60	0.00	2.43	25.88	1.35
time (sec)	N/A	1.244	0.705	0.038	0.000	0.543	0.000	0.422	4.435	0.002

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	340	340	331	944	0	5167	0	822	20897	383
N.S.	1	1.00	0.97	2.78	0.00	15.20	0.00	2.42	61.46	1.13
time (sec)	N/A	1.578	1.160	0.043	0.000	7.611	0.000	0.405	8.163	1.399

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	403	402	393	1215	0	8653	0	425	29890	529
N.S.	1	1.00	0.98	3.01	0.00	21.47	0.00	1.05	74.17	1.31
time (sec)	N/A	3.075	1.540	0.048	0.000	34.642	0.000	0.547	7.365	1.998

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	607	607	587	1880	0	0	0	1121	44649	759
N.S.	1	1.00	0.97	3.10	0.00	0.00	0.00	1.85	73.56	1.25
time (sec)	N/A	3.931	2.857	0.056	0.000	0.000	0.000	0.612	8.194	3.204

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	134	186	175	176	150	249	351	0
N.S.	1	1.00	0.95	1.32	1.24	1.25	1.06	1.77	2.49	0.00
time (sec)	N/A	0.183	0.077	0.006	0.472	0.386	0.604	0.164	0.111	0.001

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	103	145	138	139	109	211	197	0
N.S.	1	1.00	0.94	1.33	1.27	1.28	1.00	1.94	1.81	0.00
time (sec)	N/A	0.136	0.054	0.005	0.446	0.410	0.479	0.209	2.586	0.001

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	73	110	97	98	70	172	127	0
N.S.	1	1.00	1.12	1.69	1.49	1.51	1.08	2.65	1.95	0.00
time (sec)	N/A	0.060	0.034	0.003	0.445	0.388	0.381	0.155	0.071	0.001

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	43	82	63	64	46	134	65	0
N.S.	1	1.00	0.86	1.64	1.26	1.28	0.92	2.68	1.30	0.00
time (sec)	N/A	0.034	0.020	0.003	0.439	0.390	0.291	0.170	2.605	0.001

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	55	107	82	76	112	81	81	0
N.S.	1	1.00	0.89	1.73	1.32	1.23	1.81	1.31	1.31	0.00
time (sec)	N/A	0.080	0.022	0.008	0.445	0.407	0.645	0.152	0.154	0.001

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	82	149	113	165	182	0	109	0
N.S.	1	1.00	0.95	1.73	1.31	1.92	2.12	0.00	1.27	0.00
time (sec)	N/A	0.086	0.047	0.010	0.456	0.396	1.002	0.000	2.696	0.001

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	87	206	149	271	185	0	100	0
N.S.	1	1.00	1.00	2.37	1.71	3.11	2.13	0.00	1.15	0.00
time (sec)	N/A	0.097	0.075	0.011	0.450	0.405	1.027	0.000	0.129	0.001

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	122	259	206	400	248	0	152	0
N.S.	1	1.00	1.08	2.29	1.82	3.54	2.19	0.00	1.35	0.00
time (sec)	N/A	0.116	0.059	0.010	0.478	0.387	1.416	0.000	2.650	0.001

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	142	312	236	511	282	0	180	0
N.S.	1	1.00	1.02	2.24	1.70	3.68	2.03	0.00	1.29	0.00
time (sec)	N/A	0.133	0.090	0.012	0.494	0.422	1.925	0.000	0.145	0.001

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	226	286	258	328	250	367	1029	0
N.S.	1	1.00	1.04	1.31	1.18	1.50	1.15	1.68	4.72	0.00
time (sec)	N/A	0.284	0.121	0.012	0.453	0.415	1.203	0.183	2.641	0.001

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	185	245	218	288	199	327	565	0
N.S.	1	1.00	1.05	1.38	1.23	1.63	1.12	1.85	3.19	0.00
time (sec)	N/A	0.232	0.120	0.010	0.448	0.392	1.014	0.182	2.615	0.001

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	154	204	182	251	162	291	316	0
N.S.	1	1.00	1.05	1.40	1.25	1.72	1.11	1.99	2.16	0.00
time (sec)	N/A	0.181	0.093	0.010	0.460	0.400	0.854	0.188	0.094	0.001

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	115	167	141	206	119	250	185	0
N.S.	1	1.00	1.07	1.56	1.32	1.93	1.11	2.34	1.73	0.00
time (sec)	N/A	0.136	0.085	0.010	0.450	0.386	0.743	0.174	0.070	0.001

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	83	138	104	157	94	212	116	0
N.S.	1	1.00	1.06	1.77	1.33	2.01	1.21	2.72	1.49	0.00
time (sec)	N/A	0.096	0.058	0.007	0.442	0.385	0.586	0.184	2.535	0.001

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	46	96	69	95	61	160	72	0
N.S.	1	1.00	0.92	1.92	1.38	1.90	1.22	3.20	1.44	0.00
time (sec)	N/A	0.057	0.049	0.007	0.439	0.378	0.402	0.170	2.561	0.001

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	91	156	114	168	182	159	111	0
N.S.	1	1.00	1.06	1.81	1.33	1.95	2.12	1.85	1.29	0.00
time (sec)	N/A	0.082	0.049	0.009	0.453	0.387	1.040	0.185	2.644	0.001

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	85	180	111	155	156	101	115	0
N.S.	1	1.00	1.15	2.43	1.50	2.09	2.11	1.36	1.55	0.00
time (sec)	N/A	0.029	0.038	0.012	0.439	0.408	0.714	0.161	2.606	0.001

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	139	253	212	417	279	0	198	0
N.S.	1	1.00	1.15	2.09	1.75	3.45	2.31	0.00	1.64	0.00
time (sec)	N/A	0.136	0.114	0.013	0.466	0.384	1.264	0.000	0.146	0.001

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	171	270	197	337	241	0	148	0
N.S.	1	1.00	1.17	1.85	1.35	2.31	1.65	0.00	1.01	0.00
time (sec)	N/A	0.155	0.095	0.015	0.479	0.384	1.360	0.000	2.632	0.001

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	195	341	298	648	376	0	274	0
N.S.	1	1.00	1.10	1.92	1.67	3.64	2.11	0.00	1.54	0.00
time (sec)	N/A	0.200	0.144	0.016	0.504	0.403	1.928	0.000	2.700	0.001

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	229	394	342	693	427	0	314	0
N.S.	1	1.00	1.09	1.88	1.63	3.30	2.03	0.00	1.50	0.00
time (sec)	N/A	0.242	0.182	0.018	0.534	0.398	2.152	0.000	2.718	0.001

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	193	263	227	336	219	364	375	0
N.S.	1	1.00	1.08	1.47	1.27	1.88	1.22	2.03	2.09	0.00
time (sec)	N/A	0.241	0.100	0.010	0.468	0.382	1.551	0.194	0.141	0.001

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	157	228	188	294	178	324	240	0
N.S.	1	1.00	1.05	1.53	1.26	1.97	1.19	2.17	1.61	0.00
time (sec)	N/A	0.197	0.083	0.009	0.460	0.380	1.372	0.196	0.105	0.001

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	118	198	149	241	151	273	161	0
N.S.	1	1.00	1.00	1.68	1.26	2.04	1.28	2.31	1.36	0.00
time (sec)	N/A	0.143	0.090	0.009	0.457	0.388	1.212	0.377	2.597	0.001

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	93	151	105	159	102	227	107	0
N.S.	1	1.00	1.15	1.86	1.30	1.96	1.26	2.80	1.32	0.00
time (sec)	N/A	0.101	0.040	0.009	0.451	0.402	0.872	0.205	2.598	0.001

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	49	105	81	100	83	195	80	0
N.S.	1	1.00	0.80	1.72	1.33	1.64	1.36	3.20	1.31	0.00
time (sec)	N/A	0.060	0.026	0.007	0.443	0.382	0.540	0.204	0.069	0.001

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	90	218	150	271	185	197	103	0
N.S.	1	1.00	1.02	2.48	1.70	3.08	2.10	2.24	1.17	0.00
time (sec)	N/A	0.099	0.077	0.011	0.463	0.406	1.011	0.174	0.134	0.001

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	140	257	211	417	277	191	198	0
N.S.	1	1.00	1.15	2.11	1.73	3.42	2.27	1.57	1.62	0.00
time (sec)	N/A	0.115	0.102	0.030	0.471	0.398	1.325	0.168	2.638	0.001

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	110	298	152	252	144	127	114	0
N.S.	1	1.00	0.87	2.35	1.20	1.98	1.13	1.00	0.90	0.00
time (sec)	N/A	0.061	0.045	0.015	0.449	0.402	0.998	0.166	0.103	0.001

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	197	348	308	662	321	0	249	0
N.S.	1	1.00	1.05	1.85	1.64	3.52	1.71	0.00	1.32	0.00
time (sec)	N/A	0.210	0.155	0.015	0.497	0.401	1.831	0.000	2.679	0.001

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	235	235	244	421	359	793	372	0	296	0
N.S.	1	1.00	1.04	1.79	1.53	3.37	1.58	0.00	1.26	0.00
time (sec)	N/A	0.273	0.174	0.017	0.510	0.405	2.146	0.000	2.639	0.001

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	193	1308	1579	807	0	537	-1	385
N.S.	1	1.00	0.72	4.86	5.87	3.00	0.00	2.00	-0.00	1.43
time (sec)	N/A	0.975	0.968	0.059	1.054	0.494	0.000	0.661	0.000	1.982

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	168	1030	1178	624	0	411	-1	294
N.S.	1	1.00	0.78	4.79	5.48	2.90	0.00	1.91	-0.00	1.37
time (sec)	N/A	0.666	0.741	0.014	1.044	0.573	0.000	0.405	0.000	1.574

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	182	713	891	454	0	309	-1	223
N.S.	1	1.00	0.99	3.90	4.87	2.48	0.00	1.69	-0.01	1.22
time (sec)	N/A	0.403	0.807	0.013	1.032	0.440	0.000	0.380	0.000	1.117

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	110	131	583	279	0	198	125	129
N.S.	1	1.00	0.76	0.90	4.02	1.92	0.00	1.37	0.86	0.89
time (sec)	N/A	0.224	0.393	0.011	0.467	0.418	0.000	0.340	2.871	0.767

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	83	85	373	183	0	139	79	83
N.S.	1	1.00	0.71	0.73	3.19	1.56	0.00	1.19	0.68	0.71
time (sec)	N/A	0.059	0.237	0.009	0.464	0.390	0.000	0.332	2.791	0.550

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	58	55	101	106	0	70	49	53
N.S.	1	1.00	0.56	0.53	0.98	1.03	0.00	0.68	0.48	0.51
time (sec)	N/A	0.049	0.060	0.008	0.440	0.401	0.000	0.304	2.701	0.001

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	225	3961	0	1767	0	2966	-1	0
N.S.	1	1.00	0.93	16.37	0.00	7.30	0.00	12.26	-0.00	0.00
time (sec)	N/A	0.618	0.394	0.050	0.000	0.481	0.000	0.456	0.000	180.980

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	311	311	341	6760	0	3305	0	4343	-1	0
N.S.	1	1.00	1.10	21.74	0.00	10.63	0.00	13.96	-0.00	0.00
time (sec)	N/A	1.264	0.610	0.029	0.000	1.017	0.000	2.968	0.000	180.048

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	398	398	387	9593	0	5361	0	6017	-1	0
N.S.	1	1.00	0.97	24.10	0.00	13.47	0.00	15.12	-0.00	0.00
time (sec)	N/A	2.568	1.141	0.032	0.000	3.547	0.000	2.091	0.000	180.613

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	91	114	0	499	107	116	124	118
N.S.	1	1.00	0.81	1.02	0.00	4.46	0.96	1.04	1.11	1.05
time (sec)	N/A	0.215	0.089	0.019	0.000	0.434	87.879	0.187	0.234	0.221

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	207	365	326	324	0	378	222	427
N.S.	1	1.00	0.86	1.52	1.36	1.35	0.00	1.58	0.92	1.78
time (sec)	N/A	0.345	0.245	0.007	0.448	0.400	0.000	0.181	0.119	0.171

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	149	215	197	197	673	243	159	250
N.S.	1	1.00	0.85	1.23	1.13	1.13	3.85	1.39	0.91	1.43
time (sec)	N/A	0.237	0.150	0.007	0.448	0.394	108.501	0.172	2.580	0.113

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	94	101	104	100	374	134	100	117
N.S.	1	1.00	0.83	0.89	0.92	0.88	3.31	1.19	0.88	1.04
time (sec)	N/A	0.074	0.087	0.005	0.444	0.388	61.129	0.179	0.073	0.068

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	44	41	53	40	150	53	44	48
N.S.	1	1.00	0.72	0.67	0.87	0.66	2.46	0.87	0.72	0.79
time (sec)	N/A	0.026	0.026	0.004	0.433	0.400	13.098	0.154	2.558	0.035

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	92	132	0	297	100	107	107	105
N.S.	1	1.00	0.88	1.27	0.00	2.86	0.96	1.03	1.03	1.01
time (sec)	N/A	0.124	0.160	0.015	0.000	0.415	48.655	0.200	0.107	0.165

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	171	237	0	539	0	148	128	179
N.S.	1	1.00	1.40	1.94	0.00	4.42	0.00	1.21	1.05	1.47
time (sec)	N/A	0.203	0.288	0.017	0.000	0.409	0.000	0.174	2.682	0.512

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	207	384	0	896	0	278	224	225
N.S.	1	1.00	1.16	2.16	0.00	5.03	0.00	1.56	1.26	1.26
time (sec)	N/A	0.303	0.816	0.021	0.000	0.428	0.000	0.209	2.909	0.855

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	207	365	334	333	328	453	292	427
N.S.	1	1.00	0.87	1.53	1.40	1.40	1.38	1.90	1.23	1.79
time (sec)	N/A	0.268	0.240	0.008	0.460	0.393	110.873	0.209	0.091	0.181

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	149	215	205	206	204	275	199	250
N.S.	1	1.00	0.86	1.24	1.18	1.19	1.18	1.59	1.15	1.45
time (sec)	N/A	0.204	0.154	0.006	0.449	0.382	50.851	0.326	2.658	0.121

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	92	101	112	110	112	143	111	117
N.S.	1	1.00	0.83	0.91	1.01	0.99	1.01	1.29	1.00	1.05
time (sec)	N/A	0.066	0.083	0.004	0.445	0.396	25.285	0.213	0.075	0.067

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	43	41	54	49	58	56	44	47
N.S.	1	1.00	0.73	0.69	0.92	0.83	0.98	0.95	0.75	0.80
time (sec)	N/A	0.026	0.028	0.004	0.443	0.379	10.144	0.181	0.054	0.034

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	90	165	0	492	104	101	141	128
N.S.	1	1.00	0.80	1.47	0.00	4.39	0.93	0.90	1.26	1.14
time (sec)	N/A	0.174	0.068	0.014	0.000	0.425	41.279	0.200	0.136	0.221

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	118	269	0	906	0	225	187	210
N.S.	1	1.00	0.82	1.87	0.00	6.29	0.00	1.56	1.30	1.46
time (sec)	N/A	0.271	0.082	0.021	0.000	0.433	0.000	0.191	3.287	0.569

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	140	546	0	1539	0	361	310	369
N.S.	1	1.00	0.65	2.55	0.00	7.19	0.00	1.69	1.45	1.72
time (sec)	N/A	0.505	0.096	0.025	0.000	0.448	0.000	0.221	3.367	1.061

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	155	306	0	336	0	155	569	216
N.S.	1	1.00	1.05	2.08	0.00	2.29	0.00	1.05	3.87	1.47
time (sec)	N/A	0.141	0.558	0.038	0.000	0.470	0.000	0.270	20.128	0.404

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	C	A	C	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	16	16	66	13	9	12	129	12	16	46
N.S.	1	1.00	4.12	0.81	0.56	0.75	8.06	0.75	1.00	2.88
time (sec)	N/A	0.009	0.094	0.005	0.430	0.392	43.482	0.169	2.798	0.052

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	411	411	410	2497	0	0	0	0	-1	580
N.S.	1	1.00	1.00	6.08	0.00	0.00	0.00	0.00	-0.00	1.41
time (sec)	N/A	2.511	2.436	0.111	0.000	0.000	0.000	0.000	0.000	1.437

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	342	342	339	1569	0	0	0	0	-1	378
N.S.	1	1.00	0.99	4.59	0.00	0.00	0.00	0.00	-0.00	1.11
time (sec)	N/A	2.086	1.273	0.028	0.000	0.000	0.000	0.000	0.000	1.080

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	229	1383	0	1921	0	0	-1	1248
N.S.	1	1.00	0.95	5.76	0.00	8.00	0.00	0.00	-0.00	5.20
time (sec)	N/A	0.341	0.347	0.038	0.000	10.390	0.000	0.000	0.000	172.246

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	351	351	265	5383	0	5816	0	0	-1	401
N.S.	1	1.00	0.75	15.34	0.00	16.57	0.00	0.00	-0.00	1.14
time (sec)	N/A	2.153	0.696	0.062	0.000	52.328	0.000	0.000	0.000	1.247

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	613	613	353	14861	0	0	0	0	-1	498
N.S.	1	1.00	0.58	24.24	0.00	0.00	0.00	0.00	-0.00	0.81
time (sec)	N/A	3.160	2.864	0.087	0.000	0.000	0.000	0.000	0.000	1.776

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	337	337	339	2336	0	0	0	0	-1	378
N.S.	1	1.00	1.01	6.93	0.00	0.00	0.00	0.00	-0.00	1.12
time (sec)	N/A	2.463	1.091	0.037	0.000	0.000	0.000	0.000	0.000	1.161

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	229	1387	0	1913	0	0	-1	1253
N.S.	1	1.00	0.95	5.78	0.00	7.97	0.00	0.00	-0.00	5.22
time (sec)	N/A	0.334	0.334	0.033	0.000	9.630	0.000	0.000	0.000	116.508

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	225	1415	0	4325	0	0	-1	310
N.S.	1	1.00	0.98	6.15	0.00	18.80	0.00	0.00	-0.00	1.35
time (sec)	N/A	0.204	0.189	0.042	0.000	23.238	0.000	0.000	0.000	1.203

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	354	354	287	10977	0	11846	0	0	-1	393
N.S.	1	1.00	0.81	31.01	0.00	33.46	0.00	0.00	-0.00	1.11
time (sec)	N/A	0.614	0.695	0.093	0.000	80.356	0.000	0.000	0.000	1.084

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	625	625	336	8264	0	0	0	0	-1	1541
N.S.	1	1.00	0.54	13.22	0.00	0.00	0.00	0.00	-0.00	2.47
time (sec)	N/A	2.531	0.732	0.072	0.000	0.000	0.000	0.000	0.000	92.442

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	351	351	265	5383	0	5844	0	0	-1	401
N.S.	1	1.00	0.75	15.34	0.00	16.65	0.00	0.00	-0.00	1.14
time (sec)	N/A	1.806	0.641	0.056	0.000	66.392	0.000	0.000	0.000	1.276

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	354	354	287	10977	0	12028	0	0	-1	393
N.S.	1	1.00	0.81	31.01	0.00	33.98	0.00	0.00	-0.00	1.11
time (sec)	N/A	0.763	0.778	0.090	0.000	126.494	0.000	0.000	0.000	1.053

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	549	543	521	30656	0	0	0	0	-1	492
N.S.	1	0.99	0.95	55.84	0.00	0.00	0.00	0.00	-0.00	0.90
time (sec)	N/A	1.324	2.065	0.227	0.000	0.000	0.000	0.000	0.000	1.692

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	63	305	0	744	0	0	1610	59
N.S.	1	1.00	0.97	4.69	0.00	11.45	0.00	0.00	24.77	0.91
time (sec)	N/A	0.049	0.061	0.174	0.000	0.480	0.000	0.000	8.490	0.094

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	91	125	0	193	0	266	164	98
N.S.	1	1.00	1.14	1.56	0.00	2.41	0.00	3.32	2.05	1.22
time (sec)	N/A	0.143	0.136	0.018	0.000	0.399	0.000	0.350	2.955	0.354

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	148	1178	0	318	0	208	148	971
N.S.	1	1.00	1.38	11.01	0.00	2.97	0.00	1.94	1.38	9.07
time (sec)	N/A	0.241	0.296	0.038	0.000	0.543	0.000	0.588	0.286	2.556

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	120	524	0	318	0	131	148	971
N.S.	1	1.00	1.12	4.90	0.00	2.97	0.00	1.22	1.38	9.07
time (sec)	N/A	0.179	0.108	0.021	0.000	0.558	0.000	0.461	0.125	1.777

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	136	188	218	193	0	0	218	394
N.S.	1	1.00	0.51	0.70	0.81	0.72	0.00	0.00	0.81	1.46
time (sec)	N/A	0.423	0.127	0.009	0.656	0.421	0.000	0.000	3.658	0.849

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	89	116	133	123	0	0	142	199
N.S.	1	1.00	0.44	0.58	0.66	0.62	0.00	0.00	0.71	1.00
time (sec)	N/A	0.233	0.076	0.008	0.613	0.441	0.000	0.000	3.404	0.439

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	53	67	65	71	0	0	88	92
N.S.	1	1.00	0.42	0.54	0.52	0.57	0.00	0.00	0.70	0.74
time (sec)	N/A	0.092	0.047	0.007	0.551	0.399	0.000	0.000	3.226	0.234

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	35	50	18	49	0	0	54	57
N.S.	1	1.00	0.76	1.09	0.39	1.07	0.00	0.00	1.17	1.24
time (sec)	N/A	0.021	0.017	0.005	0.503	0.407	0.000	0.000	3.196	0.004

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	93	87	0	252	0	0	-1	609
N.S.	1	1.00	1.16	1.09	0.00	3.15	0.00	0.00	-0.01	7.61
time (sec)	N/A	0.131	0.042	0.030	0.000	0.424	0.000	0.000	0.000	5.269

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	136	168	0	703	0	0	-1	0
N.S.	1	1.00	0.97	1.20	0.00	5.02	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.191	0.106	0.023	0.000	0.426	0.000	0.000	0.000	180.128

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	77	285	0	1283	0	0	-1	0
N.S.	1	1.00	0.36	1.34	0.00	6.02	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.314	0.045	0.023	0.000	0.447	0.000	0.000	0.000	180.006

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	280	280	77	450	0	2027	0	0	-1	0
N.S.	1	1.00	0.28	1.61	0.00	7.24	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.424	0.044	0.024	0.000	0.472	0.000	0.000	0.000	180.011

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	257	257	134	187	165	216	0	0	252	202
N.S.	1	1.00	0.52	0.73	0.64	0.84	0.00	0.00	0.98	0.79
time (sec)	N/A	0.331	0.099	0.009	0.697	0.418	0.000	0.000	3.614	3.644

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	181	181	88	116	98	147	0	0	178	119
N.S.	1	1.00	0.49	0.64	0.54	0.81	0.00	0.00	0.98	0.66
time (sec)	N/A	0.185	0.068	0.007	0.632	0.411	0.000	0.000	3.433	2.125

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	51	66	48	96	0	0	118	63
N.S.	1	1.00	0.34	0.44	0.32	0.64	0.00	0.00	0.79	0.42
time (sec)	N/A	0.144	0.040	0.007	0.570	0.404	0.000	0.000	3.368	1.362

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	35	50	18	74	0	0	82	43
N.S.	1	1.00	0.76	1.09	0.39	1.61	0.00	0.00	1.78	0.93
time (sec)	N/A	0.022	0.012	0.003	0.512	0.414	0.000	0.000	3.265	0.005

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	71	128	0	553	0	0	-1	0
N.S.	1	1.00	0.53	0.96	0.00	4.16	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.176	0.028	0.026	0.000	0.436	0.000	0.000	0.000	180.017

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	73	225	0	1067	0	0	-1	0
N.S.	1	1.00	0.36	1.11	0.00	5.28	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.257	0.033	0.030	0.000	0.448	0.000	0.000	0.000	180.015

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	274	274	77	379	0	1863	0	0	-1	0
N.S.	1	1.00	0.28	1.38	0.00	6.80	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.352	0.038	0.035	0.000	0.470	0.000	0.000	0.000	180.004

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	131	187	219	251	0	0	278	201
N.S.	1	1.00	0.55	0.78	0.92	1.05	0.00	0.00	1.16	0.84
time (sec)	N/A	0.280	0.110	0.009	0.744	0.419	0.000	0.000	3.773	4.476

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	87	116	138	180	0	0	206	120
N.S.	1	1.00	0.41	0.55	0.65	0.85	0.00	0.00	0.98	0.57
time (sec)	N/A	0.220	0.070	0.009	0.673	0.408	0.000	0.000	3.609	2.987

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	52	66	73	129	0	0	149	66
N.S.	1	1.00	0.34	0.43	0.47	0.84	0.00	0.00	0.97	0.43
time (sec)	N/A	0.134	0.048	0.007	0.602	0.411	0.000	0.000	3.504	1.960

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	37	50	28	107	0	0	110	45
N.S.	1	1.00	0.77	1.04	0.58	2.23	0.00	0.00	2.29	0.94
time (sec)	N/A	0.022	0.029	0.003	0.519	0.407	0.000	0.000	3.316	0.003

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	73	219	0	1015	0	0	-1	0
N.S.	1	1.00	0.39	1.16	0.00	5.40	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.270	0.036	0.030	0.000	0.431	0.000	0.000	0.000	180.056

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	75	424	0	1907	0	0	-1	0
N.S.	1	1.00	0.28	1.58	0.00	7.12	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.340	0.045	0.035	0.000	0.482	0.000	0.000	0.000	180.635

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	342	342	79	670	0	2935	0	0	-1	0
N.S.	1	1.00	0.23	1.96	0.00	8.58	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.538	0.051	0.038	0.000	0.490	0.000	0.000	0.000	180.027

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	195	283	320	375	0	0	347	7594
N.S.	1	1.00	0.58	0.84	0.95	1.12	0.00	0.00	1.03	22.60
time (sec)	N/A	0.607	0.177	0.009	0.708	0.402	0.000	0.000	3.599	23.689

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	136	188	218	264	0	0	242	676
N.S.	1	1.00	0.51	0.70	0.81	0.98	0.00	0.00	0.90	2.51
time (sec)	N/A	0.390	0.117	0.010	0.646	0.415	0.000	0.000	3.371	1.254

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	90	116	133	173	0	0	157	365
N.S.	1	1.00	0.45	0.58	0.66	0.86	0.00	0.00	0.78	1.82
time (sec)	N/A	0.228	0.079	0.007	0.592	0.414	0.000	0.000	3.255	0.636

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	54	67	65	102	0	0	93	169
N.S.	1	1.00	0.43	0.54	0.52	0.82	0.00	0.00	0.74	1.35
time (sec)	N/A	0.095	0.049	0.006	0.541	0.415	0.000	0.000	3.131	0.321

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	37	50	18	57	0	0	49	82
N.S.	1	1.00	0.77	1.04	0.38	1.19	0.00	0.00	1.02	1.71
time (sec)	N/A	0.021	0.024	0.003	0.493	0.431	0.000	0.000	3.047	0.003

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	101	153	0	318	0	0	-1	931
N.S.	1	1.00	0.81	1.23	0.00	2.56	0.00	0.00	-0.01	7.51
time (sec)	N/A	0.186	0.131	0.022	0.000	0.441	0.000	0.000	0.000	4.696

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	110	161	0	562	0	0	-1	1241
N.S.	1	1.00	0.83	1.22	0.00	4.26	0.00	0.00	-0.01	9.40
time (sec)	N/A	0.161	0.192	0.025	0.000	0.451	0.000	0.000	0.000	12.699

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	207	79	285	0	1056	0	0	-1	0
N.S.	1	1.00	0.38	1.38	0.00	5.10	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.273	0.040	0.030	0.000	0.450	0.000	0.000	0.000	180.007

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	277	277	79	453	0	1732	0	0	-1	0
N.S.	1	1.00	0.29	1.64	0.00	6.25	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.350	0.043	0.036	0.000	0.504	0.000	0.000	0.000	180.023

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	347	347	79	696	0	2610	0	0	-1	0
N.S.	1	1.00	0.23	2.01	0.00	7.52	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.454	0.043	0.035	0.000	0.476	0.000	0.000	0.000	180.018

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	195	283	413	472	0	0	445	0
N.S.	1	1.00	0.58	0.84	1.23	1.40	0.00	0.00	1.32	0.00
time (sec)	N/A	0.607	0.231	0.010	0.741	0.430	0.000	0.000	3.800	180.097

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	137	188	294	340	0	0	310	0
N.S.	1	1.00	0.51	0.70	1.09	1.26	0.00	0.00	1.15	0.00
time (sec)	N/A	0.407	0.166	0.009	0.687	0.443	0.000	0.000	3.646	180.007

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	90	116	192	230	0	0	206	120
N.S.	1	1.00	0.45	0.58	0.96	1.15	0.00	0.00	1.03	0.60
time (sec)	N/A	0.233	0.119	0.009	0.625	0.418	0.000	0.000	3.426	2.269

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	54	67	107	137	0	0	109	67
N.S.	1	1.00	0.43	0.54	0.86	1.10	0.00	0.00	0.87	0.54
time (sec)	N/A	0.101	0.074	0.005	0.571	0.413	0.000	0.000	3.250	1.313

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	37	50	43	74	0	0	62	45
N.S.	1	1.00	0.77	1.04	0.90	1.54	0.00	0.00	1.29	0.94
time (sec)	N/A	0.023	0.032	0.005	0.505	0.397	0.000	0.000	3.077	0.003

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	132	263	0	408	0	0	-1	151
N.S.	1	1.00	0.74	1.47	0.00	2.28	0.00	0.00	-0.01	0.84
time (sec)	N/A	0.304	0.264	0.023	0.000	0.436	0.000	0.000	0.000	7.975

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	75	306	0	444	0	0	-1	159
N.S.	1	1.00	0.42	1.72	0.00	2.49	0.00	0.00	-0.01	0.89
time (sec)	N/A	0.251	0.051	0.030	0.000	0.524	0.000	0.000	0.000	103.687

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	135	276	0	840	0	0	-1	0
N.S.	1	1.00	0.69	1.42	0.00	4.31	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.256	0.323	0.029	0.000	0.453	0.000	0.000	0.000	180.003

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	265	265	79	453	0	1434	0	0	-1	0
N.S.	1	1.00	0.30	1.71	0.00	5.41	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.349	0.062	0.032	0.000	0.468	0.000	0.000	0.000	180.027

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	335	335	79	665	0	2238	0	0	-1	0
N.S.	1	1.00	0.24	1.99	0.00	6.68	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.452	0.061	0.041	0.000	0.485	0.000	0.000	0.000	180.005

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	405	405	79	955	0	3204	0	0	-1	0
N.S.	1	1.00	0.20	2.36	0.00	7.91	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.563	0.064	0.039	0.000	0.531	0.000	0.000	0.000	180.335

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	205	283	498	567	0	0	523	0
N.S.	1	1.00	0.61	0.84	1.48	1.69	0.00	0.00	1.56	0.00
time (sec)	N/A	0.618	0.209	0.010	0.761	0.419	0.000	0.000	4.086	180.021

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	147	188	362	416	0	0	379	0
N.S.	1	1.00	0.55	0.70	1.35	1.55	0.00	0.00	1.41	0.00
time (sec)	N/A	0.397	0.159	0.011	0.697	0.404	0.000	0.000	3.805	180.508

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	100	116	243	284	0	0	259	0
N.S.	1	1.00	0.50	0.58	1.22	1.42	0.00	0.00	1.30	0.00
time (sec)	N/A	0.235	0.112	0.009	0.637	0.418	0.000	0.000	3.562	180.007

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	64	67	141	173	0	0	134	67
N.S.	1	1.00	0.51	0.54	1.13	1.38	0.00	0.00	1.07	0.54
time (sec)	N/A	0.100	0.079	0.005	0.567	0.422	0.000	0.000	3.373	1.733

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	37	50	60	91	0	0	79	45
N.S.	1	1.00	0.77	1.04	1.25	1.90	0.00	0.00	1.65	0.94
time (sec)	N/A	0.021	0.040	0.005	0.508	0.406	0.000	0.000	3.160	0.003

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	145	431	0	587	0	0	-1	189
N.S.	1	1.00	0.61	1.83	0.00	2.49	0.00	0.00	-0.00	0.80
time (sec)	N/A	0.467	0.363	0.023	0.000	0.444	0.000	0.000	0.000	11.678

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	235	235	75	523	0	672	0	0	-1	215
N.S.	1	1.00	0.32	2.23	0.00	2.86	0.00	0.00	-0.00	0.91
time (sec)	N/A	0.378	0.068	0.031	0.000	0.486	0.000	0.000	0.000	104.169

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	79	526	0	683	0	0	-1	0
N.S.	1	1.00	0.32	2.14	0.00	2.78	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.342	0.072	0.033	0.000	0.624	0.000	0.000	0.000	180.036

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	171	441	0	1140	0	0	-1	0
N.S.	1	1.00	0.68	1.74	0.00	4.51	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.336	0.371	0.033	0.000	0.459	0.000	0.000	0.000	180.028

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	323	323	79	665	0	1862	0	0	-1	0
N.S.	1	1.00	0.24	2.06	0.00	5.76	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.473	0.075	0.035	0.000	0.477	0.000	0.000	0.000	180.022

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	393	393	79	924	0	2750	0	0	-1	0
N.S.	1	1.00	0.20	2.35	0.00	7.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.572	0.080	0.043	0.000	0.531	0.000	0.000	0.000	180.316

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	463	463	79	1261	0	3872	0	0	-1	0
N.S.	1	1.00	0.17	2.72	0.00	8.36	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.716	0.082	0.042	0.000	0.555	0.000	0.000	0.000	180.021

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	269	511	0	841	0	0	-1	238
N.S.	1	1.00	0.86	1.63	0.00	2.69	0.00	0.00	-0.00	0.76
time (sec)	N/A	0.562	0.606	0.045	0.000	1.560	0.000	0.000	0.000	8.641

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	234	328	0	655	0	0	-1	206
N.S.	1	1.00	0.96	1.34	0.00	2.68	0.00	0.00	-0.00	0.84
time (sec)	N/A	0.366	0.473	0.029	0.000	1.236	0.000	0.000	0.000	7.753

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	213	201	0	521	0	0	-1	171
N.S.	1	1.00	1.26	1.19	0.00	3.08	0.00	0.00	-0.01	1.01
time (sec)	N/A	0.222	0.192	0.025	0.000	1.145	0.000	0.000	0.000	0.710

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	160	120	0	343	0	0	-1	167
N.S.	1	1.00	1.52	1.14	0.00	3.27	0.00	0.00	-0.01	1.59
time (sec)	N/A	0.118	0.112	0.030	0.000	1.044	0.000	0.000	0.000	0.748

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	50	63	0	114	0	0	100	143
N.S.	1	1.00	0.82	1.03	0.00	1.87	0.00	0.00	1.64	2.34
time (sec)	N/A	0.065	0.031	0.007	0.000	0.426	0.000	0.000	4.638	0.641

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	69	98	0	288	0	0	147	196
N.S.	1	1.00	0.53	0.76	0.00	2.23	0.00	0.00	1.14	1.52
time (sec)	N/A	0.143	0.055	0.008	0.000	0.420	0.000	0.000	4.898	3.441

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	105	169	0	572	0	0	242	137
N.S.	1	1.00	0.53	0.85	0.00	2.89	0.00	0.00	1.22	0.69
time (sec)	N/A	0.219	0.086	0.010	0.000	0.450	0.000	0.000	5.170	8.502

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	152	260	0	953	0	0	357	169
N.S.	1	1.00	0.57	0.97	0.00	3.57	0.00	0.00	1.34	0.63
time (sec)	N/A	0.313	0.120	0.012	0.000	0.456	0.000	0.000	5.511	8.732

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	301	301	100	648	0	971	0	0	-1	342
N.S.	1	1.00	0.33	2.15	0.00	3.23	0.00	0.00	-0.00	1.14
time (sec)	N/A	0.469	0.118	0.038	0.000	1.226	0.000	0.000	0.000	3.856

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	227	100	396	0	725	0	0	-1	238
N.S.	1	1.00	0.44	1.74	0.00	3.19	0.00	0.00	-0.00	1.05
time (sec)	N/A	0.314	0.084	0.030	0.000	1.139	0.000	0.000	0.000	2.257

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	176	210	0	569	0	0	-1	162
N.S.	1	1.00	1.09	1.30	0.00	3.53	0.00	0.00	-0.01	1.01
time (sec)	N/A	0.196	0.369	0.027	0.000	1.100	0.000	0.000	0.000	0.973

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	50	63	0	125	0	0	147	118
N.S.	1	1.00	0.82	1.03	0.00	2.05	0.00	0.00	2.41	1.93
time (sec)	N/A	0.069	0.027	0.006	0.000	0.441	0.000	0.000	4.678	0.853

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	64	97	0	325	0	0	151	132
N.S.	1	1.00	0.52	0.78	0.00	2.62	0.00	0.00	1.22	1.06
time (sec)	N/A	0.154	0.049	0.007	0.000	0.436	0.000	0.000	4.976	0.883

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	105	168	0	649	0	0	268	198
N.S.	1	1.00	0.55	0.88	0.00	3.38	0.00	0.00	1.40	1.03
time (sec)	N/A	0.247	0.068	0.010	0.000	0.458	0.000	0.000	5.333	1.088

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	262	262	150	259	0	1062	0	0	414	289
N.S.	1	1.00	0.57	0.99	0.00	4.05	0.00	0.00	1.58	1.10
time (sec)	N/A	0.329	0.094	0.013	0.000	0.513	0.000	0.000	5.701	1.309

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	102	652	0	1055	0	0	-1	319
N.S.	1	1.00	0.35	2.26	0.00	3.65	0.00	0.00	-0.00	1.10
time (sec)	N/A	0.432	0.140	0.044	0.000	1.153	0.000	0.000	0.000	4.553

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	102	343	0	755	0	0	-1	227
N.S.	1	1.00	0.47	1.57	0.00	3.45	0.00	0.00	-0.00	1.04
time (sec)	N/A	0.289	0.105	0.030	0.000	1.126	0.000	0.000	0.000	2.673

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	52	63	0	193	0	0	169	99
N.S.	1	1.00	0.83	1.00	0.00	3.06	0.00	0.00	2.68	1.57
time (sec)	N/A	0.066	0.031	0.008	0.000	0.414	0.000	0.000	4.322	1.092

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	68	99	0	318	0	0	246	119
N.S.	1	1.00	0.53	0.77	0.00	2.48	0.00	0.00	1.92	0.93
time (sec)	N/A	0.141	0.055	0.008	0.000	0.453	0.000	0.000	5.059	1.031

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	194	103	169	0	667	0	0	255	198
N.S.	1	1.00	0.53	0.87	0.00	3.44	0.00	0.00	1.31	1.02
time (sec)	N/A	0.220	0.066	0.010	0.000	0.453	0.000	0.000	5.282	1.203

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	260	260	152	258	0	1065	0	0	416	0
N.S.	1	1.00	0.58	0.99	0.00	4.10	0.00	0.00	1.60	0.00
time (sec)	N/A	0.313	0.097	0.012	0.000	0.510	0.000	0.000	5.859	180.094

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	385	385	300	870	0	1065	0	0	-1	259
N.S.	1	1.00	0.78	2.26	0.00	2.77	0.00	0.00	-0.00	0.67
time (sec)	N/A	0.718	1.156	0.031	0.000	2.691	0.000	0.000	0.000	8.584

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	255	602	0	847	0	0	-1	231
N.S.	1	1.00	0.81	1.92	0.00	2.71	0.00	0.00	-0.00	0.74
time (sec)	N/A	0.518	0.844	0.028	0.000	1.501	0.000	0.000	0.000	7.679

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	215	385	0	657	0	0	-1	227
N.S.	1	1.00	0.89	1.60	0.00	2.73	0.00	0.00	-0.00	0.94
time (sec)	N/A	0.352	0.594	0.022	0.000	1.202	0.000	0.000	0.000	0.781

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	173	198	0	516	0	0	-1	164
N.S.	1	1.00	1.04	1.19	0.00	3.09	0.00	0.00	-0.01	0.98
time (sec)	N/A	0.209	0.814	0.023	0.000	1.140	0.000	0.000	0.000	0.714

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	169	197	0	521	0	0	-1	213
N.S.	1	1.00	1.07	1.25	0.00	3.30	0.00	0.00	-0.01	1.35
time (sec)	N/A	0.188	0.789	0.031	0.000	1.073	0.000	0.000	0.000	1.972

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	52	63	0	169	0	0	136	145
N.S.	1	1.00	0.83	1.00	0.00	2.68	0.00	0.00	2.16	2.30
time (sec)	N/A	0.065	0.033	0.007	0.000	0.427	0.000	0.000	3.923	2.661

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	69	99	0	402	0	0	187	196
N.S.	1	1.00	0.53	0.77	0.00	3.12	0.00	0.00	1.45	1.52
time (sec)	N/A	0.139	0.054	0.008	0.000	0.435	0.000	0.000	4.084	3.125

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	105	169	0	748	0	0	289	137
N.S.	1	1.00	0.53	0.85	0.00	3.78	0.00	0.00	1.46	0.69
time (sec)	N/A	0.220	0.092	0.009	0.000	0.455	0.000	0.000	4.290	8.341

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	152	260	0	1179	0	0	409	169
N.S.	1	1.00	0.57	0.97	0.00	4.42	0.00	0.00	1.53	0.63
time (sec)	N/A	0.308	0.139	0.013	0.000	0.468	0.000	0.000	4.501	8.876

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	382	382	302	870	0	1059	0	0	-1	431
N.S.	1	1.00	0.79	2.28	0.00	2.77	0.00	0.00	-0.00	1.13
time (sec)	N/A	0.711	1.170	0.025	0.000	2.676	0.000	0.000	0.000	2.050

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	254	602	0	847	0	0	-1	316
N.S.	1	1.00	0.82	1.94	0.00	2.73	0.00	0.00	-0.00	1.02
time (sec)	N/A	0.533	0.829	0.024	0.000	1.513	0.000	0.000	0.000	1.787

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	193	325	0	651	0	0	-1	222
N.S.	1	1.00	0.81	1.37	0.00	2.74	0.00	0.00	-0.00	0.93
time (sec)	N/A	0.345	0.771	0.027	0.000	1.209	0.000	0.000	0.000	1.381

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	222	102	383	0	663	0	0	-1	178
N.S.	1	1.00	0.46	1.73	0.00	2.99	0.00	0.00	-0.00	0.80
time (sec)	N/A	0.303	0.158	0.031	0.000	1.124	0.000	0.000	0.000	1.450

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	188	331	0	685	0	0	-1	173
N.S.	1	1.00	0.88	1.55	0.00	3.20	0.00	0.00	-0.00	0.81
time (sec)	N/A	0.278	1.063	0.034	0.000	1.078	0.000	0.000	0.000	1.542

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	52	63	0	232	0	0	232	168
N.S.	1	1.00	0.83	1.00	0.00	3.68	0.00	0.00	3.68	2.67
time (sec)	N/A	0.070	0.052	0.008	0.000	0.439	0.000	0.000	4.067	1.396

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	69	99	0	526	0	0	247	249
N.S.	1	1.00	0.53	0.77	0.00	4.08	0.00	0.00	1.91	1.93
time (sec)	N/A	0.148	0.085	0.008	0.000	0.466	0.000	0.000	4.305	1.714

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	105	169	0	918	0	0	377	0
N.S.	1	1.00	0.53	0.85	0.00	4.64	0.00	0.00	1.90	0.00
time (sec)	N/A	0.228	0.129	0.010	0.000	0.464	0.000	0.000	4.478	180.007

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	152	260	0	1420	0	0	519	0
N.S.	1	1.00	0.57	0.97	0.00	5.32	0.00	0.00	1.94	0.00
time (sec)	N/A	0.324	0.176	0.013	0.000	0.503	0.000	0.000	4.833	180.016

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	448	448	285	1191	0	1331	0	0	-1	0
N.S.	1	1.00	0.64	2.66	0.00	2.97	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.889	6.013	0.031	0.000	5.788	0.000	0.000	0.000	180.024

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	376	376	299	870	0	1065	0	0	-1	431
N.S.	1	1.00	0.80	2.31	0.00	2.83	0.00	0.00	-0.00	1.15
time (sec)	N/A	0.678	1.135	0.025	0.000	2.711	0.000	0.000	0.000	2.590

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	304	304	229	508	0	837	0	0	-1	310
N.S.	1	1.00	0.75	1.67	0.00	2.75	0.00	0.00	-0.00	1.02
time (sec)	N/A	0.487	0.998	0.031	0.000	1.555	0.000	0.000	0.000	1.837

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	294	294	112	635	0	915	0	0	-1	255
N.S.	1	1.00	0.38	2.16	0.00	3.11	0.00	0.00	-0.00	0.87
time (sec)	N/A	0.428	0.104	0.033	0.000	1.197	0.000	0.000	0.000	2.065

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	284	284	112	638	0	973	0	0	-1	240
N.S.	1	1.00	0.39	2.25	0.00	3.43	0.00	0.00	-0.00	0.85
time (sec)	N/A	0.404	0.123	0.032	0.000	1.146	0.000	0.000	0.000	2.235

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	274	274	224	511	0	933	0	0	-1	230
N.S.	1	1.00	0.82	1.86	0.00	3.41	0.00	0.00	-0.00	0.84
time (sec)	N/A	0.366	1.369	0.037	0.000	1.097	0.000	0.000	0.000	2.428

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	52	63	0	299	0	0	325	248
N.S.	1	1.00	0.83	1.00	0.00	4.75	0.00	0.00	5.16	3.94
time (sec)	N/A	0.069	0.077	0.008	0.000	0.442	0.000	0.000	4.343	1.877

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	79	99	0	639	0	0	315	0
N.S.	1	1.00	0.61	0.77	0.00	4.95	0.00	0.00	2.44	0.00
time (sec)	N/A	0.149	0.082	0.009	0.000	0.471	0.000	0.000	4.543	180.011

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	115	169	0	1101	0	0	465	0
N.S.	1	1.00	0.58	0.85	0.00	5.56	0.00	0.00	2.35	0.00
time (sec)	N/A	0.230	0.108	0.011	0.000	0.479	0.000	0.000	4.822	180.014

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	162	260	0	1648	0	0	627	0
N.S.	1	1.00	0.61	0.97	0.00	6.17	0.00	0.00	2.35	0.00
time (sec)	N/A	0.324	0.141	0.014	0.000	0.536	0.000	0.000	5.119	180.022

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	134	527	331	705	0	2024	615	0
N.S.	1	1.00	0.39	1.54	0.97	2.06	0.00	5.90	1.79	0.00
time (sec)	N/A	0.449	0.172	0.011	0.606	0.442	0.000	0.331	3.753	0.690

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	131	235	193	350	0	981	327	0
N.S.	1	1.00	0.53	0.96	0.78	1.42	0.00	3.99	1.33	0.00
time (sec)	N/A	0.205	0.108	0.010	0.556	0.460	0.000	0.280	3.518	0.407

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	67	89	94	145	0	369	139	0
N.S.	1	1.00	0.45	0.59	0.63	0.97	0.00	2.46	0.93	0.00
time (sec)	N/A	0.082	0.062	0.005	0.517	0.422	0.000	0.254	3.362	0.260

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	42	57	33	57	0	87	57	0
N.S.	1	1.00	0.78	1.06	0.61	1.06	0.00	1.61	1.06	0.00
time (sec)	N/A	0.015	0.022	0.003	0.473	0.424	0.000	0.225	3.248	0.003

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	53	64	49	66	0	114	63	0
N.S.	1	1.00	0.82	0.98	0.75	1.02	0.00	1.75	0.97	0.00
time (sec)	N/A	0.044	0.029	0.003	0.495	0.421	0.000	0.247	3.543	0.162

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	64	0	32	35	0	0	-1	64
N.S.	1	1.00	0.82	0.00	0.41	0.45	0.00	0.00	-0.01	0.82
time (sec)	N/A	0.150	0.035	0.242	0.504	0.428	0.000	0.000	0.000	0.393

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	501	501	246	641	693	597	0	0	653	8325
N.S.	1	1.00	0.49	1.28	1.38	1.19	0.00	0.00	1.30	16.62
time (sec)	N/A	0.894	0.419	0.011	0.739	0.422	0.000	0.000	4.088	38.390

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	412	412	264	425	484	408	0	0	438	676
N.S.	1	1.00	0.64	1.03	1.17	0.99	0.00	0.00	1.06	1.64
time (sec)	N/A	0.627	0.273	0.010	0.685	0.423	0.000	0.000	3.864	1.395

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	321	321	169	255	309	256	0	0	279	365
N.S.	1	1.00	0.53	0.79	0.96	0.80	0.00	0.00	0.87	1.14
time (sec)	N/A	0.420	0.180	0.010	0.627	0.426	0.000	0.000	3.711	0.801

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	96	131	168	141	0	0	152	170
N.S.	1	1.00	0.46	0.63	0.80	0.67	0.00	0.00	0.73	0.81
time (sec)	N/A	0.198	0.096	0.005	0.570	0.406	0.000	0.000	3.484	0.411

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	54	69	65	73	0	0	85	80
N.S.	1	1.00	0.50	0.63	0.60	0.67	0.00	0.00	0.78	0.73
time (sec)	N/A	0.059	0.042	0.003	0.504	0.406	0.000	0.000	3.364	0.002

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	140	163	0	511	0	0	-1	2866
N.S.	1	1.00	1.01	1.17	0.00	3.68	0.00	0.00	-0.01	20.62
time (sec)	N/A	0.192	0.108	0.024	0.000	0.436	0.000	0.000	0.000	20.818

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	155	347	0	896	0	0	-1	0
N.S.	1	1.00	0.91	2.04	0.00	5.27	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.234	0.147	0.028	0.000	0.443	0.000	0.000	0.000	180.016

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	189	673	0	1704	0	0	-1	0
N.S.	1	1.00	0.72	2.58	0.00	6.53	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.355	0.423	0.042	0.000	0.470	0.000	0.000	0.000	180.006

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	351	351	132	1142	0	2736	0	0	-1	0
N.S.	1	1.00	0.38	3.25	0.00	7.79	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.558	0.103	0.047	0.000	0.489	0.000	0.000	0.000	180.018

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	324	324	229	602	365	251	0	412	1768	1219
N.S.	1	1.00	0.71	1.86	1.13	0.77	0.00	1.27	5.46	3.76
time (sec)	N/A	0.934	0.264	0.059	0.978	0.421	0.000	0.618	31.330	0.628

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	114	291	171	134	0	196	897	446
N.S.	1	1.00	0.69	1.75	1.03	0.81	0.00	1.18	5.40	2.69
time (sec)	N/A	0.319	0.116	0.027	0.973	0.424	0.000	0.422	13.854	0.289

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	45	117	57	67	282	76	232	117
N.S.	1	1.00	0.71	1.86	0.90	1.06	4.48	1.21	3.68	1.86
time (sec)	N/A	0.063	0.033	0.018	0.970	0.419	49.711	0.268	7.760	0.002

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	282	282	260	1759	0	4313	0	684	33018	372
N.S.	1	1.00	0.92	6.24	0.00	15.29	0.00	2.43	117.09	1.32
time (sec)	N/A	0.522	0.558	0.141	0.000	0.614	0.000	1.505	82.367	1.998

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	571	571	508	41837	0	0	0	0	-1	1064
N.S.	1	1.00	0.89	73.27	0.00	0.00	0.00	0.00	-0.00	1.86
time (sec)	N/A	5.235	1.282	0.814	0.000	0.000	0.000	0.000	0.000	50.062

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	239	755	371	376	0	732	-1	1332
N.S.	1	1.00	0.87	2.74	1.34	1.36	0.00	2.65	-0.00	4.83
time (sec)	N/A	0.598	0.241	0.040	0.980	0.422	0.000	0.611	0.000	0.603

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	127	380	176	204	0	387	-1	504
N.S.	1	1.00	0.94	2.81	1.30	1.51	0.00	2.87	-0.01	3.73
time (sec)	N/A	0.195	0.117	0.030	0.969	0.412	0.000	0.394	0.000	0.292

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	39	151	61	101	0	182	-1	115
N.S.	1	1.00	0.98	3.78	1.52	2.52	0.00	4.55	-0.02	2.88
time (sec)	N/A	0.051	0.048	0.026	0.967	0.423	0.000	0.303	0.000	0.131

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	443	443	335	11141	0	0	0	0	-1	615
N.S.	1	1.00	0.76	25.15	0.00	0.00	0.00	0.00	-0.00	1.39
time (sec)	N/A	1.443	2.394	0.125	0.000	0.000	0.000	0.000	0.000	9.492

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	939	938	890	108974	0	0	0	0	-1	0
N.S.	1	1.00	0.95	116.05	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	11.844	9.219	4.138	0.000	0.000	0.000	0.000	0.000	180.024

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	275	275	249	2017	811	2032	0	3760	1943	0
N.S.	1	1.00	0.91	7.33	2.95	7.39	0.00	13.67	7.07	0.00
time (sec)	N/A	0.263	0.313	0.019	0.597	0.482	0.000	0.585	3.901	0.155

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	187	1048	512	1122	11946	2114	1133	0
N.S.	1	1.00	0.90	5.04	2.46	5.39	57.43	10.16	5.45	0.00
time (sec)	N/A	0.188	0.199	0.013	0.543	0.443	11.821	0.230	3.522	0.106

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	141	449	289	549	4952	1018	572	0
N.S.	1	1.00	0.97	3.08	1.98	3.76	33.92	6.97	3.92	0.00
time (sec)	N/A	0.112	0.280	0.007	0.505	0.429	5.269	0.387	3.288	0.083

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	73	147	135	218	1489	373	211	0
N.S.	1	1.00	0.87	1.75	1.61	2.60	17.73	4.44	2.51	0.00
time (sec)	N/A	0.063	0.100	0.005	0.475	0.409	2.212	0.188	3.072	0.056

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	85	142	87	99	420	88	84	0
N.S.	1	1.00	1.02	1.71	1.05	1.19	5.06	1.06	1.01	0.00
time (sec)	N/A	0.100	0.049	0.008	0.445	0.398	9.516	0.176	3.420	0.001

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	177	444	255	313	0	281	266	0
N.S.	1	1.00	0.96	2.41	1.39	1.70	0.00	1.53	1.45	0.00
time (sec)	N/A	0.312	0.148	0.012	0.450	0.713	0.000	0.169	3.505	0.001

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	531	531	476	1232	721	736	0	907	794	0
N.S.	1	1.00	0.90	2.32	1.36	1.39	0.00	1.71	1.50	0.00
time (sec)	N/A	0.988	0.423	0.020	0.488	3.897	0.000	0.171	4.195	0.001

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	246	606	0	0	0	392	12173	0
N.S.	1	1.00	1.00	2.46	0.00	0.00	0.00	1.59	49.48	0.00
time (sec)	N/A	0.468	0.324	0.012	0.000	0.000	0.000	0.185	19.247	0.001

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	644	644	710	9103	0	0	0	3315	130035	0
N.S.	1	1.00	1.10	14.14	0.00	0.00	0.00	5.15	201.92	0.00
time (sec)	N/A	2.052	2.632	0.050	0.000	0.000	0.000	0.276	32.634	0.001

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	287	287	249	540	429	429	1544	565	283	634
N.S.	1	1.00	0.87	1.88	1.49	1.49	5.38	1.97	0.99	2.21
time (sec)	N/A	0.495	0.412	0.009	0.459	0.558	164.697	0.200	0.150	0.347

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	184	315	261	260	1001	363	204	368
N.S.	1	1.00	0.87	1.49	1.23	1.23	4.72	1.71	0.96	1.74
time (sec)	N/A	0.339	0.341	0.009	0.449	0.540	105.539	0.233	3.169	0.215

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	131	144	129	125	549	199	125	168
N.S.	1	1.00	0.96	1.05	0.94	0.91	4.01	1.45	0.91	1.23
time (sec)	N/A	0.109	0.193	0.006	0.457	0.561	55.828	0.192	0.077	0.133

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	54	53	77	54	223	77	58	62
N.S.	1	1.00	0.74	0.73	1.05	0.74	3.05	1.05	0.79	0.85
time (sec)	N/A	0.042	0.048	0.003	0.442	0.721	10.865	0.171	3.119	0.038

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	118	189	0	341	112	128	117	117
N.S.	1	1.00	1.02	1.63	0.00	2.94	0.97	1.10	1.01	1.01
time (sec)	N/A	0.168	0.192	0.013	0.000	0.566	37.629	0.167	0.142	0.170

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	150	371	0	637	0	175	146	200
N.S.	1	1.00	1.07	2.65	0.00	4.55	0.00	1.25	1.04	1.43
time (sec)	N/A	0.290	0.560	0.019	0.000	0.651	0.000	0.182	0.233	0.546

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	206	297	538	0	1096	0	373	270	293
N.S.	1	1.00	1.44	2.61	0.00	5.32	0.00	1.81	1.31	1.42
time (sec)	N/A	0.386	0.646	0.021	0.000	0.661	0.000	0.200	0.281	0.896

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	285	285	249	540	437	438	452	669	394	634
N.S.	1	1.00	0.87	1.89	1.53	1.54	1.59	2.35	1.38	2.22
time (sec)	N/A	0.405	0.729	0.009	0.462	0.620	158.555	0.235	0.116	0.303

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	184	315	269	269	272	404	270	368
N.S.	1	1.00	0.88	1.50	1.28	1.28	1.30	1.92	1.29	1.75
time (sec)	N/A	0.289	0.356	0.008	0.445	0.608	79.487	0.230	3.127	0.203

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	128	144	137	135	141	204	147	168
N.S.	1	1.00	0.95	1.07	1.01	1.00	1.04	1.51	1.09	1.24
time (sec)	N/A	0.098	0.179	0.005	0.443	0.393	34.525	0.181	3.134	0.112

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	54	53	66	63	70	74	58	61
N.S.	1	1.00	0.76	0.75	0.93	0.89	0.99	1.04	0.82	0.86
time (sec)	N/A	0.042	0.055	0.004	0.441	0.398	13.121	0.153	0.060	0.045

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	124	237	0	540	116	112	162	139
N.S.	1	1.00	1.02	1.94	0.00	4.43	0.95	0.92	1.33	1.14
time (sec)	N/A	0.221	0.291	0.014	0.000	0.439	52.234	0.255	3.209	0.195

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	176	418	0	1088	0	282	218	268
N.S.	1	1.00	1.07	2.53	0.00	6.59	0.00	1.71	1.32	1.62
time (sec)	N/A	0.367	0.407	0.022	0.000	0.456	0.000	0.229	0.300	0.688

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	290	847	0	1883	0	462	363	497
N.S.	1	1.00	1.17	3.42	0.00	7.59	0.00	1.86	1.46	2.00
time (sec)	N/A	0.629	1.102	0.026	0.000	0.497	0.000	0.257	3.409	1.199

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	B	F	B	F	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	91	191	113	231	0	214	0	16	916	110
N.S.	1	2.10	1.24	2.54	0.00	2.35	0.00	0.18	10.07	1.21
time (sec)	N/A	0.141	0.212	0.086	0.000	0.432	0.000	0.198	5.018	0.306

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	173	425	0	380	0	179	833	229
N.S.	1	1.00	1.05	2.59	0.00	2.32	0.00	1.09	5.08	1.40
time (sec)	N/A	0.179	0.775	0.029	0.000	0.497	0.000	0.261	22.379	0.388

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	333	333	313	1207	0	852	0	448	-1	643
N.S.	1	1.00	0.94	3.62	0.00	2.56	0.00	1.35	-0.00	1.93
time (sec)	N/A	0.353	1.538	0.032	0.000	0.658	0.000	0.415	0.000	0.734

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	225	763	0	576	0	291	1832	357
N.S.	1	1.00	0.91	3.10	0.00	2.34	0.00	1.18	7.45	1.45
time (sec)	N/A	0.256	1.008	0.023	0.000	0.522	0.000	0.349	74.336	1.093

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	173	425	0	380	0	179	833	229
N.S.	1	1.00	1.05	2.59	0.00	2.32	0.00	1.09	5.08	1.40
time (sec)	N/A	0.143	0.622	0.000	0.000	0.509	0.000	0.252	0.002	0.002

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	222	697	0	588	0	201	-1	216
N.S.	1	1.00	1.72	5.40	0.00	4.56	0.00	1.56	-0.01	1.67
time (sec)	N/A	0.135	0.592	0.029	0.000	1.436	0.000	0.387	0.000	0.341

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	173	773	0	792	0	504	-1	161
N.S.	1	1.00	1.08	4.83	0.00	4.95	0.00	3.15	-0.01	1.01
time (sec)	N/A	0.180	0.230	0.028	0.000	3.553	0.000	0.685	0.000	0.226

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	178	238	0	353	0	1080	260	177
N.S.	1	1.00	0.90	1.20	0.00	1.78	0.00	5.45	1.31	0.89
time (sec)	N/A	0.212	0.228	0.010	0.000	10.806	0.000	0.853	4.304	0.172

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	332	468	0	641	0	1868	452	311
N.S.	1	1.00	1.18	1.67	0.00	2.28	0.00	6.65	1.61	1.11
time (sec)	N/A	0.291	0.353	0.010	0.000	36.001	0.000	1.272	4.654	0.230

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	249	196	834	0	580	0	237	-1	382
N.S.	1	1.00	0.79	3.35	0.00	2.33	0.00	0.95	-0.00	1.53
time (sec)	N/A	0.280	1.091	0.042	0.000	0.980	0.000	0.440	0.000	0.500

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	204	571	0	546	0	717	-1	367
N.S.	1	1.00	0.85	2.38	0.00	2.28	0.00	2.99	-0.00	1.53
time (sec)	N/A	0.225	0.848	0.044	0.000	0.463	0.000	0.519	0.000	0.459

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	163	392	0	414	0	441	1797	220
N.S.	1	1.00	0.93	2.23	0.00	2.35	0.00	2.51	10.21	1.25
time (sec)	N/A	0.171	0.437	0.025	0.000	0.457	0.000	0.382	73.154	0.670

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	135	247	0	308	0	145	893	196
N.S.	1	1.00	1.11	2.02	0.00	2.52	0.00	1.19	7.32	1.61
time (sec)	N/A	0.124	0.408	0.029	0.000	0.545	0.000	0.244	20.635	0.294

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	134	438	0	463	0	193	-1	146
N.S.	1	1.00	1.24	4.06	0.00	4.29	0.00	1.79	-0.01	1.35
time (sec)	N/A	0.120	0.337	0.032	0.000	0.541	0.000	0.364	0.000	0.239

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	128	601	0	665	0	0	-1	99
N.S.	1	1.00	1.10	5.18	0.00	5.73	0.00	0.00	-0.01	0.85
time (sec)	N/A	0.121	0.296	0.032	0.000	0.604	0.000	0.000	0.000	0.170

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	110	150	0	293	0	0	268	115
N.S.	1	1.00	0.83	1.13	0.00	2.20	0.00	0.00	2.02	0.86
time (sec)	N/A	0.130	0.084	0.012	0.000	1.055	0.000	0.000	4.301	0.139

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	173	248	0	487	0	0	389	185
N.S.	1	1.00	0.92	1.31	0.00	2.58	0.00	0.00	2.06	0.98
time (sec)	N/A	0.192	0.121	0.013	0.000	2.230	0.000	0.000	4.506	0.166

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	417	417	401	11688	0	0	0	0	-1	479
N.S.	1	1.00	0.96	28.03	0.00	0.00	0.00	0.00	-0.00	1.15
time (sec)	N/A	3.141	1.789	0.179	0.000	0.000	0.000	0.000	0.000	6.482

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	285	285	266	5482	0	4471	0	0	-1	0
N.S.	1	1.00	0.93	19.24	0.00	15.69	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.526	0.874	0.047	0.000	37.790	0.000	0.000	0.000	180.362

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	287	287	267	5507	0	0	0	0	-1	566
N.S.	1	1.00	0.93	19.19	0.00	0.00	0.00	0.00	-0.00	1.97
time (sec)	N/A	0.415	0.752	0.065	0.000	0.000	0.000	0.000	0.000	5.448

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	429	429	334	47351	0	0	0	0	-1	651
N.S.	1	1.00	0.78	110.38	0.00	0.00	0.00	0.00	-0.00	1.52
time (sec)	N/A	1.356	2.062	0.343	0.000	0.000	0.000	0.000	0.000	4.103

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	532	532	559	3941	0	0	0	0	-1	788
N.S.	1	1.00	1.05	7.41	0.00	0.00	0.00	0.00	-0.00	1.48
time (sec)	N/A	1.704	1.013	0.038	0.000	0.000	0.000	0.000	0.000	4.588

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	325	372	2602	0	0	0	0	-1	428
N.S.	1	1.00	1.14	8.01	0.00	0.00	0.00	0.00	-0.00	1.32
time (sec)	N/A	0.708	0.415	0.016	0.000	0.000	0.000	0.000	0.000	2.034

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	216	1559	0	0	0	0	-1	229
N.S.	1	1.00	0.99	7.12	0.00	0.00	0.00	0.00	-0.00	1.05
time (sec)	N/A	0.324	0.347	0.010	0.000	0.000	0.000	0.000	0.000	1.005

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	145	715	0	992	0	0	-1	191
N.S.	1	1.00	0.95	4.70	0.00	6.53	0.00	0.00	-0.01	1.26
time (sec)	N/A	0.151	0.155	0.006	0.000	1.984	0.000	0.000	0.000	0.001

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	228	228	218	1529	0	0	0	0	-1	315
N.S.	1	1.00	0.96	6.71	0.00	0.00	0.00	0.00	-0.00	1.38
time (sec)	N/A	0.331	0.341	0.031	0.000	0.000	0.000	0.000	0.000	0.700

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	490	490	222	3162	0	0	0	0	-1	351
N.S.	1	1.00	0.45	6.45	0.00	0.00	0.00	0.00	-0.00	0.72
time (sec)	N/A	0.695	0.517	0.020	0.000	0.000	0.000	0.000	0.000	1.259

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	673	673	609	6714	0	0	0	1844	-1	0
N.S.	1	1.00	0.90	9.98	0.00	0.00	0.00	2.74	-0.00	0.00
time (sec)	N/A	0.862	1.373	0.021	0.000	0.000	0.000	4.636	0.000	180.485

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	933	933	858	11995	0	0	0	0	-1	0
N.S.	1	1.00	0.92	12.86	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.225	4.159	0.024	0.000	0.000	0.000	0.000	0.000	180.019

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1098	1098	743	10058	0	0	0	0	-1	0
N.S.	1	1.00	0.68	9.16	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	3.864	2.385	0.027	0.000	0.000	0.000	0.000	0.000	180.100

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	662	662	536	6860	0	0	0	0	-1	27845
N.S.	1	1.00	0.81	10.36	0.00	0.00	0.00	0.00	-0.00	42.06
time (sec)	N/A	1.554	1.249	0.017	0.000	0.000	0.000	0.000	0.000	85.153

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	441	441	420	4188	0	0	0	0	-1	687
N.S.	1	1.00	0.95	9.50	0.00	0.00	0.00	0.00	-0.00	1.56
time (sec)	N/A	0.853	1.128	0.009	0.000	0.000	0.000	0.000	0.000	3.468

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	236	1946	0	0	0	0	-1	278
N.S.	1	1.00	0.94	7.72	0.00	0.00	0.00	0.00	-0.00	1.10
time (sec)	N/A	0.346	0.412	0.007	0.000	0.000	0.000	0.000	0.000	0.003

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	491	491	323	4226	0	0	0	0	-1	2557
N.S.	1	1.00	0.66	8.61	0.00	0.00	0.00	0.00	-0.00	5.21
time (sec)	N/A	0.841	1.068	0.018	0.000	0.000	0.000	0.000	0.000	78.213

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	787	787	357	7959	0	0	0	0	-1	3744
N.S.	1	1.00	0.45	10.11	0.00	0.00	0.00	0.00	-0.00	4.76
time (sec)	N/A	1.376	1.399	0.021	0.000	0.000	0.000	0.000	0.000	149.462

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1066	1066	1036	15927	0	0	0	0	-1	0
N.S.	1	1.00	0.97	14.94	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.706	3.273	0.024	0.000	0.000	0.000	0.000	0.000	180.189

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	886	886	647	9052	0	0	0	0	-1	0
N.S.	1	1.00	0.73	10.22	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.800	2.539	0.022	0.000	0.000	0.000	0.000	0.000	180.139

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	431	431	553	1597	0	0	0	0	-1	509
N.S.	1	1.00	1.28	3.71	0.00	0.00	0.00	0.00	-0.00	1.18
time (sec)	N/A	1.370	0.888	0.029	0.000	0.000	0.000	0.000	0.000	2.714

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	358	1007	0	0	0	0	-1	308
N.S.	1	1.00	1.33	3.73	0.00	0.00	0.00	0.00	-0.00	1.14
time (sec)	N/A	0.708	0.380	0.016	0.000	0.000	0.000	0.000	0.000	1.414

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	170	613	0	0	0	0	-1	210
N.S.	1	1.00	0.97	3.48	0.00	0.00	0.00	0.00	-0.01	1.19
time (sec)	N/A	0.305	0.483	0.015	0.000	0.000	0.000	0.000	0.000	0.766

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	126	349	0	1071	0	0	-1	152
N.S.	1	1.00	0.96	2.66	0.00	8.18	0.00	0.00	-0.01	1.16
time (sec)	N/A	0.093	0.151	0.008	0.000	31.643	0.000	0.000	0.000	0.526

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	78	157	0	343	0	72	-1	138
N.S.	1	1.00	0.99	1.99	0.00	4.34	0.00	0.91	-0.01	1.75
time (sec)	N/A	0.038	0.014	0.006	0.000	0.554	0.000	0.229	0.000	0.002

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	169	327	0	1952	0	0	-1	299
N.S.	1	1.00	0.93	1.80	0.00	10.73	0.00	0.00	-0.01	1.64
time (sec)	N/A	0.219	0.249	0.017	0.000	123.751	0.000	0.000	0.000	0.867

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	340	340	256	788	0	0	0	0	-1	2266
N.S.	1	1.00	0.75	2.32	0.00	0.00	0.00	0.00	-0.00	6.66
time (sec)	N/A	0.394	0.955	0.020	0.000	0.000	0.000	0.000	0.000	21.822

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	587	587	549	1817	0	0	0	2256	-1	0
N.S.	1	1.00	0.94	3.10	0.00	0.00	0.00	3.84	-0.00	0.00
time (sec)	N/A	0.810	2.429	0.024	0.000	0.000	0.000	3.260	0.000	180.019

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	496	496	587	4453	0	0	0	0	-1	5425
N.S.	1	1.00	1.18	8.98	0.00	0.00	0.00	0.00	-0.00	10.94
time (sec)	N/A	1.199	2.457	0.025	0.000	0.000	0.000	0.000	0.000	21.859

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	357	357	373	3127	0	0	0	0	-1	437
N.S.	1	1.00	1.04	8.76	0.00	0.00	0.00	0.00	-0.00	1.22
time (sec)	N/A	0.528	1.044	0.016	0.000	0.000	0.000	0.000	0.000	6.466

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	265	2123	0	2023	0	757	-1	323
N.S.	1	1.00	1.10	8.85	0.00	8.43	0.00	3.15	-0.00	1.35
time (sec)	N/A	0.305	0.633	0.016	0.000	7.789	0.000	0.351	0.000	1.360

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	183	1261	0	1663	0	568	-1	233
N.S.	1	1.00	0.98	6.74	0.00	8.89	0.00	3.04	-0.01	1.25
time (sec)	N/A	0.135	0.161	0.009	0.000	5.000	0.000	0.323	0.000	0.986

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	162	603	0	1349	0	447	-1	215
N.S.	1	1.00	1.05	3.89	0.00	8.70	0.00	2.88	-0.01	1.39
time (sec)	N/A	0.101	0.256	0.008	0.000	0.912	0.000	0.318	0.000	0.004

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	352	352	317	1343	0	0	0	0	-1	463
N.S.	1	1.00	0.90	3.82	0.00	0.00	0.00	0.00	-0.00	1.32
time (sec)	N/A	0.443	1.224	0.021	0.000	0.000	0.000	0.000	0.000	5.379

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	642	642	623	2807	0	0	0	0	-1	0
N.S.	1	1.00	0.97	4.37	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.911	5.084	0.026	0.000	0.000	0.000	0.000	0.000	180.075

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1064	1064	1013	5459	0	0	0	14731	-1	0
N.S.	1	1.00	0.95	5.13	0.00	0.00	0.00	13.84	-0.00	0.00
time (sec)	N/A	1.898	5.738	0.035	0.000	0.000	0.000	21.913	0.000	180.079

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	220	220	198	1347	684	1381	15757	2740	1354	0
N.S.	1	1.00	0.90	6.12	3.11	6.28	71.62	12.45	6.15	0.00
time (sec)	N/A	0.215	0.273	0.016	0.577	0.658	14.705	0.271	3.945	0.108

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	180	503	352	613	5930	1162	602	0
N.S.	1	1.00	1.25	3.49	2.44	4.26	41.18	8.07	4.18	0.00
time (sec)	N/A	0.113	0.331	0.008	0.520	0.430	5.863	0.198	3.594	0.073

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	525	525	492	5890	2034	4747	0	10489	4871	0
N.S.	1	1.00	0.94	11.22	3.87	9.04	0.00	19.98	9.28	0.00
time (sec)	N/A	0.615	0.773	0.035	0.787	0.526	0.000	0.547	5.383	0.631

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	311	311	655	2563	1118	2368	0	4940	2307	0
N.S.	1	1.00	2.11	8.24	3.59	7.61	0.00	15.88	7.42	0.00
time (sec)	N/A	0.392	1.523	0.020	0.669	0.453	0.000	0.355	4.380	0.145

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [236] had the largest ratio of [.6364]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	5	1.00	25	0.200
2	A	8	5	1.00	25	0.200
3	A	7	5	1.00	25	0.200
4	A	12	5	1.00	25	0.200
5	A	5	4	1.00	23	0.174
6	A	5	4	1.00	23	0.174
7	A	8	7	1.00	25	0.280
8	A	8	8	1.00	25	0.320
9	A	8	7	1.00	25	0.280
10	A	8	8	1.00	25	0.320
11	A	8	7	1.00	25	0.280
12	A	6	5	1.00	25	0.200
13	A	7	6	1.00	25	0.240
14	A	8	6	1.00	25	0.240
15	A	9	6	1.00	25	0.240
16	A	8	5	1.00	25	0.200
17	A	5	5	1.00	25	0.200
18	A	3	3	1.00	25	0.120
19	A	6	4	1.00	25	0.160
20	A	6	4	1.00	25	0.160
21	A	5	4	1.00	25	0.160
22	A	4	3	1.00	25	0.120
23	A	3	3	1.00	25	0.120
24	A	3	3	1.00	25	0.120
25	A	3	3	1.00	23	0.130
26	A	3	3	1.00	22	0.136
27	A	7	5	1.00	25	0.200
28	A	7	5	1.00	25	0.200
29	A	8	6	1.00	25	0.240
30	A	4	4	1.00	25	0.160
31	A	5	4	1.00	25	0.160
32	A	4	4	1.00	24	0.167
33	A	7	5	1.00	27	0.185
34	A	6	5	1.00	27	0.185
35	A	5	5	1.00	27	0.185
36	A	4	4	1.00	25	0.160
37	A	4	4	1.00	24	0.167
38	A	7	7	1.00	27	0.259
39	A	7	7	1.00	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	5	5	1.00	27	0.185
41	A	6	6	1.00	27	0.222
42	A	7	6	1.00	27	0.222
43	A	8	6	1.00	27	0.222
44	A	6	5	1.00	27	0.185
45	A	6	5	1.00	27	0.185
46	A	3	2	1.00	27	0.074
47	A	3	3	1.00	27	0.111
48	A	3	3	1.00	25	0.120
49	A	3	3	1.00	24	0.125
50	A	7	6	1.00	27	0.222
51	A	7	5	1.00	27	0.185
52	A	8	6	1.00	27	0.222
53	A	9	6	1.00	27	0.222
54	A	5	4	1.00	20	0.200
55	A	4	4	1.00	20	0.200
56	A	3	3	1.00	18	0.167
57	A	3	3	1.00	17	0.176
58	A	6	6	1.00	20	0.300
59	A	6	6	1.00	20	0.300
60	A	5	5	1.00	20	0.250
61	A	6	6	1.00	20	0.300
62	A	7	6	1.00	20	0.300
63	A	8	6	1.00	20	0.300
64	A	9	8	1.00	27	0.296
65	A	12	6	1.00	27	0.222
66	A	11	6	1.00	27	0.222
67	A	10	6	1.00	27	0.222
68	A	9	6	1.00	27	0.222
69	A	9	6	1.00	25	0.240
70	A	8	5	1.00	24	0.208
71	A	11	8	1.00	27	0.296
72	A	11	9	1.00	27	0.333
73	A	11	8	1.00	27	0.296
74	A	11	9	1.00	27	0.333
75	A	11	9	1.00	27	0.333
76	A	11	8	1.00	27	0.296
77	A	11	9	1.00	27	0.333
78	A	11	9	1.00	27	0.333
79	A	11	8	1.00	27	0.296
80	A	9	6	1.00	27	0.222
81	A	10	7	1.00	27	0.259
82	A	11	7	1.00	27	0.259
83	A	7	5	1.00	27	0.185
84	A	6	4	1.00	27	0.148
85	A	5	4	1.00	27	0.148
86	A	3	3	1.00	27	0.111
87	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
88	A	4	3	1.00	24	0.125
89	A	7	6	1.00	27	0.222
90	A	7	5	1.00	27	0.185
91	A	8	6	1.00	27	0.222
92	A	7	5	1.00	27	0.185
93	A	6	5	1.00	27	0.185
94	A	6	6	1.00	27	0.222
95	A	4	4	1.00	25	0.160
96	A	3	3	1.00	24	0.125
97	A	7	7	1.00	27	0.259
98	A	5	5	1.00	27	0.185
99	A	6	6	1.00	27	0.222
100	A	7	6	1.00	27	0.222
101	A	8	6	1.00	27	0.222
102	A	7	7	1.00	27	0.259
103	A	9	6	1.00	27	0.222
104	A	8	6	1.00	27	0.222
105	A	8	7	1.00	27	0.259
106	A	6	5	1.00	25	0.200
107	A	5	4	1.00	24	0.167
108	A	9	8	1.00	27	0.296
109	A	9	9	1.00	27	0.333
110	A	9	8	1.00	27	0.296
111	A	9	9	1.00	27	0.333
112	A	9	8	1.00	27	0.296
113	A	7	6	1.00	27	0.222
114	A	8	7	1.00	27	0.259
115	A	9	7	1.00	27	0.259
116	A	10	7	1.00	27	0.259
117	A	3	3	1.00	18	0.167
118	A	7	7	1.00	26	0.269
119	A	6	6	1.00	27	0.222
120	A	5	5	1.00	27	0.185
121	A	5	5	1.00	27	0.185
122	A	3	3	1.00	25	0.120
123	A	1	1	1.00	24	0.042
124	A	5	5	1.00	27	0.185
125	A	5	5	1.00	27	0.185
126	A	6	6	1.00	27	0.222
127	A	6	5	1.00	27	0.185
128	A	6	5	1.00	27	0.185
129	A	5	5	1.00	27	0.185
130	A	3	3	1.00	27	0.111
131	A	2	2	1.00	25	0.080
132	A	2	2	1.00	24	0.083
133	A	6	6	1.00	27	0.222
134	A	6	6	1.00	27	0.222
135	A	7	7	1.00	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
136	A	7	5	1.00	27	0.185
137	A	7	5	1.00	27	0.185
138	A	6	5	1.00	27	0.185
139	A	5	4	1.00	27	0.148
140	A	4	4	1.00	27	0.148
141	A	3	3	1.00	27	0.111
142	A	3	3	1.00	25	0.120
143	A	3	3	1.00	24	0.125
144	A	7	6	1.00	27	0.222
145	A	7	6	1.00	27	0.222
146	A	8	7	1.00	27	0.259
147	A	9	7	1.00	27	0.259
148	A	5	5	1.00	27	0.185
149	A	4	4	1.00	27	0.148
150	A	4	4	1.00	25	0.160
151	A	4	4	1.00	25	0.160
152	A	2	2	1.00	23	0.087
153	A	1	1	1.00	22	0.045
154	A	5	5	1.00	26	0.192
155	A	5	5	1.00	26	0.192
156	A	6	6	1.00	26	0.231
157	A	10	7	1.00	27	0.259
158	A	9	7	1.00	27	0.259
159	A	8	7	1.00	27	0.259
160	A	7	7	1.00	27	0.259
161	A	6	5	1.00	25	0.200
162	A	6	6	1.00	24	0.250
163	A	9	9	1.00	27	0.333
164	A	9	9	1.00	27	0.333
165	A	9	9	1.00	27	0.333
166	A	9	9	1.00	27	0.333
167	A	7	7	1.00	27	0.259
168	A	8	8	1.00	27	0.296
169	A	9	8	1.00	27	0.296
170	A	10	8	1.00	27	0.296
171	A	7	6	1.00	27	0.222
172	A	4	3	1.00	27	0.111
173	A	4	4	1.00	27	0.148
174	A	3	3	1.00	25	0.120
175	A	3	2	1.00	24	0.083
176	A	8	7	1.00	27	0.259
177	A	8	6	1.00	27	0.222
178	A	9	7	1.00	27	0.259
179	A	8	6	1.00	27	0.222
180	A	7	5	1.00	27	0.185
181	A	6	5	1.00	27	0.185
182	A	4	4	1.00	27	0.148
183	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	3	2	1.00	24	0.083
185	A	8	7	1.00	27	0.259
186	A	8	6	1.00	27	0.222
187	A	9	7	1.00	27	0.259
188	A	9	6	1.00	27	0.222
189	A	7	5	1.00	27	0.185
190	A	9	7	1.00	27	0.259
191	A	8	6	1.00	27	0.222
192	A	2	2	1.00	25	0.080
193	A	2	2	1.00	24	0.083
194	A	8	7	1.00	27	0.259
195	A	8	6	1.00	27	0.222
196	A	9	7	1.00	27	0.259
197	A	10	7	1.00	27	0.259
198	A	11	6	1.00	27	0.222
199	A	10	6	1.00	27	0.222
200	A	9	6	1.00	27	0.222
201	A	8	7	1.00	27	0.259
202	A	6	6	1.00	25	0.240
203	A	5	4	1.00	24	0.167
204	A	9	9	1.00	27	0.333
205	A	9	9	1.00	27	0.333
206	A	7	7	1.00	27	0.259
207	A	8	7	1.00	27	0.259
208	A	9	7	1.00	27	0.259
209	A	10	7	1.00	27	0.259
210	A	7	5	1.00	26	0.192
211	A	4	4	1.00	26	0.154
212	A	9	5	1.00	27	0.185
213	A	8	5	1.00	27	0.185
214	A	7	4	1.00	25	0.160
215	A	7	3	1.00	24	0.125
216	A	12	7	1.00	27	0.259
217	A	12	6	1.00	27	0.222
218	A	4	4	1.00	29	0.138
219	A	2	2	1.00	29	0.069
220	A	3	3	1.00	16	0.188
221	A	4	4	1.00	29	0.138
222	A	3	3	1.00	15	0.200
223	A	4	4	1.00	30	0.133
224	A	4	3	1.00	16	0.188
225	A	5	4	1.00	29	0.138
226	A	11	7	1.00	16	0.438
227	A	9	6	1.00	22	0.273
228	A	8	6	1.00	22	0.273
229	A	8	7	1.00	22	0.318
230	A	6	5	1.00	20	0.250
231	A	6	5	1.00	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
232	A	9	8	1.00	22	0.364
233	A	15	11	1.00	22	0.500
234	A	19	12	1.00	22	0.546
235	A	20	13	1.00	22	0.591
236	A	25	14	1.00	22	0.636
237	A	8	5	1.00	22	0.227
238	A	7	5	1.00	22	0.227
239	A	7	6	1.00	22	0.273
240	A	5	4	1.00	20	0.200
241	A	2	2	1.00	19	0.105
242	A	7	6	1.00	22	0.273
243	A	8	7	1.00	22	0.318
244	A	12	8	1.00	22	0.364
245	A	7	6	1.00	22	0.273
246	A	6	5	1.00	22	0.227
247	A	4	4	1.00	22	0.182
248	A	4	4	1.00	20	0.200
249	A	4	4	1.00	19	0.210
250	A	10	9	1.00	22	0.409
251	A	12	11	1.00	22	0.500
252	A	17	11	1.00	22	0.500
253	A	9	6	1.00	22	0.273
254	A	8	6	1.00	22	0.273
255	A	7	6	1.00	22	0.273
256	A	6	5	1.00	22	0.227
257	A	3	3	1.00	20	0.150
258	A	3	3	1.00	19	0.158
259	A	10	7	1.00	22	0.318
260	A	11	8	1.00	22	0.364
261	A	15	9	1.00	22	0.409
262	A	2	1	1.00	18	0.056
263	A	2	1	1.00	16	0.062
264	A	2	1	1.00	15	0.067
265	A	2	1	1.00	20	0.050
266	A	2	1	1.00	18	0.056
267	A	2	1	1.00	17	0.059
268	A	2	1	1.00	20	0.050
269	A	2	1	1.00	18	0.056
270	A	2	1	1.00	17	0.059
271	A	6	5	1.00	40	0.125
272	A	5	5	1.00	40	0.125
273	A	4	4	1.00	38	0.105
274	A	3	3	1.00	37	0.081
275	A	6	5	1.00	40	0.125
276	A	4	4	1.00	40	0.100
277	A	5	5	1.00	40	0.125
278	A	6	5	1.00	40	0.125
279	A	7	5	1.00	40	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
280	A	7	6	1.00	40	0.150
281	A	6	6	1.00	40	0.150
282	A	5	5	1.00	38	0.132
283	A	4	4	1.00	37	0.108
284	A	7	6	1.00	40	0.150
285	A	7	6	1.00	40	0.150
286	A	7	6	1.00	40	0.150
287	A	5	5	1.00	40	0.125
288	A	6	6	1.00	40	0.150
289	A	7	6	1.00	40	0.150
290	A	8	6	1.00	40	0.150
291	A	8	6	1.00	40	0.150
292	A	7	6	1.00	40	0.150
293	A	6	5	1.00	38	0.132
294	A	5	4	1.00	37	0.108
295	A	8	6	1.00	40	0.150
296	A	8	7	1.00	40	0.175
297	A	8	6	1.00	40	0.150
298	A	8	7	1.00	40	0.175
299	A	8	6	1.00	40	0.150
300	A	6	5	1.00	40	0.125
301	A	7	6	1.00	40	0.150
302	A	8	6	1.00	40	0.150
303	A	9	6	1.00	40	0.150
304	A	5	5	1.10	40	0.125
305	A	4	4	1.00	40	0.100
306	A	3	3	1.00	38	0.079
307	A	1	1	1.00	37	0.027
308	A	5	5	1.00	40	0.125
309	A	5	5	1.00	40	0.125
310	A	6	6	1.00	40	0.150
311	A	6	5	1.00	40	0.125
312	A	6	5	1.00	40	0.125
313	A	5	5	1.00	40	0.125
314	A	3	3	1.00	40	0.075
315	A	2	2	1.00	38	0.053
316	A	2	2	1.00	37	0.054
317	A	6	5	1.00	40	0.125
318	A	6	5	1.00	40	0.125
319	A	7	6	1.00	40	0.150
320	A	8	6	1.00	40	0.150
321	A	3	3	1.00	40	0.075
322	A	4	4	1.00	40	0.100
323	A	1	1	1.00	23	0.043
324	A	5	5	1.00	23	0.217
325	A	1	1	1.00	23	0.043
326	A	6	5	1.00	23	0.217
327	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
328	A	4	4	1.00	23	0.174
329	A	1	1	1.00	23	0.043
330	A	5	5	1.00	23	0.217
331	A	1	1	1.00	23	0.043
332	A	6	5	1.00	23	0.217
333	A	7	6	1.00	19	0.316
334	A	6	4	1.00	25	0.160
335	A	6	4	1.22	25	0.160
336	A	6	4	1.00	25	0.160
337	A	5	4	1.00	23	0.174
338	A	4	3	1.00	22	0.136
339	A	7	5	1.00	25	0.200
340	A	9	6	0.97	25	0.240
341	A	12	6	1.00	25	0.240
342	A	6	4	1.00	25	0.160
343	A	6	4	1.00	25	0.160
344	A	6	4	1.00	25	0.160
345	A	6	4	1.00	23	0.174
346	A	5	4	1.00	22	0.182
347	A	7	5	1.00	25	0.200
348	A	9	6	1.00	25	0.240
349	A	12	6	1.00	25	0.240
350	A	3	2	1.00	29	0.069
351	A	3	2	1.00	29	0.069
352	A	3	2	1.00	29	0.069
353	A	3	2	1.00	27	0.074
354	A	5	3	1.00	22	0.136
355	A	3	2	1.00	29	0.069
356	A	4	3	1.00	29	0.103
357	A	4	3	1.00	29	0.103
358	A	4	3	1.00	29	0.103
359	A	3	2	1.00	29	0.069
360	A	3	2	1.00	29	0.069
361	A	3	2	1.00	29	0.069
362	A	3	2	1.00	29	0.069
363	A	3	2	1.00	29	0.069
364	A	3	2	1.00	29	0.069
365	A	3	2	1.00	27	0.074
366	A	2	2	1.00	22	0.091
367	A	4	3	1.00	29	0.103
368	A	4	3	1.00	29	0.103
369	A	4	3	1.00	29	0.103
370	A	4	3	1.00	29	0.103
371	A	3	2	1.00	29	0.069
372	A	3	2	1.00	29	0.069
373	A	3	2	1.00	29	0.069
374	A	3	2	1.00	29	0.069
375	A	3	2	1.00	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	4	3	1.00	29	0.103
377	A	4	3	1.00	27	0.111
378	A	3	3	1.00	22	0.136
379	A	4	3	1.00	29	0.103
380	A	4	3	1.00	29	0.103
381	A	7	5	1.00	31	0.161
382	A	6	4	1.00	31	0.129
383	A	5	4	1.00	31	0.129
384	A	3	3	1.00	31	0.097
385	A	3	3	1.00	29	0.103
386	A	4	3	1.00	24	0.125
387	A	6	5	1.00	31	0.161
388	A	6	4	1.00	31	0.129
389	A	7	5	1.00	31	0.161
390	A	4	3	1.00	24	0.125
391	A	3	2	1.00	24	0.083
392	A	3	2	1.00	24	0.083
393	A	2	1	1.00	22	0.045
394	A	2	1	1.00	17	0.059
395	A	4	3	1.00	24	0.125
396	A	4	4	1.00	24	0.167
397	A	4	4	1.00	24	0.167
398	A	3	2	1.00	24	0.083
399	A	3	2	1.00	24	0.083
400	A	2	1	1.00	22	0.045
401	A	2	1	1.00	17	0.059
402	A	4	3	1.00	24	0.125
403	A	4	4	1.00	24	0.167
404	A	5	5	1.00	24	0.208
405	A	5	5	1.00	26	0.192
406	A	1	1	1.00	22	0.045
407	A	11	8	1.00	28	0.286
408	A	10	7	1.00	28	0.250
409	A	6	3	1.00	28	0.107
410	A	8	5	1.00	28	0.179
411	A	11	7	1.00	28	0.250
412	A	11	7	1.00	28	0.250
413	A	6	3	1.00	28	0.107
414	A	6	3	1.00	28	0.107
415	A	8	4	1.00	28	0.143
416	A	19	9	1.00	28	0.321
417	A	8	5	1.00	28	0.179
418	A	8	4	1.00	28	0.143
419	A	12	6	0.99	28	0.214
420	A	6	3	1.00	20	0.150
421	A	5	5	1.00	26	0.192
422	A	6	6	1.00	30	0.200
423	A	5	5	1.00	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
424	A	4	3	1.00	46	0.065
425	A	3	3	1.00	46	0.065
426	A	2	2	1.00	44	0.045
427	A	1	1	1.00	39	0.026
428	A	2	2	1.00	46	0.043
429	A	3	3	1.00	46	0.065
430	A	4	3	1.00	46	0.065
431	A	5	3	1.00	46	0.065
432	A	4	4	1.00	46	0.087
433	A	3	3	1.00	46	0.065
434	A	2	2	1.00	44	0.045
435	A	1	1	1.00	39	0.026
436	A	3	3	1.00	46	0.065
437	A	4	4	1.00	46	0.087
438	A	5	4	1.00	46	0.087
439	A	4	3	1.00	46	0.065
440	A	3	3	1.00	46	0.065
441	A	2	2	1.00	44	0.045
442	A	1	1	1.00	39	0.026
443	A	4	3	1.00	46	0.065
444	A	5	4	1.00	46	0.087
445	A	6	4	1.00	46	0.087
446	A	5	3	1.00	46	0.065
447	A	4	3	1.00	46	0.065
448	A	3	3	1.00	46	0.065
449	A	2	2	1.00	44	0.045
450	A	1	1	1.00	39	0.026
451	A	3	3	1.00	46	0.065
452	A	3	3	1.00	46	0.065
453	A	4	4	1.00	46	0.087
454	A	5	4	1.00	46	0.087
455	A	6	4	1.00	46	0.087
456	A	5	3	1.00	46	0.065
457	A	4	3	1.00	46	0.065
458	A	3	3	1.00	46	0.065
459	A	2	2	1.00	44	0.045
460	A	1	1	1.00	39	0.026
461	A	4	3	1.00	46	0.065
462	A	4	4	1.00	46	0.087
463	A	4	3	1.00	46	0.065
464	A	5	4	1.00	46	0.087
465	A	6	4	1.00	46	0.087
466	A	7	4	1.00	46	0.087
467	A	5	3	1.00	46	0.065
468	A	4	3	1.00	46	0.065
469	A	3	3	1.00	46	0.065
470	A	2	2	1.00	44	0.045
471	A	1	1	1.00	39	0.026

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
472	A	5	3	1.00	46	0.065
473	A	5	4	1.00	46	0.087
474	A	5	4	1.00	46	0.087
475	A	5	3	1.00	46	0.065
476	A	6	4	1.00	46	0.087
477	A	7	4	1.00	46	0.087
478	A	8	4	1.00	46	0.087
479	A	7	5	1.00	48	0.104
480	A	6	5	1.00	48	0.104
481	A	5	5	1.00	48	0.104
482	A	4	4	1.00	48	0.083
483	A	1	1	1.00	48	0.021
484	A	2	2	1.00	48	0.042
485	A	3	2	1.00	48	0.042
486	A	4	2	1.00	48	0.042
487	A	7	6	1.00	48	0.125
488	A	6	6	1.00	48	0.125
489	A	5	5	1.00	48	0.104
490	A	1	1	1.00	48	0.021
491	A	2	2	1.00	48	0.042
492	A	3	3	1.00	48	0.062
493	A	4	3	1.00	48	0.062
494	A	7	6	1.00	48	0.125
495	A	6	5	1.00	48	0.104
496	A	1	1	1.00	48	0.021
497	A	2	2	1.00	48	0.042
498	A	3	2	1.00	48	0.042
499	A	4	3	1.00	48	0.062
500	A	8	6	1.00	48	0.125
501	A	7	6	1.00	48	0.125
502	A	6	6	1.00	48	0.125
503	A	5	5	1.00	48	0.104
504	A	5	5	1.00	48	0.104
505	A	1	1	1.00	48	0.021
506	A	2	2	1.00	48	0.042
507	A	3	2	1.00	48	0.042
508	A	4	2	1.00	48	0.042
509	A	8	6	1.00	48	0.125
510	A	7	6	1.00	48	0.125
511	A	6	5	1.00	48	0.104
512	A	6	6	1.00	48	0.125
513	A	6	5	1.00	48	0.104
514	A	1	1	1.00	48	0.021
515	A	2	2	1.00	48	0.042
516	A	3	2	1.00	48	0.042
517	A	4	2	1.00	48	0.042
518	A	9	6	1.00	48	0.125
519	A	8	6	1.00	48	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
520	A	7	5	1.00	48	0.104
521	A	7	6	1.00	48	0.125
522	A	7	6	1.00	48	0.125
523	A	7	5	1.00	48	0.104
524	A	1	1	1.00	48	0.021
525	A	2	2	1.00	48	0.042
526	A	3	2	1.00	48	0.042
527	A	4	2	1.00	48	0.042
528	A	4	3	1.00	44	0.068
529	A	3	3	1.00	44	0.068
530	A	2	2	1.00	42	0.048
531	A	1	1	1.00	37	0.027
532	A	1	1	1.00	47	0.021
533	A	3	3	1.00	73	0.041
534	A	6	4	1.00	46	0.087
535	A	5	4	1.00	46	0.087
536	A	4	4	1.00	46	0.087
537	A	3	3	1.00	44	0.068
538	A	2	2	1.00	39	0.051
539	A	3	3	1.00	46	0.065
540	A	3	3	1.00	46	0.065
541	A	4	4	1.00	46	0.087
542	A	5	4	1.00	46	0.087
543	A	8	4	1.00	32	0.125
544	A	6	4	1.00	32	0.125
545	A	4	4	1.00	30	0.133
546	A	6	4	1.00	32	0.125
547	A	7	5	1.00	32	0.156
548	A	7	5	1.00	32	0.156
549	A	5	5	1.00	32	0.156
550	A	4	4	1.00	30	0.133
551	A	7	5	1.00	32	0.156
552	A	8	6	1.00	32	0.188
553	A	2	1	1.00	28	0.036
554	A	2	1	1.00	28	0.036
555	A	2	1	1.00	26	0.038
556	A	2	1	1.00	21	0.048
557	A	2	1	1.00	25	0.040
558	A	2	1	1.00	27	0.037
559	A	2	1	1.00	27	0.037
560	A	6	5	1.00	27	0.185
561	A	9	6	1.00	27	0.222
562	A	3	2	1.00	27	0.074
563	A	3	2	1.00	27	0.074
564	A	2	1	1.00	25	0.040
565	A	2	1	1.00	20	0.050
566	A	4	3	1.00	27	0.111
567	A	4	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
568	A	4	4	1.00	27	0.148
569	A	3	2	1.00	27	0.074
570	A	3	2	1.00	27	0.074
571	A	2	1	1.00	25	0.040
572	A	2	1	1.00	20	0.050
573	A	4	3	1.00	27	0.111
574	A	4	4	1.00	27	0.148
575	A	5	5	1.00	27	0.185
576	B	9	7	2.10	25	0.280
577	A	5	5	1.00	29	0.172
578	A	7	6	1.00	29	0.207
579	A	6	6	1.00	29	0.207
580	A	5	5	1.00	29	0.172
581	A	5	5	1.00	29	0.172
582	A	5	5	1.00	29	0.172
583	A	3	3	1.00	29	0.103
584	A	4	4	1.00	29	0.138
585	A	6	6	1.00	29	0.207
586	A	7	6	1.00	38	0.158
587	A	6	6	1.00	38	0.158
588	A	5	5	1.00	38	0.132
589	A	5	5	1.00	38	0.132
590	A	5	5	1.00	38	0.132
591	A	3	3	1.00	38	0.079
592	A	4	4	1.00	38	0.105
593	A	11	7	1.00	31	0.226
594	A	6	3	1.00	31	0.097
595	A	6	3	1.00	31	0.097
596	A	8	4	1.00	31	0.129
597	A	8	6	1.00	29	0.207
598	A	7	6	1.00	29	0.207
599	A	6	5	1.00	27	0.185
600	A	6	5	1.00	22	0.227
601	A	8	5	1.00	29	0.172
602	A	20	7	1.00	29	0.241
603	A	23	8	1.00	29	0.276
604	A	27	9	1.00	29	0.310
605	A	9	6	1.00	29	0.207
606	A	8	6	1.00	29	0.207
607	A	7	5	1.00	27	0.185
608	A	7	6	1.00	22	0.273
609	A	13	7	1.00	29	0.241
610	A	23	8	1.00	29	0.276
611	A	30	9	1.00	29	0.310
612	A	15	7	1.00	29	0.241
613	A	8	5	1.00	29	0.172
614	A	7	5	1.00	29	0.172
615	A	6	5	1.00	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
616	A	5	4	1.00	27	0.148
617	A	2	2	1.00	22	0.091
618	A	6	3	1.00	29	0.103
619	A	9	4	1.00	29	0.138
620	A	13	6	1.00	29	0.207
621	A	7	6	1.00	29	0.207
622	A	6	5	1.00	29	0.172
623	A	4	4	1.00	29	0.138
624	A	4	4	1.00	27	0.148
625	A	4	4	1.00	22	0.182
626	A	10	5	1.00	29	0.172
627	A	14	6	1.00	29	0.207
628	A	19	7	1.00	29	0.241
629	A	2	1	1.00	25	0.040
630	A	2	1	1.00	23	0.043
631	A	2	1	1.00	27	0.037
632	A	2	1	1.00	25	0.040

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=132

$$-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}+\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}$$

Rubi **[A]** time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {797, 641, 195, 217, 203}

$$\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - (d^2*(d^2 - e^2*x^2)^(3/2))/(3*e^3) -
 (d*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = -\frac{\int (d+ex)\left(d^2-e^2x^2\right)^{3/2}dx}{e^2}+\frac{d^2\int (d+ex)\sqrt{d^2-e^2x^2}\,dx}{e^2}$$
$$= -\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{d\int \left(d^2-e^2x^2\right)^{3/2}dx}{e^2}+\frac{d^3\int \sqrt{d^2-e^2x^2}\,d}{e^2}$$
$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{(3d^3)}{}$$
$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{(3d^5)}{}$$
$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan}{}$$
$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan}{}$$

Mathematica [A] time = 0.12, size = 112, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(15d^4\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-16d^4-15d^3ex-8d^2e^2x^2+30de^3x^3+24e^4x^4\right)\right)}{120e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-16*d^4 - 15*d^3*e*x - 8*d^2
*e^2*x^2 + 30*d*e^3*x^3 + 24*e^4*x^4) + 15*d^4*ArcSin[(e*x)/d]))/(120*e^3*S
qrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.89, size = 114, normalized size = 0.86

$$\frac{d^5\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e^4}+\frac{\sqrt{d^2-e^2x^2}\left(-16d^4-15d^3ex-8d^2e^2x^2+30de^3x^3+24e^4x^4\right)}{120e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^4 - 15*d^3*e*x - 8*d^2*e^2*x^2 + 30*d*e^3*x^3 +
 24*e^4*x^4))/(120*e^3) + (d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 -
e^2*x^2]])/(8*e^4)
```

fricas [A] time = 0.40, size = 95, normalized size = 0.72

$$-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,e^4x^4+30\,de^3x^3-8\,d^2e^2x^2-15\,d^3ex-16\,d^4\right)\sqrt{-e^2x^2+d^2}}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (24*e^4*x^4 + 30
*d*e^3*x^3 - 8*d^2*e^2*x^2 - 15*d^3*e*x - 16*d^4)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.26, size = 74, normalized size = 0.56

$$\frac{1}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{120}\left(16\,d^4e^{(-3)}+\left(15\,d^3e^{(-2)}+2\left(4\,d^2e^{(-1)}-3\,(4\,xe+5\,d)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^5*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/120*(16*d^4*e^(-3) + (15*d^3*e^(-2)
 + 2*(4*d^2*e^(-1) - 3*(4*x*e + 5*d)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.06, size = 125, normalized size = 0.95

$$\frac{d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2}+\frac{\sqrt{-e^2x^2+d^2}\,d^3x}{8e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x^2}{5e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4e^2}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{15e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*x^2*(-e^2*x^2+d^2)^(3/2)/e-2/15*d^2*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*d*x*(
-e^2*x^2+d^2)^(3/2)/e^2+1/8*d^3*x*(-e^2*x^2+d^2)^(1/2)/e^2+1/8*d^5/e^2/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

maxima [A] time = 0.98, size = 104, normalized size = 0.79

$$\frac{d^5\arcsin\left(\frac{ex}{d}\right)}{8\,e^3}+\frac{\sqrt{-e^2x^2+d^2}\,d^3x}{8\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x^2}{5\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4\,e^2}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{15\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*d^5*arcsin(e*x/d)/e^3 + 1/8*sqrt(-e^2*x^2 + d^2)*d^3*x/e^2 - 1/5*(-e^2*
x^2 + d^2)^(3/2)*x^2/e - 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e^2 - 2/15*(-e^2*x^
2 + d^2)^(3/2)*d^2/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x),x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x), x)
```

sympy [C] time = 5.48, size = 279, normalized size = 2.11

$$d\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**
2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d
```

```
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8
*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e*
*2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + e*Piece
wise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2
*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**
2)/4, True))
```

3.2 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=201

$$-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}+\frac{3d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}+\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e\right.}{64e^4}$$

Rubi [A] time = 0.15, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {833, 780, 195, 217, 203}

$$\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}-\frac{d^3(128d+315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}+\frac{3d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) - (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) - (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) - (d^3*(128*d + 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx &= -\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{\int x^3\left(-4d^2e-9de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{9e^2} \\
&= -\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} + \frac{\int x^2\left(27d^3e^2+32d^2e^3x\right)\left(d^2-e^2x^2\right.}{72e^4} \\
&= -\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{\int x\left(-64d^4e\right.}{} \\
&= -\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{d^3(128d+3}{} \\
&= \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} - \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(d^2-e^2\right.}{9e} \\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} - \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2\right.}{8e^2} \\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} - \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2\right.}{8e^2} \\
&= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} - \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2\right.}{8e^2}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 157, normalized size = 0.78

$$\frac{\sqrt{d^2-e^2x^2}\left(945d^8\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(1024d^8+945d^7ex+512d^6e^2x^2+630d^5e^3x^3+384d^4e^4x^4-7560d^3e^5x^5-6400d^2e^6x^6+5040de^7x^7+4480e^8x^8\right)\right)}{40320e^5\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(1024*d^8 + 945*d^7*e*x + 5
12*d^6*e^2*x^2 + 630*d^5*e^3*x^3 + 384*d^4*e^4*x^4 - 7560*d^3*e^5*x^5 - 640
0*d^2*e^6*x^6 + 5040*d*e^7*x^7 + 4480*e^8*x^8)) + 945*d^8*ArcSin[(e*x)/d]))
/(40320*e^5*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.50, size = 158, normalized size = 0.79

$$\frac{3d^9\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{128e^6}+\frac{\sqrt{d^2-e^2x^2}\left(-1024d^8-945d^7ex-512d^6e^2x^2-630d^5e^3x^3-384d^4e^4x^4+7560d^3e^5x^5+6400d^2e^6x^6-5040de^7x^7-4480e^8x^8\right)}{40320e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-1024*d^8 - 945*d^7*e*x - 512*d^6*e^2*x^2 - 630*d^5*e
^3*x^3 - 384*d^4*e^4*x^4 + 7560*d^3*e^5*x^5 + 6400*d^2*e^6*x^6 - 5040*d*e^7
*x^7 - 4480*e^8*x^8))/(40320*e^5) + (3*d^9*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) +
 Sqrt[d^2 - e^2*x^2]])/(128*e^6)
```

fricas [A] time = 0.40, size = 138, normalized size = 0.69

$$-\frac{1890\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4480\,e^8x^8+5040\,de^7x^7-6400\,d^2e^6x^6-7560\,d^3e^5x^5+384\,d^4e^4x^4+630\,d^5e^3x^3+512\,d^6e^2x^2+945\,d^7ex+1024\,d^8\right)\sqrt{-e^2x^2+d^2}}{40320\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/40320*(1890*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4480*e^8*x^
8 + 5040*d*e^7*x^7 - 6400*d^2*e^6*x^6 - 7560*d^3*e^5*x^5 + 384*d^4*e^4*x^4
+ 630*d^5*e^3*x^3 + 512*d^6*e^2*x^2 + 945*d^7*e*x + 1024*d^8)*sqrt(-e^2*x^2
 + d^2))/e^5
```

giac [A] time = 0.22, size = 117, normalized size = 0.58

$$\frac{3}{128} d^9 \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \mathrm{sgn}(d) - \frac{1}{40320} \left(1024 d^8 e^{(-5)} + \left(945 d^7 e^{(-4)} + 2 \left(256 d^6 e^{(-3)} + \left(315 d^5 e^{(-2)} + 4 \left(48 d^4 e^{(-1)} - 5 \left(189 d^3 + 2 \left(80 d^2 e - 7 \left(8 x e^3 + 9 d e^2\right) x\right) x\right) x\right) x\right) x\right) x\right) \sqrt{-x^2 e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/128*d^9*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/40320*(1024*d^8*e^(-5) + (945*d^7
*e^(-4) + 2*(256*d^6*e^(-3) + (315*d^5*e^(-2) + 4*(48*d^4*e^(-1) - 5*(189*d
^3 + 2*(80*d^2*e - 7*(8*x*e^3 + 9*d*e^2)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

maple [A] time = 0.04, size = 198, normalized size = 0.99

$$\frac{3 d^9 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2 x^2 + d^2}}\right)}{128 \sqrt{e^2}\, e^4} + \frac{3 \sqrt{-e^2 x^2 + d^2}\, d^7 x}{128 e^4} - \frac{\left(-e^2 x^2 + d^2\right)^{\frac{5}{2}} x^4}{9 e} + \frac{\left(-e^2 x^2 + d^2\right)^{\frac{3}{2}} d^5 x}{64 e^4} - \frac{\left(-e^2 x^2 + d^2\right)^{\frac{5}{2}} d x^3}{8 e^2} - \frac{4 \left(-e^2 x^2 + d^2\right)^{\frac{5}{2}} d^2 x^2}{63 e^3} - \frac{\left(-e^2 x^2 + d^2\right)^{\frac{5}{2}} d^3 x}{16 e^4} - \frac{8 \left(-e^2 x^2 + d^2\right)^{\frac{5}{2}} d^4}{315 e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e-4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-8/315
*d^4/e^5*(-e^2*x^2+d^2)^(5/2)-1/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2-1/16*d^3/e
^4*x*(-e^2*x^2+d^2)^(5/2)+1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4+3/128*d^7*x*(
-e^2*x^2+d^2)^(1/2)/e^4+3/128*d^9/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*
x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 177, normalized size = 0.88

$$-\frac{\left(-e^2 x^2 + d^2\right)^{\frac{5}{2}} x^4}{9 e} + \frac{3 d^9 \arcsin\left(\frac{e x}{d}\right)}{128 e^5} + \frac{3 \sqrt{-e^2 x^2 + d^2}\, d^7 x}{128 e^4} - \frac{\left(-e^2 x^2 + d^2\right)^{\frac{5}{2}} d x^3}{8 e^2} + \frac{\left(-e^2 x^2 + d^2\right)^{\frac{3}{2}} d^5 x}{64 e^4} - \frac{4 \left(-e^2 x^2 + d^2\right)^{\frac{5}{2}} d^2 x^2}{63 e^3} - \frac{\left(-e^2 x^2 + d^2\right)^{\frac{5}{2}} d^3 x}{16 e^4} - \frac{8 \left(-e^2 x^2 + d^2\right)^{\frac{5}{2}} d^4}{315 e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/9*(-e^2*x^2 + d^2)^(5/2)*x^4/e + 3/128*d^9*arcsin(e*x/d)/e^5 + 3/128*sqr
t(-e^2*x^2 + d^2)*d^7*x/e^4 - 1/8*(-e^2*x^2 + d^2)^(5/2)*d*x^3/e^2 + 1/64*(
-e^2*x^2 + d^2)^(3/2)*d^5*x/e^4 - 4/63*(-e^2*x^2 + d^2)^(5/2)*d^2*x^2/e^3 -
 1/16*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^4 - 8/315*(-e^2*x^2 + d^2)^(5/2)*d^4/e
^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(d^2 - e^2 x^2\right)^{3/2} (d + e x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x^4*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

sympy [C] time = 17.83, size = 830, normalized size = 4.13

$$d^3 \left(\begin{cases} - \frac{i d^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16 e^5} + \frac{i d^5 x}{16 e^4 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{i d^3 x^3}{48 e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{5 i d x^5}{24 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{i e^2 x^7}{6 d \sqrt{-1 + \frac{e^2 x^2}{d^2}}} & \text{for}\: \left|\frac{e^2 x^2}{d^2}\right| > 1 \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16 e^5} - \frac{d^5 x}{16 e^4 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^3 x^3}{48 e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{5 d x^5}{24 \sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^7}{6 d \sqrt{1 - \frac{e^2 x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + d^2 e \left(\begin{cases} - \frac{8 d^6 \sqrt{d^2 - e^2 x^2}}{105 e^6} - \frac{4 d^4 x^2 \sqrt{d^2 - e^2 x^2}}{105 e^4} - \frac{d^2 x^4 \sqrt{d^2 - e^2 x^2}}{35 e^2} + \frac{x^6 \sqrt{d^2 - e^2 x^2}}{7} & \text{for}\: e \neq 0 \\ \frac{x^6 \sqrt{d^2}}{6} & \text{otherwise} \end{cases}\right) - d e^2 \left(\begin{cases} - \frac{5 i d^8 \operatorname{acosh}\left(\frac{ex}{d}\right)}{128 e^7} + \frac{5 i d^7 x}{128 e^6 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{5 i d^5 x^3}{384 e^4 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{i d^3 x^5}{192 e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{7 i d x^7}{48 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{i e^2 x^9}{8 d \sqrt{-1 + \frac{e^2 x^2}{d^2}}} & \text{for}\: \left|\frac{e^2 x^2}{d^2}\right| > 1 \\ \frac{5 d^8 \operatorname{asin}\left(\frac{ex}{d}\right)}{128 e^7} - \frac{5 d^7 x}{128 e^6 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{5 d^5 x^3}{384 e^4 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^3 x^5}{192 e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{7 d x^7}{48 \sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^9}{8 d \sqrt{1 - \frac{e^2 x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - e^3 \left(\begin{cases} - \frac{16 d^8 \sqrt{d^2 - e^2 x^2}}{315 e^8} - \frac{8 d^6 x^2 \sqrt{d^2 - e^2 x^2}}{315 e^6} - \frac{2 d^4 x^4 \sqrt{d^2 - e^2 x^2}}{105 e^4} - \frac{d^2 x^6 \sqrt{d^2 - e^2 x^2}}{63 e^2} + \frac{x^8 \sqrt{d^2 - e^2 x^2}}{9} & \text{for}\: e \neq 0 \\ \frac{x^8 \sqrt{d^2}}{8} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) + d**2*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(1
05*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d
**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**
6*sqrt(d**2)/6, True)) - d*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7
) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e*
*4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d
**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e*
*7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*
sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) +
 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2
/d**2)), True)) - e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8
) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 -
 e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*
sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

3.3 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}+\frac{3d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}+\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+ d$$

Rubi [A] time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 780, 195, 217, 203}

$$\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}+\frac{3d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) + (d^4*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) - (x^3*(d^2 - e^2*x^2)^(5/2))
/(8*e) - (d^2*(32*d + 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) + (3*d^8*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{\int x^2\left(-3d^2e-8de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{8e^2}$$
$$= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} + \frac{\int x\left(16d^3e^2+21d^2e^3x\right)\left(d^2-e^2x^2\right)}{56e^4}$$
$$= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} + \frac{d^4}{}$$
$$= \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d+35ex)}{560}$$
$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/}}{8e}$$
$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/}}{8e}$$
$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} - \frac{x^3\left(d^2-e^2x^2\right)^{5/}}{8e}$$

Mathematica [A] time = 0.19, size = 146, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(105d^7\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(256d^7+105d^6ex+128d^5e^2x^2+70d^4e^3x^3-1024d^3e^4x^4-840d^2e^5x^5+640de^6x^6+560e^7x^7\right)\right)}{4480e^4\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(256*d^7 + 105*d^6*e*x + 12
8*d^5*e^2*x^2 + 70*d^4*e^3*x^3 - 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 + 640*d
*e^6*x^6 + 560*e^7*x^7)) + 105*d^7*ArcSin[(e*x)/d]))/(4480*e^4*Sqrt[1 - (e^
2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.46, size = 147, normalized size = 0.85

$$\frac{3d^8\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{128e^5}+\frac{\sqrt{d^2-e^2x^2}\left(-256d^7-105d^6ex-128d^5e^2x^2-70d^4e^3x^3+1024d^3e^4x^4+840d^2e^5x^5-640de^6x^6-560e^7x^7\right)}{4480e^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-256*d^7 - 105*d^6*e*x - 128*d^5*e^2*x^2 - 70*d^4*e^3
*x^3 + 1024*d^3*e^4*x^4 + 840*d^2*e^5*x^5 - 640*d*e^6*x^6 - 560*e^7*x^7))/(
4480*e^4) + (3*d^8*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(
128*e^5)
```

fricas [A] time = 0.41, size = 127, normalized size = 0.74

$$-\frac{210\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(560\,e^7x^7+640\,de^6x^6-840\,d^2e^5x^5-1024\,d^3e^4x^4+70\,d^4e^3x^3+128\,d^5e^2x^2+105\,d^6ex+256\,d^7\right)\sqrt{-e^2x^2+d^2}}{4480\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4480*(210*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (560*e^7*x^7 +
 640*d*e^6*x^6 - 840*d^2*e^5*x^5 - 1024*d^3*e^4*x^4 + 70*d^4*e^3*x^3 + 128*
d^5*e^2*x^2 + 105*d^6*e*x + 256*d^7)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [A] time = 0.23, size = 106, normalized size = 0.62

$$\frac{3}{128}\,d^8\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d) - \frac{1}{4480}\left(256\,d^7e^{(-4)} + \left(105\,d^6e^{(-3)} + 2\left(64\,d^5e^{(-2)} + \left(35\,d^4e^{(-1)} - 4\left(128\,d^3 + 5\left(21\,d^2e - 2\left(7\,xe^3 + 8\,de^2\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/128*d^8*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/4480*(256*d^7*e^(-4) + (105*d^6*e
^(-3) + 2*(64*d^5*e^(-2) + (35*d^4*e^(-1) - 4*(128*d^3 + 5*(21*d^2*e - 2*(7
*x*e^3 + 8*d*e^2)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.02, size = 173, normalized size = 1.01

$$\frac{3d^8\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128\sqrt{e^2}\,e^3} + \frac{3\sqrt{-e^2x^2+d^2}\,d^6x}{128e^3} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4x}{64e^3} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^3}{8e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,x^2}{7e^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}{16e^3} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{35e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/8*x^3*(-e^2*x^2+d^2)^(5/2)/e-1/16*d^2/e^3*x*(-e^2*x^2+d^2)^(5/2)+1/64*d^
4*x*(-e^2*x^2+d^2)^(3/2)/e^3+3/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^3+3/128*d^8
/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/7*d*x^2*(-e^2
*x^2+d^2)^(5/2)/e^2-2/35*d^3/e^4*(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.98, size = 152, normalized size = 0.88

$$\frac{3\,d^8\arcsin\left(\frac{ex}{d}\right)}{128\,e^4} + \frac{3\,\sqrt{-e^2x^2+d^2}\,d^6x}{128\,e^3} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^3}{8\,e} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4x}{64\,e^3} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^2}{7\,e^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}{16\,e^3} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{35\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/128*d^8*arcsin(e*x/d)/e^4 + 3/128*sqrt(-e^2*x^2 + d^2)*d^6*x/e^3 - 1/8*(-
e^2*x^2 + d^2)^(5/2)*x^3/e + 1/64*(-e^2*x^2 + d^2)^(3/2)*d^4*x/e^3 - 1/7*(-
e^2*x^2 + d^2)^(5/2)*d*x^2/e^2 - 1/16*(-e^2*x^2 + d^2)^(5/2)*d^2*x/e^3 - 2/
35*(-e^2*x^2 + d^2)^(5/2)*d^3/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

sympy [A] time = 17.17, size = 775, normalized size = 4.51

$$d^3\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right) + d^2e\left(\begin{cases} -\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - de^2\left(\begin{cases} -\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6} - \frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4} - \frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2} + \frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\ e \neq 0 \\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise} \end{cases}\right) - e^3\left(\begin{cases} -\frac{5id^8\operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^7} + \frac{5id^7x}{128e^6\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5id^5x^3}{384e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{id^3x^5}{192e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{7idx^7}{48\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^9}{8d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{5d^8\operatorname{asin}\left(\frac{ex}{d}\right)}{128e^7} - \frac{5d^7x}{128e^6\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5d^5x^3}{384e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3x^5}{192e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{7dx^7}{48\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^9}{8d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) + d**2*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
 I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-
1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x
**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e
*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48
*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) -
e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - d*e**2*Piecewise((-8*d**
6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(1
05*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e*
*2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - e**3*Piecewise((-5*I*d*
*8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)
) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e
**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2))
+ I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5
*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2))
 + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sq
rt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**
9/(8*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.4 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}+\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}$$

Rubi [A] time = 0.11, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {797, 641, 195, 217, 203}

$$\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) - (d^2*(d^2 - e^2*x^2)^(5/2))/(5*e^3) - (d*x*(d^2 - e^2*x^2)^(5/2))/(6*e
^2) + (d^2 - e^2*x^2)^(7/2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(16*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{\int (d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx}{e^2} + \frac{d^2\int (d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}$$
$$= -\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3} + \frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3} - \frac{d\int \left(d^2-e^2x^2\right)^{5/2}\,dx}{e^2} + \frac{d^3\int \left(d^2-e^2x^2\right.}{e^2}$$
$$= \frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{4e^2} - \frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3} - \frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2} + \frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3} -$$
$$= \frac{3d^5x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2} - \frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3} - \frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}$$
$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2} - \frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3} - \frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2} -$$
$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2} - \frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3} - \frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2} -$$
$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2} - \frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3} - \frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2} -$$

Mathematica [A] time = 0.17, size = 135, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(105d^6\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(96d^6+105d^5ex+48d^4e^2x^2-490d^3e^3x^3-384d^2e^4x^4+280de^5x^5+240e^6x^6\right)\right)}{1680e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(96*d^6 + 105*d^5*e*x + 48*
d^4*e^2*x^2 - 490*d^3*e^3*x^3 - 384*d^2*e^4*x^4 + 280*d*e^5*x^5 + 240*e^6*x
^6)) + 105*d^6*ArcSin[(e*x)/d]))/(1680*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.45, size = 136, normalized size = 0.86

$$\frac{d^7\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{16e^4}+\frac{\sqrt{d^2-e^2x^2}\left(-96d^6-105d^5ex-48d^4e^2x^2+490d^3e^3x^3+384d^2e^4x^4-280de^5x^5-240e^6x^6\right)}{1680e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-96*d^6 - 105*d^5*e*x - 48*d^4*e^2*x^2 + 490*d^3*e^3*
x^3 + 384*d^2*e^4*x^4 - 280*d*e^5*x^5 - 240*e^6*x^6))/(1680*e^3) + (d^7*Sqr
t[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^4)
```

fricas [A] time = 0.40, size = 116, normalized size = 0.73

$$-\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(240\,e^6x^6+280\,de^5x^5-384\,d^2e^4x^4-490\,d^3e^3x^3+48\,d^4e^2x^2+105\,d^5ex+96\,d^6\right)\sqrt{-e^2x^2+d^2}}{1680\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (240*e^6*x^6 +
 280*d*e^5*x^5 - 384*d^2*e^4*x^4 - 490*d^3*e^3*x^3 + 48*d^4*e^2*x^2 + 105*d
^5*e*x + 96*d^6)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.23, size = 96, normalized size = 0.60

$$\frac{1}{16}\,d^7\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{1680}\left(96\,d^6e^{(-3)}+\left(105\,d^5e^{(-2)}+2\left(24\,d^4e^{(-1)}-\left(245\,d^3+4\left(48\,d^2e-5\left(6\,xe^3+7\,de^2\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^7*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/1680*(96*d^6*e^(-3) + (105*d^5*e^(
-2) + 2*(24*d^4*e^(-1) - (245*d^3 + 4*(48*d^2*e - 5*(6*x*e^3 + 7*d*e^2)*x)*
x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.04, size = 148, normalized size = 0.93

$$\frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e^2}+\frac{\sqrt{-e^2x^2+d^2}\,d^5x}{16e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{24e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}{7e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{6e^2}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{35e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/7*x^2*(-e^2*x^2+d^2)^(5/2)/e-2/35*d^2*(-e^2*x^2+d^2)^(5/2)/e^3-1/6*d*x*(
-e^2*x^2+d^2)^(5/2)/e^2+1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/16*d^5*x*(-e^
2*x^2+d^2)^(1/2)/e^2+1/16*d^7/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+
d^2)^(1/2)*x)
```

maxima [A] time = 0.95, size = 127, normalized size = 0.80

$$\frac{d^7\arcsin\left(\frac{ex}{d}\right)}{16\,e^3}+\frac{\sqrt{-e^2x^2+d^2}\,d^5x}{16\,e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{24\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}{7\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{6\,e^2}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{35\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^7*arcsin(e*x/d)/e^3 + 1/16*sqrt(-e^2*x^2 + d^2)*d^5*x/e^2 + 1/24*(-e
^2*x^2 + d^2)^(3/2)*d^3*x/e^2 - 1/7*(-e^2*x^2 + d^2)^(5/2)*x^2/e - 1/6*(-e^
2*x^2 + d^2)^(5/2)*d*x/e^2 - 2/35*(-e^2*x^2 + d^2)^(5/2)*d^2/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

sympy [C] time = 12.27, size = 653, normalized size = 4.11

$$d^3\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6}-\frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2}+\frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\ e\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
```

```
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2
*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**
2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*
sqrt(d**2)/4, True)) - d*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I
*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1
+ e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**
7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x
/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e
**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e*
*2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - e**3*Piecewise((-8*d**6*sq
rt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e
**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x
**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

3.5 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}+\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

Rubi [A] time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {780, 195, 217, 203}

$$\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rubi steps

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{6e}$$
$$=\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\int\frac{1}{\sqrt{d^2-e^2}}}{16e}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\,\text{Subst}\Big(}{}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\Big(}{16e}$$

Mathematica [A] time = 0.14, size = 124, normalized size = 1.07

$$\frac{\sqrt{d^2-e^2x^2}\left(15d^5\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(48d^5+15d^4ex-96d^3e^2x^2-70d^2e^3x^3+48de^4x^4+40e^5x^5\right)\right)}{240e^2\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(48*d^5 + 15*d^4*e*x - 96*d
^3*e^2*x^2 - 70*d^2*e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5)) + 15*d^5*ArcSin[(
e*x)/d]))/(240*e^2*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.41, size = 125, normalized size = 1.08

$$\frac{d^6\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{16e^3}+\frac{\sqrt{d^2-e^2x^2}\left(-48d^5-15d^4ex+96d^3e^2x^2+70d^2e^3x^3-48de^4x^4-40e^5x^5\right)}{240e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 - 15*d^4*e*x + 96*d^3*e^2*x^2 + 70*d^2*e^3*x^
3 - 48*d*e^4*x^4 - 40*e^5*x^5))/(240*e^2) + (d^6*Sqrt[-e^2]*Log[-(Sqrt[-e^2
]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

fricas [A] time = 0.40, size = 105, normalized size = 0.91

$$-\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+48\,de^4x^4-70\,d^2e^3x^3-96\,d^3e^2x^2+15\,d^4ex+48\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 48
*d*e^4*x^4 - 70*d^2*e^3*x^3 - 96*d^3*e^2*x^2 + 15*d^4*e*x + 48*d^5)*sqrt(-e
^2*x^2 + d^2))/e^2
```

giac [A] time = 0.21, size = 84, normalized size = 0.72

$$\frac{1}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)-\frac{1}{240}\left(48\,d^5e^{(-2)}+\left(15\,d^4e^{(-1)}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,xe^3+6\,de^2\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^6*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/240*(48*d^5*e^(-2) + (15*d^4*e^(-1
) - 2*(48*d^3 + (35*d^2*e - 4*(5*x*e^3 + 6*d*e^2)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

maple [A] time = 0.02, size = 123, normalized size = 1.06

$$\frac{d^6 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\, e}+\frac{\sqrt{-e^2x^2+d^2}\, d^4x}{16e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^2x}{24e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} x}{6e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/6*x*(-e^2*x^2+d^2)^(5/2)/e+1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e+1/16*d^4*x*
(-e^2*x^2+d^2)^(1/2)/e+1/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+
d^2)^(1/2)*x)-1/5*d/e^2*(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.98, size = 102, normalized size = 0.88

$$\frac{d^6 \arcsin\left(\frac{ex}{d}\right)}{16\, e^2}+\frac{\sqrt{-e^2x^2+d^2}\, d^4x}{16\, e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^2x}{24\, e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} x}{6\, e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d}{5\, e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^6*arcsin(e*x/d)/e^2 + 1/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e + 1/24*(-e^2
*x^2 + d^2)^(3/2)*d^2*x/e - 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e - 1/5*(-e^2*x^2
+ d^2)^(5/2)*d/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

sympy [A] time = 12.06, size = 580, normalized size = 5.00

$$d^3\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\: e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\: \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\: e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\: \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
 + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**
```

```
2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - e**3*Piecewise((-I*d**6*acosh
(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**
2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 -
e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.6 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}+\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

Rubi [A] time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {780, 195, 217, 203}

$$\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx = -\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}dx}{6e}$$
$$=\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\int\frac{1}{\sqrt{d^2-e^2}}}{16e}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\,\mathrm{Subst}\Big(}{}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\Big(}{16e}$$

Mathematica [A] time = 0.04, size = 124, normalized size = 1.07

$$\frac{\sqrt{d^2-e^2x^2}\left(15d^5\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(48d^5+15d^4ex-96d^3e^2x^2-70d^2e^3x^3+48de^4x^4+40e^5x^5\right)\right)}{240e^2\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(48*d^5 + 15*d^4*e*x - 96*d
^3*e^2*x^2 - 70*d^2*e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5)) + 15*d^5*ArcSin[(
e*x)/d]))/(240*e^2*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.00, size = 125, normalized size = 1.08

$$\frac{d^6\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{16e^3}+\frac{\sqrt{d^2-e^2x^2}\left(-48d^5-15d^4ex+96d^3e^2x^2+70d^2e^3x^3-48de^4x^4-40e^5x^5\right)}{240e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 - 15*d^4*e*x + 96*d^3*e^2*x^2 + 70*d^2*e^3*x^
3 - 48*d*e^4*x^4 - 40*e^5*x^5))/(240*e^2) + (d^6*Sqrt[-e^2]*Log[-(Sqrt[-e^2
]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

fricas [A] time = 0.41, size = 105, normalized size = 0.91

$$-\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+48\,de^4x^4-70\,d^2e^3x^3-96\,d^3e^2x^2+15\,d^4ex+48\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 48
*d*e^4*x^4 - 70*d^2*e^3*x^3 - 96*d^3*e^2*x^2 + 15*d^4*e*x + 48*d^5)*sqrt(-e
^2*x^2 + d^2))/e^2
```

giac [A] time = 0.34, size = 84, normalized size = 0.72

$$\frac{1}{16}d^6\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)-\frac{1}{240}\left(48\,d^5e^{(-2)}+\left(15\,d^4e^{(-1)}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,xe^3+6\,de^2\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^6*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/240*(48*d^5*e^(-2) + (15*d^4*e^(-1
) - 2*(48*d^3 + (35*d^2*e - 4*(5*x*e^3 + 6*d*e^2)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

maple [A] time = 0.00, size = 123, normalized size = 1.06

$$\frac{d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e}+\frac{\sqrt{-e^2x^2+d^2}\,d^4x}{16e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/16/(e^2)^(1/2)*d^6/e*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/16*(-e^
2*x^2+d^2)^(1/2)*d^4/e*x+1/24*(-e^2*x^2+d^2)^(3/2)*d^2/e*x-1/6*(-e^2*x^2+d^
2)^(5/2)/e*x-1/5*(-e^2*x^2+d^2)^(5/2)*d/e^2
```

maxima [A] time = 0.98, size = 102, normalized size = 0.88

$$\frac{d^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^2}+\frac{\sqrt{-e^2x^2+d^2}\,d^4x}{16\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^6*arcsin(e*x/d)/e^2 + 1/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e + 1/24*(-e^2
*x^2 + d^2)^(3/2)*d^2*x/e - 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e - 1/5*(-e^2*x^2
+ d^2)^(5/2)*d/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(d^2-e^2x^2\right)^{3/2}(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

sympy [A] time = 12.18, size = 580, normalized size = 5.00

$$d^3\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
 + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**
```

```
2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - e**3*Piecewise((-I*d**6*acosh
(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**
2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 -
e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.7 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=113

$$\frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

```
[Out] (d^2*(8*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d + 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 + (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
```

```
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-4d^3e^2-3d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{4e^2}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{8d^5e^4+3d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{8e^4}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + d^5\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,d$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^5\,\mathrm{Subst}\left(\int \frac{}{x\sqrt{d^2}}\right.$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2}}\right.$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2}}\right.$$

Mathematica [A] time = 0.18, size = 124, normalized size = 1.10

$$d^4\left(-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{3d^3\sqrt{d^2-e^2x^2}\,\sin^{-1}\left(\frac{ex}{d}\right)}{8\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3+15d^2ex-8de^2x^2-6e^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 + 15*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))/24 +
(3*d^3*Sqrt[d^2 - e^2*x^2]*ArcSin[(e*x)/d])/(8*Sqrt[1 - (e^2*x^2)/d^2]) - d
^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

IntegrateAlgebraic [A] time = 0.51, size = 142, normalized size = 1.26

$$\frac{3d^4\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e}+2d^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3+15d^2ex-8de^2x^2-6e^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 + 15*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))/24 +
2*d^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + (3*d^4*Sqrt[-e^2]
*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*e)
```

fricas [A] time = 0.41, size = 107, normalized size = 0.95

$$-\frac{3}{4}\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\frac{1}{24}\left(6\,e^3x^3+8\,de^2x^2-15\,d^2ex-32\,d^3\right)\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -3/4*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^4*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) - 1/24*(6*e^3*x^3 + 8*d*e^2*x^2 - 15*d^2*e*x - 32*d^3)*sqr
t(-e^2*x^2 + d^2)
```

giac [A] time = 0.27, size = 99, normalized size = 0.88

$$\frac{3}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)\mathrm{sgn}(d)-d^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{1}{24}\left(32\,d^3+\left(15\,d^2e-2\left(3\,xe^3+4\,de^2\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 3/8*d^4*arcsin(x*e/d)*sgn(d) - d^4*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d
^2)*e)*e^(-2)/abs(x)) + 1/24*(32*d^3 + (15*d^2*e - 2*(3*x*e^3 + 4*d*e^2)*x)
*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.02, size = 151, normalized size = 1.34

$$-\frac{d^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{3d^4e\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{3\sqrt{-e^2x^2+d^2}\,d^2ex}{8}+\sqrt{-e^2x^2+d^2}\,d^3+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}ex}{4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x)
```

```
[Out] 1/4*e*x*(-e^2*x^2+d^2)^(3/2)+3/8*e*d^2*x*(-e^2*x^2+d^2)^(1/2)+3/8*e*d^4/(e^
2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/3*d*(-e^2*x^2+d^2)^(3
/2)+d^3*(-e^2*x^2+d^2)^(1/2)-d^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 124, normalized size = 1.10

$$\frac{3}{8}\,d^4\arcsin\left(\frac{ex}{d}\right)-d^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{3}{8}\,\sqrt{-e^2x^2+d^2}\,d^2ex+\sqrt{-e^2x^2+d^2}\,d^3+\frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}ex+\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 3/8*d^4*arcsin(e*x/d) - d^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs
(x)) + 3/8*sqrt(-e^2*x^2 + d^2)*d^2*e*x + sqrt(-e^2*x^2 + d^2)*d^3 + 1/4*(-
e^2*x^2 + d^2)^(3/2)*e*x + 1/3*(-e^2*x^2 + d^2)^(3/2)*d
```

mupad [B] time = 2.90, size = 107, normalized size = 0.95

$$\frac{d\left(d^2-e^2\,x^2\right)^{3/2}}{3}-d^4\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)+d^3\,\sqrt{d^2-e^2\,x^2}+\frac{e\,x\left(d^2-e^2\,x^2\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ \frac{e^2\,x^2}{d^2}\right)}{\left(1-\frac{e^2\,x^2}{d^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x,x)
```

```
[Out] (d*(d^2 - e^2*x^2)^(3/2))/3 - d^4*atanh((d^2 - e^2*x^2)^(1/2)/d) + d^3*(d^2
 - e^2*x^2)^(1/2) + (e*x*(d^2 - e^2*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2,
(e^2*x^2)/d^2))/(1 - (e^2*x^2)/d^2)^(3/2)
```

sympy [C] time = 22.41, size = 469, normalized size = 4.15

$$d^3\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for } e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x,x)
```

```
[Out] d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) + d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*
sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), A
bs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d
**2)/2, True)) - d*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2
 - e**2*x**2)**(3/2)/(3*e**2), True)) - e**3*Piecewise((-I*d**4*acosh(e*x/d
)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sq
rt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs
(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 -
 e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*
sqrt(1 - e**2*x**2/d**2)), True))
```

3.8 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=117

$$\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {813, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

```
[Out] (d*e*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d - e*x)*(d^2 - e^2*x^2)^(3
/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - d^3*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
```

```
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx &= -\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{1}{2}\int \frac{\left(-2d^2e+6de^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx\\
&= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}+\frac{\int \frac{4d^4e^3-6d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{4e^2}\\
&= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}+\left(d^4e\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}+\frac{1}{2}\left(d^4e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2}}\right.\\
&= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right.\\
&= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.17, size = 124, normalized size = 1.06

$$
-\frac{d^5\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{d^2-e^2x^2}}-\frac{1}{3}e\left(\sqrt{d^2-e^2x^2}\left(e^2x^2-4d^2\right)+3d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

```
[Out] -1/3*(e*(Sqrt[d^2 - e^2*x^2]*(-4*d^2 + e^2*x^2) + 3*d^3*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])) - (d^5*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[-3/2, -1/2,
1/2, (e^2*x^2)/d^2])/(x*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.48, size = 143, normalized size = 1.22

$$-\frac{3}{2}d^3\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+2d^3e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{\sqrt{d^2-e^2x^2}\left(-6d^3+8d^2ex-3de^2x^2-2e^3x^3\right)}{6x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 3*d*e^2*x^2 - 2*e^3*x^3))/(6*x)
+ 2*d^3*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (3*d^3*Sqrt[-
e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

fricas [A] time = 0.41, size = 124, normalized size = 1.06

$$\frac{18\,d^3ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+6\,d^3ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+8\,d^3ex-\left(2\,e^3x^3+3\,de^2x^2-8\,d^2ex+6\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(18*d^3*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 6*d^3*e*x*log(-
(d - sqrt(-e^2*x^2 + d^2))/x) + 8*d^3*e*x - (2*e^3*x^3 + 3*d*e^2*x^2 - 8*d^
2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/x
```

giac [A] time = 0.25, size = 157, normalized size = 1.34

$$-\frac{3}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e\mathrm{sgn}(d)-d^3e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{d^3xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3e^{(-1)}}{2\,x}+\frac{1}{6}\,\sqrt{-x^2e^2+d^2}\left(8\,d^2e-\left(2\,xe^3+3\,de^2\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -3/2*d^3*arcsin(x*e/d)*e*sgn(d) - d^3*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^
2 + d^2)*e)*e^(-2)/abs(x)) + 1/2*d^3*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e) -
 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*e^(-1)/x + 1/6*sqrt(-x^2*e^2 + d^2)
*(8*d^2*e - (2*x*e^3 + 3*d*e^2)*x)
```

maple [A] time = 0.03, size = 182, normalized size = 1.56

$$-\frac{d^4e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{3d^3e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{3\sqrt{-e^2x^2+d^2}\,d\,e^2x}{2}+\sqrt{-e^2x^2+d^2}\,d^2e-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2x}{d}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x)
```

```
[Out] -1/d/x*(-e^2*x^2+d^2)^(5/2)-e^2/d*x*(-e^2*x^2+d^2)^(3/2)-3/2*d*e^2*x*(-e^2*
x^2+d^2)^(1/2)-3/2*e^2*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1
/2)*x)+1/3*e*(-e^2*x^2+d^2)^(3/2)+e*d^2*(-e^2*x^2+d^2)^(1/2)-e*d^4/(d^2)^(1
/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 129, normalized size = 1.10

$$-\frac{3}{2}\,d^3e\arcsin\left(\frac{ex}{d}\right)-d^3e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{3}{2}\,\sqrt{-e^2x^2+d^2}\,de^2x+\sqrt{-e^2x^2+d^2}\,d^2e+\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*d^3*e*arcsin(e*x/d) - d^3*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*
d/abs(x)) - 3/2*sqrt(-e^2*x^2 + d^2)*d*e^2*x + sqrt(-e^2*x^2 + d^2)*d^2*e +
 1/3*(-e^2*x^2 + d^2)^(3/2)*e - (-e^2*x^2 + d^2)^(3/2)*d/x
```

mupad [B] time = 3.51, size = 114, normalized size = 0.97

$$\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3}+d^2e\sqrt{d^2-e^2x^2}-d^3e\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{d^3\sqrt{d^2-e^2x^2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ \frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^2,x)
```

```
[Out] (e*(d^2 - e^2*x^2)^(3/2))/3 + d^2*e*(d^2 - e^2*x^2)^(1/2) - d^3*e*atanh((d^
2 - e^2*x^2)^(1/2)/d) - (d^3*(d^2 - e^2*x^2)^(1/2)*hypergeom([-3/2, -1/2],
1/2, (e^2*x^2)/d^2))/(x*(1 - (e^2*x^2)/d^2)^(1/2))
```

sympy [C] time = 8.18, size = 386, normalized size = 3.30

$$d^3\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\,\mathrm{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\,\mathrm{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\,\mathrm{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\,\mathrm{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{id^2\,\mathrm{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\,\mathrm{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**2,x)
```

```
[Out] d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) + d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(
d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*
d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
*2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e)
- I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 -
e**2*x**2/d**2)/2, True)) - e**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0))
, (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

3.9 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3} dx$

Optimal. Leaf size=121

$$-\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {813, 844, 217, 203, 266, 63, 208}

$$-\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d - e*x)*(d^2 - e^2*x^2)^(
3/2))/(2*x^2) - (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^
2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
```

```
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = -\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{8}\int \frac{\left(-4d^2e+4de^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{16}\int \frac{-8d^3e^2-8d^2e^3x}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{1}{2}\left(3d^3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{1}{4}\left(3d^3e^2\right)\text{Subst}\left(\int \frac{}{x\sqrt{d^2}}\right.$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)$$

Mathematica [C] time = 0.08, size = 110, normalized size = 0.91

$$-\frac{d^2e\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] -((d^2*e*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, (e^2*x^2)/d
^2])/(x*Sqrt[1 - (e^2*x^2)/d^2])) - (e^2*(d^2 - e^2*x^2)^(5/2)*Hypergeometr
ic2F1[2, 5/2, 7/2, 1 - (e^2*x^2)/d^2])/(5*d^3)
```

IntegrateAlgebraic [A] time = 0.69, size = 146, normalized size = 1.21

$$-\frac{3}{2}d^2e\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)-3d^2e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{\sqrt{d^2-e^2x^2}\left(-d^3-2d^2ex-2de^2x^2-e^3x^3\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^3 - 2*d^2*e*x - 2*d*e^2*x^2 - e^3*x^3))/(2*x^2) -
3*d^2*e^2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (3*d^2*e*Sqrt
[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

fricas [A] time = 0.41, size = 133, normalized size = 1.10

$$\frac{6\,d^2e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-3\,d^2e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-2\,d^2e^2x^2-\left(e^3x^3+2\,de^2x^2+2\,d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^2*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 3*d^2*e^2*x^
2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 2*d^2*e^2*x^2 - (e^3*x^3 + 2*d*e^2*x
^2 + 2*d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2))/x^2
```

giac [B] time = 0.28, size = 217, normalized size = 1.79

$$-\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^2\mathrm{sgn}(d)+\frac{3}{2}\,d^2e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)-\frac{1}{8}\left(\frac{4\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^8}{x}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^6}{x^2}\right)e^{(-8)}-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\left(xe^3+2\,de^2\right)+\frac{\left(d^2e^6+\frac{4\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^4}{x}\right)x^2}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -3/2*d^2*arcsin(x*e/d)*e^2*sgn(d) + 3/2*d^2*e^2*log(1/2*abs(-2*d*e - 2*sqrt
(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 1/8*(4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
d^2*e^8/x + (d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^6/x^2)*e^(-8) - 1/2*sqrt
(-x^2*e^2 + d^2)*(x*e^3 + 2*d*e^2) + 1/8*(d^2*e^6 + 4*(d*e + sqrt(-x^2*e^2
+ d^2)*e)*d^2*e^4/x)*x^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^2
```

maple [B] time = 0.02, size = 212, normalized size = 1.75

$$\frac{3d^3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{3d^2e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{3\sqrt{-e^2x^2+d^2}\,e^3x}{2}-\frac{3\sqrt{-e^2x^2+d^2}\,d\,e^2}{2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3x}{d^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{2d}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{d^2x}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{2d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x)
```

```
[Out] -e/d^2/x*(-e^2*x^2+d^2)^(5/2)-e^3/d^2*x*(-e^2*x^2+d^2)^(3/2)-3/2*e^3*x*(-e^
2*x^2+d^2)^(1/2)-3/2*e^3*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^
(1/2)*x)-1/2/d/x^2*(-e^2*x^2+d^2)^(5/2)-1/2*e^2/d*(-e^2*x^2+d^2)^(3/2)-3/2*
d*e^2*(-e^2*x^2+d^2)^(1/2)+3/2*e^2*d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 160, normalized size = 1.32

$$-\frac{3}{2}\,d^2e^2\arcsin\left(\frac{ex}{d}\right)+\frac{3}{2}\,d^2e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{3}{2}\,\sqrt{-e^2x^2+d^2}\,e^3x-\frac{3}{2}\,\sqrt{-e^2x^2+d^2}\,de^2-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{2\,d}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{x}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*d^2*e^2*arcsin(e*x/d) + 3/2*d^2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2
 + d^2)*d/abs(x)) - 3/2*sqrt(-e^2*x^2 + d^2)*e^3*x - 3/2*sqrt(-e^2*x^2 + d^
2)*d*e^2 - 1/2*(-e^2*x^2 + d^2)^(3/2)*e^2/d - (-e^2*x^2 + d^2)^(3/2)*e/x -
1/2*(-e^2*x^2 + d^2)^(5/2)/(d*x^2)
```

mupad [B] time = 3.74, size = 120, normalized size = 0.99

$$\frac{3\,d^2\,e^2\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{2}-\frac{d^3\,\sqrt{d^2-e^2\,x^2}}{2\,x^2}-d\,e^2\,\sqrt{d^2-e^2\,x^2}-\frac{e\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ \frac{e^2\,x^2}{d^2}\right)}{x\,{\left(1-\frac{e^2\,x^2}{d^2}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^3,x)
```

```
[Out] (3*d^2*e^2*atanh((d^2 - e^2*x^2)^(1/2)/d))/2 - (d^3*(d^2 - e^2*x^2)^(1/2))/
(2*x^2) - d*e^2*(d^2 - e^2*x^2)^(1/2) - (e*(d^2 - e^2*x^2)^(3/2)*hypergeom(
[-3/2, -1/2], 1/2, (e^2*x^2)/d^2))/(x*(1 - (e^2*x^2)/d^2)^(3/2))
```

sympy [C] time = 9.53, size = 461, normalized size = 3.81

$$d^3\left(\begin{cases}-\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**3,x)
```

```
[Out] d**3*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), T
rue)) + d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x
/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d
/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1))
- d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2))
> 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*
x/sqrt(-d**2/(e**2*x**2) + 1), True)) - e**3*Piecewise((-I*d**2*acosh(e*x/d
)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 +
e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*s
qrt(1 - e**2*x**2/d**2)/2, True))
```

3.10 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=120

$$\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {811, 813, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] (e^2*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d + 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
```

```
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx &= -\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}-\frac{\int \frac{\left(4d^3e^2+6d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{4d^2}\\
&= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+\frac{\int \frac{-12d^4e^3+8d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}\\
&= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}-\frac{1}{2}\left(3d^2e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x}}\\
&= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}-\frac{1}{4}\left(3d^2e^3\right)\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.\\
&= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\\
&= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 111, normalized size = 0.92

$$
-\frac{e^3\left(d^2-e^2x^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)}{5d^4}-\frac{d^3\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3x^3\sqrt{1-\frac{e^2x^2}{d^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] -1/3*(d^3*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (e^2*x^2)
/d^2])/(x^3*Sqrt[1 - (e^2*x^2)/d^2]) - (e^3*(d^2 - e^2*x^2)^(5/2)*Hypergeom
etric2F1[2, 5/2, 7/2, 1 - (e^2*x^2)/d^2])/(5*d^4)
```

IntegrateAlgebraic [A] time = 0.59, size = 141, normalized size = 1.18

$$d\sqrt{-e^2}\,e^2\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)-3de^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{\sqrt{d^2-e^2x^2}\left(-2d^3-3d^2ex+8de^2x^2-6e^3x^3\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 - 3*d^2*e*x + 8*d*e^2*x^2 - 6*e^3*x^3))/(6*x^3
) - 3*d*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + d*e^2*Sqrt[
-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.42, size = 129, normalized size = 1.08

$$-\frac{12\,de^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+9\,de^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+6\,de^3x^3+\left(6\,e^3x^3-8\,de^2x^2+3\,d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(12*d*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 9*d*e^3*x^3*
log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 6*d*e^3*x^3 + (6*e^3*x^3 - 8*d*e^2*x^2
 + 3*d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/x^3
```

giac [B] time = 0.23, size = 261, normalized size = 2.18

$$d\arcsin\left(\frac{xe}{d}\right)e^3\mathrm{sgn}(d)+\frac{3}{2}\,de^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(de^8+\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de^6}{x}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^4}{x^2}\right)x^3e}{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3}+\frac{1}{24}\left(\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de^{16}}{x}-\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^{14}}{x^2}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3de^{12}}{x^3}\right)e^{(-15)}-\sqrt{-x^2e^2+d^2}\,e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] d*arcsin(x*e/d)*e^3*sgn(d) + 3/2*d*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/abs(x)) + 1/24*(d*e^8 + 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
d*e^6/x - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^4/x^2)*x^3*e/(d*e + sqrt(
-x^2*e^2 + d^2)*e)^3 + 1/24*(15*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^16/x - 3
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^14/x^2 - (d*e + sqrt(-x^2*e^2 + d^2)*
e)^3*d*e^12/x^3)*e^(-15) - sqrt(-x^2*e^2 + d^2)*e^3
```

maple [B] time = 0.02, size = 235, normalized size = 1.96

$$\frac{3d^2e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}+\frac{d\,e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{\sqrt{-e^2x^2+d^2}\,e^4x}{d}-\frac{3\sqrt{-e^2x^2+d^2}\,e^3}{2}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4x}{3d^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{2d^2}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{3d^3x}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{2d^2x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{3d\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x)
```

```
[Out] -1/3/d/x^3*(-e^2*x^2+d^2)^(5/2)+2/3*e^2/d^3/x*(-e^2*x^2+d^2)^(5/2)+2/3*e^4/
d^3*x*(-e^2*x^2+d^2)^(3/2)+e^4/d*x*(-e^2*x^2+d^2)^(1/2)+d*e^4/(e^2)^(1/2)*a
rctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2*e/d^2/x^2*(-e^2*x^2+d^2)^(5/2
)-1/2*e^3/d^2*(-e^2*x^2+d^2)^(3/2)-3/2*e^3*(-e^2*x^2+d^2)^(1/2)+3/2*e^3*d^2
/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.98, size = 184, normalized size = 1.53

$$de^3\arcsin\left(\frac{ex}{d}\right)+\frac{3}{2}\,de^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{\sqrt{-e^2x^2+d^2}\,e^4x}{d}-\frac{3}{2}\sqrt{-e^2x^2+d^2}\,e^3-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{2\,d^2}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{3\,dx}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{2\,d^2x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{3\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] d*e^3*arcsin(e*x/d) + 3/2*d*e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d
/abs(x)) + sqrt(-e^2*x^2 + d^2)*e^4*x/d - 3/2*sqrt(-e^2*x^2 + d^2)*e^3 - 1/
2*(-e^2*x^2 + d^2)^(3/2)*e^3/d^2 + 2/3*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x) - 1
/2*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^2) - 1/3*(-e^2*x^2 + d^2)^(5/2)/(d*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^4, x)
```

sympy [C] time = 8.93, size = 457, normalized size = 3.81

$$d^{3}\left(\begin{cases} - \frac{e \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 x^{2}} + \frac{e^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 d^{2}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 x^{2}} + \frac{i e^{3} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 d^{2}} & \text{otherwise} \end{cases}\right) + d^{2} e \left(\begin{cases} - \frac{d^{2}}{2 e x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e}{2 x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{2} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{2 d} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{2 x} - \frac{i e^{2} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{2 d} & \text{otherwise} \end{cases}\right) - d e^{2} \left(\begin{cases} \frac{i d}{x \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + i e \operatorname{acosh}{\left(\frac{e x}{d} \right)} - \frac{i e^{2} x}{d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\- \frac{d}{x \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - e \operatorname{asin}{\left(\frac{e x}{d} \right)} + \frac{e^{2} x}{d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right) - e^{3} \left(\begin{cases} \frac{d^{2}}{e x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} - d \operatorname{acosh}{\left(\frac{d}{e x} \right)} - \frac{e x}{\sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i d^{2}}{e x \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} + i d \operatorname{asin}{\left(\frac{d}{e x} \right)} + \frac{i e x}{\sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**4,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) +
 d**2*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d)
, True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(
e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
(-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*
x**2/d**2)), True)) - e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1))
 - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2))
 > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e
*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

3.11 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^5} dx$

Optimal. Leaf size=118

$$\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {811, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] (e^2*(3*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d + 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) + e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
```

```
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx &= -\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - \frac{\int \frac{\left(6d^3e^2+8d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2}\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{\int \frac{12d^5e^4+32d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{32d^4}\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{8}\left(3de^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{16}\left(3de^4\right)\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) -\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) -
\end{aligned}$$

Mathematica [C] time = 0.09, size = 133, normalized size = 1.13

$$-\frac{\sqrt{d^2-e^2x^2}\left(3d^2\left(2d^2-5e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}+9e^4x^4\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)+8d^3ex\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)\right)}{24dx^4\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] -1/24*(Sqrt[d^2 - e^2*x^2]*(3*d^2*(2*d^2 - 5*e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^
2] + 9*e^4*x^4*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]] + 8*d^3*e*x*Hypergeometric2
F1[-3/2, -3/2, -1/2, (e^2*x^2)/d^2]))/(d*x^4*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.59, size = 141, normalized size = 1.19

$$\frac{3}{4}e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\sqrt{-e^2}\,e^3\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+\frac{\sqrt{d^2-e^2x^2}\left(-6d^3-8d^2ex+15de^2x^2+32e^3x^3\right)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 8*d^2*e*x + 15*d*e^2*x^2 + 32*e^3*x^3))/(24*
x^4) + (3*e^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/4 + e^3*Sq
rt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.41, size = 119, normalized size = 1.01

$$-\frac{48\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(32\,e^3x^3+15\,de^2x^2-8\,d^2ex-6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(48*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 9*e^4*x^4*log
(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 + 15*d*e^2*x^2 - 8*d^2*e*x -
6*d^3)*sqrt(-e^2*x^2 + d^2))/x^4
```

giac [B] time = 0.23, size = 297, normalized size = 2.52

$$\arcsin\left(\frac{xe}{d}\right)e^4\mathrm{sgn}(d)+\frac{x^4\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^8}{x}-\frac{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^6}{x^2}-\frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^4}{x^3}+3\,e^{10}\right)e^2}{192\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4}+\frac{1}{192}\left(\frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{26}}{x}+\frac{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{24}}{x^2}-\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{22}}{x^3}-\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{20}}{x^4}\right)e^{(-24)}-\frac{3}{8}\,e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e^4*sgn(d) + 1/192*x^4*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^8/
x - 24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^6/x^2 - 120*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^3*e^4/x^3 + 3*e^10)*e^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^4 + 1/192*
(120*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^26/x + 24*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^2*e^24/x^2 - 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^22/x^3 - 3*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^4*e^20/x^4)*e^(-24) - 3/8*e^4*log(1/2*abs(-2*d*e - 2*s
qrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))
```

maple [B] time = 0.02, size = 260, normalized size = 2.20

$$-\frac{3d\,e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}+\frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{\sqrt{-e^2x^2+d^2}\,e^5x}{d^2}+\frac{3\sqrt{-e^2x^2+d^2}\,e^4}{8d}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5x}{3d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}{8d^3}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{3d^4x}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{8d^3x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{3d^2x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{4d\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x)
```

```
[Out] -1/4/d/x^4*(-e^2*x^2+d^2)^(5/2)+1/8*e^2/d^3/x^2*(-e^2*x^2+d^2)^(5/2)+1/8*e^
4/d^3*(-e^2*x^2+d^2)^(3/2)+3/8*e^4/d*(-e^2*x^2+d^2)^(1/2)-3/8*d*e^4/(d^2)^(
1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3*e/d^2/x^3*(-e^2*x
^2+d^2)^(5/2)+2/3*e^3/d^4/x*(-e^2*x^2+d^2)^(5/2)+2/3*e^5/d^4*x*(-e^2*x^2+d^
2)^(3/2)+e^5/d^2*x*(-e^2*x^2+d^2)^(1/2)+e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/
(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [B] time = 0.98, size = 210, normalized size = 1.78

$$e^4\arcsin\left(\frac{ex}{d}\right)-\frac{3}{8}\,e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{\sqrt{-e^2x^2+d^2}\,e^5x}{d^2}+\frac{3\,\sqrt{-e^2x^2+d^2}\,e^4}{8\,d}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}{8\,d^3}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{3\,d^2x}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{8\,d^3x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{3\,d^2x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{4\,dx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] e^4*arcsin(e*x/d) - 3/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs
(x)) + sqrt(-e^2*x^2 + d^2)*e^5*x/d^2 + 3/8*sqrt(-e^2*x^2 + d^2)*e^4/d + 1/
8*(-e^2*x^2 + d^2)^(3/2)*e^4/d^3 + 2/3*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x) +
 1/8*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^2) - 1/3*(-e^2*x^2 + d^2)^(5/2)*e/(d
^2*x^3) - 1/4*(-e^2*x^2 + d^2)^(5/2)/(d*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2x^2\right)^{3/2}(d+ex)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^5, x)
```

sympy [C] time = 11.06, size = 541, normalized size = 4.58

$$d^3\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases} + d^2e\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases} - de^2\begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases} - e^3\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**5,x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e*
*3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*s
qrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3
*d**2), True)) - d*e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) -
1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d
**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asi
n(d/(e*x))/(2*d), True)) - e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)
) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**
2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*
sqrt(1 - e**2*x**2/d**2)), True))
```

3.12 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}+\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}$$

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {807, 266, 47, 63, 208}

$$\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

```
[Out] (3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) - (
d^2 - e^2*x^2)^(5/2)/(5*d*x^5) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*
d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{1}{2}e\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right)\\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{1}{16}\left(3e^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2\ldots}}\right.\\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right.\\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 133, normalized size = 1.23

$$-\frac{8d^6+10d^5ex-24d^4e^2x^2-35d^3e^3x^3+24d^2e^4x^4+15de^5x^5\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)+25de^5x^5-8e^6x^6}{40dx^5\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

```
[Out] -1/40*(8*d^6 + 10*d^5*e*x - 24*d^4*e^2*x^2 - 35*d^3*e^3*x^3 + 24*d^2*e^4*x^
4 + 25*d*e^5*x^5 - 8*e^6*x^6 + 15*d*e^5*x^5*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh
[Sqrt[1 - (e^2*x^2)/d^2]])/(d*x^5*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.62, size = 155, normalized size = 1.44

$$-\frac{3e^5\log\left(\sqrt{d^2-e^2x^2}+d-\sqrt{-e^2}\,x\right)}{8d}+\frac{3e^5\log\left(-d\sqrt{d^2-e^2x^2}+d^2+d\sqrt{-e^2}\,x\right)}{8d}+\frac{\sqrt{d^2-e^2x^2}\left(-8d^4-10d^3ex+16d^2e^2x^2+25de^3x^3-8e^4x^4\right)}{40dx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 - 10*d^3*e*x + 16*d^2*e^2*x^2 + 25*d*e^3*x^3 -
 8*e^4*x^4))/(40*d*x^5) - (3*e^5*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]
])/(8*d) + (3*e^5*Log[d^2 + d*Sqrt[-e^2]*x - d*Sqrt[d^2 - e^2*x^2]])/(8*d)
```

fricas [A] time = 0.39, size = 98, normalized size = 0.91

$$\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(8\,e^4x^4-25\,de^3x^3-16\,d^2e^2x^2+10\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{40\,dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (8*e^4*x^4 - 25*d*e^3
*x^3 - 16*d^2*e^2*x^2 + 10*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*x^5)
```

giac [B] time = 0.25, size = 368, normalized size = 3.41

$$\frac{x^5\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{10}}{x}-\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^8}{x^2}-\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^6}{x^3}+\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^4}{x^4}+2\,e^{12}\right)e^3}{320\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d}-\frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{8\,d}-\frac{\left(\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^4e^{38}}{x}-\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^4e^{36}}{x^2}-\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^4e^{34}}{x^3}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^4e^{32}}{x^4}+\frac{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^4e^{30}}{x^5}\right)e^{(-35)}}{320\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/320*x^5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^10/x - 10*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*e^8/x^2 - 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^6/x^3 + 20*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^4/x^4 + 2*e^12)*e^3/((d*e + sqrt(-x^2*e^2
+ d^2)*e)^5*d) - 3/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(
-2)/abs(x))/d - 1/320*(20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^38/x - 40*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^36/x^2 - 10*(d*e + sqrt(-x^2*e^2 + d^2
)*e)^3*d^4*e^34/x^3 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4*e^32/x^4 + 2*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^4*e^30/x^5)*e^(-35)/d^5
```

maple [A] time = 0.03, size = 158, normalized size = 1.46

$$-\frac{3e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}+\frac{3\sqrt{-e^2x^2+d^2}\,e^5}{8d^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}{8d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{8d^4x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{4d^2x^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5d\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x)
```

```
[Out] -1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5-1/4*e/d^2/x^4*(-e^2*x^2+d^2)^(5/2)+1/8*e^3/
d^4/x^2*(-e^2*x^2+d^2)^(5/2)+1/8*e^5/d^4*(-e^2*x^2+d^2)^(3/2)+3/8*e^5/d^2*(
-e^2*x^2+d^2)^(1/2)-3/8*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d
^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 155, normalized size = 1.44

$$-\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8\,d}+\frac{3\,\sqrt{-e^2x^2+d^2}\,e^5}{8\,d^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}{8\,d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{8\,d^4x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{4\,d^2x^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5\,dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] -3/8*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d + 3/8*sqrt(-
e^2*x^2 + d^2)*e^5/d^2 + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^5/d^4 + 1/8*(-e^2*x^2
 + d^2)^(5/2)*e^3/(d^4*x^2) - 1/4*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^4) - 1/5*
(-e^2*x^2 + d^2)^(5/2)/(d*x^5)
```

mupad [B] time = 4.26, size = 93, normalized size = 0.86

$$\frac{3\,d^2\,e\,\sqrt{d^2-e^2\,x^2}}{8\,x^4}-\frac{\left(d^2-e^2\,x^2\right)^{5/2}}{5\,d\,x^5}-\frac{3\,e^5\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{8\,d}-\frac{5\,e\,\left(d^2-e^2\,x^2\right)^{3/2}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^6,x)
```

```
[Out] (3*d^2*e*(d^2 - e^2*x^2)^(1/2))/(8*x^4) - (d^2 - e^2*x^2)^(5/2)/(5*d*x^5) -
 (3*e^5*atanh((d^2 - e^2*x^2)^(1/2)/d))/(8*d) - (5*e*(d^2 - e^2*x^2)^(3/2))
/(8*x^4)
```

sympy [C] time = 11.36, size = 774, normalized size = 7.17

$$d^{3}\left(\begin{cases} \frac{3id^{3}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}} - \frac{4ide^{2}x^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}} + \frac{2ie^{6}x^{6}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{5}x^{5}+15d^{3}e^{2}x^{7}} - \frac{ie^{4}x^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{3}x^{5}+15de^{2}x^{7}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1 \\ \frac{3d^{3}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}} - \frac{4de^{2}x^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}} + \frac{2e^{6}x^{6}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{5}x^{5}+15d^{3}e^{2}x^{7}} - \frac{e^{4}x^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{3}x^{5}+15de^{2}x^{7}} & \text{otherwise} \end{cases}\right) + d^{2}e\left(\begin{cases} -\frac{d^{2}}{4ex^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} + \frac{3e}{8x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} - \frac{e^{3}}{8d^{2}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} + \frac{e^{4}\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ \frac{id^{2}}{4ex^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} - \frac{3ie}{8x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} + \frac{ie^{3}}{8d^{2}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} - \frac{ie^{4}\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{otherwise} \end{cases}\right) - de^{2}\left(\begin{cases} -\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3x^{2}} + \frac{e^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3d^{2}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3x^{2}} + \frac{ie^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3d^{2}} & \text{otherwise} \end{cases}\right) - e^{3}\left(\begin{cases} -\frac{d^{2}}{2ex^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} + \frac{e}{2x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} + \frac{e^{2}\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{2x} - \frac{ie^{2}\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**6,x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) + d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2
*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sq
rt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e
**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**
2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sq
rt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - e**3*Piecewise((-d**2/(2*e*x**
3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*a
cosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**
2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True))
```

3.13 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=143

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}+\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}$$

Rubi [A] time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {835, 807, 266, 47, 63, 208}

$$\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx = -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{\int \frac{\left(-6d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{6d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} + \frac{e^2\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} + \frac{e^2\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx, x, x^2\right)}{12d}$$

$$= -\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^4\,\mathrm{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,\right.}{16d}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} + \frac{e^6\,\mathrm{Subs}}{}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^4\,\mathrm{Subs}}{}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^6\tanh^{}}{}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.41

$$-\frac{e\left(d^2-e^2x^2\right)^{5/2}\left(d^5+e^5x^5\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)\right)}{5d^7x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

```
[Out] -1/5*(e*(d^2 - e^2*x^2)^(5/2)*(d^5 + e^5*x^5*Hypergeometric2F1[5/2, 4, 7/2,
 1 - (e^2*x^2)/d^2]))/(d^7*x^5)
```

IntegrateAlgebraic [A] time = 0.62, size = 126, normalized size = 0.88

$$\frac{e^6\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^2}+\frac{\sqrt{d^2-e^2x^2}\left(-40d^5-48d^4ex+70d^3e^2x^2+96d^2e^3x^3-15de^4x^4-48e^5x^5\right)}{240d^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^5 - 48*d^4*e*x + 70*d^3*e^2*x^2 + 96*d^2*e^3*x^
3 - 15*d*e^4*x^4 - 48*e^5*x^5))/(240*d^2*x^6) + (e^6*ArcTanh[(Sqrt[-e^2]*x)
/d - Sqrt[d^2 - e^2*x^2]/d])/(8*d^2)
```

fricas [A] time = 0.43, size = 109, normalized size = 0.76

$$\frac{15\,e^{6}x^{6}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(48\,e^{5}x^{5}+15\,de^{4}x^{4}-96\,d^{2}e^{3}x^{3}-70\,d^{3}e^{2}x^{2}+48\,d^{4}ex+40\,d^{5}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{240\,d^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/240*(15*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (48*e^5*x^5 + 15*d*e
^4*x^4 - 96*d^2*e^3*x^3 - 70*d^3*e^2*x^2 + 48*d^4*e*x + 40*d^5)*sqrt(-e^2*x
^2 + d^2))/(d^2*x^6)
```

giac [B] time = 0.28, size = 431, normalized size = 3.01

$$\frac{x^{6}\left(\frac{12\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{12}}{x}-\frac{15\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}e^{10}}{x^{2}}-\frac{60\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}e^{8}}{x^{3}}-\frac{15\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}e^{6}}{x^{4}}+\frac{120\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{5}e^{4}}{x^{5}}+5\,e^{14}\right)e^{4}}{1920\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{6}d^{2}}-\frac{e^{6}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)}{16\,d^{2}}-\frac{\left(\frac{120\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)d^{10}e^{52}}{x}-\frac{15\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}d^{10}e^{50}}{x^{2}}-\frac{60\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}d^{10}e^{48}}{x^{3}}-\frac{15\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}d^{10}e^{46}}{x^{4}}+\frac{12\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{5}d^{10}e^{44}}{x^{5}}+\frac{5\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{6}d^{10}e^{42}}{x^{6}}\right)e^{(-48)}}{1920\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/1920*x^6*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^12/x - 15*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^2*e^10/x^2 - 60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^8/x^3 - 15
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^6/x^4 + 120*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^5*e^4/x^5 + 5*e^14)*e^4/((d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^2) - 1/16*e
^6*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 - 1/19
20*(120*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^10*e^52/x - 15*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*d^10*e^50/x^2 - 60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^10*e^48
/x^3 - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^10*e^46/x^4 + 12*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^5*d^10*e^44/x^5 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^10
*e^42/x^6)*e^(-48)/d^12
```

maple [A] time = 0.03, size = 186, normalized size = 1.30

$$-\frac{e^{6}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{16\sqrt{d^{2}}\,d}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}{16d^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}{48d^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}{48d^{5}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}{24d^{3}x^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e}{5d^{2}x^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6d\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x)
```

```
[Out] -1/5*e*(-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6-1/24*e^2
/d^3/x^4*(-e^2*x^2+d^2)^(5/2)+1/48*e^4/d^5/x^2*(-e^2*x^2+d^2)^(5/2)+1/48*e^
6/d^5*(-e^2*x^2+d^2)^(3/2)+1/16*e^6/d^3*(-e^2*x^2+d^2)^(1/2)-1/16*e^6/d/(d^
2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 180, normalized size = 1.26

$$-\frac{e^{6}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{16\,d^{2}}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}{16\,d^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}{48\,d^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}{48\,d^{5}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}{24\,d^{3}x^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e}{5\,d^{2}x^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6\,dx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/16*sq
rt(-e^2*x^2 + d^2)*e^6/d^3 + 1/48*(-e^2*x^2 + d^2)^(3/2)*e^6/d^5 + 1/48*(-e
```

```
^2*x^2 + d^2)^(5/2)*e^4/(d^5*x^2) - 1/24*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^
4) - 1/5*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^5) - 1/6*(-e^2*x^2 + d^2)^(5/2)/(d
*x^6)
```

mupad [B] time = 4.66, size = 118, normalized size = 0.83

$$\frac{d^3\sqrt{d^2-e^2x^2}}{16x^6}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{6x^6}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{16dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^6\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2x^2}\,1\mathrm{i}}{d}\right)1\mathrm{i}}{16d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^7,x)
```

```
[Out] (d^3*(d^2 - e^2*x^2)^(1/2))/(16*x^6) - (d*(d^2 - e^2*x^2)^(3/2))/(6*x^6) -
(d^2 - e^2*x^2)^(5/2)/(16*d*x^6) + (e^6*atan(((d^2 - e^2*x^2)^(1/2)*1i)/d)*
1i)/(16*d^2) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5)
```

sympy [C] time = 15.25, size = 918, normalized size = 6.42

$$d^3\left(\begin{cases}-\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**7,x)
```

```
[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**2*e*Piecewise((3*I*d**3*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2
) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4
*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e*
*6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4
*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - d
*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5
*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1))
+ I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d
**3), True)) - e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True))
```

3.14 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=172

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}+\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Rubi [A] time = 0.13, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {835, 807, 266, 47, 63, 208}

$$\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (e^3*(d^2 - e^2*x^2)^(3/2))/(24*d^
2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) - (e*(d^2 - e^2*x^2)^(5/2))/(6*d^2
*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) - (e^7*ArcTanh[Sqrt[d^2
- e^2*x^2]/d])/(16*d^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{\int \frac{\left(-7d^2e-2de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} + \frac{\int \frac{\left(12d^3e^2+7d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} + \frac{e^3\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} + \frac{e^3\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\right.}{12d^2}$$

$$= -\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} - \frac{e^5}{}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-\right.}{35}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-\right.}{35}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-\right.}{35}$$

Mathematica [C] time = 0.02, size = 72, normalized size = 0.42

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}\left(5d^7+2d^5e^2x^2+7e^7x^7\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)\right)}{35d^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] -1/35*((d^2 - e^2*x^2)^(5/2)*(5*d^7 + 2*d^5*e^2*x^2 + 7*e^7*x^7*Hypergeomet
ric2F1[5/2, 4, 7/2, 1 - (e^2*x^2)/d^2]))/(d^8*x^7)
```

IntegrateAlgebraic [A] time = 0.68, size = 137, normalized size = 0.80

$$\frac{e^7\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}+\frac{\sqrt{d^2-e^2x^2}\left(-240d^6-280d^5ex+384d^4e^2x^2+490d^3e^3x^3-48d^2e^4x^4-105de^5x^5-96e^6x^6\right)}{1680d^3x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-240*d^6 - 280*d^5*e*x + 384*d^4*e^2*x^2 + 490*d^3*e^
3*x^3 - 48*d^2*e^4*x^4 - 105*d*e^5*x^5 - 96*e^6*x^6))/(1680*d^3*x^7) + (e^7
*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(8*d^3)
```

fricas [A] time = 0.42, size = 120, normalized size = 0.70

$$\frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(96\,e^6x^6+105\,de^5x^5+48\,d^2e^4x^4-490\,d^3e^3x^3-384\,d^4e^2x^2+280\,d^5ex+240\,d^6\right)\sqrt{-e^2x^2+d^2}}{1680\,d^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/1680*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (96*e^6*x^6 + 105*
d*e^5*x^5 + 48*d^2*e^4*x^4 - 490*d^3*e^3*x^3 - 384*d^4*e^2*x^2 + 280*d^5*e*
x + 240*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*x^7)
```

giac [B] time = 0.26, size = 494, normalized size = 2.87

$$\frac{x^7\left(\frac{35\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{14}}{x}-\frac{21\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{12}}{x^2}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{10}}{x^3}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^8}{x^4}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^5e^6}{x^5}+\frac{315\left(de+\sqrt{-x^2e^2+d^2}e\right)^6e^4}{x^6}+15\,e^{16}\right)e^5}{13440\left(de+\sqrt{-x^2e^2+d^2}e\right)^7d^3}-\frac{e^7\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{16\,d^3}-\frac{\left(\frac{315\left(de+\sqrt{-x^2e^2+d^2}e\right)d^{18}e^{68}}{x}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^2d^{18}e^{66}}{x^2}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^3d^{18}e^{64}}{x^3}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^4d^{18}e^{62}}{x^4}-\frac{21\left(de+\sqrt{-x^2e^2+d^2}e\right)^5d^{18}e^{60}}{x^5}+\frac{35\left(de+\sqrt{-x^2e^2+d^2}e\right)^6d^{18}e^{58}}{x^6}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}e\right)^7d^{18}e^{56}}{x^7}\right)e^{(-63)}}{13440\,d^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/13440*x^7*(35*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^14/x - 21*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^2*e^12/x^2 - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^10/x^3 -
 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^8/x^4 - 105*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^5*e^6/x^5 + 315*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^4/x^6 + 15*e^16)
*e^5/((d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^3) - 1/16*e^7*log(1/2*abs(-2*d*e -
 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 1/13440*(315*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*d^18*e^68/x - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^18*e^
66/x^2 - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^18*e^64/x^3 - 105*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^4*d^18*e^62/x^4 - 21*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5
*d^18*e^60/x^5 + 35*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^18*e^58/x^6 + 15*(d*
e + sqrt(-x^2*e^2 + d^2)*e)^7*d^18*e^56/x^7)*e^(-63)/d^21
```

maple [A] time = 0.04, size = 211, normalized size = 1.23

$$-\frac{e^7\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16\sqrt{d^2}\,d^2}+\frac{\sqrt{-e^2x^2+d^2}\,e^7}{16d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7}{48d^6}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}{48d^6x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{24d^4x^4}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{35d^3x^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{6d^2x^6}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{7d\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x)
```

```
[Out] -1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7-2/35*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^5-1/6*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^6-1/24*e^3/d^4/x^4*(-e^2*x^2+d^2)^(5/2)+1/48*e^
5/d^6/x^2*(-e^2*x^2+d^2)^(5/2)+1/48*e^7/d^6*(-e^2*x^2+d^2)^(3/2)+1/16*e^7/d
^4*(-e^2*x^2+d^2)^(1/2)-1/16*e^7/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.99, size = 205, normalized size = 1.19

$$-\frac{e^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{16\,d^3}+\frac{\sqrt{-e^2x^2+d^2}\,e^7}{16\,d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7}{48\,d^6}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}{48\,d^6x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{24\,d^4x^4}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{35\,d^3x^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{6\,d^2x^6}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{7\,dx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/16*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 + 1/16*sq
rt(-e^2*x^2 + d^2)*e^7/d^4 + 1/48*(-e^2*x^2 + d^2)^(3/2)*e^7/d^6 + 1/48*(-e
^2*x^2 + d^2)^(5/2)*e^5/(d^6*x^2) - 1/24*(-e^2*x^2 + d^2)^(5/2)*e^3/(d^4*x^
4) - 2/35*(-e^2*x^2 + d^2)^(5/2)*e^2/(d^3*x^5) - 1/6*(-e^2*x^2 + d^2)^(5/2)
*e/(d^2*x^6) - 1/7*(-e^2*x^2 + d^2)^(5/2)/(d*x^7)
```

mupad [B] time = 5.33, size = 192, normalized size = 1.12

$$\frac{8\,d\,e^2\,\sqrt{d^2-e^2\,x^2}}{35\,x^5}-\frac{d^3\,\sqrt{d^2-e^2\,x^2}}{7\,x^7}-\frac{e^4\,\sqrt{d^2-e^2\,x^2}}{35\,d\,x^3}-\frac{2\,e^6\,\sqrt{d^2-e^2\,x^2}}{35\,d^3\,x}-\frac{e\,{\left(d^2-e^2\,x^2\right)}^{3/2}}{6\,x^6}+\frac{d^2\,e\,\sqrt{d^2-e^2\,x^2}}{16\,x^6}-\frac{e\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{16\,d^2\,x^6}+\frac{e^7\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1{}\mathrm{i}}{d}\right)\,1{}\mathrm{i}}{16\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^8,x)
```

```
[Out] (e^7*atan(((d^2 - e^2*x^2)^(1/2)*1i)/d)*1i)/(16*d^3) - (d^3*(d^2 - e^2*x^2)
^(1/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(3/2))/(6*x^6) - (e^4*(d^2 - e^2*x^2)^
(1/2))/(35*d*x^3) - (2*e^6*(d^2 - e^2*x^2)^(1/2))/(35*d^3*x) + (8*d*e^2*(d^
2 - e^2*x^2)^(1/2))/(35*x^5) + (d^2*e*(d^2 - e^2*x^2)^(1/2))/(16*x^6) - (e*
(d^2 - e^2*x^2)^(5/2))/(16*d^2*x^6)
```

sympy [C] time = 16.62, size = 1037, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**8,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + d**2*e*P
iecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d
**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e*
*5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1))
- 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-
d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I
*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewise((3*I*d**3*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 +
e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1)
, (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**
2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**
6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*
sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - e**3*Pi
ecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*aco
sh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d
**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3
/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), Tr
ue))
```

3.15 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=201

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{3e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}-\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}+\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-e$$

Rubi [A] time = 0.16, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {835, 807, 266, 47, 63, 208}

$$\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4}-\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{3e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) - (e*(d^2 - e^2*x^2)^(5/2))/(7*
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) - (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{\left(-8d^2e-3de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx}{8d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} + \frac{\int \frac{\left(21d^3e^2+16d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{\int \frac{\left(-96d^4e^3-21d^3e^4x\right)\left(d^2-e^2x\right.}{x^6}}{336d^6}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4}{}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4}{}$$

$$= -\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - 2e^3$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-\right.}{16}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-\right.}{16}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-\right.}{16}$$

Mathematica [C] time = 0.02, size = 73, normalized size = 0.36

$$-\frac{e\left(d^2-e^2x^2\right)^{5/2}\left(5d^7+2d^5e^2x^2+7e^7x^7\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)\right)}{35d^9x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

```
[Out] -1/35*(e*(d^2 - e^2*x^2)^(5/2)*(5*d^7 + 2*d^5*e^2*x^2 + 7*e^7*x^7*Hypergeom
etric2F1[5/2, 5, 7/2, 1 - (e^2*x^2)/d^2]))/(d^9*x^7)
```

IntegrateAlgebraic [A] time = 0.74, size = 148, normalized size = 0.74

$$\frac{3e^8\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{64d^4}+\frac{\sqrt{d^2-e^2x^2}\left(-560d^7-640d^6ex+840d^5e^2x^2+1024d^4e^3x^3-70d^3e^4x^4-128d^2e^5x^5-105de^6x^6-256e^7x^7\right)}{4480d^4x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 - 640*d^6*e*x + 840*d^5*e^2*x^2 + 1024*d^4*e
^3*x^3 - 70*d^3*e^4*x^4 - 128*d^2*e^5*x^5 - 105*d*e^6*x^6 - 256*e^7*x^7))/(
4480*d^4*x^8) + (3*e^8*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(
64*d^4)
```

fricas [A] time = 0.45, size = 131, normalized size = 0.65

$$\frac{105\,e^{8}x^{8}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(256\,e^{7}x^{7}+105\,de^{6}x^{6}+128\,d^{2}e^{5}x^{5}+70\,d^{3}e^{4}x^{4}-1024\,d^{4}e^{3}x^{3}-840\,d^{5}e^{2}x^{2}+640\,d^{6}ex+560\,d^{7}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{4480\,d^{4}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/4480*(105*e^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (256*e^7*x^7 + 105
*d*e^6*x^6 + 128*d^2*e^5*x^5 + 70*d^3*e^4*x^4 - 1024*d^4*e^3*x^3 - 840*d^5*
e^2*x^2 + 640*d^6*e*x + 560*d^7)*sqrt(-e^2*x^2 + d^2))/(d^4*x^8)
```

giac [B] time = 0.26, size = 431, normalized size = 2.14

$$\frac{x^{8}\left(\frac{80\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)e^{16}}{x}-\frac{112\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{3}e^{12}}{x^{3}}-\frac{280\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{4}e^{10}}{x^{4}}-\frac{560\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{5}e^{8}}{x^{5}}+\frac{1680\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{7}e^{4}}{x^{7}}+35\,e^{18}\right)e^{6}}{71680\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{8}d^{4}}-\frac{3\,e^{8}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}e\right|e^{(-2)}}{2\,|x|}\right)}{128\,d^{4}}-\frac{\left(\frac{1680\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)d^{28}e^{86}}{x}-\frac{560\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{3}d^{28}e^{82}}{x^{3}}-\frac{280\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{4}d^{28}e^{80}}{x^{4}}-\frac{112\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{5}d^{28}e^{78}}{x^{5}}+\frac{80\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{7}d^{28}e^{74}}{x^{7}}+\frac{35\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{8}d^{28}e^{72}}{x^{8}}\right)e^{(-80)}}{71680\,d^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/71680*x^8*(80*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^16/x - 112*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^3*e^12/x^3 - 280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^10/x^4
- 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^8/x^5 + 1680*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^7*e^4/x^7 + 35*e^18)*e^6/((d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^4) -
 3/128*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^
4 - 1/71680*(1680*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^28*e^86/x - 560*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^3*d^28*e^82/x^3 - 280*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^4*d^28*e^80/x^4 - 112*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^28*e^78/x^5 + 80*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^28*e^74/x^7 + 35*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^8*d^28*e^72/x^8)*e^(-80)/d^32
```

maple [A] time = 0.07, size = 236, normalized size = 1.17

$$-\frac{3e^{8}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{128\sqrt{d^{2}}\,d^{3}}+\frac{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{8}}{128d^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{8}}{128d^{7}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}{128d^{7}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}{64d^{5}x^{4}}-\frac{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{3}}{35d^{4}x^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}{16d^{3}x^{6}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e}{7d^{2}x^{7}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{8d\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x)
```

```
[Out] -1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6-1/64*
e^4/d^5/x^4*(-e^2*x^2+d^2)^(5/2)+1/128*e^6/d^7/x^2*(-e^2*x^2+d^2)^(5/2)+1/1
28*e^8/d^7*(-e^2*x^2+d^2)^(3/2)+3/128*e^8/d^5*(-e^2*x^2+d^2)^(1/2)-3/128*e^
8/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/7*e*(-
e^2*x^2+d^2)^(5/2)/d^2/x^7-2/35*e^3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5
```

maxima [A] time = 0.99, size = 230, normalized size = 1.14

$$-\frac{3\,e^{8}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{128\,d^{4}}+\frac{3\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{8}}{128\,d^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{8}}{128\,d^{7}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}{128\,d^{7}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}{64\,d^{5}x^{4}}-\frac{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{3}}{35\,d^{4}x^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}{16\,d^{3}x^{6}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e}{7\,d^{2}x^{7}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{8\,dx^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -3/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 3/128*
sqrt(-e^2*x^2 + d^2)*e^8/d^5 + 1/128*(-e^2*x^2 + d^2)^(3/2)*e^8/d^7 + 1/128
*(-e^2*x^2 + d^2)^(5/2)*e^6/(d^7*x^2) - 1/64*(-e^2*x^2 + d^2)^(5/2)*e^4/(d^
5*x^4) - 2/35*(-e^2*x^2 + d^2)^(5/2)*e^3/(d^4*x^5) - 1/16*(-e^2*x^2 + d^2)^
(5/2)*e^2/(d^3*x^6) - 1/7*(-e^2*x^2 + d^2)^(5/2)*e/(d^2*x^7) - 1/8*(-e^2*x^
2 + d^2)^(5/2)/(d*x^8)
```

mupad [B] time = 6.04, size = 212, normalized size = 1.05

$$\frac{3\,d^3\,\sqrt{d^2-e^2\,x^2}}{128\,x^8}-\frac{11\,d\,{\left(d^2-e^2\,x^2\right)}^{3/2}}{128\,x^8}-\frac{11\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{128\,d\,x^8}+\frac{3\,{\left(d^2-e^2\,x^2\right)}^{7/2}}{128\,d^3\,x^8}+\frac{8\,e^3\,\sqrt{d^2-e^2\,x^2}}{35\,x^5}-\frac{e^5\,\sqrt{d^2-e^2\,x^2}}{35\,d^2\,x^3}-\frac{2\,e^7\,\sqrt{d^2-e^2\,x^2}}{35\,d^4\,x}-\frac{d^2\,e\,\sqrt{d^2-e^2\,x^2}}{7\,x^7}+\frac{e^8\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1{}\mathrm{i}}{d}\right)\,3{}\mathrm{i}}{128\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^9,x)
```

```
[Out] (3*d^3*(d^2 - e^2*x^2)^(1/2))/(128*x^8) - (11*d*(d^2 - e^2*x^2)^(3/2))/(128
*x^8) - (11*(d^2 - e^2*x^2)^(5/2))/(128*d*x^8) + (3*(d^2 - e^2*x^2)^(7/2))/
(128*d^3*x^8) + (8*e^3*(d^2 - e^2*x^2)^(1/2))/(35*x^5) + (e^8*atan(((d^2 -
e^2*x^2)^(1/2)*1i)/d)*3i)/(128*d^4) - (e^5*(d^2 - e^2*x^2)^(1/2))/(35*d^2*x
^3) - (2*e^7*(d^2 - e^2*x^2)^(1/2))/(35*d^4*x) - (d^2*e*(d^2 - e^2*x^2)^(1/
2))/(7*x^7)
```

sympy [C] time = 22.55, size = 1159, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**9,x)
```

```
[Out] d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) +
 e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x*
*2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x
**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6)
, True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) +
5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e*
*2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d
/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2
/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(
48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - e**3*Piecewise((
3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e
**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**
6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e*
*4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x
**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x
**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7),
 True))
```

3.16 $\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {797, 641, 195, 217, 203}

$$-\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((d^2*Sqrt[d^2 - e^2*x^2])/e^3) - (d*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + (d^2
 - e^2*x^2)^(3/2)/(3*e^3) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\int (d+ex)\sqrt{d^2-e^2x^2}\,dx}{e^2} + \frac{d^2\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$

$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} - \frac{d\int \sqrt{d^2-e^2x^2}\,dx}{e^2} + \frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$

$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} - \frac{dx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} - \frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{1+e^2x \cdots}\right.}{e \cdots}$$

$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} - \frac{dx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3} - \frac{d^3\,\text{Subst}\left(\int \frac{\cdots}{1-\cdots}\right.}{\cdots}$$

$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} - \frac{dx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.68

$$\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(4d^2+3dex+2e^2x^2\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(4*d^2 + 3*d*e*x + 2*e^2*x^2)) + 3*d^3*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]])/(6*e^3)
```

IntegrateAlgebraic [A] time = 0.24, size = 92, normalized size = 0.89

$$\frac{\left(-4d^2-3dex-2e^2x^2\right)\sqrt{d^2-e^2x^2}}{6e^3} + \frac{d^3\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] ((-4*d^2 - 3*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(6*e^3) + (d^3*Sqrt[-e
^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

fricas [A] time = 0.40, size = 72, normalized size = 0.70

$$-\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(2\,e^2x^2+3\,dex+4\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (2*e^2*x^2 + 3*d*e*
x + 4*d^2)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.24, size = 54, normalized size = 0.52

$$\frac{1}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\text{sgn}(d) - \frac{1}{6}\,\sqrt{-x^2e^2+d^2}\left(4\,d^2e^{(-3)}+\left(2\,xe^{(-1)}+3\,de^{(-2)}\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/6*sqrt(-x^2*e^2 + d^2)*(4*d^2*e^(-3
) + (2*x*e^(-1) + 3*d*e^(-2))*x)
```

maple [A] time = 0.02, size = 102, normalized size = 0.99

$$\frac{d^3 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^2} - \frac{\sqrt{-e^2x^2+d^2}\,x^2}{3e} - \frac{\sqrt{-e^2x^2+d^2}\,dx}{2e^2} - \frac{2\sqrt{-e^2x^2+d^2}\,d^2}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*x^2/e*(-e^2*x^2+d^2)^(1/2)-2/3*d^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*d*x*(-
e^2*x^2+d^2)^(1/2)/e^2+1/2*d^3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2
+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 81, normalized size = 0.79

$$-\frac{\sqrt{-e^2x^2+d^2}\,x^2}{3\,e} + \frac{d^3 \arcsin\left(\frac{ex}{d}\right)}{2\,e^3} - \frac{\sqrt{-e^2x^2+d^2}\,dx}{2\,e^2} - \frac{2\,\sqrt{-e^2x^2+d^2}\,d^2}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*x^2/e + 1/2*d^3*arcsin(e*x/d)/e^3 - 1/2*sqrt(-e^2
*x^2 + d^2)*d*x/e^2 - 2/3*sqrt(-e^2*x^2 + d^2)*d^2/e^3
```

mupad [B] time = 3.14, size = 112, normalized size = 1.09

$$\begin{cases} \frac{d\,x^3}{3\,\sqrt{d^2}} & \text{if } e=0 \\ -\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+e^2\,x^2\right)}{3\,e^3} - \frac{d^3\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}} - \frac{d\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2} & \text{if } e\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (d*x^3)/(3*(d^2)^(1/2)), e ~= 0, - ((d^2 - e^2*x^2)^(1/2)
*(2*d^2 + e^2*x^2))/(3*e^3) - (d^3*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)
^(1/2)))/(2*(-e^2)^(3/2)) - (d*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2))
```

sympy [C] time = 5.25, size = 177, normalized size = 1.72

$$d\left(\begin{cases} -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3} - \frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3} - \frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} -\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4} - \frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for } e\neq 0 \\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2
)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e
**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))
 + e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 -
e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True))
```

3.17 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}} dx$

Optimal. Leaf size=73

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {797, 641, 217, 203, 637}

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d*(d + e*x))/(e^3*Sqrt[d^2 - e^2*x^2]) + Sqrt[d^2 - e^2*x^2]/e^3 - (d*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx}{e^2} + \frac{d^2\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{e^2}$$
$$= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$
$$= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$
$$= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.05

$$\frac{-d\sqrt{d^2-e^2x^2}\,\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+2d^2+dex-e^2x^2}{e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (2*d^2 + d*e*x - e^2*x^2 - d*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]])/(e^3*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.47, size = 84, normalized size = 1.15

$$-\frac{d\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^4}-\frac{\sqrt{d^2-e^2x^2}\,(2d-ex)}{e^3(ex-d)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] -(((2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(e^3*(-d + e*x))) - (d*Sqrt[-e^2]*Log[-
(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^4
```

fricas [A] time = 0.40, size = 87, normalized size = 1.19

$$\frac{2\,dex-2\,d^2+2\left(dex-d^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}\,(ex-2\,d)}{e^4x-de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (2*d*e*x - 2*d^2 + 2*(d*e*x - d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + sqrt(-e^2*x^2 + d^2)*(e*x - 2*d))/(e^4*x - d*e^3)
```

giac [A] time = 0.25, size = 66, normalized size = 0.90

$$-d\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\text{sgn}(d)-\frac{\sqrt{-x^2e^2+d^2}\left(2\,d^2e^{(-3)}-\left(xe^{(-1)}-de^{(-2)}\right)x\right)}{x^2e^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -d*arcsin(x*e/d)*e^(-3)*sgn(d) - sqrt(-x^2*e^2 + d^2)*(2*d^2*e^(-3) - (x*e^
(-1) - d*e^(-2))*x)/(x^2*e^2 - d^2)
```

maple [A] time = 0.02, size = 99, normalized size = 1.36

$$-\frac{x^2}{\sqrt{-e^2x^2+d^2}\,e}+\frac{dx}{\sqrt{-e^2x^2+d^2}\,e^2}-\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^2}+\frac{2d^2}{\sqrt{-e^2x^2+d^2}\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -x^2/e/(-e^2*x^2+d^2)^(1/2)+2*d^2/e^3/(-e^2*x^2+d^2)^(1/2)+d*x/e^2/(-e^2*x^
2+d^2)^(1/2)-d/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.98, size = 78, normalized size = 1.07

$$-\frac{x^2}{\sqrt{-e^2x^2+d^2}\,e}+\frac{dx}{\sqrt{-e^2x^2+d^2}\,e^2}-\frac{d\arcsin\left(\frac{ex}{d}\right)}{e^3}+\frac{2\,d^2}{\sqrt{-e^2x^2+d^2}\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/(sqrt(-e^2*x^2 + d^2)*e) + d*x/(sqrt(-e^2*x^2 + d^2)*e^2) - d*arcsin(e
*x/d)/e^3 + 2*d^2/(sqrt(-e^2*x^2 + d^2)*e^3)
```

mupad [B] time = 2.96, size = 87, normalized size = 1.19

$$\frac{2\,d^2-e^2\,x^2}{e^3\,\sqrt{d^2-e^2\,x^2}}+\frac{d\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{{\left(-e^2\right)}^{3/2}}+\frac{d\,x}{e^2\,\sqrt{d^2-e^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x)
```

```
[Out] (2*d^2 - e^2*x^2)/(e^3*(d^2 - e^2*x^2)^(1/2)) + (d*log(x*(-e^2)^(1/2) + (d^
2 - e^2*x^2)^(1/2)))/(-e^2)^(3/2) + (d*x)/(e^2*(d^2 - e^2*x^2)^(1/2))
```

sympy [C] time = 9.71, size = 184, normalized size = 2.52

$$d\left(\begin{cases}\frac{i\operatorname{acosh}\left(\frac{ex}{d}\right)}{e^3}-\frac{ix}{de^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{\operatorname{asin}\left(\frac{ex}{d}\right)}{e^3}+\frac{x}{de^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}\tilde{\infty}x^4 & \text{for}\ \left(d=0\vee d=-\sqrt{e^2x^2}\vee d=\sqrt{e^2x^2}\right)\wedge\left(d=-\sqrt{e^2x^2}\vee d=\sqrt{e^2x^2}\vee e=0\right)\\ \frac{x^4}{4\left(d^2\right)^{\frac{3}{2}}} & \text{for}\ e=0\\ \frac{2d^2}{e^4\sqrt{d^2-e^2x^2}}-\frac{x^2}{e^2\sqrt{d^2-e^2x^2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d*Piecewise((I*acosh(e*x/d)/e**3 - I*x/(d*e**2*sqrt(-1 + e**2*x**2/d**2)),
Abs(e**2*x**2/d**2) > 1), (-asin(e*x/d)/e**3 + x/(d*e**2*sqrt(1 - e**2*x**2
/d**2)), True)) + e*Piecewise((zoo*x**4, (Eq(d, 0) | Eq(d, sqrt(e**2*x**2))
 | Eq(d, -sqrt(e**2*x**2))) & (Eq(e, 0) | Eq(d, sqrt(e**2*x**2)) | Eq(d, -s
qrt(e**2*x**2)))), (x**4/(4*(d**2)**(3/2)), Eq(e, 0)), (2*d**2/(e**4*sqrt(d
**2 - e**2*x**2)) - x**2/(e**2*sqrt(d**2 - e**2*x**2)), True))
```

3.18 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=58

$$\frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {796, 12, 261}

$$\frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (x^2*(d + e*x))/(3*d*e*(d^2 - e^2*x^2)^(3/2)) - 2/(3*e^3*Sqrt[d^2 - e^2*x^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{5/2}} \, dx &= \frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{2d^2ex}{\left(d^2-e^2x^2\right)^{3/2}} \, dx}{3d^2e^2}\\
&= \frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{2\int \frac{x}{\left(d^2-e^2x^2\right)^{3/2}} \, dx}{3e}\\
&= \frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.90

$$\frac{-2d^2+2dex+e^2x^2}{3de^3(d-ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (-2*d^2 + 2*d*e*x + e^2*x^2)/(3*d*e^3*(d - e*x)*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.43, size = 59, normalized size = 1.02

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^2+2dex+e^2x^2\right)}{3de^3(d-ex)^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^2 + 2*d*e*x + e^2*x^2))/(3*d*e^3*(d - e*x)^2*(d
+ e*x))
```

fricas [B] time = 0.39, size = 104, normalized size = 1.79

$$-\frac{2\,e^3x^3-2\,de^2x^2-2\,d^2ex+2\,d^3-\left(e^2x^2+2\,dex-2\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(de^6x^3-d^2e^5x^2-d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*e^3*x^3 - 2*d*e^2*x^2 - 2*d^2*e*x + 2*d^3 - (e^2*x^2 + 2*d*e*x - 2*
d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 - d^2*e^5*x^2 - d^3*e^4*x + d^4*e^3)
```

giac [A] time = 0.30, size = 51, normalized size = 0.88

$$\frac{\left(x^2\left(\frac{x}{d}+3\,e^{(-1)}\right)-2\,d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{3\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2*(x/d + 3*e^(-1)) - 2*d^2*e^(-3))*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d
^2)^2
```

maple [A] time = 0.01, size = 55, normalized size = 0.95

$$-\frac{(-ex+d)\,(ex+d)^2\left(-e^2x^2-2dex+2d^2\right)}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/3*(-e*x+d)*(e*x+d)^2*(-e^2*x^2-2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.44, size = 88, normalized size = 1.52

$$\frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}+\frac{dx}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}-\frac{x}{3\sqrt{-e^2x^2+d^2}\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] x^2/((-e^2*x^2 + d^2)^(3/2)*e) + 1/3*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2/3
*d^2/((-e^2*x^2 + d^2)^(3/2)*e^3) - 1/3*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)
```

mupad [B] time = 2.59, size = 55, normalized size = 0.95

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^2+2\,d\,e\,x+e^2\,x^2\right)}{3\,d\,e^3\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(e^2*x^2 - 2*d^2 + 2*d*e*x))/(3*d*e^3*(d + e*x)*(d -
 e*x)^2)
```

sympy [C] time = 9.88, size = 231, normalized size = 3.98

$$d\left(\begin{cases} \frac{ix^3}{-3d^5\sqrt{-1+\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{x^3}{-3d^5\sqrt{1-\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)+e\left(\begin{cases} \frac{2d^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}}-\frac{3e^2x^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}} & \text{for}\ e\neq 0 \\ \frac{x^4}{4\left(d^2\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d*Piecewise((I*x**3/(-3*d**5*sqrt(-1 + e**2*x**2/d**2) + 3*d**3*e**2*x**2*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-x**3/(-3*d**5*sqrt(1
 - e**2*x**2/d**2) + 3*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)), True)) + e
*Piecewise((2*d**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2) + 3*e**6*x**2*sqrt(
d**2 - e**2*x**2)) - 3*e**2*x**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2) + 3*e
**6*x**2*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(5/2)), True)
)
```

3.19 $\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=161

$$\frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8}-\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}+\frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}}-\frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.14, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {819, 780, 217, 203}

$$\frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8}-\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^6*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d + 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d + 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
) + ((32*d + 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) - (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^7(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^5\left(6d^3+7d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^3\left(24d^5+35d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x\left(48d^7+105d^6ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} -$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} -$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} -$$

Mathematica [A] time = 0.10, size = 155, normalized size = 0.96

$$\frac{96d^6+9d^5ex-249d^4e^2x^2+4d^3e^3x^3+176d^2e^4x^4-105d^2(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\,\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-15de^5x^5-15e^6x^6}{30e^8(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (96*d^6 + 9*d^5*e*x - 249*d^4*e^2*x^2 + 4*d^3*e^3*x^3 + 176*d^2*e^4*x^4 - 1
5*d*e^5*x^5 - 15*e^6*x^6 - 105*d^2*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2
]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(30*e^8*(d - e*x)^2*(d + e*x)*Sqrt[d^2
 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.78, size = 152, normalized size = 0.94

$$-\frac{7d^2\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^9}-\frac{\sqrt{d^2-e^2x^2}\,\left(96d^6+9d^5ex-249d^4e^2x^2+4d^3e^3x^3+176d^2e^4x^4-15de^5x^5-15e^6x^6\right)}{30e^8(ex-d)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/30*(Sqrt[d^2 - e^2*x^2]*(96*d^6 + 9*d^5*e*x - 249*d^4*e^2*x^2 + 4*d^3*e^
3*x^3 + 176*d^2*e^4*x^4 - 15*d*e^5*x^5 - 15*e^6*x^6))/(e^8*(-d + e*x)^3*(d
+ e*x)^2) - (7*d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(
2*e^9)
```

fricas [A] time = 0.47, size = 278, normalized size = 1.73

$$\frac{96\,d^2e^5x^5-96\,d^3e^4x^4-192\,d^4e^3x^3+192\,d^5e^2x^2+96\,d^6ex-96\,d^7+210\left(d^2e^5x^5-d^3e^4x^4-2\,d^4e^3x^3+2\,d^5e^2x^2+d^6ex-d^7\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(15\,e^6x^6+15\,de^5x^5-176\,d^2e^4x^4-4\,d^3e^3x^3+249\,d^4e^2x^2-9\,d^5ex-96\,d^6\right)\sqrt{-e^2x^2+d^2}}{30\left(e^{13}x^5-de^{12}x^4-2\,d^2e^{11}x^3+2\,d^3e^{10}x^2+d^4e^9x-d^5e^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(96*d^2*e^5*x^5 - 96*d^3*e^4*x^4 - 192*d^4*e^3*x^3 + 192*d^5*e^2*x^2 +
 96*d^6*e*x - 96*d^7 + 210*(d^2*e^5*x^5 - d^3*e^4*x^4 - 2*d^4*e^3*x^3 + 2*d
^5*e^2*x^2 + d^6*e*x - d^7)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15
*e^6*x^6 + 15*d*e^5*x^5 - 176*d^2*e^4*x^4 - 4*d^3*e^3*x^3 + 249*d^4*e^2*x^2
 - 9*d^5*e*x - 96*d^6)*sqrt(-e^2*x^2 + d^2))/(e^13*x^5 - d*e^12*x^4 - 2*d^2
*e^11*x^3 + 2*d^3*e^10*x^2 + d^4*e^9*x - d^5*e^8)
```

giac [A] time = 0.28, size = 120, normalized size = 0.75

$$-\frac{7}{2}\,d^{2}\arcsin\left(\frac{xe}{d}\right)e^{(-8)}\mathrm{sgn}(d)-\frac{\left(96\,d^{7}e^{(-8)}+\left(105\,d^{6}e^{(-7)}-\left(240\,d^{5}e^{(-6)}+\left(245\,d^{4}e^{(-5)}-\left(180\,d^{3}e^{(-4)}+\left(161\,d^{2}e^{(-3)}-15\left(xe^{(-1)}+2\,de^{(-2)}\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2}+d^{2}}}{30\left(x^{2}e^{2}-d^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -7/2*d^2*arcsin(x*e/d)*e^(-8)*sgn(d) - 1/30*(96*d^7*e^(-8) + (105*d^6*e^(-7
) - (240*d^5*e^(-6) + (245*d^4*e^(-5) - (180*d^3*e^(-4) + (161*d^2*e^(-3) -
 15*(x*e^(-1) + 2*d*e^(-2))*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2
 - d^2)^3
```

maple [A] time = 0.07, size = 227, normalized size = 1.41

$$-\frac{x^{7}}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e}-\frac{d\,x^{6}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}+\frac{7d^{2}x^{5}}{10\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{3}}+\frac{6d^{3}x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}-\frac{8d^{5}x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}-\frac{7d^{2}x^{3}}{6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}+\frac{16d^{7}}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{8}}+\frac{7d^{2}x}{2\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}-\frac{7d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}\,e^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/2*x^7/e/(-e^2*x^2+d^2)^(5/2)+7/10*d^2/e^3*x^5/(-e^2*x^2+d^2)^(5/2)-7/6*d
^2/e^5*x^3/(-e^2*x^2+d^2)^(3/2)+7/2*d^2/e^7*x/(-e^2*x^2+d^2)^(1/2)-7/2*d^2/
e^7/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-d*x^6/e^2/(-e^2*
x^2+d^2)^(5/2)+6*d^3/e^4*x^4/(-e^2*x^2+d^2)^(5/2)-8*d^5/e^6*x^2/(-e^2*x^2+d
^2)^(5/2)+16/5*d^7/e^8/(-e^2*x^2+d^2)^(5/2)
```

maxima [B] time = 1.03, size = 312, normalized size = 1.94

$$-\frac{x^{7}}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e}+\frac{7\,d^{2}x\left(\frac{15\,x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}-\frac{20\,d^{2}x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}+\frac{8\,d^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}\right)}{30\,e}-\frac{dx^{6}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}-\frac{7\,d^{2}x\left(\frac{3\,x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}-\frac{2\,d^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}\right)}{6\,e^{3}}+\frac{6\,d^{3}x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}-\frac{8\,d^{5}x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}+\frac{16\,d^{7}}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{8}}+\frac{14\,d^{4}x}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}}-\frac{49\,d^{2}x}{30\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}-\frac{7\,d^{2}\arcsin\left(\frac{ex}{d}\right)}{2\,e^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^7/((-e^2*x^2 + d^2)^(5/2)*e) + 7/30*d^2*x*(15*x^4/((-e^2*x^2 + d^2)^
(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d
^2)^(5/2)*e^6))/e - d*x^6/((-e^2*x^2 + d^2)^(5/2)*e^2) - 7/6*d^2*x*(3*x^2/(
(-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^3 + 6*d
^3*x^4/((-e^2*x^2 + d^2)^(5/2)*e^4) - 8*d^5*x^2/((-e^2*x^2 + d^2)^(5/2)*e^6
) + 16/5*d^7/((-e^2*x^2 + d^2)^(5/2)*e^8) + 14/15*d^4*x/((-e^2*x^2 + d^2)^(
3/2)*e^7) - 49/30*d^2*x/(sqrt(-e^2*x^2 + d^2)*e^7) - 7/2*d^2*arcsin(e*x/d)/
e^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7}\,\left(d+e\,x\right)}{{\left(d^{2}-e^{2}\,x^{2}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [B] time = 66.42, size = 2004, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x*
*2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 40*d**4*
e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**
2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) + 30*d**2*e**4*x**4/(
5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x*
*2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x**6/(5*d**4*e**8*sqrt(
d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**
4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)**(7/2)), True)) + e*P
iecewise((-210*I*d**7*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-60*d**5*e**9
*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2)
- 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 105*pi*d**7*sqrt(-1 + e**2*x
**2/d**2)/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x**2*sq
rt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 210*
I*d**6*e*x/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x**2*s
qrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 420
*I*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-60*d**5*e**9*sqr
t(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) - 60
*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 210*pi*d**5*e**2*x**2*sqrt(-1 +
e**2*x**2/d**2)/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x
**2*sqrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2))
- 490*I*d**4*e**3*x**3/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*
e**11*x**2*sqrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/
d**2)) - 210*I*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-60*d
**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**
2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 105*pi*d**3*e**4*x**
4*sqrt(-1 + e**2*x**2/d**2)/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) + 120*
d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1 + e**2*
x**2/d**2)) + 322*I*d**2*e**5*x**5/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2)
 + 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1
+ e**2*x**2/d**2)) - 30*I*e**7*x**7/(-60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2
) + 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) - 60*d*e**13*x**4*sqrt(-1
 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-105*d**7*sqrt(1 - e**2*x**
2/d**2)*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*
x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) +
 105*d**6*e*x/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*s
qrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 210*d
**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e
**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x
**4*sqrt(1 - e**2*x**2/d**2)) - 245*d**4*e**3*x**3/(30*d**5*e**9*sqrt(1 - e
**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x
**4*sqrt(1 - e**2*x**2/d**2)) - 105*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2)
*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sq
rt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 161*d*
*2*e**5*x**5/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sq
rt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*e**
7*x**7/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 -
 e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

3.20 $\int \frac{x^6(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=147

$$\frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {819, 641, 217, 203}

$$\frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^5*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d + 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d + 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) + (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^6(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^4\left(5d^3+6d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^2\left(15d^5+24d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^7+48d^6ex}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\ldots}{e^6}$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\,\text{Subst}\left(\int \frac{\ldots}{1\ldots}\ldots\right.}{\ldots}$$

$$= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\tan^{-1}\left(\frac{e\ldots}{\sqrt{d^2-\ldots}}\right.}{e^7}$$

Mathematica [A] time = 0.09, size = 142, normalized size = 0.97

$$\frac{48d^5-33d^4ex-87d^3e^2x^2+52d^2e^3x^3-15d(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+38de^4x^4-15e^5x^5}{15e^7(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (48*d^5 - 33*d^4*e*x - 87*d^3*e^2*x^2 + 52*d^2*e^3*x^3 + 38*d*e^4*x^4 - 15*
e^5*x^5 - 15*d*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(15*e^7*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.73, size = 137, normalized size = 0.93

$$-\frac{d\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^8} - \frac{\sqrt{d^2-e^2x^2}\left(48d^5-33d^4ex-87d^3e^2x^2+52d^2e^3x^3+38de^4x^4-15e^5x^5\right)}{15e^7(ex-d)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(48*d^5 - 33*d^4*e*x - 87*d^3*e^2*x^2 + 52*d^2*e
^3*x^3 + 38*d*e^4*x^4 - 15*e^5*x^5))/(e^7*(-d + e*x)^3*(d + e*x)^2) - (d*Sq
rt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^8
```

fricas [B] time = 0.42, size = 263, normalized size = 1.79

$$\frac{48\,de^5x^5-48\,d^2e^4x^4-96\,d^3e^3x^3+96\,d^4e^2x^2+48\,d^5ex-48\,d^6+30\left(de^5x^5-d^2e^4x^4-2\,d^3e^3x^3+2\,d^4e^2x^2+d^5ex-d^6\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(15\,e^5x^5-38\,de^4x^4-52\,d^2e^3x^3+87\,d^3e^2x^2+33\,d^4ex-48\,d^5\right)\sqrt{-e^2x^2+d^2}}{15\left(e^{12}x^5-de^{11}x^4-2\,d^2e^{10}x^3+2\,d^3e^9x^2+d^4e^8x-d^5e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(48*d*e^5*x^5 - 48*d^2*e^4*x^4 - 96*d^3*e^3*x^3 + 96*d^4*e^2*x^2 + 48*
d^5*e*x - 48*d^6 + 30*(d*e^5*x^5 - d^2*e^4*x^4 - 2*d^3*e^3*x^3 + 2*d^4*e^2*
x^2 + d^5*e*x - d^6)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^5*x^
5 - 38*d*e^4*x^4 - 52*d^2*e^3*x^3 + 87*d^3*e^2*x^2 + 33*d^4*e*x - 48*d^5)*s
qrt(-e^2*x^2 + d^2))/(e^12*x^5 - d*e^11*x^4 - 2*d^2*e^10*x^3 + 2*d^3*e^9*x^
2 + d^4*e^8*x - d^5*e^7)
```

giac [A] time = 0.27, size = 109, normalized size = 0.74

$$-d\arcsin\left(\frac{xe}{d}\right)e^{(-7)}\mathrm{sgn}(d)-\frac{\left(48\,d^6e^{(-7)}+\left(15\,d^5e^{(-6)}-\left(120\,d^4e^{(-5)}+\left(35\,d^3e^{(-4)}-\left(90\,d^2e^{(-3)}-\left(15\,xe^{(-1)}-23\,de^{(-2)}\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -d*arcsin(x*e/d)*e^(-7)*sgn(d) - 1/15*(48*d^6*e^(-7) + (15*d^5*e^(-6) - (12
0*d^4*e^(-5) + (35*d^3*e^(-4) - (90*d^2*e^(-3) - (15*x*e^(-1) - 23*d*e^(-2)
)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.02, size = 195, normalized size = 1.33

$$-\frac{x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d\,x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}+\frac{6d^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{8d^4x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}-\frac{d\,x^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}+\frac{16d^6}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^7}+\frac{dx}{\sqrt{-e^2x^2+d^2}\,e^6}-\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -x^6/e/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^3*x^4/(-e^2*x^2+d^2)^(5/2)-8*d^4/e^5*x^
2/(-e^2*x^2+d^2)^(5/2)+16/5*d^6/e^7/(-e^2*x^2+d^2)^(5/2)+1/5*d*x^5/e^2/(-e^
2*x^2+d^2)^(5/2)-1/3*d/e^4*x^3/(-e^2*x^2+d^2)^(3/2)+d/e^6*x/(-e^2*x^2+d^2)^
(1/2)-d/e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [B] time = 1.03, size = 278, normalized size = 1.89

$$\frac{1}{15}\,dx\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{dx\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)}{3\,e^2}+\frac{6\,d^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{8\,d^4x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}+\frac{16\,d^6}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^7}+\frac{4\,d^3x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6}-\frac{7\,dx}{15\sqrt{-e^2x^2+d^2}\,e^6}-\frac{d\arcsin\left(\frac{ex}{d}\right)}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*d*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2
)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - x^6/((-e^2*x^2 + d^2)^
(5/2)*e) - 1/3*d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 +
 d^2)^(3/2)*e^4))/e^2 + 6*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^3) - 8*d^4*x^2/
((-e^2*x^2 + d^2)^(5/2)*e^5) + 16/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^7) + 4/15
*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^6) - 7/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^6) -
 d*arcsin(e*x/d)/e^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6\,\left(d+e\,x\right)}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [C] time = 62.08, size = 1821, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-30*d**5*e*
*7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2)
- 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 15*pi*d**5*sqrt(-1 + e**2*x*
*2/d**2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(
-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*I*d*
*4*e*x/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1
 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 60*I*d**3
*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-30*d**5*e**7*sqrt(-1 +
e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*
x**4*sqrt(-1 + e**2*x**2/d**2)) - 30*pi*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/
d**2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1
+ e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 70*I*d**2*
e**3*x**3/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt
(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 30*I*d
*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-30*d**5*e**7*sqrt(-1 +
e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*
x**4*sqrt(-1 + e**2*x**2/d**2)) + 15*pi*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**
2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 46*I*e**5*x**
5/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e*
*2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d
**2) > 1), (-15*d**5*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqr
t(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e
**11*x**4*sqrt(1 - e**2*x**2/d**2)) + 15*d**4*e*x/(15*d**5*e**7*sqrt(1 - e*
*2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**
4*sqrt(1 - e**2*x**2/d**2)) + 30*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)*as
in(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1
 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 35*d**2*e*
*3*x**3/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 -
 e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*d*e**4*x*
*4*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d*
*2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 -
 e**2*x**2/d**2)) + 23*e**5*x**5/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 3
0*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x
**2/d**2)), True)) + e*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**
2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e
**2*x**2)) - 40*d**4*e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**
2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2))
+ 30*d**2*e**4*x**4/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**
2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x*
*6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**
2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)*
*(7/2)), True))
```

3.21 $\int \frac{x^5(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=122

$$\frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {819, 778, 217, 203}

$$\frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^4*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d + 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d + 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) - Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^5(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^3\left(4d^3+5d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x\left(8d^5+15d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Mathematica [A] time = 0.08, size = 130, normalized size = 1.07

$$\frac{8d^4+7d^3ex-27d^2e^2x^2-15(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-8de^3x^3+23e^4x^4}{15e^6(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (8*d^4 + 7*d^3*e*x - 27*d^2*e^2*x^2 - 8*d*e^3*x^3 + 23*e^4*x^4 - 15*(d - e*
x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e
^6*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.68, size = 125, normalized size = 1.02

$$-\frac{\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^7}-\frac{\sqrt{d^2-e^2x^2}\left(8d^4+7d^3ex-27d^2e^2x^2-8de^3x^3+23e^4x^4\right)}{15e^6(ex-d)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(8*d^4 + 7*d^3*e*x - 27*d^2*e^2*x^2 - 8*d*e^3*x^
3 + 23*e^4*x^4))/(e^6*(-d + e*x)^3*(d + e*x)^2) - (Sqrt[-e^2]*Log[-(Sqrt[-e
^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^7
```

fricas [B] time = 0.41, size = 247, normalized size = 2.02

$$\frac{8\,e^5x^5-8\,de^4x^4-16\,d^2e^3x^3+16\,d^3e^2x^2+8\,d^4ex-8\,d^5+30\left(e^5x^5-de^4x^4-2\,d^2e^3x^3+2\,d^3e^2x^2+d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(23\,e^4x^4-8\,de^3x^3-27\,d^2e^2x^2+7\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(e^{11}x^5-de^{10}x^4-2\,d^2e^9x^3+2\,d^3e^8x^2+d^4e^7x-d^5e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 - 8*d*e^4*x^4 - 16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 + 8*d^4*e*x
 - 8*d^5 + 30*(e^5*x^5 - d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 + d^4*e*
```

```
x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (23*e^4*x^4 - 8*d*e^3*
x^3 - 27*d^2*e^2*x^2 + 7*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(e^11*x^5 -
 d*e^10*x^4 - 2*d^2*e^9*x^3 + 2*d^3*e^8*x^2 + d^4*e^7*x - d^5*e^6)
```

giac [A] time = 0.31, size = 97, normalized size = 0.80

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d)-\frac{\left(8\,d^5e^{(-6)}+\left(15\,d^4e^{(-5)}-\left(20\,d^3e^{(-4)}+\left(35\,d^2e^{(-3)}-\left(23\,xe^{(-1)}+15\,de^{(-2)}\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-6)*sgn(d) - 1/15*(8*d^5*e^(-6) + (15*d^4*e^(-5) - (20*d^
3*e^(-4) + (35*d^2*e^(-3) - (23*x*e^(-1) + 15*d*e^(-2))*x)*x)*x)*x)*sqrt(-x
^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.02, size = 166, normalized size = 1.36

$$\frac{x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{4d^3x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}-\frac{x^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}+\frac{8d^5}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}+\frac{x}{\sqrt{-e^2x^2+d^2}\,e^5}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*x^5/e/(-e^2*x^2+d^2)^(5/2)-1/3/e^3*x^3/(-e^2*x^2+d^2)^(3/2)+1/e^5*x/(-e
^2*x^2+d^2)^(1/2)-1/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)+d*x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4/3*d^3/e^4*x^2/(-e^2*x^2+d^2)^(5/2)+8/1
5*d^5/e^6/(-e^2*x^2+d^2)^(5/2)
```

maxima [B] time = 1.01, size = 250, normalized size = 2.05

$$\frac{1}{15}\,ex\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)}{3\,e}+\frac{dx^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{4\,d^3x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^5}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}+\frac{4\,d^2x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}-\frac{7\,x}{15\sqrt{-e^2x^2+d^2}\,e^5}-\frac{\arcsin\left(\frac{ex}{d}\right)}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2
)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*x*(3*x^2/((-e^2*x^
2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + d*x^4/((-e^2*
x^2 + d^2)^(5/2)*e^2) - 4/3*d^3*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8/15*d^5
/((-e^2*x^2 + d^2)^(5/2)*e^6) + 4/15*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 7
/15*x/(sqrt(-e^2*x^2 + d^2)*e^5) - arcsin(e*x/d)/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(d+e\,x\right)}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [B] time = 73.14, size = 1739, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**
2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) - 20*d**2*
e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**
2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) + 15*e**4*x**4/(15*d
**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**2 - e**2*x**2)
+ 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6/(6*(d**2)**(7/2))
, True)) + e*Piecewise((-30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(
-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*
x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 15*pi*d**5*sqrt(-
1 + e**2*x**2/d**2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9
*x**2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)
) + 30*I*d**4*e*x/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x
**2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2))
+ 60*I*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-30*d**5*e**7
*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) -
30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 30*pi*d**3*e**2*x**2*sqrt(-1 +
 e**2*x**2/d**2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x*
*2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) -
 70*I*d**2*e**3*x**3/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**
9*x**2*sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2
)) - 30*I*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(-30*d**5*e**7
*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) -
30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 15*pi*d*e**4*x**4*sqrt(-1 + e*
*2*x**2/d**2)/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*
sqrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 46
*I*e**5*x**5/(-30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) + 60*d**3*e**9*x**2*s
qrt(-1 + e**2*x**2/d**2) - 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(
e**2*x**2/d**2) > 1), (-15*d**5*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d*
*5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**
2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) + 15*d**4*e*x/(15*d**5*e**7*
sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*
d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) + 30*d**3*e**2*x**2*sqrt(1 - e**2*x*
*2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*
x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) -
 35*d**2*e**3*x**3/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x*
*2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 1
5*d*e**4*x**4*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e
**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x*
*4*sqrt(1 - e**2*x**2/d**2)) + 23*e**5*x**5/(15*d**5*e**7*sqrt(1 - e**2*x**
2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt
(1 - e**2*x**2/d**2)), True))
```

3.22 $\int \frac{x^4(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=84

$$\frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{5e^5\sqrt{d^2-e^2x^2}}-\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {805, 266, 43}

$$\frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{5e^5\sqrt{d^2-e^2x^2}}-\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^4*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (4*d^2)/(15*e^5*(d^2 - e^2*
x^2)^(3/2)) + 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{x^3}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}$$
$$= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{x}{\left(d^2-e^2x\right)^{5/2}}\,dx, x, x^2\right)}{5e}$$
$$= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2\,\mathrm{Subst}\left(\int \left(\frac{d^2}{e^2\left(d^2-e^2x\right)^{5/2}} - \frac{1}{e^2\left(d^2-e^2x\right)^{3/2}}\right)dx, x, x^2\right)}{5e}$$
$$= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 0.98

$$\frac{8d^4-8d^3ex-12d^2e^2x^2+12de^3x^3+3e^4x^4}{15de^5(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (8*d^4 - 8*d^3*e*x - 12*d^2*e^2*x^2 + 12*d*e^3*x^3 + 3*e^4*x^4)/(15*d*e^5*(
d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.52, size = 82, normalized size = 0.98

$$\frac{\sqrt{d^2-e^2x^2}\left(8d^4-8d^3ex-12d^2e^2x^2+12de^3x^3+3e^4x^4\right)}{15de^5(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 - 8*d^3*e*x - 12*d^2*e^2*x^2 + 12*d*e^3*x^3 + 3
*e^4*x^4))/(15*d*e^5*(d - e*x)^3*(d + e*x)^2)
```

fricas [B] time = 0.39, size = 171, normalized size = 2.04

$$\frac{8\,e^5x^5-8\,de^4x^4-16\,d^2e^3x^3+16\,d^3e^2x^2+8\,d^4ex-8\,d^5-\left(3\,e^4x^4+12\,de^3x^3-12\,d^2e^2x^2-8\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(de^{10}x^5-d^2e^9x^4-2\,d^3e^8x^3+2\,d^4e^7x^2+d^5e^6x-d^6e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 - 8*d*e^4*x^4 - 16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 + 8*d^4*e*x
 - 8*d^5 - (3*e^4*x^4 + 12*d*e^3*x^3 - 12*d^2*e^2*x^2 - 8*d^3*e*x + 8*d^4)*
sqrt(-e^2*x^2 + d^2))/(d*e^10*x^5 - d^2*e^9*x^4 - 2*d^3*e^8*x^3 + 2*d^4*e^7
*x^2 + d^5*e^6*x - d^6*e^5)
```

giac [A] time = 0.27, size = 64, normalized size = 0.76

$$-\frac{\left(8\,d^4e^{(-5)}+\left(3\,x^2\left(\frac{x}{d}+5\,e^{(-1)}\right)-20\,d^2e^{(-3)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(8*d^4*e^(-5) + (3*x^2*(x/d + 5*e^(-1)) - 20*d^2*e^(-3))*x^2)*sqrt(-x
^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.92

$$\frac{(-ex+d)(ex+d)^2\left(3x^4e^4+12x^3d\,e^3-12d^2x^2e^2-8d^3xe+8d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(3*e^4*x^4+12*d*e^3*x^3-12*d^2*e^2*x^2-8*d^3*e*x+8*
d^4)/d/e^5/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.45, size = 159, normalized size = 1.89

$$\frac{x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{dx^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{4\,d^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{3\,d^3x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}+\frac{dx}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}+\frac{x}{5\sqrt{-e^2x^2+d^2}\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4
/3*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^3*x/((-e^2*x^2 + d^2)^(5/2
)*e^4) + 8/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^5) + 1/10*d*x/((-e^2*x^2 + d^2)
^(3/2)*e^4) + 1/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^4)
```

mupad [B] time = 2.70, size = 78, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(8\,d^4-8\,d^3\,e\,x-12\,d^2\,e^2\,x^2+12\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d\,e^5\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(8*d^4 + 3*e^4*x^4 + 12*d*e^3*x^3 - 12*d^2*e^2*x^2 -
 8*d^3*e*x))/(15*d*e^5*(d + e*x)^2*(d - e*x)^3)
```

sympy [C] time = 63.87, size = 418, normalized size = 4.98

$$d\left(\begin{cases}-\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}\frac{8d^4}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}}-\frac{20d^2e^2x^2}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}}+\frac{15e^4x^4}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}} & \text{for}\ e\neq 0\\ \frac{x^6}{6\left(d^2\right)^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) - 10*d**5*e**2*x**2*
sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2/d**2) - 10*d**5*e*
*2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2
)), True)) + e*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*
d**2*e**8*x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2
)) - 20*d**2*e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*
```

```
x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) + 15*e*
*4*x**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**2
- e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6/(6*(
d**2)**(7/2)), True))
```

3.23 $\int \frac{x^3(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=90

$$\frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {819, 778, 191}

$$\frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^2*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d + 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) + x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{x\left(2d^3+3d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}\\ &= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e^3}\\ &= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.91

$$\frac{-2d^4+2d^3ex+3d^2e^2x^2-3de^3x^3+3e^4x^4}{15d^2e^4(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*e^4*x^4)/(15*d^2*e^4*
(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.48, size = 82, normalized size = 0.91

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^4+2d^3ex+3d^2e^2x^2-3de^3x^3+3e^4x^4\right)}{15d^2e^4(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*
e^4*x^4))/(15*d^2*e^4*(d - e*x)^3*(d + e*x)^2)
```

fricas [B] time = 0.41, size = 172, normalized size = 1.91

$$-\frac{2\,e^5x^5-2\,de^4x^4-4\,d^2e^3x^3+4\,d^3e^2x^2+2\,d^4ex-2\,d^5+\left(3\,e^4x^4-3\,de^3x^3+3\,d^2e^2x^2+2\,d^3ex-2\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^9x^5-d^3e^8x^4-2\,d^4e^7x^3+2\,d^5e^6x^2+d^6e^5x-d^7e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 - 2*d*e^4*x^4 - 4*d^2*e^3*x^3 + 4*d^3*e^2*x^2 + 2*d^4*e*x
- 2*d^5 + (3*e^4*x^4 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + 2*d^3*e*x - 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^2*e^9*x^5 - d^3*e^8*x^4 - 2*d^4*e^7*x^3 + 2*d^5*e^6*x
^2 + d^6*e^5*x - d^7*e^4)
```

giac [A] time = 0.28, size = 58, normalized size = 0.64

$$\frac{\left(2\,d^3e^{(-4)}-\left(\frac{3\,x^3e}{d^2}+5\,de^{(-2)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*d^3*e^(-4) - (3*x^3*e/d^2 + 5*d*e^(-2))*x^2)*sqrt(-x^2*e^2 + d^2)/(
x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.86

$$-\frac{(-ex+d)\,(ex+d)^2\left(-3x^4e^4+3x^3d\,e^3-3d^2x^2e^2-2d^3xe+2d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^2*(-3*e^4*x^4+3*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*
d^4)/d^2/e^4/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.45, size = 134, normalized size = 1.49

$$\frac{x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{dx^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{3\,d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{2\,d^3}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}+\frac{x}{5\sqrt{-e^2x^2+d^2}\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) + 1/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2)
 - 3/10*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^3) - 2/15*d^3/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 1/10*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 1/5*x/(sqrt(-e^2*x^2 + d^2)
*d^2*e^3)
```

mupad [B] time = 2.66, size = 78, normalized size = 0.87

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4+2\,d^3\,e\,x+3\,d^2\,e^2\,x^2-3\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d^2\,e^4\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*e^4*x^4 - 2*d^4 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^2*e^4*(d + e*x)^2*(d - e*x)^3)
```

sympy [B] time = 20.55, size = 337, normalized size = 3.74

$$d\left(\begin{cases}-\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}}+\frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} & \text{for}\ e\neq 0\\ \frac{x^4}{4\left(d^2\right)^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}-\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x*
*2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)) + 5*e**2*x
**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*sqrt(d**2 - e*
*2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)
**(7/2)), True)) + e*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) -
 10*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2
/d**2) - 10*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt
(1 - e**2*x**2/d**2)), True))
```

3.24 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=94

$$\frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {796, 778, 191}

$$\frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^2*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (2*(d - e*x))/(15*d*e^3*(d^
2 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}} \, dx &= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{x\left(2d^2e-2de^2x\right)}{\left(d^2-e^2x^2\right)^{5/2}} \, dx}{5d^2e^2}\\
&= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}} \, dx}{15de^2}\\
&= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.87

$$\frac{-2d^4+2d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4}{15d^3e^3(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*e^4*x^4)/(15*d^3*e^3*
(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.52, size = 82, normalized size = 0.87

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^4+2d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4\right)}{15d^3e^3(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*
e^4*x^4))/(15*d^3*e^3*(d - e*x)^3*(d + e*x)^2)
```

fricas [B] time = 0.42, size = 173, normalized size = 1.84

$$-\frac{2\,e^5x^5-2\,de^4x^4-4\,d^2e^3x^3+4\,d^3e^2x^2+2\,d^4ex-2\,d^5-\left(2\,e^4x^4-2\,de^3x^3-3\,d^2e^2x^2-2\,d^3ex+2\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^8x^5-d^4e^7x^4-2\,d^5e^6x^3+2\,d^6e^5x^2+d^7e^4x-d^8e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 - 2*d*e^4*x^4 - 4*d^2*e^3*x^3 + 4*d^3*e^2*x^2 + 2*d^4*e*x
- 2*d^5 - (2*e^4*x^4 - 2*d*e^3*x^3 - 3*d^2*e^2*x^2 - 2*d^3*e*x + 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^3*e^8*x^5 - d^4*e^7*x^4 - 2*d^5*e^6*x^3 + 2*d^6*e^5*x
^2 + d^7*e^4*x - d^8*e^3)
```

giac [A] time = 0.27, size = 64, normalized size = 0.68

$$\frac{\left(\left(x\left(\frac{2\,x^2e^2}{d^3}-\frac{5}{d}\right)-5\,e^{(-1)}\right)x^2+2\,d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((x*(2*x^2*e^2/d^3 - 5/d) - 5*e^(-1))*x^2 + 2*d^2*e^(-3))*sqrt(-x^2*e^
2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.82

$$-\frac{(-ex+d)\,(ex+d)^2\left(2x^4e^4-2x^3d\,e^3-3d^2x^2e^2-2d^3xe+2d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^2*(2*e^4*x^4-2*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*d
^4)/d^3/e^3/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.44, size = 112, normalized size = 1.19

$$\frac{x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{2\,d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2}-\frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d*x/((-e^2*x^2 + d^2)^(5/2)*e^2) -
 2/15*d^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d*e
^2) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e^2)
```

mupad [B] time = 2.61, size = 78, normalized size = 0.83

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4+2\,d^3\,e\,x+3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d*e^3*x^3 - 2*e^4*x^4 - 2*d^4 + 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^3*e^3*(d + e*x)^2*(d - e*x)^3)
```

sympy [C] time = 21.31, size = 513, normalized size = 5.46

$$d\left(\begin{cases} -\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)+e\left(\begin{cases} -\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}}+\frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} & \text{for}\ e\neq 0 \\ \frac{x^4}{4\left(d^2\right)^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e**2*x**
2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/(15*d**9*sqrt(1 - e**2*x**2/d**2) -
 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e*
*2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e*
*2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**
2)), True)) + e*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 3
0*d**2*e**6*x**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**
2)) + 5*e**2*x**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*
sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (
x**4/(4*(d**2)**(7/2)), True))
```

3.25 $\int \frac{x(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {778, 192, 191}

$$-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)
) - (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}\\ &= \frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}-\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2e}\\ &= \frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 0.99

$$\frac{3d^4-3d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4}{15d^4e^2(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (3*d^4 - 3*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*e^4*x^4)/(15*d^4*e^2*(
d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.51, size = 82, normalized size = 0.99

$$\frac{\sqrt{d^2-e^2x^2}\left(3d^4-3d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4\right)}{15d^4e^2(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*e
^4*x^4))/(15*d^4*e^2*(d - e*x)^3*(d + e*x)^2)
```

fricas [B] time = 0.42, size = 172, normalized size = 2.07

$$\frac{3\,e^5x^5-3\,de^4x^4-6\,d^2e^3x^3+6\,d^3e^2x^2+3\,d^4ex-3\,d^5+\left(2\,e^4x^4-2\,de^3x^3-3\,d^2e^2x^2+3\,d^3ex-3\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^4e^7x^5-d^5e^6x^4-2\,d^6e^5x^3+2\,d^7e^4x^2+d^8e^3x-d^9e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x -
 3*d^5 + (2*e^4*x^4 - 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 3*d^3*e*x - 3*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^4*e^7*x^5 - d^5*e^6*x^4 - 2*d^6*e^5*x^3 + 2*d^7*e^4*x^
2 + d^8*e^3*x - d^9*e^2)
```

giac [A] time = 0.27, size = 57, normalized size = 0.69

$$\frac{\left(x^3\left(\frac{2\,x^2e^3}{d^4}-\frac{5\,e}{d^2}\right)-3\,de^{(-2)}\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^3*(2*x^2*e^3/d^4 - 5*e/d^2) - 3*d*e^(-2))*sqrt(-x^2*e^2 + d^2)/(x^2
*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.93

$$\frac{(-ex+d)\,(ex+d)^2\left(-2x^4e^4+2x^3d\,e^3+3d^2x^2e^2-3d^3xe+3d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}\,d^4e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(-2*e^4*x^4+2*d*e^3*x^3+3*d^2*e^2*x^2-3*d^3*e*x+3*d
^4)/d^4/e^2/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.44, size = 87, normalized size = 1.05

$$\frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e}-\frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/1
5*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^4*e)
```

mupad [B] time = 2.62, size = 78, normalized size = 0.94

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4-3\,d^3\,e\,x+3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^4\,e^2\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 - 2*e^4*x^4 + 2*d*e^3*x^3 + 3*d^2*e^2*x^2 - 3
*d^3*e*x))/(15*d^4*e^2*(d + e*x)^2*(d - e*x)^3)
```

sympy [A] time = 22.68, size = 432, normalized size = 5.20

$$d\left(\begin{cases}\frac{1}{5d^4e^2\sqrt{d^2-e^2x^2}-10d^2e^4x^2\sqrt{d^2-e^2x^2}+5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for}\: e\neq 0\\ \frac{x^2}{2\left(d^2\right)^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}-\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\: \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**4*x**2*sqrt
(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**2/
(2*(d**2)**(7/2)), True)) + e*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 +
e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x
**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/(15*d**
9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1
5*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt(1 -
e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4
*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

3.26 $\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=80

$$\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {639, 192, 191}

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/
2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3}\\
&= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 1.02

$$\frac{3d^4+12d^3ex-12d^2e^2x^2-8de^3x^3+8e^4x^4}{15d^5e(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (3*d^4 + 12*d^3*e*x - 12*d^2*e^2*x^2 - 8*d*e^3*x^3 + 8*e^4*x^4)/(15*d^5*e*(
d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.00, size = 82, normalized size = 1.02

$$\frac{\sqrt{d^2-e^2x^2}\left(3d^4+12d^3ex-12d^2e^2x^2-8de^3x^3+8e^4x^4\right)}{15d^5e(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 + 12*d^3*e*x - 12*d^2*e^2*x^2 - 8*d*e^3*x^3 + 8
*e^4*x^4))/(15*d^5*e*(d - e*x)^3*(d + e*x)^2)
```

fricas [B] time = 0.42, size = 171, normalized size = 2.14

$$\frac{3\,e^5x^5-3\,de^4x^4-6\,d^2e^3x^3+6\,d^3e^2x^2+3\,d^4ex-3\,d^5-\left(8\,e^4x^4-8\,de^3x^3-12\,d^2e^2x^2+12\,d^3ex+3\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^5e^6x^5-d^6e^5x^4-2\,d^7e^4x^3+2\,d^8e^3x^2+d^9e^2x-d^{10}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x -
 3*d^5 - (8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 + 12*d^3*e*x + 3*d^4)*sq
rt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 - d^6*e^5*x^4 - 2*d^7*e^4*x^3 + 2*d^8*e^3*
x^2 + d^9*e^2*x - d^10*e)
```

giac [A] time = 0.26, size = 65, normalized size = 0.81

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(4\,x^2\left(\frac{2\,x^2e^4}{d^5}-\frac{5\,e^2}{d^3}\right)+\frac{15}{d}\right)x+3\,e^{(-1)}\right)}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((4*x^2*(2*x^2*e^4/d^5 - 5*e^2/d^3) + 15/d)*x +
3*e^(-1))/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.96

$$\frac{(-ex+d)\,(ex+d)^2\left(8x^4e^4-8x^3d\,e^3-12d^2x^2e^2+12d^3xe+3d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(8*e^4*x^4-8*d*e^3*x^3-12*d^2*e^2*x^2+12*d^3*e*x+3*
d^4)/d^5/e/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.43, size = 80, normalized size = 1.00

$$\frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{4\,x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{8\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-e^2*x^2 + d^2)^(5/2)*d) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e) + 4/15*x/
((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

mupad [B] time = 2.58, size = 78, normalized size = 0.98

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4+12\,d^3\,e\,x-12\,d^2\,e^2\,x^2-8\,d\,e^3\,x^3+8\,e^4\,x^4\right)}{15\,d^5\,e\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 +
12*d^3*e*x))/(15*d^5*e*(d + e*x)^2*(d - e*x)^3)
```

sympy [C] time = 24.41, size = 604, normalized size = 7.55

$$d\left(\begin{cases} -\frac{15id^4x}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{20id^2e^2x^3}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{8ie^4x^5}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{15d^4x}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{20d^2e^2x^3}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{8e^4x^5}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)+e\left(\begin{cases} \frac{1}{5d^4e^2\sqrt{d^2-e^2x^2}-10d^2e^4x^2\sqrt{d^2-e^2x^2}+5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for } e\neq 0 \\ \frac{x^2}{2\left(d^2\right)^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d*
*2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2))
, Abs(e**2*x**2/d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/d**2) -
 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e*
*2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*
d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x
**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*
x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2))
, True)) + e*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**
4*x**2*sqrt(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e,
0)), (x**2/(2*(d**2)**(7/2)), True))
```

3.27 $\int \frac{d+ex}{x(d^2-e^2x^2)^{7/2}} \, dx$

Optimal. Leaf size=117

$$\frac{d+ex}{5d^2(d^2-e^2x^2)^{5/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{5d+4ex}{15d^4(d^2-e^2x^2)^{3/2}}$$

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {823, 12, 266, 63, 208}

$$\frac{d+ex}{5d^2(d^2-e^2x^2)^{5/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{5d+4ex}{15d^4(d^2-e^2x^2)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (5*d + 4*e*x)/(15*d^4*(d^2 - e^2*
x^2)^(3/2)) + (15*d + 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^
2 - e^2*x^2]/d]/d^6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rubi steps

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{5d^3e^2+4d^2e^3x}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^5e^4+8d^4e^5x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{15d^7e^6}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^5}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2 \right.}{2d^5}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2} \right.}{d^5e^2}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 1.12

$$\frac{23d^4-8d^3ex-27d^2e^2x^2-15(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)+7de^3x^3+8e^4x^4}{15d^6(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (23*d^4 - 8*d^3*e*x - 27*d^2*e^2*x^2 + 7*d*e^3*x^3 + 8*e^4*x^4 - 15*(d - e*
x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(15*d^6*
(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.72, size = 122, normalized size = 1.04

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{\sqrt{d^2-e^2x^2}\left(23d^4-8d^3ex-27d^2e^2x^2+7de^3x^3+8e^4x^4\right)}{15d^6(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(23*d^4 - 8*d^3*e*x - 27*d^2*e^2*x^2 + 7*d*e^3*x^3 + 8
*e^4*x^4))/(15*d^6*(d - e*x)^3*(d + e*x)^2) + (2*ArcTanh[(Sqrt[-e^2]*x)/d -
 Sqrt[d^2 - e^2*x^2]/d])/d^6
```

fricas [B] time = 0.42, size = 244, normalized size = 2.09

$$\frac{23\,e^5x^5 - 23\,de^4x^4 - 46\,d^2e^3x^3 + 46\,d^3e^2x^2 + 23\,d^4ex - 23\,d^5 + 15\left(e^5x^5 - de^4x^4 - 2\,d^2e^3x^3 + 2\,d^3e^2x^2 + d^4ex - d^5\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \left(8\,e^4x^4 + 7\,de^3x^3 - 27\,d^2e^2x^2 - 8\,d^3ex + 23\,d^4\right)\sqrt{-e^2x^2 + d^2}}{15\left(d^6e^5x^5 - d^7e^4x^4 - 2\,d^8e^3x^3 + 2\,d^9e^2x^2 + d^{10}ex - d^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^5*x^5 - 23*d*e^4*x^4 - 46*d^2*e^3*x^3 + 46*d^3*e^2*x^2 + 23*d^4*
e*x - 23*d^5 + 15*(e^5*x^5 - d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 + d^
4*e*x - d^5)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (8*e^4*x^4 + 7*d*e^3*x^3
- 27*d^2*e^2*x^2 - 8*d^3*e*x + 23*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*x^5 -
 d^7*e^4*x^4 - 2*d^8*e^3*x^3 + 2*d^9*e^2*x^2 + d^10*e*x - d^11)
```

giac [A] time = 0.27, size = 122, normalized size = 1.04

$$-\frac{\sqrt{-x^2e^2 + d^2}\left(\left(\left(\left(x\left(\frac{8\,xe^5}{d^6} + \frac{15\,e^4}{d^5}\right) - \frac{20\,e^3}{d^4}\right)x - \frac{35\,e^2}{d^3}\right)x + \frac{15\,e}{d^2}\right)x + \frac{23}{d}\right)}{15\left(x^2e^2 - d^2\right)^3} - \frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(8*x*e^5/d^6 + 15*e^4/d^5) - 20*e^3/d^4)*x
 - 35*e^2/d^3)*x + 15*e/d^2)*x + 23/d)/(x^2*e^2 - d^2)^3 - log(1/2*abs(-2*d
*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6
```

maple [A] time = 0.01, size = 163, normalized size = 1.39

$$\frac{ex}{5\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}d^2} + \frac{1}{5\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}d} + \frac{4ex}{15\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}d^4} + \frac{1}{3\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}d^3} - \frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^5} + \frac{8ex}{15\sqrt{-e^2x^2 + d^2}\,d^6} + \frac{1}{\sqrt{-e^2x^2 + d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*e*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/15*e/d^4*x/(-e^2*x^2+d^2)^(3/2)+8/15*e/d
^6*x/(-e^2*x^2+d^2)^(1/2)+1/5/d/(-e^2*x^2+d^2)^(5/2)+1/3/d^3/(-e^2*x^2+d^2)
^(3/2)+1/d^5/(-e^2*x^2+d^2)^(1/2)-1/d^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)
*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.45, size = 157, normalized size = 1.34

$$\frac{ex}{5\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}d^2} + \frac{1}{5\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}d} + \frac{4\,ex}{15\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}d^4} + \frac{1}{3\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}d^3} + \frac{8\,ex}{15\,\sqrt{-e^2x^2 + d^2}\,d^6} - \frac{\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d^6} + \frac{1}{\sqrt{-e^2x^2 + d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*e*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 1/5/((-e^2*x^2 + d^2)^(5/2)*d) + 4/1
5*e*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/1
5*e*x/(sqrt(-e^2*x^2 + d^2)*d^6) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2
)*d/abs(x))/d^6 + 1/(sqrt(-e^2*x^2 + d^2)*d^5)
```

mupad [B] time = 3.08, size = 127, normalized size = 1.09

$$\frac{\frac{d^2-e^2x^2}{3d^3}+\frac{\left(d^2-e^2x^2\right)^2}{d^5}+\frac{1}{5d}}{\left(d^2-e^2x^2\right)^{5/2}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{e\,x\,\left(15\,d^4-20\,d^2\,e^2\,x^2+8\,e^4\,x^4\right)}{15\,d^6\left(d^2-e^2\,x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] ((d^2 - e^2*x^2)/(3*d^3) + (d^2 - e^2*x^2)^2/d^5 + 1/(5*d))/(d^2 - e^2*x^2)
^(5/2) - atanh((d^2 - e^2*x^2)^(1/2)/d)/d^6 + (e*x*(15*d^4 + 8*e^4*x^4 - 20
*d^2*e^2*x^2))/(15*d^6*(d^2 - e^2*x^2)^(5/2))
```

sympy [C] time = 41.14, size = 2378, normalized size = 20.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*
x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 15*d**6*log(e**2*x**2/d**2)
/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) -
30*d**6*log(e*x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*
d**7*e**6*x**6) + 30*I*d**6*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2 +
90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 70*I*d**4*e**2*x**2*sqrt(-1 + e**2
*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e*
*6*x**6) - 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*
x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 90*d**4*e**2*x**2*log(e*x/d
)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) -
 90*I*d**4*e**2*x**2*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9
*e**4*x**4 - 30*d**7*e**6*x**6) + 30*I*d**2*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6
) + 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 +
90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 90*d**2*e**4*x**4*log(e*x/d)/(30*d
**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 90*I*d
**2*e**4*x**4*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x
**4 - 30*d**7*e**6*x**6) - 15*e**6*x**6*log(e**2*x**2/d**2)/(30*d**13 - 90*
d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*e**6*x**6*log
(e*x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x
**6) - 30*I*e**6*x**6*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**
9*e**4*x**4 - 30*d**7*e**6*x**6), Abs(e**2*x**2/d**2) > 1), (46*d**6*sqrt(1
 - e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*
d**7*e**6*x**6) + 15*d**6*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**
2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 30*d**6*log(sqrt(1 - e**2*x**2
/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e*
*6*x**6) + 15*I*pi*d**6/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4
- 30*d**7*e**6*x**6) - 70*d**4*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(30*d**13
 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 45*d**4*e*
*2*x**2*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x
**4 - 30*d**7*e**6*x**6) + 90*d**4*e**2*x**2*log(sqrt(1 - e**2*x**2/d**2) +
 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6)
 - 45*I*pi*d**4*e**2*x**2/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**
4 - 30*d**7*e**6*x**6) + 30*d**2*e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(30*d**
13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 45*d**2*
e**4*x**4*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4
*x**4 - 30*d**7*e**6*x**6) - 90*d**2*e**4*x**4*log(sqrt(1 - e**2*x**2/d**2)
 + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**
6) + 45*I*pi*d**2*e**4*x**4/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x
```

```
**4 - 30*d**7*e**6*x**6) - 15*e**6*x**6*log(e**2*x**2/d**2)/(30*d**13 - 90*
d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*e**6*x**6*log
(sqrt(1 - e**2*x**2/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**
4*x**4 - 30*d**7*e**6*x**6) - 15*I*pi*e**6*x**6/(30*d**13 - 90*d**11*e**2*x
**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6), True)) + e*Piecewise((-15*I*d
**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(-1 + e**
2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)) + 20*I*d**2*e**
2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(-1 + e*
*2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)) - 8*I*e**4*x**
5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(-1 + e**2*x*
*2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2
) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*x**2*s
qrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) - 20*
d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(
1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) + 8*e**4*
x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x
**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

3.28 $\int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=153

$$\frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {823, 807, 266, 63, 208}

$$\frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*x*(d^2 - e^2*x^2)^(5/2)) + (6*d + 5*e*x)/(15*d^4*x*(d^2 -
e^2*x^2)^(3/2)) + (8*d + 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[d^
2 - e^2*x^2])/(5*d^7*x) - (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{6d^3e^2+5d^2e^3x}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{24d^5e^4+15d^4e^5x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} + \frac{\int \frac{48d^7e^6+15d^6e^7x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} + \frac{e\int}{}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} + \frac{e\,\mathrm{S}}{}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} - \frac{\mathrm{Su}}{}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} - \frac{e\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.07, size = 147, normalized size = 0.96

$$\frac{-15d^5+38d^4ex+52d^3e^2x^2-87d^2e^3x^3-15ex(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-33de^4x^4+48e^5x^5}{15d^7x(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-15*d^5 + 38*d^4*e*x + 52*d^3*e^2*x^2 - 87*d^2*e^3*x^3 - 33*d*e^4*x^4 + 48
*e^5*x^5 - 15*e*x*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^
2 - e^2*x^2]/d])/(15*d^7*x*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.61, size = 137, normalized size = 0.90

$$\frac{2e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{\sqrt{d^2-e^2x^2}\left(-15d^5+38d^4ex+52d^3e^2x^2-87d^2e^3x^3-33de^4x^4+48e^5x^5\right)}{15d^7x(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-15*d^5 + 38*d^4*e*x + 52*d^3*e^2*x^2 - 87*d^2*e^3*x^
3 - 33*d*e^4*x^4 + 48*e^5*x^5))/(15*d^7*x*(d - e*x)^3*(d + e*x)^2) + (2*e*A
rcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^7
```

fricas [A] time = 0.44, size = 270, normalized size = 1.76

$$\frac{23\,e^{6}x^{6} - 23\,de^{5}x^{5} - 46\,d^{2}e^{4}x^{4} + 46\,d^{3}e^{3}x^{3} + 23\,d^{4}e^{2}x^{2} - 23\,d^{5}ex + 15\left(e^{6}x^{6} - de^{5}x^{5} - 2\,d^{2}e^{4}x^{4} + 2\,d^{3}e^{3}x^{3} + d^{4}e^{2}x^{2} - d^{5}ex\right)\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right) - \left(48\,e^{5}x^{5} - 33\,de^{4}x^{4} - 87\,d^{2}e^{3}x^{3} + 52\,d^{3}e^{2}x^{2} + 38\,d^{4}ex - 15\,d^{5}\right)\sqrt{-e^{2}x^{2} + d^{2}}}{15\left(d^{7}e^{5}x^{6} - d^{8}e^{4}x^{5} - 2\,d^{9}e^{3}x^{4} + 2\,d^{10}e^{2}x^{3} + d^{11}ex^{2} - d^{12}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^6*x^6 - 23*d*e^5*x^5 - 46*d^2*e^4*x^4 + 46*d^3*e^3*x^3 + 23*d^4*
e^2*x^2 - 23*d^5*e*x + 15*(e^6*x^6 - d*e^5*x^5 - 2*d^2*e^4*x^4 + 2*d^3*e^3*
x^3 + d^4*e^2*x^2 - d^5*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (48*e^5*x
^5 - 33*d*e^4*x^4 - 87*d^2*e^3*x^3 + 52*d^3*e^2*x^2 + 38*d^4*e*x - 15*d^5)*
sqrt(-e^2*x^2 + d^2))/(d^7*e^5*x^6 - d^8*e^4*x^5 - 2*d^9*e^3*x^4 + 2*d^10*e
^2*x^3 + d^11*e*x^2 - d^12*x)
```

giac [A] time = 0.29, size = 189, normalized size = 1.24

$$-\frac{\sqrt{-x^{2}e^{2} + d^{2}}\left(\left(\left(3\left(x\left(\frac{11\,xe^{6}}{d^{7}} + \frac{5\,e^{5}}{d^{6}}\right) - \frac{25\,e^{4}}{d^{5}}\right)x - \frac{35\,e^{3}}{d^{4}}\right)x + \frac{45\,e^{2}}{d^{3}}\right)x + \frac{23\,e}{d^{2}}\right)}{15\left(x^{2}e^{2} - d^{2}\right)^{3}} - \frac{e\log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^{7}} + \frac{xe^{3}}{2\left(de + \sqrt{-x^{2}e^{2} + d^{2}}\,e\right)d^{7}} - \frac{\left(de + \sqrt{-x^{2}e^{2} + d^{2}}\,e\right)e^{(-1)}}{2\,d^{7}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(((3*(x*(11*x*e^6/d^7 + 5*e^5/d^6) - 25*e^4/d^5)
*x - 35*e^3/d^4)*x + 45*e^2/d^3)*x + 23*e/d^2)/(x^2*e^2 - d^2)^3 - e*log(1/
2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^7 + 1/2*x*e^3/((d
*e + sqrt(-x^2*e^2 + d^2)*e)*d^7) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-
1)/(d^7*x)
```

maple [A] time = 0.02, size = 195, normalized size = 1.27

$$\frac{6e^{2}x}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}} + \frac{e}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}} - \frac{1}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}dx} + \frac{8e^{2}x}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{5}} + \frac{e}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{4}} - \frac{e\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}\,d^{6}} + \frac{16e^{2}x}{5\sqrt{-e^{2}x^{2}+d^{2}}\,d^{7}} + \frac{e}{\sqrt{-e^{2}x^{2}+d^{2}}\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*e/d^2/(-e^2*x^2+d^2)^(5/2)+1/3*e/d^4/(-e^2*x^2+d^2)^(3/2)+e/d^6/(-e^2*x
^2+d^2)^(1/2)-e/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2
))/x)-1/d/x/(-e^2*x^2+d^2)^(5/2)+6/5*e^2/d^3*x/(-e^2*x^2+d^2)^(5/2)+8/5*e^2
/d^5*x/(-e^2*x^2+d^2)^(3/2)+16/5*e^2/d^7*x/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 0.46, size = 189, normalized size = 1.24

$$\frac{6\,e^{2}x}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}} + \frac{e}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}} + \frac{8\,e^{2}x}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{5}} + \frac{e}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{4}} - \frac{1}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}dx} + \frac{16\,e^{2}x}{5\,\sqrt{-e^{2}x^{2}+d^{2}}\,d^{7}} - \frac{e\log\left(\frac{2\,d^{2}}{|x|} + \frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{d^{7}} + \frac{e}{\sqrt{-e^{2}x^{2}+d^{2}}\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 6/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d^3) + 1/5*e/((-e^2*x^2 + d^2)^(5/2)*d^2)
 + 8/5*e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^5) + 1/3*e/((-e^2*x^2 + d^2)^(3/2)*d
^4) - 1/((-e^2*x^2 + d^2)^(5/2)*d*x) + 16/5*e^2*x/(sqrt(-e^2*x^2 + d^2)*d^7
) - e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^7 + e/(sqrt(-e^
2*x^2 + d^2)*d^6)
```

mupad [B] time = 3.31, size = 141, normalized size = 0.92

$$\frac{\frac{e}{5\,d^2}+\frac{e\left(d^2-e^2\,x^2\right)^2}{d^6}+\frac{e\left(d^2-e^2\,x^2\right)}{3\,d^4}}{\left(d^2-e^2\,x^2\right)^{5/2}}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{d^7}-\frac{d^6-6\,d^4\,e^2\,x^2+8\,d^2\,e^4\,x^4-\frac{16\,e^6\,x^6}{5}}{d^7\,x\left(d^2-e^2\,x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] (e/(5*d^2) + (e*(d^2 - e^2*x^2)^2)/d^6 + (e*(d^2 - e^2*x^2))/(3*d^4))/(d^2
- e^2*x^2)^(5/2) - (e*atanh((d^2 - e^2*x^2)^(1/2)/d))/d^7 - (d^6 - (16*e^6*
x^6)/5 - 6*d^4*e^2*x^2 + 8*d^2*e^4*x^4)/(d^7*x*(d^2 - e^2*x^2)^(5/2))
```

sympy [C] time = 31.22, size = 2404, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((5*d**6*e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*
x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d**2
/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d
**8*e**6*x**6) + 40*d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 1
5*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*s
qrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x*
*4 + 5*d**8*e**6*x**6), Abs(d**2/(e**2*x**2)) > 1), (5*I*d**6*e*sqrt(-d**2/
(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d*
*8*e**6*x**6) - 30*I*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 +
 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*I*d**2*e*
*5*x**4*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**
10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**2*x**2) +
1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6),
 True)) + e*Piecewise((46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(30*d**13 - 90*d
**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 15*d**6*log(e**2*
x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**
6*x**6) - 30*d**6*log(e*x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*
x**4 - 30*d**7*e**6*x**6) + 30*I*d**6*asin(d/(e*x))/(30*d**13 - 90*d**11*e*
*2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 70*I*d**4*e**2*x**2*sqrt
(-1 + e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 -
30*d**7*e**6*x**6) - 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(30*d**13 - 90*d
**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 90*d**4*e**2*x**2
*log(e*x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e*
*6*x**6) - 90*I*d**4*e**2*x**2*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2
 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*I*d**2*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7
*e**6*x**6) + 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e*
*2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 90*d**2*e**4*x**4*log(e*
x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6
) + 90*I*d**2*e**4*x**4*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**2 + 90*d
**9*e**4*x**4 - 30*d**7*e**6*x**6) - 15*e**6*x**6*log(e**2*x**2/d**2)/(30*d
**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*e**
6*x**6*log(e*x/d)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d
**7*e**6*x**6) - 30*I*e**6*x**6*asin(d/(e*x))/(30*d**13 - 90*d**11*e**2*x**
2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6), Abs(e**2*x**2/d**2) > 1), (46*d
**6*sqrt(1 - e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*
x**4 - 30*d**7*e**6*x**6) + 15*d**6*log(e**2*x**2/d**2)/(30*d**13 - 90*d**1
1*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 30*d**6*log(sqrt(1 -
 e**2*x**2/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 -
```

```
30*d**7*e**6*x**6) + 15*I*pi*d**6/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*
e**4*x**4 - 30*d**7*e**6*x**6) - 70*d**4*e**2*x**2*sqrt(1 - e**2*x**2/d**2)
/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) -
45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d
**9*e**4*x**4 - 30*d**7*e**6*x**6) + 90*d**4*e**2*x**2*log(sqrt(1 - e**2*x*
*2/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*
e**6*x**6) - 45*I*pi*d**4*e**2*x**2/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**
9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*d**2*e**4*x**4*sqrt(1 - e**2*x**2/d**
2)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6)
+ 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(30*d**13 - 90*d**11*e**2*x**2 + 90
*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 90*d**2*e**4*x**4*log(sqrt(1 - e**2*
x**2/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**
7*e**6*x**6) + 45*I*pi*d**2*e**4*x**4/(30*d**13 - 90*d**11*e**2*x**2 + 90*d
**9*e**4*x**4 - 30*d**7*e**6*x**6) - 15*e**6*x**6*log(e**2*x**2/d**2)/(30*d
**13 - 90*d**11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6) + 30*e**
6*x**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(30*d**13 - 90*d**11*e**2*x**2 + 9
0*d**9*e**4*x**4 - 30*d**7*e**6*x**6) - 15*I*pi*e**6*x**6/(30*d**13 - 90*d*
*11*e**2*x**2 + 90*d**9*e**4*x**4 - 30*d**7*e**6*x**6), True))
```

3.29 $\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=184

$$\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d+6}{15d^4x^2\left(d^2-\right.}$$

Rubi [A] time = 0.16, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {823, 835, 807, 266, 63, 208}

$$-\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*x^2*(d^2 - e^2*x^2)^(5/2)) + (7*d + 6*e*x)/(15*d^4*x^2*(d^
2 - e^2*x^2)^(3/2)) + (35*d + 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) - (7
*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) - (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x) -
(7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{d+ex}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{7d^3e^2+6d^2e^3x}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{35d^5e^4+24d^4e^5x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} + \frac{\int \frac{105d^7e^6+48d^6e^7x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - \int$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - 16$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - 16$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - 16$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - 16$$

Mathematica [A] time = 0.13, size = 183, normalized size = 0.99

$$\frac{105e^2x^2(d+ex)^2(ex-d)^3\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)+d\sqrt{1-\frac{e^2x^2}{d^2}}\left(-15d^6-15d^5ex+176d^4e^2x^2+4d^3e^3x^3-249d^2e^4x^4+9de^5x^5+96e^6x^6\right)}{30d^9x^2(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d*Sqrt[1 - (e^2*x^2)/d^2]*(-15*d^6 - 15*d^5*e*x + 176*d^4*e^2*x^2 + 4*d^3*
e^3*x^3 - 249*d^2*e^4*x^4 + 9*d*e^5*x^5 + 96*e^6*x^6) + 105*e^2*x^2*(-d + e
*x)^3*(d + e*x)^2*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]])/(30*d^9*x^2*(d - e*x)^2
*(d + e*x)*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.81, size = 150, normalized size = 0.82

$$\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}+\frac{\sqrt{d^2-e^2x^2}\left(-15d^6-15d^5ex+176d^4e^2x^2+4d^3e^3x^3-249d^2e^4x^4+9de^5x^5+96e^6x^6\right)}{30d^8x^2(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-15*d^6 - 15*d^5*e*x + 176*d^4*e^2*x^2 + 4*d^3*e^3*x^
3 - 249*d^2*e^4*x^4 + 9*d*e^5*x^5 + 96*e^6*x^6))/(30*d^8*x^2*(d - e*x)^3*(d
 + e*x)^2) + (7*e^2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^8
```

fricas [A] time = 0.51, size = 291, normalized size = 1.58

$$\frac{116\,e^7x^7-116\,de^6x^6-232\,d^2e^5x^5+232\,d^3e^4x^4+116\,d^4e^3x^3-116\,d^5e^2x^2+105\left(e^7x^7-de^6x^6-2\,d^2e^5x^5+2\,d^3e^4x^4+d^4e^3x^3-d^5e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(96\,e^6x^6+9\,de^5x^5-249\,d^2e^4x^4+4\,d^3e^3x^3+176\,d^4e^2x^2-15\,d^5ex-15\,d^6\right)\sqrt{-e^2x^2+d^2}}{30\left(d^8e^5x^7-d^9e^4x^6-2\,d^{10}e^3x^5+2\,d^{11}e^2x^4+d^{12}ex^3-d^{13}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(116*e^7*x^7 - 116*d*e^6*x^6 - 232*d^2*e^5*x^5 + 232*d^3*e^4*x^4 + 116
*d^4*e^3*x^3 - 116*d^5*e^2*x^2 + 105*(e^7*x^7 - d*e^6*x^6 - 2*d^2*e^5*x^5 +
 2*d^3*e^4*x^4 + d^4*e^3*x^3 - d^5*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2))
/x) - (96*e^6*x^6 + 9*d*e^5*x^5 - 249*d^2*e^4*x^4 + 4*d^3*e^3*x^3 + 176*d^4
*e^2*x^2 - 15*d^5*e*x - 15*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^5*x^7 - d^9*e^
4*x^6 - 2*d^10*e^3*x^5 + 2*d^11*e^2*x^4 + d^12*e*x^3 - d^13*x^2)
```

giac [A] time = 0.36, size = 260, normalized size = 1.41

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(3\left(x\left(\frac{11\,xe^7}{d^8}+\frac{15\,e^6}{d^7}\right)-\frac{25\,e^5}{d^6}\right)x-\frac{100\,e^4}{d^5}\right)x+\frac{45\,e^3}{d^4}\right)x+\frac{58\,e^2}{d^3}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{7\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^8}+\frac{x^2\left(\frac{4\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^4}{x}+e^6\right)}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^8}-\frac{\left(\frac{4\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^8e^8}{x}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^8e^6}{x^2}\right)e^{(-8)}}{8\,d^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(((3*(x*(11*x*e^7/d^8 + 15*e^6/d^7) - 25*e^5/d^6
)*x - 100*e^4/d^5)*x + 45*e^3/d^4)*x + 58*e^2/d^3)/(x^2*e^2 - d^2)^3 - 7/2*
e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^8 + 1/8
*x^2*(4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*d^8) - 1/8*(4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^8*e^8/x + (d*e +
sqrt(-x^2*e^2 + d^2)*e)^2*d^8*e^6/x^2)*e^(-8)/d^16
```

maple [A] time = 0.02, size = 227, normalized size = 1.23

$$\frac{6e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}+\frac{7e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}-\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}+\frac{8e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}-\frac{1}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,x^2}+\frac{7e^2}{6\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}-\frac{7e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^7}+\frac{16e^3x}{5\sqrt{-e^2x^2+d^2}\,d^8}+\frac{7e^2}{2\sqrt{-e^2x^2+d^2}\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/2/d/x^2/(-e^2*x^2+d^2)^(5/2)+7/10*e^2/d^3/(-e^2*x^2+d^2)^(5/2)+7/6*e^2/d
^5/(-e^2*x^2+d^2)^(3/2)+7/2*e^2/d^7/(-e^2*x^2+d^2)^(1/2)-7/2*e^2/d^7/(d^2)^
(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-e/d^2/x/(-e^2*x^2+d^
```

```
2)^(5/2)+6/5*e^3/d^4*x/(-e^2*x^2+d^2)^(5/2)+8/5*e^3/d^6*x/(-e^2*x^2+d^2)^(3
/2)+16/5*e^3/d^8*x/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 0.47, size = 221, normalized size = 1.20

$$\frac{6\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}+\frac{7\,e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}+\frac{8\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}+\frac{7\,e^2}{6\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}-\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}+\frac{16\,e^3x}{5\sqrt{-e^2x^2+d^2}\,d^8}-\frac{7\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{2\,d^8}+\frac{7\,e^2}{2\sqrt{-e^2x^2+d^2}\,d^7}-\frac{1}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 6/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^4) + 7/10*e^2/((-e^2*x^2 + d^2)^(5/2)*d
^3) + 8/5*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^6) + 7/6*e^2/((-e^2*x^2 + d^2)^(3
/2)*d^5) - e/((-e^2*x^2 + d^2)^(5/2)*d^2*x) + 16/5*e^3*x/(sqrt(-e^2*x^2 + d
^2)*d^8) - 7/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^8
+ 7/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^7) - 1/2/((-e^2*x^2 + d^2)^(5/2)*d*x^2)
```

mupad [B] time = 3.43, size = 181, normalized size = 0.98

$$\frac{161\,e^2}{30\,d^3\left(d^2-e^2\,x^2\right)^{5/2}}-\frac{1}{2\,d\,x^2\left(d^2-e^2\,x^2\right)^{5/2}}-\frac{7\,e^2\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{2\,d^8}-\frac{49\,e^4\,x^2}{6\,d^5\left(d^2-e^2\,x^2\right)^{5/2}}+\frac{7\,e^6\,x^4}{2\,d^7\left(d^2-e^2\,x^2\right)^{5/2}}-\frac{e\left(5\,d^6-30\,d^4\,e^2\,x^2+40\,d^2\,e^4\,x^4-16\,e^6\,x^6\right)}{5\,d^8\,x\left(d^2-e^2\,x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] (161*e^2)/(30*d^3*(d^2 - e^2*x^2)^(5/2)) - 1/(2*d*x^2*(d^2 - e^2*x^2)^(5/2)
) - (7*e^2*atanh((d^2 - e^2*x^2)^(1/2)/d))/(2*d^8) - (49*e^4*x^2)/(6*d^5*(d
^2 - e^2*x^2)^(5/2)) + (7*e^6*x^4)/(2*d^7*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d^
6 - 16*e^6*x^6 - 30*d^4*e^2*x^2 + 40*d^2*e^4*x^4))/(5*d^8*x*(d^2 - e^2*x^2)
^(5/2))
```

sympy [C] time = 35.06, size = 2691, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-30*I*d**8*sqrt(-1 + e**2*x**2/d**2)/(60*d**15*x**2 - 180*d**1
3*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) + 322*I*d**6*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2)/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**
11*e**4*x**6 - 60*d**9*e**6*x**8) + 105*d**6*e**2*x**2*log(e**2*x**2/d**2)/
(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x
**8) - 210*d**6*e**2*x**2*log(e*x/d)/(60*d**15*x**2 - 180*d**13*e**2*x**4 +
 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) + 210*I*d**6*e**2*x**2*asin(d/(e*
x))/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e*
*6*x**8) - 490*I*d**4*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(60*d**15*x**2 -
180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) - 315*d**4*e
**4*x**4*log(e**2*x**2/d**2)/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**
11*e**4*x**6 - 60*d**9*e**6*x**8) + 630*d**4*e**4*x**4*log(e*x/d)/(60*d**15
*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) - 63
0*I*d**4*e**4*x**4*asin(d/(e*x))/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180
*d**11*e**4*x**6 - 60*d**9*e**6*x**8) + 210*I*d**2*e**6*x**6*sqrt(-1 + e**2
*x**2/d**2)/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60
*d**9*e**6*x**8) + 315*d**2*e**6*x**6*log(e**2*x**2/d**2)/(60*d**15*x**2 -
180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) - 630*d**2*e
**6*x**6*log(e*x/d)/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x
**6 - 60*d**9*e**6*x**8) + 630*I*d**2*e**6*x**6*asin(d/(e*x))/(60*d**15*x**
2 - 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) - 105*e*
*8*x**8*log(e**2*x**2/d**2)/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**1
```

```
1*e**4*x**6 - 60*d**9*e**6*x**8) + 210*e**8*x**8*log(e*x/d)/(60*d**15*x**2
- 180*d**13*e**2*x**4 + 180*d**11*e**4*x**6 - 60*d**9*e**6*x**8) - 210*I*e*
*8*x**8*asin(d/(e*x))/(60*d**15*x**2 - 180*d**13*e**2*x**4 + 180*d**11*e**4
*x**6 - 60*d**9*e**6*x**8), Abs(e**2*x**2/d**2) > 1), (30*d**8*sqrt(1 - e**
2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 +
60*d**9*e**6*x**8) - 322*d**6*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-60*d**15
*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 10
5*d**6*e**2*x**2*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4
- 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 210*d**6*e**2*x**2*log(sqrt(1
- e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e*
*4*x**6 + 60*d**9*e**6*x**8) - 105*I*pi*d**6*e**2*x**2/(-60*d**15*x**2 + 18
0*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 490*d**4*e**
4*x**4*sqrt(1 - e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180
*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 315*d**4*e**4*x**4*log(e**2*x**2/d*
*2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e
**6*x**8) - 630*d**4*e**4*x**4*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15
*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 31
5*I*pi*d**4*e**4*x**4/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**
4*x**6 + 60*d**9*e**6*x**8) - 210*d**2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(
-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x
**8) - 315*d**2*e**6*x**6*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e
**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 630*d**2*e**6*x**6*lo
g(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180
*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 315*I*pi*d**2*e**6*x**6/(-60*d**15*
x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 105
*e**8*x**8*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*
d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*e**8*x**8*log(sqrt(1 - e**2*x**2
/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 6
0*d**9*e**6*x**8) + 105*I*pi*e**8*x**8/(-60*d**15*x**2 + 180*d**13*e**2*x**
4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8), True)) + e*Piecewise((5*d**6*
e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4
*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d**2/(e**2*x**2) - 1)/(-
5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*
d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 -
15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*sqrt(d**2/(e**2*x**2)
 - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**
6), Abs(d**2/(e**2*x**2)) > 1), (5*I*d**6*e*sqrt(-d**2/(e**2*x**2) + 1)/(-5
*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*I
*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2
- 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*I*d**2*e**5*x**4*sqrt(-d**2/(
e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**
8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d*
*12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6), True))
```

3.30 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx$

Optimal. Leaf size=121

$$\frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}}-\frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {796, 778, 192, 191}

$$\frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

```
[Out] (x^2*(d + e*x))/(7*d*e*(d^2 - e^2*x^2)^(7/2)) - (2*(d - 2*e*x))/(35*d*e^3*(
d^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(
105*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx = \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{\int \frac{x\left(2d^2e-4de^2x\right)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7d^2e^2}$$

$$= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35de^2}$$

$$= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{105d^3e^2}$$

$$= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.04, size = 104, normalized size = 0.86

$$\frac{-6d^6+6d^5ex+15d^4e^2x^2+20d^3e^3x^3-20d^2e^4x^4-8de^5x^5+8e^6x^6}{105d^5e^3(d-ex)^3(d+ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

```
[Out] (-6*d^6 + 6*d^5*e*x + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 - 20*d^2*e^4*x^4 - 8*
d*e^5*x^5 + 8*e^6*x^6)/(105*d^5*e^3*(d - e*x)^3*(d + e*x)^2*Sqrt[d^2 - e^2*
x^2])
```

IntegrateAlgebraic [A] time = 0.55, size = 104, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\left(-6d^6+6d^5ex+15d^4e^2x^2+20d^3e^3x^3-20d^2e^4x^4-8de^5x^5+8e^6x^6\right)}{105d^5e^3(d-ex)^4(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^6 + 6*d^5*e*x + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3
- 20*d^2*e^4*x^4 - 8*d*e^5*x^5 + 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^4*(d +
e*x)^3)
```

fricas [B] time = 0.47, size = 239, normalized size = 1.98

$$-\frac{6e^7x^7-6de^6x^6-18d^2e^5x^5+18d^3e^4x^4+18d^4e^3x^3-18d^5e^2x^2-6d^6ex+6d^7-\left(8e^6x^6-8de^5x^5-20d^2e^4x^4+20d^3e^3x^3+15d^4e^2x^2+6d^5ex-6d^6\right)\sqrt{-e^2x^2+d^2}}{105\left(d^5e^{10}x^7-d^6e^9x^6-3d^7e^8x^5+3d^8e^7x^4+3d^9e^6x^3-3d^{10}e^5x^2-d^{11}e^4x+d^{12}e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(6*e^7*x^7 - 6*d*e^6*x^6 - 18*d^2*e^5*x^5 + 18*d^3*e^4*x^4 + 18*d^4*
e^3*x^3 - 18*d^5*e^2*x^2 - 6*d^6*e*x + 6*d^7 - (8*e^6*x^6 - 8*d*e^5*x^5 - 2
0*d^2*e^4*x^4 + 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 + 6*d^5*e*x - 6*d^6)*sqrt(-
e^2*x^2 + d^2))/(d^5*e^10*x^7 - d^6*e^9*x^6 - 3*d^7*e^8*x^5 + 3*d^8*e^7*x^4
 + 3*d^9*e^6*x^3 - 3*d^10*e^5*x^2 - d^11*e^4*x + d^12*e^3)
```

giac [A] time = 0.27, size = 77, normalized size = 0.64

$$\frac{\left(\left(\left(4\,x^2\left(\frac{2\,x^2e^4}{d^5}-\frac{7\,e^2}{d^3}\right)+\frac{35}{d}\right)x+21\,e^{(-1)}\right)x^2-6\,d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{105\left(x^2e^2-d^2\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*x^2*e^4/d^5 - 7*e^2/d^3) + 35/d)*x + 21*e^(-1))*x^2 - 6*d
^2*e^(-3))*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^4
```

maple [A] time = 0.01, size = 99, normalized size = 0.82

$$-\frac{(-ex+d)\,(ex+d)^2\left(-8e^6x^6+8e^5x^5d+20e^4x^4d^2-20x^3d^3e^3-15x^2d^4e^2-6x\,d^5e+6d^6\right)}{105\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x)
```

```
[Out] -1/105*(-e*x+d)*(e*x+d)^2*(-8*e^6*x^6+8*d*e^5*x^5+20*d^2*e^4*x^4-20*d^3*e^3
*x^3-15*d^4*e^2*x^2-6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(9/2)
```

maxima [A] time = 0.44, size = 135, normalized size = 1.12

$$\frac{x^2}{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}+\frac{dx}{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}-\frac{2\,d^2}{35\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}-\frac{x}{35\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2}-\frac{4\,x}{105\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2}-\frac{8\,x}{105\,\sqrt{-e^2x^2+d^2}\,d^5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^2/((-e^2*x^2 + d^2)^(7/2)*e) + 1/7*d*x/((-e^2*x^2 + d^2)^(7/2)*e^2) -
 2/35*d^2/((-e^2*x^2 + d^2)^(7/2)*e^3) - 1/35*x/((-e^2*x^2 + d^2)^(5/2)*d*e
^2) - 4/105*x/((-e^2*x^2 + d^2)^(3/2)*d^3*e^2) - 8/105*x/(sqrt(-e^2*x^2 + d
^2)*d^5*e^2)
```

mupad [B] time = 2.69, size = 164, normalized size = 1.36

$$\frac{\sqrt{d^2-e^2\,x^2}}{56\,d^2\,e^3\,(d-e\,x)^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2}{35\,e^3}-\frac{3\,x}{70\,d\,e^2}\right)}{(d+e\,x)^3\,(d-e\,x)^3}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d^2\,e^3}+\frac{4\,x}{105\,d^3\,e^2}\right)}{(d+e\,x)^2\,(d-e\,x)^2}-\frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{105\,d^5\,e^2\,(d+e\,x)\,(d-e\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(56*d^2*e^3*(d - e*x)^4) - ((d^2 - e^2*x^2)^(1/2)*(2/
(35*e^3) - (3*x)/(70*d*e^2)))/((d + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^
(1/2)*(1/(56*d^2*e^3) + (4*x)/(105*d^3*e^2)))/((d + e*x)^2*(d - e*x)^2) - (
8*x*(d^2 - e^2*x^2)^(1/2))/(105*d^5*e^2*(d + e*x)*(d - e*x))
```

sympy [C] time = 22.73, size = 903, normalized size = 7.46

$$d\left(\begin{cases}\frac{35id^4x^3}{-105d^{13}\sqrt{-1+\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{28id^2e^2x^5}{-105d^{13}\sqrt{-1+\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{8ie^4x^7}{-105d^{13}\sqrt{-1+\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{35d^4x^3}{-105d^{13}\sqrt{1-\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{28d^2e^2x^5}{-105d^{13}\sqrt{1-\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{8e^4x^7}{-105d^{13}\sqrt{1-\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}\frac{2d^2}{-35d^6e^4\sqrt{d^2-e^2x^2}+105d^4e^6x^2\sqrt{d^2-e^2x^2}-105d^2e^8x^4\sqrt{d^2-e^2x^2}+35e^{10}x^6\sqrt{d^2-e^2x^2}}-\frac{7e^2x^2}{-35d^6e^4\sqrt{d^2-e^2x^2}+105d^4e^6x^2\sqrt{d^2-e^2x^2}-105d^2e^8x^4\sqrt{d^2-e^2x^2}+35e^{10}x^6\sqrt{d^2-e^2x^2}} & \text{for}\ e\neq 0\\ \frac{x^4}{4\left(d^2\right)^{\frac{9}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(9/2),x)
```

```
[Out] d*Piecewise((35*I*d**4*x**3/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**
11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*
x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) - 28*I*d**2*e**2
*x**5/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 +
 e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*
e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) + 8*I*e**4*x**7/(-105*d**13*sqrt(-1 +
e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*
e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (-35*d**4*x**3/(-105*d**13*sqrt(1 - e**
2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9*e**4
*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2/d**2
)) + 28*d**2*e**2*x**5/(-105*d**13*sqrt(1 - e**2*x**2/d**2) + 315*d**11*e**
2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**
2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2/d**2)) - 8*e**4*x**7/(-105*d**13
*sqrt(1 - e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) -
315*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e
**2*x**2/d**2)), True)) + e*Piecewise((2*d**2/(-35*d**6*e**4*sqrt(d**2 - e*
*2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2 - e**2*x**2) - 105*d**2*e**8*x**4*s
qrt(d**2 - e**2*x**2) + 35*e**10*x**6*sqrt(d**2 - e**2*x**2)) - 7*e**2*x**2
/(-35*d**6*e**4*sqrt(d**2 - e**2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2 - e**
2*x**2) - 105*d**2*e**8*x**4*sqrt(d**2 - e**2*x**2) + 35*e**10*x**6*sqrt(d*
*2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(9/2)), True))
```

3.31 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}-\frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}-\frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}$$

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {796, 778, 192, 191}

$$\frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}-\frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

```
[Out] (x^2*(d + e*x))/(9*d*e*(d^2 - e^2*x^2)^(9/2)) - (2*(d - 3*e*x))/(63*d*e^3*(
d^2 - e^2*x^2)^(7/2)) - (2*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(5/2)) - (8*x)/(
315*d^5*e^2*(d^2 - e^2*x^2)^(3/2)) - (16*x)/(315*d^7*e^2*Sqrt[d^2 - e^2*x^2
])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx = \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{x\left(2d^2e-6de^2x\right)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{9d^2e^2}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{21de^2}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{105d^3e^2}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8x}{315d^5e^2\left(d^2-e^2x\right.}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8x}{315d^5e^2\left(d^2-e^2x\right.}$$

Mathematica [A] time = 0.05, size = 126, normalized size = 0.85

$$\frac{-10d^8+10d^7ex+35d^6e^2x^2+70d^5e^3x^3-70d^4e^4x^4-56d^3e^5x^5+56d^2e^6x^6+16de^7x^7-16e^8x^8}{315d^7e^3(d-ex)^4(d+ex)^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

```
[Out] (-10*d^8 + 10*d^7*e*x + 35*d^6*e^2*x^2 + 70*d^5*e^3*x^3 - 70*d^4*e^4*x^4 -
56*d^3*e^5*x^5 + 56*d^2*e^6*x^6 + 16*d*e^7*x^7 - 16*e^8*x^8)/(315*d^7*e^3*(
d - e*x)^4*(d + e*x)^3*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.62, size = 126, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(-10d^8+10d^7ex+35d^6e^2x^2+70d^5e^3x^3-70d^4e^4x^4-56d^3e^5x^5+56d^2e^6x^6+16de^7x^7-16e^8x^8\right)}{315d^7e^3(d-ex)^5(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-10*d^8 + 10*d^7*e*x + 35*d^6*e^2*x^2 + 70*d^5*e^3*x^
3 - 70*d^4*e^4*x^4 - 56*d^3*e^5*x^5 + 56*d^2*e^6*x^6 + 16*d*e^7*x^7 - 16*e^
8*x^8))/(315*d^7*e^3*(d - e*x)^5*(d + e*x)^4)
```

fricas [B] time = 0.76, size = 305, normalized size = 2.06

$$-\frac{10e^9x^9-10de^8x^8-40d^2e^7x^7+40d^3e^6x^6+60d^4e^5x^5-60d^5e^4x^4-40d^6e^3x^3+40d^7e^2x^2+10d^8ex-10d^9-\left(16e^8x^8-16de^7x^7-56d^2e^6x^6+56d^3e^5x^5+70d^4e^4x^4-70d^5e^3x^3-35d^6e^2x^2-10d^7ex+10d^8\right)\sqrt{-e^2x^2+d^2}}{315\left(d^7e^{12}x^9-d^8e^{11}x^8-4d^9e^{10}x^7+4d^{10}e^9x^6+6d^{11}e^8x^5-6d^{12}e^7x^4-4d^{13}e^6x^3+4d^{14}e^5x^2+d^{15}e^4x-d^{16}e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/315*(10*e^9*x^9 - 10*d*e^8*x^8 - 40*d^2*e^7*x^7 + 40*d^3*e^6*x^6 + 60*d^
4*e^5*x^5 - 60*d^5*e^4*x^4 - 40*d^6*e^3*x^3 + 40*d^7*e^2*x^2 + 10*d^8*e*x -
 10*d^9 - (16*e^8*x^8 - 16*d*e^7*x^7 - 56*d^2*e^6*x^6 + 56*d^3*e^5*x^5 + 70
*d^4*e^4*x^4 - 70*d^5*e^3*x^3 - 35*d^6*e^2*x^2 - 10*d^7*e*x + 10*d^8)*sqrt(
-e^2*x^2 + d^2))/(d^7*e^12*x^9 - d^8*e^11*x^8 - 4*d^9*e^10*x^7 + 4*d^10*e^9
```

```
*x^6 + 6*d^11*e^8*x^5 - 6*d^12*e^7*x^4 - 4*d^13*e^6*x^3 + 4*d^14*e^5*x^2 +
d^15*e^4*x - d^16*e^3)
```

giac [A] time = 0.30, size = 90, normalized size = 0.61

$$\frac{\left(\left(\left(2\left(4x^2\left(\frac{2x^2e^6}{d^7} - \frac{9e^4}{d^5}\right) + \frac{63e^2}{d^3}\right)x^2 - \frac{105}{d}\right)x - 45e^{(-1)}\right)x^2 + 10d^2e^{(-3)}\right)\sqrt{-x^2e^2 + d^2}}{315\left(x^2e^2 - d^2\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((2*(4*x^2*(2*x^2*e^6/d^7 - 9*e^4/d^5) + 63*e^2/d^3)*x^2 - 105/d)*x
- 45*e^(-1))*x^2 + 10*d^2*e^(-3))*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^5
```

maple [A] time = 0.01, size = 121, normalized size = 0.82

$$-\frac{(-ex+d)(ex+d)^2\left(16e^8x^8 - 16e^7x^7d - 56e^6x^6d^2 + 56e^5x^5d^3 + 70e^4x^4d^4 - 70x^3d^5e^3 - 35x^2d^6e^2 - 10x\,d^7e + 10d^8\right)}{315\left(-e^2x^2 + d^2\right)^{\frac{11}{2}} d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x)
```

```
[Out] -1/315*(-e*x+d)*(e*x+d)^2*(16*e^8*x^8-16*d*e^7*x^7-56*d^2*e^6*x^6+56*d^3*e^
5*x^5+70*d^4*e^4*x^4-70*d^5*e^3*x^3-35*d^6*e^2*x^2-10*d^7*e*x+10*d^8)/d^7/e
^3/(-e^2*x^2+d^2)^(11/2)
```

maxima [A] time = 0.45, size = 158, normalized size = 1.07

$$\frac{x^2}{7\left(-e^2x^2 + d^2\right)^{\frac{9}{2}}e} + \frac{dx}{9\left(-e^2x^2 + d^2\right)^{\frac{9}{2}}e^2} - \frac{2d^2}{63\left(-e^2x^2 + d^2\right)^{\frac{9}{2}}e^3} - \frac{x}{63\left(-e^2x^2 + d^2\right)^{\frac{7}{2}}de^2} - \frac{2x}{105\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}d^3e^2} - \frac{8x}{315\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}d^5e^2} - \frac{16x}{315\sqrt{-e^2x^2 + d^2}\,d^7e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/7*x^2/((-e^2*x^2 + d^2)^(9/2)*e) + 1/9*d*x/((-e^2*x^2 + d^2)^(9/2)*e^2) -
 2/63*d^2/((-e^2*x^2 + d^2)^(9/2)*e^3) - 1/63*x/((-e^2*x^2 + d^2)^(7/2)*d*e
^2) - 2/105*x/((-e^2*x^2 + d^2)^(5/2)*d^3*e^2) - 8/315*x/((-e^2*x^2 + d^2)^
(3/2)*d^5*e^2) - 16/315*x/(sqrt(-e^2*x^2 + d^2)*d^7*e^2)
```

mupad [B] time = 2.74, size = 202, normalized size = 1.36

$$\frac{\sqrt{d^2 - e^2x^2}}{144\,d^3e^3(d - e\,x)^5} - \frac{\sqrt{d^2 - e^2x^2}\left(\frac{1}{252\,e^3} - \frac{17\,x}{252\,d\,e^2}\right)}{(d + e\,x)^4(d - e\,x)^4} - \frac{\sqrt{d^2 - e^2x^2}\left(\frac{5}{144\,d^2e^3} + \frac{131\,x}{5040\,d^3e^2}\right)}{(d + e\,x)^3(d - e\,x)^3} - \frac{8\,x\sqrt{d^2 - e^2x^2}}{315\,d^5e^2(d + e\,x)^2(d - e\,x)^2} - \frac{16\,x\sqrt{d^2 - e^2x^2}}{315\,d^7e^2(d + e\,x)(d - e\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(144*d^3*e^3*(d - e*x)^5) - ((d^2 - e^2*x^2)^(1/2)*(1
/(252*e^3) - (17*x)/(252*d*e^2)))/((d + e*x)^4*(d - e*x)^4) - ((d^2 - e^2*x
^2)^(1/2)*(5/(144*d^2*e^3) + (131*x)/(5040*d^3*e^2)))/((d + e*x)^3*(d - e*x
)^3) - (8*x*(d^2 - e^2*x^2)^(1/2))/(315*d^5*e^2*(d + e*x)^2*(d - e*x)^2) -
(16*x*(d^2 - e^2*x^2)^(1/2))/(315*d^7*e^2*(d + e*x)*(d - e*x))
```

sympy [C] time = 48.46, size = 1401, normalized size = 9.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(11/2),x)
```

```
[Out] d*Piecewise((-105*I*d**6*x**3/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d
**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*
e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) + 126*I*d**4*e**2*x**5/(315*d**17*sqrt
(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 18
90*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1
 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) - 72*I*d
**2*e**4*x**7/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*s
qrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) -
 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-
1 + e**2*x**2/d**2)) + 16*I*e**6*x**9/(315*d**17*sqrt(-1 + e**2*x**2/d**2)
- 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 3
15*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (10
5*d**6*x**3/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt
(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260
*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**
2*x**2/d**2)) - 126*d**4*e**2*x**5/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 12
60*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 -
 e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9
*e**8*x**8*sqrt(1 - e**2*x**2/d**2)) + 72*d**2*e**4*x**7/(315*d**17*sqrt(1
- e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d*
*13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**2
*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d**2)) - 16*e**6*x**9/(
315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**
2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x
**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d**2))
, True)) + e*Piecewise((-2*d**2/(63*d**8*e**4*sqrt(d**2 - e**2*x**2) - 252*
d**6*e**6*x**2*sqrt(d**2 - e**2*x**2) + 378*d**4*e**8*x**4*sqrt(d**2 - e**2
*x**2) - 252*d**2*e**10*x**6*sqrt(d**2 - e**2*x**2) + 63*e**12*x**8*sqrt(d*
*2 - e**2*x**2)) + 9*e**2*x**2/(63*d**8*e**4*sqrt(d**2 - e**2*x**2) - 252*d
**6*e**6*x**2*sqrt(d**2 - e**2*x**2) + 378*d**4*e**8*x**4*sqrt(d**2 - e**2*
x**2) - 252*d**2*e**10*x**6*sqrt(d**2 - e**2*x**2) + 63*e**12*x**8*sqrt(d**
2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(11/2)), True))
```

3.32 $\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{\sin^{-1}(ax)}{a^3}-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {797, 641, 216, 637}

$$-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -((1 - a*x)/(a^3*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/a^3 - ArcSin[a*x]/
a^3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx &= \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2}-\frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&=-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&=-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.93

$$\frac{a^2x^2-\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+ax-2}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-2 + a*x + a^2*x^2 - Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(a^3*Sqrt[1 - a^2*x^2]
)
```

IntegrateAlgebraic [A] time = 0.41, size = 74, normalized size = 1.37

$$\frac{(-ax-2)\sqrt{1-a^2x^2}}{a^3(ax+1)}-\frac{\sqrt{-a^2}\,\log\left(\sqrt{1-a^2x^2}-\sqrt{-a^2}\,x\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((-2 - a*x)*Sqrt[1 - a^2*x^2])/(a^3*(1 + a*x)) - (Sqrt[-a^2]*Log[-(Sqrt[-a^
2]*x) + Sqrt[1 - a^2*x^2]])/a^4
```

fricas [A] time = 0.40, size = 66, normalized size = 1.22

$$-\frac{2\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,(ax+2)+2}{a^4x+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

giac [A] time = 0.21, size = 70, normalized size = 1.30

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^2|a|}-\frac{\sqrt{-a^2x^2+1}}{a^3}+\frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [A] time = 0.02, size = 85, normalized size = 1.57

$$\frac{x^2}{\sqrt{-a^2x^2+1}\,a}+\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] x^2/a/(-a^2*x^2+1)^(1/2)-2/a^3/(-a^2*x^2+1)^(1/2)+x/a^2/(-a^2*x^2+1)^(1/2)-
1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.96, size = 63, normalized size = 1.17

$$\frac{x^2}{\sqrt{-a^2x^2+1}\,a} + \frac{x}{\sqrt{-a^2x^2+1}\,a^2} - \frac{\arcsin(ax)}{a^3} - \frac{2}{\sqrt{-a^2x^2+1}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(-a^2*x^2 + 1)*a) + x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3 -
 2/(sqrt(-a^2*x^2 + 1)*a^3)
```

mupad [B] time = 0.09, size = 84, normalized size = 1.56

$$\frac{\sqrt{1-a^2\,x^2}}{\left(a\,\sqrt{-a^2}\,+a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}} - \frac{\operatorname{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}} - \frac{\sqrt{1-a^2\,x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x - 1))/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/((a*(-a^2)^(1/2) + a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/a^3
```

sympy [A] time = 8.33, size = 102, normalized size = 1.89

$$-a\left(\begin{cases} -\frac{x^2}{a^2\sqrt{-a^2x^2+1}} + \frac{2}{a^4\sqrt{-a^2x^2+1}} & \text{for}\ a \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{ix}{a^2\sqrt{a^2x^2-1}} + \frac{i\operatorname{acosh}(ax)}{a^3} & \text{for}\ \left|a^2x^2\right| > 1 \\ \frac{x}{a^2\sqrt{-a^2x^2+1}} - \frac{\operatorname{asin}(ax)}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*x+1)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -a*Piecewise((-x**2/(a**2*sqrt(-a**2*x**2 + 1)) + 2/(a**4*sqrt(-a**2*x**2 +
 1)), Ne(a, 0)), (x**4/4, True)) + Piecewise((-I*x/(a**2*sqrt(a**2*x**2 - 1
)) + I*acosh(a*x)/a**3, Abs(a**2*x**2) > 1), (x/(a**2*sqrt(-a**2*x**2 + 1))
 - asin(a*x)/a**3, True))
```

3.33 $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx$

Optimal. Leaf size=173

$$-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}-\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e}-\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}+\frac{11d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}-\frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5}$$

Rubi [A] time = 0.23, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1809, 833, 780, 217, 203}

$$-\frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5}-\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e}-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}+\frac{11d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-8*d^3*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (11*d^2*x^3*Sqrt[d^2 - e^2*x^2]
)/(24*e^2) - (2*d*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) - (x^5*Sqrt[d^2 - e^2*x^2]
)/6 - (d^4*(256*d + 165*e*x)*Sqrt[d^2 - e^2*x^2])/(240*e^5) + (11*d^6*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
```

```
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}dx = -\frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^4\left(-11d^2e^2-12de^3x\right)}{\sqrt{d^2-e^2x^2}}dx}{6e^2}$$

$$= -\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{x^3\left(48d^3e^3+55d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}dx}{30e^4}$$

$$= -\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^2\left(-165d^4e^4-192d^3e^5x\right)}{\sqrt{d^2-e^2x^2}}dx}{120e^6}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{x(384d^5e^5+}{\sqrt{d^2-}}}{36}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d+1}{}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d+1}{}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d+1}{}$$

Mathematica [A] time = 0.10, size = 103, normalized size = 0.60

$$\frac{165d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\sqrt{d^2-e^2x^2}\left(256d^5+165d^4ex+128d^3e^2x^2+110d^2e^3x^3+96de^4x^4+40e^5x^5\right)}{240e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(256*d^5 + 165*d^4*e*x + 128*d^3*e^2*x^2 + 110*d^2*e
^3*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5)) + 165*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*
x^2]])/(240*e^5)
```

IntegrateAlgebraic [A] time = 0.42, size = 125, normalized size = 0.72

$$\frac{11d^6\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{16e^6}+\frac{\sqrt{d^2-e^2x^2}\left(-256d^5-165d^4ex-128d^3e^2x^2-110d^2e^3x^3-96de^4x^4-40e^5x^5\right)}{240e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-256*d^5 - 165*d^4*e*x - 128*d^3*e^2*x^2 - 110*d^2*e^
3*x^3 - 96*d*e^4*x^4 - 40*e^5*x^5))/(240*e^5) + (11*d^6*Sqrt[-e^2]*Log[-(Sq
rt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^6)
```

fricas [A] time = 0.41, size = 105, normalized size = 0.61

$$-\frac{330\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+96\,de^4x^4+110\,d^2e^3x^3+128\,d^3e^2x^2+165\,d^4ex+256\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/240*(330*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 9
6*d*e^4*x^4 + 110*d^2*e^3*x^3 + 128*d^3*e^2*x^2 + 165*d^4*e*x + 256*d^5)*sq
rt(-e^2*x^2 + d^2))/e^5
```

giac [A] time = 0.27, size = 84, normalized size = 0.49

$$\frac{11}{16}\,d^6 \arcsin\left(\frac{xe}{d}\right) e^{(-5)}\mathrm{sgn}(d) - \frac{1}{240}\left(256\,d^5 e^{(-5)} + \left(165\,d^4 e^{(-4)} + 2\left(64\,d^3 e^{(-3)} + \left(55\,d^2 e^{(-2)} + 4\left(12\,d e^{(-1)} + 5\,x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 11/16*d^6*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/240*(256*d^5*e^(-5) + (165*d^4*e^
(-4) + 2*(64*d^3*e^(-3) + (55*d^2*e^(-2) + 4*(12*d*e^(-1) + 5*x)*x)*x)*x)*x
)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.03, size = 174, normalized size = 1.01

$$-\frac{\sqrt{-e^2x^2+d^2}\,x^5}{6} - \frac{2\sqrt{-e^2x^2+d^2}\,d\,x^4}{5e} + \frac{11d^6 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e^4} - \frac{11\sqrt{-e^2x^2+d^2}\,d^2x^3}{24e^2} - \frac{8\sqrt{-e^2x^2+d^2}\,d^3x^2}{15e^3} - \frac{11\sqrt{-e^2x^2+d^2}\,d^4x}{16e^4} - \frac{16\sqrt{-e^2x^2+d^2}\,d^5}{15e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/6*x^5*(-e^2*x^2+d^2)^(1/2)-11/24*d^2*x^3*(-e^2*x^2+d^2)^(1/2)/e^2-11/16*
d^4*x*(-e^2*x^2+d^2)^(1/2)/e^4+11/16/e^4*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2)
/(-e^2*x^2+d^2)^(1/2)*x)-2/5*d*x^4*(-e^2*x^2+d^2)^(1/2)/e-8/15*d^3*x^2*(-e^
2*x^2+d^2)^(1/2)/e^3-16/15*d^5*(-e^2*x^2+d^2)^(1/2)/e^5
```

maxima [A] time = 0.98, size = 153, normalized size = 0.88

$$-\frac{1}{6}\,\sqrt{-e^2x^2+d^2}\,x^5 - \frac{2\,\sqrt{-e^2x^2+d^2}\,dx^4}{5\,e} - \frac{11\,\sqrt{-e^2x^2+d^2}\,d^2x^3}{24\,e^2} - \frac{8\,\sqrt{-e^2x^2+d^2}\,d^3x^2}{15\,e^3} + \frac{11\,d^6 \arcsin\left(\frac{ex}{d}\right)}{16\,e^5} - \frac{11\,\sqrt{-e^2x^2+d^2}\,d^4x}{16\,e^4} - \frac{16\,\sqrt{-e^2x^2+d^2}\,d^5}{15\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-e^2*x^2 + d^2)*x^5 - 2/5*sqrt(-e^2*x^2 + d^2)*d*x^4/e - 11/24*sq
rt(-e^2*x^2 + d^2)*d^2*x^3/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*d^3*x^2/e^3 + 11
/16*d^6*arcsin(e*x/d)/e^5 - 11/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e^4 - 16/15*sq
rt(-e^2*x^2 + d^2)*d^5/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [C] time = 13.48, size = 558, normalized size = 3.23

$$d^2\left(\begin{cases} -\frac{3id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5} + \frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{3d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5} - \frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} -\frac{8d^4\sqrt{d^2-e^2x^2}}{15e^6} - \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^4} - \frac{x^4\sqrt{d^2-e^2x^2}}{5e^2} & \text{for } e \neq 0 \\ \frac{x^6}{6\sqrt{d^2}} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{5id^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^7} + \frac{5id^5x}{16e^6\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5id^3x^3}{48e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{idx^5}{24e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{ix^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{5d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^7} - \frac{5d^5x}{16e^6\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5d^3x^3}{48e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{dx^5}{24e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5
/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*asin(e*
x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2
*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) +
2*d*e*Piecewise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**6) - 4*d**2*x**2*sqr
t(d**2 - e**2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x**2)/(5*e**2), Ne(e,
 0)), (x**6/(6*sqrt(d**2)), True)) + e**2*Piecewise((-5*I*d**6*acosh(e*x/d)
/(16*e**7) + 5*I*d**5*x/(16*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**3*x**3
/(48*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**5/(24*e**2*sqrt(-1 + e**2*x**
2/d**2)) - I*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1)
, (5*d**6*asin(e*x/d)/(16*e**7) - 5*d**5*x/(16*e**6*sqrt(1 - e**2*x**2/d**2
)) + 5*d**3*x**3/(48*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**5/(24*e**2*sqrt(
1 - e**2*x**2/d**2)) + x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.34 $\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=144

$$-\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}$$

Rubi [A] time = 0.19, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1809, 833, 780, 217, 203}

$$-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}-\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-3*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(2*e
) - (x^4*Sqrt[d^2 - e^2*x^2])/5 - (3*d^3*(8*d + 5*e*x)*Sqrt[d^2 - e^2*x^2])
/(20*e^4) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^3\left(-9d^2e^2-10de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2} \\
&= -\frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{x^2\left(30d^3e^3+36d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4} \\
&= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{x\left(-72d^4e^4-90d^3e^5x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6} \\
&= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{\left(3d^5\right) \cdots}{\cdots} \\
&= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{\left(3d^5\right) \cdots}{\cdots} \\
&= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{3d^5\ \mathrm{ta}\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 92, normalized size = 0.64

$$
\frac{15d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(24d^4+15d^3ex+12d^2e^2x^2+10de^3x^3+4e^4x^4\right)}{20e^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(24*d^4 + 15*d^3*e*x + 12*d^2*e^2*x^2 + 10*d*e^3*x^3
 + 4*e^4*x^4)) + 15*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(20*e^4)
```

IntegrateAlgebraic [A] time = 0.42, size = 114, normalized size = 0.79

$$
\frac{3d^5\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{4e^5} + \frac{\sqrt{d^2-e^2x^2}\left(-24d^4-15d^3ex-12d^2e^2x^2-10de^3x^3-4e^4x^4\right)}{20e^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^4 - 15*d^3*e*x - 12*d^2*e^2*x^2 - 10*d*e^3*x^3
- 4*e^4*x^4))/(20*e^4) + (3*d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 -
 e^2*x^2]])/(4*e^5)
```

fricas [A] time = 0.41, size = 94, normalized size = 0.65

$$
-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(4\,e^4x^4+10\,de^3x^3+12\,d^2e^2x^2+15\,d^3ex+24\,d^4\right)\sqrt{-e^2x^2+d^2}}{20\,e^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*e^4*x^4 + 10*d
*e^3*x^3 + 12*d^2*e^2*x^2 + 15*d^3*e*x + 24*d^4)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [A] time = 0.26, size = 73, normalized size = 0.51

$$\frac{3}{4}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{1}{20}\left(24\,d^4e^{(-4)}+\left(15\,d^3e^{(-3)}+2\left(6\,d^2e^{(-2)}+\left(5\,de^{(-1)}+2\,x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*d^5*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/20*(24*d^4*e^(-4) + (15*d^3*e^(-3)
+ 2*(6*d^2*e^(-2) + (5*d*e^(-1) + 2*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 149, normalized size = 1.03

$$-\frac{\sqrt{-e^2x^2+d^2}\,x^4}{5}+\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}\,e^3}-\frac{\sqrt{-e^2x^2+d^2}\,d\,x^3}{2e}-\frac{3\sqrt{-e^2x^2+d^2}\,d^2x^2}{5e^2}-\frac{3\sqrt{-e^2x^2+d^2}\,d^3x}{4e^3}-\frac{6\sqrt{-e^2x^2+d^2}\,d^4}{5e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*x^4*(-e^2*x^2+d^2)^(1/2)-3/5*d^2*x^2*(-e^2*x^2+d^2)^(1/2)/e^2-6/5*d^4*
(-e^2*x^2+d^2)^(1/2)/e^4-1/2*d*x^3*(-e^2*x^2+d^2)^(1/2)/e-3/4*d^3*x*(-e^2*x
^2+d^2)^(1/2)/e^3+3/4*d^5/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)
^(1/2)*x)
```

maxima [A] time = 0.97, size = 128, normalized size = 0.89

$$-\frac{1}{5}\sqrt{-e^2x^2+d^2}\,x^4-\frac{\sqrt{-e^2x^2+d^2}\,dx^3}{2\,e}-\frac{3\,\sqrt{-e^2x^2+d^2}\,d^2x^2}{5\,e^2}+\frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)}{4\,e^4}-\frac{3\,\sqrt{-e^2x^2+d^2}\,d^3x}{4\,e^3}-\frac{6\,\sqrt{-e^2x^2+d^2}\,d^4}{5\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*x^4 - 1/2*sqrt(-e^2*x^2 + d^2)*d*x^3/e - 3/5*sqrt
(-e^2*x^2 + d^2)*d^2*x^2/e^2 + 3/4*d^5*arcsin(e*x/d)/e^4 - 3/4*sqrt(-e^2*x^
2 + d^2)*d^3*x/e^3 - 6/5*sqrt(-e^2*x^2 + d^2)*d^4/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [A] time = 7.87, size = 357, normalized size = 2.48

$$d^2\left(\begin{cases} -\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases} -\frac{3id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5}+\frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5}-\frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases} -\frac{8d^4\sqrt{d^2-e^2x^2}}{15e^6}-\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^4}-\frac{x^4\sqrt{d^2-e^2x^2}}{5e^2} & \text{for}\ e\neq 0\\ \frac{x^6}{6\sqrt{d^2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 -
e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + 2*d*e*Piecew
ise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-1 + e**2*x*
*2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5/(4*d*sqrt(
-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*asin(e*x/d)/(8*e**
5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2*sqrt(1 - e
**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*Piecew
ise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**6) - 4*d**2*x**2*sqrt(d**2 - e**
2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x**2)/(5*e**2), Ne(e, 0)), (x**6/
(6*sqrt(d**2)), True))
```

3.35 $\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx$

Optimal. Leaf size=115

$$-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}+\frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1809, 833, 780, 217, 203}

$$-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}+\frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-2*d*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) - (x^3*Sqrt[d^2 - e^2*x^2])/4 - (d^2*(
32*d + 21*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) + (7*d^4*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(8*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^2\left(-7d^2e^2-8de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2} \\
&= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} + \frac{\int \frac{x\left(16d^3e^3+21d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4} \\
&= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{\left(7d^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2} \\
&= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{\left(7d^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,d\right.}{8e^2} \\
&= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 0.70

$$\frac{21d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\sqrt{d^2-e^2x^2}\left(32d^3+21d^2ex+16de^2x^2+6e^3x^3\right)}{24e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3)) +
21*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e^3)
```

IntegrateAlgebraic [A] time = 0.39, size = 103, normalized size = 0.90

$$\frac{7d^4\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e^4}+\frac{\sqrt{d^2-e^2x^2}\left(-32d^3-21d^2ex-16de^2x^2-6e^3x^3\right)}{24e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-32*d^3 - 21*d^2*e*x - 16*d*e^2*x^2 - 6*e^3*x^3))/(24
*e^3) + (7*d^4*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*e^
4)
```

fricas [A] time = 0.40, size = 83, normalized size = 0.72

$$-\frac{42\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3+16\,de^2x^2+21\,d^2ex+32\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(42*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 + 16*d
*e^2*x^2 + 21*d^2*e*x + 32*d^3)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.25, size = 63, normalized size = 0.55

$$\frac{7}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{24}\left(32\,d^3e^{(-3)}+\left(21\,d^2e^{(-2)}+2\left(8\,de^{(-1)}+3\,x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 7/8*d^4*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24*(32*d^3*e^(-3) + (21*d^2*e^(-2)
+ 2*(8*d*e^(-1) + 3*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 124, normalized size = 1.08

$$\frac{7d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,x^3}{4}-\frac{2\sqrt{-e^2x^2+d^2}\,d\,x^2}{3e}-\frac{7\sqrt{-e^2x^2+d^2}\,d^2x}{8e^2}-\frac{4\sqrt{-e^2x^2+d^2}\,d^3}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/4*x^3*(-e^2*x^2+d^2)^(1/2)-7/8/e^2*d^2*x*(-e^2*x^2+d^2)^(1/2)+7/8/e^2*d^
4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-2/3*d*x^2*(-e^2*x^
2+d^2)^(1/2)/e-4/3*d^3*(-e^2*x^2+d^2)^(1/2)/e^3
```

maxima [A] time = 0.97, size = 103, normalized size = 0.90

$$-\frac{1}{4}\,\sqrt{-e^2x^2+d^2}\,x^3-\frac{2\,\sqrt{-e^2x^2+d^2}\,dx^2}{3\,e}+\frac{7\,d^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^3}-\frac{7\,\sqrt{-e^2x^2+d^2}\,d^2x}{8\,e^2}-\frac{4\,\sqrt{-e^2x^2+d^2}\,d^3}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*x^3 - 2/3*sqrt(-e^2*x^2 + d^2)*d*x^2/e + 7/8*d^4*
arcsin(e*x/d)/e^3 - 7/8*sqrt(-e^2*x^2 + d^2)*d^2*x/e^2 - 4/3*sqrt(-e^2*x^2
+ d^2)*d^3/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [C] time = 9.33, size = 386, normalized size = 3.36

$$d^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{3id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5}+\frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5}-\frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d
**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(
2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), Tru
```

```
e)) + 2*d*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(
d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + e**2*
Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-1 + e
**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5/(4*d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*asin(e*x/d)/
(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2*sqrt
(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.36 $\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}} \, dx$

Optimal. Leaf size=83

$$-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1809, 780, 217, 203}

$$-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(x^2*Sqrt[d^2 - e^2*x^2])/3 - (d*(5*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(3*e^2
) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{\int \frac{x\left(-5d^2e^2-6de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{3e^2} \\
&= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e} \\
&= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e} \\
&= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}
\end{aligned}
$$

Mathematica [**A**] time = 0.05, size = 69, normalized size = 0.83

$$
\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(5d^2+3dex+e^2x^2\right)}{3e^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(5*d^2 + 3*d*e*x + e^2*x^2)) + 3*d^3*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(3*e^2)
```

IntegrateAlgebraic [**A**] time = 0.39, size = 89, normalized size = 1.07

$$
\frac{\sqrt{d^2-e^2x^2}\left(-5d^2-3dex-e^2x^2\right)}{3e^2} + \frac{d^3\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^2 - 3*d*e*x - e^2*x^2))/(3*e^2) + (d^3*Sqrt[-e^2
]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^3
```

fricas [**A**] time = 0.40, size = 71, normalized size = 0.86

$$
-\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(e^2x^2+3\,dex+5\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\,e^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (e^2*x^2 + 3*d*e*x
+ 5*d^2)*sqrt(-e^2*x^2 + d^2))/e^2
```

giac [**A**] time = 0.25, size = 49, normalized size = 0.59

$$
d^3\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\text{sgn}(d) - \frac{1}{3}\sqrt{-x^2e^2+d^2}\left(5\,d^2e^{(-2)} + \left(3\,de^{(-1)}+x\right)x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] d^3*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/3*sqrt(-x^2*e^2 + d^2)*(5*d^2*e^(-2) +
(3*d*e^(-1) + x)*x)
```

maple [A] time = 0.01, size = 98, normalized size = 1.18

$$\frac{d^{3}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\,e}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,x^{2}}{3}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,dx}{e}-\frac{5\sqrt{-e^{2}x^{2}+d^{2}}\,d^{2}}{3e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*x^2*(-e^2*x^2+d^2)^(1/2)-5/3*d^2/e^2*(-e^2*x^2+d^2)^(1/2)-d*x*(-e^2*x^
2+d^2)^(1/2)/e+d^3/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.97, size = 77, normalized size = 0.93

$$-\frac{1}{3}\sqrt{-e^{2}x^{2}+d^{2}}\,x^{2}+\frac{d^{3}\arcsin\left(\frac{ex}{d}\right)}{e^{2}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,dx}{e}-\frac{5\sqrt{-e^{2}x^{2}+d^{2}}\,d^{2}}{3\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*x^2 + d^3*arcsin(e*x/d)/e^2 - sqrt(-e^2*x^2 + d^2
)*d*x/e - 5/3*sqrt(-e^2*x^2 + d^2)*d^2/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(d+e\,x)^{2}}{\sqrt{d^{2}-e^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [A] time = 5.62, size = 218, normalized size = 2.63

$$d^{2}\left(\begin{cases}\frac{x^{2}}{2\sqrt{d^{2}}} & \text{for}\: e^{2}=0\\-\frac{\sqrt{d^{2}-e^{2}x^{2}}}{e^{2}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{id^{2}\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^{3}}-\frac{idx\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{2e^{2}} & \text{for}\: \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\\frac{d^{2}\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^{3}}-\frac{dx}{2e^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}+\frac{x^{3}}{2d\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise}\end{cases}\right)+e^{2}\left(\begin{cases}-\frac{2d^{2}\sqrt{d^{2}-e^{2}x^{2}}}{3e^{4}}-\frac{x^{2}\sqrt{d^{2}-e^{2}x^{2}}}{3e^{2}} & \text{for}\: e\neq 0\\\frac{x^{4}}{4\sqrt{d^{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)
/e**2, True)) + 2*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt
(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)
/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2
*x**2/d**2)), True)) + e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e*
*4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2))
, True))
```

3.37 $\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {671, 641, 217, 203}

$$-\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-3*d*Sqrt[d^2 - e^2*x^2])/(2*e) - ((d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) +
(3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}(3d)\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}\left(3d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{1}{2}\left(3d^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$
$$= -\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.70

$$\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - (4d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
[Out] (-((4*d + e*x)*Sqrt[d^2 - e^2*x^2]) + 3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/(2*e)
```

IntegrateAlgebraic [A] time = 0.01, size = 81, normalized size = 0.98

$$\frac{(-4d-ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{3d^2\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2} - \sqrt{-e^2}\,x\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
[Out] ((-4*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) + (3*d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^
2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^2)
```

fricas [A] time = 0.41, size = 60, normalized size = 0.72

$$-\frac{6\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \sqrt{-e^2x^2+d^2}\,(ex+4\,d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*(6*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2
)*(e*x + 4*d))/e
```

giac [A] time = 0.25, size = 40, normalized size = 0.48

$$\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\text{sgn}(d) - \frac{1}{2}\,\sqrt{-x^2e^2+d^2}\left(4\,de^{(-1)}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/2*sqrt(-x^2*e^2 + d^2)*(4*d*e^(-1)
+ x)
```

maple [A] time = 0.01, size = 71, normalized size = 0.86

$$\frac{3d^2 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+d^2}\,x}{2} - \frac{2\sqrt{-e^2x^2+d^2}\,d}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*x*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^
2+d^2)^(1/2)*x)-2*d*(-e^2*x^2+d^2)^(1/2)/e
```

maxima [A] time = 0.97, size = 53, normalized size = 0.64

$$\frac{3\,d^2 \arcsin\left(\frac{ex}{d}\right)}{2\,e} - \frac{1}{2}\sqrt{-e^2x^2+d^2}\,x - \frac{2\sqrt{-e^2x^2+d^2}\,d}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*arcsin(e*x/d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*x - 2*sqrt(-e^2*x^2 + d^
2)*d/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [A] time = 5.04, size = 269, normalized size = 3.24

$$d^2\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0 \wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0 \wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0 \wedge e^2<0\end{cases}\right) + 2de\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right) + e^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3} - \frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3} - \frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2),
(d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d
**2), (d**2 < 0) & (e**2 < 0))) + 2*d*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(
e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + e**2*Piecewise((-I*d**2*
acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*
x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**
2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.38 $\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=66

$$-\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1809, 844, 217, 203, 266, 63, 208}

$$-\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx = -\sqrt{d^2-e^2x^2}-\frac{\int \frac{-d^2e^2-2de^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$
$$= -\sqrt{d^2-e^2x^2}+d^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+(2de)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\sqrt{d^2-e^2x^2}+\frac{1}{2}d^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)+(2de)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{}{\sqrt{d^2}}\right.$$
$$= -\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d^2\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2}$$
$$= -\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] time = 0.03, size = 66, normalized size = 1.00

$$-\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d]
```

IntegrateAlgebraic [A] time = 0.38, size = 104, normalized size = 1.58

$$-\sqrt{d^2-e^2x^2}+\frac{2d\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e}+2d\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d
] + (2*d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e
```

fricas [A] time = 0.40, size = 73, normalized size = 1.11

$$-4\,d\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -4*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d*log(-(d - sqrt(-e^2*x^2
+ d^2))/x) - sqrt(-e^2*x^2 + d^2)
```

giac [A] time = 0.26, size = 65, normalized size = 0.98

$$2\,d\arcsin\left(\frac{xe}{d}\right)\mathrm{sgn}(d) - d\log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2 + d^2}\,e\right|e^{(-2)}}{2\,|x|}\right) - \sqrt{-x^2e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d*arcsin(x*e/d)*sgn(d) - d*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)
*e^(-2)/abs(x)) - sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 91, normalized size = 1.38

$$-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}} + \frac{2de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \sqrt{-e^2x^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)+2*e*d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(
1/2)*x)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.97, size = 62, normalized size = 0.94

$$2\,d\arcsin\left(\frac{ex}{d}\right) - d\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2 + d^2}\,d}{|x|}\right) - \sqrt{-e^2x^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*d*arcsin(e*x/d) - d*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) -
 sqrt(-e^2*x^2 + d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{(d+e\,x)}^2}{x\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(1/2)), x)
```

sympy [C] time = 6.96, size = 184, normalized size = 2.79

$$d^2\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise}\end{cases}\right) + 2de\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2 > 0 \wedge e^2 > 0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2 > 0 \wedge e^2 < 0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for}\ d^2 < 0 \wedge e^2 < 0\end{cases}\right) + e^2\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for}\ e^2 = 0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(e
*x))/d, True)) + 2*d*e*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/s
qrt(d**2), (d**2 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d
**2))/sqrt(d**2), (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e
**2/d**2))/sqrt(-d**2), (d**2 < 0) & (e**2 < 0))) + e**2*Piecewise((x**2/(2
*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True))
```

3.39 $\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1807, 844, 217, 203, 266, 63, 208}

$$-\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*e*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{x} - \frac{\int \frac{-2d^3e-d^2e^2x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x} + (2de)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + e^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x} + (de)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + e^2\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-}}\right.\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x} + e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x} + e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.00

$$-\frac{\sqrt{d^2-e^2x^2}}{x} + e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*e*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d]
```

IntegrateAlgebraic [A] time = 0.40, size = 102, normalized size = 1.50

$$-\frac{\sqrt{d^2-e^2x^2}}{x} + \sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2} - \sqrt{-e^2}\,x\right) + 4e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + 4*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^
2]/d] + Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.40, size = 79, normalized size = 1.16

$$-\frac{2\,ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 2\,ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \sqrt{-e^2x^2+d^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 2*e*x*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2))/x
```

giac [A] time = 0.26, size = 107, normalized size = 1.57

$$\arcsin\left(\frac{xe}{d}\right)e\mathrm{sgn}(d) - 2\,e\log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2 + d^2}\,e\right|e^{(-2)}}{2\,|x|}\right) + \frac{xe^3}{2\left(de + \sqrt{-x^2e^2 + d^2}\,e\right)} - \frac{\left(de + \sqrt{-x^2e^2 + d^2}\,e\right)e^{(-1)}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e*sgn(d) - 2*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)
*e^(-2)/abs(x)) + 1/2*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e) - 1/2*(d*e + sqr
t(-x^2*e^2 + d^2)*e)*e^(-1)/x
```

maple [A] time = 0.01, size = 93, normalized size = 1.37

$$-\frac{2de\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}} + \frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+d^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-(-e^2*x^2+d^2)^(
1/2)/x-2*d*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.96, size = 64, normalized size = 0.94

$$e\arcsin\left(\frac{ex}{d}\right) - 2\,e\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2 + d^2}\,d}{|x|}\right) - \frac{\sqrt{-e^2x^2 + d^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] e*arcsin(e*x/d) - 2*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)) -
 sqrt(-e^2*x^2 + d^2)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{e^2\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}} - \frac{\sqrt{d^2-e^2\,x^2}}{x} - \frac{2\,d\,e\,\ln\left(\frac{\sqrt{d^2}+\sqrt{d^2-e^2\,x^2}}{x}\right)}{\sqrt{d^2}} & \text{if } e^2 < 0 \\ \int \frac{e^2}{\sqrt{d^2-e^2\,x^2}} + \frac{d^2}{x^2\,\sqrt{d^2-e^2\,x^2}} + \frac{2\,d\,e}{x\,\sqrt{d^2-e^2\,x^2}}\,dx & \text{if } \neg e^2 < 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] piecewise(e^2 < 0, - (d^2 - e^2*x^2)^(1/2)/x + (e^2*log(x*(-e^2)^(1/2) + (d
^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (2*d*e*log(((d^2)^(1/2) + (d^2 - e^2*x
^2)^(1/2))/x))/(d^2)^(1/2), ~e^2 < 0, int(e^2/(d^2 - e^2*x^2)^(1/2) + d^2/(
x^2*(d^2 - e^2*x^2)^(1/2)) + (2*d*e)/(x*(d^2 - e^2*x^2)^(1/2)), x))
```

sympy [C] time = 4.30, size = 207, normalized size = 3.04

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases} -\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases} \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**2, Abs(d**2/(e**2*x**2)) >
 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True)) + 2*d*e*Piecewise((-aco
sh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(e*x))/d, True)) + e**
2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 > 0)
 & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (d*
*2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d**2
), (d**2 < 0) & (e**2 < 0)))
```

3.40 $\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1807, 807, 266, 63, 208}

$$-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(2*x^2) - (2*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (3*e^2*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{\int \frac{-4d^3e-3d^2e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}+\frac{1}{2}\left(3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}+\frac{1}{4}\left(3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 122, normalized size = 1.52

$$\frac{e\left(-\frac{4d\sqrt{d^2-e^2x^2}}{x}-2de\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-e\sqrt{d^2-e^2x^2}\left(\frac{d^2}{e^2x^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{\sqrt{1-\frac{e^2x^2}{d^2}}}\right)\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (e*((-4*d*Sqrt[d^2 - e^2*x^2])/x - 2*d*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d] - e
*Sqrt[d^2 - e^2*x^2]*(d^2/(e^2*x^2) + ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]]/Sqrt
[1 - (e^2*x^2)/d^2])))/(2*d^2)
```

IntegrateAlgebraic [A] time = 0.46, size = 122, normalized size = 1.52

$$\frac{(-d-4ex)\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{3e^2\log\left(\sqrt{d^2-e^2x^2}+d-\sqrt{-e^2}\,x\right)}{2d}+\frac{3e^2\log\left(-d\sqrt{d^2-e^2x^2}+d^2+d\sqrt{-e^2}\,x\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((-d - 4*e*x)*Sqrt[d^2 - e^2*x^2])/(2*d*x^2) - (3*e^2*Log[d - Sqrt[-e^2]*x
+ Sqrt[d^2 - e^2*x^2]])/(2*d) + (3*e^2*Log[d^2 + d*Sqrt[-e^2]*x - d*Sqrt[d^
2 - e^2*x^2]])/(2*d)
```

fricas [A] time = 0.39, size = 63, normalized size = 0.79

$$\frac{3\,e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\sqrt{-e^2x^2+d^2}\,(4\,ex+d)}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - sqrt(-e^2*x^2 + d^2)*(4
*e*x + d))/(d*x^2)
```

giac [**B**] time = 0.27, size = 170, normalized size = 2.12

$$-\frac{3\,e^{2}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d}+\frac{x^{2}\left(\frac{8\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{4}}{x}+e^{6}\right)}{8\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}d}-\frac{\left(\frac{8\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)de^{8}}{x}+\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}de^{6}}{x^{2}}\right)e^{(-8)}}{8\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -3/2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d +
1/8*x^2*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*d) - 1/8*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^8/x + (d*e + s
qrt(-x^2*e^2 + d^2)*e)^2*d*e^6/x^2)*e^(-8)/d^2
```

maple [**A**] time = 0.01, size = 86, normalized size = 1.08

$$-\frac{3e^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{2\sqrt{d^{2}}}-\frac{2\sqrt{-e^{2}x^{2}+d^{2}}\,e}{dx}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)/x^2-3/2*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(
-e^2*x^2+d^2)^(1/2))/x)-2*e*(-e^2*x^2+d^2)^(1/2)/d/x
```

maxima [**A**] time = 0.95, size = 83, normalized size = 1.04

$$-\frac{3\,e^{2}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{2\,d}-\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,e}{dx}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 2*sqrt(-e^
2*x^2 + d^2)*e/(d*x) - 1/2*sqrt(-e^2*x^2 + d^2)/x^2
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^{2}}{x^{3}\,\sqrt{d^{2}-e^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(1/2)), x)
```

sympy [**C**] time = 6.72, size = 214, normalized size = 2.68

$$d^{2}\left(\begin{cases}-\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{2d^{2}x}-\frac{e^{2}\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^{3}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ \frac{i}{2ex^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie}{2d^{2}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{2}\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^{3}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{d^{2}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ -\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{d^{2}} & \text{otherwise}\end{cases}\right)+e^{2}\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) - e**2*acosh(d/(e*
x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**2*asin(d/(e*x)
)/(2*d**3), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**2, A
bs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True)) +
 e**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(
e*x))/d, True))
```

3.41 $\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 835, 807, 266, 63, 208}

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(3*x^3) - (e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (5*e^2*Sqr
t[d^2 - e^2*x^2])/(3*d^2*x) - (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{\int \frac{-6d^3e-5d^2e^2x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}+\frac{\int \frac{10d^4e^2+6d^3e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{e^3\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{e^3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 87, normalized size = 0.81

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{d\left(d^2+3dex+5e^2x^2\right)}{x^3}-\frac{3e^3\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-((d*(d^2 + 3*d*e*x + 5*e^2*x^2))/x^3) - (3*e^3*ArcTa
nh[Sqrt[1 - (e^2*x^2)/d^2]])/Sqrt[1 - (e^2*x^2)/d^2]))/(3*d^3)
```

IntegrateAlgebraic [A] time = 0.47, size = 91, normalized size = 0.85

$$\frac{\left(-d^2-3dex-5e^2x^2\right)\sqrt{d^2-e^2x^2}}{3d^2x^3}+\frac{2e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((-d^2 - 3*d*e*x - 5*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(3*d^2*x^3) + (2*e^3*Arc
Tanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^2
```

fricas [A] time = 0.40, size = 74, normalized size = 0.69

$$\frac{3\,e^{3}x^{3}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(5\,e^{2}x^{2}+3\,dex+d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{3\,d^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (5*e^2*x^2 + 3*d*e*x +
d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*x^3)
```

giac [B] time = 0.30, size = 239, normalized size = 2.23

$$\frac{x^{3}\left(\frac{6\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{6}}{x}+\frac{21\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}e^{4}}{x^{2}}+e^{8}\right)e}{24\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}d^{2}}-\frac{e^{3}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^{2}}-\frac{\left(\frac{21\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)d^{4}e^{16}}{x}+\frac{6\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}d^{4}e^{14}}{x^{2}}+\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}d^{4}e^{12}}{x^{3}}\right)e^{(-15)}}{24\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*x^3*(6*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^6/x + 21*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^2*e^4/x^2 + e^8)*e/((d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2) - e^3*l
og(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 - 1/24*(21
*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^16/x + 6*(d*e + sqrt(-x^2*e^2 + d^2)*
e)^2*d^4*e^14/x^2 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*e^12/x^3)*e^(-15)/
d^6
```

maple [A] time = 0.01, size = 114, normalized size = 1.07

$$-\frac{e^{3}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}\,d}-\frac{5\sqrt{-e^{2}x^{2}+d^{2}}\,e^{2}}{3d^{2}x}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e}{d\,x^{2}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -e*(-e^2*x^2+d^2)^(1/2)/d/x^2-1/d*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(
-e^2*x^2+d^2)^(1/2))/x)-5/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x-1/3*(-e^2*x^2+d^
2)^(1/2)/x^3
```

maxima [A] time = 0.97, size = 108, normalized size = 1.01

$$-\frac{e^{3}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{d^{2}}-\frac{5\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{2}}{3\,d^{2}x}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e}{dx^{2}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 - 5/3*sqrt(-e^
2*x^2 + d^2)*e^2/(d^2*x) - sqrt(-e^2*x^2 + d^2)*e/(d*x^2) - 1/3*sqrt(-e^2*x
^2 + d^2)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^{2}}{x^{4}\,\sqrt{d^{2}-e^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(1/2)), x)
```

sympy [C] time = 6.09, size = 303, normalized size = 2.83

$$d^{2}\left(\begin{cases} -\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3d^{2}x^{2}}-\frac{2e^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3d^{4}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3d^{2}x^{2}}-\frac{2ie^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3d^{4}} & \text{otherwise} \end{cases}\right)+2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{2d^{2}x}-\frac{e^{2}\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^{3}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ \frac{i}{2ex^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie}{2d^{2}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{2}\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^{3}} & \text{otherwise} \end{cases}\right)+e^{2}\left(\begin{cases} -\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{d^{2}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{d^{2}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sqrt(d
**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2
/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d
**4), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) -
e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1), (I/(2*e*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e
**2*asin(d/(e*x))/(2*d**3), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**
2) - 1)/d**2, Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)
/d**2, True))
```

3.42 $\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=140

$$-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}$$

Rubi [A] time = 0.17, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 835, 807, 266, 63, 208}

$$-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(4*x^4) - (2*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (7*e^2
*Sqrt[d^2 - e^2*x^2])/(8*d^2*x^2) - (4*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) -
 (7*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{\int \frac{-8d^3e-7d^2e^2x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{\int \frac{21d^4e^2+16d^3e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{\int \frac{-32d^5e^3-21d^4e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^6}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{\left(7e^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{\left(7e^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-}}\right.}{16d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{\left(7e^2\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right.}{8d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

Mathematica [C] time = 0.15, size = 155, normalized size = 1.11

$$-\frac{e\sqrt{d^2-e^2x^2}\left(d\left(4d^2+3dex+8e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}+6e^3x^3\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+3e^3x^3\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\right)}{6d^4x^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/6*(e*Sqrt[d^2 - e^2*x^2]*(d*(4*d^2 + 3*d*e*x + 8*e^2*x^2)*Sqrt[1 - (e^2*
x^2)/d^2] + 3*e^3*x^3*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]] + 6*e^3*x^3*Sqrt[1 -
 (e^2*x^2)/d^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (e^2*x^2)/d^2]))/(d^4*x^
3*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.55, size = 104, normalized size = 0.74

$$\frac{7e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^3} + \frac{\sqrt{d^2-e^2x^2}\left(-6d^3-16d^2ex-21de^2x^2-32e^3x^3\right)}{24d^3x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 16*d^2*e*x - 21*d*e^2*x^2 - 32*e^3*x^3))/(24
*d^3*x^4) + (7*e^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(4*d^
3)
```

fricas [A] time = 0.40, size = 87, normalized size = 0.62

$$\frac{21\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(32\,e^3x^3+21\,de^2x^2+16\,d^2ex+6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(21*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 + 21*d*e^
2*x^2 + 16*d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*x^4)
```

giac [B] time = 0.29, size = 305, normalized size = 2.18

$$\frac{x^4\left(\frac{16\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^8}{x}+\frac{48\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^6}{x^2}+\frac{144\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^4}{x^3}+3\,e^{10}\right)e^2}{192\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^3}-\frac{7\,e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{8\,d^3}-\frac{\left(\frac{144\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^9e^{26}}{x}+\frac{48\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^9e^{24}}{x^2}+\frac{16\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^9e^{22}}{x^3}+\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^9e^{20}}{x^4}\right)e^{(-24)}}{192\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*x^4*(16*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^8/x + 48*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*e^6/x^2 + 144*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^4/x^3 + 3*e^
10)*e^2/((d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^3) - 7/8*e^4*log(1/2*abs(-2*d*e
 - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 1/192*(144*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*d^9*e^26/x + 48*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^9*e^24/
x^2 + 16*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^9*e^22/x^3 + 3*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^4*d^9*e^20/x^4)*e^(-24)/d^12
```

maple [A] time = 0.02, size = 139, normalized size = 0.99

$$-\frac{7e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}\,d^2}-\frac{4\sqrt{-e^2x^2+d^2}\,e^3}{3d^3x}-\frac{7\sqrt{-e^2x^2+d^2}\,e^2}{8d^2x^2}-\frac{2\sqrt{-e^2x^2+d^2}\,e}{3d\,x^3}-\frac{\sqrt{-e^2x^2+d^2}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -2/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-4/3*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x-7/8*e^
2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-7/8*e^4/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1
/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/4*(-e^2*x^2+d^2)^(1/2)/x^4
```

maxima [A] time = 0.97, size = 133, normalized size = 0.95

$$-\frac{7\,e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8\,d^3}-\frac{4\,\sqrt{-e^2x^2+d^2}\,e^3}{3\,d^3x}-\frac{7\,\sqrt{-e^2x^2+d^2}\,e^2}{8\,d^2x^2}-\frac{2\,\sqrt{-e^2x^2+d^2}\,e}{3\,dx^3}-\frac{\sqrt{-e^2x^2+d^2}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -7/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 4/3*sqrt
(-e^2*x^2 + d^2)*e^3/(d^3*x) - 7/8*sqrt(-e^2*x^2 + d^2)*e^2/(d^2*x^2) - 2/3
*sqrt(-e^2*x^2 + d^2)*e/(d*x^3) - 1/4*sqrt(-e^2*x^2 + d^2)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^5\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^5*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^5*(d^2 - e^2*x^2)^(1/2)), x)
```

sympy [C] time = 10.32, size = 449, normalized size = 3.21

$$d^2\left(\begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x} - \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**5/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-1/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*s
qrt(d**2/(e**2*x**2) - 1)) + 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) -
 3*e**4*acosh(d/(e*x))/(8*d**5), Abs(d**2/(e**2*x**2)) > 1), (I/(4*e*x**5*s
qrt(-d**2/(e**2*x**2) + 1)) + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1))
 - 3*I*e**3/(8*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(e*x))
/(8*d**5), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*
x**2) - 2*e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) >
 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/
(e**2*x**2) + 1)/(3*d**4), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2
) - 1)/(2*d**2*x) - e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1
), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e*
*2*x**2) + 1)) + I*e**2*asin(d/(e*x))/(2*d**3), True))
```

3.43 $\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=169

$$-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}$$

Rubi [A] time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 835, 807, 266, 63, 208}

$$-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(5*x^5) - (e*Sqrt[d^2 - e^2*x^2])/(2*d*x^4) - (3*e^2*S
qrt[d^2 - e^2*x^2])/(5*d^2*x^3) - (3*e^3*Sqrt[d^2 - e^2*x^2])/(4*d^3*x^2) -
 (6*e^4*Sqrt[d^2 - e^2*x^2])/(5*d^4*x) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d])/(4*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{\int \frac{-10d^3e-9d^2e^2x}{x^5\sqrt{d^2-e^2x^2}}\,dx}{5d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} + \frac{\int \frac{36d^4e^2+30d^3e^3x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{20d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{\int \frac{-90d^5e^3-72d^4e^4x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{60d^6}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} + \frac{\int \frac{144d^6e^4+90d^5e^5x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{120d^8}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \frac{(3e^5 \ldots}{\ldots}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} + \frac{(3e^5 \ldots}{\ldots}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} - \frac{(3e^3 \ldots}{\ldots}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x} - \frac{3e^5\,\mathrm{t}\ldots}{\ldots}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.47

$$-\frac{\sqrt{d^2-e^2x^2}\left(d^5+3d^3e^2x^2+10e^5x^5\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+6de^4x^4\right)}{5d^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*(Sqrt[d^2 - e^2*x^2]*(d^5 + 3*d^3*e^2*x^2 + 6*d*e^4*x^4 + 10*e^5*x^5*H
ypergeometric2F1[1/2, 3, 3/2, 1 - (e^2*x^2)/d^2]))/(d^5*x^5)
```

IntegrateAlgebraic [A] time = 0.64, size = 115, normalized size = 0.68

$$\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}+\frac{\sqrt{d^2-e^2x^2}\left(-4d^4-10d^3ex-12d^2e^2x^2-15de^3x^3-24e^4x^4\right)}{20d^4x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^4 - 10*d^3*e*x - 12*d^2*e^2*x^2 - 15*d*e^3*x^3 -
 24*e^4*x^4))/(20*d^4*x^5) + (3*e^5*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e
^2*x^2]/d])/(2*d^4)
```

fricas [A] time = 0.40, size = 98, normalized size = 0.58

$$\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(24\,e^4x^4+15\,de^3x^3+12\,d^2e^2x^2+10\,d^3ex+4\,d^4\right)\sqrt{-e^2x^2+d^2}}{20\,d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (24*e^4*x^4 + 15*d*e^
3*x^3 + 12*d^2*e^2*x^2 + 10*d^3*e*x + 4*d^4)*sqrt(-e^2*x^2 + d^2))/(d^4*x^5
)
```

giac [B] time = 0.27, size = 365, normalized size = 2.16

$$\frac{x^5\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{10}}{x}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^8}{x^2}+\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^6}{x^3}+\frac{110\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^4}{x^4}+e^{12}\right)e^3}{160\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^4}-\frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{4\,d^4}-\frac{\left(\frac{110\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^{16}e^{38}}{x}+\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^{16}e^{36}}{x^2}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^{16}e^{34}}{x^3}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^{16}e^{32}}{x^4}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^{16}e^{30}}{x^5}\right)e^{(-35)}}{160\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/160*x^5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^10/x + 15*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*e^8/x^2 + 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^6/x^3 + 110*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^4/x^4 + e^12)*e^3/((d*e + sqrt(-x^2*e^2 +
 d^2)*e)^5*d^4) - 3/4*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^
(-2)/abs(x))/d^4 - 1/160*(110*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^16*e^38/x +
40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^16*e^36/x^2 + 15*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^3*d^16*e^34/x^3 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^16*e^32/x
^4 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^16*e^30/x^5)*e^(-35)/d^20
```

maple [A] time = 0.02, size = 164, normalized size = 0.97

$$-\frac{3e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{4\sqrt{d^2}\,d^3}-\frac{6\sqrt{-e^2x^2+d^2}\,e^4}{5d^4x}-\frac{3\sqrt{-e^2x^2+d^2}\,e^3}{4d^3x^2}-\frac{3\sqrt{-e^2x^2+d^2}\,e^2}{5d^2x^3}-\frac{\sqrt{-e^2x^2+d^2}\,e}{2d\,x^4}-\frac{\sqrt{-e^2x^2+d^2}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -3/5*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-6/5*e^4*(-e^2*x^2+d^2)^(1/2)/d^4/x-1/
2*e*(-e^2*x^2+d^2)^(1/2)/d/x^4-3/4*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-3/4/d^3
*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/5*(-e^2
*x^2+d^2)^(1/2)/x^5
```

maxima [A] time = 0.98, size = 158, normalized size = 0.93

$$-\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{4\,d^4}-\frac{6\,\sqrt{-e^2x^2+d^2}\,e^4}{5\,d^4x}-\frac{3\,\sqrt{-e^2x^2+d^2}\,e^3}{4\,d^3x^2}-\frac{3\,\sqrt{-e^2x^2+d^2}\,e^2}{5\,d^2x^3}-\frac{\sqrt{-e^2x^2+d^2}\,e}{2\,dx^4}-\frac{\sqrt{-e^2x^2+d^2}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/4*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 - 6/5*sqrt
(-e^2*x^2 + d^2)*e^4/(d^4*x) - 3/4*sqrt(-e^2*x^2 + d^2)*e^3/(d^3*x^2) - 3/5
*sqrt(-e^2*x^2 + d^2)*e^2/(d^2*x^3) - 1/2*sqrt(-e^2*x^2 + d^2)*e/(d*x^4) -
1/5*sqrt(-e^2*x^2 + d^2)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^6\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^6*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^6*(d^2 - e^2*x^2)^(1/2)), x)
```

sympy [C] time = 8.96, size = 510, normalized size = 3.02

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{5d^2x^4} - \frac{4e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^4x^2} - \frac{8e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^6} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{5d^2x^4} - \frac{4ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^4x^2} - \frac{8ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^6} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**6/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(5*d**2*x**4) - 4*e**3*sqrt(d
**2/(e**2*x**2) - 1)/(15*d**4*x**2) - 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(15
*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(5*d*
*2*x**4) - 4*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(15*d**4*x**2) - 8*I*e**5*s
qrt(-d**2/(e**2*x**2) + 1)/(15*d**6), True)) + 2*d*e*Piecewise((-1/(4*e*x**
5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1))
+ 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) - 3*e**4*acosh(d/(e*x))/(8*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1))
 + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e**3/(8*d**4*x*sqrt(
-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(e*x))/(8*d**5), True)) + e**2*Pi
ecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**4), T
rue))
```

3.44 $\int \frac{x^5(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^6}-\frac{2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}+\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.27, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1635, 1814, 641, 217, 203}

$$\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^6}-\frac{2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^4*(d + e*x)^2)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (22*d^3*(d + e*x))/(15*e^
6*(d^2 - e^2*x^2)^(3/2)) + (2*d*(30*d + 23*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + Sqrt[d^2 - e^2*x^2]/e^6 - (2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^5(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)\left(\frac{2d^5}{e^5}+\frac{5d^4x}{e^4}+\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}+\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{16d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{30d^5}{e^5}+\frac{15d^4x}{e^4}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^4}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{(2d)\int \frac{1}{\sqrt{d^2-e^2x^2}}}{e^5}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{(2d)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{2d\tan^{-1}\left(\frac{e}{\sqrt{d^2-}}\right.}{e^6}$$

Mathematica [A] time = 0.21, size = 111, normalized size = 0.78

$$\frac{56d^4-82d^3ex-32d^2e^2x^2-\frac{30(d-ex)^3(d+ex)\sin^{-1}\left(\frac{ex}{d}\right)}{\sqrt{1-\frac{e^2x^2}{d^2}}}+76de^3x^3-15e^4x^4}{15e^6(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (56*d^4 - 82*d^3*e*x - 32*d^2*e^2*x^2 + 76*d*e^3*x^3 - 15*e^4*x^4 - (30*(d
- e*x)^3*(d + e*x)*ArcSin[(e*x)/d])/Sqrt[1 - (e^2*x^2)/d^2])/(15*e^6*(d - e
*x)^2*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.65, size = 126, normalized size = 0.88

$$-\frac{2d\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^7}-\frac{\sqrt{d^2-e^2x^2}\,\left(56d^4-82d^3ex-32d^2e^2x^2+76de^3x^3-15e^4x^4\right)}{15e^6(ex-d)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(56*d^4 - 82*d^3*e*x - 32*d^2*e^2*x^2 + 76*d*e^3
*x^3 - 15*e^4*x^4))/(e^6*(-d + e*x)^3*(d + e*x)) - (2*d*Sqrt[-e^2]*Log[-(Sq
rt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^7
```

fricas [A] time = 0.40, size = 188, normalized size = 1.31

$$\frac{56\,de^{4}x^{4}-112\,d^{2}e^{3}x^{3}+112\,d^{4}ex-56\,d^{5}+60\,\left(de^{4}x^{4}-2\,d^{2}e^{3}x^{3}+2\,d^{4}ex-d^{5}\right)\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)+\left(15\,e^{4}x^{4}-76\,de^{3}x^{3}+32\,d^{2}e^{2}x^{2}+82\,d^{3}ex-56\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15\,\left(e^{10}x^{4}-2\,de^{9}x^{3}+2\,d^{3}e^{7}x-d^{4}e^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(56*d*e^4*x^4 - 112*d^2*e^3*x^3 + 112*d^4*e*x - 56*d^5 + 60*(d*e^4*x^4
 - 2*d^2*e^3*x^3 + 2*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + (15*e^4*x^4 - 76*d*e^3*x^3 + 32*d^2*e^2*x^2 + 82*d^3*e*x - 56*d^4)*sqr
t(-e^2*x^2 + d^2))/(e^10*x^4 - 2*d*e^9*x^3 + 2*d^3*e^7*x - d^4*e^6)
```

giac [A] time = 0.30, size = 106, normalized size = 0.74

$$-2\,d\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d)-\frac{\left(56\,d^{6}e^{(-6)}+\left(30\,d^{5}e^{(-5)}-\left(140\,d^{4}e^{(-4)}+\left(70\,d^{3}e^{(-3)}-\left(105\,d^{2}e^{(-2)}+\left(46\,de^{(-1)}-15\,x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2}+d^{2}}}{15\,\left(x^{2}e^{2}-d^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2*d*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/15*(56*d^6*e^(-6) + (30*d^5*e^(-5) - (
140*d^4*e^(-4) + (70*d^3*e^(-3) - (105*d^2*e^(-2) + (46*d*e^(-1) - 15*x)*x)
*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 193, normalized size = 1.35

$$-\frac{x^{6}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{2d\,x^{5}}{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e}+\frac{7d^{2}x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}-\frac{28d^{4}x^{2}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}-\frac{2d\,x^{3}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}+\frac{56d^{6}}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}+\frac{2dx}{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{5}}-\frac{2d\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\,e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -x^6/(-e^2*x^2+d^2)^(5/2)+7/e^2*d^2*x^4/(-e^2*x^2+d^2)^(5/2)-28/3/e^4*d^4*x
^2/(-e^2*x^2+d^2)^(5/2)+56/15/e^6*d^6/(-e^2*x^2+d^2)^(5/2)+2/5/e*d*x^5/(-e^
2*x^2+d^2)^(5/2)-2/3/e^3*d*x^3/(-e^2*x^2+d^2)^(3/2)+2/e^5*d*x/(-e^2*x^2+d^2
)^(1/2)-2/e^5*d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [B] time = 1.02, size = 276, normalized size = 1.93

$$\frac{2}{15}\,dex\left(\frac{15\,x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}-\frac{20\,d^{2}x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}+\frac{8\,d^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}\right)-\frac{x^{6}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2\,dx\left(\frac{3\,x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}-\frac{2\,d^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}\right)}{3\,e}+\frac{7\,d^{2}x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}-\frac{28\,d^{4}x^{2}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{4}}+\frac{56\,d^{6}}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}+\frac{8\,d^{3}x}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}-\frac{14\,dx}{15\sqrt{-e^{2}x^{2}+d^{2}}\,e^{5}}-\frac{2\,d\arcsin\left(\frac{ex}{d}\right)}{e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*d*e*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - x^6/(-e^2*x^2 + d^2)
^(5/2) - 2/3*d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d
^2)^(3/2)*e^4))/e + 7*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 28/3*d^4*x^2/(
(-e^2*x^2 + d^2)^(5/2)*e^4) + 56/15*d^6/((-e^2*x^2 + d^2)^(5/2)*e^6) + 8/15
*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 14/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^5)
 - 2*d*arcsin(e*x/d)/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5}\,{\left(d+e\,x\right)}^{2}}{{\left(d^{2}-e^{2}\,x^{2}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} (d + ex)^{2}}{(-(-d + ex)(d + ex))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x**5*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.45 $\int \frac{x^4(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

Rubi [A] time = 0.21, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1635, 1814, 12, 217, 203}

$$\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3*(d + e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (17*d^2*(d + e*x))/(15*e^
5*(d^2 - e^2*x^2)^(3/2)) + (2*(15*d + 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^2])
 - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)\left(\frac{2d^4}{e^4}+\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}+\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d} \\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{11d^4}{e^4}+\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2} \\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}\,dx}{15d^4} \\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4} \\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x}}\right.}{e^4} \\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}
\end{aligned}
$$

Mathematica [A] time = 0.20, size = 96, normalized size = 0.79

$$
\frac{16d^3-15d(d-ex)^2\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)-17d^2ex-22de^2x^2+26e^3x^3}{15e^5(d-ex)^2\sqrt{d^2-e^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (16*d^3 - 17*d^2*e*x - 22*d*e^2*x^2 + 26*e^3*x^3 - 15*d*(d - e*x)^2*Sqrt[1
- (e^2*x^2)/d^2]*ArcSin[(e*x)/d])/(15*e^5*(d - e*x)^2*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.62, size = 114, normalized size = 0.94

$$
-\frac{\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^6}-\frac{\sqrt{d^2-e^2x^2}\left(16d^3-17d^2ex-22de^2x^2+26e^3x^3\right)}{15e^5(ex-d)^3(d+ex)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(16*d^3 - 17*d^2*e*x - 22*d*e^2*x^2 + 26*e^3*x^3
))/(e^5*(-d + e*x)^3*(d + e*x)) - (Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^
2 - e^2*x^2]])/e^6
```

fricas [A] time = 0.42, size = 172, normalized size = 1.42

$$
\frac{16\,e^4x^4-32\,de^3x^3+32\,d^3ex-16\,d^4+30\left(e^4x^4-2\,de^3x^3+2\,d^3ex-d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(26\,e^3x^3-22\,de^2x^2-17\,d^2ex+16\,d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(e^9x^4-2\,de^8x^3+2\,d^3e^6x-d^4e^5\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(16*e^4*x^4 - 32*d*e^3*x^3 + 32*d^3*e*x - 16*d^4 + 30*(e^4*x^4 - 2*d*e
^3*x^3 + 2*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (26*e
^3*x^3 - 22*d*e^2*x^2 - 17*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^9*x^4
 - 2*d*e^8*x^3 + 2*d^3*e^6*x - d^4*e^5)
```

giac [A] time = 0.28, size = 95, normalized size = 0.79

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d) - \frac{\left(16\,d^5e^{(-5)} + \left(15\,d^4e^{(-4)} - \left(40\,d^3e^{(-3)} + \left(35\,d^2e^{(-2)} - 2\left(15\,de^{(-1)} + 13\,x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}}{15\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-5)*sgn(d) - 1/15*(16*d^5*e^(-5) + (15*d^4*e^(-4) - (40*d
^3*e^(-3) + (35*d^2*e^(-2) - 2*(15*d*e^(-1) + 13*x)*x)*x)*x)*x)*sqrt(-x^2*e
^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.02, size = 236, normalized size = 1.95

$$\frac{x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{2d\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} e} + \frac{d^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}} e^2} - \frac{8d^3x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}} e^3} - \frac{3d^4x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}} e^4} - \frac{x^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^2} + \frac{16d^5}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}} e^5} + \frac{d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}} e^4} + \frac{6x}{5\sqrt{-e^2x^2+d^2}\, e^4} - \frac{\arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\, e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*x^5/(-e^2*x^2+d^2)^(5/2)-1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)+6/5/e^4*x/(-e
^2*x^2+d^2)^(1/2)-1/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)+2*d/e*x^4/(-e^2*x^2+d^2)^(5/2)-8/3*d^3/e^3*x^2/(-e^2*x^2+d^2)^(5/2)+16/
15*d^5/e^5/(-e^2*x^2+d^2)^(5/2)+1/2*d^2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/10*d
^4/e^4*x/(-e^2*x^2+d^2)^(5/2)+1/10*d^2/e^4*x/(-e^2*x^2+d^2)^(3/2)
```

maxima [B] time = 1.00, size = 298, normalized size = 2.46

$$\frac{1}{15}e^2x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right) - \frac{1}{3}x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right) + \frac{2\,dx^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{d^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{8\,d^3x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} - \frac{3\,d^4x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{16\,d^5}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5} + \frac{11\,d^2x}{30\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4} - \frac{4\,x}{15\sqrt{-e^2x^2+d^2}\,e^4} - \frac{\arcsin\left(\frac{ex}{d}\right)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*x*(3*x^2/((-e^2*
x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 2*d*x^4/((-e^
2*x^2 + d^2)^(5/2)*e) + 1/2*d^2*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 8/3*d^3*
x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^4*x/((-e^2*x^2 + d^2)^(5/2)*e^4)
+ 16/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e^5) + 11/30*d^2*x/((-e^2*x^2 + d^2)^(3
/2)*e^4) - 4/15*x/(sqrt(-e^2*x^2 + d^2)*e^4) - arcsin(e*x/d)/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(d+e\,x\right)}^2}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (d + ex)^2}{(-(-d + ex)(d + ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**4*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.46 $\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.17, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1635, 637}

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^2*(d + e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d + e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d + 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)\left(\frac{2d^3}{e^3}+\frac{5d^2x}{e^2}+\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{\frac{6d^3}{e^3}+\frac{15d^2x}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}\\
&= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.65

$$\frac{2d^3-4d^2ex+de^2x^2+2e^3x^3}{5de^4(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 2*e^3*x^3)/(5*d*e^4*(d - e*x)^2*Sqrt[d^2 -
 e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.49, size = 70, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^3-4d^2ex+de^2x^2+2e^3x^3\right)}{5de^4(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 2*e^3*x^3))/(5*d*e^4*
(d - e*x)^3*(d + e*x))
```

fricas [A] time = 0.40, size = 116, normalized size = 1.20

$$\frac{2\,e^4x^4-4\,de^3x^3+4\,d^3ex-2\,d^4-\left(2\,e^3x^3+de^2x^2-4\,d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(de^8x^4-2\,d^2e^7x^3+2\,d^4e^5x-d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 + d*e^2*x^2 -
 4*d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d*e^8*x^4 - 2*d^2*e^7*x^3 + 2*d^
4*e^5*x - d^5*e^4)
```

giac [A] time = 0.29, size = 63, normalized size = 0.65

$$-\frac{\left(2\,d^4e^{(-4)}+\left(x^2\left(\frac{2\,xe}{d}+5\right)-5\,d^2e^{(-2)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{5\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*(2*d^4*e^(-4) + (x^2*(2*x*e/d + 5) - 5*d^2*e^(-2))*x^2)*sqrt(-x^2*e^2
+ d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 65, normalized size = 0.67

$$\frac{(-ex+d)\,(ex+d)^3\left(2e^3x^3+d\,e^2x^2-4d^2ex+2d^3\right)}{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}\,d\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3+d*e^2*x^2-4*d^2*e*x+2*d^3)/d/e^4/(-e^2*x^
2+d^2)^(7/2)
```

maxima [A] time = 0.45, size = 155, normalized size = 1.60

$$\frac{x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{dx^3}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{3\,d^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}+\frac{2\,d^4}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{dx}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}+\frac{2\,x}{5\sqrt{-e^2x^2+d^2}\,de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] x^4/(-e^2*x^2 + d^2)^(5/2) + d*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - d^2*x^2/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 3/5*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^3) + 2/5*d^
4/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/5*d*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 2/5
*x/(sqrt(-e^2*x^2 + d^2)*d*e^3)
```

mupad [B] time = 2.89, size = 66, normalized size = 0.68

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3-4\,d^2\,e\,x+d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d\,e^4\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 + d*e^2*x^2 - 4*d^2*e*x))/(5*d*e^
4*(d + e*x)*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(d+ex)^2}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.47 $\int \frac{x^2(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=87

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}+\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1635, 778, 191}

$$\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d*(d + e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (7*(d + e*x))/(15*e^3*(d^2
 - e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{\left(\frac{2d^2}{e^2}+\frac{5dx}{e}\right)(d+ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15e^2}\\
&= \frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.72

$$\frac{-4d^3+8d^2ex-2de^2x^2+e^3x^3}{15d^2e^3(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (-4*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)/(15*d^2*e^3*(d - e*x)^2*Sqrt[d
^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.48, size = 70, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\left(-4d^3+8d^2ex-2de^2x^2+e^3x^3\right)}{15d^2e^3(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*
e^3*(d - e*x)^3*(d + e*x))
```

fricas [A] time = 0.40, size = 117, normalized size = 1.34

$$-\frac{4\,e^4x^4-8\,de^3x^3+8\,d^3ex-4\,d^4+\left(e^3x^3-2\,de^2x^2+8\,d^2ex-4\,d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^7x^4-2\,d^3e^6x^3+2\,d^5e^4x-d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(4*e^4*x^4 - 8*d*e^3*x^3 + 8*d^3*e*x - 4*d^4 + (e^3*x^3 - 2*d*e^2*x^2
 + 8*d^2*e*x - 4*d^3)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^4 - 2*d^3*e^6*x^3 +
2*d^5*e^4*x - d^6*e^3)
```

giac [A] time = 0.28, size = 61, normalized size = 0.70

$$\frac{\left(4\,d^3e^{(-3)}-\left(x\left(\frac{x^2e^2}{d^2}+5\right)+10\,de^{(-1)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(4*d^3*e^(-3) - (x*(x^2*e^2/d^2 + 5) + 10*d*e^(-1))*x^2)*sqrt(-x^2*e^2
 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 66, normalized size = 0.76

$$-\frac{(-ex+d)\,(ex+d)^3\left(-e^3x^3+2d\,e^2x^2-8d^2ex+4d^3\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^3*(-e^3*x^3+2*d*e^2*x^2-8*d^2*e*x+4*d^3)/d^2/e^3/(-e
^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.45, size = 131, normalized size = 1.51

$$\frac{x^{3}}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{2\,dx^{2}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e}-\frac{d^{2}x}{10\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}-\frac{4\,d^{3}}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{3}}+\frac{x}{30\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}+\frac{x}{15\sqrt{-e^{2}x^{2}+d^{2}}\,d^{2}e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(-e^2*x^2 + d^2)^(5/2) + 2/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e) - 1/1
0*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4/15*d^3/((-e^2*x^2 + d^2)^(5/2)*e^3
) + 1/30*x/((-e^2*x^2 + d^2)^(3/2)*e^2) + 1/15*x/(sqrt(-e^2*x^2 + d^2)*d^2*
e^2)
```

mupad [B] time = 2.87, size = 67, normalized size = 0.77

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^3-8\,d^2\,e\,x+2\,d\,e^2\,x^2-e^3\,x^3\right)}{15\,d^2\,e^3\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(4*d^3 - e^3*x^3 + 2*d*e^2*x^2 - 8*d^2*e*x))/(15*d^
2*e^3*(d + e*x)*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(d + ex\right)^{2}}{\left(- \left(- d + ex\right) \left(d + ex\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.48 $\int \frac{x(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=89

$$\frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {789, 639, 191}

$$\frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^2/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^(3/2)) - (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}\\
&= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}\\
&= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.70

$$\frac{d^3-2d^2ex+8de^2x^2-4e^3x^3}{15d^3e^2(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3 - 2*d^2*e*x + 8*d*e^2*x^2 - 4*e^3*x^3)/(15*d^3*e^2*(d - e*x)^2*Sqrt[d^
2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.55, size = 69, normalized size = 0.78

$$\frac{\sqrt{d^2-e^2x^2}\,\left(d^3-2d^2ex+8de^2x^2-4e^3x^3\right)}{15d^3e^2(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^3 - 2*d^2*e*x + 8*d*e^2*x^2 - 4*e^3*x^3))/(15*d^3*e
^2*(d - e*x)^3*(d + e*x))
```

fricas [A] time = 0.41, size = 117, normalized size = 1.31

$$\frac{e^4x^4-2\,de^3x^3+2\,d^3ex-d^4+\left(4\,e^3x^3-8\,de^2x^2+2\,d^2ex-d^3\right)\sqrt{-e^2x^2+d^2}}{15\,\left(d^3e^6x^4-2\,d^4e^5x^3+2\,d^6e^3x-d^7e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x - d^4 + (4*e^3*x^3 - 8*d*e^2*x^2 +
2*d^2*e*x - d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^4 - 2*d^4*e^5*x^3 + 2*d^6
*e^3*x - d^7*e^2)
```

giac [A] time = 0.28, size = 64, normalized size = 0.72

$$\frac{\left(\left(2\,x\left(\frac{2\,x^2e^3}{d^3}-\frac{5\,e}{d}\right)-5\right)x^2-d^2e^{(-2)}\right)\sqrt{-x^2e^2+d^2}}{15\,\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((2*x*(2*x^2*e^3/d^3 - 5*e/d) - 5)*x^2 - d^2*e^(-2))*sqrt(-x^2*e^2 + d
^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 64, normalized size = 0.72

$$\frac{(-ex+d)\,(ex+d)^3\,\left(-4e^3x^3+8d\,e^2x^2-2d^2ex+d^3\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}\,d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^3*(-4*e^3*x^3+8*d*e^2*x^2-2*d^2*e*x+d^3)/d^3/e^2/(-e^
2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.44, size = 109, normalized size = 1.22

$$\frac{x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{2\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{2\,x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de}-\frac{4\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2/(-e^2*x^2 + d^2)^(5/2) + 2/5*d*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*
d^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 2/15*x/((-e^2*x^2 + d^2)^(3/2)*d*e) - 4/
15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e)
```

mupad [B] time = 2.86, size = 65, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^3-2\,d^2\,e\,x+8\,d\,e^2\,x^2-4\,e^3\,x^3\right)}{15\,d^3\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^3 - 4*e^3*x^3 + 8*d*e^2*x^2 - 2*d^2*e*x))/(15*d^3
*e^2*(d + e*x)*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.49 $\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=77

$$\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {653, 192, 191}

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]

[Out] (2*(d + e*x))/(5*e*(d^2 - e^2*x^2)^(5/2)) + x/(5*d^2*(d^2 - e^2*x^2)^(3/2))
 + (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}} \, dx &= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{3}{5}\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}} \, dx\\ &= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}} \, dx}{5d^2}\\ &= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.82

$$\frac{2d^3+d^2ex-4de^2x^2+2e^3x^3}{5d^4e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)/(5*d^4*e*(d - e*x)^2*Sqrt[d^2 -
 e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.05, size = 70, normalized size = 0.91

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^3+d^2ex-4de^2x^2+2e^3x^3\right)}{5d^4e(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e*
(d - e*x)^3*(d + e*x))
```

fricas [A] time = 0.40, size = 116, normalized size = 1.51

$$\frac{2\,e^4x^4-4\,de^3x^3+4\,d^3ex-2\,d^4-\left(2\,e^3x^3-4\,de^2x^2+d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\,\left(d^4e^5x^4-2\,d^5e^4x^3+2\,d^7e^2x-d^8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 - 4*d*e^2*x^2
 + d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 - 2*d^5*e^4*x^3 + 2*
d^7*e^2*x - d^8*e)
```

giac [A] time = 0.31, size = 61, normalized size = 0.79

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(x^2\left(\frac{2\,x^2e^4}{d^4}-\frac{5\,e^2}{d^2}\right)+5\right)x+2\,de^{(-1)}\right)}{5\,\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*((x^2*(2*x^2*e^4/d^4 - 5*e^2/d^2) + 5)*x + 2*d*e^
(-1))/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 65, normalized size = 0.84

$$\frac{(-ex+d)\,(ex+d)^3\left(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3\right)}{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3-4*d*e^2*x^2+d^2*e*x+2*d^3)/d^4/e/(-e^2*x^
2+d^2)^(7/2)
```

maxima [A] time = 0.44, size = 78, normalized size = 1.01

$$\frac{2\,x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{2\,d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}+\frac{2\,x}{5\,\sqrt{-e^2x^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*x/(-e^2*x^2 + d^2)^(5/2) + 2/5*d/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*x/((-
e^2*x^2 + d^2)^(3/2)*d^2) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

mupad [B] time = 2.81, size = 66, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3+d^2\,e\,x-4\,d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d^4\,e\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 - 4*d*e^2*x^2 + d^2*e*x))/(5*d^4*
e*(d + e*x)*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.50 $\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=117

$$\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 823, 12, 266, 63, 208}

$$\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d + 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d + 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-8dex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^4e^2-16d^3e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^6e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-\right.}{d^4e^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Mathematica [C] time = 0.04, size = 81, normalized size = 0.69

$$\frac{3d^5+30d^4ex-40d^2e^3x^3+3d^5\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+16e^5x^5}{15d^5\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (3*d^5 + 30*d^4*e*x - 40*d^2*e^3*x^3 + 16*e^5*x^5 + 3*d^5*Hypergeometric2F1
[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^5*(d^2 - e^2*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.65, size = 111, normalized size = 0.95

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{\sqrt{d^2-e^2x^2}\,\left(26d^3-22d^2ex-17de^2x^2+16e^3x^3\right)}{15d^5(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(26*d^3 - 22*d^2*e*x - 17*d*e^2*x^2 + 16*e^3*x^3))/(15
*d^5*(d - e*x)^3*(d + e*x)) + (2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*
x^2]/d])/d^5
```

fricas [A] time = 0.39, size = 169, normalized size = 1.44

$$\frac{26\,e^4x^4-52\,de^3x^3+52\,d^3ex-26\,d^4+15\left(e^4x^4-2\,de^3x^3+2\,d^3ex-d^4\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(16\,e^3x^3-17\,de^2x^2-22\,d^2ex+26\,d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^5e^4x^4-2\,d^6e^3x^3+2\,d^8ex-d^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(26*e^4*x^4 - 52*d*e^3*x^3 + 52*d^3*e*x - 26*d^4 + 15*(e^4*x^4 - 2*d*e
^3*x^3 + 2*d^3*e*x - d^4)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (16*e^3*x^3
- 17*d*e^2*x^2 - 22*d^2*e*x + 26*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^4*x^4 -
2*d^6*e^3*x^3 + 2*d^8*e*x - d^9)
```

giac [A] time = 0.29, size = 118, normalized size = 1.01

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{16\,xe^5}{d^5}+\frac{15\,e^4}{d^4}\right)-\frac{40\,e^3}{d^3}\right)x-\frac{35\,e^2}{d^2}\right)x+\frac{30\,e}{d}\right)x+26\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(16*x*e^5/d^5 + 15*e^4/d^4) - 40*e^3/d^3)*
x - 35*e^2/d^2)*x + 30*e/d)*x + 26)/(x^2*e^2 - d^2)^3 - log(1/2*abs(-2*d*e
- 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^5
```

maple [A] time = 0.01, size = 160, normalized size = 1.37

$$\frac{2ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{8ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^4}+\frac{16ex}{15\sqrt{-e^2x^2+d^2}\,d^5}+\frac{1}{\sqrt{-e^2x^2+d^2}\,d^4}+\frac{2}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 2/5/(-e^2*x^2+d^2)^(5/2)+2/5/d*e*x/(-e^2*x^2+d^2)^(5/2)+8/15/d^3*e*x/(-e^2*
x^2+d^2)^(3/2)+16/15/d^5*e*x/(-e^2*x^2+d^2)^(1/2)+1/3/d^2/(-e^2*x^2+d^2)^(3
/2)+1/d^4/(-e^2*x^2+d^2)^(1/2)-1/d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.46, size = 154, normalized size = 1.32

$$\frac{2\,ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{2}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{8\,ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}+\frac{16\,ex}{15\sqrt{-e^2x^2+d^2}\,d^5}-\frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d^5}+\frac{1}{\sqrt{-e^2x^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*e*x/((-e^2*x^2 + d^2)^(5/2)*d) + 2/5/(-e^2*x^2 + d^2)^(5/2) + 8/15*e*x/
((-e^2*x^2 + d^2)^(3/2)*d^3) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d^2) + 16/15*e*x
/(sqrt(-e^2*x^2 + d^2)*d^5) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/a
bs(x))/d^5 + 1/(sqrt(-e^2*x^2 + d^2)*d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{x\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.51 $\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.28, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1805, 807, 266, 63, 208}

$$\frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e*(10*d + 13*e*x))/(15*d^4
*(d^2 - e^2*x^2)^(3/2)) + (e*(30*d + 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
- Sqrt[d^2 - e^2*x^2]/(d^6*x) - (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-8e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+26e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{(2e)\int \frac{}{x\sqrt{d^2 \ldots}}}{d^5 \ldots}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{e\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{2\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{2e\tanh^{-1}\left(\ldots\right.}{d \ldots}$$

Mathematica [C] time = 0.05, size = 90, normalized size = 0.62

$$\frac{-15d^6+105d^4e^2x^2-140d^2e^4x^4+6d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+56e^6x^6}{15d^6x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-15*d^6 + 105*d^4*e^2*x^2 - 140*d^2*e^4*x^4 + 56*e^6*x^6 + 6*d^5*e*x*Hyper
geometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^6*x*(d^2 - e^2*x^2)^(
5/2))
```

IntegrateAlgebraic [A] time = 0.73, size = 126, normalized size = 0.87

$$\frac{4e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{\sqrt{d^2-e^2x^2}\left(-15d^4+76d^3ex-32d^2e^2x^2-82de^3x^3+56e^4x^4\right)}{15d^6x(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

[Out] (Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 76*d^3*e*x - 32*d^2*e^2*x^2 - 82*d*e^3*x^3 + 56*e^4*x^4))/(15*d^6*x*(d - e*x)^3*(d + e*x)) + (4*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^6

fricas [A] time = 0.42, size = 195, normalized size = 1.34

$$\frac{46\,e^5x^5 - 92\,de^4x^4 + 92\,d^3e^2x^2 - 46\,d^4ex + 30\left(e^5x^5 - 2\,de^4x^4 + 2\,d^3e^2x^2 - d^4ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \left(56\,e^4x^4 - 82\,de^3x^3 - 32\,d^2e^2x^2 + 76\,d^3ex - 15\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^6e^4x^5 - 2\,d^7e^3x^4 + 2\,d^9ex^2 - d^{10}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(46*e^5*x^5 - 92*d*e^4*x^4 + 92*d^3*e^2*x^2 - 46*d^4*e*x + 30*(e^5*x^5
 - 2*d*e^4*x^4 + 2*d^3*e^2*x^2 - d^4*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x
) - (56*e^4*x^4 - 82*d*e^3*x^3 - 32*d^2*e^2*x^2 + 76*d^3*e*x - 15*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^6*e^4*x^5 - 2*d^7*e^3*x^4 + 2*d^9*e*x^2 - d^10*x)
```

giac [A] time = 0.29, size = 188, normalized size = 1.30

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{41\,xe^6}{d^6}+\frac{30\,e^5}{d^5}\right)-\frac{95\,e^4}{d^4}\right)x-\frac{70\,e^3}{d^3}\right)x+\frac{60\,e^2}{d^2}\right)x+\frac{46\,e}{d}\right)}{15\left(x^2e^2-d^2\right)^3} - \frac{2\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^6} + \frac{xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^6} - \frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{2\,d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(41*x*e^6/d^6 + 30*e^5/d^5) - 95*e^4/d^4)*
x - 70*e^3/d^3)*x + 60*e^2/d^2)*x + 46*e/d)/(x^2*e^2 - d^2)^3 - 2*e*log(1/2
*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6 + 1/2*x*e^3/((d*
e + sqrt(-x^2*e^2 + d^2)*e)*d^6) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1
)/(d^6*x)
```

maple [A] time = 0.01, size = 193, normalized size = 1.33

$$\frac{7e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2} + \frac{2e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d} + \frac{28e^2x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4} - \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x} + \frac{2e}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3} - \frac{2e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^5} + \frac{56e^2x}{15\sqrt{-e^2x^2+d^2}\,d^6} + \frac{2e}{\sqrt{-e^2x^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 7/5*e^2*x/d^2/(-e^2*x^2+d^2)^(5/2)+28/15*e^2/d^4*x/(-e^2*x^2+d^2)^(3/2)+56/
15*e^2/d^6*x/(-e^2*x^2+d^2)^(1/2)+2/5/d*e/(-e^2*x^2+d^2)^(5/2)+2/3/d^3*e/(-
e^2*x^2+d^2)^(3/2)+2/d^5*e/(-e^2*x^2+d^2)^(1/2)-2/d^5*e/(d^2)^(1/2)*ln((2*d
^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/x/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.47, size = 187, normalized size = 1.29

$$\frac{7\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2} + \frac{2\,e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d} + \frac{28\,e^2x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4} + \frac{2\,e}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3} - \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x} + \frac{56\,e^2x}{15\,\sqrt{-e^2x^2+d^2}\,d^6} - \frac{2\,e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d^6} + \frac{2\,e}{\sqrt{-e^2x^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 7/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 2/5*e/((-e^2*x^2 + d^2)^(5/2)*d) +
 28/15*e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 2/3*e/((-e^2*x^2 + d^2)^(3/2)*d
^3) - 1/((-e^2*x^2 + d^2)^(5/2)*x) + 56/15*e^2*x/(sqrt(-e^2*x^2 + d^2)*d^6)
 - 2*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^6 + 2*e/(sqrt(
-e^2*x^2 + d^2)*d^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^2\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{x^2\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**2/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.52 $\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=182

$$\frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}}-\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

Rubi [A] time = 0.36, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1805, 1807, 807, 266, 63, 208}

$$\frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e^2*(d + e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d + 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d + 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) - (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-10e^2x^2-\frac{8e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+45e^2x^2+\frac{36e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex-60e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{\int \frac{60d^3e+135d^2}{x^2\sqrt{d^2-e^2x}}}{30d^8}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x}}{d^7x}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x}}{d^7x}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x}}{d^7x}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x}}{d^7x}$$

Mathematica [C] time = 0.06, size = 117, normalized size = 0.64

$$\frac{e\left(-10d^6+60d^4e^2x^2-80d^2e^4x^4+d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+d^5ex\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+32e^6x^6\right)}{5d^7x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (e*(-10*d^6 + 60*d^4*e^2*x^2 - 80*d^2*e^4*x^4 + 32*e^6*x^6 + d^5*e*x*Hyperg
eometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2] + d^5*e*x*Hypergeometric2F1[-
5/2, 2, -3/2, 1 - (e^2*x^2)/d^2]))/(5*d^7*x*(d^2 - e^2*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.93, size = 139, normalized size = 0.76

$$\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{\sqrt{d^2-e^2x^2}\left(-5d^5-10d^4ex+94d^3e^2x^2-58d^2e^3x^3-83de^4x^4+64e^5x^5\right)}{10d^7x^2(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^5 - 10*d^4*e*x + 94*d^3*e^2*x^2 - 58*d^2*e^3*x^3
 - 83*d*e^4*x^4 + 64*e^5*x^5))/(10*d^7*x^2*(d - e*x)^3*(d + e*x)) + (9*e^2*
ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^7
```

fricas [A] time = 0.43, size = 216, normalized size = 1.19

$$\frac{54\,e^6x^6-108\,de^5x^5+108\,d^3e^3x^3-54\,d^4e^2x^2+45\left(e^6x^6-2\,de^5x^5+2\,d^3e^3x^3-d^4e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(64\,e^5x^5-83\,de^4x^4-58\,d^2e^3x^3+94\,d^3e^2x^2-10\,d^4ex-5\,d^5\right)\sqrt{-e^2x^2+d^2}}{10\left(d^7e^4x^6-2\,d^8e^3x^5+2\,d^{10}ex^3-d^{11}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(54*e^6*x^6 - 108*d*e^5*x^5 + 108*d^3*e^3*x^3 - 54*d^4*e^2*x^2 + 45*(e
^6*x^6 - 2*d*e^5*x^5 + 2*d^3*e^3*x^3 - d^4*e^2*x^2)*log(-(d - sqrt(-e^2*x^2
 + d^2))/x) - (64*e^5*x^5 - 83*d*e^4*x^4 - 58*d^2*e^3*x^3 + 94*d^3*e^2*x^2
- 10*d^4*e*x - 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^4*x^6 - 2*d^8*e^3*x^5 +
2*d^10*e*x^3 - d^11*x^2)
```

giac [A] time = 0.30, size = 260, normalized size = 1.43

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(2\left(x\left(\frac{11\,xe^7}{d^7}+\frac{10\,e^6}{d^6}\right)-\frac{25\,e^5}{d^5}\right)x-\frac{45\,e^4}{d^4}\right)x+\frac{30\,e^3}{d^3}\right)x+\frac{27\,e^2}{d^2}\right)}{5\left(x^2e^2-d^2\right)^3}-\frac{9\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^7}+\frac{x^2\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^4}{x}+e^6\right)}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^7}-\frac{\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^7e^8}{x}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^7e^6}{x^2}\right)e^{(-8)}}{8\,d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*(((2*(x*(11*x*e^7/d^7 + 10*e^6/d^6) - 25*e^5/d^5)
*x - 45*e^4/d^4)*x + 30*e^3/d^3)*x + 27*e^2/d^2)/(x^2*e^2 - d^2)^3 - 9/2*e^
2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^7 + 1/8*x
^2*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^2 + d
^2)*e)^2*d^7) - 1/8*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^7*e^8/x + (d*e + sq
rt(-x^2*e^2 + d^2)*e)^2*d^7*e^6/x^2)*e^(-8)/d^14
```

maple [A] time = 0.02, size = 224, normalized size = 1.23

$$\frac{12e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}+\frac{9e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}-\frac{2e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}+\frac{16e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}+\frac{3e^2}{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}-\frac{1}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}-\frac{9e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^6}+\frac{32e^3x}{5\sqrt{-e^2x^2+d^2}\,d^7}+\frac{9e^2}{2\sqrt{-e^2x^2+d^2}\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 9/10*e^2/d^2/(-e^2*x^2+d^2)^(5/2)+3/2*e^2/d^4/(-e^2*x^2+d^2)^(3/2)+9/2*e^2/
d^6/(-e^2*x^2+d^2)^(1/2)-9/2*e^2/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)-2/d*e/x/(-e^2*x^2+d^2)^(5/2)+12/5/d^3*e^3*x/(-e^2*x^
2+d^2)^(5/2)+16/5/d^5*e^3*x/(-e^2*x^2+d^2)^(3/2)+32/5/d^7*e^3*x/(-e^2*x^2+d
^2)^(1/2)-1/2/x^2/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.47, size = 218, normalized size = 1.20

$$\frac{12\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}+\frac{9\,e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{16\,e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}+\frac{3\,e^2}{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}-\frac{2\,e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}+\frac{32\,e^3x}{5\sqrt{-e^2x^2+d^2}\,d^7}-\frac{9\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{2\,d^7}+\frac{9\,e^2}{2\sqrt{-e^2x^2+d^2}\,d^6}-\frac{1}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 12/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^3) + 9/10*e^2/((-e^2*x^2 + d^2)^(5/2)*
d^2) + 16/5*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^5) + 3/2*e^2/((-e^2*x^2 + d^2)^
(3/2)*d^4) - 2*e/((-e^2*x^2 + d^2)^(5/2)*d*x) + 32/5*e^3*x/(sqrt(-e^2*x^2 +
 d^2)*d^7) - 9/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^
7 + 9/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^6) - 1/2/((-e^2*x^2 + d^2)^(5/2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^3\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{x^3\,(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.53 $\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}+\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}-\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+$$

Rubi [A] time = 0.47, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 1807, 807, 266, 63, 208}

$$\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}-\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e^3*(d + e*x))/(5*d^4*(d^2 - e^2*x^2)^(5/2)) + (e^3*(20*d + 23*e*x))/(15
*d^6*(d^2 - e^2*x^2)^(3/2)) + (2*e^3*(45*d + 53*e*x))/(15*d^8*Sqrt[d^2 - e^
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^6*x^3) - (e*Sqrt[d^2 - e^2*x^2])/(d^7*x^
2) - (14*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^8*x) - (7*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-10e^2x^2-\frac{10e^3x^3}{d}-\frac{8e^4x^4}{d^2}}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+45e^2x^2+\frac{60e^3x^3}{d}+\frac{46e^4x^4}{d^2}}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex-60e^2x^2-\frac{90e^3x^3}{d}}{x^4\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} + \frac{\int \frac{90d^3e+210}{x^3}}{}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x}}{d^7x^2}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x}}{d^7x^2}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x}}{d^7x^2}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x}}{d^7x^2}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x}}{d^7x^2}$$

Mathematica [C] time = 0.05, size = 105, normalized size = 0.50

$$\frac{-5d^8-55d^6e^2x^2+330d^4e^4x^4-440d^2e^6x^6+6d^5e^3x^3\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+176e^8x^8}{15d^8x^3\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-5*d^8 - 55*d^6*e^2*x^2 + 330*d^4*e^4*x^4 - 440*d^2*e^6*x^6 + 176*e^8*x^8
+ 6*d^5*e^3*x^3*Hypergeometric2F1[-5/2, 2, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^
8*x^3*(d^2 - e^2*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 1.05, size = 150, normalized size = 0.72

$$\frac{14e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}+\frac{\sqrt{d^2-e^2x^2}\left(-5d^6-5d^5ex-40d^4e^2x^2+246d^3e^3x^3-122d^2e^4x^4-247de^5x^5+176e^6x^6\right)}{15d^8x^3(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^6 - 5*d^5*e*x - 40*d^4*e^2*x^2 + 246*d^3*e^3*x^3
 - 122*d^2*e^4*x^4 - 247*d*e^5*x^5 + 176*e^6*x^6))/(15*d^8*x^3*(d - e*x)^3*
(d + e*x)) + (14*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^8
```

fricas [A] time = 0.47, size = 227, normalized size = 1.09

$$\frac{116\,e^7x^7-232\,de^6x^6+232\,d^3e^4x^4-116\,d^4e^3x^3+105\left(e^7x^7-2\,de^6x^6+2\,d^3e^4x^4-d^4e^3x^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(176\,e^6x^6-247\,de^5x^5-122\,d^2e^4x^4+246\,d^3e^3x^3-40\,d^4e^2x^2-5\,d^5ex-5\,d^6\right)\sqrt{-e^2x^2+d^2}}{15\left(d^8e^4x^7-2\,d^9e^3x^6+2\,d^{11}ex^4-d^{12}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(116*e^7*x^7 - 232*d*e^6*x^6 + 232*d^3*e^4*x^4 - 116*d^4*e^3*x^3 + 105
*(e^7*x^7 - 2*d*e^6*x^6 + 2*d^3*e^4*x^4 - d^4*e^3*x^3)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) - (176*e^6*x^6 - 247*d*e^5*x^5 - 122*d^2*e^4*x^4 + 246*d^3*e
^3*x^3 - 40*d^4*e^2*x^2 - 5*d^5*e*x - 5*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^4
*x^7 - 2*d^9*e^3*x^6 + 2*d^11*e*x^4 - d^12*x^3)
```

giac [A] time = 0.35, size = 325, normalized size = 1.56

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(2\,x\left(\frac{53\,xe^8}{d^8}+\frac{45\,e^7}{d^7}\right)-\frac{235\,e^6}{d^6}\right)x-\frac{200\,e^5}{d^5}\right)x+\frac{135\,e^4}{d^4}\right)x+\frac{116\,e^3}{d^3}\right)}{15\left(x^2e^2-d^2\right)^3}+\frac{x^3\left(\frac{6\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^6}{x}+\frac{57\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^4}{x^2}+e^8\right)e}{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^8}-\frac{7\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^8}-\frac{\left(\frac{57\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^{16}e^{16}}{x}+\frac{6\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^{16}e^{14}}{x^2}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^{16}e^{12}}{x^3}\right)e^{(-15)}}{24\,d^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((2*x*(53*x*e^8/d^8 + 45*e^7/d^7) - 235*e^6/d^
6)*x - 200*e^5/d^5)*x + 135*e^4/d^4)*x + 116*e^3/d^3)/(x^2*e^2 - d^2)^3 + 1
/24*x^3*(6*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^6/x + 57*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*e^4/x^2 + e^8)*e/((d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^8) - 7*e^3*
log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^8 - 1/24*(5
7*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^16*e^16/x + 6*(d*e + sqrt(-x^2*e^2 + d^2
)*e)^2*d^16*e^14/x^2 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^16*e^12/x^3)*e^(-
15)/d^24
```

maple [A] time = 0.02, size = 249, normalized size = 1.19

$$\frac{22e^4x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}+\frac{7e^3}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}-\frac{11e^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}+\frac{88e^4x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}-\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,x^2}+\frac{7e^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}-\frac{7e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^7}+\frac{176e^4x}{15\sqrt{-e^2x^2+d^2}\,d^8}-\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^3}+\frac{7e^3}{\sqrt{-e^2x^2+d^2}\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x)
```

[Out] -1/d*e/x^2/(-e^2*x^2+d^2)^(5/2)+7/5/d^3*e^3/(-e^2*x^2+d^2)^(5/2)+7/3/d^5*e^3/(-e^2*x^2+d^2)^(3/2)+7/d^7*e^3/(-e^2*x^2+d^2)^(1/2)-7/d^7*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-11/3*e^2/d^2/x/(-e^2*x^2+d^2)^(5/2)+22/5*e^4/d^4*x/(-e^2*x^2+d^2)^(5/2)+88/15*e^4/d^6*x/(-e^2*x^2+d^2)^(3/2)+176/15*e^4/d^8*x/(-e^2*x^2+d^2)^(1/2)-1/3/x^3/(-e^2*x^2+d^2)^(5/2)

maxima [A] time = 0.48, size = 243, normalized size = 1.16

$$\frac{22\,e^4x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}+\frac{7\,e^3}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}+\frac{88\,e^4x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}+\frac{7\,e^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}-\frac{11\,e^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}+\frac{176\,e^4x}{15\sqrt{-e^2x^2+d^2}\,d^8}-\frac{7\,e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d^8}+\frac{7\,e^3}{\sqrt{-e^2x^2+d^2}\,d^7}-\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^2}-\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")

[Out] 22/5*e^4*x/((-e^2*x^2 + d^2)^(5/2)*d^4) + 7/5*e^3/((-e^2*x^2 + d^2)^(5/2)*d^3) + 88/15*e^4*x/((-e^2*x^2 + d^2)^(3/2)*d^6) + 7/3*e^3/((-e^2*x^2 + d^2)^(3/2)*d^5) - 11/3*e^2/((-e^2*x^2 + d^2)^(5/2)*d^2*x) + 176/15*e^4*x/(sqrt(-e^2*x^2 + d^2)*d^8) - 7*e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^8 + 7*e^3/(sqrt(-e^2*x^2 + d^2)*d^7) - e/((-e^2*x^2 + d^2)^(5/2)*d*x^2) - 1/3/((-e^2*x^2 + d^2)^(5/2)*x^3)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x\right)}^2}{x^4\,{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x)

[Out] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex\right)^{2}}{x^{4} \left(- \left(- d + ex\right) \left(d + ex\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(7/2),x)

[Out] Integral((d + e*x)**2/(x**4*(-(-d + e*x)*(d + e*x))**(7/2)), x)

3.54 $\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=81

$$-\frac{3}{5}\sqrt{1-x^2}\,x^2-\frac{3}{20}(5x+8)\sqrt{1-x^2}-\frac{1}{5}\sqrt{1-x^2}\,x^4-\frac{1}{2}\sqrt{1-x^2}\,x^3+\frac{3}{4}\sin^{-1}(x)$$

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1809, 833, 780, 216}

$$-\frac{1}{5}\sqrt{1-x^2}\,x^4-\frac{1}{2}\sqrt{1-x^2}\,x^3-\frac{3}{5}\sqrt{1-x^2}\,x^2-\frac{3}{20}(5x+8)\sqrt{1-x^2}+\frac{3}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2*Sqrt[1 - x^2])/5 - (x^3*Sqrt[1 - x^2])/2 - (x^4*Sqrt[1 - x^2])/5 -
(3*(8 + 5*x)*Sqrt[1 - x^2])/20 + (3*ArcSin[x])/4
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{5}x^4\sqrt{1-x^2} - \frac{1}{5}\int \frac{(-9-10x)x^3}{\sqrt{1-x^2}}\,dx \\ &= -\frac{1}{2}x^3\sqrt{1-x^2} - \frac{1}{5}x^4\sqrt{1-x^2} + \frac{1}{20}\int \frac{x^2(30+36x)}{\sqrt{1-x^2}}\,dx \\ &= -\frac{3}{5}x^2\sqrt{1-x^2} - \frac{1}{2}x^3\sqrt{1-x^2} - \frac{1}{5}x^4\sqrt{1-x^2} - \frac{1}{60}\int \frac{(-72-90x)x}{\sqrt{1-x^2}}\,dx \\ &= -\frac{3}{5}x^2\sqrt{1-x^2} - \frac{1}{2}x^3\sqrt{1-x^2} - \frac{1}{5}x^4\sqrt{1-x^2} - \frac{3}{20}(8+5x)\sqrt{1-x^2} + \frac{3}{4}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= -\frac{3}{5}x^2\sqrt{1-x^2} - \frac{1}{2}x^3\sqrt{1-x^2} - \frac{1}{5}x^4\sqrt{1-x^2} - \frac{3}{20}(8+5x)\sqrt{1-x^2} + \frac{3}{4}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.52

$$\frac{3}{4}\sin^{-1}(x) - \frac{1}{20}\sqrt{1-x^2}\left(4x^4+10x^3+12x^2+15x+24\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] -1/20*(Sqrt[1 - x^2]*(24 + 15*x + 12*x^2 + 10*x^3 + 4*x^4)) + (3*ArcSin[x])
/4
```

IntegrateAlgebraic [A] time = 0.23, size = 58, normalized size = 0.72

$$\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right) + \frac{1}{20}\sqrt{1-x^2}\left(-4x^4-10x^3-12x^2-15x-24\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (Sqrt[1 - x^2]*(-24 - 15*x - 12*x^2 - 10*x^3 - 4*x^4))/20 + (3*ArcTan[x/(-1
 + Sqrt[1 - x^2])])/2
```

fricas [A] time = 0.40, size = 50, normalized size = 0.62

$$-\frac{1}{20}\left(4\,x^4+10\,x^3+12\,x^2+15\,x+24\right)\sqrt{-x^2+1} - \frac{3}{2}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*(4*x^4 + 10*x^3 + 12*x^2 + 15*x + 24)*sqrt(-x^2 + 1) - 3/2*arctan((sq
rt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.18, size = 34, normalized size = 0.42

$$-\frac{1}{20}\left((2\left((2\,x+5)x+6\right)x+15)x+24\right)\sqrt{-x^2+1} + \frac{3}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/20*((2*((2*x + 5)*x + 6)*x + 15)*x + 24)*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

maple [A] time = 0.01, size = 71, normalized size = 0.88

$$-\frac{\sqrt{-x^2+1}\,x^4}{5}-\frac{\sqrt{-x^2+1}\,x^3}{2}-\frac{3\sqrt{-x^2+1}\,x^2}{5}-\frac{3\sqrt{-x^2+1}\,x}{4}+\frac{3\arcsin(x)}{4}-\frac{6\sqrt{-x^2+1}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -1/5*x^4*(-x^2+1)^(1/2)-3/5*x^2*(-x^2+1)^(1/2)-6/5*(-x^2+1)^(1/2)-1/2*x^3*(
-x^2+1)^(1/2)-3/4*x*(-x^2+1)^(1/2)+3/4*arcsin(x)
```

maxima [A] time = 0.97, size = 70, normalized size = 0.86

$$-\frac{1}{5}\sqrt{-x^2+1}\,x^4-\frac{1}{2}\sqrt{-x^2+1}\,x^3-\frac{3}{5}\sqrt{-x^2+1}\,x^2-\frac{3}{4}\sqrt{-x^2+1}\,x-\frac{6}{5}\sqrt{-x^2+1}+\frac{3}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2 + 1)*x^4 - 1/2*sqrt(-x^2 + 1)*x^3 - 3/5*sqrt(-x^2 + 1)*x^2 -
 3/4*sqrt(-x^2 + 1)*x - 6/5*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

mupad [B] time = 2.50, size = 36, normalized size = 0.44

$$\frac{3\,\mathrm{asin}(x)}{4}-\sqrt{1-x^2}\,\left(\frac{x^4}{5}+\frac{x^3}{2}+\frac{3\,x^2}{5}+\frac{3\,x}{4}+\frac{6}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

```
[Out] (3*asin(x))/4 - (1 - x^2)^(1/2)*((3*x)/4 + (3*x^2)/5 + x^3/2 + x^4/5 + 6/5)
```

sympy [A] time = 1.41, size = 73, normalized size = 0.90

$$-\frac{x^4\sqrt{1-x^2}}{5}-\frac{x^3\sqrt{1-x^2}}{2}-\frac{3x^2\sqrt{1-x^2}}{5}-\frac{3x\sqrt{1-x^2}}{4}-\frac{6\sqrt{1-x^2}}{5}+\frac{3\,\mathrm{asin}\,(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**4*sqrt(1 - x**2)/5 - x**3*sqrt(1 - x**2)/2 - 3*x**2*sqrt(1 - x**2)/5 -
3*x*sqrt(1 - x**2)/4 - 6*sqrt(1 - x**2)/5 + 3*asin(x)/4
```

3.55 $\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=63

$$-\frac{2}{3}\sqrt{1-x^2}\,x^2-\frac{1}{24}(21x+32)\sqrt{1-x^2}\;-\frac{1}{4}\sqrt{1-x^2}\,x^3+\frac{7}{8}\sin^{-1}(x)$$

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1809, 833, 780, 216}

$$-\frac{1}{4}\sqrt{1-x^2}\,x^3-\frac{2}{3}\sqrt{1-x^2}\,x^2-\frac{1}{24}(21x+32)\sqrt{1-x^2}\;+\frac{7}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (-2*x^2*Sqrt[1 - x^2])/3 - (x^3*Sqrt[1 - x^2])/4 - ((32 + 21*x)*Sqrt[1 - x^
2])/24 + (7*ArcSin[x])/8
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{4}x^3\sqrt{1-x^2} - \frac{1}{4}\int \frac{(-7-8x)x^2}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2}{3}x^2\sqrt{1-x^2} - \frac{1}{4}x^3\sqrt{1-x^2} + \frac{1}{12}\int \frac{x(16+21x)}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2}{3}x^2\sqrt{1-x^2} - \frac{1}{4}x^3\sqrt{1-x^2} - \frac{1}{24}(32+21x)\sqrt{1-x^2} + \frac{7}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2}{3}x^2\sqrt{1-x^2} - \frac{1}{4}x^3\sqrt{1-x^2} - \frac{1}{24}(32+21x)\sqrt{1-x^2} + \frac{7}{8}\sin^{-1}(x)$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.59

$$\frac{7}{8}\sin^{-1}(x) - \frac{1}{24}\sqrt{1-x^2}\left(6x^3+16x^2+21x+32\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] -1/24*(Sqrt[1 - x^2]*(32 + 21*x + 16*x^2 + 6*x^3)) + (7*ArcSin[x])/8
```

IntegrateAlgebraic [A] time = 0.22, size = 53, normalized size = 0.84

$$\frac{7}{4}\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right) + \frac{1}{24}\sqrt{1-x^2}\left(-6x^3-16x^2-21x-32\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] (Sqrt[1 - x^2]*(-32 - 21*x - 16*x^2 - 6*x^3))/24 + (7*ArcTan[x/(-1 + Sqrt[1
 - x^2])])/4
```

fricas [A] time = 0.38, size = 45, normalized size = 0.71

$$-\frac{1}{24}\left(6x^3+16x^2+21x+32\right)\sqrt{-x^2+1} - \frac{7}{4}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/24*(6*x^3 + 16*x^2 + 21*x + 32)*sqrt(-x^2 + 1) - 7/4*arctan((sqrt(-x^2 +
 1) - 1)/x)
```

giac [A] time = 0.19, size = 30, normalized size = 0.48

$$-\frac{1}{24}((2(3x+8)x+21)x+32)\sqrt{-x^2+1} + \frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/24*((2*(3*x + 8)*x + 21)*x + 32)*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

maple [A] time = 0.01, size = 57, normalized size = 0.90

$$-\frac{\sqrt{-x^2+1}\,x^3}{4} - \frac{2\sqrt{-x^2+1}\,x^2}{3} - \frac{7\sqrt{-x^2+1}\,x}{8} + \frac{7\arcsin(x)}{8} - \frac{4\sqrt{-x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-x^2+1)^(1/2)*x^3-7/8*(-x^2+1)^(1/2)*x+7/8*arcsin(x)-2/3*(-x^2+1)^(1/
2)*x^2-4/3*(-x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 56, normalized size = 0.89

$$-\frac{1}{4}\sqrt{-x^2+1}\,x^3-\frac{2}{3}\sqrt{-x^2+1}\,x^2-\frac{7}{8}\sqrt{-x^2+1}\,x-\frac{4}{3}\sqrt{-x^2+1}+\frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^2 + 1)*x^3 - 2/3*sqrt(-x^2 + 1)*x^2 - 7/8*sqrt(-x^2 + 1)*x - 4
/3*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

mupad [B] time = 0.03, size = 31, normalized size = 0.49

$$\frac{7\,\mathrm{asin}(x)}{8}-\sqrt{1-x^2}\,\left(\frac{x^3}{4}+\frac{2\,x^2}{3}+\frac{7\,x}{8}+\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

```
[Out] (7*asin(x))/8 - (1 - x^2)^(1/2)*((7*x)/8 + (2*x^2)/3 + x^3/4 + 4/3)
```

sympy [A] time = 0.81, size = 60, normalized size = 0.95

$$-\frac{x^3\sqrt{1-x^2}}{4}-\frac{2x^2\sqrt{1-x^2}}{3}-\frac{7x\sqrt{1-x^2}}{8}-\frac{4\sqrt{1-x^2}}{3}+\frac{7\,\mathrm{asin}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**3*sqrt(1 - x**2)/4 - 2*x**2*sqrt(1 - x**2)/3 - 7*x*sqrt(1 - x**2)/8 - 4
*sqrt(1 - x**2)/3 + 7*asin(x)/8
```

3.56 $\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{3}\sqrt{1-x^2}\,x^2-\frac{1}{3}(3x+5)\sqrt{1-x^2}+\sin^{-1}(x)$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1809, 780, 216}

$$-\frac{1}{3}\sqrt{1-x^2}\,x^2-\frac{1}{3}(3x+5)\sqrt{1-x^2}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] -(x^2*Sqrt[1 - x^2])/3 - ((5 + 3*x)*Sqrt[1 - x^2])/3 + ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}\int \frac{(-5-6x)x}{\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}(5+3x)\sqrt{1-x^2}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}(5+3x)\sqrt{1-x^2}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.63

$$\sin^{-1}(x)-\frac{1}{3}\sqrt{1-x^2}\left(x^2+3x+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] -1/3*(Sqrt[1 - x^2]*(5 + 3*x + x^2)) + ArcSin[x]
```

IntegrateAlgebraic [A] time = 0.19, size = 46, normalized size = 1.12

$$\frac{1}{3}\sqrt{1-x^2}\left(-x^2-3x-5\right)+2\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] (Sqrt[1 - x^2]*(-5 - 3*x - x^2))/3 + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

fricas [A] time = 0.40, size = 38, normalized size = 0.93

$$-\frac{1}{3}\left(x^2+3x+5\right)\sqrt{-x^2+1}-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*(x^2 + 3*x + 5)*sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.22, size = 21, normalized size = 0.51

$$-\frac{1}{3}\left((x+3)x+5\right)\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/3*((x + 3)*x + 5)*sqrt(-x^2 + 1) + arcsin(x)
```

maple [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{\sqrt{-x^2+1}\,x^2}{3}-\sqrt{-x^2+1}\,x+\arcsin(x)-\frac{5\sqrt{-x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^2/(-x^2+1)^(1/2),x)
[Out] -1/3*(-x^2+1)^(1/2)*x^2-5/3*(-x^2+1)^(1/2)-(-x^2+1)^(1/2)*x+arcsin(x)
```

maxima [A] time = 0.97, size = 40, normalized size = 0.98

$$-\frac{1}{3}\sqrt{-x^2+1}\,x^2-\sqrt{-x^2+1}\,x-\frac{5}{3}\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - sqrt(-x^2 + 1)*x - 5/3*sqrt(-x^2 + 1) + arcsin(x)
```

mupad [B] time = 0.03, size = 22, normalized size = 0.54

$$\mathrm{asin}(x)-\sqrt{1-x^2}\left(\frac{x^2}{3}+x+\frac{5}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

```
[Out] asin(x) - (1 - x^2)^(1/2)*(x + x^2/3 + 5/3)
```

sympy [A] time = 0.41, size = 37, normalized size = 0.90

$$-\frac{x^2\sqrt{1-x^2}}{3} - x\sqrt{1-x^2} - \frac{5\sqrt{1-x^2}}{3} + \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**2*sqrt(1 - x**2)/3 - x*sqrt(1 - x**2) - 5*sqrt(1 - x**2)/3 + asin(x)
```

3.57 $\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{2}\sqrt{1-x^2}\,(x+1)-\frac{3\sqrt{1-x^2}}{2}+\frac{3}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {671, 641, 216}

$$-\frac{1}{2}\sqrt{1-x^2}\,(x+1)-\frac{3\sqrt{1-x^2}}{2}+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/Sqrt[1 - x^2],x]
```

```
[Out] (-3*Sqrt[1 - x^2])/2 - ((1 + x)*Sqrt[1 - x^2])/2 + (3*ArcSin[x])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\int \frac{1+x}{\sqrt{1-x^2}}\,dx\\ &= -\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.62

$$\frac{1}{2}\left(3\sin^{-1}(x)-(x+4)\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/Sqrt[1 - x^2],x]
```

```
[Out] (-((4 + x)*Sqrt[1 - x^2]) + 3*ArcSin[x])/2
```

IntegrateAlgebraic [A] time = 0.19, size = 41, normalized size = 1.02

$$\frac{1}{2}(-x-4)\sqrt{1-x^2} - 3\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^2/Sqrt[1 - x^2],x]
[Out] ((-4 - x)*Sqrt[1 - x^2])/2 - 3*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

fricas [A] time = 0.40, size = 33, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-x^2+1}\,(x+4) - 3\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) - 3*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.19, size = 19, normalized size = 0.48

$$-\frac{1}{2}\sqrt{-x^2+1}\,(x+4) + \frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) + 3/2*arcsin(x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.72

$$-\frac{\sqrt{-x^2+1}\,x}{2} + \frac{3\arcsin(x)}{2} - 2\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^2+1)^(1/2),x)
[Out] -1/2*(-x^2+1)^(1/2)*x+3/2*arcsin(x)-2*(-x^2+1)^(1/2)
```

maxima [A] time = 0.96, size = 28, normalized size = 0.70

$$-\frac{1}{2}\sqrt{-x^2+1}\,x - 2\sqrt{-x^2+1} + \frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.52

$$\frac{3\,\mathrm{asin}(x)}{2} - \left(\frac{x}{2}+2\right)\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(1 - x^2)^(1/2),x)
```

```
[Out] (3*asin(x))/2 - (x/2 + 2)*(1 - x^2)^(1/2)
```

sympy [**A**] time = 0.24, size = 27, normalized size = 0.68

$$-\frac{x\sqrt{1-x^2}}{2} - 2\sqrt{1-x^2} + \frac{3\,\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x*sqrt(1 - x**2)/2 - 2*sqrt(1 - x**2) + 3*asin(x)/2
```

3.58 $\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=32

$$-\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\sin^{-1}(x)$$

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1809, 844, 216, 266, 63, 206}

$$-\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2] + 2*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx &= -\sqrt{1-x^2} - \int \frac{-1-2x}{x\sqrt{1-x^2}}\,dx \\
&= -\sqrt{1-x^2} + 2\int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= -\sqrt{1-x^2} + 2\sin^{-1}(x) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^2\right) \\
&= -\sqrt{1-x^2} + 2\sin^{-1}(x) - \operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\sqrt{1-x^2} + 2\sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$-\sqrt{1-x^2} - \tanh^{-1}\left(\sqrt{1-x^2}\right) + 2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
[Out] -Sqrt[1 - x^2] + 2*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

IntegrateAlgebraic [A] time = 0.18, size = 52, normalized size = 1.62

$$-\sqrt{1-x^2} + \log\left(\sqrt{1-x^2}-1\right) + 4\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right) - \log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
[Out] -Sqrt[1 - x^2] + 4*ArcTan[x/(-1 + Sqrt[1 - x^2])] - Log[x] + Log[-1 + Sqrt[
1 - x^2]]
```

fricas [A] time = 0.39, size = 46, normalized size = 1.44

$$-\sqrt{-x^2+1} - 4\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + \log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 1) - 4*arctan((sqrt(-x^2 + 1) - 1)/x) + log((sqrt(-x^2 + 1) -
1)/x)
```

giac [A] time = 0.18, size = 34, normalized size = 1.06

$$-\sqrt{-x^2+1} + 2\arcsin(x) + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 1) + 2*arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)+2\arcsin(x)-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x/(-x^2+1)^(1/2),x)
[Out] -(-x^2+1)^(1/2)+2*arcsin(x)-arctanh(1/(-x^2+1)^(1/2))
```

maxima [A] time = 0.98, size = 41, normalized size = 1.28

$$-\sqrt{-x^2+1}+2\arcsin(x)-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1) + 2*arcsin(x) - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.05, size = 32, normalized size = 1.00

$$2\operatorname{asin}(x)+\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x*(1 - x^2)^(1/2)),x)
[Out] 2*asin(x) + log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)
```

sympy [A] time = 6.30, size = 31, normalized size = 0.97

$$-\sqrt{1-x^2}+\begin{cases}-\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}+2\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x/(-x**2+1)**(1/2),x)
[Out] -sqrt(1 - x**2) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), T
rue)) + 2*asin(x)
```

3.59 $\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\sqrt{1-x^2}}{x}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sin^{-1}(x)$$

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 844, 216, 266, 63, 206}

$$-\frac{\sqrt{1-x^2}}{x}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2*ArcTanh[Sqrt[1 - x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{x}-\int \frac{-2-x}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{x}+2\int \frac{1}{x\sqrt{1-x^2}}\,dx+\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{x}+\sin^{-1}(x)+\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{x}+\sin^{-1}(x)-2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{x}+\sin^{-1}(x)-2\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{\sqrt{1-x^2}}{x}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2*ArcTanh[Sqrt[1 - x^2]]
```

IntegrateAlgebraic [A] time = 0.17, size = 57, normalized size = 1.73

$$-\frac{\sqrt{1-x^2}}{x}+2\log\left(\sqrt{1-x^2}-1\right)+2\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
[Out] -(Sqrt[1 - x^2]/x) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])] - 2*Log[x] + 2*Log[-1
 + Sqrt[1 - x^2]]
```

fricas [A] time = 0.40, size = 53, normalized size = 1.61

$$-\frac{2\,x\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-2\,x\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(2*x*arctan((sqrt(-x^2 + 1) - 1)/x) - 2*x*log((sqrt(-x^2 + 1) - 1)/x) + sq
rt(-x^2 + 1))/x
```

giac [A] time = 0.18, size = 55, normalized size = 1.67

$$\frac{x}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\sqrt{-x^2+1}-1}{2\,x}+\arcsin(x)+2\,\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x + arcsin(x) + 2*log
(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$-2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)+\arcsin(x)-\frac{\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^2/(-x^2+1)^(1/2),x)
```

```
[Out] arcsin(x)-(-x^2+1)^(1/2)/x-2*arctanh(1/(-x^2+1)^(1/2))
```

maxima [A] time = 0.97, size = 42, normalized size = 1.27

$$-\frac{\sqrt{-x^2+1}}{x}+\arcsin(x)-2\,\log\left(\frac{2\,\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/x + arcsin(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.08, size = 35, normalized size = 1.06

$$\mathrm{asin}(x)+2\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\frac{\sqrt{1-x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^2*(1 - x^2)^(1/2)),x)
```

```
[Out] asin(x) + 2*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)/x
```

sympy [C] time = 4.68, size = 51, normalized size = 1.55

$$\begin{cases}-\frac{i\sqrt{x^2-1}}{x} & \text{for}\ \left|x^2\right|>1\\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise}\end{cases}+2\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}\right)+\mathrm{asin}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**2/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))
+ 2*Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) + asin(x
)
```

3.60 $\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{2x^2}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1807, 807, 266, 63, 206}

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{2x^2}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
[Out] -Sqrt[1 - x^2]/(2*x^2) - (2*Sqrt[1 - x^2])/x - (3*ArcTanh[Sqrt[1 - x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{1}{2}\int \frac{-4-3x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}+\frac{3}{2}\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 40, normalized size = 0.78

$$-\frac{\sqrt{1-x^2}\,(4x+1)}{2x^2}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
```

```
[Out] -1/2*((1 + 4*x)*Sqrt[1 - x^2])/x^2 - (3*ArcTanh[Sqrt[1 - x^2]])/2
```

IntegrateAlgebraic [**A**] time = 0.17, size = 48, normalized size = 0.94

$$\frac{\sqrt{1-x^2}\,(-4x-1)}{2x^2}+\frac{3}{2}\log\left(\sqrt{1-x^2}-1\right)-\frac{3\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
```

```
[Out] ((-1 - 4*x)*Sqrt[1 - x^2])/(2*x^2) - (3*Log[x])/2 + (3*Log[-1 + Sqrt[1 - x^
2]])/2
```

fricas [**A**] time = 0.40, size = 43, normalized size = 0.84

$$\frac{3\,x^2\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}\,(4\,x+1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*x^2*log((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*(4*x + 1))/x^2
```

giac [**B**] time = 0.18, size = 91, normalized size = 1.78

$$\frac{x^2\left(\frac{8\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{8\left(\sqrt{-x^2+1}-1\right)^2}-\frac{\sqrt{-x^2+1}-1}{x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{8\,x^2}+\frac{3}{2}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*x^2*(8*(sqrt(-x^2 + 1) - 1)/x - 1)/(sqrt(-x^2 + 1) - 1)^2 - (sqrt(-x^2
+ 1) - 1)/x + 1/8*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3/2*log(-(sqrt(-x^2 + 1) - 1
)/abs(x))
```

maple [A] time = 0.01, size = 42, normalized size = 0.82

$$-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{2}-\frac{2\sqrt{-x^2+1}}{x}-\frac{\sqrt{-x^2+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^3/(-x^2+1)^(1/2),x)
[Out] -2*(-x^2+1)^(1/2)/x-3/2*arctanh(1/(-x^2+1)^(1/2))-1/2*(-x^2+1)^(1/2)/x^2
```

maxima [A] time = 0.97, size = 54, normalized size = 1.06

$$-\frac{2\sqrt{-x^2+1}}{x}-\frac{\sqrt{-x^2+1}}{2\,x^2}-\frac{3}{2}\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-x^2 + 1)/x - 1/2*sqrt(-x^2 + 1)/x^2 - 3/2*log(2*sqrt(-x^2 + 1)/abs
(x) + 2/abs(x))
```

mupad [B] time = 2.49, size = 47, normalized size = 0.92

$$\frac{3\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{2}-\frac{2\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^3*(1 - x^2)^(1/2)),x)
[Out] (3*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/2 - (2*(1 - x^2)^(1/2))/x - (1 -
 x^2)^(1/2)/(2*x^2)
```

sympy [C] time = 7.03, size = 116, normalized size = 2.27

$$2\left(\begin{cases}-\frac{i\sqrt{x^2-1}}{x} & \text{for } \left|x^2\right|>1\\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{\mathrm{acosh}\left(\frac{1}{x}\right)}{2}-\frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for } \frac{1}{|x^2|}>1\\ \frac{i\,\mathrm{asin}\left(\frac{1}{x}\right)}{2}-\frac{i}{2x\sqrt{1-\frac{1}{x^2}}}+\frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise}\end{cases}+\begin{cases}-\mathrm{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**3/(-x**2+1)**(1/2),x)
[Out] 2*Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True)
) + Piecewise((-acosh(1/x)/2 - sqrt(-1 + x**(-2))/(2*x), 1/Abs(x**2) > 1),
(I*asin(1/x)/2 - I/(2*x*sqrt(1 - 1/x**2)) + I/(2*x**3*sqrt(1 - 1/x**2)), Tr
ue)) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True))
```

3.61 $\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=67

$$-\frac{5\sqrt{1-x^2}}{3x}-\frac{\sqrt{1-x^2}}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{3x^3}$$

Rubi [A] time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 835, 807, 266, 63, 206}

$$-\frac{5\sqrt{1-x^2}}{3x}-\frac{\sqrt{1-x^2}}{x^2}-\frac{\sqrt{1-x^2}}{3x^3}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2]/(3*x^3) - Sqrt[1 - x^2]/x^2 - (5*Sqrt[1 - x^2])/(3*x) - ArcT
anh[Sqrt[1 - x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{1}{3}\int \frac{-6-5x}{x^3\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{6}\int \frac{10+6x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}+\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.64

$$
-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}\left(5x^2+3x+1\right)}{3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
[Out] -1/3*(Sqrt[1 - x^2]*(1 + 3*x + 5*x^2))/x^3 - ArcTanh[Sqrt[1 - x^2]]
```

IntegrateAlgebraic [A] time = 0.17, size = 47, normalized size = 0.70

$$
\log\left(\sqrt{1-x^2}-1\right)+\frac{\sqrt{1-x^2}\left(-5x^2-3x-1\right)}{3x^3}-\log(x)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
[Out] ((-1 - 3*x - 5*x^2)*Sqrt[1 - x^2])/(3*x^3) - Log[x] + Log[-1 + Sqrt[1 - x^2
]]
```

fricas [A] time = 0.40, size = 48, normalized size = 0.72

$$
\frac{3\,x^3\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(5\,x^2+3\,x+1\right)\sqrt{-x^2+1}}{3\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*log((sqrt(-x^2 + 1) - 1)/x) - (5*x^2 + 3*x + 1)*sqrt(-x^2 + 1))/
x^3
```

giac [B] time = 0.19, size = 125, normalized size = 1.87

$$-\frac{x^3\left(\frac{6\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{21\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{24\left(\sqrt{-x^2+1}-1\right)^3}-\frac{7\left(\sqrt{-x^2+1}-1\right)}{8\,x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{24\,x^3}+\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*x^3*(6*(sqrt(-x^2 + 1) - 1)/x - 21*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(s
qrt(-x^2 + 1) - 1)^3 - 7/8*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1
)^2/x^2 - 1/24*(sqrt(-x^2 + 1) - 1)^3/x^3 + log(-(sqrt(-x^2 + 1) - 1)/abs(x
))
```

maple [A] time = 0.01, size = 56, normalized size = 0.84

$$-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\frac{5\sqrt{-x^2+1}}{3x}-\frac{\sqrt{-x^2+1}}{x^2}-\frac{\sqrt{-x^2+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^4/(-x^2+1)^(1/2),x)
```

```
[Out] -5/3*(-x^2+1)^(1/2)/x-1/3*(-x^2+1)^(1/2)/x^3-(-x^2+1)^(1/2)/x^2-arctanh(1/(
-x^2+1)^(1/2))
```

maxima [A] time = 0.96, size = 68, normalized size = 1.01

$$-\frac{5\sqrt{-x^2+1}}{3\,x}-\frac{\sqrt{-x^2+1}}{x^2}-\frac{\sqrt{-x^2+1}}{3\,x^3}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/3*sqrt(-x^2 + 1)/x - sqrt(-x^2 + 1)/x^2 - 1/3*sqrt(-x^2 + 1)/x^3 - log(2
*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.03, size = 67, normalized size = 1.00

$$\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\sqrt{1-x^2}\,\left(\frac{2}{3\,x}+\frac{1}{3\,x^3}\right)-\frac{\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^4*(1 - x^2)^(1/2)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)*(2/(3*x) + 1/(3*x^
3)) - (1 - x^2)^(1/2)/x - (1 - x^2)^(1/2)/x^2
```

sympy [C] time = 8.35, size = 128, normalized size = 1.91

$$\begin{cases}-\frac{\sqrt{1-x^2}}{x}-\frac{\left(1-x^2\right)^{\frac{3}{2}}}{3x^3} & \text{for}\ x>-1\wedge x<1\end{cases}+\begin{cases}-\frac{i\sqrt{x^2-1}}{x} & \text{for}\ \left|x^2\right|>1\\-\frac{\sqrt{1-x^2}}{x} & \text{otherwise}\end{cases}+2\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2}-\frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for}\ \frac{1}{\left|x^2\right|}>1\\\frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2}-\frac{i}{2x\sqrt{1-\frac{1}{x^2}}}+\frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**4/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1))) + Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x,
True)) + 2*Piecewise((-acosh(1/x)/2 - sqrt(-1 + x**(-2))/(2*x), 1/Abs(x**2)
 > 1), (I*asin(1/x)/2 - I/(2*x*sqrt(1 - 1/x**2)) + I/(2*x**3*sqrt(1 - 1/x**
2)), True))
```

3.62 $\int \frac{(1+x)^2}{x^5 \sqrt{1-x^2}} \, dx$

Optimal. Leaf size=89

$$-\frac{4\sqrt{1-x^2}}{3x}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}$$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 835, 807, 266, 63, 206}

$$-\frac{4\sqrt{1-x^2}}{3x}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{4x^4}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2]/(4*x^4) - (2*Sqrt[1 - x^2])/(3*x^3) - (7*Sqrt[1 - x^2])/(8*x
^2) - (4*Sqrt[1 - x^2])/(3*x) - (7*ArcTanh[Sqrt[1 - x^2]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{1}{4}\int \frac{-8-7x}{x^4\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}+\frac{1}{12}\int \frac{21+16x}{x^3\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{1}{24}\int \frac{-32-21x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}+\frac{7}{8}\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}+\frac{7}{16}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.82

$$-\sqrt{1-x^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}\left(8x^2+3x+4\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

```
[Out] -1/6*(Sqrt[1 - x^2]*(4 + 3*x + 8*x^2))/x^3 - ArcTanh[Sqrt[1 - x^2]]/2 - Sqr
t[1 - x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - x^2]
```

IntegrateAlgebraic [A] time = 0.17, size = 58, normalized size = 0.65

$$\frac{7}{8}\log\left(\sqrt{1-x^2}-1\right)+\frac{\sqrt{1-x^2}\left(-32x^3-21x^2-16x-6\right)}{24x^4}-\frac{7\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*(-6 - 16*x - 21*x^2 - 32*x^3))/(24*x^4) - (7*Log[x])/8 + (7*
Log[-1 + Sqrt[1 - x^2]])/8
```

fricas [A] time = 0.40, size = 53, normalized size = 0.60

$$\frac{21\,x^4\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(32\,x^3+21\,x^2+16\,x+6\right)\sqrt{-x^2+1}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(21*x^4*log((sqrt(-x^2 + 1) - 1)/x) - (32*x^3 + 21*x^2 + 16*x + 6)*sqr
t(-x^2 + 1))/x^4
```

giac [B] time = 0.18, size = 163, normalized size = 1.83

$$\frac{x^4\left(\frac{16\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{48\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{144\left(\sqrt{-x^2+1}-1\right)^3}{x^3}-3\right)}{192\left(\sqrt{-x^2+1}-1\right)^4}-\frac{3\left(\sqrt{-x^2+1}-1\right)}{4\,x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{12\,x^3}+\frac{\left(\sqrt{-x^2+1}-1\right)^4}{64\,x^4}+\frac{7}{8}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*x^4*(16*(sqrt(-x^2 + 1) - 1)/x - 48*(sqrt(-x^2 + 1) - 1)^2/x^2 + 144*
(sqrt(-x^2 + 1) - 1)^3/x^3 - 3)/(sqrt(-x^2 + 1) - 1)^4 - 3/4*(sqrt(-x^2 + 1
) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1/12*(sqrt(-x^2 + 1) - 1)^3/x^3
 + 1/64*(sqrt(-x^2 + 1) - 1)^4/x^4 + 7/8*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

maple [A] time = 0.01, size = 70, normalized size = 0.79

$$-\frac{7\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{8}-\frac{4\sqrt{-x^2+1}}{3x}-\frac{7\sqrt{-x^2+1}}{8x^2}-\frac{2\sqrt{-x^2+1}}{3x^3}-\frac{\sqrt{-x^2+1}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^5/(-x^2+1)^(1/2),x)
```

```
[Out] -7/8*(-x^2+1)^(1/2)/x^2-7/8*arctanh(1/(-x^2+1)^(1/2))-2/3*(-x^2+1)^(1/2)/x^
3-4/3*(-x^2+1)^(1/2)/x-1/4*(-x^2+1)^(1/2)/x^4
```

maxima [A] time = 0.97, size = 82, normalized size = 0.92

$$-\frac{4\sqrt{-x^2+1}}{3\,x}-\frac{7\sqrt{-x^2+1}}{8\,x^2}-\frac{2\sqrt{-x^2+1}}{3\,x^3}-\frac{\sqrt{-x^2+1}}{4\,x^4}-\frac{7}{8}\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(-x^2 + 1)/x - 7/8*sqrt(-x^2 + 1)/x^2 - 2/3*sqrt(-x^2 + 1)/x^3 - 1
/4*sqrt(-x^2 + 1)/x^4 - 7/8*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.03, size = 77, normalized size = 0.87

$$\frac{7\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{8}-\sqrt{1-x^2}\,\left(\frac{4}{3\,x}+\frac{2}{3\,x^3}\right)-\sqrt{1-x^2}\,\left(\frac{3}{8\,x^2}+\frac{1}{4\,x^4}\right)-\frac{\sqrt{1-x^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^5*(1 - x^2)^(1/2)),x)
```

```
[Out] (7*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/8 - (1 - x^2)^(1/2)*(4/(3*x) + 2
/(3*x^3)) - (1 - x^2)^(1/2)*(3/(8*x^2) + 1/(4*x^4)) - (1 - x^2)^(1/2)/(2*x^
2)
```

sympy [A] time = 11.06, size = 223, normalized size = 2.51

$$2\left(\begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x > -1 \wedge x < 1 \end{cases}\right) + \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2} - \frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2} - \frac{i}{2x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases} + \begin{cases} -\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8} + \frac{3}{8x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{8x^3\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{4x^5\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8} - \frac{3i}{8x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{8x^3\sqrt{1-\frac{1}{x^2}}} + \frac{i}{4x^5\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**5/(-x**2+1)**(1/2),x)
```

```
[Out] 2*Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x
< 1))) + Piecewise((-acosh(1/x)/2 - sqrt(-1 + x**(-2))/(2*x), 1/Abs(x**2) >
 1), (I*asin(1/x)/2 - I/(2*x*sqrt(1 - 1/x**2)) + I/(2*x**3*sqrt(1 - 1/x**2)
), True)) + Piecewise((-3*acosh(1/x)/8 + 3/(8*x*sqrt(-1 + x**(-2))) - 1/(8*
x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqrt(-1 + x**(-2))), 1/Abs(x**2) > 1),
 (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/x**2)) + I/(8*x**3*sqrt(1 - 1/x**2)
) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

3.63 $\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{6\sqrt{1-x^2}}{5x}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}$$

Rubi [A] time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1807, 835, 807, 266, 63, 206}

$$-\frac{6\sqrt{1-x^2}}{5x}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{\sqrt{1-x^2}}{5x^5}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2]/(5*x^5) - Sqrt[1 - x^2]/(2*x^4) - (3*Sqrt[1 - x^2])/(5*x^3)
- (3*Sqrt[1 - x^2])/(4*x^2) - (6*Sqrt[1 - x^2])/(5*x) - (3*ArcTanh[Sqrt[1 -
 x^2]])/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{1}{5}\int \frac{-10-9x}{x^5\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}+\frac{1}{20}\int \frac{36+30x}{x^4\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{1}{60}\int \frac{-90-72x}{x^3\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}+\frac{1}{120}\int \frac{144+90x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}+\frac{3}{4}\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}+\frac{3}{8}\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x,\right.\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{}\right.\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.47

$$
-\frac{\sqrt{1-x^2}\left(10x^5\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x^2\right)+6x^4+3x^2+1\right)}{5x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

```
[Out] -1/5*(Sqrt[1 - x^2]*(1 + 3*x^2 + 6*x^4 + 10*x^5*Hypergeometric2F1[1/2, 3, 3
/2, 1 - x^2]))/x^5
```

IntegrateAlgebraic [A] time = 0.18, size = 63, normalized size = 0.59

$$
\frac{3}{4}\log\left(\sqrt{1-x^2}-1\right)+\frac{\sqrt{1-x^2}\left(-24x^4-15x^3-12x^2-10x-4\right)}{20x^5}-\frac{3\log(x)}{4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*(-4 - 10*x - 12*x^2 - 15*x^3 - 24*x^4))/(20*x^5) - (3*Log[x]
)/4 + (3*Log[-1 + Sqrt[1 - x^2]])/4
```

fricas [A]   time = 0.40, size = 58, normalized size = 0.54

$$\frac{15\,x^5\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(24\,x^4+15\,x^3+12\,x^2+10\,x+4\right)\sqrt{-x^2+1}}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(15*x^5*log((sqrt(-x^2 + 1) - 1)/x) - (24*x^4 + 15*x^3 + 12*x^2 + 10*x
 + 4)*sqrt(-x^2 + 1))/x^5
```

giac [B]   time = 0.20, size = 199, normalized size = 1.86

$$-\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{15\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{40\left(\sqrt{-x^2+1}-1\right)^3}{x^3}-\frac{110\left(\sqrt{-x^2+1}-1\right)^4}{x^4}-1\right)}{160\left(\sqrt{-x^2+1}-1\right)^5}-\frac{11\left(\sqrt{-x^2+1}-1\right)}{16\,x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2}-\frac{3\left(\sqrt{-x^2+1}-1\right)^3}{32\,x^3}+\frac{\left(\sqrt{-x^2+1}-1\right)^4}{32\,x^4}-\frac{\left(\sqrt{-x^2+1}-1\right)^5}{160\,x^5}+\frac{3}{4}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/160*x^5*(5*(sqrt(-x^2 + 1) - 1)/x - 15*(sqrt(-x^2 + 1) - 1)^2/x^2 + 40*(
sqrt(-x^2 + 1) - 1)^3/x^3 - 110*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1)/(sqrt(-x^2
+ 1) - 1)^5 - 11/16*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2
 - 3/32*(sqrt(-x^2 + 1) - 1)^3/x^3 + 1/32*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1/16
0*(sqrt(-x^2 + 1) - 1)^5/x^5 + 3/4*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

maple [A]   time = 0.01, size = 84, normalized size = 0.79

$$-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{4}-\frac{6\sqrt{-x^2+1}}{5x}-\frac{3\sqrt{-x^2+1}}{4x^2}-\frac{3\sqrt{-x^2+1}}{5x^3}-\frac{\sqrt{-x^2+1}}{2x^4}-\frac{\sqrt{-x^2+1}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^6/(-x^2+1)^(1/2),x)
```

```
[Out] -1/5*(-x^2+1)^(1/2)/x^5-3/5*(-x^2+1)^(1/2)/x^3-6/5*(-x^2+1)^(1/2)/x-1/2*(-x
^2+1)^(1/2)/x^4-3/4*(-x^2+1)^(1/2)/x^2-3/4*arctanh(1/(-x^2+1)^(1/2))
```

maxima [A]   time = 0.97, size = 96, normalized size = 0.90

$$-\frac{6\sqrt{-x^2+1}}{5\,x}-\frac{3\sqrt{-x^2+1}}{4\,x^2}-\frac{3\sqrt{-x^2+1}}{5\,x^3}-\frac{\sqrt{-x^2+1}}{2\,x^4}-\frac{\sqrt{-x^2+1}}{5\,x^5}-\frac{3}{4}\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -6/5*sqrt(-x^2 + 1)/x - 3/4*sqrt(-x^2 + 1)/x^2 - 3/5*sqrt(-x^2 + 1)/x^3 - 1
/2*sqrt(-x^2 + 1)/x^4 - 1/5*sqrt(-x^2 + 1)/x^5 - 3/4*log(2*sqrt(-x^2 + 1)/a
bs(x) + 2/abs(x))
```

mupad [B]   time = 0.04, size = 90, normalized size = 0.84

$$\frac{3\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{4}-\sqrt{1-x^2}\,\left(\frac{2}{3\,x}+\frac{1}{3\,x^3}\right)-\sqrt{1-x^2}\,\left(\frac{3}{4\,x^2}+\frac{1}{2\,x^4}\right)-\sqrt{1-x^2}\,\left(\frac{8}{15\,x}+\frac{4}{15\,x^3}+\frac{1}{5\,x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^6*(1 - x^2)^(1/2)),x)
```

```
[Out] (3*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/4 - (1 - x^2)^(1/2)*(2/(3*x) + 1
/(3*x^3)) - (1 - x^2)^(1/2)*(3/(4*x^2) + 1/(2*x^4)) - (1 - x^2)^(1/2)*(8/(1
5*x) + 4/(15*x^3) + 1/(5*x^5))
```

sympy [C] time = 12.69, size = 201, normalized size = 1.88

$$\begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x > -1 \wedge x < 1 \end{cases} + \begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{2(1-x^2)^{\frac{3}{2}}}{3x^3} - \frac{(1-x^2)^{\frac{5}{2}}}{5x^5} & \text{for } x > -1 \wedge x < 1 \end{cases} + 2\left(\begin{cases} -\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8} + \frac{3}{8x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{8x^3\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{4x^5\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8} - \frac{3i}{8x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{8x^3\sqrt{1-\frac{1}{x^2}}} + \frac{i}{4x^5\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**6/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1))) + Piecewise((-sqrt(1 - x**2)/x - 2*(1 - x**2)**(3/2)/(3*x**3) - (1 - x
**2)**(5/2)/(5*x**5), (x > -1) & (x < 1))) + 2*Piecewise((-3*acosh(1/x)/8 +
 3/(8*x*sqrt(-1 + x**(-2))) - 1/(8*x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqr
t(-1 + x**(-2))), 1/Abs(x**2) > 1), (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/
x**2)) + I/(8*x**3*sqrt(1 - 1/x**2)) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

3.64 $\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$

Optimal. Leaf size=134

$$-\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{13}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 811, 844, 217, 203, 266, 63, 208}

$$-\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{13}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] -(e^2*(13*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (d*(d^2 - e^2*x^2)^(3/2
))/(4*x^4) - (e*(d^2 - e^2*x^2)^(3/2))/x^3 - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]] + (13*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
```

```
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx &= -\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\int \frac{\sqrt{d^2-e^2x^2}\left(-12d^4e-13d^3e^2x-4d^2e^3x^2\right)}{x^4}\,dx}{4d^2}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}+\frac{\int \frac{\left(39d^5e^2+12d^4e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{12d^4}\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{\int \frac{78d^7e^4+48d^6e^5}{x\sqrt{d^2-e^2x^2}}}{48d^6}\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{1}{8}\left(13de^4\right)\int -\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{1}{16}\left(13de^4\right)\text{Su}\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-e^4\tan^{-1}\left(\frac{}{\sqrt{d^2}}\right.\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-e^4\tan^{-1}\left(\frac{}{\sqrt{d^2}}\right.
\end{aligned}$$

Mathematica [C] time = 0.24, size = 196, normalized size = 1.46

$$-\frac{e\sqrt{d^2-e^2x^2}\left(6d^2e^3x^3\sin^{-1}\left(\frac{ex}{d}\right)+2e^3x^3\left(d^2-e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{e^2x^2}{d^2}\right)-9d^2e^3x^3\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)+6d^5\sqrt{1-\frac{e^2x^2}{d^2}}+9d^4ex\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{6d^3x^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] -1/6*(e*Sqrt[d^2 - e^2*x^2]*(6*d^5*Sqrt[1 - (e^2*x^2)/d^2] + 9*d^4*e*x*Sqrt
[1 - (e^2*x^2)/d^2] + 6*d^2*e^3*x^3*ArcSin[(e*x)/d] - 9*d^2*e^3*x^3*ArcTanh
[Sqrt[1 - (e^2*x^2)/d^2]] + 2*e^3*x^3*(d^2 - e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^
```

```
2]*Hypergeometric2F1[3/2, 3, 5/2, 1 - (e^2*x^2)/d^2]))/(d^3*x^3*Sqrt[1 - (e
^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.56, size = 134, normalized size = 1.00

$$-\frac{13}{4}e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\sqrt{-e^2}\,e^3\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+\frac{\sqrt{d^2-e^2x^2}\left(-2d^3-8d^2ex-11de^2x^2\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 - 8*d^2*e*x - 11*d*e^2*x^2))/(8*x^4) - (13*e^4
*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/4 - e^3*Sqrt[-e^2]*Log[
-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.42, size = 111, normalized size = 0.83

$$\frac{16\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-13\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(11\,de^2x^2+8\,d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(16*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 13*e^4*x^4*log(
-(d - sqrt(-e^2*x^2 + d^2))/x) - (11*d*e^2*x^2 + 8*d^2*e*x + 2*d^3)*sqrt(-e
^2*x^2 + d^2))/x^4
```

giac [B] time = 0.27, size = 295, normalized size = 2.20

$$-\arcsin\left(\frac{xe}{d}\right)e^4\mathrm{sgn}(d)+\frac{x^4\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^8}{x}+\frac{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^6}{x^2}+\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^4}{x^3}+e^{10}\right)e^2}{64\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4}-\frac{1}{64}\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{26}}{x}+\frac{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{24}}{x^2}+\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{22}}{x^3}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{20}}{x^4}\right)e^{(-24)}+\frac{13}{8}e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^4*sgn(d) + 1/64*x^4*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^8/
x + 24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^6/x^2 + 8*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^3*e^4/x^3 + e^10)*e^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^4 - 1/64*(8*(d
*e + sqrt(-x^2*e^2 + d^2)*e)*e^26/x + 24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e
^24/x^2 + 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^22/x^3 + (d*e + sqrt(-x^2*e^
2 + d^2)*e)^4*e^20/x^4)*e^(-24) + 13/8*e^4*log(1/2*abs(-2*d*e - 2*sqrt(-x^2
*e^2 + d^2)*e)*e^(-2)/abs(x))
```

maple [A] time = 0.02, size = 212, normalized size = 1.58

$$\frac{13d\,e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}-\frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{\sqrt{-e^2x^2+d^2}\,e^5x}{d^2}-\frac{13\sqrt{-e^2x^2+d^2}\,e^4}{8d}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{d^2x}-\frac{13\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{8d\,x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x)
```

```
[Out] -1/4*d*(-e^2*x^2+d^2)^(3/2)/x^4-13/8/d*e^2/x^2*(-e^2*x^2+d^2)^(3/2)-13/8*(-
e^2*x^2+d^2)^(1/2)/d*e^4+13/8/(d^2)^(1/2)*d*e^4*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-e^3/d^2/x*(-e^2*x^2+d^2)^(3/2)-(-e^2*x^2+d^2)^(1/2)/d
^2*e^5*x-1/(e^2)^(1/2)*e^5*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-e*(-e
^2*x^2+d^2)^(3/2)/x^3
```

maxima [A] time = 0.98, size = 159, normalized size = 1.19

$$-e^{4} \arcsin\left(\frac{ex}{d}\right) + \frac{13}{8} e^{4} \log\left(\frac{2 d^{2}}{|x|} + \frac{2 \sqrt{-e^{2} x^{2} + d^{2}} d}{|x|}\right) - \frac{13 \sqrt{-e^{2} x^{2} + d^{2}} e^{4}}{8 d} - \frac{\sqrt{-e^{2} x^{2} + d^{2}} e^{3}}{x} - \frac{13 \left(-e^{2} x^{2} + d^{2}\right)^{\frac{3}{2}} e^{2}}{8 d x^{2}} - \frac{\left(-e^{2} x^{2} + d^{2}\right)^{\frac{3}{2}} e}{x^{3}} - \frac{\left(-e^{2} x^{2} + d^{2}\right)^{\frac{3}{2}} d}{4 x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -e^4*arcsin(e*x/d) + 13/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/a
bs(x)) - 13/8*sqrt(-e^2*x^2 + d^2)*e^4/d - sqrt(-e^2*x^2 + d^2)*e^3/x - 13/
8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^2) - (-e^2*x^2 + d^2)^(3/2)*e/x^3 - 1/4*(
-e^2*x^2 + d^2)^(3/2)*d/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2\,x^2}\,\left(d + e\,x\right)^3}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3)/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3)/x^5, x)
```

sympy [C] time = 10.03, size = 544, normalized size = 4.06

$$d^{3} \left(\begin{cases} - \frac{d^{2}}{4 e x^{5} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{3 e}{8 x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} - \frac{e^{3}}{8 d^{2} x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{4} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{8 d^{3}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\\frac{i d^{2}}{4 e x^{5} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} - \frac{3 i e}{8 x^{3} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} + \frac{i e^{3}}{8 d^{2} x \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} - \frac{i e^{4} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{8 d^{3}} & \text{otherwise} \end{cases}\right) + 3 d^{2} e \left(\begin{cases} - \frac{e \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 x^{2}} + \frac{e^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 d^{2}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 x^{2}} + \frac{i e^{3} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 d^{2}} & \text{otherwise} \end{cases}\right) + 3 d e^{2} \left(\begin{cases} - \frac{d^{2}}{2 e x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e}{2 x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{2} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{2 d} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{2 x} - \frac{i e^{2} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{2 d} & \text{otherwise} \end{cases}\right) + e^{3} \left(\begin{cases} \frac{i d}{x \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + i e \operatorname{acosh}{\left(\frac{e x}{d} \right)} - \frac{i e^{2} x}{d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\- \frac{d}{x \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - e \operatorname{asin}{\left(\frac{e x}{d} \right)} + \frac{e^{2} x}{d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(1/2)/x**5,x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + 3*d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) +
e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e
*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/
(3*d**2), True)) + 3*d*e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2
) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), A
bs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2
*asin(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d
**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2
*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x
/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.65 $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=310

$$-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}+\frac{35d^{14}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2048e^6}+\frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+$$

Rubi [A] time = 0.49, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6}-\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}+\frac{35d^{14}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2048e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (35*d^12*x*Sqrt[d^2 - e^2*x^2])/(2048*e^5) + (35*d^10*x*(d^2 - e^2*x^2)^(3/
2))/(3072*e^5) + (7*d^8*x*(d^2 - e^2*x^2)^(5/2))/(768*e^5) - (124*d^5*x^2*(
d^2 - e^2*x^2)^(7/2))/(1287*e^4) - (7*d^4*x^3*(d^2 - e^2*x^2)^(7/2))/(48*e^
3) - (31*d^3*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e^2) - (7*d^2*x^5*(d^2 - e^2*x
^2)^(7/2))/(24*e) - (3*d*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (e*x^7*(d^2 - e^2*
x^2)^(7/2))/14 - (d^6*(31744*d + 63063*e*x)*(d^2 - e^2*x^2)^(7/2))/(1153152
*e^6) + (35*d^14*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2048*e^6)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
```

```
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx &= -\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^5\left(d^2-e^2x^2\right)^{5/2}\left(-14d^3e^2-49d^2e^3x-42de^4x^2\right)}{14e^2} \\
&= -\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^5\left(434d^3e^4+637d^2e^5x\right)}{182e^4} \\
&= -\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}- \\
&= -\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}- \\
&= -\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}- \\
&= -\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}- \\
&= -\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}- \\
&= \frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}- \\
&= \frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}- \\
&= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}- \\
&= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}- \\
&= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-
\end{aligned}
$$

Mathematica [A] time = 0.36, size = 212, normalized size = 0.68

$$
\frac{\sqrt{d^2-e^2x^2}\left(315315d^{13}\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(507904d^{13}+315315d^{12}ex+253952d^{11}e^2x^2+210210d^{10}e^3x^3+190464d^9e^4x^4+168168d^8e^5x^5-2916352d^7e^6x^6-7763184d^6e^7x^7-2551808d^5e^8x^8+9499776d^4e^9x^9+8773632d^3e^{10}x^{10}-1427712d^2e^{11}x^{11}-4257792de^{12}x^{12}-1317888e^{13}x^{13}\right)\right)}{18450432e^6\sqrt{1-\frac{e^2x^2}{d^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(507904*d^13 + 315315*d^12*
e*x + 253952*d^11*e^2*x^2 + 210210*d^10*e^3*x^3 + 190464*d^9*e^4*x^4 + 1681
68*d^8*e^5*x^5 - 2916352*d^7*e^6*x^6 - 7763184*d^6*e^7*x^7 - 2551808*d^5*e^
8*x^8 + 9499776*d^4*e^9*x^9 + 8773632*d^3*e^10*x^10 - 1427712*d^2*e^11*x^11
 - 4257792*d*e^12*x^12 - 1317888*e^13*x^13)) + 315315*d^13*ArcSin[(e*x)/d])
)/(18450432*e^6*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.88, size = 213, normalized size = 0.69

$$\frac{35d^{14}\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2048e^7}+\frac{\sqrt{d^2-e^2x^2}\left(-507904d^{13}-315315d^{12}ex-253952d^{11}e^2x^2-210210d^{10}e^3x^3-190464d^9e^4x^4-168168d^8e^5x^5+2916352d^7e^6x^6+7763184d^6e^7x^7+2551808d^5e^8x^8-9499776d^4e^9x^9-8773632d^3e^{10}x^{10}+1427712d^2e^{11}x^{11}+4257792de^{12}x^{12}+1317888e^{13}x^{13}\right)}{18450432e^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-507904*d^13 - 315315*d^12*e*x - 253952*d^11*e^2*x^2
- 210210*d^10*e^3*x^3 - 190464*d^9*e^4*x^4 - 168168*d^8*e^5*x^5 + 2916352*d
^7*e^6*x^6 + 7763184*d^6*e^7*x^7 + 2551808*d^5*e^8*x^8 - 9499776*d^4*e^9*x^
9 - 8773632*d^3*e^10*x^10 + 1427712*d^2*e^11*x^11 + 4257792*d*e^12*x^12 + 1
317888*e^13*x^13))/(18450432*e^6) + (35*d^14*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x)
 + Sqrt[d^2 - e^2*x^2]])/(2048*e^7)
```

fricas [A] time = 0.43, size = 194, normalized size = 0.63

$$-\frac{630630\,d^{14}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(1317888\,e^{13}x^{13}+4257792\,de^{12}x^{12}+1427712\,d^2e^{11}x^{11}-8773632\,d^3e^{10}x^{10}-9499776\,d^4e^9x^9+2551808\,d^5e^8x^8+7763184\,d^6e^7x^7+2916352\,d^7e^6x^6-168168\,d^8e^5x^5-190464\,d^9e^4x^4-210210\,d^{10}e^3x^3-253952\,d^{11}e^2x^2-315315\,d^{12}ex-507904\,d^{13}\right)\sqrt{-e^2x^2+d^2}}{18450432\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/18450432*(630630*d^14*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (13178
88*e^13*x^13 + 4257792*d*e^12*x^12 + 1427712*d^2*e^11*x^11 - 8773632*d^3*e^
10*x^10 - 9499776*d^4*e^9*x^9 + 2551808*d^5*e^8*x^8 + 7763184*d^6*e^7*x^7 +
 2916352*d^7*e^6*x^6 - 168168*d^8*e^5*x^5 - 190464*d^9*e^4*x^4 - 210210*d^1
0*e^3*x^3 - 253952*d^11*e^2*x^2 - 315315*d^12*e*x - 507904*d^13)*sqrt(-e^2*
x^2 + d^2))/e^6
```

giac [A] time = 0.26, size = 170, normalized size = 0.55

$$\frac{35}{2048}\,d^{14}\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d)-\frac{1}{18450432}\left(507904\,d^{13}e^{(-6)}+\left(315315\,d^{12}e^{(-5)}+2\left(126976\,d^{11}e^{(-4)}+\left(105105\,d^{10}e^{(-3)}+4\left(23808\,d^9e^{(-2)}+\left(21021\,d^8e^{(-1)}-2\left(182272\,d^7+\left(485199\,d^6e+8\left(19936\,d^5e^2-3\left(24739\,d^4e^3+2\left(11424\,d^3e^4-11\left(169\,d^2e^5+12\left(13\,xe^7+42\,de^6\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 35/2048*d^14*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/18450432*(507904*d^13*e^(-6) +
 (315315*d^12*e^(-5) + 2*(126976*d^11*e^(-4) + (105105*d^10*e^(-3) + 4*(238
08*d^9*e^(-2) + (21021*d^8*e^(-1) - 2*(182272*d^7 + (485199*d^6*e + 8*(1993
6*d^5*e^2 - 3*(24739*d^4*e^3 + 2*(11424*d^3*e^4 - 11*(169*d^2*e^5 + 12*(13*
x*e^7 + 42*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.11, size = 291, normalized size = 0.94

$$\frac{35d^{14}\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2048\sqrt{e^2}\,e^5}+\frac{35\sqrt{-e^2x^2+d^2}\,d^{12}x}{2048e^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e\,x^7}{14}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d\,x^6}{13}+\frac{35\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^{10}x}{3072e^5}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^5}{24e}-\frac{31\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^4}{143e^2}+\frac{7\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^8x}{768e^5}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x^3}{48e^3}-\frac{124\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5x^2}{1287e^4}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^6x}{128e^5}-\frac{248\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^7}{9009e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/14*e*x^7*(-e^2*x^2+d^2)^(7/2)-7/24*d^2*x^5*(-e^2*x^2+d^2)^(7/2)/e-7/48*d
^4*x^3*(-e^2*x^2+d^2)^(7/2)/e^3-7/128/e^5*d^6*x*(-e^2*x^2+d^2)^(7/2)+7/768*
d^8*x*(-e^2*x^2+d^2)^(5/2)/e^5+35/3072*d^10*x*(-e^2*x^2+d^2)^(3/2)/e^5+35/2
048*d^12*x*(-e^2*x^2+d^2)^(1/2)/e^5+35/2048/e^5*d^14/(e^2)^(1/2)*arctan((e^
```

```
2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-3/13*d*x^6*(-e^2*x^2+d^2)^(7/2)-31/143*d^3
*x^4*(-e^2*x^2+d^2)^(7/2)/e^2-124/1287*d^5*x^2*(-e^2*x^2+d^2)^(7/2)/e^4-248
/9009/e^6*d^7*(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 1.00, size = 270, normalized size = 0.87

$$-\frac{1}{14}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^7-\frac{3}{13}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^6-\frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^5}{24\,e}+\frac{35\,d^{14}\arcsin\left(\frac{ex}{d}\right)}{2048\,e^6}+\frac{35\,\sqrt{-e^2x^2+d^2}\,d^{12}x}{2048\,e^5}-\frac{31\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^4}{143\,e^2}+\frac{35\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^{10}x}{3072\,e^5}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x^3}{48\,e^3}+\frac{7\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^8x}{768\,e^5}-\frac{124\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5x^2}{1287\,e^4}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^6x}{128\,e^5}-\frac{248\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^7}{9009\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/14*(-e^2*x^2 + d^2)^(7/2)*e*x^7 - 3/13*(-e^2*x^2 + d^2)^(7/2)*d*x^6 - 7/
24*(-e^2*x^2 + d^2)^(7/2)*d^2*x^5/e + 35/2048*d^14*arcsin(e*x/d)/e^6 + 35/2
048*sqrt(-e^2*x^2 + d^2)*d^12*x/e^5 - 31/143*(-e^2*x^2 + d^2)^(7/2)*d^3*x^4
/e^2 + 35/3072*(-e^2*x^2 + d^2)^(3/2)*d^10*x/e^5 - 7/48*(-e^2*x^2 + d^2)^(7
/2)*d^4*x^3/e^3 + 7/768*(-e^2*x^2 + d^2)^(5/2)*d^8*x/e^5 - 124/1287*(-e^2*x
^2 + d^2)^(7/2)*d^5*x^2/e^4 - 7/128*(-e^2*x^2 + d^2)^(7/2)*d^6*x/e^5 - 248/
9009*(-e^2*x^2 + d^2)^(7/2)*d^7/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^5*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [A] time = 101.43, size = 2273, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqr
t(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2)
 + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) + 3
*d**6*e*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6
*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d
**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*s
qrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6
*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)
) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e
**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**5*e
**2*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqr
t(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e*
*4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x*
*2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) - 5*d**4*e**3*Piecewise((-7*I*
d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d*
*2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(
1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2
*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10
*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*asin(e*x/
d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3/
(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2*
x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*
sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), Tru
e)) - 5*d**3*e**4*Piecewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10)
```

```
 - 64*d**8*x**2*sqrt(d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2
 - e**2*x**2)/(1155*e**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) -
 d**2*x**8*sqrt(d**2 - e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/
11, Ne(e, 0)), (x**10*sqrt(d**2)/10, True)) + d**2*e**5*Piecewise((-21*I*d*
*12*acosh(e*x/d)/(1024*e**11) + 21*I*d**11*x/(1024*e**10*sqrt(-1 + e**2*x**
2/d**2)) - 7*I*d**9*x**3/(1024*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x
**5/(2560*e**6*sqrt(-1 + e**2*x**2/d**2)) - I*d**5*x**7/(640*e**4*sqrt(-1 +
 e**2*x**2/d**2)) - I*d**3*x**9/(960*e**2*sqrt(-1 + e**2*x**2/d**2)) - 11*I
*d*x**11/(120*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**13/(12*d*sqrt(-1 + e**
2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (21*d**12*asin(e*x/d)/(1024*e**11)
 - 21*d**11*x/(1024*e**10*sqrt(1 - e**2*x**2/d**2)) + 7*d**9*x**3/(1024*e**
8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**5/(2560*e**6*sqrt(1 - e**2*x**2/d**
2)) + d**5*x**7/(640*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**9/(960*e**2*s
qrt(1 - e**2*x**2/d**2)) + 11*d*x**11/(120*sqrt(1 - e**2*x**2/d**2)) - e**2
*x**13/(12*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewise((-256*d
**12*sqrt(d**2 - e**2*x**2)/(9009*e**12) - 128*d**10*x**2*sqrt(d**2 - e**2*
x**2)/(9009*e**10) - 32*d**8*x**4*sqrt(d**2 - e**2*x**2)/(3003*e**8) - 80*d
**6*x**6*sqrt(d**2 - e**2*x**2)/(9009*e**6) - 10*d**4*x**8*sqrt(d**2 - e**2
*x**2)/(1287*e**4) - d**2*x**10*sqrt(d**2 - e**2*x**2)/(143*e**2) + x**12*s
qrt(d**2 - e**2*x**2)/13, Ne(e, 0)), (x**12*sqrt(d**2)/12, True)) + e**7*Pi
ecewise((-33*I*d**14*acosh(e*x/d)/(2048*e**13) + 33*I*d**13*x/(2048*e**12*s
qrt(-1 + e**2*x**2/d**2)) - 11*I*d**11*x**3/(2048*e**10*sqrt(-1 + e**2*x**2
/d**2)) - 11*I*d**9*x**5/(5120*e**8*sqrt(-1 + e**2*x**2/d**2)) - 11*I*d**7*
x**7/(8960*e**6*sqrt(-1 + e**2*x**2/d**2)) - 11*I*d**5*x**9/(13440*e**4*sqr
t(-1 + e**2*x**2/d**2)) - I*d**3*x**11/(1680*e**2*sqrt(-1 + e**2*x**2/d**2)
) - 13*I*d*x**13/(168*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**15/(14*d*sqrt(
-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (33*d**14*asin(e*x/d)/(204
8*e**13) - 33*d**13*x/(2048*e**12*sqrt(1 - e**2*x**2/d**2)) + 11*d**11*x**3
/(2048*e**10*sqrt(1 - e**2*x**2/d**2)) + 11*d**9*x**5/(5120*e**8*sqrt(1 - e
**2*x**2/d**2)) + 11*d**7*x**7/(8960*e**6*sqrt(1 - e**2*x**2/d**2)) + 11*d*
*5*x**9/(13440*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**11/(1680*e**2*sqrt(
1 - e**2*x**2/d**2)) + 13*d*x**13/(168*sqrt(1 - e**2*x**2/d**2)) - e**2*x**
15/(14*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.66 $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=281

$$-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}+\frac{27d^{13}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^5}+\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}$$

Rubi [A] time = 0.41, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5}-\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}+\frac{27d^{13}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (27*d^11*x*Sqrt[d^2 - e^2*x^2])/(1024*e^4) + (9*d^9*x*(d^2 - e^2*x^2)^(3/2)
)/(512*e^4) + (9*d^7*x*(d^2 - e^2*x^2)^(5/2))/(640*e^4) - (20*d^4*x^2*(d^2
- e^2*x^2)^(7/2))/(143*e^3) - (9*d^3*x^3*(d^2 - e^2*x^2)^(7/2))/(40*e^2) -
(45*d^2*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e) - (d*x^5*(d^2 - e^2*x^2)^(7/2))/
4 - (e*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (d^5*(12800*d + 27027*e*x)*(d^2 - e^
2*x^2)^(7/2))/(320320*e^5) + (27*d^13*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1
024*e^5)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
```

```
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx &= -\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^4\left(d^2-e^2x^2\right)^{5/2}\left(-13d^3e^2-45d^2e^3x-39de^4x^2\right)dx}{13e^2}\\
&= -\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^4\left(351d^3e^4+540d^2e^5x\right)\left(d^2-\right.}{156e^4}\\
&= -\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^3\left(-\right.}{}\\
&= -\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(\right.\\
&= -\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\\
&= -\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\\
&= \frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(\right.}{1}\\
&= \frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2\right.}{40}\\
&= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-\right.}{143e^3}\\
&= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-\right.}{143e^3}\\
&= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-\right.}{143e^3}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 200, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(135135d^{12}\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-204800d^{12}-135135d^{11}ex-102400d^{10}e^2x^2-90090d^9e^3x^3-76800d^8e^4x^4+952952d^7e^5x^5+2498560d^6e^6x^6+816816d^5e^7x^7-2938880d^4e^8x^8-2690688d^3e^9x^9+430080d^2e^{10}x^{10}+1281280de^{11}x^{11}+394240e^{12}x^{12}\right)\right)}{5125120e^5\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-204800*d^12 - 135135*d^11*e
*x - 102400*d^10*e^2*x^2 - 90090*d^9*e^3*x^3 - 76800*d^8*e^4*x^4 + 952952*d
^7*e^5*x^5 + 2498560*d^6*e^6*x^6 + 816816*d^5*e^7*x^7 - 2938880*d^4*e^8*x^8
```

```
 - 2690688*d^3*e^9*x^9 + 430080*d^2*e^10*x^10 + 1281280*d*e^11*x^11 + 39424
0*e^12*x^12) + 135135*d^12*ArcSin[(e*x)/d]))/(5125120*e^5*Sqrt[1 - (e^2*x^2
)/d^2])
```

IntegrateAlgebraic [A] time = 0.63, size = 202, normalized size = 0.72

$$\frac{27d^{13}\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{1024e^6}+\frac{\sqrt{d^2-e^2x^2}\left(-204800d^{12}-135135d^{11}ex-102400d^{10}e^2x^2-90090d^9e^3x^3-76800d^8e^4x^4+952952d^7e^5x^5+2498560d^6e^6x^6+816816d^5e^7x^7-2938880d^4e^8x^8-2690688d^3e^9x^9+430080d^2e^{10}x^{10}+1281280de^{11}x^{11}+394240e^{12}x^{12}\right)}{5125120e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-204800*d^12 - 135135*d^11*e*x - 102400*d^10*e^2*x^2
- 90090*d^9*e^3*x^3 - 76800*d^8*e^4*x^4 + 952952*d^7*e^5*x^5 + 2498560*d^6*
e^6*x^6 + 816816*d^5*e^7*x^7 - 2938880*d^4*e^8*x^8 - 2690688*d^3*e^9*x^9 +
430080*d^2*e^10*x^10 + 1281280*d*e^11*x^11 + 394240*e^12*x^12))/(5125120*e^
5) + (27*d^13*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(1024*
e^6)
```

fricas [A] time = 0.41, size = 183, normalized size = 0.65

$$-\frac{270270\,d^{13}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(394240\,e^{12}x^{12}+1281280\,de^{11}x^{11}+430080\,d^2e^{10}x^{10}-2690688\,d^3e^9x^9-2938880\,d^4e^8x^8+816816\,d^5e^7x^7+2498560\,d^6e^6x^6+952952\,d^7e^5x^5-76800\,d^8e^4x^4-90090\,d^9e^3x^3-102400\,d^{10}e^2x^2-135135\,d^{11}ex-204800\,d^{12}\right)\sqrt{-e^2x^2+d^2}}{5125120\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5125120*(270270*d^13*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (394240
*e^12*x^12 + 1281280*d*e^11*x^11 + 430080*d^2*e^10*x^10 - 2690688*d^3*e^9*x
^9 - 2938880*d^4*e^8*x^8 + 816816*d^5*e^7*x^7 + 2498560*d^6*e^6*x^6 + 95295
2*d^7*e^5*x^5 - 76800*d^8*e^4*x^4 - 90090*d^9*e^3*x^3 - 102400*d^10*e^2*x^2
 - 135135*d^11*e*x - 204800*d^12)*sqrt(-e^2*x^2 + d^2))/e^5
```

giac [A] time = 0.26, size = 160, normalized size = 0.57

$$\frac{27}{1024}\,d^{13}\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)-\frac{1}{5125120}\left(204800\,d^{12}e^{(-5)}+\left(135135\,d^{11}e^{(-4)}+2\left(51200\,d^{10}e^{(-3)}+\left(45045\,d^9e^{(-2)}+4\left(9600\,d^8e^{(-1)}-\left(119119\,d^7+2\left(156160\,d^6e+7\left(7293\,d^5e^2-8\left(3280\,d^4e^3+\left(3003\,d^3e^4-10\left(48\,d^2e^5+11\left(4\,xe^7+13\,de^6\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 27/1024*d^13*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/5125120*(204800*d^12*e^(-5) +
(135135*d^11*e^(-4) + 2*(51200*d^10*e^(-3) + (45045*d^9*e^(-2) + 4*(9600*d^
8*e^(-1) - (119119*d^7 + 2*(156160*d^6*e + 7*(7293*d^5*e^2 - 8*(3280*d^4*e^
3 + (3003*d^3*e^4 - 10*(48*d^2*e^5 + 11*(4*x*e^7 + 13*d*e^6)*x)*x)*x)*x)*x)
*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.02, size = 266, normalized size = 0.95

$$\frac{27d^{13}\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{1024\sqrt{e^2}\,e^4}+\frac{27\sqrt{-e^2x^2+d^2}\,d^{11}x}{1024e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^6}{13}+\frac{9\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^9x}{512e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^5}{4}-\frac{45\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^4}{143e}+\frac{9\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^7x}{640e^4}-\frac{9\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^3}{40e^2}-\frac{20\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x^2}{143e^3}-\frac{27\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5x}{320e^4}-\frac{40\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^6}{1001e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/13*e*x^6*(-e^2*x^2+d^2)^(7/2)-45/143*d^2*x^4*(-e^2*x^2+d^2)^(7/2)/e-20/1
43*d^4*x^2*(-e^2*x^2+d^2)^(7/2)/e^3-40/1001/e^5*d^6*(-e^2*x^2+d^2)^(7/2)-1/
4*d*x^5*(-e^2*x^2+d^2)^(7/2)-9/40*d^3*x^3*(-e^2*x^2+d^2)^(7/2)/e^2-27/320/e
^4*d^5*x*(-e^2*x^2+d^2)^(7/2)+9/640*d^7*x*(-e^2*x^2+d^2)^(5/2)/e^4+9/512*d^
9*x*(-e^2*x^2+d^2)^(3/2)/e^4+27/1024*d^11*x*(-e^2*x^2+d^2)^(1/2)/e^4+27/102
4/e^4*d^13/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 245, normalized size = 0.87

$$-\frac{1}{13}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^6-\frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^5+\frac{27\,d^{13}\arcsin\left(\frac{ex}{d}\right)}{1024\,e^5}+\frac{27\,\sqrt{-e^2x^2+d^2}\,d^{11}x}{1024\,e^4}-\frac{45\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^4}{143\,e}+\frac{9\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^9x}{512\,e^4}-\frac{9\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^3}{40\,e^2}+\frac{9\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^7x}{640\,e^4}-\frac{20\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x^2}{143\,e^3}-\frac{27\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5x}{320\,e^4}-\frac{40\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^6}{1001\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/13*(-e^2*x^2 + d^2)^(7/2)*e*x^6 - 1/4*(-e^2*x^2 + d^2)^(7/2)*d*x^5 + 27/
1024*d^13*arcsin(e*x/d)/e^5 + 27/1024*sqrt(-e^2*x^2 + d^2)*d^11*x/e^4 - 45/
143*(-e^2*x^2 + d^2)^(7/2)*d^2*x^4/e + 9/512*(-e^2*x^2 + d^2)^(3/2)*d^9*x/e
^4 - 9/40*(-e^2*x^2 + d^2)^(7/2)*d^3*x^3/e^2 + 9/640*(-e^2*x^2 + d^2)^(5/2)
*d^7*x/e^4 - 20/143*(-e^2*x^2 + d^2)^(7/2)*d^4*x^2/e^3 - 27/320*(-e^2*x^2 +
 d^2)^(7/2)*d^5*x/e^4 - 40/1001*(-e^2*x^2 + d^2)^(7/2)*d^6/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\left(d^2-e^2\,x^2\right)^{5/2}\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [C] time = 64.64, size = 2028, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) + 3*d**6*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/
(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt
(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x
**6*sqrt(d**2)/6, True)) + d**5*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128
*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(3
84*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x
**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(1
28*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*
e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**
2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2
*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**
2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4
*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e
**2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
 - 5*d**3*e**4*Piecewise((-7*I*d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(
256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**
2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**
3*x**7/(480*e**2*sqrt(-1 + e**2*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2
*x**2/d**2)) + I*e**2*x**11/(10*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2
/d**2) > 1), (7*d**10*asin(e*x/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 -
e**2*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5
*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e
```

```
**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d
*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*Piecewise((-128*d**10*sqrt(d
**2 - e**2*x**2)/(3465*e**10) - 64*d**8*x**2*sqrt(d**2 - e**2*x**2)/(3465*e
**8) - 16*d**6*x**4*sqrt(d**2 - e**2*x**2)/(1155*e**6) - 8*d**4*x**6*sqrt(d
**2 - e**2*x**2)/(693*e**4) - d**2*x**8*sqrt(d**2 - e**2*x**2)/(99*e**2) +
x**10*sqrt(d**2 - e**2*x**2)/11, Ne(e, 0)), (x**10*sqrt(d**2)/10, True)) +
3*d*e**6*Piecewise((-21*I*d**12*acosh(e*x/d)/(1024*e**11) + 21*I*d**11*x/(1
024*e**10*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**9*x**3/(1024*e**8*sqrt(-1 + e
**2*x**2/d**2)) - 7*I*d**7*x**5/(2560*e**6*sqrt(-1 + e**2*x**2/d**2)) - I*d
**5*x**7/(640*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**9/(960*e**2*sqrt(
-1 + e**2*x**2/d**2)) - 11*I*d*x**11/(120*sqrt(-1 + e**2*x**2/d**2)) + I*e*
*2*x**13/(12*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (21*d*
*12*asin(e*x/d)/(1024*e**11) - 21*d**11*x/(1024*e**10*sqrt(1 - e**2*x**2/d*
*2)) + 7*d**9*x**3/(1024*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**5/(2560
*e**6*sqrt(1 - e**2*x**2/d**2)) + d**5*x**7/(640*e**4*sqrt(1 - e**2*x**2/d*
*2)) + d**3*x**9/(960*e**2*sqrt(1 - e**2*x**2/d**2)) + 11*d*x**11/(120*sqrt
(1 - e**2*x**2/d**2)) - e**2*x**13/(12*d*sqrt(1 - e**2*x**2/d**2)), True))
+ e**7*Piecewise((-256*d**12*sqrt(d**2 - e**2*x**2)/(9009*e**12) - 128*d**1
0*x**2*sqrt(d**2 - e**2*x**2)/(9009*e**10) - 32*d**8*x**4*sqrt(d**2 - e**2*
x**2)/(3003*e**8) - 80*d**6*x**6*sqrt(d**2 - e**2*x**2)/(9009*e**6) - 10*d*
*4*x**8*sqrt(d**2 - e**2*x**2)/(1287*e**4) - d**2*x**10*sqrt(d**2 - e**2*x*
*2)/(143*e**2) + x**12*sqrt(d**2 - e**2*x**2)/13, Ne(e, 0)), (x**12*sqrt(d*
*2)/12, True))
```

3.67 $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=252

$$-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}-\frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e}+\frac{41d^{12}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^4}+\frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}+$$

Rubi [A] time = 0.36, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}+\frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3}+\frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3}-\frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4}-\frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2}-\frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e}-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}+\frac{41d^{12}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (41*d^10*x*Sqrt[d^2 - e^2*x^2])/(1024*e^3) + (41*d^8*x*(d^2 - e^2*x^2)^(3/2
))/(1536*e^3) + (41*d^6*x*(d^2 - e^2*x^2)^(5/2))/(1920*e^3) - (23*d^3*x^2*(
d^2 - e^2*x^2)^(7/2))/(99*e^2) - (41*d^2*x^3*(d^2 - e^2*x^2)^(7/2))/(120*e)
 - (3*d*x^4*(d^2 - e^2*x^2)^(7/2))/11 - (e*x^5*(d^2 - e^2*x^2)^(7/2))/12 -
(d^4*(14720*d + 28413*e*x)*(d^2 - e^2*x^2)^(7/2))/(221760*e^4) + (41*d^12*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1024*e^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^3\left(d^2-e^2x^2\right)^{5/2}\left(-12d^3e^2-41d^2e^3x-36de^4x^2\right) \cdots}{12e^2}$$

$$= -\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^3\left(276d^3e^4+451d^2e^5x\right)\cdots}{132e^4}$$

$$= -\frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^2 \cdots}{\cdots}$$

$$= -\frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}\cdots$$

$$= -\frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}\cdots$$

$$= \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}d\cdots$$

$$= \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2\cdots}{\cdots}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\cdots}{\cdots}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\cdots}{\cdots}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\cdots}{\cdots}$$

Mathematica [A] time = 0.30, size = 189, normalized size = 0.75

$$\frac{\sqrt{d^2-e^2x^2}\left(142065d^{11}\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-235520d^{11}-142065d^{10}ex-117760d^9e^2x^2-94710d^8e^3x^3+798720d^7e^4x^4+2053128d^6e^5x^5+665600d^5e^6x^6-2295216d^4e^7x^7-2078720d^3e^8x^8+325248d^2e^9x^9+967680de^{10}x^{10}+295680e^{11}x^{11}\right)\right)}{3548160e^4\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-235520*d^11 - 142065*d^10*e
*x - 117760*d^9*e^2*x^2 - 94710*d^8*e^3*x^3 + 798720*d^7*e^4*x^4 + 2053128*
d^6*e^5*x^5 + 665600*d^5*e^6*x^6 - 2295216*d^4*e^7*x^7 - 2078720*d^3*e^8*x^
8 + 325248*d^2*e^9*x^9 + 967680*d*e^10*x^10 + 295680*e^11*x^11) + 142065*d^
11*ArcSin[(e*x)/d]))/(3548160*e^4*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.68, size = 191, normalized size = 0.76

$$\frac{41d^{12}\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{1024e^5}+\frac{\sqrt{d^2-e^2x^2}\left(-235520d^{11}-142065d^{10}ex-117760d^9e^2x^2-94710d^8e^3x^3+798720d^7e^4x^4+2053128d^6e^5x^5+665600d^5e^6x^6-2295216d^4e^7x^7-2078720d^3e^8x^8+325248d^2e^9x^9+967680de^{10}x^{10}+295680e^{11}x^{11}\right)}{3548160e^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-235520*d^11 - 142065*d^10*e*x - 117760*d^9*e^2*x^2 -
 94710*d^8*e^3*x^3 + 798720*d^7*e^4*x^4 + 2053128*d^6*e^5*x^5 + 665600*d^5*
e^6*x^6 - 2295216*d^4*e^7*x^7 - 2078720*d^3*e^8*x^8 + 325248*d^2*e^9*x^9 +
967680*d*e^10*x^10 + 295680*e^11*x^11))/(3548160*e^4) + (41*d^12*Sqrt[-e^2]
*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(1024*e^5)
```

fricas [A] time = 0.41, size = 172, normalized size = 0.68

$$-\frac{284130\,d^{12}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(295680\,e^{11}x^{11}+967680\,de^{10}x^{10}+325248\,d^2e^9x^9-2078720\,d^3e^8x^8-2295216\,d^4e^7x^7+665600\,d^5e^6x^6+2053128\,d^6e^5x^5+798720\,d^7e^4x^4-94710\,d^8e^3x^3-117760\,d^9e^2x^2-142065\,d^{10}ex-235520\,d^{11}\right)\sqrt{-e^2x^2+d^2}}{3548160\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3548160*(284130*d^12*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (295680
*e^11*x^11 + 967680*d*e^10*x^10 + 325248*d^2*e^9*x^9 - 2078720*d^3*e^8*x^8
- 2295216*d^4*e^7*x^7 + 665600*d^5*e^6*x^6 + 2053128*d^6*e^5*x^5 + 798720*d
^7*e^4*x^4 - 94710*d^8*e^3*x^3 - 117760*d^9*e^2*x^2 - 142065*d^10*e*x - 235
520*d^11)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [A] time = 0.24, size = 149, normalized size = 0.59

$$\frac{41}{1024}\,d^{12}\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{1}{3548160}\left(235520\,d^{11}e^{(-4)}+\left(142065\,d^{10}e^{(-3)}+2\left(58880\,d^9e^{(-2)}+\left(47355\,d^8e^{(-1)}-4\left(99840\,d^7+\left(256641\,d^6e+2\left(41600\,d^5e^2-7\left(20493\,d^4e^3+8\left(2320\,d^3e^4-3\left(121\,d^2e^5+10\left(11\,xe^7+36\,de^6\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 41/1024*d^12*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/3548160*(235520*d^11*e^(-4) +
(142065*d^10*e^(-3) + 2*(58880*d^9*e^(-2) + (47355*d^8*e^(-1) - 4*(99840*d^
7 + (256641*d^6*e + 2*(41600*d^5*e^2 - 7*(20493*d^4*e^3 + 8*(2320*d^3*e^4 -
 3*(121*d^2*e^5 + 10*(11*x*e^7 + 36*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sq
rt(-x^2*e^2 + d^2)
```

maple [A] time = 0.02, size = 241, normalized size = 0.96

$$\frac{41d^{12}\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{1024\sqrt{e^2}\,e^3}+\frac{41\sqrt{-e^2x^2+d^2}\,d^{10}x}{1024e^3}+\frac{41\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^8x}{1536e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e\,x^5}{12}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d\,x^4}{11}+\frac{41\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^6x}{1920e^3}-\frac{41\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^3}{120e}-\frac{23\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^2}{99e^2}-\frac{41\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x}{320e^3}-\frac{46\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5}{693e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/12*e*x^5*(-e^2*x^2+d^2)^(7/2)-41/120*d^2*x^3*(-e^2*x^2+d^2)^(7/2)/e-41/3
20/e^3*d^4*x*(-e^2*x^2+d^2)^(7/2)+41/1920*d^6*x*(-e^2*x^2+d^2)^(5/2)/e^3+41
/1536*d^8*x*(-e^2*x^2+d^2)^(3/2)/e^3+41/1024*d^10*x*(-e^2*x^2+d^2)^(1/2)/e^
3+41/1024/e^3*d^12/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-3
/11*d*x^4*(-e^2*x^2+d^2)^(7/2)-23/99*d^3*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-46/69
3/e^4*d^5*(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.99, size = 220, normalized size = 0.87

$$-\frac{1}{12}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^5+\frac{41\,d^{12}\arcsin\left(\frac{ex}{d}\right)}{1024\,e^4}+\frac{41\sqrt{-e^2x^2+d^2}\,d^{10}x}{1024\,e^3}-\frac{3}{11}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^4+\frac{41\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^8x}{1536\,e^3}-\frac{41\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^3}{120\,e}+\frac{41\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^6x}{1920\,e^3}-\frac{23\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x^2}{99\,e^2}-\frac{41\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4x}{320\,e^3}-\frac{46\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5}{693\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-e^2*x^2 + d^2)^(7/2)*e*x^5 + 41/1024*d^12*arcsin(e*x/d)/e^4 + 41/10
24*sqrt(-e^2*x^2 + d^2)*d^10*x/e^3 - 3/11*(-e^2*x^2 + d^2)^(7/2)*d*x^4 + 41
/1536*(-e^2*x^2 + d^2)^(3/2)*d^8*x/e^3 - 41/120*(-e^2*x^2 + d^2)^(7/2)*d^2*
x^3/e + 41/1920*(-e^2*x^2 + d^2)^(5/2)*d^6*x/e^3 - 23/99*(-e^2*x^2 + d^2)^(
7/2)*d^3*x^2/e^2 - 41/320*(-e^2*x^2 + d^2)^(7/2)*d^4*x/e^3 - 46/693*(-e^2*x
^2 + d^2)^(7/2)*d^5/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(d^2 - e^2 x^2\right)^{5/2} (d + e\, x)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [A] time = 59.74, size = 1919, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) + 3*d**6*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5)
 + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt
(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2
*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin
(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(
48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2))
- e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**5*e**2*Piecewise((-
8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**
2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2
 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 5*d**4*e**3*Piecew
ise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**
2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3
*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*
x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d*
*2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2
*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/
(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)
) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**4*Piecewis
e((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**
2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x
**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e,
 0)), (x**8*sqrt(d**2)/8, True)) + d**2*e**5*Piecewise((-7*I*d**10*acosh(e*
x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**
7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt
(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x**2/d**2)) -
 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10*d*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*asin(e*x/d)/(256*e**9)
- 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt
(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) +
d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*
x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6
*Piecewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10) - 64*d**8*x**2*s
qrt(d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2 - e**2*x**2)/(11
```

```
55*e**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) - d**2*x**8*sqrt(d
**2 - e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/11, Ne(e, 0)), (x
**10*sqrt(d**2)/10, True)) + e**7*Piecewise((-21*I*d**12*acosh(e*x/d)/(1024
*e**11) + 21*I*d**11*x/(1024*e**10*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**9*x*
*3/(1024*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**5/(2560*e**6*sqrt(-1
 + e**2*x**2/d**2)) - I*d**5*x**7/(640*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*
d**3*x**9/(960*e**2*sqrt(-1 + e**2*x**2/d**2)) - 11*I*d*x**11/(120*sqrt(-1
+ e**2*x**2/d**2)) + I*e**2*x**13/(12*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**
2*x**2/d**2) > 1), (21*d**12*asin(e*x/d)/(1024*e**11) - 21*d**11*x/(1024*e*
*10*sqrt(1 - e**2*x**2/d**2)) + 7*d**9*x**3/(1024*e**8*sqrt(1 - e**2*x**2/d
**2)) + 7*d**7*x**5/(2560*e**6*sqrt(1 - e**2*x**2/d**2)) + d**5*x**7/(640*e
**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**9/(960*e**2*sqrt(1 - e**2*x**2/d**2
)) + 11*d*x**11/(120*sqrt(1 - e**2*x**2/d**2)) - e**2*x**13/(12*d*sqrt(1 -
e**2*x**2/d**2)), True))
```

3.68 $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=223

$$-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}+\frac{19d^{11}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^3}+\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}$$

Rubi [A] time = 0.31, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3}-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}+\frac{19d^{11}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (19*d^9*x*Sqrt[d^2 - e^2*x^2])/(256*e^2) + (19*d^7*x*(d^2 - e^2*x^2)^(3/2))
/(384*e^2) + (19*d^5*x*(d^2 - e^2*x^2)^(5/2))/(480*e^2) - (37*d^2*x^2*(d^2
- e^2*x^2)^(7/2))/(99*e) - (3*d*x^3*(d^2 - e^2*x^2)^(7/2))/10 - (e*x^4*(d^2
 - e^2*x^2)^(7/2))/11 - (d^3*(5920*d + 13167*e*x)*(d^2 - e^2*x^2)^(7/2))/(5
5440*e^3) + (19*d^11*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx &= -\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^2\left(d^2-e^2x^2\right)^{5/2}\left(-11d^3e^2-37d^2e^3x-33de^4x^2\right)dx}{11e^2} \\
&= -\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^2\left(209d^3e^4+370d^2e^5x\right)\left(d^2\cdots\right.}{110e^4} \\
&= -\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x\left(-\cdots\right.}{\cdots} \\
&= -\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}-\frac{d^3(592\cdots}{\cdots} \\
&= \frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{11}ex^4\left(d\cdots\right. \\
&= \frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{3}{10}dx^3\left(\cdots\right. \\
&= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2\cdots\right.}{99\cdots} \\
&= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2\cdots\right.}{99\cdots} \\
&= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2\cdots\right.}{99\cdots}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 178, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\left(65835d^{10}\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-94720d^{10}-65835d^9ex-47360d^8e^2x^2+251790d^7e^3x^3+629760d^6e^4x^4+201432d^5e^5x^5-657920d^4e^6x^6-587664d^3e^7x^7+89600d^2e^8x^8+266112de^9x^9+80640e^{10}x^{10}\right)\right)}{887040e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-94720*d^10 - 65835*d^9*e*x
- 47360*d^8*e^2*x^2 + 251790*d^7*e^3*x^3 + 629760*d^6*e^4*x^4 + 201432*d^5*
e^5*x^5 - 657920*d^4*e^6*x^6 - 587664*d^3*e^7*x^7 + 89600*d^2*e^8*x^8 + 266
112*d*e^9*x^9 + 80640*e^10*x^10) + 65835*d^10*ArcSin[(e*x)/d]))/(887040*e^3
*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.69, size = 180, normalized size = 0.81

$$\frac{19d^{11}\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{256e^4}+\frac{\sqrt{d^2-e^2x^2}\left(-94720d^{10}-65835d^9ex-47360d^8e^2x^2+251790d^7e^3x^3+629760d^6e^4x^4+201432d^5e^5x^5-657920d^4e^6x^6-587664d^3e^7x^7+89600d^2e^8x^8+266112de^9x^9+80640e^{10}x^{10}\right)}{887040e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-94720*d^10 - 65835*d^9*e*x - 47360*d^8*e^2*x^2 + 251
790*d^7*e^3*x^3 + 629760*d^6*e^4*x^4 + 201432*d^5*e^5*x^5 - 657920*d^4*e^6*
x^6 - 587664*d^3*e^7*x^7 + 89600*d^2*e^8*x^8 + 266112*d*e^9*x^9 + 80640*e^1
0*x^10))/(887040*e^3) + (19*d^11*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2
- e^2*x^2]])/(256*e^4)
```

fricas [A] time = 0.42, size = 161, normalized size = 0.72

$$-\frac{131670\,d^{11}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(80640\,e^{10}x^{10}+266112\,de^9x^9+89600\,d^2e^8x^8-587664\,d^3e^7x^7-657920\,d^4e^6x^6+201432\,d^5e^5x^5+629760\,d^6e^4x^4+251790\,d^7e^3x^3-47360\,d^8e^2x^2-65835\,d^9ex-94720\,d^{10}\right)\sqrt{-e^2x^2+d^2}}{887040\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/887040*(131670*d^11*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (80640*e
^10*x^10 + 266112*d*e^9*x^9 + 89600*d^2*e^8*x^8 - 587664*d^3*e^7*x^7 - 6579
20*d^4*e^6*x^6 + 201432*d^5*e^5*x^5 + 629760*d^6*e^4*x^4 + 251790*d^7*e^3*x
^3 - 47360*d^8*e^2*x^2 - 65835*d^9*e*x - 94720*d^10)*sqrt(-e^2*x^2 + d^2))/
e^3
```

giac [A] time = 0.29, size = 139, normalized size = 0.62

$$\frac{19}{256}\,d^{11}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{887040}\left(94720\,d^{10}e^{(-3)}+\left(65835\,d^9e^{(-2)}+2\left(23680\,d^8e^{(-1)}-\left(125895\,d^7+4\left(78720\,d^6e+\left(25179\,d^5e^2-2\left(41120\,d^4e^3+7\left(5247\,d^3e^4-8\left(100\,d^2e^5+9\left(10\,xe^7+33\,de^6\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 19/256*d^11*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/887040*(94720*d^10*e^(-3) + (65
835*d^9*e^(-2) + 2*(23680*d^8*e^(-1) - (125895*d^7 + 4*(78720*d^6*e + (2517
9*d^5*e^2 - 2*(41120*d^4*e^3 + 7*(5247*d^3*e^4 - 8*(100*d^2*e^5 + 9*(10*x*e
^7 + 33*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 216, normalized size = 0.97

$$\frac{19d^{11}\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{256\sqrt{e^2}\,e^2}+\frac{19\sqrt{-e^2x^2+d^2}\,d^9x}{256e^2}+\frac{19\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^7x}{384e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e\,x^4}{11}+\frac{19\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5x}{480e^2}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d\,x^3}{10}-\frac{37\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^2}{99e}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x}{80e^2}-\frac{74\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4}{693e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/11*e*x^4*(-e^2*x^2+d^2)^(7/2)-37/99*d^2*x^2*(-e^2*x^2+d^2)^(7/2)/e-74/69
3/e^3*d^4*(-e^2*x^2+d^2)^(7/2)-3/10*d*x^3*(-e^2*x^2+d^2)^(7/2)-19/80/e^2*d^
3*x*(-e^2*x^2+d^2)^(7/2)+19/480*d^5*x*(-e^2*x^2+d^2)^(5/2)/e^2+19/384*d^7*x
*(-e^2*x^2+d^2)^(3/2)/e^2+19/256*d^9*x*(-e^2*x^2+d^2)^(1/2)/e^2+19/256/e^2*
d^11/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 195, normalized size = 0.87

$$\frac{19\,d^{11}\arcsin\left(\frac{ex}{d}\right)}{256\,e^3}+\frac{19\,\sqrt{-e^2x^2+d^2}\,d^9x}{256\,e^2}-\frac{1}{11}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^4+\frac{19\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^7x}{384\,e^2}-\frac{3}{10}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^3+\frac{19\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5x}{480\,e^2}-\frac{37\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^2}{99\,e}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x}{80\,e^2}-\frac{74\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4}{693\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 19/256*d^11*arcsin(e*x/d)/e^3 + 19/256*sqrt(-e^2*x^2 + d^2)*d^9*x/e^2 - 1/1
1*(-e^2*x^2 + d^2)^(7/2)*e*x^4 + 19/384*(-e^2*x^2 + d^2)^(3/2)*d^7*x/e^2 -
3/10*(-e^2*x^2 + d^2)^(7/2)*d*x^3 + 19/480*(-e^2*x^2 + d^2)^(5/2)*d^5*x/e^2
```

```
 - 37/99*(-e^2*x^2 + d^2)^(7/2)*d^2*x^2/e - 19/80*(-e^2*x^2 + d^2)^(7/2)*d^
3*x/e^2 - 74/693*(-e^2*x^2 + d^2)^(7/2)*d^4/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(d^2 - e^2 x^2\right)^{5/2} (d + e x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [C] time = 40.61, size = 1681, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*
*6*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) + d**5*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5
) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqr
t(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asi
n(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/
(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2))
 - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise
((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*
x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d
**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 5*d**3*e**4*Pie
cewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 +
e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d
**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e*
*2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2
/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e
**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x*
*5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d*
*2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*Piecewi
se((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e*
*2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*
x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e
, 0)), (x**8*sqrt(d**2)/8, True)) + 3*d*e**6*Piecewise((-7*I*d**10*acosh(e*
x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**
7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt
(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x**2/d**2)) -
 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10*d*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*asin(e*x/d)/(256*e**9)
- 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt
(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) +
d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*
x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Pie
cewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10) - 64*d**8*x**2*sqrt(
d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2 - e**2*x**2)/(1155*e
**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) - d**2*x**8*sqrt(d**2
```

```
- e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/11, Ne(e, 0)), (x**10
*sqrt(d**2)/10, True))
```

3.69 $\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=230

$$-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{33d^{10}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2}+\frac{33d^8x\cdots}{2\cdots}$$

Rubi [A] time = 0.12, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {795, 671, 641, 195, 217, 203}

$$\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{33d^{10}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (33*d^8*x*Sqrt[d^2 - e^2*x^2])/(256*e) + (11*d^6*x*(d^2 - e^2*x^2)^(3/2))/(
128*e) + (11*d^4*x*(d^2 - e^2*x^2)^(5/2))/(160*e) - (33*d^3*(d^2 - e^2*x^2)
^(7/2))/(560*e^2) - (11*d^2*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(240*e^2) - (d
*(d + e*x)^2*(d^2 - e^2*x^2)^(7/2))/(30*e^2) - ((d + e*x)^3*(d^2 - e^2*x^2)
^(7/2))/(10*e^2) + (33*d^10*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
```

```
x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$
\begin{aligned}
\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx &= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(3d)\int(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx}{10e} \\
&= -\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\cdots \\
&= -\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\cdots \\
&= -\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}\cdots \\
&= \frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\cdots \\
&= \frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\cdots \\
&= \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\cdots \\
&= \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\cdots \\
&= \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\cdots
\end{aligned}
$$

Mathematica [A]   time = 0.38, size = 167, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2x^2}\left(3465d^9\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-6400d^9-3465d^8ex+10240d^7e^2x^2+24570d^6e^3x^3+7680d^5e^4x^4-23352d^4e^5x^5-20480d^3e^6x^6+3024d^2e^7x^7+8960de^8x^8+2688e^9x^9\right)\right)}{26880e^2\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-6400*d^9 - 3465*d^8*e*x + 1
0240*d^7*e^2*x^2 + 24570*d^6*e^3*x^3 + 7680*d^5*e^4*x^4 - 23352*d^4*e^5*x^5
 - 20480*d^3*e^6*x^6 + 3024*d^2*e^7*x^7 + 8960*d*e^8*x^8 + 2688*e^9*x^9) +
3465*d^9*ArcSin[(e*x)/d]))/(26880*e^2*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A]   time = 0.59, size = 169, normalized size = 0.73

$$\frac{33d^{10}\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{256e^3}+\frac{\sqrt{d^2-e^2x^2}\left(-6400d^9-3465d^8ex+10240d^7e^2x^2+24570d^6e^3x^3+7680d^5e^4x^4-23352d^4e^5x^5-20480d^3e^6x^6+3024d^2e^7x^7+8960de^8x^8+2688e^9x^9\right)}{26880e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6400*d^9 - 3465*d^8*e*x + 10240*d^7*e^2*x^2 + 24570*
d^6*e^3*x^3 + 7680*d^5*e^4*x^4 - 23352*d^4*e^5*x^5 - 20480*d^3*e^6*x^6 + 30
24*d^2*e^7*x^7 + 8960*d*e^8*x^8 + 2688*e^9*x^9))/(26880*e^2) + (33*d^10*Sqr
t[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(256*e^3)
```

fricas [A] time = 0.40, size = 150, normalized size = 0.65

$$-\frac{6930\,d^{10}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(2688\,e^9x^9+8960\,de^8x^8+3024\,d^2e^7x^7-20480\,d^3e^6x^6-23352\,d^4e^5x^5+7680\,d^5e^4x^4+24570\,d^6e^3x^3+10240\,d^7e^2x^2-3465\,d^8ex-6400\,d^9\right)\sqrt{-e^2x^2+d^2}}{26880\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/26880*(6930*d^10*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (2688*e^9*x
^9 + 8960*d*e^8*x^8 + 3024*d^2*e^7*x^7 - 20480*d^3*e^6*x^6 - 23352*d^4*e^5*
x^5 + 7680*d^5*e^4*x^4 + 24570*d^6*e^3*x^3 + 10240*d^7*e^2*x^2 - 3465*d^8*e
*x - 6400*d^9)*sqrt(-e^2*x^2 + d^2))/e^2
```

giac [A] time = 0.24, size = 128, normalized size = 0.56

$$\frac{33}{256}\,d^{10}\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)-\frac{1}{26880}\left(6400\,d^9e^{(-2)}+\left(3465\,d^8e^{(-1)}-2\left(5120\,d^7+\left(12285\,d^6e+4\left(960\,d^5e^2-\left(2919\,d^4e^3+2\left(1280\,d^3e^4-7\left(27\,d^2e^5+8\left(3\,xe^7+10\,de^6\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 33/256*d^10*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/26880*(6400*d^9*e^(-2) + (3465*
d^8*e^(-1) - 2*(5120*d^7 + (12285*d^6*e + 4*(960*d^5*e^2 - (2919*d^4*e^3 +
2*(1280*d^3*e^4 - 7*(27*d^2*e^5 + 8*(3*x*e^7 + 10*d*e^6)*x)*x)*x)*x)*x)*
x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 191, normalized size = 0.83

$$\frac{33d^{10}\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{256\sqrt{e^2}\,e}+\frac{33\sqrt{-e^2x^2+d^2}\,d^8x}{256e}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6x}{128e}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4x}{160e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^3}{10}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^2}{3}-\frac{33\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x}{80e}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3}{21e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/10*e*x^3*(-e^2*x^2+d^2)^(7/2)-33/80/e*d^2*x*(-e^2*x^2+d^2)^(7/2)+11/160*
d^4*x*(-e^2*x^2+d^2)^(5/2)/e+11/128*d^6*x*(-e^2*x^2+d^2)^(3/2)/e+33/256*d^8
*x*(-e^2*x^2+d^2)^(1/2)/e+33/256/e*d^10/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^
2*x^2+d^2)^(1/2)*x)-1/3*d*x^2*(-e^2*x^2+d^2)^(7/2)-5/21*d^3*(-e^2*x^2+d^2)^
(7/2)/e^2
```

maxima [A] time = 0.99, size = 170, normalized size = 0.74

$$\frac{33\,d^{10}\arcsin\left(\frac{ex}{d}\right)}{256\,e^2}+\frac{33\,\sqrt{-e^2x^2+d^2}\,d^8x}{256\,e}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6x}{128\,e}-\frac{1}{10}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^3+\frac{11\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4x}{160\,e}-\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^2-\frac{33\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x}{80\,e}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3}{21\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 33/256*d^10*arcsin(e*x/d)/e^2 + 33/256*sqrt(-e^2*x^2 + d^2)*d^8*x/e + 11/12
8*(-e^2*x^2 + d^2)^(3/2)*d^6*x/e - 1/10*(-e^2*x^2 + d^2)^(7/2)*e*x^3 + 11/1
60*(-e^2*x^2 + d^2)^(5/2)*d^4*x/e - 1/3*(-e^2*x^2 + d^2)^(7/2)*d*x^2 - 33/8
0*(-e^2*x^2 + d^2)^(7/2)*d^2*x/e - 5/21*(-e^2*x^2 + d^2)^(7/2)*d^3/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(d^2-e^2x^2\right)^{5/2}(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [A] time = 40.18, size = 1554, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + 3*d**6*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*
d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x*
*2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2
) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2
)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x
**2/d**2)), True)) + d**5*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(1
5*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**
2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - 5*d**4*e**3*Piecewise((-
I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)
) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-
1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**
2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e
**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(
24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), Tr
ue)) - 5*d**3*e**4*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4
*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x
**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)
/6, True)) + d**2*e**5*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d
**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7
*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*
x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d
**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 -
e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7
/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)),
True)) + 3*d*e**6*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8
*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2
*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(
d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) + e**7*Piecewise
((-7*I*d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*
x**2/d**2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5
*x**5/(1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1
 + e**2*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x*
*11/(10*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*as
in(e*x/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**
7*x**3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1
- e**2*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x*
*9/(80*sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2
)), True))
```

3.70 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal. Leaf size=188

$$-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{55d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e}+\frac{55}{128}d^7x\sqrt{d^2-e^2x^2}$$

Rubi [A] time = 0.08, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {671, 641, 195, 217, 203}

$$\frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{55d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (55*d^7*x*Sqrt[d^2 - e^2*x^2])/128 + (55*d^5*x*(d^2 - e^2*x^2)^(3/2))/192 +
 (11*d^3*x*(d^2 - e^2*x^2)^(5/2))/48 - (11*d^2*(d^2 - e^2*x^2)^(7/2))/(56*e
) - (11*d*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(72*e) - ((d + e*x)^2*(d^2 - e^2
*x^2)^(7/2))/(9*e) + (55*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = -\frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{9e} + \frac{1}{9}(11d) \int (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} dx$$
$$= -\frac{11d(d+ex) \left(d^2-e^2x^2\right)^{7/2}}{72e} - \frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{9e} + \frac{1}{8}\left(11d^2\right) \int (d+ex) \left(d\right.$$
$$= -\frac{11d^2 \left(d^2-e^2x^2\right)^{7/2}}{56e} - \frac{11d(d+ex) \left(d^2-e^2x^2\right)^{7/2}}{72e} - \frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{9e} +$$
$$= \frac{11}{48}d^3x \left(d^2-e^2x^2\right)^{5/2} - \frac{11d^2 \left(d^2-e^2x^2\right)^{7/2}}{56e} - \frac{11d(d+ex) \left(d^2-e^2x^2\right)^{7/2}}{72e} - \frac{(d+}{}$$
$$= \frac{55}{192}d^5x \left(d^2-e^2x^2\right)^{3/2} + \frac{11}{48}d^3x \left(d^2-e^2x^2\right)^{5/2} - \frac{11d^2 \left(d^2-e^2x^2\right)^{7/2}}{56e} - \frac{11d(d+e}{}$$
$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2} + \frac{55}{192}d^5x \left(d^2-e^2x^2\right)^{3/2} + \frac{11}{48}d^3x \left(d^2-e^2x^2\right)^{5/2} - \frac{11d^2 \left(d^2\right.}{5}$$
$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2} + \frac{55}{192}d^5x \left(d^2-e^2x^2\right)^{3/2} + \frac{11}{48}d^3x \left(d^2-e^2x^2\right)^{5/2} - \frac{11d^2 \left(d^2\right.}{5}$$
$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2} + \frac{55}{192}d^5x \left(d^2-e^2x^2\right)^{3/2} + \frac{11}{48}d^3x \left(d^2-e^2x^2\right)^{5/2} - \frac{11d^2 \left(d^2\right.}{5}$$

Mathematica [A] time = 0.33, size = 156, normalized size = 0.83

$$\frac{\sqrt{d^2-e^2x^2}\left(3465d^8\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(-3712d^8+4599d^7ex+10240d^6e^2x^2+3066d^5e^3x^3-8448d^4e^4x^4-7224d^3e^5x^5+1024d^2e^6x^6+3024de^7x^7+896e^8x^8\right)\right)}{8064e\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-3712*d^8 + 4599*d^7*e*x + 1
0240*d^6*e^2*x^2 + 3066*d^5*e^3*x^3 - 8448*d^4*e^4*x^4 - 7224*d^3*e^5*x^5 +
 1024*d^2*e^6*x^6 + 3024*d*e^7*x^7 + 896*e^8*x^8) + 3465*d^8*ArcSin[(e*x)/d
]))/(8064*e*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.48, size = 158, normalized size = 0.84

$$\frac{55d^9\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{128e^2}+\frac{\sqrt{d^2-e^2x^2}\left(-3712d^8+4599d^7ex+10240d^6e^2x^2+3066d^5e^3x^3-8448d^4e^4x^4-7224d^3e^5x^5+1024d^2e^6x^6+3024de^7x^7+896e^8x^8\right)}{8064e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-3712*d^8 + 4599*d^7*e*x + 10240*d^6*e^2*x^2 + 3066*d
^5*e^3*x^3 - 8448*d^4*e^4*x^4 - 7224*d^3*e^5*x^5 + 1024*d^2*e^6*x^6 + 3024*
d*e^7*x^7 + 896*e^8*x^8))/(8064*e) + (55*d^9*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x)
 + Sqrt[d^2 - e^2*x^2]])/(128*e^2)
```

fricas [A] time = 0.41, size = 139, normalized size = 0.74

$$-\frac{6930\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(896\,e^8x^8+3024\,de^7x^7+1024\,d^2e^6x^6-7224\,d^3e^5x^5-8448\,d^4e^4x^4+3066\,d^5e^3x^3+10240\,d^6e^2x^2+4599\,d^7ex-3712\,d^8\right)\sqrt{-e^2x^2+d^2}}{8064\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8064*(6930*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (896*e^8*x^8
+ 3024*d*e^7*x^7 + 1024*d^2*e^6*x^6 - 7224*d^3*e^5*x^5 - 8448*d^4*e^4*x^4 +
 3066*d^5*e^3*x^3 + 10240*d^6*e^2*x^2 + 4599*d^7*e*x - 3712*d^8)*sqrt(-e^2*
x^2 + d^2))/e
```

giac [A] time = 0.25, size = 117, normalized size = 0.62

$$\frac{55}{128}\,d^9\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}(d)-\frac{1}{8064}\left(3712\,d^8e^{(-1)}-\left(4599\,d^7+2\left(5120\,d^6e+\left(1533\,d^5e^2-4\left(1056\,d^4e^3+\left(903\,d^3e^4-2\left(64\,d^2e^5+7\left(8\,xe^7+27\,de^6\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 55/128*d^9*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/8064*(3712*d^8*e^(-1) - (4599*d^
7 + 2*(5120*d^6*e + (1533*d^5*e^2 - 4*(1056*d^4*e^3 + (903*d^3*e^4 - 2*(64*
d^2*e^5 + 7*(8*x*e^7 + 27*d*e^6)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 154, normalized size = 0.82

$$\frac{55d^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128\sqrt{e^2}}+\frac{55\sqrt{-e^2x^2+d^2}\,d^7x}{128}+\frac{55\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x}{192}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x}{48}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e\,x^2}{9}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx}{8}-\frac{29\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2}{63e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/9*e*x^2*(-e^2*x^2+d^2)^(7/2)-29/63*d^2*(-e^2*x^2+d^2)^(7/2)/e-3/8*d*x*(-
e^2*x^2+d^2)^(7/2)+11/48*d^3*x*(-e^2*x^2+d^2)^(5/2)+55/192*d^5*x*(-e^2*x^2+
d^2)^(3/2)+55/128*d^7*x*(-e^2*x^2+d^2)^(1/2)+55/128*d^9/(e^2)^(1/2)*arctan(
(e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.98, size = 136, normalized size = 0.72

$$\frac{55\,d^9\arcsin\left(\frac{ex}{d}\right)}{128\,e}+\frac{55}{128}\sqrt{-e^2x^2+d^2}\,d^7x+\frac{55}{192}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x+\frac{11}{48}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x-\frac{1}{9}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^2-\frac{3}{8}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx-\frac{29\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2}{63\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 55/128*d^9*arcsin(e*x/d)/e + 55/128*sqrt(-e^2*x^2 + d^2)*d^7*x + 55/192*(-e
^2*x^2 + d^2)^(3/2)*d^5*x + 11/48*(-e^2*x^2 + d^2)^(5/2)*d^3*x - 1/9*(-e^2*
x^2 + d^2)^(7/2)*e*x^2 - 3/8*(-e^2*x^2 + d^2)^(7/2)*d*x - 29/63*(-e^2*x^2 +
 d^2)^(7/2)*d^2/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

sympy [C] time = 25.88, size = 1284, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
```

```
1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + 3*d*
*6*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + d**5*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I
*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x
**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**
2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**
2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*
x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)
/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 -
e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - 5*d**3*e**4*Piecewise
((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d*
*2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqr
t(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(
e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1
- e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**
5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)),
 True)) + d**2*e**5*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) -
4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*
x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2
)/6, True)) + 3*d*e**6*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d
**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7
*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*
x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d
**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 -
e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7
/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)),
True)) + e**7*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**
6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**
2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2
 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

3.71 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=190

$$\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}+\frac{125}{128}d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^8\tanh^{-1}\left(\right.$$

Rubi [A] time = 0.31, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1809, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}+\frac{125}{128}d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

```
[Out] (d^6*(128*d + 125*e*x)*Sqrt[d^2 - e^2*x^2])/128 + (d^4*(64*d + 125*e*x)*(d^
2 - e^2*x^2)^(3/2))/192 + (d^2*(48*d + 125*e*x)*(d^2 - e^2*x^2)^(5/2))/240
- (3*d*(d^2 - e^2*x^2)^(7/2))/7 - (e*x*(d^2 - e^2*x^2)^(7/2))/8 + (125*d^8*
ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/128 - d^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
```

```
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx=-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-8d^3e^2-25d^2e^3x-24de^4x^2\right)}{x}\,dx}{8e^2}$$

$$=-\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}+\frac{\int \frac{\left(56d^3e^4+175d^2e^5x\right)\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx}{56e^4}$$

$$=\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}-$$

$$=\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d$$

$$=\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}$$

$$=\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}$$

$$=\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}$$

$$=\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}$$

$$=\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}$$

Mathematica [A] time = 0.36, size = 168, normalized size = 0.88

$$d^8\left(-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)+\frac{125d^7\sqrt{d^2-e^2x^2}\sin^{-1}\left(\frac{ex}{d}\right)}{128\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{\sqrt{d^2-e^2x^2}\left(14848d^7+27195d^6ex+7424d^5e^2x^2-17710d^4e^3x^3-14592d^3e^4x^4+1960d^2e^5x^5+5760de^6x^6+1680e^7x^7\right)}{13440}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(14848*d^7 + 27195*d^6*e*x + 7424*d^5*e^2*x^2 - 17710*
d^4*e^3*x^3 - 14592*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 + 5760*d*e^6*x^6 + 1680*
e^7*x^7))/13440 + (125*d^7*Sqrt[d^2 - e^2*x^2]*ArcSin[(e*x)/d])/(128*Sqrt[1
 - (e^2*x^2)/d^2]) - d^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

IntegrateAlgebraic [A]   time = 0.58, size = 186, normalized size = 0.98

$$\frac{125d^8\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{128e}+2d^8\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{\sqrt{d^2-e^2x^2}\left(14848d^7+27195d^6ex+7424d^5e^2x^2-17710d^4e^3x^3-14592d^3e^4x^4+1960d^2e^5x^5+5760de^6x^6+1680e^7x^7\right)}{13440}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(14848*d^7 + 27195*d^6*e*x + 7424*d^5*e^2*x^2 - 17710*
d^4*e^3*x^3 - 14592*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 + 5760*d*e^6*x^6 + 1680*
e^7*x^7))/13440 + 2*d^8*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] +
 (125*d^8*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(128*e)
```

fricas [A]   time = 0.43, size = 151, normalized size = 0.79

$$-\frac{125}{64}\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\frac{1}{13440}\left(1680\,e^7x^7+5760\,de^6x^6+1960\,d^2e^5x^5-14592\,d^3e^4x^4-17710\,d^4e^3x^3+7424\,d^5e^2x^2+27195\,d^6ex+14848\,d^7\right)\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -125/64*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^8*log(-(d - sqrt(
-e^2*x^2 + d^2))/x) + 1/13440*(1680*e^7*x^7 + 5760*d*e^6*x^6 + 1960*d^2*e^5
*x^5 - 14592*d^3*e^4*x^4 - 17710*d^4*e^3*x^3 + 7424*d^5*e^2*x^2 + 27195*d^6
*e*x + 14848*d^7)*sqrt(-e^2*x^2 + d^2)
```

giac [A]   time = 0.26, size = 143, normalized size = 0.75

$$\frac{125}{128}\,d^8\arcsin\left(\frac{xe}{d}\right)\mathrm{sgn}(d)-d^8\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{1}{13440}\left(14848\,d^7+\left(27195\,d^6e+2\left(3712\,d^5e^2-\left(8855\,d^4e^3+4\left(1824\,d^3e^4-5\left(49\,d^2e^5+6\left(7\,xe^7+24\,de^6\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 125/128*d^8*arcsin(x*e/d)*sgn(d) - d^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/abs(x)) + 1/13440*(14848*d^7 + (27195*d^6*e + 2*(3712*d^5
*e^2 - (8855*d^4*e^3 + 4*(1824*d^3*e^4 - 5*(49*d^2*e^5 + 6*(7*x*e^7 + 24*d*
e^6)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A]   time = 0.01, size = 231, normalized size = 1.22

$$-\frac{d^9\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{125d^8e\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128\sqrt{e^2}}+\frac{125\sqrt{-e^2x^2+d^2}\,d^6ex}{128}+\sqrt{-e^2x^2+d^2}\,d^7+\frac{125\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4ex}{192}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}{3}+\frac{25\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2ex}{48}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex}{8}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x)
```

```
[Out] -1/8*e*x*(-e^2*x^2+d^2)^(7/2)+25/48*d^2*e*x*(-e^2*x^2+d^2)^(5/2)+125/192*e*
d^4*x*(-e^2*x^2+d^2)^(3/2)+125/128*e*d^6*x*(-e^2*x^2+d^2)^(1/2)+125/128*e*d
^8/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-3/7*d*(-e^2*x^2+d
^2)^(7/2)+1/5*d^3*(-e^2*x^2+d^2)^(5/2)+1/3*d^5*(-e^2*x^2+d^2)^(3/2)+d^7*(-e
^2*x^2+d^2)^(1/2)-d^9/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1
/2))/x)
```

maxima [A]   time = 1.00, size = 204, normalized size = 1.07

$$\frac{125}{128}\,d^8\arcsin\left(\frac{ex}{d}\right)-d^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{125}{128}\,\sqrt{-e^2x^2+d^2}\,d^6ex+\sqrt{-e^2x^2+d^2}\,d^7+\frac{125}{192}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4ex+\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5+\frac{25}{48}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2ex+\frac{1}{5}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3-\frac{1}{8}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex-\frac{3}{7}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 125/128*d^8*arcsin(e*x/d) - d^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d
/abs(x)) + 125/128*sqrt(-e^2*x^2 + d^2)*d^6*e*x + sqrt(-e^2*x^2 + d^2)*d^7
+ 125/192*(-e^2*x^2 + d^2)^(3/2)*d^4*e*x + 1/3*(-e^2*x^2 + d^2)^(3/2)*d^5 +
 25/48*(-e^2*x^2 + d^2)^(5/2)*d^2*e*x + 1/5*(-e^2*x^2 + d^2)^(5/2)*d^3 - 1/
8*(-e^2*x^2 + d^2)^(7/2)*e*x - 3/7*(-e^2*x^2 + d^2)^(7/2)*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x, x)
```

sympy [C] time = 47.72, size = 1263, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x,x)
```

```
[Out] d**7*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) + 3*d**6*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(
2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2
/d**2)/2, True)) + d**5*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-
(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) - 5*d**4*e**3*Piecewise((-I*d**4
*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*
d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2
/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e
**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**
2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**4*Piecewise((-2*d
**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15
*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True
)) + d**2*e**5*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**
4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**
2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1
+ e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) -
 d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e*
*2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sq
rt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewise((-8*d**6*sqrt(d**2 - e
**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2
*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(
e, 0)), (x**6*sqrt(d**2)/6, True)) + e**7*Piecewise((-5*I*d**8*acosh(e*x/d)
/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x*
*3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e
**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(
8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/
d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/
(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**
2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 -
 e**2*x**2/d**2)), True))
```

3.72 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=193

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2}-\frac{15}{16}d^7e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-3d^7e\tanh^{-1}\left(\frac{\sqrt{d^2}}{}\right.$$

Rubi [A] time = 0.31, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 1809, 815, 844, 217, 203, 266, 63, 208}

$$\frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2}-\frac{15}{16}d^7e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-3d^7e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

```
[Out] (3*d^5*e*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^3*e*(8*d - 5*e*x)*(d^2
 - e^2*x^2)^(3/2))/8 + (d*e*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/10 - (e*(d
^2 - e^2*x^2)^(7/2))/7 - (d*(d^2 - e^2*x^2)^(7/2))/x - (15*d^7*e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]])/16 - 3*d^7*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^2}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-3d^4e+3d^3e^2x-d^2e^3x^2\right)}{x}dx}{d^2}$$

$$= -\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{x} + \frac{\int \frac{\left(21d^4e^3-21d^3e^4x\right)\left(d^2-e^2x^2\right)^{5/2}}{x}dx}{7d^2e^2}$$

$$= \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{x} - \frac{\int \frac{\left(-126d^6e^5\right.}{}}{}$$

$$= \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2}$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(\right.$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(\right.$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(\right.$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(\right.$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(\right.$$

Mathematica [C] time = 0.53, size = 221, normalized size = 1.15

$$-\frac{d^7\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - 3d^7e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{15d^6e\sqrt{d^2-e^2x^2}\sin^{-1}\left(\frac{ex}{d}\right)}{16\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{1}{560}e\sqrt{d^2-e^2x^2}\left(2496d^6+1155d^5ex-992d^4e^2x^2-910d^3e^3x^3+96d^2e^4x^4+280de^5x^5+80e^6x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(2496*d^6 + 1155*d^5*e*x - 992*d^4*e^2*x^2 - 910*d^3
*e^3*x^3 + 96*d^2*e^4*x^4 + 280*d*e^5*x^5 + 80*e^6*x^6))/560 + (15*d^6*e*Sq
rt[d^2 - e^2*x^2]*ArcSin[(e*x)/d])/(16*Sqrt[1 - (e^2*x^2)/d^2]) - 3*d^7*e*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d] - (d^7*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[
-5/2, -1/2, 1/2, (e^2*x^2)/d^2])/(x*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.57, size = 187, normalized size = 0.97

$$-\frac{15}{16}d^7\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right) + 6d^7e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{\sqrt{d^2-e^2x^2}\left(-560d^7+2496d^6ex+525d^5e^2x^2-992d^4e^3x^3-770d^3e^4x^4+96d^2e^5x^5+280de^6x^6+80e^7x^7\right)}{560x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 2496*d^6*e*x + 525*d^5*e^2*x^2 - 992*d^4*e
^3*x^3 - 770*d^3*e^4*x^4 + 96*d^2*e^5*x^5 + 280*d*e^6*x^6 + 80*e^7*x^7))/(5
60*x) + 6*d^7*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (15*d^7
*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/16
```

fricas [A] time = 0.43, size = 167, normalized size = 0.87

$$\frac{1050\,d^7ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+1680\,d^7ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+2496\,d^7ex+\left(80\,e^7x^7+280\,de^6x^6+96\,d^2e^5x^5-770\,d^3e^4x^4-992\,d^4e^3x^3+525\,d^5e^2x^2+2496\,d^6ex-560\,d^7\right)\sqrt{-e^2x^2+d^2}}{560\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/560*(1050*d^7*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 1680*d^7*e*
x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 2496*d^7*e*x + (80*e^7*x^7 + 280*d*e
^6*x^6 + 96*d^2*e^5*x^5 - 770*d^3*e^4*x^4 - 992*d^4*e^3*x^3 + 525*d^5*e^2*x
^2 + 2496*d^6*e*x - 560*d^7)*sqrt(-e^2*x^2 + d^2))/x
```

giac [A] time = 0.24, size = 199, normalized size = 1.03

$$-\frac{15}{16}\,d^7\arcsin\left(\frac{xe}{d}\right)e\mathrm{sgn}(d)-3\,d^7e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{d^7xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^7e^{(-1)}}{2\,x}+\frac{1}{560}\left(2496\,d^6e+\left(525\,d^5e^2-2\left(496\,d^4e^3+\left(385\,d^3e^4-4\left(12\,d^2e^5+5\left(2\,xe^7+7\,de^6\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] -15/16*d^7*arcsin(x*e/d)*e*sgn(d) - 3*d^7*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^
2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/2*d^7*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*
e) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^7*e^(-1)/x + 1/560*(2496*d^6*e +
(525*d^5*e^2 - 2*(496*d^4*e^3 + (385*d^3*e^4 - 4*(12*d^2*e^5 + 5*(2*x*e^7 +
 7*d*e^6)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 243, normalized size = 1.26

$$-\frac{3d^8e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{15d^7e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}}-\frac{15\sqrt{-e^2x^2+d^2}\,d^5e^2x}{16}+3\sqrt{-e^2x^2+d^2}\,d^6e-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2x}{8}+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4e-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,e^2x}{2}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e}{5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{7}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x)
```

```
[Out] -1/7*e*(-e^2*x^2+d^2)^(7/2)-1/2*d*e^2*x*(-e^2*x^2+d^2)^(5/2)-5/8*d^3*e^2*x*
(-e^2*x^2+d^2)^(3/2)-15/16*d^5*e^2*x*(-e^2*x^2+d^2)^(1/2)-15/16*d^7*e^2/(e^
2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-d*(-e^2*x^2+d^2)^(7/2)/
x+3/5*d^2*e*(-e^2*x^2+d^2)^(5/2)+d^4*e*(-e^2*x^2+d^2)^(3/2)+3*d^6*e*(-e^2*x
^2+d^2)^(1/2)-3*d^8*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1
/2))/x)
```

maxima [A] time = 1.00, size = 217, normalized size = 1.12

$$-\frac{15}{16}\,d^7e\arcsin\left(\frac{ex}{d}\right)-3\,d^7e\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{15}{16}\sqrt{-e^2x^2+d^2}\,d^5e^2x+3\sqrt{-e^2x^2+d^2}\,d^6e-\frac{5}{8}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4e+\frac{1}{2}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2x+\frac{3}{5}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e-\frac{1}{7}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] -15/16*d^7*e*arcsin(e*x/d) - 3*d^7*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d
^2)*d/abs(x)) - 15/16*sqrt(-e^2*x^2 + d^2)*d^5*e^2*x + 3*sqrt(-e^2*x^2 + d^
2)*d^6*e - 5/8*(-e^2*x^2 + d^2)^(3/2)*d^3*e^2*x + (-e^2*x^2 + d^2)^(3/2)*d^
4*e + 1/2*(-e^2*x^2 + d^2)^(5/2)*d*e^2*x + 3/5*(-e^2*x^2 + d^2)^(5/2)*d^2*e
 - 1/7*(-e^2*x^2 + d^2)^(7/2)*e - (-e^2*x^2 + d^2)^(5/2)*d^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^2, x)
```

sympy [C] time = 19.88, size = 1057, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**2,x)
```

```
[Out] d**7*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) + 3*d**6*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acos
h(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-
I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-
d**2/(e**2*x**2) + 1), True)) + d**5*e**2*Piecewise((-I*d**2*acosh(e*x/d)/(
2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**
2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt
(1 - e**2*x**2/d**2)/2, True)) - 5*d**4*e**3*Piecewise((x**2*sqrt(d**2)/2,
Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) - 5*d**3*e**4*Pi
ecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x*
*2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqr
t(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**
3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x
**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*P
iecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 -
e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt
(d**2)/4, True)) + 3*d*e**6*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d
**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/
(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d
)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**
2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2
*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((-8*d**6*sqrt
(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**
4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**
2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

3.73 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=207

$$\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{85}{16}d^6e^2\,\mathrm{tar}$$

Rubi [A] time = 0.31, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}+\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{85}{16}d^6e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}d^6e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

```
[Out] (d^4*e^2*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^2*e^2*(4*d - 85*e*x)*(
d^2 - e^2*x^2)^(3/2))/24 + (e^2*(3*d - 85*e*x)*(d^2 - e^2*x^2)^(5/2))/30 -
(d*(d^2 - e^2*x^2)^(7/2))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(7/2))/x - (85*d^6
*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/16 - (d^6*e^2*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
```

```
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-6d^4e-d^3e^2x-2d^2e^3x^2\right)}{x^2}\,dx}{2d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x} + \frac{\int \frac{\left(d^5e^2-34d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx}{2d^4}$$

$$= \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x} - \int \frac{\left(-6d^7e^4+\right.}{}$$

$$= \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d\left(d^2-e^2x^2\right.}{2x^2}$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-8$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-8$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-8$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-8$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-8$$

Mathematica [C] time = 0.64, size = 259, normalized size = 1.25

$$-\frac{e\left(5040d^9\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)+ex\left(240\left(d^2-e^2x^2\right)^4\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)-7d\left(1104d^7+165d^6ex-1632d^5e^2x^2-295d^4e^3x^3+672d^3e^4x^4+170d^2e^5x^5+75d^7\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)-720d^6\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-144de^6x^6-40e^7x^7\right)\right)\right)}{1680dx\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

```
[Out] -1/1680*(e*(5040*d^9*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[-5/2, -1/2,
1/2, (e^2*x^2)/d^2] + e*x*(-7*d*(1104*d^7 + 165*d^6*e*x - 1632*d^5*e^2*x^2
- 295*d^4*e^3*x^3 + 672*d^3*e^4*x^4 + 170*d^2*e^5*x^5 - 144*d*e^6*x^6 - 40*
e^7*x^7 + 75*d^7*Sqrt[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d] - 720*d^6*Sqrt[d^2
 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]) + 240*(d^2 - e^2*x^2)^4*Hyperge
ometric2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2])))/(d*x*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.72, size = 189, normalized size = 0.91

$$-\frac{85}{16}d^6e\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+d^6e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{\sqrt{d^2-e^2x^2}\left(-120d^7-720d^6ex+544d^5e^2x^2-645d^4e^3x^3-448d^3e^4x^4+50d^2e^5x^5+144de^6x^6+40e^7x^7\right)}{240x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-120*d^7 - 720*d^6*e*x + 544*d^5*e^2*x^2 - 645*d^4*e^
3*x^3 - 448*d^3*e^4*x^4 + 50*d^2*e^5*x^5 + 144*d*e^6*x^6 + 40*e^7*x^7))/(24
0*x^2) + d^6*e^2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (85*d^
6*e*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/16
```

fricas [A] time = 0.43, size = 179, normalized size = 0.86

$$\frac{2550\,d^6e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+120\,d^6e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+544\,d^6e^2x^2+\left(40\,e^7x^7+144\,de^6x^6+50\,d^2e^5x^5-448\,d^3e^4x^4-645\,d^4e^3x^3+544\,d^5e^2x^2-720\,d^6ex-120\,d^7\right)\sqrt{-e^2x^2+d^2}}{240\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/240*(2550*d^6*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 120*d^6
*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 544*d^6*e^2*x^2 + (40*e^7*x^7
 + 144*d*e^6*x^6 + 50*d^2*e^5*x^5 - 448*d^3*e^4*x^4 - 645*d^4*e^3*x^3 + 544
*d^5*e^2*x^2 - 720*d^6*e*x - 120*d^7)*sqrt(-e^2*x^2 + d^2))/x^2
```

giac [A] time = 0.25, size = 262, normalized size = 1.27

$$-\frac{85}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^2\mathrm{sgn}(d)-\frac{1}{2}\,d^6e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)-\frac{1}{8}\left(\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^6e^8}{x}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^6e^6}{x^2}\right)e^{(-8)}+\frac{1}{240}\left(544\,d^5e^2-\left(645\,d^4e^3+2\left(224\,d^3e^4-\left(25\,d^2e^5+4\left(5\,xe^7+18\,de^6\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}+\frac{\left(d^6e^6+\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^6e^4}{x}\right)x^2}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -85/16*d^6*arcsin(x*e/d)*e^2*sgn(d) - 1/2*d^6*e^2*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 1/8*(12*(d*e + sqrt(-x^2*e^2 + d^2)*
e)*d^6*e^8/x + (d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*e^6/x^2)*e^(-8) + 1/240
*(544*d^5*e^2 - (645*d^4*e^3 + 2*(224*d^3*e^4 - (25*d^2*e^5 + 4*(5*x*e^7 +
18*d*e^6)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2) + 1/8*(d^6*e^6 + 12*(d*e + sqrt(
-x^2*e^2 + d^2)*e)*d^6*e^4/x)*x^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^2
```

maple [A] time = 0.02, size = 252, normalized size = 1.22

$$-\frac{d^7e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{85d^6e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}}-\frac{85\sqrt{-e^2x^2+d^2}\,d^4e^3x}{16}+\frac{\sqrt{-e^2x^2+d^2}\,d^5e^2}{2}-\frac{85\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^3x}{24}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2}{6}-\frac{17\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3x}{6}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,e^2}{10}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{x}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x)
```

```
[Out] -17/6*e^3*x*(-e^2*x^2+d^2)^(5/2)-85/24*e^3*d^2*x*(-e^2*x^2+d^2)^(3/2)-85/16
*e^3*d^4*x*(-e^2*x^2+d^2)^(1/2)-85/16*e^3*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2
```

```
)/(-e^2*x^2+d^2)^(1/2)*x)-1/2*d*(-e^2*x^2+d^2)^(7/2)/x^2+1/10*d*e^2*(-e^2*x
^2+d^2)^(5/2)+1/6*d^3*e^2*(-e^2*x^2+d^2)^(3/2)+1/2*d^5*e^2*(-e^2*x^2+d^2)^(
1/2)-1/2*d^7*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/
x)-3*e*(-e^2*x^2+d^2)^(7/2)/x
```

maxima [A] time = 1.00, size = 229, normalized size = 1.11

$$-\frac{85}{16}\,d^6e^2\arcsin\left(\frac{ex}{d}\right)-\frac{1}{2}\,d^6e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{85}{16}\,\sqrt{-e^2x^2+d^2}\,d^4e^3x+\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\,d^5e^2-\frac{85}{24}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^3x+\frac{1}{6}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2+\frac{1}{6}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3x+\frac{1}{10}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2-\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e}{x}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -85/16*d^6*e^2*arcsin(e*x/d) - 1/2*d^6*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x
^2 + d^2)*d/abs(x)) - 85/16*sqrt(-e^2*x^2 + d^2)*d^4*e^3*x + 1/2*sqrt(-e^2*
x^2 + d^2)*d^5*e^2 - 85/24*(-e^2*x^2 + d^2)^(3/2)*d^2*e^3*x + 1/6*(-e^2*x^2
 + d^2)^(3/2)*d^3*e^2 + 1/6*(-e^2*x^2 + d^2)^(5/2)*e^3*x + 1/10*(-e^2*x^2 +
 d^2)^(5/2)*d*e^2 - 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e/x - 1/2*(-e^2*x^2 + d^2)
^(7/2)*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^3, x)
```

sympy [C] time = 22.22, size = 1059, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**3,x)
```

```
[Out] d**7*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), T
rue)) + 3*d**6*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e
*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (
-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x
**2/d**2)), True)) + d**5*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) -
 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x*
*2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) +
 I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**4*e**3*Piecewise((-I*d**2
*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*
d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(
2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - 5*d**3*e**4*Piecewise((x**2
*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) +
 d**2*e**5*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt
(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*
x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(
e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sq
rt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True))
+ 3*d*e**6*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*
sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0))
, (x**4*sqrt(d**2)/4, True)) + e**7*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**
5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sq
rt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e*
```

```
*2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*as
in(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3
/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)
) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.74 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=210

$$-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}-\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{25}{8}d^5e^3\tan$$

Rubi [A] time = 0.31, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}-\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x}-\frac{25}{8}d^5e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{13}{2}d^5e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

```
[Out] -(d^3*e^3*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2])/8 - (d*e^3*(26*d + 25*e*x)*(
d^2 - e^2*x^2)^(3/2))/12 - (e^2*(50*d + 39*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*
x) - (d*(d^2 - e^2*x^2)^(7/2))/(3*x^3) - (3*e*(d^2 - e^2*x^2)^(7/2))/(2*x^2
) - (25*d^5*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (13*d^5*e^3*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
```

```
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx &= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-9d^4e-5d^3e^2x-3d^2e^3x^2\right)}{x^3}\,dx}{3d^2} \\
&= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2} + \frac{\int \frac{\left(10d^5e^2-39d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^2}\,dx}{6d^4} \\
&= -\frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{\int \frac{\left(78d^6e^3+\right.}{} \\
&= -\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d\left(d^2-e^2x^2\right.}{3x^3} \\
&= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+3}{} \\
&= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+3}{} \\
&= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+3}{} \\
&= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+3}{} \\
&= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+3}{}
\end{aligned}
$$

Mathematica [C] time = 0.27, size = 251, normalized size = 1.20

$$
-\frac{3e^3\left(d^2-e^2x^2\right)^{7/2}{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^2} - \frac{d^7\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3x^3\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{3d^5e^2\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{1}{15}e^3\left(\sqrt{d^2-e^2x^2}\left(23d^4-11d^2e^2x^2+3e^4x^4\right)-15d^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

```
[Out] (e^3*(Sqrt[d^2 - e^2*x^2]*(23*d^4 - 11*d^2*e^2*x^2 + 3*e^4*x^4) - 15*d^5*Ar
cTanh[Sqrt[d^2 - e^2*x^2]/d]))/15 - (d^7*Sqrt[d^2 - e^2*x^2]*Hypergeometric
2F1[-5/2, -3/2, -1/2, (e^2*x^2)/d^2])/(3*x^3*Sqrt[1 - (e^2*x^2)/d^2]) - (3*
d^5*e^2*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, (e^2*x^2)/d^
2])/(x*Sqrt[1 - (e^2*x^2)/d^2]) - (3*e^3*(d^2 - e^2*x^2)^(7/2)*Hypergeometr
ic2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2])/(7*d^2)
```

IntegrateAlgebraic [A] time = 0.62, size = 192, normalized size = 0.91

$$
-\frac{25}{8}d^5e^2\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right) - 13d^5e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{\sqrt{d^2-e^2x^2}\left(-40d^7-180d^6ex-80d^5e^2x^2-656d^4e^3x^3-345d^3e^4x^4+32d^2e^5x^5+90de^6x^6+24e^7x^7\right)}{120x^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^7 - 180*d^6*e*x - 80*d^5*e^2*x^2 - 656*d^4*e^3*
x^3 - 345*d^3*e^4*x^4 + 32*d^2*e^5*x^5 + 90*d*e^6*x^6 + 24*e^7*x^7))/(120*x
^3) - 13*d^5*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (25*d^
5*e^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/8
```

fricas [A] time = 0.42, size = 179, normalized size = 0.85

$$\frac{750\,d^5e^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-780\,d^5e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-656\,d^5e^3x^3+\left(24\,e^7x^7+90\,de^6x^6+32\,d^2e^5x^5-345\,d^3e^4x^4-656\,d^4e^3x^3-80\,d^5e^2x^2-180\,d^6ex-40\,d^7\right)\sqrt{-e^2x^2+d^2}}{120\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/120*(750*d^5*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 780*d^5*
e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 656*d^5*e^3*x^3 + (24*e^7*x^7
+ 90*d*e^6*x^6 + 32*d^2*e^5*x^5 - 345*d^3*e^4*x^4 - 656*d^4*e^3*x^3 - 80*d^
5*e^2*x^2 - 180*d^6*e*x - 40*d^7)*sqrt(-e^2*x^2 + d^2))/x^3
```

giac [A] time = 0.29, size = 318, normalized size = 1.51

$$-\frac{25}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^3\mathrm{sgn}(d)+\frac{13}{2}\,d^5e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(d^5e^8+\frac{9\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^5e^6}{x}+\frac{9\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^5e^4}{x^2}\right)x^3e}{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3}-\frac{1}{24}\left(\frac{9\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^5e^{16}}{x}+\frac{9\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^5e^{14}}{x^2}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^5e^{12}}{x^3}\right)e^{(-15)}-\frac{1}{120}\left(656\,d^4e^3+\left(345\,d^3e^4-2\left(16\,d^2e^5+3\left(4\,xe^7+15\,de^6\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] -25/8*d^5*arcsin(x*e/d)*e^3*sgn(d) + 13/2*d^5*e^3*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/24*(d^5*e^8 + 9*(d*e + sqrt(-x^2*e
^2 + d^2)*e)*d^5*e^6/x + 9*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*e^4/x^2)*x^
3*e/(d*e + sqrt(-x^2*e^2 + d^2)*e)^3 - 1/24*(9*(d*e + sqrt(-x^2*e^2 + d^2)*
e)*d^5*e^16/x + 9*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*e^14/x^2 + (d*e + sq
rt(-x^2*e^2 + d^2)*e)^3*d^5*e^12/x^3)*e^(-15) - 1/120*(656*d^4*e^3 + (345*d
^3*e^4 - 2*(16*d^2*e^5 + 3*(4*x*e^7 + 15*d*e^6)*x)*x)*x)*sqrt(-x^2*e^2 + d^
2)
```

maple [A] time = 0.02, size = 277, normalized size = 1.32

$$\frac{13d^6e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{25d^5e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}-\frac{25\sqrt{-e^2x^2+d^2}\,d^3e^4x}{8}-\frac{13\sqrt{-e^2x^2+d^2}\,d^4e^3}{2}-\frac{25\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d\,e^4x}{12}-\frac{13\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^3}{6}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4x}{3d}-\frac{13\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{10}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{3dx}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{2x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x)
```

```
[Out] -1/3*d*(-e^2*x^2+d^2)^(7/2)/x^3-5/3/d*e^2/x*(-e^2*x^2+d^2)^(7/2)-5/3/d*e^4*
x*(-e^2*x^2+d^2)^(5/2)-25/12*d*e^4*x*(-e^2*x^2+d^2)^(3/2)-25/8*d^3*e^4*x*(-
e^2*x^2+d^2)^(1/2)-25/8*d^5*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^
2)^(1/2)*x)-3/2*e*(-e^2*x^2+d^2)^(7/2)/x^2-13/10*e^3*(-e^2*x^2+d^2)^(5/2)-1
3/6*d^2*e^3*(-e^2*x^2+d^2)^(3/2)-13/2*d^4*e^3*(-e^2*x^2+d^2)^(1/2)+13/2*d^6
*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 1.01, size = 226, normalized size = 1.08

$$-\frac{25}{8}\,d^5e^3\arcsin\left(\frac{ex}{d}\right)+\frac{13}{2}\,d^5e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{25}{8}\,\sqrt{-e^2x^2+d^2}\,d^3e^4x-\frac{13}{2}\,\sqrt{-e^2x^2+d^2}\,d^4e^3-\frac{25}{12}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^4x-\frac{13}{6}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^3-\frac{13}{10}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3-\frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2}{3\,x}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{2\,x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] -25/8*d^5*e^3*arcsin(e*x/d) + 13/2*d^5*e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x
^2 + d^2)*d/abs(x)) - 25/8*sqrt(-e^2*x^2 + d^2)*d^3*e^4*x - 13/2*sqrt(-e^2*
x^2 + d^2)*d^4*e^3 - 25/12*(-e^2*x^2 + d^2)^(3/2)*d*e^4*x - 13/6*(-e^2*x^2
+ d^2)^(3/2)*d^2*e^3 - 13/10*(-e^2*x^2 + d^2)^(5/2)*e^3 - 5/3*(-e^2*x^2 + d
^2)^(5/2)*d*e^2/x - 3/2*(-e^2*x^2 + d^2)^(7/2)*e/x^2 - 1/3*(-e^2*x^2 + d^2)
^(7/2)*d/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2 - e^2 x^2\right)^{5/2} (d + e x)^3}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^4, x)
```

sympy [C] time = 15.74, size = 911, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**4,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) +
 3*d**6*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*s
qrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2
)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*
d), True)) + d**5*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*a
cosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 -
e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2
*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/
(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/
(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**3*e**4*Piecewise(
(-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*
x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(
e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + d**2*e**5*Piecewise
((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), Tr
ue)) + 3*d*e**6*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2
*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*
e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*
asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/
(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), Tr
ue)) + e**7*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2
*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)
), (x**4*sqrt(d**2)/4, True))
```

3.75 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=209

$$-\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3}-\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2}+\frac{15de^3(2d-ex)\left(d\right.}{8x}$$

Rubi [A] time = 0.32, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1807, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2}+\frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x}-\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}+\frac{45}{8}d^4e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{45}{8}d^4e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

```
[Out] (-45*d^2*e^4*(d - e*x)*Sqrt[d^2 - e^2*x^2])/8 + (15*d*e^3*(2*d - e*x)*(d^2
- e^2*x^2)^(3/2))/(8*x) - (3*e^2*(3*d + 2*e*x)*(d^2 - e^2*x^2)^(5/2))/(8*x^
2) - (d*(d^2 - e^2*x^2)^(7/2))/(4*x^4) - (e*(d^2 - e^2*x^2)^(7/2))/x^3 + (4
5*d^4*e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (45*d^4*e^4*ArcTanh[Sqrt[d
^2 - e^2*x^2]/d])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
```

```
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-12d^4e-9d^3e^2x-4d^2e^3x^2\right)}{x^4}dx}{4d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} + \frac{\int \frac{\left(27d^5e^2-36d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^3}dx}{12d^4}$$

$$= -\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} - 5\int \frac{(144d\cdots}{\cdots}$$

$$= \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}\cdots$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(\cdots\right.}{8x^2}\cdots$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(\cdots\right.}{8x^2}\cdots$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(\cdots\right.}{8x^2}\cdots$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(\cdots\right.}{8x^2}\cdots$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(\cdots\right.}{8x^2}\cdots$$

Mathematica [C] time = 0.10, size = 195, normalized size = 0.93

$$\frac{e\sqrt{d^2-e^2x^2}\left(3\left(e^3x^2-d^2e\right)^3\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+\left(e^3x^2-d^2e\right)^3\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)-\frac{7d^9\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{7d^7e^2\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*((-7*d^9*Hypergeometric2F1[-5/2, -3/2, -1/2, (e^2*x^
2)/d^2])/(x^3*Sqrt[1 - (e^2*x^2)/d^2]) - (7*d^7*e^2*Hypergeometric2F1[-5/2,
 -1/2, 1/2, (e^2*x^2)/d^2])/(x*Sqrt[1 - (e^2*x^2)/d^2]) + 3*(-(d^2*e) + e^3
*x^2)^3*Hypergeometric2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2] + (-(d^2*e) + e^3
*x^2)^3*Hypergeometric2F1[3, 7/2, 9/2, 1 - (e^2*x^2)/d^2]))/(7*d^3)
```

IntegrateAlgebraic [A] time = 0.68, size = 194, normalized size = 0.93

$$-\frac{45}{4}d^4e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{45}{8}d^4e^3\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+\frac{\sqrt{d^2-e^2x^2}\left(-2d^7-8d^6ex-3d^5e^2x^2+48d^4e^3x^3-48d^3e^4x^4+3d^2e^5x^5+8de^6x^6+2e^7x^7\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^7 - 8*d^6*e*x - 3*d^5*e^2*x^2 + 48*d^4*e^3*x^3 -
 48*d^3*e^4*x^4 + 3*d^2*e^5*x^5 + 8*d*e^6*x^6 + 2*e^7*x^7))/(8*x^4) - (45*d
^4*e^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/4 + (45*d^4*e^3*S
qrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/8
```

fricas [A] time = 0.42, size = 180, normalized size = 0.86

$$-\frac{90\,d^4e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+45\,d^4e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+48\,d^4e^4x^4-\left(2\,e^7x^7+8\,de^6x^6+3\,d^2e^5x^5-48\,d^3e^4x^4+48\,d^4e^3x^3-3\,d^5e^2x^2-8\,d^6ex-2\,d^7\right)\sqrt{-e^2x^2+d^2}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(90*d^4*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 45*d^4*e^4
*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 48*d^4*e^4*x^4 - (2*e^7*x^7 + 8*d
*e^6*x^6 + 3*d^2*e^5*x^5 - 48*d^3*e^4*x^4 + 48*d^4*e^3*x^3 - 3*d^5*e^2*x^2
- 8*d^6*e*x - 2*d^7)*sqrt(-e^2*x^2 + d^2))/x^4
```

giac [B] time = 0.27, size = 374, normalized size = 1.79

$$\frac{45}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^4\mathrm{sgn}\left(d\right)+\frac{45}{8}\,d^4e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(d^4e^{10}+\frac{8\left(de+\sqrt{-x^2e^2+d^2}e\right)d^4e^8}{x}+\frac{8\left(de+\sqrt{-x^2e^2+d^2}e\right)^2d^4e^6}{x^2}-\frac{184\left(de+\sqrt{-x^2e^2+d^2}e\right)^3d^4e^4}{x^3}\right)x^4e^2}{64\left(de+\sqrt{-x^2e^2+d^2}e\right)^4}+\frac{1}{64}\left(\frac{184\left(de+\sqrt{-x^2e^2+d^2}e\right)d^4e^{26}}{x}-\frac{8\left(de+\sqrt{-x^2e^2+d^2}e\right)^2d^4e^{24}}{x^2}-\frac{8\left(de+\sqrt{-x^2e^2+d^2}e\right)^3d^4e^{22}}{x^3}-\frac{\left(de+\sqrt{-x^2e^2+d^2}e\right)^4d^4e^{20}}{x^4}\right)e^{(-24)}-\frac{1}{8}\left(48\,d^3e^4-\left(3\,d^2e^5+2\left(xe^7+4\,de^6\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 45/8*d^4*arcsin(x*e/d)*e^4*sgn(d) + 45/8*d^4*e^4*log(1/2*abs(-2*d*e - 2*sqr
t(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/64*(d^4*e^10 + 8*(d*e + sqrt(-x^2*e
^2 + d^2)*e)*d^4*e^8/x + 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^6/x^2 - 1
84*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*e^4/x^3)*x^4*e^2/(d*e + sqrt(-x^2*e
^2 + d^2)*e)^4 + 1/64*(184*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^26/x - 8*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^24/x^2 - 8*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^3*d^4*e^22/x^3 - (d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4*e^20/x^4)*e^(-24)
 - 1/8*(48*d^3*e^4 - (3*d^2*e^5 + 2*(x*e^7 + 4*d*e^6)*x)*x)*sqrt(-x^2*e^2 +
 d^2)
```

maple [A] time = 0.02, size = 302, normalized size = 1.44

$$\frac{45d^5e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}+\frac{45d^4e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{45\sqrt{-e^2x^2+d^2}\,d^2e^5x}{8}-\frac{45\sqrt{-e^2x^2+d^2}\,d^3e^4}{8}+\frac{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5x}{4}-\frac{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^4}{8}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5x}{d^2}-\frac{9\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{8d}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{d^2x}-\frac{9\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{8dx^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x)
```

```
[Out] -e*(-e^2*x^2+d^2)^(7/2)/x^3+3/d^2*e^3/x*(-e^2*x^2+d^2)^(7/2)+3/d^2*e^5*x*(-
e^2*x^2+d^2)^(5/2)+15/4*e^5*x*(-e^2*x^2+d^2)^(3/2)+45/8*d^2*e^5*x*(-e^2*x^2
+d^2)^(1/2)+45/8*d^4*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2
)*x)-9/8/d*e^2/x^2*(-e^2*x^2+d^2)^(7/2)-9/8/d*e^4*(-e^2*x^2+d^2)^(5/2)-15/8
*d*e^4*(-e^2*x^2+d^2)^(3/2)-45/8*d^3*e^4*(-e^2*x^2+d^2)^(1/2)+45/8*d^5*e^4/
(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/4*d*(-e^2*x^
2+d^2)^(7/2)/x^4
```

maxima [A] time = 0.99, size = 250, normalized size = 1.20

$$\frac{45}{8}\,d^4e^4\arcsin\left(\frac{ex}{d}\right)+\frac{45}{8}\,d^4e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{45}{8}\,\sqrt{-e^2x^2+d^2}\,d^2e^5x-\frac{45}{8}\,\sqrt{-e^2x^2+d^2}\,d^3e^4+\frac{15}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5x-\frac{15}{8}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^4-\frac{9\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{8\,d}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{x}-\frac{9\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{8\,dx^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 45/8*d^4*e^4*arcsin(e*x/d) + 45/8*d^4*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^
2 + d^2)*d/abs(x)) + 45/8*sqrt(-e^2*x^2 + d^2)*d^2*e^5*x - 45/8*sqrt(-e^2*x
^2 + d^2)*d^3*e^4 + 15/4*(-e^2*x^2 + d^2)^(3/2)*e^5*x - 15/8*(-e^2*x^2 + d^
2)^(3/2)*d*e^4 - 9/8*(-e^2*x^2 + d^2)^(5/2)*e^4/d + 3*(-e^2*x^2 + d^2)^(5/2
)*e^3/x - 9/8*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^2) - (-e^2*x^2 + d^2)^(7/2)*e
/x^3 - 1/4*(-e^2*x^2 + d^2)^(7/2)*d/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}(d+ex)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^5, x)
```

sympy [C] time = 20.10, size = 1028, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**5,x)
```

```
[Out] d**7*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) +
e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e
*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/
(3*d**2), True)) + d**5*e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**
2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d),
Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**
2*asin(d/(e*x))/(2*d), True)) - 5*d**4*e**3*Piecewise((I*d/(x*sqrt(-1 + e**
2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)),
Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d)
+ e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**4*Piecewise((d**2
/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*
x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2
) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) + d
**2*e**5*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x*
*2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2
) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) +
3*d*e**6*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**
(3/2)/(3*e**2), True)) + e**7*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*
d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x*
*2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2
) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2
)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x
**2/d**2)), True))
```

3.76 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=216

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4}-\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3}+\frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x}+\frac{de^3(25d-52e}{2}$$

Rubi [A] time = 0.31, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 813, 844, 217, 203, 266, 63, 208}

$$\frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x}+\frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2}-\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}+\frac{13}{2}d^3e^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{25}{8}d^3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

```
[Out] (d^2*e^4*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x) + (d*e^3*(25*d - 52*e*x
)*(d^2 - e^2*x^2)^(3/2))/(24*x^2) - (e^2*(52*d + 25*e*x)*(d^2 - e^2*x^2)^(5
/2))/(60*x^3) - (d*(d^2 - e^2*x^2)^(7/2))/(5*x^5) - (3*e*(d^2 - e^2*x^2)^(7
/2))/(4*x^4) + (13*d^3*e^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (25*d^3*e
^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
```

```
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^6}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-15d^4e-13d^3e^2x-5d^2e^3x^2\right)}{x^5}\,dx}{5d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} + \frac{\int \frac{\left(52d^5e^2-25d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx}{20d^4}$$

$$= -\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \int \frac{(150d}{}$$

$$= \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right.}{5x^5}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25e}{}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25e}{}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25e}{}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25e}{}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25e}{}$$

Mathematica [C] time = 0.09, size = 199, normalized size = 0.92

$$\frac{\sqrt{d^2-e^2x^2}\left(5e^5\left(e^2x^2-d^2\right)^3\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+15e^5\left(e^2x^2-d^2\right)^3\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)-\frac{7d^{11}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{x^5\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{35d^9e^2\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{35d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((-7*d^11*Hypergeometric2F1[-5/2, -5/2, -3/2, (e^2*x^2
)/d^2])/(x^5*Sqrt[1 - (e^2*x^2)/d^2]) - (35*d^9*e^2*Hypergeometric2F1[-5/2,
 -3/2, -1/2, (e^2*x^2)/d^2])/(x^3*Sqrt[1 - (e^2*x^2)/d^2]) + 5*e^5*(-d^2 +
e^2*x^2)^3*Hypergeometric2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2] + 15*e^5*(-d^2
 + e^2*x^2)^3*Hypergeometric2F1[3, 7/2, 9/2, 1 - (e^2*x^2)/d^2]))/(35*d^4)
```

IntegrateAlgebraic [A] time = 0.75, size = 194, normalized size = 0.90

$$\frac{25}{4}d^3e^5\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{13}{2}d^3\sqrt{-e^2}\,e^4\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+\frac{\sqrt{d^2-e^2x^2}\left(-24d^7-90d^6ex-32d^5e^2x^2+345d^4e^3x^3+656d^3e^4x^4+80d^2e^5x^5+180de^6x^6+40e^7x^7\right)}{120x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^7 - 90*d^6*e*x - 32*d^5*e^2*x^2 + 345*d^4*e^3*x
^3 + 656*d^3*e^4*x^4 + 80*d^2*e^5*x^5 + 180*d*e^6*x^6 + 40*e^7*x^7))/(120*x
^5) + (25*d^3*e^5*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/4 + (1
3*d^3*e^4*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

fricas [A] time = 0.43, size = 180, normalized size = 0.83

$$-\frac{1560\,d^3e^5x^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-375\,d^3e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-80\,d^3e^5x^5-\left(40\,e^7x^7+180\,de^6x^6+80\,d^2e^5x^5+656\,d^3e^4x^4+345\,d^4e^3x^3-32\,d^5e^2x^2-90\,d^6ex-24\,d^7\right)\sqrt{-e^2x^2+d^2}}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/120*(1560*d^3*e^5*x^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 375*d^
3*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 80*d^3*e^5*x^5 - (40*e^7*x^7
 + 180*d*e^6*x^6 + 80*d^2*e^5*x^5 + 656*d^3*e^4*x^4 + 345*d^4*e^3*x^3 - 32*
d^5*e^2*x^2 - 90*d^6*e*x - 24*d^7)*sqrt(-e^2*x^2 + d^2))/x^5
```

giac [B] time = 0.28, size = 430, normalized size = 1.99

$$\frac{13}{2}d^3\arcsin\left(\frac{xe}{d}\right)e^5\mathrm{sgn}(d)-\frac{25}{8}d^3e^5\log\left(\frac{\left|-2de-2\sqrt{-x^2e^2+d^2}\,e\right|e^{-2}}{2|x|}\right)+\frac{\left(6d^3e^{12}+\frac{45\left(de+\sqrt{-x^2e^2+d^2}e\right)d^3e^{10}}{x}+\frac{50\left(de+\sqrt{-x^2e^2+d^2}e\right)^2d^3e^8}{x^2}-\frac{600\left(de+\sqrt{-x^2e^2+d^2}e\right)^3d^3e^6}{x^3}-\frac{2580\left(de+\sqrt{-x^2e^2+d^2}e\right)^4d^3e^4}{x^4}\right)x^5e^3}{960\left(de+\sqrt{-x^2e^2+d^2}e\right)^5}+\frac{1}{960}\left(\frac{2580\left(de+\sqrt{-x^2e^2+d^2}e\right)d^3e^{38}}{x}+\frac{600\left(de+\sqrt{-x^2e^2+d^2}e\right)^2d^3e^{36}}{x^2}-\frac{50\left(de+\sqrt{-x^2e^2+d^2}e\right)^3d^3e^{34}}{x^3}-\frac{45\left(de+\sqrt{-x^2e^2+d^2}e\right)^4d^3e^{32}}{x^4}-\frac{6\left(de+\sqrt{-x^2e^2+d^2}e\right)^5d^3e^{30}}{x^5}\right)e^{(-35)}+\frac{1}{6}\left(4d^2e^5+\left(2xe^7+9de^6\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 13/2*d^3*arcsin(x*e/d)*e^5*sgn(d) - 25/8*d^3*e^5*log(1/2*abs(-2*d*e - 2*sqr
t(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/960*(6*d^3*e^12 + 45*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*d^3*e^10/x + 50*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*e^8/x
^2 - 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*e^6/x^3 - 2580*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^4*d^3*e^4/x^4)*x^5*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e)^5 +
1/960*(2580*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*e^38/x + 600*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^2*d^3*e^36/x^2 - 50*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*e^
34/x^3 - 45*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^3*e^32/x^4 - 6*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^5*d^3*e^30/x^5)*e^(-35) + 1/6*(4*d^2*e^5 + (2*x*e^7 + 9*d
*e^6)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.03, size = 327, normalized size = 1.51

$$-\frac{25d^4e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}+\frac{13d^3e^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{13\sqrt{-e^2x^2+d^2}\,d\,e^6x}{2}+\frac{25\sqrt{-e^2x^2+d^2}\,d^2e^5}{8}+\frac{13\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6x}{3d}+\frac{25\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}{24}+\frac{52\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6x}{15d^3}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}{8d^2}+\frac{52\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{15d^3x}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{8d^2x^2}-\frac{13\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{15d\,x^3}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{4x^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x)
```

```
[Out] -3/4*e*(-e^2*x^2+d^2)^(7/2)/x^4+5/8/d^2*e^3/x^2*(-e^2*x^2+d^2)^(7/2)+5/8/d^
2*e^5*(-e^2*x^2+d^2)^(5/2)+25/24*e^5*(-e^2*x^2+d^2)^(3/2)+25/8*d^2*e^5*(-e^
2*x^2+d^2)^(1/2)-25/8*d^4*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2
+d^2)^(1/2))/x)-13/15/d*e^2/x^3*(-e^2*x^2+d^2)^(7/2)+52/15/d^3*e^4/x*(-e^2*
x^2+d^2)^(7/2)+52/15/d^3*e^6*x*(-e^2*x^2+d^2)^(5/2)+13/3/d*e^6*x*(-e^2*x^2+
d^2)^(3/2)+13/2*d*e^6*x*(-e^2*x^2+d^2)^(1/2)+13/2*d^3*e^6/(e^2)^(1/2)*arcta
n((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/5*d*(-e^2*x^2+d^2)^(7/2)/x^5
```

maxima [A] time = 0.99, size = 278, normalized size = 1.29

$$\frac{13}{2}d^3e^5\arcsin\left(\frac{ex}{d}\right)-\frac{25}{8}d^3e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{13}{2}\sqrt{-e^2x^2+d^2}\,de^6x+\frac{25}{8}\sqrt{-e^2x^2+d^2}\,d^2e^5+\frac{13\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6x}{3\,d}+\frac{25}{24}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5+\frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}{8\,d^2}+\frac{52\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{15\,dx}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{8\,d^2x^2}-\frac{13\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{15\,dx^3}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{4\,x^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 13/2*d^3*e^5*arcsin(e*x/d) - 25/8*d^3*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^
2 + d^2)*d/abs(x)) + 13/2*sqrt(-e^2*x^2 + d^2)*d*e^6*x + 25/8*sqrt(-e^2*x^2
 + d^2)*d^2*e^5 + 13/3*(-e^2*x^2 + d^2)^(3/2)*e^6*x/d + 25/24*(-e^2*x^2 + d
^2)^(3/2)*e^5 + 5/8*(-e^2*x^2 + d^2)^(5/2)*e^5/d^2 + 52/15*(-e^2*x^2 + d^2)
^(5/2)*e^4/(d*x) + 5/8*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^2) - 13/15*(-e^2*x
^2 + d^2)^(7/2)*e^2/(d*x^3) - 3/4*(-e^2*x^2 + d^2)^(7/2)*e/x^4 - 1/5*(-e^2*
x^2 + d^2)^(7/2)*d/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^6,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^6, x)
```

sympy [C] time = 20.70, size = 1178, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**6,x)
```

```
[Out] d**7*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) + 3*d**6*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e*
*2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*s
qrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x
**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*
```

```
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)
) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**5*e**2*Piecewise((-e*sqrt(d*
*2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - 5*d**4*e**3*Piecewise((-d
**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) -
1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d*
*2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) - 5*d**3*e**
4*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*
x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 -
e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), Tru
e)) + d**2*e**5*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(
d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*
d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
*2/(e**2*x**2) + 1), True)) + 3*d*e**6*Piecewise((-I*d**2*acosh(e*x/d)/(2*e
) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x
**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1
- e**2*x**2/d**2)/2, True)) + e**7*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0
)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

3.77 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=214

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}-\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4}-\frac{1}{2}d^2e^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{85}{16}d^2e^6\tanh^{-1}$$

Rubi [A] time = 0.31, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1807, 813, 811, 844, 217, 203, 266, 63, 208}

$$-\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3}-\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}-\frac{1}{2}d^2e^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{85}{16}d^2e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] -(d*e^5*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/(16*x) + (d*e^3*(8*d + 85*e*x)*
(d^2 - e^2*x^2)^(3/2))/(48*x^3) - (e^2*(85*d + 12*e*x)*(d^2 - e^2*x^2)^(5/2
))/(120*x^4) - (d*(d^2 - e^2*x^2)^(7/2))/(6*x^6) - (3*e*(d^2 - e^2*x^2)^(7/
2))/(5*x^5) - (d^2*e^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (85*d^2*e^6*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d])/16
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
```

```
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^7} dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-18d^4e-17d^3e^2x-6d^2e^3x^2\right)}{x^6} dx}{6d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5} + \frac{\int \frac{\left(85d^5e^2-6d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^5} dx}{30d^4}$$

$$= -\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \ldots$$

$$= \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{d\left(d^2-e^2x^2\right)^{7}\ldots}{6x^6}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\ldots}{12\ldots}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\ldots}{12\ldots}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\ldots}{12\ldots}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\ldots}{12\ldots}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\ldots}{12\ldots}$$

Mathematica [C] time = 0.22, size = 286, normalized size = 1.34

$$-\frac{3d^6e\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{5x^5\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{3e^6\left(d^2-e^2x^2\right)^{7/2}{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^5} - \frac{d^4e^3\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3x^3\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{-8d^9+34d^7e^2x^2-59d^5e^4x^4+33d^3e^6x^6+15d^3e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{48x^6\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] (-8*d^9 + 34*d^7*e^2*x^2 - 59*d^5*e^4*x^4 + 33*d^3*e^6*x^6 + 15*d^3*e^6*x^6
*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]])/(48*x^6*Sqrt[d^2
 - e^2*x^2]) - (3*d^6*e*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, -5/2, -
3/2, (e^2*x^2)/d^2])/(5*x^5*Sqrt[1 - (e^2*x^2)/d^2]) - (d^4*e^3*Sqrt[d^2 -
e^2*x^2]*Hypergeometric2F1[-5/2, -3/2, -1/2, (e^2*x^2)/d^2])/(3*x^3*Sqrt[1
- (e^2*x^2)/d^2]) - (3*e^6*(d^2 - e^2*x^2)^(7/2)*Hypergeometric2F1[3, 7/2,
9/2, 1 - (e^2*x^2)/d^2])/(7*d^5)
```

IntegrateAlgebraic [A] time = 0.80, size = 194, normalized size = 0.91

$$\frac{85}{8}d^2e^6\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{1}{2}d^2\sqrt{-e^2}\,e^5\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right) + \frac{\sqrt{d^2-e^2x^2}\left(-40d^7-144d^6ex-50d^5e^2x^2+448d^4e^3x^3+645d^3e^4x^4-544d^2e^5x^5+720de^6x^6+120e^7x^7\right)}{240x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^7 - 144*d^6*e*x - 50*d^5*e^2*x^2 + 448*d^4*e^3*
x^3 + 645*d^3*e^4*x^4 - 544*d^2*e^5*x^5 + 720*d*e^6*x^6 + 120*e^7*x^7))/(24
0*x^6) + (85*d^2*e^6*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/8 -
 (d^2*e^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

fricas [A] time = 0.44, size = 179, normalized size = 0.84

$$\frac{240\,d^2e^6x^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+1275\,d^2e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+720\,d^2e^6x^6+\left(120\,e^7x^7+720\,de^6x^6-544\,d^2e^5x^5+645\,d^3e^4x^4+448\,d^4e^3x^3-50\,d^5e^2x^2-144\,d^6ex-40\,d^7\right)\sqrt{-e^2x^2+d^2}}{240\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/240*(240*d^2*e^6*x^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 1275*d^2
*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 720*d^2*e^6*x^6 + (120*e^7*x^
7 + 720*d*e^6*x^6 - 544*d^2*e^5*x^5 + 645*d^3*e^4*x^4 + 448*d^4*e^3*x^3 - 5
0*d^5*e^2*x^2 - 144*d^6*e*x - 40*d^7)*sqrt(-e^2*x^2 + d^2))/x^6
```

giac [B] time = 0.31, size = 485, normalized size = 2.27

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^6\mathrm{sgn}(d)-\frac{85}{16}\,d^2e^6\log\left(\frac{\left|-2\,de-2\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(5\,d^2e^{14}+\frac{36\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^{12}}{x}+\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^{10}}{x^2}-\frac{340\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2e^8}{x^3}-\frac{1215\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^2e^6}{x^4}+\frac{1800\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^2e^4}{x^5}\right)x^6e^4}{1920\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6}-\frac{1}{1920}\left(\frac{1800\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^{52}}{x}-\frac{1215\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^{50}}{x^2}-\frac{340\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2e^{48}}{x^3}+\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^2e^{46}}{x^4}+\frac{36\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^2e^{44}}{x^5}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6d^2e^{42}}{x^6}\right)e^{(-48)}+\frac{1}{2}\sqrt{-x^2e^2+d^2}\left(xe^7+6\,de^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e^6*sgn(d) - 85/16*d^2*e^6*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/1920*(5*d^2*e^14 + 36*(d*e + sqrt(
-x^2*e^2 + d^2)*e)*d^2*e^12/x + 45*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^1
0/x^2 - 340*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*e^8/x^3 - 1215*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^4*d^2*e^6/x^4 + 1800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d
^2*e^4/x^5)*x^6*e^4/(d*e + sqrt(-x^2*e^2 + d^2)*e)^6 - 1/1920*(1800*(d*e +
sqrt(-x^2*e^2 + d^2)*e)*d^2*e^52/x - 1215*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*
d^2*e^50/x^2 - 340*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*e^48/x^3 + 45*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^4*d^2*e^46/x^4 + 36*(d*e + sqrt(-x^2*e^2 + d^2)*e
)^5*d^2*e^44/x^5 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^2*e^42/x^6)*e^(-48)
 + 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^7 + 6*d*e^6)
```

maple [A] time = 0.03, size = 352, normalized size = 1.64

$$-\frac{85d^3e^6\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16\sqrt{d^2}}-\frac{d^2e^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{\sqrt{-e^2x^2+d^2}\,e^7x}{2}+\frac{85\sqrt{-e^2x^2+d^2}\,d\,e^6}{16}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7x}{3d^2}+\frac{85\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6}{48d}-\frac{4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^7x}{15d^4}+\frac{17\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}{16d^3}-\frac{4\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^5}{15d^4x}+\frac{17\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{16d^3x^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{15d^2x^3}-\frac{17\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{24d\,x^4}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{5x^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x)
```

```
[Out] 1/15*e^3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/15*e^5/d^4/x*(-e^2*x^2+d^2)^(7/2)-4
/15*e^7/d^4*x*(-e^2*x^2+d^2)^(5/2)-1/3*e^7/d^2*x*(-e^2*x^2+d^2)^(3/2)-1/2*e
^7*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-17/24/d*e^2/x
^4*(-e^2*x^2+d^2)^(7/2)+17/16/d^3*e^4/x^2*(-e^2*x^2+d^2)^(7/2)-85/16*d^3*e^
6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-3/5*e*(-e^2*
x^2+d^2)^(7/2)/x^5-1/6*d*(-e^2*x^2+d^2)^(7/2)/x^6-1/2*e^7*x*(-e^2*x^2+d^2)^
(1/2)+17/16/d^3*e^6*(-e^2*x^2+d^2)^(5/2)+85/48/d*e^6*(-e^2*x^2+d^2)^(3/2)+8
5/16*d*e^6*(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 1.00, size = 303, normalized size = 1.42

$$-\frac{1}{2}\,d^2e^6\arcsin\left(\frac{ex}{d}\right)-\frac{85}{16}\,d^2e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{1}{2}\sqrt{-e^2x^2+d^2}\,e^7x+\frac{85}{16}\sqrt{-e^2x^2+d^2}\,de^6-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7x}{3\,d^2}+\frac{85\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6}{48\,d}+\frac{17\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}{16\,d^3}-\frac{4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}{15\,d^2x}+\frac{17\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{16\,d^3x^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{15\,d^2x^3}-\frac{17\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{24\,dx^4}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{5\,x^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*d^2*e^6*arcsin(e*x/d) - 85/16*d^2*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x
^2 + d^2)*d/abs(x)) - 1/2*sqrt(-e^2*x^2 + d^2)*e^7*x + 85/16*sqrt(-e^2*x^2
+ d^2)*d*e^6 - 1/3*(-e^2*x^2 + d^2)^(3/2)*e^7*x/d^2 + 85/48*(-e^2*x^2 + d^2
)^(3/2)*e^6/d + 17/16*(-e^2*x^2 + d^2)^(5/2)*e^6/d^3 - 4/15*(-e^2*x^2 + d^2
```

```
)^(5/2)*e^5/(d^2*x) + 17/16*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3*x^2) + 1/15*(-e
^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^3) - 17/24*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^4
) - 3/5*(-e^2*x^2 + d^2)^(7/2)*e/x^5 - 1/6*(-e^2*x^2 + d^2)^(7/2)*d/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}(d+ex)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^7,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^7, x)
```

sympy [C] time = 21.71, size = 1397, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**7,x)
```

```
[Out] d**7*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + 3*d**6*e*Piecewise((3*I*d**3*
sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*
sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*s
qrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d*
*2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) -
 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*
e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e*
*4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) +
 d**5*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*
x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1
)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e
*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) +
 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))
/(8*d**3), True)) - 5*d**4*e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3
*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x*
*2) + 1)/(3*d**2), True)) - 5*d**3*e**4*Piecewise((-d**2/(2*e*x**3*sqrt(d**
2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*
x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2
*x) - I*e**2*asin(d/(e*x))/(2*d), True)) + d**2*e**5*Piecewise((I*d/(x*sqrt
(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2
/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asi
n(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewise
((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/
(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**
2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True
)) + e**7*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x
**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True))
```

3.78 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=206

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}-\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}-3de^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{15}{16}de^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.31, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 811, 813, 844, 217, 203, 266, 63, 208}

$$-\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-3de^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{15}{16}de^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

```
[Out] (-3*e^6*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/(16*x) + (e^4*(16*d + 5*e*x)*(d
^2 - e^2*x^2)^(3/2))/(16*x^3) - (e^2*(24*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/
(40*x^5) - (d*(d^2 - e^2*x^2)^(7/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(7/2))/(2
*x^6) - 3*d*e^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (15*d*e^7*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/16
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
```

```
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-21d^4e-21d^3e^2x-7d^2e^3x^2\right)}{x^7}dx}{7d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6} + \frac{\int \frac{\left(126d^5e^2+21d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^6}dx}{42d^4}$$

$$= -\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6} - \frac{\int \frac{\left(1008d^7e^4+2\right.}{}}{}$$

$$= \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e}{}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-\right.}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-\right.}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-\right.}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-\right.}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-\right.}{40x^5}$$

Mathematica [C] time = 0.15, size = 247, normalized size = 1.20

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e^7\left(d^2-e^2x^2\right)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^6} - \frac{3d^5e^2\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{5x^5\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{-8d^8e+34d^6e^3x^2-59d^4e^5x^4+33d^2e^7x^6+15d^2e^7x^6\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{16x^6\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

```
[Out] -1/7*(d*(d^2 - e^2*x^2)^(7/2))/x^7 + (-8*d^8*e + 34*d^6*e^3*x^2 - 59*d^4*e^
5*x^4 + 33*d^2*e^7*x^6 + 15*d^2*e^7*x^6*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqr
t[1 - (e^2*x^2)/d^2]])/(16*x^6*Sqrt[d^2 - e^2*x^2]) - (3*d^5*e^2*Sqrt[d^2 -
 e^2*x^2]*Hypergeometric2F1[-5/2, -5/2, -3/2, (e^2*x^2)/d^2])/(5*x^5*Sqrt[1
 - (e^2*x^2)/d^2]) - (e^7*(d^2 - e^2*x^2)^(7/2)*Hypergeometric2F1[3, 7/2, 9
/2, 1 - (e^2*x^2)/d^2])/(7*d^6)
```

IntegrateAlgebraic [A] time = 0.80, size = 188, normalized size = 0.91

$$\frac{15}{8}de^7\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right) - 3d\sqrt{-e^2}\,e^6\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right) + \frac{\sqrt{d^2-e^2x^2}\left(-80d^7-280d^6ex-96d^5e^2x^2+770d^4e^3x^3+992d^3e^4x^4-525d^2e^5x^5-2496de^6x^6+560e^7x^7\right)}{560x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-80*d^7 - 280*d^6*e*x - 96*d^5*e^2*x^2 + 770*d^4*e^3*
x^3 + 992*d^3*e^4*x^4 - 525*d^2*e^5*x^5 - 2496*d*e^6*x^6 + 560*e^7*x^7))/(5
60*x^7) + (15*d*e^7*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/8 -
3*d*e^6*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.41, size = 173, normalized size = 0.84

$$\frac{3360\,de^7x^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+525\,de^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+560\,de^7x^7+\left(560\,e^7x^7-2496\,de^6x^6-525\,d^2e^5x^5+992\,d^3e^4x^4+770\,d^4e^3x^3-96\,d^5e^2x^2-280\,d^6ex-80\,d^7\right)\sqrt{-e^2x^2+d^2}}{560\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/560*(3360*d*e^7*x^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 525*d*e^7
*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 560*d*e^7*x^7 + (560*e^7*x^7 - 24
96*d*e^6*x^6 - 525*d^2*e^5*x^5 + 992*d^3*e^4*x^4 + 770*d^4*e^3*x^3 - 96*d^5
*e^2*x^2 - 280*d^6*e*x - 80*d^7)*sqrt(-e^2*x^2 + d^2))/x^7
```

giac [B] time = 0.31, size = 510, normalized size = 2.48

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^7\mathrm{sgn}(d)-\frac{15}{16}\,de^7\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(5\,de^{16}+\frac{35\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de^{14}}{x}+\frac{49\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^{12}}{x^2}-\frac{245\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3de^{10}}{x^3}-\frac{875\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4de^{8}}{x^4}+\frac{455\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5de^{6}}{x^5}+\frac{9065\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6de^{4}}{x^6}\right)x^7e^5}{4480\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^7}-\frac{1}{4480}\left(\frac{9065\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de^{68}}{x}+\frac{455\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^{66}}{x^2}-\frac{875\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3de^{64}}{x^3}-\frac{245\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4de^{62}}{x^4}+\frac{49\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5de^{60}}{x^5}+\frac{35\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6de^{58}}{x^6}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^7de^{56}}{x^7}\right)e^{(-63)}+\sqrt{-x^2e^2+d^2}\,e^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] -3*d*arcsin(x*e/d)*e^7*sgn(d) - 15/16*d*e^7*log(1/2*abs(-2*d*e - 2*sqrt(-x^
2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/4480*(5*d*e^16 + 35*(d*e + sqrt(-x^2*e^2
 + d^2)*e)*d*e^14/x + 49*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^12/x^2 - 245*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*e^10/x^3 - 875*(d*e + sqrt(-x^2*e^2 + d^
2)*e)^4*d*e^8/x^4 + 455*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d*e^6/x^5 + 9065*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d*e^4/x^6)*x^7*e^5/(d*e + sqrt(-x^2*e^2 + d
^2)*e)^7 - 1/4480*(9065*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^68/x + 455*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^2*d*e^66/x^2 - 875*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^3*d*e^64/x^3 - 245*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d*e^62/x^4 + 49*(d*e +
 sqrt(-x^2*e^2 + d^2)*e)^5*d*e^60/x^5 + 35*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6
*d*e^58/x^6 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d*e^56/x^7)*e^(-63) + sqrt
(-x^2*e^2 + d^2)*e^7
```

maple [B] time = 0.05, size = 377, normalized size = 1.83

$$-\frac{15d^2e^7\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16\sqrt{d^2}}-\frac{3d\,e^8\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{3\sqrt{-e^2x^2+d^2}\,e^8x}{d}+\frac{15\sqrt{-e^2x^2+d^2}\,e^7}{16}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8x}{d^3}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7}{16d^2}-\frac{8\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^8x}{5d^5}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^7}{16d^4}-\frac{8\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^6}{5d^5x}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^5}{16d^4x^2}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{5d^3x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{8d^2x^4}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{5d\,x^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{2x^6}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x)
```

```
[Out] -3/5/d*e^2/x^5*(-e^2*x^2+d^2)^(7/2)+2/5/d^3*e^4/x^3*(-e^2*x^2+d^2)^(7/2)-8/
5/d^5*e^6/x*(-e^2*x^2+d^2)^(7/2)-8/5/d^5*e^8*x*(-e^2*x^2+d^2)^(5/2)-2/d^3*e
^8*x*(-e^2*x^2+d^2)^(3/2)-3/d*e^8*x*(-e^2*x^2+d^2)^(1/2)-3*d*e^8/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2*e*(-e^2*x^2+d^2)^(7/2)/x^6
-1/8/d^2*e^3/x^4*(-e^2*x^2+d^2)^(7/2)+3/16/d^4*e^5/x^2*(-e^2*x^2+d^2)^(7/2)
+3/16/d^4*e^7*(-e^2*x^2+d^2)^(5/2)+5/16/d^2*e^7*(-e^2*x^2+d^2)^(3/2)+15/16*
e^7*(-e^2*x^2+d^2)^(1/2)-15/16*d^2*e^7/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x)-1/7*d*(-e^2*x^2+d^2)^(7/2)/x^7
```

maxima [A] time = 1.01, size = 326, normalized size = 1.58

$$-3\,de^7\arcsin\left(\frac{ex}{d}\right)-\frac{15}{16}\,de^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{3\,\sqrt{-e^2x^2+d^2}\,e^8x}{d}+\frac{15}{16}\,\sqrt{-e^2x^2+d^2}\,e^7-\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8x}{d^3}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^7}{16\,d^2}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^7}{16\,d^4}-\frac{8\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}{5\,d^3x}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^5}{16\,d^4x^2}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{5\,d^3x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{8\,d^2x^4}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{5\,dx^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{2\,x^6}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] -3*d*e^7*arcsin(e*x/d) - 15/16*d*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d
^2)*d/abs(x)) - 3*sqrt(-e^2*x^2 + d^2)*e^8*x/d + 15/16*sqrt(-e^2*x^2 + d^2)
*e^7 - 2*(-e^2*x^2 + d^2)^(3/2)*e^8*x/d^3 + 5/16*(-e^2*x^2 + d^2)^(3/2)*e^7
```

```
/d^2 + 3/16*(-e^2*x^2 + d^2)^(5/2)*e^7/d^4 - 8/5*(-e^2*x^2 + d^2)^(5/2)*e^6
/(d^3*x) + 3/16*(-e^2*x^2 + d^2)^(7/2)*e^5/(d^4*x^2) + 2/5*(-e^2*x^2 + d^2)
^(7/2)*e^4/(d^3*x^3) - 1/8*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^4) - 3/5*(-e^2
*x^2 + d^2)^(7/2)*e^2/(d*x^5) - 1/2*(-e^2*x^2 + d^2)^(7/2)*e/x^6 - 1/7*(-e^
2*x^2 + d^2)^(7/2)*d/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2 - e^2\, x^2\right)^{5/2}\, (d + e\, x)^3}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^8,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^8, x)
```

sympy [C] time = 22.29, size = 1513, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**8,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + 3*d**6*e
*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt
(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) -
e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5)
, Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)
) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) -
 I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**5*e**2*Piecewise((3*I*d**3*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(
-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2)
 > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*
d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**
6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*
x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - 5*
d**4*e**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x
**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)
) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*
x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) +
1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/
(8*d**3), True)) - 5*d**3*e**4*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*
x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1
), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**
2) + 1)/(3*d**2), True)) + d**2*e**5*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(
e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))
/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x)
 - I*e**2*asin(d/(e*x))/(2*d), True)) + 3*d*e**6*Piecewise((I*d/(x*sqrt(-1
+ e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**
2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*
x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((d**2/(
e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x*
```

```
*2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2)
+ 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

3.79 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=204

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6}+e^8\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{125}{128}e^8\tanh^{-1}\Big($$

Rubi [A] time = 0.30, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1807, 811, 844, 217, 203, 266, 63, 208}

$$-\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}+e^8\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{125}{128}e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

```
[Out] -(e^6*(125*d + 128*e*x)*Sqrt[d^2 - e^2*x^2])/(128*x^2) + (e^4*(125*d + 64*e
*x)*(d^2 - e^2*x^2)^(3/2))/(192*x^4) - (e^2*(125*d + 48*e*x)*(d^2 - e^2*x^2
)^(5/2))/(240*x^6) - (d*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (3*e*(d^2 - e^2*x^
2)^(7/2))/(7*x^7) - e^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (125*e^8*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d])/128
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
```

```
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-24d^4e-25d^3e^2x-8d^2e^3x^2\right)}{x^8}\,dx}{8d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}+\frac{\int \frac{\left(175d^5e^2+56d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\int \cdots$$

$$= \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6}-\frac{d\left(d^2-e^2x^2\right)^{\cdots}}{8x^8}\cdots$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+\cdots}{\cdots}$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+\cdots}{\cdots}$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+\cdots}{\cdots}$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+\cdots}{\cdots}$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+\cdots}{\cdots}$$

Mathematica [C] time = 0.15, size = 245, normalized size = 1.20

$$-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\frac{e^8\left(d^2-e^2x^2\right)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^7}-\frac{d^4e^3\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{5x^5\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{-8d^7e^2+34d^5e^4x^2-59d^3e^6x^4+15de^8x^6\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)+33de^8x^6}{16x^6\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

```
[Out] (-3*e*(d^2 - e^2*x^2)^(7/2))/(7*x^7) + (-8*d^7*e^2 + 34*d^5*e^4*x^2 - 59*d^
3*e^6*x^4 + 33*d*e^8*x^6 + 15*d*e^8*x^6*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqr
t[1 - (e^2*x^2)/d^2]])/(16*x^6*Sqrt[d^2 - e^2*x^2]) - (d^4*e^3*Sqrt[d^2 - e
^2*x^2]*Hypergeometric2F1[-5/2, -5/2, -3/2, (e^2*x^2)/d^2])/(5*x^5*Sqrt[1 -
 (e^2*x^2)/d^2]) - (e^8*(d^2 - e^2*x^2)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2
, 1 - (e^2*x^2)/d^2])/(7*d^7)
```

IntegrateAlgebraic [A] time = 0.83, size = 186, normalized size = 0.91

$$-\frac{125}{64}e^8\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\sqrt{-e^2}\,e^7\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+\frac{\sqrt{d^2-e^2x^2}\left(-1680d^7-5760d^6ex-1960d^5e^2x^2+14592d^4e^3x^3+17710d^3e^4x^4-7424d^2e^5x^5-27195de^6x^6-14848e^7x^7\right)}{13440x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-1680*d^7 - 5760*d^6*e*x - 1960*d^5*e^2*x^2 + 14592*d
^4*e^3*x^3 + 17710*d^3*e^4*x^4 - 7424*d^2*e^5*x^5 - 27195*d*e^6*x^6 - 14848
*e^7*x^7))/(13440*x^8) - (125*e^8*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2
*x^2]/d])/64 - e^7*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.48, size = 163, normalized size = 0.80

$$\frac{26880\,e^8x^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-13125\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(14848\,e^7x^7+27195\,de^6x^6+7424\,d^2e^5x^5-17710\,d^3e^4x^4-14592\,d^4e^3x^3+1960\,d^5e^2x^2+5760\,d^6ex+1680\,d^7\right)\sqrt{-e^2x^2+d^2}}{13440\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/13440*(26880*e^8*x^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 13125*e^
8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (14848*e^7*x^7 + 27195*d*e^6*x^6
 + 7424*d^2*e^5*x^5 - 17710*d^3*e^4*x^4 - 14592*d^4*e^3*x^3 + 1960*d^5*e^2*
x^2 + 5760*d^6*e*x + 1680*d^7)*sqrt(-e^2*x^2 + d^2))/x^8
```

giac [B] time = 0.31, size = 538, normalized size = 2.64

$$-\arcsin\left(\frac{xe}{d}\right)e^8\mathrm{sgn}(d)+\frac{x^8\left(\frac{720\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{16}}{x}+\frac{1120\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{14}}{x^2}-\frac{3696\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{12}}{x^3}-\frac{14280\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^{10}}{x^4}-\frac{560\left(de+\sqrt{-x^2e^2+d^2}e\right)^5e^8}{x^5}+\frac{77280\left(de+\sqrt{-x^2e^2+d^2}e\right)^6e^6}{x^6}+\frac{122640\left(de+\sqrt{-x^2e^2+d^2}e\right)^7e^4}{x^7}+105\,e^{18}\right)e^6}{215040\left(de+\sqrt{-x^2e^2+d^2}e\right)^8}-\frac{1}{215040}\left(\frac{122640\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{86}}{x}+\frac{77280\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{84}}{x^2}-\frac{560\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{82}}{x^3}-\frac{14280\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^{80}}{x^4}-\frac{3696\left(de+\sqrt{-x^2e^2+d^2}e\right)^5e^{78}}{x^5}+\frac{1120\left(de+\sqrt{-x^2e^2+d^2}e\right)^6e^{76}}{x^6}+\frac{720\left(de+\sqrt{-x^2e^2+d^2}e\right)^7e^{74}}{x^7}+\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^8e^{72}}{x^8}\right)e^{(-80)}+\frac{125}{128}e^8\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^8*sgn(d) + 1/215040*x^8*(720*(d*e + sqrt(-x^2*e^2 + d^2)*e
)*e^16/x + 1120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^14/x^2 - 3696*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^3*e^12/x^3 - 14280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^
10/x^4 - 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^8/x^5 + 77280*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^6*e^6/x^6 + 122640*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^4/x
^7 + 105*e^18)*e^6/(d*e + sqrt(-x^2*e^2 + d^2)*e)^8 - 1/215040*(122640*(d*e
 + sqrt(-x^2*e^2 + d^2)*e)*e^86/x + 77280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*
e^84/x^2 - 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^82/x^3 - 14280*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^4*e^80/x^4 - 3696*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^7
8/x^5 + 1120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^76/x^6 + 720*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^7*e^74/x^7 + 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*e^72/x^8
)*e^(-80) + 125/128*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-
2)/abs(x))
```

maple [B] time = 0.06, size = 402, normalized size = 1.97

$$\frac{125d\,e^8\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{128\sqrt{d^2}}-\frac{e^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{\sqrt{-e^2x^2+d^2}\,e^9x}{d^2}-\frac{125\sqrt{-e^2x^2+d^2}\,e^8}{128d}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^9x}{3d^4}-\frac{125\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8}{384d^3}-\frac{8\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^9x}{15d^6}-\frac{25\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^8}{128d^5}-\frac{8\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^7}{15d^6x}-\frac{25\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^6}{128d^5x^2}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^5}{15d^4x^3}+\frac{25\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{192d^3x^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{5d^2x^5}-\frac{25\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{48d\,x^6}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{7x^7}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x)
```

```
[Out] -1/8*d*(-e^2*x^2+d^2)^(7/2)/x^8-25/48/d*e^2/x^6*(-e^2*x^2+d^2)^(7/2)+25/192
/d^3*e^4/x^4*(-e^2*x^2+d^2)^(7/2)-25/128/d^5*e^6/x^2*(-e^2*x^2+d^2)^(7/2)-2
5/128/d^5*e^8*(-e^2*x^2+d^2)^(5/2)-125/384/d^3*e^8*(-e^2*x^2+d^2)^(3/2)-125
/128/d*e^8*(-e^2*x^2+d^2)^(1/2)+125/128*d*e^8/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-3/7*e*(-e^2*x^2+d^2)^(7/2)/x^7-1/5*e^3/d^2/
x^5*(-e^2*x^2+d^2)^(7/2)+2/15*e^5/d^4/x^3*(-e^2*x^2+d^2)^(7/2)-8/15*e^7/d^6
/x*(-e^2*x^2+d^2)^(7/2)-8/15*e^9/d^6*x*(-e^2*x^2+d^2)^(5/2)-2/3*e^9/d^4*x*(
-e^2*x^2+d^2)^(3/2)-e^9/d^2*x*(-e^2*x^2+d^2)^(1/2)-e^9/(e^2)^(1/2)*arctan((
e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 1.01, size = 352, normalized size = 1.73

$$-e^{8}\arcsin\left(\frac{ex}{d}\right)+\frac{125}{128}e^{8}\log\left(\frac{2d^{2}}{|x|}+\frac{2\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{9}x}{d^{2}}-\frac{125\sqrt{-e^{2}x^{2}+d^{2}}\,e^{8}}{128\,d}-\frac{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{9}x}{3\,d^{4}}-\frac{125\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{8}}{384\,d^{3}}-\frac{25\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{8}}{128\,d^{5}}-\frac{8\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{7}}{15\,d^{4}x}-\frac{25\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{6}}{128\,d^{5}x^{2}}+\frac{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{5}}{15\,d^{4}x^{3}}+\frac{25\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{4}}{192\,d^{3}x^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{3}}{5\,d^{2}x^{5}}-\frac{25\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{2}}{48\,dx^{6}}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e}{7\,x^{7}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d}{8\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] -e^8*arcsin(e*x/d) + 125/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*
d/abs(x)) - sqrt(-e^2*x^2 + d^2)*e^9*x/d^2 - 125/128*sqrt(-e^2*x^2 + d^2)*e
^8/d - 2/3*(-e^2*x^2 + d^2)^(3/2)*e^9*x/d^4 - 125/384*(-e^2*x^2 + d^2)^(3/2
)*e^8/d^3 - 25/128*(-e^2*x^2 + d^2)^(5/2)*e^8/d^5 - 8/15*(-e^2*x^2 + d^2)^(
5/2)*e^7/(d^4*x) - 25/128*(-e^2*x^2 + d^2)^(7/2)*e^6/(d^5*x^2) + 2/15*(-e^2
*x^2 + d^2)^(7/2)*e^5/(d^4*x^3) + 25/192*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3*x^
4) - 1/5*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^5) - 25/48*(-e^2*x^2 + d^2)^(7/2
)*e^2/(d*x^6) - 3/7*(-e^2*x^2 + d^2)^(7/2)*e/x^7 - 1/8*(-e^2*x^2 + d^2)^(7/
2)*d/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^9} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^9,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^9, x)
```

sympy [C] time = 31.40, size = 1719, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**9,x)
```

```
[Out] d**7*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6)
 + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*
x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6),
Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*
```

```
e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2
*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**
6), True)) + d**5*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*ac
osh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(
-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e
**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d*
*2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 5*d**4*e**3
*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**
7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**
7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*
x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x*
*7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x
**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5
 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15
*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d
*e**2*x**7), True)) - 5*d**3*e**4*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**
2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sq
rt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x*
*2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*s
qrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
 - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**2*e**5*Piecewise((-e*sqrt(d**
2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs
(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**
3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + 3*d*e**6*Piecewise((-d**2/
(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1))
+ e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(
e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) + e**7*Piecewise
((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d
**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.80 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=187

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}+\frac{55e^9\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d}-\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2\right.}{19}$$

Rubi [A] time = 0.26, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 807, 266, 47, 63, 208}

$$-\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}+\frac{55e^9\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] (-55*e^7*Sqrt[d^2 - e^2*x^2])/(128*x^2) + (55*e^5*(d^2 - e^2*x^2)^(3/2))/(1
92*x^4) - (11*e^3*(d^2 - e^2*x^2)^(5/2))/(48*x^6) - (d*(d^2 - e^2*x^2)^(7/2
))/(9*x^9) - (3*e*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (29*e^2*(d^2 - e^2*x^2)^
(7/2))/(63*d*x^7) + (55*e^9*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-27d^4e-29d^3e^2x-9d^2e^3x^2\right)}{x^9}\,dx}{9d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8} + \frac{\int \frac{\left(232d^5e^2+99d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx}{72d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7} + \frac{1}{8}\left(11e^3\right)\int \frac{\left(d^2-e^2\right.}{x^7}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7} + \frac{1}{16}\left(11e^3\right)\text{Subst}\Bigg(\int$$

$$= -\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7} -$$

$$= \frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8} -$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2} + \frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} -$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2} + \frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} -$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2} + \frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} -$$

Mathematica [C] time = 0.17, size = 218, normalized size = 1.17

$$\frac{-112d^{10}-16d^8e^2x^2-168d^7e^3x^3+1184d^6e^4x^4+714d^5e^5x^5-2336d^4e^6x^6-1239d^3e^7x^7+1744d^2e^8x^8+315de^9x^9\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)+693de^9x^9-464e^{10}x^{10}}{1008dx^9\sqrt{d^2-e^2x^2}} - \frac{3e^9\left(d^2-e^2x^2\right)^{7/2}{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] (-112*d^10 - 16*d^8*e^2*x^2 - 168*d^7*e^3*x^3 + 1184*d^6*e^4*x^4 + 714*d^5*
e^5*x^5 - 2336*d^4*e^6*x^6 - 1239*d^3*e^7*x^7 + 1744*d^2*e^8*x^8 + 693*d*e^
9*x^9 - 464*e^10*x^10 + 315*d*e^9*x^9*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[
1 - (e^2*x^2)/d^2]])/(1008*d*x^9*Sqrt[d^2 - e^2*x^2]) - (3*e^9*(d^2 - e^2*x
^2)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, 1 - (e^2*x^2)/d^2])/(7*d^8)
```

IntegrateAlgebraic [A] time = 0.88, size = 159, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(-896d^8-3024d^7ex-1024d^6e^2x^2+7224d^5e^3x^3+8448d^4e^4x^4-3066d^3e^5x^5-10240d^2e^6x^6-4599de^7x^7+3712e^8x^8\right)}{8064dx^9} - \frac{55e^9\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-896*d^8 - 3024*d^7*e*x - 1024*d^6*e^2*x^2 + 7224*d^5
*e^3*x^3 + 8448*d^4*e^4*x^4 - 3066*d^3*e^5*x^5 - 10240*d^2*e^6*x^6 - 4599*d
*e^7*x^7 + 3712*e^8*x^8))/(8064*d*x^9) - (55*e^9*ArcTanh[(Sqrt[-e^2]*x)/d -
 Sqrt[d^2 - e^2*x^2]/d])/(64*d)
```

fricas [A] time = 0.46, size = 142, normalized size = 0.76

$$-\frac{3465\,e^{9}x^{9}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(3712\,e^{8}x^{8}-4599\,de^{7}x^{7}-10240\,d^{2}e^{6}x^{6}-3066\,d^{3}e^{5}x^{5}+8448\,d^{4}e^{4}x^{4}+7224\,d^{5}e^{3}x^{3}-1024\,d^{6}e^{2}x^{2}-3024\,d^{7}ex-896\,d^{8}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{8064\,dx^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/8064*(3465*e^9*x^9*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (3712*e^8*x^8 -
4599*d*e^7*x^7 - 10240*d^2*e^6*x^6 - 3066*d^3*e^5*x^5 + 8448*d^4*e^4*x^4 +
7224*d^5*e^3*x^3 - 1024*d^6*e^2*x^2 - 3024*d^7*e*x - 896*d^8)*sqrt(-e^2*x^2
 + d^2))/(d*x^9)
```

giac [B] time = 0.47, size = 620, normalized size = 3.32

$$\frac{x^{9}\left(\frac{189\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{18}}{x}+\frac{324\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}e^{16}}{x^{2}}-\frac{672\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}e^{14}}{x^{3}}-\frac{3024\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}e^{12}}{x^{4}}-\frac{1512\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{5}e^{10}}{x^{5}}+\frac{9744\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{6}e^{8}}{x^{6}}+\frac{18144\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{7}e^{6}}{x^{7}}-\frac{16632\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{8}e^{4}}{x^{8}}+28\,e^{20}\right)e^{7}}{129024\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{9}d}+\frac{55\,e^{9}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)}{128\,d}+\frac{\left(\frac{16632\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)d^{8}e^{106}}{x}-\frac{18144\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}d^{8}e^{104}}{x^{2}}-\frac{9744\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}d^{8}e^{102}}{x^{3}}+\frac{1512\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}d^{8}e^{100}}{x^{4}}+\frac{3024\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{5}d^{8}e^{98}}{x^{5}}+\frac{672\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{6}d^{8}e^{96}}{x^{6}}-\frac{324\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{7}d^{8}e^{94}}{x^{7}}-\frac{189\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{8}d^{8}e^{92}}{x^{8}}-\frac{28\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{9}d^{8}e^{90}}{x^{9}}\right)e^{(-99)}}{129024\,d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/129024*x^9*(189*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^18/x + 324*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^2*e^16/x^2 - 672*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^14/x^
3 - 3024*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^12/x^4 - 1512*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^5*e^10/x^5 + 9744*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^8/x^6 +
18144*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^6/x^7 - 16632*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^8*e^4/x^8 + 28*e^20)*e^7/((d*e + sqrt(-x^2*e^2 + d^2)*e)^9*d) +
55/128*e^9*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d
+ 1/129024*(16632*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^8*e^106/x - 18144*(d*e +
 sqrt(-x^2*e^2 + d^2)*e)^2*d^8*e^104/x^2 - 9744*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^3*d^8*e^102/x^3 + 1512*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^8*e^100/x^4 +
 3024*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^8*e^98/x^5 + 672*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^6*d^8*e^96/x^6 - 324*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^8*e^9
4/x^7 - 189*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^8*e^92/x^8 - 28*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^9*d^8*e^90/x^9)*e^(-99)/d^9
```

maple [A] time = 0.10, size = 250, normalized size = 1.34

$$\frac{55e^{9}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{128\sqrt{d^{2}}}-\frac{55\sqrt{-e^{2}x^{2}+d^{2}}\,e^{9}}{128d^{2}}-\frac{55\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{9}}{384d^{4}}-\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{9}}{128d^{6}}-\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{7}}{128d^{6}x^{2}}+\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{5}}{192d^{4}x^{4}}-\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{3}}{48d^{2}x^{6}}-\frac{29\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{2}}{63d\,x^{7}}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e}{8x^{8}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x)
```

```
[Out] -29/63*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^7-3/8*e*(-e^2*x^2+d^2)^(7/2)/x^8-11/48/
d^2*e^3/x^6*(-e^2*x^2+d^2)^(7/2)+11/192/d^4*e^5/x^4*(-e^2*x^2+d^2)^(7/2)-11
/128/d^6*e^7/x^2*(-e^2*x^2+d^2)^(7/2)-11/128/d^6*e^9*(-e^2*x^2+d^2)^(5/2)-5
5/384/d^4*e^9*(-e^2*x^2+d^2)^(3/2)-55/128/d^2*e^9*(-e^2*x^2+d^2)^(1/2)+55/1
28*e^9/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/9*d*(
-e^2*x^2+d^2)^(7/2)/x^9
```

maxima [A] time = 1.00, size = 247, normalized size = 1.32

$$\frac{55\,e^{9}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{128\,d}-\frac{55\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{9}}{128\,d^{2}}-\frac{55\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{9}}{384\,d^{4}}-\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{9}}{128\,d^{6}}-\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{7}}{128\,d^{6}x^{2}}+\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{5}}{192\,d^{4}x^{4}}-\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{3}}{48\,d^{2}x^{6}}-\frac{29\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{2}}{63\,dx^{7}}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e}{8\,x^{8}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 55/128*e^9*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 55/128*s
qrt(-e^2*x^2 + d^2)*e^9/d^2 - 55/384*(-e^2*x^2 + d^2)^(3/2)*e^9/d^4 - 11/12
8*(-e^2*x^2 + d^2)^(5/2)*e^9/d^6 - 11/128*(-e^2*x^2 + d^2)^(7/2)*e^7/(d^6*x
^2) + 11/192*(-e^2*x^2 + d^2)^(7/2)*e^5/(d^4*x^4) - 11/48*(-e^2*x^2 + d^2)^
(7/2)*e^3/(d^2*x^6) - 29/63*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^7) - 3/8*(-e^2*
x^2 + d^2)^(7/2)*e/x^8 - 1/9*(-e^2*x^2 + d^2)^(7/2)*d/x^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^{10}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^10,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^10, x)
```

sympy [C] time = 36.71, size = 1889, normalized size = 10.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**10,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**4) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d*
*2/(e**2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**
2/(e**2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x
**6) + 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt
(-d**2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2)
+ 1)/(315*d**8), True)) + 3*d**6*e*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e*
*2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*
x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x*
*2) - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d
/(e*x))/(128*d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**
2/(e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/
(192*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(
-d**2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1))
 - 5*I*e**8*asin(d/(e*x))/(128*d**7), True)) + d**5*e**2*Piecewise((-e*sqrt
(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*
x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**
2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2
/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x*
*4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(
-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 5*d**4*e**3*Piecewise((-d**2/(6
*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) -
1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*s
qrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) +
 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x)
)/(16*d**5), True)) - 5*d**3*e**4*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d
**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d
**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**
2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d*
```

```
*2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqr
t(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1
 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e*
*2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2
*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + d**2*e**5*Piecewise(
(-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*
x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*
x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2
*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + 3
*d*e**6*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/
(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e*
*2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)
) + e**7*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sq
rt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)
) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d
), True))
```

3.81 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=225

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}-\frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8}+\frac{33e^{10}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^2}-\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2}+\frac{11e^6\left(d^2-e^2\right.}{128dx^4}$$

Rubi [A] time = 0.30, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1807, 835, 807, 266, 47, 63, 208}

$$-\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2}+\frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4}-\frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6}-\frac{5e^3\left(d^2-e^2x^2\right)^{7/2}}{21d^2x^7}-\frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}+\frac{33e^{10}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] (-33*e^8*Sqrt[d^2 - e^2*x^2])/(256*d*x^2) + (11*e^6*(d^2 - e^2*x^2)^(3/2))/
(128*d*x^4) - (11*e^4*(d^2 - e^2*x^2)^(5/2))/(160*d*x^6) - (d*(d^2 - e^2*x^
2)^(7/2))/(10*x^10) - (e*(d^2 - e^2*x^2)^(7/2))/(3*x^9) - (33*e^2*(d^2 - e^
2*x^2)^(7/2))/(80*d*x^8) - (5*e^3*(d^2 - e^2*x^2)^(7/2))/(21*d^2*x^7) + (33
*e^10*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(256*d^2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{11}}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-30d^4e-33d^3e^2x-10d^2e^3x^2\right)}{x^{10}}\,dx}{10d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} + \frac{\int \frac{\left(297d^5e^2+150d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx}{90d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{\int \frac{\left(-1200d^6e^3-297d^5e^4 \ldots}{x^8}}{720d \ldots}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{5e^3\left(d^2-e^2x^2\right)^{7/2}}{21d^2x^7}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{5e^3\left(d^2-e^2x^2\right)^{7/2}}{21d^2x^7}$$

$$= -\frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8}$$

$$= \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/\ldots}}{10x^{10}}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/\ldots}}{10x^{10}}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/\ldots}}{10x^{10}}$$

Mathematica [C] time = 0.06, size = 102, normalized size = 0.45

$$-\frac{e\left(d^2-e^2x^2\right)^{7/2}\left(7d^9+5d^7e^2x^2+9e^9x^9\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+3e^9x^9\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)\right)}{21d^9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] -1/21*(e*(d^2 - e^2*x^2)^(7/2)*(7*d^9 + 5*d^7*e^2*x^2 + 9*e^9*x^9*Hypergeom
etric2F1[7/2, 5, 9/2, 1 - (e^2*x^2)/d^2] + 3*e^9*x^9*Hypergeometric2F1[7/2,
 6, 9/2, 1 - (e^2*x^2)/d^2]))/(d^9*x^9)
```

IntegrateAlgebraic [A] time = 0.99, size = 170, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(-2688d^9-8960d^8ex-3024d^7e^2x^2+20480d^6e^3x^3+23352d^5e^4x^4-7680d^4e^5x^5-24570d^3e^6x^6-10240d^2e^7x^7+3465de^8x^8+6400e^9x^9\right)}{26880d^2x^{10}}-\frac{33e^{10}\tanh^{-1}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2688*d^9 - 8960*d^8*e*x - 3024*d^7*e^2*x^2 + 20480*d
^6*e^3*x^3 + 23352*d^5*e^4*x^4 - 7680*d^4*e^5*x^5 - 24570*d^3*e^6*x^6 - 102
40*d^2*e^7*x^7 + 3465*d*e^8*x^8 + 6400*e^9*x^9))/(26880*d^2*x^10) - (33*e^1
0*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(128*d^2)
```

fricas [A] time = 0.52, size = 153, normalized size = 0.68

$$-\frac{3465\,e^{10}x^{10}\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(6400\,e^9x^9+3465\,de^8x^8-10240\,d^2e^7x^7-24570\,d^3e^6x^6-7680\,d^4e^5x^5+23352\,d^5e^4x^4+20480\,d^6e^3x^3-3024\,d^7e^2x^2-8960\,d^8ex-2688\,d^9\right)\sqrt{-e^2x^2+d^2}}{26880\,d^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/26880*(3465*e^10*x^10*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (6400*e^9*x^9
 + 3465*d*e^8*x^8 - 10240*d^2*e^7*x^7 - 24570*d^3*e^6*x^6 - 7680*d^4*e^5*x^
5 + 23352*d^5*e^4*x^4 + 20480*d^6*e^3*x^3 - 3024*d^7*e^2*x^2 - 8960*d^8*e*x
 - 2688*d^9)*sqrt(-e^2*x^2 + d^2))/(d^2*x^10)
```

giac [B] time = 0.36, size = 683, normalized size = 3.04

$$\frac{x^{10}\left(\frac{280\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{20}}{x}+\frac{525\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{18}}{x^2}-\frac{600\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{16}}{x^3}-\frac{3570\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^{14}}{x^4}-\frac{3360\left(de+\sqrt{-x^2e^2+d^2}e\right)^5e^{12}}{x^5}+\frac{5880\left(de+\sqrt{-x^2e^2+d^2}e\right)^6e^{10}}{x^6}+\frac{16800\left(de+\sqrt{-x^2e^2+d^2}e\right)^7e^{8}}{x^7}+\frac{10500\left(de+\sqrt{-x^2e^2+d^2}e\right)^8e^{6}}{x^8}-\frac{31920\left(de+\sqrt{-x^2e^2+d^2}e\right)^9e^{4}}{x^9}+42\,e^{22}\right)e^8}{430080\left(de+\sqrt{-x^2e^2+d^2}e\right)^{10}d^2}+\frac{33\,e^{10}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{256\,d^2}+\frac{\left(\frac{31920\left(de+\sqrt{-x^2e^2+d^2}e\right)d^{18}e^{128}}{x}-\frac{10500\left(de+\sqrt{-x^2e^2+d^2}e\right)^2d^{18}e^{126}}{x^2}-\frac{16800\left(de+\sqrt{-x^2e^2+d^2}e\right)^3d^{18}e^{124}}{x^3}-\frac{5880\left(de+\sqrt{-x^2e^2+d^2}e\right)^4d^{18}e^{122}}{x^4}+\frac{3360\left(de+\sqrt{-x^2e^2+d^2}e\right)^5d^{18}e^{120}}{x^5}+\frac{3570\left(de+\sqrt{-x^2e^2+d^2}e\right)^6d^{18}e^{118}}{x^6}+\frac{600\left(de+\sqrt{-x^2e^2+d^2}e\right)^7d^{18}e^{116}}{x^7}-\frac{525\left(de+\sqrt{-x^2e^2+d^2}e\right)^8d^{18}e^{114}}{x^8}-\frac{280\left(de+\sqrt{-x^2e^2+d^2}e\right)^9d^{18}e^{112}}{x^9}-\frac{42\left(de+\sqrt{-x^2e^2+d^2}e\right)^{10}d^{18}e^{110}}{x^{10}}\right)e^{(-120)}}{430080\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/430080*x^10*(280*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^20/x + 525*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^2*e^18/x^2 - 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^16/x
^3 - 3570*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^14/x^4 - 3360*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^5*e^12/x^5 + 5880*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^10/x^6
+ 16800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^8/x^7 + 10500*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^8*e^6/x^8 - 31920*(d*e + sqrt(-x^2*e^2 + d^2)*e)^9*e^4/x^9 + 4
2*e^22)*e^8/((d*e + sqrt(-x^2*e^2 + d^2)*e)^10*d^2) + 33/256*e^10*log(1/2*a
bs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 + 1/430080*(31920*
(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^18*e^128/x - 10500*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^2*d^18*e^126/x^2 - 16800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^18*e^12
4/x^3 - 5880*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^18*e^122/x^4 + 3360*(d*e +
sqrt(-x^2*e^2 + d^2)*e)^5*d^18*e^120/x^5 + 3570*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^6*d^18*e^118/x^6 + 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^18*e^116/x^7
- 525*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^18*e^114/x^8 - 280*(d*e + sqrt(-x^
```

```
2*e^2 + d^2)*e)^9*d^18*e^112/x^9 - 42*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*d^1
8*e^110/x^10)*e^(-120)/d^20
```

maple [A] time = 0.15, size = 278, normalized size = 1.24

$$\frac{33e^{10}\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{256\sqrt{d^2}\,d}-\frac{33\sqrt{-e^2x^2+d^2}\,e^{10}}{256d^3}-\frac{11\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^{10}}{256d^5}-\frac{33\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^{10}}{1280d^7}-\frac{33\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^8}{1280d^7x^2}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^6}{640d^5x^4}-\frac{11\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{160d^3x^6}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{21d^2x^7}-\frac{33\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{80d\,x^8}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{3x^9}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x)
```

```
[Out] -1/3*e*(-e^2*x^2+d^2)^(7/2)/x^9-5/21*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^7-33/80
*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^8-11/160/d^3*e^4/x^6*(-e^2*x^2+d^2)^(7/2)+11/
640/d^5*e^6/x^4*(-e^2*x^2+d^2)^(7/2)-33/1280/d^7*e^8/x^2*(-e^2*x^2+d^2)^(7/
2)-33/1280/d^7*e^10*(-e^2*x^2+d^2)^(5/2)-11/256/d^5*e^10*(-e^2*x^2+d^2)^(3/
2)-33/256/d^3*e^10*(-e^2*x^2+d^2)^(1/2)+33/256/d*e^10/(d^2)^(1/2)*ln((2*d^2
+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/10*d*(-e^2*x^2+d^2)^(7/2)/x^10
```

maxima [A] time = 1.01, size = 272, normalized size = 1.21

$$\frac{33\,e^{10}\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{256\,d^2}-\frac{33\,\sqrt{-e^2x^2+d^2}\,e^{10}}{256\,d^3}-\frac{11\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^{10}}{256\,d^5}-\frac{33\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^{10}}{1280\,d^7}-\frac{33\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^8}{1280\,d^7x^2}+\frac{11\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^6}{640\,d^5x^4}-\frac{11\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{160\,d^3x^6}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{21\,d^2x^7}-\frac{33\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{80\,dx^8}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{3\,x^9}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] 33/256*e^10*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 - 33/25
6*sqrt(-e^2*x^2 + d^2)*e^10/d^3 - 11/256*(-e^2*x^2 + d^2)^(3/2)*e^10/d^5 -
33/1280*(-e^2*x^2 + d^2)^(5/2)*e^10/d^7 - 33/1280*(-e^2*x^2 + d^2)^(7/2)*e^
8/(d^7*x^2) + 11/640*(-e^2*x^2 + d^2)^(7/2)*e^6/(d^5*x^4) - 11/160*(-e^2*x^
2 + d^2)^(7/2)*e^4/(d^3*x^6) - 5/21*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^7) -
33/80*(-e^2*x^2 + d^2)^(7/2)*e^2/(d*x^8) - 1/3*(-e^2*x^2 + d^2)^(7/2)*e/x^9
 - 1/10*(-e^2*x^2 + d^2)^(7/2)*d/x^10
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^11,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^11, x)
```

sympy [C] time = 49.87, size = 2159, normalized size = 9.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**11,x)
```

```
[Out] d**7*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9*e/(80*x**
9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d**2/(e**2*x**2) -
 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**7/(768*d**
6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sqrt(d**2/(e**2*x**
2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2/(e**2*x**2)) > 1), (
I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*sqrt(-d**2
/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 7
*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**7/(768*d**6*x
**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d**8*x*sqrt(-d**2/(e**2*x*
*2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), True)) + 3*d**6*e*Piecewise
```

```
((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**2*x**2) - 1)/
(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**4) + 8*e**7
*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**2/(e**2*x**2)
 - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
+ 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x**6) + 2*I*e**
5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt(-d**2/(e**2*x
**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**8
), True)) + d**5*e**2*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)
) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**
2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5
*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*
d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2)
 + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**
5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x
**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*a
sin(d/(e*x))/(128*d**7), True)) - 5*d**4*e**3*Piecewise((-e*sqrt(d**2/(e**2
*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e
**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**
2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2
) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e
**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2
*x**2) + 1)/(105*d**6), True)) - 5*d**3*e**4*Piecewise((-d**2/(6*e*x**7*sqr
t(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/
(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2
*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (
I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(
e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e*
*5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5)
, True)) + d**2*e**5*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**
2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**
2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*
x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3
*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x*
*2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/
d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)
/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(
-15*d**3*x**5 + 15*d*e**2*x**7), True)) + 3*d*e**6*Piecewise((-d**2/(4*e*x*
*5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) -
e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1))
- 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/
(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**7*Piecewise(
(-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(
3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x
**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

3.82 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=254

$$-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}-\frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9}-\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2}+\frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4}-\frac{19e^5\left(d^2-e^2x\right.}{480d^2x^6}$$

Rubi [A] time = 0.33, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1807, 835, 807, 266, 47, 63, 208}

$$-\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2}+\frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4}-\frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6}-\frac{74e^4\left(d^2-e^2x^2\right)^{7/2}}{693d^3x^7}-\frac{19e^3\left(d^2-e^2x^2\right)^{7/2}}{80d^2x^8}-\frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}}+\frac{19e^{11}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

```
[Out] (-19*e^9*Sqrt[d^2 - e^2*x^2])/(256*d^2*x^2) + (19*e^7*(d^2 - e^2*x^2)^(3/2)
)/(384*d^2*x^4) - (19*e^5*(d^2 - e^2*x^2)^(5/2))/(480*d^2*x^6) - (d*(d^2 -
e^2*x^2)^(7/2))/(11*x^11) - (3*e*(d^2 - e^2*x^2)^(7/2))/(10*x^10) - (37*e^2
*(d^2 - e^2*x^2)^(7/2))/(99*d*x^9) - (19*e^3*(d^2 - e^2*x^2)^(7/2))/(80*d^2
*x^8) - (74*e^4*(d^2 - e^2*x^2)^(7/2))/(693*d^3*x^7) + (19*e^11*ArcTanh[Sqr
t[d^2 - e^2*x^2]/d])/(256*d^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-33d^4e-37d^3e^2x-11d^2e^3x^2\right)}{x^{11}}dx}{11d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} + \frac{\int \frac{\left(370d^5e^2+209d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}dx}{110d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{\int \frac{\left(-1881d^6e^3-740d^5\right.}{x^9}}{990}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{19e^3\left(d^2-e^2x^2\right)^{7}}{80d^2x^8}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{19e^3\left(d^2-e^2x^2\right)^{7}}{80d^2x^8}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{19e^3\left(d^2-e^2x^2\right)^{7}}{80d^2x^8}$$

$$= -\frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7}}{99dx^9}$$

$$= \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7,}}{11x^{11}}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7,}}{11x^{11}}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7,}}{11x^{11}}$$

Mathematica [C] time = 0.06, size = 112, normalized size = 0.44

$$-\frac{\left(d^2-e^2x^2\right)^{7/2}\left(63d^{11}+259d^9e^2x^2+74d^7e^4x^4+99e^{11}x^{11}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+297e^{11}x^{11}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)\right)}{693d^{10}x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

```
[Out] -1/693*((d^2 - e^2*x^2)^(7/2)*(63*d^11 + 259*d^9*e^2*x^2 + 74*d^7*e^4*x^4 +
 99*e^11*x^11*Hypergeometric2F1[7/2, 5, 9/2, 1 - (e^2*x^2)/d^2] + 297*e^11*
x^11*Hypergeometric2F1[7/2, 6, 9/2, 1 - (e^2*x^2)/d^2]))/(d^10*x^11)
```

IntegrateAlgebraic [A] time = 1.05, size = 181, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(-80640d^{10}-266112d^9ex-89600d^8e^2x^2+587664d^7e^3x^3+657920d^6e^4x^4-201432d^5e^5x^5-629760d^4e^6x^6-251790d^3e^7x^7+47360d^2e^8x^8+65835de^9x^9+94720e^{10}x^{10}\right)}{887040d^3x^{11}}-\frac{19e^{11}\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-80640*d^10 - 266112*d^9*e*x - 89600*d^8*e^2*x^2 + 58
7664*d^7*e^3*x^3 + 657920*d^6*e^4*x^4 - 201432*d^5*e^5*x^5 - 629760*d^4*e^6
*x^6 - 251790*d^3*e^7*x^7 + 47360*d^2*e^8*x^8 + 65835*d*e^9*x^9 + 94720*e^1
0*x^10))/(887040*d^3*x^11) - (19*e^11*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 -
 e^2*x^2]/d])/(128*d^3)
```

fricas [A] time = 0.58, size = 164, normalized size = 0.65

$$-\frac{65835\,e^{11}x^{11}\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(94720\,e^{10}x^{10}+65835\,de^9x^9+47360\,d^2e^8x^8-251790\,d^3e^7x^7-629760\,d^4e^6x^6-201432\,d^5e^5x^5+657920\,d^6e^4x^4+587664\,d^7e^3x^3-89600\,d^8e^2x^2-266112\,d^9ex-80640\,d^{10}\right)\sqrt{-e^2x^2+d^2}}{887040\,d^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/887040*(65835*e^11*x^11*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (94720*e^10
*x^10 + 65835*d*e^9*x^9 + 47360*d^2*e^8*x^8 - 251790*d^3*e^7*x^7 - 629760*d
^4*e^6*x^6 - 201432*d^5*e^5*x^5 + 657920*d^6*e^4*x^4 + 587664*d^7*e^3*x^3 -
 89600*d^8*e^2*x^2 - 266112*d^9*e*x - 80640*d^10)*sqrt(-e^2*x^2 + d^2))/(d^
3*x^11)
```

giac [B] time = 0.36, size = 746, normalized size = 2.94

$$\frac{x^{11}\left(\frac{4158\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{22}}{x}+\frac{8470\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{20}}{x^2}-\frac{3465\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{18}}{x^3}-\frac{40590\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{16}}{x^4}-\frac{57750\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{14}}{x^5}+\frac{6930\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6e^{12}}{x^6}+\frac{138600\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^7e^{10}}{x^7}+\frac{244860\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^8e^{8}}{x^8}+\frac{152460\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^9e^{6}}{x^9}-\frac{568260\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^{10}e^{4}}{x^{10}}+630\,e^{24}\right)e^9}{14192640\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^{11}d^3}+\frac{19\,e^{11}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{256\,d^3}+\frac{\left(\frac{568260\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^{30}e^{152}}{x}-\frac{152460\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^{30}e^{150}}{x^2}-\frac{244860\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^{30}e^{148}}{x^3}-\frac{138600\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^{30}e^{146}}{x^4}-\frac{6930\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^{30}e^{144}}{x^5}+\frac{57750\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6d^{30}e^{142}}{x^6}+\frac{40590\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^7d^{30}e^{140}}{x^7}+\frac{3465\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^8d^{30}e^{138}}{x^8}-\frac{8470\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^9d^{30}e^{136}}{x^9}-\frac{4158\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^{10}d^{30}e^{134}}{x^{10}}-\frac{630\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^{11}d^{30}e^{132}}{x^{11}}\right)e^{(-143)}}{14192640\,d^{33}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/14192640*x^11*(4158*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^22/x + 8470*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^2*e^20/x^2 - 3465*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e
^18/x^3 - 40590*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^16/x^4 - 57750*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^5*e^14/x^5 + 6930*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^
12/x^6 + 138600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^10/x^7 + 244860*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^8*e^8/x^8 + 152460*(d*e + sqrt(-x^2*e^2 + d^2)*e)^9*
e^6/x^9 - 568260*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*e^4/x^10 + 630*e^24)*e^9
/((d*e + sqrt(-x^2*e^2 + d^2)*e)^11*d^3) + 19/256*e^11*log(1/2*abs(-2*d*e -
 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 + 1/14192640*(568260*(d*e + s
qrt(-x^2*e^2 + d^2)*e)*d^30*e^152/x - 152460*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^2*d^30*e^150/x^2 - 244860*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^30*e^148/x^3
- 138600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^30*e^146/x^4 - 6930*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^5*d^30*e^144/x^5 + 57750*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^6*d^30*e^142/x^6 + 40590*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^30*e^140/x^7 +
 3465*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^30*e^138/x^8 - 8470*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^9*d^30*e^136/x^9 - 4158*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*
d^30*e^134/x^10 - 630*(d*e + sqrt(-x^2*e^2 + d^2)*e)^11*d^30*e^132/x^11)*e^
(-143)/d^33
```

maple [A] time = 0.24, size = 303, normalized size = 1.19

$$\frac{19 e^{11}\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{256\sqrt{d^2}\,d^2}-\frac{19\sqrt{-e^2x^2+d^2}\,e^{11}}{256d^4}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^{11}}{768d^6}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^{11}}{1280d^8}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^9}{1280d^8x^2}+\frac{19\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^7}{1920d^6x^4}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^5}{480d^4x^6}-\frac{74\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{693d^3x^7}-\frac{19\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{80d^2x^8}-\frac{37\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{99d\,x^9}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{10x^{10}}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x)
```

```
[Out] -37/99*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^9-74/693*e^4*(-e^2*x^2+d^2)^(7/2)/d^3/x
^7-19/80*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^8-19/480*e^5/d^4/x^6*(-e^2*x^2+d^2)
^(7/2)+19/1920*e^7/d^6/x^4*(-e^2*x^2+d^2)^(7/2)-19/1280*e^9/d^8/x^2*(-e^2*x
^2+d^2)^(7/2)-19/1280*e^11/d^8*(-e^2*x^2+d^2)^(5/2)-19/768*e^11/d^6*(-e^2*x
^2+d^2)^(3/2)-19/256*e^11/d^4*(-e^2*x^2+d^2)^(1/2)+19/256*e^11/d^2/(d^2)^(1
/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/11*d*(-e^2*x^2+d^2)^
(7/2)/x^11-3/10*e*(-e^2*x^2+d^2)^(7/2)/x^10
```

maxima [A] time = 1.01, size = 297, normalized size = 1.17

$$\frac{19\,e^{11}\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{256\,d^3}-\frac{19\,\sqrt{-e^2x^2+d^2}\,e^{11}}{256\,d^4}-\frac{19\,\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^{11}}{768\,d^6}-\frac{19\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^{11}}{1280\,d^8}-\frac{19\,\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^9}{1280\,d^8x^2}+\frac{19\,\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^7}{1920\,d^6x^4}-\frac{19\,\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^5}{480\,d^4x^6}-\frac{74\,\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{693\,d^3x^7}-\frac{19\,\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{80\,d^2x^8}-\frac{37\,\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{99\,dx^9}-\frac{3\,\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{10\,x^{10}}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] 19/256*e^11*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 19/25
6*sqrt(-e^2*x^2 + d^2)*e^11/d^4 - 19/768*(-e^2*x^2 + d^2)^(3/2)*e^11/d^6 -
19/1280*(-e^2*x^2 + d^2)^(5/2)*e^11/d^8 - 19/1280*(-e^2*x^2 + d^2)^(7/2)*e^
9/(d^8*x^2) + 19/1920*(-e^2*x^2 + d^2)^(7/2)*e^7/(d^6*x^4) - 19/480*(-e^2*x
^2 + d^2)^(7/2)*e^5/(d^4*x^6) - 74/693*(-e^2*x^2 + d^2)^(7/2)*e^4/(d^3*x^7)
 - 19/80*(-e^2*x^2 + d^2)^(7/2)*e^3/(d^2*x^8) - 37/99*(-e^2*x^2 + d^2)^(7/2
)*e^2/(d*x^9) - 3/10*(-e^2*x^2 + d^2)^(7/2)*e/x^10 - 1/11*(-e^2*x^2 + d^2)^
(7/2)*d/x^11
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^{12}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^12,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^12, x)
```

sympy [C] time = 74.52, size = 2397, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**12,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(11*x**10) + e**3*sqrt(d**2/(
e**2*x**2) - 1)/(99*d**2*x**8) + 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(693*d**
4*x**6) + 16*e**7*sqrt(d**2/(e**2*x**2) - 1)/(1155*d**6*x**4) + 64*e**9*sqr
t(d**2/(e**2*x**2) - 1)/(3465*d**8*x**2) + 128*e**11*sqrt(d**2/(e**2*x**2)
- 1)/(3465*d**10), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
 + 1)/(11*x**10) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(99*d**2*x**8) + 8*I*
e**5*sqrt(-d**2/(e**2*x**2) + 1)/(693*d**4*x**6) + 16*I*e**7*sqrt(-d**2/(e*
*2*x**2) + 1)/(1155*d**6*x**4) + 64*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(346
5*d**8*x**2) + 128*I*e**11*sqrt(-d**2/(e**2*x**2) + 1)/(3465*d**10), True))
 + 3*d**6*e*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9*e/
(80*x**9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d**2/(e**2*
x**2) - 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**7/(
768*d**6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sqrt(d**2/(e
**2*x**2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2/(e**2*x**2))
> 1), (I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*sqr
t(-d**2/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2) +
1)) - 7*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**7/(768
*d**6*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d**8*x*sqrt(-d**2/(
e**2*x**2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), True)) + d**5*e**2*P
iecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**2*x**
2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**4)
+ 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**2/(e*
*2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**
2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x**6) +
 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt(-d**2
/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(
```

```
315*d**8), True)) - 5*d**4*e**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*
x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**
5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2)
 - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e
*x))/(128*d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(
e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(19
2*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d*
*2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) -
5*I*e**8*asin(d/(e*x))/(128*d**7), True)) - 5*d**3*e**4*Piecewise((-e*sqrt(
d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x
**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2
/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/
(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**
4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-
d**2/(e**2*x**2) + 1)/(105*d**6), True)) + d**2*e**5*Piecewise((-d**2/(6*e*
x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1))
 + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d*
*2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2))
 > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)
) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(
16*d**5), True)) + 3*d*e**6*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(
-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(
-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-1
5*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-
15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 -
e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**
2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**
2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/
d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e**7*Piecewise((-d**2/(4*e
*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1))
 - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3
), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1
)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d*
*2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True))
```

3.83 $\int \frac{x^5(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} dx$

Optimal. Leaf size=174

$$\frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}-\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.40, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1635, 1815, 641, 217, 203}

$$\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}+\frac{x\sqrt{d^2-e^2x^2}}{2e^5}-\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^4*(d + e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d + e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d + e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 + (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
```

```
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^5}{e^5}+\frac{5d^4x}{e^4}+\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}+\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{37d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{90d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{15d^3x^2}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{\int \frac{-\frac{195d^5}{e^3}-\frac{90d^4x}{e^2}}{\sqrt{d^2-e^2x^2}}}{30d^3e^2}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5}$$

Mathematica [A] time = 0.24, size = 131, normalized size = 0.75

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(304d^4-717d^3ex+479d^2e^2x^2-45de^3x^3-15e^4x^4\right)-195d(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{30e^6(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(Sqrt[1 - (e^2*x^2)/d^2]*(304*d^4 - 717*d^3*e*x + 479*d^2*e^2*x^
2 - 45*d*e^3*x^3 - 15*e^4*x^4) - 195*d*(d - e*x)^3*ArcSin[(e*x)/d]))/(30*e^
6*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.58, size = 123, normalized size = 0.71

$$-\frac{13d^2\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^7} - \frac{\sqrt{d^2-e^2x^2}\left(304d^4-717d^3ex+479d^2e^2x^2-45de^3x^3-15e^4x^4\right)}{30e^6(ex-d)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/30*(Sqrt[d^2 - e^2*x^2]*(304*d^4 - 717*d^3*e*x + 479*d^2*e^2*x^2 - 45*d*
e^3*x^3 - 15*e^4*x^4))/(e^6*(-d + e*x)^3) - (13*d^2*Sqrt[-e^2]*Log[-(Sqrt[-
e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^7)
```

fricas [A] time = 0.43, size = 192, normalized size = 1.10

$$\frac{304\,d^2e^3x^3 - 912\,d^3e^2x^2 + 912\,d^4ex - 304\,d^5 + 390\left(d^2e^3x^3 - 3\,d^3e^2x^2 + 3\,d^4ex - d^5\right)\arctan\left(-\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) + \left(15\,e^4x^4 + 45\,de^3x^3 - 479\,d^2e^2x^2 + 717\,d^3ex - 304\,d^4\right)\sqrt{-e^2x^2 + d^2}}{30\left(e^9x^3 - 3\,de^8x^2 + 3\,d^2e^7x - d^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(304*d^2*e^3*x^3 - 912*d^3*e^2*x^2 + 912*d^4*e*x - 304*d^5 + 390*(d^2*
e^3*x^3 - 3*d^3*e^2*x^2 + 3*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) + (15*e^4*x^4 + 45*d*e^3*x^3 - 479*d^2*e^2*x^2 + 717*d^3*e*x - 30
4*d^4)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 - 3*d*e^8*x^2 + 3*d^2*e^7*x - d^3*e^6
)
```

giac [A] time = 0.30, size = 118, normalized size = 0.68

$$-\frac{13}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d) - \frac{\left(304\,d^7e^{(-6)} + \left(195\,d^6e^{(-5)} - \left(760\,d^5e^{(-4)} + \left(455\,d^4e^{(-3)} - \left(570\,d^3e^{(-2)} + \left(299\,d^2e^{(-1)} - 15\,(xe + 6\,d)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}}{30\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -13/2*d^2*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/30*(304*d^7*e^(-6) + (195*d^6*e^(
-5) - (760*d^5*e^(-4) + (455*d^4*e^(-3) - (570*d^3*e^(-2) + (299*d^2*e^(-1)
 - 15*(x*e + 6*d)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 222, normalized size = 1.28

$$-\frac{e\,x^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} - \frac{3d\,x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{13d^2x^5}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{19d^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{76d^5x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} - \frac{13d^2x^3}{6\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} + \frac{152d^7}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6} + \frac{13d^2x}{2\sqrt{-e^2x^2+d^2}\,e^5} - \frac{13d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/2*e*x^7/(-e^2*x^2+d^2)^(5/2)+13/10/e*d^2*x^5/(-e^2*x^2+d^2)^(5/2)-13/6/e
^3*d^2*x^3/(-e^2*x^2+d^2)^(3/2)+13/2/e^5*d^2*x/(-e^2*x^2+d^2)^(1/2)-13/2/e^
5*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-3*d*x^6/(-e^2*
x^2+d^2)^(5/2)+19/e^2*d^3*x^4/(-e^2*x^2+d^2)^(5/2)-76/3/e^4*d^5*x^2/(-e^2*x
^2+d^2)^(5/2)+152/15/e^6*d^7/(-e^2*x^2+d^2)^(5/2)
```

maxima [B] time = 1.03, size = 305, normalized size = 1.75

$$-\frac{ex^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{13}{30}\,d^2ex\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right) - \frac{3\,dx^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} - \frac{13\,d^2x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)}{6\,e} + \frac{19\,d^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{76\,d^5x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{152\,d^7}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6} + \frac{26\,d^4x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5} - \frac{91\,d^2x}{30\sqrt{-e^2x^2+d^2}\,e^5} - \frac{13\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*e*x^7/(-e^2*x^2 + d^2)^(5/2) + 13/30*d^2*e*x*(15*x^4/((-e^2*x^2 + d^2)
^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 +
d^2)^(5/2)*e^6)) - 3*d*x^6/(-e^2*x^2 + d^2)^(5/2) - 13/6*d^2*x*(3*x^2/((-e^
2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + 19*d^3*x^
4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 76/3*d^5*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4)
+ 152/15*d^7/((-e^2*x^2 + d^2)^(5/2)*e^6) + 26/15*d^4*x/((-e^2*x^2 + d^2)^(
3/2)*e^5) - 91/30*d^2*x/(sqrt(-e^2*x^2 + d^2)*e^5) - 13/2*d^2*arcsin(e*x/d)
/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,(d+e\,x)^3}{\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\,(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**5*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.84 $\int \frac{x^4(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

Rubi [A] time = 0.32, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1635, 641, 217, 203}

$$\frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3*(d + e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (6*d^2*(d + e*x)^2)/(5*e^
5*(d^2 - e^2*x^2)^(3/2)) + (24*d*(d + e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2]) + S
qrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^4}{e^4}+\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}+\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{27d^4}{e^4}+\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{45d^4}{e^4}+\frac{15d^3x}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,d}{e^4}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\,\text{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2}}\right.}{e^5}$$

Mathematica [A] time = 0.20, size = 119, normalized size = 0.84

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(24d^3-57d^2ex+39de^2x^2-5e^3x^3\right)-15(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{5e^5(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(Sqrt[1 - (e^2*x^2)/d^2]*(24*d^3 - 57*d^2*e*x + 39*d*e^2*x^2 - 5
*e^3*x^3) - 15*(d - e*x)^3*ArcSin[(e*x)/d]))/(5*e^5*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.54, size = 108, normalized size = 0.76

$$-\frac{3d\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^6}-\frac{\sqrt{d^2-e^2x^2}\left(24d^3-57d^2ex+39de^2x^2-5e^3x^3\right)}{5e^5(ex-d)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/5*(Sqrt[d^2 - e^2*x^2]*(24*d^3 - 57*d^2*e*x + 39*d*e^2*x^2 - 5*e^3*x^3))
/(e^5*(-d + e*x)^3) - (3*d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*
x^2]])/e^6
```

fricas [A] time = 0.42, size = 177, normalized size = 1.25

$$\frac{24\,de^3x^3-72\,d^2e^2x^2+72\,d^3ex-24\,d^4+30\left(de^3x^3-3\,d^2e^2x^2+3\,d^3ex-d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(5\,e^3x^3-39\,de^2x^2+57\,d^2ex-24\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(e^8x^3-3\,de^7x^2+3\,d^2e^6x-d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(24*d*e^3*x^3 - 72*d^2*e^2*x^2 + 72*d^3*e*x - 24*d^4 + 30*(d*e^3*x^3 -
3*d^2*e^2*x^2 + 3*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
+ (5*e^3*x^3 - 39*d*e^2*x^2 + 57*d^2*e*x - 24*d^3)*sqrt(-e^2*x^2 + d^2))/(e
^8*x^3 - 3*d*e^7*x^2 + 3*d^2*e^6*x - d^3*e^5)
```

giac [A] time = 0.29, size = 107, normalized size = 0.75

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)-\frac{\left(24\,d^6e^{(-5)}+\left(15\,d^5e^{(-4)}-\left(60\,d^4e^{(-3)}+\left(35\,d^3e^{(-2)}-\left(45\,d^2e^{(-1)}-(5\,xe-24\,d)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}}{5\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -3*d*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/5*(24*d^6*e^(-5) + (15*d^5*e^(-4) - (6
0*d^4*e^(-3) + (35*d^3*e^(-2) - (45*d^2*e^(-1) - (5*x*e - 24*d)*x)*x)*x)*x)
*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.01, size = 262, normalized size = 1.85

$$-\frac{e\,x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3d\,x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{9d^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d^3x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{12d^4x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{3d^5x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}-\frac{d\,x^3}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}+\frac{24d^6}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}+\frac{d^3x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}+\frac{16dx}{5\sqrt{-e^2x^2+d^2}\,e^4}-\frac{3d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -e*x^6/(-e^2*x^2+d^2)^(5/2)+9/e*d^2*x^4/(-e^2*x^2+d^2)^(5/2)-12/e^3*d^4*x^2
/(-e^2*x^2+d^2)^(5/2)+24/5/e^5*d^6/(-e^2*x^2+d^2)^(5/2)+3/5*d*x^5/(-e^2*x^2
+d^2)^(5/2)-d/e^2*x^3/(-e^2*x^2+d^2)^(3/2)+16/5*d/e^4*x/(-e^2*x^2+d^2)^(1/2
)-3*d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/2*d^3*x^
3/e^2/(-e^2*x^2+d^2)^(5/2)-3/10*d^5/e^4*x/(-e^2*x^2+d^2)^(5/2)+1/10*d^3/e^4
*x/(-e^2*x^2+d^2)^(3/2)
```

maxima [B] time = 1.02, size = 324, normalized size = 2.28

$$\frac{1}{5}\,de^2x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{ex^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-dx\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)+\frac{9\,d^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d^3x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{12\,d^4x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{3\,d^5x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{24\,d^6}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}+\frac{9\,d^3x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}-\frac{6\,dx}{5\sqrt{-e^2x^2+d^2}\,e^4}-\frac{3\,d\arcsin\left(\frac{ex}{d}\right)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*d*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - e*x^6/(-e^2*x^2 + d
^2)^(5/2) - d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^
2)^(3/2)*e^4)) + 9*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d^3*x^3/((-e^2*
x^2 + d^2)^(5/2)*e^2) - 12*d^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^5*
x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 24/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^5) + 9/
10*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 6/5*d*x/(sqrt(-e^2*x^2 + d^2)*e^4)
- 3*d*arcsin(e*x/d)/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(d+e\,x\right)}^3}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} (d + ex)^{3}}{(-(-d + ex)(d + ex))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**4*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.85 $\int \frac{x^3(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1635, 778, 217, 203}

$$\frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^2*(d + e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d + e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d + e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) - Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^3}{e^3}+\frac{5d^2x}{e^2}+\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\left(\frac{17d^3}{e^3}+\frac{15d^2x}{e^2}\right)(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x}}\right.}{e^3}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Mathematica [A] time = 0.15, size = 112, normalized size = 0.95

$$\frac{(d+ex)\left(d\left(22d^2-51dex+32e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}-15(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{15de^4(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(d*(22*d^2 - 51*d*e*x + 32*e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^2] - 15
*(d - e*x)^3*ArcSin[(e*x)/d]))/(15*d*e^4*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sq
rt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.62, size = 96, normalized size = 0.81

$$-\frac{\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^5}-\frac{\sqrt{d^2-e^2x^2}\left(22d^2-51dex+32e^2x^2\right)}{15e^4(ex-d)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(22*d^2 - 51*d*e*x + 32*e^2*x^2))/(e^4*(-d + e*x
)^3) - (Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^5
```

fricas [A] time = 0.40, size = 161, normalized size = 1.36

$$\frac{22\,e^3x^3-66\,de^2x^2+66\,d^2ex-22\,d^3+30\left(e^3x^3-3\,de^2x^2+3\,d^2ex-d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(32\,e^2x^2-51\,dex+22\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(e^7x^3-3\,de^6x^2+3\,d^2e^5x-d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(22*e^3*x^3 - 66*d*e^2*x^2 + 66*d^2*e*x - 22*d^3 + 30*(e^3*x^3 - 3*d*e
^2*x^2 + 3*d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (32*e
^2*x^2 - 51*d*e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 - 3*d*e^6*x^2 +
3*d^2*e^5*x - d^3*e^4)
```

giac [A] time = 0.29, size = 95, normalized size = 0.81

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d) - \frac{\left(22\,d^5e^{(-4)} + \left(15\,d^4e^{(-3)} - \left(55\,d^3e^{(-2)} + \left(35\,d^2e^{(-1)} - (32\,xe + 45\,d)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}}{15\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-4)*sgn(d) - 1/15*(22*d^5*e^(-4) + (15*d^4*e^(-3) - (55*d
^3*e^(-2) + (35*d^2*e^(-1) - (32*x*e + 45*d)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^
2)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.01, size = 234, normalized size = 1.98

$$\frac{e\,x^5}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{3d\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{3d^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} - \frac{11d^3x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{9d^4x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} - \frac{x^3}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e} + \frac{22d^5}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{3d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} + \frac{8x}{5\sqrt{-e^2x^2+d^2}\,e^3} - \frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*e*x^5/(-e^2*x^2+d^2)^(5/2)-1/3/e*x^3/(-e^2*x^2+d^2)^(3/2)+8/5/e^3*x/(-e
^2*x^2+d^2)^(1/2)-1/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)+3*d*x^4/(-e^2*x^2+d^2)^(5/2)-11/3/e^2*d^3*x^2/(-e^2*x^2+d^2)^(5/2)+22/1
5/e^4*d^5/(-e^2*x^2+d^2)^(5/2)+3/2/e*d^2*x^3/(-e^2*x^2+d^2)^(5/2)-9/10/e^3*
d^4*x/(-e^2*x^2+d^2)^(5/2)+3/10/e^3*d^2*x/(-e^2*x^2+d^2)^(3/2)
```

maxima [B] time = 1.02, size = 296, normalized size = 2.51

$$\frac{1}{15}e^3x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right) - \frac{1}{3}ex\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right) + \frac{3\,dx^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{3\,d^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} - \frac{11\,d^3x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{9\,d^4x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} + \frac{22\,d^5}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{17\,d^2x}{30\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} + \frac{2\,x}{15\sqrt{-e^2x^2+d^2}\,e^3} - \frac{\arcsin\left(\frac{ex}{d}\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^3*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e*x*(3*x^2/((-e^
2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 3*d*x^4/(-e
^2*x^2 + d^2)^(5/2) + 3/2*d^2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - 11/3*d^3*x^2
/((-e^2*x^2 + d^2)^(5/2)*e^2) - 9/10*d^4*x/((-e^2*x^2 + d^2)^(5/2)*e^3) + 2
2/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e^4) + 17/30*d^2*x/((-e^2*x^2 + d^2)^(3/2)
*e^3) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*e^3) - arcsin(e*x/d)/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d+e\,x\right)}^3}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.86 $\int \frac{x^2(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=93

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1635, 789, 637}

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d*(d + e*x)^3)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (8*(d + e*x)^2)/(15*e^3*(d^
2 - e^2*x^2)^(3/2)) + (7*(d + e*x))/(15*d*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d(d+ex)^3}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{\left(\frac{3d^2}{e^2}+\frac{5dx}{e}\right)(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$
$$= \frac{d(d+ex)^3}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{8(d+ex)^2}{15e^3(d^2-e^2x^2)^{3/2}} + \frac{7\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx}{15e^2}$$
$$= \frac{d(d+ex)^3}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{8(d+ex)^2}{15e^3(d^2-e^2x^2)^{3/2}} + \frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.08, size = 58, normalized size = 0.62

$$\frac{(d+ex)\left(2d^2-6dex+7e^2x^2\right)}{15de^3(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(2*d^2 - 6*d*e*x + 7*e^2*x^2))/(15*d*e^3*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.44, size = 53, normalized size = 0.57

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^2-6dex+7e^2x^2\right)}{15de^3(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 - 6*d*e*x + 7*e^2*x^2))/(15*d*e^3*(d - e*x)^3)
```

fricas [A] time = 0.40, size = 106, normalized size = 1.14

$$\frac{2\,e^3x^3-6\,de^2x^2+6\,d^2ex-2\,d^3-\left(7\,e^2x^2-6\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(de^6x^3-3\,d^2e^5x^2+3\,d^3e^4x-d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*e^3*x^3 - 6*d*e^2*x^2 + 6*d^2*e*x - 2*d^3 - (7*e^2*x^2 - 6*d*e*x +
2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 - 3*d^2*e^5*x^2 + 3*d^3*e^4*x - d^4
*e^3)
```

giac [A] time = 0.28, size = 72, normalized size = 0.77

$$-\frac{\left(2\,d^4e^{(-3)}-\left(5\,d^2e^{(-1)}-\left(x\left(\frac{7\,xe^2}{d}+15\,e\right)+5\,d\right)x\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(2*d^4*e^(-3) - (5*d^2*e^(-1) - (x*(7*x*e^2/d + 15*e) + 5*d)*x)*x^2)*
sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 55, normalized size = 0.59

$$\frac{(-ex+d)(ex+d)^4 \left(7e^2x^2-6dex+2d^2\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}} d\, e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(7*e^2*x^2-6*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(7/2
)
```

maxima [A] time = 0.45, size = 154, normalized size = 1.66

$$\frac{ex^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3\,dx^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{7\,d^3x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}+\frac{2\,d^4}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}+\frac{7\,dx}{30\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}+\frac{7\,x}{15\sqrt{-e^2x^2+d^2}\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] e*x^4/(-e^2*x^2 + d^2)^(5/2) + 3/2*d*x^3/(-e^2*x^2 + d^2)^(5/2) - 1/3*d^2*x
^2/((-e^2*x^2 + d^2)^(5/2)*e) - 7/10*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 2
/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^3) + 7/30*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2
) + 7/15*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)
```

mupad [B] time = 2.69, size = 49, normalized size = 0.53

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2-6\,d\,e\,x+7\,e^2\,x^2\right)}{15\,d\,e^3\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^2 + 7*e^2*x^2 - 6*d*e*x))/(15*d*e^3*(d - e*x)^3
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.87 $\int \frac{x(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} dx$

Optimal. Leaf size=86

$$\frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {789, 653, 191}

$$\frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^3/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(5*e^2*(d^2 - e^2
*x^2)^(3/2)) - x/(5*d^2*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} dx &= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{3\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}} dx}{5e}\\
&= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}} dx}{5e}\\
&= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 55, normalized size = 0.64

$$-\frac{(d+ex)\left(d^2-3dex+e^2x^2\right)}{5d^2e^2(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
[Out] -1/5*((d + e*x)*(d^2 - 3*d*e*x + e^2*x^2))/(d^2*e^2*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.42, size = 53, normalized size = 0.62

$$\frac{\sqrt{d^2-e^2x^2}\left(-d^2+3dex-e^2x^2\right)}{5d^2e^2(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 + 3*d*e*x - e^2*x^2))/(5*d^2*e^2*(d - e*x)^3)
```

fricas [A] time = 0.39, size = 104, normalized size = 1.21

$$-\frac{e^3x^3-3\,de^2x^2+3\,d^2ex-d^3-\left(e^2x^2-3\,dex+d^2\right)\sqrt{-e^2x^2+d^2}}{5\left(d^2e^5x^3-3\,d^3e^4x^2+3\,d^4e^3x-d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
[Out] -1/5*(e^3*x^3 - 3*d*e^2*x^2 + 3*d^2*e*x - d^3 - (e^2*x^2 - 3*d*e*x + d^2)*s
qrt(-e^2*x^2 + d^2))/(d^2*e^5*x^3 - 3*d^3*e^4*x^2 + 3*d^4*e^3*x - d^5*e^2)
```

giac [A] time = 0.31, size = 60, normalized size = 0.70

$$\frac{\left(d^3e^{(-2)}+\left(x\left(\frac{x^2e^3}{d^2}-5\,e\right)-5\,d\right)x^2\right)\sqrt{-x^2e^2+d^2}}{5\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] 1/5*(d^3*e^(-2) + (x*(x^2*e^3/d^2 - 5*e) - 5*d)*x^2)*sqrt(-x^2*e^2 + d^2)/(
x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 52, normalized size = 0.60

$$-\frac{(-ex+d)\,(ex+d)^4\left(e^2x^2-3dex+d^2\right)}{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
[Out] -1/5*(-e*x+d)*(e*x+d)^4*(e^2*x^2-3*d*e*x+d^2)/d^2/e^2/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.44, size = 128, normalized size = 1.49

$$\frac{ex^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{dx^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3\,d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{d^3}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}-\frac{x}{5\sqrt{-e^2x^2+d^2}\,d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^3/(-e^2*x^2 + d^2)^(5/2) + d*x^2/(-e^2*x^2 + d^2)^(5/2) + 3/10*d^2*
x/((-e^2*x^2 + d^2)^(5/2)*e) - 1/5*d^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/10*
x/((-e^2*x^2 + d^2)^(3/2)*e) - 1/5*x/(sqrt(-e^2*x^2 + d^2)*d^2*e)
```

mupad [B] time = 2.66, size = 46, normalized size = 0.53

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2-3\,d\,e\,x+e^2\,x^2\right)}{5\,d^2\,e^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 - 3*d*e*x))/(5*d^2*e^2*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(d+ex\right)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.88 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {655, 659, 651}

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} \, dx &= \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}} \, dx\\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}} \, dx}{5d}\\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}} \, dx}{15d^2}\\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.56

$$\frac{(d+ex)\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.00, size = 53, normalized size = 0.51

$$\frac{\sqrt{d^2-e^2x^2}\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^3)
```

fricas [A] time = 0.40, size = 106, normalized size = 1.03

$$\frac{7\,e^3x^3-21\,de^2x^2+21\,d^2ex-7\,d^3-\left(2\,e^2x^2-6\,dex+7\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^4x^3-3\,d^4e^3x^2+3\,d^5e^2x-d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
+ 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x -
 d^6*e)
```

giac [A] time = 0.29, size = 70, normalized size = 0.68

$$-\frac{\sqrt{-x^2e^2+d^2}\left(7\,d^2e^{(-1)}+\left(\left(x\left(\frac{2\,x^2e^4}{d^3}-\frac{5\,e^2}{d}\right)+5\,e\right)x+15\,d\right)x\right)}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(7*d^2*e^(-1) + ((x*(2*x^2*e^4/d^3 - 5*e^2/d) +
5*e)*x + 15*d)*x)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 55, normalized size = 0.53

$$\frac{(-ex+d)\,(ex+d)^4\left(2e^2x^2-6dex+7d^2\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/e/(-e^2*x^2+d^2)^(7/2
)
```

maxima [A] time = 0.44, size = 101, normalized size = 0.98

$$\frac{ex^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{4\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{7\,d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}+\frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^
2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

mupad [B] time = 2.66, size = 49, normalized size = 0.48

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^3
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.89 $\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=114

$$\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Rubi [A] time = 0.16, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 823, 12, 266, 63, 208}

$$\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}}+\frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*(d + e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d + 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d + 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-11d^2ex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^5e^2-22d^4e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^7e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x\right.}{2d^3} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2}\right.}{d^3e^2} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 81, normalized size = 0.71

$$
\frac{9d^5+45d^4ex-55d^2e^3x^3+3d^5\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+22e^5x^5}{15d^4\left(d^2-e^2x^2\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (9*d^5 + 45*d^4*e*x - 55*d^2*e^3*x^3 + 22*e^5*x^5 + 3*d^5*Hypergeometric2F1
[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^4*(d^2 - e^2*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.67, size = 93, normalized size = 0.82

$$\frac{\sqrt{d^2-e^2x^2}\left(32d^2-51dex+22e^2x^2\right)}{15d^4(d-ex)^3}+\frac{2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^2 - 51*d*e*x + 22*e^2*x^2))/(15*d^4*(d - e*x)^3)
 + (2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^4
```

fricas [A] time = 0.41, size = 158, normalized size = 1.39

$$\frac{32\,e^3x^3-96\,de^2x^2+96\,d^2ex-32\,d^3+15\left(e^3x^3-3\,de^2x^2+3\,d^2ex-d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(22\,e^2x^2-51\,dex+32\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(d^4e^3x^3-3\,d^5e^2x^2+3\,d^6ex-d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(32*e^3*x^3 - 96*d*e^2*x^2 + 96*d^2*e*x - 32*d^3 + 15*(e^3*x^3 - 3*d*e
^2*x^2 + 3*d^2*e*x - d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (22*e^2*x^2
- 51*d*e*x + 32*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 - 3*d^5*e^2*x^2 + 3
*d^6*e*x - d^7)
```

giac [A] time = 0.29, size = 117, normalized size = 1.03

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{22\,xe^5}{d^4}+\frac{15\,e^4}{d^3}\right)-\frac{55\,e^3}{d^2}\right)x-\frac{35\,e^2}{d}\right)x+45\,e\right)x+32\,d\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(22*x*e^5/d^4 + 15*e^4/d^3) - 55*e^3/d^2)*
x - 35*e^2/d)*x + 45*e)*x + 32*d)/(x^2*e^2 - d^2)^3 - log(1/2*abs(-2*d*e -
2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^4
```

maple [A] time = 0.01, size = 158, normalized size = 1.39

$$\frac{4ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{4d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{11ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^3}+\frac{22ex}{15\sqrt{-e^2x^2+d^2}\,d^4}+\frac{1}{\sqrt{-e^2x^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*e*x/(-e^2*x^2+d^2)^(5/2)+11/15*e/d^2*x/(-e^2*x^2+d^2)^(3/2)+22/15*e/d^4
*x/(-e^2*x^2+d^2)^(1/2)+4/5*d/(-e^2*x^2+d^2)^(5/2)+1/3/d/(-e^2*x^2+d^2)^(3/
2)+1/d^3/(-e^2*x^2+d^2)^(1/2)-1/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.46, size = 152, normalized size = 1.33

$$\frac{4\,ex}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{4\,d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{11\,ex}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}+\frac{22\,ex}{15\,\sqrt{-e^2x^2+d^2}\,d^4}-\frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d^4}+\frac{1}{\sqrt{-e^2x^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 4/5*e*x/(-e^2*x^2 + d^2)^(5/2) + 4/5*d/(-e^2*x^2 + d^2)^(5/2) + 11/15*e*x/(
(-e^2*x^2 + d^2)^(3/2)*d^2) + 1/3/((-e^2*x^2 + d^2)^(3/2)*d) + 22/15*e*x/(s
qrt(-e^2*x^2 + d^2)*d^4) - log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(
x))/d^4 + 1/(sqrt(-e^2*x^2 + d^2)*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^3}{x\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{x\,(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.90 $\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.29, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1805, 807, 266, 63, 208}

$$\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*e*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (e*(5*d + 7*e*x))/(5*d^3*(d^2
 - e^2*x^2)^(3/2)) + (e*(15*d + 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqrt
[d^2 - e^2*x^2]/(d^5*x) - (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-15d^2ex-16de^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3+45d^2ex+42de^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3-45d^2ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{(3e)\int \frac{}{x\sqrt{d^2}}}{d^4}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{(3e)\,\mathrm{Subst}}{}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{3\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{3e\tanh^{-1}\left(\right.}{d^5}$$

Mathematica [C] time = 0.06, size = 96, normalized size = 0.66

$$\frac{-5d^6+d^5ex+45d^4e^2x^2-60d^2e^4x^4+3d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+24e^6x^6}{5d^5x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-5*d^6 + d^5*e*x + 45*d^4*e^2*x^2 - 60*d^2*e^4*x^4 + 24*e^6*x^6 + 3*d^5*e*
x*Hypergeometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(5*d^5*x*(d^2 - e^2*
x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.59, size = 108, normalized size = 0.74

$$\frac{6e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{\sqrt{d^2-e^2x^2}\left(-5d^3+39d^2ex-57de^2x^2+24e^3x^3\right)}{5d^5x(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^3 + 39*d^2*e*x - 57*d*e^2*x^2 + 24*e^3*x^3))/(5*
d^5*x*(d - e*x)^3) + (6*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d]
)/d^5
```

fricas [A] time = 0.42, size = 184, normalized size = 1.27

$$\frac{24\,e^4x^4 - 72\,de^3x^3 + 72\,d^2e^2x^2 - 24\,d^3ex + 15\left(e^4x^4 - 3\,de^3x^3 + 3\,d^2e^2x^2 - d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \left(24\,e^3x^3 - 57\,de^2x^2 + 39\,d^2ex - 5\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(d^5e^3x^4 - 3\,d^6e^2x^3 + 3\,d^7ex^2 - d^8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(24*e^4*x^4 - 72*d*e^3*x^3 + 72*d^2*e^2*x^2 - 24*d^3*e*x + 15*(e^4*x^4
- 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x)
 - (24*e^3*x^3 - 57*d*e^2*x^2 + 39*d^2*e*x - 5*d^3)*sqrt(-e^2*x^2 + d^2))/(
d^5*e^3*x^4 - 3*d^6*e^2*x^3 + 3*d^7*e*x^2 - d^8*x)
```

giac [A] time = 0.29, size = 185, normalized size = 1.28

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{19\,xe^6}{d^5}+\frac{15\,e^5}{d^4}\right)-\frac{45\,e^4}{d^3}\right)x-\frac{35\,e^3}{d^2}\right)x+\frac{30\,e^2}{d}\right)x+24\,e\right)}{5\left(x^2e^2-d^2\right)^3} - \frac{3\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^5} + \frac{xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^5} - \frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{2\,d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*((((x*(19*x*e^6/d^5 + 15*e^5/d^4) - 45*e^4/d^3)*x
 - 35*e^3/d^2)*x + 30*e^2/d)*x + 24*e)/(x^2*e^2 - d^2)^3 - 3*e*log(1/2*abs(
-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^5 + 1/2*x*e^3/((d*e + s
qrt(-x^2*e^2 + d^2)*e)*d^5) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1)/(d^
5*x)
```

maple [A] time = 0.01, size = 190, normalized size = 1.31

$$\frac{9e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d} + \frac{4e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} - \frac{d}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x} + \frac{12e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3} + \frac{e}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2} - \frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^4} + \frac{24e^2x}{5\sqrt{-e^2x^2+d^2}\,d^5} + \frac{3e}{\sqrt{-e^2x^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*e/(-e^2*x^2+d^2)^(5/2)+9/5*e^2/d*x/(-e^2*x^2+d^2)^(5/2)+12/5*e^2/d^3*x/
(-e^2*x^2+d^2)^(3/2)+24/5*e^2/d^5*x/(-e^2*x^2+d^2)^(1/2)-d/x/(-e^2*x^2+d^2)
^(5/2)+e/d^2/(-e^2*x^2+d^2)^(3/2)+3*e/d^4/(-e^2*x^2+d^2)^(1/2)-3*e/d^4/(d^2
)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

maxima [A] time = 0.47, size = 184, normalized size = 1.27

$$\frac{9\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d} + \frac{4\,e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{12\,e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3} + \frac{e}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2} - \frac{d}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x} + \frac{24\,e^2x}{5\,\sqrt{-e^2x^2+d^2}\,d^5} - \frac{3\,e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d^5} + \frac{3\,e}{\sqrt{-e^2x^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 9/5*e^2*x/((-e^2*x^2 + d^2)^(5/2)*d) + 4/5*e/(-e^2*x^2 + d^2)^(5/2) + 12/5*
e^2*x/((-e^2*x^2 + d^2)^(3/2)*d^3) + e/((-e^2*x^2 + d^2)^(3/2)*d^2) - d/((-
e^2*x^2 + d^2)^(5/2)*x) + 24/5*e^2*x/(sqrt(-e^2*x^2 + d^2)*d^5) - 3*e*log(2
*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 + 3*e/(sqrt(-e^2*x^2 + d
^2)*d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^3}{x^2\,{\left(d^2-e^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{x^2\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.91 $\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=182

$$\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.36, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 1807, 807, 266, 63, 208}

$$\frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*e^2*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d + 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d + 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) - (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-15d^2ex-20de^2x^2-16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3+45d^2ex+75de^2x^2+62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3-45d^2ex-90de^2x^2 \cdots}{x^3\sqrt{d^2-e^2x^2}}\,d\cdots}{15d^6}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{90d^4e+\cdots}{x^2\sqrt{\cdots}}}{3\cdots}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-\cdots}}{d^6\cdots}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-\cdots}}{d^6\cdots}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-\cdots}}{d^6\cdots}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-\cdots}}{d^6\cdots}$$

Mathematica [C] time = 0.07, size = 119, normalized size = 0.65

$$\frac{e\left(-45d^6+285d^4e^2x^2-380d^2e^4x^4+9d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+3d^5ex\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+152e^6x^6\right)}{15d^6x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (e*(-45*d^6 + 285*d^4*e^2*x^2 - 380*d^2*e^4*x^4 + 152*e^6*x^6 + 9*d^5*e*x*H
ypergeometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2] + 3*d^5*e*x*Hypergeometr
ic2F1[-5/2, 2, -3/2, 1 - (e^2*x^2)/d^2]))/(15*d^6*x*(d^2 - e^2*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.72, size = 121, normalized size = 0.66

$$\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{\sqrt{d^2-e^2x^2}\left(-15d^4-45d^3ex+479d^2e^2x^2-717de^3x^3+304e^4x^4\right)}{30d^6x^2(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-15*d^4 - 45*d^3*e*x + 479*d^2*e^2*x^2 - 717*d*e^3*x^
3 + 304*e^4*x^4))/(30*d^6*x^2*(d - e*x)^3) + (13*e^2*ArcTanh[(Sqrt[-e^2]*x)
/d - Sqrt[d^2 - e^2*x^2]/d])/d^6
```

fricas [A] time = 0.42, size = 205, normalized size = 1.13

$$\frac{254\,e^5x^5-762\,de^4x^4+762\,d^2e^3x^3-254\,d^3e^2x^2+195\left(e^5x^5-3\,de^4x^4+3\,d^2e^3x^3-d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(304\,e^4x^4-717\,de^3x^3+479\,d^2e^2x^2-45\,d^3ex-15\,d^4\right)\sqrt{-e^2x^2+d^2}}{30\left(d^6e^3x^5-3\,d^7e^2x^4+3\,d^8ex^3-d^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(254*e^5*x^5 - 762*d*e^4*x^4 + 762*d^2*e^3*x^3 - 254*d^3*e^2*x^2 + 195
*(e^5*x^5 - 3*d*e^4*x^4 + 3*d^2*e^3*x^3 - d^3*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) - (304*e^4*x^4 - 717*d*e^3*x^3 + 479*d^2*e^2*x^2 - 45*d^3*e*
x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^5 - 3*d^7*e^2*x^4 + 3*d^8*e*x^
3 - d^9*x^2)
```

giac [A] time = 0.33, size = 259, normalized size = 1.42

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{107\,xe^7}{d^6}+\frac{90\,e^6}{d^5}\right)-\frac{245\,e^5}{d^4}\right)x-\frac{205\,e^4}{d^3}\right)x+\frac{150\,e^3}{d^2}\right)x+\frac{127\,e^2}{d}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{13\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^6}+\frac{x^2\left(\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^4}{x}+e^6\right)}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^6}-\frac{\left(\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^6e^8}{x}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^6e^6}{x^2}\right)e^{(-8)}}{8\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(107*x*e^7/d^6 + 90*e^6/d^5) - 245*e^5/d^4
)*x - 205*e^4/d^3)*x + 150*e^3/d^2)*x + 127*e^2/d)/(x^2*e^2 - d^2)^3 - 13/2
*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6 + 1/
8*x^2*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^2
 + d^2)*e)^2*d^6) - 1/8*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6*e^8/x + (d*e
 + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*e^6/x^2)*e^(-8)/d^12
```

maple [A] time = 0.01, size = 222, normalized size = 1.22

$$\frac{19e^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}+\frac{13e^2}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{76e^3x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}-\frac{3e}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}-\frac{d}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}+\frac{13e^2}{6\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}-\frac{13e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^5}+\frac{152e^3x}{15\sqrt{-e^2x^2+d^2}\,d^6}+\frac{13e^2}{2\sqrt{-e^2x^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 19/5*e^3*x/d^2/(-e^2*x^2+d^2)^(5/2)+76/15*e^3/d^4*x/(-e^2*x^2+d^2)^(3/2)+15
2/15*e^3/d^6*x/(-e^2*x^2+d^2)^(1/2)-3*e/x/(-e^2*x^2+d^2)^(5/2)+13/10*e^2/d/
(-e^2*x^2+d^2)^(5/2)+13/6*e^2/d^3/(-e^2*x^2+d^2)^(3/2)+13/2*e^2/d^5/(-e^2*x
^2+d^2)^(1/2)-13/2*e^2/d^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^
2)^(1/2))/x)-1/2*d/x^2/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.48, size = 216, normalized size = 1.19

$$\frac{19\,e^3x}{5\,{\left(-e^2x^2+d^2\right)}^{\frac{5}{2}}d^2}+\frac{13\,e^2}{10\,{\left(-e^2x^2+d^2\right)}^{\frac{5}{2}}d}+\frac{76\,e^3x}{15\,{\left(-e^2x^2+d^2\right)}^{\frac{3}{2}}d^4}+\frac{13\,e^2}{6\,{\left(-e^2x^2+d^2\right)}^{\frac{3}{2}}d^3}-\frac{3\,e}{{\left(-e^2x^2+d^2\right)}^{\frac{5}{2}}x}+\frac{152\,e^3x}{15\,\sqrt{-e^2x^2+d^2}\,d^6}-\frac{13\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{2\,d^6}+\frac{13\,e^2}{2\,\sqrt{-e^2x^2+d^2}\,d^5}-\frac{d}{2\,{\left(-e^2x^2+d^2\right)}^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 19/5*e^3*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 13/10*e^2/((-e^2*x^2 + d^2)^(5/2)
*d) + 76/15*e^3*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 13/6*e^2/((-e^2*x^2 + d^2)
^(3/2)*d^3) - 3*e/((-e^2*x^2 + d^2)^(5/2)*x) + 152/15*e^3*x/(sqrt(-e^2*x^2
+ d^2)*d^6) - 13/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/
d^6 + 13/2*e^2/(sqrt(-e^2*x^2 + d^2)*d^5) - 1/2*d/((-e^2*x^2 + d^2)^(5/2)*x
^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^3}{x^3\,{\left(d^2-e^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

3.92 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=147

$$\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2}+\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5}+\frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5}$$

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 833, 780, 217, 203}

$$\frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5}+\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (4*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(4*e
^2) + (x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (d^3*(64*d - 45*e*x)*Sqrt[d^2 - e^2
*x^2])/(120*e^5) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= \int \frac{x^4(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{\int \frac{x^3(4d^2e-5de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2} \\
&= -\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{\int \frac{x^2(15d^3e^2-16d^2e^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4} \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{\int \frac{x(32d^4e^3-45d^3e^4x)}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6} \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \cdots \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \cdots \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \cdots
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 91, normalized size = 0.62

$$\frac{45d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(64d^4-45d^3ex+32d^2e^2x^2-30de^3x^3+24e^4x^4\right)}{120e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(64*d^4 - 45*d^3*e*x + 32*d^2*e^2*x^2 - 30*d*e^3*x^3 +
 24*e^4*x^4) + 45*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(120*e^5)
```

IntegrateAlgebraic [A] time = 0.26, size = 114, normalized size = 0.78

$$\frac{3d^5\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e^6}+\frac{\sqrt{d^2-e^2x^2}\left(64d^4-45d^3ex+32d^2e^2x^2-30de^3x^3+24e^4x^4\right)}{120e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(64*d^4 - 45*d^3*e*x + 32*d^2*e^2*x^2 - 30*d*e^3*x^3 +
 24*e^4*x^4))/(120*e^5) + (3*d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2
- e^2*x^2]])/(8*e^6)
```

fricas [A] time = 0.41, size = 95, normalized size = 0.65

$$-\frac{90\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,e^4x^4-30\,de^3x^3+32\,d^2e^2x^2-45\,d^3ex+64\,d^4\right)\sqrt{-e^2x^2+d^2}}{120\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/120*(90*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (24*e^4*x^4 - 30
*d*e^3*x^3 + 32*d^2*e^2*x^2 - 45*d^3*e*x + 64*d^4)*sqrt(-e^2*x^2 + d^2))/e^
5
```

giac [A] time = 0.20, size = 77, normalized size = 0.52

$$\frac{3}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)+\frac{1}{120}\left(64\,d^4e^{(-5)}-\left(45\,d^3e^{(-4)}-2\left(16\,d^2e^{(-3)}+3\left(4\,xe^{(-1)}-5\,de^{(-2)}\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 3/8*d^5*arcsin(x*e/d)*e^(-5)*sgn(d) + 1/120*(64*d^4*e^(-5) - (45*d^3*e^(-4)
 - 2*(16*d^2*e^(-3) + 3*(4*x*e^(-1) - 5*d*e^(-2))*x)*x)*x)*sqrt(-x^2*e^2 +
d^2)
```

maple [A] time = 0.02, size = 208, normalized size = 1.41

$$\frac{d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e^4}-\frac{5d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^4}-\frac{5\sqrt{-e^2x^2+d^2}\,d^3x}{8e^4}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^4}{e^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x^2}{5e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4e^4}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{15e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/5/e^3*x^2*(-e^2*x^2+d^2)^(3/2)-7/15*d^2/e^5*(-e^2*x^2+d^2)^(3/2)+1/4*d/e
^4*x*(-e^2*x^2+d^2)^(3/2)-5/8*d^3/e^4*x*(-e^2*x^2+d^2)^(1/2)-5/8*d^5/e^4/(e
^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+d^4/e^5*(-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(1/2)+d^5/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(1/2))
```

maxima [A] time = 0.99, size = 125, normalized size = 0.85

$$\frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)}{8\,e^5}-\frac{5\sqrt{-e^2x^2+d^2}\,d^3x}{8\,e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x^2}{5\,e^3}+\frac{\sqrt{-e^2x^2+d^2}\,d^4}{e^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4\,e^4}-\frac{7\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{15\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*d^5*arcsin(e*x/d)/e^5 - 5/8*sqrt(-e^2*x^2 + d^2)*d^3*x/e^4 - 1/5*(-e^2*
x^2 + d^2)^(3/2)*x^2/e^3 + sqrt(-e^2*x^2 + d^2)*d^4/e^5 + 1/4*(-e^2*x^2 + d
^2)^(3/2)*d*x/e^4 - 7/15*(-e^2*x^2 + d^2)^(3/2)*d^2/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.93 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=118

$$-\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}-\frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 833, 780, 217, 203}

$$-\frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}-\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] -(d*x^2*Sqrt[d^2 - e^2*x^2])/(3*e^2) + (x^3*Sqrt[d^2 - e^2*x^2])/(4*e) - (d
^2*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^4) - (3*d^4*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(8*e^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^3(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{\int \frac{x^2\left(3d^2e-4de^2x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$
$$= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} + \frac{\int \frac{x\left(8d^3e^2-9d^2e^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4}$$
$$= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{\left(3d^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^3}$$
$$= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{\left(3d^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,\right.}{8e^3}$$
$$= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

Mathematica [A] time = 0.10, size = 80, normalized size = 0.68

$$\frac{\sqrt{d^2-e^2x^2}\left(-16d^3+9d^2ex-8de^2x^2+6e^3x^3\right)-9d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{24e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 + 9*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3) - 9*d^
4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e^4)
```

IntegrateAlgebraic [A] time = 0.27, size = 103, normalized size = 0.87

$$\frac{\sqrt{d^2-e^2x^2}\left(-16d^3+9d^2ex-8de^2x^2+6e^3x^3\right)}{24e^4} - \frac{3d^4\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 + 9*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3))/(24*e
^4) - (3*d^4*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*e^5)
```

fricas [A] time = 0.40, size = 83, normalized size = 0.70

$$\frac{18\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3-8\,de^2x^2+9\,d^2ex-16\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(18*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 - 8*d*e
^2*x^2 + 9*d^2*e*x - 16*d^3)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [A] time = 0.21, size = 66, normalized size = 0.56

$$-\frac{3}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{1}{24}\left(16\,d^3e^{(-4)}-\left(9\,d^2e^{(-3)}+2\left(3\,xe^{(-1)}-4\,de^{(-2)}\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -3/8*d^4*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/24*(16*d^3*e^(-4) - (9*d^2*e^(-3)
+ 2*(3*x*e^(-1) - 4*d*e^(-2))*x)*x)*sqrt(-x^2*e^2 + d^2)
```

maple [A] time = 0.01, size = 185, normalized size = 1.57

$$-\frac{d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e^3}+\frac{5d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^3}+\frac{5\sqrt{-e^2x^2+d^2}\,d^2x}{8e^3}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3}{e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x}{4e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{3e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/4/e^3*x*(-e^2*x^2+d^2)^(3/2)+5/8*d^2/e^3*x*(-e^2*x^2+d^2)^(1/2)+5/8/e^3*
d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/3*d/e^4*(-e^2*
x^2+d^2)^(3/2)-d^3/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-d^4/e^3/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 101, normalized size = 0.86

$$-\frac{3\,d^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^4}+\frac{5\,\sqrt{-e^2x^2+d^2}\,d^2x}{8\,e^3}-\frac{\sqrt{-e^2x^2+d^2}\,d^3}{e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}x}{4\,e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{3\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*d^4*arcsin(e*x/d)/e^4 + 5/8*sqrt(-e^2*x^2 + d^2)*d^2*x/e^3 - sqrt(-e^2
*x^2 + d^2)*d^3/e^4 - 1/4*(-e^2*x^2 + d^2)^(3/2)*x/e^3 + 1/3*(-e^2*x^2 + d^
2)^(3/2)*d/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.94 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=86

$$\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1639, 12, 785, 780, 217, 203}

$$\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (d*(2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (d^2 - e^2*x^2)^(3/2)/(3*e^3)
 + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] && !ILtQ[p - 1/2, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
```

```
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{\int \frac{3de^3x\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{3e^4}\\ &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{d\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{e}\\ &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{\int \frac{x\left(d^2e-de^2x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\ &= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\ &= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}\\ &= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}\end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^2-3dex+2e^2x^2\right)+3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^2 - 3*d*e*x + 2*e^2*x^2) + 3*d^3*ArcTan[(e*x)/Sqr
t[d^2 - e^2*x^2]])/(6*e^3)
```

IntegrateAlgebraic [A] time = 0.25, size = 92, normalized size = 1.07

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^2-3dex+2e^2x^2\right)}{6e^3}+\frac{d^3\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^2 - 3*d*e*x + 2*e^2*x^2))/(6*e^3) + (d^3*Sqrt[-e^
2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

fricas [A] time = 0.40, size = 73, normalized size = 0.85

$$-\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(2\,e^2x^2-3\,dex+4\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (2*e^2*x^2 - 3*d*e*
x + 4*d^2)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.20, size = 54, normalized size = 0.63

$$\frac{1}{2}\, d^3 \arcsin\left(\frac{xe}{d}\right) e^{(-3)}\mathrm{sgn}(d) + \frac{1}{6}\, \sqrt{-x^2e^2 + d^2}\left(4\, d^2e^{(-3)} + \left(2\, xe^{(-1)} - 3\, de^{(-2)}\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/6*sqrt(-x^2*e^2 + d^2)*(4*d^2*e^(-3
) + (2*x*e^(-1) - 3*d*e^(-2))*x)
```

maple [B] time = 0.01, size = 160, normalized size = 1.86

$$\frac{d^3 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\, e^2} - \frac{d^3 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\, e^2} - \frac{\sqrt{-e^2x^2 + d^2}\, dx}{2e^2} + \frac{\sqrt{2\left(x + \frac{d}{e}\right) de - \left(x + \frac{d}{e}\right)^2 e^2}\, d^2}{e^3} - \frac{\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)/e^3-1/2*(-e^2*x^2+d^2)^(1/2)*d/e^2*x-1/2/(e^2)^(1
/2)*d^3/e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+d^2/e^3*(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(1/2)+d^3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 77, normalized size = 0.90

$$\frac{d^3 \arcsin\left(\frac{ex}{d}\right)}{2\, e^3} - \frac{\sqrt{-e^2x^2 + d^2}\, dx}{2\, e^2} + \frac{\sqrt{-e^2x^2 + d^2}\, d^2}{e^3} - \frac{\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}}{3\, e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*arcsin(e*x/d)/e^3 - 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^2 + sqrt(-e^2*x^
2 + d^2)*d^2/e^3 - 1/3*(-e^2*x^2 + d^2)^(3/2)/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\, \sqrt{d^2 - e^2\, x^2}}{d + e\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{-\left(-d + ex\right)\left(d + ex\right)}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.95 $\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=62

$$-\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {785, 780, 217, 203}

$$-\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] -((2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (d^2*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(2*e^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{\int \frac{x(d^2e-de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{de}$$
$$= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e}$$
$$= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$
$$= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.92

$$\frac{(ex-2d)\sqrt{d^2-e^2x^2} - d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] ((-2*d + e*x)*Sqrt[d^2 - e^2*x^2] - d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
(2*e^2)
```

IntegrateAlgebraic [A] time = 0.22, size = 80, normalized size = 1.29

$$\frac{(ex-2d)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] ((-2*d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^
2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

fricas [A] time = 0.40, size = 60, normalized size = 0.97

$$\frac{2\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}\,(ex-2\,d)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2)
*(e*x - 2*d))/e^2
```

giac [A] time = 0.22, size = 43, normalized size = 0.69

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,\left(xe^{(-1)}-2\,de^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e^(-2)*sgn(d) + 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^(-1) -
 2*d*e^(-2))
```

maple [B] time = 0.01, size = 140, normalized size = 2.26

$$-\frac{d^2 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\, e}+\frac{d^2 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\, e}+\frac{\sqrt{-e^2x^2+d^2}\, x}{2e}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\, d}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*x*(-e^2*x^2+d^2)^(1/2)+1/2/e*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2
*x^2+d^2)^(1/2)*x)-d/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-d^2/e/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.97, size = 56, normalized size = 0.90

$$-\frac{d^2 \arcsin\left(\frac{ex}{d}\right)}{2\,e^2}+\frac{\sqrt{-e^2x^2+d^2}\,x}{2\,e}-\frac{\sqrt{-e^2x^2+d^2}\,d}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*d^2*arcsin(e*x/d)/e^2 + 1/2*sqrt(-e^2*x^2 + d^2)*x/e - sqrt(-e^2*x^2 +
 d^2)*d/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-(-d+ex)(d+ex)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.96 $\int \frac{\sqrt{d^2-e^2x^2}}{d+ex} dx$

Optimal. Leaf size=46

$$\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {665, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/e + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{d+ex} dx &= \frac{\sqrt{d^2-e^2x^2}}{e}+d\int \frac{1}{\sqrt{d^2-e^2x^2}} dx\\
&= \frac{\sqrt{d^2-e^2x^2}}{e}+d\,\text{Subst}\left(\int \frac{1}{1+e^2x^2} dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)\\
&= \frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2x^2}+d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
[Out] (Sqrt[d^2 - e^2*x^2] + d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

IntegrateAlgebraic [A] time = 0.20, size = 65, normalized size = 1.41

$$\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
[Out] Sqrt[d^2 - e^2*x^2]/e + (d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*
x^2]])/e^2
```

fricas [A] time = 0.39, size = 52, normalized size = 1.13

$$-\frac{2\,d\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\sqrt{-e^2x^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] -(2*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - sqrt(-e^2*x^2 + d^2))/e
```

giac [A] time = 0.21, size = 31, normalized size = 0.67

$$d\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}(d)+\sqrt{-x^2e^2+d^2}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] d*arcsin(x*e/d)*e^(-1)*sgn(d) + sqrt(-x^2*e^2 + d^2)*e^(-1)
```

maple [A] time = 0.00, size = 77, normalized size = 1.67

$$\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
[Out] 1/e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.97, size = 31, normalized size = 0.67

$$\frac{d\arcsin\left(\frac{ex}{d}\right)}{e}+\frac{\sqrt{-e^2x^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] d*arcsin(e*x/d)/e + sqrt(-e^2*x^2 + d^2)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d^2 - e^2 x^2}}{d + e x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{d + ex} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

3.97 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx$

Optimal. Leaf size=46

$$-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 844, 217, 203, 266, 63, 208}

$$-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

```
[Out] -ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx &= \int \frac{d-ex}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= d\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - e\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)\\
&= -\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2}\\
&= -\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.00

$$-\log\left(\sqrt{d^2-e^2x^2}+d\right) - \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

```
[Out] -ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + Log[x] - Log[d + Sqrt[d^2 - e^2*x^2]]
```

IntegrateAlgebraic [A] time = 0.21, size = 84, normalized size = 1.83

$$2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

```
[Out] 2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (Sqrt[-e^2]*Log[-(Sqr
t[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e
```

fricas [A] time = 0.43, size = 54, normalized size = 1.17

$$2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + log(-(d - sqrt(-e^2*x^2 + d^2
))/x)
```

giac [A] time = 0.21, size = 48, normalized size = 1.04

$$-\arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d) - \log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2 + d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*sgn(d) - log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-
2)/abs(x))
```

maple [B] time = 0.01, size = 137, normalized size = 2.98

$$-\frac{d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}} - \frac{e\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}} + \frac{\sqrt{-e^2x^2+d^2}}{d} - \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x)
```

```
[Out] (-e^2*x^2+d^2)^(1/2)/d-d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-1/d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-e/(e^2)^(1/2)*arctan((e^
2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 56, normalized size = 1.22

$$-\frac{e\left(\frac{d\arcsin\left(\frac{ex}{d}\right)}{e} + \frac{d\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{e}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -e*(d*arcsin(e*x/d)/e + d*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x
))/e)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d^2 - e^2\,x^2}}{x\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d + ex\right)\left(d + ex\right)}}{x\left(d + ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)), x)
```

3.98 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=51

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 807, 266, 63, 208}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*x)) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx = \int \frac{d-ex}{x^2\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{dx} - e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{dx} - \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{dx} + \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{dx} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}$$

Mathematica [**A**] time = 0.06, size = 53, normalized size = 1.04

$$-\frac{\sqrt{d^2-e^2x^2} - ex\log\left(\sqrt{d^2-e^2x^2}+d\right) + ex\log(x)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2] + e*x*Log[x] - e*x*Log[d + Sqrt[d^2 - e^2*x^2]])/(d*
x))
```

IntegrateAlgebraic [**B**] time = 0.26, size = 103, normalized size = 2.02

$$-\frac{\sqrt{d^2-e^2x^2}}{dx} + \frac{e\log\left(\sqrt{d^2-e^2x^2}+d-\sqrt{-e^2}\,x\right)}{d} - \frac{e\log\left(-d\sqrt{d^2-e^2x^2}+d^2+d\sqrt{-e^2}\,x\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*x)) + (e*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2
]])/d - (e*Log[d^2 + d*Sqrt[-e^2]*x - d*Sqrt[d^2 - e^2*x^2]])/d
```

fricas [**A**] time = 0.40, size = 50, normalized size = 0.98

$$-\frac{ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \sqrt{-e^2x^2+d^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] -(e*x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2))/(d*x)
```

giac [**B**] time = 0.21, size = 102, normalized size = 2.00

$$\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d} + \frac{xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d} - \frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d + 1/2*x*e
^3/((d*e + sqrt(-x^2*e^2 + d^2)*e)*d) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
e^(-1)/(d*x)
```

maple [B] time = 0.01, size = 222, normalized size = 4.35

$$\frac{e^2 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\, d}-\frac{e^2 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\, d}+\frac{e \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{\sqrt{-e^2x^2+d^2}\, e^2 x}{d^3}-\frac{\sqrt{-e^2x^2+d^2}\, e}{d^2}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\, e}{d^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{d^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x)
```

```
[Out] -1/d^3/x*(-e^2*x^2+d^2)^(3/2)-1/d^3*e^2*x*(-e^2*x^2+d^2)^(1/2)-1/d*e^2/(e^2
)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-e/d^2*(-e^2*x^2+d^2)^(1/
2)+e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+e/d^2*(2*
(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+e^2/d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x
+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^2\,\left(d+e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^2\left(d+ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)), x)
```

3.99 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=82

$$\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 835, 807, 266, 63, 208}

$$\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(2*d*x^2) + (e*Sqrt[d^2 - e^2*x^2])/(d^2*x) - (e^2*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx &= \int \frac{d-ex}{x^3\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} - \frac{\int \frac{2d^2e-de^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^2} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} + \frac{e^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} + \frac{e^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} - \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} - \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 70, normalized size = 0.85

$$-\frac{(d-2ex)\sqrt{d^2-e^2x^2}+e^2x^2\log\left(\sqrt{d^2-e^2x^2}+d\right)-e^2x^2\log(x)}{2d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*((d - 2*e*x)*Sqrt[d^2 - e^2*x^2] - e^2*x^2*Log[x] + e^2*x^2*Log[d + Sq
rt[d^2 - e^2*x^2]])/(d^2*x^2)
```

IntegrateAlgebraic [A] time = 0.29, size = 79, normalized size = 0.96

$$\frac{(2ex-d)\sqrt{d^2-e^2x^2}}{2d^2x^2} + \frac{e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] ((-d + 2*e*x)*Sqrt[d^2 - e^2*x^2])/(2*d^2*x^2) + (e^2*ArcTanh[(Sqrt[-e^2]*x
)/d - Sqrt[d^2 - e^2*x^2]/d])/d^2
```

fricas [A] time = 0.39, size = 63, normalized size = 0.77

$$\frac{e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}\,(2\,ex-d)}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2)*(2*e
*x - d))/(d^2*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/8*(
exp(2)^3+2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^3/x/exp(2))/d
^2/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2/exp(1)^4+1
/16*(-2*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*e
xp(1)^4*exp(2)^5-4*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6
*exp(2)^4/x/exp(2))/d^4/exp(1)^6/exp(2)^3+1/2*(exp(2)^3-2*exp(1)^4*exp(2))*
ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^2/ex
p(1)^3/exp(1)+1/2*(4*exp(1)^3*exp(2)-4*exp(1)*exp(2)^2)*atan((-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/d^2
/sqrt(-exp(1)^4+exp(2)^2)/exp(1)
```

maple [B] time = 0.01, size = 254, normalized size = 3.10

$$-\frac{e^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 \sqrt{d^{2}}\, d}-\frac{e^{3} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{2 \left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}\right)}{\sqrt{e^{2}}\, d^{2}}+\frac{e^{3} \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\, d^{2}}+\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{3} x}{d^{4}}+\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{2}}{2 d^{3}}-\frac{\sqrt{2 \left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, e^{2}}{d^{3}}+\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} e}{d^{4} x}-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{2 d^{3} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x)
```

```
[Out] e/d^4/x*(-e^2*x^2+d^2)^(3/2)+e^3/d^4*x*(-e^2*x^2+d^2)^(1/2)+e^3/d^2/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2/d^3/x^2*(-e^2*x^2+d^2)^
(3/2)+1/2/d^3*e^2*(-e^2*x^2+d^2)^(1/2)-1/2/d*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d
^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/d^3*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)-1/d^2*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2
)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^{2} x^{2}+d^{2}}}{(e x+d) x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^{3}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d),x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)), x)
```

3.100 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=114

$$\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 835, 807, 266, 63, 208}

$$-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(3*d*x^3) + (e*Sqrt[d^2 - e^2*x^2])/(2*d^2*x^2) - (2*e
^2*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) + (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2
*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx &= \int \frac{d-ex}{x^4\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{\int \frac{3d^2e-2de^2x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}+\frac{\int \frac{4d^3e^2-3d^2e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{e^3\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{e^3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{4d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 84, normalized size = 0.74

$$
\frac{\left(-2d^2+3dex-4e^2x^2\right)\sqrt{d^2-e^2x^2}+3e^3x^3\log\left(\sqrt{d^2-e^2x^2}+d\right)-3e^3x^3\log(x)}{6d^3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] ((-2*d^2 + 3*d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2] - 3*e^3*x^3*Log[x] + 3*
e^3*x^3*Log[d + Sqrt[d^2 - e^2*x^2]])/(6*d^3*x^3)
```

IntegrateAlgebraic [A] time = 0.38, size = 137, normalized size = 1.20

$$
\frac{\left(-2d^2+3dex-4e^2x^2\right)\sqrt{d^2-e^2x^2}}{6d^3x^3}+\frac{e^3\log\left(\sqrt{d^2-e^2x^2}+d-\sqrt{-e^2}\,x\right)}{2d^3}-\frac{e^3\log\left(d^4+d^3\sqrt{-e^2}\,x-d^3\sqrt{d^2-e^2x^2}\right)}{2d^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] ((-2*d^2 + 3*d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(6*d^3*x^3) + (e^3*Log
[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]])/(2*d^3) - (e^3*Log[d^4 + d^3*Sqrt
[-e^2]*x - d^3*Sqrt[d^2 - e^2*x^2]])/(2*d^3)
```

fricas [A] time = 0.40, size = 75, normalized size = 0.66

$$-\frac{3\,e^{3}x^{3}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(4\,e^{2}x^{2}-3\,dex+2\,d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{6\,d^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(3*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (4*e^2*x^2 - 3*d*e*x +
 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/24*
((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*(12*exp(1)^4
*exp(2)^2-3*exp(2)^4)+exp(2)^4+3/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))*exp(2)^4/x/exp(2))/d^3/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^3/exp(1)^5+1/512*(64*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^7-64/3*d^6*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^8+96*d^6*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^8/x/exp(2)-128*d^6*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^7/x/exp(2)+128*d^6*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^6/x/exp(2))/d^9/
exp(1)^15/exp(2)^3+1/2*(4*exp(2)^3-4*exp(1)^4*exp(2))*atan((-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/d^3/s
qrt(-exp(1)^4+exp(2)^2)/exp(1)+1/2*(-exp(2)^3+2*exp(1)^4*exp(2))*ln(1/2*abs
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^3/exp(1)/exp(2
)
```

maple [B] time = 0.01, size = 280, normalized size = 2.46

$$\frac{e^{3}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{2\sqrt{d^{2}}\,d^{2}}+\frac{e^{4}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{\sqrt{e^{2}}\,d^{3}}-\frac{e^{4}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\,d^{3}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{4}x}{d^{5}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{3}}{2d^{4}}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{3}}{d^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{d^{5}x}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e}{2d^{4}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3d^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x)
```

```
[Out] -1/d^5*e^2/x*(-e^2*x^2+d^2)^(3/2)-1/d^5*e^4*x*(-e^2*x^2+d^2)^(1/2)-1/d^3*e^
4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/2*e/d^4/x^2*(-e^
2*x^2+d^2)^(3/2)-1/2*e^3/d^4*(-e^2*x^2+d^2)^(1/2)+1/2*e^3/d^2/(d^2)^(1/2)*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3/d^3/x^3*(-e^2*x^2+d^2)^
(3/2)+1/d^4*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+1/d^3*e^4/(e^2)^(1/2)*a
rctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^{2}x^{2}+d^{2}}}{(ex+d)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2 x^2}}{x^4 (d + e x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^4 (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)), x)
```

3.101 $\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}$$

Rubi [A] time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 835, 807, 266, 63, 208}

$$\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}-\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(4*d*x^4) + (e*Sqrt[d^2 - e^2*x^2])/(3*d^2*x^3) - (3*e
^2*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) + (2*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^4*x)
 - (3*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx &= \int \frac{d-ex}{x^5\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} - \frac{\int \frac{4d^2e-3de^2x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} + \frac{\int \frac{9d^3e^2-8d^2e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{\int \frac{16d^4e^3-9d^3e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^6}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} + \frac{\left(3e^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^3}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} + \frac{\left(3e^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\right.}{16d^3}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} - \frac{\left(3e^2\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,\right.}{8d^3}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} - \frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 95, normalized size = 0.66

$$
\frac{-9e^4x^4\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-6d^3+8d^2ex-9de^2x^2+16e^3x^3\right)+9e^4x^4\log(x)}{24d^4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 9*d*e^2*x^2 + 16*e^3*x^3) + 9*e^
4*x^4*Log[x] - 9*e^4*x^4*Log[d + Sqrt[d^2 - e^2*x^2]])/(24*d^4*x^4)
```

IntegrateAlgebraic [A] time = 0.41, size = 104, normalized size = 0.73

$$
\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}+\frac{\sqrt{d^2-e^2x^2}\left(-6d^3+8d^2ex-9de^2x^2+16e^3x^3\right)}{24d^4x^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 9*d*e^2*x^2 + 16*e^3*x^3))/(24*d
^4*x^4) + (3*e^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(4*d^4)
```

fricas [A] time = 0.39, size = 86, normalized size = 0.60

$$\frac{9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16\,e^3x^3-9\,de^2x^2+8\,d^2ex-6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(9*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3 - 9*d*e^2*
x^2 + 8*d^2*e*x - 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/192
*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-96*exp(1)
^6*exp(2)^2+96*exp(1)^4*exp(2)^3-72*exp(2)^5)+24*exp(1)^4*exp(2)^3*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2+3*exp(2)^5+4*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2))/d^4/(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4/exp(1)^6+1/65536*(-8192*d^12*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^22*exp
(2)^7+8192/3*d^12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^3*exp(1)^20*exp(2)^8-1024*d^12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^4*exp(1)^18*exp(2)^9+8192*d^12*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^20*exp(2)^8-8192*d^12*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^18*exp(2)^9-122
88*d^12*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^20*exp(2)^8/x/ex
p(2)+16384*d^12*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^22*exp(2
)^7/x/exp(2)-16384*d^12*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^
24*exp(2)^6/x/exp(2))/d^16/exp(1)^24/exp(2)^4+1/2*(-4*exp(1)^3*exp(2)^2+4*e
xp(1)^5*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+ex
p(2))/sqrt(-exp(1)^4+exp(2)^2))/d^4/sqrt(-exp(1)^4+exp(2)^2)/exp(1)+1/8*(8*
exp(1)^6*exp(2)^2-4*exp(1)^4*exp(2)^3+exp(2)^5-8*exp(1)^8*exp(2))*ln(1/2*ab
s(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^4/exp(1)^5/ex
p(1)
```

maple [B] time = 0.01, size = 304, normalized size = 2.13

$$-\frac{3e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}\,d^3}-\frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,d^4}+\frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,d^4}+\frac{\sqrt{-e^2x^2+d^2}\,e^5x}{d^6}+\frac{3\sqrt{-e^2x^2+d^2}\,e^4}{8d^5}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^4}{d^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{d^6x}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{8d^5x^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{3d^4x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x)
```

```
[Out] -1/4/d^3/x^4*(-e^2*x^2+d^2)^(3/2)-5/8/d^5*e^2/x^2*(-e^2*x^2+d^2)^(3/2)+3/8/
d^5*e^4*(-e^2*x^2+d^2)^(1/2)-3/8/d^3*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2
)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^6*e^3/x*(-e^2*x^2+d^2)^(3/2)+1/d^6*e^5*x*(-e
^2*x^2+d^2)^(1/2)+1/d^4*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(
1/2)*x)+1/3*e/d^4/x^3*(-e^2*x^2+d^2)^(3/2)-1/d^5*e^4*(2*(x+d/e)*d*e-(x+d/e)
^2*e^2)^(1/2)-1/d^4*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2 + d^2}}{(ex + d)x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2 x^2}}{x^5 (d + e x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^5 (d + ex)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**5*(d + e*x)), x)
```

3.102 $\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=113

$$\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}+\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}$$

Rubi [A] time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1639, 12, 785, 780, 195, 217, 203}

$$\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) + (d*(4*d - 3*e*x)*(d^2 - e^2*x^2)^(3/2
))/(12*e^3) - (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(8*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
```

```
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{\int \frac{5de^3x\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx}{5e^4}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{d\int \frac{x\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx}{e}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{\int x\left(d^2e-de^2x\right)\sqrt{d^2-e^2x^2}\,dx}{e^2}\\
&= \frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{4e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x
}{8e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 112, normalized size = 0.99

$$\frac{\sqrt{d^2-e^2x^2}\left(15d^4\sin^{-1}\left(\frac{ex}{d}\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(16d^4-15d^3ex+8d^2e^2x^2+30de^3x^3-24e^4x^4\right)\right)}{120e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(16*d^4 - 15*d^3*e*x + 8*d^2*
e^2*x^2 + 30*d*e^3*x^3 - 24*e^4*x^4) + 15*d^4*ArcSin[(e*x)/d]))/(120*e^3*Sq
rt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 0.24, size = 114, normalized size = 1.01

$$\frac{d^5\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e^4}+\frac{\sqrt{d^2-e^2x^2}\left(16d^4-15d^3ex+8d^2e^2x^2+30de^3x^3-24e^4x^4\right)}{120e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^4 - 15*d^3*e*x + 8*d^2*e^2*x^2 + 30*d*e^3*x^3 -
24*e^4*x^4))/(120*e^3) + (d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e
^2*x^2]])/(8*e^4)
```

fricas [A]   time = 0.39, size = 94, normalized size = 0.83

$$-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(24\,e^4x^4-30\,de^3x^3-8\,d^2e^2x^2+15\,d^3ex-16\,d^4\right)\sqrt{-e^2x^2+d^2}}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/120*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*e^4*x^4 - 30
*d*e^3*x^3 - 8*d^2*e^2*x^2 + 15*d^3*e*x - 16*d^4)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
4*d^5*exp(2)^3-4*d^5*exp(1)^4*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)
^2)/exp(1)^6/exp(1)+1/8*d^5*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)/exp(2)+2
*((((-192*exp(1)^7*1/1920/exp(1)^6*x+240*exp(1)^6*d*1/1920/exp(1)^6)*x+64*e
xp(1)^5*d^2*1/1920/exp(1)^6)*x-120*exp(1)^4*d^3*1/1920/exp(1)^6)*x+128*exp(
1)^3*d^4*1/1920/exp(1)^6)*sqrt(d^2-x^2*exp(2))
```

maple [B]   time = 0.02, size = 222, normalized size = 1.96

$$\frac{d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{2\sqrt{e^2}\,e^2}-\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2}-\frac{3\sqrt{-e^2x^2+d^2}\,d^3x}{8e^2}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3x}{2e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4e^2}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^2}{3e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x)
```

```
[Out] -1/5*(-e^2*x^2+d^2)^(5/2)/e^3-1/4*(-e^2*x^2+d^2)^(3/2)*d/e^2*x-3/8*(-e^2*x^
2+d^2)^(1/2)*d^3/e^2*x-3/8/(e^2)^(1/2)*d^5/e^2*arctan((e^2)^(1/2)/(-e^2*x^2
+d^2)^(1/2)*x)+1/3*d^2/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+1/2*d^3/e^2*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+1/2*d^5/e^2/(e^2)^(1/2)*arctan((e^2)^
(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C]   time = 1.01, size = 174, normalized size = 1.54

$$-\frac{i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^3}-\frac{3\,d^5\arcsin\left(\frac{ex}{d}\right)}{8\,e^3}+\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^3x}{2\,e^2}-\frac{3\,\sqrt{-e^2x^2+d^2}\,d^3x}{8\,e^2}+\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^4}{e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4\,e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{3\,e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*I*d^5*arcsin(e*x/d + 2)/e^3 - 3/8*d^5*arcsin(e*x/d)/e^3 + 1/2*sqrt(e^2
*x^2 + 4*d*e*x + 3*d^2)*d^3*x/e^2 - 3/8*sqrt(-e^2*x^2 + d^2)*d^3*x/e^2 + sq
```

```
rt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e^3 - 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e^2
+ 1/3*(-e^2*x^2 + d^2)^(3/2)*d^2/e^3 - 1/5*(-e^2*x^2 + d^2)^(5/2)/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(d^2 - e^2 x^2\right)^{3/2}}{d + e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x), x)
```

sympy [C] time = 7.85, size = 279, normalized size = 2.47

$$d \left(\begin{cases} -\frac{i d^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{i d^3 x}{8e^2 \sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - e \left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(3/2)/(e*x+d),x)
```

```
[Out] d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**
2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8
*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e*
*2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - e*Piece
wise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2
*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**
2)/4, True))
```

3.103 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex} \, dx$

Optimal. Leaf size=201

$$\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}+\frac{3d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}+\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-\right.}{64}$$

Rubi [A] time = 0.16, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {850, 833, 780, 195, 217, 203}

$$\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}+\frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}+\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e}+\frac{3d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) + (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) + (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) + (d^3*(128*d - 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4 \left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \int x^4(d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$$
$$= \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{\int x^3\left(4d^2e-9de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{9e^2}$$
$$= -\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} + \frac{\int x^2\left(27d^3e^2-32d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{72e^4}$$
$$= \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{\int x\left(64d^4e^3-189d^3e^4x\right)\cdots}{504e^6}$$
$$= \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} + \frac{d^3(128d-315ex)\left(d^2-\cdots\right.}{5040e^5}$$
$$= \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} + \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} + \frac{d^3(\cdots}{\cdots}$$
$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} + \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4(\cdots}{\cdots}$$
$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} + \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4(\cdots}{\cdots}$$
$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} + \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4(\cdots}{\cdots}$$

Mathematica [A] time = 0.17, size = 135, normalized size = 0.67

$$\frac{945d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(1024d^8-945d^7ex+512d^6e^2x^2-630d^5e^3x^3+384d^4e^4x^4+7560d^3e^5x^5-6400d^2e^6x^6-5040de^7x^7+4480e^8x^8\right)}{40320e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(1024*d^8 - 945*d^7*e*x + 512*d^6*e^2*x^2 - 630*d^5*e^
3*x^3 + 384*d^4*e^4*x^4 + 7560*d^3*e^5*x^5 - 6400*d^2*e^6*x^6 - 5040*d*e^7*
x^7 + 4480*e^8*x^8) + 945*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(40320*e^5
)
```

IntegrateAlgebraic [A] time = 0.38, size = 158, normalized size = 0.79

$$\frac{3d^9\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{128e^6}+\frac{\sqrt{d^2-e^2x^2}\left(1024d^8-945d^7ex+512d^6e^2x^2-630d^5e^3x^3+384d^4e^4x^4+7560d^3e^5x^5-6400d^2e^6x^6-5040de^7x^7+4480e^8x^8\right)}{40320e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(1024*d^8 - 945*d^7*e*x + 512*d^6*e^2*x^2 - 630*d^5*e^
3*x^3 + 384*d^4*e^4*x^4 + 7560*d^3*e^5*x^5 - 6400*d^2*e^6*x^6 - 5040*d*e^7*
x^7 + 4480*e^8*x^8))/(40320*e^5) + (3*d^9*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) +
Sqrt[d^2 - e^2*x^2]])/(128*e^6)
```

fricas [A] time = 0.41, size = 139, normalized size = 0.69

$$-\frac{1890\,d^{9}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-\left(4480\,e^{8}x^{8}-5040\,de^{7}x^{7}-6400\,d^{2}e^{6}x^{6}+7560\,d^{3}e^{5}x^{5}+384\,d^{4}e^{4}x^{4}-630\,d^{5}e^{3}x^{3}+512\,d^{6}e^{2}x^{2}-945\,d^{7}ex+1024\,d^{8}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{40320\,e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40320*(1890*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (4480*e^8*x^
8 - 5040*d*e^7*x^7 - 6400*d^2*e^6*x^6 + 7560*d^3*e^5*x^5 + 384*d^4*e^4*x^4
- 630*d^5*e^3*x^3 + 512*d^6*e^2*x^2 - 945*d^7*e*x + 1024*d^8)*sqrt(-e^2*x^2
 + d^2))/e^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
12*d^9*exp(1)^4*exp(2)^2-8*d^9*exp(2)^4-4*d^9*exp(1)^6*exp(2))*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^10/exp(1)+3/128*d^9*sign(d)*asin(x*exp(
2)/d/exp(1))/exp(1)^5+2*(((((((((322560*exp(1)^17*1/5806080/exp(1)^14*x-3628
80*exp(1)^16*d*1/5806080/exp(1)^14)*x-460800*exp(1)^15*d^2*1/5806080/exp(1)
^14)*x+544320*exp(1)^14*d^3*1/5806080/exp(1)^14)*x+27648*exp(1)^13*d^4*1/58
06080/exp(1)^14)*x-45360*exp(1)^12*d^5*1/5806080/exp(1)^14)*x+36864*exp(1)^
11*d^6*1/5806080/exp(1)^14)*x-68040*exp(1)^10*d^7*1/5806080/exp(1)^14)*x+73
728*exp(1)^9*d^8*1/5806080/exp(1)^14)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.02, size = 330, normalized size = 1.64

$$\frac{3d^{9}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,e^{4}}-\frac{45d^{9}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{128\sqrt{e^{2}}\,e^{4}}-\frac{45\sqrt{-e^{2}x^{2}+d^{2}}\,d^{7}x}{128e^{4}}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{7}x}{8e^{4}}-\frac{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{5}x}{64e^{4}}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}d^{5}x}{4e^{4}}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}x}{16e^{4}}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}d^{4}}{5e^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}x^{2}}{9e^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}dx}{8e^{4}}-\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d^{2}}{63e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] -1/9/e^3*x^2*(-e^2*x^2+d^2)^(7/2)-11/63*d^2/e^5*(-e^2*x^2+d^2)^(7/2)+1/8*d/
e^4*x*(-e^2*x^2+d^2)^(7/2)-3/16*(-e^2*x^2+d^2)^(5/2)*d^3/e^4*x-15/64*(-e^2*
x^2+d^2)^(3/2)*d^5/e^4*x-45/128*(-e^2*x^2+d^2)^(1/2)*d^7/e^4*x-45/128/(e^2)
^(1/2)*d^9/e^4*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/5*d^4/e^5*(2*(x
+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+1/4*d^5/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3
/2)*x+3/8*d^7/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+3/8*d^9/e^4/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 1.04, size = 246, normalized size = 1.22

$$-\frac{3i\,d^{9}\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^{5}}-\frac{45\,d^{9}\arcsin\left(\frac{ex}{d}\right)}{128\,e^{5}}+\frac{3\,\sqrt{e^{2}x^{2}+4\,dex+3\,d^{2}}\,d^{7}x}{8\,e^{4}}-\frac{45\,\sqrt{-e^{2}x^{2}+d^{2}}\,d^{7}x}{128\,e^{4}}+\frac{3\,\sqrt{e^{2}x^{2}+4\,dex+3\,d^{2}}\,d^{8}}{4\,e^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{5}x}{64\,e^{4}}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}x}{16\,e^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}x^{2}}{9\,e^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}}{5\,e^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}dx}{8\,e^{4}}-\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d^{2}}{63\,e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^9*arcsin(e*x/d + 2)/e^5 - 45/128*d^9*arcsin(e*x/d)/e^5 + 3/8*sqrt(
e^2*x^2 + 4*d*e*x + 3*d^2)*d^7*x/e^4 - 45/128*sqrt(-e^2*x^2 + d^2)*d^7*x/e^
4 + 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^8/e^5 + 1/64*(-e^2*x^2 + d^2)^(3/
2)*d^5*x/e^4 - 3/16*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^4 - 1/9*(-e^2*x^2 + d^2)
^(7/2)*x^2/e^3 + 1/5*(-e^2*x^2 + d^2)^(5/2)*d^4/e^5 + 1/8*(-e^2*x^2 + d^2)^
(7/2)*d*x/e^4 - 11/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(d^2 - e^2 x^2\right)^{5/2}}{d + e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

sympy [C] time = 25.15, size = 830, normalized size = 4.13

$$d^3 \left(\begin{cases} -\frac{i d^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16 e^5} + \frac{i d^5 x}{16 e^4 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{i d^3 x^3}{48 e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{5 i d x^5}{24 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{i e^2 x^7}{6 d \sqrt{-1 + \frac{e^2 x^2}{d^2}}} & \text{for}\: \left|\frac{e^2 x^2}{d^2}\right| > 1 \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16 e^5} - \frac{d^5 x}{16 e^4 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^3 x^3}{48 e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{5 d x^5}{24 \sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^7}{6 d \sqrt{1 - \frac{e^2 x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - d^2 e \left(\begin{cases} -\frac{8 d^6 \sqrt{d^2 - e^2 x^2}}{105 e^6} - \frac{4 d^4 x^2 \sqrt{d^2 - e^2 x^2}}{105 e^4} - \frac{d^2 x^4 \sqrt{d^2 - e^2 x^2}}{35 e^2} + \frac{x^6 \sqrt{d^2 - e^2 x^2}}{7} & \text{for}\: e \neq 0 \\ \frac{x^6 \sqrt{d^2}}{6} & \text{otherwise} \end{cases}\right) - d e^2 \left(\begin{cases} -\frac{5 i d^8 \operatorname{acosh}\left(\frac{ex}{d}\right)}{128 e^7} + \frac{5 i d^7 x}{128 e^6 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{5 i d^5 x^3}{384 e^4 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{i d^3 x^5}{192 e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{7 i d x^7}{48 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{i e^2 x^9}{8 d \sqrt{-1 + \frac{e^2 x^2}{d^2}}} & \text{for}\: \left|\frac{e^2 x^2}{d^2}\right| > 1 \\ \frac{5 d^8 \operatorname{asin}\left(\frac{ex}{d}\right)}{128 e^7} - \frac{5 d^7 x}{128 e^6 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{5 d^5 x^3}{384 e^4 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^3 x^5}{192 e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{7 d x^7}{48 \sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^9}{8 d \sqrt{1 - \frac{e^2 x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^3 \left(\begin{cases} -\frac{16 d^8 \sqrt{d^2 - e^2 x^2}}{315 e^8} - \frac{8 d^6 x^2 \sqrt{d^2 - e^2 x^2}}{315 e^6} - \frac{2 d^4 x^4 \sqrt{d^2 - e^2 x^2}}{105 e^4} - \frac{d^2 x^6 \sqrt{d^2 - e^2 x^2}}{63 e^2} + \frac{x^8 \sqrt{d^2 - e^2 x^2}}{9} & \text{for}\: e \neq 0 \\ \frac{x^8 \sqrt{d^2}}{8} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) - d**2*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(1
05*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d
**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**
6*sqrt(d**2)/6, True)) - d*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7
) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e*
*4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d
**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e*
*7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*
sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) +
 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2
/d**2)), True)) + e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8
) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 -
 e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*
sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

3.104 $\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=172

$$-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}+\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}-\frac{3d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}-\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}$$

Rubi [A] time = 0.12, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 833, 780, 195, 217, 203}

$$-\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}-\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}+\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{3d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (-3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) - (d^4*x*(d^2 - e^2*x^2)^(3/2))/(6
4*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) + (x^3*(d^2 - e^2*x^2)^(5/2)
)/(8*e) - (d^2*(32*d - 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) - (3*d^8*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \int x^3(d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$$
$$= \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{\int x^2\left(3d^2e-8de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{8e^2}$$
$$= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} + \frac{\int x\left(16d^3e^2-21d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{56e^4}$$
$$= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} - \frac{d^4\int\left(d^2-e^2x\right.}{16e^3}$$
$$= -\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x\right.}{560e^4}$$
$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32}{}$$
$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32}{}$$
$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32}{}$$

Mathematica [A] time = 0.13, size = 124, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\left(-256d^7+105d^6ex-128d^5e^2x^2+70d^4e^3x^3+1024d^3e^4x^4-840d^2e^5x^5-640de^6x^6+560e^7x^7\right)-105d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4480e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-256*d^7 + 105*d^6*e*x - 128*d^5*e^2*x^2 + 70*d^4*e^3
*x^3 + 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 - 640*d*e^6*x^6 + 560*e^7*x^7) -
105*d^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4480*e^4)
```

IntegrateAlgebraic [A] time = 0.39, size = 147, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(-256d^7+105d^6ex-128d^5e^2x^2+70d^4e^3x^3+1024d^3e^4x^4-840d^2e^5x^5-640de^6x^6+560e^7x^7\right)}{4480e^4} - \frac{3d^8\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{128e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-256*d^7 + 105*d^6*e*x - 128*d^5*e^2*x^2 + 70*d^4*e^3
*x^3 + 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 - 640*d*e^6*x^6 + 560*e^7*x^7))/(
```

```
4480*e^4) - (3*d^8*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(
128*e^5)
```

fricas [A] time = 0.40, size = 127, normalized size = 0.74

$$\frac{210\,d^{8}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)+\left(560\,e^{7}x^{7}-640\,de^{6}x^{6}-840\,d^{2}e^{5}x^{5}+1024\,d^{3}e^{4}x^{4}+70\,d^{4}e^{3}x^{3}-128\,d^{5}e^{2}x^{2}+105\,d^{6}ex-256\,d^{7}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{4480\,e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4480*(210*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (560*e^7*x^7 -
640*d*e^6*x^6 - 840*d^2*e^5*x^5 + 1024*d^3*e^4*x^4 + 70*d^4*e^3*x^3 - 128*d
^5*e^2*x^2 + 105*d^6*e*x - 256*d^7)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
-12*d^8*exp(1)^4*exp(2)^2+8*d^8*exp(2)^4+4*d^8*exp(1)^6*exp(2))*atan((-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)
^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^9/exp(1)-3/128*d^8*sign(d)*asin(x*exp(
2)/d/exp(1))/exp(1)^4+2*(((((((40320*exp(1)^15*1/645120/exp(1)^12*x-46080*e
xp(1)^14*d*1/645120/exp(1)^12)*x-60480*exp(1)^13*d^2*1/645120/exp(1)^12)*x+
73728*exp(1)^12*d^3*1/645120/exp(1)^12)*x+5040*exp(1)^11*d^4*1/645120/exp(1
)^12)*x-9216*exp(1)^10*d^5*1/645120/exp(1)^12)*x+7560*exp(1)^9*d^6*1/645120
/exp(1)^12)*x-18432*exp(1)^8*d^7*1/645120/exp(1)^12)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 305, normalized size = 1.77

$$-\frac{3d^{8}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,e^{3}}+\frac{45d^{8}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{128\sqrt{e^{2}}\,e^{3}}+\frac{45\sqrt{-e^{2}x^{2}+d^{2}}\,d^{6}x}{128e^{3}}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{6}x}{8e^{3}}+\frac{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{4}x}{64e^{3}}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}d^{4}x}{4e^{3}}+\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}x}{16e^{3}}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}d^{3}}{5e^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}x}{8e^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d}{7e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] -1/8/e^3*x*(-e^2*x^2+d^2)^(7/2)+3/16*(-e^2*x^2+d^2)^(5/2)*d^2/e^3*x+15/64*(
-e^2*x^2+d^2)^(3/2)*d^4/e^3*x+45/128*(-e^2*x^2+d^2)^(1/2)*d^6/e^3*x+45/128/
(e^2)^(1/2)*d^8/e^3*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/7*d/e^4*(-
e^2*x^2+d^2)^(7/2)-1/5*d^3/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/4*d^4/
e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-3/8*d^6/e^3*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(1/2)*x-3/8*d^8/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-
(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 1.03, size = 221, normalized size = 1.28

$$\frac{3i\,d^{8}\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^{4}}+\frac{45\,d^{8}\arcsin\left(\frac{ex}{d}\right)}{128\,e^{4}}-\frac{3\,\sqrt{e^{2}x^{2}+4\,dex+3\,d^{2}}\,d^{6}x}{8\,e^{3}}+\frac{45\,\sqrt{-e^{2}x^{2}+d^{2}}\,d^{6}x}{128\,e^{3}}-\frac{3\,\sqrt{e^{2}x^{2}+4\,dex+3\,d^{2}}\,d^{7}}{4\,e^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{4}x}{64\,e^{3}}+\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}x}{16\,e^{3}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}}{5\,e^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}x}{8\,e^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d}{7\,e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*I*d^8*arcsin(e*x/d + 2)/e^4 + 45/128*d^8*arcsin(e*x/d)/e^4 - 3/8*sqrt(e
^2*x^2 + 4*d*e*x + 3*d^2)*d^6*x/e^3 + 45/128*sqrt(-e^2*x^2 + d^2)*d^6*x/e^3
 - 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^7/e^4 - 1/64*(-e^2*x^2 + d^2)^(3/2
)*d^4*x/e^3 + 3/16*(-e^2*x^2 + d^2)^(5/2)*d^2*x/e^3 - 1/5*(-e^2*x^2 + d^2)^
```

```
(5/2)*d^3/e^4 - 1/8*(-e^2*x^2 + d^2)^(7/2)*x/e^3 + 1/7*(-e^2*x^2 + d^2)^(7/
2)*d/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left(d^2 - e^2 x^2\right)^{5/2}}{d + e x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

sympy [A] time = 23.01, size = 775, normalized size = 4.51

$$d^3 \left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right) - d^2 e \left(\begin{cases} -\frac{i d^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{i d^5 x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{i d^3 x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5 i d x^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{i e^2 x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5 x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - d e^2 \left(\begin{cases} -\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6} - \frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4} - \frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2} + \frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\ e \neq 0 \\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise} \end{cases}\right) + e^3 \left(\begin{cases} -\frac{5 i d^8 \operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^7} + \frac{5 i d^7 x}{128e^6\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5 i d^5 x^3}{384e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{i d^3 x^5}{192e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{7 i d x^7}{48\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{i e^2 x^9}{8d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{5d^8 \operatorname{asin}\left(\frac{ex}{d}\right)}{128e^7} - \frac{5d^7x}{128e^6\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5d^5x^3}{384e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3x^5}{192e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{7dx^7}{48\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^9}{8d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) - d**2*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
 I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-
1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x
**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e
*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48
*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) -
e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - d*e**2*Piecewise((-8*d**
6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(1
05*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e*
*2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) + e**3*Piecewise((-5*I*d*
*8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)
) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e
**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2))
+ I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5
*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2))
 + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sq
rt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**
9/(8*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.105 $\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=140

$$\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}+\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}$$

Rubi [A] time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1639, 12, 785, 780, 195, 217, 203}

$$\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) + (d*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^3) - (d^2 - e^2*x^2)^(7/
2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
```

```
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{\int \frac{7de^3x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx}{7e^4}\\
&= -\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{d\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx}{e}\\
&= -\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{\int x\left(d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}\\
&= \frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^3\int \left(d^2-e^2x^2\right)^{3/2}\,dx}{6e^2}\\
&= \frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^5\int \sqrt{d^2-e^2x^2}\,dx}{8e^2}\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7}{\ }\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7}{\ }\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7}{\ }
\end{aligned}$$

Mathematica [A] time = 0.10, size = 113, normalized size = 0.81

$$\frac{105d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(96d^6-105d^5ex+48d^4e^2x^2+490d^3e^3x^3-384d^2e^4x^4-280de^5x^5+240e^6x^6\right)}{1680e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(96*d^6 - 105*d^5*e*x + 48*d^4*e^2*x^2 + 490*d^3*e^3*x
^3 - 384*d^2*e^4*x^4 - 280*d*e^5*x^5 + 240*e^6*x^6) + 105*d^7*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]])/(1680*e^3)
```

IntegrateAlgebraic [A] time = 0.39, size = 136, normalized size = 0.97

$$\frac{d^7\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{16e^4}+\frac{\sqrt{d^2-e^2x^2}\left(96d^6-105d^5ex+48d^4e^2x^2+490d^3e^3x^3-384d^2e^4x^4-280de^5x^5+240e^6x^6\right)}{1680e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(96*d^6 - 105*d^5*e*x + 48*d^4*e^2*x^2 + 490*d^3*e^3*x
^3 - 384*d^2*e^4*x^4 - 280*d*e^5*x^5 + 240*e^6*x^6))/(1680*e^3) + (d^7*Sqrt
[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^4)
```

fricas [A] time = 0.41, size = 117, normalized size = 0.84

$$-\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(240\,e^6x^6-280\,de^5x^5-384\,d^2e^4x^4+490\,d^3e^3x^3+48\,d^4e^2x^2-105\,d^5ex+96\,d^6\right)\sqrt{-e^2x^2+d^2}}{1680\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/1680*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (240*e^6*x^6 -
 280*d*e^5*x^5 - 384*d^2*e^4*x^4 + 490*d^3*e^3*x^3 + 48*d^4*e^2*x^2 - 105*d
^5*e*x + 96*d^6)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
12*d^7*exp(1)^4*exp(2)^2-8*d^7*exp(2)^4-4*d^7*exp(1)^6*exp(2))*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^8/exp(1)+1/16*d^7*sign(d)*asin(x*exp(2)
/d/exp(1))/exp(1)/exp(2)+2*((((((5760*exp(1)^13*1/80640/exp(1)^10*x-6720*ex
p(1)^12*d*1/80640/exp(1)^10)*x-9216*exp(1)^11*d^2*1/80640/exp(1)^10)*x+1176
0*exp(1)^10*d^3*1/80640/exp(1)^10)*x+1152*exp(1)^9*d^4*1/80640/exp(1)^10)*x
-2520*exp(1)^8*d^5*1/80640/exp(1)^10)*x+2304*exp(1)^7*d^6*1/80640/exp(1)^10
)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 282, normalized size = 2.01

$$\frac{3d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}\,e^2}-\frac{5d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e^2}-\frac{5\sqrt{-e^2x^2+d^2}\,d^5x}{16e^2}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^5x}{8e^2}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{24e^2}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^3x}{4e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{6e^2}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^2}{5e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] -1/7*(-e^2*x^2+d^2)^(7/2)/e^3-1/6*(-e^2*x^2+d^2)^(5/2)*d/e^2*x-5/24*(-e^2*x
^2+d^2)^(3/2)*d^3/e^2*x-5/16*(-e^2*x^2+d^2)^(1/2)*d^5/e^2*x-5/16/(e^2)^(1/2
)*d^7/e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/5*d^2/e^3*(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(5/2)+1/4*d^3/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x
+3/8*d^5/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+3/8*d^7/e^2/(e^2)^(1/2)*
arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 1.04, size = 198, normalized size = 1.41

$$-\frac{3i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^3}-\frac{5\,d^7\arcsin\left(\frac{ex}{d}\right)}{16\,e^3}+\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^5x}{8\,e^2}-\frac{5\,\sqrt{-e^2x^2+d^2}\,d^5x}{16\,e^2}+\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^6}{4\,e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{24\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{6\,e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{5\,e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{7\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^7*arcsin(e*x/d + 2)/e^3 - 5/16*d^7*arcsin(e*x/d)/e^3 + 3/8*sqrt(e^
2*x^2 + 4*d*e*x + 3*d^2)*d^5*x/e^2 - 5/16*sqrt(-e^2*x^2 + d^2)*d^5*x/e^2 +
3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^6/e^3 + 1/24*(-e^2*x^2 + d^2)^(3/2)*d
^3*x/e^2 - 1/6*(-e^2*x^2 + d^2)^(5/2)*d*x/e^2 + 1/5*(-e^2*x^2 + d^2)^(5/2)*
d^2/e^3 - 1/7*(-e^2*x^2 + d^2)^(7/2)/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(d^2 - e^2 x^2\right)^{5/2}}{d + e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

sympy [C] time = 16.66, size = 653, normalized size = 4.66

$$d^3 \left(\begin{cases} -\frac{i d^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{i d^3 x}{8e^2 \sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3 i d x^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{i e^2 x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3 x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3 d x^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2 x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - d^2 e \left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right) - d e^2 \left(\begin{cases} -\frac{i d^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{i d^5 x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{i d^3 x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5 i d x^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{i e^2 x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5 x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3 x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5 d x^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2 x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^3 \left(\begin{cases} -\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6} - \frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4} - \frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2} + \frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for } e \neq 0 \\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - d**2
*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**
2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*
sqrt(d**2)/4, True)) - d*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I
*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1
+ e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**
7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x
/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e
**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e*
*2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**3*Piecewise((-8*d**6*sq
rt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e
**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x
**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

3.106 $\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=116

$$-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}-\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {785, 780, 195, 217, 203}

$$-\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] -(d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) - (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e)
- ((6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) - (d^6*ArcTan[(e*x)/Sqrt[d
^2 - e^2*x^2]])/(16*e^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] && !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{\int x\left(d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}dx}{de}$$
$$= -\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2} - \frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}dx}{6e}$$
$$= -\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e} - \frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2} - \frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}$$
$$= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e} - \frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e} - \frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2} - \frac{d^6\int\frac{1}{\sqrt{d^2-e^2x^2}}dx}{16e}$$
$$= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e} - \frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e} - \frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2} - \frac{d^6\,\text{Subst}\left(\int\frac{1}{1+e^2x^2}dx\right.}{16e}$$
$$= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e} - \frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e} - \frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2} - \frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Mathematica [A] time = 0.09, size = 102, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\left(-48d^5+15d^4ex+96d^3e^2x^2-70d^2e^3x^3-48de^4x^4+40e^5x^5\right)-15d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{240e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 + 15*d^4*e*x + 96*d^3*e^2*x^2 - 70*d^2*e^3*x^
3 - 48*d*e^4*x^4 + 40*e^5*x^5) - 15*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
(240*e^2)
```

IntegrateAlgebraic [A] time = 0.38, size = 125, normalized size = 1.08

$$\frac{\sqrt{d^2-e^2x^2}\left(-48d^5+15d^4ex+96d^3e^2x^2-70d^2e^3x^3-48de^4x^4+40e^5x^5\right)}{240e^2} - \frac{d^6\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 + 15*d^4*e*x + 96*d^3*e^2*x^2 - 70*d^2*e^3*x^
3 - 48*d*e^4*x^4 + 40*e^5*x^5))/(240*e^2) - (d^6*Sqrt[-e^2]*Log[-(Sqrt[-e^2
]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

fricas [A] time = 0.40, size = 105, normalized size = 0.91

$$\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5-48\,de^4x^4-70\,d^2e^3x^3+96\,d^3e^2x^2+15\,d^4ex-48\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 - 48*
d*e^4*x^4 - 70*d^2*e^3*x^3 + 96*d^3*e^2*x^2 + 15*d^4*e*x - 48*d^5)*sqrt(-e^
2*x^2 + d^2))/e^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
-12*d^6*exp(1)^4*exp(2)^2+8*d^6*exp(2)^4+4*d^6*exp(1)^6*exp(2))*atan((-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)
^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^7/exp(1)-1/16*d^6*sign(d)*asin(x*exp(2
)/d/exp(1))/exp(1)^2+2*(((((960*exp(1)^11*1/11520/exp(1)^8*x-1152*exp(1)^10
*d*1/11520/exp(1)^8)*x-1680*exp(1)^9*d^2*1/11520/exp(1)^8)*x+2304*exp(1)^8*
d^3*1/11520/exp(1)^8)*x+360*exp(1)^7*d^4*1/11520/exp(1)^8)*x-1152*exp(1)^6*
d^5*1/11520/exp(1)^8)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 260, normalized size = 2.24

$$-\frac{3d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{8\sqrt{e^2}\,e}+\frac{5d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e}+\frac{5\sqrt{-e^2x^2+d^2}\,d^4x}{16e}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\,d^4x}{8e}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24e}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}}d^2x}{4e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6e}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{5}{2}}d}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] 1/6*(-e^2*x^2+d^2)^(5/2)/e*x+5/24*(-e^2*x^2+d^2)^(3/2)*d^2/e*x+5/16*(-e^2*x
^2+d^2)^(1/2)*d^4/e*x+5/16/(e^2)^(1/2)*d^6/e*arctan((e^2)^(1/2)/(-e^2*x^2+d
^2)^(1/2)*x)-1/5*d/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/4*d^2/e*(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-3/8*d^4/e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/
2)*x-3/8*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(1/2)*x)
```

maxima [C] time = 1.02, size = 176, normalized size = 1.52

$$\frac{3i\,d^6\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^2}+\frac{5\,d^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^2}-\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^4x}{8\,e}+\frac{5\,\sqrt{-e^2x^2+d^2}\,d^4x}{16\,e}-\frac{3\,\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^5}{4\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*I*d^6*arcsin(e*x/d + 2)/e^2 + 5/16*d^6*arcsin(e*x/d)/e^2 - 3/8*sqrt(e^2
*x^2 + 4*d*e*x + 3*d^2)*d^4*x/e + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e - 3/4*s
qrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^5/e^2 - 1/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/
e + 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e - 1/5*(-e^2*x^2 + d^2)^(5/2)*d/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

sympy [A] time = 16.12, size = 580, normalized size = 5.00

$$d^3\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\: e^2=0\\-\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\:\left|\frac{e^2x^2}{d^2}\right|>1\\\frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\: e\neq 0\\\frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\:\left|\frac{e^2x^2}{d^2}\right|>1\\\frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) - d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
 + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**
2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + e**3*Piecewise((-I*d**6*acosh
(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**
2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 -
e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.107 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=100

$$\frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}+\frac{3}{8}d^3x\sqrt{d^2-e^2x^2}$$

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {665, 195, 217, 203}

$$\frac{3}{8}d^3x\sqrt{d^2-e^2x^2}+\frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (3*d^3*x*Sqrt[d^2 - e^2*x^2])/8 + (d*x*(d^2 - e^2*x^2)^(3/2))/4 + (d^2 - e^
2*x^2)^(5/2)/(5*e) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + d\int \left(d^2-e^2x^2\right)^{3/2}\,dx$$
$$= \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{1}{4}\left(3d^3\right)\int \sqrt{d^2-e^2x^2}\,dx$$
$$= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{1}{8}\left(3d^5\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{1}{8}\left(3d^5\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x\right.$$
$$= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.91

$$\frac{15d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \sqrt{d^2-e^2x^2}\left(8d^4+25d^3ex-16d^2e^2x^2-10de^3x^3+8e^4x^4\right)}{40e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 + 25*d^3*e*x - 16*d^2*e^2*x^2 - 10*d*e^3*x^3 +
8*e^4*x^4) + 15*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(40*e)
```

IntegrateAlgebraic [A] time = 0.38, size = 114, normalized size = 1.14

$$\frac{3d^5\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e^2} + \frac{\sqrt{d^2-e^2x^2}\left(8d^4+25d^3ex-16d^2e^2x^2-10de^3x^3+8e^4x^4\right)}{40e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 + 25*d^3*e*x - 16*d^2*e^2*x^2 - 10*d*e^3*x^3 +
8*e^4*x^4))/(40*e) + (3*d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2
*x^2]])/(8*e^2)
```

fricas [A] time = 0.40, size = 95, normalized size = 0.95

$$-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(8\,e^4x^4-10\,de^3x^3-16\,d^2e^2x^2+25\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{40\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (8*e^4*x^4 - 10*d
*e^3*x^3 - 16*d^2*e^2*x^2 + 25*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/e
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
12*d^5*exp(1)^4*exp(2)^2-8*d^5*exp(2)^4-4*d^5*exp(1)^6*exp(2))*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^6/exp(1)+3/8*d^5*sign(d)*asin(x*exp(2)/
d/exp(1))/exp(1)+2*(((((192*exp(1)^9*1/1920/exp(1)^6*x-240*exp(1)^8*d*1/1920
/exp(1)^6)*x-384*exp(1)^7*d^2*1/1920/exp(1)^6)*x+600*exp(1)^6*d^3*1/1920/ex
p(1)^6)*x+192*exp(1)^5*d^4*1/1920/exp(1)^6)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.01, size = 147, normalized size = 1.47

$$\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3x}{8}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}dx}{4}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}}{5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] 1/5/e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+1/4*d*(2*(x+d/e)*d*e-(x+d/e)^2*e^
2)^(3/2)*x+3/8*d^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+3/8*d^5/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 0.99, size = 109, normalized size = 1.09

$$-\frac{3i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{8\,e}+\frac{3}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^3x+\frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^4}{4\,e}+\frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^5*arcsin(e*x/d + 2)/e + 3/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
+ 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*
x + 1/5*(-e^2*x^2 + d^2)^(5/2)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x), x)
```

sympy [C] time = 10.46, size = 435, normalized size = 4.35

$$d^3\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - d**2
*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/
(3*e**2), True)) - d*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3
```

```
*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) +
 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/
d**2)), True)) + e**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) -
 d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5
, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

3.108 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=113

$$\frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {850, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (d^2*(8*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d - 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 - (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
```

```
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx$$

$$= \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-4d^3e^2+3d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{4e^2}$$

$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{8d^5e^4-3d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{8e^4}$$

$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + d^5\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{8}\left(3d^4e\right.$$

$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^5\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x\right.$$

$$d^5\,\mathrm{S}$$

$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - $$

$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4\,\mathrm{ta}$$

Mathematica [A] time = 0.09, size = 108, normalized size = 0.96

$$d^4\log(x) - d^4\log\left(\sqrt{d^2-e^2x^2}+d\right) - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3-15d^2ex-8de^2x^2+6e^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 - 15*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3))/24 -
(3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + d^4*Log[x] - d^4*Log[d + Sqrt
[d^2 - e^2*x^2]]
```

IntegrateAlgebraic [A] time = 0.44, size = 142, normalized size = 1.26

$$-\frac{3d^4\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e} + 2d^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3-15d^2ex-8de^2x^2+6e^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 - 15*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3))/24 +
2*d^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (3*d^4*Sqrt[-e^2]
*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*e)
```

fricas [A] time = 0.41, size = 107, normalized size = 0.95

$$\frac{3}{4}\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\frac{1}{24}\left(6\,e^3x^3-8\,de^2x^2-15\,d^2ex+32\,d^3\right)\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] 3/4*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^4*log(-(d - sqrt(-e^2
*x^2 + d^2))/x) + 1/24*(6*e^3*x^3 - 8*d*e^2*x^2 - 15*d^2*e*x + 32*d^3)*sqrt
(-e^2*x^2 + d^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -3/8*
d^4*sign(d)*asin(x*exp(2)/d/exp(1))+1/2*(-12*d^4*exp(1)^4*exp(2)^2+8*d^4*ex
p(2)^4+4*d^4*exp(1)^6*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(
1)^5/exp(1)-d^4*exp(2)*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/abs(x)/exp(2))/exp(1)^2+2*(((24*exp(1)^7*1/192/exp(1)^4*x-32*exp(1)^6*d*1
/192/exp(1)^4)*x-60*exp(1)^5*d^2*1/192/exp(1)^4)*x+128*exp(1)^4*d^3*1/192/e
xp(1)^4)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 245, normalized size = 2.17

$$-\frac{d^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{3d^4e\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2ex}{8}+\sqrt{-e^2x^2+d^2}\,d^3-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}ex}{4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5d}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x)
```

```
[Out] 1/5/d*(-e^2*x^2+d^2)^(5/2)+1/3*(-e^2*x^2+d^2)^(3/2)*d+(-e^2*x^2+d^2)^(1/2)*
d^3-1/(d^2)^(1/2)*d^5*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/5/
d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/4*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(
3/2)*x-3/8*d^2*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-3/8*d^4*e/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 124, normalized size = 1.10

$$-\frac{3}{8}\,d^4\arcsin\left(\frac{ex}{d}\right)-d^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{3}{8}\,\sqrt{-e^2x^2+d^2}\,d^2ex+\sqrt{-e^2x^2+d^2}\,d^3-\frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}ex+\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*d^4*arcsin(e*x/d) - d^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/ab
s(x)) - 3/8*sqrt(-e^2*x^2 + d^2)*d^2*e*x + sqrt(-e^2*x^2 + d^2)*d^3 - 1/4*(
-e^2*x^2 + d^2)^(3/2)*e*x + 1/3*(-e^2*x^2 + d^2)^(3/2)*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2 x^2\right)^{5/2}}{x\, (d+e\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)), x)
```

sympy [C] time = 25.65, size = 469, normalized size = 4.15

$$d^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right) - d^2e\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right) - de^2\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2=0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) + e^3\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d),x)
```

```
[Out] d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) - d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*
sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), A
bs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d
**2)/2, True)) - d*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2
 - e**2*x**2)**(3/2)/(3*e**2), True)) + e**3*Piecewise((-I*d**4*acosh(e*x/d
)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sq
rt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs
(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 -
 e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*
sqrt(1 - e**2*x**2/d**2)), True))
```

3.109 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=115

$$-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)(d^2-e^2x^2)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {850, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)(d^2-e^2x^2)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] -(d*e*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d + e*x)*(d^2 - e^2*x^2)^(
3/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + d^3*e*ArcTanh
[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
```

```
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx$$

$$= -\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{\left(2d^2e+6de^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} + \frac{\int \frac{-4d^4e^3-6d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \left(d^4e\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}\left(3d^3\right.$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{1}{2}\left(d^4e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,\right.$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{d^4\,\text{Su}}{}$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + d^3e\,\text{ta}$$

Mathematica [A] time = 0.13, size = 114, normalized size = 0.99

$$-d^3e\log(x)+d^3e\log\left(\sqrt{d^2-e^2x^2}+d\right)-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(-\frac{d^3}{x}-\frac{4d^2e}{3}-\frac{1}{2}de^2x+\frac{e^3x^2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((-4*d^2*e)/3 - d^3/x - (d*e^2*x)/2 + (e^3*x^2)/3) - (3
*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - d^3*e*Log[x] + d^3*e*Log[d +
Sqrt[d^2 - e^2*x^2]]
```

IntegrateAlgebraic [A] time = 0.45, size = 143, normalized size = 1.24

$$-\frac{3}{2}d^3\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)-2d^3e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{\sqrt{d^2-e^2x^2}\left(-6d^3-8d^2ex-3de^2x^2+2e^3x^3\right)}{6x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 8*d^2*e*x - 3*d*e^2*x^2 + 2*e^3*x^3))/(6*x)
- 2*d^3*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (3*d^3*Sqrt[-
e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

fricas [A] time = 0.39, size = 123, normalized size = 1.07

$$\frac{18\,d^3ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-6\,d^3ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-8\,d^3ex+\left(2\,e^3x^3-3\,de^2x^2-8\,d^2ex-6\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/6*(18*d^3*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 6*d^3*e*x*log(-
(d - sqrt(-e^2*x^2 + d^2))/x) - 8*d^3*e*x + (2*e^3*x^3 - 3*d*e^2*x^2 - 8*d^
2*e*x - 6*d^3)*sqrt(-e^2*x^2 + d^2))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
12*d^3*exp(1)^4*exp(2)^2-8*d^3*exp(2)^4-4*d^3*exp(1)^6*exp(2))*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^4/exp(1)-d^3*x*exp(2)^3/(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/exp(1)/exp(2)+1/4*d^3*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))*exp(2)^4/exp(1)^4/x/exp(1)/exp(2)^2+d^3*exp(2)*ln(1/2*a
bs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)-3/2*d^3
*sign(d)*asin(x*exp(2)/d/exp(1))*exp(2)/exp(1)+2*((4*exp(1)^5*1/24/exp(1)^2
*x-6*exp(1)^4*d*1/24/exp(1)^2)*x-16*exp(1)^3*d^2*1/24/exp(1)^2)*sqrt(d^2-x^
2*exp(2))
```

maple [B] time = 0.01, size = 380, normalized size = 3.30

$$\frac{d^4e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{3d^3e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{15d^3e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}-\frac{15\sqrt{-e^2x^2+d^2}\,d\,e^2x}{8}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d\,e^2x}{8}-\sqrt{-e^2x^2+d^2}\,d^2e-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2x}{4d}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^2x}{4d}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2x}{d^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{5d^2}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e}{5d^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x)
```

```
[Out] -1/d^3/x*(-e^2*x^2+d^2)^(7/2)-1/d^3*e^2*x*(-e^2*x^2+d^2)^(5/2)-5/4*(-e^2*x^
2+d^2)^(3/2)/d*e^2*x-15/8*(-e^2*x^2+d^2)^(1/2)*d*e^2*x-15/8/(e^2)^(1/2)*d^3
*e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/5*e/d^2*(-e^2*x^2+d^2)^(5
/2)-1/3*(-e^2*x^2+d^2)^(3/2)*e-(-e^2*x^2+d^2)^(1/2)*d^2*e+1/(d^2)^(1/2)*d^4
*e*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5*e/d^2*(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(5/2)+1/4*e^2/d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+3/8*
e^2*d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+3/8*e^2*d^3/(e^2)^(1/2)*arctan(
(e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 131, normalized size = 1.14

$$-\frac{3}{2}\,d^3e\arcsin\left(\frac{ex}{d}\right)+d^3e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\,de^2x-\sqrt{-e^2x^2+d^2}\,d^2e-\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e-\frac{\sqrt{-e^2x^2+d^2}\,d^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/2*d^3*e*arcsin(e*x/d) + d^3*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*
d/abs(x)) - 1/2*sqrt(-e^2*x^2 + d^2)*d*e^2*x - sqrt(-e^2*x^2 + d^2)*d^2*e -
 1/3*(-e^2*x^2 + d^2)^(3/2)*e - sqrt(-e^2*x^2 + d^2)*d^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^2\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

sympy [C] time = 10.20, size = 386, normalized size = 3.36

$$d^3\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)-d^2e\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right)-de^2\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right)+e^3\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d),x)
```

```
[Out] d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) - d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(
d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*
d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
*2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e)
- I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 -
e**2*x**2/d**2)/2, True)) + e**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0))
, (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

3.110 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=121

$$\frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {850, 813, 844, 217, 203, 266, 63, 208}

$$\frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (3*d*e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d + e*x)*(d^2 - e^2*x^2)^(3
/2))/(2*x^2) + (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^2
*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
```

```
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx$$

$$= -\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{\left(4d^2e+4de^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{-8d^3e^2+8d^2e^3x}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{2}\left(3d^3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \frac{1}{2}\left(3d^2e\right.$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{4}\left(3d^3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x,\right.$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{2}\left(3d^3\right)\text{S}$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2\tan$$

Mathematica [A] time = 0.16, size = 119, normalized size = 0.98

$$\frac{1}{2}\left(3d^2e^2\log\left(\sqrt{d^2-e^2x^2}+d\right)+3d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-3d^2e^2\log(x)+\frac{\sqrt{d^2-e^2x^2}\left(-d^3+2d^2ex-2de^2x^2+e^3x^3\right)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^3 + 2*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/x^2 + 3*d
^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 3*d^2*e^2*Log[x] + 3*d^2*e^2*Log
[d + Sqrt[d^2 - e^2*x^2]])/2
```

IntegrateAlgebraic [A] time = 0.66, size = 145, normalized size = 1.20

$$\frac{3}{2}d^2e\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)-3d^2e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{\sqrt{d^2-e^2x^2}\left(-d^3+2d^2ex-2de^2x^2+e^3x^3\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^3 + 2*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/(2*x^2) -
3*d^2*e^2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + (3*d^2*e*Sqrt
[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

fricas [A] time = 0.42, size = 135, normalized size = 1.12

$$-\frac{6\,d^2e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+3\,d^2e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+2\,d^2e^2x^2-\left(e^3x^3-2\,de^2x^2+2\,d^2ex-d^3\right)\sqrt{-e^2x^2+d^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*(6*d^2*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*d^2*e^2*x
^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 2*d^2*e^2*x^2 - (e^3*x^3 - 2*d*e^2*
x^2 + 2*d^2*e*x - d^3)*sqrt(-e^2*x^2 + d^2))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/8*(
d^2*exp(2)^3+2*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^3/x/e
xp(2))/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2/exp(1)
^4+1/16*(-2*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^2*exp(1)^4*exp(2)^5-4*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(
1)^6*exp(2)^4/x/exp(2))/exp(1)^6/exp(2)^3+1/2*(5*d^2*exp(2)^3-2*d^2*exp(1)^
4*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(
2))/exp(1)^3/exp(1)+1/2*(-12*d^2*exp(1)^4*exp(2)^2+8*d^2*exp(2)^4+4*d^2*exp
(1)^6*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(
2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^3/exp(1)+3/2*
d^2*sign(d)*asin(x*exp(2)/d/exp(1))*exp(1)^2+2*(2*exp(1)^3/8*x-4*exp(1)^2*d
/8)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 411, normalized size = 3.40

$$\frac{3d^3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{3d^2e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}+\frac{15d^2e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{15\sqrt{-e^2x^2+d^2}\,e^3x}{8}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^3x}{8}-\frac{3\sqrt{-e^2x^2+d^2}\,d\,e^2}{2}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3x}{4d^2}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^3x}{4d^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{2d}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3x}{d^4}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{10d^3}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e^2}{5d^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{d^4x}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{2d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x)
```

```
[Out] e/d^4/x*(-e^2*x^2+d^2)^(7/2)+e^3/d^4*x*(-e^2*x^2+d^2)^(5/2)+5/4*(-e^2*x^2+d
^2)^(3/2)/d^2*e^3*x+15/8*(-e^2*x^2+d^2)^(1/2)*e^3*x+15/8/(e^2)^(1/2)*d^2*e^
3*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2/d^3/x^2*(-e^2*x^2+d^2)^(7/
2)-3/10/d^3*e^2*(-e^2*x^2+d^2)^(5/2)-1/2*(-e^2*x^2+d^2)^(3/2)/d*e^2-3/2*(-e
^2*x^2+d^2)^(1/2)*d*e^2+3/2/(d^2)^(1/2)*d^3*e^2*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-1/5/d^3*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/4/d
^2*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-3/8*e^3*(2*(x+d/e)*d*e-(x+d/e)
```

```
^2*e^2)^(1/2)*x-3/8*d^2*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(
x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.97, size = 138, normalized size = 1.14

$$\frac{3}{2}\,d^{2}e^{2}\arcsin\left(\frac{ex}{d}\right)+\frac{3}{2}\,d^{2}e^{2}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)+\frac{1}{2}\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{3}x-\frac{3}{2}\,\sqrt{-e^{2}x^{2}+d^{2}}\,de^{2}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,d^{2}e}{x}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*e^2*arcsin(e*x/d) + 3/2*d^2*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2
+ d^2)*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^3*x - 3/2*sqrt(-e^2*x^2 + d^2
)*d*e^2 + sqrt(-e^2*x^2 + d^2)*d^2*e/x - 1/2*(-e^2*x^2 + d^2)^(3/2)*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

sympy [C] time = 13.35, size = 461, normalized size = 3.81

$$d^{3}\left(\begin{cases} -\frac{d^{2}}{2ex^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e}{2x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{2}\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\: \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{2x}-\frac{ie^{2}\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)-d^{2}e\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^{2}x}{d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\: \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^{2}x}{d\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right)-de^{2}\left(\begin{cases} \frac{d^{2}}{ex\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} & \text{for}\: \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ -\frac{id^{2}}{ex\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} & \text{otherwise} \end{cases}\right)+e^{3}\left(\begin{cases} -\frac{id^{2}\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+\frac{ie^{2}x^{3}}{2d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\: \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1 \\ \frac{d^{2}\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), T
rue)) - d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x
/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d
/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1))
- d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2))
> 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*
x/sqrt(-d**2/(e**2*x**2) + 1), True)) + e**3*Piecewise((-I*d**2*acosh(e*x/d
)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 +
e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*s
qrt(1 - e**2*x**2/d**2)/2, True))
```

3.111 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=120

$$\frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {850, 811, 813, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (e^2*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d - 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
```

```
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx\\
&= -\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}-\frac{\int \frac{\left(4d^3e^2-6d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{4d^2}\\
&= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+\frac{\int \frac{12d^4e^3+8d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}\\
&= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+\frac{1}{2}\left(3d^2e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+\left(de^4\right.\\
&= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+\frac{1}{4}\left(3d^2e^3\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx\right.\\
&= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}\left(3d^2e\right)\\
&= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{2}de^3\tan
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 116, normalized size = 0.97

$$-\frac{3}{2}de^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\left(-\frac{d^3}{3x^3}+\frac{d^2e}{2x^2}+\frac{4de^2}{3x}+e^3\right)\sqrt{d^2-e^2x^2}+\frac{3}{2}de^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (e^3 - d^3/(3*x^3) + (d^2*e)/(2*x^2) + (4*d*e^2)/(3*x))*Sqrt[d^2 - e^2*x^2]
 + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (3*d*e^3*Log[x])/2 - (3*d*e^3*
Log[d + Sqrt[d^2 - e^2*x^2]])/2
```

IntegrateAlgebraic [A] time = 0.50, size = 141, normalized size = 1.18

$$d\sqrt{-e^2}\,e^2\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+3de^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{\sqrt{d^2-e^2x^2}\left(-2d^3+3d^2ex+8de^2x^2+6e^3x^3\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 + 3*d^2*e*x + 8*d*e^2*x^2 + 6*e^3*x^3))/(6*x^3
) + 3*d*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + d*e^2*Sqrt[
-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.41, size = 130, normalized size = 1.08

$$-\frac{12\,de^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-9\,de^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-6\,de^3x^3-\left(6\,e^3x^3+8\,de^2x^2+3\,d^2ex-2\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(12*d*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 9*d*e^3*x^3*
log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 6*d*e^3*x^3 - (6*e^3*x^3 + 8*d*e^2*x^2
 + 3*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/24*
((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*(12*d*exp(1)
^4*exp(2)^2-27*d*exp(2)^4)+d*exp(2)^4+3/2*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))*exp(2)^4/x/exp(2))/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^3/exp(1)^5+1/512*(64*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^7-64/3*d*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^8+96*d*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^8/x/exp(2)-384*d*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^7/x/exp(2)+128*d*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^6/x/exp(2))/exp(1)^15
/exp(2)^3+1/2*(-5*d*exp(2)^3+2*d*exp(1)^4*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)/exp(2)+1/2*(12*d*exp(1)^
4*exp(2)^2-8*d*exp(2)^4-4*d*exp(1)^6*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4
+exp(2)^2)/exp(1)/exp(2)+d*sign(d)*asin(x*exp(2)/d/exp(1))*exp(1)^3+4*exp(1
)^3/4*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 439, normalized size = 3.66

$$-\frac{3d^2e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}+\frac{3d\,e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}+\frac{5d\,e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{5\sqrt{-e^2x^2+d^2}\,e^4x}{8d}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^4x}{8d}+\frac{3\sqrt{-e^2x^2+d^2}\,e^3}{2}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4x}{12d^3}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^4x}{4d^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{2d^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4x}{3d^5}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{10d^4}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e^3}{5d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{3d^5x}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{2d^4x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{3d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x)
```

```
[Out] 1/3/d^5*e^2/x*(-e^2*x^2+d^2)^(7/2)+1/3/d^5*e^4*x*(-e^2*x^2+d^2)^(5/2)+5/12*
(-e^2*x^2+d^2)^(3/2)/d^3*e^4*x+5/8*(-e^2*x^2+d^2)^(1/2)/d*e^4*x+5/8/(e^2)^(
1/2)*d*e^4*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/2*e/d^4/x^2*(-e^2*x
^2+d^2)^(7/2)+3/10*e^3/d^4*(-e^2*x^2+d^2)^(5/2)+1/2*(-e^2*x^2+d^2)^(3/2)/d^
2*e^3+3/2*(-e^2*x^2+d^2)^(1/2)*e^3-3/2/(d^2)^(1/2)*d^2*e^3*ln((2*d^2+2*(d^2
)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3/d^3/x^3*(-e^2*x^2+d^2)^(7/2)+1/5/d^4*e
^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+1/4/d^3*e^4*(2*(x+d/e)*d*e-(x+d/e)^2
*e^2)^(3/2)*x+3/8/d*e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+3/8*d*e^4/(e^
2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 132, normalized size = 1.10

$$de^3\arcsin\left(\frac{ex}{d}\right)-\frac{3}{2}\,de^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{3}{2}\,\sqrt{-e^2x^2+d^2}\,e^3+\frac{\sqrt{-e^2x^2+d^2}\,de^2}{x}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{2\,x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] d*e^3*arcsin(e*x/d) - 3/2*d*e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d
/abs(x)) + 3/2*sqrt(-e^2*x^2 + d^2)*e^3 + sqrt(-e^2*x^2 + d^2)*d*e^2/x + 1/
2*(-e^2*x^2 + d^2)^(3/2)*e/x^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)*d/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^4\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

sympy [C] time = 11.68, size = 457, normalized size = 3.81

$$d^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}-\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) -
 d**2*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d)
, True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(
e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
(-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*
x**2/d**2)), True)) + e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1))
 - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2))
 > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e
*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

3.112 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=119

$$\frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {850, 811, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (e^2*(3*d - 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d - 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
```

```
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 && !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx$$

$$= -\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - \frac{\int \frac{\left(6d^3e^2-8d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2}$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{\int \frac{12d^5e^4-32d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{32d^4}$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{8}\left(3de^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^5\int$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{16}\left(3de^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx\right.$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{8}\left(3de^2\right)\text{S}$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4\tanh^{-1}$$

Mathematica [A] time = 0.19, size = 111, normalized size = 0.93

$$\frac{1}{24}\left(-9e^4\log\left(\sqrt{d^2-e^2x^2}+d\right) - 24e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}\left(-6d^3+8d^2ex+15de^2x^2-32e^3x^3\right)}{x^4} + 9e^4\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x + 15*d*e^2*x^2 - 32*e^3*x^3))/x^4
 - 24*e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + 9*e^4*Log[x] - 9*e^4*Log[d +
Sqrt[d^2 - e^2*x^2]])/24
```

IntegrateAlgebraic [A] time = 0.58, size = 142, normalized size = 1.19

$$\frac{3}{4}e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\sqrt{-e^2}\,e^3\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+\frac{\sqrt{d^2-e^2x^2}\,\left(-6d^3+8d^2ex+15de^2x^2-32e^3x^3\right)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x + 15*d*e^2*x^2 - 32*e^3*x^3))/(24*
x^4) + (3*e^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/4 - e^3*Sq
rt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.41, size = 119, normalized size = 1.00

$$\frac{48\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(32\,e^3x^3-15\,de^2x^2-8\,d^2ex+6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(48*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 9*e^4*x^4*log(
-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 - 15*d*e^2*x^2 - 8*d^2*e*x + 6
*d^3)*sqrt(-e^2*x^2 + d^2))/x^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/192
*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-96*exp(1)
^6*exp(2)^2+288*exp(1)^4*exp(2)^3-72*exp(2)^5)+(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*(24*exp(1)^4*exp(2)^3-48*exp(2)^5)+3*exp(
2)^5+4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2))/(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4/exp(1)^6+1/65536*(
-8192*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^
22*exp(2)^7+8192/3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^3*exp(1)^20*exp(2)^8-1024*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^4*exp(1)^18*exp(2)^9+24576*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))/x/exp(2))^2*exp(1)^20*exp(2)^8-8192*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^18*exp(2)^9-12288*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^20*exp(2)^8/x/exp(2)+49152*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^22*exp(2)^7/x/exp(2)-16384*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^24*exp(2)^6/x/exp(2))/exp(1)
^24/exp(2)^4+1/2*(-12*exp(1)^4*exp(2)^2+8*exp(2)^4+4*exp(1)^6*exp(2))*atan(
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+
exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^2+1/8*(24*exp(1)^6*exp(2)^2-28*e
xp(1)^4*exp(2)^3+9*exp(2)^5-8*exp(1)^8*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)^5/exp(1)-sign(d)*asin(x*exp
(2)/d/exp(1))*exp(1)^4
```

maple [B] time = 0.01, size = 463, normalized size = 3.89

$$-\frac{3d\,e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}-\frac{3e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{5e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^5x}{8d^2}-\frac{5\sqrt{-e^2x^2+d^2}\,e^5x}{8d^2}+\frac{3\sqrt{-e^2x^2+d^2}\,e^4}{8d}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^5x}{4d^4}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5x}{12d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}{8d^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5x}{3d^6}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e^4}{5d^5}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{40d^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{3d^6x}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{8d^5x^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{3d^4x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{4d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x)
```

```
[Out] -5/8/(e^2)^(1/2)*e^5*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/8*(-e^2*x
^2+d^2)^(3/2)/d^3*e^4+3/8*(-e^2*x^2+d^2)^(1/2)/d*e^4-1/4/d^4*e^5*(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(3/2)*x-3/8/d^2*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
*x+1/3*e/d^4/x^3*(-e^2*x^2+d^2)^(7/2)-1/3/d^6*e^3/x*(-e^2*x^2+d^2)^(7/2)-1/
3/d^6*e^5*x*(-e^2*x^2+d^2)^(5/2)-1/8/d^5*e^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/12*
(-e^2*x^2+d^2)^(3/2)/d^4*e^5*x-5/8*(-e^2*x^2+d^2)^(1/2)/d^2*e^5*x-3/8/(d^2)
^(1/2)*d*e^4*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/5/d^5*e^4*(
2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-3/8*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(
2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/4/d^3/x^4*(-e^2*x^2+d^2)^(7/2)+3/40
/d^5*e^4*(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 1.01, size = 159, normalized size = 1.34

$$-e^4\arcsin\left(\frac{ex}{d}\right)-\frac{3}{8}e^4\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{3\sqrt{-e^2x^2+d^2}\,e^4}{8d}-\frac{\sqrt{-e^2x^2+d^2}\,e^3}{x}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{8dx^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{3x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] -e^4*arcsin(e*x/d) - 3/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/ab
s(x)) + 3/8*sqrt(-e^2*x^2 + d^2)*e^4/d - sqrt(-e^2*x^2 + d^2)*e^3/x + 3/8*(
-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^2) + 1/3*(-e^2*x^2 + d^2)^(3/2)*e/x^3 - 1/4*
(-e^2*x^2 + d^2)^(3/2)*d/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^5\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

sympy [C] time = 14.37, size = 541, normalized size = 4.55

$$d^3\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e*
*3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*s
qrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3
*d**2), True)) - d*e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) -
1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d
**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asi
n(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)
) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**
2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*
sqrt(1 - e**2*x**2/d**2)), True))
```

3.113 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)} dx$

Optimal. Leaf size=108

$$-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}+\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}+\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}-\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}$$

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 807, 266, 47, 63, 208}

$$-\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}+\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}+\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (-3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) + (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) -
(d^2 - e^2*x^2)^(5/2)/(5*d*x^5) + (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8
*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} - e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} - \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx, x, x^2\right) \\
&= \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} + \frac{1}{8}\left(3e^3\right)\mathrm{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} - \frac{1}{16}\left(3e^5\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, \right. \\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} + \frac{1}{8}\left(3e^3\right)\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2 \right. \\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} + \frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 106, normalized size = 0.98

$$\frac{15e^5x^5\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-8d^4+10d^3ex+16d^2e^2x^2-25de^3x^3-8e^4x^4\right)-15e^5x^5\log(x)}{40dx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 + 10*d^3*e*x + 16*d^2*e^2*x^2 - 25*d*e^3*x^3 -
 8*e^4*x^4) - 15*e^5*x^5*Log[x] + 15*e^5*x^5*Log[d + Sqrt[d^2 - e^2*x^2]])/
(40*d*x^5)
```

IntegrateAlgebraic [A] time = 0.59, size = 155, normalized size = 1.44

$$\frac{3e^5\log\left(\sqrt{d^2-e^2x^2}+d-\sqrt{-e^2}\,x\right)}{8d}-\frac{3e^5\log\left(-d\sqrt{d^2-e^2x^2}+d^2+d\sqrt{-e^2}\,x\right)}{8d}+\frac{\sqrt{d^2-e^2x^2}\left(-8d^4+10d^3ex+16d^2e^2x^2-25de^3x^3-8e^4x^4\right)}{40dx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 + 10*d^3*e*x + 16*d^2*e^2*x^2 - 25*d*e^3*x^3 -
 8*e^4*x^4))/(40*d*x^5) + (3*e^5*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]
])/(8*d) - (3*e^5*Log[d^2 + d*Sqrt[-e^2]*x - d*Sqrt[d^2 - e^2*x^2]])/(8*d)
```

fricas [A] time = 0.40, size = 97, normalized size = 0.90

$$-\frac{15\,e^{5}x^{5}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(8\,e^{4}x^{4}+25\,de^{3}x^{3}-16\,d^{2}e^{2}x^{2}-10\,d^{3}ex+8\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{40\,dx^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^4*x^4 + 25*d*e^
3*x^3 - 16*d^2*e^2*x^2 - 10*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*x^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/960
*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(480*exp(1)
^8*exp(2)^2-1440*exp(1)^6*exp(2)^3+1800*exp(1)^4*exp(2)^4-780*exp(2)^6)+(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-120*exp(1)^6*e
xp(2)^3+360*exp(1)^4*exp(2)^4-120*exp(2)^6)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^2*(40*exp(1)^4*exp(2)^4-70*exp(2)^6)+6*exp(2)^
6+15/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^6/x/exp(2))/(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5/exp(1)^6/d/exp(1)+
1/33554432*(4194304*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^34*exp(2)^8-4194304/3*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^32*exp(2)^9+524288*d^4*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^30*exp(2)^10-1048576
/5*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)
^28*exp(2)^11-12582912*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^2*exp(1)^32*exp(2)^9+4194304*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^30*exp(2)^10+4194304*d^4*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^30*exp(2)^10-5242
880/3*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^28*exp(2)^11+5242880*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*ex
p(1)^28*exp(2)^11/x/exp(2)-18874368*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))*exp(1)^30*exp(2)^10/x/exp(2)+31457280*d^4*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))*exp(1)^32*exp(2)^9/x/exp(2)-25165824*d^4*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^34*exp(2)^8/x/exp(2)+8388608*d^4*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^36*exp(2)^7/x/exp(2))/d^5/exp
(1)^35/exp(2)^5+1/2*(12*exp(1)^4*exp(2)^2-8*exp(2)^4-4*exp(1)^6*exp(2))*ata
n((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^
4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/d/exp(1)+1/8*(-24*exp(1)^6*exp(2)^2+2
8*exp(1)^4*exp(2)^3-9*exp(2)^5+8*exp(1)^8*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)^4/d/exp(1)
```

maple [B] time = 0.02, size = 493, normalized size = 4.56

$$\frac{3e^{6}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,d}-\frac{3e^{6}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8\sqrt{e^{2}}\,d}+\frac{3e^{5}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{8\sqrt{d^{2}}}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{6}x}{8d^{3}}-\frac{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}x}{8d^{3}}-\frac{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{5}}{8d^{2}}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}e^{6}x}{4d^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}x}{4d^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}{8d^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}x}{5d^{7}}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}e^{5}}{5d^{6}}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{5}}{40d^{6}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{4}}{5d^{7}x}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{3}}{8d^{6}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{2}}{5d^{5}x^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e}{4d^{4}x^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{5d^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x)
```

```
[Out] -1/8*(-e^2*x^2+d^2)^(3/2)/d^4*e^5-3/8*(-e^2*x^2+d^2)^(1/2)/d^2*e^5+3/8/(d^2
)^(1/2)*e^5*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/4/d^5*e^6*(2
```

```
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+3/8/d^3*e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^
2)^(1/2)*x+3/8/d*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^
2*e^2)^(1/2)*x)-1/5/d^5*e^2/x^3*(-e^2*x^2+d^2)^(7/2)-1/5/d^7*e^4/x*(-e^2*x^
2+d^2)^(7/2)-1/5/d^7*e^6*x*(-e^2*x^2+d^2)^(5/2)-1/4/d^5*e^6*x*(-e^2*x^2+d^2
)^(3/2)-3/8/d^3*e^6*x*(-e^2*x^2+d^2)^(1/2)-3/8/d*e^6/(e^2)^(1/2)*arctan((e^
2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/4*e/d^4/x^4*(-e^2*x^2+d^2)^(7/2)+1/8*e^3
/d^6/x^2*(-e^2*x^2+d^2)^(7/2)+1/5/d^6*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/
2)-1/5/d^3/x^5*(-e^2*x^2+d^2)^(7/2)-3/40*e^5/d^6*(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 1.00, size = 153, normalized size = 1.42

$$\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8\,d}-\frac{3\,\sqrt{-e^2x^2+d^2}\,e^5}{8\,d^2}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{8\,d^2x^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{5\,dx^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{4\,x^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 3/8*sqrt(-e
^2*x^2 + d^2)*e^5/d^2 - 3/8*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^2) + 1/5*(-e^
2*x^2 + d^2)^(3/2)*e^2/(d*x^3) + 1/4*(-e^2*x^2 + d^2)^(3/2)*e/x^4 - 1/5*(-e
^2*x^2 + d^2)^(3/2)*d/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^6\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

sympy [C] time = 13.94, size = 774, normalized size = 7.17

$$d^3\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\: \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases} - d^2e\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\: \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases} - de^2\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\: \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases} + e^3\begin{cases}-\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\: \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d),x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) - d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2
*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sq
rt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e
**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**
2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sq
rt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + e**3*Piecewise((-d**2/(2*e*x**
3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*a
cosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**
2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True))
```

3.114 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=143

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}+\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}$$

Rubi [A] time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 835, 807, 266, 47, 63, 208}

$$\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) + (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \int \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{x^7}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{\int \frac{(6d^2e-de^2x)(d^2-e^2x^2)^{3/2}}{x^6}\,dx}{6d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} + \frac{e^2\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}\,dx}{6d}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} + \frac{e^2\,\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x, x^2\right)}{12d}$$

$$= -\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^4\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{16d}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} + \frac{e^6\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2}}\right.}{32d}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^4\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,d\right.}{16}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right.}{16d^2}$$

Mathematica [A] time = 0.18, size = 117, normalized size = 0.82

$$\frac{-15e^6x^6\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-40d^5+48d^4ex+70d^3e^2x^2-96d^2e^3x^3-15de^4x^4+48e^5x^5\right)+15e^6x^6\log(x)}{240d^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^5 + 48*d^4*e*x + 70*d^3*e^2*x^2 - 96*d^2*e^3*x^
3 - 15*d*e^4*x^4 + 48*e^5*x^5) + 15*e^6*x^6*Log[x] - 15*e^6*x^6*Log[d + Sqr
t[d^2 - e^2*x^2]])/(240*d^2*x^6)
```

IntegrateAlgebraic [A] time = 0.61, size = 126, normalized size = 0.88

$$\frac{e^6\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^2}+\frac{\sqrt{d^2-e^2x^2}\left(-40d^5+48d^4ex+70d^3e^2x^2-96d^2e^3x^3-15de^4x^4+48e^5x^5\right)}{240d^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^5 + 48*d^4*e*x + 70*d^3*e^2*x^2 - 96*d^2*e^3*x^
3 - 15*d*e^4*x^4 + 48*e^5*x^5))/(240*d^2*x^6) + (e^6*ArcTanh[(Sqrt[-e^2]*x)
/d - Sqrt[d^2 - e^2*x^2]/d])/(8*d^2)
```

fricas [A] time = 0.42, size = 108, normalized size = 0.76

$$\frac{15\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(48\,e^5x^5-15\,de^4x^4-96\,d^2e^3x^3+70\,d^3e^2x^2+48\,d^4ex-40\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/240*(15*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (48*e^5*x^5 - 15*d*e
^4*x^4 - 96*d^2*e^3*x^3 + 70*d^3*e^2*x^2 + 48*d^4*e*x - 40*d^5)*sqrt(-e^2*x
^2 + d^2))/(d^2*x^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/192
0*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(-960*exp(
1)^10*exp(2)^2+2880*exp(1)^8*exp(2)^3-3600*exp(1)^6*exp(2)^4+2160*exp(1)^4*
exp(2)^5-600*exp(2)^7)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^4*(240*exp(1)^8*exp(2)^3-720*exp(1)^6*exp(2)^4+960*exp(1)^4*exp(2)^
5-495*exp(2)^7)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^3*(-80*exp(1)^6*exp(2)^4+240*exp(1)^4*exp(2)^5-100*exp(2)^7)+(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*(30*exp(1)^4*exp(2)^5-45*e
xp(2)^7)+5*exp(2)^7+6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^7/
x/exp(2))/d^2/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6
/exp(1)^8+1/68719476736*(-8589934592*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))/x/exp(2))^2*exp(1)^48*exp(2)^9+8589934592/3*d^10*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^46*exp(2)^10-10
73741824*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4
*exp(1)^44*exp(2)^11+2147483648/5*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^5*exp(1)^42*exp(2)^12-536870912/3*d^10*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^40*exp(2)^13+25769
803776*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*e
xp(1)^46*exp(2)^10-8589934592*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^3*exp(1)^44*exp(2)^11+3221225472*d^10*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^42*exp(2)^12-3435973836
8*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)
^44*exp(2)^11+10737418240/3*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^3*exp(1)^42*exp(2)^12-1610612736*d^10*(-1/2*(-2*d*exp(1)-
2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^40*exp(2)^13+25769803776*
d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4
```

```
2*exp(2)^12-8053063680*d^10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)^40*exp(2)^13-10737418240*d^10*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))*exp(1)^42*exp(2)^12/x/exp(2)+38654705664*d^10*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^44*exp(2)^11/x/exp(2)-6442450944
0*d^10*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^46*exp(2)^10/x/ex
p(2)+51539607552*d^10*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^48
*exp(2)^9/x/exp(2)-17179869184*d^10*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))*exp(1)^50*exp(2)^8/x/exp(2))/d^12/exp(1)^48/exp(2)^6+1/2*(-12*exp(1)^5
*exp(2)^2+12*exp(1)^3*exp(2)^3+4*exp(1)^7*exp(2)-4*exp(1)*exp(2)^4)*atan((-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+ex
p(2)^2))/d^2/sqrt(-exp(1)^4+exp(2)^2)/exp(1)+1/16*(48*exp(1)^10*exp(2)^2-56
*exp(1)^8*exp(2)^3+40*exp(1)^6*exp(2)^4-30*exp(1)^4*exp(2)^5+13*exp(2)^7-16
*exp(1)^12*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/ab
s(x)/exp(2))/d^2/exp(1)^7/exp(1)
```

maple [B] time = 0.01, size = 521, normalized size = 3.64

$$-\frac{e^{6}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{16\sqrt{d^{2}}\,d}-\frac{3e^{7}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,d^{2}}+\frac{3e^{7}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8\sqrt{e^{2}}\,d^{2}}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{7}x}{8d^{4}}+\frac{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}x}{8d^{4}}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}{16d^{3}}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}e^{7}x}{4d^{6}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}x}{4d^{6}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}{48d^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{7}x}{5d^{8}}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}e^{6}}{5d^{7}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}{80d^{7}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{5}}{5d^{8}x}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{4}}{16d^{7}x^{2}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{3}}{5d^{6}x^{3}}-\frac{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{2}}{24d^{5}x^{4}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e}{5d^{4}x^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{6d^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x)
```

```
[Out] 1/48*(-e^2*x^2+d^2)^(3/2)/d^5*e^6+1/16*(-e^2*x^2+d^2)^(1/2)/d^3*e^6+1/5*e/d
^4/x^5*(-e^2*x^2+d^2)^(7/2)-1/4/d^6*e^7*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
*x-3/8/d^4*e^7*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-3/8/d^2*e^7/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+1/5/d^6*e^3/x^3
*(-e^2*x^2+d^2)^(7/2)+1/5/d^8*e^5/x*(-e^2*x^2+d^2)^(7/2)+1/5/d^8*e^7*x*(-e^
2*x^2+d^2)^(5/2)+1/4/d^6*e^7*x*(-e^2*x^2+d^2)^(3/2)+3/8/d^4*e^7*x*(-e^2*x^2
+d^2)^(1/2)+3/8/d^2*e^7/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)-5/24/d^5*e^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/16/d^7*e^4/x^2*(-e^2*x^2+d^2)^(
7/2)-1/16/(d^2)^(1/2)/d*e^6*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x
)-1/5/d^7*e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/6/d^3/x^6*(-e^2*x^2+d^2
)^(7/2)+1/80/d^7*e^6*(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 1.00, size = 178, normalized size = 1.24

$$-\frac{e^{6}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{16\,d^{2}}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}{16\,d^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}{16\,d^{3}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}{5\,d^{2}x^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{8\,dx^{4}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e}{5\,x^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/16*sq
rt(-e^2*x^2 + d^2)*e^6/d^3 + 1/16*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^3*x^2) - 1/
5*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^3) + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*
x^4) + 1/5*(-e^2*x^2 + d^2)^(3/2)*e/x^5 - 1/6*(-e^2*x^2 + d^2)^(3/2)*d/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^7\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

sympy [C] time = 18.69, size = 918, normalized size = 6.42

$$d^{3}\left(\begin{cases}-\frac{d^{2}}{6ex^{7}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{5e}{24x^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{3}}{48d^{2}x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{e^{5}}{16d^{4}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{6}\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^{5}} & \text{for}\: \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\\frac{id^{2}}{6ex^{7}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{5ie}{24x^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{3}}{48d^{2}x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{5}}{16d^{4}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{6}\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^{5}} & \text{otherwise}\end{cases}\right)-d^{2}e\left(\begin{cases}\frac{3id^{3}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}-\frac{4ide^{2}x^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}+\frac{2ie^{6}x^{6}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{5}x^{5}+15d^{3}e^{2}x^{7}}-\frac{ie^{4}x^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{3}x^{5}+15de^{2}x^{7}} & \text{for}\: \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\\frac{3d^{3}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}-\frac{4de^{2}x^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}+\frac{2e^{6}x^{6}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{5}x^{5}+15d^{3}e^{2}x^{7}}-\frac{e^{4}x^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{3}x^{5}+15de^{2}x^{7}} & \text{otherwise}\end{cases}\right)-de^{2}\left(\begin{cases}-\frac{d^{2}}{4ex^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{3e}{8x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{e^{3}}{8d^{2}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{4}\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{for}\: \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\\frac{id^{2}}{4ex^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{3ie}{8x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{3}}{8d^{2}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{4}\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{otherwise}\end{cases}\right)+e^{3}\left(\begin{cases}-\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3x^{2}}+\frac{e^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3d^{2}} & \text{for}\: \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\-\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3x^{2}}+\frac{ie^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3d^{2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - d**2*e*Piecewise((3*I*d**3*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2
) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4
*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e*
*6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4
*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - d
*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5
*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1))
+ I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d
**3), True)) + e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True))
```

3.115 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx$

Optimal. Leaf size=172

$$-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}+\frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6}-\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4}-\frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5}+\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Rubi [A] time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 835, 807, 266, 47, 63, 208}

$$-\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4}-\frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5}+\frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6}-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}+\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] -(e^5*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) + (e^3*(d^2 - e^2*x^2)^(3/2))/(24*d
^2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) + (e*(d^2 - e^2*x^2)^(5/2))/(6*d^
2*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) + (e^7*ArcTanh[Sqrt[d^2
 - e^2*x^2]/d])/(16*d^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{\int \frac{\left(7d^2e-2de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} + \frac{\int \frac{\left(12d^3e^2-7d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} - \frac{e^3\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} - \frac{e^3\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx, x, x^2\right.}{12d^2}$$

$$= \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} + \frac{e^5\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)}{35d^3x^5}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)}{35d^3x^5}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)}{35d^3x^5}$$

Mathematica [A] time = 0.19, size = 128, normalized size = 0.74

$$\frac{105e^7x^7\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-240d^6+280d^5ex+384d^4e^2x^2-490d^3e^3x^3-48d^2e^4x^4+105de^5x^5-96e^6x^6\right)-105e^7x^7\log(x)}{1680d^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-240*d^6 + 280*d^5*e*x + 384*d^4*e^2*x^2 - 490*d^3*e^
3*x^3 - 48*d^2*e^4*x^4 + 105*d*e^5*x^5 - 96*e^6*x^6) - 105*e^7*x^7*Log[x] +
 105*e^7*x^7*Log[d + Sqrt[d^2 - e^2*x^2]])/(1680*d^3*x^7)
```

IntegrateAlgebraic [A] time = 0.68, size = 137, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\left(-240d^6+280d^5ex+384d^4e^2x^2-490d^3e^3x^3-48d^2e^4x^4+105de^5x^5-96e^6x^6\right)}{1680d^3x^7}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-240*d^6 + 280*d^5*e*x + 384*d^4*e^2*x^2 - 490*d^3*e^
3*x^3 - 48*d^2*e^4*x^4 + 105*d*e^5*x^5 - 96*e^6*x^6))/(1680*d^3*x^7) - (e^7
*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(8*d^3)
```

fricas [A] time = 0.43, size = 119, normalized size = 0.69

$$-\frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(96\,e^6x^6-105\,de^5x^5+48\,d^2e^4x^4+490\,d^3e^3x^3-384\,d^4e^2x^2-280\,d^5ex+240\,d^6\right)\sqrt{-e^2x^2+d^2}}{1680\,d^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/1680*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (96*e^6*x^6 - 105
*d*e^5*x^5 + 48*d^2*e^4*x^4 + 490*d^3*e^3*x^3 - 384*d^4*e^2*x^2 - 280*d^5*e
*x + 240*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*x^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/134
40*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*(6720*exp
(1)^12*exp(2)^2-20160*exp(1)^10*exp(2)^3+25200*exp(1)^8*exp(2)^4-21840*exp(
1)^6*exp(2)^5+19320*exp(1)^4*exp(2)^6-8925*exp(2)^8)+(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(-1680*exp(1)^10*exp(2)^3+5040*exp(
1)^8*exp(2)^4-6720*exp(1)^6*exp(2)^5+5040*exp(1)^4*exp(2)^6-1575*exp(2)^8)+
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(560*exp(1)^8
*exp(2)^4-1680*exp(1)^6*exp(2)^5+2380*exp(1)^4*exp(2)^6-1365*exp(2)^8)+(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-210*exp(1)^6*ex
p(2)^5+630*exp(1)^4*exp(2)^6-315*exp(2)^8)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^2*(84*exp(1)^4*exp(2)^6-105*exp(2)^8)+15*exp(2)
^8+35/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2))/d^3/
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7/exp(1)^9+1/56
2949953421312*(70368744177664*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^2*exp(1)^64*exp(2)^10-70368744177664/3*d^18*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^62*exp(2)^11+8796
093022208*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
4*exp(1)^60*exp(2)^12-17592186044416/5*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^58*exp(2)^13+4398046511104/3*d^18*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^56*exp(2)
^14-4398046511104/7*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^7*exp(1)^54*exp(2)^15-211106232532992*d^18*(-1/2*(-2*d*exp(1)-2*s
```

```
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^62*exp(2)^11+70368744177664*
d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6
0*exp(2)^12-26388279066624*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^4*exp(1)^58*exp(2)^13+52776558133248/5*d^18*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^56*exp(2)^14+2814749
76710656*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2
*exp(1)^60*exp(2)^12-299067162755072/3*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^58*exp(2)^13+13194139533312*d^18*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^56*exp(2)^
14-30786325577728/5*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^5*exp(1)^54*exp(2)^15-211106232532992*d^18*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^58*exp(2)^13+87960930222080*
d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^5
6*exp(2)^14+65970697666560*d^18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^2*exp(1)^56*exp(2)^14-30786325577728*d^18*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^54*exp(2)^15+769658139
44320*d^18*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^54*exp(2)^15/
x/exp(2)-263882790666240*d^18*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*e
xp(1)^56*exp(2)^14/x/exp(2)+404620279021568*d^18*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))*exp(1)^58*exp(2)^13/x/exp(2)-457396837154816*d^18*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^60*exp(2)^12/x/exp(2)+527765581
332480*d^18*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^62*exp(2)^11
/x/exp(2)-422212465065984*d^18*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*
exp(1)^64*exp(2)^10/x/exp(2)+140737488355328*d^18*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))*exp(1)^66*exp(2)^9/x/exp(2))/d^21/exp(1)^63/exp(2)^7+1/2
*(12*exp(1)^6*exp(2)^2-12*exp(1)^4*exp(2)^3+4*exp(2)^5-4*exp(1)^8*exp(2))*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/d^3/sqrt(-exp(1)^4+exp(2)^2)/exp(1)+1/16*(-48*exp(1)^10*exp(
2)^2+56*exp(1)^8*exp(2)^3-40*exp(1)^6*exp(2)^4+30*exp(1)^4*exp(2)^5-13*exp(
2)^7+16*exp(1)^12*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/abs(x)/exp(2))/d^3/exp(1)^6/exp(1)
```

maple [B] time = 0.02, size = 546, normalized size = 3.17

$$\frac{e^{7}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{16\sqrt{d^{2}}\,d^{2}}+\frac{3e^{8}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,d^{3}}-\frac{3e^{8}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8\sqrt{e^{2}}\,d^{3}}+\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{8}x}{8d^{5}}-\frac{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{8}x}{8d^{5}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}{16d^{4}}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}e^{8}x}{4d^{7}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{8}x}{4d^{7}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}}{48d^{6}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{8}x}{5d^{9}}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}e^{7}}{5d^{8}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{7}}{80d^{8}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{6}}{5d^{9}x}+\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{5}}{16d^{8}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{4}}{5d^{7}x^{3}}+\frac{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{3}}{24d^{6}x^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{2}}{5d^{5}x^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e}{6d^{4}x^{6}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{7d^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x)
```

```
[Out] 1/4/d^7*e^8*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+3/8/d^5*e^8*(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(1/2)*x+3/8/d^3*e^8/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d
/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/5/d^5*e^2/x^5*(-e^2*x^2+d^2)^(7/2)+1/6*e/
d^4/x^6*(-e^2*x^2+d^2)^(7/2)-1/5/d^7*e^4/x^3*(-e^2*x^2+d^2)^(7/2)-1/5/d^9*e
^6/x*(-e^2*x^2+d^2)^(7/2)-1/5/d^9*e^8*x*(-e^2*x^2+d^2)^(5/2)-1/4/d^7*e^8*x*
(-e^2*x^2+d^2)^(3/2)-3/8/d^5*e^8*x*(-e^2*x^2+d^2)^(1/2)-3/8/d^3*e^8/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+5/24/d^6*e^3/x^4*(-e^2*x^2+
d^2)^(7/2)+3/16/d^8*e^5/x^2*(-e^2*x^2+d^2)^(7/2)+1/16/(d^2)^(1/2)/d^2*e^7*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^8*e^7*(2*(x+d/e)*d*e-
(x+d/e)^2*e^2)^(5/2)-1/80/d^8*e^7*(-e^2*x^2+d^2)^(5/2)-1/7/d^3/x^7*(-e^2*x^
2+d^2)^(7/2)-1/48*(-e^2*x^2+d^2)^(3/2)/d^6*e^7-1/16*(-e^2*x^2+d^2)^(1/2)/d^
4*e^7
```

maxima [A] time = 0.99, size = 203, normalized size = 1.18

$$\frac{e^{7}\log\left(\frac{2d^{2}}{|x|}+\frac{2\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{16d^{3}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}{16d^{4}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}{16d^{4}x^{2}}+\frac{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}{35d^{3}x^{3}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}{8d^{2}x^{4}}+\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{35dx^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e}{6x^{6}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/16*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 1/16*sqr
t(-e^2*x^2 + d^2)*e^7/d^4 - 1/16*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^4*x^2) + 2/3
5*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^3*x^3) - 1/8*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^
2*x^4) + 3/35*(-e^2*x^2 + d^2)^(3/2)*e^2/(d*x^5) + 1/6*(-e^2*x^2 + d^2)^(3/
2)*e/x^6 - 1/7*(-e^2*x^2 + d^2)^(3/2)*d/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^8\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

sympy [C] time = 18.79, size = 1037, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - d**2*e*P
iecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d
**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e*
*5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1))
- 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-
d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I
*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewise((3*I*d**3*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 +
e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1)
, (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**
2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**
6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*
sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e**3*Pi
ecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*aco
sh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d
**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3
/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), Tr
ue))
```

3.116 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^9(d+ex)}\,dx$

Optimal. Leaf size=201

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{3e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}+\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}+\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}$$

Rubi [A] time = 0.19, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 835, 807, 266, 47, 63, 208}

$$\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4}+\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{3e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) + (e*(d^2 - e^2*x^2)^(5/2))/(7*
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) + (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^9(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{\left(8d^2e-3de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx}{8d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} + \frac{\int \frac{\left(21d^3e^2-16d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{\int \frac{\left(96d^4e^3-21d^3e^4x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{336d^6}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4\int \frac{\left(d^2-e^2x^2\right)^{3/\cdots}}{x^5}\cdots}{16d^3}\cdots$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4\,\mathrm{Subst}\left(\int \cdots\right)}{\cdots}\cdots$$

$$= -\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \frac{2e^3\left(d^2-e^2x^2\right)^{\cdots}}{35d^4x^5}\cdots$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \cdots$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \cdots$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} + \cdots$$

Mathematica [A] time = 0.21, size = 139, normalized size = 0.69

$$\frac{-105e^8x^8\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-560d^7+640d^6ex+840d^5e^2x^2-1024d^4e^3x^3-70d^3e^4x^4+128d^2e^5x^5-105de^6x^6+256e^7x^7\right)+105e^8x^8\log(x)}{4480d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 640*d^6*e*x + 840*d^5*e^2*x^2 - 1024*d^4*e
^3*x^3 - 70*d^3*e^4*x^4 + 128*d^2*e^5*x^5 - 105*d*e^6*x^6 + 256*e^7*x^7) +
105*e^8*x^8*Log[x] - 105*e^8*x^8*Log[d + Sqrt[d^2 - e^2*x^2]])/(4480*d^4*x^
8)
```

IntegrateAlgebraic [A] time = 0.74, size = 148, normalized size = 0.74

$$\frac{3e^8\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{64d^4}+\frac{\sqrt{d^2-e^2x^2}\left(-560d^7+640d^6ex+840d^5e^2x^2-1024d^4e^3x^3-70d^3e^4x^4+128d^2e^5x^5-105de^6x^6+256e^7x^7\right)}{4480d^4x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 640*d^6*e*x + 840*d^5*e^2*x^2 - 1024*d^4*e
^3*x^3 - 70*d^3*e^4*x^4 + 128*d^2*e^5*x^5 - 105*d*e^6*x^6 + 256*e^7*x^7))/(
4480*d^4*x^8) + (3*e^8*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(
64*d^4)
```

fricas [A] time = 0.46, size = 130, normalized size = 0.65

$$\frac{105\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(256\,e^7x^7-105\,de^6x^6+128\,d^2e^5x^5-70\,d^3e^4x^4-1024\,d^4e^3x^3+840\,d^5e^2x^2+640\,d^6ex-560\,d^7\right)\sqrt{-e^2x^2+d^2}}{4480\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4480*(105*e^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (256*e^7*x^7 - 105
*d*e^6*x^6 + 128*d^2*e^5*x^5 - 70*d^3*e^4*x^4 - 1024*d^4*e^3*x^3 + 840*d^5*
e^2*x^2 + 640*d^6*e*x - 560*d^7)*sqrt(-e^2*x^2 + d^2))/(d^4*x^8)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/215
040*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*(-107520
*exp(1)^14*exp(2)^2+322560*exp(1)^12*exp(2)^3-403200*exp(1)^10*exp(2)^4+349
440*exp(1)^8*exp(2)^5-309120*exp(1)^6*exp(2)^6+201600*exp(1)^4*exp(2)^7-588
00*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*
(26880*exp(1)^12*exp(2)^3-80640*exp(1)^10*exp(2)^4+107520*exp(1)^8*exp(2)^5
-107520*exp(1)^6*exp(2)^6+105840*exp(1)^4*exp(2)^7-52080*exp(2)^9)+(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(-8960*exp(1)^10*exp(
2)^4+26880*exp(1)^8*exp(2)^5-38080*exp(1)^6*exp(2)^6+33600*exp(1)^4*exp(2)^
7-11760*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^4*(3360*exp(1)^8*exp(2)^5-10080*exp(1)^6*exp(2)^6+15120*exp(1)^4*exp(2)^
7-9240*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^3*(-1344*exp(1)^6*exp(2)^6+4032*exp(1)^4*exp(2)^7-2352*exp(2)^9)+(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*(560*exp(1)^4*exp(2)^
7-560*exp(2)^9)+105*exp(2)^9+120*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)*exp(2)^9/x/exp(2))/d^4/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^8/exp(1)^10+1/18446744073709551616*(-2305843009213693952*d^28*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^82*exp(2)
```

```
^11+2305843009213693952/3*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^3*exp(1)^80*exp(2)^12-2882303761517117440*d^28*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^78*exp(2)^13+576460
752303423488/5*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^5*exp(1)^76*exp(2)^14-144115188075855872/3*d^28*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^74*exp(2)^15+144115188075855
872/7*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*ex
p(1)^72*exp(2)^16-9007199254740992*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^8*exp(1)^70*exp(2)^17+6917529027641081856*d^28*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^80*exp(2)
^12-2305843009213693952*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^3*exp(1)^78*exp(2)^13+864691128455135232*d^28*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^76*exp(2)^14-17293822
56910270464/5*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^5*exp(1)^74*exp(2)^15+144115188075855872*d^28*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^72*exp(2)^16-922337203685477580
8*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)
^78*exp(2)^13+9799832789158199296/3*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^3*exp(1)^76*exp(2)^14-1297036692682702848*d^28*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^74*exp(2
)^15+1008806316530991104/5*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^5*exp(1)^72*exp(2)^16-288230376151711744/3*d^28*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^70*exp(2)^17+922
3372036854775808*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(1)^76*exp(2)^14-2882303761517117440*d^28*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^74*exp(2)^15+12970366926827
02848*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*ex
p(1)^72*exp(2)^16-9079256848778919936*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^74*exp(2)^15+1008806316530991104*d^28*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^72*exp
(2)^16-504403158265495552*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^4*exp(1)^70*exp(2)^17+6485183463413514240*d^28*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^72*exp(2)^16-20176
12633061982208*d^28*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^2*exp(1)^70*exp(2)^17-2522015791327477760*d^28*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))*exp(1)^72*exp(2)^16/x/exp(2)+8646911284551352320*d^28*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^74*exp(2)^15/x/exp(2)-13
258597302978740224*d^28*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^
76*exp(2)^14/x/exp(2)+14987979559889010688*d^28*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))*exp(1)^78*exp(2)^13/x/exp(2)-17293822569102704640*d^28*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^80*exp(2)^12/x/exp(2)+13835
058055282163712*d^28*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^82*
exp(2)^11/x/exp(2)-4611686018427387904*d^28*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))*exp(1)^84*exp(2)^10/x/exp(2))/d^32/exp(1)^80/exp(2)^8+1/2*(-12
*exp(1)^7*exp(2)^2+12*exp(1)^5*exp(2)^3-4*exp(1)^3*exp(2)^4+4*exp(1)^9*exp(
2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-
exp(1)^4+exp(2)^2))/d^4/sqrt(-exp(1)^4+exp(2)^2)/exp(1)+1/128*(384*exp(1)^1
4*exp(2)^2-448*exp(1)^12*exp(2)^3+320*exp(1)^10*exp(2)^4-240*exp(1)^8*exp(2
)^5+208*exp(1)^6*exp(2)^6-184*exp(1)^4*exp(2)^7+85*exp(2)^9-128*exp(1)^16*e
xp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))
/d^4/exp(1)^9/exp(1)
```

maple [B] time = 0.02, size = 571, normalized size = 2.84

$$-\frac{3e^8\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{128\sqrt{d^2}\,d^3}-\frac{3e^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}\,d^4}+\frac{3e^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,d^4}-\frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^9x}{8d^6}+\frac{3\sqrt{-e^2x^2+d^2}\,e^9x}{8d^6}+\frac{3\sqrt{-e^2x^2+d^2}\,e^8}{128d^5}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^9x}{4d^8}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^9x}{4d^8}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^8}{128d^7}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^9x}{5d^{10}}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e^8}{5d^9}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^8}{640d^9}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^7}{5d^{10}x}-\frac{25\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^6}{128d^9x^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^5}{5d^8x^3}-\frac{13\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{64d^7x^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{5d^6x^5}-\frac{3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{16d^5x^6}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{7d^4x^7}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8d^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x)
```

```
[Out] -1/4/d^8*e^9*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-3/8/d^6*e^9*(2*(x+d/e)*d
```

```
*e-(x+d/e)^2*e^2)^(1/2)*x-3/8/d^4*e^9/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+1/5/d^6*e^3/x^5*(-e^2*x^2+d^2)^(7/2)-3/16/
d^5*e^2/x^6*(-e^2*x^2+d^2)^(7/2)+1/5/d^8*e^5/x^3*(-e^2*x^2+d^2)^(7/2)+1/5/d
^10*e^7/x*(-e^2*x^2+d^2)^(7/2)+1/5/d^10*e^9*x*(-e^2*x^2+d^2)^(5/2)+1/4/d^8*
e^9*x*(-e^2*x^2+d^2)^(3/2)+3/8/d^6*e^9*x*(-e^2*x^2+d^2)^(1/2)+3/8/d^4*e^9/(
e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-13/64/d^7*e^4/x^4*(-e
^2*x^2+d^2)^(7/2)-25/128/d^9*e^6/x^2*(-e^2*x^2+d^2)^(7/2)+1/7*e/d^4/x^7*(-e
^2*x^2+d^2)^(7/2)-3/128/(d^2)^(1/2)/d^3*e^8*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x
^2+d^2)^(1/2))/x)-1/5/d^9*e^8*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/8/d^3/x
^8*(-e^2*x^2+d^2)^(7/2)+3/640/d^9*e^8*(-e^2*x^2+d^2)^(5/2)+1/128*(-e^2*x^2+
d^2)^(3/2)/d^7*e^8+3/128*(-e^2*x^2+d^2)^(1/2)/d^5*e^8
```

maxima [A] time = 1.00, size = 228, normalized size = 1.13

$$-\frac{3\,e^{8}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{128\,d^{4}}+\frac{3\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{8}}{128\,d^{5}}+\frac{3\,\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}{128\,d^{5}x^{2}}-\frac{2\,\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}{35\,d^{4}x^{3}}+\frac{3\,\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}{64\,d^{3}x^{4}}-\frac{3\,\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}{35\,d^{2}x^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{16\,dx^{6}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e}{7\,x^{7}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d}{8\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 3/128*
sqrt(-e^2*x^2 + d^2)*e^8/d^5 + 3/128*(-e^2*x^2 + d^2)^(3/2)*e^6/(d^5*x^2) -
 2/35*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^4*x^3) + 3/64*(-e^2*x^2 + d^2)^(3/2)*e^
4/(d^3*x^4) - 3/35*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^2*x^5) + 1/16*(-e^2*x^2 +
d^2)^(3/2)*e^2/(d*x^6) + 1/7*(-e^2*x^2 + d^2)^(3/2)*e/x^7 - 1/8*(-e^2*x^2 +
 d^2)^(3/2)*d/x^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^9\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

sympy [C] time = 27.71, size = 1159, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**9/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) +
 e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x*
*2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x
**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6)
, True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) +
5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e*
```

```
*2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d
/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2
/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(
48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + e**3*Piecewise((
3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e
**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**
6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e*
*4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x
**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x
**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7),
 True))
```

3.117 $\int \frac{x\sqrt{1-x^2}}{1+x}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {785, 780, 216}

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(1 + x),x]
```

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{1-x^2}}{1+x}\,dx &= \int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 0.96

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(1 + x),x]
```

```
[Out] (-1 + x/2)*Sqrt[1 - x^2] - ArcSin[x]/2
```

IntegrateAlgebraic [A] time = 0.18, size = 37, normalized size = 1.37

$$\frac{1}{2}\sqrt{1-x^2}\,(x-2)+\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[1 - x^2])/(1 + x),x]
[Out] ((-2 + x)*Sqrt[1 - x^2])/2 + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

fricas [A] time = 0.41, size = 31, normalized size = 1.15

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.16, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

maple [A] time = 0.01, size = 34, normalized size = 1.26

$$\frac{\sqrt{-x^2+1}\,x}{2}-\frac{\arcsin(x)}{2}-\sqrt{2x-(x+1)^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(1+x),x)
[Out] 1/2*(-x^2+1)^(1/2)*x-1/2*arcsin(x)-(-(1+x)^2+2+2*x)^(1/2)
```

maxima [A] time = 0.98, size = 28, normalized size = 1.04

$$\frac{1}{2}\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.74

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\mathrm{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^2)^(1/2))/(x + 1),x)
```

```
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

sympy [**A**] time = 3.35, size = 29, normalized size = 1.07

$$\begin{cases} \frac{x\sqrt{1-x^2}}{2} - \sqrt{1-x^2} - \frac{\operatorname{asin}(x)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(1+x),x)
```

```
[Out] Piecewise((x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2, (x > -1) & (x <
 1)))
```

3.118 $\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)} dx$

Optimal. Leaf size=51

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - a\sin^{-1}(ax)$$

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {850, 813, 844, 216, 266, 63, 208}

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - a\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

```
[Out] -(((1 - a*x)*Sqrt[1 - a^2*x^2])/x) - a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2
*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^2(1-ax)}\,dx &= \int \frac{(1+ax)\sqrt{1-a^2x^2}}{x^2}\,dx \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x}-\frac{1}{2}\int \frac{-2a+2a^2x}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x}+a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx-a^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x}-a\sin^{-1}(ax)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right) \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x}-a\sin^{-1}(ax)-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x}-a\sin^{-1}(ax)-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 49, normalized size = 0.96

$$\frac{\sqrt{1-a^2x^2}\,(ax-1)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-a\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

```
[Out] ((-1 + a*x)*Sqrt[1 - a^2*x^2])/x - a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2*x
^2]]
```

IntegrateAlgebraic [A] time = 0.44, size = 95, normalized size = 1.86

$$\frac{\sqrt{1-a^2x^2}\,(ax-1)}{x}-\sqrt{-a^2}\,\log\left(\sqrt{1-a^2x^2}-\sqrt{-a^2}\,x\right)+2a\tanh^{-1}\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

```
[Out] ((-1 + a*x)*Sqrt[1 - a^2*x^2])/x + 2*a*ArcTanh[Sqrt[-a^2]*x - Sqrt[1 - a^2*
x^2]] - Sqrt[-a^2]*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x^2]]
```

fricas [A] time = 0.41, size = 74, normalized size = 1.45

$$\frac{2\,ax\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+ax+\sqrt{-a^2x^2+1}\,(ax-1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a*x*log((sqrt(-a^2*x^2 + 1)
 - 1)/x) + a*x + sqrt(-a^2*x^2 + 1)*(a*x - 1))/x
```

giac [B] time = 0.19, size = 125, normalized size = 2.45

$$\frac{a^4x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{a^2\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}-\frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{|a|}+\sqrt{-a^2x^2+1}\,a-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*arcsin(a*x)*sgn(a)
/abs(a) - a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))
/abs(a) + sqrt(-a^2*x^2 + 1)*a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs
(a))
```

maple [B] time = 0.02, size = 238, normalized size = 4.67

$$\frac{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}\;a^2x}{2}-\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2x-\frac{3\sqrt{-a^2x^2+1}\;a^2x}{2}+\frac{a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}\right)}{2\sqrt{a^2}}-\frac{3a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}-a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\left(-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a\right)^{\frac{3}{2}}a}{3}+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}a}{3}+\sqrt{-a^2x^2+1}\;a-\frac{\left(-a^2x^2+1\right)^{\frac{5}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x)
```

```
[Out] -1/3*a*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+1/2*a^2*(-(x-1/a)^2*a^2-2*(x-1/a)
*a)^(1/2)*x+1/2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x-1/a)^2*a^2-2*(x-1
/a)*a)^(1/2))-1/x*(-a^2*x^2+1)^(5/2)-a^2*x*(-a^2*x^2+1)^(3/2)-3/2*a^2*x*(-a
^2*x^2+1)^(1/2)-3/2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*x^2+1)^(1/2)*x
)+1/3*a*(-a^2*x^2+1)^(3/2)+a*(-a^2*x^2+1)^(1/2)-a*arctanh(1/(-a^2*x^2+1)^(1
/2))
```

maxima [A] time = 0.99, size = 68, normalized size = 1.33

$$-a\arcsin(ax)-a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\sqrt{-a^2x^2+1}\,a-\frac{\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="maxima")
```

```
[Out] -a*arcsin(a*x) - a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*
x^2 + 1)*a - sqrt(-a^2*x^2 + 1)/x
```

mupad [B] time = 0.05, size = 74, normalized size = 1.45

$$a\sqrt{1-a^2\,x^2}-\frac{\sqrt{1-a^2\,x^2}}{x}-\frac{a^2\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}+a\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\;1{}\mathrm{i}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1 - a^2*x^2)^(3/2)/(x^2*(a*x - 1)),x)
```

```
[Out] a*atan((1 - a^2*x^2)^(1/2)*1i)*1i + a*(1 - a^2*x^2)^(1/2) - (1 - a^2*x^2)^(
1/2)/x - (a^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

sympy [C] time = 6.53, size = 170, normalized size = 3.33

$$a\left(\begin{cases} i\sqrt{a^2x^2-1} - \log(ax) + \frac{\log(a^2x^2)}{2} + i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{for } |a^2x^2| > 1 \\ \sqrt{-a^2x^2+1} + \frac{\log(a^2x^2)}{2} - \log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{ia^2x}{\sqrt{a^2x^2-1}} + ia\operatorname{acosh}(ax) + \frac{i}{x\sqrt{a^2x^2-1}} & \text{for } |a^2x^2| > 1 \\ \frac{a^2x}{\sqrt{-a^2x^2+1}} - a\operatorname{asin}(ax) - \frac{1}{x\sqrt{-a^2x^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)/x**2/(-a*x+1),x)
```

```
[Out] a*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(1
/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 - lo
g(sqrt(-a**2*x**2 + 1) + 1), True)) + Piecewise((-I*a**2*x/sqrt(a**2*x**2 -
 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1), (a**
2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1)), True))
```

3.119 $\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=118

$$\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}-\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 819, 833, 780, 217, 203}

$$-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}+\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (x^3*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) - (4*x^2*Sqrt[d^2 - e^2*x^2])/(3*
e^3) - (d*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^5) - (3*d^3*ArcTan[(e*x)
/Sqrt[d^2 - e^2*x^2]])/(2*e^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
```

```
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x^2\left(3d^3-4d^2ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} + \frac{\int \frac{x\left(8d^4e-9d^3e^2x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{3d^2e^4}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{\left(3d^3\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^4}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{\left(3d^3\right)\text{Subst}\left(\int \frac{1}{1+e^2 \dots}\right.}{2e \dots}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right.}{2e^5}$$

Mathematica [A] time = 0.09, size = 91, normalized size = 0.77

$$\frac{\sqrt{d^2-e^2x^2}\left(-16d^3-7d^2ex+de^2x^2-2e^3x^3\right)-9d^3(d+ex)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 - 7*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3) - 9*d^3*
(d + e*x)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(6*e^5*(d + e*x))
```

IntegrateAlgebraic [A] time = 0.46, size = 109, normalized size = 0.92

$$\frac{\sqrt{d^2-e^2x^2}\left(-16d^3-7d^2ex+de^2x^2-2e^3x^3\right)}{6e^5(d+ex)} - \frac{3d^3\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 - 7*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(6*e^5*
(d + e*x)) - (3*d^3*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/
(2*e^6)
```

fricas [A] time = 0.41, size = 112, normalized size = 0.95

$$-\frac{16\,d^3ex+16\,d^4-18\left(d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,e^3x^3-de^2x^2+7\,d^2ex+16\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\left(e^6x+de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(16*d^3*e*x + 16*d^4 - 18*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) + (2*e^3*x^3 - d*e^2*x^2 + 7*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2
+ d^2))/(e^6*x + d*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -3/2*
d^3*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^5-2*d^3*exp(2)*atan((-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/s
qrt(-exp(1)^4+exp(2)^2)/exp(1)^4/exp(1)+2*((-16*exp(1)^13*1/96/exp(1)^16*x+
24*exp(1)^12*d*1/96/exp(1)^16)*x-80*exp(1)^11*d^2*1/96/exp(1)^16)*sqrt(-exp
(2)*x^2+d^2)
```

maple [A] time = 0.01, size = 147, normalized size = 1.25

$$-\frac{3d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^4}-\frac{\sqrt{-e^2x^2+d^2}\,x^2}{3e^3}+\frac{\sqrt{-e^2x^2+d^2}\,dx}{2e^4}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3}{\left(x+\frac{d}{e}\right)e^6}-\frac{5\sqrt{-e^2x^2+d^2}\,d^2}{3e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3-5/3/e^5*d^2*(-e^2*x^2+d^2)^(1/2)+1/2*d/e^
4*x*(-e^2*x^2+d^2)^(1/2)-3/2*d^3/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x
^2+d^2)^(1/2)*x)-d^3/e^6/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 1.01, size = 113, normalized size = 0.96

$$-\frac{\sqrt{-e^2x^2+d^2}\,d^3}{e^6x+de^5}-\frac{\sqrt{-e^2x^2+d^2}\,x^2}{3\,e^3}-\frac{3\,d^3\arcsin\left(\frac{ex}{d}\right)}{2\,e^5}+\frac{\sqrt{-e^2x^2+d^2}\,dx}{2\,e^4}-\frac{5\,\sqrt{-e^2x^2+d^2}\,d^2}{3\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*d^3/(e^6*x + d*e^5) - 1/3*sqrt(-e^2*x^2 + d^2)*x^2/e^
3 - 3/2*d^3*arcsin(e*x/d)/e^5 + 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^4 - 5/3*sqrt
(-e^2*x^2 + d^2)*d^2/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.120 $\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=91

$$\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 780, 217, 203}

$$\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}+\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (x^2*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) + ((4*d - 3*e*x)*Sqrt[d^2 - e^2*x
^2])/(2*e^4) + (3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}\\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{\left(3d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^3}\\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}\\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^2+dex-e^2x^2\right)+3d^2(d+ex)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^2 + d*e*x - e^2*x^2) + 3*d^2*(d + e*x)*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4*(d + e*x))
```

IntegrateAlgebraic [A] time = 0.36, size = 98, normalized size = 1.08

$$\frac{3d^2\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^5}+\frac{\sqrt{d^2-e^2x^2}\left(4d^2+dex-e^2x^2\right)}{2e^4(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^2 + d*e*x - e^2*x^2))/(2*e^4*(d + e*x)) + (3*d^2*
Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^5)
```

fricas [A] time = 0.40, size = 101, normalized size = 1.11

$$\frac{4\,d^2ex+4\,d^3-6\left(d^2ex+d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(e^2x^2-dex-4\,d^2\right)\sqrt{-e^2x^2+d^2}}{2\left(e^5x+de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*d^2*e*x + 4*d^3 - 6*(d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (e^2*x^2 - d*e*x - 4*d^2)*sqrt(-e^2*x^2 + d^2))/(e^5*x + d*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 3/2*d
^2*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^4+2*d^2*exp(2)*atan((-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sq
rt(-exp(1)^4+exp(2)^2)/exp(1)^3/exp(1)+2*(-4*exp(1)^7*1/16/exp(1)^10*x+8*ex
p(1)^6*d*1/16/exp(1)^10)*sqrt(-exp(2)*x^2+d^2)
```

maple [A] time = 0.01, size = 120, normalized size = 1.32

$$\frac{3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^3}-\frac{\sqrt{-e^2x^2+d^2}\,x}{2e^3}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2}{\left(x+\frac{d}{e}\right)e^5}+\frac{\sqrt{-e^2x^2+d^2}\,d}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2/e^3*x*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/
(-e^2*x^2+d^2)^(1/2)*x)+d/e^4*(-e^2*x^2+d^2)^(1/2)+d^2/e^5/(x+d/e)*(2*(x+d/
e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 1.00, size = 86, normalized size = 0.95

$$\frac{\sqrt{-e^2x^2+d^2}\,d^2}{e^5x+de^4}+\frac{3\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^4}-\frac{\sqrt{-e^2x^2+d^2}\,x}{2\,e^3}+\frac{\sqrt{-e^2x^2+d^2}\,d}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-e^2*x^2 + d^2)*d^2/(e^5*x + d*e^4) + 3/2*d^2*arcsin(e*x/d)/e^4 - 1/2*
sqrt(-e^2*x^2 + d^2)*x/e^3 + sqrt(-e^2*x^2 + d^2)*d/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.121 $\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=77

$$-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1639, 12, 793, 217, 203}

$$-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/e^3) - (d*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{\int \frac{de^3x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.77

$$-\frac{\frac{\sqrt{d^2-e^2x^2}(2d+ex)}{d+ex}+d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((((2*d + e*x)*Sqrt[d^2 - e^2*x^2])/(d + e*x) + d*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/e^3)
```

IntegrateAlgebraic [A] time = 0.34, size = 81, normalized size = 1.05

$$\frac{(-2d-ex)\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((-2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*Sqrt[-e^2]*Log[-(Sq
rt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^4
```

fricas [A] time = 0.41, size = 85, normalized size = 1.10

$$-\frac{2\,dex+2\,d^2-2\left(dex+d^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}\,(ex+2\,d)}{e^4x+de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*d*e*x + 2*d^2 - 2*(d*e*x + d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + sqrt(-e^2*x^2 + d^2)*(e*x + 2*d))/(e^4*x + d*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -d*si
gn(d)*asin(x*exp(2)/d/exp(1))/exp(1)/exp(2)-2*d*exp(2)*atan((-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt
(-exp(1)^4+exp(2)^2)/exp(1)/exp(2)-4*exp(1)^2*1/4/exp(1)^5*sqrt(-exp(2)*x^2
+d^2)
```

maple [A] time = 0.01, size = 97, normalized size = 1.26

$$-\frac{d \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\, e^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2}\, d}{\left(x+\frac{d}{e}\right) e^4}-\frac{\sqrt{-e^2x^2+d^2}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e^3-1/(e^2)^(1/2)*d/e^2*arctan((e^2)^(1/2)/(-e^2*x^2+
d^2)^(1/2)*x)-d/e^4/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 0.98, size = 63, normalized size = 0.82

$$-\frac{\sqrt{-e^2x^2+d^2}\, d}{e^4x+de^3}-\frac{d \arcsin\left(\frac{ex}{d}\right)}{e^3}-\frac{\sqrt{-e^2x^2+d^2}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*d/(e^4*x + d*e^3) - d*arcsin(e*x/d)/e^3 - sqrt(-e^2*x
^2 + d^2)/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.122 $\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {793, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]

[Out] Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx&=\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e}\\&=\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e}\\&=\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.94

$$\frac{\frac{\sqrt{d^2-e^2x^2}}{d+ex}+\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]/(d + e*x) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

IntegrateAlgebraic [A] time = 0.29, size = 71, normalized size = 1.37

$$\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) + (Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqr
t[d^2 - e^2*x^2]])/e^3
```

fricas [A] time = 0.40, size = 67, normalized size = 1.29

$$\frac{ex-2\,(ex+d)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d+\sqrt{-e^2x^2+d^2}}{e^3x+de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x - 2*(e*x + d)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d + sqrt(-e^
2*x^2 + d^2))/(e^3*x + d*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: sign(
d)*asin(x*exp(2)/d/exp(1))/exp(1)^2+2*exp(2)*atan((-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4
+exp(2)^2)/exp(1)^2
```

maple [A] time = 0.01, size = 74, normalized size = 1.42

$$\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{\left(x+\frac{d}{e}\right)e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] 1/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/e^3/(x+d/e)*(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 0.97, size = 40, normalized size = 0.77

$$\frac{\sqrt{-e^2x^2+d^2}}{e^3x+de^2}+\frac{\arcsin\left(\frac{ex}{d}\right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-e^2*x^2 + d^2)/(e^3*x + d*e^2) + arcsin(e*x/d)/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.123 $\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.03

$$-\frac{\sqrt{d^2-e^2x^2}}{d^2e+de^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d^2*e + d*e^2*x))
```

IntegrateAlgebraic [A] time = 0.00, size = 31, normalized size = 1.00

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

fricas [A] time = 0.38, size = 35, normalized size = 1.13

$$-\frac{ex+d+\sqrt{-e^2x^2+d^2}}{de^2x+d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(e*x + d + sqrt(-e^2*x^2 + d^2))/(d*e^2*x + d^2*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -2*ex
p(2)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(
-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/d/exp(1)
```

maple [A] time = 0.01, size = 29, normalized size = 0.94

$$-\frac{-ex+d}{\sqrt{-e^{2}x^{2}+d^{2}}\, de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e*x+d)/d/e/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 0.98, size = 30, normalized size = 0.97

$$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{de^{2}x+d^{2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)/(d*e^2*x + d^2*e)
```

mupad [B] time = 2.64, size = 29, normalized size = 0.94

$$-\frac{\sqrt{d^2-e^2\,x^2}}{d\,e\,\left(d+e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] -(d^2 - e^2*x^2)^(1/2)/(d*e*(d + e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.124 $\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {857, 12, 266, 63, 208}

$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(d^2*(d + e*x)) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} + \frac{\int \frac{de^2}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{de^2}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.96

$$\frac{\frac{\sqrt{d^2-e^2x^2}}{d+ex} - \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]/(d + e*x) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

IntegrateAlgebraic [A] time = 0.40, size = 70, normalized size = 1.30

$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} + \frac{2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(d^2*(d + e*x)) + (2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^
2 - e^2*x^2]/d])/d^2
```

fricas [A] time = 0.38, size = 62, normalized size = 1.15

$$\frac{ex + (ex+d)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + d + \sqrt{-e^2x^2+d^2}}{d^2ex+d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + (e*x + d)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + d + sqrt(-e^2*x^2 + d
^2))/(d^2*e*x + d^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -exp(
2)*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^2
/exp(1)^2+2*exp(1)*exp(2)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/d^2/sqrt(-exp(1)^4+exp(2)^2)/exp(
1)
```

maple [A] time = 0.01, size = 88, normalized size = 1.63

$$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{\left(x+\frac{d}{e}\right)d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^2/e/(
x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-e^2x^2+d^2}\,(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{x\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.125 $\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {857, 807, 266, 63, 208}

$$-\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(d^3*x) + Sqrt[d^2 - e^2*x^2]/(d^2*x*(d + e*x)) +
(e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{\int \frac{-2de^2+e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$

$$= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{e\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^2}$$

$$= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^2e}$$

$$= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.77

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{(d+2ex)\sqrt{d^2-e^2x^2}}{x(d+ex)}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-(((d + 2*e*x)*Sqrt[d^2 - e^2*x^2])/(x*(d + e*x))) + e*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])/d^3
```

IntegrateAlgebraic [A] time = 0.36, size = 82, normalized size = 1.01

$$\frac{(-d-2ex)\sqrt{d^2-e^2x^2}}{d^3x(d+ex)} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((-d - 2*e*x)*Sqrt[d^2 - e^2*x^2])/(d^3*x*(d + e*x)) - (2*e*ArcTanh[(Sqrt[-
e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^3
```

fricas [A] time = 0.41, size = 88, normalized size = 1.09

$$-\frac{e^2x^2 + dex + \left(e^2x^2 + dex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \sqrt{-e^2x^2+d^2}\,(2\,ex+d)}{d^3ex^2 + d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(e^2*x^2 + d*e*x + (e^2*x^2 + d*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) +
sqrt(-e^2*x^2 + d^2)*(2*e*x + d))/(d^3*e*x^2 + d^4*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -x*ex
p(2)^3/d^3/(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/exp(1)/exp(2)-2*exp(
2)^2*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(
-exp(1)^4+exp(2)^2))/d^3/sqrt(-exp(1)^4+exp(2)^2)/exp(1)+1/4*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^3/d^3/x/exp(1)/exp(2)^3+exp(2)*ln(1/2*
abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^3/exp(1)
```

maple [A] time = 0.01, size = 108, normalized size = 1.33

$$\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{\left(x+\frac{d}{e}\right)d^3}-\frac{\sqrt{-e^2x^2+d^2}}{d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^3/x+e/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x)-1/d^3/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-e^2x^2+d^2}\,(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^2\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.126 $\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}+\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}-\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 835, 807, 266, 63, 208}

$$\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}-\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^4*x) +
Sqrt[d^2 - e^2*x^2]/(d^2*x^2*(d + e*x)) - (3*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2
]/d])/(2*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{\int \frac{-3de^2+2e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{\int \frac{-4d^2e^3+3de^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^4e^2}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{\left(3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{\left(3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right.}{4d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2\right.}{2d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Mathematica [A] time = 0.32, size = 127, normalized size = 1.12

$$-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4} - \frac{d^3 + de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right) - 2d^2ex - 3de^2x^2 + 4e^3x^3}{2d^4x^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^4) - (d^3 - 2*d^2*e*x - 3*d*e^2*x^
2 + 4*e^3*x^3 + d*e^2*x^2*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2
)/d^2]])/(2*d^4*x^2*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.44, size = 97, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\left(-d^2+dex+4e^2x^2\right)}{2d^4x^2(d+ex)} + \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 + d*e*x + 4*e^2*x^2))/(2*d^4*x^2*(d + e*x)) + (3
*e^2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^4
```

fricas [A] time = 0.41, size = 113, normalized size = 1.00

$$\frac{2\,e^3x^3 + 2\,de^2x^2 + 3\left(e^3x^3 + de^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \left(4\,e^2x^2 + dex - d^2\right)\sqrt{-e^2x^2 + d^2}}{2\left(d^4ex^3 + d^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^3*x^3 + 2*d*e^2*x^2 + 3*(e^3*x^3 + d*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) + (4*e^2*x^2 + d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e*x^3
 + d^5*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/8*(
exp(2)^3+2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^3/x/exp(2))/d
^4/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2/exp(1)^4+1
/16*(-2*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*e
xp(1)^4*exp(2)^5-4*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6
*exp(2)^4/x/exp(2))/d^8/exp(1)^6/exp(2)^3+1/2*(-exp(2)^3-2*exp(1)^4*exp(2))
*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^4/e
xp(1)^3/exp(1)+2*exp(1)^3*exp(2)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/d^4/sqrt(-exp(1)^4+exp(2)^
2)/exp(1)
```

maple [A] time = 0.01, size = 133, normalized size = 1.18

$$-\frac{3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^3} + \frac{\sqrt{2\left(x+\frac{d}{e}\right)de - \left(x+\frac{d}{e}\right)^2 e^2}\;e}{\left(x+\frac{d}{e}\right)d^4} + \frac{\sqrt{-e^2x^2+d^2}\;e}{d^4x} - \frac{\sqrt{-e^2x^2+d^2}}{2d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] e*(-e^2*x^2+d^2)^(1/2)/d^4/x-1/2*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-3/2/d^3*e^2/(
d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^4*e/(x+d/e)
*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-e^2x^2 + d^2}\,(ex + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.127 $\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6}-\frac{5d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}-\frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 780, 217, 203}

$$\frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}-\frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6}-\frac{5d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^4*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x^2*(4*d - 5*e*x))/(3*e^4*
Sqrt[d^2 - e^2*x^2]) - ((16*d - 15*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^6) - (5*d
^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
```

```
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{x^5(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx \\
&= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x^3\left(4d^3-5d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2} \\
&= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{x\left(8d^5-15d^4ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{3d^4e^4} \\
&= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{\left(5d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{\left(5d^2\right)\text{Subst}\left(\int \right.}{} \\
&= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{5d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e}}\right.}{2e^6}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 106, normalized size = 0.83

$$
\frac{\frac{\sqrt{d^2-e^2x^2}\left(16d^4+d^3ex-23d^2e^2x^2-3de^3x^3+3e^4x^4\right)}{(ex-d)(d+ex)^2}-15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(16*d^4 + d^3*e*x - 23*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*
e^4*x^4))/((-d + e*x)*(d + e*x)^2) - 15*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/(6*e^6)
```

IntegrateAlgebraic [A] time = 0.52, size = 130, normalized size = 1.02

$$
-\frac{5d^2\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^7}-\frac{\sqrt{d^2-e^2x^2}\left(-16d^4-d^3ex+23d^2e^2x^2+3de^3x^3-3e^4x^4\right)}{6e^6(ex-d)(d+ex)^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -1/6*(Sqrt[d^2 - e^2*x^2]*(-16*d^4 - d^3*e*x + 23*d^2*e^2*x^2 + 3*d*e^3*x^3
 - 3*e^4*x^4))/(e^6*(-d + e*x)*(d + e*x)^2) - (5*d^2*Sqrt[-e^2]*Log[-(Sqrt[
-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^7)
```

fricas [A] time = 0.40, size = 190, normalized size = 1.48

$$
-\frac{16\,d^2e^3x^3+16\,d^3e^2x^2-16\,d^4ex-16\,d^5-30\left(d^2e^3x^3+d^3e^2x^2-d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(3\,e^4x^4-3\,de^3x^3-23\,d^2e^2x^2+d^3ex+16\,d^4\right)\sqrt{-e^2x^2+d^2}}{6\left(e^9x^3+de^8x^2-d^2e^7x-d^3e^6\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 - 16*d^4*e*x - 16*d^5 - 30*(d^2*e^3*x
^3 + d^3*e^2*x^2 - d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
 - (3*e^4*x^4 - 3*d*e^3*x^3 - 23*d^2*e^2*x^2 + d^3*e*x + 16*d^4)*sqrt(-e^2*
x^2 + d^2))/(e^9*x^3 + d*e^8*x^2 - d^2*e^7*x - d^3*e^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.02, size = 208, normalized size = 1.62

$$-\frac{x^{3}}{2\sqrt{-e^{2}x^{2}+d^{2}}\,e^{3}}+\frac{d\,x^{2}}{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{4}}+\frac{7d^{2}x}{2\sqrt{-e^{2}x^{2}+d^{2}}\,e^{5}}-\frac{2d^{2}x}{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{5}}-\frac{5d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}\,e^{5}}+\frac{d^{4}}{3\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{7}}-\frac{3d^{3}}{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/2/e^3*x^3/(-e^2*x^2+d^2)^(1/2)+7/2/(-e^2*x^2+d^2)^(1/2)*d^2/e^5*x-5/2/(e
^2)^(1/2)*d^2/e^5*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+d/e^4*x^2/(-e^
2*x^2+d^2)^(1/2)-3*d^3/e^6/(-e^2*x^2+d^2)^(1/2)+1/3*d^4/e^7/(x+d/e)/(2*(x+d
/e)*d*e-(x+d/e)^2*e^2)^(1/2)-2/3*d^2/e^5/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2
)*x
```

maxima [A] time = 1.01, size = 151, normalized size = 1.18

$$\frac{d^{4}}{3\left(\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}x+\sqrt{-e^{2}x^{2}+d^{2}}\,de^{6}\right)}-\frac{x^{3}}{2\sqrt{-e^{2}x^{2}+d^{2}}\,e^{3}}+\frac{dx^{2}}{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{4}}+\frac{17\,d^{2}x}{6\sqrt{-e^{2}x^{2}+d^{2}}\,e^{5}}-\frac{5\,d^{2}\arcsin\left(\frac{ex}{d}\right)}{2\,e^{6}}-\frac{3\,d^{3}}{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*d^4/(sqrt(-e^2*x^2 + d^2)*e^7*x + sqrt(-e^2*x^2 + d^2)*d*e^6) - 1/2*x^3
/(sqrt(-e^2*x^2 + d^2)*e^3) + d*x^2/(sqrt(-e^2*x^2 + d^2)*e^4) + 17/6*d^2*x
/(sqrt(-e^2*x^2 + d^2)*e^5) - 5/2*d^2*arcsin(e*x/d)/e^6 - 3*d^3/(sqrt(-e^2*
x^2 + d^2)*e^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**5/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.128 $\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 641, 217, 203}

$$\frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^3*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x*(3*d - 4*e*x))/(3*e^4*Sq
rt[d^2 - e^2*x^2]) + (8*Sqrt[d^2 - e^2*x^2])/(3*e^5) + (d*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/e^5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x^2\left(3d^3-4d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{3d^5-8d^4ex}{\sqrt{d^2-e^2x^2}}\,dx}{3d^4e^4}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] time = 0.14, size = 93, normalized size = 0.82

$$\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}\left(8d^3+5d^2ex-7de^2x^2-3e^3x^3\right)}{(d-ex)(d+ex)^2}}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(8*d^3 + 5*d^2*e*x - 7*d*e^2*x^2 - 3*e^3*x^3))/((d -
e*x)*(d + e*x)^2) + 3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(3*e^5)
```

IntegrateAlgebraic [A] time = 0.46, size = 114, normalized size = 1.01

$$\frac{d\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^6} - \frac{\sqrt{d^2-e^2x^2}\left(8d^3+5d^2ex-7de^2x^2-3e^3x^3\right)}{3e^5(ex-d)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -1/3*(Sqrt[d^2 - e^2*x^2]*(8*d^3 + 5*d^2*e*x - 7*d*e^2*x^2 - 3*e^3*x^3))/(e
^5*(-d + e*x)*(d + e*x)^2) + (d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 -
 e^2*x^2]])/e^6
```

fricas [A] time = 0.42, size = 175, normalized size = 1.55

$$\frac{8\,de^3x^3+8\,d^2e^2x^2-8\,d^3ex-8\,d^4-6\left(de^3x^3+d^2e^2x^2-d^3ex-d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(3\,e^3x^3+7\,de^2x^2-5\,d^2ex-8\,d^3\right)\sqrt{-e^2x^2+d^2}}{3\left(e^8x^3+de^7x^2-d^2e^6x-d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(8*d*e^3*x^3 + 8*d^2*e^2*x^2 - 8*d^3*e*x - 8*d^4 - 6*(d*e^3*x^3 + d^2*e
^2*x^2 - d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (3*e^3*
x^3 + 7*d*e^2*x^2 - 5*d^2*e*x - 8*d^3)*sqrt(-e^2*x^2 + d^2))/(e^8*x^3 + d*e
^7*x^2 - d^2*e^6*x - d^3*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 179, normalized size = 1.58

$$-\frac{x^{2}}{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{3}}-\frac{2 d x}{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{4}}+\frac{2 d x}{3 \sqrt{2 \left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, e^{4}}+\frac{d \arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\, e^{4}}-\frac{d^{3}}{3 \left(x+\frac{d}{e}\right) \sqrt{2 \left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, e^{6}}+\frac{3 d^{2}}{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/e^3*x^2/(-e^2*x^2+d^2)^(1/2)+3*d^2/e^5/(-e^2*x^2+d^2)^(1/2)-2/(-e^2*x^2+
d^2)^(1/2)*d/e^4*x+1/(e^2)^(1/2)*d/e^4*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1
/2)*x)-1/3*d^3/e^6/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+2/3*d/e^4/(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [A] time = 1.01, size = 124, normalized size = 1.10

$$-\frac{d^{3}}{3 \left(\sqrt{-e^{2} x^{2}+d^{2}}\, e^{6} x+\sqrt{-e^{2} x^{2}+d^{2}}\, d e^{5}\right)}-\frac{x^{2}}{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{3}}-\frac{4 \, d x}{3 \, \sqrt{-e^{2} x^{2}+d^{2}}\, e^{4}}+\frac{d \arcsin \left(\frac{e x}{d}\right)}{e^{5}}+\frac{3 \, d^{2}}{\sqrt{-e^{2} x^{2}+d^{2}}\, e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*d^3/(sqrt(-e^2*x^2 + d^2)*e^6*x + sqrt(-e^2*x^2 + d^2)*d*e^5) - x^2/(s
qrt(-e^2*x^2 + d^2)*e^3) - 4/3*d*x/(sqrt(-e^2*x^2 + d^2)*e^4) + d*arcsin(e*
x/d)/e^5 + 3*d^2/(sqrt(-e^2*x^2 + d^2)*e^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(- \left(- d + e x\right) \left(d + e x\right)\right)^{\frac{3}{2}} \left(d + e x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.129 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 778, 217, 203}

$$\frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^2*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (2*d - 3*e*x)/(3*e^4*Sqrt[d
^2 - e^2*x^2]) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx\\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}\\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}\\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3}\\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}\end{aligned}$$

Mathematica [A] time = 0.12, size = 80, normalized size = 0.90

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-2d^2+dex+4e^2x^2\right)}{(d-ex)(d+ex)^2} - 3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{3e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] ((Sqrt[d^2 - e^2*x^2]*(-2*d^2 + d*e*x + 4*e^2*x^2))/((d - e*x)*(d + e*x)^2)
 - 3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(3*e^4)
```

IntegrateAlgebraic [A] time = 0.44, size = 102, normalized size = 1.15

$$-\frac{\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^5} - \frac{\sqrt{d^2-e^2x^2}\left(-2d^2+dex+4e^2x^2\right)}{3e^4(ex-d)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] -1/3*(Sqrt[d^2 - e^2*x^2]*(-2*d^2 + d*e*x + 4*e^2*x^2))/(e^4*(-d + e*x)*(d
+ e*x)^2) - (Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^5
```

fricas [A] time = 0.42, size = 157, normalized size = 1.76

$$-\frac{2\,e^3x^3+2\,de^2x^2-2\,d^2ex-2\,d^3-6\left(e^3x^3+de^2x^2-d^2ex-d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4\,e^2x^2+dex-2\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(e^7x^3+de^6x^2-d^2e^5x-d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
[Out] -1/3*(2*e^3*x^3 + 2*d*e^2*x^2 - 2*d^2*e*x - 2*d^3 - 6*(e^3*x^3 + d*e^2*x^2
- d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*e^2*x^2 + d
```

```
*e*x - 2*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 + d*e^6*x^2 - d^2*e^5*x - d^3*
e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 153, normalized size = 1.72

$$\frac{2x}{\sqrt{-e^2x^2+d^2}\, e^3}-\frac{2x}{3\sqrt{2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2}\, e^3}-\frac{\arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\, e^3}+\frac{d^2}{3\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2}\, e^5}-\frac{d}{\sqrt{-e^2x^2+d^2}\, e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 2/(-e^2*x^2+d^2)^(1/2)/e^3*x-1/(e^2)^(1/2)/e^3*arctan((e^2)^(1/2)/(-e^2*x^2
+d^2)^(1/2)*x)-d/e^4/(-e^2*x^2+d^2)^(1/2)+1/3*d^2/e^5/(x+d/e)/(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(1/2)-2/3/e^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [A] time = 1.00, size = 99, normalized size = 1.11

$$\frac{d^2}{3\left(\sqrt{-e^2x^2+d^2}\, e^5x+\sqrt{-e^2x^2+d^2}\, de^4\right)}+\frac{4\,x}{3\sqrt{-e^2x^2+d^2}\, e^3}-\frac{\arcsin\left(\frac{ex}{d}\right)}{e^4}-\frac{d}{\sqrt{-e^2x^2+d^2}\, e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*d^2/(sqrt(-e^2*x^2 + d^2)*e^5*x + sqrt(-e^2*x^2 + d^2)*d*e^4) + 4/3*x/(
sqrt(-e^2*x^2 + d^2)*e^3) - arcsin(e*x/d)/e^4 - d/(sqrt(-e^2*x^2 + d^2)*e^4
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.130 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {855, 12, 261}

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] 2/(3*e^3*Sqrt[d^2 - e^2*x^2]) - x^2/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{\int \frac{2dx}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3de}\\ &= -\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{x}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3e}\\ &= \frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 1.00

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^2+2dex-e^2x^2\right)}{3de^3(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 + 2*d*e*x - e^2*x^2))/(3*d*e^3*(d - e*x)*(d + e
*x)^2)
```

IntegrateAlgebraic [A] time = 0.39, size = 60, normalized size = 1.00

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^2+2dex-e^2x^2\right)}{3de^3(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 + 2*d*e*x - e^2*x^2))/(3*d*e^3*(d - e*x)*(d + e
*x)^2)
```

fricas [A] time = 0.40, size = 103, normalized size = 1.72

$$\frac{2\,e^3x^3+2\,de^2x^2-2\,d^2ex-2\,d^3+\left(e^2x^2-2\,dex-2\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(de^6x^3+d^2e^5x^2-d^3e^4x-d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*e^3*x^3 + 2*d*e^2*x^2 - 2*d^2*e*x - 2*d^3 + (e^2*x^2 - 2*d*e*x - 2*d
^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 + d^2*e^5*x^2 - d^3*e^4*x - d^4*e^3)
```

giac [A] time = 0.24, size = 1, normalized size = 0.02

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [A] time = 0.01, size = 48, normalized size = 0.80

$$\frac{(-ex+d)\left(-e^2x^2+2dex+2d^2\right)}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/3*(-e*x+d)*(-e^2*x^2+2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(3/2)
```

maxima [A] time = 0.46, size = 86, normalized size = 1.43

$$-\frac{d}{3\left(\sqrt{-e^2x^2+d^2}\,e^4x+\sqrt{-e^2x^2+d^2}\,de^3\right)}-\frac{x}{3\sqrt{-e^2x^2+d^2}\,de^2}+\frac{1}{\sqrt{-e^2x^2+d^2}\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*d/(sqrt(-e^2*x^2 + d^2)*e^4*x + sqrt(-e^2*x^2 + d^2)*d*e^3) - 1/3*x/(s
qrt(-e^2*x^2 + d^2)*d*e^2) + 1/(sqrt(-e^2*x^2 + d^2)*e^3)
```

mupad [**B**] time = 2.71, size = 56, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+2\,d\,e\,x-e^2\,x^2\right)}{3\,d\,e^3\,{\left(d+e\,x\right)}^2\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^2 - e^2*x^2 + 2*d*e*x))/(3*d*e^3*(d + e*x)^2*(d
 - e*x))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.131 $\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {793, 191}

$$\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] x/(3*d^2*e*Sqrt[d^2 - e^2*x^2]) + 1/(3*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3e}\\ &= \frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(d^2+dex+e^2x^2\right)}{3d^2e^2(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^2 + d*e*x + e^2*x^2))/(3*d^2*e^2*(d - e*x)*(d + e*x
)^2)
```

IntegrateAlgebraic [A] time = 0.37, size = 56, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(d^2+dex+e^2x^2\right)}{3d^2e^2(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^2 + d*e*x + e^2*x^2))/(3*d^2*e^2*(d - e*x)*(d + e*x
)^2)
```

fricas [B] time = 0.41, size = 101, normalized size = 1.74

$$\frac{e^3x^3+de^2x^2-d^2ex-d^3-\left(e^2x^2+dex+d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(d^2e^5x^3+d^3e^4x^2-d^4e^3x-d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 - (e^2*x^2 + d*e*x + d^2)*sqrt(-e^
2*x^2 + d^2))/(d^2*e^5*x^3 + d^3*e^4*x^2 - d^4*e^3*x - d^5*e^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.01, size = 44, normalized size = 0.76

$$\frac{(-ex+d)\left(e^2x^2+dex+d^2\right)}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/3*(-e*x+d)*(e^2*x^2+d*e*x+d^2)/d^2/e^2/(-e^2*x^2+d^2)^(3/2)
```

maxima [A] time = 0.46, size = 67, normalized size = 1.16

$$\frac{1}{3\left(\sqrt{-e^2x^2+d^2}\,e^3x+\sqrt{-e^2x^2+d^2}\,de^2\right)}+\frac{x}{3\sqrt{-e^2x^2+d^2}\,d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3/(sqrt(-e^2*x^2 + d^2)*e^3*x + sqrt(-e^2*x^2 + d^2)*d*e^2) + 1/3*x/(sqrt
(-e^2*x^2 + d^2)*d^2*e)
```

mupad [B] time = 2.71, size = 52, normalized size = 0.90

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+d\,e\,x+e^2\,x^2\right)}{3\,d^2\,e^2\,{\left(d+e\,x\right)}^2\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 + d*e*x))/(3*d^2*e^2*(d + e*x)^2*(d -
 e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.132 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 191}

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d}\\ &= \frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.00

$$-\frac{\left(d^2-2dex-2e^2x^2\right)\sqrt{d^2-e^2x^2}}{3d^3e(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -1/3*((d^2 - 2*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(d^3*e*(d - e*x)*(d
 + e*x)^2)
```

IntegrateAlgebraic [A] time = 0.38, size = 60, normalized size = 1.03

$$\frac{\sqrt{d^2-e^2x^2}\left(-d^2+2dex+2e^2x^2\right)}{3d^3e(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 + 2*d*e*x + 2*e^2*x^2))/(3*d^3*e*(d - e*x)*(d +
e*x)^2)
```

fricas [B] time = 0.40, size = 102, normalized size = 1.76

$$-\frac{e^3x^3 + de^2x^2 - d^2ex - d^3 + \left(2\,e^2x^2 + 2\,dex - d^2\right)\sqrt{-e^2x^2 + d^2}}{3\,\left(d^3e^4x^3 + d^4e^3x^2 - d^5e^2x - d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 + (2*e^2*x^2 + 2*d*e*x - d^2)*sqr
t(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + d^4*e^3*x^2 - d^5*e^2*x - d^6*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.01, size = 46, normalized size = 0.79

$$-\frac{(-ex + d)\left(-2e^2x^2 - 2dex + d^2\right)}{3\left(-e^2x^2 + d^2\right)^{\frac{3}{2}} d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/3*(-e*x+d)*(-2*e^2*x^2-2*d*e*x+d^2)/d^3/e/(-e^2*x^2+d^2)^(3/2)
```

maxima [A] time = 0.45, size = 65, normalized size = 1.12

$$-\frac{1}{3\left(\sqrt{-e^2x^2 + d^2}\,de^2x + \sqrt{-e^2x^2 + d^2}\,d^2e\right)} + \frac{2\,x}{3\,\sqrt{-e^2x^2 + d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3/(sqrt(-e^2*x^2 + d^2)*d*e^2*x + sqrt(-e^2*x^2 + d^2)*d^2*e) + 2/3*x/(s
qrt(-e^2*x^2 + d^2)*d^3)
```

mupad [B] time = 2.71, size = 56, normalized size = 0.97

$$\frac{\sqrt{d^2 - e^2\,x^2}\,\left(-d^2 + 2\,d\,e\,x + 2\,e^2\,x^2\right)}{3\,d^3\,e\,{\left(d + e\,x\right)}^2\,\left(d - e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*e^2*x^2 - d^2 + 2*d*e*x))/(3*d^3*e*(d + e*x)^2*(d
 - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.133 $\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 12, 266, 63, 208}

$$\frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (3*d - 2*e*x)/(3*d^4*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*(d + e*x)*Sqrt[d^2 - e
^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-3de^2+2e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2} \\
&= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{3d^3e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4} \\
&= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3} \\
&= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^3} \\
&= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^3e^2} \\
&= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 83, normalized size = 0.94

$$
\frac{\frac{\sqrt{d^2-e^2x^2}\left(4d^2+dex-2e^2x^2\right)}{(d-ex)(d+ex)^2} - 3\log\left(\sqrt{d^2-e^2x^2}+d\right) + 3\log(x)}{3d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (((4*d^2 + d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/((d - e*x)*(d + e*x)^2)
+ 3*Log[x] - 3*Log[d + Sqrt[d^2 - e^2*x^2]])/(3*d^4)
```

IntegrateAlgebraic [A] time = 0.53, size = 99, normalized size = 1.12

$$
\frac{\sqrt{d^2-e^2x^2}\left(4d^2+dex-2e^2x^2\right)}{3d^4(d-ex)(d+ex)^2} + \frac{2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((4*d^2 + d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(3*d^4*(d - e*x)*(d + e*x
)^2) + (2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^4
```

fricas [A] time = 0.40, size = 155, normalized size = 1.76

$$\frac{4e^3x^3 + 4de^2x^2 - 4d^2ex - 4d^3 + 3\left(e^3x^3 + de^2x^2 - d^2ex - d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \left(2e^2x^2 - dex - 4d^2\right)\sqrt{-e^2x^2 + d^2}}{3\left(d^4e^3x^3 + d^5e^2x^2 - d^6ex - d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(4*e^3*x^3 + 4*d*e^2*x^2 - 4*d^2*e*x - 4*d^3 + 3*(e^3*x^3 + d*e^2*x^2 -
 d^2*e*x - d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (2*e^2*x^2 - d*e*x - 4
*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 + d^5*e^2*x^2 - d^6*e*x - d^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 142, normalized size = 1.61

$$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^3} - \frac{2ex}{3\sqrt{2\left(x+\frac{d}{e}\right)de - \left(x+\frac{d}{e}\right)^2 e^2}\,d^4} + \frac{1}{3\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de - \left(x+\frac{d}{e}\right)^2 e^2}\,d^2e} + \frac{1}{\sqrt{-e^2x^2 + d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/(-e^2*x^2+d^2)^(1/2)/d^3-1/(d^2)^(1/2)/d^3*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)+1/3/d^2/e/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-2/
3/d^4*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}(ex + d)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(d^2 - e^2\,x^2\right)^{3/2}\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.134 $\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{3d^5x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 807, 266, 63, 208}

$$\frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{3d^5x}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (4*d - 3*e*x)/(3*d^4*x*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x*(d + e*x)*Sqrt[d^2
 - e^2*x^2]) - (8*Sqrt[d^2 - e^2*x^2])/(3*d^5*x) + (e*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/d^5
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-4de^2+3e^3x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-8d^3e^4+3d^2e^5x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} - \frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,d}{d^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} - \frac{e\,\mathrm{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{2}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-}\right.}{}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{e\tanh^{-1}\left(\sqrt{d^2}\right.}{d^5}$$

Mathematica [A] time = 0.12, size = 101, normalized size = 0.84

$$\frac{3e\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(3d^3+7d^2ex-5de^2x^2-8e^3x^3\right)}{x(ex-d)(d+ex)^2}-3e\log(x)}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^3 + 7*d^2*e*x - 5*d*e^2*x^2 - 8*e^3*x^3))/(x*(-d
 + e*x)*(d + e*x)^2) - 3*e*Log[x] + 3*e*Log[d + Sqrt[d^2 - e^2*x^2]])/(3*d^
5)
```

IntegrateAlgebraic [A] time = 0.49, size = 115, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\left(-3d^3-7d^2ex+5de^2x^2+8e^3x^3\right)}{3d^5x(d-ex)(d+ex)^2} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-3*d^3 - 7*d^2*e*x + 5*d*e^2*x^2 + 8*e^3*x^3))/(3*d^5
*x*(d - e*x)*(d + e*x)^2) - (2*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*
x^2]/d])/d^5
```

fricas [A] time = 0.39, size = 181, normalized size = 1.51

$$-\frac{4\,e^{4}x^{4}+4\,de^{3}x^{3}-4\,d^{2}e^{2}x^{2}-4\,d^{3}ex+3\left(e^{4}x^{4}+de^{3}x^{3}-d^{2}e^{2}x^{2}-d^{3}ex\right)\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(8\,e^{3}x^{3}+5\,de^{2}x^{2}-7\,d^{2}ex-3\,d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{3\left(d^{5}e^{3}x^{4}+d^{6}e^{2}x^{3}-d^{7}ex^{2}-d^{8}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*e^4*x^4 + 4*d*e^3*x^3 - 4*d^2*e^2*x^2 - 4*d^3*e*x + 3*(e^4*x^4 + d*
e^3*x^3 - d^2*e^2*x^2 - d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^
3*x^3 + 5*d*e^2*x^2 - 7*d^2*e*x - 3*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^3*x^4
 + d^6*e^2*x^3 - d^7*e*x^2 - d^8*x)
```

giac [A] time = 0.25, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [A] time = 0.02, size = 188, normalized size = 1.57

$$\frac{e\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}\,d^{4}}+\frac{2e^{2}x}{\sqrt{-e^{2}x^{2}+d^{2}}\,d^{5}}+\frac{2e^{2}x}{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{5}}-\frac{1}{3\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{3}}-\frac{e}{\sqrt{-e^{2}x^{2}+d^{2}}\,d^{4}}-\frac{1}{\sqrt{-e^{2}x^{2}+d^{2}}\,d^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/d^3/x/(-e^2*x^2+d^2)^(1/2)+2/(-e^2*x^2+d^2)^(1/2)/d^5*e^2*x-1/(-e^2*x^2+
d^2)^(1/2)/d^4*e+1/(d^2)^(1/2)/d^4*e*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-1/3/d^3/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+2/3*e^2/d^5/
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}(ex+d)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2}\left(d^{2}-e^{2}\,x^{2}\right)^{3/2}\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.135 $\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}-\frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.13, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {857, 823, 835, 807, 266, 63, 208}

$$\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{5e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (5*d - 4*e*x)/(3*d^4*x^2*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x^2*(d + e*x)*Sqrt
[d^2 - e^2*x^2]) - (5*Sqrt[d^2 - e^2*x^2])/(2*d^5*x^2) + (8*e*Sqrt[d^2 - e^
2*x^2])/(3*d^6*x) - (5*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-5de^2+4e^3x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3e^4+8d^2e^5x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{-16d^4e^5+15}{x^2\sqrt{d^2-e^2}}}{6d^8e}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2}}{3d^6x}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2}}{3d^6x}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2}}{3d^6x}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2}}{3d^6x}$$

Mathematica [A] time = 0.10, size = 115, normalized size = 0.76

$$\frac{-15e^2\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(3d^4-3d^3ex-23d^2e^2x^2+de^3x^3+16e^4x^4\right)}{x^2(ex-d)(d+ex)^2}+15e^2\log(x)}{6d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x - 23*d^2*e^2*x^2 + d*e^3*x^3 + 16*
e^4*x^4))/(x^2*(-d + e*x)*(d + e*x)^2) + 15*e^2*Log[x] - 15*e^2*Log[d + Sqr
t[d^2 - e^2*x^2]])/(6*d^6)
```

IntegrateAlgebraic [A] time = 0.57, size = 128, normalized size = 0.84

$$\frac{5e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{\sqrt{d^2-e^2x^2}\left(-3d^4+3d^3ex+23d^2e^2x^2-de^3x^3-16e^4x^4\right)}{6d^6x^2(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-3*d^4 + 3*d^3*e*x + 23*d^2*e^2*x^2 - d*e^3*x^3 - 16*
e^4*x^4))/(6*d^6*x^2*(d - e*x)*(d + e*x)^2) + (5*e^2*ArcTanh[(Sqrt[-e^2]*x)
/d - Sqrt[d^2 - e^2*x^2]/d])/d^6
```

fricas [A] time = 0.42, size = 201, normalized size = 1.32

$$\frac{14\,e^5x^5+14\,de^4x^4-14\,d^2e^3x^3-14\,d^3e^2x^2+15\left(e^5x^5+de^4x^4-d^2e^3x^3-d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16\,e^4x^4+de^3x^3-23\,d^2e^2x^2-3\,d^3ex+3\,d^4\right)\sqrt{-e^2x^2+d^2}}{6\left(d^6e^3x^5+d^7e^2x^4-d^8ex^3-d^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(14*e^5*x^5 + 14*d*e^4*x^4 - 14*d^2*e^3*x^3 - 14*d^3*e^2*x^2 + 15*(e^5*
x^5 + d*e^4*x^4 - d^2*e^3*x^3 - d^3*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2)
)/x) + (16*e^4*x^4 + d*e^3*x^3 - 23*d^2*e^2*x^2 - 3*d^3*e*x + 3*d^4)*sqrt(-
e^2*x^2 + d^2))/(d^6*e^3*x^5 + d^7*e^2*x^4 - d^8*e*x^3 - d^9*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.02, size = 216, normalized size = 1.42

$$-\frac{5e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^5}-\frac{2e^3x}{\sqrt{-e^2x^2+d^2}\,d^6}-\frac{2e^3x}{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^6}+\frac{e}{3\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^4}+\frac{5e^2}{2\sqrt{-e^2x^2+d^2}\,d^5}+\frac{e}{\sqrt{-e^2x^2+d^2}\,d^4x}-\frac{1}{2\sqrt{-e^2x^2+d^2}\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] e/d^4/x/(-e^2*x^2+d^2)^(1/2)-2/(-e^2*x^2+d^2)^(1/2)/d^6*e^3*x-1/2/d^3/x^2/(
-e^2*x^2+d^2)^(1/2)+5/2/(-e^2*x^2+d^2)^(1/2)/d^5*e^2-5/2/(d^2)^(1/2)/d^5*e^
2*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/3/d^4*e/(x+d/e)/(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-2/3/d^6*e^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/
2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

3.136 $\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}} dx$

Optimal. Leaf size=162

$$\frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}+\frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}}-\frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.16, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 780, 217, 203}

$$\frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^6*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d - 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d - 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
) + ((32*d - 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) + (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
```

```
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^7(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^5\left(6d^3-7d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^3\left(24d^5-35d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x\left(48d^7-105d^6ex\right)}{\sqrt{d^2-e^2x^2}}\,d}{15d^6e^6}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2}}{10e^8}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2}}{10e^8}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2}}{10e^8}$$

Mathematica [A] time = 0.24, size = 128, normalized size = 0.79

$$\frac{105d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}\left(96d^6-9d^5ex-249d^4e^2x^2-4d^3e^3x^3+176d^2e^4x^4+15de^5x^5-15e^6x^6\right)}{(d-ex)^2(d+ex)^3}}{30e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(96*d^6 - 9*d^5*e*x - 249*d^4*e^2*x^2 - 4*d^3*e^3*x^3
 + 176*d^2*e^4*x^4 + 15*d*e^5*x^5 - 15*e^6*x^6))/((d - e*x)^2*(d + e*x)^3)
+ 105*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(30*e^8)
```

IntegrateAlgebraic [A] time = 0.64, size = 152, normalized size = 0.94

$$\frac{7d^2\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^9} + \frac{\sqrt{d^2-e^2x^2}\left(96d^6-9d^5ex-249d^4e^2x^2-4d^3e^3x^3+176d^2e^4x^4+15de^5x^5-15e^6x^6\right)}{30e^8(ex-d)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(96*d^6 - 9*d^5*e*x - 249*d^4*e^2*x^2 - 4*d^3*e^3*x^3
+ 176*d^2*e^4*x^4 + 15*d*e^5*x^5 - 15*e^6*x^6))/(30*e^8*(-d + e*x)^2*(d + e
```

```
*x)^3) + (7*d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e
^9)
```

fricas [A] time = 0.46, size = 274, normalized size = 1.69

$$\frac{96\,d^{2}e^{5}x^{5}+96\,d^{3}e^{4}x^{4}-192\,d^{4}e^{3}x^{3}-192\,d^{5}e^{2}x^{2}+96\,d^{6}ex+96\,d^{7}-210\left(d^{2}e^{5}x^{5}+d^{3}e^{4}x^{4}-2\,d^{4}e^{3}x^{3}-2\,d^{5}e^{2}x^{2}+d^{6}ex+d^{7}\right)\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-\left(15\,e^{6}x^{6}-15\,de^{5}x^{5}-176\,d^{2}e^{4}x^{4}+4\,d^{3}e^{3}x^{3}+249\,d^{4}e^{2}x^{2}+9\,d^{5}ex-96\,d^{6}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{30\left(e^{13}x^{5}+de^{12}x^{4}-2\,d^{2}e^{11}x^{3}-2\,d^{3}e^{10}x^{2}+d^{4}e^{9}x+d^{5}e^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(96*d^2*e^5*x^5 + 96*d^3*e^4*x^4 - 192*d^4*e^3*x^3 - 192*d^5*e^2*x^2 +
 96*d^6*e*x + 96*d^7 - 210*(d^2*e^5*x^5 + d^3*e^4*x^4 - 2*d^4*e^3*x^3 - 2*d
^5*e^2*x^2 + d^6*e*x + d^7)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (15
*e^6*x^6 - 15*d*e^5*x^5 - 176*d^2*e^4*x^4 + 4*d^3*e^3*x^3 + 249*d^4*e^2*x^2
 + 9*d^5*e*x - 96*d^6)*sqrt(-e^2*x^2 + d^2))/(e^13*x^5 + d*e^12*x^4 - 2*d^2
*e^11*x^3 - 2*d^3*e^10*x^2 + d^4*e^9*x + d^5*e^8)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.05, size = 318, normalized size = 1.96

$$-\frac{x^{5}}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}+\frac{d\,x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}+\frac{7d^{2}x^{3}}{6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}-\frac{5d^{3}x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}+\frac{2d^{4}x}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}}-\frac{4d^{4}x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}e^{7}}+\frac{d^{6}}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}e^{9}}+\frac{3d^{5}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{8}}-\frac{19d^{2}x}{6\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}-\frac{8d^{2}x}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{7}}+\frac{7d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}\,e^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/2/e^3*x^5/(-e^2*x^2+d^2)^(3/2)+7/6/(-e^2*x^2+d^2)^(3/2)*d^2/e^5*x^3-19/6
/(-e^2*x^2+d^2)^(1/2)*d^2/e^7*x+7/2/(e^2)^(1/2)*d^2/e^7*arctan((e^2)^(1/2)/
(-e^2*x^2+d^2)^(1/2)*x)+d/e^4*x^4/(-e^2*x^2+d^2)^(3/2)-5*d^3/e^6*x^2/(-e^2*
x^2+d^2)^(3/2)+3*d^5/e^8/(-e^2*x^2+d^2)^(3/2)+2/3*d^4/e^7*x/(-e^2*x^2+d^2)^
(3/2)+1/5*d^6/e^9/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-4/15*d^4/e^7/
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-8/15*d^2/e^7/(2*(x+d/e)*d*e-(x+d/e)^2
*e^2)^(1/2)*x
```

maxima [B] time = 1.09, size = 289, normalized size = 1.78

$$\frac{d^{6}}{5\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{9}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}de^{8}\right)}-\frac{x^{5}}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}+\frac{dx^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}+\frac{25\,d^{2}x^{3}}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}-\frac{65\,d^{3}x^{2}}{6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}-\frac{164\,d^{4}x}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}}-\frac{7\,dx^{2}}{6\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}+\frac{53\,d^{5}}{6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{8}}+\frac{229\,d^{2}x}{30\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}+\frac{7\,d^{2}\arcsin\left(\frac{ex}{d}\right)}{2\,e^{8}}-\frac{14\,d^{3}}{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{8}}-\frac{7\sqrt{-e^{2}x^{2}+d^{2}}\,d}{6\,e^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^6/((-e^2*x^2 + d^2)^(3/2)*e^9*x + (-e^2*x^2 + d^2)^(3/2)*d*e^8) - 1/2
*x^5/((-e^2*x^2 + d^2)^(3/2)*e^3) + d*x^4/((-e^2*x^2 + d^2)^(3/2)*e^4) + 25
/2*d^2*x^3/((-e^2*x^2 + d^2)^(3/2)*e^5) - 65/6*d^3*x^2/((-e^2*x^2 + d^2)^(3
/2)*e^6) - 164/15*d^4*x/((-e^2*x^2 + d^2)^(3/2)*e^7) - 7/6*d*x^2/(sqrt(-e^2
*x^2 + d^2)*e^6) + 53/6*d^5/((-e^2*x^2 + d^2)^(3/2)*e^8) + 229/30*d^2*x/(sq
rt(-e^2*x^2 + d^2)*e^7) + 7/2*d^2*arcsin(e*x/d)/e^8 - 14/3*d^3/(sqrt(-e^2*x
^2 + d^2)*e^8) - 7/6*sqrt(-e^2*x^2 + d^2)*d/e^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(d^2 - e^2 x^2\right)^{5/2} (d + e x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(x^7/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-(-d + ex)(d + ex))^{\frac{5}{2}} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**7/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.137 $\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}} dx$

Optimal. Leaf size=148

$$\frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}+\frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 641, 217, 203}

$$\frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^5*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d - 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d - 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) - (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^6(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^4\left(5d^3-6d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^2\left(15d^5-24d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^7-48d^6ex}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d}{-}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d}{-}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d}{-}$$

Mathematica [A] time = 0.19, size = 115, normalized size = 0.78

$$-\frac{15d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(48d^5+33d^4ex-87d^3e^2x^2-52d^2e^3x^3+38de^4x^4+15e^5x^5\right)}{(d-ex)^2(d+ex)^3}}{15e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(48*d^5 + 33*d^4*e*x - 87*d^3*e^2*x^2 - 52*d^2*
e^3*x^3 + 38*d*e^4*x^4 + 15*e^5*x^5))/((d - e*x)^2*(d + e*x)^3) + 15*d*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

IntegrateAlgebraic [A] time = 0.57, size = 137, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2x^2}\left(-48d^5-33d^4ex+87d^3e^2x^2+52d^2e^3x^3-38de^4x^4-15e^5x^5\right)}{15e^7(ex-d)^2(d+ex)^3}-\frac{d\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 - 33*d^4*e*x + 87*d^3*e^2*x^2 + 52*d^2*e^3*x^
3 - 38*d*e^4*x^4 - 15*e^5*x^5))/(15*e^7*(-d + e*x)^2*(d + e*x)^3) - (d*Sqrt
[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^8
```

fricas [A] time = 0.44, size = 258, normalized size = 1.74

$$-\frac{48\,de^5x^5+48\,d^2e^4x^4-96\,d^3e^3x^3-96\,d^4e^2x^2+48\,d^5ex+48\,d^6-30\left(de^5x^5+d^2e^4x^4-2\,d^3e^3x^3-2\,d^4e^2x^2+d^5ex+d^6\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(15\,e^5x^5+38\,de^4x^4-52\,d^2e^3x^3-87\,d^3e^2x^2+33\,d^4ex+48\,d^5\right)\sqrt{-e^2x^2+d^2}}{15\left(e^{12}x^5+de^{11}x^4-2\,d^2e^{10}x^3-2\,d^3e^9x^2+d^4e^8x+d^5e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(48*d*e^5*x^5 + 48*d^2*e^4*x^4 - 96*d^3*e^3*x^3 - 96*d^4*e^2*x^2 + 48
*d^5*e*x + 48*d^6 - 30*(d*e^5*x^5 + d^2*e^4*x^4 - 2*d^3*e^3*x^3 - 2*d^4*e^2
*x^2 + d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^5*x
^5 + 38*d*e^4*x^4 - 52*d^2*e^3*x^3 - 87*d^3*e^2*x^2 + 33*d^4*e*x + 48*d^5)*
sqrt(-e^2*x^2 + d^2))/(e^12*x^5 + d*e^11*x^4 - 2*d^2*e^10*x^3 - 2*d^3*e^9*x
^2 + d^4*e^8*x + d^5*e^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.01, size = 288, normalized size = 1.95

$$-\frac{x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}} e^{3}}-\frac{d\,x^{3}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}} e^{4}}+\frac{5d^{2}x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}} e^{5}}-\frac{2d^{3}x}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}} e^{6}}+\frac{4d^{3}x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}} e^{6}}-\frac{d^{5}}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}} e^{8}}-\frac{3d^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}} e^{7}}+\frac{2dx}{3\sqrt{-e^{2}x^{2}+d^{2}}\, e^{6}}+\frac{8dx}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\, e^{6}}-\frac{d\arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\, e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/e^3*x^4/(-e^2*x^2+d^2)^(3/2)+5/e^5*d^2*x^2/(-e^2*x^2+d^2)^(3/2)-3*d^4/e^
7/(-e^2*x^2+d^2)^(3/2)-1/3/(-e^2*x^2+d^2)^(3/2)*d/e^4*x^3+2/3/(-e^2*x^2+d^2
)^(1/2)*d/e^6*x-1/(e^2)^(1/2)*d/e^6*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)-2/3*d^3/e^6*x/(-e^2*x^2+d^2)^(3/2)-1/5*d^5/e^8/(x+d/e)/(2*(x+d/e)*d*e-(
x+d/e)^2*e^2)^(3/2)+4/15*d^3/e^6/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+8/15
*d/e^6/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [A] time = 1.09, size = 259, normalized size = 1.75

$$-\frac{d^{5}}{5\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{8}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}de^{7}\right)}-\frac{x^{4}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}-\frac{5\,dx^{3}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}+\frac{20\,d^{2}x^{2}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}+\frac{64\,d^{3}x}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}+\frac{x^{2}}{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{5}}-\frac{14\,d^{4}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}}-\frac{52\,dx}{15\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}-\frac{d\arcsin\left(\frac{ex}{d}\right)}{e^{7}}+\frac{4\,d^{2}}{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{3\,e^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*d^5/((-e^2*x^2 + d^2)^(3/2)*e^8*x + (-e^2*x^2 + d^2)^(3/2)*d*e^7) - x^
4/((-e^2*x^2 + d^2)^(3/2)*e^3) - 5*d*x^3/((-e^2*x^2 + d^2)^(3/2)*e^4) + 20/
3*d^2*x^2/((-e^2*x^2 + d^2)^(3/2)*e^5) + 64/15*d^3*x/((-e^2*x^2 + d^2)^(3/2
)*e^6) + 1/3*x^2/(sqrt(-e^2*x^2 + d^2)*e^5) - 14/3*d^4/((-e^2*x^2 + d^2)^(3
/2)*e^7) - 52/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^6) - d*arcsin(e*x/d)/e^7 + 4/3
*d^2/(sqrt(-e^2*x^2 + d^2)*e^7) + 1/3*sqrt(-e^2*x^2 + d^2)/e^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(d^2-e^2\,x^2\right)}^{5/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(x^6/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(x**6/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.138 $\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}-\frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 778, 217, 203}

$$\frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^4*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d - 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d - 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) + Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
```

```
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= \int \frac{x^5(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx \\
&= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^3\left(4d^3-5d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2} \\
&= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x\left(8d^5-15d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4} \\
&= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5} \\
&= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,d\right.}{e^5} \\
&= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 103, normalized size = 0.84

$$
\frac{15\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}\left(8d^4-7d^3ex-27d^2e^2x^2+8de^3x^3+23e^4x^4\right)}{(d-ex)^2(d+ex)^3}}{15e^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(8*d^4 - 7*d^3*e*x - 27*d^2*e^2*x^2 + 8*d*e^3*x^3 + 2
3*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) + 15*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(15*e^6)
```

IntegrateAlgebraic [A] time = 0.53, size = 124, normalized size = 1.02

$$
\frac{\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^7} + \frac{\sqrt{d^2-e^2x^2}\left(8d^4-7d^3ex-27d^2e^2x^2+8de^3x^3+23e^4x^4\right)}{15e^6(ex-d)^2(d+ex)^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 - 7*d^3*e*x - 27*d^2*e^2*x^2 + 8*d*e^3*x^3 + 23
*e^4*x^4))/(15*e^6*(-d + e*x)^2*(d + e*x)^3) + (Sqrt[-e^2]*Log[-(Sqrt[-e^2]
*x) + Sqrt[d^2 - e^2*x^2]])/e^7
```

fricas [B] time = 0.43, size = 241, normalized size = 1.98

$$
\frac{8\,e^5x^5 + 8\,de^4x^4 - 16\,d^2e^3x^3 - 16\,d^3e^2x^2 + 8\,d^4ex + 8\,d^5 - 30\left(e^5x^5 + de^4x^4 - 2\,d^2e^3x^3 - 2\,d^3e^2x^2 + d^4ex + d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(23\,e^4x^4 + 8\,de^3x^3 - 27\,d^2e^2x^2 - 7\,d^3ex + 8\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(e^{11}x^5 + de^{10}x^4 - 2\,d^2e^9x^3 - 2\,d^3e^8x^2 + d^4e^7x + d^5e^6\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 + 8*d*e^4*x^4 - 16*d^2*e^3*x^3 - 16*d^3*e^2*x^2 + 8*d^4*e*x
 + 8*d^5 - 30*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*e*
x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (23*e^4*x^4 + 8*d*e^3*
x^3 - 27*d^2*e^2*x^2 - 7*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(e^11*x^5 +
 d*e^10*x^4 - 2*d^2*e^9*x^3 - 2*d^3*e^8*x^2 + d^4*e^7*x + d^5*e^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.01, size = 259, normalized size = 2.12

$$\frac{x^{3}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}} e^{3}}-\frac{d\,x^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}} e^{4}}+\frac{2d^{2}x}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}} e^{5}}-\frac{4d^{2}x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}} e^{5}}+\frac{d^{4}}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}} e^{7}}+\frac{d^{3}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}} e^{6}}-\frac{2x}{3\sqrt{-e^{2}x^{2}+d^{2}}\, e^{5}}-\frac{8x}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\, e^{5}}+\frac{\arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\, e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/3/(-e^2*x^2+d^2)^(3/2)/e^3*x^3-2/3/(-e^2*x^2+d^2)^(1/2)/e^5*x+1/(e^2)^(1/
2)/e^5*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-d/e^4*x^2/(-e^2*x^2+d^2)^
(3/2)+1/3*d^3/e^6/(-e^2*x^2+d^2)^(3/2)+2/3*d^2/e^5*x/(-e^2*x^2+d^2)^(3/2)+1
/5*d^4/e^7/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-4/15*d^2/e^5/(2*(x+d
/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-8/15/e^5/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
*x
```

maxima [B] time = 1.06, size = 234, normalized size = 1.92

$$\frac{d^{4}}{5\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}de^{6}\right)}+\frac{x^{3}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}-\frac{8\,dx^{2}}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}-\frac{4\,d^{2}x}{15\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}-\frac{x^{2}}{3\sqrt{-e^{2}x^{2}+d^{2}}\,de^{4}}+\frac{2\,d^{3}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}-\frac{8\,x}{15\sqrt{-e^{2}x^{2}+d^{2}}\,e^{5}}+\frac{\arcsin\left(\frac{ex}{d}\right)}{e^{6}}-\frac{4\,d}{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{3\,de^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^4/((-e^2*x^2 + d^2)^(3/2)*e^7*x + (-e^2*x^2 + d^2)^(3/2)*d*e^6) + x^3
/((-e^2*x^2 + d^2)^(3/2)*e^3) - 8/3*d*x^2/((-e^2*x^2 + d^2)^(3/2)*e^4) - 4/
15*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 1/3*x^2/(sqrt(-e^2*x^2 + d^2)*d*e^4
) + 2*d^3/((-e^2*x^2 + d^2)^(3/2)*e^6) - 8/15*x/(sqrt(-e^2*x^2 + d^2)*e^5)
+ arcsin(e*x/d)/e^6 - 4/3*d/(sqrt(-e^2*x^2 + d^2)*e^6) - 1/3*sqrt(-e^2*x^2
+ d^2)/(d*e^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{{\left(d^2-e^2\,x^2\right)}^{5/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**5/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.139 $\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {850, 805, 266, 43}

$$-\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -(x^4*(d - e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*d^2)/(15*e^5*(d^2 - e^2
*x^2)^(3/2)) - 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4\int \frac{x^3}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5e}$$

$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\left(d^2-e^2x\right)^{5/2}}dx, x, x^2\right)}{5e}$$

$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \left(\frac{d^2}{e^2\left(d^2-e^2x\right)^{5/2}} - \frac{1}{e^2\left(d^2-e^2x\right)^{3/2}}\right)dx, x, x^2\right)}{5e}$$

$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.09, size = 82, normalized size = 0.96

$$-\frac{\sqrt{d^2-e^2x^2}\left(8d^4+8d^3ex-12d^2e^2x^2-12de^3x^3+3e^4x^4\right)}{15de^5(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(8*d^4 + 8*d^3*e*x - 12*d^2*e^2*x^2 - 12*d*e^3*x
^3 + 3*e^4*x^4))/(d*e^5*(d - e*x)^2*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.45, size = 82, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\left(-8d^4-8d^3ex+12d^2e^2x^2+12de^3x^3-3e^4x^4\right)}{15de^5(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 - 8*d^3*e*x + 12*d^2*e^2*x^2 + 12*d*e^3*x^3 -
3*e^4*x^4))/(15*d*e^5*(d - e*x)^2*(d + e*x)^3)
```

fricas [B] time = 0.41, size = 168, normalized size = 1.98

$$-\frac{8\,e^5x^5+8\,de^4x^4-16\,d^2e^3x^3-16\,d^3e^2x^2+8\,d^4ex+8\,d^5+\left(3\,e^4x^4-12\,de^3x^3-12\,d^2e^2x^2+8\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(de^{10}x^5+d^2e^9x^4-2\,d^3e^8x^3-2\,d^4e^7x^2+d^5e^6x+d^6e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*e^5*x^5 + 8*d*e^4*x^4 - 16*d^2*e^3*x^3 - 16*d^3*e^2*x^2 + 8*d^4*e*
x + 8*d^5 + (3*e^4*x^4 - 12*d*e^3*x^3 - 12*d^2*e^2*x^2 + 8*d^3*e*x + 8*d^4)
*sqrt(-e^2*x^2 + d^2))/(d*e^10*x^5 + d^2*e^9*x^4 - 2*d^3*e^8*x^3 - 2*d^4*e^
7*x^2 + d^5*e^6*x + d^6*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 70, normalized size = 0.82

$$-\frac{(-ex+d)\left(3x^4e^4-12x^3d\,e^3-12d^2x^2e^2+8d^3xe+8d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/15*(-e*x+d)*(3*e^4*x^4-12*d*e^3*x^3-12*d^2*e^2*x^2+8*d^3*e*x+8*d^4)/d/e^
5/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.50, size = 134, normalized size = 1.58

$$-\frac{d^3}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^5\right)}+\frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}-\frac{2\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}-\frac{d^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}+\frac{x}{5\sqrt{-e^2x^2+d^2}\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*d^3/((-e^2*x^2 + d^2)^(3/2)*e^6*x + (-e^2*x^2 + d^2)^(3/2)*d*e^5) + x^
2/((-e^2*x^2 + d^2)^(3/2)*e^3) - 2/5*d*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/3
*d^2/((-e^2*x^2 + d^2)^(3/2)*e^5) + 1/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^4)
```

mupad [B] time = 2.95, size = 78, normalized size = 0.92

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(8\,d^4+8\,d^3\,e\,x-12\,d^2\,e^2\,x^2-12\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d\,e^5\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(8*d^4 + 3*e^4*x^4 - 12*d*e^3*x^3 - 12*d^2*e^2*x^2
+ 8*d^3*e*x))/(15*d*e^5*(d + e*x)^3*(d - e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.140 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}} dx$

Optimal. Leaf size=91

$$\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {850, 819, 778, 191}

$$\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^2*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d - 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) - x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$
$$= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$
$$= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e^3}$$
$$= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [**A**] time = 0.07, size = 82, normalized size = 0.90

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^4-2d^3ex+3d^2e^2x^2+3de^3x^3+3e^4x^4\right)}{15d^2e^4(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 - 2*d^3*e*x + 3*d^2*e^2*x^2 + 3*d*e^3*x^3 + 3*
e^4*x^4))/(15*d^2*e^4*(d - e*x)^2*(d + e*x)^3)
```

IntegrateAlgebraic [**A**] time = 0.44, size = 82, normalized size = 0.90

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^4-2d^3ex+3d^2e^2x^2+3de^3x^3+3e^4x^4\right)}{15d^2e^4(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 - 2*d^3*e*x + 3*d^2*e^2*x^2 + 3*d*e^3*x^3 + 3*
e^4*x^4))/(15*d^2*e^4*(d - e*x)^2*(d + e*x)^3)
```

fricas [**B**] time = 0.41, size = 171, normalized size = 1.88

$$-\frac{2\,e^5x^5+2\,de^4x^4-4\,d^2e^3x^3-4\,d^3e^2x^2+2\,d^4ex+2\,d^5-\left(3\,e^4x^4+3\,de^3x^3+3\,d^2e^2x^2-2\,d^3ex-2\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^9x^5+d^3e^8x^4-2\,d^4e^7x^3-2\,d^5e^6x^2+d^6e^5x+d^7e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 + 2*d*e^4*x^4 - 4*d^2*e^3*x^3 - 4*d^3*e^2*x^2 + 2*d^4*e*x
+ 2*d^5 - (3*e^4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - 2*d^3*e*x - 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^2*e^9*x^5 + d^3*e^8*x^4 - 2*d^4*e^7*x^3 - 2*d^5*e^6*x
^2 + d^6*e^5*x + d^7*e^4)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 70, normalized size = 0.77

$$-\frac{(-ex+d)\left(-3x^4e^4-3x^3d\,e^3-3d^2x^2e^2+2d^3xe+2d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/15*(-e*x+d)*(-3*e^4*x^4-3*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^2/e
^4/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.47, size = 110, normalized size = 1.21

$$\frac{d^2}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^4\right)}+\frac{2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}-\frac{d}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}-\frac{x}{5\sqrt{-e^2x^2+d^2}\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^2/((-e^2*x^2 + d^2)^(3/2)*e^5*x + (-e^2*x^2 + d^2)^(3/2)*d*e^4) + 2/5
*x/((-e^2*x^2 + d^2)^(3/2)*e^3) - 1/3*d/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*
x/(sqrt(-e^2*x^2 + d^2)*d^2*e^3)
```

mupad [B] time = 2.84, size = 78, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4-2\,d^3\,e\,x+3\,d^2\,e^2\,x^2+3\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d^2\,e^4\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*e^4*x^4 - 2*d^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - 2
*d^3*e*x))/(15*d^2*e^4*(d + e*x)^3*(d - e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.141 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {855, 778, 191}

$$-\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -x^2/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (2*(d + e*x))/(15*d*e^3*(d^2
 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{x(2d+2ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5de}\\
&= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de^2}\\
&= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^4+2d^3ex-3d^2e^2x^2+2de^3x^3+2e^4x^4\right)}{15d^3e^3(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^4 + 2*d^3*e*x - 3*d^2*e^2*x^2 + 2*d*e^3*x^3 + 2*e
^4*x^4))/(15*d^3*e^3*(d - e*x)^2*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.44, size = 82, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^4+2d^3ex-3d^2e^2x^2+2de^3x^3+2e^4x^4\right)}{15d^3e^3(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^4 + 2*d^3*e*x - 3*d^2*e^2*x^2 + 2*d*e^3*x^3 + 2*e
^4*x^4))/(15*d^3*e^3*(d - e*x)^2*(d + e*x)^3)
```

fricas [B] time = 0.41, size = 170, normalized size = 1.79

$$\frac{2\,e^5x^5+2\,de^4x^4-4\,d^2e^3x^3-4\,d^3e^2x^2+2\,d^4ex+2\,d^5+\left(2\,e^4x^4+2\,de^3x^3-3\,d^2e^2x^2+2\,d^3ex+2\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^8x^5+d^4e^7x^4-2\,d^5e^6x^3-2\,d^6e^5x^2+d^7e^4x+d^8e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*e^5*x^5 + 2*d*e^4*x^4 - 4*d^2*e^3*x^3 - 4*d^3*e^2*x^2 + 2*d^4*e*x +
 2*d^5 + (2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 2*d^3*e*x + 2*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^3*e^8*x^5 + d^4*e^7*x^4 - 2*d^5*e^6*x^3 - 2*d^6*e^5*x^
2 + d^7*e^4*x + d^8*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 70, normalized size = 0.74

$$\frac{(-ex+d)\left(2x^4e^4+2x^3d\,e^3-3d^2x^2e^2+2d^3xe+2d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/15*(-e*x+d)*(2*e^4*x^4+2*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^3/e^3
/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.48, size = 110, normalized size = 1.16

$$-\frac{d}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^3\right)}-\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}-\frac{2x}{15\sqrt{-e^2x^2+d^2}\,d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*d/((-e^2*x^2 + d^2)^(3/2)*e^4*x + (-e^2*x^2 + d^2)^(3/2)*d*e^3) - 1/15
*x/((-e^2*x^2 + d^2)^(3/2)*d*e^2) + 1/3/((-e^2*x^2 + d^2)^(3/2)*e^3) - 2/15
*x/(sqrt(-e^2*x^2 + d^2)*d^3*e^2)
```

mupad [B] time = 2.79, size = 78, normalized size = 0.82

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^4+2\,d^3\,e\,x-3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3+2\,e^4\,x^4\right)}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^4 + 2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^3*e^3*(d + e*x)^3*(d - e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.142 $\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {793, 192, 191}

$$\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}+\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)) + 1/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/
2)) + (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}\\ &= \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2e}\\ &= \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\left(3d^4+3d^3ex+3d^2e^2x^2-2de^3x^3-2e^4x^4\right)}{15d^4e^2(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 + 3*d^3*e*x + 3*d^2*e^2*x^2 - 2*d*e^3*x^3 - 2*e
^4*x^4))/(15*d^4*e^2*(d - e*x)^2*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.42, size = 82, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\left(3d^4+3d^3ex+3d^2e^2x^2-2de^3x^3-2e^4x^4\right)}{15d^4e^2(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 + 3*d^3*e*x + 3*d^2*e^2*x^2 - 2*d*e^3*x^3 - 2*e
^4*x^4))/(15*d^4*e^2*(d - e*x)^2*(d + e*x)^3)
```

fricas [B] time = 0.42, size = 171, normalized size = 2.01

$$\frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5-\left(2\,e^4x^4+2\,de^3x^3-3\,d^2e^2x^2-3\,d^3ex-3\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^4e^7x^5+d^5e^6x^4-2\,d^6e^5x^3-2\,d^7e^4x^2+d^8e^3x+d^9e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x +
 3*d^5 - (2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 - 3*d^3*e*x - 3*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^4*e^7*x^5 + d^5*e^6*x^4 - 2*d^6*e^5*x^3 - 2*d^7*e^4*x^
2 + d^8*e^3*x + d^9*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 70, normalized size = 0.82

$$\frac{(-ex+d)\left(-2x^4e^4-2x^3d\,e^3+3d^2x^2e^2+3d^3xe+3d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/15*(-e*x+d)*(-2*e^4*x^4-2*d*e^3*x^3+3*d^2*e^2*x^2+3*d^3*e*x+3*d^4)/d^4/e^
2/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.49, size = 90, normalized size = 1.06

$$\frac{1}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2\right)}+\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e}+\frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}\,d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5/((-e^2*x^2 + d^2)^(3/2)*e^3*x + (-e^2*x^2 + d^2)^(3/2)*d*e^2) + 1/15*x/
((-e^2*x^2 + d^2)^(3/2)*d^2*e) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^4*e)
```

mupad [B] time = 2.78, size = 78, normalized size = 0.92

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4+3\,d^3\,e\,x+3\,d^2\,e^2\,x^2-2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^4\,e^2\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 - 2*e^4*x^4 - 2*d*e^3*x^3 + 3*d^2*e^2*x^2 + 3
*d^3*e*x))/(15*d^4*e^2*(d + e*x)^3*(d - e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.143 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/2)) - 1/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(
3/2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= -\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\ &= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3}\\ &= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 1.00

$$-\frac{\sqrt{d^2-e^2x^2}\left(3d^4-12d^3ex-12d^2e^2x^2+8de^3x^3+8e^4x^4\right)}{15d^5e(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(3*d^4 - 12*d^3*e*x - 12*d^2*e^2*x^2 + 8*d*e^3*x
^3 + 8*e^4*x^4))/(d^5*e*(d - e*x)^2*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.00, size = 82, normalized size = 1.00

$$\frac{\sqrt{d^2-e^2x^2}\left(-3d^4+12d^3ex+12d^2e^2x^2-8de^3x^3-8e^4x^4\right)}{15d^5e(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-3*d^4 + 12*d^3*e*x + 12*d^2*e^2*x^2 - 8*d*e^3*x^3 -
8*e^4*x^4))/(15*d^5*e*(d - e*x)^2*(d + e*x)^3)
```

fricas [B] time = 0.41, size = 168, normalized size = 2.05

$$-\frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5+\left(8\,e^4x^4+8\,de^3x^3-12\,d^2e^2x^2-12\,d^3ex+3\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^5e^6x^5+d^6e^5x^4-2\,d^7e^4x^3-2\,d^8e^3x^2+d^9e^2x+d^{10}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x
+ 3*d^5 + (8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 - 12*d^3*e*x + 3*d^4)*s
qrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 + d^6*e^5*x^4 - 2*d^7*e^4*x^3 - 2*d^8*e^3
*x^2 + d^9*e^2*x + d^10*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 70, normalized size = 0.85

$$-\frac{(-ex+d)\left(8x^4e^4+8x^3d\,e^3-12d^2x^2e^2-12d^3xe+3d^4\right)}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/15*(-e*x+d)*(8*e^4*x^4+8*d*e^3*x^3-12*d^2*e^2*x^2-12*d^3*e*x+3*d^4)/d^5/
e/(-e^2*x^2+d^2)^(5/2)
```

maxima [A] time = 0.45, size = 85, normalized size = 1.04

$$-\frac{1}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e\right)}+\frac{4\,x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{8\,x}{15\sqrt{-e^2x^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5/((-e^2*x^2 + d^2)^(3/2)*d*e^2*x + (-e^2*x^2 + d^2)^(3/2)*d^2*e) + 4/15
*x/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

mupad [B] time = 2.76, size = 78, normalized size = 0.95

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4-12\,d^3\,e\,x-12\,d^2\,e^2\,x^2+8\,d\,e^3\,x^3+8\,e^4\,x^4\right)}{15\,d^5\,e\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 -
 12*d^3*e*x))/(15*d^5*e*(d + e*x)^3*(d - e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.144 $\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 12, 266, 63, 208}

$$\frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (5*d - 4*e*x)/(15*d^4*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*(d + e*x)*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-5de^2+4e^3x}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^3e^4+8d^2e^5x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^5e^6}{x\sqrt{d^2-e^2}}}{15d^{10}e}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x}}}{d^5}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{}\right.}{d^6}$$

Mathematica [A] time = 0.09, size = 106, normalized size = 0.89

$$\frac{-15\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(23d^4+8d^3ex-27d^2e^2x^2-7de^3x^3+8e^4x^4\right)}{(d-ex)^2(d+ex)^3}+15\log(x)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(23*d^4 + 8*d^3*e*x - 27*d^2*e^2*x^2 - 7*d*e^3*x^3 +
8*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) + 15*Log[x] - 15*Log[d + Sqrt[d^2 - e
^2*x^2]])/(15*d^6)
```

IntegrateAlgebraic [A] time = 0.66, size = 122, normalized size = 1.03

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{\sqrt{d^2-e^2x^2}\,\left(23d^4+8d^3ex-27d^2e^2x^2-7de^3x^3+8e^4x^4\right)}{15d^6(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(23*d^4 + 8*d^3*e*x - 27*d^2*e^2*x^2 - 7*d*e^3*x^3 + 8
*e^4*x^4))/(15*d^6*(d - e*x)^2*(d + e*x)^3) + (2*ArcTanh[(Sqrt[-e^2]*x)/d -
 Sqrt[d^2 - e^2*x^2]/d])/d^6
```

fricas [B] time = 0.42, size = 237, normalized size = 1.99

$$\frac{23\,e^5x^5+23\,de^4x^4-46\,d^2e^3x^3-46\,d^3e^2x^2+23\,d^4ex+23\,d^5+15\left(e^5x^5+de^4x^4-2\,d^2e^3x^3-2\,d^3e^2x^2+d^4ex+d^5\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(8\,e^4x^4-7\,de^3x^3-27\,d^2e^2x^2+8\,d^3ex+23\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^6e^5x^5+d^7e^4x^4-2\,d^8e^3x^3-2\,d^9e^2x^2+d^{10}ex+d^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^5*x^5 + 23*d*e^4*x^4 - 46*d^2*e^3*x^3 - 46*d^3*e^2*x^2 + 23*d^4*
e*x + 23*d^5 + 15*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^
4*e*x + d^5)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^4*x^4 - 7*d*e^3*x^3
- 27*d^2*e^2*x^2 + 8*d^3*e*x + 23*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*x^5 +
 d^7*e^4*x^4 - 2*d^8*e^3*x^3 - 2*d^9*e^2*x^2 + d^10*e*x + d^11)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.02, size = 196, normalized size = 1.65

$$-\frac{4ex}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^4}+\frac{1}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^2e}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^5}-\frac{8ex}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^6}+\frac{1}{\sqrt{-e^2x^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/3/(-e^2*x^2+d^2)^(3/2)/d^3+1/(-e^2*x^2+d^2)^(1/2)/d^5-1/(d^2)^(1/2)/d^5*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^2/e/(x+d/e)/(2*(x+d/e
)*d*e-(x+d/e)^2*e^2)^(3/2)-4/15/d^4*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x
-8/15/d^6*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(d^2-e^2x^2\right)^{5/2}(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.145 $\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 807, 266, 63, 208}

$$\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (6*d - 5*e*x)/(15*d^4*x*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x*(d + e*x)*(d^2
- e^2*x^2)^(3/2)) + (8*d - 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[
d^2 - e^2*x^2])/(5*d^7*x) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-6de^2+5e^3x}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-24d^3e^4+15d^2e^5x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-48d^5\cdots}{x^2\cdots}}{1\cdots}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2\cdots}}{5d\cdots}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2\cdots}}{5d\cdots}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2\cdots}}{5d\cdots}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2\cdots}}{5d\cdots}$$

Mathematica [A] time = 0.12, size = 122, normalized size = 0.79

$$-\frac{-15e\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(15d^5+38d^4ex-52d^3e^2x^2-87d^2e^3x^3+33de^4x^4+48e^5x^5\right)}{x(d-ex)^2(d+ex)^3}+15e\log(x)}{15d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(15*d^5 + 38*d^4*e*x - 52*d^3*e^2*x^2 - 87*d^2*
e^3*x^3 + 33*d*e^4*x^4 + 48*e^5*x^5))/(x*(d - e*x)^2*(d + e*x)^3) + 15*e*Lo
g[x] - 15*e*Log[d + Sqrt[d^2 - e^2*x^2]])/d^7
```

IntegrateAlgebraic [A] time = 0.76, size = 137, normalized size = 0.89

$$\frac{\sqrt{d^2-e^2x^2}\left(-15d^5-38d^4ex+52d^3e^2x^2+87d^2e^3x^3-33de^4x^4-48e^5x^5\right)}{15d^7x(d-ex)^2(d+ex)^3}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-15*d^5 - 38*d^4*e*x + 52*d^3*e^2*x^2 + 87*d^2*e^3*x^
3 - 33*d*e^4*x^4 - 48*e^5*x^5))/(15*d^7*x*(d - e*x)^2*(d + e*x)^3) - (2*e*A
rcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^7
```

fricas [A] time = 0.45, size = 265, normalized size = 1.72

$$-\frac{23\,e^6x^6+23\,de^5x^5-46\,d^2e^4x^4-46\,d^3e^3x^3+23\,d^4e^2x^2+23\,d^5ex+15\left(e^6x^6+de^5x^5-2\,d^2e^4x^4-2\,d^3e^3x^3+d^4e^2x^2+d^5ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(48\,e^5x^5+33\,de^4x^4-87\,d^2e^3x^3-52\,d^3e^2x^2+38\,d^4ex+15\,d^5\right)\sqrt{-e^2x^2+d^2}}{15\left(d^7e^5x^6+d^8e^4x^5-2\,d^9e^3x^4-2\,d^{10}e^2x^3+d^{11}ex^2+d^{12}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(23*e^6*x^6 + 23*d*e^5*x^5 - 46*d^2*e^4*x^4 - 46*d^3*e^3*x^3 + 23*d^4
*e^2*x^2 + 23*d^5*e*x + 15*(e^6*x^6 + d*e^5*x^5 - 2*d^2*e^4*x^4 - 2*d^3*e^3
*x^3 + d^4*e^2*x^2 + d^5*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (48*e^5*
x^5 + 33*d*e^4*x^4 - 87*d^2*e^3*x^3 - 52*d^3*e^2*x^2 + 38*d^4*e*x + 15*d^5)
*sqrt(-e^2*x^2 + d^2))/(d^7*e^5*x^6 + d^8*e^4*x^5 - 2*d^9*e^3*x^4 - 2*d^10*
e^2*x^3 + d^11*e*x^2 + d^12*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.02, size = 268, normalized size = 1.74

$$\frac{4e^2x}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}+\frac{4e^2x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^5}-\frac{1}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^3}-\frac{e}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}-\frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}+\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^6}+\frac{8e^2x}{3\sqrt{-e^2x^2+d^2}\,d^7}+\frac{8e^2x}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^7}-\frac{e}{\sqrt{-e^2x^2+d^2}\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/d^3/x/(-e^2*x^2+d^2)^(3/2)+4/3/(-e^2*x^2+d^2)^(3/2)/d^5*e^2*x+8/3/(-e^2*
x^2+d^2)^(1/2)/d^7*e^2*x-1/3/(-e^2*x^2+d^2)^(3/2)/d^4*e-1/(-e^2*x^2+d^2)^(1
/2)/d^6*e+1/(d^2)^(1/2)/d^6*e*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))
/x)-1/5/d^3/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+4/15*e^2/d^5/(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+8/15*e^2/d^7/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(d^2-e^2x^2\right)^{5/2}(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.146 $\int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+$$

Rubi [A] time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {857, 823, 835, 807, 266, 63, 208}

$$\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (7*d - 6*e*x)/(15*d^4*x^2*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^2*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (35*d - 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) -
(7*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) + (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x)
- (7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-7de^2+6e^3x}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-35d^3e^4+24d^2e^5x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \int$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{}}{}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{}}{}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{}}{}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{}}{}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{}}{}$$

Mathematica [A] time = 0.13, size = 137, normalized size = 0.74

$$\frac{-105e^2\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(-15d^6+15d^5ex+176d^4e^2x^2-4d^3e^3x^3-249d^2e^4x^4-9de^5x^5+96e^6x^6\right)}{x^2(d-ex)^2(d+ex)^3}+105e^2\log(x)}{30d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^6 + 15*d^5*e*x + 176*d^4*e^2*x^2 - 4*d^3*e^3*x
^3 - 249*d^2*e^4*x^4 - 9*d*e^5*x^5 + 96*e^6*x^6))/(x^2*(d - e*x)^2*(d + e*x
)^3) + 105*e^2*Log[x] - 105*e^2*Log[d + Sqrt[d^2 - e^2*x^2]])/(30*d^8)
```

IntegrateAlgebraic [A] time = 1.04, size = 150, normalized size = 0.81

$$\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}+\frac{\sqrt{d^2-e^2x^2}\left(-15d^6+15d^5ex+176d^4e^2x^2-4d^3e^3x^3-249d^2e^4x^4-9de^5x^5+96e^6x^6\right)}{30d^8x^2(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-15*d^6 + 15*d^5*e*x + 176*d^4*e^2*x^2 - 4*d^3*e^3*x^
3 - 249*d^2*e^4*x^4 - 9*d*e^5*x^5 + 96*e^6*x^6))/(30*d^8*x^2*(d - e*x)^2*(d
 + e*x)^3) + (7*e^2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^8
```

fricas [A] time = 0.51, size = 286, normalized size = 1.54

$$\frac{116\,e^7x^7+116\,de^6x^6-232\,d^2e^5x^5-232\,d^3e^4x^4+116\,d^4e^3x^3+116\,d^5e^2x^2+105\left(e^7x^7+de^6x^6-2\,d^2e^5x^5-2\,d^3e^4x^4+d^4e^3x^3+d^5e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(96\,e^6x^6-9\,de^5x^5-249\,d^2e^4x^4-4\,d^3e^3x^3+176\,d^4e^2x^2+15\,d^5ex-15\,d^6\right)\sqrt{-e^2x^2+d^2}}{30\left(d^8e^5x^7+d^9e^4x^6-2\,d^{10}e^3x^5-2\,d^{11}e^2x^4+d^{12}ex^3+d^{13}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(116*e^7*x^7 + 116*d*e^6*x^6 - 232*d^2*e^5*x^5 - 232*d^3*e^4*x^4 + 116
*d^4*e^3*x^3 + 116*d^5*e^2*x^2 + 105*(e^7*x^7 + d*e^6*x^6 - 2*d^2*e^5*x^5 -
 2*d^3*e^4*x^4 + d^4*e^3*x^3 + d^5*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2))
/x) + (96*e^6*x^6 - 9*d*e^5*x^5 - 249*d^2*e^4*x^4 - 4*d^3*e^3*x^3 + 176*d^4
*e^2*x^2 + 15*d^5*e*x - 15*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^5*x^7 + d^9*e^
4*x^6 - 2*d^10*e^3*x^5 - 2*d^11*e^2*x^4 + d^12*e*x^3 + d^13*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.02, size = 298, normalized size = 1.60

$$-\frac{4e^3x}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}-\frac{4e^3x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^6}+\frac{e}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^4}+\frac{7e^2}{6\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}+\frac{e}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4x}-\frac{7e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^7}-\frac{8e^3x}{3\sqrt{-e^2x^2+d^2}\,d^8}-\frac{8e^3x}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^8}-\frac{1}{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x^2}+\frac{7e^2}{2\sqrt{-e^2x^2+d^2}\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] e/d^4/x/(-e^2*x^2+d^2)^(3/2)-4/3/(-e^2*x^2+d^2)^(3/2)/d^6*e^3*x-8/3/(-e^2*x
^2+d^2)^(1/2)/d^8*e^3*x-1/2/d^3/x^2/(-e^2*x^2+d^2)^(3/2)+7/6/(-e^2*x^2+d^2)
^(3/2)/d^5*e^2+7/2/(-e^2*x^2+d^2)^(1/2)/d^7*e^2-7/2/(d^2)^(1/2)/d^7*e^2*ln(
(2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^4*e/(x+d/e)/(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(3/2)-4/15/d^6*e^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x
-8/15/d^8*e^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(d^2-e^2\,x^2\right)}^{5/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.147 $\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=215

$$\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}+\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{4}{15d^6}$$

Rubi [A] time = 0.21, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {857, 823, 835, 807, 266, 63, 208}

$$-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}+\frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (8*d - 7*e*x)/(15*d^4*x^3*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^3*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (48*d - 35*e*x)/(15*d^6*x^3*Sqrt[d^2 - e^2*x^2]) -
(64*Sqrt[d^2 - e^2*x^2])/(15*d^7*x^3) + (7*e*Sqrt[d^2 - e^2*x^2])/(2*d^8*x^
2) - (128*e^2*Sqrt[d^2 - e^2*x^2])/(15*d^9*x) + (7*e^3*ArcTanh[Sqrt[d^2 - e
^2*x^2]/d])/(2*d^9)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-8de^2+7e^3x}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-48d^3e^4+35d^2e^5x}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} -$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} -$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} -$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} -$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} -$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} -$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} -$$

Mathematica [A] time = 0.16, size = 148, normalized size = 0.69

$$-\frac{-105e^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(10d^7-5d^6ex+75d^5e^2x^2+236d^4e^3x^3-244d^3e^4x^4-489d^2e^5x^5+151de^6x^6+256e^7x^7\right)}{x^3(d-ex)^2(d+ex)^3}+105e^3\log(x)}{30d^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/30*((Sqrt[d^2 - e^2*x^2]*(10*d^7 - 5*d^6*e*x + 75*d^5*e^2*x^2 + 236*d^4*
e^3*x^3 - 244*d^3*e^4*x^4 - 489*d^2*e^5*x^5 + 151*d*e^6*x^6 + 256*e^7*x^7))
/(x^3*(d - e*x)^2*(d + e*x)^3) + 105*e^3*Log[x] - 105*e^3*Log[d + Sqrt[d^2
- e^2*x^2]])/d^9
```

IntegrateAlgebraic [A] time = 1.42, size = 161, normalized size = 0.75

$$\frac{\sqrt{d^2-e^2x^2}\left(-10d^7+5d^6ex-75d^5e^2x^2-236d^4e^3x^3+244d^3e^4x^4+489d^2e^5x^5-151de^6x^6-256e^7x^7\right)}{30d^9x^3(d-ex)^2(d+ex)^3}-\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-10*d^7 + 5*d^6*e*x - 75*d^5*e^2*x^2 - 236*d^4*e^3*x^
3 + 244*d^3*e^4*x^4 + 489*d^2*e^5*x^5 - 151*d*e^6*x^6 - 256*e^7*x^7))/(30*d
^9*x^3*(d - e*x)^2*(d + e*x)^3) - (7*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^
2 - e^2*x^2]/d])/d^9
```

fricas [A]   time = 0.52, size = 297, normalized size = 1.38

$$-\frac{116\,e^{8}x^{8}+116\,de^{7}x^{7}-232\,d^{2}e^{6}x^{6}-232\,d^{3}e^{5}x^{5}+116\,d^{4}e^{4}x^{4}+116\,d^{5}e^{3}x^{3}+105\left(e^{8}x^{8}+de^{7}x^{7}-2\,d^{2}e^{6}x^{6}-2\,d^{3}e^{5}x^{5}+d^{4}e^{4}x^{4}+d^{5}e^{3}x^{3}\right)\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(256\,e^{7}x^{7}+151\,de^{6}x^{6}-489\,d^{2}e^{5}x^{5}-244\,d^{3}e^{4}x^{4}+236\,d^{4}e^{3}x^{3}+75\,d^{5}e^{2}x^{2}-5\,d^{6}ex+10\,d^{7}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{30\left(d^{9}e^{5}x^{8}+d^{10}e^{4}x^{7}-2\,d^{11}e^{3}x^{6}-2\,d^{12}e^{2}x^{5}+d^{13}ex^{4}+d^{14}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(116*e^8*x^8 + 116*d*e^7*x^7 - 232*d^2*e^6*x^6 - 232*d^3*e^5*x^5 + 11
6*d^4*e^4*x^4 + 116*d^5*e^3*x^3 + 105*(e^8*x^8 + d*e^7*x^7 - 2*d^2*e^6*x^6
 - 2*d^3*e^5*x^5 + d^4*e^4*x^4 + d^5*e^3*x^3)*log(-(d - sqrt(-e^2*x^2 + d^2)
)/x) + (256*e^7*x^7 + 151*d*e^6*x^6 - 489*d^2*e^5*x^5 - 244*d^3*e^4*x^4 + 2
36*d^4*e^3*x^3 + 75*d^5*e^2*x^2 - 5*d^6*e*x + 10*d^7)*sqrt(-e^2*x^2 + d^2))
/(d^9*e^5*x^8 + d^10*e^4*x^7 - 2*d^11*e^3*x^6 - 2*d^12*e^2*x^5 + d^13*e*x^4
 + d^14*x^3)
```

giac [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A]   time = 0.02, size = 326, normalized size = 1.52

$$\frac{4e^{4}x}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{7}}+\frac{4e^{4}x}{15\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}d^{7}}-\frac{e^{2}}{5\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}d^{5}}-\frac{7e^{3}}{6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{6}}-\frac{3e^{2}}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{5}x}+\frac{7e^{3}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{2\sqrt{d^{2}}\,d^{8}}+\frac{8e^{4}x}{\sqrt{-e^{2}x^{2}+d^{2}}\,d^{9}}+\frac{8e^{4}x}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{9}}+\frac{e}{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{4}x^{2}}-\frac{7e^{3}}{2\sqrt{-e^{2}x^{2}+d^{2}}\,d^{8}}-\frac{1}{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -3/d^5*e^2/x/(-e^2*x^2+d^2)^(3/2)+4/d^7*e^4*x/(-e^2*x^2+d^2)^(3/2)+8/d^9*e^
4*x/(-e^2*x^2+d^2)^(1/2)+1/2*e/d^4/x^2/(-e^2*x^2+d^2)^(3/2)-7/6*e^3/d^6/(-e
^2*x^2+d^2)^(3/2)-7/2*e^3/d^8/(-e^2*x^2+d^2)^(1/2)+7/2*e^3/d^8/(d^2)^(1/2)*
ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3/d^3/x^3/(-e^2*x^2+d^2)
^(3/2)-1/5/d^5*e^2/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+4/15/d^7*e^4
/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+8/15/d^9*e^4/(2*(x+d/e)*d*e-(x+d/e)^
2*e^2)^(1/2)*x
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}(ex+d)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(d^2-e^2x^2\right)^{5/2}(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(1/(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(1/(x**4*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

3.148 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}}-\frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 778, 192, 191}

$$\frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}}-\frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}-\frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x^2*(d - e*x))/(7*e^2*(d^2 - e^2*x^2)^(7/2)) - (2*d - 3*e*x)/(35*e^4*(d^2
- e^2*x^2)^(5/2)) - x/(35*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) - (2*x)/(35*d^4*e^
3*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
```

```
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx \\
&= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7d^2e^2} \\
&= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{3\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35e^3} \\
&= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/}}}{35d^2e^3} \\
&= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2x}{35d^4e^3\sqrt{d^2-}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 104, normalized size = 0.88

$$-\frac{\sqrt{d^2-e^2x^2}\left(2d^6+2d^5ex-5d^4e^2x^2-5d^3e^3x^3-5d^2e^4x^4+2de^5x^5+2e^6x^6\right)}{35d^4e^4(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] -1/35*(Sqrt[d^2 - e^2*x^2]*(2*d^6 + 2*d^5*e*x - 5*d^4*e^2*x^2 - 5*d^3*e^3*x
^3 - 5*d^2*e^4*x^4 + 2*d*e^5*x^5 + 2*e^6*x^6))/(d^4*e^4*(d - e*x)^3*(d + e*
x)^4)
```

IntegrateAlgebraic [A] time = 0.78, size = 104, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^6-2d^5ex+5d^4e^2x^2+5d^3e^3x^3+5d^2e^4x^4-2de^5x^5-2e^6x^6\right)}{35d^4e^4(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^6 - 2*d^5*e*x + 5*d^4*e^2*x^2 + 5*d^3*e^3*x^3 +
5*d^2*e^4*x^4 - 2*d*e^5*x^5 - 2*e^6*x^6))/(35*d^4*e^4*(d - e*x)^3*(d + e*x)
^4)
```

fricas [B] time = 0.49, size = 239, normalized size = 2.03

$$-\frac{2\,e^7x^7+2\,de^6x^6-6\,d^2e^5x^5-6\,d^3e^4x^4+6\,d^4e^3x^3+6\,d^5e^2x^2-2\,d^6ex-2\,d^7-\left(2\,e^6x^6+2\,de^5x^5-5\,d^2e^4x^4-5\,d^3e^3x^3-5\,d^4e^2x^2+2\,d^5ex+2\,d^6\right)\sqrt{-e^2x^2+d^2}}{35\left(d^4e^{11}x^7+d^5e^{10}x^6-3\,d^6e^9x^5-3\,d^7e^8x^4+3\,d^8e^7x^3+3\,d^9e^6x^2-d^{10}e^5x-d^{11}e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/35*(2*e^7*x^7 + 2*d*e^6*x^6 - 6*d^2*e^5*x^5 - 6*d^3*e^4*x^4 + 6*d^4*e^3*
x^3 + 6*d^5*e^2*x^2 - 2*d^6*e*x - 2*d^7 - (2*e^6*x^6 + 2*d*e^5*x^5 - 5*d^2*
e^4*x^4 - 5*d^3*e^3*x^3 - 5*d^4*e^2*x^2 + 2*d^5*e*x + 2*d^6)*sqrt(-e^2*x^2
+ d^2))/(d^4*e^11*x^7 + d^5*e^10*x^6 - 3*d^6*e^9*x^5 - 3*d^7*e^8*x^4 + 3*d^
8*e^7*x^3 + 3*d^9*e^6*x^2 - d^10*e^5*x - d^11*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 92, normalized size = 0.78

$$-\frac{(-ex+d)\left(2e^6x^6+2e^5x^5d-5e^4x^4d^2-5x^3d^3e^3-5x^2d^4e^2+2d^5xe+2d^6\right)}{35\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^4e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/35*(-e*x+d)*(2*e^6*x^6+2*d*e^5*x^5-5*d^2*e^4*x^4-5*d^3*e^3*x^3-5*d^4*e^2
*x^2+2*d^5*e*x+2*d^6)/d^4/e^4/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.51, size = 133, normalized size = 1.13

$$\frac{d^2}{7\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^4\right)}+\frac{8x}{35\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}-\frac{x}{35\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^3}-\frac{2x}{35\sqrt{-e^2x^2+d^2}\,d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/7*d^2/((-e^2*x^2 + d^2)^(5/2)*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d*e^4) + 8/3
5*x/((-e^2*x^2 + d^2)^(5/2)*e^3) - 1/5*d/((-e^2*x^2 + d^2)^(5/2)*e^4) - 1/3
5*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^3) - 2/35*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^3
)
```

mupad [B] time = 2.95, size = 161, normalized size = 1.36

$$\frac{\sqrt{d^2-e^2\,x^2}}{56\,d\,e^4\,{\left(d+e\,x\right)}^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d\,e^4}+\frac{x}{35\,d^2\,e^3}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2\,d}{35\,e^4}-\frac{11\,x}{70\,e^3}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{2\,x\,\sqrt{d^2-e^2\,x^2}}{35\,d^4\,e^3\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(56*d*e^4*(d + e*x)^4) - ((d^2 - e^2*x^2)^(1/2)*(1/(5
6*d*e^4) + x/(35*d^2*e^3)))/((d + e*x)^2*(d - e*x)^2) - ((d^2 - e^2*x^2)^(1
/2)*((2*d)/(35*e^4) - (11*x)/(70*e^3)))/((d + e*x)^3*(d - e*x)^3) - (2*x*(d
^2 - e^2*x^2)^(1/2))/(35*d^4*e^3*(d + e*x)*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

3.149 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {855, 778, 192, 191}

$$-\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] -x^2/(7*d*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (2*(d + 2*e*x))/(35*d*e^3*(d
^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(1
05*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{x(2d+4ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7de}$$

$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35de^2}$$

$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} -$$

$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} -$$

Mathematica [A] time = 0.08, size = 104, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(6d^6+6d^5ex-15d^4e^2x^2+20d^3e^3x^3+20d^2e^4x^4-8de^5x^5-8e^6x^6\right)}{105d^5e^3(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(6*d^6 + 6*d^5*e*x - 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 +
 20*d^2*e^4*x^4 - 8*d*e^5*x^5 - 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^3*(d + e
*x)^4)
```

IntegrateAlgebraic [A] time = 0.68, size = 104, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(6d^6+6d^5ex-15d^4e^2x^2+20d^3e^3x^3+20d^2e^4x^4-8de^5x^5-8e^6x^6\right)}{105d^5e^3(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(6*d^6 + 6*d^5*e*x - 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 +
 20*d^2*e^4*x^4 - 8*d*e^5*x^5 - 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^3*(d + e
*x)^4)
```

fricas [B] time = 0.49, size = 238, normalized size = 1.93

$$\frac{6e^7x^7+6de^6x^6-18d^2e^5x^5-18d^3e^4x^4+18d^4e^3x^3+18d^5e^2x^2-6d^6ex-6d^7+\left(8e^6x^6+8de^5x^5-20d^2e^4x^4-20d^3e^3x^3+15d^4e^2x^2-6d^5ex-6d^6\right)\sqrt{-e^2x^2+d^2}}{105\left(d^5e^{10}x^7+d^6e^9x^6-3d^7e^8x^5-3d^8e^7x^4+3d^9e^6x^3+3d^{10}e^5x^2-d^{11}e^4x-d^{12}e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/105*(6*e^7*x^7 + 6*d*e^6*x^6 - 18*d^2*e^5*x^5 - 18*d^3*e^4*x^4 + 18*d^4*e
^3*x^3 + 18*d^5*e^2*x^2 - 6*d^6*e*x - 6*d^7 + (8*e^6*x^6 + 8*d*e^5*x^5 - 20
*d^2*e^4*x^4 - 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 - 6*d^5*e*x - 6*d^6)*sqrt(-e
^2*x^2 + d^2))/(d^5*e^10*x^7 + d^6*e^9*x^6 - 3*d^7*e^8*x^5 - 3*d^8*e^7*x^4
+ 3*d^9*e^6*x^3 + 3*d^10*e^5*x^2 - d^11*e^4*x - d^12*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 92, normalized size = 0.75

$$\frac{(-ex+d)\left(-8e^6x^6-8e^5x^5d+20e^4x^4d^2+20x^3d^3e^3-15x^2d^4e^2+6d^5xe+6d^6\right)}{105\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/105*(-e*x+d)*(-8*e^6*x^6-8*d*e^5*x^5+20*d^2*e^4*x^4+20*d^3*e^3*x^3-15*d^4
*e^2*x^2+6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(7/2)
```

maxima [A] time = 0.48, size = 133, normalized size = 1.08

$$-\frac{d}{7\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^3\right)}-\frac{x}{35\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2}+\frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}-\frac{4x}{105\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2}-\frac{8x}{105\sqrt{-e^2x^2+d^2}\,d^5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/7*d/((-e^2*x^2 + d^2)^(5/2)*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d*e^3) - 1/35
*x/((-e^2*x^2 + d^2)^(5/2)*d*e^2) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e^3) - 4/10
5*x/((-e^2*x^2 + d^2)^(3/2)*d^3*e^2) - 8/105*x/(sqrt(-e^2*x^2 + d^2)*d^5*e^
2)
```

mupad [B] time = 2.88, size = 161, normalized size = 1.31

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d^2\,e^3}-\frac{4\,x}{105\,d^3\,e^2}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}+\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2}{35\,e^3}+\frac{3\,x}{70\,d\,e^2}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}}{56\,d^2\,e^3\,{\left(d+e\,x\right)}^4}-\frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{105\,d^5\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(1/(56*d^2*e^3) - (4*x)/(105*d^3*e^2)))/((d + e*x)^2
*(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*(2/(35*e^3) + (3*x)/(70*d*e^2)))/((d
 + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(56*d^2*e^3*(d + e*x)^4) - (
8*x*(d^2 - e^2*x^2)^(1/2))/(105*d^5*e^2*(d + e*x)*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

3.150 $\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=66

$$\frac{3\sin^{-1}(ax)}{2a^4}+\frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}}+\frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}$$

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {850, 819, 780, 216}

$$\frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}}+\frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}+\frac{3\sin^{-1}(ax)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (x^2*(1 - a*x))/(a^2*Sqrt[1 - a^2*x^2]) + ((4 - 3*a*x)*Sqrt[1 - a^2*x^2])/(
2*a^4) + (3*ArcSin[a*x])/(2*a^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx$$
$$= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} - \frac{\int \frac{x(2-3ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^3}$$
$$= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\sin^{-1}(ax)}{2a^4}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(-a^2x^2+ax+4\right)+3(ax+1)\sin^{-1}(ax)}{2a^4(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(4 + a*x - a^2*x^2) + 3*(1 + a*x)*ArcSin[a*x])/(2*a^4*(1
 + a*x))
```

IntegrateAlgebraic [A] time = 0.43, size = 86, normalized size = 1.30

$$\frac{3\sqrt{-a^2}\log\left(\sqrt{1-a^2x^2}-\sqrt{-a^2}\,x\right)}{2a^5}+\frac{\sqrt{1-a^2x^2}\left(-a^2x^2+ax+4\right)}{2a^4(ax+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(4 + a*x - a^2*x^2))/(2*a^4*(1 + a*x)) + (3*Sqrt[-a^2]*L
og[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x^2]])/(2*a^5)
```

fricas [A] time = 0.42, size = 75, normalized size = 1.14

$$\frac{4\,ax-6\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(a^2x^2-ax-4\right)\sqrt{-a^2x^2+1}+4}{2\left(a^5x+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x - 6*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (a^2*x^2
 - a*x - 4)*sqrt(-a^2*x^2 + 1) + 4)/(a^5*x + a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 100, normalized size = 1.52

$$-\frac{\sqrt{-a^2x^2+1}\,x}{2a^3}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}\,a^3}+\frac{\sqrt{-a^2x^2+1}}{a^4}+\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2+2\left(x+\frac{1}{a}\right)a}}{\left(x+\frac{1}{a}\right)a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/a^3*x*(-a^2*x^2+1)^(1/2)+3/2/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*
x^2+1)^(1/2)*x)+1/a^4*(-a^2*x^2+1)^(1/2)+1/a^5/(x+1/a)*(-(x+1/a)^2*a^2+2*(x
+1/a)*a)^(1/2)
```

maxima [A] time = 0.98, size = 68, normalized size = 1.03

$$\frac{\sqrt{-a^2x^2+1}}{a^5x+a^4}-\frac{\sqrt{-a^2x^2+1}\,x}{2\,a^3}+\frac{3\,\arcsin\left(ax\right)}{2\,a^4}+\frac{\sqrt{-a^2x^2+1}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a^5*x + a^4) - 1/2*sqrt(-a^2*x^2 + 1)*x/a^3 + 3/2*arcsi
n(a*x)/a^4 + sqrt(-a^2*x^2 + 1)/a^4
```

mupad [B] time = 0.07, size = 116, normalized size = 1.76

$$\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a^3\,\sqrt{-a^2}}-\frac{\left(\frac{1}{a^2\,\sqrt{-a^2}}+\frac{x\,\sqrt{-a^2}}{2\,a^3}\right)\sqrt{1-a^2\,x^2}}{\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a^3\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] (3*asinh(x*(-a^2)^(1/2)))/(2*a^3*(-a^2)^(1/2)) - ((1/(a^2*(-a^2)^(1/2)) + (
x*(-a^2)^(1/2))/(2*a^3))*(1 - a^2*x^2)^(1/2))/(-a^2)^(1/2) - (1 - a^2*x^2)^
(1/2)/(a^3*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.151 $\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=55

$$-\frac{\sin^{-1}(ax)}{a^3}-\frac{\sqrt{1-a^2x^2}}{a^3(ax+1)}-\frac{\sqrt{1-a^2x^2}}{a^3}$$

Rubi [A] time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1639, 12, 793, 216}

$$-\frac{\sqrt{1-a^2x^2}}{a^3(ax+1)}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a^3) - Sqrt[1 - a^2*x^2]/(a^3*(1 + a*x)) - ArcSin[a*x]/
a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\int \frac{a^3x}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{a^4}$$
$$= -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{a}$$
$$= -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\sqrt{1-a^2x^2}}{a^3(1+ax)} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= -\frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\sqrt{1-a^2x^2}}{a^3(1+ax)} - \frac{\sin^{-1}(ax)}{a^3}$$

Mathematica [A] time = 0.05, size = 37, normalized size = 0.67

$$-\frac{\frac{\sqrt{1-a^2x^2}\,(ax+2)}{ax+1} + \sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] -((((2 + a*x)*Sqrt[1 - a^2*x^2])/(1 + a*x) + ArcSin[a*x])/a^3)
```

IntegrateAlgebraic [A] time = 0.32, size = 74, normalized size = 1.35

$$\frac{(-ax-2)\sqrt{1-a^2x^2}}{a^3(ax+1)} - \frac{\sqrt{-a^2}\,\log\left(\sqrt{1-a^2x^2} - \sqrt{-a^2}\,x\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] ((-2 - a*x)*Sqrt[1 - a^2*x^2])/(a^3*(1 + a*x)) - (Sqrt[-a^2]*Log[-(Sqrt[-a^
2]*x) + Sqrt[1 - a^2*x^2]])/a^4
```

fricas [A] time = 0.40, size = 66, normalized size = 1.20

$$-\frac{2\,ax - 2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}\,(ax+2) + 2}{a^4x + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

giac [A] time = 0.20, size = 70, normalized size = 1.27

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^2|a|} - \frac{\sqrt{-a^2x^2+1}}{a^3} + \frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [A] time = 0.01, size = 84, normalized size = 1.53

$$-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}\,a^2}-\frac{\sqrt{-a^2x^2+1}}{a^3}-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2+2\left(x+\frac{1}{a}\right)a}}{\left(x+\frac{1}{a}\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a^3-1/(a^2)^(1/2)/a^2*arctan((a^2)^(1/2)/(-a^2*x^2+1)^(
1/2)*x)-1/a^4/(x+1/a)*(-(x+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

maxima [A] time = 0.98, size = 52, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+1}}{a^4x+a^3}-\frac{\arcsin\left(ax\right)}{a^3}-\frac{\sqrt{-a^2x^2+1}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-a^2*x^2 + 1)/(a^4*x + a^3) - arcsin(a*x)/a^3 - sqrt(-a^2*x^2 + 1)/a^
3
```

mupad [B] time = 0.07, size = 84, normalized size = 1.53

$$\frac{\sqrt{1-a^2\,x^2}}{\left(a\,\sqrt{-a^2}+a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/((a*(-a^2)^(1/2) + a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.152 $\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=34

$$\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}+\frac{\sin^{-1}(ax)}{a^2}$$

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {793, 216}

$$\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}+\frac{\sin^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + ArcSin[a*x]/a^2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a}\\ &= \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\sin^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.91

$$\frac{\frac{\sqrt{1-a^2x^2}}{ax+1}+\sin^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]/(1 + a*x) + ArcSin[a*x])/a^2
```

IntegrateAlgebraic [A] time = 0.34, size = 67, normalized size = 1.97

$$\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}+\frac{\sqrt{-a^2}\log\left(\sqrt{1-a^2x^2}-\sqrt{-a^2}\,x\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + (Sqrt[-a^2]*Log[-(Sqrt[-a^2]*x) + Sqrt[
1 - a^2*x^2]])/a^3
```

fricas [A] time = 0.41, size = 58, normalized size = 1.71

$$\frac{ax - 2\,(ax + 1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2 + 1} + 1}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 +
 1) + 1)/(a^3*x + a^2)
```

giac [A] time = 0.22, size = 52, normalized size = 1.53

$$\frac{\arcsin\left(ax\right)\operatorname{sgn}(a)}{a|a|} - \frac{2}{a\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(a*abs(a)) - 2/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*
x) + 1)*abs(a))
```

maple [A] time = 0.01, size = 65, normalized size = 1.91

$$\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}\,a} + \frac{\sqrt{-\left(x + \frac{1}{a}\right)^2 a^2 + 2\left(x + \frac{1}{a}\right)a}}{\left(x + \frac{1}{a}\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*x^2+1)^(1/2)*x)+1/a^3/(x+1/a)*(-(x
+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

maxima [A] time = 0.99, size = 33, normalized size = 0.97

$$\frac{\sqrt{-a^2x^2 + 1}}{a^3x + a^2} + \frac{\arcsin\left(ax\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a^3*x + a^2) + arcsin(a*x)/a^2
```

mupad [B] time = 2.60, size = 57, normalized size = 1.68

$$\frac{1}{a^2\sqrt{1 - a^2\,x^2}} - \frac{x}{a\sqrt{1 - a^2\,x^2}} - \frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)\sqrt{-a^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] 1/(a^2*(1 - a^2*x^2)^(1/2)) - x/(a*(1 - a^2*x^2)^(1/2)) - (asinh(x*(-a^2)^(
1/2))*(-a^2)^(1/2))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.153 $\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {651}

$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] -(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.96

$$-\frac{\sqrt{1-a^2x^2}}{a^2x+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] -(Sqrt[1 - a^2*x^2]/(a + a^2*x))
```

IntegrateAlgebraic [A] time = 0.32, size = 26, normalized size = 1.00

$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] -(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

fricas [A] time = 0.39, size = 28, normalized size = 1.08

$$-\frac{ax+\sqrt{-a^2x^2+1}+1}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(a*x + sqrt(-a^2*x^2 + 1) + 1)/(a^2*x + a)
```

giac [A] time = 0.20, size = 34, normalized size = 1.31

$$\frac{2}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] 2/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [A] time = 0.01, size = 22, normalized size = 0.85

$$\frac{ax-1}{\sqrt{-a^2x^2+1}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
[Out] (a*x-1)/a/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 23, normalized size = 0.88

$$-\frac{\sqrt{-a^2x^2+1}}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-a^2*x^2 + 1)/(a^2*x + a)
```

mupad [B] time = 2.59, size = 23, normalized size = 0.88

$$-\frac{\sqrt{1-a^2\,x^2}}{x\,a^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
[Out] -(1 - a^2*x^2)^(1/2)/(a + a^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\;\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.154 $\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {857, 12, 266, 63, 208}

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{1-ax}+\frac{\int \frac{a^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2}\\ &= \frac{\sqrt{1-a^2x^2}}{1-ax}+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\ &= \frac{\sqrt{1-a^2x^2}}{1-ax}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{1-a^2x^2}}{1-ax}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\ &= \frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) - ArcTanh[Sqrt[1 - a^2*x^2]]
```

IntegrateAlgebraic [A] time = 0.45, size = 55, normalized size = 1.34

$$\frac{\sqrt{1-a^2x^2}}{1-ax}+2\tanh^{-1}\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) + 2*ArcTanh[Sqrt[-a^2]*x - Sqrt[1 - a^2*x^2]]
```

fricas [A] time = 0.40, size = 52, normalized size = 1.27

$$\frac{ax+(ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}-1}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (a*x + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1) - 1)/
(a*x - 1)
```

giac [A] time = 0.21, size = 74, normalized size = 1.80

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{|a|}+\frac{2a}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 2
*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [A] time = 0.01, size = 58, normalized size = 1.41

$$-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}{\left(x-\frac{1}{a}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/a/(x-1/a)*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{\sqrt{-a^2x^2+1}\,(ax-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x), x)
```

mupad [B] time = 2.65, size = 58, normalized size = 1.41

$$\frac{a\sqrt{1-a^2x^2}}{\sqrt{-a^2}\left(\frac{a}{\sqrt{-a^2}}+x\sqrt{-a^2}\right)}-\operatorname{atanh}\left(\sqrt{1-a^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] (a*(1 - a^2*x^2)^(1/2))/((-a^2)^(1/2)*(a/(-a^2)^(1/2) + x*(-a^2)^(1/2))) -
atanh((1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{ax^2\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x)
```

3.155 $\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=64

$$-\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {857, 807, 266, 63, 208}

$$-\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x^2]/(x*(1 - a*x)) - a*ArcTanh[Sqrt
[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - \frac{\int \frac{-2a^2-a^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)$$
$$= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a}$$
$$= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.78

$$\frac{(1-2ax)\sqrt{1-a^2x^2}}{x(ax-1)} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((1 - 2*a*x)*Sqrt[1 - a^2*x^2])/(x*(-1 + a*x)) - a*ArcTanh[Sqrt[1 - a^2*x^2
]]
```

IntegrateAlgebraic [A] time = 0.44, size = 64, normalized size = 1.00

$$\frac{\sqrt{1-a^2x^2}\,(1-2ax)}{x(ax-1)} + 2a\tanh^{-1}\left(\sqrt{-a^2}\,x - \sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((1 - 2*a*x)*Sqrt[1 - a^2*x^2])/(x*(-1 + a*x)) + 2*a*ArcTanh[Sqrt[-a^2]*x -
 Sqrt[1 - a^2*x^2]]
```

fricas [A] time = 0.40, size = 76, normalized size = 1.19

$$\frac{a^2x^2 - ax + \left(a^2x^2 - ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}\,(2\,ax-1)}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - a*x + (a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^
2*x^2 + 1)*(2*a*x - 1))/(a*x^2 - x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 73, normalized size = 1.14

$$-a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\sqrt{-a^2x^2+1}}{x}-\frac{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}{x-\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/(x-1/a)*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)-(-a^2*x^2+1)^(1/2)/x-a*arctan
h(1/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{\sqrt{-a^2x^2+1}\,(ax-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^2), x)
```

mupad [B] time = 2.59, size = 81, normalized size = 1.27

$$\frac{a^2\sqrt{1-a^2x^2}}{\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}-\frac{\sqrt{1-a^2x^2}}{x}-a\,\mathrm{atanh}\left(\sqrt{1-a^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] (a^2*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (1 - a^2*x^2)^(1/2)/x - a*atanh((1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x)
```

3.156 $\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=90

$$-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2x^2}-\frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {857, 835, 807, 266, 63, 208}

$$-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2x^2}-\frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x
^2]/(x^2*(1 - a*x)) - (3*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{\int \frac{-3a^2-2a^3x}{x^3\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{\int \frac{4a^3+3a^4x}{x^2\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{1}{2}\left(3a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,\right.\\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2}\right.\\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.70

$$\frac{1}{2}\left(\frac{\left(-4a^2x^2+ax+1\right)\sqrt{1-a^2x^2}}{x^2(ax-1)} - 3a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] (((1 + a*x - 4*a^2*x^2)*Sqrt[1 - a^2*x^2])/(x^2*(-1 + a*x)) - 3*a^2*ArcTanh
[Sqrt[1 - a^2*x^2]])/2
```

IntegrateAlgebraic [A] time = 0.55, size = 76, normalized size = 0.84

$$\frac{\left(-4a^2x^2+ax+1\right)\sqrt{1-a^2x^2}}{2x^2(ax-1)} + 3a^2\tanh^{-1}\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] ((1 + a*x - 4*a^2*x^2)*Sqrt[1 - a^2*x^2])/(2*x^2*(-1 + a*x)) + 3*a^2*ArcTan
h[Sqrt[-a^2]*x - Sqrt[1 - a^2*x^2]]
```

fricas [A] time = 0.40, size = 97, normalized size = 1.08

$$\frac{2\,a^3x^3-2\,a^2x^2+3\left(a^3x^3-a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(4\,a^2x^2-ax-1\right)\sqrt{-a^2x^2+1}}{2\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*x^3 - 2*a^2*x^2 + 3*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (4*a^2*x^2 - a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*x^3 - x^2)
```

giac [B] time = 0.21, size = 213, normalized size = 2.37

$$-\frac{\left(a^3+\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{x}-\frac{20\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}-\frac{3\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\frac{4\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(a^3 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 20*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-a
^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 3/2*a^3*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(4*(sqrt(-a^2*x^2 +
1)*abs(a) + a)*a*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^2
))/a^2
```

maple [A] time = 0.01, size = 94, normalized size = 1.04

$$-\frac{3a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}\;a}{x-\frac{1}{a}}-\frac{\sqrt{-a^2x^2+1}\;a}{x}-\frac{\sqrt{-a^2x^2+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -a/(x-1/a)*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)-a*(-a^2*x^2+1)^(1/2)/x-1/2*(-
a^2*x^2+1)^(1/2)/x^2-3/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{-a^2x^2+1}\,(ax-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^3), x)
```

mupad [B] time = 2.61, size = 105, normalized size = 1.17

$$\frac{a^3\sqrt{1-a^2\,x^2}}{\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}-\frac{a\sqrt{1-a^2\,x^2}}{x}-\frac{\sqrt{1-a^2\,x^2}}{2\,x^2}+\frac{a^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\;1\mathrm{i}\right)\,3\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/2 - (1 - a^2*x^2)^(1/2)/(2*x^2) - (a*
(1 - a^2*x^2)^(1/2))/x + (a^3*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2
)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ax^4\sqrt{-a^2x^2+1} - x^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x)
```

3.157 $\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=229

$$-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}+\frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2}-\frac{5d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{64e^6}-\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{d^5(256d-$$

Rubi [A] time = 0.31, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$-\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{d^5(256d-315ex)\left(d^2-e^2x^2\right)^{3/2}}{2016e^6}-\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4}+\frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3}-\frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2}+\frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}-\frac{5d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{64e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (-5*d^7*x*Sqrt[d^2 - e^2*x^2])/(64*e^5) - (4*d^4*x^2*(d^2 - e^2*x^2)^(3/2))
/(21*e^4) + (5*d^3*x^3*(d^2 - e^2*x^2)^(3/2))/(24*e^3) - (5*d^2*x^4*(d^2 -
e^2*x^2)^(3/2))/(21*e^2) + (d*x^5*(d^2 - e^2*x^2)^(3/2))/(4*e) - (x^6*(d^2
- e^2*x^2)^(3/2))/9 - (d^5*(256*d - 315*e*x)*(d^2 - e^2*x^2)^(3/2))/(2016*e
^6) - (5*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(64*e^6)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx &= \int x^5(d-ex)^2\sqrt{d^2-e^2x^2}\,dx\\
&= -\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}-\frac{\int x^5\left(-15d^2e^2+18de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{9e^2}\\
&= \frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}+\frac{\int x^4\left(-90d^3e^3+120d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{72e^4}\\
&= -\frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2}+\frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}-\frac{\int x^3\left(-480d^4e^4+\cdots\right.}{}\\
&= \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3}-\frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2}+\frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e}-\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2}\\
&= -\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4}+\frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3}-\frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2}+\frac{dx^5\left(d^2-e^2x^2\right.}{4e}\\
&= -\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4}+\frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3}-\frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2}+\frac{dx^5\left(d^2-e^2x^2\right.}{4e}\\
&= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4}+\frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3}-\frac{5d^2x^4\left(d^2-e^2x^2\right)}{21e^2}\\
&= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4}+\frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3}-\frac{5d^2x^4\left(d^2-e^2x^2\right)}{21e^2}\\
&= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4}+\frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3}-\frac{5d^2x^4\left(d^2-e^2x^2\right)}{21e^2}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 135, normalized size = 0.59

$$
\frac{\sqrt{d^2-e^2x^2}\left(-512d^8+315d^7ex-256d^6e^2x^2+210d^5e^3x^3-192d^4e^4x^4+168d^3e^5x^5+512d^2e^6x^6-1008de^7x^7+448e^8x^8\right)-315d^9\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4032e^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-512*d^8 + 315*d^7*e*x - 256*d^6*e^2*x^2 + 210*d^5*e^
3*x^3 - 192*d^4*e^4*x^4 + 168*d^3*e^5*x^5 + 512*d^2*e^6*x^6 - 1008*d*e^7*x^
7 + 448*e^8*x^8) - 315*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4032*e^6)
```

IntegrateAlgebraic [A] time = 0.73, size = 158, normalized size = 0.69

$$\frac{\sqrt{d^2-e^2x^2}\left(-512d^8+315d^7ex-256d^6e^2x^2+210d^5e^3x^3-192d^4e^4x^4+168d^3e^5x^5+512d^2e^6x^6-1008de^7x^7+448e^8x^8\right)}{4032e^6}-\frac{5d^9\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{64e^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-512*d^8 + 315*d^7*e*x - 256*d^6*e^2*x^2 + 210*d^5*e^
3*x^3 - 192*d^4*e^4*x^4 + 168*d^3*e^5*x^5 + 512*d^2*e^6*x^6 - 1008*d*e^7*x^
7 + 448*e^8*x^8))/(4032*e^6) - (5*d^9*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt
[d^2 - e^2*x^2]])/(64*e^7)
```

fricas [A] time = 0.40, size = 138, normalized size = 0.60

$$\frac{630\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(448\,e^8x^8-1008\,de^7x^7+512\,d^2e^6x^6+168\,d^3e^5x^5-192\,d^4e^4x^4+210\,d^5e^3x^3-256\,d^6e^2x^2+315\,d^7ex-512\,d^8\right)\sqrt{-e^2x^2+d^2}}{4032\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4032*(630*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (448*e^8*x^8 -
1008*d*e^7*x^7 + 512*d^2*e^6*x^6 + 168*d^3*e^5*x^5 - 192*d^4*e^4*x^4 + 210*
d^5*e^3*x^3 - 256*d^6*e^2*x^2 + 315*d^7*e*x - 512*d^8)*sqrt(-e^2*x^2 + d^2)
)/e^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 375, normalized size = 1.64

$$\frac{5d^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{4\sqrt{e^2}\,e^5}-\frac{85d^9\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{64\sqrt{e^2}\,e^5}-\frac{85\sqrt{-e^2x^2+d^2}\,d^7x}{64e^5}+\frac{5\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^7x}{4e^5}-\frac{85\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x}{96e^5}+\frac{5\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^5x}{6e^5}-\frac{17\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x}{24e^5}+\frac{2\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^4}{3e^6}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}x^2}{9e^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx}{4e^5}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}d^4}{3\left(x+\frac{d}{e}\right)^2e^8}-\frac{29\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2}{63e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -1/9/e^4*x^2*(-e^2*x^2+d^2)^(7/2)-29/63*d^2/e^6*(-e^2*x^2+d^2)^(7/2)+1/4/e^
5*d*x*(-e^2*x^2+d^2)^(7/2)-17/24/e^5*d^3*x*(-e^2*x^2+d^2)^(5/2)-85/96/e^5*d
^5*x*(-e^2*x^2+d^2)^(3/2)-85/64*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^5-85/64/e^5*d^
9/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+2/3/e^6*d^4*(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+5/6/e^5*d^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/
2)*x+5/4/e^5*d^7*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+5/4/e^5*d^9/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3*d^4/e^8/(
x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)
```

maxima [C] time = 1.07, size = 299, normalized size = 1.31

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5}{4\left(e^7x+de^6\right)}-\frac{5i\,d^9\arcsin\left(\frac{ex}{d}+2\right)}{4\,e^6}-\frac{85\,d^9\arcsin\left(\frac{ex}{d}\right)}{64\,e^6}+\frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^7x}{4\,e^5}-\frac{85\sqrt{-e^2x^2+d^2}\,d^7x}{64\,e^5}+\frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^8}{2\,e^6}+\frac{35\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x}{96\,e^5}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6}{12\,e^6}-\frac{17\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x}{24\,e^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}x^2}{9\,e^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}{e^6}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx}{4\,e^5}-\frac{29\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2}{63\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(-e^2*x^2 + d^2)^(5/2)*d^5/(e^7*x + d*e^6) - 5/4*I*d^9*arcsin(e*x/d +
2)/e^6 - 85/64*d^9*arcsin(e*x/d)/e^6 + 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*
d^7*x/e^5 - 85/64*sqrt(-e^2*x^2 + d^2)*d^7*x/e^5 + 5/2*sqrt(e^2*x^2 + 4*d*e
*x + 3*d^2)*d^8/e^6 + 35/96*(-e^2*x^2 + d^2)^(3/2)*d^5*x/e^5 - 5/12*(-e^2*x
^2 + d^2)^(3/2)*d^6/e^6 - 17/24*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^5 - 1/9*(-e^
2*x^2 + d^2)^(7/2)*x^2/e^4 + (-e^2*x^2 + d^2)^(5/2)*d^4/e^6 + 1/4*(-e^2*x^2
 + d^2)^(7/2)*d*x/e^5 - 29/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

sympy [A] time = 17.48, size = 571, normalized size = 2.49

$$d^2\left(\begin{cases} -\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6} - \frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4} - \frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2} + \frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\ e \neq 0 \\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{5id^8\operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^7} + \frac{5id^7x}{128e^6\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5id^5x^3}{384e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{id^3x^5}{192e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{7idx^7}{48\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^9}{8d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{5d^8\operatorname{asin}\left(\frac{ex}{d}\right)}{128e^7} - \frac{5d^7x}{128e^6\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5d^5x^3}{384e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3x^5}{192e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{7dx^7}{48\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^9}{8d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{16d^8\sqrt{d^2-e^2x^2}}{315e^8} - \frac{8d^6x^2\sqrt{d^2-e^2x^2}}{315e^6} - \frac{2d^4x^4\sqrt{d^2-e^2x^2}}{105e^4} - \frac{d^2x^6\sqrt{d^2-e^2x^2}}{63e^2} + \frac{x^8\sqrt{d^2-e^2x^2}}{9} & \text{for}\ e \neq 0 \\ \frac{x^8\sqrt{d^2}}{8} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqr
t(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2)
 + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 2
*d*e*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sq
rt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2
)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt
(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e
**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sq
rt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) +
 d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2
*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*Piec
ewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 -
 e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d*
*2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, N
e(e, 0)), (x**8*sqrt(d**2)/8, True))
```

3.158 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=200

$$-\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}+\frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2}+\frac{13d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}+\frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^4(1\ldots$$

Rubi [A] time = 0.27, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$\frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^4(1024d-1365ex)\left(d^2-e^2x^2\right)^{3/2}}{6720e^5}+\frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3}-\frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2}+\frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e}-\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}+\frac{13d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (13*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (8*d^3*x^2*(d^2 - e^2*x^2)^(3/2)
)/(35*e^3) - (13*d^2*x^3*(d^2 - e^2*x^2)^(3/2))/(48*e^2) + (2*d*x^4*(d^2 -
e^2*x^2)^(3/2))/(7*e) - (x^5*(d^2 - e^2*x^2)^(3/2))/8 + (d^4*(1024*d - 1365
*e*x)*(d^2 - e^2*x^2)^(3/2))/(6720*e^5) + (13*d^8*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(128*e^5)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int x^4(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^4\left(-13d^2e^2+16de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{8e^2}$$

$$= \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^3\left(-64d^3e^3+91d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{56e^4}$$

$$= -\frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^2\left(-273d^4e^4\right.}{}$$

$$= \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)$$

$$= \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)}{7e}$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)}{7e}$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)}{7e}$$

Mathematica [A] time = 0.15, size = 124, normalized size = 0.62

$$\frac{1365d^8\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(2048d^7-1365d^6ex+1024d^5e^2x^2-910d^4e^3x^3+768d^3e^4x^4+1960d^2e^5x^5-3840de^6x^6+1680e^7x^7\right)}{13440e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2048*d^7 - 1365*d^6*e*x + 1024*d^5*e^2*x^2 - 910*d^4*
e^3*x^3 + 768*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 - 3840*d*e^6*x^6 + 1680*e^7*x^
7) + 1365*d^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(13440*e^5)
```

IntegrateAlgebraic [A] time = 0.65, size = 147, normalized size = 0.74

$$\frac{13d^8\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{128e^6}+\frac{\sqrt{d^2-e^2x^2}\left(2048d^7-1365d^6ex+1024d^5e^2x^2-910d^4e^3x^3+768d^3e^4x^4+1960d^2e^5x^5-3840de^6x^6+1680e^7x^7\right)}{13440e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2048*d^7 - 1365*d^6*e*x + 1024*d^5*e^2*x^2 - 910*d^4*
e^3*x^3 + 768*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 - 3840*d*e^6*x^6 + 1680*e^7*x^
7))/(13440*e^5) + (13*d^8*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x
^2]])/(128*e^6)
```

fricas [A] time = 0.40, size = 128, normalized size = 0.64

$$-\frac{2730\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(1680\,e^7x^7-3840\,de^6x^6+1960\,d^2e^5x^5+768\,d^3e^4x^4-910\,d^4e^3x^3+1024\,d^5e^2x^2-1365\,d^6ex+2048\,d^7\right)\sqrt{-e^2x^2+d^2}}{13440\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/13440*(2730*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (1680*e^7*x^
7 - 3840*d*e^6*x^6 + 1960*d^2*e^5*x^5 + 768*d^3*e^4*x^4 - 910*d^4*e^3*x^3 +
 1024*d^5*e^2*x^2 - 1365*d^6*e*x + 2048*d^7)*sqrt(-e^2*x^2 + d^2))/e^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 350, normalized size = 1.75

$$-\frac{7d^8\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}\,e^4}+\frac{125d^8\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{128\sqrt{e^2}\,e^4}+\frac{125\sqrt{-e^2x^2+d^2}\,d^6x}{128e^4}-\frac{7\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^6x}{8e^4}+\frac{125\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4x}{192e^4}-\frac{7\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^4x}{12e^4}+\frac{25\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}{48e^4}-\frac{7\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^3}{15e^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}x}{8e^4}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}d^3}{3\left(x+\frac{d}{e}\right)^2e^7}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{7e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -1/8/e^4*x*(-e^2*x^2+d^2)^(7/2)+25/48*d^2/e^4*x*(-e^2*x^2+d^2)^(5/2)+125/19
2/e^4*d^4*x*(-e^2*x^2+d^2)^(3/2)+125/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^4+125
/128/e^4*d^8/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+2/7*d/e
^5*(-e^2*x^2+d^2)^(7/2)-7/15/e^5*d^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-7/
12/e^4*d^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-7/8/e^4*d^6*(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(1/2)*x-7/8/e^4*d^8/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/
e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+1/3*d^3/e^7/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)
^2*e^2)^(7/2)
```

maxima [C] time = 1.04, size = 275, normalized size = 1.38

$$\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4}{4\left(e^6x+de^5\right)}+\frac{7i\,d^8\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^5}+\frac{125\,d^8\arcsin\left(\frac{ex}{d}\right)}{128\,e^5}-\frac{7\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^6x}{8\,e^4}+\frac{125\sqrt{-e^2x^2+d^2}\,d^6x}{128\,e^4}-\frac{7\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^7}{4\,e^5}-\frac{67\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4x}{192\,e^4}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5}{12\,e^5}+\frac{25\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2x}{48\,e^4}-\frac{4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{5\,e^5}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}x}{8\,e^4}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d}{7\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(-e^2*x^2 + d^2)^(5/2)*d^4/(e^6*x + d*e^5) + 7/8*I*d^8*arcsin(e*x/d + 2
)/e^5 + 125/128*d^8*arcsin(e*x/d)/e^5 - 7/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)
*d^6*x/e^4 + 125/128*sqrt(-e^2*x^2 + d^2)*d^6*x/e^4 - 7/4*sqrt(e^2*x^2 + 4*
d*e*x + 3*d^2)*d^7/e^5 - 67/192*(-e^2*x^2 + d^2)^(3/2)*d^4*x/e^4 + 5/12*(-e
^2*x^2 + d^2)^(3/2)*d^5/e^5 + 25/48*(-e^2*x^2 + d^2)^(5/2)*d^2*x/e^4 - 4/5*
(-e^2*x^2 + d^2)^(5/2)*d^3/e^5 - 1/8*(-e^2*x^2 + d^2)^(7/2)*x/e^4 + 2/7*(-e
^2*x^2 + d^2)^(7/2)*d/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(d^2-e^2 x^2\right)^{5/2}}{(d+e x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

sympy [C] time = 21.40, size = 690, normalized size = 3.45

$$d^{2} \left(\begin{cases} - \frac{i d^{6} \operatorname{acosh}{\left(\frac{e x}{d} \right)}}{16 e^{5}} + \frac{i d^{5} x}{16 e^{4} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} - \frac{i d^{3} x^{3}}{48 e^{2} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} - \frac{5 i d x^{5}}{24 \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + \frac{i e^{2} x^{7}}{6 d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\\frac{d^{6} \operatorname{asin}{\left(\frac{e x}{d} \right)}}{16 e^{5}} - \frac{d^{5} x}{16 e^{4} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} + \frac{d^{3} x^{3}}{48 e^{2} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} + \frac{5 d x^{5}}{24 \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - \frac{e^{2} x^{7}}{6 d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right) - 2 d e \left(\begin{cases} - \frac{8 d^{6} \sqrt{d^{2} - e^{2} x^{2}}}{105 e^{6}} - \frac{4 d^{4} x^{2} \sqrt{d^{2} - e^{2} x^{2}}}{105 e^{4}} - \frac{d^{2} x^{4} \sqrt{d^{2} - e^{2} x^{2}}}{35 e^{2}} + \frac{x^{6} \sqrt{d^{2} - e^{2} x^{2}}}{7} & \text{for}\: e \neq 0 \\\frac{x^{6} \sqrt{d^{2}}}{6} & \text{otherwise} \end{cases}\right) + e^{2} \left(\begin{cases} - \frac{5 i d^{8} \operatorname{acosh}{\left(\frac{e x}{d} \right)}}{128 e^{7}} + \frac{5 i d^{7} x}{128 e^{6} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} - \frac{5 i d^{5} x^{3}}{384 e^{4} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} - \frac{i d^{3} x^{5}}{192 e^{2} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} - \frac{7 i d x^{7}}{48 \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + \frac{i e^{2} x^{9}}{8 d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\\frac{5 d^{8} \operatorname{asin}{\left(\frac{e x}{d} \right)}}{128 e^{7}} - \frac{5 d^{7} x}{128 e^{6} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} + \frac{5 d^{5} x^{3}}{384 e^{4} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} + \frac{d^{3} x^{5}}{192 e^{2} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} + \frac{7 d x^{7}}{48 \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - \frac{e^{2} x^{9}}{8 d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) - 2*d*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(10
5*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d*
*2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6
*sqrt(d**2)/6, True)) + e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) +
 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*
sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2
)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 +
 e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7)
 - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqr
t(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*
d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d*
*2)), True))
```

3.159 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx$

Optimal. Leaf size=171

$$-\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2}-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}+\frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}-\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{d^3(88d-1\mathrm{C}}{}$$

Rubi [A] time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$-\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{d^3(88d-105ex)\left(d^2-e^2x^2\right)^{3/2}}{420e^4}-\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2}+\frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e}-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}-\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -(d^5*x*Sqrt[d^2 - e^2*x^2])/(8*e^3) - (11*d^2*x^2*(d^2 - e^2*x^2)^(3/2))/(
35*e^2) + (d*x^3*(d^2 - e^2*x^2)^(3/2))/(3*e) - (x^4*(d^2 - e^2*x^2)^(3/2))
/7 - (d^3*(88*d - 105*e*x)*(d^2 - e^2*x^2)^(3/2))/(420*e^4) - (d^7*ArcTan[(
e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int x^3(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^3\left(-11d^2e^2+14de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{7e^2}$$

$$= \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^2\left(-42d^3e^3+66d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{42e^4}$$

$$= -\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x\left(-132d^4e^4+\right.}{}$$

$$= -\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{d^3(88d-105ex)\left(\right.}{420e^4}$$

$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} -$$

$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} -$$

$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} -$$

Mathematica [A] time = 0.13, size = 113, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(-176d^6+105d^5ex-88d^4e^2x^2+70d^3e^3x^3+144d^2e^4x^4-280de^5x^5+120e^6x^6\right)-105d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{840e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-176*d^6 + 105*d^5*e*x - 88*d^4*e^2*x^2 + 70*d^3*e^3*
x^3 + 144*d^2*e^4*x^4 - 280*d*e^5*x^5 + 120*e^6*x^6) - 105*d^7*ArcTan[(e*x)
/Sqrt[d^2 - e^2*x^2]])/(840*e^4)
```

IntegrateAlgebraic [A] time = 0.65, size = 136, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\left(-176d^6+105d^5ex-88d^4e^2x^2+70d^3e^3x^3+144d^2e^4x^4-280de^5x^5+120e^6x^6\right)}{840e^4}-\frac{d^7\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-176*d^6 + 105*d^5*e*x - 88*d^4*e^2*x^2 + 70*d^3*e^3*
x^3 + 144*d^2*e^4*x^4 - 280*d*e^5*x^5 + 120*e^6*x^6))/(840*e^4) - (d^7*Sqrt
[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*e^5)
```

fricas [A] time = 0.41, size = 116, normalized size = 0.68

$$\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(120\,e^6x^6-280\,de^5x^5+144\,d^2e^4x^4+70\,d^3e^3x^3-88\,d^4e^2x^2+105\,d^5ex-176\,d^6\right)\sqrt{-e^2x^2+d^2}}{840\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (120*e^6*x^6 - 2
80*d*e^5*x^5 + 144*d^2*e^4*x^4 + 70*d^3*e^3*x^3 - 88*d^4*e^2*x^2 + 105*d^5*
e*x - 176*d^6)*sqrt(-e^2*x^2 + d^2))/e^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 327, normalized size = 1.91

$$\frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{2\sqrt{e^2}\,e^3}-\frac{5d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^3}-\frac{5\sqrt{-e^2x^2+d^2}\,d^5x}{8e^3}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^5x}{2e^3}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{12e^3}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^3x}{3e^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{3e^3}+\frac{4\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^2}{15e^4}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}d^2}{3\left(x+\frac{d}{e}\right)^2e^6}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{7e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -1/7/e^4*(-e^2*x^2+d^2)^(7/2)-1/3*d/e^3*x*(-e^2*x^2+d^2)^(5/2)-5/12/e^3*d^3
*x*(-e^2*x^2+d^2)^(3/2)-5/8*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^3-5/8/e^3*d^7/(e^2
)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+4/15/e^4*d^2*(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(5/2)+1/3/e^3*d^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+
1/2/e^3*d^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+1/2/e^3*d^7/(e^2)^(1/2)*a
rctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3*d^2/e^6/(x+d/e
)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)
```

maxima [C] time = 1.04, size = 251, normalized size = 1.47

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3}{4\left(e^5x+de^4\right)}-\frac{i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{2\,e^4}-\frac{5\,d^7\arcsin\left(\frac{ex}{d}\right)}{8\,e^4}+\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^5x}{2\,e^3}-\frac{5\sqrt{-e^2x^2+d^2}\,d^5x}{8\,e^3}+\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^6}{e^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{3\,e^3}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4}{12\,e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{3\,e^3}+\frac{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{5\,e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{7\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^5*x + d*e^4) - 1/2*I*d^7*arcsin(e*x/d +
2)/e^4 - 5/8*d^7*arcsin(e*x/d)/e^4 + 1/2*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^
```

```
5*x/e^3 - 5/8*sqrt(-e^2*x^2 + d^2)*d^5*x/e^3 + sqrt(e^2*x^2 + 4*d*e*x + 3*d
^2)*d^6/e^4 + 1/3*(-e^2*x^2 + d^2)^(3/2)*d^3*x/e^3 - 5/12*(-e^2*x^2 + d^2)^
(3/2)*d^4/e^4 - 1/3*(-e^2*x^2 + d^2)^(5/2)*d*x/e^3 + 3/5*(-e^2*x^2 + d^2)^(
5/2)*d^2/e^4 - 1/7*(-e^2*x^2 + d^2)^(7/2)/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left(d^2 - e^2 x^2\right)^{5/2}}{(d + e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

sympy [A] time = 12.03, size = 450, normalized size = 2.63

$$d^2\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{i d^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{i d^5 x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{i d^3 x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5 i d x^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{i e^2 x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5 x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6} - \frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4} - \frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2} + \frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\ e \neq 0 \\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) - 2*d*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1
 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x*
*7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*
x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*
e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e
**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*Piecewise((-8*d**6*s
qrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*
e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*
x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

3.160 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=142

$$\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{3d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}+\frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}$$

Rubi [A] time = 0.18, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$\frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{3d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (3*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (2*d*x^2*(d^2 - e^2*x^2)^(3/2))/(5
*e) - (x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^2*(32*d - 45*e*x)*(d^2 - e^2*x^2)^
(3/2))/(120*e^3) + (3*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int x^2(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^2\left(-9d^2e^2+12de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{6e^2}$$

$$= \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x\left(-24d^3e^3+45d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{30e^4}$$

$$= \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3} + \frac{\left(3d^4\right)\int \sqrt{}}{}$$

$$= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{d^2(32d-45ex)\left(d^2-e^2\right.}{120e^3}$$

$$= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{d^2(32d-45ex)\left(d^2-e\right.}{120e^3}$$

$$= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{d^2(32d-45ex)\left(d^2-e\right.}{120e^3}$$

Mathematica [A] time = 0.11, size = 102, normalized size = 0.72

$$\frac{45d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(64d^5-45d^4ex+32d^3e^2x^2+50d^2e^3x^3-96de^4x^4+40e^5x^5\right)}{240e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(64*d^5 - 45*d^4*e*x + 32*d^3*e^2*x^2 + 50*d^2*e^3*x^3
 - 96*d*e^4*x^4 + 40*e^5*x^5) + 45*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(
240*e^3)
```

IntegrateAlgebraic [A] time = 0.57, size = 125, normalized size = 0.88

$$\frac{3d^6\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{16e^4}+\frac{\sqrt{d^2-e^2x^2}\left(64d^5-45d^4ex+32d^3e^2x^2+50d^2e^3x^3-96de^4x^4+40e^5x^5\right)}{240e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(64*d^5 - 45*d^4*e*x + 32*d^3*e^2*x^2 + 50*d^2*e^3*x^3
 - 96*d*e^4*x^4 + 40*e^5*x^5))/(240*e^3) + (3*d^6*Sqrt[-e^2]*Log[-(Sqrt[-e^
2]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^4)
```

fricas [A] time = 0.41, size = 106, normalized size = 0.75

$$-\frac{90\,d^{6}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-\left(40\,e^{5}x^{5}-96\,de^{4}x^{4}+50\,d^{2}e^{3}x^{3}+32\,d^{3}e^{2}x^{2}-45\,d^{4}ex+64\,d^{5}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{240\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(90*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (40*e^5*x^5 - 96
*d*e^4*x^4 + 50*d^2*e^3*x^3 + 32*d^3*e^2*x^2 - 45*d^4*e*x + 64*d^5)*sqrt(-e
^2*x^2 + d^2))/e^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.01, size = 303, normalized size = 2.13

$$-\frac{d^{6}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,e^{2}}+\frac{5d^{6}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{16\sqrt{e^{2}}\,e^{2}}+\frac{5\sqrt{-e^{2}x^{2}+d^{2}}\,d^{4}x}{16e^{2}}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{4}x}{8e^{2}}+\frac{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{2}x}{24e^{2}}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}d^{2}x}{12e^{2}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}x}{6e^{2}}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}d}{15e^{3}}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{7}{2}}d}{3\left(x+\frac{d}{e}\right)^{2}e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] 1/6*x*(-e^2*x^2+d^2)^(5/2)/e^2+5/24/e^2*d^2*x*(-e^2*x^2+d^2)^(3/2)+5/16*d^4
*x*(-e^2*x^2+d^2)^(1/2)/e^2+5/16/e^2*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e
^2*x^2+d^2)^(1/2)*x)-1/15*d/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/12*d^
2/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-1/8*d^4/e^2*(2*(x+d/e)*d*e-(x+d
/e)^2*e^2)^(1/2)*x-1/8*d^6/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(1/2)*x)+1/3*d/e^5/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(7/2)
```

maxima [C] time = 1.03, size = 230, normalized size = 1.62

$$\frac{i\,d^{6}\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^{3}}+\frac{5\,d^{6}\arcsin\left(\frac{ex}{d}\right)}{16\,e^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}}{4\left(e^{4}x+de^{3}\right)}-\frac{\sqrt{e^{2}x^{2}+4\,dex+3\,d^{2}}\,d^{4}x}{8\,e^{2}}+\frac{5\sqrt{-e^{2}x^{2}+d^{2}}\,d^{4}x}{16\,e^{2}}-\frac{\sqrt{e^{2}x^{2}+4\,dex+3\,d^{2}}\,d^{5}}{4\,e^{3}}-\frac{7\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{2}x}{24\,e^{2}}+\frac{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{3}}{12\,e^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}x}{6\,e^{2}}-\frac{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d}{5\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*I*d^6*arcsin(e*x/d + 2)/e^3 + 5/16*d^6*arcsin(e*x/d)/e^3 + 1/4*(-e^2*x^
2 + d^2)^(5/2)*d^2/(e^4*x + d*e^3) - 1/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^
4*x/e^2 + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e^2 - 1/4*sqrt(e^2*x^2 + 4*d*e*x
+ 3*d^2)*d^5/e^3 - 7/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e^2 + 5/12*(-e^2*x^2 +
 d^2)^(3/2)*d^3/e^3 + 1/6*(-e^2*x^2 + d^2)^(5/2)*x/e^2 - 2/5*(-e^2*x^2 + d^
2)^(5/2)*d/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(d^2 - e^2 x^2\right)^{5/2}}{\left(d + e x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

sympy [C] time = 14.45, size = 541, normalized size = 3.81

$$d^2 \left(\begin{cases} -\frac{i d^4 \operatorname{acosh}{\left(\frac{e x}{d} \right)}}{8 e^3} + \frac{i d^3 x}{8 e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{3 i d x^3}{8 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{i e^2 x^5}{4 d \sqrt{-1 + \frac{e^2 x^2}{d^2}}} & \text{for}\: \left|{\frac{e^2 x^2}{d^2}}\right| > 1 \\ \frac{d^4 \operatorname{asin}{\left(\frac{e x}{d} \right)}}{8 e^3} - \frac{d^3 x}{8 e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{3 d x^3}{8 \sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^5}{4 d \sqrt{1 - \frac{e^2 x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - 2 d e \left(\begin{cases} - \frac{2 d^4 \sqrt{d^2 - e^2 x^2}}{15 e^4} - \frac{d^2 x^2 \sqrt{d^2 - e^2 x^2}}{15 e^2} + \frac{x^4 \sqrt{d^2 - e^2 x^2}}{5} & \text{for}\: e \neq 0 \\ \frac{x^4 \sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right) + e^2 \left(\begin{cases} - \frac{i d^6 \operatorname{acosh}{\left(\frac{e x}{d} \right)}}{16 e^5} + \frac{i d^5 x}{16 e^4 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{i d^3 x^3}{48 e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{5 i d x^5}{24 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{i e^2 x^7}{6 d \sqrt{-1 + \frac{e^2 x^2}{d^2}}} & \text{for}\: \left|{\frac{e^2 x^2}{d^2}}\right| > 1 \\ \frac{d^6 \operatorname{asin}{\left(\frac{e x}{d} \right)}}{16 e^5} - \frac{d^5 x}{16 e^4 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^3 x^3}{48 e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{5 d x^5}{24 \sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^7}{6 d \sqrt{1 - \frac{e^2 x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - 2*d*
e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2
 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*s
qrt(d**2)/4, True)) + e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d*
*5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e
**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(
6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)
/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2
*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*
x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.161 $\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx$

Optimal. Leaf size=136

$$-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}-\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}$$

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {793, 665, 195, 217, 203}

$$-\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -(d^3*x*Sqrt[d^2 - e^2*x^2])/(4*e) - (d*x*(d^2 - e^2*x^2)^(3/2))/(6*e) - (2
*(d^2 - e^2*x^2)^(5/2))/(15*e^2) - (d^2 - e^2*x^2)^(7/2)/(3*e^2*(d + e*x)^2
) - (d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rubi steps

$$\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = -\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2} - \frac{2\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx}{3e}$$
$$= -\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2} - \frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2} - \frac{(2d)\int \left(d^2-e^2x^2\right)^{3/2}\,dx}{3e}$$
$$= -\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e} - \frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2} - \frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2} - \frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{2e}$$
$$= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e} - \frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e} - \frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2} - \frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2} - \frac{d^5\int \frac{1}{\sqrt{d^2-e^2}}}{4e}$$
$$= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e} - \frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e} - \frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2} - \frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2} - \frac{d^5\,\mathrm{Subst}\Big(}{}$$
$$= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e} - \frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e} - \frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2} - \frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2} - \frac{d^5\tan^{-1}\Big(}{4e^2}$$

Mathematica [A] time = 0.08, size = 91, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2}\left(-28d^4+15d^3ex+16d^2e^2x^2-30de^3x^3+12e^4x^4\right)-15d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{60e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-28*d^4 + 15*d^3*e*x + 16*d^2*e^2*x^2 - 30*d*e^3*x^3
+ 12*e^4*x^4) - 15*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(60*e^2)
```

IntegrateAlgebraic [A] time = 0.46, size = 114, normalized size = 0.84

$$\frac{\sqrt{d^2-e^2x^2}\left(-28d^4+15d^3ex+16d^2e^2x^2-30de^3x^3+12e^4x^4\right)}{60e^2} - \frac{d^5\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{4e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-28*d^4 + 15*d^3*e*x + 16*d^2*e^2*x^2 - 30*d*e^3*x^3
+ 12*e^4*x^4))/(60*e^2) - (d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 -
e^2*x^2]])/(4*e^3)
```

fricas [A] time = 0.39, size = 94, normalized size = 0.69

$$\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(12\,e^4x^4-30\,de^3x^3+16\,d^2e^2x^2+15\,d^3ex-28\,d^4\right)\sqrt{-e^2x^2+d^2}}{60\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (12*e^4*x^4 - 30*d
*e^3*x^3 + 16*d^2*e^2*x^2 + 15*d^3*e*x - 28*d^4)*sqrt(-e^2*x^2 + d^2))/e^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 198, normalized size = 1.46

$$-\frac{d^5 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{4\sqrt{e^2}\, e}-\frac{\sqrt{2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2}\, d^3 x}{4e}-\frac{\left(2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} dx}{6e}-\frac{2\left(2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{5}{2}}}{15e^2}-\frac{\left(2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{7}{2}}}{3\left(x+\frac{d}{e}\right)^2 e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -2/15/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/6/e*d*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(3/2)*x-1/4/e*d^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-1/4/e*d^5/
(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3/e
^4/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)
```

maxima [C] time = 1.00, size = 167, normalized size = 1.23

$$\frac{i\, d^5 \arcsin\left(\frac{ex}{d}+2\right)}{4\, e^2}-\frac{\sqrt{e^2x^2+4\, dex+3\, d^2}\, d^3 x}{4\, e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d}{4\left(e^3x+de^2\right)}-\frac{\sqrt{e^2x^2+4\, dex+3\, d^2}\, d^4}{2\, e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}} dx}{4\, e}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^2}{12\, e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5\, e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*d^5*arcsin(e*x/d + 2)/e^2 - 1/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
/e - 1/4*(-e^2*x^2 + d^2)^(5/2)*d/(e^3*x + d*e^2) - 1/2*sqrt(e^2*x^2 + 4*d*
e*x + 3*d^2)*d^4/e^2 + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e - 5/12*(-e^2*x^2 +
d^2)^(3/2)*d^2/e^2 + 1/5*(-e^2*x^2 + d^2)^(5/2)/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(d^2-e^2\, x^2\right)^{5/2}}{\left(d+e\, x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

sympy [A] time = 8.57, size = 321, normalized size = 2.36

$$d^2\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for}\: e^2 = 0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\: \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\: e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) - 2*d*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**
3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/
d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
+ 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2
/d**2)), True)) + e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4)
- d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/
5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

3.162 $\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=108

$$\frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {655, 671, 641, 195, 217, 203}

$$\frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (5*d^2*x*Sqrt[d^2 - e^2*x^2])/8 + (5*d*(d^2 - e^2*x^2)^(3/2))/(12*e) + ((d
- e*x)*(d^2 - e^2*x^2)^(3/2))/(4*e) + (5*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(8*e)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
```

```
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx &= \int (d-ex)^2\sqrt{d^2-e^2x^2}\,dx\\
&= \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{4}(5d)\int (d-ex)\sqrt{d^2-e^2x^2}\,dx\\
&= \frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{4}\left(5d^2\right)\int \sqrt{d^2-e^2x^2}\,dx\\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{8}\left(5d^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{8}\left(5d^4\right)\text{Subst}\left(\int \frac{}{1+}\right.\\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.74

$$\frac{15d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\sqrt{d^2-e^2x^2}\left(16d^3+9d^2ex-16de^2x^2+6e^3x^3\right)}{24e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^3 + 9*d^2*e*x - 16*d*e^2*x^2 + 6*e^3*x^3) + 15*d
^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e)
```

IntegrateAlgebraic [A] time = 0.44, size = 103, normalized size = 0.95

$$\frac{5d^4\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e^2}+\frac{\sqrt{d^2-e^2x^2}\left(16d^3+9d^2ex-16de^2x^2+6e^3x^3\right)}{24e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^3 + 9*d^2*e*x - 16*d*e^2*x^2 + 6*e^3*x^3))/(24*e
) + (5*d^4*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*e^2)
```

fricas [A] time = 0.40, size = 84, normalized size = 0.78

$$-\frac{30\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(6\,e^3x^3-16\,de^2x^2+9\,d^2ex+16\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(30*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (6*e^3*x^3 - 16*d
*e^2*x^2 + 9*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/e
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.01, size = 194, normalized size = 1.80

$$\frac{5d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}+\frac{5\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2x}{8}+\frac{5\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}x}{12}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}}{3de}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{3\left(x+\frac{d}{e}\right)^2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] 1/3/e^3/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+1/3/e/d*(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(5/2)+5/12*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+5/8*d^2
*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+5/8*d^4/(e^2)^(1/2)*arctan((e^2)^(1/
2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [C] time = 0.99, size = 119, normalized size = 1.10

$$-\frac{5i\,d^4\arcsin\left(\frac{ex}{d}+2\right)}{8\,e}+\frac{5}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^2x+\frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}\,d^3}{4\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{4\left(e^2x+de\right)}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{12\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -5/8*I*d^4*arcsin(e*x/d + 2)/e + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^2*x
+ 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3/e + 1/4*(-e^2*x^2 + d^2)^(5/2)/(e
^2*x + d*e) + 5/12*(-e^2*x^2 + d^2)^(3/2)*d/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^2, x)
```

sympy [C] time = 9.35, size = 350, normalized size = 3.24

$$d^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for } e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - 2*d*
e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(
3*e**2), True)) + e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/
(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2
)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
 (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*
d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**
2)), True))
```

3.163 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx$

Optimal. Leaf size=96

$$d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}+d^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.16, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1809, 815, 844, 217, 203, 266, 63, 208}

$$d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}+d^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

```
[Out] d*(d - e*x)*Sqrt[d^2 - e^2*x^2] - (d^2 - e^2*x^2)^(3/2)/3 - d^3*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
```

```
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x}\,dx\\
&= -\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}-\frac{\int \frac{\left(-3d^2e^2+6de^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{3e^2}\\
&= d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}+\frac{\int \frac{6d^4e^4-6d^3e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{6e^4}\\
&= d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}+d^4\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-\left(d^3e\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}
\\
&= d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{2}d^4\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)-\left(d^3e\right)
\\
&= d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}-d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d^4\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,d\right.}{e^2}\\
&= d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}-d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2}}{d}\right.
\end{aligned}$$

Mathematica [A] time = 0.10, size = 96, normalized size = 1.00

$$d^3\log(x)+\sqrt{d^2-e^2x^2}\left(\frac{2d^2}{3}-dex+\frac{e^2x^2}{3}\right)-d^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+d^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((2*d^2)/3 - d*e*x + (e^2*x^2)/3) - d^3*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]] + d^3*Log[x] - d^3*Log[d + Sqrt[d^2 - e^2*x^2]]
```

IntegrateAlgebraic [A]   time = 0.48, size = 128, normalized size = 1.33

$$\frac{1}{3}\sqrt{d^2-e^2x^2}\left(2d^2-3dex+e^2x^2\right)-\frac{d^3\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e}+2d^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 - 3*d*e*x + e^2*x^2))/3 + 2*d^3*ArcTanh[(Sqrt[-
e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (d^3*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) +
Sqrt[d^2 - e^2*x^2]])/e
```

fricas [A]   time = 0.41, size = 95, normalized size = 0.99

$$2\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\frac{1}{3}\left(e^2x^2-3\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 2*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^3*log(-(d - sqrt(-e^2*x
^2 + d^2))/x) + 1/3*(e^2*x^2 - 3*d*e*x + 2*d^2)*sqrt(-e^2*x^2 + d^2)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B]   time = 0.01, size = 290, normalized size = 3.02

$$-\frac{d^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}-\frac{d^3e\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}}-\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,dex+\sqrt{-e^2x^2+d^2}\,d^2-\frac{2\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}ex}{3d}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5d^2}-\frac{8\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}}{15d^2}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{3\left(x+\frac{d}{e}\right)^2d^2e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x)
```

```
[Out] 1/5/d^2*(-e^2*x^2+d^2)^(5/2)+1/3*(-e^2*x^2+d^2)^(3/2)+d^2*(-e^2*x^2+d^2)^(1
/2)-d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-8/15/d
^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-2/3/d*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2
)^(3/2)*x-d*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-d^3*e/(e^2)^(1/2)*arcta
n((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3/d^2/e^2/(x+d/e)^2*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)
```

maxima [A]   time = 0.99, size = 103, normalized size = 1.07

$$-d^3\arcsin\left(\frac{ex}{d}\right)-d^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)-\sqrt{-e^2x^2+d^2}\,dex+\sqrt{-e^2x^2+d^2}\,d^2-\frac{1}{3}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -d^3*arcsin(e*x/d) - d^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)
) - sqrt(-e^2*x^2 + d^2)*d*e*x + sqrt(-e^2*x^2 + d^2)*d^2 - 1/3*(-e^2*x^2 +
 d^2)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2), x)
```

sympy [C] time = 14.83, size = 267, normalized size = 2.78

$$d^2\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\,\mathrm{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\,\mathrm{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{id^2\,\mathrm{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2\,\mathrm{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2 = 0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) - 2*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*s
qrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d*
*2)/2, True)) + e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 -
e**2*x**2)**(3/2)/(3*e**2), True))
```

3.164 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2} \, dx$

Optimal. Leaf size=105

$$-\frac{(d^2-e^2x^2)^{3/2}}{x}-\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1807, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{(d^2-e^2x^2)^{3/2}}{x}-\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

```
[Out] -(e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2])/2 - (d^2 - e^2*x^2)^(3/2)/x - (d^2*e*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + 2*d^2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d
]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^2}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{x} - \frac{\int \frac{\left(2d^3e+d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{d^2}$$

$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{x} + \frac{\int \frac{-4d^5e^3-d^4e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^2e^2}$$

$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{x} - \left(2d^3e\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}\left(d^2e^2\right)\int \frac{}{\sqrt{}}$$

$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{x} - \left(d^3e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right) - \frac{1}{2}$$

$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{x} - \frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\left(2d^3\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{x} - \frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 2d^2e\tanh^{-1}\left(\right.$$

Mathematica [A] time = 0.13, size = 100, normalized size = 0.95

$$\left(-\frac{d^2}{x}-2de+\frac{e^2x}{2}\right)\sqrt{d^2-e^2x^2}+2d^2e\log\left(\sqrt{d^2-e^2x^2}+d\right)-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2d^2e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

```
[Out] (-2*d*e - d^2/x + (e^2*x)/2)*Sqrt[d^2 - e^2*x^2] - (d^2*e*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/2 - 2*d^2*e*Log[x] + 2*d^2*e*Log[d + Sqrt[d^2 - e^2*x^2]]
```

IntegrateAlgebraic [A] time = 0.60, size = 131, normalized size = 1.25

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^2-4dex+e^2x^2\right)}{2x}-\frac{1}{2}d^2\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)-4d^2e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^2 - 4*d*e*x + e^2*x^2))/(2*x) - 4*d^2*e*ArcTanh[
(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2
]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

fricas [A] time = 0.40, size = 111, normalized size = 1.06

$$\frac{2\,d^2ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-4\,d^2ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-4\,d^2ex+\left(e^2x^2-4\,dex-2\,d^2\right)\sqrt{-e^2x^2+d^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 4*d^2*e*x*log(-(
d - sqrt(-e^2*x^2 + d^2))/x) - 4*d^2*e*x + (e^2*x^2 - 4*d*e*x - 2*d^2)*sqrt
(-e^2*x^2 + d^2))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.01, size = 425, normalized size = 4.05

$$\frac{2d^3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{11d^2e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{15d^2e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}-\frac{15\sqrt{-e^2x^2+d^2}\,e^2x}{8}+\frac{11\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^2x}{8}-2\sqrt{-e^2x^2+d^2}\,de-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2x}{4d^2}+\frac{11\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^2x}{12d^2}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{3d}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2x}{d^4}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{5d^3}+\frac{11\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e}{15d^3}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{3\left(x+\frac{d}{e}\right)^2d^3e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x)
```

```
[Out] -1/d^4/x*(-e^2*x^2+d^2)^(7/2)-1/d^4*e^2*x*(-e^2*x^2+d^2)^(5/2)-5/4/d^2*e^2*
x*(-e^2*x^2+d^2)^(3/2)-15/8*e^2*x*(-e^2*x^2+d^2)^(1/2)-15/8*d^2*e^2/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-2/5/d^3*e*(-e^2*x^2+d^2)^(5
/2)-2/3/d*e*(-e^2*x^2+d^2)^(3/2)-2*d*e*(-e^2*x^2+d^2)^(1/2)+2*d^3*e/(d^2)^(
1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+11/15/d^3*e*(2*(x+d/e
)*d*e-(x+d/e)^2*e^2)^(5/2)+11/12/d^2*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2
```

```
)*x+11/8*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+11/8*d^2*e^2/(e^2)^(1/2)
*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+1/3/d^3/e/(x+d/e
)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)
```

maxima [A] time = 1.12, size = 112, normalized size = 1.07

$$-\frac{1}{2}\,d^2e\arcsin\left(\frac{ex}{d}\right)+2\,d^2e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)+\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\,e^2x-2\,\sqrt{-e^2x^2+d^2}\,de-\frac{\sqrt{-e^2x^2+d^2}\,d^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*d^2*e*arcsin(e*x/d) + 2*d^2*e*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2
)*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^2*x - 2*sqrt(-e^2*x^2 + d^2)*d*e -
 sqrt(-e^2*x^2 + d^2)*d^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^2\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2), x)
```

sympy [C] time = 9.85, size = 347, normalized size = 3.30

$$d^2\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) - 2*d*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d
/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d
**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**
2/(e**2*x**2) + 1), True)) + e**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I
*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d
**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**
2*x**2/d**2)/2, True))
```

3.165 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx$

Optimal. Leaf size=110

$$\frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d^2-e^2x^2)^{3/2}}{2x^2}+2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1807, 813, 844, 217, 203, 266, 63, 208}

$$\frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d^2-e^2x^2)^{3/2}}{2x^2}+2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] (e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - (d^2 - e^2*x^2)^(3/2)/(2*x^2) +
 2*d*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (d*e^2*ArcTanh[Sqrt[d^2 - e^2*
x^2]/d])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
```

```
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^3}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{\int \frac{\left(4d^3e-d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{2d^2}$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{\int \frac{2d^4e^2+8d^3e^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{4d^2}$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{1}{2}\left(d^2e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \left(2de^3\right)\int \frac{}{\sqrt{d^2}}$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{1}{4}\left(d^2e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + (2$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}d^2\,\text{Subst}\left(\int \frac{}{\frac{d^2}{e^2}}\right.$$

$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2}}{}\right.$$

Mathematica [A] time = 0.14, size = 102, normalized size = 0.93

$$\left(-\frac{d^2}{2x^2}+\frac{2de}{x}+e^2\right)\sqrt{d^2-e^2x^2} - \frac{1}{2}de^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{2}de^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] (e^2 - d^2/(2*x^2) + (2*d*e)/x)*Sqrt[d^2 - e^2*x^2] + 2*d*e^2*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]] + (d*e^2*Log[x])/2 - (d*e^2*Log[d + Sqrt[d^2 - e^2*x^2
]])/2
```

IntegrateAlgebraic [A] time = 0.50, size = 128, normalized size = 1.16

$$\frac{\sqrt{d^2-e^2x^2}\left(-d^2+4dex+2e^2x^2\right)}{2x^2}+2d\sqrt{-e^2}\,e\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+de^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 + 4*d*e*x + 2*e^2*x^2))/(2*x^2) + d*e^2*ArcTanh[
(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + 2*d*e*Sqrt[-e^2]*Log[-(Sqrt[-e^
2]*x) + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.41, size = 119, normalized size = 1.08

$$-\frac{8\,de^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-de^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-2\,de^2x^2-\left(2\,e^2x^2+4\,dex-d^2\right)\sqrt{-e^2x^2+d^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(8*d*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - d*e^2*x^2*log
(-(d - sqrt(-e^2*x^2 + d^2))/x) - 2*d*e^2*x^2 - (2*e^2*x^2 + 4*d*e*x - d^2)
*sqrt(-e^2*x^2 + d^2))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.01, size = 456, normalized size = 4.15

$$-\frac{d^2e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}-\frac{7d\,e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{4\sqrt{e^2}}+\frac{15d\,e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}}+\frac{15\sqrt{-e^2x^2+d^2}\,e^3x}{4d}-\frac{7\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^3x}{4d}+\frac{\sqrt{-e^2x^2+d^2}\,e^2}{2}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3x}{2d^3}-\frac{7\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^3x}{6d^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{6d^2}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3x}{d^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}{10d^4}-\frac{14\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e^2}{15d^4}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{3\left(x+\frac{d}{e}\right)^2d^4}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{d^5x}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{2d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x)
```

```
[Out] 2/d^5*e/x*(-e^2*x^2+d^2)^(7/2)+2/d^5*e^3*x*(-e^2*x^2+d^2)^(5/2)+5/2/d^3*e^3
*x*(-e^2*x^2+d^2)^(3/2)+15/4/d*e^3*x*(-e^2*x^2+d^2)^(1/2)+15/4*d*e^3/(e^2)^
(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2/d^4/x^2*(-e^2*x^2+d^2)
^(7/2)+1/10/d^4*e^2*(-e^2*x^2+d^2)^(5/2)+1/6/d^2*e^2*(-e^2*x^2+d^2)^(3/2)+1
/2*e^2*(-e^2*x^2+d^2)^(1/2)-1/2*d^2*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)
*(-e^2*x^2+d^2)^(1/2))/x)-14/15/d^4*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)
-7/6/d^3*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-7/4/d*e^3*(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(1/2)*x-7/4*d*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3/d^4/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2
)^(7/2)
```

maxima [A] time = 0.99, size = 111, normalized size = 1.01

$$2\,de^{2}\arcsin\left(\frac{ex}{d}\right)-\frac{1}{2}\,de^{2}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)+\frac{1}{2}\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{2}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,de}{x}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 2*d*e^2*arcsin(e*x/d) - 1/2*d*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)
*d/abs(x)) + 1/2*sqrt(-e^2*x^2 + d^2)*e^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e/x -
1/2*(-e^2*x^2 + d^2)^(3/2)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^3\,\left(d+e\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2), x)
```

sympy [C] time = 10.17, size = 347, normalized size = 3.15

$$d^{2}\left(\begin{cases} - \frac{d^{2}}{2 e x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e}{2 x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{2} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{2 d} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{2 x} - \frac{i e^{2} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{2 d} & \text{otherwise} \end{cases}\right) - 2 d e \left(\begin{cases} \frac{i d}{x \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + i e \operatorname{acosh}{\left(\frac{e x}{d} \right)} - \frac{i e^{2} x}{d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\- \frac{d}{x \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - e \operatorname{asin}{\left(\frac{e x}{d} \right)} + \frac{e^{2} x}{d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right) + e^{2} \left(\begin{cases} \frac{d^{2}}{e x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} - d \operatorname{acosh}{\left(\frac{d}{e x} \right)} - \frac{e x}{\sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i d^{2}}{e x \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} + i d \operatorname{asin}{\left(\frac{d}{e x} \right)} + \frac{i e x}{\sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), T
rue)) - 2*d*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/
d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/
(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2
/d**2)), True)) + e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d
*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1
), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/s
qrt(-d**2/(e**2*x**2) + 1), True))
```

3.166 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^2}\,dx$

Optimal. Leaf size=102

$$\frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+e^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1807, 811, 844, 217, 203, 266, 63, 208}

$$\frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+e^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] (e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/x^2 - (d^2 - e^2*x^2)^(3/2)/(3*x^3) - e^3
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
```

```
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^4}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} - \frac{\int \frac{\left(6d^3e-3d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{3d^2} \\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} + \frac{\int \frac{12d^5e^3-12d^4e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{12d^4} \\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} + \left(de^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^4\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} + \frac{1}{2}\left(de^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right) - e^4\,\text{Su} \\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} - e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - (de)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right. \\
&= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} - e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right.
\end{aligned}$$

Mathematica [A] time = 0.18, size = 96, normalized size = 0.94

$$-\frac{\sqrt{d^2-e^2x^2}\left(d^2-3dex+2e^2x^2\right)}{3x^3} - e^3\log\left(\sqrt{d^2-e^2x^2}+d\right) + e^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right) + e^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] -1/3*(Sqrt[d^2 - e^2*x^2]*(d^2 - 3*d*e*x + 2*e^2*x^2))/x^3 - e^3*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]] + e^3*Log[x] - e^3*Log[d + Sqrt[d^2 - e^2*x^2]]
```

IntegrateAlgebraic [A] time = 0.54, size = 129, normalized size = 1.26

$$-\sqrt{-e^2}\,e^2\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)+\frac{\left(-d^2+3dex-2e^2x^2\right)\sqrt{d^2-e^2x^2}}{3x^3}+2e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] ((-d^2 + 3*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(3*x^3) + 2*e^3*ArcTanh[
(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - e^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]
*x) + Sqrt[d^2 - e^2*x^2]]
```

fricas [A] time = 0.42, size = 106, normalized size = 1.04

$$\frac{6\,e^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(2\,e^2x^2-3\,dex+d^2\right)\sqrt{-e^2x^2+d^2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*e^3*x^3*log(-(
d - sqrt(-e^2*x^2 + d^2))/x) - (2*e^2*x^2 - 3*d*e*x + d^2)*sqrt(-e^2*x^2 +
d^2))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.71index.cc index_m i_lex_is_greater Error: Bad Ar
gument Value
```

maple [B] time = 0.02, size = 479, normalized size = 4.70

$$-\frac{d\,e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{17e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{25e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{17\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^4x}{8d^2}-\frac{25\sqrt{-e^2x^2+d^2}\,e^4x}{8d^2}+\frac{\sqrt{-e^2x^2+d^2}\,e^3}{d}+\frac{17\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^4x}{12d^4}-\frac{25\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4x}{12d^4}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{3d^3}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4x}{3d^6}+\frac{17\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e^3}{15d^5}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{5d^5}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}e}{3\left(x+\frac{d}{e}\right)^2d^5}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{3d^6x}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{d^5x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{3d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x)
```

```
[Out] 17/12/d^4*e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+17/8/d^2*e^4*(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(1/2)*x+1/3/d^5*e/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^
2)^(7/2)+1/d^5*e/x^2*(-e^2*x^2+d^2)^(7/2)-d*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^
2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-5/3/d^6*e^2/x*(-e^2*x^2+d^2)^(7/2)-5/3/d^
6*e^4*x*(-e^2*x^2+d^2)^(5/2)-25/12/d^4*e^4*x*(-e^2*x^2+d^2)^(3/2)-25/8/d^2*
e^4*x*(-e^2*x^2+d^2)^(1/2)+17/15/d^5*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2
)+17/8*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/
```

```
2)*x)-1/3/d^4/x^3*(-e^2*x^2+d^2)^(7/2)+1/5/d^5*e^3*(-e^2*x^2+d^2)^(5/2)+1/3
/d^3*e^3*(-e^2*x^2+d^2)^(3/2)+1/d*e^3*(-e^2*x^2+d^2)^(1/2)-25/8*e^4/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 134, normalized size = 1.31

$$-e^{3} \arcsin\left(\frac{ex}{d}\right) - e^{3} \log\left(\frac{2 d^{2}}{|x|} + \frac{2 \sqrt{-e^{2} x^{2} + d^{2}} d}{|x|}\right) + \frac{\sqrt{-e^{2} x^{2} + d^{2}} e^{3}}{d} - \frac{\sqrt{-e^{2} x^{2} + d^{2}} e^{2}}{x} + \frac{\left(-e^{2} x^{2} + d^{2}\right)^{\frac{3}{2}} e}{d x^{2}} - \frac{\left(-e^{2} x^{2} + d^{2}\right)^{\frac{3}{2}}}{3 x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -e^3*arcsin(e*x/d) - e^3*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x)
) + sqrt(-e^2*x^2 + d^2)*e^3/d - sqrt(-e^2*x^2 + d^2)*e^2/x + (-e^2*x^2 + d
^2)^(3/2)*e/(d*x^2) - 1/3*(-e^2*x^2 + d^2)^(3/2)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2\,x^2\right)^{5/2}}{x^4\,\left(d + e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2), x)
```

sympy [C] time = 9.73, size = 338, normalized size = 3.31

$$d^{2} \left(\begin{cases} - \frac{e \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 x^{2}} + \frac{e^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 d^{2}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 x^{2}} + \frac{i e^{3} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 d^{2}} & \text{otherwise} \end{cases}\right) - 2 d e \left(\begin{cases} - \frac{d^{2}}{2 e x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e}{2 x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{2} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{2 d} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{2 x} - \frac{i e^{2} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{2 d} & \text{otherwise} \end{cases}\right) + e^{2} \left(\begin{cases} \frac{i d}{x \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + i e \operatorname{acosh}{\left(\frac{e x}{d} \right)} - \frac{i e^{2} x}{d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\- \frac{d}{x \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - e \operatorname{asin}{\left(\frac{e x}{d} \right)} + \frac{e^{2} x}{d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) -
 2*d*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2))
> 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d),
 True)) + e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x
/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d
/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**
2/d**2)), True))
```

3.167 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx$

Optimal. Leaf size=108

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(d^2-e^2x^2)^{3/2}}{4x^4}+\frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3}+\frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Rubi [A] time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1807, 807, 266, 47, 63, 208}

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}+\frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3}-\frac{(d^2-e^2x^2)^{3/2}}{4x^4}+\frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

```
[Out] (-5*e^2*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (d^2 - e^2*x^2)^(3/2)/(4*x^4) + (2*e
*(d^2 - e^2*x^2)^(3/2))/(3*d*x^3) + (5*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
(8*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^5}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\int \frac{\left(8d^3e-5d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{4d^2} \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3} + \frac{1}{4}\left(5e^2\right)\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3} + \frac{1}{8}\left(5e^2\right)\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3} - \frac{1}{16}\left(5e^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}} \right. \\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3} + \frac{1}{8}\left(5e^2\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \right. \\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3} + \frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 95, normalized size = 0.88

$$
-\frac{-15e^4x^4\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(6d^3-16d^2ex+9de^2x^2+16e^3x^3\right)+15e^4x^4\log(x)}{24dx^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

```
[Out] -1/24*(Sqrt[d^2 - e^2*x^2]*(6*d^3 - 16*d^2*e*x + 9*d*e^2*x^2 + 16*e^3*x^3)
+ 15*e^4*x^4*Log[x] - 15*e^4*x^4*Log[d + Sqrt[d^2 - e^2*x^2]])/(d*x^4)
```

IntegrateAlgebraic [A] time = 0.72, size = 144, normalized size = 1.33

$$
\frac{5e^4\log\left(\sqrt{d^2-e^2x^2}+d-\sqrt{-e^2}\,x\right)}{8d}-\frac{5e^4\log\left(-d\sqrt{d^2-e^2x^2}+d^2+d\sqrt{-e^2}\,x\right)}{8d}+\frac{\sqrt{d^2-e^2x^2}\left(-6d^3+16d^2ex-9de^2x^2-16e^3x^3\right)}{24dx^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 16*d^2*e*x - 9*d*e^2*x^2 - 16*e^3*x^3))/(24*
d*x^4) + (5*e^4*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]])/(8*d) - (5*e^4
*Log[d^2 + d*Sqrt[-e^2]*x - d*Sqrt[d^2 - e^2*x^2]])/(8*d)
```

fricas [A] time = 0.39, size = 86, normalized size = 0.80

$$-\frac{15\, e^{4} x^{4} \log\left(-\frac{d - \sqrt{-e^{2} x^{2} + d^{2}}}{x}\right) + \left(16\, e^{3} x^{3} + 9\, d e^{2} x^{2} - 16\, d^{2} e x + 6\, d^{3}\right) \sqrt{-e^{2} x^{2} + d^{2}}}{24\, d x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(15*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3 + 9*d*e^
2*x^2 - 16*d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d*x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument ValueEvaluation time: 0.69Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 513, normalized size = 4.75

$$-\frac{5 e^{5} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}\right)}{2 \sqrt{e^{2}}\, d}+\frac{5 e^{5} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}\, d}+\frac{5 e^{4} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{8 \sqrt{d^{2}}}-\frac{5 \sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, e^{5} x}{2 d^{3}}+\frac{5 \sqrt{-e^{2} x^{2}+d^{2}}\, e^{5} x}{2 d^{3}}-\frac{5 \sqrt{-e^{2} x^{2}+d^{2}}\, e^{4}}{8 d^{2}}-\frac{5 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} e^{5} x}{3 d^{5}}+\frac{5 \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} e^{5} x}{3 d^{5}}-\frac{5 \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} e^{4}}{24 d^{4}}+\frac{4 \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} e^{5} x}{3 d^{7}}-\frac{4 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{5}{2}} e^{4}}{3 d^{6}}-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} e^{4}}{8 d^{6}}-\frac{\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{7}{2}} e^{2}}{3 \left(x+\frac{d}{e}\right)^{2} d^{6}}+\frac{4 \left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}} e^{3}}{3 d^{7} x}-\frac{9 \left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}} e^{2}}{8 d^{6} x^{2}}+\frac{2 \left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}} e}{3 d^{5} x^{3}}-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{4 d^{4} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x)
```

```
[Out] -1/3/d^6*e^2/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-5/3/d^5*e^5*(2*(
x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-5/2/d^3*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(1/2)*x-5/2/d*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*
e^2)^(1/2)*x)+2/3/d^5*e/x^3*(-e^2*x^2+d^2)^(7/2)+4/3/d^7*e^5*x*(-e^2*x^2+d^
2)^(5/2)+5/3/d^5*e^5*x*(-e^2*x^2+d^2)^(3/2)+5/2/d^3*e^5*x*(-e^2*x^2+d^2)^(1
/2)+5/2/d*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+4/3/d^
7*e^3/x*(-e^2*x^2+d^2)^(7/2)-9/8/d^6*e^2/x^2*(-e^2*x^2+d^2)^(7/2)-4/3/d^6*e
^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-5/8/d^2*e^4*(-e^2*x^2+d^2)^(1/2)+5/8
*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/4/d^4/x
^4*(-e^2*x^2+d^2)^(7/2)-1/8/d^6*e^4*(-e^2*x^2+d^2)^(5/2)-5/24/d^4*e^4*(-e^2
*x^2+d^2)^(3/2)
```

maxima [A] time = 1.01, size = 130, normalized size = 1.20

$$\frac{5\, e^{4} \log\left(\frac{2\, d^{2}}{{\left| x \right|}} + \frac{2\, \sqrt{-e^{2} x^{2} + d^{2}} d}{{\left| x \right|}}\right)}{8\, d} - \frac{5\, \sqrt{-e^{2} x^{2} + d^{2}} e^{4}}{8\, d^{2}} - \frac{5\, {\left(-e^{2} x^{2} + d^{2}\right)}^{\frac{3}{2}} e^{2}}{8\, d^{2} x^{2}} + \frac{2\, {\left(-e^{2} x^{2} + d^{2}\right)}^{\frac{3}{2}} e}{3\, d x^{3}} - \frac{{\left(-e^{2} x^{2} + d^{2}\right)}^{\frac{3}{2}}}{4\, x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 5/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 5/8*sqrt(-e
^2*x^2 + d^2)*e^4/d^2 - 5/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^2) + 2/3*(-e^
2*x^2 + d^2)^(3/2)*e/(d*x^3) - 1/4*(-e^2*x^2 + d^2)^(3/2)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^5\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2), x)
```

sympy [C] time = 12.36, size = 422, normalized size = 3.91

$$d^{2}\left(\begin{cases} - \frac{d^{2}}{4 e x^{5} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{3 e}{8 x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} - \frac{e^{3}}{8 d^{2} x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{4} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{8 d^{3}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\\frac{i d^{2}}{4 e x^{5} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} - \frac{3 i e}{8 x^{3} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} + \frac{i e^{3}}{8 d^{2} x \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} - \frac{i e^{4} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{8 d^{3}} & \text{otherwise} \end{cases}\right) - 2 d e \left(\begin{cases} - \frac{e \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 x^{2}} + \frac{e^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 d^{2}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 x^{2}} + \frac{i e^{3} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 d^{2}} & \text{otherwise} \end{cases}\right) + e^{2} \left(\begin{cases} - \frac{d^{2}}{2 e x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e}{2 x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{2} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{2 d} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{2 x} - \frac{i e^{2} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{2 d} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) - 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True)) + e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1))
 + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2
/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d
/(e*x))/(2*d), True))
```

3.168 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx$

Optimal. Leaf size=140

$$-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}+\frac{e(d^2-e^2x^2)^{3/2}}{2dx^4}-\frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3}-\frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}+\frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2}$$

Rubi [A] time = 0.18, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {852, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2}-\frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3}+\frac{e(d^2-e^2x^2)^{3/2}}{2dx^4}-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}-\frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (e^3*Sqrt[d^2 - e^2*x^2])/(4*d*x^2) - (d^2 - e^2*x^2)^(3/2)/(5*x^5) + (e*(d
^2 - e^2*x^2)^(3/2))/(2*d*x^4) - (7*e^2*(d^2 - e^2*x^2)^(3/2))/(15*d^2*x^3)
 - (e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(4*d^2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^6}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} - \frac{\int \frac{\left(10d^3e-7d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^5}\,dx}{5d^2}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} + \frac{\int \frac{\left(28d^4e^2-10d^3e^3x\right)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{20d^4}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} - \frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3} - \frac{e^3\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx}{2d}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} - \frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3} - \frac{e^3\,\mathrm{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{4d}\\
&= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} - \frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3} + \frac{e^5\,\mathrm{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{8}\\
&= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} - \frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3} - \frac{e^3\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}}\right.}{}\\
&= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} - \frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3} - \frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-}}{}\right.}{4d^2}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 106, normalized size = 0.76

$$\frac{-15e^5x^5\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-12d^4+30d^3ex-16d^2e^2x^2-15de^3x^3+28e^4x^4\right)+15e^5x^5\log(x)}{60d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-12*d^4 + 30*d^3*e*x - 16*d^2*e^2*x^2 - 15*d*e^3*x^3
+ 28*e^4*x^4) + 15*e^5*x^5*Log[x] - 15*e^5*x^5*Log[d + Sqrt[d^2 - e^2*x^2]]
)/(60*d^2*x^5)
```

IntegrateAlgebraic [A] time = 0.71, size = 115, normalized size = 0.82

$$\frac{e^5\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}+\frac{\sqrt{d^2-e^2x^2}\left(-12d^4+30d^3ex-16d^2e^2x^2-15de^3x^3+28e^4x^4\right)}{60d^2x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-12*d^4 + 30*d^3*e*x - 16*d^2*e^2*x^2 - 15*d*e^3*x^3
+ 28*e^4*x^4))/(60*d^2*x^5) + (e^5*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^
2*x^2]/d])/(2*d^2)
```

fricas [A] time = 0.41, size = 97, normalized size = 0.69

$$\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(28\,e^4x^4-15\,de^3x^3-16\,d^2e^2x^2+30\,d^3ex-12\,d^4\right)\sqrt{-e^2x^2+d^2}}{60\,d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (28*e^4*x^4 - 15*d*e^
3*x^3 - 16*d^2*e^2*x^2 + 30*d^3*e*x - 12*d^4)*sqrt(-e^2*x^2 + d^2))/(d^2*x^
5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument ValueEvaluation time: 0.69Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 541, normalized size = 3.86

$$-\frac{e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{4\sqrt{d^2}\,d}+\frac{23e^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}\,d^2}-\frac{23e^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,d^2}+\frac{23\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^6x}{8d^4}-\frac{23\sqrt{-e^2x^2+d^2}\,e^6x}{8d^4}+\frac{\sqrt{-e^2x^2+d^2}\,e^5}{4d^3}+\frac{23\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^6x}{12d^6}-\frac{23\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^6x}{12d^6}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5}{12d^5}-\frac{23\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6x}{15d^8}+\frac{23\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e^5}{15d^7}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5}{20d^7}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}e^3}{3\left(x+\frac{d}{e}\right)^2d^7}-\frac{23\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^4}{15d^8x}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{4d^7x^2}-\frac{13\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{15d^6x^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{2d^5x^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{5d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x)
```

```
[Out] 1/3/d^7*e^3/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+23/12/d^6*e^6*(2*
(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+23/8/d^4*e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^
2)^(1/2)*x+23/8/d^2*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(1/2)*x)-13/15/d^6*e^2/x^3*(-e^2*x^2+d^2)^(7/2)-23/15/d^8*e^4/x*(
-e^2*x^2+d^2)^(7/2)-23/15/d^8*e^6*x*(-e^2*x^2+d^2)^(5/2)-23/12/d^6*e^6*x*(-
e^2*x^2+d^2)^(3/2)-23/8/d^4*e^6*x*(-e^2*x^2+d^2)^(1/2)-23/8/d^2*e^6/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/2/d^5*e/x^4*(-e^2*x^2+d^2
)^(7/2)+5/4/d^7*e^3/x^2*(-e^2*x^2+d^2)^(7/2)-1/4/d*e^5/(d^2)^(1/2)*ln((2*d^
2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+23/15/d^7*e^5*(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(5/2)-1/5/d^4/x^5*(-e^2*x^2+d^2)^(7/2)+1/20/d^7*e^5*(-e^2*x^2+d^2
)^(5/2)+1/12/d^5*e^5*(-e^2*x^2+d^2)^(3/2)+1/4/d^3*e^5*(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 0.99, size = 155, normalized size = 1.11

$$-\frac{e^5 \log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{4\,d^2} + \frac{\sqrt{-e^2x^2+d^2}\,e^5}{4\,d^3} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{4\,d^3x^2} - \frac{7\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}{15\,d^2x^3} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{2\,dx^4} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^5*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^2 + 1/4*sqrt
(-e^2*x^2 + d^2)*e^5/d^3 + 1/4*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^3*x^2) - 7/15*
(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^3) + 1/2*(-e^2*x^2 + d^2)^(3/2)*e/(d*x^4)
 - 1/5*(-e^2*x^2 + d^2)^(3/2)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^6\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2), x)
```

sympy [C] time = 13.42, size = 660, normalized size = 4.71

$$d^2\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) - 2*d*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*
x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2
)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqr
t(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) -
 I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2
```

```
*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(
e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(
-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

3.169 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{(d^2-e^2x^2)^{3/2}}{6x^6}+\frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5}-\frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4}-\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{3e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}+\frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3}$$

Rubi [A] time = 0.21, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {852, 1807, 835, 807, 266, 47, 63, 208}

$$-\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3}-\frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4}+\frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5}-\frac{(d^2-e^2x^2)^{3/2}}{6x^6}+\frac{3e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] (-3*e^4*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (d^2 - e^2*x^2)^(3/2)/(6*x^6) +
 (2*e*(d^2 - e^2*x^2)^(3/2))/(5*d*x^5) - (3*e^2*(d^2 - e^2*x^2)^(3/2))/(8*d
^2*x^4) + (4*e^3*(d^2 - e^2*x^2)^(3/2))/(15*d^3*x^3) + (3*e^6*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/(16*d^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^7}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} - \frac{\int \frac{\left(12d^3e-9d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^6}\,dx}{6d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} + \frac{\int \frac{\left(45d^4e^2-24d^3e^3x\right)\sqrt{d^2-e^2x^2}}{x^5}\,dx}{30d^4}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} - \frac{\int \frac{\left(96d^5e^3-45d^4e^4x\right)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{120d^6}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3} + \frac{\left(3e^4\right)\int}{}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3} + \frac{\left(3e^4\right)\mathrm{Su}}{}$$
$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e\right.}{15d^3x}$$
$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e\right.}{15d^3x}$$
$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e\right.}{15d^3x}$$

Mathematica [A] time = 0.23, size = 117, normalized size = 0.69

$$-\frac{-45e^6x^6\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(40d^5-96d^4ex+50d^3e^2x^2+32d^2e^3x^3-45de^4x^4+64e^5x^5\right)+45e^6x^6\log(x)}{240d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] -1/240*(Sqrt[d^2 - e^2*x^2]*(40*d^5 - 96*d^4*e*x + 50*d^3*e^2*x^2 + 32*d^2*
e^3*x^3 - 45*d*e^4*x^4 + 64*e^5*x^5) + 45*e^6*x^6*Log[x] - 45*e^6*x^6*Log[d
 + Sqrt[d^2 - e^2*x^2]])/(d^3*x^6)
```

IntegrateAlgebraic [A] time = 0.75, size = 126, normalized size = 0.75

$$\frac{\sqrt{d^2-e^2x^2}\left(-40d^5+96d^4ex-50d^3e^2x^2-32d^2e^3x^3+45de^4x^4-64e^5x^5\right)}{240d^3x^6} - \frac{3e^6\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^5 + 96*d^4*e*x - 50*d^3*e^2*x^2 - 32*d^2*e^3*x^
3 + 45*d*e^4*x^4 - 64*e^5*x^5))/(240*d^3*x^6) - (3*e^6*ArcTanh[(Sqrt[-e^2]*
x)/d - Sqrt[d^2 - e^2*x^2]/d])/(8*d^3)
```

fricas [A] time = 0.42, size = 108, normalized size = 0.64

$$-\frac{45\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(64\,e^5x^5-45\,de^4x^4+32\,d^2e^3x^3+50\,d^3e^2x^2-96\,d^4ex+40\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(45*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (64*e^5*x^5 - 45*d*
e^4*x^4 + 32*d^2*e^3*x^3 + 50*d^3*e^2*x^2 - 96*d^4*e*x + 40*d^5)*sqrt(-e^2*
x^2 + d^2))/(d^3*x^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument ValueEvaluation time: 1.04Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 566, normalized size = 3.35

$$\frac{3e^{6}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{16\sqrt{d^{2}}\,d^{2}}-\frac{13e^{7}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{4\sqrt{e^{2}}\,d^{3}}+\frac{13e^{7}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{4\sqrt{e^{2}}\,d^{3}}-\frac{13\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{7}x}{4d^{5}}+\frac{13\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}x}{4d^{5}}-\frac{3\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}{16d^{4}}-\frac{13\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}e^{7}x}{6d^{7}}+\frac{13\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}x}{6d^{7}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{6}}{16d^{6}}+\frac{26\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{7}x}{15d^{9}}-\frac{26\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}e^{6}}{15d^{8}}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}}{80d^{8}}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{7}{2}}e^{4}}{3\left(x+\frac{d}{e}\right)^{2}d^{8}}+\frac{26\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{5}}{15d^{9}x}-\frac{23\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{4}}{16d^{8}x^{2}}+\frac{16\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{3}}{15d^{7}x^{3}}-\frac{17\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{2}}{24d^{6}x^{4}}+\frac{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e}{5d^{5}x^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{6d^{4}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x)
```

```
[Out] -13/6/d^7*e^7*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-13/4/d^5*e^7*(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(1/2)*x-13/4/d^3*e^7/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*
(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/3/d^8*e^4/(x+d/e)^2*(2*(x+d/e)*d*e-(x
+d/e)^2*e^2)^(7/2)+2/5/d^5*e/x^5*(-e^2*x^2+d^2)^(7/2)+16/15/d^7*e^3/x^3*(-e
^2*x^2+d^2)^(7/2)+13/4/d^5*e^7*x*(-e^2*x^2+d^2)^(1/2)+13/4/d^3*e^7/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+26/15/d^9*e^5/x*(-e^2*x^2+d^
2)^(7/2)+26/15/d^9*e^7*x*(-e^2*x^2+d^2)^(5/2)+13/6/d^7*e^7*x*(-e^2*x^2+d^2)
^(3/2)-17/24/d^6*e^2/x^4*(-e^2*x^2+d^2)^(7/2)-23/16/d^8*e^4/x^2*(-e^2*x^2+d
^2)^(7/2)+3/16/d^2*e^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(
1/2))/x)-26/15/d^8*e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-1/6/d^4/x^6*(-e^
2*x^2+d^2)^(7/2)-3/80/d^8*e^6*(-e^2*x^2+d^2)^(5/2)-1/16/d^6*e^6*(-e^2*x^2+d
^2)^(3/2)-3/16/d^4*e^6*(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 1.01, size = 180, normalized size = 1.07

$$\frac{3\,e^{6}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{16\,d^{3}}-\frac{3\,\sqrt{-e^{2}x^{2}+d^{2}}\,e^{6}}{16\,d^{4}}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}{16\,d^{4}x^{2}}+\frac{4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}{15\,d^{3}x^{3}}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{8\,d^{2}x^{4}}+\frac{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e}{5\,dx^{5}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 3/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/16*sqr
t(-e^2*x^2 + d^2)*e^6/d^4 - 3/16*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^4*x^2) + 4/1
5*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^3*x^3) - 3/8*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^
2*x^4) + 2/5*(-e^2*x^2 + d^2)^(3/2)*e/(d*x^5) - 1/6*(-e^2*x^2 + d^2)^(3/2)/
x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^7\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2), x)
```

sympy [C] time = 19.72, size = 808, normalized size = 4.78

$$d^{2}\left(\begin{cases} -\frac{d^{2}}{6ex^{7}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} + \frac{5e}{24x^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} + \frac{e^{3}}{48d^{2}x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} - \frac{e^{5}}{16d^{4}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} + \frac{e^{6}\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^{5}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right| > 1 \\ \frac{id^{2}}{6ex^{7}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} - \frac{5ie}{24x^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} - \frac{ie^{3}}{48d^{2}x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} + \frac{ie^{5}}{16d^{4}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} - \frac{ie^{6}\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^{5}} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} \frac{3id^{3}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}} - \frac{4ide^{2}x^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}} + \frac{2ie^{6}x^{6}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{5}x^{5}+15d^{3}e^{2}x^{7}} - \frac{ie^{4}x^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{3}x^{5}+15de^{2}x^{7}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right| > 1 \\ \frac{3d^{3}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}} - \frac{4de^{2}x^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}} + \frac{2e^{6}x^{6}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{5}x^{5}+15d^{3}e^{2}x^{7}} - \frac{e^{4}x^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{3}x^{5}+15de^{2}x^{7}} & \text{otherwise} \end{cases}\right) + e^{2}\left(\begin{cases} -\frac{d^{2}}{4ex^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} + \frac{3e}{8x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} - \frac{e^{3}}{8d^{2}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} + \frac{e^{4}\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right| > 1 \\ \frac{id^{2}}{4ex^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} - \frac{3ie}{8x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} + \frac{ie^{3}}{8d^{2}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} - \frac{ie^{4}\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 2*d*e*Piecewise((3*I*d**3*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(
-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2)
 > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*
d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**
6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*
x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e*
*2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqr
t(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**
4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sq
rt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I
*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3
), True))
```

3.170 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx$

Optimal. Leaf size=198

$$-\frac{(d^2-e^2x^2)^{3/2}}{7x^7}+\frac{e(d^2-e^2x^2)^{3/2}}{3dx^6}-\frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}-\frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3}+\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}+$$

Rubi [A] time = 0.24, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {852, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}-\frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3}+\frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4}-\frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5}+\frac{e(d^2-e^2x^2)^{3/2}}{3dx^6}-\frac{(d^2-e^2x^2)^{3/2}}{7x^7}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) - (d^2 - e^2*x^2)^(3/2)/(7*x^7) + (e*
(d^2 - e^2*x^2)^(3/2))/(3*d*x^6) - (11*e^2*(d^2 - e^2*x^2)^(3/2))/(35*d^2*x
^5) + (e^3*(d^2 - e^2*x^2)^(3/2))/(4*d^3*x^4) - (22*e^4*(d^2 - e^2*x^2)^(3/
2))/(105*d^4*x^3) - (e^7*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^8}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7} - \frac{\int \frac{\left(14d^3e-11d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6} + \frac{\int \frac{\left(66d^4e^2-42d^3e^3x\right)\sqrt{d^2-e^2x^2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6} - \frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5} - \frac{\int \frac{\left(210d^5e^3-132d^4e^4x\right)\sqrt{d^2-e^2x^2}}{x^5}\,dx}{210d^6}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6} - \frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4} + \frac{\int \frac{\left(528d^6e^4-210\right.}{}}{8}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6} - \frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4} - \frac{22e^4\left(d^2-e^2\right.}{105d^4x^3}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6} - \frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4} - \frac{22e^4\left(d^2-e^2\right.}{105d^4x^3}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6} - \frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6} - \frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6} - \frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}$$

Mathematica [A] time = 0.23, size = 128, normalized size = 0.65

$$\frac{-105e^7x^7\log\left(\sqrt{d^2-e^2x^2}+d\right)+\sqrt{d^2-e^2x^2}\left(-120d^6+280d^5ex-144d^4e^2x^2-70d^3e^3x^3+88d^2e^4x^4-105de^5x^5+176e^6x^6\right)+105e^7x^7\log(x)}{840d^4x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-120*d^6 + 280*d^5*e*x - 144*d^4*e^2*x^2 - 70*d^3*e^3
*x^3 + 88*d^2*e^4*x^4 - 105*d*e^5*x^5 + 176*e^6*x^6) + 105*e^7*x^7*Log[x] -
 105*e^7*x^7*Log[d + Sqrt[d^2 - e^2*x^2]])/(840*d^4*x^7)
```

IntegrateAlgebraic [A] time = 0.85, size = 137, normalized size = 0.69

$$\frac{e^7\tanh^{-1}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}+\frac{\sqrt{d^2-e^2x^2}\left(-120d^6+280d^5ex-144d^4e^2x^2-70d^3e^3x^3+88d^2e^4x^4-105de^5x^5+176e^6x^6\right)}{840d^4x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-120*d^6 + 280*d^5*e*x - 144*d^4*e^2*x^2 - 70*d^3*e^3
*x^3 + 88*d^2*e^4*x^4 - 105*d*e^5*x^5 + 176*e^6*x^6))/(840*d^4*x^7) + (e^7*
ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(4*d^4)
```

fricas [A] time = 0.40, size = 119, normalized size = 0.60

$$\frac{105\,e^{7}x^{7}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(176\,e^{6}x^{6}-105\,de^{5}x^{5}+88\,d^{2}e^{4}x^{4}-70\,d^{3}e^{3}x^{3}-144\,d^{4}e^{2}x^{2}+280\,d^{5}ex-120\,d^{6}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{840\,d^{4}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (176*e^6*x^6 - 105*
d*e^5*x^5 + 88*d^2*e^4*x^4 - 70*d^3*e^3*x^3 - 144*d^4*e^2*x^2 + 280*d^5*e*x
 - 120*d^6)*sqrt(-e^2*x^2 + d^2))/(d^4*x^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument ValueEvaluation time: 1.08Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 591, normalized size = 2.98

$$-\frac{e^{7}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{8\sqrt{d^{2}}\,d^{3}}+\frac{29e^{8}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{8\sqrt{e^{2}}\,d^{4}}-\frac{29e^{8}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8\sqrt{e^{2}}\,d^{4}}+\frac{29\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{8}x}{8d^{6}}-\frac{29\sqrt{-e^{2}x^{2}+d^{2}}\,e^{8}x}{8d^{6}}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}{8d^{5}}+\frac{29\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}e^{8}x}{12d^{8}}-\frac{29\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{8}x}{12d^{8}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{7}}{24d^{7}}-\frac{29\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{8}x}{15d^{10}}+\frac{29\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}e^{7}}{15d^{9}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{7}}{40d^{9}}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{7}{2}}e^{5}}{3\left(x+\frac{d}{e}\right)^{2}d^{9}}-\frac{29\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{6}}{15d^{10}x}+\frac{13\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{5}}{8d^{9}x^{2}}-\frac{19\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{4}}{15d^{8}x^{3}}+\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{3}}{12d^{7}x^{4}}-\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e^{2}}{5d^{6}x^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e}{3d^{5}x^{6}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{7d^{4}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x)
```

```
[Out] 1/3/d^9*e^5/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+29/12/d^8*e^8*(2*
(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+29/8/d^6*e^8*(2*(x+d/e)*d*e-(x+d/e)^2*e^
2)^(1/2)*x+29/8/d^4*e^8/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(1/2)*x)-3/5/d^6*e^2/x^5*(-e^2*x^2+d^2)^(7/2)+1/3/d^5*e/x^6*(-e^2
*x^2+d^2)^(7/2)-19/15/d^8*e^4/x^3*(-e^2*x^2+d^2)^(7/2)-29/15/d^10*e^6/x*(-e
^2*x^2+d^2)^(7/2)-29/15/d^10*e^8*x*(-e^2*x^2+d^2)^(5/2)-29/12/d^8*e^8*x*(-e
^2*x^2+d^2)^(3/2)-29/8/d^6*e^8*x*(-e^2*x^2+d^2)^(1/2)-29/8/d^4*e^8/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+11/12/d^7*e^3/x^4*(-e^2*x^2+
d^2)^(7/2)+13/8/d^9*e^5/x^2*(-e^2*x^2+d^2)^(7/2)-1/8/d^3*e^7/(d^2)^(1/2)*ln
((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+29/15/d^9*e^7*(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(5/2)+1/40/d^9*e^7*(-e^2*x^2+d^2)^(5/2)+1/24/d^7*e^7*(-e^2*
x^2+d^2)^(3/2)+1/8/d^5*e^7*(-e^2*x^2+d^2)^(1/2)-1/7/d^4/x^7*(-e^2*x^2+d^2)^
(7/2)
```

maxima [A] time = 0.99, size = 205, normalized size = 1.04

$$-\frac{e^{7}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{8\,d^{4}}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,e^{7}}{8\,d^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{5}}{8\,d^{5}x^{2}}-\frac{22\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}{105\,d^{4}x^{3}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}{4\,d^{3}x^{4}}-\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{35\,d^{2}x^{5}}+\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e}{3\,dx^{6}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*e^7*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^4 + 1/8*sqrt
(-e^2*x^2 + d^2)*e^7/d^5 + 1/8*(-e^2*x^2 + d^2)^(3/2)*e^5/(d^5*x^2) - 22/10
5*(-e^2*x^2 + d^2)^(3/2)*e^4/(d^4*x^3) + 1/4*(-e^2*x^2 + d^2)^(3/2)*e^3/(d^
```

```
3*x^4) - 11/35*(-e^2*x^2 + d^2)^(3/2)*e^2/(d^2*x^5) + 1/3*(-e^2*x^2 + d^2)^
(3/2)*e/(d*x^6) - 1/7*(-e^2*x^2 + d^2)^(3/2)/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2\,x^2\right)^{5/2}}{x^8\,\left(d + e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2), x)
```

sympy [C] time = 18.18, size = 835, normalized size = 4.22

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{7x^6} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{35d^2x^4} + \frac{4e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^4x^2} + \frac{8e^7\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^6} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{7x^6} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{35d^2x^4} + \frac{4ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^4x^2} + \frac{8ie^7\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^6} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} \frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} - \frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7} + \frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7} - \frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 2*d*e*Pi
ecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d*
*2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**
5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), A
bs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) -
 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d
**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*
e**6*asin(d/(e*x))/(16*d**5), True)) + e**2*Piecewise((3*I*d**3*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**
2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e*
*2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (
3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x
**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*s
qrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqr
t(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True))
```

3.171 $\int \frac{x^4}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}-\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

Rubi [A] time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1814, 12, 217, 203}

$$-\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -(d^3*(d - e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (17*d^2*(d - e*x))/(15*e
^5*(d^2 - e^2*x^2)^(3/2)) - (2*(15*d - 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^2]
) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^4(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)\left(\frac{2d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{11d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}\,dx}{15d^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx\right.}{e^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] time = 0.16, size = 106, normalized size = 0.86

$$\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{10e^5(d+ex)^3} + \frac{31d}{60e^5(d+ex)^2} - \frac{1}{8e^5(ex-d)} - \frac{193}{120e^5(d+ex)}\right) - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*(-1/8*1/(e^5*(-d + e*x)) - d^2/(10*e^5*(d + e*x)^3) + (
31*d)/(60*e^5*(d + e*x)^2) - 193/(120*e^5*(d + e*x))) - ArcTan[(e*x)/Sqrt[d
^2 - e^2*x^2]]/e^5
```

IntegrateAlgebraic [A] time = 0.67, size = 114, normalized size = 0.93

$$-\frac{\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^6} - \frac{\sqrt{d^2-e^2x^2}\left(-16d^3-17d^2ex+22de^2x^2+26e^3x^3\right)}{15e^5(ex-d)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(-16*d^3 - 17*d^2*e*x + 22*d*e^2*x^2 + 26*e^3*x^
3))/(e^5*(-d + e*x)*(d + e*x)^3) - (Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d
^2 - e^2*x^2]])/e^6
```

fricas [A] time = 0.41, size = 171, normalized size = 1.39

$$-\frac{16 e^4 x^4 + 32 d e^3 x^3 - 32 d^3 e x - 16 d^4 - 30 \left(e^4 x^4 + 2 d e^3 x^3 - 2 d^3 e x - d^4\right) \arctan\left(-\frac{d - \sqrt{-e^2 x^2 + d^2}}{e x}\right) + \left(26 e^3 x^3 + 22 d e^2 x^2 - 17 d^2 e x - 16 d^3\right) \sqrt{-e^2 x^2 + d^2}}{15 \left(e^9 x^4 + 2 d e^8 x^3 - 2 d^3 e^6 x - d^4 e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(16*e^4*x^4 + 32*d*e^3*x^3 - 32*d^3*e*x - 16*d^4 - 30*(e^4*x^4 + 2*d*
e^3*x^3 - 2*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (26*
e^3*x^3 + 22*d*e^2*x^2 - 17*d^2*e*x - 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^9*x^
4 + 2*d*e^8*x^3 - 2*d^3*e^6*x - d^4*e^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 198, normalized size = 1.61

$$\frac{4x}{\sqrt{-e^2 x^2 + d^2}\, e^4} - \frac{34x}{15 \sqrt{2 \left(x + \frac{d}{e}\right) d e - \left(x + \frac{d}{e}\right)^2 e^2}\, e^4} - \frac{\arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2 x^2 + d^2}}\right)}{\sqrt{e^2}\, e^4} - \frac{d^3}{5 \left(x + \frac{d}{e}\right)^2 \sqrt{2 \left(x + \frac{d}{e}\right) d e - \left(x + \frac{d}{e}\right)^2 e^2}\, e^7} + \frac{17 d^2}{15 \left(x + \frac{d}{e}\right) \sqrt{2 \left(x + \frac{d}{e}\right) d e - \left(x + \frac{d}{e}\right)^2 e^2}\, e^6} - \frac{2d}{\sqrt{-e^2 x^2 + d^2}\, e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 4/(-e^2*x^2+d^2)^(1/2)/e^4*x-1/(e^2)^(1/2)/e^4*arctan((e^2)^(1/2)/(-e^2*x^2
+d^2)^(1/2)*x)-2*d/e^5/(-e^2*x^2+d^2)^(1/2)+17/15/e^6*d^2/(x+d/e)/(2*(x+d/e
)*d*e-(x+d/e)^2*e^2)^(1/2)-34/15/e^4/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-
1/5*d^3/e^7/(x+d/e)^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 1.02, size = 170, normalized size = 1.38

$$-\frac{d^3}{5 \left(\sqrt{-e^2 x^2 + d^2}\, e^7 x^2 + 2 \sqrt{-e^2 x^2 + d^2}\, d e^6 x + \sqrt{-e^2 x^2 + d^2}\, d^2 e^5\right)} + \frac{17 d^2}{15 \left(\sqrt{-e^2 x^2 + d^2}\, e^6 x + \sqrt{-e^2 x^2 + d^2}\, d e^5\right)} + \frac{26 x}{15 \sqrt{-e^2 x^2 + d^2}\, e^4} - \frac{\arcsin\left(\frac{e x}{d}\right)}{e^5} - \frac{2 d}{\sqrt{-e^2 x^2 + d^2}\, e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*d^3/(sqrt(-e^2*x^2 + d^2)*e^7*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^6*x + s
qrt(-e^2*x^2 + d^2)*d^2*e^5) + 17/15*d^2/(sqrt(-e^2*x^2 + d^2)*e^6*x + sqrt
(-e^2*x^2 + d^2)*d*e^5) + 26/15*x/(sqrt(-e^2*x^2 + d^2)*e^4) - arcsin(e*x/d
)/e^5 - 2*d/(sqrt(-e^2*x^2 + d^2)*e^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(d^2 - e^2 x^2\right)}^{3/2} \left(d + e x\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-(-d+ex)(d+ex))^{\frac{3}{2}}(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.172 $\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.20, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {852, 1635, 637}

$$\frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (d^2*(d - e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d - e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d - 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^2 \left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^3(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)\left(-\frac{2d^3}{e^3}+\frac{5d^2x}{e^2}-\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$

$$= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{-\frac{6d^3}{e^3}+\frac{15d^2x}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}dx}{15d^2}$$

$$= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.08, size = 70, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^3+4d^2ex+de^2x^2-2e^3x^3\right)}{5de^4(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 + 4*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(5*d*e^4*
(d - e*x)*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.54, size = 70, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^3+4d^2ex+de^2x^2-2e^3x^3\right)}{5de^4(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 + 4*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(5*d*e^4*
(d - e*x)*(d + e*x)^3)
```

fricas [A] time = 0.40, size = 116, normalized size = 1.17

$$\frac{2\,e^4x^4+4\,de^3x^3-4\,d^3ex-2\,d^4+\left(2\,e^3x^3-de^2x^2-4\,d^2ex-2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(de^8x^4+2\,d^2e^7x^3-2\,d^4e^5x-d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 - d*e^2*x^2 -
 4*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d*e^8*x^4 + 2*d^2*e^7*x^3 - 2*d^
4*e^5*x - d^5*e^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 65, normalized size = 0.66

$$\frac{(-ex+d)\left(-2e^3x^3+d\,e^2x^2+4d^2ex+2d^3\right)}{5\,(ex+d)\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\,d\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/5*(-e*x+d)*(-2*e^3*x^3+d*e^2*x^2+4*d^2*e*x+2*d^3)/(e*x+d)/d/e^4/(-e^2*x^2
+d^2)^(3/2)
```

maxima [A] time = 0.47, size = 157, normalized size = 1.59

$$\frac{d^2}{5\left(\sqrt{-e^2x^2+d^2}\,e^6x^2+2\sqrt{-e^2x^2+d^2}\,de^5x+\sqrt{-e^2x^2+d^2}\,d^2e^4\right)}-\frac{4\,d}{5\left(\sqrt{-e^2x^2+d^2}\,e^5x+\sqrt{-e^2x^2+d^2}\,de^4\right)}-\frac{2\,x}{5\sqrt{-e^2x^2+d^2}\,de^3}+\frac{1}{\sqrt{-e^2x^2+d^2}\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^2/(sqrt(-e^2*x^2 + d^2)*e^6*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^5*x + sq
rt(-e^2*x^2 + d^2)*d^2*e^4) - 4/5*d/(sqrt(-e^2*x^2 + d^2)*e^5*x + sqrt(-e^2
*x^2 + d^2)*d*e^4) - 2/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^3) + 1/(sqrt(-e^2*x^2
+ d^2)*e^4)
```

mupad [B] time = 2.97, size = 66, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3+4\,d^2\,e\,x+d\,e^2\,x^2-2\,e^3\,x^3\right)}{5\,d\,e^4\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 - 2*e^3*x^3 + d*e^2*x^2 + 4*d^2*e*x))/(5*d*e^
4*(d + e*x)^3*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.173 $\int \frac{x^2}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}-\frac{d}{5e^3(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{7}{15e^3(d+ex)\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {852, 1635, 778, 191}

$$-\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -(d*(d - e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) + (7*(d - e*x))/(15*e^3*(d^2
 - e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^2(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$
$$= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{\left(\frac{2d^2}{e^2}-\frac{5dx}{e}\right)(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$
$$= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{15e^2}$$
$$= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.79

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^3+8d^2ex+2de^2x^2+e^3x^3\right)}{15d^2e^3(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^3 + 8*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*e
^3*(d - e*x)*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.54, size = 70, normalized size = 0.79

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^3+8d^2ex+2de^2x^2+e^3x^3\right)}{15d^2e^3(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^3 + 8*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*e
^3*(d - e*x)*(d + e*x)^3)
```

fricas [A] time = 0.39, size = 118, normalized size = 1.33

$$\frac{4\,e^4x^4+8\,de^3x^3-8\,d^3ex-4\,d^4-\left(e^3x^3+2\,de^2x^2+8\,d^2ex+4\,d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^7x^4+2\,d^3e^6x^3-2\,d^5e^4x-d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(4*e^4*x^4 + 8*d*e^3*x^3 - 8*d^3*e*x - 4*d^4 - (e^3*x^3 + 2*d*e^2*x^2
+ 8*d^2*e*x + 4*d^3)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^4 + 2*d^3*e^6*x^3 - 2
*d^5*e^4*x - d^6*e^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 65, normalized size = 0.73

$$\frac{(-ex+d)\left(e^3x^3+2d\,e^2x^2+8d^2ex+4d^3\right)}{15\,(ex+d)\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e^3*x^3+2*d*e^2*x^2+8*d^2*e*x+4*d^3)/(e*x+d)/d^2/e^3/(-e^2*x
^2+d^2)^(3/2)
```

maxima [A] time = 0.47, size = 136, normalized size = 1.53

$$-\frac{d}{5\left(\sqrt{-e^2x^2+d^2}\,e^5x^2+2\sqrt{-e^2x^2+d^2}\,de^4x+\sqrt{-e^2x^2+d^2}\,d^2e^3\right)}+\frac{7}{15\left(\sqrt{-e^2x^2+d^2}\,e^4x+\sqrt{-e^2x^2+d^2}\,de^3\right)}+\frac{x}{15\sqrt{-e^2x^2+d^2}\,d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*d/(sqrt(-e^2*x^2 + d^2)*e^5*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^4*x + sqr
t(-e^2*x^2 + d^2)*d^2*e^3) + 7/15/(sqrt(-e^2*x^2 + d^2)*e^4*x + sqrt(-e^2*x
^2 + d^2)*d*e^3) + 1/15*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^2)
```

mupad [B] time = 2.90, size = 66, normalized size = 0.74

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^3+8\,d^2\,e\,x+2\,d\,e^2\,x^2+e^3\,x^3\right)}{15\,d^2\,e^3\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(4*d^3 + e^3*x^3 + 2*d*e^2*x^2 + 8*d^2*e*x))/(15*d^2
*e^3*(d + e*x)^3*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.174 $\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {793, 659, 191}

$$\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2]) + 1/(5*e^2*(d + e*x)^2*Sqrt[d^2 - e^2*
x^2]) - 2/(15*d*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx&=\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e}\\&=\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}\\&=\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(d^3+2d^2ex+8de^2x^2+4e^3x^3\right)}{15d^3e^2(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 8*d*e^2*x^2 + 4*e^3*x^3))/(15*d^3*e
^2*(d - e*x)*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.48, size = 69, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(d^3+2d^2ex+8de^2x^2+4e^3x^3\right)}{15d^3e^2(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 8*d*e^2*x^2 + 4*e^3*x^3))/(15*d^3*e
^2*(d - e*x)*(d + e*x)^3)
```

fricas [A] time = 0.40, size = 116, normalized size = 1.27

$$\frac{e^4x^4+2\,de^3x^3-2\,d^3ex-d^4-\left(4\,e^3x^3+8\,de^2x^2+2\,d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^6x^4+2\,d^4e^5x^3-2\,d^6e^3x-d^7e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(e^4*x^4 + 2*d*e^3*x^3 - 2*d^3*e*x - d^4 - (4*e^3*x^3 + 8*d*e^2*x^2 +
2*d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^4 + 2*d^4*e^5*x^3 - 2*d^6
*e^3*x - d^7*e^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 64, normalized size = 0.70

$$\frac{(-ex+d)\left(4e^3x^3+8d\,e^2x^2+2d^2ex+d^3\right)}{15\,(ex+d)\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\,d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/15*(-e*x+d)*(4*e^3*x^3+8*d*e^2*x^2+2*d^2*e*x+d^3)/(e*x+d)/d^3/e^2/(-e^2*x
^2+d^2)^(3/2)
```

maxima [A] time = 0.46, size = 138, normalized size = 1.52

$$\frac{1}{5\left(\sqrt{-e^2x^2+d^2}\,e^4x^2+2\sqrt{-e^2x^2+d^2}\,de^3x+\sqrt{-e^2x^2+d^2}\,d^2e^2\right)}-\frac{2}{15\left(\sqrt{-e^2x^2+d^2}\,de^3x+\sqrt{-e^2x^2+d^2}\,d^2e^2\right)}+\frac{4\,x}{15\sqrt{-e^2x^2+d^2}\,d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5/(sqrt(-e^2*x^2 + d^2)*e^4*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^3*x + sqrt(-
e^2*x^2 + d^2)*d^2*e^2) - 2/15/(sqrt(-e^2*x^2 + d^2)*d*e^3*x + sqrt(-e^2*x^
2 + d^2)*d^2*e^2) + 4/15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e)
```

mupad [B] time = 2.88, size = 65, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^3+2\,d^2\,e\,x+8\,d\,e^2\,x^2+4\,e^3\,x^3\right)}{15\,d^3\,e^2\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^3 + 4*e^3*x^3 + 8*d*e^2*x^2 + 2*d^2*e*x))/(15*d^3
*e^2*(d + e*x)^3*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.175 $\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 191}

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2]) - 1/(5*d*e*(d + e*x)^2*Sqrt[d^2 - e^2*x^2
]) - 1/(5*d^2*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{3\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d}\\ &= -\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d^2}\\ &= \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.77

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^3+d^2ex+4de^2x^2+2e^3x^3\right)}{5d^4e(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 + d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e
*(d - e*x)*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.52, size = 70, normalized size = 0.77

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^3+d^2ex+4de^2x^2+2e^3x^3\right)}{5d^4e(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 + d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e
*(d - e*x)*(d + e*x)^3)
```

fricas [A] time = 0.39, size = 115, normalized size = 1.26

$$-\frac{2\,e^4x^4+4\,de^3x^3-4\,d^3ex-2\,d^4+\left(2\,e^3x^3+4\,de^2x^2+d^2ex-2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(d^4e^5x^4+2\,d^5e^4x^3-2\,d^7e^2x-d^8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 + 4*d*e^2*x^
2 + d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 + 2*d^5*e^4*x^3 - 2
*d^7*e^2*x - d^8*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 66, normalized size = 0.73

$$-\frac{(-ex+d)\left(-2e^3x^3-4d\,e^2x^2-d^2ex+2d^3\right)}{5\,(ex+d)\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/5*(-e*x+d)*(-2*e^3*x^3-4*d*e^2*x^2-d^2*e*x+2*d^3)/(e*x+d)/d^4/e/(-e^2*x^
2+d^2)^(3/2)
```

maxima [A] time = 0.44, size = 136, normalized size = 1.49

$$-\frac{1}{5\left(\sqrt{-e^2x^2+d^2}\,de^3x^2+2\sqrt{-e^2x^2+d^2}\,d^2e^2x+\sqrt{-e^2x^2+d^2}\,d^3e\right)}-\frac{1}{5\left(\sqrt{-e^2x^2+d^2}\,d^2e^2x+\sqrt{-e^2x^2+d^2}\,d^3e\right)}+\frac{2\,x}{5\sqrt{-e^2x^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5/(sqrt(-e^2*x^2 + d^2)*d*e^3*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d^2*e^2*x + s
qrt(-e^2*x^2 + d^2)*d^3*e) - 1/5/(sqrt(-e^2*x^2 + d^2)*d^2*e^2*x + sqrt(-e^
2*x^2 + d^2)*d^3*e) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

mupad [B] time = 2.85, size = 66, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^3+d^2\,e\,x+4\,d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d^4\,e\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*e^3*x^3 - 2*d^3 + 4*d*e^2*x^2 + d^2*e*x))/(5*d^4*
e*(d + e*x)^3*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.176 $\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.18, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d - 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{(d-ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2+8dex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^4e^2+16d^3e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^6e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\right.}{2d^4}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx\right.}{d^4e}$$

$$= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Mathematica [A] time = 0.09, size = 95, normalized size = 0.81

$$\frac{-15\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(26d^3+22d^2ex-17de^2x^2-16e^3x^3\right)}{(d-ex)(d+ex)^3}+15\log(x)}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(26*d^3 + 22*d^2*e*x - 17*d*e^2*x^2 - 16*e^3*x^3))/((
d - e*x)*(d + e*x)^3) + 15*Log[x] - 15*Log[d + Sqrt[d^2 - e^2*x^2]])/(15*d^
5)
```

IntegrateAlgebraic [A] time = 0.76, size = 111, normalized size = 0.94

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{\sqrt{d^2-e^2x^2}\left(26d^3+22d^2ex-17de^2x^2-16e^3x^3\right)}{15d^5(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(26*d^3 + 22*d^2*e*x - 17*d*e^2*x^2 - 16*e^3*x^3))/(15
*d^5*(d - e*x)*(d + e*x)^3) + (2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*
x^2]/d])/d^5
```

fricas [A] time = 0.40, size = 168, normalized size = 1.42

$$\frac{26\,e^4x^4+52\,de^3x^3-52\,d^3ex-26\,d^4+15\left(e^4x^4+2\,de^3x^3-2\,d^3ex-d^4\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16\,e^3x^3+17\,de^2x^2-22\,d^2ex-26\,d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^5e^4x^4+2\,d^6e^3x^3-2\,d^8ex-d^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(26*e^4*x^4 + 52*d*e^3*x^3 - 52*d^3*e*x - 26*d^4 + 15*(e^4*x^4 + 2*d*e
^3*x^3 - 2*d^3*e*x - d^4)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3
+ 17*d*e^2*x^2 - 22*d^2*e*x - 26*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^4*x^4 +
2*d^6*e^3*x^3 - 2*d^8*e*x - d^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 187, normalized size = 1.58

$$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^4}-\frac{16ex}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^5}+\frac{1}{5\left(x+\frac{d}{e}\right)^2\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2e^2}+\frac{8}{15\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3e}+\frac{1}{\sqrt{-e^2x^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/(-e^2*x^2+d^2)^(1/2)/d^4-1/(d^2)^(1/2)/d^4*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)+8/15/d^3/e/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-1
```

```
6/15/d^5*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+1/5/d^2/e^2/(x+d/e)^2/(2*(
x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(d^2-e^2\,x^2\right)^{3/2}\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
[Out] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.177 $\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.30, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (e*(10*d - 13*e*x))/(15*d^
4*(d^2 - e^2*x^2)^(3/2)) - (e*(30*d - 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
 - Sqrt[d^2 - e^2*x^2]/(d^6*x) + (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
&& !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{(d-ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$=-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{-5d^2+10dex-8e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$=-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{15d^2-30dex+26e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$=-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-15d^2+30dex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$=-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}-$$

$$=-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}-$$

$$=-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+$$

$$=-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+$$

Mathematica [A] time = 0.11, size = 112, normalized size = 0.77

$$\frac{30e\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(15d^4+76d^3ex+32d^2e^2x^2-82de^3x^3-56e^4x^4\right)}{x(ex-d)(d+ex)^3}-30e\log(x)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(15*d^4 + 76*d^3*e*x + 32*d^2*e^2*x^2 - 82*d*e^3*x^3
- 56*e^4*x^4))/(x*(-d + e*x)*(d + e*x)^3) - 30*e*Log[x] + 30*e*Log[d + Sqrt
[d^2 - e^2*x^2]])/(15*d^6)
```

IntegrateAlgebraic [A] time = 0.78, size = 126, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\,\left(-15d^4-76d^3ex-32d^2e^2x^2+82de^3x^3+56e^4x^4\right)}{15d^6x(d-ex)(d+ex)^3}-\frac{4e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-15*d^4 - 76*d^3*e*x - 32*d^2*e^2*x^2 + 82*d*e^3*x^3
+ 56*e^4*x^4))/(15*d^6*x*(d - e*x)*(d + e*x)^3) - (4*e*ArcTanh[(Sqrt[-e^2]*
x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^6
```

fricas [A] time = 0.42, size = 194, normalized size = 1.33

$$-\frac{46\,e^5x^5+92\,de^4x^4-92\,d^3e^2x^2-46\,d^4ex+30\left(e^5x^5+2\,de^4x^4-2\,d^3e^2x^2-d^4ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(56\,e^4x^4+82\,de^3x^3-32\,d^2e^2x^2-76\,d^3ex-15\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^6e^4x^5+2\,d^7e^3x^4-2\,d^9ex^2-d^{10}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(46*e^5*x^5 + 92*d*e^4*x^4 - 92*d^3*e^2*x^2 - 46*d^4*e*x + 30*(e^5*x^
5 + 2*d*e^4*x^4 - 2*d^3*e^2*x^2 - d^4*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/
x) + (56*e^4*x^4 + 82*d*e^3*x^3 - 32*d^2*e^2*x^2 - 76*d^3*e*x - 15*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^6*e^4*x^5 + 2*d^7*e^3*x^4 - 2*d^9*e*x^2 - d^10*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 234, normalized size = 1.60

$$\frac{2e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^5}+\frac{2e^2x}{\sqrt{-e^2x^2+d^2}\,d^6}+\frac{26e^2x}{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^6}-\frac{1}{5\left(x+\frac{d}{e}\right)^2\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3e}-\frac{13}{15\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^4}-\frac{2e}{\sqrt{-e^2x^2+d^2}\,d^5}-\frac{1}{\sqrt{-e^2x^2+d^2}\,d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/d^4/x/(-e^2*x^2+d^2)^(1/2)+2/(-e^2*x^2+d^2)^(1/2)/d^6*e^2*x-2/(-e^2*x^2+
d^2)^(1/2)/d^5*e+2/(d^2)^(1/2)/d^5*e*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-13/15/d^4/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+26/15/d^6*
e^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-1/5/d^3/e/(x+d/e)^2/(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \left(d^2-e^2 x^2\right)^{3/2} (d+e x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}} \left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.178 $\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=183

$$\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}$$

Rubi [A] time = 0.37, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*e^2*(d - e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d - 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d - 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{(d-ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2+10dex-10e^2x^2+\frac{8e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2-30dex+45e^2x^2-\frac{36e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2+30dex-6}{x^3\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} +$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} +$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} +$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} +$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} +$$

Mathematica [A] time = 0.13, size = 127, normalized size = 0.69

$$\frac{-45e^2\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(5d^5-10d^4ex-94d^3e^2x^2-58d^2e^3x^3+83de^4x^4+64e^5x^5\right)}{x^2(ex-d)(d+ex)^3}+45e^2\log(x)}{10d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(5*d^5 - 10*d^4*e*x - 94*d^3*e^2*x^2 - 58*d^2*e^3*x^3
 + 83*d*e^4*x^4 + 64*e^5*x^5))/(x^2*(-d + e*x)*(d + e*x)^3) + 45*e^2*Log[x]
 - 45*e^2*Log[d + Sqrt[d^2 - e^2*x^2]])/(10*d^7)
```

IntegrateAlgebraic [A] time = 0.95, size = 139, normalized size = 0.76

$$\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{\sqrt{d^2-e^2x^2}\left(-5d^5+10d^4ex+94d^3e^2x^2+58d^2e^3x^3-83de^4x^4-64e^5x^5\right)}{10d^7x^2(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^5 + 10*d^4*e*x + 94*d^3*e^2*x^2 + 58*d^2*e^3*x^3
 - 83*d*e^4*x^4 - 64*e^5*x^5))/(10*d^7*x^2*(d - e*x)*(d + e*x)^3) + (9*e^2*
ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^7
```

fricas [A] time = 0.43, size = 215, normalized size = 1.17

$$\frac{54\,e^{6}x^{6} + 108\,de^{5}x^{5} - 108\,d^{3}e^{3}x^{3} - 54\,d^{4}e^{2}x^{2} + 45\left(e^{6}x^{6} + 2\,de^{5}x^{5} - 2\,d^{3}e^{3}x^{3} - d^{4}e^{2}x^{2}\right)\log\left(-\frac{d - \sqrt{-e^{2}x^{2} + d^{2}}}{x}\right) + \left(64\,e^{5}x^{5} + 83\,de^{4}x^{4} - 58\,d^{2}e^{3}x^{3} - 94\,d^{3}e^{2}x^{2} - 10\,d^{4}ex + 5\,d^{5}\right)\sqrt{-e^{2}x^{2} + d^{2}}}{10\left(d^{7}e^{4}x^{6} + 2\,d^{8}e^{3}x^{5} - 2\,d^{10}ex^{3} - d^{11}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*(54*e^6*x^6 + 108*d*e^5*x^5 - 108*d^3*e^3*x^3 - 54*d^4*e^2*x^2 + 45*(e
^6*x^6 + 2*d*e^5*x^5 - 2*d^3*e^3*x^3 - d^4*e^2*x^2)*log(-(d - sqrt(-e^2*x^2
 + d^2))/x) + (64*e^5*x^5 + 83*d*e^4*x^4 - 58*d^2*e^3*x^3 - 94*d^3*e^2*x^2
- 10*d^4*e*x + 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^4*x^6 + 2*d^8*e^3*x^5 -
2*d^10*e*x^3 - d^11*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 259, normalized size = 1.42

$$-\frac{9e^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{2\sqrt{d^{2}}\,d^{6}} - \frac{4e^{3}x}{\sqrt{-e^{2}x^{2}+d^{2}}\,d^{7}} - \frac{12e^{3}x}{5\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{7}} + \frac{1}{5\left(x+\frac{d}{e}\right)^{2}\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{4}} + \frac{6e}{5\left(x+\frac{d}{e}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,d^{5}} + \frac{9e^{2}}{2\sqrt{-e^{2}x^{2}+d^{2}}\,d^{6}} + \frac{2e}{\sqrt{-e^{2}x^{2}+d^{2}}\,d^{5}x} - \frac{1}{2\sqrt{-e^{2}x^{2}+d^{2}}\,d^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 2/d^5*e/x/(-e^2*x^2+d^2)^(1/2)-4/(-e^2*x^2+d^2)^(1/2)/d^7*e^3*x-1/2/d^4/x^2
/(-e^2*x^2+d^2)^(1/2)+9/2/(-e^2*x^2+d^2)^(1/2)/d^6*e^2-9/2/(d^2)^(1/2)/d^6*
e^2*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+6/5/d^5*e/(x+d/e)/(2*(
x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-12/5/d^7*e^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)*x+1/5/d^4/(x+d/e)^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^{2}x^{2} + d^{2}\right)^{\frac{3}{2}}\left(ex + d\right)^{2}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3}\,\left(d^{2}-e^{2}\,x^{2}\right)^{3/2}\,\left(d+e\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

3.179 $\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=177

$$\frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}+\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}-\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.44, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}-\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (d^4*(d - e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d - e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d - e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 - (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) + (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^5(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(-\frac{3d^5}{e^5}+\frac{5d^4x}{e^4}-\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}-\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)\left(-\frac{37d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-\frac{90d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{15d^3x^2}{e^3}}{\sqrt{d^2-e^2x^2}}}{15d^3}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \int$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - x$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - x$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - x$$

Mathematica [A] time = 0.19, size = 98, normalized size = 0.55

$$\frac{195d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(304d^4+717d^3ex+479d^2e^2x^2+45de^3x^3-15e^4x^4\right)}{(d+ex)^3}}{30e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(304*d^4 + 717*d^3*e*x + 479*d^2*e^2*x^2 + 45*d*e^3*x
^3 - 15*e^4*x^4))/(d + e*x)^3 + 195*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
(30*e^6)
```

IntegrateAlgebraic [A] time = 0.69, size = 121, normalized size = 0.68

$$\frac{13d^2\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^7}+\frac{\sqrt{d^2-e^2x^2}\,\left(304d^4+717d^3ex+479d^2e^2x^2+45de^3x^3-15e^4x^4\right)}{30e^6(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(304*d^4 + 717*d^3*e*x + 479*d^2*e^2*x^2 + 45*d*e^3*x^
3 - 15*e^4*x^4))/(30*e^6*(d + e*x)^3) + (13*d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]
*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^7)
```

fricas [A] time = 0.43, size = 190, normalized size = 1.07

$$\frac{304\,d^2e^3x^3+912\,d^3e^2x^2+912\,d^4ex+304\,d^5-390\left(d^2e^3x^3+3\,d^3e^2x^2+3\,d^4ex+d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(15\,e^4x^4-45\,de^3x^3-479\,d^2e^2x^2-717\,d^3ex-304\,d^4\right)\sqrt{-e^2x^2+d^2}}{30\left(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(304*d^2*e^3*x^3 + 912*d^3*e^2*x^2 + 912*d^4*e*x + 304*d^5 - 390*(d^2*
e^3*x^3 + 3*d^3*e^2*x^2 + 3*d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (15*e^4*x^4 - 45*d*e^3*x^3 - 479*d^2*e^2*x^2 - 717*d^3*e*x - 30
4*d^4)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (3*d^
2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*ex
p(2)^3-18*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2
*exp(1)^8*exp(2)-8*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^3*exp(1)^6*exp(2)^2+5*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^3*exp(2)^5-9*d^2*exp(1)^4*exp(2)^3+6*d^2*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^5+6*d^2*exp(2)^5-19/2
*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2)+14*d^2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^2/x/exp(2))/((-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^2/(-exp(1)^12+2*exp(1)^8*exp(2
)^2-exp(1)^4*exp(2)^4)+1/2*(-58*d^2*exp(1)^4*exp(2)^3+24*d^2*exp(2)^5+40*d^
2*exp(1)^8*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(exp(1)^14-2*ex
p(1)^10*exp(2)^2+exp(1)^6*exp(2)^4)+13/2*d^2*sign(d)*asin(x*exp(2)/d/exp(1)
)/exp(1)^6+2*(-2*exp(1)^11*1/8/exp(1)^16*x+12*exp(1)^10*d*1/8/exp(1)^16)*sq
rt(-exp(2)*x^2+d^2)
```

maple [A] time = 0.02, size = 212, normalized size = 1.20

$$\frac{13d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^5}-\frac{\sqrt{-e^2x^2+d^2}\,x}{2e^5}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^4}{5\left(x+\frac{d}{e}\right)^3e^9}-\frac{23\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3}{15\left(x+\frac{d}{e}\right)^2e^8}+\frac{127\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2}{15\left(x+\frac{d}{e}\right)e^7}+\frac{3\sqrt{-e^2x^2+d^2}\,d}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*x*(-e^2*x^2+d^2)^(1/2)/e^5+13/2/(e^2)^(1/2)*d^2/e^5*arctan((e^2)^(1/2)
/(-e^2*x^2+d^2)^(1/2)*x)+3*d*(-e^2*x^2+d^2)^(1/2)/e^6+1/5*d^4/e^9/(x+d/e)^3
*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-23/15*d^3/e^8/(x+d/e)^2*(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(1/2)+127/15/e^7*d^2/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)
```

maxima [A] time = 1.00, size = 185, normalized size = 1.05

$$\frac{\sqrt{-e^2x^2+d^2}\,d^4}{5\left(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6\right)}-\frac{23\,\sqrt{-e^2x^2+d^2}\,d^3}{15\left(e^8x^2+2\,de^7x+d^2e^6\right)}+\frac{127\,\sqrt{-e^2x^2+d^2}\,d^2}{15\left(e^7x+de^6\right)}+\frac{13\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^6}-\frac{\sqrt{-e^2x^2+d^2}\,x}{2\,e^5}+\frac{3\,\sqrt{-e^2x^2+d^2}\,d}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-e^2*x^2 + d^2)*d^4/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6
) - 23/15*sqrt(-e^2*x^2 + d^2)*d^3/(e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 127/15
*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x + d*e^6) + 13/2*d^2*arcsin(e*x/d)/e^6 - 1/
2*sqrt(-e^2*x^2 + d^2)*x/e^5 + 3*sqrt(-e^2*x^2 + d^2)*d/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.180 $\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=146

$$\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

Rubi [A] time = 0.37, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1635, 641, 217, 203}

$$-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(d^3*(d - e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (6*d^2*(d - e*x)^2)/(5*e
^5*(d^2 - e^2*x^2)^(3/2)) - (24*d*(d - e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2]) -
Sqrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(\frac{3d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)\left(\frac{27d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{45d^4}{e^4}-\frac{15d^3x}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\ \cdots}{\cdots}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\ \cdots}{\cdots}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\tan\cdots}{\cdots}$$

Mathematica [A] time = 0.15, size = 85, normalized size = 0.58

$$-\frac{15d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(24d^3+57d^2ex+39de^2x^2+5e^3x^3\right)}{(d+ex)^3}}{5e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*((Sqrt[d^2 - e^2*x^2]*(24*d^3 + 57*d^2*e*x + 39*d*e^2*x^2 + 5*e^3*x^3)
)/(d + e*x)^3 + 15*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

IntegrateAlgebraic [A] time = 0.72, size = 106, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2x^2}\left(-24d^3-57d^2ex-39de^2x^2-5e^3x^3\right)}{5e^5(d+ex)^3} - \frac{3d\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^3 - 57*d^2*e*x - 39*d*e^2*x^2 - 5*e^3*x^3))/(5*
e^5*(d + e*x)^3) - (3*d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2
]])/e^6
```

fricas [A] time = 0.41, size = 174, normalized size = 1.19

$$-\frac{24\,de^3x^3 + 72\,d^2e^2x^2 + 72\,d^3ex + 24\,d^4 - 30\left(de^3x^3 + 3\,d^2e^2x^2 + 3\,d^3ex + d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(5\,e^3x^3 + 39\,de^2x^2 + 57\,d^2ex + 24\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(e^8x^3 + 3\,de^7x^2 + 3\,d^2e^6x + d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(24*d*e^3*x^3 + 72*d^2*e^2*x^2 + 72*d^3*e*x + 24*d^4 - 30*(d*e^3*x^3 +
 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
 + (5*e^3*x^3 + 39*d*e^2*x^2 + 57*d^2*e*x + 24*d^3)*sqrt(-e^2*x^2 + d^2))/(
e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-d*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2
)^3+14*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(
1)^8*exp(2)+6*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^3*exp(1)^6*exp(2)^2-3*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^3*exp(2)^5+7*d*exp(1)^4*exp(2)^3-4*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^5-4*d*exp(2)^5+13/2*d*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2)-11*d*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^2/x/exp(2))/((-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x+exp(2))^2/(-exp(1)^11+2*exp(1)^7*exp(2)^2-exp(1)*exp(2)^5
)+1/2*(30*d*exp(1)^4*exp(2)^3-12*d*exp(2)^5-24*d*exp(1)^8*exp(2))*atan((-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(
2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(exp(1)^13-2*exp(1)^9*exp(2)^2+exp(1)^5*exp
(2)^4)-3*d*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^5-4*exp(1)^4*1/4/exp(1)^9
*sqrt(-exp(2)*x^2+d^2)
```

maple [A] time = 0.02, size = 187, normalized size = 1.28

$$-\frac{3d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^4} - \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^3}{5\left(x+\frac{d}{e}\right)^3e^8} + \frac{6\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^2}{5\left(x+\frac{d}{e}\right)^2e^7} - \frac{24\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d}{5\left(x+\frac{d}{e}\right)e^6} - \frac{\sqrt{-e^2x^2+d^2}}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e^5-3/(e^2)^(1/2)*d/e^4*arctan((e^2)^(1/2)/(-e^2*x^2+
d^2)^(1/2)*x)-1/5*d^3/e^8/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+6/5
*d^2/e^7/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-24/5/e^6*d/(x+d/e)*(
2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 0.98, size = 160, normalized size = 1.10

$$-\frac{\sqrt{-e^2x^2+d^2}\,d^3}{5\left(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5\right)} + \frac{6\sqrt{-e^2x^2+d^2}\,d^2}{5\left(e^7x^2+2\,de^6x+d^2e^5\right)} - \frac{24\sqrt{-e^2x^2+d^2}\,d}{5\left(e^6x+de^5\right)} - \frac{3\,d\arcsin\left(\frac{ex}{d}\right)}{e^5} - \frac{\sqrt{-e^2x^2+d^2}}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*d^3/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^
5) + 6/5*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) - 24/5*sq
rt(-e^2*x^2 + d^2)*d/(e^6*x + d*e^5) - 3*d*arcsin(e*x/d)/e^5 - sqrt(-e^2*x^
2 + d^2)/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{d^2 - e^2 x^2}\,(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-(-d + ex)(d + ex)}\,(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.181 $\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=120

$$\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Rubi [A] time = 0.26, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1635, 778, 217, 203}

$$\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (d^2*(d - e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d - e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d - e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) + Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
```

```
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$
$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(-\frac{3d^3}{e^3}+\frac{5d^2x}{e^2}-\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$
$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\left(-\frac{17d^3}{e^3}+\frac{15d^2x}{e^2}\right)(d-ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$
$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}$$
$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,\right.}{e^3}$$
$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Mathematica [A] time = 0.11, size = 73, normalized size = 0.61

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(22d^2+51dex+32e^2x^2\right)}{(d+ex)^3}+15\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] ((Sqrt[d^2 - e^2*x^2]*(22*d^2 + 51*d*e*x + 32*e^2*x^2))/(d + e*x)^3 + 15*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^4)
```

IntegrateAlgebraic [A] time = 0.60, size = 93, normalized size = 0.78

$$\frac{\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^5}+\frac{\sqrt{d^2-e^2x^2}\,\left(22d^2+51dex+32e^2x^2\right)}{15e^4(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(22*d^2 + 51*d*e*x + 32*e^2*x^2))/(15*e^4*(d + e*x)^3)
 + (Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^5
```

fricas [A] time = 0.41, size = 157, normalized size = 1.31

$$\frac{22\,e^3x^3+66\,de^2x^2+66\,d^2ex+22\,d^3-30\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(32\,e^2x^2+51\,dex+22\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(22*e^3*x^3 + 66*d*e^2*x^2 + 66*d^2*e*x + 22*d^3 - 30*(e^3*x^3 + 3*d*e
^2*x^2 + 3*d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (32*e
^2*x^2 + 51*d*e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 + 3*d*e^6*x^2 +
3*d^2*e^5*x + d^3*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^
3-10*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8
*exp(2)-4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^6*exp(2)^2+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
3*exp(2)^5-5*exp(1)^4*exp(2)^3+2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^2*exp(2)^5+2*exp(2)^5-7/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))*exp(2)^5/x/exp(2)+8*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)*exp(1)^6*exp(2)^2/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(
2))^2/(-exp(1)^10+2*exp(1)^6*exp(2)^2-exp(1)^2*exp(2)^4)+1/2*(-10*exp(1)^4*
exp(2)^3+4*exp(2)^5+12*exp(1)^8*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(
2)^2)/(exp(1)^12-2*exp(1)^8*exp(2)^2+exp(1)^4*exp(2)^4)+sign(d)*asin(x*exp(
2)/d/exp(1))/exp(1)^4
```

maple [A] time = 0.01, size = 163, normalized size = 1.36

$$\frac{\arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\, e^3}+\frac{\sqrt{2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2}\, d^2}{5\left(x+\frac{d}{e}\right)^3 e^7}-\frac{13\sqrt{2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2}\, d}{15\left(x+\frac{d}{e}\right)^2 e^6}+\frac{32\sqrt{2\left(x+\frac{d}{e}\right) de-\left(x+\frac{d}{e}\right)^2 e^2}}{15\left(x+\frac{d}{e}\right) e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] 1/(e^2)^(1/2)/e^3*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/5*d^2/e^7/(x
+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-13/15*d/e^6/(x+d/e)^2*(2*(x+d/e
)*d*e-(x+d/e)^2*e^2)^(1/2)+32/15/e^5/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)
```

maxima [A] time = 0.99, size = 136, normalized size = 1.13

$$\frac{\sqrt{-e^2x^2+d^2}\, d^2}{5\left(e^7x^3+3\, de^6x^2+3\, d^2e^5x+d^3e^4\right)}-\frac{13\sqrt{-e^2x^2+d^2}\, d}{15\left(e^6x^2+2\, de^5x+d^2e^4\right)}+\frac{32\sqrt{-e^2x^2+d^2}}{15\left(e^5x+de^4\right)}+\frac{\arcsin\left(\frac{ex}{d}\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4
) - 13/15*sqrt(-e^2*x^2 + d^2)*d/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) + 32/15*sq
rt(-e^2*x^2 + d^2)/(e^5*x + d*e^4) + arcsin(e*x/d)/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.182 $\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=95

$$-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = $\{1639, 793, 659, 651\}$

$$-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(d*Sqrt[d^2 - e^2*x^2])/(5*e^3*(d + e*x)^3) + (8*Sqrt[d^2 - e^2*x^2])/(15*
e^3*(d + e*x)^2) - (7*Sqrt[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{\int \frac{2d^2e^2+de^3x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&=-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{(7d)\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5e^2}\\
&=-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}+\frac{7\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15e^2}\\
&=-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 52, normalized size = 0.55

$$
-\frac{\sqrt{d^2-e^2x^2}\left(2d^2+6dex+7e^2x^2\right)}{15de^3(d+ex)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(2*d^2 + 6*d*e*x + 7*e^2*x^2))/(d*e^3*(d + e*x)^
3)
```

IntegrateAlgebraic [A] time = 0.53, size = 52, normalized size = 0.55

$$
\frac{\left(-2d^2-6dex-7e^2x^2\right)\sqrt{d^2-e^2x^2}}{15de^3(d+ex)^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((-2*d^2 - 6*d*e*x - 7*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x)^3)
```

fricas [A] time = 0.40, size = 104, normalized size = 1.09

$$
-\frac{2\,e^3x^3+6\,de^2x^2+6\,d^2ex+2\,d^3+\left(7\,e^2x^2+6\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^3*x^3 + 6*d*e^2*x^2 + 6*d^2*e*x + 2*d^3 + (7*e^2*x^2 + 6*d*e*x +
 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x + d^
4*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (3*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)
```

```
^3+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8
*exp(2)+2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^6*exp(2)^2+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
3*exp(2)^5+3*exp(1)^4*exp(2)^3+1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))*exp(2)^5/x/exp(2)-5*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^
6*exp(2)^2/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^2/(-d
*exp(1)^9+2*d*exp(1)^5*exp(2)^2-d*exp(1)*exp(2)^4)+1/2*(-2*exp(2)^4-4*exp(1
)^6*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2)
)/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(d*exp(1)^9-2*d*exp(1)
^5*exp(2)^2+d*exp(1)*exp(2)^4)
```

maple [A] time = 0.01, size = 55, normalized size = 0.58

$$-\frac{(-ex+d)\left(7e^2x^2+6dex+2d^2\right)}{15\,(ex+d)^2\sqrt{-e^2x^2+d^2}\;d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)*(7*e^2*x^2+6*d*e*x+2*d^2)/(e*x+d)^2/d/e^3/(-e^2*x^2+d^2)^(1/
2)
```

maxima [A] time = 0.98, size = 125, normalized size = 1.32

$$-\frac{\sqrt{-e^2x^2+d^2}\,d}{5\left(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3\right)}+\frac{8\sqrt{-e^2x^2+d^2}}{15\left(e^5x^2+2\,de^4x+d^2e^3\right)}-\frac{7\sqrt{-e^2x^2+d^2}}{15\left(de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*d/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
 + 8/15*sqrt(-e^2*x^2 + d^2)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) - 7/15*sqrt(-e
^2*x^2 + d^2)/(d*e^4*x + d^2*e^3)
```

mupad [B] time = 2.76, size = 48, normalized size = 0.51

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+6\,d\,e\,x+7\,e^2\,x^2\right)}{15\,d\,e^3\,(d+e\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(2*d^2 + 7*e^2*x^2 + 6*d*e*x))/(15*d*e^3*(d + e*x)^
3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(-d+ex)(d+ex)}\;(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.183 $\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {793, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*e^2*(d + e*x)^3) - Sqrt[d^2 - e^2*x^2]/(5*d*e^2*(d +
 e*x)^2) - Sqrt[d^2 - e^2*x^2]/(5*d^2*e^2*(d + e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}+\frac{3\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5e}\\
&= \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}+\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{5de}\\
&= \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.51

$$-\frac{\sqrt{d^2-e^2x^2}\left(d^2+3dex+e^2x^2\right)}{5d^2e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] -1/5*(Sqrt[d^2 - e^2*x^2]*(d^2 + 3*d*e*x + e^2*x^2))/(d^2*e^2*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.48, size = 52, normalized size = 0.54

$$\frac{\sqrt{d^2-e^2x^2}\left(-d^2-3dex-e^2x^2\right)}{5d^2e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 - 3*d*e*x - e^2*x^2))/(5*d^2*e^2*(d + e*x)^3)
```

fricas [A] time = 0.40, size = 100, normalized size = 1.03

$$-\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3+\left(e^2x^2+3\,dex+d^2\right)\sqrt{-e^2x^2+d^2}}{5\left(d^2e^5x^3+3\,d^3e^4x^2+3\,d^4e^3x+d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -1/5*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3 + (e^2*x^2 + 3*d*e*x + d^2)*s
qrt(-e^2*x^2 + d^2))/(d^2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (2*ex
p(1)*exp(2)^5+5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^2*exp(1)^5*exp(2)^3+2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^2*exp(1)^9*exp(2)+2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)*exp(2)^5+3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^3*exp(1)^3*exp(2)^4+exp(1)^5*exp(2)^3-5/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^4/x/exp(2)-2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^2/x/exp(2))/((-1/2*(-2*d*exp(1)-
2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x+exp(2))^2/(d^2*exp(1)^9-2*d^2*exp(1)^5*exp(2)^2+d^2*ex
p(1)*exp(2)^4)+3*exp(1)^3*exp(2)^3*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)
/(d^2*exp(1)^9-2*d^2*exp(1)^5*exp(2)^2+d^2*exp(1)*exp(2)^4)
```

maple [A] time = 0.01, size = 52, normalized size = 0.54

$$-\frac{(-ex+d)\left(e^2x^2+3dex+d^2\right)}{5\,(ex+d)^2\sqrt{-e^2x^2+d^2}\;d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*(-e*x+d)*(e^2*x^2+3*d*e*x+d^2)/(e*x+d)^2/d^2/e^2/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 0.98, size = 129, normalized size = 1.33

$$\frac{\sqrt{-e^2x^2+d^2}}{5\left(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2\right)}-\frac{\sqrt{-e^2x^2+d^2}}{5\left(de^4x^2+2\,d^2e^3x+d^3e^2\right)}-\frac{\sqrt{-e^2x^2+d^2}}{5\left(d^2e^3x+d^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-e^2*x^2 + d^2)/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2) -
1/5*sqrt(-e^2*x^2 + d^2)/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) - 1/5*sqrt(-e^
2*x^2 + d^2)/(d^2*e^3*x + d^3*e^2)
```

mupad [B] time = 2.59, size = 45, normalized size = 0.46

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+3\,d\,e\,x+e^2\,x^2\right)}{5\,d^2\,e^2\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 + 3*d*e*x))/(5*d^2*e^2*(d + e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.184 $\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}$$

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(5*d*e*(d + e*x)^3) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*
e*(d + e*x)^2) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}+\frac{2\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}+\frac{2\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.52

$$-\frac{\sqrt{d^2-e^2x^2}\left(7d^2+6dex+2e^2x^2\right)}{15d^3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(7*d^2 + 6*d*e*x + 2*e^2*x^2))/(d^3*e*(d + e*x)^
3)
```

IntegrateAlgebraic [A] time = 0.00, size = 52, normalized size = 0.52

$$\frac{\left(-7d^2-6dex-2e^2x^2\right)\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((-7*d^2 - 6*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x)^3)
```

fricas [A] time = 0.41, size = 104, normalized size = 1.04

$$-\frac{7\,e^3x^3+21\,de^2x^2+21\,d^2ex+7\,d^3+\left(2\,e^2x^2+6\,dex+7\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\,\left(d^3e^4x^3+3\,d^4e^3x^2+3\,d^5e^2x+d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*e^3*x^3 + 21*d*e^2*x^2 + 21*d^2*e*x + 7*d^3 + (2*e^2*x^2 + 6*d*e*x
 + 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + 3*d^4*e^3*x^2 + 3*d^5*e^2*x
+ d^6*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (7*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)
^3-2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^1
0*exp(2)-2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*ex
p(1)^8*exp(2)^2+5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^3*exp(1)^4*exp(2)^4-exp(1)^6*exp(2)^3+4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^6+4*exp(2)^6-11/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^4/x/exp(2)+(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^2/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x+exp(2))^2/(-d^3*exp(1)^9+2*d^3*exp(1)^5*exp(2)^2-d^3*exp(1)*e
xp(2)^4)+1/2*(-2*exp(1)^4*exp(2)^3-4*exp(2)^5)*atan((-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)
^4+exp(2)^2)/(d^3*exp(1)^9-2*d^3*exp(1)^5*exp(2)^2+d^3*exp(1)*exp(2)^4)
```

maple [A] time = 0.01, size = 55, normalized size = 0.55

$$-\frac{(-ex+d)\left(2e^2x^2+6dex+7d^2\right)}{15\,(ex+d)^2\,\sqrt{-e^2x^2+d^2}\;d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)*(2*e^2*x^2+6*d*e*x+7*d^2)/(e*x+d)^2/d^3/e/(-e^2*x^2+d^2)^(1/
2)
```

maxima [A] time = 0.98, size = 128, normalized size = 1.28

$$-\frac{\sqrt{-e^2x^2+d^2}}{5\left(de^4x^3+3\,d^2e^3x^2+3\,d^3e^2x+d^4e\right)}-\frac{2\,\sqrt{-e^2x^2+d^2}}{15\left(d^2e^3x^2+2\,d^3e^2x+d^4e\right)}-\frac{2\,\sqrt{-e^2x^2+d^2}}{15\left(d^3e^2x+d^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e^2*x + d^4*e)
 - 2/15*sqrt(-e^2*x^2 + d^2)/(d^2*e^3*x^2 + 2*d^3*e^2*x + d^4*e) - 2/15*sqr
t(-e^2*x^2 + d^2)/(d^3*e^2*x + d^4*e)
```

mupad [B] time = 2.62, size = 48, normalized size = 0.48

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2+6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 + 6*d*e*x))/(15*d^3*e*(d + e*x)^
3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.185 $\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=115

$$\frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Rubi [A] time = 0.18, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$\frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}}+\frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (4*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d - 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d - 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+11d^2ex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^5e^2+22d^4e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^7e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,\right.}{2d^3}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \right.}{d^3e^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Mathematica [A] time = 0.12, size = 76, normalized size = 0.66

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(32d^2+51dex+22e^2x^2\right)}{(d+ex)^3} - 15\log\left(\sqrt{d^2-e^2x^2}+d\right) + 15\log(x)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(32*d^2 + 51*d*e*x + 22*e^2*x^2))/(d + e*x)^3 + 15*Lo
g[x] - 15*Log[d + Sqrt[d^2 - e^2*x^2]])/(15*d^4)
```

IntegrateAlgebraic [A] time = 0.91, size = 92, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\left(32d^2+51dex+22e^2x^2\right)}{15d^4(d+ex)^3}+\frac{2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^2 + 51*d*e*x + 22*e^2*x^2))/(15*d^4*(d + e*x)^3)
 + (2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^4
```

fricas [A] time = 0.40, size = 153, normalized size = 1.33

$$\frac{32\,e^3x^3+96\,de^2x^2+96\,d^2ex+32\,d^3+15\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(22\,e^2x^2+51\,dex+32\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(d^4e^3x^3+3\,d^5e^2x^2+3\,d^6ex+d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(32*e^3*x^3 + 96*d*e^2*x^2 + 96*d^2*e*x + 32*d^3 + 15*(e^3*x^3 + 3*d*e
^2*x^2 + 3*d^2*e*x + d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (22*e^2*x^2
+ 51*d*e*x + 32*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 + 3*d^5*e^2*x^2 + 3
*d^6*e*x + d^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-9*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^7*exp(2
)^3+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^
11*exp(2)+4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*e
xp(1)^9*exp(2)^2-7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^3*exp(1)^5*exp(2)^4+3*exp(1)^7*exp(2)^3-6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^3*exp(2)^5-6*exp(1)^3*exp(2)^5+17/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^4/x/exp(2)-4*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9*exp(2)^2/x/exp(2))/((-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^2/(-d^4*exp(1)^9+2*d^4*exp(1)^
5*exp(2)^2-d^4*exp(1)*exp(2)^4)+1/2*(-10*exp(1)^5*exp(2)^3+4*exp(1)^9*exp(2
)+12*exp(1)*exp(2)^5)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(d^4*exp(1)^
9-2*d^4*exp(1)^5*exp(2)^2+d^4*exp(1)*exp(2)^4)-exp(2)*ln(1/2*abs(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^4/exp(1)^2
```

maple [A] time = 0.01, size = 179, normalized size = 1.56

$$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^3}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{5\left(x+\frac{d}{e}\right)^3d^2e^3}+\frac{7\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{15\left(x+\frac{d}{e}\right)^2d^3e^2}+\frac{22\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{15\left(x+\frac{d}{e}\right)d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/(d^2)^(1/2)/d^3*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^2
/e^3/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+7/15/d^3/e^2/(x+d/e)^2*(
2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+22/15/d^4/e/(x+d/e)*(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}\,(ex+d)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.186 $\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.30, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-4*e*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d - 7*e*x))/(5*d^3*(d^
2 - e^2*x^2)^(3/2)) - (e*(15*d - 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqr
t[d^2 - e^2*x^2]/(d^5*x) + (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+15d^2ex-16de^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3-45d^2ex+42de^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3+45d^2ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{(3e)\int}{}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{(3e)\,\mathrm{Su}}{}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{3\,\mathrm{Subs}}{}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{3e\tan}{}$$

Mathematica [A] time = 0.18, size = 92, normalized size = 0.63

$$-\frac{-15e\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(5d^3+39d^2ex+57de^2x^2+24e^3x^3\right)}{x(d+ex)^3}+15e\log(x)}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*((Sqrt[d^2 - e^2*x^2]*(5*d^3 + 39*d^2*e*x + 57*d*e^2*x^2 + 24*e^3*x^3)
)/(x*(d + e*x)^3) + 15*e*Log[x] - 15*e*Log[d + Sqrt[d^2 - e^2*x^2]])/d^5
```

IntegrateAlgebraic [A] time = 0.70, size = 107, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2x^2}\,\left(-5d^3-39d^2ex-57de^2x^2-24e^3x^3\right)}{5d^5x(d+ex)^3}-\frac{6e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^3 - 39*d^2*e*x - 57*d*e^2*x^2 - 24*e^3*x^3))/(5*
d^5*x*(d + e*x)^3) - (6*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d]
)/d^5
```

fricas [A] time = 0.41, size = 181, normalized size = 1.24

$$-\frac{24\,e^4x^4+72\,de^3x^3+72\,d^2e^2x^2+24\,d^3ex+15\left(e^4x^4+3\,de^3x^3+3\,d^2e^2x^2+d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(24\,e^3x^3+57\,de^2x^2+39\,d^2ex+5\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(d^5e^3x^4+3\,d^6e^2x^3+3\,d^7ex^2+d^8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(24*e^4*x^4 + 72*d*e^3*x^3 + 72*d^2*e^2*x^2 + 24*d^3*e*x + 15*(e^4*x^4
 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x
) + (24*e^3*x^3 + 57*d*e^2*x^2 + 39*d^2*e*x + 5*d^3)*sqrt(-e^2*x^2 + d^2))/
(d^5*e^3*x^4 + 3*d^6*e^2*x^3 + 3*d^7*e*x^2 + d^8*x)
```

giac [A] time = 0.30, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [A] time = 0.01, size = 199, normalized size = 1.36

$$\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^4}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{5\left(x+\frac{d}{e}\right)^3d^3e^2}-\frac{4\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{5\left(x+\frac{d}{e}\right)^2d^4e}-\frac{19\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{5\left(x+\frac{d}{e}\right)d^5}-\frac{\sqrt{-e^2x^2+d^2}}{d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d^5/x+3/(d^2)^(1/2)/d^4*e*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)-1/5/d^3/e^2/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(
1/2)-4/5/d^4/e/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-19/5/d^5/(x+d/
e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-e^2x^2+d^2}\,(ex+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.187 $\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=183

$$\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.38, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (4*e^2*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d - 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d - 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) + (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+15d^2ex-20de^2x^2+16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3-45d^2ex+75de^2x^2-62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3+45d^2ex-90de^2}{x^3\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \int -9$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + 3e\sqrt{}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + 3e\sqrt{}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + 3e\sqrt{}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + 3e\sqrt{}$$

Mathematica [A] time = 0.17, size = 107, normalized size = 0.58

$$\frac{-195e^2\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(-15d^4+45d^3ex+479d^2e^2x^2+717de^3x^3+304e^4x^4\right)}{x^2(d+ex)^3}+195e^2\log(x)}{30d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 45*d^3*e*x + 479*d^2*e^2*x^2 + 717*d*e^3*x
^3 + 304*e^4*x^4))/(x^2*(d + e*x)^3) + 195*e^2*Log[x] - 195*e^2*Log[d + Sqr
t[d^2 - e^2*x^2]])/(30*d^6)
```

IntegrateAlgebraic [A] time = 0.96, size = 120, normalized size = 0.66

$$\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{\sqrt{d^2-e^2x^2}\left(-15d^4+45d^3ex+479d^2e^2x^2+717de^3x^3+304e^4x^4\right)}{30d^6x^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 45*d^3*e*x + 479*d^2*e^2*x^2 + 717*d*e^3*x^
3 + 304*e^4*x^4))/(30*d^6*x^2*(d + e*x)^3) + (13*e^2*ArcTanh[(Sqrt[-e^2]*x)
/d - Sqrt[d^2 - e^2*x^2]/d])/d^6
```

fricas [A] time = 0.42, size = 202, normalized size = 1.10

$$\frac{254\,e^5x^5+762\,de^4x^4+762\,d^2e^3x^3+254\,d^3e^2x^2+195\left(e^5x^5+3\,de^4x^4+3\,d^2e^3x^3+d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(304\,e^4x^4+717\,de^3x^3+479\,d^2e^2x^2+45\,d^3ex-15\,d^4\right)\sqrt{-e^2x^2+d^2}}{30\left(d^6e^3x^5+3\,d^7e^2x^4+3\,d^8ex^3+d^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(254*e^5*x^5 + 762*d*e^4*x^4 + 762*d^2*e^3*x^3 + 254*d^3*e^2*x^2 + 195
*(e^5*x^5 + 3*d*e^4*x^4 + 3*d^2*e^3*x^3 + d^3*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) + (304*e^4*x^4 + 717*d*e^3*x^3 + 479*d^2*e^2*x^2 + 45*d^3*e*
x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^5 + 3*d^7*e^2*x^4 + 3*d^8*e*x^
3 + d^9*x^2)
```

giac [A] time = 0.29, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [A] time = 0.01, size = 222, normalized size = 1.21

$$-\frac{13e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^5}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{5\left(x+\frac{d}{e}\right)^3d^4e}+\frac{17\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{15\left(x+\frac{d}{e}\right)^2d^5}+\frac{107\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e}{15\left(x+\frac{d}{e}\right)d^6}+\frac{3\sqrt{-e^2x^2+d^2}\,e}{d^6x}-\frac{\sqrt{-e^2x^2+d^2}}{2d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] 3*e*(-e^2*x^2+d^2)^(1/2)/d^6/x-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2-13/2/(d^2)^
(1/2)/d^5*e^2*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^4/e/(x
```

```
+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+17/15/d^5/(x+d/e)^2*(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(1/2)+107/15/d^6*e/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-e^2x^2 + d^2}\,(ex + d)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{d^2 - e^2\,x^2}\,\left(d + e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{-(-d + ex)(d + ex)}\,(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.188 $\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=204

$$\frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}+\frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6}-\frac{2dx\sqrt{d^2-e^2x^2}}{e^5}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{18d^3}{}$$

Rubi [A] time = 0.59, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{2dx\sqrt{d^2-e^2x^2}}{e^5}+\frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6}+\frac{18d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (d^4*(d - e*x)^4)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (8*d^3*(d - e*x)^3)/(5*e^
6*(d^2 - e^2*x^2)^(3/2)) + (10*d^2*(d - e*x)^2)/(e^6*Sqrt[d^2 - e^2*x^2]) +
 (59*d^2*Sqrt[d^2 - e^2*x^2])/(3*e^6) - (2*d*x*Sqrt[d^2 - e^2*x^2])/e^5 + (
x^2*Sqrt[d^2 - e^2*x^2])/(3*e^4) + (18*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
```

```
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^5(d-ex)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^5}{e^5}+\frac{5d^4x}{e^4}-\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}-\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)^2\left(-\frac{60d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)\left(-\frac{240d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{15d^3x^2}{e^3}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} + \frac{\int \frac{\frac{720d^6}{e^3}-\frac{885d^5x}{e^2}+1}{\sqrt{d^2-e^2x^2}}}{45d^3e^2}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-e^2}}{e^5}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-e^2}}{e^5}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-e^2}}{e^5}$$

Mathematica [A] time = 0.18, size = 109, normalized size = 0.53

$$\frac{270d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(424d^5+1002d^4ex+674d^3e^2x^2+70d^2e^3x^3-15de^4x^4+5e^5x^5\right)}{(d+ex)^3}}{15e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(424*d^5 + 1002*d^4*e*x + 674*d^3*e^2*x^2 + 70*d^2*e^
3*x^3 - 15*d*e^4*x^4 + 5*e^5*x^5))/(d + e*x)^3 + 270*d^3*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(15*e^6)
```

IntegrateAlgebraic [A] time = 0.76, size = 130, normalized size = 0.64

$$\frac{18d^3\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^7}+\frac{\sqrt{d^2-e^2x^2}\,\left(424d^5+1002d^4ex+674d^3e^2x^2+70d^2e^3x^3-15de^4x^4+5e^5x^5\right)}{15e^6(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(424*d^5 + 1002*d^4*e*x + 674*d^3*e^2*x^2 + 70*d^2*e^3
*x^3 - 15*d*e^4*x^4 + 5*e^5*x^5))/(15*e^6*(d + e*x)^3) + (18*d^3*Sqrt[-e^2]
*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^7
```

fricas [A] time = 0.44, size = 200, normalized size = 0.98

$$\frac{424\,d^3e^3x^3+1272\,d^4e^2x^2+1272\,d^5ex+424\,d^6-540\,\left(d^3e^3x^3+3\,d^4e^2x^2+3\,d^5ex+d^6\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(5\,e^5x^5-15\,de^4x^4+70\,d^2e^3x^3+674\,d^3e^2x^2+1002\,d^4ex+424\,d^5\right)\sqrt{-e^2x^2+d^2}}{15\,\left(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(424*d^3*e^3*x^3 + 1272*d^4*e^2*x^2 + 1272*d^5*e*x + 424*d^6 - 540*(d^
3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^2 + d
^2))/(e*x)) + (5*e^5*x^5 - 15*d*e^4*x^4 + 70*d^2*e^3*x^3 + 674*d^3*e^2*x^2
+ 1002*d^4*e*x + 424*d^5)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 + 3*d*e^8*x^2 + 3*
d^2*e^7*x + d^3*e^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-162
*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^1
2*exp(2)^2-36*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^5*exp(1)^10*exp(2)^3+240*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^3*exp(1)^12*exp(2)^2+228*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+54*d^3*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4-402*d^3*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)
^2+158*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*ex
p(1)^10*exp(2)^3+339*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^4*exp(1)^8*exp(2)^4+87*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5+492*d^3*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3+192*d^3*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-96*d^3
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp
(2)^5-36*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*
exp(1)^4*exp(2)^6+840*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^2*exp(1)^8*exp(2)^4+420*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^5-102*d^3*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6-48*d^3*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8-228*d^3*(-1/2*(
```

```
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-25
2*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^
4*exp(2)^6-47*d^3*exp(1)^8*exp(2)^4-102*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8-288*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6+60*d^3*exp(1)^6*exp(2)
^5-360*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*ex
p(2)^8+110*d^3*exp(1)^4*exp(2)^6-204*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-102*d^3*exp(2)^8-188*d^3*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+156*d^3*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+108*d^3*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)-573/2*d^3*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)-153*
d^3*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)+
123*d^3*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/ex
p(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)
-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^16-6*exp
(1)^12*exp(2)^2-6*exp(1)^10*exp(2)^3+3*exp(1)^8*exp(2)^4+3*exp(1)^6*exp(2)^
5+3*exp(1)^14*exp(2))+1/2*(-100*d^3*exp(1)^10*exp(2)^2-170*d^3*exp(1)^8*exp
(2)^3+152*d^3*exp(1)^6*exp(2)^4+208*d^3*exp(1)^4*exp(2)^5-144*d^3*exp(2)^7+
40*d^3*exp(1)^12*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^
18+2*exp(1)^14*exp(2)^2+2*exp(1)^12*exp(2)^3-exp(1)^10*exp(2)^4-exp(1)^8*ex
p(2)^5-exp(1)^16*exp(2))+18*d^3*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^6+2*
((2*exp(1)^16*1/12/exp(1)^20*x-12*exp(1)^15*d*1/12/exp(1)^20)*x+58*exp(1)^1
4*d^2*1/12/exp(1)^20)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.02, size = 297, normalized size = 1.46

$$\frac{20 d^{3} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}\right)}{\sqrt{e^{2}}\, e^{5}}-\frac{2 d^{3} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\, e^{5}}-\frac{2 \sqrt{-e^{2} x^{2}+d^{2}}\, d x}{e^{5}}+\frac{20 \sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, d^{2}}{e^{6}}+\frac{\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} d^{4}}{5\left(x+\frac{d}{e}\right)^{4} e^{10}}-\frac{8\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} d^{3}}{5\left(x+\frac{d}{e}\right)^{3} e^{9}}+\frac{10\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} d^{2}}{\left(x+\frac{d}{e}\right)^{2} e^{8}}-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{3 e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/3/e^6*(-e^2*x^2+d^2)^(3/2)-2*d*x*(-e^2*x^2+d^2)^(1/2)/e^5-2/e^5*d^3/(e^2
)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/5*d^4/e^10/(x+d/e)^4*(
2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-8/5/e^9*d^3/(x+d/e)^3*(2*(x+d/e)*d*e-(x+
d/e)^2*e^2)^(3/2)+20/e^6*d^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)+20/e^5*d^3
/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+10/e
^8*d^2/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**5*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.189 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=160

$$\frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}-\frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5}-\frac{19d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

Rubi [A] time = 0.42, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {852, 1635, 780, 217, 203}

$$-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}-\frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5}-\frac{19d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -(d^3*(d - e*x)^4)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (19*d^2*(d - e*x)^3)/(15
*e^5*(d^2 - e^2*x^2)^(3/2)) - (6*d*(d - e*x)^2)/(e^5*Sqrt[d^2 - e^2*x^2]) -
 ((20*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (19*d^2*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(2*e^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^4(d-ex)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)^2\left(\frac{45d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\left(\frac{135d^4}{e^4}-\frac{15d^3x}{e^3}\right)(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{(1}{}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{(1}{}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{19}{}$$

Mathematica [A] time = 0.18, size = 98, normalized size = 0.61

$$-\frac{285d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(448d^4+1059d^3ex+713d^2e^2x^2+75de^3x^3-15e^4x^4\right)}{(d+ex)^3}}{30e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/30*((Sqrt[d^2 - e^2*x^2]*(448*d^4 + 1059*d^3*e*x + 713*d^2*e^2*x^2 + 75*
d*e^3*x^3 - 15*e^4*x^4))/(d + e*x)^3 + 285*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*
x^2]])/e^5
```

IntegrateAlgebraic [A] time = 0.82, size = 121, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(-448d^4-1059d^3ex-713d^2e^2x^2-75de^3x^3+15e^4x^4\right)}{30e^5(d+ex)^3} - \frac{19d^2\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

[Out] (Sqrt[d^2 - e^2*x^2]*(-448*d^4 - 1059*d^3*e*x - 713*d^2*e^2*x^2 - 75*d*e^3*x^3 + 15*e^4*x^4))/(30*e^5*(d + e*x)^3) - (19*d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^6)

fricas [A] time = 0.43, size = 190, normalized size = 1.19

$$-\frac{448\,d^{2}e^{3}x^{3}+1344\,d^{3}e^{2}x^{2}+1344\,d^{4}ex+448\,d^{5}-570\left(d^{2}e^{3}x^{3}+3\,d^{3}e^{2}x^{2}+3\,d^{4}ex+d^{5}\right)\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-\left(15\,e^{4}x^{4}-75\,de^{3}x^{3}-713\,d^{2}e^{2}x^{2}-1059\,d^{3}ex-448\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{30\left(e^{8}x^{3}+3\,de^{7}x^{2}+3\,d^{2}e^{6}x+d^{3}e^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")

[Out] -1/30*(448*d^2*e^3*x^3 + 1344*d^3*e^2*x^2 + 1344*d^4*e*x + 448*d^5 - 570*(d^2*e^3*x^3 + 3*d^3*e^2*x^2 + 3*d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (15*e^4*x^4 - 75*d*e^3*x^3 - 713*d^2*e^2*x^2 - 1059*d^3*e*x - 448*d^4)*sqrt(-e^2*x^2 + d^2))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (84*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*exp(2)^2+18*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^10*exp(2)^3-192*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^12*exp(2)^2-180*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3-42*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+228*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2-104*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3-192*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^4-48*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-396*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-168*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4+60*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5+24*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^6-510*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^4-246*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^5+57*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+27*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8+156*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5+180*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+26*d^2*exp(1)^8*exp(2)^4+66*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8+180*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6-48*d^2*exp(1)^6*exp(2)^5+216*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^8-65*d^2*exp(1)^4*exp(2)^6+132*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8+66*d^2*exp(2)^8+104*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)-189/2*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)-78*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+171*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)+123*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-69*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((

```
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^15-6*exp(1)^11*e
xp(2)^2-6*exp(1)^9*exp(2)^3+3*exp(1)^7*exp(2)^4+3*exp(1)^5*exp(2)^5+3*exp(1
)^13*exp(2))+1/2*(64*d^2*exp(1)^10*exp(2)^2+80*d^2*exp(1)^8*exp(2)^3-88*d^2
*exp(1)^6*exp(2)^4-102*d^2*exp(1)^4*exp(2)^5+76*d^2*exp(2)^7-16*d^2*exp(1)^
12*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))
/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^17+2*exp(1)^13
*exp(2)^2+2*exp(1)^11*exp(2)^3-exp(1)^9*exp(2)^4-exp(1)^7*exp(2)^5-exp(1)^1
5*exp(2))-19/2*d^2*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^5+2*(2*exp(1)^9*1
/8/exp(1)^13*x-16*exp(1)^8*d*1/8/exp(1)^13)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.01, size = 273, normalized size = 1.71

$$-\frac{10 d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}\right)}{\sqrt{e^{2}}\, e^{4}}+\frac{d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}\, e^{4}}+\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, x}{2 e^{4}}-\frac{10 \sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, d}{e^{5}}-\frac{\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} d^{3}}{5\left(x+\frac{d}{e}\right)^{4} e^{9}}+\frac{19\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} d^{2}}{15\left(x+\frac{d}{e}\right)^{3} e^{8}}-\frac{6\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} d}{\left(x+\frac{d}{e}\right)^{2} e^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] 1/2/e^4*x*(-e^2*x^2+d^2)^(1/2)+1/2/e^4*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(
-e^2*x^2+d^2)^(1/2)*x)-1/5*d^3/e^9/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(3/2)+19/15/e^8*d^2/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-10/e^5*d*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-10/e^4*d^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-6/e^7*d/(x+d/e)^2*(2*(x+d/e)*d*e-
(x+d/e)^2*e^2)^(3/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \sqrt{- \left(- d + e x\right) \left(d + e x\right)}}{\left(d + e x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.190 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=148

$$\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}+\frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Rubi [A] time = 0.25, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1639, 1637, 659, 651, 663, 217, 203}

$$\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}+\frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (8*d*Sqrt[d^2 - e^2*x^2])/(e^4*(d + e*x)) + (d^2*(d^2 - e^2*x^2)^(3/2))/(5*
e^4*(d + e*x)^4) - (14*d*(d^2 - e^2*x^2)^(3/2))/(15*e^4*(d + e*x)^3) - (d^2
 - e^2*x^2)^(3/2)/(e^4*(d + e*x)^2) + (4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}-\frac{\int \frac{\sqrt{d^2-e^2x^2}\left(2d^3e^2+5d^2e^3x+4de^4x^2\right)}{(d+ex)^4}\,dx}{e^5}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}-\frac{\int \left(\frac{d^3e^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}-\frac{3d^2e^2\sqrt{d^2-e^2x^2}}{(d+ex)^3}+\frac{4de^2\sqrt{d^2-e^2x^2}}{(d+ex)^2}\right)dx}{e^5}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}-\frac{(4d)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^3}+\frac{\left(3d^2\right)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^3}-\frac{d^3\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^3}\\
&= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}+\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{(4d)\int \frac{1}{\sqrt{d^2-e^2x^2}}}{e^3}\\
&= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}+\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{(4d)\,\mathrm{Subst}\left(\right.}{}\\
&= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}+\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{4d\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{e^4}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 85, normalized size = 0.57

$$\frac{60d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(94d^3+222d^2ex+149de^2x^2+15e^3x^3\right)}{(d+ex)^3}}{15e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(94*d^3 + 222*d^2*e*x + 149*d*e^2*x^2 + 15*e^3*x^3))/
(d + e*x)^3 + 60*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^4)
```

IntegrateAlgebraic [A] time = 0.65, size = 106, normalized size = 0.72

$$\frac{4d\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^5}+\frac{\sqrt{d^2-e^2x^2}\,\left(94d^3+222d^2ex+149de^2x^2+15e^3x^3\right)}{15e^4(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(94*d^3 + 222*d^2*e*x + 149*d*e^2*x^2 + 15*e^3*x^3))/(
15*e^4*(d + e*x)^3) + (4*d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*
x^2]])/e^5
```

fricas [A] time = 0.42, size = 174, normalized size = 1.18

$$\frac{94\,de^{3}x^{3} + 282\,d^{2}e^{2}x^{2} + 282\,d^{3}ex + 94\,d^{4} - 120\left(de^{3}x^{3} + 3\,d^{2}e^{2}x^{2} + 3\,d^{3}ex + d^{4}\right)\arctan\left(-\frac{d - \sqrt{-e^{2}x^{2} + d^{2}}}{ex}\right) + \left(15\,e^{3}x^{3} + 149\,de^{2}x^{2} + 222\,d^{2}ex + 94\,d^{3}\right)\sqrt{-e^{2}x^{2} + d^{2}}}{15\left(e^{7}x^{3} + 3\,de^{6}x^{2} + 3\,d^{2}e^{5}x + d^{3}e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(94*d*e^3*x^3 + 282*d^2*e^2*x^2 + 282*d^3*e*x + 94*d^4 - 120*(d*e^3*x^
3 + 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*
x)) + (15*e^3*x^3 + 149*d*e^2*x^2 + 222*d^2*e*x + 94*d^3)*sqrt(-e^2*x^2 + d
^2))/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-30*
d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*e
xp(2)^2-6*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*e
xp(1)^10*exp(2)^3+144*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^3*exp(1)^12*exp(2)^2+132*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+30*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4-102*d*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2+62*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3+
87*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8
*exp(2)^4+21*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
5*exp(1)^6*exp(2)^5+300*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^2*exp(1)^10*exp(2)^3+144*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-24*d*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-12*d*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^6+264*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^4+1
20*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6
*exp(2)^5-24*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
4*exp(1)^4*exp(2)^6-12*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^5*exp(2)^8-84*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^2*exp(1)^6*exp(2)^5-108*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6-11*d*exp(1)^8*exp(2)^4-36*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8-96*d*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6
+36*d*exp(1)^6*exp(2)^5-108*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^3*exp(2)^8+32*d*exp(1)^4*exp(2)^6-72*d*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-36*d*exp(2)^8-44*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+48
*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+48*d*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)-171/2*d*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)-93*d*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)+30*d
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/(
```

```
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^14-6*exp(1)^10*
exp(2)^2-6*exp(1)^8*exp(2)^3+3*exp(1)^6*exp(2)^4+3*exp(1)^4*exp(2)^5+3*exp(
1)^12*exp(2))+1/2*(-36*d*exp(1)^10*exp(2)^2-30*d*exp(1)^8*exp(2)^3+40*d*exp
(1)^6*exp(2)^4+40*d*exp(1)^4*exp(2)^5-32*d*exp(2)^7+4*d*exp(1)^12*exp(2))*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^16+2*exp(1)^12*exp(2)^2+2*
exp(1)^10*exp(2)^3-exp(1)^8*exp(2)^4-exp(1)^6*exp(2)^5-exp(1)^14*exp(2))+4*
d*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^4+2*exp(1)^3*1/2/exp(1)^7*sqrt(d^2
-x^2*exp(2))
```

maple [A] time = 0.01, size = 212, normalized size = 1.43

$$\frac{4d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e^3}+\frac{4\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}{e^4}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^2}{5\left(x+\frac{d}{e}\right)^4e^8}-\frac{14\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d}{15\left(x+\frac{d}{e}\right)^3e^7}+\frac{3\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{\left(x+\frac{d}{e}\right)^2e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] 1/5*d^2/e^8/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-14/15*d/e^7/(x+d/
e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+4/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2
)^(1/2)+4/e^3*d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2
)^(1/2)*x)+3/e^6/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.191 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=115

$$\frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4}-\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1637, 659, 651, 663, 217, 203}

$$\frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4}-\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*(d^2 - e^2*x^2)^(3/2))/(5*e^3
*(d + e*x)^4) + (3*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^3) - ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]]/e^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx &= \int \left(\frac{d^2\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4} - \frac{2d\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3} + \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)^2}\right)dx\\ &= \frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2} - \frac{(2d)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^2} + \frac{d^2\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^2}\\ &= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4} + \frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e^3(d+ex)^3} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2} + \frac{d\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5e^2}\\ &= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4} + \frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\ &= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4} + \frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.12, size = 73, normalized size = 0.63

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(8d^2+19dex+13e^2x^2\right)}{(d+ex)^3}+5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/5*((Sqrt[d^2 - e^2*x^2]*(8*d^2 + 19*d*e*x + 13*e^2*x^2))/(d + e*x)^3 + 5
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

IntegrateAlgebraic [A] time = 0.69, size = 94, normalized size = 0.82

$$\frac{\left(-8d^2-19dex-13e^2x^2\right)\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3} - \frac{\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] ((-8*d^2 - 19*d*e*x - 13*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(5*e^3*(d + e*x)^3)
- (Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^4
```

fricas [A] time = 0.41, size = 157, normalized size = 1.37

$$-\frac{8e^3x^3+24de^2x^2+24d^2ex+8d^3-10\left(e^3x^3+3de^2x^2+3d^2ex+d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(13e^2x^2+19dex+8d^2\right)\sqrt{-e^2x^2+d^2}}{5\left(e^6x^3+3de^5x^2+3d^2e^4x+d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/5*(8*e^3*x^3 + 24*d*e^2*x^2 + 24*d^2*e*x + 8*d^3 - 10*(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (13*e^2
*x^2 + 19*d*e*x + 8*d^2)*sqrt(-e^2*x^2 + d^2))/(e^6*x^3 + 3*d*e^5*x^2 + 3*d
^2*e^4*x + d^3*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-96*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^12*exp
(2)^2-84*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(
1)^10*exp(2)^3-18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^5*exp(1)^8*exp(2)^4+24*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^2*exp(1)^12*exp(2)^2-32*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3+8*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)-24*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^4-6*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-204*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-120
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp
(2)^4-12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(
1)^6*exp(2)^5-102*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^2*exp(1)^8*exp(2)^4-42*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^3*exp(1)^6*exp(2)^5+3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^5*exp(2)^8+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5+36*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+2*exp(1)^8*exp(2)^4+12*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8+36*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2
)^6-24*exp(1)^6*exp(2)^5+36*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^3*exp(2)^8-11*exp(1)^4*exp(2)^6+24*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8+12*exp(2)^8-33/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)-18*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+30*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)+63*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-6*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x+exp(2))^3/(3*exp(1)^13-6*exp(1)^9*exp(2)^2-6*exp(1)^7*exp(2)^3+3
*exp(1)^5*exp(2)^4+3*exp(1)^11*exp(2)+3*exp(1)*exp(2)^6)+1/2*(16*exp(1)^10*
exp(2)^2+8*exp(1)^8*exp(2)^3-8*exp(1)^6*exp(2)^4-10*exp(1)^4*exp(2)^5+8*exp
(2)^7)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqr
t(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^15+2*exp(1)^11*exp
(2)^2+2*exp(1)^9*exp(2)^3-exp(1)^7*exp(2)^4-exp(1)^5*exp(2)^5-exp(1)^13*exp
(2))-sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)/exp(2)
```

maple [B] time = 0.01, size = 214, normalized size = 1.86

$$-\frac{\arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}\right)}{\sqrt{e^{2}}\, e^{2}}-\frac{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}{d\, e^{3}}-\frac{\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} d}{5\left(x+\frac{d}{e}\right)^{4} e^{7}}-\frac{\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}}}{\left(x+\frac{d}{e}\right)^{2} d\, e^{5}}+\frac{3\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}}}{5\left(x+\frac{d}{e}\right)^{3} e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/5*d/e^7/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+3/5/e^6/(x+d/e)^3*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-1/e^5/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(3/2)-1/e^3/d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-1/e^2/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2 + d^2}\, x^2}{(ex+d)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)*x^2/(e*x + d)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.192 $\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=64

$$\frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{15de^2(d+ex)^3}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {793, 651}

$$\frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (d^2 - e^2*x^2)^(3/2)/(5*e^2*(d + e*x)^4) - (4*(d^2 - e^2*x^2)^(3/2))/(15*d
*e^2*(d + e*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx &= \frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}+\frac{4\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5e}\\ &= \frac{\left(d^2-e^2x^2\right)^{3/2}}{5e^2(d+ex)^4}-\frac{4\left(d^2-e^2x^2\right)^{3/2}}{15de^2(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.78

$$-\frac{\left(d^2+3dex-4e^2x^2\right)\sqrt{d^2-e^2x^2}}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/15*((d^2 + 3*d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(d*e^2*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.59, size = 52, normalized size = 0.81

$$\frac{\sqrt{d^2-e^2x^2}\,\left(-d^2-3dex+4e^2x^2\right)}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 - 3*d*e*x + 4*e^2*x^2))/(15*d*e^2*(d + e*x)^3)
```

fricas [A] time = 0.40, size = 102, normalized size = 1.59

$$-\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3-\left(4\,e^2x^2-3\,dex-d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
[Out] -1/15*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3 - (4*e^2*x^2 - 3*d*e*x - d^2
)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (6*ex
p(1)*exp(2)^7+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^2*exp(1)*exp(2)^7+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^4*exp(1)^11*exp(2)^2+48*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^3*exp(1)^11*exp(2)^2+36*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^4*exp(1)^9*exp(2)^3+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^7*exp(2)^4+6*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^11*exp(2)^2+14*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^9*exp(2)^3+4*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^13*exp(2)+3
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^7*exp
(2)^4+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1
)*exp(2)^7+3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*
exp(1)^5*exp(2)^5+108*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(1)^9*exp(2)^3+96*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^3*exp(1)^7*exp(2)^4+48*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^4*exp(1)^5*exp(2)^5+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^3*exp(2)^6+24*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^7*exp(2)^4+12*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^5*exp(2)^5+6*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^3*exp(2)^6+6
0*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^5*ex
p(2)^5+36*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^3*exp(2)^6+exp(1)^7*exp(2)^4+12*exp(1)^5*exp(2)^5+2*exp(1)^3*exp(2)^6-1
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^6/x/exp(2)-9/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^5/x/exp(2)-33
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^4/x/exp(2)-3*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9*exp(2)^3/x/exp(2))/((-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp
```

```
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*d*exp(1)^11-6*d*exp(1)^7*
exp(2)^2-6*d*exp(1)^5*exp(2)^3+3*d*exp(1)^9*exp(2)+6*d*exp(1)*exp(2)^5)+1/2
*(4*exp(1)^7*exp(2)^2+2*exp(1)^5*exp(2)^3+8*exp(1)^3*exp(2)^4)*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/(d*exp(1)^11-2*d*exp(1)^7*exp(2)^2-2*d*exp(1)^
5*exp(2)^3+d*exp(1)^9*exp(2)+2*d*exp(1)*exp(2)^5)
```

maple [A] time = 0.01, size = 42, normalized size = 0.66

$$-\frac{(-ex+d)(4ex+d)\sqrt{-e^2x^2+d^2}}{15\,(ex+d)^3\,d\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/15*(-e*x+d)*(4*e*x+d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/e^2/d
```

maxima [B] time = 0.45, size = 125, normalized size = 1.95

$$\frac{2\sqrt{-e^2x^2+d^2}\,d}{5\left(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2\right)}-\frac{11\sqrt{-e^2x^2+d^2}}{15\left(e^4x^2+2\,de^3x+d^2e^2\right)}+\frac{4\sqrt{-e^2x^2+d^2}}{15\left(de^3x+d^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(-e^2*x^2 + d^2)*d/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2)
- 11/15*sqrt(-e^2*x^2 + d^2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 4/15*sqrt(-e
^2*x^2 + d^2)/(d*e^3*x + d^2*e^2)
```

mupad [B] time = 2.90, size = 46, normalized size = 0.72

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+3\,d\,e\,x-4\,e^2\,x^2\right)}{15\,d\,e^2\,(d+e\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 - 4*e^2*x^2 + 3*d*e*x))/(15*d*e^2*(d + e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.193 $\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} \, dx$

Optimal. Leaf size=67

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] -(d^2 - e^2*x^2)^(3/2)/(5*d*e*(d + e*x)^4) - (d^2 - e^2*x^2)^(3/2)/(15*d^2*
e*(d + e*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} \, dx &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}+\frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3} \, dx}{5d}\\ &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(-4d^2+3dex+e^2x^2\right)}{15d^2e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^2 + 3*d*e*x + e^2*x^2))/(15*d^2*e*(d + e*x)^3)
```

IntegrateAlgebraic [A] time = 0.62, size = 51, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(-4d^2+3dex+e^2x^2\right)}{15d^2e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^2 + 3*d*e*x + e^2*x^2))/(15*d^2*e*(d + e*x)^3)
```

fricas [A] time = 0.41, size = 104, normalized size = 1.55

$$-\frac{4\,e^3x^3+12\,de^2x^2+12\,d^2ex+4\,d^3-\left(e^2x^2+3\,dex-4\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^4x^3+3\,d^3e^3x^2+3\,d^4e^2x+d^5e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(4*e^3*x^3 + 12*d*e^2*x^2 + 12*d^2*e*x + 4*d^3 - (e^2*x^2 + 3*d*e*x -
 4*d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*e^4*x^3 + 3*d^3*e^3*x^2 + 3*d^4*e^2*x +
d^5*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (12*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*exp(
2)^2+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)
^10*exp(2)^3+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^4*exp(1)^10*exp(2)^3+6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^5*exp(1)^8*exp(2)^4+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2-8*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3-24*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^4-12*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-12*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-72*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2
)^4-84*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)
^6*exp(2)^5-24*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
5*exp(1)^4*exp(2)^6-30*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^2*exp(1)^8*exp(2)^4-30*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^3*exp(1)^6*exp(2)^5-3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8-132*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-108*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+2*exp(1)^8*exp(2
)^4-18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)
^8-12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^
4*exp(2)^6-5*exp(1)^4*exp(2)^6-36*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^2*exp(2)^8-18*exp(2)^8+8*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+3/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+42*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+9*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
```

```
*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)+3*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))*exp(1)^8*exp(2)^4/x/exp(2)-3*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))*exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+
exp(2))^3/(3*d^2*exp(1)^11-6*d^2*exp(1)^7*exp(2)^2-6*d^2*exp(1)^5*exp(2)^3+
3*d^2*exp(1)^9*exp(2)+6*d^2*exp(1)*exp(2)^5)+1/2*(8*exp(1)^4*exp(2)^4+6*exp
(2)^6)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqr
t(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d^2*exp(1)^11+2*d^2*exp(1
)^7*exp(2)^2+2*d^2*exp(1)^5*exp(2)^3-d^2*exp(1)^9*exp(2)-2*d^2*exp(1)*exp(2
)^5)
```

maple [A] time = 0.01, size = 43, normalized size = 0.64

$$-\frac{(-ex+d)\,(ex+4d)\,\sqrt{-e^2x^2+d^2}}{15\,(ex+d)^3\,d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/d^2/e
```

maxima [B] time = 0.44, size = 123, normalized size = 1.84

$$-\frac{2\sqrt{-e^2x^2+d^2}}{5\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}+\frac{\sqrt{-e^2x^2+d^2}}{15\left(de^3x^2+2\,d^2e^2x+d^3e\right)}+\frac{\sqrt{-e^2x^2+d^2}}{15\left(d^2e^2x+d^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(-e^2*x^2 + d^2)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) + 1
/15*sqrt(-e^2*x^2 + d^2)/(d*e^3*x^2 + 2*d^2*e^2*x + d^3*e) + 1/15*sqrt(-e^2
*x^2 + d^2)/(d^2*e^2*x + d^3*e)
```

mupad [B] time = 2.78, size = 47, normalized size = 0.70

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-4\,d^2+3\,d\,e\,x+e^2\,x^2\right)}{15\,d^2\,e\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(d + e*x)^4,x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(e^2*x^2 - 4*d^2 + 3*d*e*x))/(15*d^2*e*(d + e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

3.194 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx$

Optimal. Leaf size=110

$$-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Rubi [A] time = 0.22, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

```
[Out] (8*d*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*x)/(5*d*(d^2 - e^2*x^2)^(3
/2)) + (5*d - 8*e*x)/(5*d^3*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x
^2]/d]/d^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx \\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+12d^3ex+5d^2e^2x^2}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2} \\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-24d^3ex}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4} \\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\int \frac{15d^6e^2}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^8e^2} \\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2} \\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^2} \\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^2e^2} \\
&= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 76, normalized size = 0.69

$$\frac{\frac{\sqrt{d^2-e^2x^2}(13d^2+19dex+8e^2x^2)}{(d+ex)^3}-5\log\left(\sqrt{d^2-e^2x^2}+d\right)+5\log(x)}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(13*d^2 + 19*d*e*x + 8*e^2*x^2))/(d + e*x)^3 + 5*Log[
x] - 5*Log[d + Sqrt[d^2 - e^2*x^2]])/(5*d^3)
```

IntegrateAlgebraic [A] time = 0.91, size = 92, normalized size = 0.84

$$\frac{\sqrt{d^2-e^2x^2}\left(13d^2+19dex+8e^2x^2\right)}{5d^3(d+ex)^3}+\frac{2\tanh^{-1}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(13*d^2 + 19*d*e*x + 8*e^2*x^2))/(5*d^3*(d + e*x)^3) +
 (2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^3
```

fricas [A] time = 0.41, size = 153, normalized size = 1.39

$$\frac{13e^3x^3+39de^2x^2+39d^2ex+13d^3+5\left(e^3x^3+3de^2x^2+3d^2ex+d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(8e^2x^2+19dex+13d^2\right)\sqrt{-e^2x^2+d^2}}{5\left(d^3e^3x^3+3d^4e^2x^2+3d^5ex+d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(13*e^3*x^3 + 39*d*e^2*x^2 + 39*d^2*e*x + 13*d^3 + 5*(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^2*x^2 + 1
9*d*e*x + 13*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d^
5*e*x + d^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (6*ex
p(1)*exp(2)^8+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^2*exp(1)*exp(2)^8+54*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^4*exp(1)^13*exp(2)^2+18*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^5*exp(1)^11*exp(2)^3+48*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^3*exp(1)^13*exp(2)^2+44*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^15*exp(2)+60*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^11*exp(2)^3+6*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)*exp(2)^8+18*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^9*exp(2)^4
+78*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^13
*exp(2)^2-14*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*
exp(1)^11*exp(2)^3-87*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^4*exp(1)^9*exp(2)^4-33*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
```

```
(1))/x/exp(2))^5*exp(1)^7*exp(2)^5+84*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^2*exp(1)^11*exp(2)^3-48*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^9*exp(2)^4-120*(-1/2*(-2*d*exp(1)-
2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^7*exp(2)^5-36*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^5*exp(2)^6-120*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^9*exp(2)
^4-96*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^
7*exp(2)^5+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4
*exp(1)^5*exp(2)^6+12*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^5*exp(1)^3*exp(2)^7-204*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^2*exp(1)^7*exp(2)^5-180*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^3*exp(1)^5*exp(2)^6-30*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^3*exp(2)^7+11*exp(1)^9*exp(2)^4+3
6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^3*ex
p(2)^7+12*exp(1)^7*exp(2)^5-60*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^2*exp(1)^3*exp(2)^7-20*exp(1)^5*exp(2)^6-30*exp(1)^3*exp(2)
^7-12*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^7/x/exp(2
)+72*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^6/x/exp(2)
+87/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^5/x/exp(2
)-27*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9*exp(2)^4/x/exp(2)
-24*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^11*exp(2)^3/x/exp(2)
)/(-(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)+(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x-exp(2))^3/(3*d^3*exp(1)^11-6*d^
3*exp(1)^7*exp(2)^2-6*d^3*exp(1)^5*exp(2)^3+3*d^3*exp(1)^9*exp(2)+6*d^3*exp
(1)*exp(2)^5)+1/2*(-4*exp(1)^9*exp(2)^2+10*exp(1)^7*exp(2)^3+8*exp(1)^5*exp
(2)^4-8*exp(1)^3*exp(2)^5-4*exp(1)^11*exp(2)-16*exp(1)*exp(2)^6)*atan((-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2
)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d^3*exp(1)^11+2*d^3*exp(1)^7*exp(2)^2+2*d^
3*exp(1)^5*exp(2)^3-d^3*exp(1)^9*exp(2)-2*d^3*exp(1)*exp(2)^5)-exp(2)*ln(1/
2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^3/exp(1)^
2
```

maple [B] time = 0.01, size = 196, normalized size = 1.78

$$-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^2}+\frac{\sqrt{-e^2x^2+d^2}}{d^4}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{5\left(x+\frac{d}{e}\right)^4d^2e^4}+\frac{2\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{5\left(x+\frac{d}{e}\right)^3d^3e^3}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{\left(x+\frac{d}{e}\right)^2d^4e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x)
```

```
[Out] 1/5/d^2/e^4/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+2/5/e^3/d^3/(x+d/
e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+1/d^4*(-e^2*x^2+d^2)^(1/2)-1/d^2/(
d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^4/e^2/(x+d/
e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x\,{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)^4), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)**4), x)
```

3.195 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx$

Optimal. Leaf size=143

$$-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Rubi [A] time = 0.31, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] (-8*e*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*(5*d - 8*e*x))/(15*d^2*(d
^2 - e^2*x^2)^(3/2)) - (e*(60*d - 79*e*x))/(15*d^4*Sqrt[d^2 - e^2*x^2]) - S
qrt[d^2 - e^2*x^2]/(d^4*x) + (4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-27d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+64d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} - \frac{(4e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}}{d^3}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} - \frac{(2e)\,\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{d^3}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right.}{}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e}}{d}\right.}{d^4}$$

Mathematica [A] time = 0.21, size = 92, normalized size = 0.64

$$-\frac{-60e\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(15d^3+149d^2ex+222de^2x^2+94e^3x^3\right)}{x(d+ex)^3}+60e\log(x)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(15*d^3 + 149*d^2*e*x + 222*d*e^2*x^2 + 94*e^3*
x^3))/(x*(d + e*x)^3) + 60*e*Log[x] - 60*e*Log[d + Sqrt[d^2 - e^2*x^2]])/d^
4
```

IntegrateAlgebraic [A] time = 0.59, size = 107, normalized size = 0.75

$$\frac{\sqrt{d^2-e^2x^2}\left(-15d^3-149d^2ex-222de^2x^2-94e^3x^3\right)}{15d^4x(d+ex)^3}-\frac{8e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-15*d^3 - 149*d^2*e*x - 222*d*e^2*x^2 - 94*e^3*x^3))/
(15*d^4*x*(d + e*x)^3) - (8*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2
]/d])/d^4
```

fricas [A] time = 0.42, size = 181, normalized size = 1.27

$$-\frac{104\,e^4x^4+312\,de^3x^3+312\,d^2e^2x^2+104\,d^3ex+60\left(e^4x^4+3\,de^3x^3+3\,d^2e^2x^2+d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(94\,e^3x^3+222\,de^2x^2+149\,d^2ex+15\,d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^4e^3x^4+3\,d^5e^2x^3+3\,d^6ex^2+d^7x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(104*e^4*x^4 + 312*d*e^3*x^3 + 312*d^2*e^2*x^2 + 104*d^3*e*x + 60*(e^
4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^
2))/x) + (94*e^3*x^3 + 222*d*e^2*x^2 + 149*d^2*e*x + 15*d^3)*sqrt(-e^2*x^2
+ d^2))/(d^4*e^3*x^4 + 3*d^5*e^2*x^3 + 3*d^6*e*x^2 + d^7*x)
```

giac [A] time = 0.28, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [B] time = 0.01, size = 361, normalized size = 2.52

$$\frac{4e\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^3}+\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,d^4}-\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,d^4}-\frac{\sqrt{-e^2x^2+d^2}\,e^2x}{d^6}-\frac{4\sqrt{-e^2x^2+d^2}\,e}{d^5}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e}{d^5}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{5\left(x+\frac{d}{e}\right)^4d^3e^3}-\frac{11\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{15\left(x+\frac{d}{e}\right)^3d^4e^2}-\frac{3\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{\left(x+\frac{d}{e}\right)^2d^5e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x)
```

```
[Out] -1/5/d^3/e^3/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-11/15/d^4/e^2/(x
+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-1/d^6/x*(-e^2*x^2+d^2)^(3/2)-1/
d^6*e^2*x*(-e^2*x^2+d^2)^(1/2)-1/d^4*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e
^2*x^2+d^2)^(1/2)*x)-4/d^5*e*(-e^2*x^2+d^2)^(1/2)+4/d^3*e/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^5*e*(2*(x+d/e)*d*e-(x+d/e)^
2*e^2)^(1/2)+1/d^4*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(1/2)*x)-3/d^5/e/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2 x^2}}{x^2 (d + e x)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^2 (d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d)**4,x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)**4), x)
```

3.196 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx$

Optimal. Leaf size=183

$$\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4e\sqrt{d^2-e^2x^2}}{d^5x}-\frac{19e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5}-\frac{\sqrt{d^2-e^2x^2}}{2d^4x^2}+\frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Rubi [A] time = 0.39, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{4e\sqrt{d^2-e^2x^2}}{d^5x}-\frac{\sqrt{d^2-e^2x^2}}{2d^4x^2}-\frac{19e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

```
[Out] (8*e^2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (4*e^2*(10*d - 13*e*x))/(15
*d^3*(d^2 - e^2*x^2)^(3/2)) + (e^2*(135*d - 164*e*x))/(15*d^5*Sqrt[d^2 - e^
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^4*x^2) + (4*e*Sqrt[d^2 - e^2*x^2])/(d^5*
x) - (19*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^5)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
&& !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-35d^2e^2x^2+32de^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+120d^2e^2x^2-104de^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex-135d^2e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{\int \frac{-120d^5e+285d^4e^2}{x^2\sqrt{d^2-e^2x^2}}}{30d^8}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} +$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} +$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} -$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} -$$

Mathematica [A] time = 0.23, size = 107, normalized size = 0.58

$$\frac{-285e^2\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(-15d^4+75d^3ex+713d^2e^2x^2+1059de^3x^3+448e^4x^4\right)}{x^2(d+ex)^3}+285e^2\log(x)}{30d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 75*d^3*e*x + 713*d^2*e^2*x^2 + 1059*d*e^3*
x^3 + 448*e^4*x^4))/(x^2*(d + e*x)^3) + 285*e^2*Log[x] - 285*e^2*Log[d + Sq
rt[d^2 - e^2*x^2]])/(30*d^5)
```

IntegrateAlgebraic [A] time = 0.79, size = 120, normalized size = 0.66

$$\frac{19e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{\sqrt{d^2-e^2x^2}\left(-15d^4+75d^3ex+713d^2e^2x^2+1059de^3x^3+448e^4x^4\right)}{30d^5x^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 75*d^3*e*x + 713*d^2*e^2*x^2 + 1059*d*e^3*x
^3 + 448*e^4*x^4))/(30*d^5*x^2*(d + e*x)^3) + (19*e^2*ArcTanh[(Sqrt[-e^2]*x
)/d - Sqrt[d^2 - e^2*x^2]/d])/d^5
```

fricas [A] time = 0.42, size = 202, normalized size = 1.10

$$\frac{398\,e^5x^5+1194\,de^4x^4+1194\,d^2e^3x^3+398\,d^3e^2x^2+285\left(e^5x^5+3\,de^4x^4+3\,d^2e^3x^3+d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(448\,e^4x^4+1059\,de^3x^3+713\,d^2e^2x^2+75\,d^3ex-15\,d^4\right)\sqrt{-e^2x^2+d^2}}{30\left(d^5e^3x^5+3\,d^6e^2x^4+3\,d^7ex^3+d^8x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/30*(398*e^5*x^5 + 1194*d*e^4*x^4 + 1194*d^2*e^3*x^3 + 398*d^3*e^2*x^2 + 2
85*(e^5*x^5 + 3*d*e^4*x^4 + 3*d^2*e^3*x^3 + d^3*e^2*x^2)*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) + (448*e^4*x^4 + 1059*d*e^3*x^3 + 713*d^2*e^2*x^2 + 75*d^3
*e*x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^5*e^3*x^5 + 3*d^6*e^2*x^4 + 3*d^7*e
*x^3 + d^8*x^2)
```

giac [A] time = 0.30, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [B] time = 0.01, size = 389, normalized size = 2.13

$$-\frac{19e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}\,d^4}-\frac{4e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,d^5}+\frac{4e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,d^5}+\frac{4\sqrt{-e^2x^2+d^2}\,e^3x}{d^7}+\frac{19\sqrt{-e^2x^2+d^2}\,e^2}{2d^6}-\frac{4\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^2}{d^6}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{5\left(x+\frac{d}{e}\right)^4d^4e^2}+\frac{16\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{15\left(x+\frac{d}{e}\right)^3d^5e}+\frac{6\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}}{\left(x+\frac{d}{e}\right)^2d^6}+\frac{4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{d^7x}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{2d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x)
```

```
[Out] 1/5/d^4/e^2/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+16/15/d^5/e/(x+d/
e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)+4/d^7*e/x*(-e^2*x^2+d^2)^(3/2)+4/d
```

```
^7*e^3*x*(-e^2*x^2+d^2)^(1/2)+4/d^5*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^
2*x^2+d^2)^(1/2)*x)-1/2/d^6/x^2*(-e^2*x^2+d^2)^(3/2)+19/2/d^6*e^2*(-e^2*x^2
+d^2)^(1/2)-19/2/d^4*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-4/d^6*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)-4/d^5*e^3/(e^2)^(1
/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+6/d^6/(x+d/e)
^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2 + d^2}}{(ex + d)^4 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2\, x^2}}{x^3\, (d + e\, x)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^3 (d + ex)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)**4), x)
```

3.197 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx$

Optimal. Leaf size=210

$$-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}+\frac{18e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}-$$

Rubi [A] time = 0.49, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{18e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

```
[Out] (-8*e^3*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (4*e^3*(5*d - 6*e*x))/(5
*d^4*(d^2 - e^2*x^2)^(3/2)) - (e^3*(80*d - 93*e*x))/(5*d^6*Sqrt[d^2 - e^2*x
^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^4*x^3) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^5*x^2
) - (29*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^6*x) + (18*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^6
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-35d^2e^2x^2+40de^3x^3-32e^4x^4}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+120d^2e^2x^2-180de^3x^3+144e^4x^4}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex-135d^2e^2x^2+240de}{x^4\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{\int \frac{-180d^5e+435d}{x^3\sqrt{d}}}{4}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}$$

Mathematica [A] time = 0.27, size = 118, normalized size = 0.56

$$-\frac{-270e^3\log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}\left(5d^5-15d^4ex+70d^3e^2x^2+674d^2e^3x^3+1002de^4x^4+424e^5x^5\right)}{x^3(d+ex)^3} + 270e^3\log(x)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(5*d^5 - 15*d^4*e*x + 70*d^3*e^2*x^2 + 674*d^2*
e^3*x^3 + 1002*d*e^4*x^4 + 424*e^5*x^5))/(x^3*(d + e*x)^3) + 270*e^3*Log[x]
 - 270*e^3*Log[d + Sqrt[d^2 - e^2*x^2]])/d^6
```

IntegrateAlgebraic [A] time = 1.15, size = 131, normalized size = 0.62

$$\frac{\sqrt{d^2-e^2x^2}\left(-5d^5+15d^4ex-70d^3e^2x^2-674d^2e^3x^3-1002de^4x^4-424e^5x^5\right)}{15d^6x^3(d+ex)^3} - \frac{36e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^5 + 15*d^4*e*x - 70*d^3*e^2*x^2 - 674*d^2*e^3*x^
3 - 1002*d*e^4*x^4 - 424*e^5*x^5))/(15*d^6*x^3*(d + e*x)^3) - (36*e^3*ArcTa
nh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^6
```

fricas [A] time = 0.46, size = 213, normalized size = 1.01

$$-\frac{324\,e^{6}x^{6}+972\,de^{5}x^{5}+972\,d^{2}e^{4}x^{4}+324\,d^{3}e^{3}x^{3}+270\left(e^{6}x^{6}+3\,de^{5}x^{5}+3\,d^{2}e^{4}x^{4}+d^{3}e^{3}x^{3}\right)\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(424\,e^{5}x^{5}+1002\,de^{4}x^{4}+674\,d^{2}e^{3}x^{3}+70\,d^{3}e^{2}x^{2}-15\,d^{4}ex+5\,d^{5}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15\left(d^{6}e^{3}x^{6}+3\,d^{7}e^{2}x^{5}+3\,d^{8}ex^{4}+d^{9}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(324*e^6*x^6 + 972*d*e^5*x^5 + 972*d^2*e^4*x^4 + 324*d^3*e^3*x^3 + 27
0*(e^6*x^6 + 3*d*e^5*x^5 + 3*d^2*e^4*x^4 + d^3*e^3*x^3)*log(-(d - sqrt(-e^2
*x^2 + d^2))/x) + (424*e^5*x^5 + 1002*d*e^4*x^4 + 674*d^2*e^3*x^3 + 70*d^3*
e^2*x^2 - 15*d^4*e*x + 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^6 + 3*d^7*e^
2*x^5 + 3*d^8*e*x^4 + d^9*x^3)
```

giac [A] time = 0.37, size = 1, normalized size = 0.00

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] +Infinity
```

maple [B] time = 0.02, size = 412, normalized size = 1.96

$$\frac{18e^{3}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}\,d^{5}}+\frac{10e^{4}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{\sqrt{e^{2}}\,d^{6}}-\frac{10e^{4}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\,d^{6}}-\frac{10\sqrt{-e^{2}x^{2}+d^{2}}\,e^{4}x}{d^{8}}-\frac{18\sqrt{-e^{2}x^{2}+d^{2}}\,e^{3}}{d^{7}}+\frac{10\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{3}}{d^{7}}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}}{5\left(x+\frac{d}{e}\right)^{4}d^{5}e}-\frac{7\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}}{5\left(x+\frac{d}{e}\right)^{3}d^{6}}-\frac{10\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}e}{\left(x+\frac{d}{e}\right)^{2}d^{7}}-\frac{10\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{d^{8}x}+\frac{2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e}{d^{7}x^{2}}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3d^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x)
```

```
[Out] -1/5/d^5/e/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-7/5/d^6/(x+d/e)^3*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-10/d^8*e^2/x*(-e^2*x^2+d^2)^(3/2)-10/d^
8*e^4*x*(-e^2*x^2+d^2)^(1/2)-10/d^6*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^
2*x^2+d^2)^(1/2)*x)+2/d^7*e/x^2*(-e^2*x^2+d^2)^(3/2)-18/d^7*e^3*(-e^2*x^2+d
^2)^(1/2)+18/d^5*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/
2))/x)-1/3/d^6/x^3*(-e^2*x^2+d^2)^(3/2)+10/d^7*e^3*(2*(x+d/e)*d*e-(x+d/e)^2
*e^2)^(1/2)+10/d^6*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(1/2)*x)-10/d^7*e/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^{2}x^{2}+d^{2}}}{\left(ex+d\right)^{4}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^4\,{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^4\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)**4), x)
```

3.198 $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=252

$$\frac{1}{7}x^6\sqrt{d^2-e^2x^2}-\frac{2dx^5\sqrt{d^2-e^2x^2}}{3e}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}+\frac{65d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^6}+\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e}}{4e^5}$$

Rubi [A] time = 0.66, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}+\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}-\frac{2dx^5\sqrt{d^2-e^2x^2}}{3e}+\frac{1}{7}x^6\sqrt{d^2-e^2x^2}+\frac{65d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (d^4*(d - e*x)^4)/(e^6*Sqrt[d^2 - e^2*x^2]) + (515*d^6*Sqrt[d^2 - e^2*x^2])
/(21*e^6) - (49*d^5*x*Sqrt[d^2 - e^2*x^2])/(4*e^5) + (121*d^4*x^2*Sqrt[d^2
- e^2*x^2])/(21*e^4) - (17*d^3*x^3*Sqrt[d^2 - e^2*x^2])/(6*e^3) + (11*d^2*x
^4*Sqrt[d^2 - e^2*x^2])/(7*e^2) - (2*d*x^5*Sqrt[d^2 - e^2*x^2])/(3*e) + (x^
6*Sqrt[d^2 - e^2*x^2])/7 + (65*d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^
6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
```

```
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx &= \int \frac{x^5(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx \\
&= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^5}{e^5}+\frac{d^4x}{e^4}-\frac{d^3x^2}{e^3}+\frac{d^2x^3}{e^2}-\frac{dx^4}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d} \\
&= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{\int \frac{\frac{28d^8}{e^3}-\frac{91d^7x}{e^2}+\frac{112d^6x^2}{e}-77d^5x^3+56d^4ex^4-55d^3e^2x^5+28d^2e^3x^6}{\sqrt{d^2-e^2x^2}}}{7de^2} \\
&= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} - \frac{\int \frac{-\frac{168d^8}{e}+546d^7x-672d^6ex^2+462d^5e^2}{\sqrt{d^2-e^2x^2}}}{42de^4} \\
&= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{\int \frac{840d^8e-}{}}{} \\
&= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6 \\
&= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \\
&= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \\
&= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \\
&= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \\
&= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} -
\end{aligned}$$

Mathematica [A] time = 0.23, size = 131, normalized size = 0.52

$$\frac{1365d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(2144d^7+779d^6ex-293d^5e^2x^2+162d^4e^3x^3-106d^3e^4x^4+76d^2e^5x^5-44de^6x^6+12e^7x^7\right)}{d+ex}}{84e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(2144*d^7 + 779*d^6*e*x - 293*d^5*e^2*x^2 + 162*d^4*e
^3*x^3 - 106*d^3*e^4*x^4 + 76*d^2*e^5*x^5 - 44*d*e^6*x^6 + 12*e^7*x^7))/(d
+ e*x) + 1365*d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(84*e^6)
```

IntegrateAlgebraic [A] time = 0.60, size = 154, normalized size = 0.61

$$\frac{65d^7\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{4e^7}+\frac{\sqrt{d^2-e^2x^2}\left(2144d^7+779d^6ex-293d^5e^2x^2+162d^4e^3x^3-106d^3e^4x^4+76d^2e^5x^5-44de^6x^6+12e^7x^7\right)}{84e^6(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2144*d^7 + 779*d^6*e*x - 293*d^5*e^2*x^2 + 162*d^4*e^
3*x^3 - 106*d^3*e^4*x^4 + 76*d^2*e^5*x^5 - 44*d*e^6*x^6 + 12*e^7*x^7))/(84*
e^6*(d + e*x)) + (65*d^7*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^
2]])/(4*e^7)
```

fricas [A] time = 0.43, size = 156, normalized size = 0.62

$$\frac{2144\,d^7ex+2144\,d^8-2730\left(d^7ex+d^8\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(12\,e^7x^7-44\,de^6x^6+76\,d^2e^5x^5-106\,d^3e^4x^4+162\,d^4e^3x^3-293\,d^5e^2x^2+779\,d^6ex+2144\,d^7\right)\sqrt{-e^2x^2+d^2}}{84\left(e^7x+de^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/84*(2144*d^7*e*x + 2144*d^8 - 2730*(d^7*e*x + d^8)*arctan(-(d - sqrt(-e^2
*x^2 + d^2))/(e*x)) + (12*e^7*x^7 - 44*d*e^6*x^6 + 76*d^2*e^5*x^5 - 106*d^3
*e^4*x^4 + 162*d^4*e^3*x^3 - 293*d^5*e^2*x^2 + 779*d^6*e*x + 2144*d^7)*sqrt
(-e^2*x^2 + d^2))/(e^7*x + d*e^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-162
*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^1
2*exp(2)^2-36*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^5*exp(1)^10*exp(2)^3+720*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^3*exp(1)^12*exp(2)^2+684*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+162*d^7*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4-402*d^7*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2
)^2+350*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*e
xp(1)^10*exp(2)^3+507*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^4*exp(1)^8*exp(2)^4+123*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5+1476*d^7*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-864*d^7*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-252
*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4
*exp(2)^6+1248*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^2*exp(1)^8*exp(2)^4+84*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^3*exp(1)^6*exp(2)^5-654*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6-192*d^7*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8-1836*d^7*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-1620*d^7
```

```
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp
(2)^6-47*d^7*exp(1)^8*exp(2)^4-486*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^4*exp(2)^8-1464*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6+180*d^7*exp(1)^6*exp(2)^5-
1296*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(
2)^8+158*d^7*exp(1)^4*exp(2)^6-972*d^7*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^2*exp(2)^8-486*d^7*exp(2)^8-188*d^7*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+552*d^7*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+684*d^7*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)-825/2*d^7*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)-459*d^
7*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)+12
3*d^7*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(
2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^16+9*exp(1
)^12*exp(2)^2+3*exp(1)^10*exp(2)^3+9*exp(1)^14*exp(2))+1/2*(-300*d^7*exp(1)
^10*exp(2)^2-82*d^7*exp(1)^8*exp(2)^3+1000*d^7*exp(1)^6*exp(2)^4+464*d^7*ex
p(1)^4*exp(2)^5-1248*d^7*exp(2)^7+40*d^7*exp(1)^12*exp(2))*atan((-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/
sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^18-3*exp(1)^14*exp(2)^2-exp(1)^12*exp(2)^
3-3*exp(1)^16*exp(2))+65/4*d^7*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^6+2*(
(((((720*exp(1)^26*1/10080/exp(1)^26*x-3360*exp(1)^25*d*1/10080/exp(1)^26)*
x+7920*exp(1)^24*d^2*1/10080/exp(1)^26)*x-14280*exp(1)^23*d^3*1/10080/exp(1
)^26)*x+24000*exp(1)^22*d^4*1/10080/exp(1)^26)*x-41580*exp(1)^21*d^5*1/1008
0/exp(1)^26)*x+88320*exp(1)^20*d^6*1/10080/exp(1)^26)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.03, size = 416, normalized size = 1.65

$$\frac{35 d^7 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{2\sqrt{e^2}\, e^5}-\frac{5 d^7 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}\, e^5}+\frac{35\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2}\, d^5 x}{2e^5}-\frac{5\sqrt{-e^2x^2+d^2}\, d^5 x}{4e^5}+\frac{35\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} d^3 x}{3e^5}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^3 x}{6e^5}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d x}{3e^5}+\frac{28\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{5}{2}} d^2}{3e^6}+\frac{\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{7}{2}} d^4}{\left(x+\frac{d}{e}\right)^4 e^{10}}+\frac{8\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{7}{2}} d^3}{\left(x+\frac{d}{e}\right)^3 e^9}+\frac{22\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{7}{2}} d^2}{3\left(x+\frac{d}{e}\right)^2 e^8}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{7e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] d^4/e^10/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+8*d^3/e^9/(x+d/e)^3*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+22/3*d^2/e^8/(x+d/e)^2*(2*(x+d/e)*d*e-(
x+d/e)^2*e^2)^(7/2)+35/3*d^3/e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+35/2
*d^5/e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+35/2*d^7/e^5/(e^2)^(1/2)*arc
tan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-2/3/e^5*d*x*(-e^2*x^
2+d^2)^(5/2)-5/6/e^5*d^3*x*(-e^2*x^2+d^2)^(3/2)-5/4*d^5*x*(-e^2*x^2+d^2)^(1
/2)/e^5-5/4/e^5*d^7/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-
1/7/e^6*(-e^2*x^2+d^2)^(7/2)+28/3*d^2/e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/
2)
```

maxima [C] time = 1.06, size = 478, normalized size = 1.90

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d^5}{2\left(e^9x^3+3de^8x^2+3d^2e^7x+d^3e^6\right)}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^6}{2\left(e^8x^2+2de^7x+d^2e^6\right)}+\frac{15\sqrt{-e^2x^2+d^2}\, d^7}{e^7x+de^6}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d^4}{3\left(e^8x^2+2de^7x+d^2e^6\right)}+\frac{25\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^5}{6\left(e^7x+de^6\right)}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d^3}{2\left(e^7x+de^6\right)}+\frac{5i\, d^7 \arcsin\left(\frac{ex}{d}+2\right)}{2e^6}+\frac{75 d^7 \arcsin\left(\frac{ex}{d}\right)}{4e^6}-\frac{5\sqrt{e^2x^2+4dex+3d^2}\, d^5 x}{2e^5}-\frac{5\sqrt{-e^2x^2+d^2}\, d^5 x}{4e^5}-\frac{5\sqrt{e^2x^2+4dex+3d^2}\, d^6}{e^6}+\frac{25\sqrt{-e^2x^2+d^2}\, d^6}{2e^6}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^3 x}{3e^5}-\frac{25\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^4}{6e^6}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d x}{3e^5}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d^2}{e^6}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{7e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(5/2)*d^5/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*
e^6) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^6/(e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 15*
sqrt(-e^2*x^2 + d^2)*d^7/(e^7*x + d*e^6) + 5/3*(-e^2*x^2 + d^2)^(5/2)*d^4/(
e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 25/6*(-e^2*x^2 + d^2)^(3/2)*d^5/(e^7*x + d
*e^6) - 5/2*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^7*x + d*e^6) + 5/2*I*d^7*arcsin(e
*x/d + 2)/e^6 + 75/4*d^7*arcsin(e*x/d)/e^6 - 5/2*sqrt(e^2*x^2 + 4*d*e*x + 3
*d^2)*d^5*x/e^5 - 5/4*sqrt(-e^2*x^2 + d^2)*d^5*x/e^5 - 5*sqrt(e^2*x^2 + 4*d
*e*x + 3*d^2)*d^6/e^6 + 25/2*sqrt(-e^2*x^2 + d^2)*d^6/e^6 + 5/3*(-e^2*x^2 +
 d^2)^(3/2)*d^3*x/e^5 - 25/6*(-e^2*x^2 + d^2)^(3/2)*d^4/e^6 - 2/3*(-e^2*x^2
```

```
 + d^2)^(5/2)*d*x/e^5 + 2*(-e^2*x^2 + d^2)^(5/2)*d^2/e^6 - 1/7*(-e^2*x^2 +
d^2)^(7/2)/e^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left(d^2 - e^2 x^2\right)^{5/2}}{(d + e x)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
[Out] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(-(-d + ex)(d + ex)\right)^{\frac{5}{2}}}{(d + ex)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
[Out] Integral(x**5*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.199 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=224

$$\frac{1}{6}x^5\sqrt{d^2-e^2x^2}-\frac{4dx^4\sqrt{d^2-e^2x^2}}{5e}+\frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{239d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}-\frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5}+\frac{175d^4x\sqrt{}}{16}$$

Rubi [A] time = 0.53, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$-\frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5}+\frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4}-\frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}}+\frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{4dx^4\sqrt{d^2-e^2x^2}}{5e}+\frac{1}{6}x^5\sqrt{d^2-e^2x^2}-\frac{239d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] -((d^3*(d - e*x)^4)/(e^5*Sqrt[d^2 - e^2*x^2])) - (337*d^5*Sqrt[d^2 - e^2*x^
2])/(15*e^5) + (175*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^4) - (71*d^3*x^2*Sqrt[
d^2 - e^2*x^2])/(15*e^3) + (47*d^2*x^3*Sqrt[d^2 - e^2*x^2])/(24*e^2) - (4*d
*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (x^5*Sqrt[d^2 - e^2*x^2])/6 - (239*d^6*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^4(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{d^3x}{e^3}+\frac{d^2x^2}{e^2}-\frac{dx^3}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{-\frac{24d^7}{e^2}+\frac{78d^6x}{e}-96d^5x^2+66d^4ex^3-47d^3e^2x^4+24d^2e^3x^5}{\sqrt{d^2-e^2x^2}}\,dx}{6de^2}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{120d^7-390d^6ex+480d^5e^2x^2-426d^4e^3x^3}{\sqrt{d^2-e^2x^2}}}{30de^4}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{-480d^7e^2}{}}{}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - 4dx$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + 47d^2$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + 47d^2$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + 47d^2$$

Mathematica [A] time = 0.16, size = 125, normalized size = 0.56

$$\frac{\sqrt{d^2-e^2x^2}\left(-5632d^6-2047d^5ex+769d^4e^2x^2-426d^3e^3x^3+278d^2e^4x^4-152de^5x^5+40e^6x^6\right)-3585d^6(d+ex)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{240e^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5632*d^6 - 2047*d^5*e*x + 769*d^4*e^2*x^2 - 426*d^3*
e^3*x^3 + 278*d^2*e^4*x^4 - 152*d*e^5*x^5 + 40*e^6*x^6) - 3585*d^6*(d + e*x
)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(240*e^5*(d + e*x))
```

IntegrateAlgebraic [A] time = 0.65, size = 143, normalized size = 0.64

$$\frac{\sqrt{d^2-e^2x^2}\left(-5632d^6-2047d^5ex+769d^4e^2x^2-426d^3e^3x^3+278d^2e^4x^4-152de^5x^5+40e^6x^6\right)}{240e^5(d+ex)}-\frac{239d^6\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{16e^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5632*d^6 - 2047*d^5*e*x + 769*d^4*e^2*x^2 - 426*d^3*
e^3*x^3 + 278*d^2*e^4*x^4 - 152*d*e^5*x^5 + 40*e^6*x^6))/(240*e^5*(d + e*x)
) - (239*d^6*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^6
)
```

fricas [A] time = 0.41, size = 146, normalized size = 0.65

$$-\frac{5632\,d^6ex+5632\,d^7-7170\left(d^6ex+d^7\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(40\,e^6x^6-152\,de^5x^5+278\,d^2e^4x^4-426\,d^3e^3x^3+769\,d^4e^2x^2-2047\,d^5ex-5632\,d^6\right)\sqrt{-e^2x^2+d^2}}{240\left(e^6x+de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/240*(5632*d^6*e*x + 5632*d^7 - 7170*(d^6*e*x + d^7)*arctan(-(d - sqrt(-e
^2*x^2 + d^2))/(e*x)) - (40*e^6*x^6 - 152*d*e^5*x^5 + 278*d^2*e^4*x^4 - 426
*d^3*e^3*x^3 + 769*d^4*e^2*x^2 - 2047*d^5*e*x - 5632*d^6)*sqrt(-e^2*x^2 + d
^2))/(e^6*x + d*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (84*d
^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*
exp(2)^2+18*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^5*exp(1)^10*exp(2)^3-576*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^3*exp(1)^12*exp(2)^2-540*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3-126*d^6*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+228*d^6*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^
2-200*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^10*exp(2)^3-264*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^4*exp(1)^8*exp(2)^4-60*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-1188*d^6*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3+72*d^6*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4+756*d^6
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp
(2)^5+216*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5
*exp(1)^4*exp(2)^6-726*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^2*exp(1)^8*exp(2)^4+138*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^5+537*d^6*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+147*d^6*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8+1620*d^6*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5
+1404*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp
(1)^4*exp(2)^6+26*d^6*exp(1)^8*exp(2)^4+402*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8+1212*d^6*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6-144*d^6*exp(1)^6*
```

```
exp(2)^5+1008*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^3*exp(2)^8-89*d^6*exp(1)^4*exp(2)^6+804*d^6*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8+402*d^6*exp(2)^8+104*d^6*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)-861
/2*d^6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)-594*d^
6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+23
7*d^6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2
)+369*d^6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/e
xp(2)-69*d^6*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3
/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*e
xp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^15+
9*exp(1)^11*exp(2)^2+3*exp(1)^9*exp(2)^3+9*exp(1)^13*exp(2))+1/2*(-192*d^6*
exp(1)^10*exp(2)^2+32*d^6*exp(1)^8*exp(2)^3+712*d^6*exp(1)^6*exp(2)^4+230*d
^6*exp(1)^4*exp(2)^5-924*d^6*exp(2)^7+16*d^6*exp(1)^12*exp(2))*atan((-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^
2))/sqrt(-exp(1)^4+exp(2)^2)/(exp(1)^17+3*exp(1)^13*exp(2)^2+exp(1)^11*exp(
2)^3+3*exp(1)^15*exp(2))-239/16*d^6*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^
5+2*(((((240*exp(1)^19*1/2880/exp(1)^19*x-1152*exp(1)^18*d*1/2880/exp(1)^19
)*x+2820*exp(1)^17*d^2*1/2880/exp(1)^19)*x-5376*exp(1)^16*d^3*1/2880/exp(1)
^19)*x+9990*exp(1)^15*d^4*1/2880/exp(1)^19)*x-22272*exp(1)^14*d^5*1/2880/ex
p(1)^19)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.02, size = 393, normalized size = 1.75

$$-\frac{61 d^{6} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}\right)}{4 \sqrt{e^{2}}\, e^{4}}+\frac{5 d^{6} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{16 \sqrt{e^{2}}\, e^{4}}-\frac{61 \sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, d^{4} x}{4 e^{4}}+\frac{5 \sqrt{-e^{2} x^{2}+d^{2}}\, d^{4} x}{16 e^{4}}-\frac{61 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} d^{2} x}{6 e^{4}}+\frac{5 \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} d^{2} x}{24 e^{4}}+\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} x}{6 e^{4}}-\frac{122 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{5}{2}} d}{15 e^{5}}-\frac{\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{7}{2}} d^{3}}{\left(x+\frac{d}{e}\right)^{4} e^{9}}-\frac{7 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{7}{2}} d^{2}}{\left(x+\frac{d}{e}\right)^{3} e^{8}}-\frac{22 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{7}{2}} d}{3 \left(x+\frac{d}{e}\right)^{2} e^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -d^3/e^9/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-7*d^2/e^8/(x+d/e)^3*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-22/3*d/e^7/(x+d/e)^2*(2*(x+d/e)*d*e-(x+
d/e)^2*e^2)^(7/2)-61/6*d^2/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-61/4*d
^4/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-61/4*d^6/e^4/(e^2)^(1/2)*arcta
n((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+5/24/e^4*d^2*x*(-e^2*x
^2+d^2)^(3/2)+5/16/(e^2)^(1/2)*d^6/e^4*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1
/2)*x)+5/16*(-e^2*x^2+d^2)^(1/2)*d^4/e^4*x-122/15*d/e^5*(2*(x+d/e)*d*e-(x+d
/e)^2*e^2)^(5/2)+1/6/e^4*x*(-e^2*x^2+d^2)^(5/2)
```

maxima [C] time = 1.04, size = 456, normalized size = 2.04

$$\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} d^{4}}{2\left(e^{8} x^{3}+3 d e^{7} x^{2}+3 d^{2} e^{6} x+d^{3} e^{5}\right)}+\frac{5\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} d^{5}}{2\left(e^{7} x^{2}+2 d e^{6} x+d^{2} e^{5}\right)}-\frac{15 \sqrt{-e^{2} x^{2}+d^{2}}\, d^{6}}{e^{6} x+d e^{5}}-\frac{4\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} d^{3}}{3\left(e^{7} x^{2}+2 d e^{6} x+d^{2} e^{5}\right)}-\frac{10\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} d^{4}}{3\left(e^{6} x+d e^{5}\right)}+\frac{3\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} d^{2}}{2\left(e^{6} x+d e^{5}\right)}-\frac{9 i d^{6} \arcsin\left(\frac{e x}{d}+2\right)}{4 e^{5}}-\frac{275 d^{6} \arcsin\left(\frac{e x}{d}\right)}{16 e^{5}}+\frac{9 \sqrt{e^{2} x^{2}+4 d e x+3 d^{2}}\, d^{4} x}{4 e^{4}}+\frac{5 \sqrt{-e^{2} x^{2}+d^{2}}\, d^{4} x}{16 e^{4}}+\frac{9 \sqrt{e^{2} x^{2}+4 d e x+3 d^{2}}\, d^{5}}{2 e^{5}}-\frac{10 \sqrt{-e^{2} x^{2}+d^{2}}\, d^{5}}{e^{5}}-\frac{19\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} d^{2} x}{24 e^{4}}+\frac{5\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} d^{3}}{2 e^{5}}+\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} x}{6 e^{4}}-\frac{4\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} d}{5 e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(-e^2*x^2 + d^2)^(5/2)*d^4/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e
^5) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^5/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) - 15*s
qrt(-e^2*x^2 + d^2)*d^6/(e^6*x + d*e^5) - 4/3*(-e^2*x^2 + d^2)^(5/2)*d^3/(e
^7*x^2 + 2*d*e^6*x + d^2*e^5) - 10/3*(-e^2*x^2 + d^2)^(3/2)*d^4/(e^6*x + d*
e^5) + 3/2*(-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*x + d*e^5) - 9/4*I*d^6*arcsin(e*
x/d + 2)/e^5 - 275/16*d^6*arcsin(e*x/d)/e^5 + 9/4*sqrt(e^2*x^2 + 4*d*e*x +
3*d^2)*d^4*x/e^4 + 5/16*sqrt(-e^2*x^2 + d^2)*d^4*x/e^4 + 9/2*sqrt(e^2*x^2 +
 4*d*e*x + 3*d^2)*d^5/e^5 - 10*sqrt(-e^2*x^2 + d^2)*d^5/e^5 - 19/24*(-e^2*x
^2 + d^2)^(3/2)*d^2*x/e^4 + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/e^5 + 1/6*(-e^2*
x^2 + d^2)^(5/2)*x/e^4 - 4/5*(-e^2*x^2 + d^2)^(5/2)*d/e^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.200 $\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=192

$$\frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}+\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{e}+\frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}}+\frac{27d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}+\frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4}-$$

Rubi [A] time = 0.44, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4}-\frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3}+\frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}+\frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}}-\frac{dx^3\sqrt{d^2-e^2x^2}}{e}+\frac{1}{5}x^4\sqrt{d^2-e^2x^2}+\frac{27d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (d^2*(d - e*x)^4)/(e^4*Sqrt[d^2 - e^2*x^2]) + (101*d^4*Sqrt[d^2 - e^2*x^2])
/(5*e^4) - (19*d^3*x*Sqrt[d^2 - e^2*x^2])/(2*e^3) + (18*d^2*x^2*Sqrt[d^2 -
e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/e + (x^4*Sqrt[d^2 - e^2*x^2
])/5 + (27*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^3(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^3}{e^3}+\frac{d^2x}{e^2}-\frac{dx^2}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{\frac{20d^6}{e}-65d^5x+80d^4ex^2-54d^3e^2x^3+20d^2e^3x^4}{\sqrt{d^2-e^2x^2}}\,dx}{5de^2}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{-80d^6e+260d^5e^2x-380d^4e^3x^2+216d^3e}{\sqrt{d^2-e^2x^2}}}{20de^4}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{240d^6e^3-}{}}{}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{}}{}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{}}{}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{}}{}$$

Mathematica [A] time = 0.14, size = 109, normalized size = 0.57

$$\frac{135d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{\sqrt{d^2-e^2x^2}\left(212d^5+77d^4ex-29d^3e^2x^2+16d^2e^3x^3-8de^4x^4+2e^5x^5\right)}{d+ex}}{10e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(212*d^5 + 77*d^4*e*x - 29*d^3*e^2*x^2 + 16*d^2*e^3*x
^3 - 8*d*e^4*x^4 + 2*e^5*x^5))/(d + e*x) + 135*d^5*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(10*e^4)
```

IntegrateAlgebraic [A] time = 0.54, size = 132, normalized size = 0.69

$$\frac{27d^5\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^5}+\frac{\sqrt{d^2-e^2x^2}\left(212d^5+77d^4ex-29d^3e^2x^2+16d^2e^3x^3-8de^4x^4+2e^5x^5\right)}{10e^4(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(212*d^5 + 77*d^4*e*x - 29*d^3*e^2*x^2 + 16*d^2*e^3*x^
3 - 8*d*e^4*x^4 + 2*e^5*x^5))/(10*e^4*(d + e*x)) + (27*d^5*Sqrt[-e^2]*Log[-
(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^5)
```

fricas [A] time = 0.40, size = 134, normalized size = 0.70

$$\frac{212\,d^5ex + 212\,d^6 - 270\left(d^5ex + d^6\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(2\,e^5x^5 - 8\,de^4x^4 + 16\,d^2e^3x^3 - 29\,d^3e^2x^2 + 77\,d^4ex + 212\,d^5\right)\sqrt{-e^2x^2 + d^2}}{10\left(e^5x + de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/10*(212*d^5*e*x + 212*d^6 - 270*(d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (2*e^5*x^5 - 8*d*e^4*x^4 + 16*d^2*e^3*x^3 - 29*d^3*e^2*x
^2 + 77*d^4*e*x + 212*d^5)*sqrt(-e^2*x^2 + d^2))/(e^5*x + d*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-30*
d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12
*exp(2)^2-6*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^5*exp(1)^10*exp(2)^3+432*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^3*exp(1)^12*exp(2)^2+396*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+90*d^5*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4-102*d^5*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2
+62*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1
)^10*exp(2)^3+63*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^4*exp(1)^8*exp(2)^4+9*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5+900*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-144*d^5*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-648*d^5*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^
5-180*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp
(1)^4*exp(2)^6+288*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^2*exp(1)^8*exp(2)^4-312*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^5-432*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6-108*d^5*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8-1404*d^5*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-118
8*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^
4*exp(2)^6-11*d^5*exp(1)^8*exp(2)^4-324*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8-984*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6+108*d^5*exp(1)^6*exp(2
)^5-756*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*e
xp(2)^8+32*d^5*exp(1)^4*exp(2)^6-648*d^5*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-324*d^5*exp(2)^8-44*d^5*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+324*d^5*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+504*d^5*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)-183/2*d^5*(
```

```
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)-279*d
^5*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)+3
0*d^5*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(
2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^14+9*exp(1
)^10*exp(2)^2+3*exp(1)^8*exp(2)^3+9*exp(1)^12*exp(2))+1/2*(-108*d^5*exp(1)^
10*exp(2)^2+90*d^5*exp(1)^8*exp(2)^3+472*d^5*exp(1)^6*exp(2)^4+72*d^5*exp(1
)^4*exp(2)^5-656*d^5*exp(2)^7+4*d^5*exp(1)^12*exp(2))*atan((-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(
-exp(1)^4+exp(2)^2)/(-exp(1)^16-3*exp(1)^12*exp(2)^2-exp(1)^10*exp(2)^3-3*e
xp(1)^14*exp(2))+27/2*d^5*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)^4+2*((((24
*exp(1)^13*1/240/exp(1)^13*x-120*exp(1)^12*d*1/240/exp(1)^13)*x+312*exp(1)^
11*d^2*1/240/exp(1)^13)*x-660*exp(1)^10*d^3*1/240/exp(1)^13)*x+1584*exp(1)^
9*d^4*1/240/exp(1)^13)*sqrt(d^2-x^2*exp(2))
```

maple [A] time = 0.02, size = 285, normalized size = 1.48

$$\frac{27 d^{5} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}\right)}{2 \sqrt{e^{2}}\, e^{3}}+\frac{27 \sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, d^{3} x}{2 e^{3}}+\frac{9 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} d x}{e^{3}}+\frac{36 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{5}{2}}}{5 e^{4}}+\frac{\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{7}{2}} d^{2}}{\left(x+\frac{d}{e}\right)^{4} e^{8}}+\frac{6 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{7}{2}} d}{\left(x+\frac{d}{e}\right)^{3} e^{7}}+\frac{7 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^{2} e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] d^2/e^8/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+6*d/e^7/(x+d/e)^3*(2*
(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+7/e^6/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e
^2)^(7/2)+36/5/e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+9*d/e^3*(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(3/2)*x+27/2*d^3/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*
x+27/2*d^5/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(1/2)*x)
```

maxima [C] time = 1.03, size = 407, normalized size = 2.12

$$-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} d^{3}}{2\left(e^{7} x^{3}+3 d e^{6} x^{2}+3 d^{2} e^{5} x+d^{3} e^{4}\right)}-\frac{5\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} d^{4}}{2\left(e^{6} x^{2}+2 d e^{5} x+d^{2} e^{4}\right)}+\frac{15 \sqrt{-e^{2} x^{2}+d^{2}}\, d^{5}}{e^{5} x+d e^{4}}+\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} d^{2}}{e^{6} x^{2}+2 d e^{5} x+d^{2} e^{4}}+\frac{5\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} d^{3}}{2\left(e^{5} x+d e^{4}\right)}-\frac{3\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}} d}{4\left(e^{5} x+d e^{4}\right)}+\frac{3 i d^{5} \arcsin\left(\frac{e x}{d}+2\right)}{2 e^{4}}+\frac{15 d^{5} \arcsin\left(\frac{e x}{d}\right)}{e^{4}}-\frac{3 \sqrt{e^{2} x^{2}+4 d e x+3 d^{2}}\, d^{3} x}{2 e^{3}}-\frac{3 \sqrt{e^{2} x^{2}+4 d e x+3 d^{2}}\, d^{4}}{e^{4}}+\frac{15 \sqrt{-e^{2} x^{2}+d^{2}}\, d^{4}}{2 e^{4}}+\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} d x}{4 e^{3}}-\frac{5\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}} d^{2}}{4 e^{4}}+\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{5 e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(5/2)*d^3/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*
e^4) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^4/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) + 15*
sqrt(-e^2*x^2 + d^2)*d^5/(e^5*x + d*e^4) + (-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*
x^2 + 2*d*e^5*x + d^2*e^4) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/(e^5*x + d*e^4)
 - 3/4*(-e^2*x^2 + d^2)^(5/2)*d/(e^5*x + d*e^4) + 3/2*I*d^5*arcsin(e*x/d +
2)/e^4 + 15*d^5*arcsin(e*x/d)/e^4 - 3/2*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3
*x/e^3 - 3*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e^4 + 15/2*sqrt(-e^2*x^2 + d
^2)*d^4/e^4 + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e^3 - 5/4*(-e^2*x^2 + d^2)^(3/
2)*d^2/e^4 + 1/5*(-e^2*x^2 + d^2)^(5/2)/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.201 $\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=182

$$-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{95d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Rubi [A] time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1635, 795, 671, 641, 217, 203}

$$-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{95d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] -((d*(d - e*x)^4)/(e^3*Sqrt[d^2 - e^2*x^2])) - (95*d^3*Sqrt[d^2 - e^2*x^2])
/(8*e^3) - (95*d^2*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) - (19*d*(d - e*x
)^2*Sqrt[d^2 - e^2*x^2])/(12*e^3) - ((d - e*x)^3*Sqrt[d^2 - e^2*x^2])/(4*e^
3) - (95*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx=\int \frac{x^2(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$=-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{\int \frac{\left(\frac{4d^2}{e^2}-\frac{dx}{e}\right)(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$=-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{(19d)\int \frac{(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$=-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{\left(95d^2\right)\int \frac{(d-ex)^2}{\sqrt{d^2-e^2x^2}}}{12e^2}$$

$$=-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{(d-ex)^3\sqrt{d^2-}}{4e^3}$$

$$=-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x}}{12e^3}$$

$$=-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x}}{12e^3}$$

$$=-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x}}{12e^3}$$

Mathematica [A] time = 0.12, size = 103, normalized size = 0.57

$$\sqrt{d^2-e^2x^2}\left(-\frac{8d^4}{e^3(d+ex)}-\frac{32d^3}{3e^3}+\frac{31d^2x}{8e^2}-\frac{4dx^2}{3e}+\frac{x^3}{4}\right)-\frac{95d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((-32*d^3)/(3*e^3) + (31*d^2*x)/(8*e^2) - (4*d*x^2)/(3*
e) + x^3/4 - (8*d^4)/(e^3*(d + e*x))) - (95*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/(8*e^3)
```

IntegrateAlgebraic [A] time = 0.54, size = 121, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(-448d^4-163d^3ex+61d^2e^2x^2-26de^3x^3+6e^4x^4\right)}{24e^3(d+ex)}-\frac{95d^4\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{8e^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-448*d^4 - 163*d^3*e*x + 61*d^2*e^2*x^2 - 26*d*e^3*x^
3 + 6*e^4*x^4))/(24*e^3*(d + e*x)) - (95*d^4*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x)
 + Sqrt[d^2 - e^2*x^2]])/(8*e^4)
```

fricas [A] time = 0.41, size = 124, normalized size = 0.68

$$-\frac{448\,d^4ex+448\,d^5-570\left(d^4ex+d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(6\,e^4x^4-26\,de^3x^3+61\,d^2e^2x^2-163\,d^3ex-448\,d^4\right)\sqrt{-e^2x^2+d^2}}{24\left(e^4x+de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(448*d^4*e*x + 448*d^5 - 570*(d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x
^2 + d^2))/(e*x)) - (6*e^4*x^4 - 26*d*e^3*x^3 + 61*d^2*e^2*x^2 - 163*d^3*e*
x - 448*d^4)*sqrt(-e^2*x^2 + d^2))/(e^4*x + d*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-288
*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^1
2*exp(2)^2-252*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^4*exp(1)^10*exp(2)^3-54*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+24*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2+64*d^4*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3+8*d^4*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+96
*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8
*exp(2)^4+30*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^5*exp(1)^6*exp(2)^5-612*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^2*exp(1)^10*exp(2)^3+216*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4+540*d^4*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5+144*d^4*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^6+
66*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)
^8*exp(2)^4+438*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^3*exp(1)^6*exp(2)^5+339*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+75*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8+1188*d^4*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5+972*d^4*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+2*d^4*e
xp(1)^8*exp(2)^4+252*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^4*exp(2)^8+780*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^2*exp(1)^4*exp(2)^6-72*d^4*exp(1)^6*exp(2)^5+540*d^4*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^8+13*d^4*exp(1
```

```
)^4*exp(2)^6+504*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^2*exp(2)^8+252*d^4*exp(2)^8-465/2*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))*exp(2)^8/x/exp(2)-414*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))*exp(1)^4*exp(2)^6/x/exp(2)-24*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)+189*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-6*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x+exp(2))^3/(3*exp(1)^13+9*exp(1)^9*exp(2)^2+3*exp(1)^7*exp(2)^3+
9*exp(1)^11*exp(2))+1/2*(48*d^4*exp(1)^10*exp(2)^2-104*d^4*exp(1)^8*exp(2)^
3-280*d^4*exp(1)^6*exp(2)^4+22*d^4*exp(1)^4*exp(2)^5+440*d^4*exp(2)^7)*atan
((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4
+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^15-3*exp(1)^11*exp(2)^2-exp(1
)^9*exp(2)^3-3*exp(1)^13*exp(2))-95/8*d^4*sign(d)*asin(x*exp(2)/d/exp(1))/e
xp(1)/exp(2)+2*(((12*exp(1)^8*1/96/exp(1)^8*x-64*exp(1)^7*d*1/96/exp(1)^8)*
x+186*exp(1)^6*d^2*1/96/exp(1)^8)*x-512*exp(1)^5*d^3*1/96/exp(1)^8)*sqrt(d^
2-x^2*exp(2))
```

maple [A] time = 0.01, size = 288, normalized size = 1.58

$$-\frac{95 d^4 \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{8\sqrt{e^2}\, e^2}-\frac{95\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2}\, d^2 x}{8e^2}-\frac{95\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} x}{12e^2}-\frac{19\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{5}{2}}}{3d\, e^3}-\frac{\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{7}{2}} d}{\left(x+\frac{d}{e}\right)^4 e^7}-\frac{19\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{7}{2}}}{3\left(x+\frac{d}{e}\right)^2 d\, e^5}-\frac{5\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^3 e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -d/e^7/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-5/e^6/(x+d/e)^3*(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-19/3/d/e^5/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2
*e^2)^(7/2)-19/3/d/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-95/12/e^2*(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-95/8*d^2/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)*x-95/8*d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^
2*e^2)^(1/2)*x)
```

maxima [C] time = 1.02, size = 363, normalized size = 1.99

$$\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d^2}{2\left(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3\right)}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^3}{2\left(e^5x^2+2\,de^4x+d^2e^3\right)}-\frac{15\sqrt{-e^2x^2+d^2}\, d^4}{e^4x+de^3}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}} d}{3\left(e^5x^2+2\,de^4x+d^2e^3\right)}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d^2}{3\left(e^4x+de^3\right)}-\frac{5i\,d^4\arcsin\left(\frac{ex}{d}+2\right)}{8\,e^3}-\frac{25\,d^4\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{4\left(e^4x+de^3\right)}+\frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}\, d^2x}{8\,e^2}+\frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}\, d^3}{4\,e^3}-\frac{5\sqrt{-e^2x^2+d^2}\, d^3}{e^3}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}} d}{12\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(-e^2*x^2 + d^2)^(5/2)*d^2/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e
^3) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d^3/(e^5*x^2 + 2*d*e^4*x + d^2*e^3) - 15*s
qrt(-e^2*x^2 + d^2)*d^4/(e^4*x + d*e^3) - 2/3*(-e^2*x^2 + d^2)^(5/2)*d/(e^5
*x^2 + 2*d*e^4*x + d^2*e^3) - 5/3*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^4*x + d*e^3
) - 5/8*I*d^4*arcsin(e*x/d + 2)/e^3 - 25/2*d^4*arcsin(e*x/d)/e^3 + 1/4*(-e^
2*x^2 + d^2)^(5/2)/(e^4*x + d*e^3) + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^
2*x/e^2 + 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3/e^3 - 5*sqrt(-e^2*x^2 + d
^2)*d^3/e^3 + 5/12*(-e^2*x^2 + d^2)^(3/2)*d/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.202 $\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=130

$$\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Rubi [A] time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {793, 663, 665, 195, 217, 203}

$$\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (10*d*x*Sqrt[d^2 - e^2*x^2])/e + (20*(d^2 - e^2*x^2)^(3/2))/(3*e^2) + (8*(d
^2 - e^2*x^2)^(5/2))/(e^2*(d + e*x)^2) + (d^2 - e^2*x^2)^(7/2)/(e^2*(d + e*
x)^4) + (10*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx &= \frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{4\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx}{e}\\
&= \frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{20\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx}{e}\\
&= \frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{(20d)\int \sqrt{d^2-e^2x^2}\,dx}{e}\\
&= \frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{\left(10d^3\right)\int \cdots}{e}\\
&= \frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{\left(10d^3\right)\mathrm{Sub}\cdots}{\cdots}\\
&= \frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{10d^3\tan^{-1}\cdots}{e\cdots}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 83, normalized size = 0.64

$$\frac{1}{3}\sqrt{d^2-e^2x^2}\left(\frac{24d^3}{e^2(d+ex)}+\frac{23d^2}{e^2}-\frac{6dx}{e}+x^2\right)+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((23*d^2)/e^2 - (6*d*x)/e + x^2 + (24*d^3)/(e^2*(d + e
*x))))/3 + (10*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

IntegrateAlgebraic [A] time = 0.48, size = 107, normalized size = 0.82

$$\frac{10d^3\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^3}+\frac{\sqrt{d^2-e^2x^2}\,\left(47d^3+17d^2ex-5de^2x^2+e^3x^3\right)}{3e^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(47*d^3 + 17*d^2*e*x - 5*d*e^2*x^2 + e^3*x^3))/(3*e^2*
(d + e*x)) + (10*d^3*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])
/e^3
```

fricas [A] time = 0.40, size = 111, normalized size = 0.85

$$\frac{47\,d^3ex+47\,d^4-60\left(d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(e^3x^3-5\,de^2x^2+17\,d^2ex+47\,d^3\right)\sqrt{-e^2x^2+d^2}}{3\left(e^3x+de^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(47*d^3*e*x + 47*d^4 - 60*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) + (e^3*x^3 - 5*d*e^2*x^2 + 17*d^2*e*x + 47*d^3)*sqrt(-e^2*x^2
+ d^2))/(e^3*x + d*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (6*d^
3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*e
xp(2)^2+144*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^3*exp(1)^12*exp(2)^2+108*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^4*exp(1)^10*exp(2)^3+18*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+6*d^3*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2-178*d^3*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3-2
13*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)
^8*exp(2)^4-57*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^5*exp(1)^6*exp(2)^5+324*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^2*exp(1)^10*exp(2)^3-288*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-432*d^3*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-108*d^3*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^
6-336*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp
(1)^8*exp(2)^4-516*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^3*exp(1)^6*exp(2)^5-258*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6-48*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8-972*d^3*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-756*d^3*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+d^3*e
xp(1)^8*exp(2)^4-186*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^4*exp(2)^8-600*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^2*exp(1)^4*exp(2)^6+36*d^3*exp(1)^6*exp(2)^5-360*d^3*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^8-46*d^3*exp(1
)^4*exp(2)^6-372*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^2*exp(2)^8-186*d^3*exp(2)^8+4*d^3*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)+156*d^3*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+324*d^3*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+219/2*d^3*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(2)-99*d^3*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-3*d^3*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^12+9*exp(1)^8*exp(2)^2+3*exp(1)^6
*exp(2)^3+9*exp(1)^10*exp(2))+1/2*(12*d^3*exp(1)^10*exp(2)^2-86*d^3*exp(1)^
8*exp(2)^3-136*d^3*exp(1)^6*exp(2)^4+64*d^3*exp(1)^4*exp(2)^5+272*d^3*exp(2
)^7)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(
```

```
-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(exp(1)^14+3*exp(1)^10*exp(2)
^2+exp(1)^8*exp(2)^3+3*exp(1)^12*exp(2))+10*d^3*sign(d)*asin(x*exp(2)/d/exp
(1))/exp(1)^2+2*((2*exp(1)^4*1/12/exp(1)^4*x-12*exp(1)^3*d*1/12/exp(1)^4)*x
+46*exp(1)^2*d^2*1/12/exp(1)^4)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 290, normalized size = 2.23

$$\frac{10d^3 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\,e}+\frac{10\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\,dx}{e}+\frac{20\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} x}{3de}+\frac{16\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{5}{2}}}{3d^2e^2}+\frac{4\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^3 d\,e^5}+\frac{16\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{7}{2}}}{3\left(x+\frac{d}{e}\right)^2 d^2e^4}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^4 e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] 1/e^6/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+4/d/e^5/(x+d/e)^3*(2*(x
+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+16/3/d^2/e^4/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(7/2)+16/3/d^2/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+20/3/d/e*(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+10*d/e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1
/2)*x+10*d^3/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)
^(1/2)*x)
```

maxima [A] time = 0.99, size = 235, normalized size = 1.81

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{2\left(e^5x^3+3de^4x^2+3d^2e^3x+d^3e^2\right)}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2}{2\left(e^4x^2+2de^3x+d^2e^2\right)}+\frac{15\sqrt{-e^2x^2+d^2}\,d^3}{e^3x+de^2}+\frac{10\,d^3\arcsin\left(\frac{ex}{d}\right)}{e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{3\left(e^4x^2+2de^3x+d^2e^2\right)}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{6\left(e^3x+de^2\right)}+\frac{5\sqrt{-e^2x^2+d^2}\,d^2}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(-e^2*x^2 + d^2)^(5/2)*d/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^
2) - 5/2*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 15*sq
rt(-e^2*x^2 + d^2)*d^3/(e^3*x + d*e^2) + 10*d^3*arcsin(e*x/d)/e^2 + 1/3*(-e
^2*x^2 + d^2)^(5/2)/(e^4*x^2 + 2*d*e^3*x + d^2*e^2) + 5/6*(-e^2*x^2 + d^2)^
(3/2)*d/(e^3*x + d*e^2) + 5/2*sqrt(-e^2*x^2 + d^2)*d^2/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.203 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=113

$$-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3}-\frac{5\left(d^2-e^2x^2\right)^{3/2}}{2e(d+ex)}-\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {663, 665, 217, 203}

$$-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3}-\frac{5\left(d^2-e^2x^2\right)^{3/2}}{2e(d+ex)}-\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (-15*d*Sqrt[d^2 - e^2*x^2])/(2*e) - (5*(d^2 - e^2*x^2)^(3/2))/(2*e*(d + e*x
)) - (2*(d^2 - e^2*x^2)^(5/2))/(e*(d + e*x)^3) - (15*d^2*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(2*e)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - 5\int \frac{(d^2-e^2x^2)^{3/2}}{(d+ex)^2}\,dx$$
$$= -\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{1}{2}(15d)\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx$$
$$= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{1}{2}(15d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{1}{2}(15d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx\right.$$
$$= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.66

$$\sqrt{d^2-e^2x^2}\left(-\frac{8d^2}{e(d+ex)} - \frac{4d}{e} + \frac{x}{2}\right) - \frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((-4*d)/e + x/2 - (8*d^2)/(e*(d + e*x))) - (15*d^2*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

IntegrateAlgebraic [A] time = 0.45, size = 98, normalized size = 0.87

$$\frac{\sqrt{d^2-e^2x^2}\left(-24d^2-7dex+e^2x^2\right)}{2e(d+ex)} - \frac{15d^2\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^2 - 7*d*e*x + e^2*x^2))/(2*e*(d + e*x)) - (15*d
^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^2)
```

fricas [A] time = 0.40, size = 99, normalized size = 0.88

$$-\frac{24\,d^2ex + 24\,d^3 - 30\left(d^2ex + d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(e^2x^2 - 7\,dex - 24\,d^2\right)\sqrt{-e^2x^2+d^2}}{2\left(e^2x + de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^2*e*x + 24*d^3 - 30*(d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) - (e^2*x^2 - 7*d*e*x - 24*d^2)*sqrt(-e^2*x^2 + d^2))/(e^2*x +
 d*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (12*d
^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*
exp(2)^2+6*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
5*exp(1)^10*exp(2)^3+36*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^4*exp(1)^10*exp(2)^3+18*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+12*d^2*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2+280*d^2*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3+28
8*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^
8*exp(2)^4+72*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^5*exp(1)^6*exp(2)^5-36*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^2*exp(1)^10*exp(2)^3+360*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4+324*d^2*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5+72*d^2*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^6+5
22*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)
^8*exp(2)^4+546*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^3*exp(1)^6*exp(2)^5+189*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^6+27*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8+756*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^5+540*d^2*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+2*d^2*ex
p(1)^8*exp(2)^4+126*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^4*exp(2)^8+444*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)^4*exp(2)^6+216*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^3*exp(2)^8+67*d^2*exp(1)^4*exp(2)^6+252*d^2*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8+126*d^2*e
xp(2)^8+8*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3
*exp(1)^14*exp(2)-189/2*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp
(2)^8/x/exp(2)-234*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4
*exp(2)^6/x/exp(2)-165*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(
1)^6*exp(2)^5/x/exp(2)+9*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*ex
p(1)^8*exp(2)^4/x/exp(2)-3*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*
exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2
))^3/(3*exp(1)^11+9*exp(1)^7*exp(2)^2+3*exp(1)^5*exp(2)^3+9*exp(1)^9*exp(2)
)+1/2*(48*d^2*exp(1)^8*exp(2)^3+40*d^2*exp(1)^6*exp(2)^4-66*d^2*exp(1)^4*ex
p(2)^5-148*d^2*exp(2)^7)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(exp(1)^1
3+3*exp(1)^9*exp(2)^2+exp(1)^7*exp(2)^3+3*exp(1)^11*exp(2))-15/2*d^2*sign(d
)*asin(x*exp(2)/d/exp(1))/exp(1)+2*(2*exp(1)*1/8/exp(1)*x-16*d*1/8/exp(1))*
sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 284, normalized size = 2.51

$$-\frac{15 d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}}\right)}{2 \sqrt{e^{2}}}-\frac{15 \sqrt{2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}}\, x}{2}-\frac{5 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{3}{2}} x}{d^{2}}-\frac{4 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{5}{2}}}{d^{3} e}-\frac{\left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^{4} d\, e^{5}}-\frac{3 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^{3} d^{2} e^{4}}-\frac{4 \left(2\left(x+\frac{d}{e}\right) d e-\left(x+\frac{d}{e}\right)^{2} e^{2}\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^{2} d^{3} e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -1/e^5/d/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-3/e^4/d^2/(x+d/e)^3*
(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-4/e^3/d^3/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d
/e)^2*e^2)^(7/2)-4/e/d^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)-5/d^2*(2*(x+d/
e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-15/2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-15
```

```
/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x
)
```

maxima [A] time = 0.98, size = 134, normalized size = 1.19

$$-\frac{15\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{2\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{2\left(e^3x^2+2\,de^2x+d^2e\right)}-\frac{15\,\sqrt{-e^2x^2+d^2}\,d^2}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -15/2*d^2*arcsin(e*x/d)/e + 1/2*(-e^2*x^2 + d^2)^(5/2)/(e^4*x^3 + 3*d*e^3*x
^2 + 3*d^2*e^2*x + d^3*e) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d/(e^3*x^2 + 2*d*e^2
*x + d^2*e) - 15*sqrt(-e^2*x^2 + d^2)*d^2/(e^2*x + d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

3.204 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx$

Optimal. Leaf size=89

$$\frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.21, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1805, 1809, 844, 217, 203, 266, 63, 208}

$$\frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] (8*d*(d - e*x))/Sqrt[d^2 - e^2*x^2] + Sqrt[d^2 - e^2*x^2] + 4*d*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4-4d^3ex+d^2e^2x^2}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + \frac{\int \frac{d^4e^2+4d^3e^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$

$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + d^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + (4de)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + \frac{1}{2}d^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + (4de)\,\text{Subst}\left(\int \frac{}{1-}\right.$$

$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + 4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^2\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e}\right.}{e^2}$$

$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + 4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] time = 0.15, size = 79, normalized size = 0.89

$$\sqrt{d^2-e^2x^2}\left(\frac{8d}{d+ex}+1\right) - d\log\left(\sqrt{d^2-e^2x^2}+d\right) + 4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*(1 + (8*d)/(d + e*x)) + 4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]] + d*Log[x] - d*Log[d + Sqrt[d^2 - e^2*x^2]]
```

IntegrateAlgebraic [A] time = 0.64, size = 117, normalized size = 1.31

$$\frac{\sqrt{d^2-e^2x^2}\,(9d+ex)}{d+ex}+\frac{4d\sqrt{-e^2}\,\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e}+2d\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] ((9*d + e*x)*Sqrt[d^2 - e^2*x^2])/(d + e*x) + 2*d*ArcTanh[(Sqrt[-e^2]*x)/d
- Sqrt[d^2 - e^2*x^2]/d] + (4*d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 -
e^2*x^2]])/e
```

fricas [A] time = 0.41, size = 111, normalized size = 1.25

$$\frac{9\,dex+9\,d^2-8\left(dex+d^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(dex+d^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}\,(ex+9\,d)}{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] (9*d*e*x + 9*d^2 - 8*(d*e*x + d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + (d*e*x + d^2)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2)
*(e*x + 9*d))/(e*x + d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-54*
d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*e
xp(2)^2-18*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*
exp(1)^10*exp(2)^3-144*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^3*exp(1)^12*exp(2)^2-180*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3-54*d*(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4-78*d*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^2-370*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3
-321*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)
^8*exp(2)^4-75*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^5*exp(1)^6*exp(2)^5-252*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)^10*exp(2)^3-432*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-216*d*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-36*d*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^6-624*d*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^
4-528*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1
)^6*exp(2)^5-132*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^4*exp(1)^4*exp(2)^6-12*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
```

```
1))/x/exp(2))^5*exp(2)^8-540*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^2*exp(1)^6*exp(2)^5-324*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6-11*d*exp(1)^8*exp(2)^4-72*d*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8-312*
d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*ex
p(2)^6-36*d*exp(1)^6*exp(2)^5-108*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^3*exp(2)^8-76*d*exp(1)^4*exp(2)^6-144*d*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-72*d*exp(2)^8-44*d
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*ex
p(2)+48*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/exp(2)+144
*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^6/x/exp(2)+3
81/2*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^5/x/exp(
2)+81*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^4/x/exp
(2)+24*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^3/x/e
xp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2
)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/(3*exp(1)^10+9*ex
p(1)^6*exp(2)^2+3*exp(1)^4*exp(2)^3+9*exp(1)^8*exp(2))+4*d*sign(d)*asin(x*e
xp(2)/d/exp(1))+1/2*(-12*d*exp(1)^10*exp(2)^2+2*d*exp(1)^8*exp(2)^3-8*d*exp
(1)^6*exp(2)^4-40*d*exp(1)^4*exp(2)^5-64*d*exp(2)^7-4*d*exp(1)^12*exp(2))*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-exp(1)^12-3*exp(1)^8*exp(2)^2-exp
(1)^6*exp(2)^3-3*exp(1)^10*exp(2))-d*exp(2)*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)^2+sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.01, size = 378, normalized size = 4.25

$$-\frac{d^2 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{4de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}}+\frac{4\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,ex}{d}+\frac{8\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}ex}{3d^3}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3d^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5d^4}+\frac{32\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}}{15d^4}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^4d^2e^4}+\frac{2\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^3d^3e^3}+\frac{7\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{3\left(x+\frac{d}{e}\right)^2d^4e^2}+\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x)
```

```
[Out] -1/(d^2)^(1/2)*d^2*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^2/e
^4/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+2/d^3/e^3/(x+d/e)^3*(2*(x+
d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+7/3/d^4/e^2/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^
2*e^2)^(7/2)+8/3/d^3*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+4/d*e*(2*(x+d/
e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+(-e^2*x^2+d^2)^(1/2)+4*d*e/(e^2)^(1/2)*arctan
((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)+1/5/d^4*(-e^2*x^2+d^2)^
(5/2)+1/3/d^2*(-e^2*x^2+d^2)^(3/2)+32/15/d^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^{\frac{5}{2}}}{x(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x*(d + e*x)**4), x)
```

3.205 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^4}\,dx$

Optimal. Leaf size=94

$$-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Rubi [A] time = 0.22, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {852, 1805, 1807, 844, 217, 203, 266, 63, 208}

$$-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] (-8*e*(d - e*x))/Sqrt[d^2 - e^2*x^2] - Sqrt[d^2 - e^2*x^2]/x - e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]] + 4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex+d^2e^2x^2}{x^2\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} + \frac{\int \frac{-4d^5e-d^4e^2x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - (4de)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - (2de)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - e^2\,\text{Subst}\left(\int \frac{1}{1+e^2
\right.\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{(4d)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2
\right.}{e}\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.19, size = 84, normalized size = 0.89

$$\sqrt{d^2-e^2x^2}\left(-\frac{8e}{d+ex}-\frac{1}{x}\right)+4e\log\left(\sqrt{d^2-e^2x^2}+d\right)-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-4e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*(-x^(-1) - (8*e)/(d + e*x)) - e*ArcTan[(e*x)/Sqrt[d^2 -
 e^2*x^2]] - 4*e*Log[x] + 4*e*Log[d + Sqrt[d^2 - e^2*x^2]]
```

IntegrateAlgebraic [A] time = 0.54, size = 117, normalized size = 1.24

$$\frac{\sqrt{d^2-e^2x^2}\,(-d-9ex)}{x(d+ex)}-\sqrt{-e^2}\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)-8e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] ((-d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(x*(d + e*x)) - 8*e*ArcTanh[(Sqrt[-e^2]*
x)/d - Sqrt[d^2 - e^2*x^2]/d] - Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 -
 e^2*x^2]]
```

fricas [A] time = 0.42, size = 127, normalized size = 1.35

$$-\frac{8\,e^2x^2+8\,dex-2\left(e^2x^2+dex\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+4\left(e^2x^2+dex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}\,(9\,ex+d)}{ex^2+dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -(8*e^2*x^2 + 8*d*e*x - 2*(e^2*x^2 + d*e*x)*arctan(-(d - sqrt(-e^2*x^2 + d^
2))/(e*x)) + 4*(e^2*x^2 + d*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(
-e^2*x^2 + d^2)*(9*e*x + d))/(e*x^2 + d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/4*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^3/exp(1)^4/x/exp(1)/exp(2
)+1/2*(-48*exp(1)^10*exp(2)^2-40*exp(1)^8*exp(2)^3-8*exp(1)^6*exp(2)^4+2*ex
p(1)^4*exp(2)^5-16*exp(2)^7-16*exp(1)^12*exp(2))*atan((-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(
1)^4+exp(2)^2)/(exp(1)^11+3*exp(1)^7*exp(2)^2+exp(1)^5*exp(2)^3+3*exp(1)^9*
exp(2))+4*exp(2)*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(
x)/exp(2))/exp(1)-1/3*x*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^3*(408*exp(1)^12*exp(2)^2+1152*exp(1)^10*exp(2)^3+1392*exp(1)^8*e
xp(2)^4+780*exp(1)^6*exp(2)^5+516*exp(1)^4*exp(2)^6+132*exp(2)^8)+(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(576*exp(1)^12*exp(2)^
2+932*exp(1)^10*exp(2)^3+1116*exp(1)^8*exp(2)^4+1041*exp(1)^6*exp(2)^5+279*
exp(1)^4*exp(2)^6+108*exp(2)^8+208*exp(1)^14*exp(2))+(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(240*exp(1)^12*exp(2)^2+648*exp(1)^
10*exp(2)^3+642*exp(1)^8*exp(2)^4+270*exp(1)^6*exp(2)^5+228*exp(1)^4*exp(2)
^6+66*exp(2)^8)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^6*(72*exp(1)^10*exp(2)^3+180*exp(1)^8*exp(2)^4+135*exp(1)^6*exp(2)^5+9*exp
(1)^4*exp(2)^6+18*exp(2)^8)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*(276*exp(1)^10*exp(2)^3+792*exp(1)^8*exp(2)^4+861*exp(1)^6*e
xp(2)^5+279*exp(1)^4*exp(2)^6+102*exp(2)^8)+3*exp(1)^6*exp(2)^5+9*exp(1)^4*
exp(2)^6+12*exp(2)^8-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*(70*ex
p(1)^8*exp(2)^4+198*exp(1)^6*exp(2)^5+200*exp(1)^4*exp(2)^6+66*exp(2)^8)/x/
exp(2))*exp(2)/(2*exp(2))^3/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
```

```
1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2)
)^3/(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/exp(1)/exp(2)-sign(d)*asin(
x*exp(2)/d/exp(1))*exp(2)/exp(1)
```

maple [B] time = 0.01, size = 515, normalized size = 5.48

$$\frac{4de\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}+\frac{7e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{8\sqrt{e^2}}-\frac{15e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}-\frac{15\sqrt{-e^2x^2+d^2}\,e^2x}{8d^2}+\frac{7\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^2x}{8d^2}-\frac{4\sqrt{-e^2x^2+d^2}\,e}{d}-\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2x}{4d^4}+\frac{7\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^2x}{12d^4}-\frac{4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}{3d^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2x}{d^6}-\frac{4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}{5d^5}+\frac{7\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e}{15d^5}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^4d^3e^3}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^3d^4e^2}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{3\left(x+\frac{d}{e}\right)^2d^5e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x)
```

```
[Out] -15/8/(e^2)^(1/2)*e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/d^6*e^2*
x*(-e^2*x^2+d^2)^(5/2)-5/4/d^4*e^2*x*(-e^2*x^2+d^2)^(3/2)-15/8/d^2*e^2*x*(-
e^2*x^2+d^2)^(1/2)-1/d^3/e^3/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-
1/d^4/e^2/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-1/3/d^5/e/(x+d/e)^2
*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+7/12/d^4*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*
e^2)^(3/2)*x+7/8/d^2*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+4/(d^2)^(1/2
)*d*e*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-4/d*e*(-e^2*x^2+d^2)
^(1/2)-4/5/d^5*e*(-e^2*x^2+d^2)^(5/2)-4/3/d^3*e*(-e^2*x^2+d^2)^(3/2)-1/d^6/
x*(-e^2*x^2+d^2)^(7/2)+7/15/d^5*e*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+7/8*e
^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{\left(ex+d\right)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^2\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^2\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**2*(d + e*x)**4), x)
```

3.206 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx$

Optimal. Leaf size=110

$$\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Rubi [A] time = 0.22, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] (8*e^2*(d - e*x))/(d*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*x^2) + (
4*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (15*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2
*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{\int \frac{-8d^5e+15d^4e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^4}\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} + \frac{1}{2}\left(15e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} + \frac{1}{4}\left(15e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{15}{2}\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)\\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 85, normalized size = 0.77

$$
\frac{\frac{\sqrt{d^2-e^2x^2}\left(-d^2+7dex+24e^2x^2\right)}{x^2(d+ex)} - 15e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + 15e^2\log(x)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^2 + 7*d*e*x + 24*e^2*x^2))/(x^2*(d + e*x)) + 15*e
^2*Log[x] - 15*e^2*Log[d + Sqrt[d^2 - e^2*x^2]])/(2*d)
```

IntegrateAlgebraic [A] time = 0.62, size = 140, normalized size = 1.27

$$\frac{\sqrt{d^2-e^2x^2}\left(-d^2+7dex+24e^2x^2\right)}{2dx^2(d+ex)}-\frac{15e^2\log\left(\sqrt{d^2-e^2x^2}+d-\sqrt{-e^2}\,x\right)}{2d}+\frac{15e^2\log\left(-d\sqrt{d^2-e^2x^2}+d^2+d\sqrt{-e^2}\,x\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 + 7*d*e*x + 24*e^2*x^2))/(2*d*x^2*(d + e*x)) - (
15*e^2*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]])/(2*d) + (15*e^2*Log[d^2
 + d*Sqrt[-e^2]*x - d*Sqrt[d^2 - e^2*x^2]])/(2*d)
```

fricas [A] time = 0.41, size = 112, normalized size = 1.02

$$\frac{16\,e^3x^3+16\,de^2x^2+15\left(e^3x^3+de^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(24\,e^2x^2+7\,dex-d^2\right)\sqrt{-e^2x^2+d^2}}{2\left(dex^3+d^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(16*e^3*x^3 + 16*d*e^2*x^2 + 15*(e^3*x^3 + d*e^2*x^2)*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) + (24*e^2*x^2 + 7*d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/(d*e
*x^3 + d^2*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/16*
(-2*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^
4*exp(2)^11-16*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2
)^10/x/exp(2))/d^2/exp(1)^6/exp(2)^9+1/24*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^4*(-3216*exp(1)^14*exp(2)^2-7776*exp(1)^12*exp(
2)^3-6300*exp(1)^10*exp(2)^4-2868*exp(1)^8*exp(2)^5-2571*exp(1)^6*exp(2)^6-
225*exp(1)^4*exp(2)^7+36*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^5*(-3456*exp(1)^14*exp(2)^2-4688*exp(1)^12*exp(2)^3-633
6*exp(1)^10*exp(2)^4-4638*exp(1)^8*exp(2)^5-90*exp(1)^6*exp(2)^6-378*exp(1)
^4*exp(2)^7-126*exp(2)^9-1504*exp(1)^16*exp(2))+(-1/2*(-2*d*exp(1)-2*sqrt(d
^2-x^2*exp(2))*exp(1))/x/exp(2))^6*(-1680*exp(1)^14*exp(2)^2-3744*exp(1)^12
*exp(2)^3-2376*exp(1)^10*exp(2)^4-864*exp(1)^8*exp(2)^5-1293*exp(1)^6*exp(2
)^6-135*exp(1)^4*exp(2)^7+12*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^7*(-480*exp(1)^12*exp(2)^3-1008*exp(1)^10*exp(2)^4-
408*exp(1)^8*exp(2)^5+144*exp(1)^6*exp(2)^6-144*exp(1)^4*exp(2)^7-48*exp(2)
^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-2328*ex
p(1)^12*exp(2)^3-5832*exp(1)^10*exp(2)^4-4188*exp(1)^8*exp(2)^5-300*exp(1)^
6*exp(2)^6-324*exp(1)^4*exp(2)^7-108*exp(2)^9)+(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*(-628*exp(1)^10*exp(2)^4-1620*exp(1)^8*ex
p(2)^5-1211*exp(1)^6*exp(2)^6-81*exp(1)^4*exp(2)^7+36*exp(2)^9)+3*exp(1)^6*
exp(2)^6+9*exp(1)^4*exp(2)^7+12*exp(2)^9-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))*(-30*exp(1)^8*exp(2)^5-90*exp(1)^6*exp(2)^6-90*exp(1)^4*exp(2
)^7-30*exp(2)^9)/x/exp(2))/(2*exp(2))^3/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^2/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))
^3/exp(1)^3/d/exp(1)+1/2*(5*exp(2)^3-20*exp(1)^4*exp(2))*ln(1/2*abs(-2*d*ex
```

```
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/exp(1)^3/d/exp(1)+1/2*(-
108*exp(1)^7*exp(2)^2-66*exp(1)^5*exp(2)^3+40*exp(1)^3*exp(2)^4-40*exp(1)^9
*exp(2)+48*exp(1)*exp(2)^5)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d*ex
p(1)^7-3*d*exp(1)^5*exp(2)-4*d*exp(1)*exp(2)^3)
```

maple [B] time = 0.01, size = 504, normalized size = 4.58

$$-\frac{15e^{3}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}}\right)}{2\sqrt{e^{2}}\,d}+\frac{15e^{3}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}\,d}-\frac{15e^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{2\sqrt{d^{2}}}+\frac{15\sqrt{-e^{2}x^{2}+d^{2}}\,e^{3}x}{2d^{3}}-\frac{15\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}}\,e^{3}x}{2d^{3}}+\frac{15\sqrt{-e^{2}x^{2}+d^{2}}\,e^{2}}{2d^{2}}+\frac{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}x}{d^{5}}-\frac{5\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{3}{2}}e^{3}x}{d^{5}}+\frac{5\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{2d^{4}}+\frac{4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{3}x}{d^{7}}+\frac{3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}{2d^{6}}-\frac{4\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{5}{2}}e^{2}}{d^{6}}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^{4}d^{4}e^{2}}-\frac{2\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}e^{2}\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^{2}d^{6}}+\frac{4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}e}{d^{7}x}-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{2d^{6}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x)
```

```
[Out] -15/2/(d^2)^(1/2)*e^2*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+4/d^
7*e/x*(-e^2*x^2+d^2)^(7/2)+4/d^7*e^3*x*(-e^2*x^2+d^2)^(5/2)+5/d^5*e^3*x*(-e
^2*x^2+d^2)^(3/2)+15/2/d^3*e^3*x*(-e^2*x^2+d^2)^(1/2)+15/2/d*e^3/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/d^4/e^2/(x+d/e)^4*(2*(x+d/e)
*d*e-(x+d/e)^2*e^2)^(7/2)-5/d^5*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-1
5/2/d^3*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-15/2/d*e^3/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/2/d^6/x^2*(-e^2*x
^2+d^2)^(7/2)+3/2/d^6*e^2*(-e^2*x^2+d^2)^(5/2)+5/2/d^4*e^2*(-e^2*x^2+d^2)^(
3/2)+15/2/d^2*e^2*(-e^2*x^2+d^2)^(1/2)-2/d^6/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(7/2)-4/d^6*e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{(ex+d)^{4}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^{2}-e^{2}\,x^{2}\right)^{5/2}}{x^{3}\,\left(d+e\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^{3}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**3*(d + e*x)**4), x)
```

3.207 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^4}\,dx$

Optimal. Leaf size=137

$$-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}+\frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Rubi [A] time = 0.30, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

```
[Out] (-8*e^3*(d - e*x))/(d^2*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*x^3)
+ (2*e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (23*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^2*x
) + (10*e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^4}\,dx=\int \frac{(d-ex)^4}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$=-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$=-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{\int \frac{-12d^5e+23d^4e^2x-24d^3e^3x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^4}$$

$$=-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\int \frac{-46d^6e^2+60d^5e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^6}$$

$$=-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{\left(10e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$=-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{\left(5e^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-\dots}}\dots\right)}{d}$$

$$=-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{(10e)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\dots\right)}{d}$$

$$=-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Mathematica [A] time = 0.25, size = 94, normalized size = 0.69

$$-\frac{-30e^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(d^3-5d^2ex+17de^2x^2+47e^3x^3\right)}{x^3(d+ex)}+30e^3\log(x)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

```
[Out] -1/3*((Sqrt[d^2 - e^2*x^2]*(d^3 - 5*d^2*e*x + 17*d*e^2*x^2 + 47*e^3*x^3))/(
x^3*(d + e*x)) + 30*e^3*Log[x] - 30*e^3*Log[d + Sqrt[d^2 - e^2*x^2]])/d^2
```

IntegrateAlgebraic [A] time = 0.76, size = 109, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\left(-d^3+5d^2ex-17de^2x^2-47e^3x^3\right)}{3d^2x^3(d+ex)}-\frac{20e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^3 + 5*d^2*e*x - 17*d*e^2*x^2 - 47*e^3*x^3))/(3*d^2
*x^3*(d + e*x)) - (20*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d]
)/d^2
```

fricas [A] time = 0.40, size = 123, normalized size = 0.90

$$-\frac{24\,e^4x^4+24\,de^3x^3+30\left(e^4x^4+de^3x^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(47\,e^3x^3+17\,de^2x^2-5\,d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}}{3\left(d^2ex^4+d^3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(24*e^4*x^4 + 24*d*e^3*x^3 + 30*(e^4*x^4 + d*e^3*x^3)*log(-(d - sqrt(-
e^2*x^2 + d^2))/x) + (47*e^3*x^3 + 17*d*e^2*x^2 - 5*d^2*e*x + d^3)*sqrt(-e^
2*x^2 + d^2))/(d^2*e*x^4 + d^3*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/512
*(256*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp
(1)^10*exp(2)^16-64/3*d^4*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^3*exp(1)^8*exp(2)^17+96*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))*exp(1)^8*exp(2)^17/x/exp(2)-384*d^4*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))*exp(1)^10*exp(2)^16/x/exp(2)+1280*d^4*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^15/x/exp(2))/d^6/exp(1)^15/exp(2)^12+1
/72*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(16416*e
xp(1)^16*exp(2)^2+34560*exp(1)^14*exp(2)^3+20376*exp(1)^12*exp(2)^4+8712*ex
p(1)^10*exp(2)^5+9234*exp(1)^8*exp(2)^6-1674*exp(1)^6*exp(2)^7-594*exp(1)^4
*exp(2)^8+234*exp(2)^10)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^6*(13824*exp(1)^16*exp(2)^2+17952*exp(1)^14*exp(2)^3+27216*exp(1)
^12*exp(2)^4+14724*exp(1)^10*exp(2)^5-4104*exp(1)^8*exp(2)^6+1380*exp(1)^6*
exp(2)^7+468*exp(1)^4*exp(2)^8+12*exp(2)^10+7104*exp(1)^18*exp(2))+(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*(7776*exp(1)^16*exp(2
)^2+14688*exp(1)^14*exp(2)^3+6192*exp(1)^12*exp(2)^4+3240*exp(1)^10*exp(2)^
5+5454*exp(1)^8*exp(2)^6-702*exp(1)^6*exp(2)^7-270*exp(1)^4*exp(2)^8+126*ex
p(2)^10)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*(216
0*exp(1)^14*exp(2)^3+3888*exp(1)^12*exp(2)^4+648*exp(1)^10*exp(2)^5-756*exp
(1)^8*exp(2)^6+855*exp(1)^6*exp(2)^7+117*exp(1)^4*exp(2)^8)+(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(12528*exp(1)^14*exp(2)^3+27
648*exp(1)^12*exp(2)^4+13392*exp(1)^10*exp(2)^5-3204*exp(1)^8*exp(2)^6+774*
exp(1)^6*exp(2)^7+306*exp(1)^4*exp(2)^8+36*exp(2)^10)+(-1/2*(-2*d*exp(1)-2*
```

```
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(3528*exp(1)^12*exp(2)^4+8064*exp(
1)^10*exp(2)^5+4146*exp(1)^8*exp(2)^6-1218*exp(1)^6*exp(2)^7-378*exp(1)^4*e
xp(2)^8+90*exp(2)^10)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*(180*exp(1)^10*exp(2)^5+432*exp(1)^8*exp(2)^6+252*exp(1)^6*exp(2)^
7-36*exp(1)^4*exp(2)^8+36*exp(2)^10)+3*exp(1)^6*exp(2)^7+9*exp(1)^4*exp(2)^
8+12*exp(2)^10-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*(-18*exp(1)^
8*exp(2)^6-54*exp(1)^6*exp(2)^7-54*exp(1)^4*exp(2)^8-18*exp(2)^10)/x/exp(2)
)/d^2/(2*exp(2))^3/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^3/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)
-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/exp(1)^5-(10*exp(2
)^3-20*exp(1)^4*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/abs(x)/exp(2))/d^2/exp(1)/exp(2)+1/2*(-192*exp(1)^8*exp(2)^2-64*exp(1)^6
*exp(2)^3+136*exp(1)^4*exp(2)^4+74*exp(2)^6-80*exp(1)^10*exp(2))*atan((-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2
)^2))/sqrt(-exp(1)^4+exp(2)^2)/(d^2*exp(1)^7+3*d^2*exp(1)^5*exp(2)+4*d^2*ex
p(1)*exp(2)^3)
```

maple [B] time = 0.02, size = 575, normalized size = 4.20

$$\frac{10e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d}+\frac{65e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{4\sqrt{e^2}\,d^2}-\frac{65e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}\,d^2}-\frac{65\sqrt{-e^2x^2+d^2}\,e^4x}{4d^4}+\frac{65\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^4x}{4d^4}-\frac{10\sqrt{-e^2x^2+d^2}\,e^3}{d^3}-\frac{65\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4x}{6d^6}+\frac{65\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^4x}{6d^6}-\frac{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}{3d^5}-\frac{26\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4x}{3d^8}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}{d^7}+\frac{26\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e^3}{3d^7}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^4d^5e}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^3d^6}+\frac{14\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}e}{3\left(x+\frac{d}{e}\right)^2d^7}-\frac{26\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{3d^8x}+\frac{2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{d^7x^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{3d^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x)
```

```
[Out] -26/3/d^8*e^4*x*(-e^2*x^2+d^2)^(5/2)-65/6/d^6*e^4*x*(-e^2*x^2+d^2)^(3/2)-65
/4/d^4*e^4*x*(-e^2*x^2+d^2)^(1/2)-65/4/d^2*e^4/(e^2)^(1/2)*arctan((e^2)^(1/
2)/(-e^2*x^2+d^2)^(1/2)*x)+2/d^7*e/x^2*(-e^2*x^2+d^2)^(7/2)-1/d^5/e/(x+d/e)
^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+14/3/d^7*e/(x+d/e)^2*(2*(x+d/e)*d*e-
(x+d/e)^2*e^2)^(7/2)+65/6/d^6*e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+65/
4/d^4*e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x+65/4/d^2*e^4/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-26/3/d^8*e^2/x*(-e^
2*x^2+d^2)^(7/2)+10/(d^2)^(1/2)/d*e^3*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2
)^(1/2))/x)-1/3/d^6/x^3*(-e^2*x^2+d^2)^(7/2)-2/d^7*e^3*(-e^2*x^2+d^2)^(5/2)
-10/3/d^5*e^3*(-e^2*x^2+d^2)^(3/2)-10/d^3*e^3*(-e^2*x^2+d^2)^(1/2)+26/3/d^7
*e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+1/d^6/(x+d/e)^3*(2*(x+d/e)*d*e-(x+
d/e)^2*e^2)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{\left(ex+d\right)^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^4\,{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^{4}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**4*(d + e*x)**4), x)
```

3.208 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx$

Optimal. Leaf size=170

$$-\frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{4e\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}}-\frac{95e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}+\frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x}$$

Rubi [A] time = 0.39, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}}+\frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{95e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] (8*e^4*(d - e*x))/(d^3*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(4*x^4) +
 (4*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (31*e^2*Sqrt[d^2 - e^2*x^2])/(8*d^2*
x^2) + (32*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) - (95*e^4*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])/(8*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^5\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3-8e^4x^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{\int \frac{-16d^5e+31d^4e^2x-32d^3e^3x^2+32d^2e^4x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^4}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{\int \frac{-93d^6e^2+128d^5e^3x-96d^4e^4x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^6}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{\int \frac{-256d^7e^3+285d^6e^4x}{x^2\sqrt{d^2-e^2x^2}}\,d}{24d^8}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} +$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} +$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} -$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} -$$

Mathematica [A] time = 0.27, size = 107, normalized size = 0.63

$$\frac{-285e^4\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(-6d^4+26d^3ex-61d^2e^2x^2+163de^3x^3+448e^4x^4\right)}{x^4(d+ex)}+285e^4\log(x)}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-6*d^4 + 26*d^3*e*x - 61*d^2*e^2*x^2 + 163*d*e^3*x^3
 + 448*e^4*x^4))/(x^4*(d + e*x)) + 285*e^4*Log[x] - 285*e^4*Log[d + Sqrt[d^
2 - e^2*x^2]])/(24*d^3)
```

IntegrateAlgebraic [A] time = 0.94, size = 122, normalized size = 0.72

$$\frac{95e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^3}+\frac{\sqrt{d^2-e^2x^2}\left(-6d^4+26d^3ex-61d^2e^2x^2+163de^3x^3+448e^4x^4\right)}{24d^3x^4(d+ex)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^4 + 26*d^3*e*x - 61*d^2*e^2*x^2 + 163*d*e^3*x^3
+ 448*e^4*x^4))/(24*d^3*x^4*(d + e*x)) + (95*e^4*ArcTanh[(Sqrt[-e^2]*x)/d -
 Sqrt[d^2 - e^2*x^2]/d])/(4*d^3)
```

fricas [A] time = 0.40, size = 136, normalized size = 0.80

$$\frac{192\,e^5x^5+192\,de^4x^4+285\left(e^5x^5+de^4x^4\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(448\,e^4x^4+163\,de^3x^3-61\,d^2e^2x^2+26\,d^3ex-6\,d^4\right)\sqrt{-e^2x^2+d^2}}{24\left(d^3ex^5+d^4x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(192*e^5*x^5 + 192*d*e^4*x^4 + 285*(e^5*x^5 + d*e^4*x^4)*log(-(d - sqr
t(-e^2*x^2 + d^2))/x) + (448*e^4*x^4 + 163*d*e^3*x^3 - 61*d^2*e^2*x^2 + 26*
d^3*e*x - 6*d^4)*sqrt(-e^2*x^2 + d^2))/(d^3*e*x^5 + d^4*x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/655
36*(-81920*d^9*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
2*exp(1)^22*exp(2)^19+32768/3*d^9*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^3*exp(1)^20*exp(2)^20-1024*d^9*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^18*exp(2)^21+24576*d^9*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^20*exp(2)^20-8
192*d^9*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1
)^18*exp(2)^21-49152*d^9*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)
^20*exp(2)^20/x/exp(2)+196608*d^9*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))*exp(1)^22*exp(2)^19/x/exp(2)-327680*d^9*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))*exp(1)^24*exp(2)^18/x/exp(2))/d^12/exp(1)^24/exp(2)^16+1/192*((
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*(-67968*exp(1)
^18*exp(2)^2-126720*exp(1)^16*exp(2)^3-53184*exp(1)^14*exp(2)^4-21408*exp(1
```

)^12*exp(2)^5-23472*exp(1)^10*exp(2)^6+19800*exp(1)^8*exp(2)^7+3699*exp(1)^6*exp(2)^8-3063*exp(1)^4*exp(2)^9+84*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*(-46080*exp(1)^18*exp(2)^2-62080*exp(1)^16*exp(2)^3-101376*exp(1)^14*exp(2)^4-33888*exp(1)^12*exp(2)^5+32688*exp(1)^10*exp(2)^6-3488*exp(1)^8*exp(2)^7-960*exp(1)^6*exp(2)^8+768*exp(1)^4*exp(2)^9-752*exp(2)^11-27392*exp(1)^20*exp(2))+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*(-29568*exp(1)^18*exp(2)^2-48384*exp(1)^16*exp(2)^3-13632*exp(1)^14*exp(2)^4-13824*exp(1)^12*exp(2)^5-16848*exp(1)^10*exp(2)^6+10440*exp(1)^8*exp(2)^7+1872*exp(1)^6*exp(2)^8-1632*exp(1)^4*exp(2)^9+24*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*(-8064*exp(1)^16*exp(2)^3-12672*exp(1)^14*exp(2)^4+192*exp(1)^12*exp(2)^5+2304*exp(1)^10*exp(2)^6-3360*exp(1)^8*exp(2)^7+672*exp(1)^6*exp(2)^8+288*exp(1)^4*exp(2)^9-288*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(-54144*exp(1)^16*exp(2)^3-106560*exp(1)^14*exp(2)^4-29184*exp(1)^12*exp(2)^5+29280*exp(1)^10*exp(2)^6-750*exp(1)^8*exp(2)^7-714*exp(1)^6*exp(2)^8+630*exp(1)^4*exp(2)^9-654*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(-15744*exp(1)^14*exp(2)^4-32160*exp(1)^12*exp(2)^5-9100*exp(1)^10*exp(2)^6+11588*exp(1)^8*exp(2)^7+1893*exp(1)^6*exp(2)^8-1473*exp(1)^4*exp(2)^9+108*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-840*exp(1)^12*exp(2)^5-1800*exp(1)^10*exp(2)^6-564*exp(1)^8*exp(2)^7+708*exp(1)^6*exp(2)^8+108*exp(1)^4*exp(2)^9-204*exp(2)^11)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*(84*exp(1)^10*exp(2)^6+180*exp(1)^8*exp(2)^7+69*exp(1)^6*exp(2)^8-33*exp(1)^4*exp(2)^9+60*exp(2)^11)+3*exp(1)^6*exp(2)^8+9*exp(1)^4*exp(2)^9+12*exp(2)^11-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*(-14*exp(1)^8*exp(2)^7-42*exp(1)^6*exp(2)^8-42*exp(1)^4*exp(2)^9-14*exp(2)^11)/x/exp(2))/d^3/(2*exp(2))^3/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/exp(1)^6+1/8*(240*exp(1)^6*exp(2)^2-64*exp(1)^4*exp(2)^3+9*exp(2)^5-280*exp(1)^8*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^3/exp(1)^5/exp(1)+1/2*(-300*exp(1)^9*exp(2)^2-22*exp(1)^7*exp(2)^3+280*exp(1)^5*exp(2)^4+104*exp(1)^3*exp(2)^5-140*exp(1)^11*exp(2)-48*exp(1)*exp(2)^6)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d^3*exp(1)^7-3*d^3*exp(1)^5*exp(2)-4*d^3*exp(1)*exp(2)^3)

maple [B] time = 0.02, size = 600, normalized size = 3.53

$$-\frac{95 e^4 \ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}\,d^2}-\frac{55 e^5 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{2\sqrt{e^2}\,d^3}+\frac{55 e^5 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,d^3}+\frac{55\sqrt{-e^2x^2+d^2}\,e^5x}{2d^5}-\frac{55\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,e^5x}{2d^5}+\frac{95\sqrt{-e^2x^2+d^2}\,e^4}{8d^4}+\frac{55\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^5x}{3d^7}-\frac{55\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}e^5x}{3d^7}+\frac{95\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}{24d^6}+\frac{44\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5x}{3d^9}+\frac{19\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}{8d^8}-\frac{44\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}e^4}{3d^8}+\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}}{\left(x+\frac{d}{e}\right)^4d^6}-\frac{2\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}e}{\left(x+\frac{d}{e}\right)^3d^7}-\frac{23\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{7}{2}}e^2}{3\left(x+\frac{d}{e}\right)^2d^8}+\frac{44\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}{3d^9x}-\frac{37\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}{8d^8x^2}+\frac{4\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}{3d^7x^3}-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{4d^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x)

[Out] 4/3/d^7*e/x^3*(-e^2*x^2+d^2)^(7/2)-37/8/d^8*e^2/x^2*(-e^2*x^2+d^2)^(7/2)+44/3/d^9*e^3/x*(-e^2*x^2+d^2)^(7/2)+44/3/d^9*e^5*x*(-e^2*x^2+d^2)^(5/2)+55/3/d^7*e^5*x*(-e^2*x^2+d^2)^(3/2)+55/2/d^5*e^5*x*(-e^2*x^2+d^2)^(1/2)+55/2/d^3*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-2/d^7*e/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-23/3/d^8*e^2/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-55/3/d^7*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-55/2/d^5*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x-55/2/d^3*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-95/8/(d^2)^(1/2)/d^2*e^4*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/4/d^6/x^4*(-e^2*x^2+d^2)^(7/2)+19/8/d^8*e^4*(-e^2*x^2+d^2)^(5/2)+95/24/d^6*e^4*(-e^2*x^2+d^2)^(3/2)+95/8/d^4*e^4*(-e^2*x^2+d^2)^(1/2)+1/d^6/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)-44/3/d^8*e^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^5\,{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^5\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**5*(d + e*x)**4), x)
```

3.209 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx$

Optimal. Leaf size=196

$$-\frac{\sqrt{d^2-e^2x^2}}{5x^5}+\frac{e\sqrt{d^2-e^2x^2}}{dx^4}-\frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}}+\frac{27e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}-\frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x}+\frac{11e$$

Rubi [A] time = 0.52, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$-\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}}-\frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x}+\frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2}-\frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}+\frac{e\sqrt{d^2-e^2x^2}}{dx^4}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}+\frac{27e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

```
[Out] (-8*e^5*(d - e*x))/(d^4*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(5*x^5)
+ (e*Sqrt[d^2 - e^2*x^2])/(d*x^4) - (13*e^2*Sqrt[d^2 - e^2*x^2])/(5*d^2*x^3
) + (11*e^3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) - (66*e^4*Sqrt[d^2 - e^2*x^2])
/(5*d^4*x) + (27*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^6\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3-8e^4x^4+\frac{8e^5x^5}{d}}{x^6\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{\int \frac{-20d^5e+39d^4e^2x-40d^3e^3x^2+40d^2e^4x^3-40de^5x^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{5d^4}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{\int \frac{-156d^6e^2+220d^5e^3x-160d^4e^4x^2+160d^3e^5x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{20d^6}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{\int \frac{-660d^7e^3+792d^6e^4x-480d\cdots}{x^3\sqrt{d^2-e^2x^2}}}{60d^8}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \int \cdots$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - 66e\cdots$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - 66e\cdots$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - 66e\cdots$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - 66e\cdots$$

Mathematica [A] time = 0.33, size = 118, normalized size = 0.60

$$-\frac{-135e^5\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(2d^5-8d^4ex+16d^3e^2x^2-29d^2e^3x^3+77de^4x^4+212e^5x^5\right)}{x^5(d+ex)}+135e^5\log(x)}{10d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

```
[Out] -1/10*((Sqrt[d^2 - e^2*x^2]*(2*d^5 - 8*d^4*e*x + 16*d^3*e^2*x^2 - 29*d^2*e^
3*x^3 + 77*d*e^4*x^4 + 212*e^5*x^5))/(x^5*(d + e*x)) + 135*e^5*Log[x] - 135
*e^5*Log[d + Sqrt[d^2 - e^2*x^2]])/d^4
```

IntegrateAlgebraic [A] time = 1.06, size = 131, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^5+8d^4ex-16d^3e^2x^2+29d^2e^3x^3-77de^4x^4-212e^5x^5\right)}{10d^4x^5(d+ex)}-\frac{27e^5\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^5 + 8*d^4*e*x - 16*d^3*e^2*x^2 + 29*d^2*e^3*x^3
- 77*d*e^4*x^4 - 212*e^5*x^5))/(10*d^4*x^5*(d + e*x)) - (27*e^5*ArcTanh[(Sq
rt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^4
```

fricas [A] time = 0.41, size = 147, normalized size = 0.75

$$-\frac{80\,e^6x^6+80\,de^5x^5+135\left(e^6x^6+de^5x^5\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(212\,e^5x^5+77\,de^4x^4-29\,d^2e^3x^3+16\,d^3e^2x^2-8\,d^4ex+2\,d^5\right)\sqrt{-e^2x^2+d^2}}{10\left(d^4ex^6+d^5x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/10*(80*e^6*x^6 + 80*d*e^5*x^5 + 135*(e^6*x^6 + d*e^5*x^5)*log(-(d - sqrt
(-e^2*x^2 + d^2))/x) + (212*e^5*x^5 + 77*d*e^4*x^4 - 29*d^2*e^3*x^3 + 16*d^
3*e^2*x^2 - 8*d^4*e*x + 2*d^5)*sqrt(-e^2*x^2 + d^2))/(d^4*e*x^6 + d^5*x^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/480
*((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*(247680*exp
(1)^20*exp(2)^2+414720*exp(1)^18*exp(2)^3+115200*exp(1)^16*exp(2)^4+45600*e
xp(1)^14*exp(2)^5+43920*exp(1)^12*exp(2)^6-106200*exp(1)^10*exp(2)^7-10680*
exp(1)^8*exp(2)^8+15720*exp(1)^6*exp(2)^9-1080*exp(1)^4*exp(2)^10+1200*exp(
2)^12)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*(13824
0*exp(1)^20*exp(2)^2+203200*exp(1)^18*exp(2)^3+342720*exp(1)^16*exp(2)^4+46
320*exp(1)^14*exp(2)^5-158400*exp(1)^12*exp(2)^6+12440*exp(1)^10*exp(2)^7-3
60*exp(1)^8*exp(2)^8-3965*exp(1)^6*exp(2)^9+4505*exp(1)^4*exp(2)^10+220*exp
(2)^12+93440*exp(1)^22*exp(2))+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^9*(99840*exp(1)^20*exp(2)^2+144000*exp(1)^18*exp(2)^3+27360
*exp(1)^16*exp(2)^4+56160*exp(1)^14*exp(2)^5+39840*exp(1)^12*exp(2)^6-60120
*exp(1)^10*exp(2)^7-5460*exp(1)^8*exp(2)^8+8340*exp(1)^6*exp(2)^9-540*exp(1
```

```
)^4*exp(2)^10+660*exp(2)^12)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^10*(26880*exp(1)^18*exp(2)^3+37440*exp(1)^16*exp(2)^4-6000*ex
p(1)^14*exp(2)^5-5040*exp(1)^12*exp(2)^6+10440*exp(1)^10*exp(2)^7-6180*exp(
1)^8*exp(2)^8-1110*exp(1)^6*exp(2)^9+1350*exp(1)^4*exp(2)^10+60*exp(2)^12)+
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*(204480*exp(1
)^18*exp(2)^3+362880*exp(1)^16*exp(2)^4+30960*exp(1)^14*exp(2)^5-144240*exp
(1)^12*exp(2)^6+3600*exp(1)^10*exp(2)^7+840*exp(1)^8*exp(2)^8-3432*exp(1)^6
*exp(2)^9+3384*exp(1)^4*exp(2)^10+312*exp(2)^12)+(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*(60960*exp(1)^16*exp(2)^4+112320*exp(1)
^14*exp(2)^5+9120*exp(1)^12*exp(2)^6-56840*exp(1)^10*exp(2)^7-4512*exp(1)^8
*exp(2)^8+6704*exp(1)^6*exp(2)^9-576*exp(1)^4*exp(2)^10+408*exp(2)^12)+(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*(3360*exp(1)^14*e
xp(2)^5+6480*exp(1)^12*exp(2)^6+936*exp(1)^10*exp(2)^7-3192*exp(1)^8*exp(2)
^8-558*exp(1)^6*exp(2)^9+198*exp(1)^4*exp(2)^10+216*exp(2)^12)+(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*(-336*exp(1)^12*exp(2)^6-
648*exp(1)^10*exp(2)^7-72*exp(1)^8*exp(2)^8+312*exp(1)^6*exp(2)^9-72*exp(1)
^4*exp(2)^10-144*exp(2)^12)+(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*(56*exp(1)^10*exp(2)^7+108*exp(1)^8*exp(2)^8+22*exp(1)^6*exp
(2)^9-22*exp(1)^4*exp(2)^10+76*exp(2)^12)+3*exp(1)^6*exp(2)^9+9*exp(1)^4*ex
p(2)^10+12*exp(2)^12-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*(-12*e
xp(1)^8*exp(2)^8-36*exp(1)^6*exp(2)^9-36*exp(1)^4*exp(2)^10-12*exp(2)^12)/x
/exp(2))/d^4/(2*exp(2))^3/(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^5/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2
*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^3/exp(1)^7+1/
33554432*(83886080*d^16*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^34*exp(2)^23-41943040/3*d^16*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^32*exp(2)^24+2097152*d^16*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^30*exp(2)^25-1
048576/5*d^16*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5
*exp(1)^28*exp(2)^26-50331648*d^16*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^2*exp(1)^32*exp(2)^24+4194304*d^16*(-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^30*exp(2)^25+16777216*d^16
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^30*ex
p(2)^25-5242880/3*d^16*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^3*exp(1)^28*exp(2)^26+5242880*d^16*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))*exp(1)^28*exp(2)^26/x/exp(2)-18874368*d^16*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))*exp(1)^30*exp(2)^25/x/exp(2)+88080384*d^16*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^32*exp(2)^24/x/exp(2)-251658240*
d^16*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^34*exp(2)^23/x/exp(
2)+293601280*d^16*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^36*exp
(2)^22/x/exp(2))/d^20/exp(1)^35/exp(2)^20+1/2*(-120*exp(1)^6*exp(2)^2+44*ex
p(1)^4*exp(2)^3-9*exp(2)^5+112*exp(1)^8*exp(2))*ln(1/2*abs(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/abs(x)/exp(2))/d^4/exp(1)^4/exp(1)+1/2*(-432*exp
(1)^10*exp(2)^2+72*exp(1)^8*exp(2)^3+472*exp(1)^6*exp(2)^4+90*exp(1)^4*exp(
2)^5-104*exp(2)^7-224*exp(1)^12*exp(2))*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(
2)^2)/(d^4*exp(1)^7+3*d^4*exp(1)^5*exp(2)+4*d^4*exp(1)*exp(2)^3)
```

maple [B] time = 0.02, size = 628, normalized size = 3.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x)
```

```
[Out] 1/d^7*e/x^4*(-e^2*x^2+d^2)^(7/2)-16/5/d^8*e^2/x^3*(-e^2*x^2+d^2)^(7/2)-111/
5/d^10*e^4/x*(-e^2*x^2+d^2)^(7/2)-111/5/d^10*e^6*x*(-e^2*x^2+d^2)^(5/2)-111
/4/d^8*e^6*x*(-e^2*x^2+d^2)^(3/2)-333/8/d^6*e^6*x*(-e^2*x^2+d^2)^(1/2)-333/
8/d^4*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+17/2/d^9*e
^3/x^2*(-e^2*x^2+d^2)^(7/2)+333/8/d^4*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2
```

```
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)-1/d^7*e/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/
e)^2*e^2)^(7/2)+3/d^8*e^2/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+11/
d^9*e^3/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(7/2)+111/4/d^8*e^6*(2*(x+d
/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+333/8/d^6*e^6*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)*x+27/2/(d^2)^(1/2)/d^3*e^5*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/
2))/x)-1/5/d^6/x^5*(-e^2*x^2+d^2)^(7/2)-27/10/d^9*e^5*(-e^2*x^2+d^2)^(5/2)-
9/2/d^7*e^5*(-e^2*x^2+d^2)^(3/2)-27/2/d^5*e^5*(-e^2*x^2+d^2)^(1/2)+111/5/d^
9*e^5*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^6\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^6\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**6*(d + e*x)**4), x)
```

3.210 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx$

Optimal. Leaf size=95

$$-\frac{\sin^{-1}(ax)}{a^3}-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}$$

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1637, 659, 651, 663, 216}

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}-\frac{\sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^3*(1 - a*x
)^4) - (3*(1 - a^2*x^2)^(3/2))/(5*a^3*(1 - a*x)^3) - ArcSin[a*x]/a^3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx = \int \left(\frac{\sqrt{1-a^2x^2}}{a^2(-1+ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^2(-1+ax)^3}+\frac{\sqrt{1-a^2x^2}}{a^2(-1+ax)^2}\right)dx$$
$$= \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^2}+\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^2}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^2}$$
$$= \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}-\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^3(1-ax)^3}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^3}-\frac{\sin^{-1}(ax)}{a^3}$$

Mathematica [A] time = 0.11, size = 50, normalized size = 0.53

$$\frac{\frac{\left(-13a^2x^2+19ax-8\right)\sqrt{1-a^2x^2}}{(ax-1)^3}-5\sin^{-1}(ax)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

```
[Out] (((-8 + 19*a*x - 13*a^2*x^2)*Sqrt[1 - a^2*x^2])/(-1 + a*x)^3 - 5*ArcSin[a*x
])/(5*a^3)
```

IntegrateAlgebraic [A] time = 0.69, size = 85, normalized size = 0.89

$$\frac{\left(-13a^2x^2+19ax-8\right)\sqrt{1-a^2x^2}}{5a^3(ax-1)^3}-\frac{\sqrt{-a^2}\,\log\left(\sqrt{1-a^2x^2}-\sqrt{-a^2}\,x\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

```
[Out] ((-8 + 19*a*x - 13*a^2*x^2)*Sqrt[1 - a^2*x^2])/(5*a^3*(-1 + a*x)^3) - (Sqrt
[-a^2]*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x^2]])/a^4
```

fricas [A] time = 0.40, size = 126, normalized size = 1.33

$$\frac{8\,a^3x^3-24\,a^2x^2+24\,ax+10\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(13\,a^2x^2-19\,ax+8\right)\sqrt{-a^2x^2+1}-8}{5\left(a^6x^3-3\,a^5x^2+3\,a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(8*a^3*x^3 - 24*a^2*x^2 + 24*a*x + 10*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (13*a^2*x^2 - 19*a*x + 8)*sqrt(-a
^2*x^2 + 1) - 8)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 200, normalized size = 2.11

$$-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2 a^2-2\left(x-\frac{1}{a}\right)a}}\right)}{\sqrt{a^2}\,a^2}+\frac{\sqrt{-\left(x-\frac{1}{a}\right)^2 a^2-2\left(x-\frac{1}{a}\right)a}}{a^3}+\frac{\left(-\left(x-\frac{1}{a}\right)^2 a^2-2\left(x-\frac{1}{a}\right)a\right)^{\frac{3}{2}}}{\left(x-\frac{1}{a}\right)^2 a^5}+\frac{3\left(-\left(x-\frac{1}{a}\right)^2 a^2-2\left(x-\frac{1}{a}\right)a\right)^{\frac{3}{2}}}{5\left(x-\frac{1}{a}\right)^3 a^6}+\frac{\left(-\left(x-\frac{1}{a}\right)^2 a^2-2\left(x-\frac{1}{a}\right)a\right)^{\frac{3}{2}}}{5\left(x-\frac{1}{a}\right)^4 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x)

[Out] 1/a^5/(x-1/a)^2*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+1/a^3*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)*x)+1/5/a^7/(x-1/a)^4*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+3/5/a^6/(x-1/a)^3*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,x^2}{(ax-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="maxima")

[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2/(a*x - 1)^4, x)

mupad [B] time = 2.70, size = 220, normalized size = 2.32

$$\frac{4\,a^2\,\sqrt{1-a^2\,x^2}}{15\,\left(a^7\,x^2-2\,a^6\,x+a^5\right)}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{2\,\sqrt{1-a^2\,x^2}}{5\,\sqrt{-a^2}\,\left(a\,\sqrt{-a^2}-3\,a^2\,x\,\sqrt{-a^2}+3\,a^3\,x^2\,\sqrt{-a^2}-a^4\,x^3\,\sqrt{-a^2}\right)}-\frac{13\,\sqrt{1-a^2\,x^2}}{5\,\left(a\,\sqrt{-a^2}-a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{5\,\sqrt{1-a^2\,x^2}}{3\,\left(a^5\,x^2-2\,a^4\,x+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(1 - a^2*x^2)^(1/2))/(a*x - 1)^4,x)

[Out] (4*a^2*(1 - a^2*x^2)^(1/2))/(15*(a^5 - 2*a^6*x + a^7*x^2)) - asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(a*(-a^2)^(1/2) - 3*a^2*x*(-a^2)^(1/2) + 3*a^3*x^2*(-a^2)^(1/2) - a^4*x^3*(-a^2)^(1/2))) - (13*(1 - a^2*x^2)^(1/2))/(5*(a*(-a^2)^(1/2) - a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (5*(1 - a^2*x^2)^(1/2))/(3*(a^3 - 2*a^4*x + a^5*x^2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(ax-1)(ax+1)}}{(ax-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**4,x)

[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/(a*x - 1)**4, x)

3.211 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx$

Optimal. Leaf size=88

$$\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3(1-ax)^3}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}$$

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1639, 793, 659, 651}

$$\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3(1-ax)^3}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^3*(1 - a*x)^5) - (12*(1 - a^2*x^2)^(3/2))/(35*a^3*
(1 - a*x)^4) + (23*(1 - a^2*x^2)^(3/2))/(105*a^3*(1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx &= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4} + \frac{\int \frac{\left(4a^2-3a^3x\right)\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx}{a^4} \\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5} - \frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4} + \frac{23\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx}{7a^2} \\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5} - \frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4} + \frac{23\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^3}\,dx}{35a^2} \\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5} - \frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4} + \frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.07, size = 50, normalized size = 0.57

$$\frac{\sqrt{1-a^2x^2}\left(23a^3x^3+13a^2x^2-8ax+2\right)}{105a^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2 - 8*a*x + 13*a^2*x^2 + 23*a^3*x^3))/(105*a^3*(-1 + a*
x)^4)
```

IntegrateAlgebraic [A] time = 0.72, size = 50, normalized size = 0.57

$$\frac{\sqrt{1-a^2x^2}\left(23a^3x^3+13a^2x^2-8ax+2\right)}{105a^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2 - 8*a*x + 13*a^2*x^2 + 23*a^3*x^3))/(105*a^3*(-1 + a*
x)^4)
```

fricas [A] time = 0.40, size = 102, normalized size = 1.16

$$\frac{2\,a^4x^4-8\,a^3x^3+12\,a^2x^2-8\,ax+\left(23\,a^3x^3+13\,a^2x^2-8\,ax+2\right)\sqrt{-a^2x^2+1}+2}{105\left(a^7x^4-4\,a^6x^3+6\,a^5x^2-4\,a^4x+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/105*(2*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 8*a*x + (23*a^3*x^3 + 13*a^2*x^
2 - 8*a*x + 2)*sqrt(-a^2*x^2 + 1) + 2)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4
*a^4*x + a^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 44, normalized size = 0.50

$$\frac{\sqrt{-a^{2}x^{2}+1}\,\left(23a^{2}x^{2}-10ax+2\right)(ax+1)}{105\,(ax-1)^{4}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x)
```

```
[Out] 1/105*(-a^2*x^2+1)^(1/2)*(23*a^2*x^2-10*a*x+2)*(a*x+1)/(a*x-1)^4/a^3
```

maxima [B] time = 0.44, size = 153, normalized size = 1.74

$$\frac{2\sqrt{-a^{2}x^{2}+1}}{7\left(a^{7}x^{4}-4\,a^{6}x^{3}+6\,a^{5}x^{2}-4\,a^{4}x+a^{3}\right)}+\frac{29\sqrt{-a^{2}x^{2}+1}}{35\left(a^{6}x^{3}-3\,a^{5}x^{2}+3\,a^{4}x-a^{3}\right)}+\frac{82\sqrt{-a^{2}x^{2}+1}}{105\left(a^{5}x^{2}-2\,a^{4}x+a^{3}\right)}+\frac{23\sqrt{-a^{2}x^{2}+1}}{105\left(a^{4}x-a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4*a^4*x + a^3) +
29/35*sqrt(-a^2*x^2 + 1)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3) + 82/105*sqr
t(-a^2*x^2 + 1)/(a^5*x^2 - 2*a^4*x + a^3) + 23/105*sqrt(-a^2*x^2 + 1)/(a^4*
x - a^3)
```

mupad [B] time = 0.06, size = 287, normalized size = 3.26

$$\frac{2\sqrt{1-a^{2}x^{2}}}{7\left(a^{7}x^{4}-4\,a^{6}x^{3}+6\,a^{5}x^{2}-4\,a^{4}x+a^{3}\right)}+\frac{4\sqrt{1-a^{2}x^{2}}}{3\left(a^{5}x^{2}-2\,a^{4}x+a^{3}\right)}+\frac{4\,a\sqrt{1-a^{2}x^{2}}}{35\left(a^{6}x^{2}-2\,a^{5}x+a^{4}\right)}+\frac{29\sqrt{1-a^{2}x^{2}}}{35\sqrt{-a^{2}}\left(a\sqrt{-a^{2}}-3\,a^{2}x\sqrt{-a^{2}}+3\,a^{3}x^{2}\sqrt{-a^{2}}-a^{4}x^{3}\sqrt{-a^{2}}\right)}+\frac{23\sqrt{1-a^{2}x^{2}}}{105\left(a\sqrt{-a^{2}}-a^{2}x\sqrt{-a^{2}}\right)\sqrt{-a^{2}}}-\frac{2\,a^{2}\sqrt{1-a^{2}x^{2}}}{3\left(a^{7}x^{2}-2\,a^{6}x+a^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(1 - a^2*x^2)^(1/2))/(a*x - 1)^5,x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(7*(a^3 - 4*a^4*x + 6*a^5*x^2 - 4*a^6*x^3 + a^7*x^4
)) + (4*(1 - a^2*x^2)^(1/2))/(3*(a^3 - 2*a^4*x + a^5*x^2)) + (4*a*(1 - a^2*
x^2)^(1/2))/(35*(a^4 - 2*a^5*x + a^6*x^2)) + (29*(1 - a^2*x^2)^(1/2))/(35*(
-a^2)^(1/2)*(a*(-a^2)^(1/2) - 3*a^2*x*(-a^2)^(1/2) + 3*a^3*x^2*(-a^2)^(1/2)
 - a^4*x^3*(-a^2)^(1/2))) + (23*(1 - a^2*x^2)^(1/2))/(105*(a*(-a^2)^(1/2) -
 a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (2*a^2*(1 - a^2*x^2)^(1/2))/(3*(a^5 -
2*a^6*x + a^7*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{2}\sqrt{-a^{2}x^{2}+1}}{a^{5}x^{5}-5a^{4}x^{4}+10a^{3}x^{3}-10a^{2}x^{2}+5ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**5,x)
```

```
[Out] -Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5 - 5*a**4*x**4 + 10*a**3*x**3
 - 10*a**2*x**2 + 5*a*x - 1), x)
```

3.212 $\int \frac{x^3}{(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx$

Optimal. Leaf size=209

$$\frac{d^2}{13e^4(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{21}{143e^4(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}} + \frac{4}{1001de^4(d+ex) \left(d^2 - \right.}$$

Rubi [A] time = 0.31, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1639, 793, 659, 192, 191}

$$\frac{d^2}{13e^4(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{21}{143e^4(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}} + \frac{4}{1001de^4(d+ex) \left(d^2-e^2x^2\right)^{5/2}} - \frac{24x}{5005d^3e^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{32x}{5005d^5e^3 \left(d^2-e^2x^2\right)^{3/2}} - \frac{64x}{5005d^7e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-24*x)/(5005*d^3*e^3*(d^2 - e^2*x^2)^(5/2)) + d^2/(13*e^4*(d + e*x)^4*(d^2
 - e^2*x^2)^(5/2)) - (30*d)/(143*e^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) + 2
1/(143*e^4*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) + 4/(1001*d*e^4*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) - (32*x)/(5005*d^5*e^3*(d^2 - e^2*x^2)^(3/2)) - (64*x)/
(5005*d^7*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{2d^3e^2-3d^2e^3x-12de^4x^2}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7e^5}$$

$$= -\frac{3d}{14e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{-20d^3e^6+36d^2e^7\cdots}{(d+ex)^4\left(d^2-e^2x^2\right)\cdots}}{56e^9}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{3d}{14e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{7e^4(d+ex)^2\left(d^2\cdots\right.}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{7e^4(d+ex)^2\left(d^2\cdots\right.}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{21\cdots}{143e^4(d+ex)^2\cdots}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{21\cdots}{143e^4(d+ex)^2\cdots}$$

$$= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2\cdots\right.}$$

$$= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2\cdots\right.}$$

$$= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2\cdots\right.}$$

Mathematica [A] time = 0.18, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(90d^9+360d^8ex+315d^7e^2x^2-540d^6e^3x^3+160d^5e^4x^4+776d^4e^5x^5+384d^3e^6x^6-224d^2e^7x^7-256de^8x^8-64e^9x^9\right)}{5005d^7e^4(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(90*d^9 + 360*d^8*e*x + 315*d^7*e^2*x^2 - 540*d^6*e^3*
x^3 + 160*d^5*e^4*x^4 + 776*d^4*e^5*x^5 + 384*d^3*e^6*x^6 - 224*d^2*e^7*x^7
 - 256*d*e^8*x^8 - 64*e^9*x^9))/(5005*d^7*e^4*(d - e*x)^3*(d + e*x)^7)
```

IntegrateAlgebraic [A] time = 0.98, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(90d^9+360d^8ex+315d^7e^2x^2-540d^6e^3x^3+160d^5e^4x^4+776d^4e^5x^5+384d^3e^6x^6-224d^2e^7x^7-256de^8x^8-64e^9x^9\right)}{5005d^7e^4(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(90*d^9 + 360*d^8*e*x + 315*d^7*e^2*x^2 - 540*d^6*e^3*
x^3 + 160*d^5*e^4*x^4 + 776*d^4*e^5*x^5 + 384*d^3*e^6*x^6 - 224*d^2*e^7*x^7
 - 256*d*e^8*x^8 - 64*e^9*x^9))/(5005*d^7*e^4*(d - e*x)^3*(d + e*x)^7)
```

fricas [A] time = 0.92, size = 316, normalized size = 1.51

$$\frac{90e^{10}x^{10}+360de^9x^9+270d^2e^8x^8-720d^3e^7x^7-1260d^4e^6x^6+1260d^6e^4x^4+720d^7e^3x^3-270d^8e^2x^2-360d^9ex-90d^{10}+\left(64e^9x^9+256de^8x^8+224d^2e^7x^7-384d^3e^6x^6-776d^4e^5x^5-160d^5e^4x^4+540d^6e^3x^3-315d^7e^2x^2-360d^8ex-90d^9\right)\sqrt{-e^2x^2+d^2}}{5005\left(d^7e^{14}x^{10}+4d^8e^{13}x^9+3d^9e^{12}x^8-8d^{10}e^{11}x^7-14d^{11}e^{10}x^6+14d^{13}e^8x^4+8d^{14}e^7x^3-3d^{15}e^6x^2-4d^{16}e^5x-d^{17}e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5005*(90*e^10*x^10 + 360*d*e^9*x^9 + 270*d^2*e^8*x^8 - 720*d^3*e^7*x^7 -
1260*d^4*e^6*x^6 + 1260*d^6*e^4*x^4 + 720*d^7*e^3*x^3 - 270*d^8*e^2*x^2 - 3
60*d^9*e*x - 90*d^10 + (64*e^9*x^9 + 256*d*e^8*x^8 + 224*d^2*e^7*x^7 - 384*
d^3*e^6*x^6 - 776*d^4*e^5*x^5 - 160*d^5*e^4*x^4 + 540*d^6*e^3*x^3 - 315*d^7
*e^2*x^2 - 360*d^8*e*x - 90*d^9)*sqrt(-e^2*x^2 + d^2))/(d^7*e^14*x^10 + 4*d
^8*e^13*x^9 + 3*d^9*e^12*x^8 - 8*d^10*e^11*x^7 - 14*d^11*e^10*x^6 + 14*d^13
*e^8*x^4 + 8*d^14*e^7*x^3 - 3*d^15*e^6*x^2 - 4*d^16*e^5*x - d^17*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 132, normalized size = 0.63

$$\frac{(-ex+d)\left(-64e^9x^9-256e^8x^8d-224e^7x^7d^2+384e^6x^6d^3+776e^5x^5d^4+160x^4d^5e^4-540x^3d^6e^3+315x^2d^7e^2+360d^8xe+90d^9\right)}{5005\,(ex+d)^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^7e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5005*(-e*x+d)*(-64*e^9*x^9-256*d*e^8*x^8-224*d^2*e^7*x^7+384*d^3*e^6*x^6+
776*d^4*e^5*x^5+160*d^5*e^4*x^4-540*d^6*e^3*x^3+315*d^7*e^2*x^2+360*d^8*e*x
+90*d^9)/(e*x+d)^3/d^7/e^4/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.50, size = 399, normalized size = 1.91

$$\frac{d^2}{13\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^8x^4+4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^7x^3+6\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^6x^2+4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^5x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4e^4\right)}-\frac{30\,d}{143\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^7x^3+3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^6x^2+3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^5x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^4\right)}+\frac{21}{143\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6x^2+2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^5x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^4\right)}+\frac{4}{1001\left(\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^5x+\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2e^4\right)}-\frac{24\,x}{5005\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^3}-\frac{32\,x}{5005\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5e^3}-\frac{64\,x}{5005\sqrt{-e^2x^2+d^2}\,d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/13*d^2/((-e^2*x^2 + d^2)^(5/2)*e^8*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^7*x
^3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^6*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e^
```

```
5*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^4) - 30/143*d/((-e^2*x^2 + d^2)^(5/2)*e^
7*x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d*e^6*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e
^5*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^4) + 21/143/((-e^2*x^2 + d^2)^(5/2)*e^6
*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d^2*e^4) +
 4/1001/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x + (-e^2*x^2 + d^2)^(5/2)*d^2*e^4) -
 24/5005*x/((-e^2*x^2 + d^2)^(5/2)*d^3*e^3) - 32/5005*x/((-e^2*x^2 + d^2)^(
3/2)*d^5*e^3) - 64/5005*x/(sqrt(-e^2*x^2 + d^2)*d^7*e^3)
```

mupad [B] time = 3.22, size = 252, normalized size = 1.21

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{107}{4004\,d^2e^4}-\frac{1139\,x}{80080\,d^3e^3}\right)}{(d+e\,x)^3\,(d-e\,x)^3}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{23}{32032\,d^4e^4}+\frac{32\,x}{5005\,d^5e^3}\right)}{(d+e\,x)^2\,(d-e\,x)^2}+\frac{\sqrt{d^2-e^2x^2}}{104\,d\,e^4\,(d+e\,x)^7}-\frac{27\sqrt{d^2-e^2x^2}}{2288\,d^2e^4\,(d+e\,x)^6}-\frac{15\sqrt{d^2-e^2x^2}}{2288\,d^3e^4\,(d+e\,x)^5}+\frac{23\sqrt{d^2-e^2x^2}}{32032\,d^4e^4\,(d+e\,x)^4}-\frac{64\,x\sqrt{d^2-e^2x^2}}{5005\,d^7e^3\,(d+e\,x)\,(d-e\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(107/(4004*d^2*e^4) - (1139*x)/(80080*d^3*e^3)))/((d
 + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^(1/2)*(23/(32032*d^4*e^4) + (32*x
)/(5005*d^5*e^3)))/((d + e*x)^2*(d - e*x)^2) + (d^2 - e^2*x^2)^(1/2)/(104*d
*e^4*(d + e*x)^7) - (27*(d^2 - e^2*x^2)^(1/2))/(2288*d^2*e^4*(d + e*x)^6) -
 (15*(d^2 - e^2*x^2)^(1/2))/(2288*d^3*e^4*(d + e*x)^5) + (23*(d^2 - e^2*x^2
)^(1/2))/(32032*d^4*e^4*(d + e*x)^4) - (64*x*(d^2 - e^2*x^2)^(1/2))/(5005*d
^7*e^3*(d + e*x)*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.213 $\int \frac{x^2}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287de^3(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287d^2e^3(d+ex)\left(\right.}$$

Rubi [A] time = 0.21, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1639, 793, 659, 192, 191}

$$-\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287de^3(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287d^2e^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{56x}{6435d^6e^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{112x}{6435d^8e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (14*x)/(2145*d^4*e^2*(d^2 - e^2*x^2)^(5/2)) - d/(13*e^3*(d + e*x)^4*(d^2 -
e^2*x^2)^(5/2)) + 17/(143*e^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*
d*e^3*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*d^2*e^3*(d + e*x)*(d^2 -
 e^2*x^2)^(5/2)) + (56*x)/(6435*d^6*e^2*(d^2 - e^2*x^2)^(3/2)) + (112*x)/(6
435*d^8*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{3d^2e^2-5de^3x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx}{8e^4}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{(7d)\int \frac{1}{(d+ex)^3(d^2-}}{104e^2}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7\int \frac{1}{(d+ex)^2(d^2-}}{143e^2}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{}{1287de^3(d+e}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{}{1287de^3(d+e}$$

$$= \frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-\right.}$$

$$= \frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-\right.}$$

$$= \frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-\right.}$$

Mathematica [A] time = 0.10, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(200d^9+800d^8ex+700d^7e^2x^2+945d^6e^3x^3-280d^5e^4x^4-1358d^4e^5x^5-672d^3e^6x^6+392d^2e^7x^7+448de^8x^8+112e^9x^9\right)}{6435d^8e^3(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(200*d^9 + 800*d^8*e*x + 700*d^7*e^2*x^2 + 945*d^6*e^3
*x^3 - 280*d^5*e^4*x^4 - 1358*d^4*e^5*x^5 - 672*d^3*e^6*x^6 + 392*d^2*e^7*x
^7 + 448*d*e^8*x^8 + 112*e^9*x^9))/(6435*d^8*e^3*(d - e*x)^3*(d + e*x)^7)
```

IntegrateAlgebraic [A] time = 0.77, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(200d^9+800d^8ex+700d^7e^2x^2+945d^6e^3x^3-280d^5e^4x^4-1358d^4e^5x^5-672d^3e^6x^6+392d^2e^7x^7+448de^8x^8+112e^9x^9\right)}{6435d^8e^3(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(200*d^9 + 800*d^8*e*x + 700*d^7*e^2*x^2 + 945*d^6*e^3
*x^3 - 280*d^5*e^4*x^4 - 1358*d^4*e^5*x^5 - 672*d^3*e^6*x^6 + 392*d^2*e^7*x
^7 + 448*d*e^8*x^8 + 112*e^9*x^9))/(6435*d^8*e^3*(d - e*x)^3*(d + e*x)^7)
```

fricas [A] time = 0.96, size = 317, normalized size = 1.52

$$\frac{200\,e^{10}x^{10} + 800\,de^{9}x^{9} + 600\,d^{2}e^{8}x^{8} - 1600\,d^{3}e^{7}x^{7} - 2800\,d^{4}e^{6}x^{6} + 2800\,d^{6}e^{4}x^{4} + 1600\,d^{7}e^{3}x^{3} - 600\,d^{8}e^{2}x^{2} - 800\,d^{9}ex - 200\,d^{10} - \left(112\,e^{9}x^{9} + 448\,de^{8}x^{8} + 392\,d^{2}e^{7}x^{7} - 672\,d^{3}e^{6}x^{6} - 1358\,d^{4}e^{5}x^{5} - 280\,d^{5}e^{4}x^{4} + 945\,d^{6}e^{3}x^{3} + 700\,d^{7}e^{2}x^{2} + 800\,d^{8}ex + 200\,d^{9}\right)\sqrt{-e^{2}x^{2} + d^{2}}}{6435\left(d^{8}e^{13}x^{10} + 4\,d^{9}e^{12}x^{9} + 3\,d^{10}e^{11}x^{8} - 8\,d^{11}e^{10}x^{7} - 14\,d^{12}e^{9}x^{6} + 14\,d^{14}e^{7}x^{4} + 8\,d^{15}e^{6}x^{3} - 3\,d^{16}e^{5}x^{2} - 4\,d^{17}e^{4}x - d^{18}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/6435*(200*e^10*x^10 + 800*d*e^9*x^9 + 600*d^2*e^8*x^8 - 1600*d^3*e^7*x^7
- 2800*d^4*e^6*x^6 + 2800*d^6*e^4*x^4 + 1600*d^7*e^3*x^3 - 600*d^8*e^2*x^2
- 800*d^9*e*x - 200*d^10 - (112*e^9*x^9 + 448*d*e^8*x^8 + 392*d^2*e^7*x^7 -
 672*d^3*e^6*x^6 - 1358*d^4*e^5*x^5 - 280*d^5*e^4*x^4 + 945*d^6*e^3*x^3 + 7
00*d^7*e^2*x^2 + 800*d^8*e*x + 200*d^9)*sqrt(-e^2*x^2 + d^2))/(d^8*e^13*x^1
0 + 4*d^9*e^12*x^9 + 3*d^10*e^11*x^8 - 8*d^11*e^10*x^7 - 14*d^12*e^9*x^6 +
14*d^14*e^7*x^4 + 8*d^15*e^6*x^3 - 3*d^16*e^5*x^2 - 4*d^17*e^4*x - d^18*e^3
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 132, normalized size = 0.63

$$\frac{(-ex + d)\left(112e^{9}x^{9} + 448e^{8}x^{8}d + 392e^{7}x^{7}d^{2} - 672e^{6}x^{6}d^{3} - 1358e^{5}x^{5}d^{4} - 280x^{4}d^{5}e^{4} + 945x^{3}d^{6}e^{3} + 700x^{2}d^{7}e^{2} + 800d^{8}xe + 200d^{9}\right)}{6435\,(ex + d)^{3}\left(-e^{2}x^{2} + d^{2}\right)^{\frac{7}{2}}d^{8}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/6435*(-e*x+d)*(112*e^9*x^9+448*d*e^8*x^8+392*d^2*e^7*x^7-672*d^3*e^6*x^6-
1358*d^4*e^5*x^5-280*d^5*e^4*x^4+945*d^6*e^3*x^3+700*d^7*e^2*x^2+800*d^8*e*
x+200*d^9)/(e*x+d)^3/d^8/e^3/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.49, size = 401, normalized size = 1.92

$$-\frac{d}{13\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{7}x^{4} + 4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}de^{6}x^{3} + 6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{5}x^{2} + 4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{4}x + \left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{3}\right)} + \frac{17}{143\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}x^{3} + 3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}de^{5}x^{2} + 3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{4}x + \left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{3}\right)} - \frac{7}{1287\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}de^{5}x^{2} + 2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{4}x + \left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{3}\right)} - \frac{7}{1287\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{4}x + \left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{3}\right)} + \frac{14\,x}{2145\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{2}} + \frac{56\,x}{6435\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{6}e^{2}} + \frac{112\,x}{6435\sqrt{-e^{2}x^{2}+d^{2}}\,d^{8}e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/13*d/((-e^2*x^2 + d^2)^(5/2)*e^7*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^6*x^
3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^5*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e^4
*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^3) + 17/143/((-e^2*x^2 + d^2)^(5/2)*e^6*x
^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*
x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^3) - 7/1287/((-e^2*x^2 + d^2)^(5/2)*d*e^5*
x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^3)
- 7/1287/((-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x + (-e^2*x^2 + d^2)^(5/2)*d^3*e^3
```

```
) + 14/2145*x/((-e^2*x^2 + d^2)^(5/2)*d^4*e^2) + 56/6435*x/((-e^2*x^2 + d^2
)^(3/2)*d^6*e^2) + 112/6435*x/(sqrt(-e^2*x^2 + d^2)*d^8*e^2)
```

mupad [B] time = 3.19, size = 252, normalized size = 1.21

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{227}{6864d^3e^3}-\frac{353x}{17160d^4e^2}\right)}{(d+ex)^3(d-ex)^3}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{353}{41184d^5e^3}-\frac{56x}{6435d^6e^2}\right)}{(d+ex)^2(d-ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{104d^2e^3(d+ex)^7}+\frac{\sqrt{d^2-e^2x^2}}{2288d^3e^3(d+ex)^6}+\frac{37\sqrt{d^2-e^2x^2}}{5148d^4e^3(d+ex)^5}+\frac{353\sqrt{d^2-e^2x^2}}{41184d^5e^3(d+ex)^4}+\frac{112x\sqrt{d^2-e^2x^2}}{6435d^8e^2(d+ex)(d-ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(227/(6864*d^3*e^3) - (353*x)/(17160*d^4*e^2)))/((d
+ e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^(1/2)*(353/(41184*d^5*e^3) - (56*x
)/(6435*d^6*e^2)))/((d + e*x)^2*(d - e*x)^2) - (d^2 - e^2*x^2)^(1/2)/(104*d
^2*e^3*(d + e*x)^7) + (d^2 - e^2*x^2)^(1/2)/(2288*d^3*e^3*(d + e*x)^6) + (3
7*(d^2 - e^2*x^2)^(1/2))/(5148*d^4*e^3*(d + e*x)^5) + (353*(d^2 - e^2*x^2)^
(1/2))/(41184*d^5*e^3*(d + e*x)^4) + (112*x*(d^2 - e^2*x^2)^(1/2))/(6435*d^
8*e^2*(d + e*x)*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.214 $\int \frac{x}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{32}{1287d^2e^2(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{512x}{6435d^9e\sqrt{d^2-e}}$$

Rubi [A] time = 0.10, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {793, 659, 192, 191}

$$\frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}}+\frac{256x}{6435d^7e\left(d^2-e^2x^2\right)^{3/2}}+\frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^3e^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^2e^2(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (64*x)/(2145*d^5*e*(d^2 - e^2*x^2)^(5/2)) + 1/(13*e^2*(d + e*x)^4*(d^2 - e^
2*x^2)^(5/2)) - 4/(143*d*e^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*
d^2*e^2*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*d^3*e^2*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) + (256*x)/(6435*d^7*e*(d^2 - e^2*x^2)^(3/2)) + (512*x)/
(6435*d^9*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{4\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx}{13e}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{32\int \frac{1}{(d+ex)^2\left(d^2- \right.}}{143d}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{}{1287d^2e^2(d+}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{}{1287d^2e^2(d+}$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-\right.}$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-\right.}$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-\right.}$$

Mathematica [A] time = 0.09, size = 137, normalized size = 0.65

$$\frac{\sqrt{d^2-e^2x^2}\left(-5d^9-20d^8ex+3200d^7e^2x^2+4320d^6e^3x^3-1280d^5e^4x^4-6208d^4e^5x^5-3072d^3e^6x^6+1792d^2e^7x^7+2048de^8x^8+512e^9x^9\right)}{6435d^9e^2(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^9 - 20*d^8*e*x + 3200*d^7*e^2*x^2 + 4320*d^6*e^3
*x^3 - 1280*d^5*e^4*x^4 - 6208*d^4*e^5*x^5 - 3072*d^3*e^6*x^6 + 1792*d^2*e^
7*x^7 + 2048*d*e^8*x^8 + 512*e^9*x^9))/(6435*d^9*e^2*(d - e*x)^3*(d + e*x)^
7)
```

IntegrateAlgebraic [A] time = 0.90, size = 137, normalized size = 0.65

$$\frac{\sqrt{d^2-e^2x^2}\left(-5d^9-20d^8ex+3200d^7e^2x^2+4320d^6e^3x^3-1280d^5e^4x^4-6208d^4e^5x^5-3072d^3e^6x^6+1792d^2e^7x^7+2048de^8x^8+512e^9x^9\right)}{6435d^9e^2(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^9 - 20*d^8*e*x + 3200*d^7*e^2*x^2 + 4320*d^6*e^3
*x^3 - 1280*d^5*e^4*x^4 - 6208*d^4*e^5*x^5 - 3072*d^3*e^6*x^6 + 1792*d^2*e^
7*x^7 + 2048*d*e^8*x^8 + 512*e^9*x^9))/(6435*d^9*e^2*(d - e*x)^3*(d + e*x)^
7)
```

fricas [A] time = 1.04, size = 316, normalized size = 1.50

$$-\frac{5\,e^{10}x^{10}+20\,de^9x^9+15\,d^2e^8x^8-40\,d^3e^7x^7-70\,d^4e^6x^6+70\,d^6e^4x^4+40\,d^7e^3x^3-15\,d^8e^2x^2-20\,d^9ex-5\,d^{10}+\left(512\,e^9x^9+2048\,de^8x^8+1792\,d^2e^7x^7-3072\,d^3e^6x^6-6208\,d^4e^5x^5-1280\,d^5e^4x^4+4320\,d^6e^3x^3+3200\,d^7e^2x^2-20\,d^8ex-5\,d^9\right)\sqrt{-e^2x^2+d^2}}{6435\left(d^9e^{12}x^{10}+4\,d^{10}e^{11}x^9+3\,d^{11}e^{10}x^8-8\,d^{12}e^9x^7-14\,d^{13}e^8x^6+14\,d^{15}e^6x^4+8\,d^{16}e^5x^3-3\,d^{17}e^4x^2-4\,d^{18}e^3x-d^{19}e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/6435*(5*e^10*x^10 + 20*d*e^9*x^9 + 15*d^2*e^8*x^8 - 40*d^3*e^7*x^7 - 70*
d^4*e^6*x^6 + 70*d^6*e^4*x^4 + 40*d^7*e^3*x^3 - 15*d^8*e^2*x^2 - 20*d^9*e*x
 - 5*d^10 + (512*e^9*x^9 + 2048*d*e^8*x^8 + 1792*d^2*e^7*x^7 - 3072*d^3*e^6
*x^6 - 6208*d^4*e^5*x^5 - 1280*d^5*e^4*x^4 + 4320*d^6*e^3*x^3 + 3200*d^7*e^
2*x^2 - 20*d^8*e*x - 5*d^9)*sqrt(-e^2*x^2 + d^2))/(d^9*e^12*x^10 + 4*d^10*e
^11*x^9 + 3*d^11*e^10*x^8 - 8*d^12*e^9*x^7 - 14*d^13*e^8*x^6 + 14*d^15*e^6*
x^4 + 8*d^16*e^5*x^3 - 3*d^17*e^4*x^2 - 4*d^18*e^3*x - d^19*e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 132, normalized size = 0.63

$$-\frac{\left(-ex+d\right)\left(-512e^{9}x^{9}-2048e^{8}x^{8}d-1792e^{7}x^{7}d^{2}+3072e^{6}x^{6}d^{3}+6208e^{5}x^{5}d^{4}+1280x^{4}d^{5}e^{4}-4320x^{3}d^{6}e^{3}-3200x^{2}d^{7}e^{2}+20d^{8}xe+5d^{9}\right)}{6435\left(ex+d\right)^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d^{9}e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/6435*(-e*x+d)*(-512*e^9*x^9-2048*d*e^8*x^8-1792*d^2*e^7*x^7+3072*d^3*e^6
*x^6+6208*d^4*e^5*x^5+1280*d^5*e^4*x^4-4320*d^6*e^3*x^3-3200*d^7*e^2*x^2+20
*d^8*e*x+5*d^9)/(e*x+d)^3/d^9/e^2/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.49, size = 405, normalized size = 1.92

$$\frac{1}{13\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}e^{6}x^{4}+4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}de^{5}x^{3}+6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{4}x^{2}+4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{3}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{2}\right)}-\frac{4}{143\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}de^{5}x^{3}+3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{4}x^{2}+3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{3}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{2}\right)}-\frac{32}{1287\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{4}x^{2}+2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{3}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{2}\right)}-\frac{32}{1287\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{3}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{2}\right)}+\frac{64x}{2145\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{5}e}+\frac{256x}{6435\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{7}e}+\frac{512x}{6435\sqrt{-e^{2}x^{2}+d^{2}}d^{9}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/13/((-e^2*x^2 + d^2)^(5/2)*e^6*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d*e^5*x^3 +
 6*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x
+ (-e^2*x^2 + d^2)^(5/2)*d^4*e^2) - 4/143/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x^3
 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*
x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^2) - 32/1287/((-e^2*x^2 + d^2)^(5/2)*d^2*e
^4*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e^
2) - 32/1287/((-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x + (-e^2*x^2 + d^2)^(5/2)*d^4
*e^2) + 64/2145*x/((-e^2*x^2 + d^2)^(5/2)*d^5*e) + 256/6435*x/((-e^2*x^2 +
d^2)^(3/2)*d^7*e) + 512/6435*x/(sqrt(-e^2*x^2 + d^2)*d^9*e)
```

mupad [B] time = 3.19, size = 252, normalized size = 1.19

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{41}{41184\,d^6\,e^2}+\frac{256\,x}{6435\,d^7\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{47}{1716\,d^4\,e^2}-\frac{1369\,x}{34320\,d^5\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}+\frac{\sqrt{d^2-e^2\,x^2}}{104\,d^3\,e^2\,{\left(d+e\,x\right)}^7}+\frac{25\,\sqrt{d^2-e^2\,x^2}}{2288\,d^4\,e^2\,{\left(d+e\,x\right)}^6}+\frac{125\,\sqrt{d^2-e^2\,x^2}}{20592\,d^5\,e^2\,{\left(d+e\,x\right)}^5}-\frac{41\,\sqrt{d^2-e^2\,x^2}}{41184\,d^6\,e^2\,{\left(d+e\,x\right)}^4}+\frac{512\,x\,\sqrt{d^2-e^2\,x^2}}{6435\,d^9\,e\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(41/(41184*d^6*e^2) + (256*x)/(6435*d^7*e)))/((d + e
*x)^2*(d - e*x)^2) - ((d^2 - e^2*x^2)^(1/2)*(47/(1716*d^4*e^2) - (1369*x)/(
34320*d^5*e)))/((d + e*x)^3*(d - e*x)^3) + (d^2 - e^2*x^2)^(1/2)/(104*d^3*e
^2*(d + e*x)^7) + (25*(d^2 - e^2*x^2)^(1/2))/(2288*d^4*e^2*(d + e*x)^6) + (
125*(d^2 - e^2*x^2)^(1/2))/(20592*d^5*e^2*(d + e*x)^5) - (41*(d^2 - e^2*x^2
)^(1/2))/(41184*d^6*e^2*(d + e*x)^4) + (512*x*(d^2 - e^2*x^2)^(1/2))/(6435*
d^9*e*(d + e*x)*(d - e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.215 $\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{}{715d^6\left(\right.}$$

Rubi [A] time = 0.09, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (48*x)/(715*d^6*(d^2 - e^2*x^2)^(5/2)) - 1/(13*d*e*(d + e*x)^4*(d^2 - e^2*x
^2)^(5/2)) - 9/(143*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^3*e
*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^4*e*(d + e*x)*(d^2 - e^2*x^2
)^(5/2)) + (64*x)/(715*d^8*(d^2 - e^2*x^2)^(3/2)) + (128*x)/(715*d^10*Sqrt[
d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{9\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx}{13d}$$

$$= -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \cdots$$

$$= -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \cdots$$

$$= -\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \cdots$$

$$= \frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} \cdots$$

$$= \frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} \cdots$$

$$= \frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} \cdots$$

Mathematica [A] time = 0.07, size = 137, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2}\left(-180d^9-5d^8ex+800d^7e^2x^2+1080d^6e^3x^3-320d^5e^4x^4-1552d^4e^5x^5-768d^3e^6x^6+448d^2e^7x^7+512de^8x^8+128e^9x^9\right)}{715d^{10}e(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-180*d^9 - 5*d^8*e*x + 800*d^7*e^2*x^2 + 1080*d^6*e^3
*x^3 - 320*d^5*e^4*x^4 - 1552*d^4*e^5*x^5 - 768*d^3*e^6*x^6 + 448*d^2*e^7*x
^7 + 512*d*e^8*x^8 + 128*e^9*x^9))/(715*d^10*e*(d - e*x)^3*(d + e*x)^7)
```

IntegrateAlgebraic [A] time = 0.05, size = 137, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2}\left(-180d^9-5d^8ex+800d^7e^2x^2+1080d^6e^3x^3-320d^5e^4x^4-1552d^4e^5x^5-768d^3e^6x^6+448d^2e^7x^7+512de^8x^8+128e^9x^9\right)}{715d^{10}e(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-180*d^9 - 5*d^8*e*x + 800*d^7*e^2*x^2 + 1080*d^6*e^3
*x^3 - 320*d^5*e^4*x^4 - 1552*d^4*e^5*x^5 - 768*d^3*e^6*x^6 + 448*d^2*e^7*x
^7 + 512*d*e^8*x^8 + 128*e^9*x^9))/(715*d^10*e*(d - e*x)^3*(d + e*x)^7)
```

fricas [A] time = 1.14, size = 314, normalized size = 1.53

$$-\frac{180\,e^{10}x^{10}+720\,de^9x^9+540\,d^2e^8x^8-1440\,d^3e^7x^7-2520\,d^4e^6x^6+2520\,d^6e^4x^4+1440\,d^7e^3x^3-540\,d^8e^2x^2-720\,d^9ex-180\,d^{10}+\left(128\,e^9x^9+512\,de^8x^8+448\,d^2e^7x^7-768\,d^3e^6x^6-1552\,d^4e^5x^5-320\,d^5e^4x^4+1080\,d^6e^3x^3+800\,d^7e^2x^2-5\,d^8ex-180\,d^9\right)\sqrt{-e^2x^2+d^2}}{715\left(d^{10}e^{11}x^{10}+4\,d^{11}e^{10}x^9+3\,d^{12}e^9x^8-8\,d^{13}e^8x^7-14\,d^{14}e^7x^6+14\,d^{16}e^5x^4+8\,d^{17}e^4x^3-3\,d^{18}e^3x^2-4\,d^{19}e^2x-d^{20}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/715*(180*e^10*x^10 + 720*d*e^9*x^9 + 540*d^2*e^8*x^8 - 1440*d^3*e^7*x^7
- 2520*d^4*e^6*x^6 + 2520*d^6*e^4*x^4 + 1440*d^7*e^3*x^3 - 540*d^8*e^2*x^2
- 720*d^9*e*x - 180*d^10 + (128*e^9*x^9 + 512*d*e^8*x^8 + 448*d^2*e^7*x^7 -
 768*d^3*e^6*x^6 - 1552*d^4*e^5*x^5 - 320*d^5*e^4*x^4 + 1080*d^6*e^3*x^3 +
800*d^7*e^2*x^2 - 5*d^8*e*x - 180*d^9)*sqrt(-e^2*x^2 + d^2))/(d^10*e^11*x^1
0 + 4*d^11*e^10*x^9 + 3*d^12*e^9*x^8 - 8*d^13*e^8*x^7 - 14*d^14*e^7*x^6 + 1
4*d^16*e^5*x^4 + 8*d^17*e^4*x^3 - 3*d^18*e^3*x^2 - 4*d^19*e^2*x - d^20*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 132, normalized size = 0.64

$$-\frac{\left(-ex+d\right)\left(-128e^{9}x^{9}-512e^{8}x^{8}d-448e^{7}x^{7}d^{2}+768e^{6}x^{6}d^{3}+1552e^{5}x^{5}d^{4}+320x^{4}d^{5}e^{4}-1080x^{3}d^{6}e^{3}-800x^{2}d^{7}e^{2}+5d^{8}xe+180d^{9}\right)}{715\left(ex+d\right)^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}d^{10}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/715*(-e*x+d)*(-128*e^9*x^9-512*d*e^8*x^8-448*d^2*e^7*x^7+768*d^3*e^6*x^6
+1552*d^4*e^5*x^5+320*d^5*e^4*x^4-1080*d^6*e^3*x^3-800*d^7*e^2*x^2+5*d^8*e*
x+180*d^9)/(e*x+d)^3/d^10/e/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.50, size = 393, normalized size = 1.92

$$-\frac{1}{13\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}de^{5}x^{4}+4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{4}x^{3}+6\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{3}x^{2}+4\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{2}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{5}e\right)}-\frac{9}{143\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{4}x^{3}+3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{3}x^{2}+3\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{2}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{5}e\right)}-\frac{8}{143\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{3}x^{2}+2\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{2}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{5}e\right)}-\frac{8}{143\left(\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{2}x+\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{5}e\right)}+\frac{48x}{715\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{6}}+\frac{64x}{715\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{8}}+\frac{128x}{715\sqrt{-e^{2}x^{2}+d^{2}}d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/13/((-e^2*x^2 + d^2)^(5/2)*d*e^5*x^4 + 4*(-e^2*x^2 + d^2)^(5/2)*d^2*e^4*
x^3 + 6*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x^2 + 4*(-e^2*x^2 + d^2)^(5/2)*d^4*e
^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e) - 9/143/((-e^2*x^2 + d^2)^(5/2)*d^2*e^
4*x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d^3*e^3*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^4
*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e) - 8/143/((-e^2*x^2 + d^2)^(5/2)*d^3*
e^3*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^4*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e
) - 8/143/((-e^2*x^2 + d^2)^(5/2)*d^4*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^5*e)
 + 48/715*x/((-e^2*x^2 + d^2)^(5/2)*d^6) + 64/715*x/((-e^2*x^2 + d^2)^(3/2)
*d^8) + 128/715*x/(sqrt(-e^2*x^2 + d^2)*d^10)
```

mupad [B] time = 3.12, size = 242, normalized size = 1.18

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{64\,x}{715\,d^8}+\frac{189}{4576\,d^7\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}+\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1139\,x}{5720\,d^6}-\frac{427}{2288\,d^5\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}}{104\,d^4\,e\,{\left(d+e\,x\right)}^7}-\frac{51\,\sqrt{d^2-e^2\,x^2}}{2288\,d^5\,e\,{\left(d+e\,x\right)}^6}-\frac{19\,\sqrt{d^2-e^2\,x^2}}{572\,d^6\,e\,{\left(d+e\,x\right)}^5}-\frac{189\,\sqrt{d^2-e^2\,x^2}}{4576\,d^7\,e\,{\left(d+e\,x\right)}^4}+\frac{128\,x\,\sqrt{d^2-e^2\,x^2}}{715\,d^{10}\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*((64*x)/(715*d^8) + 189/(4576*d^7*e)))/((d + e*x)^2*
(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*((1139*x)/(5720*d^6) - 427/(2288*d^5*
```

```
e)))/((d + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(104*d^4*e*(d + e*x)
^7) - (51*(d^2 - e^2*x^2)^(1/2))/(2288*d^5*e*(d + e*x)^6) - (19*(d^2 - e^2*
x^2)^(1/2))/(572*d^6*e*(d + e*x)^5) - (189*(d^2 - e^2*x^2)^(1/2))/(4576*d^7
*e*(d + e*x)^4) + (128*x*(d^2 - e^2*x^2)^(1/2))/(715*d^10*(d + e*x)*(d - e*
x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.216 $\int \frac{1}{x(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}+\frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}}+\frac{273d-512ex}{819d^9\left(d^2-e^2x^2\right)^{3/2}}+\frac{273d}{1365d^7\left(\right.}$$

Rubi [A] time = 0.38, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$-\frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}}+\frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}}+\frac{273d-512ex}{819d^9\left(d^2-e^2x^2\right)^{3/2}}+\frac{273d-640ex}{1365d^7\left(d^2-e^2x^2\right)^{5/2}}+\frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}}+\frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}}+\frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (8*d*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*x)/(13*d*(d^2 - e^2*x^2)
^(11/2)) + (13*d - 40*e*x)/(117*d^3*(d^2 - e^2*x^2)^(9/2)) + (117*d - 320*e
*x)/(819*d^5*(d^2 - e^2*x^2)^(7/2)) + (273*d - 640*e*x)/(1365*d^7*(d^2 - e^
2*x^2)^(5/2)) + (273*d - 512*e*x)/(819*d^9*(d^2 - e^2*x^2)^(3/2)) + (819*d
- 1024*e*x)/(819*d^11*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
/d^11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{15/2}}\,dx$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{\int \frac{-13d^4+44d^3ex+13d^2e^2x^2}{x\left(d^2-e^2x^2\right)^{13/2}}\,dx}{13d^2}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{\int \frac{143d^4-440d^3ex}{x\left(d^2-e^2x^2\right)^{11/2}}\,dx}{143d^4}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{\int \frac{1287d^6e^2-3520\cdots}{x\left(d^2-e^2x^2\right)^{\cdots}}}{1287d^8e\cdots}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320\cdots}{819d^5\left(d^2-e^2\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320\cdots}{819d^5\left(d^2-e^2\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320\cdots}{819d^5\left(d^2-e^2\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320\cdots}{819d^5\left(d^2-e^2\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320\cdots}{819d^5\left(d^2-e^2\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320\cdots}{819d^5\left(d^2-e^2\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320\cdots}{819d^5\left(d^2-e^2\cdots\right.}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320\cdots}{819d^5\left(d^2-e^2\cdots\right.}$$

Mathematica [A] time = 0.17, size = 161, normalized size = 0.69

$$\frac{-4095\log\left(\sqrt{d^2-e^2x^2}+d\right)+\frac{\sqrt{d^2-e^2x^2}\left(9839d^9+22976d^8ex-4466d^7e^2x^2-56304d^6e^3x^3-34156d^5e^4x^4+40240d^4e^5x^5+45735d^3e^6x^6-1540d^2e^7x^7-16385de^8x^8-5120e^9x^9\right)}{(d-ex)^3(d+ex)^7}+4095\log(x)}{4095d^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(9839*d^9 + 22976*d^8*e*x - 4466*d^7*e^2*x^2 - 56304*
d^6*e^3*x^3 - 34156*d^5*e^4*x^4 + 40240*d^4*e^5*x^5 + 45735*d^3*e^6*x^6 - 1
```

```
540*d^2*e^7*x^7 - 16385*d*e^8*x^8 - 5120*e^9*x^9))/((d - e*x)^3*(d + e*x)^7
) + 4095*Log[x] - 4095*Log[d + Sqrt[d^2 - e^2*x^2]])/(4095*d^11)
```

IntegrateAlgebraic [A] time = 1.32, size = 177, normalized size = 0.76

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}}+\frac{\sqrt{d^2-e^2x^2}\left(9839d^9+22976d^8ex-4466d^7e^2x^2-56304d^6e^3x^3-34156d^5e^4x^4+40240d^4e^5x^5+45735d^3e^6x^6-1540d^2e^7x^7-16385de^8x^8-5120e^9x^9\right)}{4095d^{11}(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(9839*d^9 + 22976*d^8*e*x - 4466*d^7*e^2*x^2 - 56304*d
^6*e^3*x^3 - 34156*d^5*e^4*x^4 + 40240*d^4*e^5*x^5 + 45735*d^3*e^6*x^6 - 15
40*d^2*e^7*x^7 - 16385*d*e^8*x^8 - 5120*e^9*x^9))/(4095*d^11*(d - e*x)^3*(d
 + e*x)^7) + (2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^11
```

fricas [B] time = 1.18, size = 432, normalized size = 1.85

$$\frac{9839e^{10}x^{10}+39356de^9x^9+29517d^2e^8x^8-78712d^3e^7x^7-137746d^4e^6x^6+137746d^6e^4x^4+78712d^7e^3x^3-29517d^8e^2x^2-39356d^9ex-9839d^{10}+4095\left(e^{10}x^{10}+4de^9x^9+3d^2e^8x^8-8d^3e^7x^7-14d^4e^6x^6+14d^6e^4x^4+8d^7e^3x^3-3d^8e^2x^2-4d^9ex-d^{10}\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(5120e^9x^9+16385de^8x^8+1540d^2e^7x^7-45735d^3e^6x^6-40240d^4e^5x^5+34156d^5e^4x^4+56304d^6e^3x^3+4466d^7e^2x^2-22976d^8ex-9839d^9\right)\sqrt{-e^2x^2+d^2}}{4095\left(d^{11}e^{10}x^{10}+4d^{12}e^9x^9+3d^{13}e^8x^8-8d^{14}e^7x^7-14d^{15}e^6x^6+14d^{17}e^4x^4+8d^{18}e^3x^3-3d^{19}e^2x^2-4d^{20}ex-d^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4095*(9839*e^10*x^10 + 39356*d*e^9*x^9 + 29517*d^2*e^8*x^8 - 78712*d^3*e^
7*x^7 - 137746*d^4*e^6*x^6 + 137746*d^6*e^4*x^4 + 78712*d^7*e^3*x^3 - 29517
*d^8*e^2*x^2 - 39356*d^9*e*x - 9839*d^10 + 4095*(e^10*x^10 + 4*d*e^9*x^9 +
3*d^2*e^8*x^8 - 8*d^3*e^7*x^7 - 14*d^4*e^6*x^6 + 14*d^6*e^4*x^4 + 8*d^7*e^3
*x^3 - 3*d^8*e^2*x^2 - 4*d^9*e*x - d^10)*log(-(d - sqrt(-e^2*x^2 + d^2))/x)
 + (5120*e^9*x^9 + 16385*d*e^8*x^8 + 1540*d^2*e^7*x^7 - 45735*d^3*e^6*x^6 -
 40240*d^4*e^5*x^5 + 34156*d^5*e^4*x^4 + 56304*d^6*e^3*x^3 + 4466*d^7*e^2*x
^2 - 22976*d^8*e*x - 9839*d^9)*sqrt(-e^2*x^2 + d^2))/(d^11*e^10*x^10 + 4*d^
12*e^9*x^9 + 3*d^13*e^8*x^8 - 8*d^14*e^7*x^7 - 14*d^15*e^6*x^6 + 14*d^17*e^
4*x^4 + 8*d^18*e^3*x^3 - 3*d^19*e^2*x^2 - 4*d^20*e*x - d^21)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.03, size = 385, normalized size = 1.65

$$-\frac{128ex}{273\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^7}+\frac{1}{13\left(x+\frac{d}{e}\right)^4\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^2e^4}+\frac{2}{13\left(x+\frac{d}{e}\right)^3\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^3e^3}+\frac{29}{117\left(x+\frac{d}{e}\right)^2\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^4e^2}+\frac{320}{819\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^5e}+\frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^6}-\frac{512ex}{819\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^9}+\frac{1}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^8}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^{10}}-\frac{1024ex}{819\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^{11}}+\frac{1}{\sqrt{-e^2x^2+d^2}\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/13/d^2/e^4/(x+d/e)^4/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+2/13/d^3/e^3/(x+
d/e)^3/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+29/117/d^4/e^2/(x+d/e)^2/(2*(x+d
/e)*d*e-(x+d/e)^2*e^2)^(5/2)+320/819/d^5/e/(x+d/e)/(2*(x+d/e)*d*e-(x+d/e)^2
*e^2)^(5/2)-128/273/d^7*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)*x-512/819/d^9
*e/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x-1024/819/d^11*e/(2*(x+d/e)*d*e-(x+
d/e)^2*e^2)^(1/2)*x+1/5/d^6/(-e^2*x^2+d^2)^(5/2)+1/3/d^8/(-e^2*x^2+d^2)^(3/
```

```
2)+1/d^10/(-e^2*x^2+d^2)^(1/2)-1/d^10/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(
-e^2*x^2+d^2)^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^4 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(d^2-e^2\,x^2\right)^{7/2}\,(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.217 $\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}-\frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^{12}x}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}}-\frac{e(1201\ldots}{9009d\ldots}$$

Rubi [A] time = 0.68, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^{12}x}-\frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}}-\frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}}-\frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-8*e*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*(13*d - 24*e*x))/(143*d
^2*(d^2 - e^2*x^2)^(11/2)) - (e*(572*d - 1103*e*x))/(1287*d^4*(d^2 - e^2*x^
2)^(9/2)) - (e*(5148*d - 10111*e*x))/(9009*d^6*(d^2 - e^2*x^2)^(7/2)) - (e*
(12012*d - 23225*e*x))/(15015*d^8*(d^2 - e^2*x^2)^(5/2)) - (e*(12012*d - 21
583*e*x))/(9009*d^10*(d^2 - e^2*x^2)^(3/2)) - (e*(36036*d - 52175*e*x))/(90
09*d^12*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(d^12*x) + (4*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/d^12
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{15/2}}\,dx$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{\int \frac{-13d^4+52d^3ex-83d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{13/2}}dx}{13d^2}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} + \frac{\int \frac{143d^4-572d^3ex+960d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{11/2}}dx}{143d^4}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{-12\cdots}{\cdots}}{\cdots}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(51\cdots}{9009\cdots}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(51\cdots}{9009\cdots}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(51\cdots}{9009\cdots}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(51\cdots}{9009\cdots}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(51\cdots}{9009\cdots}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(51\cdots}{9009\cdots}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(51\cdots}{9009\cdots}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(51\cdots}{9009\cdots}$$

Mathematica [A] time = 0.21, size = 183, normalized size = 0.68

$$-\frac{4e\log(x)}{d^{12}}+\frac{4e\log\left(\sqrt{d^2-e^2x^2}+d\right)}{d^{12}}+\frac{\sqrt{d^2-e^2x^2}\left(45045d^{10}+546316d^9ex+1014094d^8e^2x^2-700504d^7e^3x^3-3157776d^6e^4x^4-1301264d^5e^5x^5+2748320d^4e^6x^6+2496180d^3e^7x^7-350000d^2e^8x^8-1043500de^9x^9-305920e^{10}x^{10}\right)}{45045d^{12}x(ex-d)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(45045*d^10 + 546316*d^9*e*x + 1014094*d^8*e^2*x^2 - 7
00504*d^7*e^3*x^3 - 3157776*d^6*e^4*x^4 - 1301264*d^5*e^5*x^5 + 2748320*d^4
```

```
*e^6*x^6 + 2496180*d^3*e^7*x^7 - 350000*d^2*e^8*x^8 - 1043500*d*e^9*x^9 - 3
05920*e^10*x^10))/(45045*d^12*x*(-d + e*x)^3*(d + e*x)^7) - (4*e*Log[x])/d^
12 + (4*e*Log[d + Sqrt[d^2 - e^2*x^2]])/d^12
```

IntegrateAlgebraic [A] time = 1.31, size = 192, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(-45045d^{10}-546316d^9ex-1014094d^8e^2x^2+700504d^7e^3x^3+3157776d^6e^4x^4+1301264d^5e^5x^5-2748320d^4e^6x^6-2496180d^3e^7x^7+350000d^2e^8x^8+1043500de^9x^9+305920e^{10}x^{10}\right)}{45045d^{12}x(d-ex)^3(d+ex)^7}-\frac{8e\tanh^{-1}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-45045*d^10 - 546316*d^9*e*x - 1014094*d^8*e^2*x^2 +
700504*d^7*e^3*x^3 + 3157776*d^6*e^4*x^4 + 1301264*d^5*e^5*x^5 - 2748320*d^
4*e^6*x^6 - 2496180*d^3*e^7*x^7 + 350000*d^2*e^8*x^8 + 1043500*d*e^9*x^9 +
305920*e^10*x^10))/(45045*d^12*x*(d - e*x)^3*(d + e*x)^7) - (8*e*ArcTanh[(S
qrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^12
```

fricas [A] time = 1.78, size = 458, normalized size = 1.69

$$-\frac{366136e^{11}x^{11}+1464544de^{10}x^{10}+1098408d^2e^9x^9-2929088d^3e^8x^8-5125904d^4e^7x^7+5125904d^6e^5x^5+2929088d^7e^4x^4-1098408d^8e^3x^3-1464544d^9e^2x^2-366136d^{10}ex+180180\left(e^{11}x^{11}+4de^{10}x^{10}+3d^2e^9x^9-8d^3e^8x^8-14d^4e^7x^7+14d^6e^5x^5+8d^7e^4x^4-3d^8e^3x^3-4d^9e^2x^2-d^{10}ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(305920e^{10}x^{10}+1043500de^9x^9+350000d^2e^8x^8-2496180d^3e^7x^7-2748320d^4e^6x^6+1301264d^5e^5x^5+3157776d^6e^4x^4+700504d^7e^3x^3-1014094d^8e^2x^2-546316d^9ex-45045d^{10}\right)\sqrt{-e^2x^2+d^2}}{45045\left(d^{12}e^{10}x^{11}+4d^{13}e^9x^{10}+3d^{14}e^8x^9-8d^{15}e^7x^8-14d^{16}e^6x^7+14d^{18}e^4x^5+8d^{19}e^3x^4-3d^{20}e^2x^3-4d^{21}ex^2-d^{22}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/45045*(366136*e^11*x^11 + 1464544*d*e^10*x^10 + 1098408*d^2*e^9*x^9 - 29
29088*d^3*e^8*x^8 - 5125904*d^4*e^7*x^7 + 5125904*d^6*e^5*x^5 + 2929088*d^7
*e^4*x^4 - 1098408*d^8*e^3*x^3 - 1464544*d^9*e^2*x^2 - 366136*d^10*e*x + 18
0180*(e^11*x^11 + 4*d*e^10*x^10 + 3*d^2*e^9*x^9 - 8*d^3*e^8*x^8 - 14*d^4*e^
7*x^7 + 14*d^6*e^5*x^5 + 8*d^7*e^4*x^4 - 3*d^8*e^3*x^3 - 4*d^9*e^2*x^2 - d^
10*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (305920*e^10*x^10 + 1043500*d*
e^9*x^9 + 350000*d^2*e^8*x^8 - 2496180*d^3*e^7*x^7 - 2748320*d^4*e^6*x^6 +
1301264*d^5*e^5*x^5 + 3157776*d^6*e^4*x^4 + 700504*d^7*e^3*x^3 - 1014094*d^
8*e^2*x^2 - 546316*d^9*e*x - 45045*d^10)*sqrt(-e^2*x^2 + d^2))/(d^12*e^10*x
^11 + 4*d^13*e^9*x^10 + 3*d^14*e^8*x^9 - 8*d^15*e^7*x^8 - 14*d^16*e^6*x^7 +
 14*d^18*e^4*x^5 + 8*d^19*e^3*x^4 - 3*d^20*e^2*x^3 - 4*d^21*e*x^2 - d^22*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.02, size = 484, normalized size = 1.79

$$\frac{6e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^8}+\frac{20222e^2x}{15015\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^8}-\frac{1}{13\left(x+\frac{d}{e}\right)^4\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^3e^3}-\frac{35}{143\left(x+\frac{d}{e}\right)^3\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^4e^2}-\frac{709}{1287\left(x+\frac{d}{e}\right)^2\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^5e}-\frac{10111}{9009\left(x+\frac{d}{e}\right)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{5}{2}}d^6}-\frac{4e}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^7}-\frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^6x}+\frac{8e^2x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^{10}}+\frac{80888e^2x}{45045\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2\right)^{\frac{3}{2}}d^{10}}-\frac{4e}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^9}+\frac{4e\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}\,d^{11}}+\frac{16e^2x}{5\sqrt{-e^2x^2+d^2}\,d^{12}}+\frac{161776e^2x}{45045\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,d^{12}}-\frac{4e}{\sqrt{-e^2x^2+d^2}\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/d^11*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+6/5/d
^8*e^2*x/(-e^2*x^2+d^2)^(5/2)+8/5/d^10*e^2*x/(-e^2*x^2+d^2)^(3/2)+16/5/d^12
*e^2*x/(-e^2*x^2+d^2)^(1/2)-35/143/d^4/e^2/(x+d/e)^3/(2*(x+d/e)*d*e-(x+d/e)
^2*e^2)^(5/2)-709/1287/d^5/e/(x+d/e)^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)+
```

```
20222/15015/d^8*e^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)*x+80888/45045/d^10*
e^2/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*x+161776/45045/d^12*e^2/(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(1/2)*x-1/13/d^3/e^3/(x+d/e)^4/(2*(x+d/e)*d*e-(x+d/e)^2*
e^2)^(5/2)-4/5/d^7*e/(-e^2*x^2+d^2)^(5/2)-4/3/d^9*e/(-e^2*x^2+d^2)^(3/2)-4/
d^11*e/(-e^2*x^2+d^2)^(1/2)-1/d^6/x/(-e^2*x^2+d^2)^(5/2)-10111/9009/d^6/(x+
d/e)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(d^2-e^2x^2\right)^{7/2}(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

3.218 $\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx$

Optimal. Leaf size=102

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}-\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {879, 865, 875, 208}

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}-\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] -((a*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]) - (c^2*(1 - a^2*x^2)^(3/2))/(x*(
c - a*c*x)^(3/2)) + a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c -
a*c*x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}dx &= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} - \frac{1}{2}(ac)\int \frac{\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}dx \\ &= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} - \frac{1}{2}a\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}dx \\ &= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} - \left(a^3c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}dx, x, \frac{\sqrt{1-a}}{\sqrt{c-a}} \right. \\ &= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} + a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\end{aligned}$$

Mathematica [A] time = 0.10, size = 93, normalized size = 0.91

$$\frac{\sqrt{1-a^2x^2}\left(a\sqrt{c}\,x\tanh^{-1}\left(\sqrt{c}\sqrt{\frac{ax+1}{c}}\right)-c(2ax+1)\sqrt{\frac{ax+1}{c}}\right)}{x\sqrt{\frac{ax+1}{c}}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-(c*Sqrt[(1 + a*x)/c]*(1 + 2*a*x)) + a*Sqrt[c]*x*ArcTan
h[Sqrt[c]*Sqrt[(1 + a*x)/c]]))/(x*Sqrt[(1 + a*x)/c]*Sqrt[c - a*c*x])
```

IntegrateAlgebraic [C] time = 0.48, size = 123, normalized size = 1.21

$$-\frac{i\sqrt{c-acx}\left(\frac{i\sqrt{c}\sqrt{-(ax-1)^2-2(ax-1)}\,(2(ax-1)+3)}{x\sqrt{ax-1}}+ia\sqrt{c}\tan^{-1}\left(\frac{\sqrt{ax-1}}{\sqrt{-(ax-1)^2-2(ax-1)}}\right)\right)}{\sqrt{c}\sqrt{ax-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] ((-I)*Sqrt[c - a*c*x]*((I*Sqrt[c]*(3 + 2*(-1 + a*x))*Sqrt[-2*(-1 + a*x) - (
-1 + a*x)^2])/(x*Sqrt[-1 + a*x]) + I*a*Sqrt[c]*ArcTan[Sqrt[-1 + a*x]/Sqrt[-
2*(-1 + a*x) - (-1 + a*x)^2]]))/(Sqrt[c]*Sqrt[-1 + a*x])
```

fricas [A] time = 0.42, size = 217, normalized size = 2.13

$$\left[\frac{\left(a^2x^2-ax\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\,(2ax+1)}{2\left(ax^2-x\right)}, \frac{\left(a^2x^2-ax\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-a^2x^2+1}\sqrt{-acx+c}\,(2ax+1)}{ax^2-x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x), ((a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt
(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 95, normalized size = 0.93

$$\frac{\left(-acx\,\mathrm{arctanh}\left(\frac{\sqrt{(ax+1)c}}{\sqrt{c}}\right)+2\sqrt{(ax+1)\,c}\;a\sqrt{c}\;x+\sqrt{(ax+1)\,c}\;\sqrt{c}\right)\sqrt{-\left(ax-1\right)c}\;\sqrt{-a^2x^2+1}}{(ax-1)\sqrt{(ax+1)\,c}\;\sqrt{c}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] (-arctanh((c*(a*x+1))^(1/2)/c^(1/2))*x*a*c+2*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+
(c*(a*x+1))^(1/2)*c^(1/2))*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(c
*(a*x+1))^(1/2)/x/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/x^2,x)
```

```
[Out] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)}\,\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/x**2, x)
```

3.219 $\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=39

$$-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {875, 208}

$$-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx &= \left(2a^2c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\ &= -2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.72

$$-\frac{2\sqrt{c}\sqrt{\frac{ax}{c}+\frac{1}{c}}\sqrt{c-acx}\tanh^{-1}\left(\sqrt{c}\sqrt{\frac{ax}{c}+\frac{1}{c}}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[c^(-1) + (a*x)/c]*Sqrt[c - a*c*x]*ArcTanh[Sqrt[c]*Sqrt[c^(
-1) + (a*x)/c]])/Sqrt[1 - a^2*x^2]
```

IntegrateAlgebraic [A] time = 0.34, size = 54, normalized size = 1.38

$$-\frac{2\sqrt{c-acx}\tan^{-1}\left(\frac{\sqrt{ax-1}}{\sqrt{-(ax-1)^2-2(ax-1)}}\right)}{\sqrt{ax-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*ArcTan[Sqrt[-1 + a*x]/Sqrt[-2*(-1 + a*x) - (-1 + a*x)^2
]])/Sqrt[-1 + a*x]
```

fricas [A] time = 0.41, size = 110, normalized size = 2.82

$$\left[\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right), -2\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{-c}}{a^2cx^2-c}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sq
rt(c) - 2*c)/(a*x^2 - x)), -2*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c)*sqrt(-c)/(a^2*c*x^2 - c))]
```

giac [A] time = 0.17, size = 57, normalized size = 1.46

$$-\frac{2c^3\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}\right)}{|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*c^3*(arctan(sqrt(2)*sqrt(c)/sqrt(-c))/(sqrt(-c)*c) - arctan(sqrt(a*c*x +
 c)/sqrt(-c))/(sqrt(-c)*c))/abs(c)
```

maple [A] time = 0.02, size = 58, normalized size = 1.49

$$\frac{2\sqrt{-(ax-1)c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{(ax+1)c}}{\sqrt{c}}\right)}{(ax-1)\sqrt{(ax+1)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2*(-(a*x-1)*c)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/((a*x+1)*c)^(1/2)*c^(1/2)*a
rctanh(((a*x+1)*c)^(1/2)/c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sqrt{c-a\,c\,x}}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}}{x\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.220 $\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx &= \sqrt{x}\sqrt{1-ax}+\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx\\
&= \sqrt{x}\sqrt{1-ax}+\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.08, size = 54, normalized size = 1.54

$$\sqrt{x}\sqrt{1-ax}+\frac{\sqrt{-a}\log\left(\sqrt{1-ax}-\sqrt{-a}\sqrt{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + (Sqrt[-a]*Log[-(Sqrt[-a]*Sqrt[x]) + Sqrt[1 - a*x]])
/a
```

fricas [A] time = 0.40, size = 92, normalized size = 2.63

$$\left[\frac{2\sqrt{-ax+1}\,a\sqrt{x}-\sqrt{-a}\log\left(-2\,ax+2\sqrt{-ax+1}\sqrt{-a}\sqrt{x}+1\right)}{2\,a},\ \frac{\sqrt{-ax+1}\,a\sqrt{x}-\sqrt{a}\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(-a*x + 1)*a*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt(-a*x + 1)*s
qrt(-a)*sqrt(x) + 1))/a, (sqrt(-a*x + 1)*a*sqrt(x) - sqrt(a)*arctan(sqrt(-a
*x + 1)/(sqrt(a)*sqrt(x))))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}
+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]
%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{
4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[1,1]%%%
}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[
0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-15.6438432182,61.793747834
9]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0
,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}
+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[
2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%
%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{
1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%
%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%
```

```
}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[
1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%
%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-29.292030761,78.649
3344628]1/abs(a)*a^2/a*(1/a*sqrt(-a*x+1)*sqrt(-a*(-a*x+1)+a)+1/sqrt(-a)*ln(
abs(sqrt(-a*(-a*x+1)+a)-sqrt(-a)*sqrt(-a*x+1))))
```

maple [B] time = 0.01, size = 62, normalized size = 1.77

$$\sqrt{-ax+1}\,\sqrt{x}+\frac{\sqrt{(-ax+1)\,x}\ \arctan\left(\frac{\left(x-\frac{1}{2a}\right)\sqrt{a}}{\sqrt{-a\,x^{2}+x}}\right)}{2\sqrt{-ax+1}\,\sqrt{a}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x+1)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(-a*x+1)^(1/2)+1/2*((-a*x+1)*x)^(1/2)/(-a*x+1)^(1/2)/x^(1/2)/a^(1/2
)*arctan(a^(1/2)*(x-1/2/a)/(-a*x^2+x)^(1/2))
```

maxima [A] time = 0.96, size = 48, normalized size = 1.37

$$-\frac{\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\,\sqrt{x}}\right)}{\sqrt{a}}+\frac{\sqrt{-ax+1}}{\left(a-\frac{ax-1}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x)))/sqrt(a) + sqrt(-a*x + 1)/((a - (a
*x - 1)/x)*sqrt(x))
```

mupad [B] time = 2.99, size = 38, normalized size = 1.09

$$\sqrt{x}\,\sqrt{1-a\,x}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{1-a\,x}-1}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(1 - a*x)^(1/2) + (2*atan((a^(1/2)*x^(1/2))/((1 - a*x)^(1/2) - 1)))
/a^(1/2)
```

sympy [A] time = 1.90, size = 83, normalized size = 2.37

$$\begin{cases} \frac{iax^{\frac{3}{2}}}{\sqrt{ax-1}}-\frac{i\sqrt{x}}{\sqrt{ax-1}}-\frac{i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}} & \text{for}\ |ax|>1\\ \sqrt{x}\,\sqrt{-ax+1}+\frac{\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*a*x**(3/2)/sqrt(a*x - 1) - I*sqrt(x)/sqrt(a*x - 1) - I*acosh(s
qrt(a)*sqrt(x))/sqrt(a), Abs(a*x) > 1), (sqrt(x)*sqrt(-a*x + 1) + asin(sqrt
(a)*sqrt(x))/sqrt(a), True))
```

3.221 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {848, 50, 54, 216}

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx &= \int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx \\
&= \sqrt{x}\sqrt{1-ax} + \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx \\
&= \sqrt{x}\sqrt{1-ax} + \text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right) \\
&= \sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

IntegrateAlgebraic [F] time = 2.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]), x]
```

fricas [B] time = 0.44, size = 199, normalized size = 5.69

$$\left[\frac{4\sqrt{-a^2x^2+1}\sqrt{ax+1}\,a\sqrt{x} - (ax+1)\sqrt{-a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}(2ax-1)\sqrt{ax+1}\sqrt{-a}\sqrt{x}-7ax+1}{ax+1}\right)}{4\left(a^2x+a\right)}, \frac{2\sqrt{-a^2x^2+1}\sqrt{ax+1}\,a\sqrt{x} - (ax+1)\sqrt{a}\arctan\left(\frac{2\sqrt{-a^2x^2+1}\sqrt{ax+1}\sqrt{a}\sqrt{x}}{2a^2x^2+ax-1}\right)}{2\left(a^2x+a\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(a*x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(-a)*log
(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*sqrt(-a)*sqrt
(x) - 7*a*x + 1)/(a*x + 1)))/(a^2*x + a), 1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt(a*
x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(a*x +
 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^2*x + a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
```

```
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}
+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]
%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{
4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[1,1]%%%
}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[
0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-15.6438432182,61.793747834
9]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0
,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}
+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[
2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%
%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{
1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%
%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%
}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[
1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%
%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-29.292030761,78.649
3344628]2/abs(a)*a^2/a*(1/2*ln(abs(-sqrt(-a)*sqrt(-a*x+1)+sqrt(-a*(-a*x+1)+
a)))/sqrt(-a)+1/2*sqrt(-a*x+1)*sqrt(-a*(-a*x+1)+a)/a+(sqrt(2)-ln(abs(-sqrt(
-a)*sqrt(2)+sqrt(-a))))/2/sqrt(-a))
```

maple [B] time = 0.02, size = 76, normalized size = 2.17

$$\frac{\sqrt{-a^{2}x^{2}+1}\,\left(\arctan\left(\frac{2ax-1}{2\sqrt{-(ax-1)x}\,\sqrt{a}}\right)+2\sqrt{-\left(ax-1\right)x}\,\sqrt{a}\right)\sqrt{x}}{2\sqrt{ax+1}\,\sqrt{-\left(ax-1\right)x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)/(a*x+1)^(1/2)*(2*a^(1/2)*(-x*(a*x-1))^(1/2)+
arctan(1/2/a^(1/2)*(2*a*x-1)/(-x*(a*x-1))^(1/2)))/(-x*(a*x-1))^(1/2)/a^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}}{\sqrt{ax+1}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(a*x + 1)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-a^{2}\,x^{2}}}{\sqrt{x}\,\sqrt{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(a*x + 1)^(1/2)),x)
[Out] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(a*x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(a*x+1)**(1/2),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(a*x + 1)), x)
```

3.222 $\int \frac{\sqrt{1+ax}}{\sqrt{x}} \, dx$

Optimal. Leaf size=34

$$\sqrt{x}\sqrt{ax+1} + \frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$\sqrt{x}\sqrt{ax+1} + \frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+ax}}{\sqrt{x}} \, dx &= \sqrt{x}\sqrt{1+ax} + \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}} \, dx \\ &= \sqrt{x}\sqrt{1+ax} + \text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}} \, dx, x, \sqrt{x}\right) \\ &= \sqrt{x}\sqrt{1+ax} + \frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\sqrt{x}\sqrt{ax+1} + \frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.06, size = 46, normalized size = 1.35

$$\sqrt{x}\sqrt{ax+1}-\frac{\log\left(\sqrt{ax+1}-\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] - Log[-(Sqrt[a]*Sqrt[x]) + Sqrt[1 + a*x]]/Sqrt[a]
```

fricas [A] time = 0.42, size = 90, normalized size = 2.65

$$\left[\frac{2\sqrt{ax+1}\,a\sqrt{x}+\sqrt{a}\log\left(2\,ax+2\sqrt{ax+1}\sqrt{a}\sqrt{x}+1\right)}{2\,a},\ \frac{\sqrt{ax+1}\,a\sqrt{x}-\sqrt{-a}\arctan\left(\frac{\sqrt{ax+1}\sqrt{-a}}{a\sqrt{x}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(a*x + 1)*a*sqrt(x) + sqrt(a)*log(2*a*x + 2*sqrt(a*x + 1)*sqrt(
a)*sqrt(x) + 1))/a, (sqrt(a*x + 1)*a*sqrt(x) - sqrt(-a)*arctan(sqrt(a*x + 1
)*sqrt(-a)/(a*sqrt(x))))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{16,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}+
%%%{6,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[
3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%}
+%%%{4,[1,3]%%%}+%%%{12,[1,2]%%%}+%%%{-52,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,
[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]%%%
}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,
[3,4]%%%}+%%%{8,[3,3]%%%}+%%%{-8,[3,2]%%%}+%%%{8,[3,1]%%%}+%%%{-4,[3,0]%%%}
+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6,[
2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{8,[1,3]%%%}+%%%{-8,[1,2]%%%}+%%%{8,[1,1]%%%}+
%%%{-4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[0
,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [85.3561567818,61.7937478349]
Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{4,[1,2]%%%}+%%%{16,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]
%%%}+%%%{6,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%
{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0
]%%%}+%%%{4,[1,3]%%%}+%%%{12,[1,2]%%%}+%%%{-52,[1,1]%%%}+%%%{4,[1,0]%%%}+%%
%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{8,[3,3]%%%}+%%%{-8,[3,2]%%%}+%%%{8,[3,1]%%%}+%%%{-4,[3,0
]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{8,[1,3]%%%}+%%%{-8,[1,2]%%%}+%%%{8,[1,1]
```

```
%%%}+%%%{-4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{
-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [71.707969239,78.6493344
628]1/abs(a)*a^2/a*(1/a*sqrt(a*x+1)*sqrt(a*(a*x+1)-a)-1/sqrt(a)*ln(abs(sqrt
(a*(a*x+1)-a)-sqrt(a)*sqrt(a*x+1))))
```

maple [B] time = 0.01, size = 57, normalized size = 1.68

$$\sqrt{ax+1}\,\sqrt{x}+\frac{\sqrt{(ax+1)\,x}\,\ln\left(\frac{ax+\frac{1}{2}}{\sqrt{a}}+\sqrt{a\,x^2+x}\right)}{2\sqrt{ax+1}\,\sqrt{a}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a*x+1)^(1/2)+1/2*((a*x+1)*x)^(1/2)/(a*x+1)^(1/2)/x^(1/2)*ln((1/2+a
*x)/a^(1/2)+(a*x^2+x)^(1/2))/a^(1/2)
```

maxima [B] time = 0.96, size = 68, normalized size = 2.00

$$-\frac{\log\left(-\frac{\sqrt{a}-\frac{\sqrt{ax+1}}{\sqrt{x}}}{\sqrt{a}+\frac{\sqrt{ax+1}}{\sqrt{x}}}\right)}{2\sqrt{a}}-\frac{\sqrt{ax+1}}{\left(a-\frac{ax+1}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(-(sqrt(a) - sqrt(a*x + 1)/sqrt(x))/(sqrt(a) + sqrt(a*x + 1)/sqrt(x
)))/sqrt(a) - sqrt(a*x + 1)/((a - (a*x + 1)/x)*sqrt(x))
```

mupad [B] time = 3.00, size = 36, normalized size = 1.06

$$\sqrt{x}\,\sqrt{a\,x+1}+\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{a\,x+1}-1}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a*x + 1)^(1/2) + (2*atanh((a^(1/2)*x^(1/2))/((a*x + 1)^(1/2) - 1))
)/a^(1/2)
```

sympy [A] time = 1.97, size = 29, normalized size = 0.85

$$\sqrt{x}\,\sqrt{ax+1}+\frac{\mathrm{asinh}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(a*x + 1) + asinh(sqrt(a)*sqrt(x))/sqrt(a)
```

3.223 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=34

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {848, 50, 54, 215}

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx &= \int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx\\ &= \sqrt{x}\sqrt{1+ax}+\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx\\ &= \sqrt{x}\sqrt{1+ax}+\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}\sqrt{1+ax}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

IntegrateAlgebraic [F] time = 2.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
[Out] Defer[IntegrateAlgebraic][Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]), x]
```

fricas [B] time = 0.45, size = 208, normalized size = 6.12

$$\left[-\frac{4\sqrt{-a^2x^2+1}\sqrt{-ax+1}\,a\sqrt{x}-(ax-1)\sqrt{a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}\,(2ax+1)\sqrt{-ax+1}\sqrt{a}\sqrt{x}-7ax-1}{ax-1}\right)}{4\left(a^2x-a\right)},-\frac{2\sqrt{-a^2x^2+1}\sqrt{-ax+1}\,a\sqrt{x}-(ax-1)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a^2x^2+1}\sqrt{-ax+1}\sqrt{-a}\sqrt{x}}{2a^2x^2-ax-1}\right)}{2\left(a^2x-a\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] [-1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(a)*lo
g(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x + 1)*sqrt(-a*x + 1)*sqrt(a)*sqr
t(x) - 7*a*x - 1)/(a*x - 1)))/(a^2*x - a), -1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(-a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(-
a*x + 1)*sqrt(-a)*sqrt(x)/(2*a^2*x^2 - a*x - 1)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
```

```
}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{16,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}+
%%%{6,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[
3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%}
+%%%{4,[1,3]%%%}+%%%{12,[1,2]%%%}+%%%{-52,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,
[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]%%%
}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,
[3,4]%%%}+%%%{8,[3,3]%%%}+%%%{-8,[3,2]%%%}+%%%{8,[3,1]%%%}+%%%{-4,[3,0]%%%}
+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6,[
2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{8,[1,3]%%%}+%%%{-8,[1,2]%%%}+%%%{8,[1,1]%%%}+
%%%{-4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[0
,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [85.3561567818,61.7937478349]
Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{4,[1,2]%%%}+%%%{16,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]
%%%}+%%%{6,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%
{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0
]%%%}+%%%{4,[1,3]%%%}+%%%{12,[1,2]%%%}+%%%{-52,[1,1]%%%}+%%%{4,[1,0]%%%}+%%
%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{8,[3,3]%%%}+%%%{-8,[3,2]%%%}+%%%{8,[3,1]%%%}+%%%{-4,[3,0
]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{8,[1,3]%%%}+%%%{-8,[1,2]%%%}+%%%{8,[1,1]
%%%}+%%%{-4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{
-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [71.707969239,78.6493344
628]-2/abs(a)*a^2/a*(1/2*ln(abs(-sqrt(a)*sqrt(a*x+1)+sqrt(a*(a*x+1)-a)))/sq
rt(a)-1/2*sqrt(a*(a*x+1)-a)*sqrt(a*x+1)/a+(sqrt(2)-ln(abs(-sqrt(2)*sqrt(a)+
sqrt(a))))/2/sqrt(a))
```

maple [B] time = 0.01, size = 86, normalized size = 2.53

$$-\frac{\sqrt{-a^{2} x^{2}+1}\,\sqrt{-a x+1}\,\left(\ln\left(\frac{2 a x+2 \sqrt{(a x+1) x}\,\sqrt{a}+1}{2 \sqrt{a}}\right)+2 \sqrt{(a x+1) x}\,\sqrt{a}\right) \sqrt{x}}{2 (a x-1) \sqrt{(a x+1) x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)*(-a*x+1)^(1/2)*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/((a*x+1)*x)
^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2} x^{2}+1}}{\sqrt{-a x+1}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a*x + 1)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-a^2\,x^2}}{\sqrt{x}\,\sqrt{1-a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(1 - a*x)^(1/2)),x)
[Out] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(1 - a*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(-a*x+1)**(1/2),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(-a*x + 1)), x)
```

3.224 $\int \sqrt{x}\sqrt{1-ax}\,dx$

Optimal. Leaf size=63

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 - a*x])/(4*a) + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{1-ax}\,dx &= \frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}\,dx\\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx}{8a}\\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx,x,\sqrt{x}\right)}{4a}\\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.78

$$\frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}\,(2ax-1)+\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + ArcSin[Sqrt[a]*Sqrt[x]])/(4*a
^(3/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 74, normalized size = 1.17

$$\frac{\sqrt{-a}\log\left(\sqrt{1-ax}-\sqrt{-a}\sqrt{x}\right)}{4a^2}+\frac{\sqrt{1-ax}\left(2ax^{3/2}-\sqrt{x}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] (Sqrt[1 - a*x]*(-Sqrt[x] + 2*a*x^(3/2)))/(4*a) + (Sqrt[-a]*Log[-(Sqrt[-a]*S
qrt[x]) + Sqrt[1 - a*x]])/(4*a^2)
```

fricas [A] time = 0.41, size = 111, normalized size = 1.76

$$\left[\frac{2\left(2a^2x-a\right)\sqrt{-ax+1}\sqrt{x}-\sqrt{-a}\log\left(-2ax+2\sqrt{-ax+1}\sqrt{-a}\sqrt{x}+1\right)}{8a^2},\frac{\left(2a^2x-a\right)\sqrt{-ax+1}\sqrt{x}-\sqrt{a}\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{4a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(2*a^2*x - a)*sqrt(-a*x + 1)*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt
(-a*x + 1)*sqrt(-a)*sqrt(x) + 1))/a^2, 1/4*((2*a^2*x - a)*sqrt(-a*x + 1)*sq
rt(x) - sqrt(a)*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x))))/a^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}
+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]
%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{
4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[1,1]%%%
}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[
0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-41.1343540126,25.838873679
7]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0
,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}
+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[
2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%
%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{
1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%
```

```
%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%
}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[
1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%
%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-67.0714422017,15.45
1549686]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{
-4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]
%%%}+%%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%
%{4,[0,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,
1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%
%%{4,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[
1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},
0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[
4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+
%%%{4,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2
,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%
%{-8,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]
%%%}+%%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-46.2420096635
,81.9516051291]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%
%}+%%%{-4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{
6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]
%%%}+%%%{4,[0,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%
{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1
]%%%}+%%%{4,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%
%%{-4,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,
0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+
%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,
1]%%%}+%%%{4,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%
%{-8,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]
%%%}+%%%{-8,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{
6,[0,2]%%%}+%%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-82.594
7937798,51.6443148847]1/a*(-2*a*abs(a)/a^2/a*(2*(1/8*sqrt(-a*x+1)*sqrt(-a*x
+1)-5/16)*sqrt(-a*x+1)*sqrt(-a*(-a*x+1)+a)+6*a/16/sqrt(-a)*ln(abs(sqrt(-a*(
-a*x+1)+a)-sqrt(-a)*sqrt(-a*x+1))))-2*abs(a)/a^2*(1/2*sqrt(-a*x+1)*sqrt(-a*
(-a*x+1)+a)-2*a/4/sqrt(-a)*ln(abs(sqrt(-a*(-a*x+1)+a)-sqrt(-a)*sqrt(-a*x+1)
))))
```

maple [A] time = 0.00, size = 79, normalized size = 1.25

$$\frac{\sqrt{-ax+1}\,x^{\frac{3}{2}}}{2}-\frac{\sqrt{-ax+1}\,\sqrt{x}}{4a}+\frac{\sqrt{(-ax+1)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{2a}\right)\sqrt{a}}{\sqrt{-a\,x^{2}+x}}\right)}{8\sqrt{-ax+1}\,a^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x+1)^(1/2)*x^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)*(-a*x+1)^(1/2)/a+1/8/a^(3/2)*((-a*x+
1)*x)^(1/2)/(-a*x+1)^(1/2)/x^(1/2)*arctan((x-1/2/a)/(-a*x^2+x)^(1/2)*a^(1/2
))
```

maxima [A] time = 0.96, size = 82, normalized size = 1.30

$$\frac{\frac{\sqrt{-ax+1}\,a}{\sqrt{x}}-\frac{(-ax+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{4\left(a^{3}-\frac{2\,(ax-1)a^{2}}{x}+\frac{(ax-1)^{2}a}{x^{2}}\right)}-\frac{\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\,\sqrt{x}}\right)}{4\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(-a*x + 1)*a/sqrt(x) - (-a*x + 1)^(3/2)/x^(3/2))/(a^3 - 2*(a*x - 1
)*a^2/x + (a*x - 1)^2*a/x^2) - 1/4*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x)))
/a^(3/2)
```

mupad [B] time = 2.60, size = 54, normalized size = 0.86

$$\sqrt{x}\,\left(\frac{x}{2}-\frac{1}{4\,a}\right)\,\sqrt{1-a\,x}-\frac{\ln\left(2\,\sqrt{-a}\,\sqrt{x}\,\sqrt{1-a\,x}-2\,a\,x+1\right)}{8\,{\left(-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1 - a*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - 1/(4*a))*(1 - a*x)^(1/2) - log(2*(-a)^(1/2)*x^(1/2)*(1 - a*x
)^(1/2) - 2*a*x + 1)/(8*(-a)^(3/2))
```

sympy [A] time = 3.39, size = 148, normalized size = 2.35

$$\begin{cases} \frac{iax^{\frac{5}{2}}}{2\sqrt{ax-1}} - \frac{3ix^{\frac{3}{2}}}{4\sqrt{ax-1}} + \frac{i\sqrt{x}}{4a\sqrt{ax-1}} - \frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{for}\ |ax| > 1 \\ -\frac{ax^{\frac{5}{2}}}{2\sqrt{-ax+1}} + \frac{3x^{\frac{3}{2}}}{4\sqrt{-ax+1}} - \frac{\sqrt{x}}{4a\sqrt{-ax+1}} + \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-a*x+1)**(1/2),x)
```

```
[Out] Piecewise((I*a*x**(5/2)/(2*sqrt(a*x - 1)) - 3*I*x**(3/2)/(4*sqrt(a*x - 1))
+ I*sqrt(x)/(4*a*sqrt(a*x - 1)) - I*acosh(sqrt(a)*sqrt(x))/(4*a**(3/2)), Ab
s(a*x) > 1), (-a*x**(5/2)/(2*sqrt(-a*x + 1)) + 3*x**(3/2)/(4*sqrt(-a*x + 1)
) - sqrt(x)/(4*a*sqrt(-a*x + 1)) + asin(sqrt(a)*sqrt(x))/(4*a**(3/2)), True
))
```

3.225 $\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx$

Optimal. Leaf size=63

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {848, 50, 54, 216}

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 - a*x])/(4*a) + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \int \sqrt{x}\sqrt{1-ax}\,dx$$
$$= \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}\,dx$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx}{8a}$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right)}{4a}$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.78

$$\frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}\,(2ax-1) + \sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + ArcSin[Sqrt[a]*Sqrt[x]])/(4*a
^(3/2))
```

IntegrateAlgebraic [F] time = 2.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

```
[Out] Defer[IntegrateAlgebraic][(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x], x]
```

fricas [B] time = 0.45, size = 221, normalized size = 3.51

$$\left[\frac{4\sqrt{-a^2x^2+1}\left(2a^2x-a\right)\sqrt{ax+1}\sqrt{x} - (ax+1)\sqrt{-a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}(2ax-1)\sqrt{ax+1}\sqrt{-a}\sqrt{x}-7ax+1}{ax+1}\right)}{16\left(a^3x+a^2\right)}, \frac{2\sqrt{-a^2x^2+1}\left(2a^2x-a\right)\sqrt{ax+1}\sqrt{x} - (ax+1)\sqrt{a}\arctan\left(\frac{2\sqrt{-a^2x^2+1}\sqrt{ax+1}\sqrt{a}\sqrt{x}}{2a^2x^2+ax-1}\right)}{8\left(a^3x+a^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-a^2*x^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)
*sqrt(-a)*log(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*
sqrt(-a)*sqrt(x) - 7*a*x + 1)/(a*x + 1)))/(a^3*x + a^2), 1/8*(2*sqrt(-a^2*x
^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sq
rt(-a^2*x^2 + 1)*sqrt(a*x + 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^3
*x + a^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.01, size = 92, normalized size = 1.46

$$\frac{\sqrt{-a^{2}x^{2}+1}\,\left(4\sqrt{-\left(ax-1\right)x}\;a^{\frac{3}{2}}x+\arctan\left(\frac{2ax-1}{2\sqrt{-\left(ax-1\right)x}\;\sqrt{a}}\right)-2\sqrt{-\left(ax-1\right)x}\;\sqrt{a}\right)\sqrt{x}}{8\sqrt{ax+1}\;\sqrt{-\left(ax-1\right)x}\;a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x)
```

```
[Out] 1/8*x^(1/2)*(-a^2*x^2+1)^(1/2)/a^(3/2)*(4*x*a^(3/2)*(-(a*x-1)*x)^(1/2)-2*(-
(a*x-1)*x)^(1/2)*a^(1/2)+arctan(1/2*(2*a*x-1)/(-(a*x-1)*x)^(1/2)/a^(1/2)))/
(a*x+1)^(1/2)/(-(a*x-1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{x}}{\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(x)/sqrt(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}\,\sqrt{1-a^{2}\,x^{2}}}{\sqrt{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1)^(1/2),x)
```

```
[Out] int((x^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}\,\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-a**2*x**2+1)**(1/2)/(a*x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(-(a*x - 1)*(a*x + 1))/sqrt(a*x + 1), x)
```

3.226 $\int \frac{x\sqrt{1+x}}{1+x^2}\,dx$

Optimal. Leaf size=214

$$2\sqrt{x+1}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\,\sqrt{x+1}+\sqrt{2}+1\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\right.$$

Rubi [A] time = 0.25, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {825, 827, 1169, 634, 618, 204, 628}

$$2\sqrt{x+1}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)+\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] + ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 +
 Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] - ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[
1 + x])/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] + (Sqrt[(1 + Sqrt[2])
/2]*Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2 - (Sqrt[(1
+ Sqrt[2])/2]*Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{1+x}}{1+x^2}\,dx &= 2\sqrt{1+x} + \int \frac{-1+x}{\sqrt{1+x}\left(1+x^2\right)}\,dx \\
&= 2\sqrt{1+x} + 2\,\text{Subst}\left(\int \frac{-2+x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right) \\
&= 2\sqrt{1+x} + \frac{\text{Subst}\left(\int \frac{-2\sqrt{2\left(1+\sqrt{2}\right)}-\left(-2-\sqrt{2}\right)x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\text{Subst}\left(\int \frac{-2\sqrt{2\left(1+\sqrt{2}\right)}+\left(-2-\sqrt{2}\right)x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx,\right.}{2\sqrt{1+\sqrt{2}}} \\
&= 2\sqrt{1+x} - \frac{1}{2}\sqrt{3-2\sqrt{2}}\,\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+x}\right) - \frac{1}{2}\sqrt{3-2\sqrt{2}}\,\text{Su} \\
&= 2\sqrt{1+x} + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\,\sqrt{1+x}\right) - \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\log\left(\right. \\
&= 2\sqrt{1+x} + \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right) - \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{2\left(1\right.}}{\sqrt{}}\right.
\end{aligned}$$

Mathematica [C] time = 0.04, size = 60, normalized size = 0.28

$$2\sqrt{x+1} - \sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right) - \sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] - Sqrt[1 - I]*ArcTanh[Sqrt[1 + x]/Sqrt[1 - I]] - Sqrt[1 + I]*
ArcTanh[Sqrt[1 + x]/Sqrt[1 + I]]
```

IntegrateAlgebraic [C] time = 0.38, size = 68, normalized size = 0.32

$$2\sqrt{x+1} - \sqrt{-1+i}\,\tan^{-1}\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\,\sqrt{x+1}\right) - \sqrt{-1-i}\,\tan^{-1}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\,\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] - Sqrt[-1 + I]*ArcTan[Sqrt[-1/2 - I/2]*Sqrt[1 + x]] - Sqrt[-1
 - I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + x]]
```

fricas [A] time = 0.42, size = 307, normalized size = 1.43

$$-\frac{1}{8}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}\log\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}+x+\sqrt{2}+1\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}\log\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}+x+\sqrt{2}+1\right)+\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{2^{\frac{1}{4}}\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}+2x+2\sqrt{2}+2}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}-\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{x+1}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}-\sqrt{2}-1\right)+\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{-2^{\frac{1}{4}}\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}+2x+2\sqrt{2}+2}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}-\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{x+1}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}+\sqrt{2}+1\right)+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*2^(1/4)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4)*log(1/2*2^(1/4)*sqrt(x + 1)
*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + x + sqrt(2) + 1) + 1/8*2^(1/4)*(sqrt(
2) + 2)*sqrt(-2*sqrt(2) + 4)*log(-1/2*2^(1/4)*sqrt(x + 1)*(sqrt(2) + 2)*sqr
t(-2*sqrt(2) + 4) + x + sqrt(2) + 1) + 1/2*2^(3/4)*sqrt(-2*sqrt(2) + 4)*arc
tan(1/4*2^(3/4)*sqrt(2^(1/4)*sqrt(x + 1)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4)
 + 2*x + 2*sqrt(2) + 2)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) - 1/2*2^(3/4)*sq
rt(x + 1)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) - sqrt(2) - 1) + 1/2*2^(3/4)*s
qrt(-2*sqrt(2) + 4)*arctan(1/4*2^(3/4)*sqrt(-2^(1/4)*sqrt(x + 1)*(sqrt(2) +
 2)*sqrt(-2*sqrt(2) + 4) + 2*x + 2*sqrt(2) + 2)*(sqrt(2) + 2)*sqrt(-2*sqrt(
2) + 4) - 1/2*2^(3/4)*sqrt(x + 1)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) + sqrt
(2) + 1) + 2*sqrt(x + 1)
```

giac [A] time = 0.90, size = 167, normalized size = 0.78

$$-\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)+\frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*
sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/2*sqrt(2*sqrt(2) - 2)*arctan(-1/2*2^(3
/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/4*s
qrt(2*sqrt(2) + 2)*log(2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(2)
+ 1) + 1/4*sqrt(2*sqrt(2) + 2)*log(-2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) +
 x + sqrt(2) + 1) + 2*sqrt(x + 1)
```

maple [A] time = 0.12, size = 240, normalized size = 1.12

$$-\frac{\sqrt{2}\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}+\frac{\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}+\frac{\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}-\frac{\sqrt{2}\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(x+1+\sqrt{2}-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}\right)}{4}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(x+1+\sqrt{2}+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}\right)}{4}+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x+1)^(1/2)/(x^2+1),x)
```

```
[Out] 2*(x+1)^(1/2)+1/4*ln(1+x+2^(1/2)-(x+1)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1
/2))^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1/2)
)/(-2+2*2^(1/2))^(1/2))*2^(1/2)+1/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2
)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*ln(1+x+2^(1/2)+(x+1)^(1/2)
*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/(-2+2*2^(1/2))^(1/2)*arctan((2*
(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/(-2+2*2^(1/2))^(1/
2)*arctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}\,x}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 1)*x/(x^2 + 1), x)
```

mupad [B] time = 0.11, size = 201, normalized size = 0.94

$$2\sqrt{x+1} + \operatorname{atanh}\left(\frac{\sqrt{x+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} - \frac{\sqrt{x+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}} - 2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right) - \operatorname{atanh}\left(\frac{\sqrt{x+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{x+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} - \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}} + 2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(1/2))/(x^2 + 1),x)
```

```
[Out] 2*(x + 1)^(1/2) + atanh((x + 1)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (x + 1)
^(1/2)/(4*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(8*(1/8 - 2^(1
/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/
8 - 2^(1/2)/8)^(1/2) - 2*(2^(1/2)/8 + 1/8)^(1/2)) - atanh((x + 1)^(1/2)/(4*
(1/8 - 2^(1/2)/8)^(1/2)) + (x + 1)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (2^(
1/2)*(x + 1)^(1/2))/(8*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(
8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/8 - 2^(1/2)/8)^(1/2) + 2*(2^(1/2)/8 + 1/8
)^(1/2))
```

sympy [A] time = 11.16, size = 68, normalized size = 0.32

$$2\sqrt{x+1} - 4\operatorname{RootSum}\left(512t^4+32t^2+1, \left(t \mapsto t\log\left(-128t^3+\sqrt{x+1}\right)\right)\right) + 2\operatorname{RootSum}\left(128t^4+16t^2+1, \left(t \mapsto t\log\left(64t^3+4t+\sqrt{x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*sqrt(x + 1) - 4*RootSum(512*_t**4 + 32*_t**2 + 1, Lambda(_t, _t*log(-128*
_t**3 + sqrt(x + 1)))) + 2*RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*
log(64*_t**3 + 4*_t + sqrt(x + 1))))
```

3.227 $\int \frac{x^4 \sqrt{a+cx^2}}{d+ex} \, dx$

Optimal. Leaf size=255

$$-\frac{d\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6}+\frac{\left(a+cx^2\right)^{3/2}\left(47cd^2-8ae^2\right)}{60c^2e^3}-\frac{d^4\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cd}{\sqrt{a+cx^2}\,\sqrt{a}}\right.}{e^6}$$

Rubi [A] time = 0.63, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1654, 815, 844, 217, 206, 725}

$$-\frac{d\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6}+\frac{\left(a+cx^2\right)^{3/2}\left(47cd^2-8ae^2\right)}{60c^2e^3}+\frac{d\sqrt{a+cx^2}\left(8cd^3-ex\left(4cd^2-ae^2\right)\right)}{8ce^5}-\frac{d^4\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^6}-\frac{13d\left(a+cx^2\right)^{3/2}(d+ex)}{20ce^3}+\frac{\left(a+cx^2\right)^{3/2}(d+ex)^2}{5ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (d*(8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(8*c*e^5) + ((47*c*d^
2 - 8*a*e^2)*(a + c*x^2)^(3/2))/(60*c^2*e^3) - (13*d*(d + e*x)*(a + c*x^2)^
(3/2))/(20*c*e^3) + ((d + e*x)^2*(a + c*x^2)^(3/2))/(5*c*e^3) - (d*(8*c^2*d
^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3
/2)*e^6) - (d^4*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e
^2]*Sqrt[a + c*x^2])])/e^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3} + \frac{\int \frac{\sqrt{a+cx^2}\left(-2ad^2e^2-de\left(3cd^2+4ae^2\right)x-e^2\left(11cd^2+2ae^2\right)x^2-13cde^3x^3\right)}{d+ex}dx}{5ce^4}$$

$$= -\frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3} + \frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3} + \frac{\int \frac{\sqrt{a+cx^2}\left(5acd^2e^5+3cde^4\left(9cd^2-ae^2\right)x+ce^5\left(47cd^2\cdots\right.\right.}{d+ex}}{20c^2e^7}$$

$$= \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3} + \frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3} + \int \frac{\left(15ac^2d^2e^7\cdots\right.}{\cdots}$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3\cdots}}{20ce^3}$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3\cdots}}{20ce^3}$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3\cdots}}{20ce^3}$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3\cdots}}{20ce^3}$$

Mathematica [A] time = 0.61, size = 259, normalized size = 1.02

$$\frac{e\sqrt{a+cx^2}\left(-16a^2e^4+ace^2\left(40d^2-15dex+8e^2x^2\right)+2c^2\left(60d^4-30d^3ex+20d^2e^2x^2-15de^3x^3+12e^4x^4\right)\right)-120c^{5/2}d^5\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)-120c^2d^4\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)+\frac{15\sqrt{a}\sqrt{c}de^2\sqrt{a+cx^2}\left(ae^2-4cd^2\right)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{\frac{cx^2}{a}+1}}}{120c^2e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(-16*a^2*e^4 + a*c*e^2*(40*d^2 - 15*d*e*x + 8*e^2*x^2) +
 2*c^2*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4))
+ (15*Sqrt[a]*Sqrt[c]*d*e^2*(-4*c*d^2 + a*e^2)*Sqrt[a + c*x^2]*ArcSinh[(Sqr
t[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a] - 120*c^(5/2)*d^5*ArcTanh[(Sqrt[c]*x)
/Sqrt[a + c*x^2]] - 120*c^2*d^4*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(
Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(120*c^2*e^6)
```

IntegrateAlgebraic [A] time = 0.91, size = 283, normalized size = 1.11

$$\frac{\sqrt{a+cx^2}\left(-16a^2e^4+40acd^2e^2-15acde^3x+8ace^4x^2+120c^2d^4-60c^2d^3ex+40c^2d^2e^2x^2-30c^2de^3x^3+24c^2e^4x^4\right)}{120c^2e^5}+\frac{\left(-a^2de^4+4acd^3e^2+8c^2d^5\right)\log\left(\sqrt{a+cx^2}-\sqrt{c}x\right)}{8c^{3/2}e^6}+\frac{2d^4\sqrt{-ae^2-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2-cd^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[a + c*x^2]*(120*c^2*d^4 + 40*a*c*d^2*e^2 - 16*a^2*e^4 - 60*c^2*d^3*e*
x - 15*a*c*d*e^3*x + 40*c^2*d^2*e^2*x^2 + 8*a*c*e^4*x^2 - 30*c^2*d*e^3*x^3
+ 24*c^2*e^4*x^4))/(120*c^2*e^5) + (2*d^4*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sq
rt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e
*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/e^6 + ((8*c^2*d^5 + 4*a*c*d^3*e^
2 - a^2*d*e^4)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*c^(3/2)*e^6)
```

fricas [A] time = 6.90, size = 1104, normalized size = 4.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/240*(120*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*
e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^
2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(24*c^
2*e^5*x^4 - 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5
+ 8*(5*c^2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(
c*x^2 + a))/(c^2*e^6), -1/240*(240*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) + 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log
(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(24*c^2*e^5*x^4 - 30*c^2*d
*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^2*e^3 +
 a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^6
), 1/120*(60*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a
^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (24*c^2*e^5*x^4 - 3
0*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^
2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(
c^2*e^6), -1/120*(120*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e
^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*
x^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)
*x/sqrt(c*x^2 + a)) - (24*c^2*e^5*x^4 - 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e +
40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d
^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^6)]
```

giac [A] time = 0.21, size = 252, normalized size = 0.99

$$\frac{2\left(cd^{6}+ad^{4}e^{2}\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^{2}+a}\right)e+\sqrt{c}d}{\sqrt{-cd^{2}-ae^{2}}}\right)e^{(-6)}}{\sqrt{-cd^{2}-ae^{2}}}+\frac{1}{120}\sqrt{cx^{2}+a}\left(\left(2\left(3\left(4xe^{(-1)}-5de^{(-2)}\right)x+\frac{4\left(5c^{3}d^{2}e^{18}+ac^{2}e^{20}\right)e^{(-21)}}{c^{3}}\right)x-\frac{15\left(4c^{3}d^{3}e^{17}+ac^{2}de^{19}\right)e^{(-21)}}{c^{3}}\right)x+\frac{8\left(15c^{3}d^{4}e^{16}+5ac^{2}d^{2}e^{18}-2a^{2}ce^{20}\right)e^{(-21)}}{c^{3}}\right)+\frac{\left(8c^{\frac{5}{2}}d^{5}+4ac^{\frac{3}{2}}d^{3}e^{2}-a^{2}\sqrt{c}de^{4}\right)e^{(-6)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^{2}+a}\right|\right)}{8c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^6 + a*d^4*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))*e^(-6)/sqrt(-c*d^2 - a*e^2) + 1/120*sqrt(c*x^2 + a)*
((2*(3*(4*x*e^(-1) - 5*d*e^(-2))*x + 4*(5*c^3*d^2*e^18 + a*c^2*e^20)*e^(-21
)/c^3)*x - 15*(4*c^3*d^3*e^17 + a*c^2*d*e^19)*e^(-21)/c^3)*x + 8*(15*c^3*d^
4*e^16 + 5*a*c^2*d^2*e^18 - 2*a^2*c*e^20)*e^(-21)/c^3) + 1/8*(8*c^(5/2)*d^5
 + 4*a*c^(3/2)*d^3*e^2 - a^2*sqrt(c)*d*e^4)*e^(-6)*log(abs(-sqrt(c)*x + sqr
t(c*x^2 + a)))/c^2
```

maple [B] time = 0.02, size = 560, normalized size = 2.20

$$-\frac{a d^{4}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^{2}+2cd^{2}}{e^{2}}+2\sqrt{\frac{ae^{2}+cd^{2}}{e^{2}}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{ae^{2}+cd^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^{2}+cd^{2}}{e^{2}}}\,e^{5}}-\frac{c d^{6}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^{2}+2cd^{2}}{e^{2}}+2\sqrt{\frac{ae^{2}+cd^{2}}{e^{2}}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{ae^{2}+cd^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^{2}+cd^{2}}{e^{2}}}\,e^{7}}+\frac{a^{2}d\ln\left(\sqrt{c}\,x+\sqrt{cx^{2}+a}\right)}{8c^{\frac{3}{2}}e^{2}}-\frac{ad^{3}\ln\left(\sqrt{c}\,x+\sqrt{cx^{2}+a}\right)}{2\sqrt{c}\,e^{4}}-\frac{\sqrt{c}\,d^{5}\ln\left(\frac{-\frac{cd}{e}+\left(x+\frac{d}{e}\right)c}{\sqrt{c}}+\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{ae^{2}+cd^{2}}{e^{2}}}\right)}{e^{6}}+\frac{\sqrt{cx^{2}+a}\,adx}{8ce^{2}}+\frac{\left(cx^{2}+a\right)^{\frac{3}{2}}x^{2}}{5ce}-\frac{\sqrt{cx^{2}+a}\,d^{3}x}{2e^{4}}-\frac{\left(cx^{2}+a\right)^{\frac{3}{2}}dx}{4ce^{2}}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{ae^{2}+cd^{2}}{e^{2}}}\,d^{4}}{e^{5}}-\frac{2\left(cx^{2}+a\right)^{\frac{3}{2}}a}{15c^{2}e}+\frac{\left(cx^{2}+a\right)^{\frac{3}{2}}d^{2}}{3ce^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/5/e*x^2*(c*x^2+a)^(3/2)/c-2/15/e*a/c^2*(c*x^2+a)^(3/2)-1/4*d/e^2*x*(c*x^2
+a)^(3/2)/c+1/8*d/e^2*a/c*x*(c*x^2+a)^(1/2)+1/8*d/e^2*a^2/c^(3/2)*ln(x*c^(1
/2)+(c*x^2+a)^(1/2))+1/3*d^2/e^3*(c*x^2+a)^(3/2)/c-1/2*d^3/e^4*x*(c*x^2+a)^
(1/2)-1/2*d^3/e^4*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+d^4/e^5*((x+d/e)^
2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-d^5/e^6*c^(1/2)*ln((-c*d/e+c*(
x+d/e))/c^(1/2)+((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))-d^4/
e^5/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a
*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2
))/(x+d/e))*a-d^6/e^7/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c
*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^
2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [A] time = 0.68, size = 249, normalized size = 0.98

$$\frac{\left(cx^2+a\right)^{\frac{3}{2}}x^2}{5\,ce}-\frac{\sqrt{cx^2+a}\,d^3x}{2\,e^4}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}dx}{4\,ce^2}+\frac{\sqrt{cx^2+a}\,adx}{8\,ce^2}-\frac{\sqrt{c}\,d^5\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^6}-\frac{ad^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}\,e^4}+\frac{a^2d\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}e^2}+\frac{\sqrt{a+\frac{cd^2}{e^2}}\,d^4\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{e^5}+\frac{\sqrt{cx^2+a}\,d^4}{e^5}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}d^2}{3\,ce^3}-\frac{2\left(cx^2+a\right)^{\frac{3}{2}}a}{15\,c^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2 + a)^(3/2)*x^2/(c*e) - 1/2*sqrt(c*x^2 + a)*d^3*x/e^4 - 1/4*(c*x^
2 + a)^(3/2)*d*x/(c*e^2) + 1/8*sqrt(c*x^2 + a)*a*d*x/(c*e^2) - sqrt(c)*d^5*
arcsinh(c*x/sqrt(a*c))/e^6 - 1/2*a*d^3*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e^4)
 + 1/8*a^2*d*arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e^2) + sqrt(a + c*d^2/e^2)*d^4
*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/e^5
 + sqrt(c*x^2 + a)*d^4/e^5 + 1/3*(c*x^2 + a)^(3/2)*d^2/(c*e^3) - 2/15*(c*x^
2 + a)^(3/2)*a/(c^2*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^4*(a + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**4*sqrt(a + c*x**2)/(d + e*x), x)
```

3.228 $\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=211

$$\frac{\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5}+\frac{d^3\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^5}-\frac{\sqrt{a+cx^2}\,\left(8cd^3-ex\left(4c\right.\right.}{8ce^4}$$

Rubi [A] time = 0.39, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1654, 815, 844, 217, 206, 725}

$$\frac{\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5}-\frac{\sqrt{a+cx^2}\,\left(8cd^3-ex\left(4cd^2-ae^2\right)\right)}{8ce^4}+\frac{d^3\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^5}-\frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2}+\frac{\left(a+cx^2\right)^{3/2}(d+ex)}{4ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -((8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(8*c*e^4) - (7*d*(a + c*x^2)^(3/2))/(12*c*e^2) + ((d + e*x)*(a + c*x^2)^(3/2))/(4*c*e^2) + ((8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3/2)*e^5) + (d^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\sqrt{a+cx^2}\left(-ade^2-e\left(3cd^2+ae^2\right)x-7cde^2x^2\right)}{d+ex}\,dx}{4ce^3}$$

$$= -\frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\left(-3acde^4+3ce^3\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{d+ex}\,dx}{12c^2e^5}$$

$$= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \int \frac{-3ac^2de^4(4\cdots}{\cdots}$$

$$= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} - \frac{\left(d^3\left(cd^2+\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\left(d^3\left(cd^2+\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\left(8c^2d^4+4\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.40, size = 225, normalized size = 1.07

$$\frac{24c^{3/2}d^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+24cd^3\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)+e\sqrt{a+cx^2}\left(ae^2(3ex-8d)+c\left(-24d^3+12d^2ex-8de^2x^2+6e^3x^3\right)\right)}{24ce^5}-\frac{\sqrt{a}\sqrt{a+cx^2}\left(ae^2-4cd^2\right)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8c^{3/2}e^3\sqrt{\frac{cx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -1/8*(Sqrt[a]*(-4*c*d^2 + a*e^2)*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a
]])/(c^(3/2)*e^3*Sqrt[1 + (c*x^2)/a]) + (e*Sqrt[a + c*x^2]*(a*e^2*(-8*d + 3
*e*x) + c*(-24*d^3 + 12*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3)) + 24*c^(3/2)*d^
4*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + 24*c*d^3*Sqrt[c*d^2 + a*e^2]*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(24*c*e^5)
```

IntegrateAlgebraic [A] time = 0.67, size = 236, normalized size = 1.12

$$\frac{\left(a^2e^4-4acd^2e^2-8c^2d^4\right)\log\left(\sqrt{a+cx^2}-\sqrt{c}x\right)}{8c^{3/2}e^5}-\frac{2d^3\sqrt{-ae^2-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2-cd^2}}\right)}{e^5}+\frac{\sqrt{a+cx^2}\left(-8ade^2+3ae^3x-24cd^3+12cd^2ex-8cde^2x^2+6ce^3x^3\right)}{24ce^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-24*c*d^3 - 8*a*d*e^2 + 12*c*d^2*e*x + 3*a*e^3*x - 8*c*d*
e^2*x^2 + 6*c*e^3*x^3))/(24*c*e^4) - (2*d^3*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(
```

```
Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] -
(e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/e^5 + ((-8*c^2*d^4 - 4*a*c*d^2
*e^2 + a^2*e^4)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*c^(3/2)*e^5)
```

fricas [A] time = 7.04, size = 963, normalized size = 4.56

$$\left[\frac{24\,\sqrt{cd^2+ae^2}\,c^2d^3\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right)-3\left(8c^2d^4+4acd^2e^2-a^2e^4\right)\sqrt{c}\log\left(-2cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}x-a\right)+2\left(6c^2e^4x^3-8c^2de^3x^2-24c^2d^3e-8acde^3+3\left(4c^2d^2e^2+ace^4\right)x\right)\sqrt{cx^2+a}}{48\,c^2e^5},\ \frac{48\,\sqrt{-cd^2-ae^2}\,c^2d^3\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-3\left(8c^2d^4+4acd^2e^2-a^2e^4\right)\sqrt{c}\log\left(-2cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}x-a\right)+2\left(6c^2e^4x^3-8c^2de^3x^2-24c^2d^3e-8acde^3+3\left(4c^2d^2e^2+ace^4\right)x\right)\sqrt{cx^2+a}}{48\,c^2e^5},\ \frac{12\,\sqrt{cd^2+ae^2}\,c^2d^3\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right)-3\left(8c^2d^4+4acd^2e^2-a^2e^4\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)+\left(6c^2e^4x^3-8c^2de^3x^2-24c^2d^3e-8acde^3+3\left(4c^2d^2e^2+ace^4\right)x\right)\sqrt{cx^2+a}}{24\,c^2e^5},\ \frac{24\,\sqrt{-cd^2-ae^2}\,c^2d^3\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-3\left(8c^2d^4+4acd^2e^2-a^2e^4\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)+\left(6c^2e^4x^3-8c^2de^3x^2-24c^2d^3e-8acde^3+3\left(4c^2d^2e^2+ace^4\right)x\right)\sqrt{cx^2+a}}{24\,c^2e^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^
2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e
^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*e^4*
x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c
*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/48*(48*sqrt(-c*d^2 - a*e^2)*c^2*d^3*
arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^
2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqr
t(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*e^4*x^3 - 8
*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c*e^4)*x
)*sqrt(c*x^2 + a))/(c^2*e^5), 1/24*(12*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a
*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 +
 a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(8*c^
2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)
) + (6*c^2*e^4*x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^
2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/24*(24*sqrt(-c*d^2 -
a*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a
*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*c^2*e^4*x^3 -
8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c*e^4)*
x)*sqrt(c*x^2 + a))/(c^2*e^5)]
```

giac [A] time = 0.22, size = 201, normalized size = 0.95

$$-\frac{2\left(cd^5+ad^3e^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-5)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(3xe^{(-1)}-4de^{(-2)}\right)x+\frac{3\left(4c^2d^2e^{12}+ace^{14}\right)e^{(-15)}}{c^2}\right)x-\frac{8\left(3c^2d^3e^{11}+acde^{13}\right)e^{(-15)}}{c^2}\right)-\frac{\left(8c^2d^4+4acd^2e^2-a^2e^4\right)e^{(-5)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^5 + a*d^3*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 - a*e^2))*e^(-5)/sqrt(-c*d^2 - a*e^2) + 1/24*sqrt(c*x^2 + a)*
((2*(3*x*e^(-1) - 4*d*e^(-2))*x + 3*(4*c^2*d^2*e^12 + a*c*e^14)*e^(-15)/c^2
)*x - 8*(3*c^2*d^3*e^11 + a*c*d*e^13)*e^(-15)/c^2) - 1/8*(8*c^2*d^4 + 4*a*c
*d^2*e^2 - a^2*e^4)*e^(-5)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

maple [B] time = 0.01, size = 515, normalized size = 2.44

$$\frac{a\,d^3\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,e^4}+\frac{c\,d^5\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,e^6}-\frac{a^2\ln\left(\sqrt{c}\,x+\sqrt{cx^2+a}\right)}{8c^{\frac{3}{2}}e}+\frac{a\,d^2\ln\left(\sqrt{c}\,x+\sqrt{cx^2+a}\right)}{2\sqrt{c}\,e^3}+\frac{\sqrt{c}\,d^4\ln\left(\frac{-\frac{cd}{e}+\left(x+\frac{d}{e}\right)c}{\sqrt{c}}+\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}\right)}{e^5}-\frac{\sqrt{cx^2+a}\,ax}{8ce}+\frac{\sqrt{cx^2+a}\,d^2x}{2e^3}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}x}{4ce}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}\,d^3}{e^4}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}d}{3ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/4/e*x*(c*x^2+a)^(3/2)/c-1/8/e*a/c*x*(c*x^2+a)^(1/2)-1/8/e*a^2/c^(3/2)*ln(
c^(1/2)*x+(c*x^2+a)^(1/2))-1/3*d*(c*x^2+a)^(3/2)/c/e^2+1/2*d^2/e^3*x*(c*x^2
+a)^(1/2)+1/2*d^2/e^3*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-d^3/e^4*(-2*(
x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+d^4/e^5*c^(1/2)*ln((-c*d/
e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)
```

```
)+d^3/e^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^
2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e
^2)^(1/2))/(x+d/e))*a+d^5/e^6/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/
e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)
^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [A] time = 0.58, size = 207, normalized size = 0.98

$$\frac{\sqrt{cx^2+a}\,d^2x}{2\,e^3}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}x}{4\,ce}-\frac{\sqrt{cx^2+a}\,ax}{8\,ce}+\frac{\sqrt{c}\,d^4\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^5}+\frac{ad^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}\,e^3}-\frac{a^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}e}-\frac{\sqrt{a+\frac{cd^2}{e^2}}\,d^3\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{e^4}-\frac{\sqrt{cx^2+a}\,d^3}{e^4}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}d}{3\,ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*d^2*x/e^3 + 1/4*(c*x^2 + a)^(3/2)*x/(c*e) - 1/8*sqrt(c*
x^2 + a)*a*x/(c*e) + sqrt(c)*d^4*arcsinh(c*x/sqrt(a*c))/e^5 + 1/2*a*d^2*arc
sinh(c*x/sqrt(a*c))/(sqrt(c)*e^3) - 1/8*a^2*arcsinh(c*x/sqrt(a*c))/(c^(3/2)
*e) - sqrt(a + c*d^2/e^2)*d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/
(sqrt(a*c)*abs(e*x + d)))/e^4 - sqrt(c*x^2 + a)*d^3/e^4 - 1/3*(c*x^2 + a)^(
3/2)*d/(c*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^3*(a + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt(a + c*x**2)/(d + e*x), x)
```

3.229 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=153

$$-\frac{d^2\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{d\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^4}+\frac{d\sqrt{a+cx^2}(2d-ex)}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}$$

Rubi [A] time = 0.21, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1654, 12, 815, 844, 217, 206, 725}

$$-\frac{d^2\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{d\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^4}+\frac{d\sqrt{a+cx^2}(2d-ex)}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (d*(2*d - e*x)*Sqrt[a + c*x^2])/(2*e^3) + (a + c*x^2)^(3/2)/(3*c*e) - (d*(2
*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^4) - (d^
2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c
*x^2])])/e^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx &= \frac{\left(a+cx^2\right)^{3/2}}{3ce}+\frac{\int -\frac{3cdex\sqrt{a+cx^2}}{d+ex}\,dx}{3ce^2}\\
&= \frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx}{e}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\int \frac{-acde+c\left(2cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}+\frac{\left(d^2\left(cd^2+ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^4}-\frac{\left(d\left(2cd^2+ae^2\right)\right)}{2e^4}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{\left(d^2\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^4}-\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\left(2cd^2+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^4}-\frac{d^2\sqrt{cd^2+ae^2}\,\tanh}{e}
\end{aligned}
$$

Mathematica [A] time = 0.32, size = 193, normalized size = 1.26

$$
\frac{-6c^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)+e\sqrt{a+cx^2}\left(2ae^2+c\left(6d^2-3dex+2e^2x^2\right)\right)-6cd^2\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)-\frac{3\sqrt{a}\sqrt{c}\,de^2\sqrt{a+cx^2}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{cx^2}{a}+1}}}{6ce^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(2*a*e^2 + c*(6*d^2 - 3*d*e*x + 2*e^2*x^2)) - (3*Sqrt[a]
*Sqrt[c]*d*e^2*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^
2)/a] - 6*c^(3/2)*d^3*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - 6*c*d^2*Sqrt[c
*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])
/(6*c*e^4)
```

IntegrateAlgebraic [A] time = 0.59, size = 203, normalized size = 1.33

$$
\frac{\left(ade^2+2cd^3\right)\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\sqrt{c}\,e^4}+\frac{2d^2\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{e^4}+\frac{\sqrt{a+cx^2}\left(2ae^2+6cd^2-3cdex+2ce^2x^2\right)}{6ce^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[a + c*x^2]*(6*c*d^2 + 2*a*e^2 - 3*c*d*e*x + 2*c*e^2*x^2))/(6*c*e^3) +
 (2*d^2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] +
(Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) -
a*e^2]])/e^4 + ((2*c*d^3 + a*d*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2
*Sqrt[c]*e^4)
```

fricas [A] time = 0.69, size = 776, normalized size = 5.07

$$\left[\frac{6 \sqrt{c d^{2} + a e^{2}} c d^{2} \log\left(\frac{2 a c d e x - a c d^{2} - 2 a^{2} e^{2} - (2 c^{2} d^{2} + a c e^{2}) x^{2} - 2 \sqrt{c d^{2} + a e^{2}} (c d x - a e) \sqrt{c x^{2} + a}}{e^{2} x^{2} + 2 d e x + d^{2}}\right) + 3 \left(2 c d^{3} + a d e^{2}\right) \sqrt{c} \log\left(-2 c x^{2} + 2 \sqrt{c x^{2} + a} \sqrt{c} x - a\right) + 2 \left(2 c e^{3} x^{2} - 3 c d e^{2} x + 6 c d^{2} e + 2 a e^{3}\right) \sqrt{c x^{2} + a}}{12 c e^{4}}, -\frac{12 \sqrt{-c d^{2} - a e^{2}} c d^{2} \arctan\left(\frac{\sqrt{-c d^{2} - a e^{2}} (c d x - a e) \sqrt{c x^{2} + a}}{a c d^{2} + a^{2} e^{2} + (c^{2} d^{2} + a c e^{2}) x^{2}}\right) - 3 \left(2 c d^{3} + a d e^{2}\right) \sqrt{c} \log\left(-2 c x^{2} + 2 \sqrt{c x^{2} + a} \sqrt{c} x - a\right) - 2 \left(2 c e^{3} x^{2} - 3 c d e^{2} x + 6 c d^{2} e + 2 a e^{3}\right) \sqrt{c x^{2} + a}}{12 c e^{4}}, \frac{3 \sqrt{c d^{2} + a e^{2}} c d^{2} \log\left(\frac{2 a c d e x - a c d^{2} - 2 a^{2} e^{2} - (2 c^{2} d^{2} + a c e^{2}) x^{2} - 2 \sqrt{c d^{2} + a e^{2}} (c d x - a e) \sqrt{c x^{2} + a}}{e^{2} x^{2} + 2 d e x + d^{2}}\right) + 3 \left(2 c d^{3} + a d e^{2}\right) \sqrt{-c} \arctan\left(\frac{\sqrt{-c} x}{\sqrt{c x^{2} + a}}\right) + \left(2 c e^{3} x^{2} - 3 c d e^{2} x + 6 c d^{2} e + 2 a e^{3}\right) \sqrt{c x^{2} + a}}{6 c e^{4}}, -\frac{6 \sqrt{-c d^{2} - a e^{2}} c d^{2} \arctan\left(\frac{\sqrt{-c d^{2} - a e^{2}} (c d x - a e) \sqrt{c x^{2} + a}}{a c d^{2} + a^{2} e^{2} + (c^{2} d^{2} + a c e^{2}) x^{2}}\right) - 3 \left(2 c d^{3} + a d e^{2}\right) \sqrt{-c} \arctan\left(\frac{\sqrt{-c} x}{\sqrt{c x^{2} + a}}\right) - \left(2 c e^{3} x^{2} - 3 c d e^{2} x + 6 c d^{2} e + 2 a e^{3}\right) \sqrt{c x^{2} + a}}{6 c e^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
 (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c
*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), -1/12*(12*sqrt(-c*d^2 - a*e^2)*
c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2*
c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6
*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), 1/6*(3*sqrt(c*d^2 + a*e^2)*c*
d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*
sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2
)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2
*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), -
1/6*(6*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^
3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c*e^3*x^2 - 3
*c*d*e^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4)]
```

giac [A] time = 0.21, size = 157, normalized size = 1.03

$$\frac{2 \left(c d^{4} + a d^{2} e^{2}\right) \arctan\left(-\frac{\left(\sqrt{c} x - \sqrt{c x^{2} + a}\right) e + \sqrt{c} d}{\sqrt{-c d^{2} - a e^{2}}}\right) e^{(-4)}}{\sqrt{-c d^{2} - a e^{2}}} + \frac{\left(2 c d^{3} + a d e^{2}\right) e^{(-4)} \log\left(\left|-\sqrt{c} x + \sqrt{c x^{2} + a}\right|\right)}{2 \sqrt{c}} + \frac{1}{6} \sqrt{c x^{2} + a} \left(\left(2 x e^{(-1)} - 3 d e^{(-2)}\right) x + \frac{2 \left(3 c d^{2} e^{7} + a e^{9}\right) e^{(-10)}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^4 + a*d^2*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))*e^(-4)/sqrt(-c*d^2 - a*e^2) + 1/2*(2*c*d^3 + a*d*e^2
)*e^(-4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/6*sqrt(c*x^2 +
a)*((2*x*e^(-1) - 3*d*e^(-2))*x + 2*(3*c*d^2*e^7 + a*e^9)*e^(-10)/c)
```

maple [B] time = 0.01, size = 448, normalized size = 2.93

$$-\frac{a d^{2} \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right) c d}{e}+\frac{2 a e^{2}+2 c d^{2}}{e^{2}}+2 \sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}} \sqrt{-\frac{2\left(x+\frac{d}{e}\right) c d}{e}+\left(x+\frac{d}{e}\right)^{2} c+\frac{a e^{2}+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\, e^{3}}-\frac{c d^{4} \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right) c d}{e}+\frac{2 a e^{2}+2 c d^{2}}{e^{2}}+2 \sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}} \sqrt{-\frac{2\left(x+\frac{d}{e}\right) c d}{e}+\left(x+\frac{d}{e}\right)^{2} c+\frac{a e^{2}+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\, e^{5}}-\frac{a d \ln\left(\sqrt{c}\, x+\sqrt{c x^{2}+a}\right)}{2 \sqrt{c}\, e^{2}}-\frac{\sqrt{c}\, d^{3} \ln\left(\frac{-\frac{c d}{e}+\left(x+\frac{d}{e}\right) c}{\sqrt{c}}+\sqrt{-\frac{2\left(x+\frac{d}{e}\right) c d}{e}+\left(x+\frac{d}{e}\right)^{2} c+\frac{a e^{2}+c d^{2}}{e^{2}}}\right)}{e^{4}}-\frac{\sqrt{c x^{2}+a}\, d x}{2 e^{2}}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right) c d}{e}+\left(x+\frac{d}{e}\right)^{2} c+\frac{a e^{2}+c d^{2}}{e^{2}}}\, d^{2}}{e^{3}}+\frac{\left(c x^{2}+a\right)^{\frac{3}{2}}}{3 c e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/3*(c*x^2+a)^(3/2)/c/e-1/2*d/e^2*x*(c*x^2+a)^(1/2)-1/2/e^2*d*a/c^(1/2)*ln(
c^(1/2)*x+(c*x^2+a)^(1/2))+d^2/e^3*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d
^2)/e^2)^(1/2)-d^3/e^4*c^(1/2)*ln((-c*d/e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*
```

```
d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))-d^2/e^3/((a*e^2+c*d^2)/e^2)^(1/2)
*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(
x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*a-d^4/e^5/((a*e
^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d
^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d
/e))*c
```

maxima [A] time = 0.52, size = 144, normalized size = 0.94

$$-\frac{\sqrt{cx^2+a}\,dx}{2\,e^2}-\frac{\sqrt{c}\,d^3\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^4}-\frac{ad\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}\,e^2}+\frac{\sqrt{a+\frac{cd^2}{e^2}}\,d^2\,\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{e^3}+\frac{\sqrt{cx^2+a}\,d^2}{e^3}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}}{3\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*d*x/e^2 - sqrt(c)*d^3*arcsinh(c*x/sqrt(a*c))/e^4 - 1/2
*a*d*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e^2) + sqrt(a + c*d^2/e^2)*d^2*arcsinh
(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/e^3 + sqrt(
c*x^2 + a)*d^2/e^3 + 1/3*(c*x^2 + a)^(3/2)/(c*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^2*(a + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x), x)
```

3.230 $\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=127

$$\frac{\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^3}+\frac{d\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^3}-\frac{\sqrt{a+cx^2}\,(2d-ex)}{2e^2}$$

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {815, 844, 217, 206, 725}

$$\frac{\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^3}+\frac{d\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^3}-\frac{\sqrt{a+cx^2}\,(2d-ex)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -((2*d - e*x)*Sqrt[a + c*x^2])/(2*e^2) + ((2*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^3) + (d*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*
e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx &= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{\int \frac{-acde+c\left(2cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^2}\\
&= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} - \frac{\left(d\left(cd^2+ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3} + \frac{\left(2cd^2+ae^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2e^3}\\
&= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{\left(d\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^3} + \frac{\left(2cd^2+ae^2\right)\text{Sub}}{}\\
&= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{\left(2cd^2+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^3} + \frac{d\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right.}{e^3}
\end{aligned}
$$

Mathematica [A] time = 0.28, size = 175, normalized size = 1.38

$$
\frac{\frac{a^{3/2}e^2\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{c}\,\sqrt{a+cx^2}} + 2d\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right) + 2\sqrt{c}\,d^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right) - 2de\sqrt{a+cx^2} + e^2x\sqrt{a+cx^2}}{2e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (-2*d*e*Sqrt[a + c*x^2] + e^2*x*Sqrt[a + c*x^2] + (a^(3/2)*e^2*Sqrt[1 + (c*
x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[a + c*x^2]) + 2*Sqrt[c]
*d^2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + 2*d*Sqrt[c*d^2 + a*e^2]*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]])/(2*e^3)
```

IntegrateAlgebraic [A] time = 0.47, size = 177, normalized size = 1.39

$$
\frac{\left(-ae^2-2cd^2\right)\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2\sqrt{c}\,e^3} - \frac{2d\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{e^3} + \frac{\sqrt{a+cx^2}\,(ex-2d)}{2e^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] ((-2*d + e*x)*Sqrt[a + c*x^2])/(2*e^2) - (2*d*Sqrt[-(c*d^2) - a*e^2]*ArcTan
[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2]
- (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/e^3 + ((-2*c*d^2 - a*e^2)*Lo
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^3)
```

fricas [A] time = 0.68, size = 684, normalized size = 5.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + (2*c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c
*e^3), 1/4*(4*sqrt(-c*d^2 - a*e^2)*c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
 a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (2*c
*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(
```

```
c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3), 1/2*(sqrt(c*d^2 + a*e^2)*c*d*l
og((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) -
(2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c*e^2*x -
2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3), 1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d*arctan(s
qrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2
*d^2 + a*c*e^2)*x^2)) - (2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3)]
```

giac [A] time = 0.20, size = 135, normalized size = 1.06

$$-\frac{2\left(cd^{3}+ade^{2}\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^{2}+a}\right)e+\sqrt{c}d}{\sqrt{-cd^{2}-ae^{2}}}\right)e^{(-3)}}{\sqrt{-cd^{2}-ae^{2}}}-\frac{\left(2c^{\frac{3}{2}}d^{2}+a\sqrt{c}e^{2}\right)e^{(-3)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^{2}+a}\right|\right)}{2c}+\frac{1}{2}\sqrt{cx^{2}+a}\left(xe^{(-1)}-2de^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^3 + a*d*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/
sqrt(-c*d^2 - a*e^2))*e^(-3)/sqrt(-c*d^2 - a*e^2) - 1/2*(2*c^(3/2)*d^2 + a*
sqrt(c)*e^2)*e^(-3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c + 1/2*sqrt(c*x
^2 + a)*(x*e^(-1) - 2*d*e^(-2))
```

maple [B] time = 0.01, size = 423, normalized size = 3.33

$$\frac{ad\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^{2}+2cd^{2}}{e^{2}}+2\sqrt{\frac{ae^{2}+cd^{2}}{e^{2}}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{ae^{2}+cd^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^{2}+cd^{2}}{e^{2}}}\,e^{2}}+\frac{cd^{3}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^{2}+2cd^{2}}{e^{2}}+2\sqrt{\frac{ae^{2}+cd^{2}}{e^{2}}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{ae^{2}+cd^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^{2}+cd^{2}}{e^{2}}}\,e^{4}}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{cx^{2}+a}\right)}{2\sqrt{c}\,e}+\frac{\sqrt{c}\,d^{2}\ln\left(\frac{-\frac{cd}{e}+\left(x+\frac{d}{e}\right)c}{\sqrt{c}}+\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{ae^{2}+cd^{2}}{e^{2}}}\right)}{e^{3}}+\frac{\sqrt{cx^{2}+a}\,x}{2e}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{ae^{2}+cd^{2}}{e^{2}}}\,d}{e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*x*(c*x^2+a)^(1/2)+1/2/e*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-d/e^2
*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+d^2/e^3*c^(1/2)*ln(
(-c*d/e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)
^(1/2))+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2
)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^
2)/e^2)^(1/2))/(x+d/e))*a+d^3/e^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*
c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+
d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [A] time = 0.51, size = 122, normalized size = 0.96

$$\frac{\sqrt{cx^{2}+a}\,x}{2\,e}+\frac{\sqrt{c}\,d^{2}\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^{3}}+\frac{a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}\,e}-\frac{\sqrt{a+\frac{cd^{2}}{e^{2}}}\,d\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{e^{2}}-\frac{\sqrt{cx^{2}+a}\,d}{e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x/e + sqrt(c)*d^2*arcsinh(c*x/sqrt(a*c))/e^3 + 1/2*a*ar
csinh(c*x/sqrt(a*c))/(sqrt(c)*e) - sqrt(a + c*d^2/e^2)*d*arcsinh(c*d*x/(sqr
t(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/e^2 - sqrt(c*x^2 + a)*
d/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c\,x^{2}+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x*(a + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x), x)
```

3.231 $\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2}-\frac{\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{\sqrt{a+cx^2}}{e}$$

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {735, 844, 217, 206, 725}

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2}-\frac{\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{\sqrt{a+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^2]/e - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^2 -
(Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])])/e^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \frac{\sqrt{a+cx^2}}{e} + \frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}$$

$$= \frac{\sqrt{a+cx^2}}{e} + \left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx - \frac{(cd)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2}$$

$$= \frac{\sqrt{a+cx^2}}{e} + \left(-a-\frac{cd^2}{e^2}\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right) - \frac{(cd)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x\right.}{e^2}$$

$$= \frac{\sqrt{a+cx^2}}{e} - \frac{\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e^2} - \frac{\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{e^2}$$

Mathematica [A] time = 0.02, size = 99, normalized size = 0.96

$$\frac{-\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right) - \sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right) + e\sqrt{a+cx^2}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2] - Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[
c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]
)/e^2
```

IntegrateAlgebraic [A] time = 0.01, size = 151, normalized size = 1.47

$$\frac{2\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{e^2} + \frac{\sqrt{c}\,d\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{e^2} + \frac{\sqrt{a+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^2]/e + (2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d
^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c*x^2])/S
qrt[-(c*d^2) - a*e^2]])/e^2 + (Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
])/e^2
```

fricas [A] time = 0.51, size = 574, normalized size = 5.57

$$\left[\frac{\sqrt{c}\,d\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\,\sqrt{cx^2+a}\,e+\sqrt{cd^2+ae^2}\,\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)}{2\,e^2}, \frac{2\,\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+2\,\sqrt{cx^2+a}\,e+\sqrt{cd^2+ae^2}\,\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)}{2\,e^2}, \frac{\sqrt{c}\,d\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\,\sqrt{cx^2+a}\,e-2\,\sqrt{-cd^2-ae^2}\,\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)}{2\,e^2}, \frac{\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+\sqrt{cx^2+a}\,e-\sqrt{-cd^2-ae^2}\,\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)}{e^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*
x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (
2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/s
qrt(c*x^2 + a)) + 2*sqrt(c*x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*
x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(sqrt(
c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*x^2 + a)*e
- 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x
```

```
^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2, (sqrt(-c)*d*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + sqrt(c*x^2 + a)*e - sqrt(-c*d^2 - a*e^2)
*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e
^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2]
```

giac [A] time = 0.19, size = 109, normalized size = 1.06

$$\sqrt{c}\,de^{(-2)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)+\frac{2\left(cd^2+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-2)}}{\sqrt{-cd^2-ae^2}}+\sqrt{cx^2+a}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] sqrt(c)*d*e^(-2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a))) + 2*(c*d^2 + a*e^2)
*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2)
)*e^(-2)/sqrt(-c*d^2 - a*e^2) + sqrt(c*x^2 + a)*e^(-1)
```

maple [B] time = 0.01, size = 381, normalized size = 3.70

$$-\frac{a\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,e}-\frac{cd^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,e^3}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+\left(x+\frac{d}{e}\right)c}{\sqrt{c}}+\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}\right)}{e^2}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-1/e^2*c^(1/2)*d*
ln((-c*d/e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e
^2)^(1/2))-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^
2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d
^2)/e^2)^(1/2))/(x+d/e))*a-1/e^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c
*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d
/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^2
```

maxima [A] time = 0.48, size = 84, normalized size = 0.82

$$-\frac{\sqrt{c}\,d\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^2}+\frac{\sqrt{a+\frac{cd^2}{e^2}}\,\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{e}+\frac{\sqrt{cx^2+a}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/e^2 + sqrt(a + c*d^2/e^2)*arcsinh(c*d*x/(
sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/e + sqrt(c*x^2 + a)
/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x), x)
```

3.232 $\int \frac{\sqrt{a+cx^2}}{x(d+ex)} dx$

Optimal. Leaf size=116

$$\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}$$

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {896, 266, 63, 208, 844, 217, 206, 725}

$$\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e + (Sqrt[c*d^2 + a*e^2]*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*e) - (Sqrt[a]
*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 896

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e
*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx &= -\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{d}+\frac{a\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d}\\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d}+\frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e}-\left(\frac{cd}{e}+\frac{ae}{d}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx\\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd}+\frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{e}-\left(-\frac{cd}{e}-\frac{ae}{d}\right)\text{Subst}\left(\int \right.\\
&= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e}+\frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 113, normalized size = 0.97

$$\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)+\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)-\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[c*d^2 + a*e^2]*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])] - Sqrt[a]*e*ArcTanh
[Sqrt[a + c*x^2]/Sqrt[a]])/(d*e)
```

IntegrateAlgebraic [A] time = 0.40, size = 181, normalized size = 1.56

$$-\frac{2\sqrt{-ae^2-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{de}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{c}\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

```
[Out] (-2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqr
t[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^
```

```
2]])/(d*e) + (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[
a]])/d - (Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/e
```

fricas [A] time = 1.15, size = 1316, normalized size = 11.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(a)*e
*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(c*d^2 + a*e^2)*
log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt
(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/
(d*e), -1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*e*lo
g(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - sqrt(c*d^2 + a*e^2)*log
((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*
d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*
e), 1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(a
)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(-c*d^2 - a
*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/2*(2*sqrt(-c)*d*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a)
+ 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e),
1/2*(2*sqrt(-a)*e*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + sqrt(c)*d*log(-2*c*x^2
 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sq
rt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - 2*sqrt(-a)*e*arctan(sqrt(-a)/
sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-a)*e*arctan(sqrt
(-a)/sqrt(c*x^2 + a)) + sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) + 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e), -(sqr
t(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(-a)*e*arctan(sqrt(-a)/sqr
t(c*x^2 + a)) - sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 420, normalized size = 3.62

$$\frac{a \ln \left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\, d}+\frac{cd \ln \left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\, e^2}-\frac{\sqrt{a}\, \ln \left(\frac{2a+2\sqrt{c x^2+a}\,\sqrt{a}}{x}\right)}{d}+\frac{\sqrt{c}\, \ln \left(\frac{-\frac{cd}{e}+\left(x+\frac{d}{e}\right)c}{\sqrt{c}}+\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}\right)}{e}+\frac{\sqrt{c x^2+a}}{d}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x/(e*x+d),x)
```

```
[Out] -1/d*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+1/d*(c*x^2+a)^(1/2)-1/d*
(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+c^(1/2)/e*ln((-c*d/e
```

```
+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))
+1/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*(
(a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(
1/2))/(x+d/e))*a+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*
e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a
*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [A] time = 0.50, size = 103, normalized size = 0.89

$$\frac{e\left(\frac{\sqrt{c}\,d\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{e^2}-\frac{\sqrt{a+\frac{cd^2}{e^2}}\,\operatorname{arsinh}\left(\frac{2\,cdx}{\sqrt{ac}\,|2\,ex+2\,d|}-\frac{2\,ae}{\sqrt{ac}\,|2\,ex+2\,d|}\right)}{e}-\frac{\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ac}\,|x|}\right)}{e}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] e*(sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/e^2 - sqrt(a + c*d^2/e^2)*arcsinh(2*c*d
*x/(sqrt(a*c)*abs(2*e*x + 2*d)) - 2*a*e/(sqrt(a*c)*abs(2*e*x + 2*d)))/e - s
qrt(a)*arcsinh(a/(sqrt(a*c)*abs(x)))/e)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x)), x)
```

3.233 $\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

Rubi [A] time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {961, 277, 217, 206, 266, 50, 63, 208, 735, 844, 725}

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^2 + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx &= \int \left(\frac{\sqrt{a+cx^2}}{dx^2} - \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e^2\sqrt{a+cx^2}}{d^2(d+ex)}\right) dx \\
&= \frac{\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^2} \\
&= \frac{e\sqrt{a+cx^2}}{d^2} - \frac{\sqrt{a+cx^2}}{dx} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d} - \frac{e\,\mathrm{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{e\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,d}{d^2} \\
&= -\frac{\sqrt{a+cx^2}}{dx} - \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x\right.}{2d^2} \\
&= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d} - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\right.}{cd} \\
&= -\frac{\sqrt{a+cx^2}}{dx} - \frac{\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{d^2} + \frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] time = 0.24, size = 178, normalized size = 1.70

$$
\frac{-\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right) - \frac{d\sqrt{a+cx^2}}{x} + \frac{\sqrt{a}\,\sqrt{c}\,d\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a+cx^2}} - \sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right) + \sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] (-((d*Sqrt[a + c*x^2])/x) + (Sqrt[a]*Sqrt[c]*d*Sqrt[1 + (c*x^2)/a]*ArcSinh[
(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a + c*x^2] - Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[
a + c*x^2]] - Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2])] + Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

IntegrateAlgebraic [A] time = 0.45, size = 167, normalized size = 1.59

$$
\frac{2\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{d^2} - \frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}} - \frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} - \frac{\sqrt{a+cx^2}}{dx}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) + (2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqr
t[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c*
x^2])/Sqrt[-(c*d^2) - a*e^2]])/d^2 - (2*Sqrt[a]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[
a] - Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

fricas [A] time = 0.48, size = 599, normalized size = 5.70

$$
\left[\frac{\sqrt{a}\,ex\log\left(-\frac{cx^2+2\sqrt{cx^2+a}\,\sqrt{a}+2a}{x^2}\right) + \sqrt{cd^2+ae^2}\,x\log\left(\frac{2acdex-acd^2-2a^2e^2-\left(2c^2d^2+ace^2\right)x^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right) - 2\sqrt{cx^2+a}\,d}{2d^2x}, \frac{\sqrt{a}\,ex\log\left(-\frac{cx^2+2\sqrt{cx^2+a}\,\sqrt{a}+2a}{x^2}\right) - 2\sqrt{-cd^2-ae^2}\,x\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right) - 2\sqrt{cx^2+a}\,d}{2d^2x}, -\frac{2\sqrt{-a}\,ex\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right) - \sqrt{cd^2+ae^2}\,x\log\left(\frac{2acdex-acd^2-2a^2e^2-\left(2c^2d^2+ace^2\right)x^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right) + 2\sqrt{cx^2+a}\,d}{2d^2x}, -\frac{\sqrt{-a}\,ex\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right) + \sqrt{-cd^2-ae^2}\,x\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right) + \sqrt{cx^2+a}\,d}{d^2x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*e*x*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqr
t(c*d^2 + a*e^2)*x*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*
```

```
c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2
+ 2*d*e*x + d^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), 1/2*(sqrt(a)*e*x*log(-(c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*x*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), -1/2*(2*sqrt(-a)*
e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*x*log((2*a*c*d*e
*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2
)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(c*x^2
+ a)*d)/(d^2*x), -(sqrt(-a)*e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + sqrt(-c*
d^2 - a*e^2)*x*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a
*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + sqrt(c*x^2 + a)*d)/(d^2*x)]
```

giac [A] time = 0.22, size = 145, normalized size = 1.38

$$-\frac{2\,a\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,d^2}+\frac{2\,a\sqrt{c}}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2-a\right)d}+\frac{2\left(cd^2+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*a*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))*e/(sqrt(-a)*d^2) + 2*a
*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*d) + 2*(c*d^2 + a*e^2)*arct
an(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/(sq
rt(-c*d^2 - a*e^2)*d^2)
```

maple [B] time = 0.01, size = 486, normalized size = 4.63

$$-\frac{ae\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,d^2}-\frac{c\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,e}+\frac{\sqrt{a}\,e\ln\left(\frac{2a+2\sqrt{cx^2+a}\,\sqrt{a}}{x}\right)}{d^2}+\frac{\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{cx^2+a}\right)}{d}-\frac{\sqrt{c}\,\ln\left(\frac{-\frac{cd}{e}+\left(x+\frac{d}{e}\right)c}{\sqrt{c}}+\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}\right)}{d}+\frac{\sqrt{cx^2+a}\,cx}{ad}-\frac{\sqrt{cx^2+a}\,e}{d^2}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}\,e}{d^2}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^2/(e*x+d),x)
```

```
[Out] -1/d/a/x*(c*x^2+a)^(3/2)+1/d*c/a*x*(c*x^2+a)^(1/2)+1/d*c^(1/2)*ln(c^(1/2)*x
+(c*x^2+a)^(1/2))+e/d^2*a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-e/d^2
*(c*x^2+a)^(1/2)+e/d^2*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/
2)-1/d*c^(1/2)*ln((-c*d/e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+
(a*e^2+c*d^2)/e^2)^(1/2))-e/d^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*
d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/
e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*a-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+
d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x^2\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x)), x)
```

3.234 $\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{a}\,e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{e\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^3}-\frac{\sqrt{a+cx^2}}{2dx^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Rubi [A] time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {961, 266, 47, 63, 208, 277, 217, 206, 50, 735, 844, 725}

$$-\frac{\sqrt{a}\,e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{a+cx^2}}{2dx^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (e*Sqrt[c*d^2 +
a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^3 -
(c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) - (Sqrt[a]*e^2*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^3} - \frac{e\sqrt{a+cx^2}}{d^2x^2} + \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{e^3\sqrt{a+cx^2}}{d^3(d+ex)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^3}$$

$$= -\frac{e^2\sqrt{a+cx^2}}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} + \frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{a-}}{x}\right.}{2d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d} + \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(ce)\,\text{Subst}\left(\int \frac{}{1}\right.}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2d} + \frac{(ce)\,\text{S}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{d^3} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d} - \frac{\sqrt{a}}{}$$

Mathematica [A] time = 0.40, size = 283, normalized size = 1.77

$$-\frac{-2ex^2\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right) + cd^2x^2\sqrt{\frac{cx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right) + 2\sqrt{a}\,\sqrt{c}\,dex^2\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right) - 2\sqrt{c}\,dex^2\sqrt{a+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right) + 2\sqrt{a}\,e^2x^2\sqrt{a+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right) + ad^2 - 2adex + cd^2x^2 - 2cdex^3}{2d^3x^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*(a*d^2 - 2*a*d*e*x + c*d^2*x^2 - 2*c*d*e*x^3 + 2*Sqrt[a]*Sqrt[c]*d*e*x
^2*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] - 2*Sqrt[c]*d*e*x^2*Sqr
t[a + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - 2*e*Sqrt[c*d^2 + a*e^2]
*x^2*Sqrt[a + c*x^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])] + 2*Sqrt[a]*e^2*x^2*Sqrt[a + c*x^2]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]
 + c*d^2*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 + (c*x^2)/a]])/(d^3*x^2*Sqr
t[a + c*x^2])
```

IntegrateAlgebraic [A] time = 0.64, size = 184, normalized size = 1.15

$$\frac{\sqrt{a+cx^2}\,(2ex-d)}{2d^2x^2} - \frac{2e\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{d^3} + \frac{\left(2ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] ((-d + 2*e*x)*Sqrt[a + c*x^2])/(2*d^2*x^2) - (2*e*Sqrt[-(c*d^2) - a*e^2]*Ar
cTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e
^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/d^3 + ((c*d^2 + 2*a*e^2)
*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(Sqrt[a]*d^3)
```

fricas [A] time = 0.49, size = 726, normalized size = 4.54

$$\left[\frac{2\sqrt{cd^2+ae^2}\,aex^2\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2+2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right) + (cd^2+2ae^2)\sqrt{a}\,x^2\log\left(-\frac{cx^2-2\sqrt{cx^2+a}\,\sqrt{a}+2a}{x^2}\right) + 2(2adex-ad^2)\sqrt{cx^2+a}}{4ad^3x^2}, \frac{4\sqrt{-cd^2-ae^2}\,aex^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right) + (cd^2+2ae^2)\sqrt{a}\,x^2\log\left(-\frac{cx^2-2\sqrt{cx^2+a}\,\sqrt{a}+2a}{x^2}\right) + 2(2adex-ad^2)\sqrt{cx^2+a}}{4ad^3x^2}, \frac{\sqrt{cd^2+ae^2}\,aex^2\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2+2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right) + (cd^2+2ae^2)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right) + (2adex-ad^2)\sqrt{cx^2+a}}{2ad^3x^2}, \frac{2\sqrt{-cd^2-ae^2}\,aex^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right) + (cd^2+2ae^2)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right) + (2adex-ad^2)\sqrt{cx^2+a}}{2ad^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*a*e*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + 2*a*e^2)*sqrt(a)*x^2*log(-(c*
x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*e*x - a*d^2)*sqrt(c*
x^2 + a))/(a*d^3*x^2), 1/4*(4*sqrt(-c*d^2 - a*e^2)*a*e*x^2*arctan(sqrt(-c*d
^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a
*c*e^2)*x^2)) + (c*d^2 + 2*a*e^2)*sqrt(a)*x^2*log(-(c*x^2 - 2*sqrt(c*x^2 +
a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)
, 1/2*(sqrt(c*d^2 + a*e^2)*a*e*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
 (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + 2*a*e^2)*sqrt(-a)*x^2*arctan(s
qrt(-a)/sqrt(c*x^2 + a)) + (2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)
, 1/2*(2*sqrt(-c*d^2 - a*e^2)*a*e*x^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^
2 + 2*a*e^2)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (2*a*d*e*x - a
*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)]
```

giac [A] time = 0.22, size = 230, normalized size = 1.44

$$-\frac{2\left(cd^2e+ae^3\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^3}+\frac{\left(cd^2+2ae^2\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,d^3}+\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3cd-2\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2a\sqrt{c}e+\left(\sqrt{c}x-\sqrt{cx^2+a}\right)acd+2a^2\sqrt{c}e}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/
sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^3) + (c*d^2 + 2*a*e^2)*arctan
(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^3) + ((sqrt(c)*x - sq
rt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*sqrt(c)*e + (sqr
t(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^
2 + a))^2 - a)^2*d^2)
```

maple [B] time = 0.01, size = 567, normalized size = 3.54

$$\frac{ae^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,d^3}+\frac{c\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,d}-\frac{\sqrt{a}\,e^2\ln\left(\frac{2a+2\sqrt{cx^2+a}\,\sqrt{a}}{x}\right)}{d^3}-\frac{c\ln\left(\frac{2a+2\sqrt{cx^2+a}\,\sqrt{a}}{x}\right)}{2\sqrt{a}\,d}-\frac{\sqrt{c}\,e\ln\left(\sqrt{c}x+\sqrt{cx^2+a}\right)}{d^2}+\frac{\sqrt{c}\,e\ln\left(\frac{-\frac{cd}{e}+\left(x+\frac{d}{e}\right)c}{\sqrt{c}}+\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}\right)}{d^2}-\frac{\sqrt{cx^2+a}\,cex}{ad^2}+\frac{\sqrt{cx^2+a}\,c}{2ad}+\frac{\sqrt{cx^2+a}\,e^2}{d^3}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}\,e^2}{d^3}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}e}{ad^2x}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}}{2adx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^3/(e*x+d),x)
```

```
[Out] e/d^2/a/x*(c*x^2+a)^(3/2)-e/d^2*c/a*x*(c*x^2+a)^(1/2)-e/d^2*c^(1/2)*ln(c^(1
/2)*x+(c*x^2+a)^(1/2))-1/2/d/a/x^2*(c*x^2+a)^(3/2)-1/2/d*c/a^(1/2)*ln((2*a+
2*(c*x^2+a)^(1/2)*a^(1/2))/x)+1/2/d*c/a*(c*x^2+a)^(1/2)-1/d^3*e^2*a^(1/2)*l
n((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)+1/d^3*e^2*(c*x^2+a)^(1/2)-1/d^3*e^2*(-
2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+1/d^2*e*c^(1/2)*ln((-c
*d/e+(x+d/e)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1
/2))+1/d^3*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^
2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d
^2)/e^2)^(1/2))/(x+d/e))*a+1/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d
/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e
)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^3*(d + e*x)),x)
[Out] int((a + c*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**3/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x)), x)
```

3.235 $\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{a}\,e^3\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}-\frac{e^2\sqrt{a+cx^2}}{d^3x}+\frac{e\sqrt{a+cx^2}}{2d^2x^2}+\frac{ce\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d^2}-\frac{e^2\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^4}$$

Rubi [A] time = 0.23, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {961, 264, 266, 47, 63, 208, 277, 217, 206, 50, 735, 844, 725}

$$-\frac{e^2\sqrt{a+cx^2}}{d^3x}+\frac{\sqrt{a}\,e^3\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}-\frac{e^2\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^4}+\frac{e\sqrt{a+cx^2}}{2d^2x^2}+\frac{ce\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d^2}-\frac{\left(a+cx^2\right)^{3/2}}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (e*Sqrt[a + c*x^2])/(2*d^2*x^2) - (e^2*Sqrt[a + c*x^2])/(d^3*x) - (a + c*x^
2)^(3/2)/(3*a*d*x^3) - (e^2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^4 + (c*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[
a]])/(2*Sqrt[a]*d^2) + (Sqrt[a]*e^3*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^4} - \frac{e\sqrt{a+cx^2}}{d^2x^3} + \frac{e^2\sqrt{a+cx^2}}{d^3x^2} - \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e^4\sqrt{a+cx^2}}{d^4(d+ex)} \right) dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^4}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^4} + \frac{e^4\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^4}$$

$$= \frac{e^3\sqrt{a+cx^2}}{d^4} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{\left(a+cx^2\right)^{3/2}}{3adx^3} - \frac{e\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^2\right)}{2d^2} + \frac{\left(ce^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,d}{d^3}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{\left(a+cx^2\right)^{3/2}}{3adx^3} - \frac{(ce)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d^2} - \frac{\left(ce^2\right)\int \frac{1}{\sqrt{a+cx^2}}}{d^3}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{\left(a+cx^2\right)^{3/2}}{3adx^3} + \frac{\sqrt{c}\,e^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d^3} - \frac{e\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,\right.}{2d^2}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{\left(a+cx^2\right)^{3/2}}{3adx^3} - \frac{e^2\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{d^4} + \frac{ce\tan}{}$$

Mathematica [A] time = 1.02, size = 301, normalized size = 1.58

$$-\frac{\frac{2d^3\left(a+cx^2\right)^{3/2}}{ax^3} + 6e^2\left(\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right) + \sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\right) - \frac{3d^2e\left(cx^2\sqrt{\frac{cx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)+a+cx^2\right)}{x^2\sqrt{a+cx^2}} + \frac{6de^2\left(-\sqrt{a}\,\sqrt{c}\,x\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)+a+cx^2\right)}{x\sqrt{a+cx^2}} - 6e^3\sqrt{a+cx^2} + 6e^3\left(\sqrt{a+cx^2} - \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -1/6*(-6*e^3*Sqrt[a + c*x^2] + (2*d^3*(a + c*x^2)^(3/2))/(a*x^3) + (6*d*e^2
*(a + c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqr
t[a]]))/(x*Sqrt[a + c*x^2]) + 6*e^2*(Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
 c*x^2]] + Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*S
qrt[a + c*x^2])]) + 6*e^3*(Sqrt[a + c*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2
]/Sqrt[a]]) - (3*d^2*e*(a + c*x^2 + c*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[
1 + (c*x^2)/a]]))/(x^2*Sqrt[a + c*x^2]))/d^4
```

IntegrateAlgebraic [A] time = 0.86, size = 214, normalized size = 1.12

$$\frac{\left(-2ae^3-cd^2e\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^4} + \frac{2e^2\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{d^4} + \frac{\sqrt{a+cx^2}\,\left(-2ad^2+3adex-6ae^2x^2-2cd^2x^2\right)}{6ad^3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-2*a*d^2 + 3*a*d*e*x - 2*c*d^2*x^2 - 6*a*e^2*x^2))/(6*a*d
^3*x^3) + (2*e^2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) -
a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(
c*d^2) - a*e^2]])/d^4 + ((-(c*d^2*e) - 2*a*e^3)*ArcTanh[(Sqrt[c]*x - Sqrt[a
 + c*x^2])/Sqrt[a]])/(Sqrt[a]*d^4)
```

fricas [A] time = 0.51, size = 824, normalized size = 4.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*a*e^2*x^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(c*d^2*e + 2*a*e^3)*sqrt(a)*x^3*l
og(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*a*d^2*e*x - 2*a*d
^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3), -1/12*(12*sqr
t(-c*d^2 - a*e^2)*a*e^2*x^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c*d^2*e + 2*
a*e^3)*sqrt(a)*x^3*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*
(3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4
*x^3), 1/6*(3*sqrt(c*d^2 + a*e^2)*a*e^2*x^3*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c*d^2*e + 2*a*e^3)*sqrt(-a)
*x^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 +
 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3), -1/6*(6*sqrt(-c*d^2 - a*e^2)
*a*e^2*x^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d
^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(c*d^2*e + 2*a*e^3)*sqrt(-a)*x
^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3
*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3)]
```

giac [A] time = 0.21, size = 309, normalized size = 1.62

$$\frac{2\left(cd^2e^2+ae^4\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^4}-\frac{\left(cd^2e+2ae^3\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,d^4}-\frac{3\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^5cde-6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^4c^{\frac{3}{2}}d^2-6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^4a\sqrt{c}e^2-3\left(\sqrt{c}x-\sqrt{cx^2+a}\right)a^2cde-2a^2c^{\frac{3}{2}}d^2+12\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2a^2\sqrt{c}e^2-6a^3\sqrt{c}e^2}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^2*e^2 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^4) - (c*d^2*e + 2*a*e^3)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^4) - 1/3*(3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*c*d*e - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(3/2)
*d^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*sqrt(c)*e^2 - 3*(sqrt(c)*x - sqr
t(c*x^2 + a))*a^2*c*d*e - 2*a^2*c^(3/2)*d^2 + 12*(sqrt(c)*x - sqrt(c*x^2 +
a))^2*a^2*sqrt(c)*e^2 - 6*a^3*sqrt(c)*e^2)/(((sqrt(c)*x - sqrt(c*x^2 + a))^
2 - a)^3*d^3)
```

maple [B] time = 0.02, size = 600, normalized size = 3.14

$$-\frac{a e^3\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,d^4}-\frac{ce\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,d^2}+\frac{\sqrt{a}\,e^3\ln\left(\frac{2a+2\sqrt{cx^2+a}\,\sqrt{a}}{x}\right)}{d^4}+\frac{ce\ln\left(\frac{2a+2\sqrt{cx^2+a}\,\sqrt{a}}{x}\right)}{2\sqrt{a}\,d^2}+\frac{\sqrt{c}\,e^2\ln\left(\sqrt{c}\,x+\sqrt{cx^2+a}\right)}{d^3}-\frac{\sqrt{c}\,e^2\ln\left(\frac{-\frac{cd}{e}+\left(x+\frac{d}{e}\right)c}{\sqrt{c}}+\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}\right)}{d^3}+\frac{\sqrt{cx^2+a}\,ce^2x}{ad^3}-\frac{\sqrt{cx^2+a}\,ce}{2ad^2}-\frac{\sqrt{cx^2+a}\,e^3}{d^4}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}\,e^3}{d^4}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}e^2}{ad^3x}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}e}{2ad^2x^2}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}}{3adx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^4/(e*x+d),x)
```

```
[Out] -1/d^3*e^2/a/x*(c*x^2+a)^(3/2)+1/d^3*e^2*c/a*x*(c*x^2+a)^(1/2)+1/d^3*e^2*c^
(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+1/2*e/d^2/a/x^2*(c*x^2+a)^(3/2)+1/2*e/d
^2*c/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-1/2*e/d^2*c/a*(c*x^2+a)^
(1/2)-1/3*(c*x^2+a)^(3/2)/a/d/x^3+1/d^4*e^3*a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/
2)*a^(1/2))/x)-1/d^4*e^3*(c*x^2+a)^(1/2)+1/d^4*e^3*(-2*(x+d/e)*c*d/e+(x+d/e
)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-1/d^3*e^2*c^(1/2)*ln((-c*d/e+(x+d/e)*c)/c^(1
/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))-1/d^4*e^3/((a*e
^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d
^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d
/e))*a-1/d^2*e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^
2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d
^2)/e^2)^(1/2))/(x+d/e))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{x^4\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^4*(d + e*x)),x)
[Out] int((a + c*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x^4\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**4/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**4*(d + e*x)), x)
```

3.236 $\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=274

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}\,e^4\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}+\frac{e^3\sqrt{a+cx^2}}{d^4x}-\frac{e^2\sqrt{a+cx^2}}{2d^3x^2}-\frac{ce^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d^3}+\frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3}+\frac{e^3\cdots}{}$$

Rubi [A] time = 0.30, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {961, 266, 47, 51, 63, 208, 264, 277, 217, 206, 50, 735, 844, 725}

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d}+\frac{e^3\sqrt{a+cx^2}}{d^4x}-\frac{e^2\sqrt{a+cx^2}}{2d^3x^2}-\frac{\sqrt{a}\,e^4\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}+\frac{e^3\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^5}-\frac{ce^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d^3}+\frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3}-\frac{c\sqrt{a+cx^2}}{8adx^2}-\frac{\sqrt{a+cx^2}}{4dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(4*d*x^4) - (c*Sqrt[a + c*x^2])/(8*a*d*x^2) - (e^2*Sqrt[a
+ c*x^2])/(2*d^3*x^2) + (e^3*Sqrt[a + c*x^2])/(d^4*x) + (e*(a + c*x^2)^(3/2
))/(3*a*d^2*x^3) + (e^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d
^2 + a*e^2]*Sqrt[a + c*x^2])])/d^5 + (c^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])
/(8*a^(3/2)*d) - (c*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d^3) -
 (Sqrt[a]*e^4*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^5
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^5} - \frac{e\sqrt{a+cx^2}}{d^2x^4} + \frac{e^2\sqrt{a+cx^2}}{d^3x^3} - \frac{e^3\sqrt{a+cx^2}}{d^4x^2} + \frac{e^4\sqrt{a+cx^2}}{d^5x} - \frac{e^5\sqrt{a+cx^2}}{d^5(d+ex)}\right) dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^5}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^4}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^4} + \frac{e^4\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^5} - \frac{e^5\int \frac{\sqrt{a+c}}{d+e}}{d^5}$$

$$= -\frac{e^4\sqrt{a+cx^2}}{d^5} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^3}\,dx, x, x^2\right)}{2d} + \frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{a+c}}{x^2}\right.}{2d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{c\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{8d} + $$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} - \frac{\sqrt{c}\,e^3\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+c}}\right.}{d^4}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2+ae^2}\,\tanh}{d}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2+ae^2}\,\tanh}{d}$$

Mathematica [C] time = 1.10, size = 344, normalized size = 1.26

$$\frac{-\frac{2c^2d^4\left(a+cx^2\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx^2}{a}+1\right)}{a^3} + \frac{2d^3e\left(a+cx^2\right)^{3/2}}{ax^3} - \frac{3d^2e^2\left(cx^2\sqrt{\frac{cx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)+a+cx^2\right)}{x^2\sqrt{a+cx^2}} + 6e^3\left(\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)+\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\right) + \frac{6de^3\left(-\sqrt{a}\sqrt{c}\,x\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)+a+cx^2\right)}{x\sqrt{a+cx^2}} - 6e^4\sqrt{a+cx^2} + 6e^4\left(\sqrt{a+cx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)\right)}{6d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (-6*e^4*Sqrt[a + c*x^2] + (2*d^3*e*(a + c*x^2)^(3/2))/(a*x^3) + (6*d*e^3*(a
 + c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a
]]))/(x*Sqrt[a + c*x^2]) + 6*e^3*(Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*
x^2]] + Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt
[a + c*x^2])]) + 6*e^4*(Sqrt[a + c*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/S
qrt[a]]) - (3*d^2*e^2*(a + c*x^2 + c*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1
 + (c*x^2)/a]]))/(x^2*Sqrt[a + c*x^2]) - (2*c^2*d^4*(a + c*x^2)^(3/2)*Hyper
geometric2F1[3/2, 3, 5/2, 1 + (c*x^2)/a])/a^3)/(6*d^5)
```

IntegrateAlgebraic [A] time = 1.13, size = 252, normalized size = 0.92

$$\frac{\left(8a^2e^4+4acd^2e^2-c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{4a^{3/2}d^5} - \frac{2e^3\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{d^5} + \frac{\sqrt{a+cx^2}\left(-6ad^3+8ad^2ex-12ade^2x^2+24ae^3x^3-3cd^3x^2+8cd^2ex^3\right)}{24ad^4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-6*a*d^3 + 8*a*d^2*e*x - 3*c*d^3*x^2 - 12*a*d*e^2*x^2 + 8
*c*d^2*e*x^3 + 24*a*e^3*x^3))/(24*a*d^4*x^4) - (2*e^3*Sqrt[-(c*d^2) - a*e^2
]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) -
 a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/d^5 + ((-(c^2*d^4) +
 4*a*c*d^2*e^2 + 8*a^2*e^4)*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])
/(4*a^(3/2)*d^5)
```

fricas [A] time = 0.58, size = 1007, normalized size = 3.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^
2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a
^2*e^4)*sqrt(a)*x^4*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2
*(8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4
+ 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), 1/48*(48*sqrt(-c*d^2
- a*e^2)*a^2*e^3*x^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c^2*d^4 - 4*a*c*d^2
*e^2 - 8*a^2*e^4)*sqrt(a)*x^4*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3
*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), 1/24*(12*sq
rt(c*d^2 + a*e^2)*a^2*e^3*x^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*sqrt
(-a)*x^4*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (8*a^2*d^3*e*x - 6*a^2*d^4 + 8*
(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2
 + a))/(a^2*d^5*x^4), 1/24*(24*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^4*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*sqrt(-a)*x^4*a
rctan(sqrt(-a)/sqrt(c*x^2 + a)) + (8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e
 + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^
2*d^5*x^4)]
```

giac [B] time = 0.26, size = 596, normalized size = 2.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e^3 + a*e^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^5) - 1/4*(c^2*d^4 - 4*a*c*d
^2*e^2 - 8*a^2*e^4)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-
a)*a*d^5) + 1/12*(3*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^2*d^3 - 24*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*a*c^(3/2)*d^2*e + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^5*
a*c^2*d^3 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c*d*e^2 + 24*(sqrt(c)*x -
sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d^2*e + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a
^2*c^2*d^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*d*e^2 - 24*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*a^2*sqrt(c)*e^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a
^3*c^(3/2)*d^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d^3 - 12*(sqrt(c
)*x - sqrt(c*x^2 + a))^3*a^3*c*d*e^2 + 72*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a
^3*sqrt(c)*e^3 + 8*a^4*c^(3/2)*d^2*e + 12*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4
```

```
*c*d*e^2 - 72*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*e^3 + 24*a^5*sqrt
(c)*e^3)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^4*a*d^4)
```

maple [B] time = 0.02, size = 703, normalized size = 2.57

$$\frac{a e^{4} \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)c d}{e}+\frac{2a e^{2}+2c d^{2}}{e^{2}}+2\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)c d}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a e^{2}+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\, d^{5}}+\frac{c e^{2} \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)c d}{e}+\frac{2a e^{2}+2c d^{2}}{e^{2}}+2\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)c d}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a e^{2}+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\, d^{3}}-\frac{\sqrt{a}\, e^{4} \ln\left(\frac{2a+2\sqrt{c x^{2}+a}\,\sqrt{a}}{x}\right)}{d^{5}}-\frac{c e^{2} \ln\left(\frac{2a+2\sqrt{c x^{2}+a}\,\sqrt{a}}{x}\right)}{2\sqrt{a}\, d^{3}}+\frac{c^{2} \ln\left(\frac{2a+2\sqrt{c x^{2}+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{3}{2}} d}-\frac{\sqrt{c}\, e^{3} \ln\left(\sqrt{c}\, x+\sqrt{c x^{2}+a}\right)}{d^{4}}+\frac{\sqrt{c}\, e^{3} \ln\left(\frac{-\frac{c d}{e}+\left(x+\frac{d}{e}\right)c}{\sqrt{c}}+\sqrt{-\frac{2\left(x+\frac{d}{e}\right)c d}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a e^{2}+c d^{2}}{e^{2}}}\right)}{d^{4}}-\frac{\sqrt{c x^{2}+a}\, c e^{3} x}{a d^{4}}+\frac{\sqrt{c x^{2}+a}\, c e^{2}}{2a d^{3}}-\frac{\sqrt{c x^{2}+a}\, c^{2}}{8a^{2} d}+\frac{\sqrt{c x^{2}+a}\, e^{4}}{d^{5}}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)c d}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a e^{2}+c d^{2}}{e^{2}}}\, e^{4}}{d^{5}}+\frac{\left(c x^{2}+a\right)^{\frac{3}{2}} e^{3}}{a d^{4} x}-\frac{\left(c x^{2}+a\right)^{\frac{3}{2}} e^{2}}{2a d^{3} x^{2}}+\frac{\left(c x^{2}+a\right)^{\frac{3}{2}} c}{8a^{2} d x^{2}}+\frac{\left(c x^{2}+a\right)^{\frac{3}{2}} e}{3a d^{2} x^{3}}-\frac{\left(c x^{2}+a\right)^{\frac{3}{2}}}{4a d x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^5/(e*x+d),x)
```

```
[Out] 1/d^4*e^3/a/x*(c*x^2+a)^(3/2)-1/d^4*e^3*c/a*x*(c*x^2+a)^(1/2)-1/d^4*e^3*c^(
1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-1/2/d^3*e^2/a/x^2*(c*x^2+a)^(3/2)-1/2/d^
3*e^2*c/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)+1/2/d^3*e^2*c/a*(c*x^
2+a)^(1/2)+1/3*e*(c*x^2+a)^(3/2)/a/d^2/x^3-1/d^5*e^4*a^(1/2)*ln((2*a+2*(c*x
^2+a)^(1/2)*a^(1/2))/x)+1/d^5*e^4*(c*x^2+a)^(1/2)-1/d^5*e^4*(-2*(x+d/e)*c*d
/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+1/d^4*e^3*c^(1/2)*ln((-c*d/e+(x+d/e
)*c)/c^(1/2)+(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))+1/d^5*
e^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((
a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1
/2))/(x+d/e))*a+1/d^3*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*
(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c
+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c-1/4/d/a/x^4*(c*x^2+a)^(3/2)+1/8/d*c/a
^2/x^2*(c*x^2+a)^(3/2)+1/8/d*c^2/a^(3/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))
/x)-1/8/d*c^2/a^2*(c*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c x^2 + a}}{(e x + d) x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{x^5\,\left(d+e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + c x^{2}}}{x^{5} \left(d + e x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**5*(d + e*x)), x)
```

3.237 $\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=195

$$-\frac{d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}+\frac{\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{6c^2e^3}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}}-\frac{7d\sqrt{a+cx^2}(d+ex)}{6ce^3}$$

Rubi [A] time = 0.48, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1654, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{6c^2e^3}-\frac{d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}}-\frac{7d\sqrt{a+cx^2}(d+ex)}{6ce^3}+\frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((11*c*d^2 - 4*a*e^2)*Sqrt[a + c*x^2])/(6*c^2*e^3) - (7*d*(d + e*x)*Sqrt[a
+ c*x^2])/(6*c*e^3) + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^3) - (d*(2*c*d^2
 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*Sqrt[c*d^2 +
 a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
```

```
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{-2ad^2e^2-de\left(cd^2+4ae^2\right)x-e^2\left(5cd^2+2ae^2\right)x^2-7cde^3x^3}{(d+ex)\sqrt{a+cx^2}}\,dx}{3ce^4}$$

$$= -\frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{3acd^2e^5+cde^4\left(5cd^2-ae^2\right)x+ce^5\left(11cd^2-4ae^2\right)x^2}{(d+ex)\sqrt{a+cx^2}}\,d}{6c^2e^7}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{3ac^2d^2e^7-3c^2a}{(d+ex)}}{6c}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{d^4\int \frac{1}{(d+ex)\sqrt{a+}}}{e^4}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} - \frac{d^4\,\mathrm{Subst}\left(\int \frac{}{c}\right.}{}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} - \frac{d\left(2cd^2-ae^2\right.}{2}$$

Mathematica [A] time = 0.23, size = 149, normalized size = 0.76

$$\frac{-\frac{3d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}+\frac{e\sqrt{a+cx^2}\left(-4ae^2+6cd^2-3cdex+2ce^2x^2\right)}{c^2}-\frac{6d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(6*c*d^2 - 4*a*e^2 - 3*c*d*e*x + 2*c*e^2*x^2))/c^2 - (3
*d*(2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/c^(3/2) - (6*d^4
*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/Sqrt[c*d^2 +
 a*e^2])/(6*e^4)
```

IntegrateAlgebraic [A] time = 0.63, size = 217, normalized size = 1.11

$$\frac{\left(2cd^3-ade^2\right)\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2c^{3/2}e^4}+\frac{\sqrt{a+cx^2}\left(-4ae^2+6cd^2-3cdex+2ce^2x^2\right)}{6c^2e^3}+\frac{2d^4\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{e^4\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[a + c*x^2]*(6*c*d^2 - 4*a*e^2 - 3*c*d*e*x + 2*c*e^2*x^2))/(6*c^2*e^3)
 + (2*d^4*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2]
+ (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2)
- a*e^2]])/(e^4*(c*d^2 + a*e^2)) + ((2*c*d^3 - a*d*e^2)*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4)
```

fricas [A] time = 2.88, size = 1060, normalized size = 5.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x
^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(2*c^2*d^5 + a*c*d^3*e^2 - a^2*d*e^
4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*d^4*e
 + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e
^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), -1/12*(12*sq
rt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(2*c^2*d^5 + a
*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
- a) - 2*(6*c^2*d^4*e + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^
5)*x^2 - 3*(c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c
^2*e^6), 1/6*(3*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(2*c^2*d^5 + a*c*d^3*e^2 - a
^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*c^2*d^4*e + 2*a*
c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e^2 + a*
c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), -1/6*(6*sqrt(-c*d^2
 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c^2*d^5 + a*c*d^3*e^
2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (6*c^2*d^4*e +
 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e^2
 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6)]
```

giac [A] time = 0.21, size = 163, normalized size = 0.84

$$\frac{2\,d^4 \arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-4)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{6}\,\sqrt{cx^2+a}\left(x\left(\frac{2\,xe^{(-1)}}{c}-\frac{3\,de^{(-2)}}{c}\right)+\frac{2\left(3\,c^2d^2e^7-2\,ace^9\right)e^{(-10)}}{c^3}\right)+\frac{\left(2\,c^{\frac{3}{2}}d^3-a\sqrt{c}\,de^2\right)e^{(-4)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))*e^(-4)/sqrt(-c*d^2 - a*e^2) + 1/6*sqrt(c*x^2 + a)*(x*(2*x*e^(-1)/c -
 3*d*e^(-2)/c) + 2*(3*c^2*d^2*e^7 - 2*a*c*e^9)*e^(-10)/c^3) + 1/2*(2*c^(3/2
)*d^3 - a*sqrt(c)*d*e^2)*e^(-4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^2
```

maple [A] time = 0.02, size = 260, normalized size = 1.33

$$\frac{\sqrt{c\,x^2+a}\;x^2}{3ce}-\frac{d^4\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^5}+\frac{ad\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}e^2}-\frac{d^3\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}\;e^4}-\frac{\sqrt{c\,x^2+a}\;dx}{2c\,e^2}-\frac{2\sqrt{c\,x^2+a}\;a}{3c^2e}+\frac{\sqrt{c\,x^2+a}\;d^2}{c\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3/e*x^2/c*(c*x^2+a)^(1/2)-2/3/e*a/c^2*(c*x^2+a)^(1/2)-1/2*d/e^2*x/c*(c*x^
2+a)^(1/2)+1/2*d/e^2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+d^2/e^3/c*(c*x
^2+a)^(1/2)-d^3/e^4*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-d^4/e^5/((a*e^2+c
*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/
e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.55, size = 171, normalized size = 0.88

$$\frac{\sqrt{cx^2+a}\,x^2}{3\,ce}-\frac{\sqrt{cx^2+a}\,dx}{2\,ce^2}-\frac{d^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^4}+\frac{ad\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}e^2}+\frac{d^4\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^5}+\frac{\sqrt{cx^2+a}\,d^2}{ce^3}-\frac{2\,\sqrt{cx^2+a}\,a}{3\,c^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2 + a)*x^2/(c*e) - 1/2*sqrt(c*x^2 + a)*d*x/(c*e^2) - d^3*arcsi
nh(c*x/sqrt(a*c))/(sqrt(c)*e^4) + 1/2*a*d*arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e
^2) + d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x +
 d)))/(sqrt(a + c*d^2/e^2)*e^5) + sqrt(c*x^2 + a)*d^2/(c*e^3) - 2/3*sqrt(c*
x^2 + a)*a/(c^2*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.238 $\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=152

$$\frac{\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}}-\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{\sqrt{a+cx^2}\,(d+ex)}{2ce^2}$$

Rubi [A] time = 0.27, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1654, 844, 217, 206, 725}

$$\frac{\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}}-\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{\sqrt{a+cx^2}\,(d+ex)}{2ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-3*d*Sqrt[a + c*x^2])/(2*c*e^2) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^2) +
((2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^3) +
(d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^3*Sqr
t[c*d^2 + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
```

```
1/2, 0]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{\int \frac{-ade^2-e\left(cd^2+ae^2\right)x-3cde^2x^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{\int \frac{-acde^4+ce^3\left(2cd^2-ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2c^2e^5}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} - \frac{d^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3} + \frac{\left(2cd^2-ae^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2ce^3}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{d^3\,\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^3} + \frac{\left(2cd^2-a\right.}{}$$

$$= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3} + \frac{d^3\tanh^{-1}\left(\frac{}{\sqrt{cd^2\ldots}}\right.}{e^3\sqrt{cd^2+\ldots}}$$

Mathematica [A] time = 0.22, size = 131, normalized size = 0.86

$$\frac{\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)+\sqrt{c}\left(\frac{2cd^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}+e\sqrt{a+cx^2}\,(ex-2d)\right)}{2c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[c]*(e*(-2*d
+ e*x)*Sqrt[a + c*x^2] + (2*c*d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2])])/Sqrt[c*d^2 + a*e^2]))/(2*c^(3/2)*e^3)
```

IntegrateAlgebraic [A] time = 0.47, size = 194, normalized size = 1.28

$$\frac{\left(ae^2-2cd^2\right)\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2c^{3/2}e^3}-\frac{2d^3\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{e^3\left(ae^2+cd^2\right)}+\frac{\sqrt{a+cx^2}\,(ex-2d)}{2ce^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-2*d + e*x)*Sqrt[a + c*x^2])/(2*c*e^2) - (2*d^3*Sqrt[-(c*d^2) - a*e^2]*Ar
cTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e
^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(e^3*(c*d^2 + a*e^2)) +
((-2*c*d^2 + a*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*c^(3/2)*e^3)
```

fricas [A] time = 2.80, size = 924, normalized size = 6.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
```

```
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sq
rt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c^2*d^3*e + 2*
a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^
2*e^5), 1/4*(4*sqrt(-c*d^2 - a*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) -
 (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 +
a)*sqrt(c)*x - a) - 2*(2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*
x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(sqrt(c*d^2 + a*e^2)*c^2
*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2
*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) - (2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c
*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(2*sqrt(-c*d^2 - a*e^2)*c^2*d^3*a
rctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2
 + (c^2*d^2 + a*c*e^2)*x^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)
*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2
*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5)]
```

giac [A] time = 0.22, size = 129, normalized size = 0.85

$$-\frac{2\,d^3\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-3)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{2}\,\sqrt{cx^2+a}\left(\frac{xe^{(-1)}}{c}-\frac{2\,de^{(-2)}}{c}\right)-\frac{\left(2\,cd^2-ae^2\right)e^{(-3)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*d^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
a*e^2))*e^(-3)/sqrt(-c*d^2 - a*e^2) + 1/2*sqrt(c*x^2 + a)*(x*e^(-1)/c - 2*d
*e^(-2)/c) - 1/2*(2*c*d^2 - a*e^2)*e^(-3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/c^(3/2)
```

maple [A] time = 0.01, size = 217, normalized size = 1.43

$$\frac{d^3\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\,e^4}-\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}e}+\frac{d^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}\,e^3}+\frac{\sqrt{c\,x^2+a}\,x}{2ce}-\frac{\sqrt{c\,x^2+a}\,d}{c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2/e*x/c*(c*x^2+a)^(1/2)-1/2/e*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-d*(
c*x^2+a)^(1/2)/c/e^2+d^2/e^3*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+d^3/e^4/
((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^
2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))
/(x+d/e))
```

maxima [A] time = 0.52, size = 130, normalized size = 0.86

$$\frac{\sqrt{cx^2+a}\,x}{2\,ce}+\frac{d^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^3}-\frac{a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}e}-\frac{d^3\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^4}-\frac{\sqrt{cx^2+a}\,d}{ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x/(c*e) + d^2*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e^3) - 1/
2*a*arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e) - d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(e
```

```
*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt(a + c*d^2/e^2)*e^4) - sqrt(c
*x^2 + a)*d/(c*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.239 $\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=109

$$-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}+\frac{\sqrt{a+cx^2}}{ce}$$

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1654, 12, 844, 217, 206, 725}

$$-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}+\frac{\sqrt{a+cx^2}}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e
^2) - (d^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e
^2*Sqrt[c*d^2 + a*e^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
```

```
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx &= \frac{\sqrt{a+cx^2}}{ce} - \frac{\int \frac{cdex}{(d+ex)\sqrt{a+cx^2}}\,dx}{ce^2} \\
&= \frac{\sqrt{a+cx^2}}{ce} - \frac{d\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx}{e} \\
&= \frac{\sqrt{a+cx^2}}{ce} - \frac{d\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2} + \frac{d^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^2} \\
&= \frac{\sqrt{a+cx^2}}{ce} - \frac{d\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^2} - \frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^2} \\
&= \frac{\sqrt{a+cx^2}}{ce} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2} - \frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{e^2\sqrt{cd^2+ae^2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.96

$$
\frac{-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}} + \frac{e\sqrt{a+cx^2}}{c}}{e^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2])/c - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c] -
 (d^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/Sqrt[c*
d^2 + a*e^2])/e^2
```

IntegrateAlgebraic [A] time = 0.45, size = 170, normalized size = 1.56

$$
\frac{2d^2\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{e^2\left(ae^2+cd^2\right)} + \frac{d\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{\sqrt{c}\,e^2} + \frac{\sqrt{a+cx^2}}{ce}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e) + (2*d^2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sq
rt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c
*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(e^2*(c*d^2 + a*e^2)) + (d*Log[-(Sqrt[c]*x)
 + Sqrt[a + c*x^2]])/(Sqrt[c]*e^2)
```

fricas [A] time = 0.53, size = 745, normalized size = 6.83

$$
\left[\frac{\sqrt{cd^2+ae^2}\,cd^2\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right)+\left(cd^3+ade^2\right)\sqrt{c}\log\left(-2cx^2+2\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\left(cd^2e+ae^3\right)\sqrt{cx^2+a}}{2\left(c^2d^2e^2+ace^4\right)}, -\frac{2\sqrt{-cd^2-ae^2}\,cd^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-\left(cd^3+ade^2\right)\sqrt{c}\log\left(-2cx^2+2\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(cd^2e+ae^3\right)\sqrt{cx^2+a}}{2\left(c^2d^2e^2+ace^4\right)}, \frac{\sqrt{cd^2+ae^2}\,cd^2\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right)+2\left(cd^3+ade^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+2\left(cd^2e+ae^3\right)\sqrt{cx^2+a}}{2\left(c^2d^2e^2+ace^4\right)}, -\frac{\sqrt{-cd^2-ae^2}\,cd^2\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-\left(cd^3+ade^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-\left(cd^2e+ae^3\right)\sqrt{cx^2+a}}{c^2d^2e^2+ace^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2
*d^2*e^2 + a*c*e^4), -1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2
- a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*
e^2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqr
t(c)*x - a) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4),
 1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-
c)*x/sqrt(c*x^2 + a)) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 +
 a*c*e^4), -(sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c*
d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*d^2*e + a*e
^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4)]
```

giac [A] time = 0.23, size = 105, normalized size = 0.96

$$\frac{2\,d^2\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-2)}}{\sqrt{-cd^2-ae^2}}+\frac{de^{(-2)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,e^{(-1)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))*e^(-2)/sqrt(-c*d^2 - a*e^2) + d*e^(-2)*log(abs(-sqrt(c)*x + sqrt(c*x
^2 + a)))/sqrt(c) + sqrt(c*x^2 + a)*e^(-1)/c
```

maple [A] time = 0.01, size = 172, normalized size = 1.58

$$-\frac{d^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^3}-\frac{d\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}\;e^2}+\frac{\sqrt{c\,x^2+a}}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/e-1/e^2*d*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-d^2/e^3/(
(a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2
+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/
(x+d/e))
```

maxima [A] time = 0.49, size = 90, normalized size = 0.83

$$-\frac{d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^2}+\frac{d^2\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^3}+\frac{\sqrt{cx^2+a}}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -d*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e^2) + d^2*arcsinh(c*d*x/(sqrt(a*c)*abs(
e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt(a + c*d^2/e^2)*e^3) + sqrt(
c*x^2 + a)/(c*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.240 $\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e}$$

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {844, 217, 206, 725}

$$\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (d*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx &= \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e}-\frac{d\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{e}+\frac{d\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e}+\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\sqrt{cd^2+ae^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 86, normalized size = 1.00

$$\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (d*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])
```

IntegrateAlgebraic [A] time = 0.37, size = 150, normalized size = 1.74

$$-\frac{2d\sqrt{-ae^2-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2-cd^2}}\right)}{e\left(ae^2+cd^2\right)}-\frac{\log\left(\sqrt{a+cx^2}-\sqrt{c}x\right)}{\sqrt{c}e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*d*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (S
qrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*
e^2]])/(e*(c*d^2 + a*e^2)) - Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/(Sqrt[c]*e
)
```

fricas [A] time = 0.53, size = 631, normalized size = 7.34

$$\left[\frac{\sqrt{cd^2+ae^2}\,cd\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^2+ae^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}x-a\right)}{2\left(c^2d^2e+ace^3\right)},\ \frac{2\,\sqrt{-cd^2-ae^2}\,cd\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)+\left(cd^2+ae^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}x-a\right)}{2\left(c^2d^2e+ace^3\right)},\ \frac{\sqrt{cd^2+ae^2}\,cd\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(cd^2+ae^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)}{2\left(c^2d^2e+ace^3\right)},\ \frac{\sqrt{-cd^2-ae^2}\,cd\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)-\left(cd^2+ae^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)}{c^2d^2e+ace^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqr
t(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(2*sqrt(-c*d^2 - a*
e^2)*c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2
 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x
^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(sqrt(c*d
^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e
^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2
*d*e*x + d^2)) - 2*(c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 +
a)))/(c^2*d^2*e + a*c*e^3), (sqrt(-c*d^2 - a*e^2)*c*d*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)))/(c^
2*d^2*e + a*c*e^3)]
```

giac [A] time = 0.20, size = 88, normalized size = 1.02

$$-\frac{2\,d\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-1)}}{\sqrt{-cd^2-ae^2}}-\frac{e^{(-1)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*d*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*
e^2))*e^(-1)/sqrt(-c*d^2 - a*e^2) - e^(-1)*log(abs(-sqrt(c)*x + sqrt(c*x^2
+ a)))/sqrt(c)
```

maple [B] time = 0.01, size = 151, normalized size = 1.76

$$\frac{d \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right) c d}{e}+\frac{2 a e^{2}+2 c d^{2}}{e^{2}}+2 \sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\, \sqrt{-\frac{2\left(x+\frac{d}{e}\right) c d}{e}+\left(x+\frac{d}{e}\right)^{2} c+\frac{a e^{2}+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\, e^{2}}+\frac{\ln\left(\sqrt{c}\, x+\sqrt{c x^{2}+a}\right)}{\sqrt{c}\, e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/e*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+
d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.49, size = 71, normalized size = 0.83

$$\frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\, e}-\frac{d \operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\, |ex+d|}-\frac{ae}{\sqrt{ac}\, |ex+d|}\right)}{\sqrt{a+\frac{cd^{2}}{e^{2}}}\, e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e) - d*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d
)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt(a + c*d^2/e^2)*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c x^{2}+a}\, (d+e x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + cx^{2}}\, (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.241 $\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=54

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {725, 206}

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx &= -\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\sqrt{cd^2+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

IntegrateAlgebraic [B] time = 0.01, size = 114, normalized size = 2.11

$$\frac{2\sqrt{-ae^2-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2-cd^2}}\right)}{ae^2+cd^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt
[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2
]])/(c*d^2 + a*e^2)
```

fricas [B] time = 0.42, size = 211, normalized size = 3.91

$$\left[\frac{\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{cd^2+ae^2}},-\frac{\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)}{cd^2+ae^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2
*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2))/sqrt(c*d^2 + a*e^2), -sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)
)/(c*d^2 + a*e^2)]
```

giac [A] time = 0.19, size = 59, normalized size = 1.09

$$\frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2
))/sqrt(-c*d^2 - a*e^2)
```

maple [B] time = 0.00, size = 127, normalized size = 2.35

$$-\frac{\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*(
(a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(
1/2))/(x+d/e))
```

maxima [A] time = 0.46, size = 52, normalized size = 0.96

$$\frac{\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|} - \frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a + \frac{cd^2}{e^2}}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqr
t(a + c*d^2/e^2)*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^2}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)), x)
```

3.242 $\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {961, 266, 63, 208, 725, 206}

$$\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*Sqrt[c*
d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{dx\sqrt{a+cx^2}} - \frac{e}{d(d+ex)\sqrt{a+cx^2}}\right)dx \\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{e\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d} \\
&= \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd} \\
&= \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 86, normalized size = 1.00

$$
\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*Sqrt[c*
d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)
```

IntegrateAlgebraic [A] time = 0.35, size = 161, normalized size = 1.87

$$
\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}} - \frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{2e\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{d\left(ae^2+cd^2\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (S
qrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*
e^2]])/(d*(c*d^2 + a*e^2)) + (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^
2]/Sqrt[a]])/(Sqrt[a]*d)
```

fricas [A] time = 0.46, size = 634, normalized size = 7.37

$$
\left[\frac{\sqrt{cd^2+ae^2}\,ae\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^2+ae^2\right)\sqrt{a}\,\log\left(-\frac{cx^2-2\,\sqrt{cx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)}{2\left(acd^3+a^2de^2\right)}, \frac{2\,\sqrt{-cd^2-ae^2}\,ae\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)+\left(cd^2+ae^2\right)\sqrt{a}\,\log\left(-\frac{cx^2-2\,\sqrt{cx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)}{2\left(acd^3+a^2de^2\right)}, \frac{\sqrt{cd^2+ae^2}\,ae\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+2\left(cd^2+ae^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)}{2\left(acd^3+a^2de^2\right)}, \frac{\sqrt{-cd^2-ae^2}\,ae\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)+\left(cd^2+ae^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)}{acd^3+a^2de^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*a*e*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + a*e^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt
(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), 1/2*(2*sqrt(-c*d^2
- a*e^2)*a*e*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c
*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(a)*log(-(
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), 1/2*(
sqrt(c*d^2 + a*e^2)*a*e*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2
 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2
*x^2 + 2*d*e*x + d^2)) + 2*(c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*
x^2 + a)))/(a*c*d^3 + a^2*d*e^2), (sqrt(-c*d^2 - a*e^2)*a*e*arctan(sqrt(-c*
d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 +
a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)))
/(a*c*d^3 + a^2*d*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 158, normalized size = 1.84

$$\frac{\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\, d}-\frac{\ln\left(\frac{2a+2\sqrt{c x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/d/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)+1/d/((a*e^2+c*d^2)/e^2)^
(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*
(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)), x)
```

3.243 $\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=111

$$-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

Rubi [A] time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {961, 264, 266, 63, 208, 725, 206}

$$-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2 + a*e^2]) + (e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{dx^2\sqrt{a+cx^2}} - \frac{e}{d^2x\sqrt{a+cx^2}} + \frac{e^2}{d^2(d+ex)\sqrt{a+cx^2}}\right) dx \\
&= \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2} \\
&= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} - \frac{e^2\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^2} \\
&= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} - \frac{e\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2} \\
&= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} + \frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 107, normalized size = 0.96

$$\frac{-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}} - \frac{d\sqrt{a+cx^2}}{ax} + \frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-((d*Sqrt[a + c*x^2])/(a*x)) - (e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*
e^2]*Sqrt[a + c*x^2])])/Sqrt[c*d^2 + a*e^2] + (e*ArcTanh[Sqrt[a + c*x^2]/Sq
rt[a]])/Sqrt[a])/d^2
```

IntegrateAlgebraic [A] time = 0.40, size = 186, normalized size = 1.68

$$\frac{2e^2\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{d^2\left(ae^2+cd^2\right)} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}} - \frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} - \frac{\sqrt{a+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) + (2*e^2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*
d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[
a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(d^2*(c*d^2 + a*e^2)) - (2*e*ArcTanh[(
Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2)
```

fricas [A] time = 0.48, size = 767, normalized size = 6.91

$$\left[\frac{\sqrt{cd^2+ae^2}\,ae^2x\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2-2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right)+(cd^2e+ae^3)\sqrt{a}\,x\log\left(-\frac{cx^2+2\sqrt{cx^2+a}\sqrt{a}+2a}{x^2}\right)-2(cd^3+ade^2)\sqrt{cx^2+a}}{2(acd^4+a^2d^2e^2)x},\ -\frac{2\sqrt{-cd^2-ae^2}\,ae^2x\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)-(cd^2e+ae^3)\sqrt{a}\,x\log\left(-\frac{cx^2+2\sqrt{cx^2+a}\sqrt{a}+2a}{x^2}\right)+2(cd^3+ade^2)\sqrt{cx^2+a}}{2(acd^4+a^2d^2e^2)x},\ \frac{\sqrt{cd^2+ae^2}\,ae^2x\log\left(\frac{2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2-2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right)-2(cd^2e+ae^3)\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)-2(cd^3+ade^2)\sqrt{cx^2+a}}{2(acd^4+a^2d^2e^2)x},\ -\frac{\sqrt{-cd^2-ae^2}\,ae^2x\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+(c^2d^2+ace^2)x^2}\right)+(cd^2e+ae^3)\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)+(cd^3+ade^2)\sqrt{cx^2+a}}{(acd^4+a^2d^2e^2)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*a*e^2*x*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
(2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
+ a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2*e + a*e^3)*sqrt(a)*x*log(-(c*x^2
+ 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2 +
a))/((a*c*d^4 + a^2*d^2*e^2)*x), -1/2*(2*sqrt(-c*d^2 - a*e^2)*a*e^2*x*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) - (c*d^2*e + a*e^3)*sqrt(a)*x*log(-(c*x^2 + 2*sqrt
(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/((a*
c*d^4 + a^2*d^2*e^2)*x), 1/2*(sqrt(c*d^2 + a*e^2)*a*e^2*x*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2*e + a*e
^3)*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - 2*(c*d^3 + a*d*e^2)*sqrt(
c*x^2 + a))/((a*c*d^4 + a^2*d^2*e^2)*x), -(sqrt(-c*d^2 - a*e^2)*a*e^2*x*arc
tan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
 (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(-a)*x*arctan(sqrt(-a)/s
qrt(c*x^2 + a)) + (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/((a*c*d^4 + a^2*d^2*e^
2)*x)]
```

giac [A] time = 0.22, size = 142, normalized size = 1.28

$$2c\left(\frac{\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^2}{\sqrt{-cd^2-ae^2}\,cd^2}-\frac{\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,cd^2}+\frac{1}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)\sqrt{c}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*(arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*
e^2))*e^2/(sqrt(-c*d^2 - a*e^2)*c*d^2) - arctan(-(sqrt(c)*x - sqrt(c*x^2 +
a))/sqrt(-a))*e/(sqrt(-a)*c*d^2) + 1/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)
*sqrt(c)*d))
```

maple [A] time = 0.01, size = 180, normalized size = 1.62

$$-\frac{e\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,d^2}+\frac{e\ln\left(\frac{2a+2\sqrt{cx^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\,d^2}-\frac{\sqrt{cx^2+a}}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -(c*x^2+a)^(1/2)/a/d/x+e/d^2/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-
1/d^2*e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+
2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2
)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}\,(ex + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{c\,x^2 + a}\,\,(d + e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a + cx^2}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)), x)
```

3.244 $\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=168

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}+\frac{e\sqrt{a+cx^2}}{ad^2x}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

Rubi [A] time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {961, 266, 51, 63, 208, 264, 725, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}+\frac{e\sqrt{a+cx^2}}{ad^2x}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (e^3*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*
e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - (e^2*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \int \left(\frac{1}{dx^3\sqrt{a+cx^2}} - \frac{e}{d^2x^2\sqrt{a+cx^2}} + \frac{e^2}{d^3x\sqrt{a+cx^2}} - \frac{e^3}{d^3(d+ex)\sqrt{a+cx^2}}\right) dx$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3} - \frac{e^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^3}$$

$$= \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{e^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^3} + \frac{e^3\,\text{Subs}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x\right.}{4ad}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3} - \frac{\text{Sul}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} + \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} - \frac{e^2\,\text{ta}}{}$$

Mathematica [A] time = 0.64, size = 163, normalized size = 0.97

$$\frac{\frac{2e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}} + \frac{d\left(cdx^2\sqrt{\frac{cx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)-\left(a+cx^2\right)(d-2ex)\right)}{ax^2\sqrt{a+cx^2}} - \frac{2e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/Sqrt[
c*d^2 + a*e^2] - (2*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a] + (d*(-((
d - 2*e*x)*(a + c*x^2)) + c*d*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 + (c*x
^2)/a]]))/(a*x^2*Sqrt[a + c*x^2]))/(2*d^3)
```

IntegrateAlgebraic [A] time = 0.69, size = 203, normalized size = 1.21

$$\frac{\left(2ae^2-cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}+\frac{\sqrt{a+cx^2}\,(2ex-d)}{2ad^2x^2}-\frac{2e^3\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2-cd^2}}\right)}{d^3\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-d + 2*e*x)*Sqrt[a + c*x^2])/(2*a*d^2*x^2) - (2*e^3*Sqrt[-(c*d^2) - a*e^2
]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) -
 a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(d^3*(c*d^2 + a*e^2)
) + ((-(c*d^2) + 2*a*e^2)*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(
a^(3/2)*d^3)
```

fricas [A] time = 0.50, size = 956, normalized size = 5.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*
e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4
)*sqrt(a)*x^2*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c*
d^4 + a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d
^5 + a^3*d^3*e^2)*x^2), 1/4*(4*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^2*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*sqrt(a)*x^2*log(-(
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c*d^4 + a^2*d^2*e^2 -
2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x^
2), 1/2*(sqrt(c*d^2 + a*e^2)*a^2*e^3*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^
4)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a*c*d^4 + a^2*d^2*e^2 -
 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x
^2), 1/2*(2*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^2*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x
^2 + a)) - (a*c*d^4 + a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2
 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x^2)]
```

giac [A] time = 0.22, size = 239, normalized size = 1.42

$$-c^{\frac{3}{2}}\left(\frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^3}{\sqrt{-cd^2-ae^2}\,c^{\frac{3}{2}}d^3}+\frac{\left(cd^2-2ae^2\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,ac^{\frac{3}{2}}d^3}-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3\sqrt{c}d-2\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2ae+\left(\sqrt{c}x-\sqrt{cx^2+a}\right)a\sqrt{c}d+2a^2e}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)^2acd^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -c^(3/2)*(2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d
^2 - a*e^2))*e^3/(sqrt(-c*d^2 - a*e^2)*c^(3/2)*d^3) + (c*d^2 - 2*a*e^2)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*c^(3/2)*d^3) - ((s
qrt(c)*x - sqrt(c*x^2 + a))^3*sqrt(c)*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*sqrt(c)*d + 2*a^2*e)/(((sqrt(c)*x -
sqrt(c*x^2 + a))^2 - a)^2*a*c*d^2))
```

maple [A] time = 0.01, size = 236, normalized size = 1.40

$$\frac{e^2 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\, d^3}-\frac{e^2 \ln\left(\frac{2a+2\sqrt{c\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\, d^3}+\frac{c\ln\left(\frac{2a+2\sqrt{c\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}d}+\frac{\sqrt{c\,x^2+a}\, e}{a\,d^2 x}-\frac{\sqrt{c\,x^2+a}}{2ad\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] e*(c*x^2+a)^(1/2)/a/d^2/x-1/2*(c*x^2+a)^(1/2)/a/d/x^2+1/2/d*c/a^(3/2)*ln((2
*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-1/d^3*e^2/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2
)*a^(1/2))/x)+1/d^3*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a
*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(
a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)), x)
```

3.245 $\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2}+\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{\sqrt{a+cx^2}}{c^2e}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}$$

Rubi [A] time = 0.31, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1647, 1654, 844, 217, 206, 725}

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2}+\frac{\sqrt{a+cx^2}}{c^2e}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(a*e + c*d*x))/(c^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + Sqrt[a + c*x^2]/(
c^2*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^2) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e
^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{\frac{a^2d^2}{cd^2+ae^2}-ax^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{\int \frac{\frac{a^2cd^2e^2}{cd^2+ae^2}+acdex}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac^2e^2}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce^2} + \frac{d^4\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^2\left(cd^2+ae^2\right)}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{ce^2} - \frac{d^4\,\text{Subst}\Big(}{}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} - \frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a-}}\right.}{e^2\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.44, size = 179, normalized size = 1.23

$$\frac{\frac{e\left(2a^2e^2+ac\left(d^2+dex+e^2x^2\right)+c^2d^2x^2\right)}{c^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)} - \frac{\sqrt{a}\,d\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}\sqrt{a+cx^2}} - \frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] ((e*(2*a^2*e^2 + c^2*d^2*x^2 + a*c*(d^2 + d*e*x + e^2*x^2)))/(c^2*(c*d^2 +
a*e^2)*Sqrt[a + c*x^2]) - (Sqrt[a]*d*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x
)/Sqrt[a]])/(c^(3/2)*Sqrt[a + c*x^2]) - (d^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*
d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2))/e^2
```

IntegrateAlgebraic [A] time = 0.85, size = 223, normalized size = 1.53

$$\frac{2a^2e^2+acd^2+acdex+ace^2x^2+c^2d^2x^2}{c^2e\sqrt{a+cx^2}\left(ae^2+cd^2\right)} + \frac{d\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{c^{3/2}e^2} + \frac{2d^4\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*c*d^2 + 2*a^2*e^2 + a*c*d*e*x + c^2*d^2*x^2 + a*c*e^2*x^2)/(c^2*e*(c*d^2
 + a*e^2)*Sqrt[a + c*x^2]) + (2*d^4*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*
d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[
a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(e^2*(c*d^2 + a*e^2)^2) + (d*Log[-(Sqr
t[c]*x) + Sqrt[a + c*x^2]])/(c^(3/2)*e^2)
```

fricas [B] time = 4.65, size = 1525, normalized size = 10.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 + 2*a*c^2*d^3*e^2
 + a^2*c*d*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^4*e + 3*a^2*c
*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c
^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^
2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c^4*d^2*e^4 + a^2*c^3*e^6)*x^2), -
1/2*(2*(c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 + 2*a*c^2*d^
3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) - 2*(a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^
2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 +
a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c
^4*d^2*e^4 + a^2*c^3*e^6)*x^2), 1/2*(2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d
*e^4 + (c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log(
(2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d
^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(
a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^2*d^2*e^3 +
a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d
^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c^4*d^2*e^4 +
 a^2*c^3*e^6)*x^2), -((c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arctan
(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c
^2*d^2 + a*c*e^2)*x^2)) - (a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d
^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*
x^2 + a)) - (a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c
^2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 +
 a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*
c^4*d^2*e^4 + a^2*c^3*e^6)*x^2)]
```

giac [B] time = 0.27, size = 299, normalized size = 2.05

$$\frac{2\,d^4\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^2e^2+ae^4\right)\sqrt{-cd^2-ae^2}}+\frac{de^{(-2)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}+\frac{\left(\frac{\left(c^4d^4e^5+2\,ac^3d^2e^7+a^2c^2e^9\right)x}{c^5d^4e^6+2\,ac^4d^2e^8+a^2c^3e^{10}}+\frac{ac^3d^3e^6+a^2c^2de^8}{c^5d^4e^6+2\,ac^4d^2e^8+a^2c^3e^{10}}\right)x+\frac{ac^3d^4e^5+3\,a^2c^2d^2e^7+2\,a^3ce^9}{c^5d^4e^6+2\,ac^4d^2e^8+a^2c^3e^{10}}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*d^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2*e^2 + a*e^4)*sqrt(-c*d^2 - a*e^2)) + d*e^(-2)*log(abs(-sqrt(
c)*x + sqrt(c*x^2 + a)))/c^(3/2) + (((c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c
```

```
^2*e^9)*x/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 + a^2*c^3*e^10) + (a*c^3*d^3*e^6 +
 a^2*c^2*d*e^8)/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 + a^2*c^3*e^10))*x + (a*c^3*
d^4*e^5 + 3*a^2*c^2*d^2*e^7 + 2*a^3*c*e^9)/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 +
 a^2*c^3*e^10))/sqrt(c*x^2 + a)
```

maple [B] time = 0.02, size = 396, normalized size = 2.71

$$\frac{c\,d^5x}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\,a\,e^4}-\frac{d^4\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e^3}+\frac{d^4}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\,e^3}+\frac{x^2}{\sqrt{c\,x^2+a}\,ce}-\frac{d^3x}{\sqrt{c\,x^2+a}\,a\,e^4}+\frac{dx}{\sqrt{c\,x^2+a}\,c\,e^2}-\frac{d\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}e^2}+\frac{2a}{\sqrt{c\,x^2+a}\,c^2e}-\frac{d^2}{\sqrt{c\,x^2+a}\,c\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/e*x^2/c/(c*x^2+a)^(1/2)+2/e*a/c^2/(c*x^2+a)^(1/2)+d/e^2*x/c/(c*x^2+a)^(1/
2)-d/e^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-d^2/e^3/c/(c*x^2+a)^(1/2)-d^
3/e^4*x/a/(c*x^2+a)^(1/2)+d^4/e^3/(a*e^2+c*d^2)/(-2*(x+d/e)*c*d/e+(x+d/e)^2
*c+(a*e^2+c*d^2)/e^2)^(1/2)+d^5/e^4/(a*e^2+c*d^2)/a/(-2*(x+d/e)*c*d/e+(x+d/
e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x-d^4/e^3/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^
2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/
2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.62, size = 251, normalized size = 1.72

$$\frac{cd^5x}{\sqrt{cx^2+a}\,acd^2e^4+\sqrt{cx^2+a}\,a^2e^6}+\frac{d^4}{\sqrt{cx^2+a}\,cd^2e^3+\sqrt{cx^2+a}\,ae^5}+\frac{x^2}{\sqrt{cx^2+a}\,ce}-\frac{d^3x}{\sqrt{cx^2+a}\,ae^4}+\frac{dx}{\sqrt{cx^2+a}\,ce^2}-\frac{d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}e^2}+\frac{d^4\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^5}-\frac{d^2}{\sqrt{cx^2+a}\,ce^3}+\frac{2\,a}{\sqrt{cx^2+a}\,c^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*d^5*x/(sqrt(c*x^2 + a)*a*c*d^2*e^4 + sqrt(c*x^2 + a)*a^2*e^6) + d^4/(sqrt
(c*x^2 + a)*c*d^2*e^3 + sqrt(c*x^2 + a)*a*e^5) + x^2/(sqrt(c*x^2 + a)*c*e)
- d^3*x/(sqrt(c*x^2 + a)*a*e^4) + d*x/(sqrt(c*x^2 + a)*c*e^2) - d*arcsinh(c
*x/sqrt(a*c))/(c^(3/2)*e^2) + d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) -
a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e^5) - d^2/(sqrt(c*x^2
 + a)*c*e^3) + 2*a/(sqrt(c*x^2 + a)*c^2*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**4/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.246 $\int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}} \, dx$

Optimal. Leaf size=123

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\,\left(ae^2+cd^2\right)}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e\left(ae^2+cd^2\right)^{3/2}}$$

Rubi [A] time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1647, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\,\left(ae^2+cd^2\right)}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(d - e*x))/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]]/(c^(3/2)*e) + (d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]
*Sqrt[a + c*x^2])])/(e*(c*d^2 + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx &= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{\int \frac{-\frac{a^2de}{cd^2+ae^2}-ax}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac}\\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce}-\frac{d^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e\left(cd^2+ae^2\right)}\\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{ce}+\frac{d^3\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,\right.}{e\left(cd^2+ae^2\right)}\\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{e\left(cd^2+ae^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.28, size = 153, normalized size = 1.24

$$
\frac{\frac{\sqrt{c}\left(ae(d-ex)\sqrt{ae^2+cd^2}+cd^3\sqrt{a+cx^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)\right)}{\left(ae^2+cd^2\right)^{3/2}}+\sqrt{a}\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}e\sqrt{a+cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] + (Sqrt[c]*(a*e*S
qrt[c*d^2 + a*e^2]*(d - e*x) + c*d^3*Sqrt[a + c*x^2]*ArcTanh[(a*e - c*d*x)/
(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]))/(c*d^2 + a*e^2)^(3/2))/(c^(3/2)*e*
Sqrt[a + c*x^2])
```

IntegrateAlgebraic [A] time = 0.69, size = 189, normalized size = 1.54

$$
-\frac{\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{c^{3/2}e}+\frac{ad-aex}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{2d^3\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{e\left(ae^2+cd^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*d - a*e*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*d^3*Sqrt[-(c*d^2) -
a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d
^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(e*(c*d^2 + a*e
^2)^2) - Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/(c^(3/2)*e)
```

fricas [B] time = 4.69, size = 1323, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
```

```
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^3*e + a^2*c*d*e^3
 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3
*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^2),
1/2*(2*(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*
e^2 + a^2*c*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x -
a) + 2*(a*c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x
^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*
c^4*d^2*e^3 + a^2*c^3*e^5)*x^2), -1/2*(2*(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log((2
*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2
 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(a*
c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x^2 + a))/(
a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^
3 + a^2*c^3*e^5)*x^2), ((c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d
^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^
2 + a)) + (a*c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(
c*x^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2
*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^2)]
```

giac [A] time = 0.24, size = 219, normalized size = 1.78

$$-\frac{2\,d^3\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^2e+ae^3\right)\sqrt{-cd^2-ae^2}}-\frac{\frac{\left(ac^2d^2e^3+a^2ce^5\right)x}{c^4d^4e^2+2\,ac^3d^2e^4+a^2c^2e^6}-\frac{ac^2d^3e^2+a^2cde^4}{c^4d^4e^2+2\,ac^3d^2e^4+a^2c^2e^6}}{\sqrt{cx^2+a}}-\frac{e^{(-1)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*d^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
a*e^2))/((c*d^2*e + a*e^3)*sqrt(-c*d^2 - a*e^2)) - ((a*c^2*d^2*e^3 + a^2*c*
e^5)*x/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6) - (a*c^2*d^3*e^2 + a^2
*c*d*e^4)/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6))/sqrt(c*x^2 + a) -
e^(-1)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

maple [B] time = 0.01, size = 354, normalized size = 2.88

$$-\frac{c\,d^4x}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;a\,e^3}+\frac{d^3\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e^2}-\frac{d^3}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;e^2}+\frac{d^2x}{\sqrt{c\,x^2+a}\;a\,e^3}-\frac{x}{\sqrt{c\,x^2+a}\;ce}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}e}+\frac{d}{\sqrt{c\,x^2+a}\;c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/e*x/c/(c*x^2+a)^(1/2)+1/e/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+d/e^2/c/
(c*x^2+a)^(1/2)+d^2/e^3*x/a/(c*x^2+a)^(1/2)-d^3/e^2/(a*e^2+c*d^2)/(-2*(x+d/
e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-d^4/e^3/(a*e^2+c*d^2)/a/(-2*(
x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x+d^3/e^2/(a*e^2+c*d^2)
/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e
^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)
)/(x+d/e))
```

maxima [A] time = 0.59, size = 211, normalized size = 1.72

$$-\frac{cd^4x}{\sqrt{cx^2+a}\,acd^2e^3+\sqrt{cx^2+a}\,a^2e^5}-\frac{d^3}{\sqrt{cx^2+a}\,cd^2e^2+\sqrt{cx^2+a}\,ae^4}+\frac{d^2x}{\sqrt{cx^2+a}\,ae^3}-\frac{x}{\sqrt{cx^2+a}\,ce}+\frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}e}-\frac{d^3\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^4}+\frac{d}{\sqrt{cx^2+a}\,ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -c*d^4*x/(sqrt(c*x^2 + a)*a*c*d^2*e^3 + sqrt(c*x^2 + a)*a^2*e^5) - d^3/(sqr
t(c*x^2 + a)*c*d^2*e^2 + sqrt(c*x^2 + a)*a*e^4) + d^2*x/(sqrt(c*x^2 + a)*a*
e^3) - x/(sqrt(c*x^2 + a)*c*e) + arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e) - d^3*a
rcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a +
 c*d^2/e^2)^(3/2)*e^4) + d/(sqrt(c*x^2 + a)*c*e^2)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+cx^2\right)^{\frac{3}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.247 $\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Rubi [A] time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1647, 12, 725, 206}

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*e + c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) - (d^2*ArcTanh[(a*e -
c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{acd^2}{\left(cd^2+ae^2\right)(d+ex)\sqrt{a+cx^2}}\,dx}{ac}$$
$$= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{d^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$
$$= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{d^2\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$
$$= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.08, size = 95, normalized size = 1.00

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*e + c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) - (d^2*ArcTanh[(a*e -
c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.51, size = 156, normalized size = 1.64

$$\frac{-ae-cdx}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{2d^2\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a*e) - c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (2*d^2*Sqrt[-(c*d^2)
 - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(
c*d^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(c*d^2 + a*e
^2)^2
```

fricas [B] time = 0.47, size = 455, normalized size = 4.79

$$\left[\frac{\left(c^2d^2x^2+acd^2\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(acd^2e+a^2e^3+\left(c^2d^3+acde^2\right)x\right)\sqrt{cx^2+a}}{2\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)x^2\right)}, -\frac{\left(c^2d^2x^2+acd^2\right)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)+\left(acd^2e+a^2e^3+\left(c^2d^3+acde^2\right)x\right)\sqrt{cx^2+a}}{ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*d^2*x^2 + a*c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(a*c*d^2*e + a^2*e^3 +
 (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 +
 a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^2), -((c^2*d^2*x^2
 + a*c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
```

```
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (a*c*d^2*e
 + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*d^4 + 2*a^2*c
^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^2)]
```

giac [A] time = 0.22, size = 174, normalized size = 1.83

$$-\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^2+ae^2\right)\sqrt{-cd^2-ae^2}}-\frac{\frac{\left(c^2d^3+acde^2\right)x}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}+\frac{acd^2e+a^2e^3}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*d^2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) - ((c^2*d^3 + a*c*d*e^2)*x/(c
^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4) + (a*c*d^2*e + a^2*e^3)/(c^3*d^4 + 2*
a*c^2*d^2*e^2 + a^2*c*e^4))/sqrt(c*x^2 + a)
```

maple [B] time = 0.01, size = 311, normalized size = 3.27

$$\frac{c\,d^3x}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;a\,e^2}-\frac{d^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e}+\frac{d^2}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;e}-\frac{dx}{\sqrt{c\,x^2+a}\;a\,e^2}-\frac{1}{\sqrt{c\,x^2+a}\;ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/e/c/(c*x^2+a)^(1/2)-1/e^2*d*x/a/(c*x^2+a)^(1/2)+d^2/e/(a*e^2+c*d^2)/(-2*
(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+d^3/e^2/(a*e^2+c*d^2)/a/
(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x-d^2/e/(a*e^2+c*d
^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((
a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1
/2))/(x+d/e))
```

maxima [A] time = 0.55, size = 171, normalized size = 1.80

$$\frac{cd^3x}{\sqrt{cx^2+a}\,acd^2e^2+\sqrt{cx^2+a}\,a^2e^4}+\frac{d^2}{\sqrt{cx^2+a}\,cd^2e+\sqrt{cx^2+a}\,ae^3}-\frac{dx}{\sqrt{cx^2+a}\,ae^2}+\frac{d^2\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^3}-\frac{1}{\sqrt{cx^2+a}\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*d^3*x/(sqrt(c*x^2 + a)*a*c*d^2*e^2 + sqrt(c*x^2 + a)*a^2*e^4) + d^2/(sqrt
(c*x^2 + a)*c*d^2*e + sqrt(c*x^2 + a)*a*e^3) - d*x/(sqrt(c*x^2 + a)*a*e^2)
+ d^2*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d))
)/((a + c*d^2/e^2)^(3/2)*e^3) - 1/(sqrt(c*x^2 + a)*c*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a + cx^2\right)^{\frac{3}{2}} (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.248 $\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{d-ex}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)}$$

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {823, 12, 725, 206}

$$\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{d-ex}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((d - e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*e*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx &= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{\int \frac{acde}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac\left(cd^2+ae^2\right)}\\ &= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{(de)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\ &= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{(de)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\ &= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 1.00

$$\frac{ex-d}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-d + e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (d*e*ArcTanh[(a*e - c*d*x)/(
Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.53, size = 149, normalized size = 1.69

$$\frac{ex-d}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{2de\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-d + e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*d*e*Sqrt[-(c*d^2) - a*e^2
]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) -
 a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(c*d^2 + a*e^2)^2
```

fricas [B] time = 0.48, size = 425, normalized size = 4.83

$$\left[\frac{\left(cdex^2+ade\right)\sqrt{cd^2+ae^2}\,\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(cd^3+ade^2-\left(cd^2e+ae^3\right)x\right)\sqrt{cx^2+a}}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4+\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)x^2\right)},\ \frac{\left(cdex^2+ade\right)\sqrt{-cd^2-ae^2}\,\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)-\left(cd^3+ade^2-\left(cd^2e+ae^3\right)x\right)\sqrt{cx^2+a}}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4+\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x^2 + a*d*e)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^3 + a*d*e^2 - (c*d^2*
e + a*e^3)*x)*sqrt(c*x^2 + a))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^
3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2), ((c*d*e*x^2 + a*d*e)*sqrt(-c*d^2
 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^
2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c*d^3 + a*d*e^2 - (c*d^2*e + a*e
```

```
^3)*x)*sqrt(c*x^2 + a))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 +
 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)]
```

giac [A] time = 0.21, size = 162, normalized size = 1.84

$$\frac{2\,d\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e}{\left(cd^2+ae^2\right)\sqrt{-cd^2-ae^2}}+\frac{\frac{\left(cd^2e+ae^3\right)x}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{cd^3+ade^2}{c^2d^4+2\,acd^2e^2+a^2e^4}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*d*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^
2))*e/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) + ((c*d^2*e + a*e^3)*x/(c^2*d^
4 + 2*a*c*d^2*e^2 + a^2*e^4) - (c*d^3 + a*d*e^2)/(c^2*d^4 + 2*a*c*d^2*e^2 +
 a^2*e^4))/sqrt(c*x^2 + a)
```

maple [B] time = 0.01, size = 283, normalized size = 3.22

$$-\frac{c\,d^2x}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;ae}+\frac{d\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{ae^2+cd^2}{e^2}}}-\frac{d}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}+\frac{x}{\sqrt{c\,x^2+a}\;ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/e*x/a/(c*x^2+a)^(1/2)-d/(a*e^2+c*d^2)/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^
2+c*d^2)/e^2)^(1/2)-d^2/e/(a*e^2+c*d^2)/a/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*
e^2+c*d^2)/e^2)^(1/2)*c*x+d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*
(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c
*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.54, size = 148, normalized size = 1.68

$$-\frac{cd^2x}{\sqrt{cx^2+a}\,acd^2e+\sqrt{cx^2+a}\,a^2e^3}-\frac{d}{\sqrt{cx^2+a}\,cd^2+\sqrt{cx^2+a}\,ae^2}+\frac{x}{\sqrt{cx^2+a}\,ae}-\frac{d\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -c*d^2*x/(sqrt(c*x^2 + a)*a*c*d^2*e + sqrt(c*x^2 + a)*a^2*e^3) - d/(sqrt(c*
x^2 + a)*c*d^2 + sqrt(c*x^2 + a)*a*e^2) + x/(sqrt(c*x^2 + a)*a*e) - d*arcsi
nh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d
^2/e^2)^(3/2)*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+cx^{2}\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.249 $\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {741, 12, 725, 206}

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{e^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{e^2\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.05, size = 94, normalized size = 1.00

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.01, size = 154, normalized size = 1.64

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{2e^2\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2-cd^2}}\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (2*e^2*Sqrt[-(c*d^2) -
a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d
^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(c*d^2 + a*e^2)
^2
```

fricas [B] time = 0.49, size = 456, normalized size = 4.85

$$\left[\frac{\left(ace^2x^2+a^2e^2\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+2\left(acd^2e+a^2e^3+\left(c^2d^3+acde^2\right)x\right)\sqrt{cx^2+a}}{2\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4\right)x^2\right)}, -\frac{\left(ace^2x^2+a^2e^2\right)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)-\left(acd^2e+a^2e^3+\left(c^2d^3+acde^2\right)x\right)\sqrt{cx^2+a}}{a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*e^2*x^2 + a^2*e^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c*d^2*e + a^2*e^3 +
 (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2), -((a*c*e^2*x^2
 + a^2*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
```

```
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (a*c*d^2*e
 + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3
*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2)]
```

giac [A] time = 0.23, size = 172, normalized size = 1.83

$$\frac{\frac{(c^2d^3+acde^2)x}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}+\frac{acd^2e+a^2e^3}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}}{\sqrt{cx^2+a}}-\frac{2\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^2}{\left(cd^2+ae^2\right)\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^2*d^3 + a*c*d*e^2)*x/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) + (a*c*d^2
*e + a^2*e^3)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))/sqrt(c*x^2 + a) - 2*
arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))*
e^2/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2))
```

maple [B] time = 0.01, size = 260, normalized size = 2.77

$$\frac{cdx}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;a}-\frac{e\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}+\frac{e}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] e/(a*e^2+c*d^2)/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+d/(a
*e^2+c*d^2)/a/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x-e/
(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2
)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^
2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.50, size = 123, normalized size = 1.31

$$\frac{cdx}{\sqrt{cx^2+a}\,acd^2+\sqrt{cx^2+a}\,a^2e^2}+\frac{1}{\frac{\sqrt{cx^2+a}\,cd^2}{e}+\sqrt{cx^2+a}\,ae}+\frac{\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*d*x/(sqrt(c*x^2 + a)*a*c*d^2 + sqrt(c*x^2 + a)*a^2*e^2) + 1/(sqrt(c*x^2 +
 a)*c*d^2/e + sqrt(c*x^2 + a)*a*e) + arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d))
 - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.250 $\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{e(ae+cdx)}{ad\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}+\frac{1}{ad\sqrt{a+cx^2}}$$

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {961, 266, 51, 63, 208, 741, 12, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{e(ae+cdx)}{ad\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}+\frac{1}{ad\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (e*(a*e + c*d*x))/(a*d*(c*d^2 + a*e^2)*Sqrt[a + c
*x^2]) + (e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])
/(d*(c*d^2 + a*e^2)^(3/2)) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx=\int \left(\frac{1}{dx\left(a+cx^2\right)^{3/2}}-\frac{e}{d(d+ex)\left(a+cx^2\right)^{3/2}}\right)dx$$

$$=\frac{\int \frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d}-\frac{e\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx}{d}$$

$$=-\frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx,x,x^2\right)}{2d}-\frac{e\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ad\left(cd^2+ae^2\right)}$$

$$=\frac{1}{ad\sqrt{a+cx^2}}-\frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2ad}-\frac{e^3\int \frac{}{(d+ex)}}{d\left(cd^2\right.}$$

$$=\frac{1}{ad\sqrt{a+cx^2}}-\frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{acd}+e^3\,\mathrm{S}$$

$$=\frac{1}{ad\sqrt{a+cx^2}}-\frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\left(cd^2+ae^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\right.}{a^{3/2}}$$

Mathematica [C] time = 0.17, size = 132, normalized size = 0.90

$$\frac{-\frac{e(ae+cdx)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((e*(a*e + c*d*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (e^3*ArcTanh[(a
*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) +
 Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/a]/(a*Sqrt[a + c*x^2]))/d
```

IntegrateAlgebraic [A] time = 1.14, size = 200, normalized size = 1.36

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{cd-cex}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{2e^3\sqrt{-ae^2-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2-cd^2}}\right)}{d\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (c*d - c*e*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*e^3*Sqrt[-(c*d^2) -
a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d
^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(d*(c*d^2 + a*e
^2)^2) + (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2
)*d)
```

fricas [B] time = 0.69, size = 1325, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*e^3*x^2 + a^3*e^3)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*
d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(a)*log
(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^4 + a^2*c*d^2
*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4
*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^
2), 1/2*(2*(a^2*c*e^3*x^2 + a^3*e^3)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*
d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) +
 2*a)/x^2) + 2*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*
sqrt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5
+ 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^2), 1/2*(2*(a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-a)*arctan(
sqrt(-a)/sqrt(c*x^2 + a)) + (a^2*c*e^3*x^2 + a^3*e^3)*sqrt(c*d^2 + a*e^2)*l
og((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) +
2*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*x^2 +
a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d
^3*e^2 + a^4*c*d*e^4)*x^2), ((a^2*c*e^3*x^2 + a^3*e^3)*sqrt(-c*d^2 - a*e^2)
*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e
^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c
*x^2 + a)) + (a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sq
rt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 +
2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 318, normalized size = 2.16

$$-\frac{cex}{\left(a e^{2}+c d^{2}\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a e^{2}+c d^{2}}{e^{2}}}\, a}+\frac{e^{2}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^{2}+2c d^{2}}{e^{2}}+2\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a e^{2}+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(a e^{2}+c d^{2}\right)\sqrt{\frac{a e^{2}+c d^{2}}{e^{2}}}\, d}-\frac{e^{2}}{\left(a e^{2}+c d^{2}\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a e^{2}+c d^{2}}{e^{2}}}\, d}-\frac{\ln\left(\frac{2a+2\sqrt{c x^{2}+a}\,\sqrt{a}}{x}\right)}{a^{\frac{3}{2}}d}+\frac{1}{\sqrt{c x^{2}+a}\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/a/d/(c*x^2+a)^(1/2)-1/d/a^(3/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-1/d
/(a*e^2+c*d^2)*e^2/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-e
/(a*e^2+c*d^2)/a/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x
+1/d/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*
e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a
*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2}+a\right)^{\frac{3}{2}}(ex+d)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(c\,x^{2}+a\right)^{3/2}\,(d+e\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+cx^{2}\right)^{\frac{3}{2}}(d+ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.251 $\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{}{adx\cdots}$$

Rubi [A] time = 0.17, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {961, 271, 191, 266, 51, 63, 208, 741, 12, 725, 206}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (e^2*(a*e + c*d*x))/(a*d^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2
]) - (e^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^
2*(c*d^2 + a*e^2)^(3/2)) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \left(\frac{1}{dx^2\left(a+cx^2\right)^{3/2}} - \frac{e}{d^2x\left(a+cx^2\right)^{3/2}} + \frac{e^2}{d^2(d+ex)\left(a+cx^2\right)^{3/2}}\right)dx$$

$$= \frac{\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}}\,dx}{d} - \frac{e\int \frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx}{d^2}$$

$$= -\frac{1}{adx\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{(2c)\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{ad} - \frac{e\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} -$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} -$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} -$$

Mathematica [C] time = 0.40, size = 163, normalized size = 0.84

$$-\frac{\frac{d\left(a+2cx^2\right)}{a^2x\sqrt{a+cx^2}} - \frac{e^2(ae+cdx)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)} + \frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{e\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((-((e^2*(a*e + c*d*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*(a + 2*c
*x^2))/(a^2*x*Sqrt[a + c*x^2]) + (e^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a
*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e*Hypergeometric2F1[-1/2,
 1, 1/2, 1 + (c*x^2)/a])/(a*Sqrt[a + c*x^2]))/d^2)
```

IntegrateAlgebraic [A] time = 0.73, size = 242, normalized size = 1.25

$$-\frac{2e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}} - \frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2} + \frac{-a^2e^2 - acd^2 - acdex - ace^2x^2 - 2c^2d^2x^2}{a^2dx\sqrt{a+cx^2}\left(ae^2+cd^2\right)} + \frac{2e^4\sqrt{-ae^2-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}ex}{\sqrt{-ae^2-cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2-cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a*c*d^2) - a^2*e^2 - a*c*d*e*x - 2*c^2*d^2*x^2 - a*c*e^2*x^2)/(a^2*d*(c*
d^2 + a*e^2)*x*Sqrt[a + c*x^2]) + (2*e^4*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqr
t[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*
Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(d^2*(c*d^2 + a*e^2)^2) - (2*e*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2)
```

fricas [B] time = 0.72, size = 1556, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*
c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + ((c^3*d^4*e + 2*a*c^
2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x)*s
qrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c^2*d^5 +
 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*
x^2 + (a*c^2*d^4*e + a^2*c*d^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a
^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*
d^2*e^4)*x), -1/2*(2*(a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(-c*d^2 - a*e^2)*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) - ((c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 +
 (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(
c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4
 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c^2*d^4*e + a^2*c*d
^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e
^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x), -1/2*(2*((c^3*d
^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 +
a^3*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^2*c*e^4*x^3 + a^
3*e^4*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^
2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))
/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 +
(2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c^2*d^4*e + a^2*c*d^2*
e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)
*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x), -((a^2*c*e^4*x^3 +
 a^3*e^4*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + ((c^3*d^4*
e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3
*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (a*c^2*d^5 + 2*a^2*c*d
^3*e^2 + a^3*d*e^4 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c
^2*d^4*e + a^2*c*d^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4
*e^2 + a^4*c*d^2*e^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x
)]
```

giac [A] time = 0.25, size = 266, normalized size = 1.37

$$-\frac{\frac{\left(ac^3d^3+a^2c^2de^2\right)x}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}+\frac{a^2c^2d^2e+a^3ce^3}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}}{\sqrt{cx^2+a}}-\frac{2\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^4}{\left(cd^4+ad^2e^2\right)\sqrt{-cd^2-ae^2}}-\frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,ad^2}+\frac{2\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((a*c^3*d^3 + a^2*c^2*d*e^2)*x/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4) +
 (a^2*c^2*d^2*e + a^3*c*e^3)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))/sqr
t(c*x^2 + a) - 2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(
-c*d^2 - a*e^2))*e^4/((c*d^4 + a*d^2*e^2)*sqrt(-c*d^2 - a*e^2)) - 2*arctan(
-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))*e/(sqrt(-a)*a*d^2) + 2*sqrt(c)/(((
sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*a*d)
```

maple [B] time = 0.01, size = 363, normalized size = 1.87

$$\frac{c\,e^2x}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;ad}-\frac{e^3\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;d^2}+\frac{e^3}{\left(a\,e^2+c\,d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\;d^2}-\frac{2cx}{\sqrt{c\,x^2+a}\;a^2d}+\frac{e\ln\left(\frac{2a+2\sqrt{c\,x^2+a}\,\sqrt{a}}{x}\right)}{a^{\frac{3}{2}}d^2}-\frac{e}{\sqrt{c\,x^2+a}\;a\,d^2}-\frac{1}{\sqrt{c\,x^2+a}\;adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/a/d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(c*x^2+a)^(1/2)-e/a/d^2/(c*x^2+a)^(1/2
)+e/d^2/a^(3/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)+e^3/d^2/(a*e^2+c*d^2)
/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+e^2/d/(a*e^2+c*d^2)
/a/(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x-e^3/d^2/(a*e^
2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2
+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^
2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.252 $\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=276

$$-\frac{e^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}+\frac{3c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{3c}{2a^2d\sqrt{a+cx^2}}+\frac{e^2}{ad^3\sqrt{a+cx^2}}+\frac{e}{ad^2x\sqrt{a+cx^2}}+\frac{e^5 \tanh}{}$$

Rubi [A] time = 0.24, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {961, 266, 51, 63, 208, 271, 191, 741, 12, 725, 206}

$$-\frac{e^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{3\sqrt{a+cx^2}}{2a^2dx^2}+\frac{3c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{e^3(ae+cdx)}{ad^3\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^2}{ad^3\sqrt{a+cx^2}}+\frac{e^5 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\left(ae^2+cd^2\right)^{3/2}}+\frac{e}{ad^2x\sqrt{a+cx^2}}+\frac{1}{adx^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] e^2/(a*d^3*Sqrt[a + c*x^2]) + 1/(a*d*x^2*Sqrt[a + c*x^2]) + e/(a*d^2*x*Sqrt
[a + c*x^2]) + (2*c*e*x)/(a^2*d^2*Sqrt[a + c*x^2]) - (e^3*(a*e + c*d*x))/(a
*d^3*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (3*Sqrt[a + c*x^2])/(2*a^2*d*x^2) +
 (e^5*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*(c
*d^2 + a*e^2)^(3/2)) + (3*c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(5/2)*d)
 - (e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \left(\frac{1}{dx^3\left(a+cx^2\right)^{3/2}} - \frac{e}{d^2x^2\left(a+cx^2\right)^{3/2}} + \frac{e^2}{d^3x\left(a+cx^2\right)^{3/2}} - \frac{e^3}{d^3(d+ex)\left(a+cx^2\right)\right.$$

$$= \frac{\int \frac{1}{x^3\left(a+cx^2\right)^{3/2}}dx}{d} - \frac{e\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}}dx}{d^2} + \frac{e^2\int \frac{1}{x\left(a+cx^2\right)^{3/2}}dx}{d^3} - \frac{e^3\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}dx}{d^3}$$

$$= \frac{e}{ad^2x\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x^2(a+cx)^{3/2}}dx, x, x^2\right)}{2d} + \frac{(2ce}{}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae}{ad^3\left(cd^2+ae\right.}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae}{ad^3\left(cd^2+ae\right.}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae}{ad^3\left(cd^2+ae\right.}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae}{ad^3\left(cd^2+ae\right.}$$

Mathematica [C] time = 0.34, size = 203, normalized size = 0.74

$$\frac{-\frac{cd^2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a^2\sqrt{a+cx^2}} + \frac{de\left(a+2cx^2\right)}{a^2x\sqrt{a+cx^2}} + \frac{e^5\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{e^3(ae+cdx)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)} + \frac{e^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((e^3*(a*e + c*d*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*e*(a + 2*c
*x^2))/(a^2*x*Sqrt[a + c*x^2]) + (e^5*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a
*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e^2*Hypergeometric2F1[-1/
2, 1, 1/2, 1 + (c*x^2)/a])/(a*Sqrt[a + c*x^2]) - (c*d^2*Hypergeometric2F1[-
1/2, 2, 1/2, 1 + (c*x^2)/a])/(a^2*Sqrt[a + c*x^2]))/d^3
```

IntegrateAlgebraic [A] time = 1.02, size = 287, normalized size = 1.04

$$\frac{\left(2ae^2-3cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{5/2}d^3} + \frac{-a^2de^2+2a^2e^3x-acd^3+2acd^2ex-acde^2x^2+2ace^3x^3-3c^2d^3x^2+4c^2d^2ex^3}{2a^2d^2x^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)} - \frac{2e^5\sqrt{-ae^2-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2-cd^2}}\right)}{d^3\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a*c*d^3) - a^2*d*e^2 + 2*a*c*d^2*e*x + 2*a^2*e^3*x - 3*c^2*d^3*x^2 - a*c
*d*e^2*x^2 + 4*c^2*d^2*e*x^3 + 2*a*c*e^3*x^3)/(2*a^2*d^2*(c*d^2 + a*e^2)*x^
2*Sqrt[a + c*x^2]) - (2*e^5*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[
-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) - a*e^2] - (e*Sqrt[a + c*x^
```

```
2])/Sqrt[-(c*d^2) - a*e^2]])/(d^3*(c*d^2 + a*e^2)^2) + ((-3*c*d^2 + 2*a*e^2
)*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(a^(5/2)*d^3)
```

fricas [A] time = 1.04, size = 1943, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
 - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - ((3*c^4*d^6 + 4*
a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c
^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c
*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*
e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 + (3*a*c^3*d^
6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e + 2*a^3*c*d^3
*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*d^5*e^2 + a
^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4)*x^2), 1/4
*(4*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
 a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e
^2)*x^2)) - ((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^6)*
x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)*sq
rt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a^2*c^2*d^6
+ 2*a^3*c*d^4*e^2 + a^4*d^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^
3*c*d*e^5)*x^3 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*
(a^2*c^2*d^5*e + 2*a^3*c*d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3
*d^7 + 2*a^4*c^2*d^5*e^2 + a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*
e^2 + a^6*d^3*e^4)*x^2), -1/2*(((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*
e^4 - 2*a^3*c*e^6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 -
 2*a^4*e^6)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^3*c*e^5*x^4
 + a^4*e^5*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d
^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 + (3*a*c^3
*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e + 2*a^3*c*
d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*d^5*e^2
+ a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4)*x^2),
1/2*(2*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) - ((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^
6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)
*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^2*c^2*d^6 + 2*a^3*c*d^4*e^2
 + a^4*d^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 +
(3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e +
2*a^3*c*d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*
d^5*e^2 + a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4
)*x^2)]
```

giac [A] time = 0.29, size = 358, normalized size = 1.30

$$\frac{\frac{\left(a^{2} c^{3} d^{2} e+a^{3} c^{2} e^{3}\right) x}{a^{4} c^{2} d^{4}+2 \, a^{5} c d^{2} e^{2}+a^{6} e^{4}}-\frac{a^{2} c^{3} d^{3}+a^{3} c^{2} d e^{2}}{a^{4} c^{2} d^{4}+2 \, a^{5} c d^{2} e^{2}+a^{6} e^{4}}}{\sqrt{c x^{2}+a}}-\frac{2 \, \arctan\left(-\frac{\left(\sqrt{c} x-\sqrt{c x^{2}+a}\right) e+\sqrt{c} d}{\sqrt{-c d^{2}-a e^{2}}}\right) e^{5}}{\left(c d^{5}+a d^{3} e^{2}\right) \sqrt{-c d^{2}-a e^{2}}}-\frac{\left(3 \, c d^{2}-2 \, a e^{2}\right) \arctan\left(-\frac{\sqrt{c} x-\sqrt{c x^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a} a^{2} d^{3}}+\frac{\left(\sqrt{c} x-\sqrt{c x^{2}+a}\right)^{3} c d-2 \, \left(\sqrt{c} x-\sqrt{c x^{2}+a}\right)^{2} a \sqrt{c} e+\left(\sqrt{c} x-\sqrt{c x^{2}+a}\right) a c d+2 \, a^{2} \sqrt{c} e}{\left(\left(\sqrt{c} x-\sqrt{c x^{2}+a}\right)^{2}-a\right)^{2} a^{2} d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^2*c^3*d^2*e + a^3*c^2*e^3)*x/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4)
- (a^2*c^3*d^3 + a^3*c^2*d*e^2)/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4))/
```

```
sqrt(c*x^2 + a) - 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/s
qrt(-c*d^2 - a*e^2))*e^5/((c*d^5 + a*d^3*e^2)*sqrt(-c*d^2 - a*e^2)) - (3*c*
d^2 - 2*a*e^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^
2*d^3) + ((sqrt(c)*x - sqrt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a*sqrt(c)*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)
/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^2*a^2*d^2)
```

maple [A] time = 0.01, size = 439, normalized size = 1.59

$$-\frac{c e^3 x}{\left(a e^2+c d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}\, a d^2}+\frac{e^4 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a e^2+c d^2\right)\sqrt{\frac{a e^2+c d^2}{e^2}}\, d^3}-\frac{e^4}{\left(a e^2+c d^2\right)\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}\, d^3}+\frac{2cex}{\sqrt{c x^2+a}\, a^2 d^2}-\frac{e^2 \ln\left(\frac{2a+2\sqrt{c x^2+a}\,\sqrt{a}}{x}\right)}{a^{\frac{3}{2}} d^3}+\frac{3c \ln\left(\frac{2a+2\sqrt{c x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{5}{2}} d}+\frac{e^2}{\sqrt{c x^2+a}\, a d^3}-\frac{3c}{2\sqrt{c x^2+a}\, a^2 d}+\frac{e}{\sqrt{c x^2+a}\, a d^2 x}-\frac{1}{2\sqrt{c x^2+a}\, a d x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] e/a/d^2/x/(c*x^2+a)^(1/2)+2*c*e*x/a^2/d^2/(c*x^2+a)^(1/2)-1/2/a/d/x^2/(c*x^
2+a)^(1/2)-3/2*c/a^2/d/(c*x^2+a)^(1/2)+3/2/d*c/a^(5/2)*ln((2*a+2*(c*x^2+a)^
(1/2)*a^(1/2))/x)+e^2/a/d^3/(c*x^2+a)^(1/2)-1/d^3*e^2/a^(3/2)*ln((2*a+2*(c*
x^2+a)^(1/2)*a^(1/2))/x)-1/d^3*e^4/(a*e^2+c*d^2)/(-2*(x+d/e)*c*d/e+(x+d/e)^
2*c+(a*e^2+c*d^2)/e^2)^(1/2)-1/d^2*e^3/(a*e^2+c*d^2)/a/(-2*(x+d/e)*c*d/e+(x
+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)*c*x+1/d^3*e^4/(a*e^2+c*d^2)/((a*e^2+c*d^
2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2
)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

3.253 $\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=244

$$-\frac{d\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5}+\frac{\sqrt{a+cx^2}\,\left(13cd^2-2ae^2\right)}{3c^2e^4}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}-\frac{d^4\left(5ae^2+4cd^2\right)\tanh^{-1}\left(\right.}{e^5\left(ae^2+cd^2\right.}$$

Rubi [A] time = 0.89, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1651, 1654, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\,\left(13cd^2-2ae^2\right)}{3c^2e^4}-\frac{d\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}-\frac{d^4\left(5ae^2+4cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^5\left(ae^2+cd^2\right)^{3/2}}-\frac{5d\sqrt{a+cx^2}\,(d+ex)}{3ce^4}+\frac{\sqrt{a+cx^2}\,(d+ex)^2}{3ce^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((13*c*d^2 - 2*a*e^2)*Sqrt[a + c*x^2])/(3*c^2*e^4) + (d^5*Sqrt[a + c*x^2])/
(e^4*(c*d^2 + a*e^2)*(d + e*x)) - (5*d*(d + e*x)*Sqrt[a + c*x^2])/(3*c*e^4)
 + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^4) - (d*(4*c*d^2 - a*e^2)*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^5) - (d^4*(4*c*d^2 + 5*a*e^2)*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^5*(c*d^2 + a*e^
2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx &= \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-\frac{ad^4}{e^3}+\frac{d^3\left(cd^2+ae^2\right)x}{e^4}-\frac{d^2\left(cd^2+ae^2\right)x^2}{e^3}+d\left(a+\frac{cd^2}{e^2}\right)x^3-\frac{\left(cd^2+ae^2\right)x^4}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{\int \frac{-ad^2e\left(cd^2-2ae^2\right)+4d\left(cd^2+ae^2\right)^2x+2e\left(cd^2+ae^2\right)\cdots}{(d+ex)\sqrt{a+cx^2}}}{3ce^4\left(cd^2+ae^2\right)}\cdots\\
&= \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{\int \frac{-6acd^2e^4\left(2cd^2+\cdots\right.}{\cdots}}{\cdots}\\
&= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\cdots}{3\cdots}\\
&= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\cdots}{3\cdots}\\
&= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\cdots}{3\cdots}\\
&= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\cdots}{3\cdots}
\end{aligned}$$

Mathematica [A] time = 0.51, size = 230, normalized size = 0.94

$$\frac{-\frac{3d\left(4cd^2-ae^2\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{3/2}}+e\sqrt{a+cx^2}\left(-\frac{2ae^2}{c^2}+\frac{3d^5}{(d+ex)\left(ae^2+cd^2\right)}+\frac{9d^2-3dex+e^2x^2}{c}\right)-\frac{3d^4\left(5ae^2+4cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}}+\frac{3d^4\left(5ae^2+4cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*((-2*a*e^2)/c^2 + (3*d^5)/((c*d^2 + a*e^2)*(d + e*x)) +
(9*d^2 - 3*d*e*x + e^2*x^2)/c) + (3*d^4*(4*c*d^2 + 5*a*e^2)*Log[d + e*x])/(
c*d^2 + a*e^2)^(3/2) - (3*d*(4*c*d^2 - a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*
x^2]])/c^(3/2) - (3*d^4*(4*c*d^2 + 5*a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 +
a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/(3*e^5)
```

IntegrateAlgebraic [A] time = 1.59, size = 291, normalized size = 1.19

$$\frac{\sqrt{a+cx^2}\left(-2a^2de^4-2a^2e^5x+7acd^3e^2+4acd^2e^3x-2acde^4x^2+ace^5x^3+12c^2d^5+6c^2d^4ex-2c^2d^3e^2x^2+c^2d^2e^3x^3\right)}{3c^2e^4(d+ex)\left(ae^2+cd^2\right)}+\frac{\left(4cd^3-ade^2\right)\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{c^{3/2}e^5}+\frac{2\sqrt{-ae^2-cd^2}\left(5ad^4e^2+4cd^6\right)\tan^{-1}\left(\frac{-e\sqrt{a+cx^2}+\sqrt{c}\,d+\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}\right)}{e^5\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[a + c*x^2]*(12*c^2*d^5 + 7*a*c*d^3*e^2 - 2*a^2*d*e^4 + 6*c^2*d^4*e*x
+ 4*a*c*d^2*e^3*x - 2*a^2*e^5*x - 2*c^2*d^3*e^2*x^2 - 2*a*c*d*e^4*x^2 + c^2
*d^2*e^3*x^3 + a*c*e^5*x^3))/(3*c^2*e^4*(c*d^2 + a*e^2)*(d + e*x)) + (2*Sqr
t[-(c*d^2) - a*e^2]*(4*c*d^6 + 5*a*d^4*e^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x
 - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(e^5*(c*d^2 + a*e^2)^2) + ((
4*c*d^3 - a*d*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(c^(3/2)*e^5)
```

fricas [B] time = 41.86, size = 2025, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^4*e^4 - a^3*d^2*e^6 + (4*c
^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 - a^3*d*e^7)*x)*sqrt(c)*log(-2
*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 3*(4*c^3*d^7 + 5*a*c^2*d^5*e^2
+ (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x -
 a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(12*c^3*d^7*e +
 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*
d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)*
x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^3*e^8)*x)*sqrt
(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e^9 + (c^4*d^4*e^6
+ 2*a*c^3*d^2*e^8 + a^2*c^2*e^10)*x), -1/6*(6*(4*c^3*d^7 + 5*a*c^2*d^5*e^2
+ (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) - 3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^4*e^4 - a^3*d^2*e
^6 + (4*c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 - a^3*d*e^7)*x)*sqrt(
c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(12*c^3*d^7*e + 19*a
*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e
^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)*x^2 +
 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^3*e^8)*x)*sqrt(c*x^
2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e^9 + (c^4*d^4*e^6 + 2*a
*c^3*d^2*e^8 + a^2*c^2*e^10)*x), 1/6*(6*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^
2*c*d^4*e^4 - a^3*d^2*e^6 + (4*c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^
5 - a^3*d*e^7)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + 3*(4*c^3*d^
7 + 5*a*c^2*d^5*e^2 + (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(c*d^2 + a*e^2
)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sq
rt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2))
 + 2*(12*c^3*d^7*e + 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^
3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3
*e^5 + a^2*c*d*e^7)*x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^
6 - a^3*e^8)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d
*e^9 + (c^4*d^4*e^6 + 2*a*c^3*d^2*e^8 + a^2*c^2*e^10)*x), -1/3*(3*(4*c^3*d^
7 + 5*a*c^2*d^5*e^2 + (4*c^3*d^6*e + 5*a*c^2*d^4*e^3)*x)*sqrt(-c*d^2 - a*e^
2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2
*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(4*c^3*d^8 + 7*a*c^2*d^6*e^2 + 2*a^2*c
*d^4*e^4 - a^3*d^2*e^6 + (4*c^3*d^7*e + 7*a*c^2*d^5*e^3 + 2*a^2*c*d^3*e^5 -
 a^3*d*e^7)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (12*c^3*d^7*e
+ 19*a*c^2*d^5*e^3 + 5*a^2*c*d^3*e^5 - 2*a^3*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2
*d^2*e^6 + a^2*c*e^8)*x^3 - 2*(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7)
```

```
*x^2 + 2*(3*c^3*d^6*e^2 + 5*a*c^2*d^4*e^4 + a^2*c*d^2*e^6 - a^3*e^8)*x)*sqr
t(c*x^2 + a))/(c^4*d^5*e^5 + 2*a*c^3*d^3*e^7 + a^2*c^2*d*e^9 + (c^4*d^4*e^6
 + 2*a*c^3*d^2*e^8 + a^2*c^2*e^10)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 474, normalized size = 1.94

$$\frac{c\,d^{6}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,e^{6}}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,d^{5}}{\left(a\,e^{2}+c\,d^{2}\right)\left(x+\frac{d}{e}\right)e^{5}}+\frac{\sqrt{c\,x^{2}+a}\,x^{2}}{3c\,e^{2}}-\frac{5d^{4}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,e^{6}}+\frac{ad\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{c^{\frac{3}{2}}e^{3}}-\frac{4d^{3}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{\sqrt{c}\,e^{5}}-\frac{\sqrt{c\,x^{2}+a}\,dx}{c\,e^{3}}-\frac{2\sqrt{c\,x^{2}+a}\,a}{3c^{2}e^{2}}+\frac{3\sqrt{c\,x^{2}+a}\,d^{2}}{c\,e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3/e^2*x^2/c*(c*x^2+a)^(1/2)-2/3/e^2*a/c^2*(c*x^2+a)^(1/2)-d/e^3*x/c*(c*x^
2+a)^(1/2)+d/e^3*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+3*d^2/e^4/c*(c*x^2
+a)^(1/2)-4*d^3/e^5*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-5/e^6*d^4/((a*e^2
+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2
)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e
))+d^5/e^5/(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2
)/e^2)^(1/2)+d^6/e^6*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/
e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+
(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.60, size = 274, normalized size = 1.12

$$\frac{\sqrt{cx^{2}+a}\,d^{5}}{cd^{2}e^{5}x+ae^{7}x+cd^{3}e^{4}+ade^{6}}+\frac{\sqrt{cx^{2}+a}\,x^{2}}{3\,ce^{2}}-\frac{\sqrt{cx^{2}+a}\,dx}{ce^{3}}-\frac{4\,d^{3}\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^{5}}+\frac{ad\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}e^{3}}-\frac{cd^{6}\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^{2}}{e^{2}}\right)^{\frac{3}{2}}e^{8}}+\frac{5\,d^{4}\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^{2}}{e^{2}}}\,e^{6}}+\frac{3\,\sqrt{cx^{2}+a}\,d^{2}}{ce^{4}}-\frac{2\,\sqrt{cx^{2}+a}\,a}{3\,c^{2}e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + a)*d^5/(c*d^2*e^5*x + a*e^7*x + c*d^3*e^4 + a*d*e^6) + 1/3*sqr
t(c*x^2 + a)*x^2/(c*e^2) - sqrt(c*x^2 + a)*d*x/(c*e^3) - 4*d^3*arcsinh(c*x/
sqrt(a*c))/(sqrt(c)*e^5) + a*d*arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e^3) - c*d^6
*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a
 + c*d^2/e^2)^(3/2)*e^8) + 5*d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a
*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt(a + c*d^2/e^2)*e^6) + 3*sqrt(c*x^2 + a)*
d^2/(c*e^4) - 2/3*sqrt(c*x^2 + a)*a/(c^2*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^5/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\sqrt{a + cx^{2}} \left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.254 $\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{\left(6cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}+\frac{d^3\left(4ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^4\left(ae^2+cd^2\right)^{3/2}}-\frac{5d\sqrt{a+cx^2}}{2ce^3}+\cdot$$

Rubi [A] time = 0.52, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1651, 1654, 844, 217, 206, 725}

$$\frac{\left(6cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}+\frac{d^3\left(4ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^4\left(ae^2+cd^2\right)^{3/2}}-\frac{5d\sqrt{a+cx^2}}{2ce^3}+\frac{\sqrt{a+cx^2}\,(d+ex)}{2ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-5*d*Sqrt[a + c*x^2])/(2*c*e^3) - (d^4*Sqrt[a + c*x^2])/(e^3*(c*d^2 + a*e^
2)*(d + e*x)) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^3) + ((6*c*d^2 - a*e^2)*
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) + (d^3*(3*c*d^2 + 4*a
*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*(c
*d^2 + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\frac{ad^3}{e^2}-\frac{d^2\left(cd^2+ae^2\right)x}{e^3}+d\left(a+\frac{cd^2}{e^2}\right)x^2-\frac{\left(cd^2+ae^2\right)x^3}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} - \frac{\int \frac{ade\left(3cd^2+ae^2\right)-\left(c^2d^4-a^2e^4\right)x+5cde\left(cd^2+a\cdots\right.}{(d+ex)\sqrt{a+cx^2}}}{2ce^3\left(cd^2+ae^2\right)}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} - \frac{\int \frac{acde^3\left(3cd^2+ae^2\right)-ce^2\left(\cdots\right.}{(d+ex)\sqrt{\cdots}}}{2c^2e^5\left(c\cdots\right.}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\int \frac{1}{\sqrt{a+\cdots}}}{2ce^4}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\text{Subst}\cdots}{\cdots}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\tanh^{-1}\cdots}{2c^{3/2}e^4}$$

Mathematica [A] time = 0.37, size = 208, normalized size = 1.02

$$\frac{\frac{\left(6cd^2-ae^2\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{3/2}}+e\sqrt{a+cx^2}\left(\frac{ex-4d}{c}-\frac{2d^4}{(d+ex)\left(ae^2+cd^2\right)}\right)+\frac{2d^3\left(4ae^2+3cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{2d^3\left(4ae^2+3cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*((-4*d + e*x)/c - (2*d^4)/((c*d^2 + a*e^2)*(d + e*x))) -
 (2*d^3*(3*c*d^2 + 4*a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) + ((6*c*d^2
 - a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/c^(3/2) + (2*d^3*(3*c*d^2 + 4
*a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*
e^2)^(3/2))/(2*e^4)
```

IntegrateAlgebraic [A] time = 1.35, size = 238, normalized size = 1.17

$$\frac{\left(ae^2-6cd^2\right)\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{2c^{3/2}e^4}-\frac{2\sqrt{-ae^2-cd^2}\left(4ad^3e^2+3cd^5\right)\tan^{-1}\left(\frac{-e\sqrt{a+cx^2}+\sqrt{c}\,d+\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}\right)}{e^4\left(ae^2+cd^2\right)^2}+\frac{\sqrt{a+cx^2}\left(-4ad^2e^2-3ade^3x+ae^4x^2-6cd^4-3cd^3ex+cd^2e^2x^2\right)}{2ce^3(d+ex)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-6*c*d^4 - 4*a*d^2*e^2 - 3*c*d^3*e*x - 3*a*d*e^3*x + c*d^
2*e^2*x^2 + a*e^4*x^2))/(2*c*e^3*(c*d^2 + a*e^2)*(d + e*x)) - (2*Sqrt[-(c*d
^2) - a*e^2]*(3*c*d^5 + 4*a*d^3*e^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x - e*Sq
rt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(e^4*(c*d^2 + a*e^2)^2) + ((-6*c*d^
2 + a*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4)
```

fricas [B] time = 66.28, size = 1786, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 + (6*c^3
*d^6*e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c)*log(-2*c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (
3*c^3*d^5*e + 4*a*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*
c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(6*c^3*d^6*e + 10*
a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^
7)*x^2 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a)
)/(c^4*d^5*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d
^2*e^7 + a^2*c^2*e^9)*x), 1/4*(4*(3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (3*c^3*d^5*
e + 4*a*c^2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- (6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 + (6*c^3*d^6*
e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c)*log(-2*c*x^2 +
 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(6*c^3*d^6*e + 10*a*c^2*d^4*e^3 + 4*a
^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x^2 + 3*(c^3*d^5
*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^4 + 2*
a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c^2*e^
9)*x), -1/2*((6*c^3*d^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 +
(6*c^3*d^6*e + 11*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (3*c^3*d^
5*e + 4*a*c^2*d^3*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (6*c^3*d^6*e + 10*a*c^2*d^4*
e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x^2 + 3
*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^5
*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a
^2*c^2*e^9)*x), 1/2*(2*(3*c^3*d^6 + 4*a*c^2*d^4*e^2 + (3*c^3*d^5*e + 4*a*c^
2*d^3*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e
)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (6*c^3*d
^7 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - a^3*d*e^6 + (6*c^3*d^6*e + 11*a*c
^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(
c*x^2 + a)) - (6*c^3*d^6*e + 10*a*c^2*d^4*e^3 + 4*a^2*c*d^2*e^5 - (c^3*d^4*
e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x^2 + 3*(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 +
 a^2*c*d*e^6)*x)*sqrt(c*x^2 + a))/(c^4*d^5*e^4 + 2*a*c^3*d^3*e^6 + a^2*c^2*
d*e^8 + (c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c^2*e^9)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

[Out] Timed out

maple [B] time = 0.01, size = 435, normalized size = 2.13

$$-\frac{c\,d^5 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e^5}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}\,d^4}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)e^4}+\frac{4d^3 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e^5}-\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}e^2}+\frac{3d^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}\,e^4}+\frac{\sqrt{c\,x^2+a}\,x}{2c\,e^2}-\frac{2\sqrt{c\,x^2+a}\,d}{c\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2/e^2*x/c*(c*x^2+a)^(1/2)-1/2/e^2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
-2*d*(c*x^2+a)^(1/2)/c/e^3+3*d^2/e^4*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+
4/e^5*d^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^
2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e
^2)^(1/2))/(x+d/e))-d^4/e^4/(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)
^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-d^5/e^5*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(
1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(
-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.57, size = 233, normalized size = 1.14

$$-\frac{\sqrt{cx^2+a}\,d^4}{cd^2e^4x+ae^6x+cd^3e^3+ade^5}+\frac{\sqrt{cx^2+a}\,x}{2\,ce^2}+\frac{3\,d^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^4}-\frac{a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}e^2}+\frac{cd^5\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^7}-\frac{4\,d^3\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^5}-\frac{2\,\sqrt{cx^2+a}\,d}{ce^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*d^4/(c*d^2*e^4*x + a*e^6*x + c*d^3*e^3 + a*d*e^5) + 1/2*sq
rt(c*x^2 + a)*x/(c*e^2) + 3*d^2*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*e^4) - 1/2*
a*arcsinh(c*x/sqrt(a*c))/(c^(3/2)*e^2) + c*d^5*arcsinh(c*d*x/(sqrt(a*c)*abs
(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e^7) - 4*
d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/
(sqrt(a + c*d^2/e^2)*e^5) - 2*sqrt(c*x^2 + a)*d/(c*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+cx^2}\,\left(d+ex\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.255 $\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=160

$$-\frac{d^2\left(3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\left(ae^2+cd^2\right)^{3/2}}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^3}+\frac{\sqrt{a+cx^2}}{ce^2}$$

Rubi [A] time = 0.33, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1651, 1654, 844, 217, 206, 725}

$$\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}-\frac{d^2\left(3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\left(ae^2+cd^2\right)^{3/2}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^3}+\frac{\sqrt{a+cx^2}}{ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e^2) + (d^3*Sqrt[a + c*x^2])/(e^2*(c*d^2 + a*e^2)*(d + e
*x)) - (2*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e^3) - (d^2*(2*c
*d^2 + 3*a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])
])/(e^3*(c*d^2 + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-\frac{ad^2}{e}+d\left(a+\frac{cd^2}{e^2}\right)x-\frac{\left(cd^2+ae^2\right)x^2}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-acd^2e+2cd\left(cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{ce^2\left(cd^2+ae^2\right)}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{(2d)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^3} + \frac{\left(d^2\left(2cd^2+3ae^2\right)\right)\int \frac{\ldots}{(d+\ldots}}{e^3\left(cd^2+ae^2\ldots\right.}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^3} - \frac{\left(d^2\left(2c\ldots\right.\right.}{\ldots}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^3} - \frac{d^2\left(2cd^2+3ae^2\right)\tanh\ldots}{e^3\left(cd^2+\ldots\right.}$$

Mathematica [A] time = 0.27, size = 184, normalized size = 1.15

$$\frac{-\frac{d^2\left(3ae^2+2cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}}+\frac{d^2\left(3ae^2+2cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}+e\sqrt{a+cx^2}\left(\frac{d^3}{(d+ex)\left(ae^2+cd^2\right)}+\frac{1}{c}\right)-\frac{2d\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{\sqrt{c}}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(c^(-1) + d^3/((c*d^2 + a*e^2)*(d + e*x))) + (d^2*(2*c*d
^2 + 3*a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) - (2*d*Log[c*x + Sqrt[c]*
Sqrt[a + c*x^2]])/Sqrt[c] - (d^2*(2*c*d^2 + 3*a*e^2)*Log[a*e - c*d*x + Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/e^3
```

IntegrateAlgebraic [A] time = 1.12, size = 197, normalized size = 1.23

$$\frac{2\sqrt{-ae^2-cd^2}\left(3ad^2e^2+2cd^4\right)\tan^{-1}\left(\frac{-e\sqrt{a+cx^2}+\sqrt{c}\,d+\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}\right)}{e^3\left(ae^2+cd^2\right)^2}+\frac{\sqrt{a+cx^2}\left(ade^2+ae^3x+2cd^3+cd^2ex\right)}{ce^2(d+ex)\left(ae^2+cd^2\right)}+\frac{2d\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{\sqrt{c}\,e^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*c*d^3 + a*d*e^2 + c*d^2*e*x + a*e^3*x)*Sqrt[a + c*x^2])/(c*e^2*(c*d^2 +
 a*e^2)*(d + e*x)) + (2*Sqrt[-(c*d^2) - a*e^2]*(2*c*d^4 + 3*a*d^2*e^2)*ArcT
an[(Sqrt[c]*d + Sqrt[c]*e*x - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(
e^3*(c*d^2 + a*e^2)^2) + (2*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(Sqrt[c]
*e^3)
```

fricas [B] time = 6.77, size = 1449, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3
 + a^2*d*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
(2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c*d^2 +
a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 -
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
d^2)) + 2*(2*c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d
^2*e^4 + a^2*e^6)*x)*sqrt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*
c*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), -((2*c^2*d^5 + 3*
a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(
sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^
2*d^2 + a*c*e^2)*x^2)) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*
e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*
sqrt(c)*x - a) - (2*c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 +
2*a*c*d^2*e^4 + a^2*e^6)*x)*sqrt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5
 + a^2*c*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), 1/2*(4*(c^
2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^
5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c^2*d^5 + 3*a*c*d^3*
e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
 - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(2*c^2*d^5*e
+ 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x)*sq
rt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7 + (c^3*d^4*e^4
+ 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), -((2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^
4*e + 3*a*c*d^2*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a
^2*d*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*d^5*e + 3
*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x)*sqrt(
c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7 + (c^3*d^4*e^4 + 2
*a*c^2*d^2*e^6 + a^2*c*e^8)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[0,1,0,0]%%
%},[4,1]%%%}+%%%{%%%{-2,[0,1,1,0]%%%},[2,1]%%%}+%%%{%%%{1,[0,1,2,0]%%%},[0,
1]%%%} / %%%{%%%{1,[1,2,0,0]%%%}+%%%{1,[0,0,1,2]%%%},[4,0]%%%}+%%%{%%%{-2,[
1,2,1,0]%%%}+%%%{-2,[0,0,2,2]%%%},[2,0]%%%}+%%%{%%%{1,[1,2,2,0]%%%}+%%%{1,[
0,0,3,2]%%%},[0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.01, size = 386, normalized size = 2.41

$$\frac{c\,d^4 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e^4}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}\,d^3}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)e^3}-\frac{3d^2 \ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e^4}-\frac{2d \ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}\,e^3}+\frac{\sqrt{c\,x^2+a}}{c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/e^2-2/e^3*d*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-3/e^4*d
^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a
*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/
2))/(x+d/e))+d^3/e^3/(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a
*e^2+c*d^2)/e^2)^(1/2)+d^4/e^4*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln
((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d
/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.54, size = 193, normalized size = 1.21

$$\frac{\sqrt{cx^2+a}\,d^3}{cd^2e^3x+ae^5x+cd^3e^2+ade^4}-\frac{2\,d\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^3}-\frac{cd^4\,\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^6}+\frac{3\,d^2\,\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^4}+\frac{\sqrt{cx^2+a}}{ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + a)*d^3/(c*d^2*e^3*x + a*e^5*x + c*d^3*e^2 + a*d*e^4) - 2*d*arc
sinh(c*x/sqrt(a*c))/(sqrt(c)*e^3) - c*d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x
+ d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e^6) + 3*d^2*a
rcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt
(a + c*d^2/e^2)*e^4) + sqrt(c*x^2 + a)/(c*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{a + cx^{2}} \left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.256 $\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=137

$$-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\left(ae^2+cd^2\right)}+\frac{d\left(2ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}$$

Rubi [A] time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1651, 844, 217, 206, 725}

$$-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\left(ae^2+cd^2\right)}+\frac{d\left(2ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((d^2*Sqrt[a + c*x^2])/(e*(c*d^2 + a*e^2)*(d + e*x))) + ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]]/(Sqrt[c]*e^2) + (d*(c*d^2 + 2*a*e^2)*ArcTanh[(a*e - c*d*
x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{ad-\frac{\left(cd^2+ae^2\right)x}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2} - \frac{\left(d\left(2a+\frac{cd^2}{e^2}\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^2} + \frac{\left(d\left(2a+\frac{cd^2}{e^2}\right)\right)\text{Subst}\left(\int \right.}{cd^2+}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2} + \frac{d\left(2a+\frac{cd^2}{e^2}\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.32, size = 172, normalized size = 1.26

$$\frac{d\left(\frac{\left(2ae^2+cd^2\right)\log\left(\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{de\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}\right) - \frac{\left(2ade^2+cd^3\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{\log\left(\sqrt{c}\,\sqrt{a+cx^2}+cx\right)}{\sqrt{c}}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-(((c*d^3 + 2*a*d*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2)) + Log[c*x + Sq
rt[c]*Sqrt[a + c*x^2]]/Sqrt[c] + d*(-((d*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2
)*(d + e*x))) + ((c*d^2 + 2*a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sq
rt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2)))/e^2
```

IntegrateAlgebraic [A] time = 0.81, size = 168, normalized size = 1.23

$$-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\left(ae^2+cd^2\right)} - \frac{2\sqrt{-ae^2-cd^2}\,\left(2ade^2+cd^3\right)\tan^{-1}\left(\frac{-e\sqrt{a+cx^2}+\sqrt{c}\,d+\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^2} - \frac{\log\left(\sqrt{a+cx^2}-\sqrt{c}\,x\right)}{\sqrt{c}\,e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((d^2*Sqrt[a + c*x^2])/(e*(c*d^2 + a*e^2)*(d + e*x))) - (2*Sqrt[-(c*d^2) -
 a*e^2]*(c*d^3 + 2*a*d*e^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x - e*Sqrt[a + c*
x^2])/Sqrt[-(c*d^2) - a*e^2]])/(e^2*(c*d^2 + a*e^2)^2) - Log[-(Sqrt[c]*x) +
 Sqrt[a + c*x^2]]/(Sqrt[c]*e^2)
```

fricas [B] time = 6.37, size = 1260, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a
^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + (c^2*d
^4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((
2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^
2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c
```

```
^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a
^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x), 1/2*(2*(c^2*d^
4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*
e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 +
2*a*c^2*d^3*e^4 + a^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)
*x), -1/2*(2*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*
e^3 + a^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^2*d^4 +
2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c
*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a
*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c^2*d^
4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*
d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x), ((c^2*d^4 + 2*a*c*d
^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d
^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a
*c*e^2)*x^2)) - (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d
^2*e^3 + a^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^2*d^4
*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d
*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.66Error: Bad Argument Type
```

maple [B] time = 0.01, size = 368, normalized size = 2.69

$$-\frac{c\,d^{3}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,e^{3}}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,d^{2}}{\left(a\,e^{2}+c\,d^{2}\right)\left(x+\frac{d}{e}\right)e^{2}}+\frac{2d\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,e^{3}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{\sqrt{c}\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/e^2*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+2*d/e^3/((a*e^2+c*d^2)/e^2)^(1/
2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2
*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))-d^2/e^2/(a*e^
2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-d^3
/e^3*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^
2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e
^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.53, size = 171, normalized size = 1.25

$$-\frac{\sqrt{cx^2+a}\,d^2}{cd^2e^2x+ae^4x+cd^3e+ade^3}+\frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,e^2}+\frac{cd^3\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^5}-\frac{2\,d\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*d^2/(c*d^2*e^2*x + a*e^4*x + c*d^3*e + a*d*e^3) + arcsinh(
c*x/sqrt(a*c))/(sqrt(c)*e^2) + c*d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d))
 - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(3/2)*e^5) - 2*d*arcsinh(
c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/(sqrt(a + c*
d^2/e^2)*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{a + cx^{2}} \left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.257 $\int \frac{x}{(d+ex)^2 \sqrt{a+cx^2}} \, dx$

Optimal. Leaf size=90

$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {807, 725, 206}

$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]

[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (a*e*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx &= \frac{d\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}+\frac{(ae)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= \frac{d\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}-\frac{(ae)\operatorname{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\
&= \frac{d\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 1.00

$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (a*e*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.61, size = 150, normalized size = 1.67

$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}+\frac{2ae\sqrt{-ae^2-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2-cd^2}}\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) + (2*a*e*Sqrt[-(c*d^2) - a*
e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2
) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(c*d^2 + a*e^2)^2
```

fricas [B] time = 0.46, size = 382, normalized size = 4.24

$$\left[\frac{\left(ae^2x+ade\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+2\left(cd^3+ade^2\right)\sqrt{cx^2+a}}{2\left(c^2d^5+2\,acd^3e^2+a^2de^4+\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)x\right)},-\frac{\left(ae^2x+ade\right)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)-\left(cd^3+ade^2\right)\sqrt{cx^2+a}}{c^2d^5+2\,acd^3e^2+a^2de^4+\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*e^2*x + a*d*e)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2
 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
a^2*e^5)*x), -((a*e^2*x + a*d*e)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^
2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 340, normalized size = 3.78

$$\frac{c\,d^2\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e^2}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}\,d}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)e}-\frac{\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2
*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)
^(1/2))/(x+d/e))+d/e/(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a
*e^2+c*d^2)/e^2)^(1/2)+d^2/e^2*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln
((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d
/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.52, size = 148, normalized size = 1.64

$$\frac{\sqrt{cx^2+a}\,d}{cd^2ex+ae^3x+cd^3+ade^2}-\frac{cd^2\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^4}+\frac{\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\sqrt{a+\frac{cd^2}{e^2}}\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + a)*d/(c*d^2*e*x + a*e^3*x + c*d^3 + a*d*e^2) - c*d^2*arcsinh(c
*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e
^2)^(3/2)*e^4) + arcsinh(c*d*x/(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*ab
s(e*x + d)))/(sqrt(a + c*d^2/e^2)*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.258 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=91

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {731, 725, 206}

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (c*d*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx &= -\frac{e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}+\frac{(cd)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= -\frac{e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}-\frac{(cd)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\
&= -\frac{e\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 115, normalized size = 1.26

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{cd\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}}+\frac{cd\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) + (c*d*Log[d + e*x])/(c*
d^2 + a*e^2)^(3/2) - (c*d*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2]])/(c*d^2 + a*e^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.01, size = 151, normalized size = 1.66

$$\frac{2cd\sqrt{-ae^2-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+cx^2}}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2-cd^2}}\right)}{\left(ae^2+cd^2\right)^2}-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) + (2*c*d*Sqrt[-(c*d^2) -
 a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*
d^2) - a*e^2] - (e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(c*d^2 + a*e^2
)^2
```

fricas [B] time = 0.46, size = 381, normalized size = 4.19

$$\left[\frac{\left(cdex+cd^2\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(cd^2e+ae^3\right)\sqrt{cx^2+a}}{2\left(c^2d^5+2\,acd^3e^2+a^2de^4+\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)x\right)},-\frac{\left(cdex+cd^2\right)\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)+\left(cd^2e+ae^3\right)\sqrt{cx^2+a}}{c^2d^5+2\,acd^3e^2+a^2de^4+\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x + c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2
 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
a^2*e^5)*x), -((c*d*e*x + c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^
2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 210, normalized size = 2.31

$$-\frac{cd\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^2+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\;e}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{a\,e^2+c\,d^2}{e^2}}}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(
1/2)-1/e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2
*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*
c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [A] time = 0.49, size = 93, normalized size = 1.02

$$-\frac{\sqrt{cx^2+a}}{cd^2x+ae^2x+\frac{cd^3}{e}+ade}+\frac{cd\,\text{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|ex+d|}-\frac{ae}{\sqrt{ac}\,|ex+d|}\right)}{\left(a+\frac{cd^2}{e^2}\right)^{\frac{3}{2}}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)/(c*d^2*x + a*e^2*x + c*d^3/e + a*d*e) + c*d*arcsinh(c*d*x/
(sqrt(a*c)*abs(e*x + d)) - a*e/(sqrt(a*c)*abs(e*x + d)))/((a + c*d^2/e^2)^(
3/2)*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.259 $\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=179

$$\frac{e^2\sqrt{a+cx^2}}{d(d+ex)\left(ae^2+cd^2\right)}+\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}$$

Rubi [A] time = 0.14, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {961, 266, 63, 208, 731, 725, 206}

$$\frac{e^2\sqrt{a+cx^2}}{d(d+ex)\left(ae^2+cd^2\right)}+\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e^2*Sqrt[a + c*x^2])/(d*(c*d^2 + a*e^2)*(d + e*x)) + (c*e*ArcTanh[(a*e - c
*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e*Ar
cTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2
 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{d^2x\sqrt{a+cx^2}} - \frac{e}{d(d+ex)^2\sqrt{a+cx^2}} - \frac{e}{d^2(d+ex)\sqrt{a+cx^2}}\right)dx \\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2} - \frac{e\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2} - \frac{e\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{d} \\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} + \frac{e\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \right.}{d^2} \\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+\right.}{cd^2} \\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}} + \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 178, normalized size = 0.99

$$
\frac{\frac{de^2\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)} + \frac{e\left(ae^2+2cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{e\left(ae^2+2cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{\sqrt{a}} + \frac{\log(x)}{\sqrt{a}}}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((d*e^2*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) + Log[x]/Sqrt[a] - (e*
(2*c*d^2 + a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) - Log[a + Sqrt[a]*Sqr
t[a + c*x^2]]/Sqrt[a] + (e*(2*c*d^2 + a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 +
 a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/d^2
```

IntegrateAlgebraic [A] time = 0.80, size = 178, normalized size = 0.99

$$
\frac{e^2\sqrt{a+cx^2}}{d(d+ex)\left(ae^2+cd^2\right)} - \frac{2\sqrt{-ae^2-cd^2}\left(ae^3+2cd^2e\right)\tan^{-1}\left(\frac{-e\sqrt{a+cx^2}+\sqrt{c}d+\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^2} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}-\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e^2*Sqrt[a + c*x^2])/(d*(c*d^2 + a*e^2)*(d + e*x)) - (2*Sqrt[-(c*d^2) - a*
e^2]*(2*c*d^2*e + a*e^3)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x - e*Sqrt[a + c*x^2
])/Sqrt[-(c*d^2) - a*e^2]])/(d^2*(c*d^2 + a*e^2)^2) + (2*ArcTanh[(Sqrt[c]*x
)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2)
```

fricas [A] time = 0.81, size = 1261, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((2*a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(c*d^2 +
a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 +
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
d^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
 a^2*e^5)*x)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) +
2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 +
 a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x), 1/2*(2*(2*
a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-c*d^2 - a*e^2)*a
rctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2
 + (c^2*d^2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*
d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*
d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*
d^2*e^5)*x), 1/2*(2*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a
*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (2*a*c
*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(c*d^2 + a*e^2)*log((
2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^
2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a
*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3
*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x), ((2*a*c*d^3*e
+ a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*
a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (a*c*
d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^
3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)]
```

giac [A] time = 0.57, size = 126, normalized size = 0.70

$$\left(\frac{\sqrt{c - \frac{2\,cd}{xe+d} + \frac{cd^2}{(xe+d)^2} + \frac{ae^2}{(xe+d)^2}}\,d^2e^2\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{cd^5\mathrm{sgn}\left(\frac{1}{xe+d}\right)^2 + ad^3e^2\mathrm{sgn}\left(\frac{1}{xe+d}\right)^2} - \frac{\sqrt{c}\,e^2\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{cd^3 + ade^2}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)*d^2*e^2*
sgn(1/(x*e + d))/(c*d^5*sgn(1/(x*e + d))^2 + a*d^3*e^2*sgn(1/(x*e + d))^2)
- sqrt(c)*e^2*sgn(1/(x*e + d))/(c*d^3 + a*d*e^2))*e^(-1)
```

maple [B] time = 0.01, size = 364, normalized size = 2.03

$$\frac{c\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(a e^2+c d^2\right)\sqrt{\frac{a e^2+c d^2}{e^2}}}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}\;e}{\left(a e^2+c d^2\right)\left(x+\frac{d}{e}\right)d}+\frac{\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a e^2+2c d^2}{e^2}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2 c+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^2+c d^2}{e^2}}\,d^2}-\frac{\ln\left(\frac{2a+2\sqrt{c x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/d^2/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)+1/d^2/((a*e^2+c*d^2)/e
^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1
/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))+1/d*e/
(a*e^2+c*d^2)/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2
)+c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c
*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+
c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}\,(ex + d)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + cx^2}\,(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.260 $\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=212

$$\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}-\frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}-\frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)\left(ae^2+cd^2\right)}-\frac{\sqrt{a+cx^2}}{ad^2x}-\frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}$$

Rubi [A] time = 0.17, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {961, 264, 266, 63, 208, 731, 725, 206}

$$-\frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)\left(ae^2+cd^2\right)}-\frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}-\frac{\sqrt{a+cx^2}}{ad^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d^2*x)) - (e^3*Sqrt[a + c*x^2])/(d^2*(c*d^2 + a*e^2)*(
d + e*x)) - (c*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^
2])])/(d*(c*d^2 + a*e^2)^(3/2)) - (2*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2
+ a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*e^2]) + (2*e*ArcTanh[Sqrt[a
 + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{d^2x^2\sqrt{a+cx^2}} - \frac{2e}{d^3x\sqrt{a+cx^2}} + \frac{e^2}{d^2(d+ex)^2\sqrt{a+cx^2}} + \frac{2e^2}{d^3(d+ex)\sqrt{a+cx}} \right.\\
&= \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(2e)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3} + \frac{\left(2e^2\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^3} + \frac{e^2\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}}{d^2}\\
&= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{d^3} - \frac{\left(2e^2\right)\mathrm{Sub}}{}\\
&= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{(2e)\,\mathrm{Subs}}{}\\
&= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\left(cd^2+ae^2\right)^{3/2}} - \frac{2e^2\tanh}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 197, normalized size = 0.93

$$\frac{-\frac{e^2\left(2ae^2+3cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{e^2\left(2ae^2+3cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}} - d\sqrt{a+cx^2}\left(\frac{e^3}{(d+ex)\left(ae^2+cd^2\right)} + \frac{1}{ax}\right) + \frac{2e\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{\sqrt{a}} - \frac{2e\log(x)}{\sqrt{a}}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-(d*Sqrt[a + c*x^2]*(1/(a*x) + e^3/((c*d^2 + a*e^2)*(d + e*x)))) - (2*e*Lo
g[x])/Sqrt[a] + (e^2*(3*c*d^2 + 2*a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2
) + (2*e*Log[a + Sqrt[a]*Sqrt[a + c*x^2]])/Sqrt[a] - (e^2*(3*c*d^2 + 2*a*e^
2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^
(3/2))/d^3
```

IntegrateAlgebraic [A] time = 1.32, size = 214, normalized size = 1.01

$$-\frac{4e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}} - \frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3} + \frac{2\sqrt{-ae^2-cd^2}\left(2ae^4+3cd^2e^2\right)\tan^{-1}\left(\frac{-e\sqrt{a+cx^2}+\sqrt{c}\,d+\sqrt{c}\,ex}{\sqrt{-ae^2-cd^2}}\right)}{d^3\left(ae^2+cd^2\right)^2} + \frac{\sqrt{a+cx^2}\left(-ade^2-2ae^3x-cd^3-cd^2ex\right)}{ad^2x(d+ex)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-(c*d^3) - a*d*e^2 - c*d^2*e*x - 2*a*e^3*x)*Sqrt[a + c*x^2])/(a*d^2*(c*d^
2 + a*e^2)*x*(d + e*x)) + (2*Sqrt[-(c*d^2) - a*e^2]*(3*c*d^2*e^2 + 2*a*e^4)
*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2
]])/(d^3*(c*d^2 + a*e^2)^2) - (4*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c
*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

fricas [A] time = 0.76, size = 1512, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e^2
 + 2*a^2*d*e^4)*x)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*
c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2
+ 2*d*e*x + d^2)) + 2*((c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c^2*d
^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5
*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^2 + a))/((a*c^2*d^7*e + 2*a^2
*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2*a^2*c*d^6*e^2 + a^3*d^4*e^4)
*x), -(sqrt(-c*d^2 - a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e
^2 + 2*a^2*d*e^4)*x)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^2*d^4*e^2 + 2*a*c*
d^2*e^4 + a^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(a)
*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + (c^2*d^6 + 2*a*c*d^4
*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^
2 + a))/((a*c^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2
*a^2*c*d^6*e^2 + a^3*d^4*e^4)*x), -1/2*(4*((c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a
^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(-a)*arctan(sq
rt(-a)/sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*
x^2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^
2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^2 + a))/((a*c
^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2*a^2*c*d^6*e^
2 + a^3*d^4*e^4)*x), -(sqrt(-c*d^2 - a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^
2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 2*((c^
2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2
*d*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (c^2*d^6 + 2*a*c*d^4
*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^
2 + a))/((a*c^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2
*a^2*c*d^6*e^2 + a^3*d^4*e^4)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%{[%%%{1,[0,0,0,
5]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[4,1]%%%}+%%%{
```

```
%%%{-4,[1,2,0,4]%%%}+%%%{-4,[0,0,1,6]%%%},[3,1]%%%}+%%%{%%{[%%%{4,[1,2,0,3]
%%%}+%%%{6,[0,0,1,5]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]
%%},[2,1]%%%}+%%%{%%%{-4,[1,2,1,4]%%%}+%%%{-4,[0,0,2,6]%%%},[1,1]%%%}+%%%{%
%{[%%%{1,[0,0,2,5]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%
},[0,1]%%%} / %%%{%%%{1,[1,2,0,2]%%%}+%%%{1,[0,0,1,4]%%%},[4,0]%%%}+%%%{%%{
poly1[%%%{-4,[1,2,0,1]%%%}+%%%{-4,[0,0,1,3]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%
}+%%%{-1,[0,0,1,2]%%%}]%%},[3,0]%%%}+%%%{%%%{4,[2,4,0,0]%%%}+%%%{10,[1,2,1,
2]%%%}+%%%{6,[0,0,2,4]%%%},[2,0]%%%}+%%%{%%{poly1[%%%{-4,[1,2,1,1]%%%}+%%%{
-4,[0,0,2,3]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[1,0
]%%%}+%%%{%%%{1,[1,2,2,2]%%%}+%%%{1,[0,0,3,4]%%%},[0,0]%%%} Error: Bad Argu
ment Value
```

maple [B] time = 0.01, size = 395, normalized size = 1.86

$$-\frac{ce\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(ae^2+cd^2\right)\sqrt{\frac{ae^2+cd^2}{e^2}}\,d}-\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}\,e^2}{\left(ae^2+cd^2\right)\left(x+\frac{d}{e}\right)d^2}-\frac{2e\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2ae^2+2cd^2}{e^2}+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^2c+\frac{ae^2+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2+cd^2}{e^2}}\,d^3}+\frac{2e\ln\left(\frac{2a+2\sqrt{cx^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\,d^3}-\frac{\sqrt{cx^2+a}}{ad^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -(c*x^2+a)^(1/2)/a/d^2/x+2/d^3*e/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))
/x)-2/d^3*e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/
e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)
/e^2)^(1/2))/(x+d/e))-1/d^2/(a*e^2+c*d^2)*e^2/(x+d/e)*(-2*(x+d/e)*c*d/e+(x+
d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)-1/d*c*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)
^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)
*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.261 $\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=268

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^4}+\frac{2e\sqrt{a+cx^2}}{ad^3x}+\frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{\sqrt{a+cx^2}}{2ad^2x^2}+\frac{3e^3\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.}{d^4\sqrt{ae^2}}$$

Rubi [A] time = 0.22, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {961, 266, 51, 63, 208, 264, 731, 725, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}+\frac{e^4\sqrt{a+cx^2}}{d^3(d+ex)\left(ae^2+cd^2\right)}+\frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4\sqrt{ae^2+cd^2}}+\frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^4}+\frac{2e\sqrt{a+cx^2}}{ad^3x}-\frac{\sqrt{a+cx^2}}{2ad^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*a*d^2*x^2) + (2*e*Sqrt[a + c*x^2])/(a*d^3*x) + (e^4*Sqr
t[a + c*x^2])/(d^3*(c*d^2 + a*e^2)*(d + e*x)) + (c*e^3*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^(3/2)) + (3*
e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^4*Sqrt
[c*d^2 + a*e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d^2) - (
3*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^4)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \left(\frac{1}{d^2x^3\sqrt{a+cx^2}} - \frac{2e}{d^3x^2\sqrt{a+cx^2}} + \frac{3e^2}{d^4x\sqrt{a+cx^2}} - \frac{e^3}{d^3(d+ex)^2\sqrt{a+cx^2}} - \cdots$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(2e)\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^3} + \frac{\left(3e^2\right)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^4} - \frac{\left(3e^3\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}}{d^4} \cdots$$

$$= \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)} + \frac{\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} + \frac{\left(3e^2\right)\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)} + \frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+c\cdots}}\right.}{d^4\sqrt{cd^2+ae^2}} \cdots$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)} + \frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+c\cdots}}\right.}{d^2\left(cd^2+ae^2\right)^{3/2}} \cdots$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)} + \frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+c\cdots}}\right.}{d^2\left(cd^2+ae^2\right)^{3/2}} \cdots$$

Mathematica [A] time = 0.45, size = 229, normalized size = 0.85

$$\frac{\frac{\left(cd^2-6ae^2\right)\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{a^{3/2}} + \frac{\log(x)\left(6ae^2-cd^2\right)}{a^{3/2}} + d\sqrt{a+cx^2}\left(\frac{2e^4}{(d+ex)\left(ae^2+cd^2\right)} - \frac{d-4ex}{ax^2}\right) + \frac{2e^3\left(3ae^2+4cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{2e^3\left(3ae^2+4cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2]*(-((d - 4*e*x)/(a*x^2)) + (2*e^4)/((c*d^2 + a*e^2)*(d +
e*x))) + ((-(c*d^2) + 6*a*e^2)*Log[x])/a^(3/2) - (2*e^3*(4*c*d^2 + 3*a*e^2)
*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) + ((c*d^2 - 6*a*e^2)*Log[a + Sqrt[a]*S
qrt[a + c*x^2]])/a^(3/2) + (2*e^3*(4*c*d^2 + 3*a*e^2)*Log[a*e - c*d*x + Sqr
t[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/(2*d^4)
```

IntegrateAlgebraic [A] time = 1.78, size = 248, normalized size = 0.93

$$\frac{\left(6ae^2-cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^4}-\frac{2\sqrt{-ae^2-cd^2}\left(3ae^5+4cd^2e^3\right)\tan^{-1}\left(\frac{-e\sqrt{a+cx^2}+\sqrt{c}d+\sqrt{c}ex}{\sqrt{-ae^2-cd^2}}\right)}{d^4\left(ae^2+cd^2\right)^2}+\frac{\sqrt{a+cx^2}\left(-ad^2e^2+3ade^3x+6ae^4x^2-cd^4+3cd^3ex+4cd^2e^2x^2\right)}{2ad^3x^2(d+ex)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-(c*d^4) - a*d^2*e^2 + 3*c*d^3*e*x + 3*a*d*e^3*x + 4*c*d^
2*e^2*x^2 + 6*a*e^4*x^2))/(2*a*d^3*(c*d^2 + a*e^2)*x^2*(d + e*x)) - (2*Sqrt
[-(c*d^2) - a*e^2]*(4*c*d^2*e^3 + 3*a*e^5)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x
- e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(d^4*(c*d^2 + a*e^2)^2) + ((-
(c*d^2) + 6*a*e^2)*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(a^(3/2)
*d^4)
```

fricas [A] time = 1.29, size = 1867, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^3 + 3*a^3*d*e^5
)*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*
d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(
e^2*x^2 + 2*d*e*x + d^2)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^
5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*
d*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*
d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5
)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3
+ (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*e^4)*x^2), 1/4*(4*((4*a^2*c*d^2*
e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^2)*sqrt(-c*d^2 - a
*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^
2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^
4 - 6*a^3*d*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a
)/x^2) - 2*(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2
+ 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a
^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4
*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*e^4)*x^2), -1/2*(((c^3
*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4
*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*d*e^6)*x^2)*sqrt(-a)*arctan(sqrt(
-a)/sqrt(c*x^2 + a)) - ((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^
3 + 3*a^3*d*e^5)*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^
2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a
^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a
*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d
^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2
+ a^4*d^5*e^4)*x^2), 1/2*(2*((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d
```

```
^3*e^3 + 3*a^3*d*e^5)*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2
)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c
^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*d*e^6)*x^2)*sqrt(-a)*ar
ctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4
 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e
 + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a
^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*
e^4)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.53Unable to divide, perhaps due to rounding error
%%%{%%%{1,[0,0,0,7]%%%},[6,1]%%%}+%%%{%%{[%%%{-6,[0,0,0,6]%%%},0]:[1,0,%%%{
-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[5,1]%%%}+%%%{%%%{12,[1,2,0,5]%%%
}+%%%{15,[0,0,1,7]%%%},[4,1]%%%}+%%%{%%{[%%%{-8,[1,2,0,4]%%%}+%%%{-20,[0,0,
1,6]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[3,1]%%%}+%%
%{%%%{12,[1,2,1,5]%%%}+%%%{15,[0,0,2,7]%%%},[2,1]%%%}+%%%{%%{[%%%{-6,[0,0,2
,6]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[1,1]%%%}+%%%
{%%%{1,[0,0,3,7]%%%},[0,1]%%%} / %%%{%%{[%%%{-1,[1,2,0,3]%%%}+%%%{-1,[0,0,1
,5]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[6,0]%%%}+%%%
{%%%{6,[2,4,0,2]%%%}+%%%{12,[1,2,1,4]%%%}+%%%{6,[0,0,2,6]%%%},[5,0]%%%}+%%%
{%%{[%%%{-12,[2,4,0,1]%%%}+%%%{-27,[1,2,1,3]%%%}+%%%{-15,[0,0,2,5]%%%},0]:[
1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[4,0]%%%}+%%%{%%%{8,[3,6,
0,0]%%%}+%%%{36,[2,4,1,2]%%%}+%%%{48,[1,2,2,4]%%%}+%%%{20,[0,0,3,6]%%%},[3,
0]%%%}+%%%{%%{[%%%{-12,[2,4,1,1]%%%}+%%%{-27,[1,2,2,3]%%%}+%%%{-15,[0,0,3,5
]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%%%{-1,[0,0,1,2]%%%}]%%},[2,0]%%%}+%%%{%
%%{6,[2,4,2,2]%%%}+%%%{12,[1,2,3,4]%%%}+%%%{6,[0,0,4,6]%%%},[1,0]%%%}+%%%{%
%{[%%%{-1,[1,2,3,3]%%%}+%%%{-1,[0,0,4,5]%%%},0]:[1,0,%%%{-1,[1,2,0,0]%%%}+%
%%{-1,[0,0,1,2]%%%}]%%},[0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.01, size = 452, normalized size = 1.69

$$\frac{c\,e^{2}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,d^{2}}+\frac{\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,e^{3}}{\left(a\,e^{2}+c\,d^{2}\right)\left(x+\frac{d}{e}\right)d^{3}}+\frac{3e^{2}\ln\left(\frac{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{-\frac{2\left(x+\frac{d}{e}\right)cd}{e}+\left(x+\frac{d}{e}\right)^{2}c+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,d^{4}}-\frac{3e^{2}\ln\left(\frac{2a+2\sqrt{c\,x^{2}+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}\,d^{4}}+\frac{c\ln\left(\frac{2a+2\sqrt{c\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}d^{2}}+\frac{2\sqrt{c\,x^{2}+a}\,e}{a\,d^{3}x}-\frac{\sqrt{c\,x^{2}+a}}{2a\,d^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*e*(c*x^2+a)^(1/2)/a/d^3/x-1/2*(c*x^2+a)^(1/2)/a/d^2/x^2+1/2/d^2*c/a^(3/2)
*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)-3/d^4*e^2/a^(1/2)*ln((2*a+2*(c*x^2+a
)^(1/2)*a^(1/2))/x)+3/d^4*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/
e+2*(a*e^2+c*d^2)/e^2+2*((a*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)
^2*c+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))+1/d^3*e^3/(a*e^2+c*d^2)/(x+d/e)*(-2
*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/2)+1/d^2*e^2*c/(a*e^2+c*d^
2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((-2*(x+d/e)*c*d/e+2*(a*e^2+c*d^2)/e^2+2*((a
*e^2+c*d^2)/e^2)^(1/2)*(-2*(x+d/e)*c*d/e+(x+d/e)^2*c+(a*e^2+c*d^2)/e^2)^(1/
2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\,(ex+d)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

3.262 $\int x^2(a+bx)^n\left(c+dx^2\right)dx$

Optimal. Leaf size=135

$$\frac{a^2\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^5(n+1)}-\frac{2a\left(2a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{\left(6a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

Rubi [A] time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {948}

$$\frac{a^2\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^5(n+1)}-\frac{2a\left(2a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{\left(6a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] (a^2*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (2*a*(b^2*c + 2*a^2
*d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + ((b^2*c + 6*a^2*d)*(a + b*x)^(3 + n)
)/(b^5*(3 + n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5
 + n))/(b^5*(5 + n))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^2\right)dx&=\int\left(\frac{\left(a^2b^2c+a^4d\right)(a+bx)^n}{b^4}-\frac{2\left(ab^2c+2a^3d\right)(a+bx)^{1+n}}{b^4}+\frac{\left(b^2c+6a^2d\right)(a+b\right.\\&=\frac{a^2\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^5(1+n)}-\frac{2a\left(b^2c+2a^2d\right)(a+bx)^{2+n}}{b^5(2+n)}+\frac{\left(b^2c+6a^2d\right)(a+bx\right.}{b^5(3+n)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 114, normalized size = 0.84

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)^2\left(6a^2d+b^2c\right)}{n+3}-\frac{2a(a+bx)\left(2a^2d+b^2c\right)}{n+2}+\frac{a^4d+a^2b^2c}{n+1}+\frac{d(a+bx)^4}{n+5}-\frac{4ad(a+bx)^3}{n+4}\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*b^2*c + a^4*d)/(1 + n) - (2*a*(b^2*c + 2*a^2*d)*(a
 + b*x))/(2 + n) + ((b^2*c + 6*a^2*d)*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*
x)^3)/(4 + n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^n\left(c+dx^2\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n*(c + d*x^2), x]
```

fricas [B] time = 0.42, size = 368, normalized size = 2.73

$$\frac{\left(2\,a^{3}b^{2}cn^{2}+18\,a^{3}b^{2}cn+40\,a^{3}b^{2}c+24\,a^{5}d+\left(b^{5}dn^{4}+10\,b^{5}dn^{3}+35\,b^{5}dn^{2}+50\,b^{5}dn+24\,b^{5}d\right)x^{5}+\left(ab^{4}dn^{4}+6\,ab^{4}dn^{3}+11\,ab^{4}dn^{2}+6\,ab^{4}dn\right)x^{4}+\left(b^{5}cn^{4}+40\,b^{5}c+4\left(3\,b^{5}c-a^{2}b^{3}d\right)n^{3}+\left(49\,b^{5}c-12\,a^{2}b^{3}d\right)n^{2}+2\left(39\,b^{5}c-4\,a^{2}b^{3}d\right)n\right)x^{3}+\left(ab^{4}cn^{4}+10\,ab^{4}cn^{3}+\left(29\,ab^{4}c+12\,a^{3}b^{2}d\right)n^{2}+4\left(5\,ab^{4}c+3\,a^{3}b^{2}d\right)n\right)x^{2}-2\left(a^{2}b^{3}cn^{3}+9\,a^{2}b^{3}cn^{2}+4\left(5\,a^{2}b^{3}c+3\,a^{4}bd\right)n\right)x\right)(bx+a)^{n}}{b^{5}n^{5}+15\,b^{5}n^{4}+85\,b^{5}n^{3}+225\,b^{5}n^{2}+274\,b^{5}n+120\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] (2*a^3*b^2*c*n^2 + 18*a^3*b^2*c*n + 40*a^3*b^2*c + 24*a^5*d + (b^5*d*n^4 +
10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5 + (a*b^4*d*n^4 + 6
*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + (b^5*c*n^4 + 40*b^5*c +
4*(3*b^5*c - a^2*b^3*d)*n^3 + (49*b^5*c - 12*a^2*b^3*d)*n^2 + 2*(39*b^5*c -
 4*a^2*b^3*d)*n)*x^3 + (a*b^4*c*n^4 + 10*a*b^4*c*n^3 + (29*a*b^4*c + 12*a^3
*b^2*d)*n^2 + 4*(5*a*b^4*c + 3*a^3*b^2*d)*n)*x^2 - 2*(a^2*b^3*c*n^3 + 9*a^2
*b^3*c*n^2 + 4*(5*a^2*b^3*c + 3*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^
5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

giac [B] time = 0.19, size = 624, normalized size = 4.62

$$\frac{(bx+a)^{n}b^{5}dn^{4}x^{5}+(bx+a)^{n}ab^{4}dn^{4}x^{4}+10\,(bx+a)^{n}b^{5}dn^{3}x^{5}+(bx+a)^{n}b^{5}cn^{4}x^{3}+6\,(bx+a)^{n}ab^{4}dn^{3}x^{4}+35\,(bx+a)^{n}b^{5}dn^{2}x^{5}+(bx+a)^{n}ab^{4}cn^{4}x^{2}+12\,(bx+a)^{n}b^{5}cn^{3}x^{3}-4\,(bx+a)^{n}a^{2}b^{3}dn^{3}x^{3}+11\,(bx+a)^{n}ab^{4}dn^{2}x^{4}+50\,(bx+a)^{n}b^{5}dnx^{5}+10\,(bx+a)^{n}ab^{4}cn^{3}x^{2}+49\,(bx+a)^{n}b^{5}cn^{2}x^{3}-12\,(bx+a)^{n}a^{2}b^{3}dn^{2}x^{3}+6\,(bx+a)^{n}ab^{4}dnx^{4}+24\,(bx+a)^{n}b^{5}dx^{5}-2\,(bx+a)^{n}a^{2}b^{3}cn^{3}x+29\,(bx+a)^{n}ab^{4}cn^{2}x^{2}+12\,(bx+a)^{n}a^{3}b^{2}dn^{2}x^{2}+78\,(bx+a)^{n}b^{5}cnx^{3}-8\,(bx+a)^{n}a^{2}b^{3}dnx^{3}-18\,(bx+a)^{n}a^{2}b^{3}cn^{2}x+20\,(bx+a)^{n}ab^{4}cnx^{2}+12\,(bx+a)^{n}a^{3}b^{2}dnx^{2}+40\,(bx+a)^{n}b^{5}cx^{3}+2\,(bx+a)^{n}a^{3}b^{2}cn^{2}-40\,(bx+a)^{n}a^{2}b^{3}cnx-24\,(bx+a)^{n}a^{4}bdnx+18\,(bx+a)^{n}a^{3}b^{2}cn+40\,(bx+a)^{n}a^{3}b^{2}c+24\,(bx+a)^{n}a^{5}d}{b^{5}n^{5}+15\,b^{5}n^{4}+85\,b^{5}n^{3}+225\,b^{5}n^{2}+274\,b^{5}n+120\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*b
^5*d*n^3*x^5 + (b*x + a)^n*b^5*c*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 +
35*(b*x + a)^n*b^5*d*n^2*x^5 + (b*x + a)^n*a*b^4*c*n^4*x^2 + 12*(b*x + a)^n
*b^5*c*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x + a)^n*a*b^4*d*n
^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + 10*(b*x + a)^n*a*b^4*c*n^3*x^2 + 49*(
b*x + a)^n*b^5*c*n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b*x + a)^n
*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 - 2*(b*x + a)^n*a^2*b^3*c*n^3*x +
 29*(b*x + a)^n*a*b^4*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2 + 78*(b*
x + a)^n*b^5*c*n*x^3 - 8*(b*x + a)^n*a^2*b^3*d*n*x^3 - 18*(b*x + a)^n*a^2*b
^3*c*n^2*x + 20*(b*x + a)^n*a*b^4*c*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n*x^2
+ 40*(b*x + a)^n*b^5*c*x^3 + 2*(b*x + a)^n*a^3*b^2*c*n^2 - 40*(b*x + a)^n*a
^2*b^3*c*n*x - 24*(b*x + a)^n*a^4*b*d*n*x + 18*(b*x + a)^n*a^3*b^2*c*n + 40
*(b*x + a)^n*a^3*b^2*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b^5*n^4 + 85*b
^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

maple [B] time = 0.01, size = 328, normalized size = 2.43

$$\frac{\left(b^{4}d\,n^{4}x^{4}+10b^{4}d\,n^{3}x^{4}-4a\,b^{3}d\,n^{3}x^{3}+b^{4}c\,n^{4}x^{2}+35b^{4}d\,n^{2}x^{4}-24a\,b^{3}d\,n^{2}x^{3}+12b^{4}c\,n^{3}x^{2}+50b^{4}dn\,x^{4}+12a^{2}b^{2}d\,n^{2}x^{2}-2a\,b^{3}c\,n^{3}x-44a\,b^{3}dn\,x^{3}+49b^{4}c\,n^{2}x^{2}+24d\,x^{4}b^{4}+36a^{2}b^{2}dn\,x^{2}-20a\,b^{3}c\,n^{2}x-24da\,x^{3}b^{3}+78b^{4}cn\,x^{2}-24a^{3}bdnx+2a^{2}b^{2}c\,n^{2}+24a^{2}b^{2}d\,x^{2}-58a\,b^{3}cnx+40b^{4}c\,x^{2}-24a^{3}bdx+18a^{2}b^{2}cn-40a\,b^{3}cx+24a^{4}d+40a^{2}b^{2}c\right)(bx+a)^{n+1}}{\left(n^{5}+15n^{4}+85n^{3}+225n^{2}+274n+120\right)b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c),x)
```

```
[Out] (b*x+a)^(1+n)*(b^4*d*n^4*x^4+10*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+b^4*c*n^4*x
^2+35*b^4*d*n^2*x^4-24*a*b^3*d*n^2*x^3+12*b^4*c*n^3*x^2+50*b^4*d*n*x^4+12*a
^2*b^2*d*n^2*x^2-2*a*b^3*c*n^3*x-44*a*b^3*d*n*x^3+49*b^4*c*n^2*x^2+24*b^4*d
*x^4+36*a^2*b^2*d*n*x^2-20*a*b^3*c*n^2*x-24*a*b^3*d*x^3+78*b^4*c*n*x^2-24*a
^3*b*d*n*x+2*a^2*b^2*c*n^2+24*a^2*b^2*d*x^2-58*a*b^3*c*n*x+40*b^4*c*x^2-24*
a^3*b*d*x+18*a^2*b^2*c*n-40*a*b^3*c*x+24*a^4*d+40*a^2*b^2*c)/b^5/(n^5+15*n^
4+85*n^3+225*n^2+274*n+120)
```

maxima [A] time = 0.47, size = 210, normalized size = 1.56

$$\frac{\left(\left(n^2+3n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^nc}{\left(n^3+6n^2+11n+6\right)b^3}+\frac{\left(\left(n^4+10n^3+35n^2+50n+24\right)b^5x^5+\left(n^4+6n^3+11n^2+6n\right)ab^4x^4-4\left(n^3+3n^2+2n\right)a^2b^3x^3+12\left(n^2+n\right)a^3b^2x^2-24a^4bnx+24a^5\right)(bx+a)^nd}{\left(n^5+15n^4+85n^3+225n^2+274n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n +
24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n
)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)
^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

mupad [B] time = 2.82, size = 363, normalized size = 2.69

$$(a+bx)^n\left(\frac{2a^3\left(12da^2+cb^2n^2+9cb^2n+20cb^2\right)}{b^5\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}+\frac{dx^5\left(n^4+10n^3+35n^2+50n+24\right)}{n^5+15n^4+85n^3+225n^2+274n+120}+\frac{x^3\left(n^2+3n+2\right)\left(-4da^2n+cb^2n^2+9cb^2n+20cb^2\right)}{b^2\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}-\frac{2a^2nx\left(12da^2+cb^2n^2+9cb^2n+20cb^2\right)}{b^4\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}+\frac{anx^2(n+1)\left(12da^2+cb^2n^2+9cb^2n+20cb^2\right)}{b^3\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}+\frac{adnx^4\left(n^3+6n^2+11n+6\right)}{b\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((2*a^3*(12*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^5*(27
4*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (d*x^5*(50*n + 35*n^2 + 10*
n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (x^3*(3*
n + n^2 + 2)*(20*b^2*c + b^2*c*n^2 - 4*a^2*d*n + 9*b^2*c*n))/(b^2*(274*n +
225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (2*a^2*n*x*(12*a^2*d + 20*b^2*c +
 b^2*c*n^2 + 9*b^2*c*n))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (a*n*x^2*(n + 1)*(12*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^3*
(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2
 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

sympy [A] time = 6.70, size = 4134, normalized size = 30.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/(
12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**
9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48
*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 4
8*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 88*a**
3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) - a**2*b**2*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**
7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b*
*9*x**4) + 108*a**2*b**2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b*
*7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 4*a*b**3*c*x/(12*a**4*b**5 + 48*
a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**
3*d*x**3*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 +
48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b
**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 6*b**4*c*x**2/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b*
*9*x**4) + 12*b**4*d*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*
a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-12*a**4*d*lo
g(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 22
*a**4*d/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 36*a*
*3*b*d*x*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3) - 54*a**3*b*d*x/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**
8*x**3) - a**2*b**2*c/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
```

```
*x**3) - 36*a**2*b**2*d*x**2*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*
a*b**7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d*x**2/(3*a**3*b**5 + 9*a**2*b**6
*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 3*a*b**3*c*x/(3*a**3*b**5 + 9*a**2*b**6
*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 12*a*b**3*d*x**3*log(a/b + x)/(3*a**3*b
**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 3*b**4*c*x**2/(3*a**3*
b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + 3*b**4*d*x**4/(3*a**3
*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3), Eq(n, -4)), (12*a**4*
d*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2
*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5
 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*
b**7*x**2) + 2*a**2*b**2*c*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*
x**2) + 3*a**2*b**2*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b*
*2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a*b**3*
c*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a*b**3*c*x/(2
*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*b**5 + 4*a
*b**6*x + 2*b**7*x**2) + 2*b**4*c*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6
*x + 2*b**7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), E
q(n, -3)), (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*
b**5 + 3*b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 6*a**
2*b**2*c*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 6*a**2*b**2*c/(3*a*b**5 + 3*b
**6*x) + 6*a**2*b**2*d*x**2/(3*a*b**5 + 3*b**6*x) - 6*a*b**3*c*x*log(a/b +
x)/(3*a*b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*x) + 3*b**4*c
*x**2/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5 + 3*b**6*x), Eq(n, -2))
, (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4 + a**2*c*log(a/b + x)/b**3 + a*
*2*d*x**2/(2*b**3) - a*c*x/b**2 - a*d*x**3/(3*b**2) + c*x**2/(2*b) + d*x**4
/(4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 2*a**3*b**2*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 18*a**3*b**2*c*n*(
a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*
b**5*n + 120*b**5) + 40*a**3*b**2*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n*
*2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n*x**2*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 2
*a**2*b**3*c*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 18*a**2*b**3*c*n**2*x*(a + b*x)**
n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 1
20*b**5) - 40*a**2*b**3*c*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b
**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3
*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 27
4*b**5*n + 120*b**5) - 12*a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 1
5*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**
2*b**3*d*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225
*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*c*n**4*x**2*(a + b*x)**n/(b**5
*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5
) + 10*a*b**4*c*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 29*a*b**4*c*n**2*x**2*(a +
b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5
*n + 120*b**5) + 20*a*b**4*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*d*n**4*x**
4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 2
74*b**5*n + 120*b**5) + 6*a*b**4*d*n**3*x**4*(a + b*x)**n/(b**5*n**5 + 15*b
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 11*a*b**
4*d*n**2*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b
**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d*n*x**4*(a + b*x)**n/(b**5*n*
*5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) +
 b**5*c*n**4*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 2
```

```
25*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*b**5*c*n**3*x**3*(a + b*x)**n/(b
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b
**5) + 49*b**5*c*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5
*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 78*b**5*c*n*x**3*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) + 40*b**5*c*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**
5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d*n**4*x**5*(a + b*x
)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
+ 120*b**5) + 10*b**5*d*n**3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 35*b**5*d*n**2*x**5
*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 27
4*b**5*n + 120*b**5) + 50*b**5*d*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n
**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*d*x**
5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 2
74*b**5*n + 120*b**5), True))
```

3.263 $\int x(a+bx)^n \left(c+dx^2\right) dx$

Optimal. Leaf size=102

$$-\frac{a\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Rubi [A] time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {772}

$$-\frac{a\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] -((a*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + ((b^2*c + 3*a^2*d)
*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n))
 + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\left(c+dx^2\right)dx &= \int\left(\frac{a\left(-b^2c-a^2d\right)(a+bx)^n}{b^3}+\frac{\left(b^2c+3a^2d\right)(a+bx)^{1+n}}{b^3}-\frac{3ad(a+bx)^{2+n}}{b^3}+\frac{d(a+bx)^{3+n}}{b^3}\right)dx\\ &= -\frac{a\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^4(1+n)}+\frac{\left(b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.10, size = 109, normalized size = 1.07

$$\frac{(a+bx)^{n+1}\left(-6a^3d+6a^2bd(n+1)x-ab^2\left(c\left(n^2+7n+12\right)+3d\left(n^2+3n+2\right)x^2\right)+b^3\left(n^2+4n+3\right)x\left(c(n+4)+d(n+2)x^2\right)\right)}{b^4(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*(-6*a^3*d + 6*a^2*b*d*(1 + n)*x + b^3*(3 + 4*n + n^2)*x*
(c*(4 + n) + d*(2 + n)*x^2) - a*b^2*(c*(12 + 7*n + n^2) + 3*d*(2 + 3*n + n^
2)*x^2)))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x(a+bx)^n\left(c+dx^2\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n*(c + d*x^2), x]
```

fricas [B] time = 0.41, size = 250, normalized size = 2.45

$$-\frac{\left(a^{2} b^{2} c n^{2} + 7 \, a^{2} b^{2} c n + 12 \, a^{2} b^{2} c + 6 \, a^{4} d - \left(b^{4} d n^{3} + 6 \, b^{4} d n^{2} + 11 \, b^{4} d n + 6 \, b^{4} d\right)x^{4} - \left(a b^{3} d n^{3} + 3 \, a b^{3} d n^{2} + 2 \, a b^{3} d n\right)x^{3} - \left(b^{4} c n^{3} + 12 \, b^{4} c + \left(8 \, b^{4} c - 3 \, a^{2} b^{2} d\right)n^{2} + \left(19 \, b^{4} c - 3 \, a^{2} b^{2} d\right)n\right)x^{2} - \left(a b^{3} c n^{3} + 7 \, a b^{3} c n^{2} + 6 \, \left(2 \, a b^{3} c + a^{3} b d\right)n\right)x\right)(b x + a)^{n}}{b^{4} n^{4} + 10 \, b^{4} n^{3} + 35 \, b^{4} n^{2} + 50 \, b^{4} n + 24 \, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -(a^2*b^2*c*n^2 + 7*a^2*b^2*c*n + 12*a^2*b^2*c + 6*a^4*d - (b^4*d*n^3 + 6*b
^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 - (a*b^3*d*n^3 + 3*a*b^3*d*n^2 + 2*a*b
^3*d*n)*x^3 - (b^4*c*n^3 + 12*b^4*c + (8*b^4*c - 3*a^2*b^2*d)*n^2 + (19*b^4
*c - 3*a^2*b^2*d)*n)*x^2 - (a*b^3*c*n^3 + 7*a*b^3*c*n^2 + 6*(2*a*b^3*c + a^
3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24
*b^4)
```

giac [B] time = 0.19, size = 410, normalized size = 4.02

$$\frac{(b x + a)^{n} b^{4} d n^{3} x^{4} + (b x + a)^{n} a b^{3} d n^{3} x^{3} + 6 \, (b x + a)^{n} b^{4} d n^{2} x^{4} + (b x + a)^{n} b^{4} c n^{3} x^{2} + 3 \, (b x + a)^{n} a b^{3} d n^{2} x^{3} + 11 \, (b x + a)^{n} b^{4} d n x^{4} + (b x + a)^{n} a b^{3} c n^{3} x + 8 \, (b x + a)^{n} b^{4} c n^{2} x^{2} - 3 \, (b x + a)^{n} a^{2} b^{2} d n^{2} x^{2} + 2 \, (b x + a)^{n} a b^{3} d n x^{3} + 6 \, (b x + a)^{n} b^{4} d x^{4} + 7 \, (b x + a)^{n} a b^{3} c n^{2} x + 19 \, (b x + a)^{n} b^{4} c n x^{2} - 3 \, (b x + a)^{n} a^{2} b^{2} d n x^{2} - (b x + a)^{n} a^{2} b^{2} c n^{2} + 12 \, (b x + a)^{n} a b^{3} c n x + 6 \, (b x + a)^{n} a^{3} b d n x + 12 \, (b x + a)^{n} b^{4} c x^{2} - 7 \, (b x + a)^{n} a^{2} b^{2} c n - 12 \, (b x + a)^{n} a^{2} b^{2} c - 6 \, (b x + a)^{n} a^{4} d}{b^{4} n^{4} + 10 \, b^{4} n^{3} + 35 \, b^{4} n^{2} + 50 \, b^{4} n + 24 \, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^
4*d*n^2*x^4 + (b*x + a)^n*b^4*c*n^3*x^2 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 1
1*(b*x + a)^n*b^4*d*n*x^4 + (b*x + a)^n*a*b^3*c*n^3*x + 8*(b*x + a)^n*b^4*c
*n^2*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a*b^3*d*n*x^3 +
6*(b*x + a)^n*b^4*d*x^4 + 7*(b*x + a)^n*a*b^3*c*n^2*x + 19*(b*x + a)^n*b^4*
c*n*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 - (b*x + a)^n*a^2*b^2*c*n^2 + 12*(b
*x + a)^n*a*b^3*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 12*(b*x + a)^n*b^4*c*x^
2 - 7*(b*x + a)^n*a^2*b^2*c*n - 12*(b*x + a)^n*a^2*b^2*c - 6*(b*x + a)^n*a^
4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

maple [A] time = 0.00, size = 195, normalized size = 1.91

$$-\frac{\left(-b^{3} d \,n^{3} x^{3}-6 b^{3} d \,n^{2} x^{3}+3 a \,b^{2} d \,n^{2} x^{2}-b^{3} c \,n^{3} x -11 b^{3} d n \,x^{3}+9 a \,b^{2} d n \,x^{2}-8 b^{3} c \,n^{2} x -6 d \,x^{3} b^{3}-6 a^{2} b d n x +a \,b^{2} c \,n^{2}+6 a d \,x^{2} b^{2}-19 b^{3} c n x -6 a^{2} b d x +7 a \,b^{2} c n -12 b^{3} c x +6 a^{3} d +12 a \,b^{2} c \right) \left(b x +a \right)^{n +1}}{\left(n^{4}+10 n^{3}+35 n^{2}+50 n +24\right) b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c),x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-b^3*c*n^3*
x-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-8*b^3*c*n^2*x-6*b^3*d*x^3-6*a^2*b*d*n*x+a*
b^2*c*n^2+6*a*b^2*d*x^2-19*b^3*c*n*x-6*a^2*b*d*x+7*a*b^2*c*n-12*b^3*c*x+6*a
^3*d+12*a*b^2*c)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

maxima [A] time = 0.47, size = 146, normalized size = 1.43

$$\frac{\left(b^{2}(n + 1)x^{2} + a b n x - a^{2}\right)(b x + a)^{n} c}{\left(n^{2} + 3 \, n + 2\right)b^{2}} + \frac{\left(\left(n^{3} + 6 \, n^{2} + 11 \, n + 6\right)b^{4} x^{4} + \left(n^{3} + 3 \, n^{2} + 2 \, n\right)a b^{3} x^{3} - 3 \, \left(n^{2} + n\right)a^{2} b^{2} x^{2} + 6 \, a^{3} b n x - 6 \, a^{4}\right)(b x + a)^{n} d}{\left(n^{4} + 10 \, n^{3} + 35 \, n^{2} + 50 \, n + 24\right)b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n
^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n
)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2
+ 50*n + 24)*b^4)
```

mupad [B] time = 2.70, size = 255, normalized size = 2.50

$$(a+bx)^n\left(\frac{d\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{a^2\,\left(6\,d\,a^2+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2\right)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{x^2\,\left(n+1\right)\,\left(-3\,d\,a^2\,n+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x\,\left(6\,d\,a^2+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2\right)}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,d\,n\,x^3\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4
 + 24) - (a^2*(6*a^2*d + 12*b^2*c + b^2*c*n^2 + 7*b^2*c*n))/(b^4*(50*n + 35
*n^2 + 10*n^3 + n^4 + 24)) + (x^2*(n + 1)*(12*b^2*c + b^2*c*n^2 - 3*a^2*d*n
 + 7*b^2*c*n))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x*(6*a^2*d
+ 12*b^2*c + b^2*c*n^2 + 7*b^2*c*n))/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (a*d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

sympy [A] time = 3.53, size = 2181, normalized size = 21.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6
*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*
log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
+ 27*a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**
3) - a*b**2*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
 + 18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*
x**2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a
*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a
*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a*
*2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(a/b +
 x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*
b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) -
 a*b**2*c/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d*x**2*log(a/
b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*c*x/(2*a**2*b**4 +
 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b*
*6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3
*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 2*a*b**2*c*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c/(2*a*b**4 + 2*
b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*x*log(a/b + x)/(
2*a*b**4 + 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**
3*d*log(a/b + x)/b**4 + a**2*d*x/b**3 - a*c*log(a/b + x)/b**2 - a*d*x**2/(2
*b**2) + c*x/b + d*x**3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n*
*4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a
 + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
- a**2*b**2*c*n**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) - 7*a**2*b**2*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 12*a**2*b**2*c*(a + b*x)**n/(b**
4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d
*n**2*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*
n + 24*b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3
+ 35*b**4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*x*(a + b*x)**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 7*a*b**3*c*n**2
*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b
**4) + 12*a*b**3*c*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**
2 + 50*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(a +
 b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
```

```
2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 5
0*b**4*n + 24*b**4) + b**4*c*n**3*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 8*b**4*c*n**2*x**2*(a + b*x)**n/
(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 19*b**4*c
*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
 24*b**4) + 12*b**4*c*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4
*n**2 + 50*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 1
0*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a +
 b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
11*b**4*d*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 +
 35*b**4*n**2 + 50*b**4*n + 24*b**4), True))
```

3.264 $\int (a+bx)^n \left(c+dx^2\right) dx$

Optimal. Leaf size=70

$$\frac{\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}$$

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$\frac{\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n
))/(b^3*(2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n \left(c+dx^2\right) dx &= \int \left(\frac{\left(b^2c+a^2d\right)(a+bx)^n}{b^2}-\frac{2ad(a+bx)^{1+n}}{b^2}+\frac{d(a+bx)^{2+n}}{b^2}\right) dx\\ &= \frac{\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.93

$$\frac{(a+bx)^{n+1}\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) +
d*(1 + n)*x^2)))/(b^3*(1 + n)*(2 + n)*(3 + n))
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (a+bx)^n \left(c+dx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x^2), x]
```

fricas [B] time = 0.41, size = 148, normalized size = 2.11

$$\frac{\left(ab^2cn^2 + 5\,ab^2cn + 6\,ab^2c + 2\,a^3d + \left(b^3dn^2 + 3\,b^3dn + 2\,b^3d\right)x^3 + \left(ab^2dn^2 + ab^2dn\right)x^2 + \left(b^3cn^2 + 6\,b^3c + \left(5\,b^3c - 2\,a^2bd\right)n\right)x\right)(bx + a)^n}{b^3n^3 + 6\,b^3n^2 + 11\,b^3n + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] (a*b^2*c*n^2 + 5*a*b^2*c*n + 6*a*b^2*c + 2*a^3*d + (b^3*d*n^2 + 3*b^3*d*n +
 2*b^3*d)*x^3 + (a*b^2*d*n^2 + a*b^2*d*n)*x^2 + (b^3*c*n^2 + 6*b^3*c + (5*b
^3*c - 2*a^2*b*d)*n)*x)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3
)
```

giac [B] time = 0.16, size = 237, normalized size = 3.39

$$\frac{(bx+a)^n b^3dn^2x^3 + (bx+a)^n ab^2dn^2x^2 + 3\,(bx+a)^n b^3dnx^3 + (bx+a)^n b^3cn^2x + (bx+a)^n ab^2dnx^2 + 2\,(bx+a)^n b^3dx^3 + (bx+a)^n ab^2cn^2 + 5\,(bx+a)^n b^3cnx - 2\,(bx+a)^n a^2bdnx + 5\,(bx+a)^n ab^2cn + 6\,(bx+a)^n b^3cx + 6\,(bx+a)^n ab^2c + 2\,(bx+a)^n a^3d}{b^3n^3 + 6\,b^3n^2 + 11\,b^3n + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*d*n^2*x^3 + (b*x + a)^n*a*b^2*d*n^2*x^2 + 3*(b*x + a)^n*b^
3*d*n*x^3 + (b*x + a)^n*b^3*c*n^2*x + (b*x + a)^n*a*b^2*d*n*x^2 + 2*(b*x +
a)^n*b^3*d*x^3 + (b*x + a)^n*a*b^2*c*n^2 + 5*(b*x + a)^n*b^3*c*n*x - 2*(b*x
 + a)^n*a^2*b*d*n*x + 5*(b*x + a)^n*a*b^2*c*n + 6*(b*x + a)^n*b^3*c*x + 6*(
b*x + a)^n*a*b^2*c + 2*(b*x + a)^n*a^3*d)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n +
 6*b^3)
```

maple [A] time = 0.00, size = 100, normalized size = 1.43

$$\frac{\left(b^2d\,n^2x^2 + 3b^2dn\,x^2 - 2abdnx + b^2c\,n^2 + 2d\,x^2b^2 - 2adxb + 5b^2cn + 2a^2d + 6b^2c\right)(bx+a)^{n+1}}{\left(n^3 + 6n^2 + 11n + 6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c),x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*d*n^2*x^2+3*b^2*d*n*x^2-2*a*b*d*n*x+b^2*c*n^2+2*b^2*d*x^
2-2*a*b*d*x+5*b^2*c*n+2*a^2*d+6*b^2*c)/b^3/(n^3+6*n^2+11*n+6)
```

maxima [A] time = 0.46, size = 89, normalized size = 1.27

$$\frac{(bx+a)^{n+1}c}{b(n+1)} + \frac{\left(\left(n^2 + 3\,n + 2\right)b^3x^3 + \left(n^2 + n\right)ab^2x^2 - 2\,a^2bnx + 2\,a^3\right)(bx+a)^n d}{\left(n^3 + 6\,n^2 + 11\,n + 6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^
2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*d/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

mupad [B] time = 2.63, size = 163, normalized size = 2.33

$$(a+b\,x)^n\left(\frac{d\,x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6} + \frac{x\,\left(-2\,d\,a^2\,b\,n + c\,b^3\,n^2 + 5\,c\,b^3\,n + 6\,c\,b^3\right)}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{a\,\left(2\,d\,a^2 + c\,b^2\,n^2 + 5\,c\,b^2\,n + 6\,c\,b^2\right)}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{a\,d\,n\,x^2\,(n+1)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (x*(6*b^3*c
 + b^3*c*n^2 + 5*b^3*c*n - 2*a^2*b*d*n))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (
a*(2*a^2*d + 6*b^2*c + b^2*c*n^2 + 5*b^2*c*n))/(b^3*(11*n + 6*n^2 + n^3 + 6
)) + (a*d*n*x^2*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6)))
```

sympy [**A**] time = 2.07, size = 952, normalized size = 13.60

$$\begin{cases} a^n\left(cx+\frac{dx^3}{3}\right) & \text{for } b=0 \\ \frac{2a^2d\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{3a^2d}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abdx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abdx}{2a^2b^3+4ab^4x+2b^5x^2}-\frac{b^2c}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{2b^2dx^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} & \text{for } n=-3 \\ -\frac{2a^2d\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{2a^2d}{ab^3+b^4x}-\frac{2abdx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{b^2c}{ab^3+b^4x}+\frac{b^2dx^2}{ab^3+b^4x} & \text{for } n=-2 \\ \frac{a^2d\log\left(\frac{a}{b}+x\right)}{b^3}-\frac{adx}{b^2}+\frac{c\log\left(\frac{a}{b}+x\right)}{b}+\frac{dx^2}{2b} & \text{for } n=-1 \\ \frac{2a^3d(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}-\frac{2a^2bdnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2cn^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{5ab^2cn(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{6ab^2c(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2dn^2x^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2dnx^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{b^3cn^2x(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{5b^3cnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{6b^3cx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{b^3dn^2x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{3b^3dnx^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{2b^3dx^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x + d*x**3/3), Eq(b, 0)), (2*a**2*d*log(a/b + x)/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2*d/(2*a**2*b**3 + 4*a*b**4*x + 2*
b**5*x**2) + 4*a*b*d*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2
) + 4*a*b*d*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**3 + 4
*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*d*log(a/b + x)/(a*b**3 + b**
4*x) - 2*a**2*d/(a*b**3 + b**4*x) - 2*a*b*d*x*log(a/b + x)/(a*b**3 + b**4*x
) - b**2*c/(a*b**3 + b**4*x) + b**2*d*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (
a**2*d*log(a/b + x)/b**3 - a*d*x/b**2 + c*log(a/b + x)/b + d*x**2/(2*b), Eq
(n, -1)), (2*a**3*d*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b
**3) - 2*a**2*b*d*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6
*b**3) + a*b**2*c*n**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n +
6*b**3) + 5*a*b**2*c*n*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n +
6*b**3) + 6*a*b**2*c*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*
b**3) + a*b**2*d*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*
n + 6*b**3) + a*b**2*d*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b*
*3*n + 6*b**3) + b**3*c*n**2*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b
**3*n + 6*b**3) + 5*b**3*c*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b
**3*n + 6*b**3) + 6*b**3*c*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**
3*n + 6*b**3) + b**3*d*n**2*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11
*b**3*n + 6*b**3) + 3*b**3*d*n*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 +
 11*b**3*n + 6*b**3) + 2*b**3*d*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
+ 11*b**3*n + 6*b**3), True))
```

3.265 $\int x^2(a+bx)^n \left(c+dx^2\right)^2 dx$

Optimal. Leaf size=232

$$\frac{a^2\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^7(n+1)}-\frac{2a\left(a^2d+b^2c\right)\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{4ad\left(5a^2d+2b^2c\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{d\left(15a^2d\right.}{}$$

Rubi [A] time = 0.14, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {948}

$$\frac{\left(12a^2b^2cd+15a^4d^2+b^4c^2\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{a^2\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^7(n+1)}-\frac{2a\left(a^2d+b^2c\right)\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{4ad\left(5a^2d+2b^2c\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{d\left(15a^2d+2b^2c\right)(a+bx)^{n+5}}{b^7(n+5)}-\frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] (a^2*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (2*a*(b^2*c + a^2
*d)*(b^2*c + 3*a^2*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) + ((b^4*c^2 + 12*a^2
*b^2*c*d + 15*a^4*d^2)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d*(2*b^2*c +
 5*a^2*d)*(a + b*x)^(4 + n))/(b^7*(4 + n)) + (d*(2*b^2*c + 15*a^2*d)*(a + b
*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d
^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^2\right)^2 dx &= \int\left(\frac{\left(ab^2c+a^3d\right)^2(a+bx)^n}{b^6}+\frac{2a\left(-b^2c-3a^2d\right)\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^6}+\frac{\left(b^4c\right.}{}\right.\\ &= \frac{a^2\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^7(1+n)}-\frac{2a\left(b^2c+a^2d\right)\left(b^2c+3a^2d\right)(a+bx)^{2+n}}{b^7(2+n)}+\frac{\left(b^4c^2\right.}{}\end{aligned}$$

Mathematica [A] time = 0.18, size = 199, normalized size = 0.86

$$\frac{(a+bx)^{n+1}\left(\frac{\left(a^3d+ab^2c\right)^2}{n+1}+\frac{d(a+bx)^4\left(15a^2d+2b^2c\right)}{n+5}-\frac{4ad(a+bx)^3\left(5a^2d+2b^2c\right)}{n+4}-\frac{2a(a+bx)\left(a^2d+b^2c\right)\left(3a^2d+b^2c\right)}{n+2}+\frac{(a+bx)^2\left(15a^4d^2+12a^2b^2cd+b^4c^2\right)}{n+3}+\frac{d^2(a+bx)^6}{n+7}-\frac{6ad^2(a+bx)^5}{n+6}\right)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*b^2*c + a^3*d)^2/(1 + n) - (2*a*(b^2*c + a^2*d)*(b^2
*c + 3*a^2*d)*(a + b*x))/(2 + n) + ((b^4*c^2 + 12*a^2*b^2*c*d + 15*a^4*d^2)
*(a + b*x)^2)/(3 + n) - (4*a*d*(2*b^2*c + 5*a^2*d)*(a + b*x)^3)/(4 + n) + (
d*(2*b^2*c + 15*a^2*d)*(a + b*x)^4)/(5 + n) - (6*a*d^2*(a + b*x)^5)/(6 + n)
 + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n*(c + d*x^2)^2, x]
```

fricas [B] time = 0.43, size = 1027, normalized size = 4.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^4*c^2*n^4 + 44*a^3*b^4*c^2*n^3 + 1680*a^3*b^4*c^2 + 2016*a^5*b^2*c
*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*n^4 + 735*b^
7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^7 + (a*b^6*d
^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^3 + 274*a*b^
6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 + 2*(b^7*c*d*n^6 + 1008*b^7*c*d + (23*b^7*
c*d - 3*a^2*b^5*d^2)*n^5 + 3*(69*b^7*c*d - 10*a^2*b^5*d^2)*n^4 + 5*(185*b^7
*c*d - 21*a^2*b^5*d^2)*n^3 + 2*(1072*b^7*c*d - 75*a^2*b^5*d^2)*n^2 + 36*(67
*b^7*c*d - 2*a^2*b^5*d^2)*n)*x^5 + 2*(a*b^6*c*d*n^6 + 19*a*b^6*c*d*n^5 + (1
31*a*b^6*c*d + 15*a^3*b^4*d^2)*n^4 + (401*a*b^6*c*d + 90*a^3*b^4*d^2)*n^3 +
 15*(36*a*b^6*c*d + 11*a^3*b^4*d^2)*n^2 + 18*(14*a*b^6*c*d + 5*a^3*b^4*d^2)
*n)*x^4 + (b^7*c^2*n^6 + 1680*b^7*c^2 + (25*b^7*c^2 - 8*a^2*b^5*c*d)*n^5 +
(247*b^7*c^2 - 128*a^2*b^5*c*d)*n^4 + (1219*b^7*c^2 - 664*a^2*b^5*c*d - 120
*a^4*b^3*d^2)*n^3 + 8*(389*b^7*c^2 - 152*a^2*b^5*c*d - 45*a^4*b^3*d^2)*n^2
+ 4*(949*b^7*c^2 - 168*a^2*b^5*c*d - 60*a^4*b^3*d^2)*n)*x^3 + 2*(179*a^3*b^
4*c^2 + 24*a^5*b^2*c*d)*n^2 + (a*b^6*c^2*n^6 + 23*a*b^6*c^2*n^5 + 3*(67*a*b
^6*c^2 + 8*a^3*b^4*c*d)*n^4 + (817*a*b^6*c^2 + 336*a^3*b^4*c*d)*n^3 + 2*(73
9*a*b^6*c^2 + 660*a^3*b^4*c*d + 180*a^5*b^2*d^2)*n^2 + 24*(35*a*b^6*c^2 + 4
2*a^3*b^4*c*d + 15*a^5*b^2*d^2)*n)*x^2 + 4*(319*a^3*b^4*c^2 + 156*a^5*b^2*c
*d)*n - 2*(a^2*b^5*c^2*n^5 + 22*a^2*b^5*c^2*n^4 + (179*a^2*b^5*c^2 + 24*a^4
*b^3*c*d)*n^3 + 2*(319*a^2*b^5*c^2 + 156*a^4*b^3*c*d)*n^2 + 24*(35*a^2*b^5*
c^2 + 42*a^4*b^3*c*d + 15*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*n^7 + 28*b^7*n^
6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n
 + 5040*b^7)
```

giac [B] time = 0.22, size = 1750, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^5 + 15*(b*x + a)^n*a*b^6*d
^2*n^5*x^6 + 175*(b*x + a)^n*b^7*d^2*n^4*x^7 + 2*(b*x + a)^n*a*b^6*c*d*n^6*
x^4 + 46*(b*x + a)^n*b^7*c*d*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^2*n^5*x^5 +
85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7 + (b*x +
 a)^n*b^7*c^2*n^6*x^3 + 38*(b*x + a)^n*a*b^6*c*d*n^5*x^4 + 414*(b*x + a)^n*
b^7*c*d*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^
6*d^2*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^2*n^2*x^7 + (b*x + a)^n*a*b^6*c^2*n^
6*x^2 + 25*(b*x + a)^n*b^7*c^2*n^5*x^3 - 8*(b*x + a)^n*a^2*b^5*c*d*n^5*x^3
+ 262*(b*x + a)^n*a*b^6*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 +
1850*(b*x + a)^n*b^7*c*d*n^3*x^5 - 210*(b*x + a)^n*a^2*b^5*d^2*n^3*x^5 + 27
4*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b^7*d^2*n*x^7 + 23*(b*x
```

```
+ a)^n*a*b^6*c^2*n^5*x^2 + 247*(b*x + a)^n*b^7*c^2*n^4*x^3 - 128*(b*x + a)^
n*a^2*b^5*c*d*n^4*x^3 + 802*(b*x + a)^n*a*b^6*c*d*n^3*x^4 + 180*(b*x + a)^n
*a^3*b^4*d^2*n^3*x^4 + 4288*(b*x + a)^n*b^7*c*d*n^2*x^5 - 300*(b*x + a)^n*a
^2*b^5*d^2*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*
d^2*x^7 - 2*(b*x + a)^n*a^2*b^5*c^2*n^5*x + 201*(b*x + a)^n*a*b^6*c^2*n^4*x
^2 + 24*(b*x + a)^n*a^3*b^4*c*d*n^4*x^2 + 1219*(b*x + a)^n*b^7*c^2*n^3*x^3
- 664*(b*x + a)^n*a^2*b^5*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^2*n^3*x^3
 + 1080*(b*x + a)^n*a*b^6*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*n^2*x^4
 + 4824*(b*x + a)^n*b^7*c*d*n*x^5 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 - 44*
(b*x + a)^n*a^2*b^5*c^2*n^4*x + 817*(b*x + a)^n*a*b^6*c^2*n^3*x^2 + 336*(b*
x + a)^n*a^3*b^4*c*d*n^3*x^2 + 3112*(b*x + a)^n*b^7*c^2*n^2*x^3 - 1216*(b*x
 + a)^n*a^2*b^5*c*d*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 504*(b*
x + a)^n*a*b^6*c*d*n*x^4 + 180*(b*x + a)^n*a^3*b^4*d^2*n*x^4 + 2016*(b*x +
a)^n*b^7*c*d*x^5 + 2*(b*x + a)^n*a^3*b^4*c^2*n^4 - 358*(b*x + a)^n*a^2*b^5*
c^2*n^3*x - 48*(b*x + a)^n*a^4*b^3*c*d*n^3*x + 1478*(b*x + a)^n*a*b^6*c^2*n
^2*x^2 + 1320*(b*x + a)^n*a^3*b^4*c*d*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2
*n^2*x^2 + 3796*(b*x + a)^n*b^7*c^2*n*x^3 - 672*(b*x + a)^n*a^2*b^5*c*d*n*x
^3 - 240*(b*x + a)^n*a^4*b^3*d^2*n*x^3 + 44*(b*x + a)^n*a^3*b^4*c^2*n^3 - 1
276*(b*x + a)^n*a^2*b^5*c^2*n^2*x - 624*(b*x + a)^n*a^4*b^3*c*d*n^2*x + 840
*(b*x + a)^n*a*b^6*c^2*n*x^2 + 1008*(b*x + a)^n*a^3*b^4*c*d*n*x^2 + 360*(b*
x + a)^n*a^5*b^2*d^2*n*x^2 + 1680*(b*x + a)^n*b^7*c^2*x^3 + 358*(b*x + a)^n
*a^3*b^4*c^2*n^2 + 48*(b*x + a)^n*a^5*b^2*c*d*n^2 - 1680*(b*x + a)^n*a^2*b^
5*c^2*n*x - 2016*(b*x + a)^n*a^4*b^3*c*d*n*x - 720*(b*x + a)^n*a^6*b*d^2*n*
x + 1276*(b*x + a)^n*a^3*b^4*c^2*n + 624*(b*x + a)^n*a^5*b^2*c*d*n + 1680*(
b*x + a)^n*a^3*b^4*c^2 + 2016*(b*x + a)^n*a^5*b^2*c*d + 720*(b*x + a)^n*a^7
*d^2)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 1
3132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

maple [B] time = 0.02, size = 1000, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c)^2,x)
```

```
[Out] (b*x+a)^(n+1)*(b^6*d^2*n^6*x^6+21*b^6*d^2*n^5*x^6-6*a*b^5*d^2*n^5*x^5+2*b^6
*c*d*n^6*x^4+175*b^6*d^2*n^4*x^6-90*a*b^5*d^2*n^4*x^5+46*b^6*c*d*n^5*x^4+73
5*b^6*d^2*n^3*x^6+30*a^2*b^4*d^2*n^4*x^4-8*a*b^5*c*d*n^5*x^3-510*a*b^5*d^2*
n^3*x^5+b^6*c^2*n^6*x^2+414*b^6*c*d*n^4*x^4+1624*b^6*d^2*n^2*x^6+300*a^2*b^
4*d^2*n^3*x^4-152*a*b^5*c*d*n^4*x^3-1350*a*b^5*d^2*n^2*x^5+25*b^6*c^2*n^5*x
^2+1850*b^6*c*d*n^3*x^4+1764*b^6*d^2*n*x^6-120*a^3*b^3*d^2*n^3*x^3+24*a^2*b
^4*c*d*n^4*x^2+1050*a^2*b^4*d^2*n^2*x^4-2*a*b^5*c^2*n^5*x-1048*a*b^5*c*d*n^
3*x^3-1644*a*b^5*d^2*n*x^5+247*b^6*c^2*n^4*x^2+4288*b^6*c*d*n^2*x^4+720*b^6
*d^2*x^6-720*a^3*b^3*d^2*n^2*x^3+384*a^2*b^4*c*d*n^3*x^2+1500*a^2*b^4*d^2*n
*x^4-46*a*b^5*c^2*n^4*x-3208*a*b^5*c*d*n^2*x^3-720*a*b^5*d^2*x^5+1219*b^6*c
^2*n^3*x^2+4824*b^6*c*d*n*x^4+360*a^4*b^2*d^2*n^2*x^2-48*a^3*b^3*c*d*n^3*x-
1320*a^3*b^3*d^2*n*x^3+2*a^2*b^4*c^2*n^4+1992*a^2*b^4*c*d*n^2*x^2+720*a^2*b
^4*d^2*x^4-402*a*b^5*c^2*n^3*x-4320*a*b^5*c*d*n*x^3+3112*b^6*c^2*n^2*x^2+20
16*b^6*c*d*x^4+1080*a^4*b^2*d^2*n*x^2-672*a^3*b^3*c*d*n^2*x-720*a^3*b^3*d^2
*x^3+44*a^2*b^4*c^2*n^3+3648*a^2*b^4*c*d*n*x^2-1634*a*b^5*c^2*n^2*x-2016*a*
b^5*c*d*x^3+3796*b^6*c^2*n*x^2-720*a^5*b*d^2*n*x+48*a^4*b^2*c*d*n^2+720*a^4
*b^2*d^2*x^2-2640*a^3*b^3*c*d*n*x+358*a^2*b^4*c^2*n^2+2016*a^2*b^4*c*d*x^2-
2956*a*b^5*c^2*n*x+1680*b^6*c^2*x^2-720*a^5*b*d^2*x+624*a^4*b^2*c*d*n-2016*
a^3*b^3*c*d*x+1276*a^2*b^4*c^2*n-1680*a*b^5*c^2*x+720*a^6*d^2+2016*a^4*b^2*
c*d+1680*a^2*b^4*c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+1
3068*n+5040)
```

maxima [A] time = 0.50, size = 447, normalized size = 1.93

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")

[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^4 + 10*n^3 + 35*n^2 + 50*
n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
+ a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + ((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n
^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x
^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*
a^6*b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 +
6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)

mupad [B] time = 3.12, size = 932, normalized size = 4.02

$$\frac{2a^3(a+bx)^n\left(360a^4d^2+24a^2b^2cdn^2+312a^2b^2cdn+1008a^2b^2cd+b^4c^2n^4+22b^4c^2n^3+179b^4c^2n^2+638b^4c^2n+840b^4c^2\right)}{b^7\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)}+\frac{d^2x^7(a+bx)^n\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}{n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040}+\frac{x^3(a+bx)^n\left(n^2+3n+2\right)\left(-120a^4d^2n-8a^2b^2cdn^3-104a^2b^2cdn^2-336a^2b^2cdn+b^4c^2n^4+22b^4c^2n^3+179b^4c^2n^2+638b^4c^2n+840b^4c^2\right)}{b^4\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)}-\frac{2a^2nx(a+bx)^n\left(360a^4d^2+24a^2b^2cdn^2+312a^2b^2cdn+1008a^2b^2cd+b^4c^2n^4+22b^4c^2n^3+179b^4c^2n^2+638b^4c^2n+840b^4c^2\right)}{b^6\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)}+\frac{2dx^5(a+bx)^n\left(-3da^2n+cb^2n^2+13cb^2n+42cb^2\right)\left(n^4+10n^3+35n^2+50n+24\right)}{b^2\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)}+\frac{ad^2nx^6(a+bx)^n\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}{b\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)}+\frac{anx^2(n+1)(a+bx)^n\left(360a^4d^2+24a^2b^2cdn^2+312a^2b^2cdn+1008a^2b^2cd+b^4c^2n^4+22b^4c^2n^3+179b^4c^2n^2+638b^4c^2n+840b^4c^2\right)}{b^5\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)}+\frac{2adnx^4(a+bx)^n\left(n^3+6n^2+11n+6\right)\left(15da^2+cb^2n^2+13cb^2n+42cb^2\right)}{b^3\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c + d*x^2)^2*(a + b*x)^n,x)

[Out] (2*a^3*(a + b*x)^n*(360*a^4*d^2 + 840*b^4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2
*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 1008*a^2*b^2*c*d + 312*a^2*b^2*c*d*n
+ 24*a^2*b^2*c*d*n^2))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 32
2*n^5 + 28*n^6 + n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 7
35*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1
960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)
*(840*b^4*c^2 - 120*a^4*d^2*n + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^
2*n^3 + b^4*c^2*n^4 - 336*a^2*b^2*c*d*n - 104*a^2*b^2*c*d*n^2 - 8*a^2*b^2*c
*d*n^3))/(b^4*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6
 + n^7 + 5040)) - (2*a^2*n*x*(a + b*x)^n*(360*a^4*d^2 + 840*b^4*c^2 + 638*b
^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 1008*a^2*b^2*c*
d + 312*a^2*b^2*c*d*n + 24*a^2*b^2*c*d*n^2))/(b^6*(13068*n + 13132*n^2 + 67
69*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^5*(a + b*x)^n*
(42*b^2*c + b^2*c*n^2 - 3*a^2*d*n + 13*b^2*c*n)*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^
6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(360*a^4*d^2 + 840*b^
4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 10
08*a^2*b^2*c*d + 312*a^2*b^2*c*d*n + 24*a^2*b^2*c*d*n^2))/(b^5*(13068*n + 1
3132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n
x^4(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(15*a^2*d + 42*b^2*c + b^2*c*n^2
+ 13*b^2*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
28*n^6 + n^7 + 5040))

sympy [A] time = 21.59, size = 14317, normalized size = 61.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x+a)**n*(d*x**2+c)**2,x)

[Out] Piecewise((a**n*(c**2*x**3/3 + 2*c*d*x**5/5 + d**2*x**7/7), Eq(b, 0)), (60*
a**6*d**2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13
*x**6) + 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13
*x**6) + 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 9

```
00*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x
 + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*
b**12*x**5 + 60*b**13*x**6) - 4*a**4*b**2*c*d/(60*a**6*b**7 + 360*a**5*b**8
*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a*
*6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900
*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d**2*
x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10
*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 24*a**3*b
**3*c*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*
b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1200
*a**3*b**3*d**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**
4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
 + 60*b**13*x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**5*b**8*
x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a
*b**12*x**5 + 60*b**13*x**6) - a**2*b**4*c**2/(60*a**6*b**7 + 360*a**5*b**8
*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) - 60*a**2*b**4*c*d*x**2/(60*a**6*b**7 + 360*a
**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**
4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**2*b**4*d**2*x**4*log(a/b + x
)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x*
*3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1350*a**2*b*
*4*d**2*x**4/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a*
*3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 6
*a*b**5*c**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*
a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) -
 80*a*b**5*c*d*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 +
1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x*
*6) + 360*a*b**5*d**2*x**5*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 9
00*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 360*a*b**5*d**2*x**5/(60*a**6*b**7 + 360*a**5*b**
8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360
*a*b**12*x**5 + 60*b**13*x**6) - 15*b**6*c**2*x**2/(60*a**6*b**7 + 360*a**5
*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 +
 360*a*b**12*x**5 + 60*b**13*x**6) - 60*b**6*c*d*x**4/(60*a**6*b**7 + 360*a
**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**
4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 60*b**6*d**2*x**6*log(a/b + x)/(60*
a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 9
00*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6), Eq(n, -7)), (-180*a
**6*d**2*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2
+ 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 411*a**6*d**2/(
30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 +
 150*a*b**11*x**4 + 30*b**12*x**5) - 900*a**5*b*d**2*x*log(a/b + x)/(30*a**
5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a
*b**11*x**4 + 30*b**12*x**5) - 1875*a**5*b*d**2*x/(30*a**5*b**7 + 150*a**4*
b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b
**12*x**5) - 12*a**4*b**2*c*d/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b*
*9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 1800*a*
*4*b**2*d**2*x**2*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b
**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 3300*a
**4*b**2*d**2*x**2/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 3
00*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 60*a**3*b**3*c*d*x
/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3
 + 150*a*b**11*x**4 + 30*b**12*x**5) - 1800*a**3*b**3*d**2*x**3*log(a/b + x
)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**
3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 2700*a**3*b**3*d**2*x**3/(30*a**5*b
**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b*
*11*x**4 + 30*b**12*x**5) - a**2*b**4*c**2/(30*a**5*b**7 + 150*a**4*b**8*x
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+ 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x*
*5) - 120*a**2*b**4*c*d*x**2/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**
9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 900*a**2
*b**4*d**2*x**4*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**
9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 900*a**2
*b**4*d**2*x**4/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*
a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 5*a*b**5*c**2*x/(30*a
**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150
*a*b**11*x**4 + 30*b**12*x**5) - 120*a*b**5*c*d*x**3/(30*a**5*b**7 + 150*a*
*4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 3
0*b**12*x**5) - 180*a*b**5*d**2*x**5*log(a/b + x)/(30*a**5*b**7 + 150*a**4*
b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b
**12*x**5) - 10*b**6*c**2*x**2/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b
**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 60*b**
6*c*d*x**4/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*
b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) + 30*b**6*d**2*x**6/(30*a**5
*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*
b**11*x**4 + 30*b**12*x**5), Eq(n, -6)), (180*a**6*d**2*log(a/b + x)/(12*a*
*4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x
**4) + 375*a**6*d**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 4
8*a*b**10*x**3 + 12*b**11*x**4) + 720*a**5*b*d**2*x*log(a/b + x)/(12*a**4*b
**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4)
 + 1320*a**5*b*d**2*x/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 +
48*a*b**10*x**3 + 12*b**11*x**4) + 24*a**4*b**2*c*d*log(a/b + x)/(12*a**4*b
**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4)
 + 50*a**4*b**2*c*d/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48
*a*b**10*x**3 + 12*b**11*x**4) + 1080*a**4*b**2*d**2*x**2*log(a/b + x)/(12*
a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11
*x**4) + 1620*a**4*b**2*d**2*x**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*
b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 96*a**3*b**3*c*d*x*log(a/b +
 x)/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 +
12*b**11*x**4) + 176*a**3*b**3*c*d*x/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a*
*2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 720*a**3*b**3*d**2*x**3*l
og(a/b + x)/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10
*x**3 + 12*b**11*x**4) + 720*a**3*b**3*d**2*x**3/(12*a**4*b**7 + 48*a**3*b*
*8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) - a**2*b**4*c**
2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12
*b**11*x**4) + 144*a**2*b**4*c*d*x**2*log(a/b + x)/(12*a**4*b**7 + 48*a**3*
b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 216*a**2*b*
*4*c*d*x**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10
*x**3 + 12*b**11*x**4) + 180*a**2*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**7
 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) -
4*a*b**5*c**2*x/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b
**10*x**3 + 12*b**11*x**4) + 96*a*b**5*c*d*x**3*log(a/b + x)/(12*a**4*b**7
+ 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 9
6*a*b**5*c*d*x**3/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a
*b**10*x**3 + 12*b**11*x**4) - 36*a*b**5*d**2*x**5/(12*a**4*b**7 + 48*a**3*
b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) - 6*b**6*c**2
*x**2/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3
+ 12*b**11*x**4) + 24*b**6*c*d*x**4*log(a/b + x)/(12*a**4*b**7 + 48*a**3*b*
*8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 + 12*b**11*x**4) + 6*b**6*d**2*x
**6/(12*a**4*b**7 + 48*a**3*b**8*x + 72*a**2*b**9*x**2 + 48*a*b**10*x**3 +
12*b**11*x**4), Eq(n, -5)), (-60*a**6*d**2*log(a/b + x)/(3*a**3*b**7 + 9*a*
*2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 110*a**6*d**2/(3*a**3*b**7 + 9*
a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**5*b*d**2*x*log(a/b + x
)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 270*a**5*b
*d**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 24*a
**4*b**2*c*d*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*
b**10*x**3) - 44*a**4*b**2*c*d/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2
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 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a*
*2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2/(3*a**3
*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 72*a**3*b**3*c*d*x*
log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) -
 108*a**3*b**3*c*d*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10
*x**3) - 60*a**3*b**3*d**2*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x +
 9*a*b**9*x**2 + 3*b**10*x**3) - a**2*b**4*c**2/(3*a**3*b**7 + 9*a**2*b**8*
x + 9*a*b**9*x**2 + 3*b**10*x**3) - 72*a**2*b**4*c*d*x**2*log(a/b + x)/(3*a
**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 72*a**2*b**4*c*d
*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 15*a**
2*b**4*d**2*x**4/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**
3) - 3*a*b**5*c**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10
*x**3) - 24*a*b**5*c*d*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a
*b**9*x**2 + 3*b**10*x**3) - 3*a*b**5*d**2*x**5/(3*a**3*b**7 + 9*a**2*b**8*
x + 9*a*b**9*x**2 + 3*b**10*x**3) - 3*b**6*c**2*x**2/(3*a**3*b**7 + 9*a**2*
b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 6*b**6*c*d*x**4/(3*a**3*b**7 + 9*a
**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + b**6*d**2*x**6/(3*a**3*b**7 +
9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3), Eq(n, -4)), (60*a**6*d**2*lo
g(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 90*a**6*d**2/(4*a**2*
b**7 + 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x*log(a/b + x)/(4*a**2*b
**7 + 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x/(4*a**2*b**7 + 8*a*b**8
*x + 4*b**9*x**2) + 48*a**4*b**2*c*d*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x
 + 4*b**9*x**2) + 72*a**4*b**2*c*d/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2)
 + 60*a**4*b**2*d**2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x
**2) + 96*a**3*b**3*c*d*x*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x
**2) + 96*a**3*b**3*c*d*x/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 20*a**
3*b**3*d**2*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 4*a**2*b**4*c**
2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 6*a**2*b**4*c**2/
(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 48*a**2*b**4*c*d*x**2*log(a/b +
x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 5*a**2*b**4*d**2*x**4/(4*a**2
*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 8*a*b**5*c**2*x*log(a/b + x)/(4*a**2*b*
*7 + 8*a*b**8*x + 4*b**9*x**2) + 8*a*b**5*c**2*x/(4*a**2*b**7 + 8*a*b**8*x
+ 4*b**9*x**2) - 16*a*b**5*c*d*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2
) - 2*a*b**5*d**2*x**5/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 4*b**6*c*
*2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 4*b**6*c*d*
x**4/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + b**6*d**2*x**6/(4*a**2*b**7
 + 8*a*b**8*x + 4*b**9*x**2), Eq(n, -3)), (-180*a**6*d**2*log(a/b + x)/(30*
a*b**7 + 30*b**8*x) - 180*a**6*d**2/(30*a*b**7 + 30*b**8*x) - 180*a**5*b*d*
*2*x*log(a/b + x)/(30*a*b**7 + 30*b**8*x) - 240*a**4*b**2*c*d*log(a/b + x)/
(30*a*b**7 + 30*b**8*x) - 240*a**4*b**2*c*d/(30*a*b**7 + 30*b**8*x) + 90*a*
*4*b**2*d**2*x**2/(30*a*b**7 + 30*b**8*x) - 240*a**3*b**3*c*d*x*log(a/b + x
)/(30*a*b**7 + 30*b**8*x) - 30*a**3*b**3*d**2*x**3/(30*a*b**7 + 30*b**8*x)
- 60*a**2*b**4*c**2*log(a/b + x)/(30*a*b**7 + 30*b**8*x) - 60*a**2*b**4*c**
2/(30*a*b**7 + 30*b**8*x) + 120*a**2*b**4*c*d*x**2/(30*a*b**7 + 30*b**8*x)
+ 15*a**2*b**4*d**2*x**4/(30*a*b**7 + 30*b**8*x) - 60*a*b**5*c**2*x*log(a/b
 + x)/(30*a*b**7 + 30*b**8*x) - 40*a*b**5*c*d*x**3/(30*a*b**7 + 30*b**8*x)
- 9*a*b**5*d**2*x**5/(30*a*b**7 + 30*b**8*x) + 30*b**6*c**2*x**2/(30*a*b**7
 + 30*b**8*x) + 20*b**6*c*d*x**4/(30*a*b**7 + 30*b**8*x) + 6*b**6*d**2*x**6
/(30*a*b**7 + 30*b**8*x), Eq(n, -2)), (a**6*d**2*log(a/b + x)/b**7 - a**5*d
**2*x/b**6 + 2*a**4*c*d*log(a/b + x)/b**5 + a**4*d**2*x**2/(2*b**5) - 2*a**
3*c*d*x/b**4 - a**3*d**2*x**3/(3*b**4) + a**2*c**2*log(a/b + x)/b**3 + a**2
*c*d*x**2/b**3 + a**2*d**2*x**4/(4*b**3) - a*c**2*x/b**2 - 2*a*c*d*x**3/(3*
b**2) - a*d**2*x**5/(5*b**2) + c**2*x**2/(2*b) + c*d*x**4/(2*b) + d**2*x**6
/(6*b), Eq(n, -1)), (720*a**7*d**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) - 720*a**6*b*d**2*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7
*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
 13068*b**7*n + 5040*b**7) + 48*a**5*b**2*c*d*n**2*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
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*7*n**2 + 13068*b**7*n + 5040*b**7) + 624*a**5*b**2*c*d*n*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2016*a**5*b**2*c*d*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5*b**2*d**2*n**2*
x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5
*b**2*d**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) - 48*a**4*b**3*c*d*n**3*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b
**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n
 + 5040*b**7) - 624*a**4*b**3*c*d*n**2*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*
n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) - 2016*a**4*b**3*c*d*n*x*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
*7*n**2 + 13068*b**7*n + 5040*b**7) - 120*a**4*b**3*d**2*n**3*x**3*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 360*a**4*b**3*d**2*n*
*2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*
n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 240*a
**4*b**3*d**2*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + 2*a**3*b**4*c**2*n**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 44*a**3*b**4*c**2*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n
**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) + 358*a**3*b**4*c**2*n**2*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
*7*n**2 + 13068*b**7*n + 5040*b**7) + 1276*a**3*b**4*c**2*n*(a + b*x)**n/(b
**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1680*a**3*b**4*c**2*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 24*a**3*b**4*c*d*n**4
*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 336*a**
3*b**4*c*d*n**3*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + 1320*a**3*b**4*c*d*n**2*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 1008*a**3*b**4*c*d*n*x**2*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 30*a**3*b**4*d**2*n**4*x**4*(a + b*x)*
*n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n
**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n**3
*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 330*a**
3*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**
5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) + 180*a**3*b**4*d**2*n*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) - 2*a**2*b**5*c**2*n**5*x*(a + b*x)**n/(b**7*n**7 + 28*
b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n*
*2 + 13068*b**7*n + 5040*b**7) - 44*a**2*b**5*c**2*n**4*x*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 358*a**2*b**5*c**2*n**3*x*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1276*a**2*b**5*c*
*2*n**2*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**
7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 168
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0*a**2*b**5*c**2*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) - 8*a**2*b**5*c*d*n**5*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
**7*n + 5040*b**7) - 128*a**2*b**5*c*d*n**4*x**3*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) - 664*a**2*b**5*c*d*n**3*x**3*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1216*a**2*b**5*c*d*n**2*
x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 672*a**2
*b**5*c*d*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1
960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7
) - 6*a**2*b**5*d**2*n**5*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322
*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7
*n + 5040*b**7) - 60*a**2*b**5*d**2*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*
b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n*
*2 + 13068*b**7*n + 5040*b**7) - 210*a**2*b**5*d**2*n**3*x**5*(a + b*x)**n/
(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 300*a**2*b**5*d**2*n**2*x*
*5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
+ 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 144*a**2*b
**5*d**2*n*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 19
60*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
 + a*b**6*c**2*n**6*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + 23*a*b**6*c**2*n**5*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 201*a*b**6*c**2*n**4*x**2*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 817*a*b**6*c**2*n**3*x**2*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1478*a*b**6*c**2*n**2*x**
2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 840*a*b**6*
c**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b
**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2
*a*b**6*c*d*n**6*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**
5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) + 38*a*b**6*c*d*n**5*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3
22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b*
*7*n + 5040*b**7) + 262*a*b**6*c*d*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b
**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7) + 802*a*b**6*c*d*n**3*x**4*(a + b*x)**n/(b**7
*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13
132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1080*a*b**6*c*d*n**2*x**4*(a +
b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b
**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 504*a*b**6*c*d*n*x
**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + a*b**6*d*
*2*n**6*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*
b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) +
15*a*b**6*d**2*n**5*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + 85*a*b**6*d**2*n**4*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 225*a*b**6*d**2*n**3*x**6*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 274*a*b**6*d**2*n**2*x**6*(a + b*x)**n
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/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 120*a*b**6*d**2*n*x**6*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*c**2*n**6*
x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 25*b**7*
c**2*n**5*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 196
0*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
+ 247*b**7*c**2*n**4*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7
*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
5040*b**7) + 1219*b**7*c**2*n**3*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**
6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 130
68*b**7*n + 5040*b**7) + 3112*b**7*c**2*n**2*x**3*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 3796*b**7*c**2*n*x**3*(a + b*x)**n/(b*
*7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1680*b**7*c**2*x**3*(a + b*x)
**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2*b**7*c*d*n**6*x**5*(
a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 67
69*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 46*b**7*c*d*n*
*5*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*
n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 414*b
**7*c*d*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 1850*b**7*c*d*n**3*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b
**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n
 + 5040*b**7) + 4288*b**7*c*d*n**2*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n
**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) + 4824*b**7*c*d*n*x**5*(a + b*x)**n/(b**7*n**7 + 2
8*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*
n**2 + 13068*b**7*n + 5040*b**7) + 2016*b**7*c*d*x**5*(a + b*x)**n/(b**7*n*
*7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132
*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*d**2*n**6*x**7*(a + b*x)**n/(
b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
+ 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 21*b**7*d**2*n**5*x**7*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 175*b**7*d**2*n**
4*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n
**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 735*b*
*7*d**2*n**3*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 1624*b**7*d**2*n**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 1764*b**7*d**2*n*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n*
*6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13
068*b**7*n + 5040*b**7) + 720*b**7*d**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b
**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7), True))
```

3.266 $\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=185

$$-\frac{a\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^6(n+1)}+\frac{\left(a^2d+b^2c\right)\left(5a^2d+b^2c\right)(a+bx)^{n+2}}{b^6(n+2)}-\frac{2ad\left(5a^2d+3b^2c\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{2d\left(5a^2d+\right.}{b}$$

Rubi [A] time = 0.10, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {772}

$$-\frac{a\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^6(n+1)}+\frac{\left(a^2d+b^2c\right)\left(5a^2d+b^2c\right)(a+bx)^{n+2}}{b^6(n+2)}-\frac{2ad\left(5a^2d+3b^2c\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{2d\left(5a^2d+b^2c\right)(a+bx)^{n+4}}{b^6(n+4)}-\frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)}+\frac{d^2(a+bx)^{n+6}}{b^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] -((a*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^6*(1 + n))) + ((b^2*c + a^2*d)
*(b^2*c + 5*a^2*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) - (2*a*d*(3*b^2*c + 5*a
^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (2*d*(b^2*c + 5*a^2*d)*(a + b*x)^(
4 + n))/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a
 + b*x)^(6 + n))/(b^6*(6 + n))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^2\right)^2 dx = \int \left(-\frac{a\left(b^2c+a^2d\right)^2(a+bx)^n}{b^5}+\frac{\left(b^2c+a^2d\right)\left(b^2c+5a^2d\right)(a+bx)^{1+n}}{b^5}-\frac{2ad\left(3b\right.}{}\right.$$

$$= -\frac{a\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^6(1+n)}+\frac{\left(b^2c+a^2d\right)\left(b^2c+5a^2d\right)(a+bx)^{2+n}}{b^6(2+n)}-\frac{2ad\left(3b^2c\right.}{}$$

Mathematica [A] time = 0.50, size = 323, normalized size = 1.75

$$\frac{(a+bx)^{n+1}\left(4(n+1)(a+bx)\left((n+5)\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+2)x+b^2(n+3)\left(c(n+4)+d(n+2)x^2\right)\right)-ad(n+2)(a+bx)\left(2a^2d-2abd(n+3)x+b^2(n+4)\left(c(n+5)+d(n+3)x^2\right)\right)\right)-a(n+6)\left(4(n+4)\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)-4ad(n+1)(a+bx)\left(2a^2d-2abd(n+2)x+b^2(n+3)\left(c(n+4)+d(n+2)x^2\right)\right)+b^4(n+1)(n+2)(n+3)(n+4)\left(c+dx^2\right)^2\right)+b^4(n+1)(n+2)(n+3)(n+4)(n+5)(a+bx)\left(c+dx^2\right)^2\right)}{b^6(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(a + b*x)*(
c + d*x^2)^2 - a*(6 + n)*(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2
 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*
(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 +
 n)*x + b^2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2))) + 4*(1 + n)*(a + b*x)*((b
^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n)
 + d*(2 + n)*x^2)) - a*d*(2 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(3 + n)*x + b
^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))))/(b^6*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(5 + n)*(6 + n))
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n*(c + d*x^2)^2, x]
```

fricas [B] time = 0.42, size = 757, normalized size = 4.09

$$-\frac{\left(a^2b^4c^2n^4+18a^2b^4c^2n^3+360a^2b^4c^2+360a^4b^2cd+120a^6d^2-\left(b^6d^2n^5+15b^6d^2n^4+85b^6d^2n^3+225b^6d^2n^2+274b^6d^2n+120b^6d^2\right)x^6-\left(ab^5d^2n^5+10ab^5d^2n^4+35ab^5d^2n^3+50ab^5d^2n^2+24ab^5d^2n\right)x^5-\left(2b^6cdn^5+360b^6cd+\left(34b^6cd-5a^2b^4d^2\right)n^4+2\left(107b^6cd-15a^2b^4d^2\right)n^3+\left(614b^6cd-55a^2b^4d^2\right)n^2+6\left(132b^6cd-5a^2b^4d^2\right)n\right)x^4-2\left(ab^5cdn^5+14ab^5cdn^4+5\left(13ab^5cd+2a^3b^3d^2\right)n^3+2\left(56ab^5cd+15a^3b^3d^2\right)n^2+20\left(3ab^5cd+a^3b^3d^2\right)n\right)x^3+\left(119a^2b^4c^2+12a^4b^2cd\right)n^2-\left(b^6c^2n^5+360b^6c^2+\left(19b^6c^2-6a^2b^4cd\right)n^4+\left(137b^6c^2-72a^2b^4cd\right)n^3+\left(461b^6c^2-246a^2b^4cd-60a^4b^2d^2\right)n^2+6\left(117b^6c^2-30a^2b^4cd-10a^4b^2d^2\right)n\right)x^2+6\left(57a^2b^4c^2+22a^4b^2cd\right)n-\left(ab^5c^2n^5+18ab^5c^2n^4+\left(119ab^5c^2+12a^3b^3cd\right)n^3+6\left(57ab^5c^2+22a^3b^3cd\right)n^2+120\left(3ab^5c^2+3a^3b^3cd+a^5bd^2\right)n\right)x\right)(bx+a)^n}{b^6n^6+21b^6n^5+175b^6n^4+735b^6n^3+1624b^6n^2+1764b^6n+720b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^4*c^2*n^4 + 18*a^2*b^4*c^2*n^3 + 360*a^2*b^4*c^2 + 360*a^4*b^2*c*d
+ 120*a^6*d^2 - (b^6*d^2*n^5 + 15*b^6*d^2*n^4 + 85*b^6*d^2*n^3 + 225*b^6*d^
2*n^2 + 274*b^6*d^2*n + 120*b^6*d^2)*x^6 - (a*b^5*d^2*n^5 + 10*a*b^5*d^2*n^
4 + 35*a*b^5*d^2*n^3 + 50*a*b^5*d^2*n^2 + 24*a*b^5*d^2*n)*x^5 - (2*b^6*c*d*
n^5 + 360*b^6*c*d + (34*b^6*c*d - 5*a^2*b^4*d^2)*n^4 + 2*(107*b^6*c*d - 15*
a^2*b^4*d^2)*n^3 + (614*b^6*c*d - 55*a^2*b^4*d^2)*n^2 + 6*(132*b^6*c*d - 5*
a^2*b^4*d^2)*n)*x^4 - 2*(a*b^5*c*d*n^5 + 14*a*b^5*c*d*n^4 + 5*(13*a*b^5*c*d
 + 2*a^3*b^3*d^2)*n^3 + 2*(56*a*b^5*c*d + 15*a^3*b^3*d^2)*n^2 + 20*(3*a*b^5
*c*d + a^3*b^3*d^2)*n)*x^3 + (119*a^2*b^4*c^2 + 12*a^4*b^2*c*d)*n^2 - (b^6*
c^2*n^5 + 360*b^6*c^2 + (19*b^6*c^2 - 6*a^2*b^4*c*d)*n^4 + (137*b^6*c^2 - 7
2*a^2*b^4*c*d)*n^3 + (461*b^6*c^2 - 246*a^2*b^4*c*d - 60*a^4*b^2*d^2)*n^2 +
 6*(117*b^6*c^2 - 30*a^2*b^4*c*d - 10*a^4*b^2*d^2)*n)*x^2 + 6*(57*a^2*b^4*c
^2 + 22*a^4*b^2*c*d)*n - (a*b^5*c^2*n^5 + 18*a*b^5*c^2*n^4 + (119*a*b^5*c^2
 + 12*a^3*b^3*c*d)*n^3 + 6*(57*a*b^5*c^2 + 22*a^3*b^3*c*d)*n^2 + 120*(3*a*b
^5*c^2 + 3*a^3*b^3*c*d + a^5*b*d^2)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5
 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

giac [B] time = 0.22, size = 1266, normalized size = 6.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^6*d^2*n^5*x^6 + (b*x + a)^n*a*b^5*d^2*n^5*x^5 + 15*(b*x + a)
^n*b^6*d^2*n^4*x^6 + 2*(b*x + a)^n*b^6*c*d*n^5*x^4 + 10*(b*x + a)^n*a*b^5*d
^2*n^4*x^5 + 85*(b*x + a)^n*b^6*d^2*n^3*x^6 + 2*(b*x + a)^n*a*b^5*c*d*n^5*x
^3 + 34*(b*x + a)^n*b^6*c*d*n^4*x^4 - 5*(b*x + a)^n*a^2*b^4*d^2*n^4*x^4 + 3
5*(b*x + a)^n*a*b^5*d^2*n^3*x^5 + 225*(b*x + a)^n*b^6*d^2*n^2*x^6 + (b*x +
a)^n*b^6*c^2*n^5*x^2 + 28*(b*x + a)^n*a*b^5*c*d*n^4*x^3 + 214*(b*x + a)^n*b
^6*c*d*n^3*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n^3*x^4 + 50*(b*x + a)^n*a*b^5*
d^2*n^2*x^5 + 274*(b*x + a)^n*b^6*d^2*n*x^6 + (b*x + a)^n*a*b^5*c^2*n^5*x +
 19*(b*x + a)^n*b^6*c^2*n^4*x^2 - 6*(b*x + a)^n*a^2*b^4*c*d*n^4*x^2 + 130*(
b*x + a)^n*a*b^5*c*d*n^3*x^3 + 20*(b*x + a)^n*a^3*b^3*d^2*n^3*x^3 + 614*(b*
x + a)^n*b^6*c*d*n^2*x^4 - 55*(b*x + a)^n*a^2*b^4*d^2*n^2*x^4 + 24*(b*x + a
)^n*a*b^5*d^2*n*x^5 + 120*(b*x + a)^n*b^6*d^2*x^6 + 18*(b*x + a)^n*a*b^5*c^
2*n^4*x + 137*(b*x + a)^n*b^6*c^2*n^3*x^2 - 72*(b*x + a)^n*a^2*b^4*c*d*n^3*
x^2 + 224*(b*x + a)^n*a*b^5*c*d*n^2*x^3 + 60*(b*x + a)^n*a^3*b^3*d^2*n^2*x^
3 + 792*(b*x + a)^n*b^6*c*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n*x^4 - (b*x
 + a)^n*a^2*b^4*c^2*n^4 + 119*(b*x + a)^n*a*b^5*c^2*n^3*x + 12*(b*x + a)^n*
a^3*b^3*c*d*n^3*x + 461*(b*x + a)^n*b^6*c^2*n^2*x^2 - 246*(b*x + a)^n*a^2*b
^4*c*d*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n^2*x^2 + 120*(b*x + a)^n*a*b^5
*c*d*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d^2*n*x^3 + 360*(b*x + a)^n*b^6*c*d*x^4
```

```
 - 18*(b*x + a)^n*a^2*b^4*c^2*n^3 + 342*(b*x + a)^n*a*b^5*c^2*n^2*x + 132*(
b*x + a)^n*a^3*b^3*c*d*n^2*x + 702*(b*x + a)^n*b^6*c^2*n*x^2 - 180*(b*x + a
)^n*a^2*b^4*c*d*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n*x^2 - 119*(b*x + a)^n*
a^2*b^4*c^2*n^2 - 12*(b*x + a)^n*a^4*b^2*c*d*n^2 + 360*(b*x + a)^n*a*b^5*c^
2*n*x + 360*(b*x + a)^n*a^3*b^3*c*d*n*x + 120*(b*x + a)^n*a^5*b*d^2*n*x + 3
60*(b*x + a)^n*b^6*c^2*x^2 - 342*(b*x + a)^n*a^2*b^4*c^2*n - 132*(b*x + a)^
n*a^4*b^2*c*d*n - 360*(b*x + a)^n*a^2*b^4*c^2 - 360*(b*x + a)^n*a^4*b^2*c*d
 - 120*(b*x + a)^n*a^6*d^2)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n
^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

maple [B] time = 0.01, size = 677, normalized size = 3.66

$$-\frac{\left(-b^5d^2n^5x^5-15b^5d^2n^4x^5+5ab^4d^2n^4x^4-2b^5cdn^5x^3-85b^5d^2n^3x^5+50ab^4d^2n^3x^4-34b^5cdn^4x^3-225b^5d^2n^2x^5-20a^2b^3d^2n^3x^3+6ab^4cdn^4x^2+175ab^4d^2n^2x^4-b^5c^2n^5x-214b^5cdn^3x^3-274b^5d^2nx^5-120a^2b^3d^2n^2x^3+84ab^4cdn^3x^2+250ab^4d^2nx^4-19b^5c^2n^4x-614b^5cdn^2x^3-120d^2x^5b^5+60a^3b^2d^2n^2x^2-12a^2b^3cdn^3x-220a^2b^3d^2nx^3+ab^4c^2n^4+390ab^4cdn^2x^2+120ad^2x^4b^4-137b^5c^2n^3x-792b^5cdnx^3+180a^3b^2d^2nx^2-144a^2b^3cdn^2x-120a^2b^3d^2x^3+18ab^4c^2n^3+672ab^4cdnx^2-461b^5c^2n^2x-360b^5cdx^3-120a^4bd^2nx+12a^3b^2cdn^2+120a^3b^2d^2x^2-492a^2b^3cdnx+119ab^4c^2n^2+360ab^4cdx^2-702b^5c^2nx-120a^4bd^2x+132a^3b^2cdn-360a^2b^3cdx+342ab^4c^2n-360b^5c^2x+120a^5d^2+360a^3b^2cd+360ab^4c^2\right)(bx+a)^{n+1}}{\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^2,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^5*d^2*n^5*x^5-15*b^5*d^2*n^4*x^5+5*a*b^4*d^2*n^4*x^4-2*b
^5*c*d*n^5*x^3-85*b^5*d^2*n^3*x^5+50*a*b^4*d^2*n^3*x^4-34*b^5*c*d*n^4*x^3-2
25*b^5*d^2*n^2*x^5-20*a^2*b^3*d^2*n^3*x^3+6*a*b^4*c*d*n^4*x^2+175*a*b^4*d^2
*n^2*x^4-b^5*c^2*n^5*x-214*b^5*c*d*n^3*x^3-274*b^5*d^2*n*x^5-120*a^2*b^3*d^
2*n^2*x^3+84*a*b^4*c*d*n^3*x^2+250*a*b^4*d^2*n*x^4-19*b^5*c^2*n^4*x-614*b^5
*c*d*n^2*x^3-120*b^5*d^2*x^5+60*a^3*b^2*d^2*n^2*x^2-12*a^2*b^3*c*d*n^3*x-22
0*a^2*b^3*d^2*n*x^3+a*b^4*c^2*n^4+390*a*b^4*c*d*n^2*x^2+120*a*b^4*d^2*x^4-1
37*b^5*c^2*n^3*x-792*b^5*c*d*n*x^3+180*a^3*b^2*d^2*n*x^2-144*a^2*b^3*c*d*n^
2*x-120*a^2*b^3*d^2*x^3+18*a*b^4*c^2*n^3+672*a*b^4*c*d*n*x^2-461*b^5*c^2*n^
2*x-360*b^5*c*d*x^3-120*a^4*b*d^2*n*x+12*a^3*b^2*c*d*n^2+120*a^3*b^2*d^2*x^
2-492*a^2*b^3*c*d*n*x+119*a*b^4*c^2*n^2+360*a*b^4*c*d*x^2-702*b^5*c^2*n*x-1
20*a^4*b*d^2*x+132*a^3*b^2*c*d*n-360*a^2*b^3*c*d*x+342*a*b^4*c^2*n-360*b^5*
c^2*x+120*a^5*d^2+360*a^3*b^2*c*d+360*a*b^4*c^2)/b^6/(n^6+21*n^5+175*n^4+73
5*n^3+1624*n^2+1764*n+720)
```

maxima [A] time = 0.50, size = 335, normalized size = 1.81

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^nc^2}{\left(n^2+3n+2\right)b^2}+\frac{2\left(\left(n^3+6n^2+11n+6\right)b^4x^4+\left(n^3+3n^2+2n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6a^3bnx-6a^4\right)(bx+a)^ncd}{\left(n^4+10n^3+35n^2+50n+24\right)b^4}+\frac{\left(\left(n^5+15n^4+85n^3+225n^2+274n+120\right)b^6x^6+\left(n^5+10n^4+35n^3+50n^2+24n\right)ab^5x^5-5\left(n^4+6n^3+11n^2+6n\right)a^2b^4x^4+20\left(n^3+3n^2+2n\right)a^3b^3x^3-60\left(n^2+n\right)a^4b^2x^2+120a^5bnx-120a^6\right)(bx+a)^nd^2}{\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2
*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 3
5*n^2 + 50*n + 24)*b^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*
b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^
3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^
2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d^2/((n^6 + 21*n^
5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

mupad [B] time = 3.05, size = 723, normalized size = 3.91

$$\frac{d^2x^6(a+bx)^n\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}{n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720}-\frac{a^2(a+bx)^n\left(120a^4d^2+12a^2b^2cdn^2+132a^2b^2cdn+360a^2b^2cd+b^4c^2n^4+18b^4c^2n^3+119b^4c^2n^2+342b^4c^2n+360b^4c^2\right)}{b^6\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}+\frac{x^2(n+1)(a+bx)^n\left(-60a^4d^2n-6a^2b^2cdn^3-66a^2b^2cdn^2-180a^2b^2cdn+b^4c^2n^4+18b^4c^2n^3+119b^4c^2n^2+342b^4c^2n+360b^4c^2\right)}{b^4\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}+\frac{dx^4(a+bx)^n\left(-5da^2n+2cb^2n^2+22cb^2n+60cb^2\right)\left(n^3+6n^2+11n+6\right)}{b^2\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}+\frac{anx(a+bx)^n\left(120a^4d^2+12a^2b^2cdn^2+132a^2b^2cdn+360a^2b^2cd+b^4c^2n^4+18b^4c^2n^3+119b^4c^2n^2+342b^4c^2n+360b^4c^2\right)}{b^5\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}+\frac{ad^2nx^5(a+bx)^n\left(n^4+10n^3+35n^2+50n+24\right)}{b\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}+\frac{2adnx^3(a+bx)^n\left(n^2+3n+2\right)\left(10da^2+cb^2n^2+11cb^2n+30cb^2\right)}{b^3\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764
*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) - (a^2*(a + b*x)^n*
(120*a^4*d^2 + 360*b^4*c^2 + 342*b^4*c^2*n + 119*b^4*c^2*n^2 + 18*b^4*c^2*n
^3 + b^4*c^2*n^4 + 360*a^2*b^2*c*d + 132*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2
```

```
))/(b^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^
2*(n + 1)*(a + b*x)^n*(360*b^4*c^2 - 60*a^4*d^2*n + 342*b^4*c^2*n + 119*b^4
*c^2*n^2 + 18*b^4*c^2*n^3 + b^4*c^2*n^4 - 180*a^2*b^2*c*d*n - 66*a^2*b^2*c*
d*n^2 - 6*a^2*b^2*c*d*n^3))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 2
1*n^5 + n^6 + 720)) + (d*x^4*(a + b*x)^n*(60*b^2*c + 2*b^2*c*n^2 - 5*a^2*d*
n + 22*b^2*c*n)*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^2 + 735*n^3
 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*n*x*(a + b*x)^n*(120*a^4*d^2 + 360*b
^4*c^2 + 342*b^4*c^2*n + 119*b^4*c^2*n^2 + 18*b^4*c^2*n^3 + b^4*c^2*n^4 + 3
60*a^2*b^2*c*d + 132*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2))/(b^5*(1764*n + 16
24*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d^2*n*x^5*(a + b*x)^
n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 17
5*n^4 + 21*n^5 + n^6 + 720)) + (2*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(10
*a^2*d + 30*b^2*c + b^2*c*n^2 + 11*b^2*c*n))/(b^3*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
```

sympy [A] time = 13.80, size = 8940, normalized size = 48.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**2/2 + c*d*x**4/2 + d**2*x**6/6), Eq(b, 0)), (60*a*
*5*d**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 +
 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d**2/(60
*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 30
0*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*d**2*x*log(a/b + x)/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**
10*x**4 + 60*b**11*x**5) + 625*a**4*b*d**2*x/(60*a**5*b**6 + 300*a**4*b**7*
x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x
**5) - 6*a**3*b**2*c*d/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d
**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
+ 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d
**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b*
*9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 30*a**2*b**3*c*d*x/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**
10*x**4 + 60*b**11*x**5) + 600*a**2*b**3*d**2*x**3*log(a/b + x)/(60*a**5*b*
*6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**1
0*x**4 + 60*b**11*x**5) + 900*a**2*b**3*d**2*x**3/(60*a**5*b**6 + 300*a**4*
b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b*
*11*x**5) - 3*a*b**4*c**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x
**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 60*a*b**4*c*
d*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9
*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d**2*x**4*log(a/b +
x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**
3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d**2*x**4/(60*a**5*b**6
+ 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x
**4 + 60*b**11*x**5) - 15*b**5*c**2*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600
*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) -
60*b**5*c*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600
*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*d**2*x**5*log
(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b
**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*d**2*lo
g(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x
**3 + 12*b**10*x**4) - 125*a**5*d**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*
*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*d**2*x*log(a/b
+ x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 +
12*b**10*x**4) - 440*a**4*b*d**2*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2
*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 6*a**3*b**2*c*d/(12*a**4*b**
```

```
6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) -
360*a**3*b**2*d**2*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*
*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 540*a**3*b**2*d**2*x**2/(1
2*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**1
0*x**4) - 24*a**2*b**3*c*d*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*
x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d**2*x**3*log(a/b +
x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12
*b**10*x**4) - 240*a**2*b**3*d**2*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*
a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - a*b**4*c**2/(12*a**4*b**
6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) -
36*a*b**4*c*d*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*
a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b*
*6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) -
 4*b**5*c**2*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b*
*9*x**3 + 12*b**10*x**4) - 24*b**5*c*d*x**3/(12*a**4*b**6 + 48*a**3*b**7*x
+ 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) + 12*b**5*d**2*x**5/(
12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**
10*x**4), Eq(n, -5)), (60*a**5*d**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**
7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5*d**2/(6*a**3*b**6 + 18*a**2*
b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*b*d**2*x*log(a/b + x)/(6*
a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 270*a**4*b*d**
2*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 12*a**3
*b**2*c*d*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b
**9*x**3) + 22*a**3*b**2*c*d/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2
 + 6*b**9*x**3) + 180*a**3*b**2*d**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a*
*2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d**2*x**2/(6*a**3
*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 36*a**2*b**3*c*d*x
*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3)
 + 54*a**2*b**3*c*d*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b*
*9*x**3) + 60*a**2*b**3*d**2*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*
x + 18*a*b**8*x**2 + 6*b**9*x**3) - a*b**4*c**2/(6*a**3*b**6 + 18*a**2*b**7
*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 36*a*b**4*c*d*x**2*log(a/b + x)/(6*a**
3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 36*a*b**4*c*d*x**
2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4
*d**2*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) -
3*b**5*c**2*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3)
 + 12*b**5*c*d*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*
x**2 + 6*b**9*x**3) + 3*b**5*d**2*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a
*b**8*x**2 + 6*b**9*x**3), Eq(n, -4)), (-60*a**5*d**2*log(a/b + x)/(6*a**2*
b**6 + 12*a*b**7*x + 6*b**8*x**2) - 90*a**5*d**2/(6*a**2*b**6 + 12*a*b**7*x
 + 6*b**8*x**2) - 120*a**4*b*d**2*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x
 + 6*b**8*x**2) - 120*a**4*b*d**2*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**
2) - 36*a**3*b**2*c*d*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2
) - 54*a**3*b**2*c*d/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b*
*2*d**2*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 72*a*
*2*b**3*c*d*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 72*a
**2*b**3*c*d*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*d**
2*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 3*a*b**4*c**2/(6*a**2*b*
*6 + 12*a*b**7*x + 6*b**8*x**2) - 36*a*b**4*c*d*x**2*log(a/b + x)/(6*a**2*b
**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*a*b**4*d**2*x**4/(6*a**2*b**6 + 12*a*b
**7*x + 6*b**8*x**2) - 6*b**5*c**2*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x*
*2) + 12*b**5*c*d*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*d
**2*x**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*d*
*2*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 60*a**5*d**2/(12*a*b**6 + 12*b**7
*x) + 60*a**4*b*d**2*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 72*a**3*b**2*
c*d*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 72*a**3*b**2*c*d/(12*a*b**6 + 12
*b**7*x) - 30*a**3*b**2*d**2*x**2/(12*a*b**6 + 12*b**7*x) + 72*a**2*b**3*c*
d*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 10*a**2*b**3*d**2*x**3/(12*a*b**
6 + 12*b**7*x) + 12*a*b**4*c**2*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 12*a
```

```
*b**4*c**2/(12*a*b**6 + 12*b**7*x) - 36*a*b**4*c*d*x**2/(12*a*b**6 + 12*b**
7*x) - 5*a*b**4*d**2*x**4/(12*a*b**6 + 12*b**7*x) + 12*b**5*c**2*x*log(a/b
+ x)/(12*a*b**6 + 12*b**7*x) + 12*b**5*c*d*x**3/(12*a*b**6 + 12*b**7*x) + 3
*b**5*d**2*x**5/(12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*d**2*log(a/b +
x)/b**6 + a**4*d**2*x/b**5 - 2*a**3*c*d*log(a/b + x)/b**4 - a**3*d**2*x**2/
(2*b**4) + 2*a**2*c*d*x/b**3 + a**2*d**2*x**3/(3*b**3) - a*c**2*log(a/b + x
)/b**2 - a*c*d*x**2/b**2 - a*d**2*x**4/(4*b**2) + c**2*x/b + 2*c*d*x**3/(3*
b) + d**2*x**5/(5*b), Eq(n, -1)), (-120*a**6*d**2*(a + b*x)**n/(b**6*n**6 +
 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*
n + 720*b**6) + 120*a**5*b*d**2*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
+ 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6)
- 12*a**4*b**2*c*d*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 132*a**4*b
**2*c*d*n*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 360*a**4*b**2*c*d*(a + b
*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**
6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d**2*n**2*x**2*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d**2*n*x**2*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) + 12*a**3*b**3*c*d*n**3*x*(a + b*x)**n/(b**6*n**6 + 21*b*
*6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 72
0*b**6) + 132*a**3*b**3*c*d*n**2*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) +
 360*a**3*b**3*c*d*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**
3*d**2*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 7
35*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 60*a**3*b**3*d**2
*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**
6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 40*a**3*b**3*d**2*n*x**
3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - a**2*b**4*c**2*n**4*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
 + 1764*b**6*n + 720*b**6) - 18*a**2*b**4*c**2*n**3*(a + b*x)**n/(b**6*n**6
 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) - 119*a**2*b**4*c**2*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) - 342*a**2*b**4*c**2*n*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b
**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 360*a
**2*b**4*c**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*
b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 6*a**2*b**4*c*d*n**4
*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**
3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 72*a**2*b**4*c*d*n**3*x**2*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) - 246*a**2*b**4*c*d*n**2*x**2*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) - 180*a**2*b**4*c*d*n*x**2*(a + b*x)**n/(b*
*6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1
764*b**6*n + 720*b**6) - 5*a**2*b**4*d**2*n**4*x**4*(a + b*x)**n/(b**6*n**6
 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) - 30*a**2*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) - 55*a**2*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) - 30*a**2*b**4*d**2*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b
**5*c**2*n**5*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 18*a*b**5*c**2*n**
4*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
```

```
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 119*a*b**5*c**2*n**3*x*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 342*a*b**5*c**2*n**2*x*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 176
4*b**6*n + 720*b**6) + 360*a*b**5*c**2*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**
6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) + 2*a*b**5*c*d*n**5*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 28
*a*b**5*c*d*n**4*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 130*a*b**5*c
*d*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b
**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 224*a*b**5*c*d*n**2*x
**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a*b**5*c*d*n*x**3*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + a*b**5*d**2*n**5*x**5*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 10*a*b**5*d**2*n**4*x**5*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 35*a*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6)
 + 50*a*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b*
*6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 24*a*b
**5*d**2*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*c**2*n**5*x**
2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 19*b**6*c**2*n**4*x**2*(a + b*x)
**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n
**2 + 1764*b**6*n + 720*b**6) + 137*b**6*c**2*n**3*x**2*(a + b*x)**n/(b**6*
n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 461*b**6*c**2*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 702*b**6*c**2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 1
75*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 3
60*b**6*c**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 +
735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 2*b**6*c*d*n**5*
x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 34*b**6*c*d*n**4*x**4*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 214*b**6*c*d*n**3*x**4*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 176
4*b**6*n + 720*b**6) + 614*b**6*c*d*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 792*b**6*c*d*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 36
0*b**6*c*d*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*d**2*n**5*x**
6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*d**2*n**4*x**6*(a + b*x)
**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n
**2 + 1764*b**6*n + 720*b**6) + 85*b**6*d**2*n**3*x**6*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 225*b**6*d**2*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
20*b**6) + 274*b**6*d**2*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 12
0*b**6*d**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 7
35*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True))
```

3.267 $\int (a+bx)^n \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=140

$$\frac{\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^5(n+1)}-\frac{4ad\left(a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{2d\left(3a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad^2(a+bx)^{n+4}}{b^5(n+4)}+\frac{d^2(a+bx}{b^5(n+}$$

Rubi [A] time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^5(n+1)}-\frac{4ad\left(a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{2d\left(3a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad^2(a+bx)^{n+4}}{b^5(n+4)}+\frac{d^2(a+bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (4*a*d*(b^2*c + a^2*d
)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (2*d*(b^2*c + 3*a^2*d)*(a + b*x)^(3 +
n))/(b^5*(3 + n)) - (4*a*d^2*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d^2*(a + b
*x)^(5 + n))/(b^5*(5 + n))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n\left(c+dx^2\right)^2\,dx &= \int \left(\frac{\left(b^2c+a^2d\right)^2(a+bx)^n}{b^4}-\frac{4ad\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^4}+\frac{2d\left(b^2c+3a^2d\right)(a+bx)^{}}{b^4}\right.\\ &= \frac{\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^5(1+n)}-\frac{4ad\left(b^2c+a^2d\right)(a+bx)^{2+n}}{b^5(2+n)}+\frac{2d\left(b^2c+3a^2d\right)(a+bx)^{3+}}{b^5(3+n)}\end{aligned}$$

Mathematica [A] time = 0.18, size = 160, normalized size = 1.14

$$\frac{(a+bx)^{n+1}\left(\frac{4\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)}{b^4(n+1)(n+2)(n+3)}-\frac{4ad(a+bx)\left(2a^2d-2abd(n+2)x+b^2(n+3)\left(c(n+4)+d(n+2)x^2\right)\right)}{b^4(n+2)(n+3)(n+4)}+\left(c+dx^2\right)^2\right)}{b(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((c + d*x^2)^2 + (4*(b^2*c + a^2*d)*(2*a^2*d - 2*a*b*d*(
1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)))/(b^4*(1 + n)*(2 + n)*(
3 + n)) - (4*a*d*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4
 + n) + d*(2 + n)*x^2)))/(b^4*(2 + n)*(3 + n)*(4 + n))))/(b*(5 + n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^n\left(c+dx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x^2)^2, x]
```

fricas [B] time = 0.42, size = 519, normalized size = 3.71

$$\frac{\left(ab^4c^2n^4 + 14\,ab^4c^2n^3 + 120\,ab^4c^2 + 80\,a^3b^2cd + 24\,a^5d^2 + \left(b^5d^2n^4 + 10\,b^5d^2n^3 + 35\,b^5d^2n^2 + 50\,b^5d^2n + 24\,b^5d^2\right)x^5 + \left(ab^4d^2n^4 + 6\,ab^4d^2n^3 + 11\,ab^4d^2n^2 + 6\,ab^4d^2n\right)x^4 + 2\left(b^5cdn^4 + 40\,b^5cd + 2\left(6\,b^5cd - a^2b^3d^2\right)n^3 + \left(49\,b^5cd - 6\,a^2b^3d^2\right)n^2 + 2\left(39\,b^5cd - 2\,a^2b^3d^2\right)n\right)x^3 + \left(71\,ab^4c^2 + 4\,a^3b^2cd\right)n^2 + 2\left(ab^4cdn^4 + 10\,ab^4cdn^3 + \left(29\,ab^4cd + 6\,a^3b^2d^2\right)n^2 + 2\left(10\,ab^4cd + 3\,a^3b^2d^2\right)n\right)x^2 + 2\left(77\,ab^4c^2 + 18\,a^3b^2cd\right)n + \left(b^5c^2n^4 + 120\,b^5c^2 + 2\left(7\,b^5c^2 - 2\,a^2b^3cd\right)n^3 + \left(71\,b^5c^2 - 36\,a^2b^3cd\right)n^2 + 2\left(77\,b^5c^2 - 40\,a^2b^3cd - 12\,a^4bd^2\right)n\right)x\right)(bx+a)^n}{b^5n^5 + 15\,b^5n^4 + 85\,b^5n^3 + 225\,b^5n^2 + 274\,b^5n + 120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^4*c^2*n^4 + 14*a*b^4*c^2*n^3 + 120*a*b^4*c^2 + 80*a^3*b^2*c*d + 24*a^5
*d^2 + (b^5*d^2*n^4 + 10*b^5*d^2*n^3 + 35*b^5*d^2*n^2 + 50*b^5*d^2*n + 24*b
^5*d^2)*x^5 + (a*b^4*d^2*n^4 + 6*a*b^4*d^2*n^3 + 11*a*b^4*d^2*n^2 + 6*a*b^4
*d^2*n)*x^4 + 2*(b^5*c*d*n^4 + 40*b^5*c*d + 2*(6*b^5*c*d - a^2*b^3*d^2)*n^3
 + (49*b^5*c*d - 6*a^2*b^3*d^2)*n^2 + 2*(39*b^5*c*d - 2*a^2*b^3*d^2)*n)*x^3
 + (71*a*b^4*c^2 + 4*a^3*b^2*c*d)*n^2 + 2*(a*b^4*c*d*n^4 + 10*a*b^4*c*d*n^3
 + (29*a*b^4*c*d + 6*a^3*b^2*d^2)*n^2 + 2*(10*a*b^4*c*d + 3*a^3*b^2*d^2)*n)
*x^2 + 2*(77*a*b^4*c^2 + 18*a^3*b^2*c*d)*n + (b^5*c^2*n^4 + 120*b^5*c^2 + 2
*(7*b^5*c^2 - 2*a^2*b^3*c*d)*n^3 + (71*b^5*c^2 - 36*a^2*b^3*c*d)*n^2 + 2*(7
7*b^5*c^2 - 40*a^2*b^3*c*d - 12*a^4*b*d^2)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*
b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

giac [B] time = 0.20, size = 851, normalized size = 6.08

$$\frac{(bx+a)^n b^5d^2n^4x^5 + (bx+a)^n ab^4d^2n^4x^4 + 10\,(bx+a)^n b^5d^2n^3x^5 + 2\,(bx+a)^n b^5cdn^4x^3 + 6\,(bx+a)^n ab^4d^2n^3x^4 + 35\,(bx+a)^n b^5d^2n^2x^5 + 2\,(bx+a)^n ab^4cdn^4x^2 + 24\,(bx+a)^n b^5cdn^3x^3 - 4\,(bx+a)^n a^2b^3d^2n^3x^3 + 11\,(bx+a)^n ab^4d^2n^2x^4 + 50\,(bx+a)^n b^5d^2nx^5 + (bx+a)^n b^5c^2n^4x + 20\,(bx+a)^n ab^4cdn^3x^2 + 98\,(bx+a)^n b^5cdn^2x^3 - 12\,(bx+a)^n a^2b^3d^2n^2x^3 + 6\,(bx+a)^n ab^4d^2nx^4 + 24\,(bx+a)^n b^5d^2x^5 + (bx+a)^n ab^4c^2n^4 + 14\,(bx+a)^n b^5c^2n^3x - 4\,(bx+a)^n a^2b^3cdn^3x + 58\,(bx+a)^n ab^4cdn^2x^2 + 12\,(bx+a)^n a^3b^2d^2n^2x^2 + 156\,(bx+a)^n b^5cdnx^3 - 8\,(bx+a)^n a^2b^3d^2nx^3 + 14\,(bx+a)^n ab^4c^2n^3 + 71\,(bx+a)^n b^5c^2n^2x - 36\,(bx+a)^n a^2b^3cdn^2x + 40\,(bx+a)^n ab^4cdnx^2 + 12\,(bx+a)^n a^3b^2d^2nx^2 + 80\,(bx+a)^n b^5cdx^3 + 71\,(bx+a)^n ab^4c^2n^2 + 4\,(bx+a)^n a^3b^2cdn^2 + 154\,(bx+a)^n b^5c^2nx - 80\,(bx+a)^n a^2b^3cdnx - 24\,(bx+a)^n a^4bd^2nx + 154\,(bx+a)^n ab^4c^2n + 36\,(bx+a)^n a^3b^2cdn + 120\,(bx+a)^n b^5c^2x + 120\,(bx+a)^n ab^4c^2 + 80\,(bx+a)^n a^3b^2cd + 24\,(bx+a)^n a^5d^2}{b^5n^5 + 15\,b^5n^4 + 85\,b^5n^3 + 225\,b^5n^2 + 274\,b^5n + 120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d^2*n^4*x^5 + (b*x + a)^n*a*b^4*d^2*n^4*x^4 + 10*(b*x + a)
^n*b^5*d^2*n^3*x^5 + 2*(b*x + a)^n*b^5*c*d*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d^
2*n^3*x^4 + 35*(b*x + a)^n*b^5*d^2*n^2*x^5 + 2*(b*x + a)^n*a*b^4*c*d*n^4*x^
2 + 24*(b*x + a)^n*b^5*c*d*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d^2*n^3*x^3 + 11
*(b*x + a)^n*a*b^4*d^2*n^2*x^4 + 50*(b*x + a)^n*b^5*d^2*n*x^5 + (b*x + a)^n
*b^5*c^2*n^4*x + 20*(b*x + a)^n*a*b^4*c*d*n^3*x^2 + 98*(b*x + a)^n*b^5*c*d*
n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d^2*n^2*x^3 + 6*(b*x + a)^n*a*b^4*d^2*n*x^
4 + 24*(b*x + a)^n*b^5*d^2*x^5 + (b*x + a)^n*a*b^4*c^2*n^4 + 14*(b*x + a)^n
*b^5*c^2*n^3*x - 4*(b*x + a)^n*a^2*b^3*c*d*n^3*x + 58*(b*x + a)^n*a*b^4*c*d
*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n^2*x^2 + 156*(b*x + a)^n*b^5*c*d*n*x
^3 - 8*(b*x + a)^n*a^2*b^3*d^2*n*x^3 + 14*(b*x + a)^n*a*b^4*c^2*n^3 + 71*(b
*x + a)^n*b^5*c^2*n^2*x - 36*(b*x + a)^n*a^2*b^3*c*d*n^2*x + 40*(b*x + a)^n
*a*b^4*c*d*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n*x^2 + 80*(b*x + a)^n*b^5*c*
d*x^3 + 71*(b*x + a)^n*a*b^4*c^2*n^2 + 4*(b*x + a)^n*a^3*b^2*c*d*n^2 + 154*
(b*x + a)^n*b^5*c^2*n*x - 80*(b*x + a)^n*a^2*b^3*c*d*n*x - 24*(b*x + a)^n*a
^4*b*d^2*n*x + 154*(b*x + a)^n*a*b^4*c^2*n + 36*(b*x + a)^n*a^3*b^2*c*d*n +
 120*(b*x + a)^n*b^5*c^2*x + 120*(b*x + a)^n*a*b^4*c^2 + 80*(b*x + a)^n*a^3
*b^2*c*d + 24*(b*x + a)^n*a^5*d^2)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225
*b^5*n^2 + 274*b^5*n + 120*b^5)
```

maple [B] time = 0.01, size = 420, normalized size = 3.00

$$\frac{\left(b^4d^2n^4x^4 + 10b^4d^2n^3x^4 - 4a\,b^3d^2n^3x^3 + 2b^4cd\,n^4x^2 + 35b^4d^2n^2x^4 - 24a\,b^3d^2n^2x^3 + 24b^4cd\,n^3x^2 + 50b^4d^2n\,x^4 + 12a^2b^2d^2n^2x^2 - 4a\,b^3cd\,n^3x - 44a\,b^3d^2n\,x^3 + b^4c^2n^4 + 98b^4cd\,n^2x^2 + 24d^2x^4b^4 + 36a^2b^2d^2n\,x^2 - 40a\,b^3cd\,n^2x - 24a\,d^2x^3b^3 + 14b^4c^2n^3 + 156b^4cdn\,x^2 - 24a^3b\,d^2nx + 4a^2b^2cd\,n^2 + 24a^2b^2d^2x^2 - 116a\,b^3cdnx + 71b^4c^2n^2 + 80b^4cd\,x^2 - 24a^3b\,d^2x + 36a^2b^2cdn - 80a\,b^3cdx + 154b^4c^2n + 24a^4d^2 + 80a^2b^2cd + 120b^4c^2\right)(bx+a)^{n+1}}{\left(n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^2,x)
```

```
[Out] (b*x+a)^(n+1)*(b^4*d^2*n^4*x^4+10*b^4*d^2*n^3*x^4-4*a*b^3*d^2*n^3*x^3+2*b^4
*c*d*n^4*x^2+35*b^4*d^2*n^2*x^4-24*a*b^3*d^2*n^2*x^3+24*b^4*c*d*n^3*x^2+50*
b^4*d^2*n*x^4+12*a^2*b^2*d^2*n^2*x^2-4*a*b^3*c*d*n^3*x-44*a*b^3*d^2*n*x^3+b
^4*c^2*n^4+98*b^4*c*d*n^2*x^2+24*b^4*d^2*x^4+36*a^2*b^2*d^2*n*x^2-40*a*b^3*
c*d*n^2*x-24*a*b^3*d^2*x^3+14*b^4*c^2*n^3+156*b^4*c*d*n*x^2-24*a^3*b*d^2*n*
x+4*a^2*b^2*c*d*n^2+24*a^2*b^2*d^2*x^2-116*a*b^3*c*d*n*x+71*b^4*c^2*n^2+80*
b^4*c*d*x^2-24*a^3*b*d^2*x+36*a^2*b^2*c*d*n-80*a*b^3*c*d*x+154*b^4*c^2*n+24
*a^4*d^2+80*a^2*b^2*c*d+120*b^4*c^2)/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+1
20)
```

maxima [A] time = 0.48, size = 235, normalized size = 1.68

$$\frac{(bx+a)^{n+1}c^2}{b(n+1)} + \frac{2\left(\left(n^2+3n+2\right)b^3x^3 + \left(n^2+n\right)ab^2x^2 - 2a^2bnx + 2a^3\right)(bx+a)^n cd}{\left(n^3+6n^2+11n+6\right)b^3} + \frac{\left(\left(n^4+10n^3+35n^2+50n+24\right)b^5x^5 + \left(n^4+6n^3+11n^2+6n\right)ab^4x^4 - 4\left(n^3+3n^2+2n\right)a^2b^3x^3 + 12\left(n^2+n\right)a^3b^2x^2 - 24a^4bnx + 24a^5\right)(bx+a)^n d^2}{\left(n^5+15n^4+85n^3+225n^2+274n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*
a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c*d/((n^3 + 6*n^2 + 11*n + 6)*
b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2
 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^
2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d^2/((n^5 + 15*n^4 + 85*n^3 + 22
5*n^2 + 274*n + 120)*b^5)
```

mupad [B] time = 2.84, size = 496, normalized size = 3.54

$$(a+bx)^n\left(\frac{a\left(24a^4d^2+4a^2b^2cdn^2+36a^2b^2cdn+80a^2b^2cd+b^4c^2n^4+14b^4c^2n^3+71b^4c^2n^2+154b^4c^2n+120b^4c^2\right)}{b^5\left(n^5+15n^4+85n^3+225n^2+274n+120\right)} + \frac{d^2x^5\left(n^4+10n^3+35n^2+50n+24\right)}{n^5+15n^4+85n^3+225n^2+274n+120} + \frac{x\left(-24a^4bd^2n-4a^2b^3cdn^3-36a^2b^3cdn^2-80a^2b^3cdn+b^5c^2n^4+14b^5c^2n^3+71b^5c^2n^2+154b^5c^2n+120b^5c^2\right)}{b^5\left(n^5+15n^4+85n^3+225n^2+274n+120\right)} + \frac{2dx^3\left(n^2+3n+2\right)\left(-2da^2n+cb^2n^2+9cb^2n+20cb^2\right)}{b^2\left(n^5+15n^4+85n^3+225n^2+274n+120\right)} + \frac{ad^2nx^4\left(n^3+6n^2+11n+6\right)}{b\left(n^5+15n^4+85n^3+225n^2+274n+120\right)} + \frac{2adnx^2(n+1)\left(6da^2+cb^2n^2+9cb^2n+20cb^2\right)}{b^3\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((a*(24*a^4*d^2 + 120*b^4*c^2 + 154*b^4*c^2*n + 71*b^4*c^2*n^2
+ 14*b^4*c^2*n^3 + b^4*c^2*n^4 + 80*a^2*b^2*c*d + 36*a^2*b^2*c*d*n + 4*a^2*
b^2*c*d*n^2))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (d^2*
x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4
 + n^5 + 120) + (x*(120*b^5*c^2 + 154*b^5*c^2*n + 71*b^5*c^2*n^2 + 14*b^5*c
^2*n^3 + b^5*c^2*n^4 - 24*a^4*b*d^2*n - 80*a^2*b^3*c*d*n - 36*a^2*b^3*c*d*n
^2 - 4*a^2*b^3*c*d*n^3))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (2*d*x^3*(3*n + n^2 + 2)*(20*b^2*c + b^2*c*n^2 - 2*a^2*d*n + 9*b^2*c*
n))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d^2*n*x^4*(1
1*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
 + (2*a*d*n*x^2*(n + 1)*(6*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^3*
(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

sympy [A] time = 7.13, size = 5097, normalized size = 36.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5), Eq(b, 0)), (12*a**4*
d**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a
*b**8*x**3 + 12*b**9*x**4) + 25*a**4*d**2/(12*a**4*b**5 + 48*a**3*b**6*x +
72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d**2*x*log(a
/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) + 88*a**3*b*d**2*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**
2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 2*a**2*b**2*c*d/(12*a**4*b**
5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 7
2*a**2*b**2*d**2*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2
```

```
*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*a**2*b**2*d**2*x**2/(12*a
**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x*
*4) - 8*a*b**3*c*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 4
8*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d**2*x**3*log(a/b + x)/(12*a**4*b
**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) +
 48*a*b**3*d**2*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 4
8*a*b**8*x**3 + 12*b**9*x**4) - 3*b**4*c**2/(12*a**4*b**5 + 48*a**3*b**6*x
+ 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 12*b**4*c*d*x**2/(12
*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*
x**4) + 12*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*
a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-12*a**4*d**2
*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) -
 22*a**4*d**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) -
 36*a**3*b*d**2*x*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2
 + 3*b**8*x**3) - 54*a**3*b*d**2*x/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*
x**2 + 3*b**8*x**3) - 2*a**2*b**2*c*d/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b*
*7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**5 +
 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d**2*x**2/(3*a
**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 6*a*b**3*c*d*x/(3
*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 12*a*b**3*d**2*
x**3*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**
3) - b**4*c**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3)
- 6*b**4*c*d*x**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**
3) + 3*b**4*d**2*x**4/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3), Eq(n, -4)), (12*a**4*d**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x +
2*b**7*x**2) + 18*a**4*d**2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a
**3*b*d**2*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**
3*b*d**2*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a**2*b**2*c*d*log(a
/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 6*a**2*b**2*c*d/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b**2*d**2*x**2*log(a/b + x)/(2*a
**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 8*a*b**3*c*d*x*log(a/b + x)/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) + 8*a*b**3*c*d*x/(2*a**2*b**5 + 4*a*b**6*x
 + 2*b**7*x**2) - 4*a*b**3*d**2*x**3/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**
2) - b**4*c**2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*b**4*c*d*x**2*l
og(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + b**4*d**2*x**4/(2*a*
*2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -3)), (-12*a**4*d**2*log(a/b + x
)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d**2/(3*a*b**5 + 3*b**6*x) - 12*a**3*b*d*
*2*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**2*b**2*c*d*log(a/b + x)/(3*
a*b**5 + 3*b**6*x) - 12*a**2*b**2*c*d/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2*d
**2*x**2/(3*a*b**5 + 3*b**6*x) - 12*a*b**3*c*d*x*log(a/b + x)/(3*a*b**5 + 3
*b**6*x) - 2*a*b**3*d**2*x**3/(3*a*b**5 + 3*b**6*x) - 3*b**4*c**2/(3*a*b**5
 + 3*b**6*x) + 6*b**4*c*d*x**2/(3*a*b**5 + 3*b**6*x) + b**4*d**2*x**4/(3*a*
b**5 + 3*b**6*x), Eq(n, -2)), (a**4*d**2*log(a/b + x)/b**5 - a**3*d**2*x/b*
*4 + 2*a**2*c*d*log(a/b + x)/b**3 + a**2*d**2*x**2/(2*b**3) - 2*a*c*d*x/b**
2 - a*d**2*x**3/(3*b**2) + c**2*log(a/b + x)/b + c*d*x**2/b + d**2*x**4/(4*
b), Eq(n, -1)), (24*a**5*d**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b
**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d**2*n*x*(a +
 b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**
5*n + 120*b**5) + 4*a**3*b**2*c*d*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n*
*4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 36*a**3*b**2*c
*d*n*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) + 80*a**3*b**2*c*d*(a + b*x)**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b
**2*d**2*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d**2*n*x**2*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) - 4*a**2*b**3*c*d*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 36*a**2*b**3*c*d*
n**2*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**
```

```
2 + 274*b**5*n + 120*b**5) - 80*a**2*b**3*c*d*n*x*(a + b*x)**n/(b**5*n**5 +
 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a
**2*b**3*d**2*n**3*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n*
*3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a**2*b**3*d**2*n**2*x**3*(
a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*
b**5*n + 120*b**5) - 8*a**2*b**3*d**2*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*c
**2*n**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n
**2 + 274*b**5*n + 120*b**5) + 14*a*b**4*c**2*n**3*(a + b*x)**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 71
*a*b**4*c**2*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 2
25*b**5*n**2 + 274*b**5*n + 120*b**5) + 154*a*b**4*c**2*n*(a + b*x)**n/(b**
5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**
5) + 120*a*b**4*c**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
+ 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 2*a*b**4*c*d*n**4*x**2*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) + 20*a*b**4*c*d*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 58*a*b**4*c*d*n*
*2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) + 40*a*b**4*c*d*n*x**2*(a + b*x)**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*
b**4*d**2*n**4*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d**2*n**3*x**4*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) + 11*a*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d**2*n
*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
 + 274*b**5*n + 120*b**5) + b**5*c**2*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 14*b**5*
c**2*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**
5*n**2 + 274*b**5*n + 120*b**5) + 71*b**5*c**2*n**2*x*(a + b*x)**n/(b**5*n*
*5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) +
 154*b**5*c**2*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + 120*b**5*c**2*x*(a + b*x)**n/(b**5
*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5
) + 2*b**5*c*d*n**4*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n
**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*c*d*n**3*x**3*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 98*b**5*c*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 156*b**5*c*d*n*
x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) + 80*b**5*c*d*x**3*(a + b*x)**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d**2
*n**4*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5
*n**2 + 274*b**5*n + 120*b**5) + 10*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**5*
n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
 + 35*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 50*b**5*d**2*n*x**5*(a + b*
x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
 + 120*b**5) + 24*b**5*d**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 8
5*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5), True))
```

3.268 $\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=343

$$-\frac{2ad^2\left(28a^2d+9b^2c\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{d^2\left(28a^2d+3b^2c\right)(a+bx)^{n+7}}{b^9(n+7)}+\frac{a^2\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^9(n+1)}-\frac{2a\left(a^2d+b^2c\right.}{}$$

Rubi [A] time = 0.21, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {948}

$$\frac{\left(a^2d+b^2c\right)\left(17a^2b^2cd+28a^4d^2+b^4c^2\right)(a+bx)^{n+3}}{b^9(n+3)}-\frac{4ad\left(15a^2b^2cd+14a^4d^2+3b^4c^2\right)(a+bx)^{n+4}}{b^9(n+4)}+\frac{d\left(45a^2b^2cd+70a^4d^2+3b^4c^2\right)(a+bx)^{n+5}}{b^9(n+5)}-\frac{2ad^2\left(28a^2d+9b^2c\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{d^2\left(28a^2d+3b^2c\right)(a+bx)^{n+7}}{b^9(n+7)}+\frac{a^2\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^9(n+1)}-\frac{2a\left(a^2d+b^2c\right)^2\left(4a^2d+b^2c\right)(a+bx)^{n+2}}{b^9(n+2)}-\frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^3(a+bx)^{n+9}}{b^9(n+9)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] (a^2*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^2*c + a^2
*d)^2*(b^2*c + 4*a^2*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^2*c + a^2*d)
*(b^4*c^2 + 17*a^2*b^2*c*d + 28*a^4*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) -
 (4*a*d*(3*b^4*c^2 + 15*a^2*b^2*c*d + 14*a^4*d^2)*(a + b*x)^(4 + n))/(b^9*(
4 + n)) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d + 70*a^4*d^2)*(a + b*x)^(5 + n))/(
b^9*(5 + n)) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)^(6 + n))/(b^9*(6 + n
)) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*
(a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx=\int\left(\frac{a^2\left(b^2c+a^2d\right)^3(a+bx)^n}{b^8}-\frac{2a\left(b^2c+a^2d\right)^2\left(b^2c+4a^2d\right)(a+bx)^{1+n}}{b^8}+\frac{\left(b^2\right.}{}\right.$$

$$=\frac{a^2\left(b^2c+a^2d\right)^3(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a\left(b^2c+a^2d\right)^2\left(b^2c+4a^2d\right)(a+bx)^{2+n}}{b^9(2+n)}+\frac{\left(b^2c\right.}{}$$

Mathematica [A] time = 0.25, size = 302, normalized size = 0.88

$$\frac{(a+bx)^{n+1}\left(\frac{d^2(a+bx)^6\left(28a^2d+3b^2c\right)}{n+7}-\frac{2ad^2(a+bx)^5\left(28a^2d+9b^2c\right)}{n+6}-\frac{2a(a+bx)\left(a^2d+b^2c\right)^2\left(4a^2d+b^2c\right)}{n+2}+\frac{a^2\left(a^2d+b^2c\right)^3}{n+1}+\frac{d(a+bx)^4\left(70a^4d^2+45a^2b^2cd+3b^4c^2\right)}{n+5}-\frac{4ad(a+bx)^3\left(14a^4d^2+15a^2b^2cd+3b^4c^2\right)}{n+4}+\frac{(a+bx)^2\left(a^2d+b^2c\right)\left(28a^4d^2+17a^2b^2cd+b^4c^2\right)}{n+3}+\frac{d^3(a+bx)^8}{n+9}-\frac{8ad^3(a+bx)^7}{n+8}\right)}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*(b^2*c + a^2*d)^3)/(1 + n) - (2*a*(b^2*c + a^2*d)^
2*(b^2*c + 4*a^2*d)*(a + b*x))/(2 + n) + ((b^2*c + a^2*d)*(b^4*c^2 + 17*a^2
*b^2*c*d + 28*a^4*d^2)*(a + b*x)^2)/(3 + n) - (4*a*d*(3*b^4*c^2 + 15*a^2*b^
2*c*d + 14*a^4*d^2)*(a + b*x)^3)/(4 + n) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d +
 70*a^4*d^2)*(a + b*x)^4)/(5 + n) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)
```

```
^5)/(6 + n) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^6)/(7 + n) - (8*a*d^3*(a
+ b*x)^7)/(8 + n) + (d^3*(a + b*x)^8)/(9 + n)))/b^9
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n*(c + d*x^2)^3, x]
```

fricas [B] time = 0.45, size = 2165, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^3*n^6 + 78*a^3*b^6*c^3*n^5 + 120960*a^3*b^6*c^3 + 217728*a^5*b
^4*c^2*d + 155520*a^7*b^2*c*d^2 + 40320*a^9*d^3 + (b^9*d^3*n^8 + 36*b^9*d^3
*n^7 + 546*b^9*d^3*n^6 + 4536*b^9*d^3*n^5 + 22449*b^9*d^3*n^4 + 67284*b^9*d
^3*n^3 + 118124*b^9*d^3*n^2 + 109584*b^9*d^3*n + 40320*b^9*d^3)*x^9 + (a*b^
8*d^3*n^8 + 28*a*b^8*d^3*n^7 + 322*a*b^8*d^3*n^6 + 1960*a*b^8*d^3*n^5 + 676
9*a*b^8*d^3*n^4 + 13132*a*b^8*d^3*n^3 + 13068*a*b^8*d^3*n^2 + 5040*a*b^8*d^
3*n)*x^8 + (3*b^9*c*d^2*n^8 + 155520*b^9*c*d^2 + 2*(57*b^9*c*d^2 - 4*a^2*b^
7*d^3)*n^7 + 12*(151*b^9*c*d^2 - 14*a^2*b^7*d^3)*n^6 + 14*(1119*b^9*c*d^2 -
 100*a^2*b^7*d^3)*n^5 + 21*(3817*b^9*c*d^2 - 280*a^2*b^7*d^3)*n^4 + 28*(881
7*b^9*c*d^2 - 464*a^2*b^7*d^3)*n^3 + 36*(12303*b^9*c*d^2 - 392*a^2*b^7*d^3)
*n^2 + 144*(2901*b^9*c*d^2 - 40*a^2*b^7*d^3)*n)*x^7 + (3*a*b^8*c*d^2*n^8 +
96*a*b^8*c*d^2*n^7 + 4*(309*a*b^8*c*d^2 + 14*a^3*b^6*d^3)*n^6 + 30*(275*a*b
^8*c*d^2 + 28*a^3*b^6*d^3)*n^5 + (30657*a*b^8*c*d^2 + 4760*a^3*b^6*d^3)*n^4
 + 6*(10489*a*b^8*c*d^2 + 2100*a^3*b^6*d^3)*n^3 + 8*(8163*a*b^8*c*d^2 + 191
8*a^3*b^6*d^3)*n^2 + 960*(27*a*b^8*c*d^2 + 7*a^3*b^6*d^3)*n)*x^6 + 3*(b^9*c
^2*d*n^8 + 72576*b^9*c^2*d + 2*(20*b^9*c^2*d - 3*a^2*b^7*c*d^2)*n^7 + 2*(33
5*b^9*c^2*d - 81*a^2*b^7*c*d^2)*n^6 + 2*(3050*b^9*c^2*d - 831*a^2*b^7*c*d^2
 - 56*a^4*b^5*d^3)*n^5 + (32773*b^9*c^2*d - 8190*a^2*b^7*c*d^2 - 1120*a^4*b
^5*d^3)*n^4 + 4*(26365*b^9*c^2*d - 5091*a^2*b^7*c*d^2 - 980*a^4*b^5*d^3)*n^
3 + 4*(49095*b^9*c^2*d - 6012*a^2*b^7*c*d^2 - 1400*a^4*b^5*d^3)*n^2 + 48*(3
975*b^9*c^2*d - 216*a^2*b^7*c*d^2 - 56*a^4*b^5*d^3)*n)*x^5 + 2*(625*a^3*b^6
*c^3 + 36*a^5*b^4*c^2*d)*n^4 + 3*(a*b^8*c^2*d*n^8 + 36*a*b^8*c^2*d*n^7 + 2*
(263*a*b^8*c^2*d + 15*a^3*b^6*c*d^2)*n^6 + 6*(666*a*b^8*c^2*d + 115*a^3*b^6
*c*d^2)*n^5 + (16789*a*b^8*c^2*d + 5550*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^
4 + 6*(6384*a*b^8*c^2*d + 3125*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^3 + 4*(10
791*a*b^8*c^2*d + 6705*a^3*b^6*c*d^2 + 1540*a^5*b^4*d^3)*n^2 + 96*(189*a*b^
8*c^2*d + 135*a^3*b^6*c*d^2 + 35*a^5*b^4*d^3)*n)*x^4 + 270*(39*a^3*b^6*c^3
+ 8*a^5*b^4*c^2*d)*n^3 + (b^9*c^3*n^8 + 120960*b^9*c^3 + 6*(7*b^9*c^3 - 2*a
^2*b^7*c^2*d)*n^7 + 12*(62*b^9*c^3 - 33*a^2*b^7*c^2*d)*n^6 + 6*(1203*b^9*c^
3 - 854*a^2*b^7*c^2*d - 60*a^4*b^5*c*d^2)*n^5 + 3*(13873*b^9*c^3 - 10860*a^
2*b^7*c^2*d - 2400*a^4*b^5*c*d^2)*n^4 + 12*(12039*b^9*c^3 - 8644*a^2*b^7*c^
2*d - 3750*a^4*b^5*c*d^2 - 560*a^6*b^3*d^3)*n^3 + 4*(72569*b^9*c^3 - 37116*
a^2*b^7*c^2*d - 22500*a^4*b^5*c*d^2 - 5040*a^6*b^3*d^3)*n^2 + 48*(6289*b^9*
c^3 - 1512*a^2*b^7*c^2*d - 1080*a^4*b^5*c*d^2 - 280*a^6*b^3*d^3)*n)*x^3 + 4
*(12287*a^3*b^6*c^3 + 6030*a^5*b^4*c^2*d + 540*a^7*b^2*c*d^2)*n^2 + (a*b^8*
c^3*n^8 + 40*a*b^8*c^3*n^7 + 4*(166*a*b^8*c^3 + 9*a^3*b^6*c^2*d)*n^6 + 62*(
95*a*b^8*c^3 + 18*a^3*b^6*c^2*d)*n^5 + (29839*a*b^8*c^3 + 13140*a^3*b^6*c^2
*d + 1080*a^5*b^4*c*d^2)*n^4 + 10*(8479*a*b^8*c^3 + 7146*a^3*b^6*c^2*d + 19
44*a^5*b^4*c*d^2)*n^3 + 24*(5029*a*b^8*c^3 + 7011*a^3*b^6*c^2*d + 4005*a^5*
b^4*c*d^2 + 840*a^7*b^2*d^3)*n^2 + 576*(105*a*b^8*c^3 + 189*a^3*b^6*c^2*d +
 135*a^5*b^4*c*d^2 + 35*a^7*b^2*d^3)*n)*x^2 + 48*(2509*a^3*b^6*c^3 + 2475*a
```

```
^5*b^4*c^2*d + 765*a^7*b^2*c*d^2)*n - 2*(a^2*b^7*c^3*n^7 + 39*a^2*b^7*c^3*n
^6 + (625*a^2*b^7*c^3 + 36*a^4*b^5*c^2*d)*n^5 + 135*(39*a^2*b^7*c^3 + 8*a^4
*b^5*c^2*d)*n^4 + 2*(12287*a^2*b^7*c^3 + 6030*a^4*b^5*c^2*d + 540*a^6*b^3*c
*d^2)*n^3 + 24*(2509*a^2*b^7*c^3 + 2475*a^4*b^5*c^2*d + 765*a^6*b^3*c*d^2)*
n^2 + 576*(105*a^2*b^7*c^3 + 189*a^4*b^5*c^2*d + 135*a^6*b^3*c*d^2 + 35*a^8
*b*d^3)*n)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^
6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 102
6576*b^9*n + 362880*b^9)
```

giac [B] time = 0.30, size = 3713, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^9*d^3*n^8*x^9 + (b*x + a)^n*a*b^8*d^3*n^8*x^8 + 36*(b*x + a)
^n*b^9*d^3*n^7*x^9 + 3*(b*x + a)^n*b^9*c*d^2*n^8*x^7 + 28*(b*x + a)^n*a*b^8
*d^3*n^7*x^8 + 546*(b*x + a)^n*b^9*d^3*n^6*x^9 + 3*(b*x + a)^n*a*b^8*c*d^2*
n^8*x^6 + 114*(b*x + a)^n*b^9*c*d^2*n^7*x^7 - 8*(b*x + a)^n*a^2*b^7*d^3*n^7
*x^7 + 322*(b*x + a)^n*a*b^8*d^3*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^3*n^5*x^9
 + 3*(b*x + a)^n*b^9*c^2*d*n^8*x^5 + 96*(b*x + a)^n*a*b^8*c*d^2*n^7*x^6 + 1
812*(b*x + a)^n*b^9*c*d^2*n^6*x^7 - 168*(b*x + a)^n*a^2*b^7*d^3*n^6*x^7 + 1
960*(b*x + a)^n*a*b^8*d^3*n^5*x^8 + 22449*(b*x + a)^n*b^9*d^3*n^4*x^9 + 3*(
b*x + a)^n*a*b^8*c^2*d*n^8*x^4 + 120*(b*x + a)^n*b^9*c^2*d*n^7*x^5 - 18*(b*
x + a)^n*a^2*b^7*c*d^2*n^7*x^5 + 1236*(b*x + a)^n*a*b^8*c*d^2*n^6*x^6 + 56*
(b*x + a)^n*a^3*b^6*d^3*n^6*x^6 + 15666*(b*x + a)^n*b^9*c*d^2*n^5*x^7 - 140
0*(b*x + a)^n*a^2*b^7*d^3*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^3*n^4*x^8 + 67
284*(b*x + a)^n*b^9*d^3*n^3*x^9 + (b*x + a)^n*b^9*c^3*n^8*x^3 + 108*(b*x +
a)^n*a*b^8*c^2*d*n^7*x^4 + 2010*(b*x + a)^n*b^9*c^2*d*n^6*x^5 - 486*(b*x +
a)^n*a^2*b^7*c*d^2*n^6*x^5 + 8250*(b*x + a)^n*a*b^8*c*d^2*n^5*x^6 + 840*(b*
x + a)^n*a^3*b^6*d^3*n^5*x^6 + 80157*(b*x + a)^n*b^9*c*d^2*n^4*x^7 - 5880*(
b*x + a)^n*a^2*b^7*d^3*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^3*n^3*x^8 + 1181
24*(b*x + a)^n*b^9*d^3*n^2*x^9 + (b*x + a)^n*a*b^8*c^3*n^8*x^2 + 42*(b*x +
a)^n*b^9*c^3*n^7*x^3 - 12*(b*x + a)^n*a^2*b^7*c^2*d*n^7*x^3 + 1578*(b*x + a
)^n*a*b^8*c^2*d*n^6*x^4 + 90*(b*x + a)^n*a^3*b^6*c*d^2*n^6*x^4 + 18300*(b*x
 + a)^n*b^9*c^2*d*n^5*x^5 - 4986*(b*x + a)^n*a^2*b^7*c*d^2*n^5*x^5 - 336*(b
*x + a)^n*a^4*b^5*d^3*n^5*x^5 + 30657*(b*x + a)^n*a*b^8*c*d^2*n^4*x^6 + 476
0*(b*x + a)^n*a^3*b^6*d^3*n^4*x^6 + 246876*(b*x + a)^n*b^9*c*d^2*n^3*x^7 -
12992*(b*x + a)^n*a^2*b^7*d^3*n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^3*n^2*x^8
 + 109584*(b*x + a)^n*b^9*d^3*n*x^9 + 40*(b*x + a)^n*a*b^8*c^3*n^7*x^2 + 74
4*(b*x + a)^n*b^9*c^3*n^6*x^3 - 396*(b*x + a)^n*a^2*b^7*c^2*d*n^6*x^3 + 119
88*(b*x + a)^n*a*b^8*c^2*d*n^5*x^4 + 2070*(b*x + a)^n*a^3*b^6*c*d^2*n^5*x^4
 + 98319*(b*x + a)^n*b^9*c^2*d*n^4*x^5 - 24570*(b*x + a)^n*a^2*b^7*c*d^2*n^
4*x^5 - 3360*(b*x + a)^n*a^4*b^5*d^3*n^4*x^5 + 62934*(b*x + a)^n*a*b^8*c*d^
2*n^3*x^6 + 12600*(b*x + a)^n*a^3*b^6*d^3*n^3*x^6 + 442908*(b*x + a)^n*b^9*
c*d^2*n^2*x^7 - 14112*(b*x + a)^n*a^2*b^7*d^3*n^2*x^7 + 5040*(b*x + a)^n*a*
b^8*d^3*n*x^8 + 40320*(b*x + a)^n*b^9*d^3*x^9 - 2*(b*x + a)^n*a^2*b^7*c^3*n
^7*x + 664*(b*x + a)^n*a*b^8*c^3*n^6*x^2 + 36*(b*x + a)^n*a^3*b^6*c^2*d*n^6
*x^2 + 7218*(b*x + a)^n*b^9*c^3*n^5*x^3 - 5124*(b*x + a)^n*a^2*b^7*c^2*d*n^
5*x^3 - 360*(b*x + a)^n*a^4*b^5*c*d^2*n^5*x^3 + 50367*(b*x + a)^n*a*b^8*c^2
*d*n^4*x^4 + 16650*(b*x + a)^n*a^3*b^6*c*d^2*n^4*x^4 + 1680*(b*x + a)^n*a^5
*b^4*d^3*n^4*x^4 + 316380*(b*x + a)^n*b^9*c^2*d*n^3*x^5 - 61092*(b*x + a)^n
*a^2*b^7*c*d^2*n^3*x^5 - 11760*(b*x + a)^n*a^4*b^5*d^3*n^3*x^5 + 65304*(b*x
 + a)^n*a*b^8*c*d^2*n^2*x^6 + 15344*(b*x + a)^n*a^3*b^6*d^3*n^2*x^6 + 41774
4*(b*x + a)^n*b^9*c*d^2*n*x^7 - 5760*(b*x + a)^n*a^2*b^7*d^3*n*x^7 - 78*(b*
x + a)^n*a^2*b^7*c^3*n^6*x + 5890*(b*x + a)^n*a*b^8*c^3*n^5*x^2 + 1116*(b*x
 + a)^n*a^3*b^6*c^2*d*n^5*x^2 + 41619*(b*x + a)^n*b^9*c^3*n^4*x^3 - 32580*(
b*x + a)^n*a^2*b^7*c^2*d*n^4*x^3 - 7200*(b*x + a)^n*a^4*b^5*c*d^2*n^4*x^3 +
 114912*(b*x + a)^n*a*b^8*c^2*d*n^3*x^4 + 56250*(b*x + a)^n*a^3*b^6*c*d^2*n
```

```
^3*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^3*n^3*x^4 + 589140*(b*x + a)^n*b^9*c^2
*d*n^2*x^5 - 72144*(b*x + a)^n*a^2*b^7*c*d^2*n^2*x^5 - 16800*(b*x + a)^n*a^
4*b^5*d^3*n^2*x^5 + 25920*(b*x + a)^n*a*b^8*c*d^2*n*x^6 + 6720*(b*x + a)^n*
a^3*b^6*d^3*n*x^6 + 155520*(b*x + a)^n*b^9*c*d^2*x^7 + 2*(b*x + a)^n*a^3*b^
6*c^3*n^6 - 1250*(b*x + a)^n*a^2*b^7*c^3*n^5*x - 72*(b*x + a)^n*a^4*b^5*c^2
*d*n^5*x + 29839*(b*x + a)^n*a*b^8*c^3*n^4*x^2 + 13140*(b*x + a)^n*a^3*b^6*
c^2*d*n^4*x^2 + 1080*(b*x + a)^n*a^5*b^4*c*d^2*n^4*x^2 + 144468*(b*x + a)^n
*b^9*c^3*n^3*x^3 - 103728*(b*x + a)^n*a^2*b^7*c^2*d*n^3*x^3 - 45000*(b*x +
a)^n*a^4*b^5*c*d^2*n^3*x^3 - 6720*(b*x + a)^n*a^6*b^3*d^3*n^3*x^3 + 129492*
(b*x + a)^n*a*b^8*c^2*d*n^2*x^4 + 80460*(b*x + a)^n*a^3*b^6*c*d^2*n^2*x^4 +
 18480*(b*x + a)^n*a^5*b^4*d^3*n^2*x^4 + 572400*(b*x + a)^n*b^9*c^2*d*n*x^5
 - 31104*(b*x + a)^n*a^2*b^7*c*d^2*n*x^5 - 8064*(b*x + a)^n*a^4*b^5*d^3*n*x
^5 + 78*(b*x + a)^n*a^3*b^6*c^3*n^5 - 10530*(b*x + a)^n*a^2*b^7*c^3*n^4*x -
 2160*(b*x + a)^n*a^4*b^5*c^2*d*n^4*x + 84790*(b*x + a)^n*a*b^8*c^3*n^3*x^2
 + 71460*(b*x + a)^n*a^3*b^6*c^2*d*n^3*x^2 + 19440*(b*x + a)^n*a^5*b^4*c*d^
2*n^3*x^2 + 290276*(b*x + a)^n*b^9*c^3*n^2*x^3 - 148464*(b*x + a)^n*a^2*b^7
*c^2*d*n^2*x^3 - 90000*(b*x + a)^n*a^4*b^5*c*d^2*n^2*x^3 - 20160*(b*x + a)^
n*a^6*b^3*d^3*n^2*x^3 + 54432*(b*x + a)^n*a*b^8*c^2*d*n*x^4 + 38880*(b*x +
a)^n*a^3*b^6*c*d^2*n*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^3*n*x^4 + 217728*(b*
x + a)^n*b^9*c^2*d*x^5 + 1250*(b*x + a)^n*a^3*b^6*c^3*n^4 + 72*(b*x + a)^n*
a^5*b^4*c^2*d*n^4 - 49148*(b*x + a)^n*a^2*b^7*c^3*n^3*x - 24120*(b*x + a)^n
*a^4*b^5*c^2*d*n^3*x - 2160*(b*x + a)^n*a^6*b^3*c*d^2*n^3*x + 120696*(b*x +
 a)^n*a*b^8*c^3*n^2*x^2 + 168264*(b*x + a)^n*a^3*b^6*c^2*d*n^2*x^2 + 96120*
(b*x + a)^n*a^5*b^4*c*d^2*n^2*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^3*n^2*x^2 +
 301872*(b*x + a)^n*b^9*c^3*n*x^3 - 72576*(b*x + a)^n*a^2*b^7*c^2*d*n*x^3 -
 51840*(b*x + a)^n*a^4*b^5*c*d^2*n*x^3 - 13440*(b*x + a)^n*a^6*b^3*d^3*n*x^
3 + 10530*(b*x + a)^n*a^3*b^6*c^3*n^3 + 2160*(b*x + a)^n*a^5*b^4*c^2*d*n^3
- 120432*(b*x + a)^n*a^2*b^7*c^3*n^2*x - 118800*(b*x + a)^n*a^4*b^5*c^2*d*n
^2*x - 36720*(b*x + a)^n*a^6*b^3*c*d^2*n^2*x + 60480*(b*x + a)^n*a*b^8*c^3*
n*x^2 + 108864*(b*x + a)^n*a^3*b^6*c^2*d*n*x^2 + 77760*(b*x + a)^n*a^5*b^4*
c*d^2*n*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^3*n*x^2 + 120960*(b*x + a)^n*b^9*
c^3*x^3 + 49148*(b*x + a)^n*a^3*b^6*c^3*n^2 + 24120*(b*x + a)^n*a^5*b^4*c^2
*d*n^2 + 2160*(b*x + a)^n*a^7*b^2*c*d^2*n^2 - 120960*(b*x + a)^n*a^2*b^7*c^
3*n*x - 217728*(b*x + a)^n*a^4*b^5*c^2*d*n*x - 155520*(b*x + a)^n*a^6*b^3*c
*d^2*n*x - 40320*(b*x + a)^n*a^8*b*d^3*n*x + 120432*(b*x + a)^n*a^3*b^6*c^3
*n + 118800*(b*x + a)^n*a^5*b^4*c^2*d*n + 36720*(b*x + a)^n*a^7*b^2*c*d^2*n
 + 120960*(b*x + a)^n*a^3*b^6*c^3 + 217728*(b*x + a)^n*a^5*b^4*c^2*d + 1555
20*(b*x + a)^n*a^7*b^2*c*d^2 + 40320*(b*x + a)^n*a^9*d^3)/(b^9*n^9 + 45*b^9
*n^8 + 870*b^9*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680
*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)
```

maple [B] time = 0.02, size = 2232, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c)^3,x)
```

```
[Out] (b*x+a)^(n+1)*(b^8*d^3*n^8*x^8+36*b^8*d^3*n^7*x^8-8*a*b^7*d^3*n^7*x^7+3*b^8
*c*d^2*n^8*x^6+546*b^8*d^3*n^6*x^8-224*a*b^7*d^3*n^6*x^7+114*b^8*c*d^2*n^7*
x^6+4536*b^8*d^3*n^5*x^8+56*a^2*b^6*d^3*n^6*x^6-18*a*b^7*c*d^2*n^7*x^5-2576
*a*b^7*d^3*n^5*x^7+3*b^8*c^2*d*n^8*x^4+1812*b^8*c*d^2*n^6*x^6+22449*b^8*d^3
*n^4*x^8+1176*a^2*b^6*d^3*n^5*x^6-576*a*b^7*c*d^2*n^6*x^5-15680*a*b^7*d^3*n
^4*x^7+120*b^8*c^2*d*n^7*x^4+15666*b^8*c*d^2*n^5*x^6+67284*b^8*d^3*n^3*x^8-
336*a^3*b^5*d^3*n^5*x^5+90*a^2*b^6*c*d^2*n^6*x^4+9800*a^2*b^6*d^3*n^4*x^6-1
2*a*b^7*c^2*d*n^7*x^3-7416*a*b^7*c*d^2*n^5*x^5-54152*a*b^7*d^3*n^3*x^7+b^8*
c^3*n^8*x^2+2010*b^8*c^2*d*n^6*x^4+80157*b^8*c*d^2*n^4*x^6+118124*b^8*d^3*n
^2*x^8-5040*a^3*b^5*d^3*n^4*x^5+2430*a^2*b^6*c*d^2*n^5*x^4+41160*a^2*b^6*d^
3*n^3*x^6-432*a*b^7*c^2*d*n^6*x^3-49500*a*b^7*c*d^2*n^4*x^5-105056*a*b^7*d^
3*n^2*x^7+42*b^8*c^3*n^7*x^2+18300*b^8*c^2*d*n^5*x^4+246876*b^8*c*d^2*n^3*x
```

```
^6+109584*b^8*d^3*n*x^8+1680*a^4*b^4*d^3*n^4*x^4-360*a^3*b^5*c*d^2*n^5*x^3-
28560*a^3*b^5*d^3*n^3*x^5+36*a^2*b^6*c^2*d*n^6*x^2+24930*a^2*b^6*c*d^2*n^4*
x^4+90944*a^2*b^6*d^3*n^2*x^6-2*a*b^7*c^3*n^7*x-6312*a*b^7*c^2*d*n^5*x^3-18
3942*a*b^7*c*d^2*n^3*x^5-104544*a*b^7*d^3*n*x^7+744*b^8*c^3*n^6*x^2+98319*b
^8*c^2*d*n^4*x^4+442908*b^8*c*d^2*n^2*x^6+40320*b^8*d^3*x^8+16800*a^4*b^4*d
^3*n^3*x^4-8280*a^3*b^5*c*d^2*n^4*x^3-75600*a^3*b^5*d^3*n^2*x^5+1188*a^2*b^
6*c^2*d*n^5*x^2+122850*a^2*b^6*c*d^2*n^3*x^4+98784*a^2*b^6*d^3*n*x^6-80*a*b
^7*c^3*n^6*x-47952*a*b^7*c^2*d*n^4*x^3-377604*a*b^7*c*d^2*n^2*x^5-40320*a*b
^7*d^3*x^7+7218*b^8*c^3*n^5*x^2+316380*b^8*c^2*d*n^3*x^4+417744*b^8*c*d^2*n
*x^6-6720*a^5*b^3*d^3*n^3*x^3+1080*a^4*b^4*c*d^2*n^4*x^2+58800*a^4*b^4*d^3*
n^2*x^4-72*a^3*b^5*c^2*d*n^5*x-66600*a^3*b^5*c*d^2*n^3*x^3-92064*a^3*b^5*d^
3*n*x^5+2*a^2*b^6*c^3*n^6+15372*a^2*b^6*c^2*d*n^4*x^2+305460*a^2*b^6*c*d^2*
n^2*x^4+40320*a^2*b^6*d^3*x^6-1328*a*b^7*c^3*n^5*x-201468*a*b^7*c^2*d*n^3*x
^3-391824*a*b^7*c*d^2*n*x^5+41619*b^8*c^3*n^4*x^2+589140*b^8*c^2*d*n^2*x^4+
155520*b^8*c*d^2*x^6-40320*a^5*b^3*d^3*n^2*x^3+21600*a^4*b^4*c*d^2*n^3*x^2+
84000*a^4*b^4*d^3*n*x^4-2232*a^3*b^5*c^2*d*n^4*x-225000*a^3*b^5*c*d^2*n^2*x
^3-40320*a^3*b^5*d^3*x^5+78*a^2*b^6*c^3*n^5+97740*a^2*b^6*c^2*d*n^3*x^2+360
720*a^2*b^6*c*d^2*n*x^4-11780*a*b^7*c^3*n^4*x-459648*a*b^7*c^2*d*n^2*x^3-15
5520*a*b^7*c*d^2*x^5+144468*b^8*c^3*n^3*x^2+572400*b^8*c^2*d*n*x^4+20160*a^
6*b^2*d^3*n^2*x^2-2160*a^5*b^3*c*d^2*n^3*x-73920*a^5*b^3*d^3*n*x^3+72*a^4*b
^4*c^2*d*n^4+135000*a^4*b^4*c*d^2*n^2*x^2+40320*a^4*b^4*d^3*x^4-26280*a^3*b
^5*c^2*d*n^3*x-321840*a^3*b^5*c*d^2*n*x^3+1250*a^2*b^6*c^3*n^4+311184*a^2*b
^6*c^2*d*n^2*x^2+155520*a^2*b^6*c*d^2*x^4-59678*a*b^7*c^3*n^3*x-517968*a*b^
7*c^2*d*n*x^3+290276*b^8*c^3*n^2*x^2+217728*b^8*c^2*d*x^4+60480*a^6*b^2*d^3
*n*x^2-38880*a^5*b^3*c*d^2*n^2*x-40320*a^5*b^3*d^3*x^3+2160*a^4*b^4*c^2*d*n
^3+270000*a^4*b^4*c*d^2*n*x^2-142920*a^3*b^5*c^2*d*n^2*x-155520*a^3*b^5*c*d
^2*x^3+10530*a^2*b^6*c^3*n^3+445392*a^2*b^6*c^2*d*n*x^2-169580*a*b^7*c^3*n^
2*x-217728*a*b^7*c^2*d*x^3+301872*b^8*c^3*n*x^2-40320*a^7*b*d^3*n*x+2160*a^
6*b^2*c*d^2*n^2+40320*a^6*b^2*d^3*x^2-192240*a^5*b^3*c*d^2*n*x+24120*a^4*b^
4*c^2*d*n^2+155520*a^4*b^4*c*d^2*x^2-336528*a^3*b^5*c^2*d*n*x+49148*a^2*b^6
*c^3*n^2+217728*a^2*b^6*c^2*d*x^2-241392*a*b^7*c^3*n*x+120960*b^8*c^3*x^2-4
0320*a^7*b*d^3*x+36720*a^6*b^2*c*d^2*n-155520*a^5*b^3*c*d^2*x+118800*a^4*b^
4*c^2*d*n-217728*a^3*b^5*c^2*d*x+120432*a^2*b^6*c^3*n-120960*a*b^7*c^3*x+40
320*a^8*d^3+155520*a^6*b^2*c*d^2+217728*a^4*b^4*c^2*d+120960*a^2*b^6*c^3)/b
^9/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4+723680*n^3+1172700*n^2
+1026576*n+362880)
```

maxima [B] time = 0.55, size = 795, normalized size = 2.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*
n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
+ a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + 3*((n^
6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 +
15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 +
35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b
^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 -
720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*
n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^8 + 36*n^7 + 546*n^
6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^9*x
^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2
+ 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764
*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 +
 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^4*b^5*x
```

```
^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4 - 6720*(n^3 + 3*n^2 + 2*
n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 40320*a^8*b*n*x + 40320*a^9)
*(b*x + a)^n*d^3/((n^9 + 45*n^8 + 870*n^7 + 9450*n^6 + 63273*n^5 + 269325*n
^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)*b^9)
```

mupad [B] time = 3.81, size = 1796, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*
n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*n^
3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880) +
(2*a^3*(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^6*c^3*n + 24574
*b^6*c^3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c^3*n^5 + b^6*c^
3*n^6 + 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a^2*b^4*c^2*d*n
+ 18360*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4*b^2*c*d^2*n^2
+ 1080*a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^9*(1026576*n + 1172700
*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
n^9 + 362880)) - (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(6720*a^6*d^3*n - 60480*b
^6*c^3 - 60216*b^6*c^3*n - 24574*b^6*c^3*n^2 - 5265*b^6*c^3*n^3 - 625*b^6*c
^3*n^4 - 39*b^6*c^3*n^5 - b^6*c^3*n^6 + 36288*a^2*b^4*c^2*d*n + 25920*a^4*b
^2*c*d^2*n + 19800*a^2*b^4*c^2*d*n^2 + 6120*a^4*b^2*c*d^2*n^2 + 4020*a^2*b^
4*c^2*d*n^3 + 360*a^4*b^2*c*d^2*n^3 + 360*a^2*b^4*c^2*d*n^4 + 12*a^2*b^4*c^
2*d*n^5))/(b^6*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n
^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (3*d*x^5*(a + b*x)^n*(5
0*n + 35*n^2 + 10*n^3 + n^4 + 24)*(3024*b^4*c^2 - 112*a^4*d^2*n + 1650*b^4*
c^2*n + 335*b^4*c^2*n^2 + 30*b^4*c^2*n^3 + b^4*c^2*n^4 - 432*a^2*b^2*c*d*n
- 102*a^2*b^2*c*d*n^2 - 6*a^2*b^2*c*d*n^3))/(b^4*(1026576*n + 1172700*n^2 +
 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 +
362880)) - (2*a^2*n*x*(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^
6*c^3*n + 24574*b^6*c^3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c
^3*n^5 + b^6*c^3*n^6 + 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a
^2*b^4*c^2*d*n + 18360*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4
*b^2*c*d^2*n^2 + 1080*a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^8*(1026
576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*
n^7 + 45*n^8 + n^9 + 362880)) + (d^2*x^7*(a + b*x)^n*(216*b^2*c + 3*b^2*c*n
^2 - 8*a^2*d*n + 51*b^2*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^
5 + n^6 + 720))/(b^2*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 6
3273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (a*n*x^2*(n + 1)*
(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^6*c^3*n + 24574*b^6*c^
3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c^3*n^5 + b^6*c^3*n^6 +
 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a^2*b^4*c^2*d*n + 18360
*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4*b^2*c*d^2*n^2 + 1080*
a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^7*(1026576*n + 1172700*n^2 +
723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 3
62880)) + (a*d^3*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n
^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b*(1026576*n + 1172700*n^2 + 723680*n
^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))
+ (a*d^2*n*x^6*(a + b*x)^n*(56*a^2*d + 216*b^2*c + 3*b^2*c*n^2 + 51*b^2*c*n
)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^3*(1026576*n + 117270
0*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
 n^9 + 362880)) + (3*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(560*a^
4*d^2 + 3024*b^4*c^2 + 1650*b^4*c^2*n + 335*b^4*c^2*n^2 + 30*b^4*c^2*n^3 +
b^4*c^2*n^4 + 2160*a^2*b^2*c*d + 510*a^2*b^2*c*d*n + 30*a^2*b^2*c*d*n^2))/(
b^5*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n
^6 + 870*n^7 + 45*n^8 + n^9 + 362880))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.269 $\int x(a+bx)^n \left(c+dx^2\right)^3 dx$

Optimal. Leaf size=282

$$-\frac{5ad^2\left(7a^2d+3b^2c\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{3d^2\left(7a^2d+b^2c\right)(a+bx)^{n+6}}{b^8(n+6)}-\frac{a\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(a^2d+b^2c\right)^2\left(7a^2d+\right.}{b^8(n+}$$

Rubi [A] time = 0.17, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {772}

$$\frac{d\left(30a^2b^2cd+35a^4d^2+3b^4c^2\right)(a+bx)^{n+4}}{b^8(n+4)}-\frac{5ad^2\left(7a^2d+3b^2c\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{3d^2\left(7a^2d+b^2c\right)(a+bx)^{n+6}}{b^8(n+6)}-\frac{a\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(a^2d+b^2c\right)^2\left(7a^2d+b^2c\right)(a+bx)^{n+2}}{b^8(n+2)}-\frac{3ad\left(a^2d+b^2c\right)\left(7a^2d+3b^2c\right)(a+bx)^{n+3}}{b^8(n+3)}-\frac{7ad^3(a+bx)^{n+7}}{b^8(n+7)}+\frac{d^3(a+bx)^{n+8}}{b^8(n+8)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] -((a*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^2*c + a^2*d)
^2*(b^2*c + 7*a^2*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) - (3*a*d*(b^2*c + a^2
*d)*(3*b^2*c + 7*a^2*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) + (d*(3*b^4*c^2 +
30*a^2*b^2*c*d + 35*a^4*d^2)*(a + b*x)^(4 + n))/(b^8*(4 + n)) - (5*a*d^2*(3
*b^2*c + 7*a^2*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (3*d^2*(b^2*c + 7*a^2*
d)*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^3*(a + b*x)^(7 + n))/(b^8*(7 +
 n)) + (d^3*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^2\right)^3 dx = \int \left(-\frac{a\left(b^2c+a^2d\right)^3(a+bx)^n}{b^7}+\frac{\left(b^2c+a^2d\right)^2\left(b^2c+7a^2d\right)(a+bx)^{1+n}}{b^7}+\frac{3ad\left(-3b^2\right.}{}\right.$$

$$= -\frac{a\left(b^2c+a^2d\right)^3(a+bx)^{1+n}}{b^8(1+n)}+\frac{\left(b^2c+a^2d\right)^2\left(b^2c+7a^2d\right)(a+bx)^{2+n}}{b^8(2+n)}-\frac{3ad\left(b^2c+\right.}{}$$

Mathematica [B] time = 1.47, size = 709, normalized size = 2.51

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7
+ n)*(a + b*x)*(c + d*x^2)^3 - a*(8 + n)*(b^6*(1 + n)*(2 + n)*(3 + n)*(4 +
n)*(5 + n)*(6 + n)*(c + d*x^2)^3 + 6*(b^2*c + a^2*d)*(6 + n)*(b^4*(1 + n)*(
2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*d -
 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1 +
n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n) + d*(2 +
 n)*x^2))) - 6*a*d*(1 + n)*(a + b*x)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(
c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2
*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d -
```

```
2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))) + 6*(1 + n)*
(a + b*x)*((b^2*c + a^2*d)*(7 + n)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(c
+ d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(
3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d - 2*
a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2))) - a*d*(2 + n)*(
a + b*x)*(b^4*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^
2*d)*(6 + n)*(2*a^2*d - 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 +
 n)*x^2)) - 4*a*d*(3 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(4 + n)*x + b^2*(5 +
 n)*(c*(6 + n) + d*(4 + n)*x^2))))))/(b^8*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(
5 + n)*(6 + n)*(7 + n)*(8 + n))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x(a+bx)^n \left(c+dx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n*(c + d*x^2)^3, x]
```

fricas [B] time = 0.45, size = 1675, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^3*n^6 + 33*a^2*b^6*c^3*n^5 + 20160*a^2*b^6*c^3 + 30240*a^4*b^4*
c^2*d + 20160*a^6*b^2*c*d^2 + 5040*a^8*d^3 - (b^8*d^3*n^7 + 28*b^8*d^3*n^6
+ 322*b^8*d^3*n^5 + 1960*b^8*d^3*n^4 + 6769*b^8*d^3*n^3 + 13132*b^8*d^3*n^2
 + 13068*b^8*d^3*n + 5040*b^8*d^3)*x^8 - (a*b^7*d^3*n^7 + 21*a*b^7*d^3*n^6
+ 175*a*b^7*d^3*n^5 + 735*a*b^7*d^3*n^4 + 1624*a*b^7*d^3*n^3 + 1764*a*b^7*d
^3*n^2 + 720*a*b^7*d^3*n)*x^7 - (3*b^8*c*d^2*n^7 + 20160*b^8*c*d^2 + (90*b^
8*c*d^2 - 7*a^2*b^6*d^3)*n^6 + 3*(366*b^8*c*d^2 - 35*a^2*b^6*d^3)*n^5 + 5*(
1404*b^8*c*d^2 - 119*a^2*b^6*d^3)*n^4 + 9*(2803*b^8*c*d^2 - 175*a^2*b^6*d^3
)*n^3 + 2*(25245*b^8*c*d^2 - 959*a^2*b^6*d^3)*n^2 + 24*(2143*b^8*c*d^2 - 35
*a^2*b^6*d^3)*n)*x^6 - 3*(a*b^7*c*d^2*n^7 + 25*a*b^7*c*d^2*n^6 + (241*a*b^7
*c*d^2 + 14*a^3*b^5*d^3)*n^5 + 5*(227*a*b^7*c*d^2 + 28*a^3*b^5*d^3)*n^4 + 2
*(1367*a*b^7*c*d^2 + 245*a^3*b^5*d^3)*n^3 + 20*(158*a*b^7*c*d^2 + 35*a^3*b^
5*d^3)*n^2 + 336*(4*a*b^7*c*d^2 + a^3*b^5*d^3)*n)*x^5 + (445*a^2*b^6*c^3 +
18*a^4*b^4*c^2*d)*n^4 - 3*(b^8*c^2*d*n^7 + 10080*b^8*c^2*d + (32*b^8*c^2*d
- 5*a^2*b^6*c*d^2)*n^6 + (418*b^8*c^2*d - 105*a^2*b^6*c*d^2)*n^5 + (2864*b^
8*c^2*d - 785*a^2*b^6*c*d^2 - 70*a^4*b^4*d^3)*n^4 + (10993*b^8*c^2*d - 2535
*a^2*b^6*c*d^2 - 420*a^4*b^4*d^3)*n^3 + 2*(11656*b^8*c^2*d - 1765*a^2*b^6*c
*d^2 - 385*a^4*b^4*d^3)*n^2 + 12*(2073*b^8*c^2*d - 140*a^2*b^6*c*d^2 - 35*a
^4*b^4*d^3)*n)*x^4 + 3*(1045*a^2*b^6*c^3 + 156*a^4*b^4*c^2*d)*n^3 - 3*(a*b^
7*c^2*d*n^7 + 29*a*b^7*c^2*d*n^6 + (331*a*b^7*c^2*d + 20*a^3*b^5*c*d^2)*n^5
 + (1871*a*b^7*c^2*d + 360*a^3*b^5*c*d^2)*n^4 + 20*(269*a*b^7*c^2*d + 103*a
^3*b^5*c*d^2 + 14*a^5*b^3*d^3)*n^3 + 4*(1793*a*b^7*c^2*d + 990*a^3*b^5*c*d^
2 + 210*a^5*b^3*d^3)*n^2 + 560*(6*a*b^7*c^2*d + 4*a^3*b^5*c*d^2 + a^5*b^3*d
^3)*n)*x^3 + 2*(6077*a^2*b^6*c^3 + 2259*a^4*b^4*c^2*d + 180*a^6*b^2*c*d^2)*
n^2 - (b^8*c^3*n^7 + 20160*b^8*c^3 + (34*b^8*c^3 - 9*a^2*b^6*c^2*d)*n^6 + (
478*b^8*c^3 - 243*a^2*b^6*c^2*d)*n^5 + (3580*b^8*c^3 - 2493*a^2*b^6*c^2*d -
 180*a^4*b^4*c*d^2)*n^4 + (15289*b^8*c^3 - 11853*a^2*b^6*c^2*d - 2880*a^4*b
^4*c*d^2)*n^3 + 2*(18353*b^8*c^3 - 12357*a^2*b^6*c^2*d - 6390*a^4*b^4*c*d^2
 - 1260*a^6*b^2*d^3)*n^2 + 72*(621*b^8*c^3 - 210*a^2*b^6*c^2*d - 140*a^4*b^
4*c*d^2 - 35*a^6*b^2*d^3)*n)*x^2 + 36*(682*a^2*b^6*c^3 + 533*a^4*b^4*c^2*d
+ 150*a^6*b^2*c*d^2)*n - (a*b^7*c^3*n^7 + 33*a*b^7*c^3*n^6 + (445*a*b^7*c^3
 + 18*a^3*b^5*c^2*d)*n^5 + 3*(1045*a*b^7*c^3 + 156*a^3*b^5*c^2*d)*n^4 + 2*(
6077*a*b^7*c^3 + 2259*a^3*b^5*c^2*d + 180*a^5*b^3*c*d^2)*n^3 + 36*(682*a*b^
```

```
7*c^3 + 533*a^3*b^5*c^2*d + 150*a^5*b^3*c*d^2)*n^2 + 5040*(4*a*b^7*c^3 + 6*
a^3*b^5*c^2*d + 4*a^5*b^3*c*d^2 + a^7*b*d^3)*n)*x)*(b*x + a)^n/(b^8*n^8 + 3
6*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 11
8124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

giac [B] time = 0.25, size = 2851, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^8*d^3*n^7*x^8 + (b*x + a)^n*a*b^7*d^3*n^7*x^7 + 28*(b*x + a)
^n*b^8*d^3*n^6*x^8 + 3*(b*x + a)^n*b^8*c*d^2*n^7*x^6 + 21*(b*x + a)^n*a*b^7
*d^3*n^6*x^7 + 322*(b*x + a)^n*b^8*d^3*n^5*x^8 + 3*(b*x + a)^n*a*b^7*c*d^2*
n^7*x^5 + 90*(b*x + a)^n*b^8*c*d^2*n^6*x^6 - 7*(b*x + a)^n*a^2*b^6*d^3*n^6*
x^6 + 175*(b*x + a)^n*a*b^7*d^3*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^3*n^4*x^8
+ 3*(b*x + a)^n*b^8*c^2*d*n^7*x^4 + 75*(b*x + a)^n*a*b^7*c*d^2*n^6*x^5 + 10
98*(b*x + a)^n*b^8*c*d^2*n^5*x^6 - 105*(b*x + a)^n*a^2*b^6*d^3*n^5*x^6 + 73
5*(b*x + a)^n*a*b^7*d^3*n^4*x^7 + 6769*(b*x + a)^n*b^8*d^3*n^3*x^8 + 3*(b*x
 + a)^n*a*b^7*c^2*d*n^7*x^3 + 96*(b*x + a)^n*b^8*c^2*d*n^6*x^4 - 15*(b*x +
a)^n*a^2*b^6*c*d^2*n^6*x^4 + 723*(b*x + a)^n*a*b^7*c*d^2*n^5*x^5 + 42*(b*x
+ a)^n*a^3*b^5*d^3*n^5*x^5 + 7020*(b*x + a)^n*b^8*c*d^2*n^4*x^6 - 595*(b*x
+ a)^n*a^2*b^6*d^3*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^3*n^3*x^7 + 13132*(b*
x + a)^n*b^8*d^3*n^2*x^8 + (b*x + a)^n*b^8*c^3*n^7*x^2 + 87*(b*x + a)^n*a*b
^7*c^2*d*n^6*x^3 + 1254*(b*x + a)^n*b^8*c^2*d*n^5*x^4 - 315*(b*x + a)^n*a^2
*b^6*c*d^2*n^5*x^4 + 3405*(b*x + a)^n*a*b^7*c*d^2*n^4*x^5 + 420*(b*x + a)^n
*a^3*b^5*d^3*n^4*x^5 + 25227*(b*x + a)^n*b^8*c*d^2*n^3*x^6 - 1575*(b*x + a)
^n*a^2*b^6*d^3*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^3*n^2*x^7 + 13068*(b*x +
a)^n*b^8*d^3*n*x^8 + (b*x + a)^n*a*b^7*c^3*n^7*x + 34*(b*x + a)^n*b^8*c^3*n
^6*x^2 - 9*(b*x + a)^n*a^2*b^6*c^2*d*n^6*x^2 + 993*(b*x + a)^n*a*b^7*c^2*d*
n^5*x^3 + 60*(b*x + a)^n*a^3*b^5*c*d^2*n^5*x^3 + 8592*(b*x + a)^n*b^8*c^2*d
*n^4*x^4 - 2355*(b*x + a)^n*a^2*b^6*c*d^2*n^4*x^4 - 210*(b*x + a)^n*a^4*b^4
*d^3*n^4*x^4 + 8202*(b*x + a)^n*a*b^7*c*d^2*n^3*x^5 + 1470*(b*x + a)^n*a^3*
b^5*d^3*n^3*x^5 + 50490*(b*x + a)^n*b^8*c*d^2*n^2*x^6 - 1918*(b*x + a)^n*a^
2*b^6*d^3*n^2*x^6 + 720*(b*x + a)^n*a*b^7*d^3*n*x^7 + 5040*(b*x + a)^n*b^8*
d^3*x^8 + 33*(b*x + a)^n*a*b^7*c^3*n^6*x + 478*(b*x + a)^n*b^8*c^3*n^5*x^2
- 243*(b*x + a)^n*a^2*b^6*c^2*d*n^5*x^2 + 5613*(b*x + a)^n*a*b^7*c^2*d*n^4*
x^3 + 1080*(b*x + a)^n*a^3*b^5*c*d^2*n^4*x^3 + 32979*(b*x + a)^n*b^8*c^2*d*
n^3*x^4 - 7605*(b*x + a)^n*a^2*b^6*c*d^2*n^3*x^4 - 1260*(b*x + a)^n*a^4*b^4
*d^3*n^3*x^4 + 9480*(b*x + a)^n*a*b^7*c*d^2*n^2*x^5 + 2100*(b*x + a)^n*a^3*
b^5*d^3*n^2*x^5 + 51432*(b*x + a)^n*b^8*c*d^2*n*x^6 - 840*(b*x + a)^n*a^2*b
^6*d^3*n*x^6 - (b*x + a)^n*a^2*b^6*c^3*n^6 + 445*(b*x + a)^n*a*b^7*c^3*n^5*
x + 18*(b*x + a)^n*a^3*b^5*c^2*d*n^5*x + 3580*(b*x + a)^n*b^8*c^3*n^4*x^2 -
 2493*(b*x + a)^n*a^2*b^6*c^2*d*n^4*x^2 - 180*(b*x + a)^n*a^4*b^4*c*d^2*n^4
*x^2 + 16140*(b*x + a)^n*a*b^7*c^2*d*n^3*x^3 + 6180*(b*x + a)^n*a^3*b^5*c*d
^2*n^3*x^3 + 840*(b*x + a)^n*a^5*b^3*d^3*n^3*x^3 + 69936*(b*x + a)^n*b^8*c^
2*d*n^2*x^4 - 10590*(b*x + a)^n*a^2*b^6*c*d^2*n^2*x^4 - 2310*(b*x + a)^n*a^
4*b^4*d^3*n^2*x^4 + 4032*(b*x + a)^n*a*b^7*c*d^2*n*x^5 + 1008*(b*x + a)^n*a
^3*b^5*d^3*n*x^5 + 20160*(b*x + a)^n*b^8*c*d^2*x^6 - 33*(b*x + a)^n*a^2*b^6
*c^3*n^5 + 3135*(b*x + a)^n*a*b^7*c^3*n^4*x + 468*(b*x + a)^n*a^3*b^5*c^2*d
*n^4*x + 15289*(b*x + a)^n*b^8*c^3*n^3*x^2 - 11853*(b*x + a)^n*a^2*b^6*c^2*
d*n^3*x^2 - 2880*(b*x + a)^n*a^4*b^4*c*d^2*n^3*x^2 + 21516*(b*x + a)^n*a*b^
7*c^2*d*n^2*x^3 + 11880*(b*x + a)^n*a^3*b^5*c*d^2*n^2*x^3 + 2520*(b*x + a)^
n*a^5*b^3*d^3*n^2*x^3 + 74628*(b*x + a)^n*b^8*c^2*d*n*x^4 - 5040*(b*x + a)^
n*a^2*b^6*c*d^2*n*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^3*n*x^4 - 445*(b*x + a)^
n*a^2*b^6*c^3*n^4 - 18*(b*x + a)^n*a^4*b^4*c^2*d*n^4 + 12154*(b*x + a)^n*a*
b^7*c^3*n^3*x + 4518*(b*x + a)^n*a^3*b^5*c^2*d*n^3*x + 360*(b*x + a)^n*a^5*
b^3*c*d^2*n^3*x + 36706*(b*x + a)^n*b^8*c^3*n^2*x^2 - 24714*(b*x + a)^n*a^2
*b^6*c^2*d*n^2*x^2 - 12780*(b*x + a)^n*a^4*b^4*c*d^2*n^2*x^2 - 2520*(b*x +
```

```
a)^n*a^6*b^2*d^3*n^2*x^2 + 10080*(b*x + a)^n*a*b^7*c^2*d*n*x^3 + 6720*(b*x
+ a)^n*a^3*b^5*c*d^2*n*x^3 + 1680*(b*x + a)^n*a^5*b^3*d^3*n*x^3 + 30240*(b*
x + a)^n*b^8*c^2*d*x^4 - 3135*(b*x + a)^n*a^2*b^6*c^3*n^3 - 468*(b*x + a)^n
*a^4*b^4*c^2*d*n^3 + 24552*(b*x + a)^n*a*b^7*c^3*n^2*x + 19188*(b*x + a)^n*
a^3*b^5*c^2*d*n^2*x + 5400*(b*x + a)^n*a^5*b^3*c*d^2*n^2*x + 44712*(b*x + a
)^n*b^8*c^3*n*x^2 - 15120*(b*x + a)^n*a^2*b^6*c^2*d*n*x^2 - 10080*(b*x + a)
^n*a^4*b^4*c*d^2*n*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^3*n*x^2 - 12154*(b*x +
a)^n*a^2*b^6*c^3*n^2 - 4518*(b*x + a)^n*a^4*b^4*c^2*d*n^2 - 360*(b*x + a)^n
*a^6*b^2*c*d^2*n^2 + 20160*(b*x + a)^n*a*b^7*c^3*n*x + 30240*(b*x + a)^n*a^
3*b^5*c^2*d*n*x + 20160*(b*x + a)^n*a^5*b^3*c*d^2*n*x + 5040*(b*x + a)^n*a^
7*b*d^3*n*x + 20160*(b*x + a)^n*b^8*c^3*x^2 - 24552*(b*x + a)^n*a^2*b^6*c^3
*n - 19188*(b*x + a)^n*a^4*b^4*c^2*d*n - 5400*(b*x + a)^n*a^6*b^2*c*d^2*n -
 20160*(b*x + a)^n*a^2*b^6*c^3 - 30240*(b*x + a)^n*a^4*b^4*c^2*d - 20160*(b
*x + a)^n*a^6*b^2*c*d^2 - 5040*(b*x + a)^n*a^8*d^3)/(b^8*n^8 + 36*b^8*n^7 +
 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 118124*b^8*n^
2 + 109584*b^8*n + 40320*b^8)
```

maple [B] time = 0.02, size = 1639, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^3,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^7*d^3*n^7*x^7-28*b^7*d^3*n^6*x^7+7*a*b^6*d^3*n^6*x^6-3*b
^7*c*d^2*n^7*x^5-322*b^7*d^3*n^5*x^7+147*a*b^6*d^3*n^5*x^6-90*b^7*c*d^2*n^6
*x^5-1960*b^7*d^3*n^4*x^7-42*a^2*b^5*d^3*n^5*x^5+15*a*b^6*c*d^2*n^6*x^4+122
5*a*b^6*d^3*n^4*x^6-3*b^7*c^2*d*n^7*x^3-1098*b^7*c*d^2*n^5*x^5-6769*b^7*d^3
*n^3*x^7-630*a^2*b^5*d^3*n^4*x^5+375*a*b^6*c*d^2*n^5*x^4+5145*a*b^6*d^3*n^3
*x^6-96*b^7*c^2*d*n^6*x^3-7020*b^7*c*d^2*n^4*x^5-13132*b^7*d^3*n^2*x^7+210*
a^3*b^4*d^3*n^4*x^4-60*a^2*b^5*c*d^2*n^5*x^3-3570*a^2*b^5*d^3*n^3*x^5+9*a*b
^6*c^2*d*n^6*x^2+3615*a*b^6*c*d^2*n^4*x^4+11368*a*b^6*d^3*n^2*x^6-b^7*c^3*n
^7*x-1254*b^7*c^2*d*n^5*x^3-25227*b^7*c*d^2*n^3*x^5-13068*b^7*d^3*n*x^7+210
0*a^3*b^4*d^3*n^3*x^4-1260*a^2*b^5*c*d^2*n^4*x^3-9450*a^2*b^5*d^3*n^2*x^5+2
61*a*b^6*c^2*d*n^5*x^2+17025*a*b^6*c*d^2*n^3*x^4+12348*a*b^6*d^3*n*x^6-34*b
^7*c^3*n^6*x-8592*b^7*c^2*d*n^4*x^3-50490*b^7*c*d^2*n^2*x^5-5040*b^7*d^3*x^
7-840*a^4*b^3*d^3*n^3*x^3+180*a^3*b^4*c*d^2*n^4*x^2+7350*a^3*b^4*d^3*n^2*x^
4-18*a^2*b^5*c^2*d*n^5*x-9420*a^2*b^5*c*d^2*n^3*x^3-11508*a^2*b^5*d^3*n*x^5
+a*b^6*c^3*n^6+2979*a*b^6*c^2*d*n^4*x^2+41010*a*b^6*c*d^2*n^2*x^4+5040*a*b^
6*d^3*x^6-478*b^7*c^3*n^5*x-32979*b^7*c^2*d*n^3*x^3-51432*b^7*c*d^2*n*x^5-5
040*a^4*b^3*d^3*n^2*x^3+3240*a^3*b^4*c*d^2*n^3*x^2+10500*a^3*b^4*d^3*n*x^4-
486*a^2*b^5*c^2*d*n^4*x-30420*a^2*b^5*c*d^2*n^2*x^3-5040*a^2*b^5*d^3*x^5+33
*a*b^6*c^3*n^5+16839*a*b^6*c^2*d*n^3*x^2+47400*a*b^6*c*d^2*n*x^4-3580*b^7*c
^3*n^4*x-69936*b^7*c^2*d*n^2*x^3-20160*b^7*c*d^2*x^5+2520*a^5*b^2*d^3*n^2*x
^2-360*a^4*b^3*c*d^2*n^3*x-9240*a^4*b^3*d^3*n*x^3+18*a^3*b^4*c^2*d*n^4+1854
0*a^3*b^4*c*d^2*n^2*x^2+5040*a^3*b^4*d^3*x^4-4986*a^2*b^5*c^2*d*n^3*x-42360
*a^2*b^5*c*d^2*n*x^3+445*a*b^6*c^3*n^4+48420*a*b^6*c^2*d*n^2*x^2+20160*a*b^
6*c*d^2*x^4-15289*b^7*c^3*n^3*x-74628*b^7*c^2*d*n*x^3+7560*a^5*b^2*d^3*n*x^
2-5760*a^4*b^3*c*d^2*n^2*x-5040*a^4*b^3*d^3*x^3+468*a^3*b^4*c^2*d*n^3+35640
*a^3*b^4*c*d^2*n*x^2-23706*a^2*b^5*c^2*d*n^2*x-20160*a^2*b^5*c*d^2*x^3+3135
*a*b^6*c^3*n^3+64548*a*b^6*c^2*d*n*x^2-36706*b^7*c^3*n^2*x-30240*b^7*c^2*d*
x^3-5040*a^6*b*d^3*n*x+360*a^5*b^2*c*d^2*n^2+5040*a^5*b^2*d^3*x^2-25560*a^4
*b^3*c*d^2*n*x+4518*a^3*b^4*c^2*d*n^2+20160*a^3*b^4*c*d^2*x^2-49428*a^2*b^5
*c^2*d*n*x+12154*a*b^6*c^3*n^2+30240*a*b^6*c^2*d*x^2-44712*b^7*c^3*n*x-5040
*a^6*b*d^3*x+5400*a^5*b^2*c*d^2*n-20160*a^4*b^3*c*d^2*x+19188*a^3*b^4*c^2*d
*n-30240*a^2*b^5*c^2*d*x+24552*a*b^6*c^3*n-20160*b^7*c^3*x+5040*a^7*d^3+201
60*a^5*b^2*c*d^2+30240*a^3*b^4*c^2*d+20160*a*b^6*c^3)/b^8/(n^8+36*n^7+546*n
^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)
```

maxima [B] time = 0.53, size = 625, normalized size = 2.22

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n c^3}{\left(n^2+3n+2\right)b^2}+\frac{3\left(\left(n^3+6n^2+11n+6\right)b^4x^4+\left(n^3+3n^2+2n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6a^3bnx-6a^4\right)(bx+a)^n c^2d}{\left(n^4+10n^3+35n^2+50n+24\right)b^4}+\frac{3\left(\left(n^5+15n^4+85n^3+225n^2+274n+120\right)b^6x^6+\left(n^5+10n^4+35n^3+50n^2+24n\right)ab^5x^5-5\left(n^4+6n^3+11n^2+6n\right)a^2b^4x^4+20\left(n^3+3n^2+2n\right)a^3b^3x^3-60\left(n^2+n\right)a^4b^2x^2+120a^5bnx-120a^6\right)(bx+a)^n cd^2}{\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)b^6}+\frac{\left(\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)b^8x^8+\left(n^7+21n^6+175n^5+735n^4+1624n^3+1764n^2+720n\right)ab^7x^7-7\left(n^6+15n^5+85n^4+225n^3+274n^2+120n\right)a^2b^6x^6+42\left(n^5+10n^4+35n^3+50n^2+24n\right)a^3b^5x^5-210\left(n^4+6n^3+11n^2+6n\right)a^4b^4x^4+840\left(n^3+3n^2+2n\right)a^5b^3x^3-2520\left(n^2+n\right)a^6b^2x^2+5040a^7bnx-5040a^8\right)(bx+a)^n d^3}{\left(n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124n^2+109584n+40320\right)b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 +
 35*n^2 + 50*n + 24)*b^4) + 3*((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 1
20)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 +
6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60
*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*c*d^2/((n^6 +
 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) + ((n^7 + 28*n^
6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^8*x^8 + (
n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 -
 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^
5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^
2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)
*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8)*(b*x + a)^n*d^3/((n^8 + 36*n^7 +
546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)
*b^8)
```

mupad [B] time = 3.48, size = 1459, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 453
6*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^3 +
20160*b^6*c^3 + 24552*b^6*c^3*n + 12154*b^6*c^3*n^2 + 3135*b^6*c^3*n^3 + 44
5*b^6*c^3*n^4 + 33*b^6*c^3*n^5 + b^6*c^3*n^6 + 30240*a^2*b^4*c^2*d + 20160*
a^4*b^2*c*d^2 + 19188*a^2*b^4*c^2*d*n + 5400*a^4*b^2*c*d^2*n + 4518*a^2*b^4
*c^2*d*n^2 + 360*a^4*b^2*c*d^2*n^2 + 468*a^2*b^4*c^2*d*n^3 + 18*a^2*b^4*c^2
*d*n^4))/(b^8*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 5
46*n^6 + 36*n^7 + n^8 + 40320)) - (x^2*(n + 1)*(a + b*x)^n*(2520*a^6*d^3*n
- 20160*b^6*c^3 - 24552*b^6*c^3*n - 12154*b^6*c^3*n^2 - 3135*b^6*c^3*n^3 -
445*b^6*c^3*n^4 - 33*b^6*c^3*n^5 - b^6*c^3*n^6 + 15120*a^2*b^4*c^2*d*n + 10
080*a^4*b^2*c*d^2*n + 9594*a^2*b^4*c^2*d*n^2 + 2700*a^4*b^2*c*d^2*n^2 + 225
9*a^2*b^4*c^2*d*n^3 + 180*a^4*b^2*c*d^2*n^3 + 234*a^2*b^4*c^2*d*n^4 + 9*a^2
*b^4*c^2*d*n^5))/(b^6*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (d^2*x^6*(a + b*x)^n*(168*b^2*c +
 3*b^2*c*n^2 - 7*a^2*d*n + 45*b^2*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 +
 n^5 + 120))/(b^2*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5
 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*d*x^4*(a + b*x)^n*(11*n + 6*n^2 +
n^3 + 6)*(1680*b^4*c^2 - 70*a^4*d^2*n + 1066*b^4*c^2*n + 251*b^4*c^2*n^2 +
26*b^4*c^2*n^3 + b^4*c^2*n^4 - 280*a^2*b^2*c*d*n - 75*a^2*b^2*c*d*n^2 - 5*a
^2*b^2*c*d*n^3))/(b^4*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n*x*(a + b*x)^n*(5040*a^6*d^3
+ 20160*b^6*c^3 + 24552*b^6*c^3*n + 12154*b^6*c^3*n^2 + 3135*b^6*c^3*n^3 +
445*b^6*c^3*n^4 + 33*b^6*c^3*n^5 + b^6*c^3*n^6 + 30240*a^2*b^4*c^2*d + 2016
0*a^4*b^2*c*d^2 + 19188*a^2*b^4*c^2*d*n + 5400*a^4*b^2*c*d^2*n + 4518*a^2*b
^4*c^2*d*n^2 + 360*a^4*b^2*c*d^2*n^2 + 468*a^2*b^4*c^2*d*n^3 + 18*a^2*b^4*c
^2*d*n^4))/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 +
 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*d^3*n*x^7*(a + b*x)^n*(1764*n + 1624
*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b*(109584*n + 118124*n^2 +
```

```
 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*a
*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(280*a^4*d^2 + 1680*b^4*c^2 + 1066*b^4
*c^2*n + 251*b^4*c^2*n^2 + 26*b^4*c^2*n^3 + b^4*c^2*n^4 + 1120*a^2*b^2*c*d
+ 300*a^2*b^2*c*d*n + 20*a^2*b^2*c*d*n^2))/(b^5*(109584*n + 118124*n^2 + 67
284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*a*d^
2*n*x^5*(a + b*x)^n*(14*a^2*d + 56*b^2*c + b^2*c*n^2 + 15*b^2*c*n)*(50*n +
35*n^2 + 10*n^3 + n^4 + 24))/(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 2244
9*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.270 $\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=223

$$-\frac{4ad^2\left(5a^2d+3b^2c\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{3d^2\left(5a^2d+b^2c\right)(a+bx)^{n+5}}{b^7(n+5)}+\frac{\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^7(n+1)}-\frac{6ad\left(a^2d+b^2c\right)^2(a+}{b^7(n+2)}$$

Rubi [A] time = 0.13, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{4ad^2\left(5a^2d+3b^2c\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{3d^2\left(5a^2d+b^2c\right)(a+bx)^{n+5}}{b^7(n+5)}+\frac{\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^7(n+1)}-\frac{6ad\left(a^2d+b^2c\right)^2(a+bx)^{n+2}}{b^7(n+2)}+\frac{3d\left(a^2d+b^2c\right)\left(5a^2d+b^2c\right)(a+bx)^{n+3}}{b^7(n+3)}-\frac{6ad^3(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^3(a+bx)^{n+7}}{b^7(n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (6*a*d*(b^2*c + a^2*d
)^2*(a + b*x)^(2 + n))/(b^7*(2 + n)) + (3*d*(b^2*c + a^2*d)*(b^2*c + 5*a^2*
d)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d^2*(3*b^2*c + 5*a^2*d)*(a + b*x
)^(4 + n))/(b^7*(4 + n)) + (3*d^2*(b^2*c + 5*a^2*d)*(a + b*x)^(5 + n))/(b^7
*(5 + n)) - (6*a*d^3*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d^3*(a + b*x)^(7 +
 n))/(b^7*(7 + n))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+bx)^n\left(c+dx^2\right)^3 dx = \int \left(\frac{\left(b^2c+a^2d\right)^3(a+bx)^n}{b^6}-\frac{6ad\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^6}+\frac{3d\left(b^2c+a^2d\right)\left(b^2c+5a\right.}{b^6}\right.$$

$$=\frac{\left(b^2c+a^2d\right)^3(a+bx)^{1+n}}{b^7(1+n)}-\frac{6ad\left(b^2c+a^2d\right)^2(a+bx)^{2+n}}{b^7(2+n)}+\frac{3d\left(b^2c+a^2d\right)\left(b^2c+5a^2\right.}{b^7(3+n)}$$

Mathematica [A] time = 0.50, size = 347, normalized size = 1.56

$$\frac{(a+bx)^{n+1}\left(\frac{6\left((n+6)\left(a^2d+b^2c\right)\left(4(n+4)\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)-4ad(n+1)(a+bx)\left(2a^2d-2abd(n+2)x+b^2(n+3)\left(c(n+4)+d(n+2)x^2\right)\right)+b^4(n+1)(n+2)(n+3)(n+4)\left(c+dx^2\right)^2\right)-ad(n+1)(a+bx)\left(4(n+5)\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+2)x+b^2(n+3)\left(c(n+4)+d(n+2)x^2\right)\right)-4ad(n+2)(a+bx)\left(2a^2d-2abd(n+3)x+b^2(n+4)\left(c(n+5)+d(n+3)x^2\right)\right)+b^4(n+2)(n+3)(n+4)(n+5)\left(c+dx^2\right)^2\right)\right)}{b^6(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)}+\left(c+dx^2\right)^3\right)}{b(n+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((c + d*x^2)^3 + (6*((b^2*c + a^2*d)*(6 + n)*(b^4*(1 + n
)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*
d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1
 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n) + d*(
2 + n)*x^2))) - a*d*(1 + n)*(a + b*x)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*
(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^
2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d -
 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))))/(b^6*(1 +
n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))))/(b*(7 + n))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x^2)^3, x]
```

fricas [B] time = 0.41, size = 1244, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^3*n^6 + 27*a*b^6*c^3*n^5 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d + 3
024*a^5*b^2*c*d^2 + 720*a^7*d^3 + (b^7*d^3*n^6 + 21*b^7*d^3*n^5 + 175*b^7*d
^3*n^4 + 735*b^7*d^3*n^3 + 1624*b^7*d^3*n^2 + 1764*b^7*d^3*n + 720*b^7*d^3)
*x^7 + (a*b^6*d^3*n^6 + 15*a*b^6*d^3*n^5 + 85*a*b^6*d^3*n^4 + 225*a*b^6*d^3
*n^3 + 274*a*b^6*d^3*n^2 + 120*a*b^6*d^3*n)*x^6 + 3*(b^7*c*d^2*n^6 + 1008*b
^7*c*d^2 + (23*b^7*c*d^2 - 2*a^2*b^5*d^3)*n^5 + (207*b^7*c*d^2 - 20*a^2*b^5
*d^3)*n^4 + 5*(185*b^7*c*d^2 - 14*a^2*b^5*d^3)*n^3 + 4*(536*b^7*c*d^2 - 25*
a^2*b^5*d^3)*n^2 + 12*(201*b^7*c*d^2 - 4*a^2*b^5*d^3)*n)*x^5 + (295*a*b^6*c
^3 + 6*a^3*b^4*c^2*d)*n^4 + 3*(a*b^6*c*d^2*n^6 + 19*a*b^6*c*d^2*n^5 + (131*
a*b^6*c*d^2 + 10*a^3*b^4*d^3)*n^4 + (401*a*b^6*c*d^2 + 60*a^3*b^4*d^3)*n^3
+ 10*(54*a*b^6*c*d^2 + 11*a^3*b^4*d^3)*n^2 + 12*(21*a*b^6*c*d^2 + 5*a^3*b^4
*d^3)*n)*x^4 + 3*(555*a*b^6*c^3 + 44*a^3*b^4*c^2*d)*n^3 + 3*(b^7*c^2*d*n^6
+ 1680*b^7*c^2*d + (25*b^7*c^2*d - 4*a^2*b^5*c*d^2)*n^5 + (247*b^7*c^2*d -
64*a^2*b^5*c*d^2)*n^4 + (1219*b^7*c^2*d - 332*a^2*b^5*c*d^2 - 40*a^4*b^3*d^
3)*n^3 + 8*(389*b^7*c^2*d - 76*a^2*b^5*c*d^2 - 15*a^4*b^3*d^3)*n^2 + 4*(949
*b^7*c^2*d - 84*a^2*b^5*c*d^2 - 20*a^4*b^3*d^3)*n)*x^3 + 2*(2552*a*b^6*c^3
+ 537*a^3*b^4*c^2*d + 36*a^5*b^2*c*d^2)*n^2 + 3*(a*b^6*c^2*d*n^6 + 23*a*b^6
*c^2*d*n^5 + 3*(67*a*b^6*c^2*d + 4*a^3*b^4*c*d^2)*n^4 + (817*a*b^6*c^2*d +
168*a^3*b^4*c*d^2)*n^3 + 2*(739*a*b^6*c^2*d + 330*a^3*b^4*c*d^2 + 60*a^5*b^
2*d^3)*n^2 + 24*(35*a*b^6*c^2*d + 21*a^3*b^4*c*d^2 + 5*a^5*b^2*d^3)*n)*x^2
+ 12*(669*a*b^6*c^3 + 319*a^3*b^4*c^2*d + 78*a^5*b^2*c*d^2)*n + (b^7*c^3*n^
6 + 5040*b^7*c^3 + 3*(9*b^7*c^3 - 2*a^2*b^5*c^2*d)*n^5 + (295*b^7*c^3 - 132
*a^2*b^5*c^2*d)*n^4 + 3*(555*b^7*c^3 - 358*a^2*b^5*c^2*d - 24*a^4*b^3*c*d^2
)*n^3 + 4*(1276*b^7*c^3 - 957*a^2*b^5*c^2*d - 234*a^4*b^3*c*d^2)*n^2 + 36*(
223*b^7*c^3 - 140*a^2*b^5*c^2*d - 84*a^4*b^3*c*d^2 - 20*a^6*b*d^3)*n)*x)*(b
*x + a)^n/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3
 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

giac [B] time = 0.24, size = 2085, normalized size = 9.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^3*n^6*x^7 + (b*x + a)^n*a*b^6*d^3*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^3*n^5*x^7 + 3*(b*x + a)^n*b^7*c*d^2*n^6*x^5 + 15*(b*x + a)^n*a*b^6
*d^3*n^5*x^6 + 175*(b*x + a)^n*b^7*d^3*n^4*x^7 + 3*(b*x + a)^n*a*b^6*c*d^2*
n^6*x^4 + 69*(b*x + a)^n*b^7*c*d^2*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^3*n^5*
x^5 + 85*(b*x + a)^n*a*b^6*d^3*n^4*x^6 + 735*(b*x + a)^n*b^7*d^3*n^3*x^7 +
3*(b*x + a)^n*b^7*c^2*d*n^6*x^3 + 57*(b*x + a)^n*a*b^6*c*d^2*n^5*x^4 + 621*
(b*x + a)^n*b^7*c*d^2*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^3*n^4*x^5 + 225*(b
*x + a)^n*a*b^6*d^3*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^3*n^2*x^7 + 3*(b*x + a
)^n*a*b^6*c^2*d*n^6*x^2 + 75*(b*x + a)^n*b^7*c^2*d*n^5*x^3 - 12*(b*x + a)^n
```

```
*a^2*b^5*c*d^2*n^5*x^3 + 393*(b*x + a)^n*a*b^6*c*d^2*n^4*x^4 + 30*(b*x + a)
^n*a^3*b^4*d^3*n^4*x^4 + 2775*(b*x + a)^n*b^7*c*d^2*n^3*x^5 - 210*(b*x + a)
^n*a^2*b^5*d^3*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^3*n^2*x^6 + 1764*(b*x + a)
^n*b^7*d^3*n*x^7 + (b*x + a)^n*b^7*c^3*n^6*x + 69*(b*x + a)^n*a*b^6*c^2*d*n
^5*x^2 + 741*(b*x + a)^n*b^7*c^2*d*n^4*x^3 - 192*(b*x + a)^n*a^2*b^5*c*d^2*
n^4*x^3 + 1203*(b*x + a)^n*a*b^6*c*d^2*n^3*x^4 + 180*(b*x + a)^n*a^3*b^4*d^
3*n^3*x^4 + 6432*(b*x + a)^n*b^7*c*d^2*n^2*x^5 - 300*(b*x + a)^n*a^2*b^5*d^
3*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^3*n*x^6 + 720*(b*x + a)^n*b^7*d^3*x^7 +
 (b*x + a)^n*a*b^6*c^3*n^6 + 27*(b*x + a)^n*b^7*c^3*n^5*x - 6*(b*x + a)^n*a
^2*b^5*c^2*d*n^5*x + 603*(b*x + a)^n*a*b^6*c^2*d*n^4*x^2 + 36*(b*x + a)^n*a
^3*b^4*c*d^2*n^4*x^2 + 3657*(b*x + a)^n*b^7*c^2*d*n^3*x^3 - 996*(b*x + a)^n
*a^2*b^5*c*d^2*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^3*n^3*x^3 + 1620*(b*x +
a)^n*a*b^6*c*d^2*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^3*n^2*x^4 + 7236*(b*x
+ a)^n*b^7*c*d^2*n*x^5 - 144*(b*x + a)^n*a^2*b^5*d^3*n*x^5 + 27*(b*x + a)^n
*a*b^6*c^3*n^5 + 295*(b*x + a)^n*b^7*c^3*n^4*x - 132*(b*x + a)^n*a^2*b^5*c^
2*d*n^4*x + 2451*(b*x + a)^n*a*b^6*c^2*d*n^3*x^2 + 504*(b*x + a)^n*a^3*b^4*
c*d^2*n^3*x^2 + 9336*(b*x + a)^n*b^7*c^2*d*n^2*x^3 - 1824*(b*x + a)^n*a^2*b
^5*c*d^2*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^3*n^2*x^3 + 756*(b*x + a)^n*a*
b^6*c*d^2*n*x^4 + 180*(b*x + a)^n*a^3*b^4*d^3*n*x^4 + 3024*(b*x + a)^n*b^7*
c*d^2*x^5 + 295*(b*x + a)^n*a*b^6*c^3*n^4 + 6*(b*x + a)^n*a^3*b^4*c^2*d*n^4
 + 1665*(b*x + a)^n*b^7*c^3*n^3*x - 1074*(b*x + a)^n*a^2*b^5*c^2*d*n^3*x -
72*(b*x + a)^n*a^4*b^3*c*d^2*n^3*x + 4434*(b*x + a)^n*a*b^6*c^2*d*n^2*x^2 +
 1980*(b*x + a)^n*a^3*b^4*c*d^2*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^3*n^2*x
^2 + 11388*(b*x + a)^n*b^7*c^2*d*n*x^3 - 1008*(b*x + a)^n*a^2*b^5*c*d^2*n*x
^3 - 240*(b*x + a)^n*a^4*b^3*d^3*n*x^3 + 1665*(b*x + a)^n*a*b^6*c^3*n^3 + 1
32*(b*x + a)^n*a^3*b^4*c^2*d*n^3 + 5104*(b*x + a)^n*b^7*c^3*n^2*x - 3828*(b
*x + a)^n*a^2*b^5*c^2*d*n^2*x - 936*(b*x + a)^n*a^4*b^3*c*d^2*n^2*x + 2520*
(b*x + a)^n*a*b^6*c^2*d*n*x^2 + 1512*(b*x + a)^n*a^3*b^4*c*d^2*n*x^2 + 360*
(b*x + a)^n*a^5*b^2*d^3*n*x^2 + 5040*(b*x + a)^n*b^7*c^2*d*x^3 + 5104*(b*x
+ a)^n*a*b^6*c^3*n^2 + 1074*(b*x + a)^n*a^3*b^4*c^2*d*n^2 + 72*(b*x + a)^n*
a^5*b^2*c*d^2*n^2 + 8028*(b*x + a)^n*b^7*c^3*n*x - 5040*(b*x + a)^n*a^2*b^5
*c^2*d*n*x - 3024*(b*x + a)^n*a^4*b^3*c*d^2*n*x - 720*(b*x + a)^n*a^6*b*d^3
*n*x + 8028*(b*x + a)^n*a*b^6*c^3*n + 3828*(b*x + a)^n*a^3*b^4*c^2*d*n + 93
6*(b*x + a)^n*a^5*b^2*c*d^2*n + 5040*(b*x + a)^n*b^7*c^3*x + 5040*(b*x + a)
^n*a*b^6*c^3 + 5040*(b*x + a)^n*a^3*b^4*c^2*d + 3024*(b*x + a)^n*a^5*b^2*c*
d^2 + 720*(b*x + a)^n*a^7*d^3)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b
^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

maple [B] time = 0.01, size = 1140, normalized size = 5.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^3,x)
```

```
[Out] (b*x+a)^(n+1)*(b^6*d^3*n^6*x^6+21*b^6*d^3*n^5*x^6-6*a*b^5*d^3*n^5*x^5+3*b^6
*c*d^2*n^6*x^4+175*b^6*d^3*n^4*x^6-90*a*b^5*d^3*n^4*x^5+69*b^6*c*d^2*n^5*x^
4+735*b^6*d^3*n^3*x^6+30*a^2*b^4*d^3*n^4*x^4-12*a*b^5*c*d^2*n^5*x^3-510*a*b
^5*d^3*n^3*x^5+3*b^6*c^2*d*n^6*x^2+621*b^6*c*d^2*n^4*x^4+1624*b^6*d^3*n^2*x
^6+300*a^2*b^4*d^3*n^3*x^4-228*a*b^5*c*d^2*n^4*x^3-1350*a*b^5*d^3*n^2*x^5+7
5*b^6*c^2*d*n^5*x^2+2775*b^6*c*d^2*n^3*x^4+1764*b^6*d^3*n*x^6-120*a^3*b^3*d
^3*n^3*x^3+36*a^2*b^4*c*d^2*n^4*x^2+1050*a^2*b^4*d^3*n^2*x^4-6*a*b^5*c^2*d*
n^5*x-1572*a*b^5*c*d^2*n^3*x^3-1644*a*b^5*d^3*n*x^5+b^6*c^3*n^6+741*b^6*c^2
*d*n^4*x^2+6432*b^6*c*d^2*n^2*x^4+720*b^6*d^3*x^6-720*a^3*b^3*d^3*n^2*x^3+5
76*a^2*b^4*c*d^2*n^3*x^2+1500*a^2*b^4*d^3*n*x^4-138*a*b^5*c^2*d*n^4*x-4812*
a*b^5*c*d^2*n^2*x^3-720*a*b^5*d^3*x^5+27*b^6*c^3*n^5+3657*b^6*c^2*d*n^3*x^2
+7236*b^6*c*d^2*n*x^4+360*a^4*b^2*d^3*n^2*x^2-72*a^3*b^3*c*d^2*n^3*x-1320*a
^3*b^3*d^3*n*x^3+6*a^2*b^4*c^2*d*n^4+2988*a^2*b^4*c*d^2*n^2*x^2+720*a^2*b^4
*d^3*x^4-1206*a*b^5*c^2*d*n^3*x-6480*a*b^5*c*d^2*n*x^3+295*b^6*c^3*n^4+9336
*b^6*c^2*d*n^2*x^2+3024*b^6*c*d^2*x^4+1080*a^4*b^2*d^3*n*x^2-1008*a^3*b^3*c
```

```
*d^2*n^2*x-720*a^3*b^3*d^3*x^3+132*a^2*b^4*c^2*d*n^3+5472*a^2*b^4*c*d^2*n*x
^2-4902*a*b^5*c^2*d*n^2*x-3024*a*b^5*c*d^2*x^3+1665*b^6*c^3*n^3+11388*b^6*c
^2*d*n*x^2-720*a^5*b*d^3*n*x+72*a^4*b^2*c*d^2*n^2+720*a^4*b^2*d^3*x^2-3960*
a^3*b^3*c*d^2*n*x+1074*a^2*b^4*c^2*d*n^2+3024*a^2*b^4*c*d^2*x^2-8868*a*b^5*
c^2*d*n*x+5104*b^6*c^3*n^2+5040*b^6*c^2*d*x^2-720*a^5*b*d^3*x+936*a^4*b^2*c
*d^2*n-3024*a^3*b^3*c*d^2*x+3828*a^2*b^4*c^2*d*n-5040*a*b^5*c^2*d*x+8028*b^
6*c^3*n+720*a^6*d^3+3024*a^4*b^2*c*d^2+5040*a^2*b^4*c^2*d+5040*b^6*c^3)/b^7
/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)
```

maxima [B] time = 0.51, size = 472, normalized size = 2.12

$$\frac{(bx+a)^{n+1}c^3}{b(n+1)}+\frac{3\left(\left(n^2+3n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^nc^2d}{\left(n^3+6n^2+11n+6\right)b^3}+\frac{3\left(\left(n^4+10n^3+35n^2+50n+24\right)b^5x^5+\left(n^4+6n^3+11n^2+6n\right)ab^4x^4-4\left(n^3+3n^2+2n\right)a^2b^3x^3+12\left(n^2+n\right)a^3b^2x^2-24a^4bnx+24a^5\right)(bx+a)^ncd^2}{\left(n^5+15n^4+85n^3+225n^2+274n+120\right)b^5}+\frac{\left(\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)b^7x^7+\left(n^6+15n^5+85n^4+225n^3+274n^2+120n\right)ab^6x^6-6\left(n^5+10n^4+35n^3+50n^2+24n\right)a^2b^5x^5+30\left(n^4+6n^3+11n^2+6n\right)a^3b^4x^4-120\left(n^3+3n^2+2n\right)a^4b^3x^3+360\left(n^2+n\right)a^5b^2x^2-720a^6bnx+720a^7\right)(bx+a)^nd^3}{\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*
a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c^2*d/((n^3 + 6*n^2 + 11*n + 6
)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11
*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^
3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d^2/((n^5 + 15*n^4 + 85*n^
3 + 225*n^2 + 274*n + 120)*b^5) + ((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624
*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 +
 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 +
 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*
b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*
d^3/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n +
5040)*b^7)
```

mupad [B] time = 3.16, size = 1144, normalized size = 5.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*(720*a^7*d^3 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d + 3024*a^5*
b^2*c*d^2 + 5104*a*b^6*c^3*n^2 + 1665*a*b^6*c^3*n^3 + 295*a*b^6*c^3*n^4 + 2
7*a*b^6*c^3*n^5 + a*b^6*c^3*n^6 + 8028*a*b^6*c^3*n + 3828*a^3*b^4*c^2*d*n +
 936*a^5*b^2*c*d^2*n + 1074*a^3*b^4*c^2*d*n^2 + 72*a^5*b^2*c*d^2*n^2 + 132*
a^3*b^4*c^2*d*n^3 + 6*a^3*b^4*c^2*d*n^4))/(b^7*(13068*n + 13132*n^2 + 6769*
n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (x*(a + b*x)^n*(720*a^6*
b*d^3*n - 8028*b^7*c^3*n - 5104*b^7*c^3*n^2 - 1665*b^7*c^3*n^3 - 295*b^7*c^
3*n^4 - 27*b^7*c^3*n^5 - b^7*c^3*n^6 - 5040*b^7*c^3 + 5040*a^2*b^5*c^2*d*n
+ 3024*a^4*b^3*c*d^2*n + 3828*a^2*b^5*c^2*d*n^2 + 936*a^4*b^3*c*d^2*n^2 + 1
074*a^2*b^5*c^2*d*n^3 + 72*a^4*b^3*c*d^2*n^3 + 132*a^2*b^5*c^2*d*n^4 + 6*a^
2*b^5*c^2*d*n^5))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5
 + 28*n^6 + n^7 + 5040)) + (d^3*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^
3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n
^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (3*d^2*x^5*(a + b*x)^n*(42*b^2*c + b^
2*c*n^2 - 2*a^2*d*n + 13*b^2*c*n)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b^2
*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040
)) + (3*d*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(840*b^4*c^2 - 40*a^4*d^2*n + 638
*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 - 168*a^2*b^2*c
*d*n - 52*a^2*b^2*c*d*n^2 - 4*a^2*b^2*c*d*n^3))/(b^4*(13068*n + 13132*n^2 +
 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (a*d^3*n*x^6*(a +
b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b*(13068*n + 13132
*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (3*a*d^2*n*x
^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(10*a^2*d + 42*b^2*c + b^2*c*n^2 +
```

```
13*b^2*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28
*n^6 + n^7 + 5040)) + (3*a*d*n*x^2*(n + 1)*(a + b*x)^n*(120*a^4*d^2 + 840*b
^4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 5
04*a^2*b^2*c*d + 156*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2))/(b^5*(13068*n + 1
3132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.271 $\int \frac{x^3 \sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=345

$$-\frac{\left(-15a^3e^6-2cdex\left(-5a^2e^4-6acd^2e^2+35c^2d^4\right)-17a^2cd^2e^4-25ac^2d^4e^2+105c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{192c^3d^3e^4}$$

Rubi [A] time = 0.51, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 832, 779, 621, 206}

$$-\frac{\left(-2cdex\left(-5a^2e^4-6acd^2e^2+35c^2d^4\right)-17a^2cd^2e^4-15a^3e^6-25ac^2d^4e^2+105c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{192c^3d^3e^4}+\frac{\left(cd^2-ae^2\right)\left(9a^2cd^2e^4+5a^3e^6+15ac^2d^4e^2+35c^3d^6\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128c^{7/2}d^{7/2}e^{9/2}}+\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}+\frac{1}{24}x^2\left(\frac{a}{cd}-\frac{7d}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] ((a/(c*d) - (7*d)/e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/24
+ (x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - ((105*c^3*d^6 -
 25*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 - 15*a^3*e^6 - 2*c*d*e*(35*c^2*d^4 - 6
*a*c*d^2*e^2 - 5*a^2*e^4)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
192*c^3*d^3*e^4) + ((c*d^2 - a*e^2)*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*
c*d^2*e^4 + 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(7/2)*d^(7
/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx=\int \frac{x^3(ae+cdx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$=\frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}+\frac{\int \frac{x^2\left(-3acd^2e-\frac{1}{2}cd\left(7cd^2-ae^2\right)x\right)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4cde}$$

$$=\frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)}}{4e}$$

$$=\frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)}}{4e}$$

$$=\frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)}}{4e}$$

$$=\frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)}}{4e}$$

Mathematica [A] time = 1.59, size = 304, normalized size = 0.88

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{3\sqrt{cd}\sqrt{cd^2-ae^2}\left(5a^3e^6+9a^2cd^2e^4+15ac^2d^4e^2+35c^3d^6\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-\sqrt{c}\sqrt{d}\sqrt{e}\left(-15a^3e^6+a^2cde^4(10ex-17d)+ac^2d^2e^2\left(-25d^2+12dex-8e^2x^2\right)+c^3d^3\left(105d^3-70d^2ex+56de^2x^2-48e^3x^3\right)\right)\right)}{192c^{7/2}d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^3*e^6 + a^
2*c*d*e^4*(-17*d + 10*e*x) + a*c^2*d^2*e^2*(-25*d^2 + 12*d*e*x - 8*e^2*x^2)
 + c^3*d^3*(105*d^3 - 70*d^2*e*x + 56*d*e^2*x^2 - 48*e^3*x^3))) + (3*Sqrt[c
*d]*Sqrt[c*d^2 - a*e^2]*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 +
5*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*S
qrt[c*d^2 - a*e^2])])/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
2)])))/(192*c^(7/2)*d^(7/2)*e^(9/2))
```

IntegrateAlgebraic [F] time = 180.36, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e
*x),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.57, size = 678, normalized size = 1.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2
*e^6 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4*e^
4*x^3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*a^3*c*d*
e^7 - 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 6*a*c^3*d
^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^4*d^4*e^5), -1/384*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^
4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2
*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(48*c^4*d^4*e^4*x^
3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7
- 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 6*a*c^3*d^4*e
^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
4*d^4*e^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Evaluation time: 1.91Error: Bad Argument Type
```

maple [B] time = 0.03, size = 946, normalized size = 2.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

```
[Out] 19/64/c^2/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+29/32/e^3*d^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/32*e^2/c^2*a^3/d*ln((1/2*a*e^2+1/2*c
```

```
*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e
)^(1/2)+5/32*e/c^2/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2-5/128*
e^4/c^3/d^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a^4+1/4/e^2*x*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)/c/d-5/24/e/c^2/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)*a+7/16/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+5/64*e^2/c^3/d^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+43/64/e^2/c*d*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)*a-3/64/c*d*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a^2-29/128/e^4*
c*d^5*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+11/32/e^2*a*d^3*ln((1/2*a*e^2+1/2*c*d^2+c*
d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)
-d^3/e^4*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-13/24/e^3/c*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+29/64/e^4*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-1/2*d^3/e^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)
+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a+1/2*d^5/e^4
*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.272 $\int \frac{x^2 \sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex} \, dx$

Optimal. Leaf size=251

$$-\frac{\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2\right.}{}$$

Rubi [A] time = 0.26, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 832, 779, 621, 206}

$$-\frac{\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cdex\left(5cd^2-ae^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{24c^2d^2e^3}+\frac{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*e) + (((5*c*d^2 - 3*a*
e^2)*(3*c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(24*c^2*d^2*e^3) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 2*
a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(5/2)*d^(5/
2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx = \int \frac{x^2(ae+cdx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e} + \frac{\int \frac{x\left(-2acd^2e-\frac{1}{2}cd\left(5cd^2-ae^2\right)x\right)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3cde}$$

$$= \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e} + \frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)-2cde\left(\right.\right.}{}$$

$$= \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e} + \frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)-2cde\left(\right.\right.}{}$$

$$= \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e} + \frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)-2cde\left(\right.\right.}{}$$

Mathematica [A]   time = 0.82, size = 245, normalized size = 0.98

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(-3a^2e^4+2acde^2(ex-2d)+c^2d^2\left(15d^2-10dex+8e^2x^2\right)\right)-\frac{3\sqrt{cd}\sqrt{cd^2-ae^2}\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right)}{24c^{5/2}d^{5/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-3*a^2*e^4 + 2*a*c
*d*e^2*(-2*d + e*x) + c^2*d^2*(15*d^2 - 10*d*e*x + 8*e^2*x^2)) - (3*Sqrt[c*
d]*Sqrt[c*d^2 - a*e^2]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcSinh[(Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sq
rt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(24*c^(5/2)*d^(5/2
)*e^(7/2))
```

IntegrateAlgebraic [B]   time = 14.05, size = 13727, normalized size = 54.69

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e
*x),x]
```

```
[Out] Result too large to show
```

fricas [A]   time = 0.47, size = 536, normalized size = 2.14

$$\left[-\frac{3\left(5c^3d^6-3ac^2d^4e^2-a^2cd^2e^4-a^3e^6\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdex+cd^2+ae^2\right)\sqrt{cde}+8\left(c^2d^3e+acde^3\right)x\right)-4\left(8c^3d^3e^3x^2+15c^3d^5e-4ac^2d^3e^3-3a^2cde^5-2\left(5c^3d^4e^2-ac^2d^2e^4\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{96c^3d^3e^4}, \frac{3\left(5c^3d^6-3ac^2d^4e^2-a^2cd^2e^4-a^3e^6\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdex+cd^2+ae^2\right)\sqrt{-cde}}{2\left(c^2d^2e^2x^2+acd^2e^2+\left(c^2d^3e+acde^3\right)x\right)}\right)+2\left(8c^3d^3e^3x^2+15c^3d^5e-4ac^2d^3e^3-3a^2cde^5-2\left(5c^3d^4e^2-ac^2d^2e^4\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{48c^3d^3e^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e - 4*a*c
^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/48*(3*(5*c^3*d^6 - 3*a
*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(
c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c^3*d^3*
e^3*x^2 + 15*c^3*d^5*e - 4*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2
 - a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*
e^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Evaluation time: 1.92Error: Bad Argument Type
```

maple [B] time = 0.01, size = 713, normalized size = 2.84

$$\frac{a^3e^3\ln\left(\frac{cdex+\frac{1}{2}ae^2+\frac{1}{2}cd^2}{\sqrt{cde}}+\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\right)}{16\sqrt{cde}\,c^2d^2}+\frac{a^2e\ln\left(\frac{cdex+\frac{1}{2}ae^2+\frac{1}{2}cd^2}{\sqrt{cde}}+\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\right)}{16\sqrt{cde}\,c}+\frac{ad^2\ln\left(\frac{\frac{ae^2}{2}-\frac{cd^2}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^2cde+\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}\,e}-\frac{5ad^2\ln\left(\frac{cdex+\frac{1}{2}ae^2+\frac{1}{2}cd^2}{\sqrt{cde}}+\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\right)}{16\sqrt{cde}\,e}-\frac{cd^4\ln\left(\frac{\frac{ae^2}{2}-\frac{cd^2}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^2cde+\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}\,e^3}+\frac{3cd^4\ln\left(\frac{cdex+\frac{1}{2}ae^2+\frac{1}{2}cd^2}{\sqrt{cde}}+\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\right)}{16\sqrt{cde}\,e^3}-\frac{\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\,ax}{4cd}-\frac{3\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\,dx}{4e^2}-\frac{\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\,a^2e}{8c^2d^2}-\frac{\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\,a}{2ce}-\frac{3\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\,d^2}{8e^3}+\frac{\sqrt{\left(x+\frac{d}{e}\right)^2cde+\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}\,d^2}{e^3}+\frac{\left(cdex^2+ade+\left(ae^2+cd^2\right)x\right)^{\frac{3}{2}}}{3cde^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d),x)
```

```
[Out] 1/3/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/c/d-1/4/c/d*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*x*a-3/4/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*x-1/8*e/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2-1/2/e/c*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a-3/8/e^3*d^2*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)+1/16*e^3/c^2/d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(
1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3+1/16*e/c*ln
((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2))/(c*d*e)^(1/2)*a^2-5/16/e*d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c
*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+3/16/e
^3*c*d^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+d^2/e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*
(x+d/e))^(1/2)+1/2*d^2/e*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/
```

```
2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a-1/2*d^4/e
^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{(d + ex)\,(ae + cdx)}}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.273 $\int \frac{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=207

$$\frac{\left(cd^2-ae^2\right)\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2cde(d+ex)}-\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right.$$

Rubi [A] time = 0.19, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {794, 664, 621, 206}

$$\frac{\left(cd^2-ae^2\right)\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2cde(d+ex)}-\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] -((a/(c*d) + (3*d)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/4 + (a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*c*d*e*(d + e*x)) + ((c*d^2 -
 a*e^2)*(3*c*d^2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(
3/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2cde(d+ex)} + \frac{1}{4}\left(-\frac{3d}{e}-\frac{ae}{cd}\right)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)}}{d+ex}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cc\right.}{2cde(d+ex)}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cc\right.}{2cde(d+ex)}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cc\right.}{2cde(d+ex)}$$

Mathematica [A] time = 0.66, size = 197, normalized size = 0.95

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{cd}\sqrt{cd^2-ae^2}\left(ae^2+3cd^2\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}+\sqrt{c}\sqrt{d}\sqrt{e}\left(ae^2+cd(2ex-3d)\right)\right)}{4c^{3/2}d^{3/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e^2 + c*d*(-3*d
+ 2*e*x)) + (Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*(3*c*d^2 + a*e^2)*ArcSinh[(Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sq
rt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(4*c^(3/2)*d^(3/2)
*e^(5/2))
```

IntegrateAlgebraic [A] time = 2.22, size = 330, normalized size = 1.59

$$-\frac{\sqrt{cde}\left(-a^2e^4-2acd^2e^2+3c^2d^4\right)\log\left(a^2e^4+8cdex\sqrt{cde}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2acd^2e^2-4acde^3x+c^2d^4-4c^2d^3ex-8c^2d^2e^2x^2\right)}{16c^2d^2e^3}+\frac{\left(-a^2e^4-2acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\left(2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2x\sqrt{cde}\right)}{ae^2+cd^2}\right)}{8c^{3/2}d^{3/2}e^{5/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2-3cd^2+2cdex\right)}{4cde^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x
),x]
```

```
[Out] ((-3*c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(4*c*d*e^2) + ((3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*Sqr
t[d]*Sqrt[e]*(-2*Sqrt[c*d*e]*x + 2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]))/(c*d^2 + a*e^2)])/(8*c^(3/2)*d^(3/2)*e^(5/2)) - (Sqrt[c*d*e]*(3*c^2*d
^4 - 2*a*c*d^2*e^2 - a^2*e^4)*Log[c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4 - 4*c^2
*d^3*e*x - 4*a*c*d*e^3*x - 8*c^2*d^2*e^2*x^2 + 8*c*d*e*Sqrt[c*d*e]*x*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(16*c^2*d^2*e^3)
```

fricas [A] time = 0.43, size = 418, normalized size = 2.02

$$\left[-\frac{\left(3c^2d^4-2acd^2e^2-a^2e^4\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4-4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdex+cd^2+ae^2\right)\sqrt{cde}+8\left(c^2d^3e+acde^3\right)x\right)-4\left(2c^2d^2e^2x-3c^2d^3e+acde^3\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{16c^2d^2e^3},-\frac{\left(3c^2d^4-2acd^2e^2-a^2e^4\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdex+cd^2+ae^2\right)\sqrt{-cde}}{2\left(c^2d^2e^2x^2+acd^2e^2+\left(c^2d^3e+acde^3\right)x\right)}\right)-2\left(2c^2d^2e^2x-3c^2d^3e+acde^3\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{8c^2d^2e^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/16*((3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2
*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*
d*e^3)*x) - 4*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/8*((3*c^2*d^4 - 2*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^
2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + a*c*d*
e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Evaluati
on time: 1.9Error: Bad Argument Type
```

maple [B] time = 0.01, size = 516, normalized size = 2.49

$$-\frac{a^2e^2\ln\left(\frac{cdex+\frac{1}{2}ae^2+\frac{1}{2}cd^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(ae^2+cd^2\right)x}\right)}{8\sqrt{cde}\,cd}-\frac{ad\ln\left(\frac{\frac{ae^2}{2}-\frac{cd^2}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^2cde+\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}}+\frac{ad\ln\left(\frac{cdex+\frac{1}{2}ae^2+\frac{1}{2}cd^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(ae^2+cd^2\right)x}\right)}{4\sqrt{cde}}+\frac{cd^3\ln\left(\frac{\frac{ae^2}{2}-\frac{cd^2}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^2cde+\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}\,e^2}-\frac{cd^3\ln\left(\frac{cdex+\frac{1}{2}ae^2+\frac{1}{2}cd^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(ae^2+cd^2\right)x}\right)}{8\sqrt{cde}\,e^2}+\frac{\sqrt{cde\,x^2+ade+\left(ae^2+cd^2\right)x}\,x}{2e}+\frac{\sqrt{cde\,x^2+ade+\left(ae^2+cd^2\right)x}\,a}{4cd}+\frac{\sqrt{cde\,x^2+ade+\left(ae^2+cd^2\right)x}\,d}{4e^2}-\frac{\sqrt{\left(x+\frac{d}{e}\right)^2cde+\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}\,d}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+1/4/c/d*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2)*a+1/4/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/
8*e^2/c/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2+1/4*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*
d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a
-1/8/e^2*c*d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-d/e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d
^2)*(x+d/e))^(1/2)-1/2*d*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/
2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a+1/2*d^3/e
^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
[Out] Integral(x*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.274 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {664, 621, 206}

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(c
*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+}}}{2e^2}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4cde-x^2}\right.}{e^2}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e} - \frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae\right.}}\right.}{2\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Mathematica [A] time = 0.75, size = 155, normalized size = 1.18

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{e}-\frac{c^{3/2}d^{3/2}\sqrt{cd^2-ae^2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right)}{e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e] - (c^(3/2)*d^(3/2)*Sqrt[c*d^2 - a*e
^2]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d
^2 - a*e^2])])/((c*d)^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 -
 a*e^2)])))/e^(3/2)
```

IntegrateAlgebraic [B] time = 0.02, size = 272, normalized size = 2.08

$$\frac{\sqrt{cde}\left(cd^2-ae^2\right)\log\left(a^2e^4+8cdex\sqrt{cde}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2acd^2e^2-4acde^3x+c^2d^4-4c^2d^3ex-8c^2d^2e^2x^2\right)}{4cde^2}+\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}}{e}+\frac{\left(ae^2-cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\left(2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2x\sqrt{cde}\right)}{ae^2+cd^2}\right)}{2\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2]/e + ((-(c*d^2) + a*e^2)*ArcTanh
[(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-2*Sqrt[c*d*e]*x + 2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]))/(c*d^2 + a*e^2)])/(2*Sqrt[c]*Sqrt[d]*e^(3/2)) + (Sqrt[c*
d*e]*(c*d^2 - a*e^2)*Log[c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4 - 4*c^2*d^3*e*x
- 4*a*c*d*e^3*x - 8*c^2*d^2*e^2*x^2 + 8*c*d*e*Sqrt[c*d*e]*x*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2]])/(4*c*d*e^2)
```

fricas [A] time = 0.44, size = 337, normalized size = 2.57

$$\left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,cde-\left(cd^2-ae^2\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdex+cd^2+ae^2\right)\sqrt{cde}+8\left(c^2d^3e+acde^3\right)x\right)}{4cde^2},\ \frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,cde+\left(cd^2-ae^2\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdex+cd^2+ae^2\right)\sqrt{-cde}}{2\left(c^2d^2e^2x^2+acd^2e^2+\left(c^2d^3e+acde^3\right)x\right)}\right)}{2cde^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2)
*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c*d*e^2), 1/2*(2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*c*d*e + (c*d^2 - a*e^2)*sqrt(-c*d*e)*arctan(1/2
```

```
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sq
rt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)))/(c
*d*e^2)]
```

giac [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A]   time = 0.01, size = 205, normalized size = 1.56

$$\frac{ae\ln\left(\frac{\frac{a e^2}{2}-\frac{c d^2}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}}-\frac{c d^2\ln\left(\frac{\frac{a e^2}{2}-\frac{c d^2}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}\, e}+\frac{\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*e*ln((1/2*a*e^2-1/2*c
*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(
1/2))/(c*d*e)^(1/2)*a-1/2/e*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^
(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c*d^2
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

3.275 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{d}}$$

Rubi [A] time = 0.17, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 843, 621, 206, 724}

$$\frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] - (Sqrt[a]*Sqrt
[e]*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[d]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
```

```
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x(d+ex)}\,dx &= \int \frac{ae+cdx}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx \\
&= (cd)\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx+(ae)\int \frac{1}{x\sqrt{ade+\left(cd^2+ae^2\right.}} \\
&= (2cd)\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)-(2ae \\
&= \frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{}{2\sqrt{a}\sqrt{\ }}\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 210, normalized size = 1.25

$$
-\frac{2\sqrt{ae+cdx}\left(\sqrt{a}\sqrt{c}\,e\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)-\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

```
[Out] (-2*Sqrt[a*e + c*d*x]*(-(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e*x))
/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt
[c*d]*Sqrt[c*d^2 - a*e^2])]) + Sqrt[a]*Sqrt[c]*e*Sqrt[d + e*x]*ArcTanh[(Sqr
t[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(Sqrt[c]*Sqrt[d]
*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [A] time = 0.68, size = 325, normalized size = 1.93

$$
-\frac{\sqrt{cde}\log\left(a^2e^4+8cdex\sqrt{cde}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2acd^2e^2-4acde^3x+c^2d^4-4c^2d^3ex-8c^2d^2e^2x^2\right)}{2e}+\frac{2\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{x\sqrt{cde}}{\sqrt{a}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{\sqrt{d}}-\frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}x\sqrt{cde}}{ae^2+cd^2}-\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{ae^2+cd^2}\right)}{\sqrt{e}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)
),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c*d*e]*x)/(Sqrt[a]*Sqrt[d]*Sqrt[e]) - Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[a]*Sqrt[d]*Sqrt[e])])/Sqrt[d]
 - (Sqrt[c]*Sqrt[d]*ArcTanh[(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[c*d*e]*x)/(c*d^
2 + a*e^2) - (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])/(c*d^2 + a*e^2)])/Sqrt[e] - (Sqrt[c*d*e]*Log[c^2*d^4 - 2*a*c*d^2*
e^2 + a^2*e^4 - 4*c^2*d^3*e*x - 4*a*c*d*e^3*x - 8*c^2*d^2*e^2*x^2 + 8*c*d*e
*Sqrt[c*d*e]*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2*e)
```

fricas [A] time = 0.66, size = 947, normalized size = 5.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/2*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
+ 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 1/2*sqrt(a*e/d)*log((8*a^
2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*
(a*c*d^3*e + a^2*d*e^3)*x)/x^2), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^
2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 1/2*sqrt(a*e/d)*log((8*a^
2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*
(a*c*d^3*e + a^2*d*e^3)*x)/x^2), sqrt(-a*e/d)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d
*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 1/2*sqrt(c*d/e)*log(8*c^
2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*
e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2
*d^3*e + a*c*d*e^3)*x), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 +
 a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + sqrt(-a*e/d)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d
)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 439, normalized size = 2.61

$$-\frac{a e^2 \ln\left(\frac{\frac{a e^2}{2}-\frac{c d^2}{2}+\left(x+\frac{d}{e}\right) c d e}{\sqrt{c d e}}+\sqrt{\left(x+\frac{d}{e}\right)^2 c d e+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}\right)}{2 \sqrt{c d e}\, d}+\frac{a e^2 \ln\left(\frac{c d e x+\frac{1}{2} a e^2+\frac{1}{2} c d^2}{\sqrt{c d e}}+\sqrt{c d e x^2+a d e+\left(a e^2+c d^2\right) x}\right)}{2 \sqrt{c d e}\, d}-\frac{a e \ln\left(\frac{2 a d e+\left(a e^2+c d^2\right) x+2 \sqrt{a d e}\, \sqrt{c d e x^2+a d e+\left(a e^2+c d^2\right) x}}{x}\right)}{\sqrt{a d e}}+\frac{c d \ln\left(\frac{\frac{a e^2}{2}-\frac{c d^2}{2}+\left(x+\frac{d}{e}\right) c d e}{\sqrt{c d e}}+\sqrt{\left(x+\frac{d}{e}\right)^2 c d e+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}\right)}{2 \sqrt{c d e}}+\frac{c d \ln\left(\frac{c d e x+\frac{1}{2} a e^2+\frac{1}{2} c d^2}{\sqrt{c d e}}+\sqrt{c d e x^2+a d e+\left(a e^2+c d^2\right) x}\right)}{2 \sqrt{c d e}}+\frac{\sqrt{c d e x^2+a d e+\left(a e^2+c d^2\right) x}}{d}-\frac{\sqrt{\left(x+\frac{d}{e}\right)^2 c d e+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/x/(e*x+d),x)
```

```
[Out] 1/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/2/d*ln((c*d*e*x+1/2*a*e^2+1/2
*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2
)*a*e^2+1/2*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c-a*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a
*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-1
/d*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/2/d*ln((1/2*a*e^2-1/2*c*
d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1
/2))/(c*d*e)^(1/2)*a*e^2+1/2*d*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{x\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x*(d + e*x)), x)
```

3.276 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=137

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\,d^{3/2}\sqrt{e}}$$

Rubi [A] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {849, 806, 724, 206}

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\,d^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d^2 - a*e^2)*Arc
Tanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^2(d+ex)}\,dx=\int \frac{ae+cdx}{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{dx}-\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\int \frac{}{x\sqrt{ade+\cdots}}}{2ade}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{dx}+\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \cdots\right.}{ac\cdots}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{dx}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{2ade+\left(cd^2\cdots\right.}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(c\cdots\right.}}\right.}{2\sqrt{a}\,d^{3/2}\sqrt{e}}$$

Mathematica [A] time = 0.14, size = 117, normalized size = 0.85

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\left(ae^2-cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}}-\frac{\sqrt{d}}{x}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[d]/x) + ((-(c*d^2) + a*e^2)*ArcTanh[
(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt
[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/d^(3/2)
```

IntegrateAlgebraic [A] time = 0.57, size = 124, normalized size = 0.91

$$\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{x\sqrt{cde}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{\sqrt{a}\,d^{3/2}\sqrt{e}}-\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*
x)),x]
```

```
[Out] -(Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2]/(d*x)) + ((c*d^2 - a*e^2)*Arc
Tanh[(Sqrt[c*d*e]*x - Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[a]
*Sqrt[d]*Sqrt[e])])/(Sqrt[a]*d^(3/2)*Sqrt[e])
```

fricas [A] time = 0.52, size = 355, normalized size = 2.59

$$\left[-\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,ade+\left(cd^2-ae^2\right)\sqrt{ade}\,x\log\left(\frac{8a^2d^2e^2+\left(c^2d^4+6acd^2e^2+a^2e^4\right)x^2+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}+8\left(acd^3e+a^2de^3\right)x}{x^2}\right)}{4ad^2ex},\ -\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,ade-\left(cd^2-ae^2\right)\sqrt{-ade}\,x\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{-ade}}{2\left(acd^2e^2x^2+a^2d^2e^2+\left(acd^3e+a^2de^3\right)x\right)}\right)}{2ad^2ex}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e + (c*d^2 - a*e^2
)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^
```

```
2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2))/(a*d^2*e*x), -1/2*(2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e - (c*d^2 - a*e^2)*sqrt(-a
*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e
+ a^2*d*e^3)*x)))/(a*d^2*e*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-
2*exp(1)*a*exp(2)+2*exp(1)^3*a)/d/2/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*a
tan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))-(-a*ex
p(2)+2*exp(1)^2*a-c*d^2)/d/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-((sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a*exp(2)+
c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)-2*d*exp(1)*sqrt(c*d*exp(1))*a)/2/d/((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a))
```

maple [B] time = 0.02, size = 594, normalized size = 4.34

$$\frac{a e^{2} \ln\left(\frac{2ade+\left(a e^{2}+c d^{2}\right)x+2\sqrt{ade}\,\sqrt{cde x^{2}+ade+\left(a e^{2}+c d^{2}\right)x}}{x}\right)}{2\sqrt{ade}\,d}+\frac{a e^{3} \ln\left(\frac{\frac{a e^{2}}{2}-\frac{c d^{2}}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^{2} cde+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}\,d^{2}}-\frac{a e^{3} \ln\left(\frac{cdex+\frac{1}{2}a e^{2}+\frac{1}{2}c d^{2}}{\sqrt{cde}}+\sqrt{cde x^{2}+ade+\left(a e^{2}+c d^{2}\right)x}\right)}{2\sqrt{cde}\,d^{2}}-\frac{cd \ln\left(\frac{2ade+\left(a e^{2}+c d^{2}\right)x+2\sqrt{ade}\,\sqrt{cde x^{2}+ade+\left(a e^{2}+c d^{2}\right)x}}{x}\right)}{2\sqrt{ade}}-\frac{ce \ln\left(\frac{\frac{a e^{2}}{2}-\frac{c d^{2}}{2}+\left(x+\frac{d}{e}\right)cde}{\sqrt{cde}}+\sqrt{\left(x+\frac{d}{e}\right)^{2} cde+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{cde}}+\frac{ce \ln\left(\frac{cdex+\frac{1}{2}a e^{2}+\frac{1}{2}c d^{2}}{\sqrt{cde}}+\sqrt{cde x^{2}+ade+\left(a e^{2}+c d^{2}\right)x}\right)}{2\sqrt{cde}}+\frac{\sqrt{cde x^{2}+ade+\left(a e^{2}+c d^{2}\right)x}\,cx}{ad}+\frac{\sqrt{cde x^{2}+ade+\left(a e^{2}+c d^{2}\right)x}\,c}{ae}+\frac{\sqrt{\left(x+\frac{d}{e}\right)^{2} cde+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\,e}{d^{2}}-\frac{\left(cde x^{2}+ade+\left(a e^{2}+c d^{2}\right)x\right)^{\frac{3}{2}}}{a d^{2} e x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/x^2/(e*x+d),x)
```

```
[Out] -1/d^2/a/e/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/a/e*(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2)*c+1/2*e*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1
/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c+1/2/d*a*e^2/(a
*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/x)-1/2*d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*
(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+1/d*c/a*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/2*e^3/d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*
d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a
+e/d^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*e^3/d^2*ln((1/2*a*
e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(
x+d/e))^(1/2))/(c*d*e)^(1/2)*a-1/2*e*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)
/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^{2} + ade + \left(cd^{2} + ae^{2}\right)x}}{(ex + d)x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{x^2\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**2/(e*x+d),x)
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**2*(d + e*x)), x)
```

3.277 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=202

$$\frac{(cd^2-ae^2)(3ae^2+cd^2)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4x}-\frac{\sqrt{x(a}}{}$$

Rubi [A] time = 0.28, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 834, 806, 724, 206}

$$\frac{(cd^2-ae^2)(3ae^2+cd^2)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4x}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*d*x^2) - ((c/(a*e) - (3*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*x) + ((c*d^2 - a*e^2)*
(c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(5/2)*e
^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3(d+ex)}\,dx=\int \frac{ae+cdx}{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2}-\frac{\int \frac{-\frac{1}{2}ae\left(cd^2-3ae^2\right)+acde^2x}{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2ade}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{4x}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{4x}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{4x}$$

Mathematica [A] time = 0.16, size = 162, normalized size = 0.80

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\left(-3a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}\sqrt{ae+cdx}}+\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(ae(3ex-2d)-cd^2x\right)}{x^2}\right)}{4a^{3/2}d^{5/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c*d^2*x) + a*e*
(-2*d + 3*e*x)))/x^2 + ((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(Sqrt
[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]
*Sqrt[d + e*x])))/(4*a^(3/2)*d^(5/2)*e^(3/2))
```

IntegrateAlgebraic [A] time = 0.89, size = 170, normalized size = 0.84

$$\frac{\left(3a^2e^4-2acd^2e^2-c^2d^4\right)\tanh^{-1}\left(\frac{x\sqrt{cde}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{4a^{3/2}d^{5/2}e^{3/2}}+\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\left(-2ade+3ae^2x-cd^2x\right)}{4ad^2ex^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*
x)),x]
```

```
[Out] ((-2*a*d*e - c*d^2*x + 3*a*e^2*x)*Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^
2])/(4*a*d^2*e*x^2) + ((-(c^2*d^4) - 2*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(Sq
```

```
rt[c*d*e]*x - Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[a]*Sqrt[d]
*Sqrt[e])])/(4*a^(3/2)*d^(5/2)*e^(3/2))
```

fricas [A] time = 0.91, size = 442, normalized size = 2.19

$$\left[-\frac{\left(c^2d^4+2\,acd^2e^2-3\,a^2e^4\right)\sqrt{ade}\,x^2\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2-4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}+8\left(acd^3e+a^2de^3\right)x}{x^2}\right)+4\left(2\,a^2d^2e^2+\left(acd^3e-3\,a^2de^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{16\,a^2d^3e^2x^2},\ -\frac{\left(c^2d^4+2\,acd^2e^2-3\,a^2e^4\right)\sqrt{-ade}\,x^2\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,ade+\left(cd^2+ae^2\right)x\right)\sqrt{-ade}}{2\left(acd^2e^2x^2+a^2d^2e^2+\left(acd^3e+a^2de^3\right)x\right)}\right)+2\left(2\,a^2d^2e^2+\left(acd^3e-3\,a^2de^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{8\,a^2d^3e^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/16*((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^
2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3
*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (a*c*d^3*e - 3*a^2*d*e^3)*x)*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^2), -1/8*((c^2*d
^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(2*a^2*d^2*e^
2 + (a*c*d^3*e - 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(a^2*d^3*e^2*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((2
*exp(1)^2*a*exp(2)-2*exp(1)^4*a)/2/d^2/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)
)*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))+(-a
^2*exp(2)^2-4*exp(1)^2*a^2*exp(2)+8*exp(1)^4*a^2-2*c*d^2*a*exp(2)-c^2*d^4)/
4/d^2/exp(1)/a/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x
+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-((sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^2*exp(2)^2-4*exp(
1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^3*a^2*exp(2)+2*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^3*a*exp(2)+c^2*d^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3-8*d*exp(1)*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^2*exp(2)
+8*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^2*a^2-8*c*d^3*exp(1)*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a+d*exp(1)
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^
3*exp(2)^2+4*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)*a^3*exp(2)+2*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^2*exp(2)+8*c*d^3*exp(1)^3*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^2+c^
2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)*a-8*d^2*exp(1)^4*sqrt(c*d*exp(1))*a^3)/8/d^2/exp(1)/a/((sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a)
^2)
```

maple [B] time = 0.02, size = 882, normalized size = 4.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/x^3/(e*x+d),x)
```

```
[Out] 5/4/d^3/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-1/4*e^2/d^3*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/d/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*
c-1/2*e^2/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c-3/8*e^3/d^2*a/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2))/x)+1/4*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-5/4*e/d^2*c/a*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2)*x-1/2/d^2/a/e/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3
/2)+1/4/d/a^2/e^2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-1/4*d/a^2/e^2
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+1/8*d^2/a/e/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2))/x)*c^2-1/4/a^2/e*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+1/2/d^
3*e^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a-1/d^3*e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2)-1/2/d^3*e^4*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^
(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a+1/2/d*
e^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(
a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{(ex + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^3\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**3*(d + e*x)), x)
```

3.278 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=286

$$\frac{(3cd^2-5ae^2)(3ae^2+cd^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24a^2d^3e^2x}-\frac{(cd^2-ae^2)(5a^2e^4+2acd^2e^2+c^2d^4)\tanh^{-1}\left(\right.}{16a^{5/2}d^{7/2}e^{5/2}}$$

Rubi [A] time = 0.40, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {849, 834, 806, 724, 206}

$$-\frac{(cd^2-ae^2)(5a^2e^4+2acd^2e^2+c^2d^4)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}}+\frac{(3cd^2-5ae^2)(3ae^2+cd^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24a^2d^3e^2x}-\frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{12x^2}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*d*x^3) - ((c/(a*e) - (5*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*x^2) + ((3*c*d^2 - 5*
a*e^2)*(c*d^2 + 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*a
^2*d^3*e^2*x) - ((c*d^2 - a*e^2)*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcT
anh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^4(d+ex)}\,dx = \int \frac{ae+cdx}{x^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3} - \frac{\int \frac{-\frac{1}{2}ae\left(cd^2-5ae^2\right)+2acde^2x}{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3ade}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{12x^2}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{12x^2}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{12x^2}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{12x^2}$$

Mathematica [A] time = 0.24, size = 210, normalized size = 0.73

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^2e^2\left(-8d^2+10dex-15e^2x^2\right)-2acd^2ex(d-2ex)+3c^2d^4x^2\right)}{x^3}-\frac{3\left(-5a^3e^6+3a^2cd^2e^4+ac^2d^4e^2+c^3d^6\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}\sqrt{ae+cdx}}\right)}{24a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^2*d^4*x^2 - 2
*a*c*d^2*e*x*(d - 2*e*x) + a^2*e^2*(-8*d^2 + 10*d*e*x - 15*e^2*x^2)))/x^3 -
 (3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(Sqrt[d
]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*S
qrt[d + e*x])))/(24*a^(5/2)*d^(7/2)*e^(5/2))
```

IntegrateAlgebraic [A] time = 1.23, size = 228, normalized size = 0.80

$$\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\left(-8a^2d^2e^2+10a^2de^3x-15a^2e^4x^2-2acd^3ex+4acd^2e^2x^2+3c^2d^4x^2\right)}{24a^2d^3e^2x^3}+\frac{\left(-5a^3e^6+3a^2cd^2e^4+ac^2d^4e^2+c^3d^6\right)\tanh^{-1}\left(\frac{x\sqrt{cde}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{8a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*
x)),x]
```

```
[Out] (Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2]*(-8*a^2*d^2*e^2 - 2*a*c*d^3*e*
x + 10*a^2*d*e^3*x + 3*c^2*d^4*x^2 + 4*a*c*d^2*e^2*x^2 - 15*a^2*e^4*x^2))/(
24*a^2*d^3*e^2*x^3) + ((c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e
^6)*ArcTanh[(Sqrt[c*d*e]*x - Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
Sqrt[a]*Sqrt[d]*Sqrt[e])])/(8*a^(5/2)*d^(7/2)*e^(5/2))
```

fricas [A] time = 2.12, size = 558, normalized size = 1.95

$$\left[-\frac{3\left(c^{3} d^{6}+a c^{2} d^{4} e^{2}+3 a^{2} c d^{2} e^{4}-5 a^{3} e^{6}\right) \sqrt{a d e}\, x^{3} \log\left(\frac{8 a^{2} d^{2} e^{2}+\left(c^{2} d^{4}+6 a c d^{2} e^{2}+a^{2} e^{4}\right) x^{2}+4 \sqrt{c d e x^{2}+a d e+\left(c d^{2}+a e^{2}\right) x}\left(2 a d e+\left(c d^{2}+a e^{2}\right) x\right) \sqrt{a d e}+8\left(a c d^{3} e+a^{2} d e^{3}\right) x}{x^{2}}\right)+4\left(8 a^{3} d^{3} e^{3}-\left(3 a c^{2} d^{5} e+4 a^{2} c d^{3} e^{3}-15 a^{3} d e^{5}\right) x^{2}+2\left(a^{2} c d^{4} e^{2}-5 a^{3} d^{2} e^{4}\right) x\right) \sqrt{c d e x^{2}+a d e+\left(c d^{2}+a e^{2}\right) x}}{96 a^{3} d^{4} e^{3} x^{3}}, \frac{3\left(c^{3} d^{6}+a c^{2} d^{4} e^{2}+3 a^{2} c d^{2} e^{4}-5 a^{3} e^{6}\right) \sqrt{-a d e}\, x^{3} \arctan\left(\frac{\sqrt{c d e x^{2}+a d e+\left(c d^{2}+a e^{2}\right) x}\left(2 a d e+\left(c d^{2}+a e^{2}\right) x\right) \sqrt{-a d e}}{2\left(a c d^{2} e^{2} x^{2}+a^{2} d^{2} e^{2}+\left(a c d^{3} e+a^{2} d e^{3}\right) x\right)}\right)-2\left(8 a^{3} d^{3} e^{3}-\left(3 a c^{2} d^{5} e+4 a^{2} c d^{3} e^{3}-15 a^{3} d e^{5}\right) x^{2}+2\left(a^{2} c d^{4} e^{2}-5 a^{3} d^{2} e^{4}\right) x\right) \sqrt{c d e x^{2}+a d e+\left(c d^{2}+a e^{2}\right) x}}{48 a^{3} d^{4} e^{3} x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(a*d*
e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 - (3*a*c^2*d
^5*e + 4*a^2*c*d^3*e^3 - 15*a^3*d*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*d^2*e
^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3), 1/48
*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(-a*d*e)*x^
3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*
d*e^3)*x)) - 2*(8*a^3*d^3*e^3 - (3*a*c^2*d^5*e + 4*a^2*c*d^3*e^3 - 15*a^3*d
*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-
2*exp(1)^3*a*exp(2)+2*exp(1)^5*a)/2/d^3/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2
))*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))-(-
a^3*exp(2)^3-2*exp(1)^2*a^3*exp(2)^2-8*exp(1)^4*a^3*exp(2)+16*exp(1)^6*a^3-
3*c*d^2*a^2*exp(2)^2-3*c^2*d^4*a*exp(2)+2*c^2*d^4*exp(1)^2*a-c^3*d^6)/8/d^3
/exp(1)^2/a^2/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))+(3*(sqrt(a*d*exp(1)+
a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)^3+6*exp
(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)^5*a^3*exp(2)^2-24*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)+9*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2*exp(2)^2+9*c^2*d^4*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a*exp(
2)-6*c^2*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-s
qrt(c*d*exp(1))*x)^5*a+3*c^3*d^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^5-48*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*ex
p(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^3*exp(2)+48
*d*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^4*a^3-8*d*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)^3+48*d*exp(1)^5*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp
(2)-24*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^3*a^3*exp(2)^2-48*c*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(
```

```
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3*exp(2)+48*c*d^3*exp(1)
^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^
3*a^3-24*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^3*a^2*exp(2)-48*c^2*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^2-8*c^3*d^7*exp(1)*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a+
48*d^2*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4*exp(2)^2+48*d^2*exp(1)^4*sqrt(c*d*exp(1
))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^
2*a^4*exp(2)-96*d^2*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4+96*c*d^4*exp(1)^2*sqrt(c*d
*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^2*a^3*exp(2)+48*c*d^4*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^3+48*c^2*d^6*exp(1)^
2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt
(c*d*exp(1))*x)^2*a^2-3*d^2*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)^3-6*d^2*exp(1)^4*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)^2-24*
d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)*a^5*exp(2)-9*c*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4*exp(2)^2-48*c*d^4*exp(1)^4*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4*exp(2)-4
8*c*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)*a^4-9*c^2*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^3*exp(2)-42*c^2*d^6*exp(1)^4*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^3-3*c^3*d^8
*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)*a^2+48*d^3*exp(1)^7*sqrt(c*d*exp(1))*a^5+16*c*d^5*exp(1)^5*sqrt(c*d*
exp(1))*a^4)/48/d^3/exp(1)^2/a^2/((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a)^3)
```

maple [B] time = 0.02, size = 1165, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/x^4/(e*x+d),x)
```

```
[Out] 3/4/d^3/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+3/8/e/a^2*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+1/4/d/a^2/e^2/x^2*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(3/2)*c-1/16*d^3/a^2/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*
x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3+1/8*d/a^3
/e^2*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/2/e/d^2/a^2/x*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c+11/8/d^3*e^2*c/a*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*x+1/d^4*e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3
/8/d^4*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/16*d/a/(a*d*e)^(1/2)*l
n((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2))/x)*c^2+1/2/d/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/3
/d^2/a/e/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-1/8/a^3/e^3/x*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^2+1/8*d^2/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*c^3-1/2/d^4*e^5*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(
1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+1/2/d^4*e^5*l
n((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a-1/2/d^2*e^3*ln((1/2*a*e^2-1/2*c*d^2
+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
)/(c*d*e)^(1/2)*c-11/8/d^4*e/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+9/
8/d^2*e/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c+1/2/d^2*e^3*ln((c*d*e*x
+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
)/(c*d*e)^(1/2)*c+5/16/d^3*e^4*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+
2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-3/16/d*e^2/(a*d
```

```
*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2))/x)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{(ex + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{x^4\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^4\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**4*(d + e*x)), x)
```

3.279 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=389

$$\frac{\left(-35a^2e^4+6acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{96a^2d^3e^2x^2}-\frac{\left(-105a^3e^6+25a^2cd^2e^4+17ac^2d^4e^2+15c^3d^6\right)\sqrt{}}{192a^3d^4e^3x}$$

Rubi [A] time = 0.59, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 834, 806, 724, 206}

$$-\frac{\left(25a^2cd^2e^4-105a^3e^6+17ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{192a^3d^4e^3x}+\frac{\left(-35a^2e^4+6acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{96a^2d^3e^2x^2}+\frac{\left(cd^2-ae^2\right)\left(15a^2cd^2e^4+35a^3e^6+9ac^2d^4e^2+5c^3d^6\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128a^{7/2}d^{9/2}e^{7/2}}-\frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{24x^3}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*d*x^4) - ((c/(a*e) - (7*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*x^3) + ((5*c^2*d^4 +
6*a*c*d^2*e^2 - 35*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(9
6*a^2*d^3*e^2*x^2) - ((15*c^3*d^6 + 17*a*c^2*d^4*e^2 + 25*a^2*c*d^2*e^4 - 1
05*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(192*a^3*d^4*e^3*x
) + ((c*d^2 - a*e^2)*(5*c^3*d^6 + 9*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 35*a
^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(7/2)*d^(9/2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
```

```
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5(d+ex)}\,dx = \int \frac{ae+cdx}{x^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\int \frac{-\frac{1}{2}ae\left(cd^2-7ae^2\right)+3acde^2x}{x^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4ade}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24x^3}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24x^3}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24x^3}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24x^3}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24x^3}$$

Mathematica [A] time = 0.36, size = 273, normalized size = 0.70

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^3e^3\left(-48d^3+56d^2ex-70de^2x^2+105e^3x^3\right)+a^2cd^2e^2x\left(-8d^2+12dex-25e^2x^2\right)+ac^2d^4ex^2(10d-17ex)-15c^3d^6x^3\right)}{x^4}+\frac{3\left(-35a^4e^8+20a^3cd^2e^6+6a^2c^2d^4e^4+4ac^3d^6e^2+5c^4d^8\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}\sqrt{ae+cdx}}\right)}{192a^{7/2}d^{9/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-15*c^3*d^6*x^3 +
 a*c^2*d^4*e*x^2*(10*d - 17*e*x) + a^2*c*d^2*e^2*x*(-8*d^2 + 12*d*e*x - 25*
e^2*x^2) + a^3*e^3*(-48*d^3 + 56*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3)))/x^
4 + (3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6
- 35*a^4*e^8)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d +
 e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*a^(7/2)*d^(9/2)*e^(7/2))
```

IntegrateAlgebraic [A] time = 1.74, size = 307, normalized size = 0.79

$$\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\left(-48a^3d^3e^3+56a^3d^2e^4x-70a^3de^5x^2+105a^3e^6x^3-8a^2cd^4e^2x+12a^2cd^3e^3x^2-25a^2cd^2e^4x^3+10ac^2d^5ex^2-17ac^2d^4e^2x^3-15c^3d^6x^3\right)}{192a^3d^4e^3x^4}+\frac{\left(35a^4e^8-20a^3cd^2e^6-6a^2c^2d^4e^4-4ac^3d^6e^2-5c^4d^8\right)\tanh^{-1}\left(\frac{x\sqrt{cde}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{64a^{7/2}d^{9/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*
x)),x]
```

```
[Out] (Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2]*(-48*a^3*d^3*e^3 - 8*a^2*c*d^4
*e^2*x + 56*a^3*d^2*e^4*x + 10*a*c^2*d^5*e*x^2 + 12*a^2*c*d^3*e^3*x^2 - 70*
a^3*d*e^5*x^2 - 15*c^3*d^6*x^3 - 17*a*c^2*d^4*e^2*x^3 - 25*a^2*c*d^2*e^4*x^
3 + 105*a^3*e^6*x^3))/(192*a^3*d^4*e^3*x^4) + ((-5*c^4*d^8 - 4*a*c^3*d^6*e^
2 - 6*a^2*c^2*d^4*e^4 - 20*a^3*c*d^2*e^6 + 35*a^4*e^8)*ArcTanh[(Sqrt[c*d*e]
*x - Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[a]*Sqrt[d]*Sqrt[e])
])/(64*a^(7/2)*d^(9/2)*e^(7/2))
```

fricas [A] time = 9.70, size = 702, normalized size = 1.80

$$\left[-\frac{3\left(5c^4d^8+4ac^3d^6e^2+6a^2c^2d^4e^4+20a^3cd^2e^6-35a^4e^8\right)\sqrt{ade}\,x^4\log\left(\frac{8a^2d^2e^2+\left(c^2d^4+6acd^2e^2+a^2e^4\right)x^2-4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}+8\left(acd^3e+a^2de^3\right)x}{x^2}\right)+4\left(48a^4d^4e^4+\left(15ac^3d^7e+17a^2c^2d^5e^3+25a^3cd^3e^5-105a^4de^7\right)x^3-2\left(5a^2c^2d^6e^2+6a^3cd^4e^4-35a^4d^2e^6\right)x^2+8\left(a^3cd^5e^3-7a^4d^3e^5\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{768a^4d^5e^4x^4}, -\frac{3\left(5c^4d^8+4ac^3d^6e^2+6a^2c^2d^4e^4+20a^3cd^2e^6-35a^4e^8\right)\sqrt{-ade}\,x^4\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{-ade}}{2\left(acd^2e^2x^2+a^2d^2e^2+\left(acd^3e+a^2de^3\right)x\right)}\right)+2\left(48a^4d^4e^4+\left(15ac^3d^7e+17a^2c^2d^5e^3+25a^3cd^3e^5-105a^4de^7\right)x^3-2\left(5a^2c^2d^6e^2+6a^3cd^4e^4-35a^4d^2e^6\right)x^2+8\left(a^3cd^5e^3-7a^4d^3e^5\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{384a^4d^5e^4x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*
e^6 - 35*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2
*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*
(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 17*a^2*c^2*d^5*e^3 + 25*a^3*c*d^3*e^5 -
 105*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 + 6*a^3*c*d^4*e^4 - 35*a^4*d^2*e
^6)*x^2 + 8*(a^3*c*d^5*e^3 - 7*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^4), -1/384*(3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2
+ 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 35*a^4*e^8)*sqrt(-a*d*e)*x^4*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*
x)) + 2*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 17*a^2*c^2*d^5*e^3 + 25*a^3*c*d
^3*e^5 - 105*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 + 6*a^3*c*d^4*e^4 - 35*a
^4*d^2*e^6)*x^2 + 8*(a^3*c*d^5*e^3 - 7*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((2
*exp(1)^4*a*exp(2)-2*exp(1)^6*a)/2/d^4/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)
)*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))+(-5
*a^4*exp(2)^4-8*exp(1)^2*a^4*exp(2)^3-16*exp(1)^4*a^4*exp(2)^2-64*exp(1)^6*
a^4*exp(2)+128*exp(1)^8*a^4-20*c*d^2*a^3*exp(2)^3-30*c^2*d^4*a^2*exp(2)^2+2
4*c^2*d^4*exp(1)^2*a^2*exp(2)-20*c^3*d^6*a*exp(2)+16*c^3*d^6*exp(1)^2*a-5*c
^4*d^8)/64/d^4/exp(1)^3/a^3/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))+(-15*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^
4*exp(2)^4-24*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^3-48*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^2+192*exp(1)^6*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*
exp(2)-60*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
```

d*exp(1))*x)^7*a^3*exp(2)^3-90*c^2*d^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2*exp(2)^2+72*c^2*d^4*exp(1)^2*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2*e
xp(2)-60*c^3*d^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^7*a*exp(2)+48*c^3*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a-15*c^4*d^8*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7+384*d*exp(1)^5*sqrt
(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^6*a^4*exp(2)-384*d*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4+55*d*exp(1)*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*ex
p(2)^4+88*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^5*a^5*exp(2)^3+48*d*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^2-576*d*exp(1)^7*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^
5*exp(2)+220*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)^3+384*c*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)-384*c*d^
3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^5*a^4+330*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)^2-264*c^2*d^5*exp(1)^3*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)
+220*c^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^5*a^2*exp(2)-176*c^3*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2+55*c^4*d^9*exp(1)*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a-38
4*d^2*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)^2-768*d^2*exp(1)^6*sqrt(c*d*exp(1
))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^
4*a^5*exp(2)+1152*d^2*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5+384*c*d^4*exp(1)^4*sqrt(
c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*ex
p(1))*x)^4*a^4*exp(2)-384*c*d^4*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+
a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4+768*c^2*d^6*ex
p(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^4*a^3-73*d^2*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^4-40*d^2*exp(1)^4*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*e
xp(2)^3+48*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^2+576*d^2*exp(1)^8*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)-292*c*d^4*ex
p(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)^3*a^5*exp(2)^3-768*c*d^4*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)^2+768*c*d^4*exp(1)^8*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5-438
*c^2*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^3*a^4*exp(2)^2-1416*c^2*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)-384*c^2*d^6*e
xp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)^3*a^4-292*c^3*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3*exp(2)-688*c^3*d^8*exp(1)^4*(sqrt(a*d*ex
p(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3-73*c^4*d^
10*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*ex
p(1))*x)^3*a^2+384*d^3*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6*exp(2)^3+384*d^3*exp(1)
^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^2*a^6*exp(2)^2+384*d^3*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6*exp(2)
-1152*d^3*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d

```
*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6+1152*c*d^5*exp(1)^3*sqrt(c*d*exp(1))
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*
a^5*exp(2)^2+1024*c*d^5*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^5*exp(2)+256*c*d^5*exp(1
)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^2*a^5+1152*c^2*d^7*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*ex
p(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4*exp(2)+64
0*c^2*d^7*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d
*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4+384*c^3*d^9*exp(1)^3*sqrt(c*d*exp(1)
)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2
*a^3-15*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)*a^7*exp(2)^4-24*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)^3-48*d^3*exp(1)^7*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*e
xp(2)^2-192*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)*a^7*exp(2)-60*c*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)^3-384*c*d^5*exp(
1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)*a^6*exp(2)^2-384*c*d^5*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)-384*c*d^5*exp(1)^9*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6-90*c^2*d^7*ex
p(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)*a^5*exp(2)^2-696*c^2*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)-384*c^2*d^7*exp(1)^7*(sqrt(a*
d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5-60*c^3*
d^9*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)*a^4*exp(2)-336*c^3*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4-15*c^4*d^11*exp(1)^3*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^3+384*d^4*ex
p(1)^10*sqrt(c*d*exp(1))*a^7+128*c*d^6*exp(1)^6*sqrt(c*d*exp(1))*a^6*exp(2)
+128*c*d^6*exp(1)^8*sqrt(c*d*exp(1))*a^6+128*c^2*d^8*exp(1)^6*sqrt(c*d*exp(
1))*a^5)/384/d^4/exp(1)^3/a^3/((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a)^4)
```

maple [B] time = 0.02, size = 1494, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/x^5/(e*x+d),x)
```

```
[Out] 19/64/e^2/d/a^3/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^2+1/32/e*d^2/a^
2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3-43/64/d^2*e/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2)*x-93/64/d^4*e^3*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*x+13/24/d^3/a/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-17/32/d/a^2*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+5/128*d^4/a^3/e^3/(a*d*e)^(1/2)*l
n((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2))/x)*c^4-5/64*d^2/a^4/e^3*c^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*x+5/64*d/a^4/e^4/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^3+5/24/d/a^2
/e^2/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-7/16/e/d^2/a^2/x^2*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-1/d^5*e^4*((x+d/e)^2*c*d*e+(a*e^2-c*d^
2)*(x+d/e))^(1/2)-29/64/d^5*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/4
/d^2/a/e/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/2/d^5*e^6*ln((c*d*e*
x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
))/(c*d*e)^(1/2)*a-1/2/d^5*e^6*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a+1/2
/d^3*e^4*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c+93/64/d^5*e^2/a/x*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-39/32/d^3*e^2/a*(c*d*e*x^2+a*d*e+(a*e^2+c*
```

```
d^2)*x)^(1/2)*c-1/2/d^3*e^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c-35/128/d^4*e^5*a/(
a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2))/x)+5/32/d^2*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d
^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-29/32/d
^4*e/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+43/64/d^3/a^2/x*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c+3/64*e/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2
+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2-7
/32/e^2*d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-19/64/e/a^3*c^3*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-5/32/a^3/e^3/x^2*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(3/2)*c^2-5/64*d^3/a^4/e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)*c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{(ex + d)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{x^5\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^5\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**5*(d + e*x)), x)
```

3.280 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=449

$$-\frac{\left(-35a^3e^6-6cdex\left(-7a^2e^4-6acd^2e^2+21c^2d^4\right)-33a^2cd^2e^4-21ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+c\right.}{960c^3d^3e^4}$$

Rubi [A] time = 0.57, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 832, 779, 612, 621, 206}

$$-\frac{\left(-6cdex\left(-7a^2e^4-6acd^2e^2+21c^2d^4\right)-33a^2cd^2e^4-35a^3e^6-21ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{960c^3d^3e^4}+\frac{\left(-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8+21c^4d^8\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^4d^4e^5}-\frac{\left(15a^2cd^2e^4+7a^3e^6+21ac^2d^4e^2+21c^3d^6\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024c^{9/2}d^{9/2}e^{11/2}}+\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}+\frac{1}{20}x^2\left(\frac{a}{cd}-\frac{3d}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((21*c^4*d^8 - 6*a^2*c^2*d^4*e^4 - 8*a^3*c*d^2*e^6 - 7*a^4*e^8)*(c*d^2 + a*
e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*c^4*d^4*
e^5) + ((a/(c*d) - (3*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/20 + (x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*e) - ((10
5*c^3*d^6 - 21*a*c^2*d^4*e^2 - 33*a^2*c*d^2*e^4 - 35*a^3*e^6 - 6*c*d*e*(21*
c^2*d^4 - 6*a*c*d^2*e^2 - 7*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(960*c^3*d^3*e^4) - ((c*d^2 - a*e^2)^3*(21*c^3*d^6 + 21*a*c^2*d
^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/
(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
1024*c^(9/2)*d^(9/2)*e^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \int x^3(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$$

$$= \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e} + \frac{\int x^2\left(-3acd^2e-\frac{3}{2}cd\left(3cd^2-ae\right.\right.}{}$$

$$= \frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} + \frac{x^3\left(ade+\left(cd^2+\right.\right.}{6}$$

$$= \frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} + \frac{x^3\left(ade+\left(cd^2+\right.\right.}{6}$$

$$= \frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+}}{512c^4d^4e^5}$$

$$= \frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+}}{512c^4d^4e^5}$$

$$= \frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+}}{512c^4d^4e^5}$$

Mathematica [A] time = 2.29, size = 425, normalized size = 0.95

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(-105a^5e^{10}+5a^4cde^8(11d+14ex)+2a^3c^2d^2e^6\left(27d^2-16dex-28e^2x^2\right)+6a^2c^3d^3e^4\left(13d^3-6d^2ex+4de^2x^2+8e^3x^3\right)+ac^4d^4e^2\left(-525d^4+336d^3ex-264d^2e^2x^2+224de^3x^3+1664e^4x^4\right)+c^5d^5\left(315d^5-210d^4ex+168d^3e^2x^2-144d^2e^3x^3+128de^4x^4+1280e^5x^5\right)\right)-\frac{15\sqrt{cd}\left(cd^2-ae^2\right)^{5/2}\left(7a^3e^6+15a^2cd^2e^4+21ac^2d^4e^2+21c^3d^6\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right)}{7680c^{9/2}d^{9/2}e^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^5*e^10 + 5*
a^4*c*d*e^8*(11*d + 14*e*x) + 2*a^3*c^2*d^2*e^6*(27*d^2 - 16*d*e*x - 28*e^2
*x^2) + 6*a^2*c^3*d^3*e^4*(13*d^3 - 6*d^2*e*x + 4*d*e^2*x^2 + 8*e^3*x^3) +
a*c^4*d^4*e^2*(-525*d^4 + 336*d^3*e*x - 264*d^2*e^2*x^2 + 224*d*e^3*x^3 + 1
664*e^4*x^4) + c^5*d^5*(315*d^5 - 210*d^4*e*x + 168*d^3*e^2*x^2 - 144*d^2*e
^3*x^3 + 128*d*e^4*x^4 + 1280*e^5*x^5)) - (15*Sqrt[c*d]*(c*d^2 - a*e^2)^(5/
```

```
2)*(21*c^3*d^6 + 21*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcSinh[(
Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]
)/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]))))/(7680*c^(9/2)
*d^(9/2)*e^(11/2))
```

IntegrateAlgebraic [F] time = 180.43, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.51, size = 1044, normalized size = 2.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 4*a^3
*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11*e -
525*a*c^5*d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*c^2*d^
3*e^9 - 105*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4 - 16*(9
*c^6*d^8*e^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c^6*d^9*e^
3 - 33*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^2 - 2*(105*
c^6*d^10*e^2 - 168*a*c^5*d^8*e^4 + 18*a^2*c^4*d^6*e^6 + 16*a^3*c^3*d^4*e^8
- 35*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5
*d^5*e^6), 1/15360*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^
4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*
sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)) + 2*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11*e - 525*a*c^5*
d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*c^2*d^3*e^9 - 10
5*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4 - 16*(9*c^6*d^8*e
^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c^6*d^9*e^3 - 33*a*c
^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^2 - 2*(105*c^6*d^10*e
^2 - 168*a*c^5*d^8*e^4 + 18*a^2*c^4*d^6*e^6 + 16*a^3*c^3*d^4*e^8 - 35*a^4*c
^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
```

```
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Warning, replacing 0 by ` u`, a substitution variab
le should perhaps be purged.Warning, replacing 0 by ` u`, a substitution va
riable should perhaps be purged.Error: Bad Argument Type
```

maple [B] time = 0.03, size = 1883, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d),x)
```

```
[Out] 1/8*d^6/e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-43/512/e^5*c*d^
6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+43/96/e^3*d^2*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(3/2)*x+21/512/e^3*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*a-7/256*e/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+29/192/c^2/d*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^2-75/512/e^3*c*d^6*ln((c*d*e*x+1/2
*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c
*d*e)^(1/2)*a+1/16*d^2*e*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16*
d^6/e^3*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2
*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/32*e^2/c^2*a^3/d*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+7/96*e/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(3/2)*x*a^2-7/256*e^4/c^3/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2)*x*a^4+7/1024*e^7/c^4/d^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/
2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^6-3/512*e^5/c^3
/d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^5-17/256*e/c*d^2*ln((c*d*e*x+1/2*a*e^2+1/
2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/
2)*a^3-3/16*d^4/e*a^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4*d^5/e^4*c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/16*d^8/e^5*c^2*ln((1/2*a*
e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(
x+d/e))^(1/2))/(c*d*e)^(1/2)-3/128/c*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*x*a^2+1/4/e^2*a*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+11/48/e/c
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a-3/1024*e^3/c^2*ln((c*d*e*x+1/2
*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c
*d*e)^(1/2)*a^4+177/1024/e*d^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/
2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2+65/192/e^2/c*
d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a+7/192*e^2/c^3/d^3*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(3/2)*a^3-43/256/e^4*c*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)*x-7/512*e^5/c^4/d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^
5-15/512*e^3/c^3/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4+29/256/e/c
*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+43/1024/e^5*c^2*d^8*ln((c*
d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/(c*d*e)^(1/2)-7/60/e/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)
*a+1/6/e^2*x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/c/d-1/3*d^3/e^4*((x+d/
```

```
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-19/60/e^3/c*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(5/2)+43/192/e^4*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-1/4
*d^3/e^2*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/8*d^2/e*a^2/c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(\left(d + e x\right) \left(a e + c d x\right)\right)^{\frac{3}{2}}}{d + e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x), x)
```

3.281 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=352

$$\frac{\left(-15a^2e^4-6cdex\left(7cd^2-3ae^2\right)-12acd^2e^2+35c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{240c^2d^2e^3}+\frac{\left(3a^2e^4+6acd^2e^2+7c^2d^4\right.}{}$$

Rubi [A] time = 0.33, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {851, 832, 779, 612, 621, 206}

$$-\frac{\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^3d^3e^4}+\frac{\left(-15a^2e^4-6cdex\left(7cd^2-3ae^2\right)-12acd^2e^2+35c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{240c^2d^2e^3}+\frac{\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256c^{7/2}d^{7/2}e^{9/2}}+\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -((c*d^2 - a*e^2)*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(c*d^2 + a*e^2 +
2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*c^3*d^3*e^4) +
 (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*e) + ((35*c^2*d^4 -
 12*a*c*d^2*e^2 - 15*a^2*e^4 - 6*c*d*e*(7*c*d^2 - 3*a*e^2)*x)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(240*c^2*d^2*e^3) + ((c*d^2 - a*e^2)^3*(7
*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(
2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2
56*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int x^2(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$$

$$=\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5e}+\frac{\int x\left(-2acd^2e-\frac{1}{2}cd\left(7cd^2-\right.\right.}{}$$

$$=\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5e}+\frac{\left(35c^2d^4-12acd^2e^2-15a^2e^4\right.}{}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade}}{128c^3d^3e^4}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade}}{128c^3d^3e^4}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade}}{128c^3d^3e^4}$$

Mathematica [A] time = 2.77, size = 497, normalized size = 1.41

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\frac{5\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(8c^3d^3e^3\sqrt{cd}\sqrt{cd^2-ae^2}(ae+cdx)^3\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}-cd\left(cd^2-ae^2\right)\left(-3c^{5/2}d^{5/2}\sqrt{e}\left(cd^2-ae^2\right)^2\sqrt{ae+cdx}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-2e^2(cd)^{5/2}\sqrt{cd^2-ae^2}(ae+cdx)^2\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}+3e(cd)^{5/2}\left(cd^2-ae^2\right)^{3/2}(ae+cdx)\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\right)\right)}{\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-48c^4d^4e^3(d+ex)\left(5ae^2+7cd^2\right)(ae+cdx)^3}{384c^5d^5e^4(ae+cdx)}+x(d+ex)(ae+cdx)^2\right)}{5cde}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(x*(a*e + c*d*x)^2*(d + e*x) + (-48*c^4*d^4*
e^3*(7*c*d^2 + 5*a*e^2)*(a*e + c*d*x)^3*(d + e*x) + (5*(7*c^2*d^4 + 6*a*c*d
^2*e^2 + 3*a^2*e^4)*(8*c^3*d^3*Sqrt[c*d]*e^3*Sqrt[c*d^2 - a*e^2]*(a*e + c*d
*x)^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - c*d*(c*d^2 - a*e^2)*(3*(c*d)^
(5/2)*e*(c*d^2 - a*e^2)^(3/2)*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a
*e^2)] - 2*(c*d)^(5/2)*e^2*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x)^2*Sqrt[(c*d*(d
 + e*x))/(c*d^2 - a*e^2)] - 3*c^(5/2)*d^(5/2)*Sqrt[e]*(c*d^2 - a*e^2)^2*Sqr
t[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*
d]*Sqrt[c*d^2 - a*e^2])])))/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e
*x))/(c*d^2 - a*e^2)]))/(384*c^5*d^5*e^4*(a*e + c*d*x))))/(5*c*d*e)
```

IntegrateAlgebraic [F] time = 181.18, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.46, size = 846, normalized size = 2.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2
*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2
+ c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3
)*x) - 4*(384*c^5*d^5*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36*a^2*
c^3*d^5*e^5 - 30*a^3*c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 11*a*
c^4*d^4*e^6)*x^3 - 8*(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)
*x^2 + 2*(35*c^5*d^8*e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 15*a^3*c^
2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5), -
1/3840*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2*d
^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c
^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(384*c^5*d^5
*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36*a^2*c^3*d^5*e^5 - 30*a^3*
c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 11*a*c^4*d^4*e^6)*x^3 - 8*
(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*x^2 + 2*(35*c^5*d^8*
e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 15*a^3*c^2*d^2*e^8)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
```

```
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Evaluation time: 0.42Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1560, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d),x)
```

```
[Out] -21/128*d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-3/32/c*d*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*a^2+9/128/e^4*c*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)-1/4/e/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a-3/64/e^2*d^3*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+1/5/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
5/2)/c/d-3/8/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/8*d*a^2/c*((
x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/16*d^3*a^2*ln((1/2*a*e^2-1/2*
c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^
(1/2))/(c*d*e)^(1/2)-1/8*d^5/e^4*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^
(1/2)-1/16*d*e^2*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)
+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/16*d^5/e^2*
a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(
a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64*e^3/c^2/d^2*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*x*a^3-3/256*e^6/c^3/d^3*ln((c*d*e*x+1/2*a*e^2+1/2*
c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)
*a^5+3/256*e^4/c^2/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4+9/128*e^2/c*d*ln((c*d*e
*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2))/(c*d*e)^(1/2)*a^3+33/256/e^2*c*d^5*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*
d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+1/16*d^
7/e^4*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c
*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/16*e/c^2/d^2*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^2+9/64/e^3*c*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)*x+3/128*e^4/c^3/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^
4-1/8/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a+3/64*e/c*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2+1/3*d^2/e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(3/2)-3/16/e^3*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/4*d
^2/e*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/4*d^4/e^3*c*((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/64*e^2/c^2/d*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2)*a^3-15/64/e*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*x*a-9/256/e^4*c^2*d^7*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
[Out] Integral(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x), x)
```

3.282 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=295

$$-\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2c\right.}{64c^2d^2e}$$

Rubi [A] time = 0.28, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {794, 664, 612, 621, 206}

$$\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64c^2d^2e^3}-\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4cde(d+ex)}-\frac{1}{24}\left(\frac{3a}{cd}+\frac{5d}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)*(5*c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c^2*d^2*e^3) - (((3*a)/(c*d) + (5*d)
/e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/24 + (a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2)/(4*c*d*e*(d + e*x)) - ((c*d^2 - a*e^2)^3*(5*c*
d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(128*c^(5/2)*d^(5/2)*e^(7
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
```

```
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4cde(d+ex)} + \frac{1}{8}\left(-\frac{5d}{e}-\frac{3ae}{cd}\right)\int \frac{\left(ade+\left(cd^2+ae\right.\right.}{d+}$$

$$= -\frac{1}{24}\left(\frac{3a}{cd}+\frac{5d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} + \frac{\left(ade+\left(cd^2+ae^2\right)\right.}{4cde(d+}$$

$$= \frac{\left(cd^2-ae^2\right)\left(5cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+}}{64c^2d^2e^3}$$

$$= \frac{\left(cd^2-ae^2\right)\left(5cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+}}{64c^2d^2e^3}$$

$$= \frac{\left(cd^2-ae^2\right)\left(5cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+}}{64c^2d^2e^3}$$

Mathematica [A] time = 1.24, size = 276, normalized size = 0.94

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(-9a^3e^6+3a^2cde^4(3d+2ex)+ac^2d^2e^2\left(-31d^2+20dex+72e^2x^2\right)+c^3d^3\left(15d^3-10d^2ex+8de^2x^2+48e^3x^3\right)\right)-\frac{3\sqrt{cd}\left(cd^2-ae^2\right)^{5/2}\left(3ae^2+5cd^2\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right)}{192c^{5/2}d^{5/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-9*a^3*e^6 + 3*a^2
*c*d*e^4*(3*d + 2*e*x) + a*c^2*d^2*e^2*(-31*d^2 + 20*d*e*x + 72*e^2*x^2) +
c^3*d^3*(15*d^3 - 10*d^2*e*x + 8*d*e^2*x^2 + 48*e^3*x^3)) - (3*Sqrt[c*d]*(c
*d^2 - a*e^2)^(5/2)*(5*c*d^2 + 3*a*e^2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sq
rt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[a*e + c*d*x]*Sqrt[
(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(192*c^(5/2)*d^(5/2)*e^(7/2))
```

IntegrateAlgebraic [F] time = 180.77, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e
*x),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.46, size = 676, normalized size = 2.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/768*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*
e^6 - 3*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*
d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4*e^4
*x^3 + 15*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e^7
+ 8*(c^4*d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3*d^4*e
^4 - 3*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
3*d^3*e^4), 1/384*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*
a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x
^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^3 + 15
*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e^7 + 8*(c^4*
d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3*d^4*e^4 - 3*a^
2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4
)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1279, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d),x)
```

```
[Out] 1/4/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/8/c/d*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(3/2)*a+1/8/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-3/
32*e^2/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2+3/16*d*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a-3/32/e^2*c*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)*x-3/64*e^3/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3
+3/64*e/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+3/64/e*d^2*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a-3/64/e^3*c*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2)+3/128*e^5/c^2/d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2
```

```
)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4-3/32*e^3/c*ln(
(c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2))/(c*d*e)^(1/2)*a^3+9/64*e*d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*
d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-3/32/
e*c*d^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+3/128/e^3*c^2*d^6*ln((c*d*e*x+1/2*a*e^
2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)
^(1/2)-1/3*d/e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-1/4*d*a*((x+
d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/8*e*a^2/c*((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2)+1/16*e^3*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c
*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^
(1/2)-3/16*d^2*e*a^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16*d^4/e*a*c*
ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4*d^3/e^2*c*((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(1/2)*x+1/8*d^4/e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d
/e))^(1/2)-1/16*d^6/e^3*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^
(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Integral(x*((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x), x)
```

3.283 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=201

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}+\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2\right.$$

Rubi [A] time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {664, 612, 621, 206}

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}+\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/8 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) +
((c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(16*c^(3/2)*d^(3/2)*e
^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int \sqrt{ade+\left(cd\right.}}{2e^2}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\right.}{}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\right.}{}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\right.}{}$$

Mathematica [A] time = 0.67, size = 264, normalized size = 1.31

$$\frac{\sqrt{c}\sqrt{d}\left(3\left(cd^2-ae^2\right)^{7/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(-3a^3e^5-a^2cde^3(8d+17ex)+ac^2d^2e\left(3d^2-10dex-22e^2x^2\right)+c^3d^3x\left(3d^2-2dex-8e^2x^2\right)\right)\right)}{24e^{5/2}(cd)^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(-3*a^3*e^5
 - a^2*c*d*e^3*(8*d + 17*e*x) + a*c^2*d^2*e*(3*d^2 - 10*d*e*x - 22*e^2*x^2)
 + c^3*d^3*x*(3*d^2 - 2*d*e*x - 8*e^2*x^2))) + 3*(c*d^2 - a*e^2)^(7/2)*Sqrt
[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d
]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(24*(c*d)^(
5/2)*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [B] time = 0.23, size = 13228, normalized size = 65.81

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),
x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.43, size = 532, normalized size = 2.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*a*c^
2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/48*(3*(c^3*d^6 - 3*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(
```

```
c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(8*c^3*d^3*
e^3*x^2 - 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 +
7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e
^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Error: Bad Argument Type
```

maple [B] time = 0.01, size = 566, normalized size = 2.82

$$-\frac{a^{3} e^{4} \ln\left(\frac{\frac{a e^{2}}{2}-\frac{c d^{2}}{2}+\left(x+\frac{d}{e}\right) c d e}{\sqrt{c d e}}+\sqrt{\left(x+\frac{d}{e}\right)^{2} c d e+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{16 \sqrt{c d e}\, c d}+\frac{3 a^{2} d e^{2} \ln\left(\frac{\frac{a e^{2}}{2}-\frac{c d^{2}}{2}+\left(x+\frac{d}{e}\right) c d e}{\sqrt{c d e}}+\sqrt{\left(x+\frac{d}{e}\right)^{2} c d e+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{16 \sqrt{c d e}}-\frac{3 a c d^{3} \ln\left(\frac{\frac{a e^{2}}{2}-\frac{c d^{2}}{2}+\left(x+\frac{d}{e}\right) c d e}{\sqrt{c d e}}+\sqrt{\left(x+\frac{d}{e}\right)^{2} c d e+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{16 \sqrt{c d e}}+\frac{c^{2} d^{5} \ln\left(\frac{\frac{a e^{2}}{2}-\frac{c d^{2}}{2}+\left(x+\frac{d}{e}\right) c d e}{\sqrt{c d e}}+\sqrt{\left(x+\frac{d}{e}\right)^{2} c d e+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{16 \sqrt{c d e}\, e^{2}}+\frac{\sqrt{\left(x+\frac{d}{e}\right)^{2} c d e+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\, a e x}{4}-\frac{\sqrt{\left(x+\frac{d}{e}\right)^{2} c d e+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\, c d^{2} x}{4 e}+\frac{\sqrt{\left(x+\frac{d}{e}\right)^{2} c d e+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\, a^{2} e^{2}}{8 c d}-\frac{\sqrt{\left(x+\frac{d}{e}\right)^{2} c d e+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\, c d^{3}}{8 e^{2}}+\frac{\left(\left(x+\frac{d}{e}\right)^{2} c d e+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/4*e*a*((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/8*e^2*a^2/c/d*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)-1/16*e^4*a^3/c/d*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)
/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
+3/16*e^2*a^2*d*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/16*a*c*d^3*ln((1/2
*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/4/e*c*d^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*
(x+d/e))^(1/2)*x-1/8/e^2*c*d^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2
)+1/16/e^2*c^2*d^5*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x
+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
```

maxima [B] time = 0.50, size = 462, normalized size = 2.30

$$\frac{3 a^{2} c d^{2} \log\left(2 c d x+\frac{c d^{2}}{e}+a e+2 \sqrt{c d e x^{2}+c d^{2} x+a e^{2} x+a d e} \sqrt{\frac{c d}{e}}\right)}{16 \left(\frac{c d}{e}\right)^{\frac{3}{2}}}+\frac{c^{3} d^{6} \log\left(2 c d x+\frac{c d^{2}}{e}+a e+2 \sqrt{c d e x^{2}+c d^{2} x+a e^{2} x+a d e} \sqrt{\frac{c d}{e}}\right)}{16 \left(\frac{c d}{e}\right)^{\frac{3}{2}} e^{4}}-\frac{3 a c^{2} d^{4} \log\left(2 c d x+\frac{c d^{2}}{e}+a e+2 \sqrt{c d e x^{2}+c d^{2} x+a e^{2} x+a d e} \sqrt{\frac{c d}{e}}\right)}{16 \left(\frac{c d}{e}\right)^{\frac{3}{2}} e^{2}}-\frac{a^{3} e^{2} \log\left(2 c d x+\frac{c d^{2}}{e}+a e+2 \sqrt{c d e x^{2}+c d^{2} x+a e^{2} x+a d e} \sqrt{\frac{c d}{e}}\right)}{16 \left(\frac{c d}{e}\right)^{\frac{3}{2}}}-\frac{\sqrt{c d e x^{2}+c d^{2} x+a e^{2} x+a d e}\, c d^{2} x}{4 e}+\frac{1}{4} \sqrt{c d e x^{2}+c d^{2} x+a e^{2} x+a d e}\, a e x-\frac{\sqrt{c d e x^{2}+c d^{2} x+a e^{2} x+a d e}\, c d^{3}}{8 e^{2}}+\frac{\sqrt{c d e x^{2}+c d^{2} x+a e^{2} x+a d e}\, a^{2} e^{2}}{8 c d}+\frac{\left(c d e x^{2}+c d^{2} x+a e^{2} x+a d e\right)^{\frac{3}{2}}}{3 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] 3/16*a^2*c*d^2*log(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e)*sqrt(c*d/e))/(c*d/e)^(3/2) + 1/16*c^3*d^6*log(2*c*d*x + c*d
```

```
^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*sqrt(c*d/e))/((c
*d/e)^(3/2)*e^4) - 3/16*a*c^2*d^4*log(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*sqrt(c*d/e))/((c*d/e)^(3/2)*e^2) - 1/16*
a^3*e^2*log(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e)*sqrt(c*d/e))/(c*d/e)^(3/2) - 1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e)*c*d^2*x/e + 1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a*e*
x - 1/8*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c*d^3/e^2 + 1/8*sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a^2*e^2/(c*d) + 1/3*(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e)^(3/2)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x), x)
```

3.284 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=251

$$-a^{3/2}\sqrt{d}\,e^{3/2}\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{}{2\sqrt{c}\sqrt{d}\sqrt{e}}\right.}{8\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Rubi [A] time = 0.28, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 814, 843, 621, 206, 724}

$$-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{c}\sqrt{d}\,e^{3/2}}-a^{3/2}\sqrt{d}\,e^{3/2}\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)+\frac{\left(5ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

```
[Out] ((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(4*e) - ((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])])/(8*Sqrt[c]*Sqrt[d]*e^(3/2)) - a^(3/2)*Sqrt[d]*e^(3/2)*ArcTanh[(2*a*
d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}\,dx$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}-\frac{\int \frac{-4a^2cd^2e^3+\frac{1}{2}cd\left(c^2 \cdots\right.}{x\sqrt{ade+\left(cd^2 \cdots\right.}}}{4 \cdots}$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}+\left(a^2de^2\right)\int \frac{\cdots}{x\sqrt{ad\cdots}}$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}-\left(2a^2de^2\right)\text{Subst}\left(\cdots\right.$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}-\frac{\left(c^2d^4-6acd^2e^2-\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.85, size = 275, normalized size = 1.10

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(-\frac{8a^{3/2}\sqrt{d}e^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}-\frac{\sqrt{c}\sqrt{d}\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}+\sqrt{e}\sqrt{ae+cdx}\left(5ae^2+cd(d+2ex)\right)\right)}{4e^{3/2}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(5*a*e^2 + c*d*(d
 + 2*e*x)) - (Sqrt[c]*Sqrt[d]*(c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcSinh
[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]
)])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) -
 (8*a^(3/2)*Sqrt[d]*e^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])]))/Sqrt[d + e*x]))/(4*e^(3/2)*Sqrt[a*e + c*d*x])
```

IntegrateAlgebraic [A] time = 4.80, size = 428, normalized size = 1.71

$$2a^{3/2}\sqrt{d}\,e^{3/2}\tanh^{-1}\left(\frac{x\sqrt{cde}}{\sqrt{a}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)+\frac{\left(3a^2e^4+6acd^2e^2-c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\left(2\sqrt{x(ae^2+cd^2)+ade+cdex^2}-2x\sqrt{cde}\right)}{ae^2+cd^2}\right)}{8\sqrt{c}\sqrt{d}\,e^{3/2}}+\frac{\left(-3a^2e^4\sqrt{cde}-6acd^2e^2\sqrt{cde}+c^2d^4\sqrt{cde}\right)\log\left(a^2e^4+8cdex\sqrt{cde}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2acd^2e^2-4acde^3x+c^2d^4-4c^2d^3ex-8c^2d^2e^2x^2\right)}{16cde^2}+\frac{\left(5ae^2+cd^2+2cdex\right)\sqrt{ade+ae^2x+cd^2x+cdex^2}}{4e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*
x)),x]
```

```
[Out] ((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2])
/(4*e) + ((-(c^2*d^4) + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(Sqrt[c]*Sqrt[d]
*Sqrt[e]*(-2*Sqrt[c*d*e]*x + 2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
)/(c*d^2 + a*e^2)])/(8*Sqrt[c]*Sqrt[d]*e^(3/2)) + 2*a^(3/2)*Sqrt[d]*e^(3/2)
*ArcTanh[(Sqrt[c*d*e]*x)/(Sqrt[a]*Sqrt[d]*Sqrt[e]) - Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]/(Sqrt[a]*Sqrt[d]*Sqrt[e])] + ((c^2*d^4*Sqrt[c*d*e] -
6*a*c*d^2*e^2*Sqrt[c*d*e] - 3*a^2*e^4*Sqrt[c*d*e])*Log[c^2*d^4 - 2*a*c*d^2*
e^2 + a^2*e^4 - 4*c^2*d^3*e*x - 4*a*c*d*e^3*x - 8*c^2*d^2*e^2*x^2 + 8*c*d*e
*Sqrt[c*d*e]*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(16*c*d*e^2)
```

fricas [A] time = 4.40, size = 1327, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/16*(8*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - (c^2*
d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4
*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(c*d*e^2), 1/8*(4*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d^2*e^2 +
 (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^
2*d*e^3)*x)/x^2) + (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^
2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)
) + 2*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(c*d*e^2), 1/16*(16*sqrt(-a*d*e)*a*c*d*e^3*arctan(1/2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - (
c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
 + 4*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(c*d*e^2), 1/8*(8*sqrt(-a*d*e)*a*c*d*e^3*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + (c^2
*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^
2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*x
+ c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*
d*e^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 1130, normalized size = 4.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x/(e*x+d),x)
```

```
[Out] 1/3/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/4/d*a*e^2*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2)*x+1/8/d^2*a^2*e^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2)+5/4*a*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/16/d^2*a^3*e^5/c*
ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2))/(c*d*e)^(1/2)+9/16*a^2*e^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c
*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+9/16*d^2
*a*e*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1/4*d*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2)*x+1/8*d^2*c/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/16*d^4*c^2/e
*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2))/(c*d*e)^(1/2)-d*a^2*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-1/3/d*((x+
d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-1/4/d*a*e^2*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(1/2)*x-1/8/d^2*a^2*e^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2)+1/16/d^2*a^3*e^5/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(
c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3
/16*a^2*e^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2
*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16*d^2*a*e*c*ln((1/2*a
*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*
(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4*d*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e
))^(1/2)*x+1/8*d^2*c/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/16*d
^4*c^2/e*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)(ae+cdx))^{\frac{3}{2}}}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x*(d + e*x)), x)
```

3.285 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=240

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(ae-cdx)}{x}+\frac{\sqrt{c}\,\sqrt{d}\,\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{e}}-\frac{\sqrt{a}\,\sqrt{e}\,(\cdots}{}$$

Rubi [A] time = 0.27, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(ae-cdx)}{x}+\frac{\sqrt{c}\,\sqrt{d}\,\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{e}}-\frac{\sqrt{a}\,\sqrt{e}\,\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

```
[Out] -(((a*e - c*d*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (Sqrt[c]
*Sqrt[d]*(c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*S
qrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[e]) -
 (Sqrt[a]*Sqrt[e]*(3*c*d^2 + a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(
2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
*Sqrt[d])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx = \int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^2}\,dx$$

$$= -\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x} - \frac{1}{2}\int \frac{-ae\left(3cd^2+ae^2\right)-}{x\sqrt{ade+\left(cd^2+\right.}}$$

$$= -\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x} + \frac{1}{2}\left(ae\left(3cd^2+ae^2\right)\right)\int \frac{-}{x}$$

$$= -\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x} - \left(ae\left(3cd^2+ae^2\right)\right)\text{Subst}$$

$$= -\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x} + \frac{\sqrt{c}\sqrt{d}\left(cd^2+3ae^2\right)\tan}{}$$

Mathematica [A] time = 1.20, size = 263, normalized size = 1.10

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{c}\,d\sqrt{cd}\left(3ae^2+cd^2\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-\frac{\sqrt{a}\sqrt{e}\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}+\frac{\sqrt{d}\sqrt{ae+cdx}\,(cdx-ae)}{x}\right)}{\sqrt{d}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[d]*(-(a*e) + c*d*x)*Sqrt[a*e + c*d*x]
)/x + (Sqrt[c]*d*Sqrt[c*d]*(c*d^2 + 3*a*e^2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[e]*Sqrt[c*d^2
 - a*e^2]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (Sqrt[a]*Sqrt[e]*(3*c*d^
2 + a*e^2)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*
x])])/Sqrt[d + e*x]))/(Sqrt[d]*Sqrt[a*e + c*d*x])
```

IntegrateAlgebraic [A] time = 2.29, size = 383, normalized size = 1.60

$$\frac{\left(a^{3/2}e^{5/2}+3\sqrt{a}\,cd^2\sqrt{e}\right)\tanh^{-1}\left(\frac{x\sqrt{cde}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{\sqrt{d}}+\frac{\left(-3ae^2\sqrt{cde}-cd^2\sqrt{cde}\right)\log\left(a^2e^4+8cdex\sqrt{cde}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2acd^2e^2-4acde^3x+c^2d^4-4c^2d^3ex-8c^2d^2e^2x^2\right)}{4e}+\frac{\left(3a\sqrt{c}\sqrt{d}\,e^2+c^{3/2}d^{5/2}\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\left(2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2x\sqrt{cde}\right)}{ae^2+cd^2}\right)}{2\sqrt{e}}+\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\,(cdx-ae)}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d +
e*x)),x]
```

```
[Out] ((-(a*e) + c*d*x)*Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2])/x + ((3*Sqrt
[a]*c*d^2*Sqrt[e] + a^(3/2)*e^(5/2))*ArcTanh[(Sqrt[c*d*e]*x - Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[a]*Sqrt[d]*Sqrt[e])]])/Sqrt[d] + ((c^(
3/2)*d^(5/2) + 3*a*Sqrt[c]*Sqrt[d]*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-
2*Sqrt[c*d*e]*x + 2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]))/(c*d^2 +
a*e^2)]])/(2*Sqrt[e]) + ((-(c*d^2*Sqrt[c*d*e]) - 3*a*e^2*Sqrt[c*d*e])*Log[c^
2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4 - 4*c^2*d^3*e*x - 4*a*c*d*e^3*x - 8*c^2*d^2
*e^2*x^2 + 8*c*d*e*Sqrt[c*d*e]*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]])/(4*e)
```

fricas [A] time = 1.80, size = 1221, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/4*((c*d^2 + 3*a*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (3*c
*d^2 + a*e^2)*sqrt(a*e/d)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e +
(c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/4*(2*(c*d^2
+ 3*a*e^2)*sqrt(-c*d/e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e +
 (c^2*d^3 + a*c*d*e^2)*x)) - (3*c*d^2 + a*e^2)*sqrt(a*e/d)*x*log((8*a^2*d^2
*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(c*d*x - a*e))/x, 1/4*(2*(3*c*d^2 + a*e^2)*sqrt(-a*e/d)*x*arctan(1/2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a
*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + (c*d^2 + 3*a
*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/2*((c*d^2 + 3*a*e^2)*sqrt(-c*d
/e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^
2)*x)) - (3*c*d^2 + a*e^2)*sqrt(-a*e/d)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2
*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1310, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^2/(e*x+d),x)
```

```
[Out] -1/4*e*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/16*d^3*c^2*ln((1
/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d
^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/d*a*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)+1/a/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c+5/4*e*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c-1/2*a^2*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+7/16
*d^3*c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-1/16*e^6/d^3*a^3/c*ln((1/2*a*e^2-1/2*c*
d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1
/2))/(c*d*e)^(1/2)-3/16*e^2*d*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c
*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/
16*e^6/d^3*a^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+27/16*d*a*e^2*ln((c*d*e*x+1/2*a
*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d
*e)^(1/2)*c-3/2*d^2*a*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)
^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+1/3*e/d^2*((x+d/e)^2*c
*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-1/8*d*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x
+d/e))^(1/2)+2/3/d^2*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+17/8*d*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c-1/4*e^3/d^2*a*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*x-3/16/d*a^2*e^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(
1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1/d*c/a*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-1/d^2/a/e/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(5/2)-1/8*e^4/d^3*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/4*e
^3/d^2*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/8*e^4/d^3*a^2/c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/16*e^4/d*a^2*ln((1/2*a*e^2-
1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(1/2))/(c*d*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^2\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d + ex\right) \left(ae + cdx\right)\right)^{\frac{3}{2}}}{x^{2} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**2*(d + e*x)), x)
```

3.286 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=256

$$-\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{a}\,d^{3/2}\sqrt{e}}+c^{3/2}d^{3/2}\sqrt{e}\tanh^{-1}\left(\frac{ae^2+cd^2 \cdots}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+\cdots\right.}}\right.$$

Rubi [A] time = 0.28, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 810, 843, 621, 206, 724}

$$-\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{a}\,d^{3/2}\sqrt{e}}+c^{3/2}d^{3/2}\sqrt{e}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(x\left(ae^2+5cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

```
[Out] -((2*a*d*e + (5*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]])/(4*d*x^2) + c^(3/2)*d^(3/2)*Sqrt[e]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])] -
 ((3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*
x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
)/(8*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
```

```
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3}\,dx$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}-\frac{\int \frac{-\frac{1}{2}ae\left(3c^2d^4+6\right.}{x\sqrt{ade+}}}{}$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}+\left(c^2d^2e\right)\int \frac{}{\sqrt{}}$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}+\left(2c^2d^2e\right)\text{Sub}$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}+c^{3/2}d^{3/2}\sqrt{e}\ \text{ta}$$

Mathematica [A] time = 2.45, size = 285, normalized size = 1.11

$$\frac{\sqrt{ae+cdx}\left(-\frac{\sqrt{d+ex}\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}}+\frac{8e(cd)^{5/2}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{c^{3/2}}-\frac{\sqrt{d}\sqrt{e}(d+ex)\sqrt{ae+cdx}\left(ae(2d+ex)+5cd^2x\right)}{x^2}\right)}{4d^{3/2}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*(-((Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(d + e*x)*(5*c*d^2
*x + a*e*(2*d + e*x)))/x^2) + (8*(c*d)^(5/2)*e*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*
d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c
*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/c^(3/2) - ((3*c^2*d^4 + 6*a*c*d^2*
e^2 - a^2*e^4)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*S
```

```
qrt[e]*Sqrt[d + e*x])])/Sqrt[a]))/(4*d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d
+ e*x)])
```

IntegrateAlgebraic [A] time = 1.89, size = 395, normalized size = 1.54

$$\frac{\left(a^2e^4-6acd^2e^2-3c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}-x\sqrt{cde}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{4\sqrt{a}d^{3/2}\sqrt{e}}-\frac{1}{2}cd\sqrt{cde}\log\left(a^2e^4+8cdex\sqrt{cde}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2acd^2e^2-4acde^3x+c^2d^4-4c^2d^3ex-8c^2d^2e^2x^2\right)-c^{3/2}d^{3/2}\sqrt{e}\tanh^{-1}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}x\sqrt{cde}}{ae^2+cd^2}-\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{ae^2+cd^2}\right)+\frac{\left(-2ade-ae^2x-5cd^2x\right)\sqrt{ade+ae^2x+cd^2x+cdex^2}}{4dx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d +
e*x)),x]
```

```
[Out] ((-2*a*d*e - 5*c*d^2*x - a*e^2*x)*Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^
2])/(4*d*x^2) + ((-3*c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(-(Sqrt[c*d
*e]*x) + Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[a]*Sqrt[d]*Sqrt
[e])])/(4*Sqrt[a]*d^(3/2)*Sqrt[e]) - c^(3/2)*d^(3/2)*Sqrt[e]*ArcTanh[(2*Sqr
t[c]*Sqrt[d]*Sqrt[e]*Sqrt[c*d*e]*x)/(c*d^2 + a*e^2) - (2*Sqrt[c]*Sqrt[d]*Sq
rt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 + a*e^2)] - (c*d*
Sqrt[c*d*e]*Log[c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4 - 4*c^2*d^3*e*x - 4*a*c*d
*e^3*x - 8*c^2*d^2*e^2*x^2 + 8*c*d*e*Sqrt[c*d*e]*x*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2]])/2
```

fricas [A] time = 2.31, size = 1375, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/16*(8*sqrt(c*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*
d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*
x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - (3*c^2*d^4
+ 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 +
6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) - 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), -1/16*(16*sqrt(-c*d*e)*a*c*d^3*e*x^
2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*
e^3)*x)) + (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2
*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^3)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), 1/8*(4*sqrt(c
*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e
^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (3*c^2*d^4 + 6*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*
d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a
^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), -1
/8*(8*sqrt(-c*d*e)*a*c*d^3*e*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*
c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*
e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2
 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^
3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.42Error: Bad Argument Typ
e
```

maple [B] time = 0.02, size = 1604, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^3/(e*x+d),x)
```

```
[Out] 1/16/d^4*e^7*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x
+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/16/d^4*e^7*a^3/
c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(1/2))/(c*d*e)^(1/2)-1/4/d/a^2/e^2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(5/2)*c-3/4*e/d^2*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-3/4*e^2
*d*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-3/8*d^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+
c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2+3/
4/d^3/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/4*e^3/d^2*a*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/4/a^2/e*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(3/2)*x+3/4*d^2/a/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2-1/3/d^3*e
^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/8*e*c*((x+d/e)^2*c*d*e+(
a*e^2-c*d^2)*(x+d/e))^(1/2)+7/8*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c
-5/12*e^2/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/4*d/a^2/e^2*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^2+3/4*d/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)*x*c^2-1/2/d^2/a/e/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/4
/d^3*e^4*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/8/d^4*e^5*a^2/
c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/16/d^2*e^5*a^2*ln((1/2*a*
e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(
x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16*e^3*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c
*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^
(1/2)+1/4/d*e^2*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/16*d^2*
e*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4/d^3*e^4*a*(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2)*x+1/8/d^4*e^5*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)+3/16*e^5/d^2*a^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+17/16*e*d^2*c^2*ln((c*
d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/(c*d*e)^(1/2)-3/16*e^3*a*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1
/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c-1/2*e^2/d*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c+1/8*e^4/d*a^2/(a*d*e)^(1/2)*ln((2*a
*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{x^3\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**3*(d + e*x)), x)
```

3.287 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=211

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8x^2}$$

Rubi [A] time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 806, 720, 724, 206}

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

```
[Out] -((c/(a*e) - e/d^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(8*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
/(3*d*x^3) + ((c*d^2 - a*e^2)^3*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sq
rt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^
(3/2)*d^(5/2)*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^4}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3dx^3}-\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\int \frac{\sqrt{a}}{}}{2ade}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2}-\frac{(a}{}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2}-\frac{(a}{}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2}-\frac{(a}{}$$

Mathematica [A] time = 0.28, size = 188, normalized size = 0.89

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{3\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}\sqrt{ae+cdx}}-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^2e^2\left(8d^2+2dex-3e^2x^2\right)+2acd^2ex(7d+4ex)+3c^2d^4x^2\right)}{x^3}\right)}{24a^{3/2}d^{5/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^2*d^4*x^2 +
 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(8*d^2 + 2*d*e*x - 3*e^2*x^2)))/x^3)
 + (3*(c*d^2 - a*e^2)^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*a^(3/2)*d^(5/2)*
e^(3/2))
```

IntegrateAlgebraic [A] time = 1.83, size = 229, normalized size = 1.09

$$\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\left(-8a^2d^2e^2-2a^2de^3x+3a^2e^4x^2-14acd^3ex-8acd^2e^2x^2-3c^2d^4x^2\right)}{24ad^2ex^3}+\frac{\left(a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6\right)\tanh^{-1}\left(\frac{x\sqrt{cde}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d +
e*x)),x]
```

```
[Out] (Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2]*(-8*a^2*d^2*e^2 - 14*a*c*d^3*e
*x - 2*a^2*d*e^3*x - 3*c^2*d^4*x^2 - 8*a*c*d^2*e^2*x^2 + 3*a^2*e^4*x^2))/(2
4*a*d^2*e*x^3) + ((-(c^3*d^6) + 3*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 + a^3*e^6
```

```
)*ArcTanh[(Sqrt[c*d*e]*x - Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sq
rt[a]*Sqrt[d]*Sqrt[e])])/(8*a^(3/2)*d^(5/2)*e^(3/2))
```

fricas [A] time = 1.55, size = 558, normalized size = 2.64

$$\left[-\frac{3\left(c^{3}d^{6}-3\,ac^{2}d^{4}e^{2}+3\,a^{2}cd^{2}e^{4}-a^{3}e^{6}\right)\sqrt{ade}\,x^{3}\log\left(\frac{8\,a^{2}d^{2}e^{2}+\left(c^{2}d^{4}+6\,acd^{2}e^{2}+a^{2}e^{4}\right)x^{2}-4\,\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\left(2\,ade+\left(cd^{2}+ae^{2}\right)x\right)\sqrt{ade}+8\left(acd^{3}e+a^{2}de^{3}\right)x}{x^{2}}\right)+4\left(8\,a^{3}d^{3}e^{3}+\left(3\,ac^{2}d^{5}e+8\,a^{2}cd^{3}e^{3}-3\,a^{3}de^{5}\right)x^{2}+2\left(7\,a^{2}cd^{4}e^{2}+a^{3}d^{2}e^{4}\right)x\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{96\,a^{2}d^{3}e^{2}x^{3}},\ -\frac{3\left(c^{3}d^{6}-3\,ac^{2}d^{4}e^{2}+3\,a^{2}cd^{2}e^{4}-a^{3}e^{6}\right)\sqrt{-ade}\,x^{3}\arctan\left(\frac{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\left(2\,ade+\left(cd^{2}+ae^{2}\right)x\right)\sqrt{-ade}}{2\left(acd^{2}e^{2}x^{2}+a^{2}d^{2}e^{2}+\left(acd^{3}e+a^{2}de^{3}\right)x\right)}\right)+2\left(8\,a^{3}d^{3}e^{3}+\left(3\,ac^{2}d^{5}e+8\,a^{2}cd^{3}e^{3}-3\,a^{3}de^{5}\right)x^{2}+2\left(7\,a^{2}cd^{4}e^{2}+a^{3}d^{2}e^{4}\right)x\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{48\,a^{2}d^{3}e^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(a*d*
e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 + (3*a*c^2*d
^5*e + 8*a^2*c*d^3*e^3 - 3*a^3*d*e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d^2*e^
4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3), -1/48
*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^
3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*
d*e^3)*x)) + 2*(8*a^3*d^3*e^3 + (3*a*c^2*d^5*e + 8*a^2*c*d^3*e^3 - 3*a^3*d*
e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((2
*exp(1)^2*a^2*exp(2)^2-4*exp(1)^4*a^2*exp(2)+2*exp(1)^6*a^2)/2/d^2/sqrt(-a*
d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*
x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(
1)^3+a*d*exp(1)*exp(2)))-(a^3*exp(2)^3+6*exp(1)^2*a^3*exp(2)^2-24*exp(1)^4*
a^3*exp(2)+16*exp(1)^6*a^3+3*c*d^2*a^2*exp(2)^2+3*c^2*d^4*a*exp(2)-6*c^2*d^
4*exp(1)^2*a+c^3*d^6)/8/d^2/exp(1)/a/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)
))+(-3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)^5*a^3*exp(2)^3+30*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)^2-24*exp(1)^4*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)-9*c*d^2*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2
*exp(2)^2-9*c^2*d^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^5*a*exp(2)-30*c^2*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a-3*c^3*d^6*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5+48*d*exp(1)*sqrt(
c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*ex
p(1))*x)^4*a^3*exp(2)^2-96*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^3*exp(2)+48*d*exp(1
)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^4*a^3+96*c*d^3*exp(1)*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^2*exp(2)+48*c^2*d
^5*exp(1)*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^4*a-8*d*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)^3-48*d*exp(1)^3*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)^
```

```
2+48*d*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^3*a^4*exp(2)-24*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3*exp(2)^2-96*c*d^3*exp(1)^3*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3*
exp(2)+48*c*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)^3*a^3-24*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^2*exp(2)-48*c^2*d^5*exp(1)^3
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*
a^2-8*c^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^3*a+144*d^2*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*
x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4*exp(2)-96*d^2*ex
p(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^2*a^4+48*c*d^4*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^3+3*d^2*exp(
1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)*a^5*exp(2)^3+18*d^2*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)^2-24*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)+9*c*d^4*ex
p(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)*a^4*exp(2)^2-48*c*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*
x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4+9*c^2*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^3*exp(2)-18*c^2*d^6*e
xp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)*a^3+3*c^3*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)*a^2-48*d^3*exp(1)^5*sqrt(c*d*exp(1))*a^5*exp(2)+48
*d^3*exp(1)^7*sqrt(c*d*exp(1))*a^5+16*c*d^5*exp(1)^5*sqrt(c*d*exp(1))*a^4)/
48/d^2/exp(1)/a/((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^2-d*exp(1)*a)^3)
```

maple [B] time = 0.02, size = 1945, normalized size = 9.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^4/(e*x+d),x)
```

```
[Out] -3/16/d*e^4*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/24*d/a^3/e^2*c^3*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/12/d/a^2/e^2/x^2*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(5/2)*c+1/16*d^4/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3-1/8*
d^2/a^2/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3-1/3/e/d^2/a^2/x*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+17/24/d^3*e^2*c/a*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(3/2)*x+3/16/d*e^4*a*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*
e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c+1/16/d^5*
e^8*a^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-1/16/d^5*e^8*a^3/c*ln((1/2*a*e^2-1/2*c
*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(
1/2))/(c*d*e)^(1/2)-1/8/d*e^2*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/
2)+7/12/d^3/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+1/8*d/a*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+5/24/e/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(3/2)*c^2+1/8/d^3*e^4*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/3/d^
4*e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+3/8/d^4*e^3*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/24/a^3/e^3/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(5/2)*c^2-1/8*d^3/a^2/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-1/
24*d^2/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^3-1/3/d^2/a/e/x^3*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/4/d^4*e^5*a*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2)*x-1/8/d^5*e^6*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)+3/16*e^3*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-17/24/d^4*e/a/x*(c*d*e*x^2+a*d*
```

```
e+(a*e^2+c*d^2)*x)^(5/2)+11/24/d^2*e/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3
/2)*c+3/8/d^2*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c-1/16/d^2*e^5*
a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-3/16*e*d^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+
c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2+1/
3/d/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/4/d^4*e^5*a*((x+d/e
)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/8/d^5*e^6*a^2/c*((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/16/d^3*e^6*a^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d
/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*
d*e)^(1/2)-1/4/d^2*e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/
16*d*e^2*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^
2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/16/d^3*e^6*a^2*ln((c*
d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/(c*d*e)^(1/2)-1/16*d*e^2*c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e
)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^4\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d + ex\right)\left(ae + cdx\right)\right)^{\frac{3}{2}}}{x^4\left(d + ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**4*(d + e*x)), x)
```

3.288 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=295

$$-\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}}+\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+\right.}{64a^2d^3}$$

Rubi [A] time = 0.39, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 834, 806, 720, 724, 206}

$$\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64a^2d^3e^2x^2}-\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}}-\frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24x^3}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

```
[Out] ((c*d^2 - a*e^2)*(3*c*d^2 + 5*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*a^2*d^3*e^2*x^2) - (a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*d*x^4) - (((3*c)/(a*e) - (5*e)/d^2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*x^3) - ((c*d^2 - a*e^2)^3*(3*
c*d^2 + 5*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(5/2)*d^(7/2)*
e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx = \int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5}\,dx$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4dx^4} - \frac{\int \frac{\left(-\frac{1}{2}ae\left(3cd^2-5ae^2\right)+acde^2x\right)\sqrt{ade+\left(cd^2+ae^2\right.}}{x^4}}{4ade}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4dx^4} - \frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{24x^3}$$

$$= \frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+}}{64a^2d^3e^2x^2}$$

$$= \frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+}}{64a^2d^3e^2x^2}$$

$$= \frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+}}{64a^2d^3e^2x^2}$$

Mathematica [A] time = 0.30, size = 253, normalized size = 0.86

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{x\left(5ae^2+3cd^2\right)\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^2e^2\left(8d^2+2dex-3e^2x^2\right)+2acd^2ex(7d+4ex)+3c^2d^4x^2\right)-3x^3\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)\right)}{a^{3/2}d^{5/2}e^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}}-48(d+ex)(ae+cdx)^2\right)}{192adex^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-48*(a*e + c*d*x)^2*(d + e*x) + ((3*c*d^2 +
 5*a*e^2)*x*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(3*c^2
*d^4*x^2 + 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(8*d^2 + 2*d*e*x - 3*e^2*x
^2)) - 3*(c*d^2 - a*e^2)^3*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]
*Sqrt[e]*Sqrt[d + e*x])]))/(a^(3/2)*d^(5/2)*e^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[
d + e*x])))/(192*a*d*e*x^4)
```

IntegrateAlgebraic [A] time = 2.19, size = 307, normalized size = 1.04

$$\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\left(-48a^3d^3e^3-8a^3d^2e^4x+10a^3de^5x^2-15a^3e^6x^3-72a^2cd^4e^2x-20a^2cd^3e^3x^2+31a^2cd^2e^4x^3-6ac^2d^5ex^2-9ac^2d^4e^2x^3+9c^3d^6x^3\right)}{192a^2d^3e^2x^4}+\frac{\left(-5a^4e^8+12a^3cd^2e^6-6a^2c^2d^4e^4-4ac^3d^6e^2+3c^4d^8\right)\tanh^{-1}\left(\frac{x\sqrt{cde}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{64a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d +
e*x)),x]
```

```
[Out] (Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2]*(-48*a^3*d^3*e^3 - 72*a^2*c*d^
4*e^2*x - 8*a^3*d^2*e^4*x - 6*a*c^2*d^5*e*x^2 - 20*a^2*c*d^3*e^3*x^2 + 10*a
^3*d*e^5*x^2 + 9*c^3*d^6*x^3 - 9*a*c^2*d^4*e^2*x^3 + 31*a^2*c*d^2*e^4*x^3 -
 15*a^3*e^6*x^3))/(192*a^2*d^3*e^2*x^4) + ((3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6
*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(Sqrt[c*d*e]*x - S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[a]*Sqrt[d]*Sqrt[e])])/(64
*a^(5/2)*d^(7/2)*e^(5/2))
```

fricas [A] time = 8.66, size = 704, normalized size = 2.39

$$\left[-\frac{3\left(3c^4d^8-4ac^3d^6e^2-6a^2c^2d^4e^4+12a^3cd^2e^6-5a^4e^8\right)\sqrt{ade}\,x^4\log\left(\frac{8a^2d^2e^2+\left(c^2d^4+6acd^2e^2+a^2e^4\right)x^2+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}+8\left(acd^3e+a^2de^3\right)x}{x^2}\right)+4\left(48a^4d^4e^4-\left(9ac^3d^7e-9a^2c^2d^5e^3+31a^3cd^3e^5-15a^4de^7\right)x^3+2\left(3a^2c^2d^6e^2+10a^3cd^4e^4-5a^4d^2e^6\right)x^2+8\left(9a^3cd^5e^3+a^4d^3e^5\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{768a^3d^4e^3x^4},\ \frac{3\left(3c^4d^8-4ac^3d^6e^2-6a^2c^2d^4e^4+12a^3cd^2e^6-5a^4e^8\right)\sqrt{-ade}\,x^4\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{-ade}}{2\left(acd^2e^2x^2+a^2d^2e^2+\left(acd^3e+a^2de^3\right)x\right)}\right)-2\left(48a^4d^4e^4-\left(9ac^3d^7e-9a^2c^2d^5e^3+31a^3cd^3e^5-15a^4de^7\right)x^3+2\left(3a^2c^2d^6e^2+10a^3cd^4e^4-5a^4d^2e^6\right)x^2+8\left(9a^3cd^5e^3+a^4d^3e^5\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{384a^3d^4e^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*
e^6 - 5*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(
48*a^4*d^4*e^4 - (9*a*c^3*d^7*e - 9*a^2*c^2*d^5*e^3 + 31*a^3*c*d^3*e^5 - 15
*a^4*d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 10*a^3*c*d^4*e^4 - 5*a^4*d^2*e^6)*
x^2 + 8*(9*a^3*c*d^5*e^3 + a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(a^3*d^4*e^3*x^4), 1/384*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a
^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-a*d*e)*x^4*arctan(1/2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) -
2*(48*a^4*d^4*e^4 - (9*a*c^3*d^7*e - 9*a^2*c^2*d^5*e^3 + 31*a^3*c*d^3*e^5 -
 15*a^4*d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 10*a^3*c*d^4*e^4 - 5*a^4*d^2*e^
6)*x^2 + 8*(9*a^3*c*d^5*e^3 + a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x))/(a^3*d^4*e^3*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-
2*exp(1)^3*a^2*exp(2)^2+4*exp(1)^5*a^2*exp(2)-2*exp(1)^7*a^2)/2/d^3/sqrt(-a
*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp
(1)^3+a*d*exp(1)*exp(2)))+(3*a^4*exp(2)^4+8*exp(1)^2*a^4*exp(2)^3+48*exp(1)
^4*a^4*exp(2)^2-192*exp(1)^6*a^4*exp(2)+128*exp(1)^8*a^4+12*c*d^2*a^3*exp(2
)^3+18*c^2*d^4*a^2*exp(2)^2-24*c^2*d^4*exp(1)^2*a^2*exp(2)+12*c^3*d^6*a*exp
(2)-16*c^3*d^6*exp(1)^2*a+3*c^4*d^8)/64/d^3/exp(1)^2/a^2/2/sqrt(-a*d*exp(1)
)*atan((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)/sqrt(-a*d*exp(1)))+(9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
```

```
))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^4+24*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^3-240*exp(1)^4*
(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a
^4*exp(2)^2+192*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)+36*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3*exp(2)^3+54*c^2*d^4*(sqrt(a*
d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2*exp(2
)^2-72*c^2*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^7*a^2*exp(2)+36*c^3*d^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a*exp(2)-48*c^3*d^6*exp(1)^2*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a+9
*c^4*d^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^7-384*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x
+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4*exp(2)^2+768*d*exp(1)^5*sqrt(c*d
*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^6*a^4*exp(2)-384*d*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4+384*c^2*d^5*exp(1)^3*
sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^6*a^2-33*d*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^4+40*d*exp(1)^3*(sqrt(a*d*exp(1)+
a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^3+624*d
*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)^5*a^5*exp(2)^2-576*d*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)-132*c*d^3*exp(1)*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)^3
-384*c*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^5*a^4*exp(2)^2+768*c*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)-384*c*d^3*exp(
1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^5*a^4-198*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)^2-888*c^2*d^5*exp(1)^3*(sqrt(a*d*exp(1
)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)-132*c
^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^5*a^2*exp(2)-464*c^3*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2-33*c^4*d^9*exp(1)*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a+384*d^2*
exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)^3+384*d^2*exp(1)^4*sqrt(c*d*exp(1))*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*
exp(2)^2-1920*d^2*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)+1152*d^2*exp(1)^8*sqr
t(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^4*a^5+1152*c*d^4*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4*exp(2)^2+768*c*d^4
*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^4*a^4*exp(2)-384*c*d^4*exp(1)^6*sqrt(c*d*exp(1))*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4
+1152*c^2*d^6*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x
+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^3*exp(2)+384*c^2*d^6*exp(1)^4*sqrt
(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^4*a^3+384*c^3*d^8*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^2-33*d^2*exp(1)^2*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6
*exp(2)^4-88*d^2*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^3-528*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^2+576*d^2*
exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^3*a^6*exp(2)-132*c*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)^3-768*c*d^4*exp(1)^4*(sqrt(
```

```
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp
(2)^2-768*c*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)+768*c*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5-198*c^2*d^6*exp(1)
^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^
3*a^4*exp(2)^2-1272*c^2*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)-384*c^2*d^6*exp(1)^6*(sqrt(a
*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4-132*
c^3*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^3*a^3*exp(2)-592*c^3*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3-33*c^4*d^10*exp(1)^2*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^2+
1536*d^3*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*
x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6*exp(2)-1152*d^3*exp(1)^9*sqrt(c*d*exp
(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^2*a^6+768*c*d^5*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^5*exp(2)+256*c*d^5*exp(1)^7*sq
rt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d
*exp(1))*x)^2*a^5+768*c^2*d^7*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*
x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^4+9*d^3*exp(1)^3*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*
exp(2)^4+24*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)*a^7*exp(2)^3+144*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)^2-192*d^3*exp(1
)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)
*a^7*exp(2)+36*c*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)^3-384*c*d^5*exp(1)^9*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6+54*c^2*d^7*exp(
1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)*a^5*exp(2)^2-72*c^2*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*
x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)-384*c^2*d^7*exp(1)^7*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5+36*c^3*d^9
*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)*a^4*exp(2)-48*c^3*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4+9*c^4*d^11*exp(1)^3*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^3-384*d^4*exp(1)^
8*sqrt(c*d*exp(1))*a^7*exp(2)+384*d^4*exp(1)^10*sqrt(c*d*exp(1))*a^7+128*c*
d^6*exp(1)^8*sqrt(c*d*exp(1))*a^6)/384/d^3/exp(1)^2/a^2/((sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a)^4)
```

maple [B] time = 0.03, size = 2427, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^5/(e*x+d),x)
```

```
[Out] -3/64/d*e^2/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2-3/32/d*e^4*a/(a
*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/x)*c-133/192/d^4*e^3*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(3/2)*x-3/16/d^2*e^5*a*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)
+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c+3/64*d^3/a^3/e^2*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^4+1/8/d/a^2/e^2/x^3*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+1/64*d^2/a^4/e^3*c^4*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(3/2)*x-1/64*d/a^4/e^4/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)
*c^3-3/128*d^5/a^2/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4+1/16/d^6*e^9*a^3/c*ln
((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-
c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16/d^2*e^5*a*c*ln((1/2*a*e^2-1/2*c*d
^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/
```

```
2))/(c*d*e)^(1/2)-1/16/d^6*e^9*a^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*
e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-13/48/e/d^2
/a^2/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+19/192/e^2/d/a^3/x*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2-91/192/d^2*e/a^2*c^2*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*x+1/8/d^2*e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2)-1/16*e^3*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/
2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+11/24/d^3/a
/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/32*e/a*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2)*c^2-29/96/d/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*
c^2+1/16*e^3*c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-5/64/d^4*e^5*a*(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2)-3/32/d^2*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*c-1/3/d^5*e^4*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-23/64/d^5*e^4
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+3/16/d^4*e^7*a^2*ln((c*d*e*x+1/2*a
*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d
*e)^(1/2)+133/192/d^5*e^2/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+3/64*
d*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2-1/32/a^3/e^3/x^2*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(5/2)*c^2+3/64*d^4/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)*c^4+1/64*d^3/a^4/e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^4-1/4/d
^2/a/e/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-53/96/d^3*e^2/a*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-21/64/d^3*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)*x*c+5/128/d^3*e^6*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*
x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-59/96/d^4*e/a
/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+91/192/d^3/a^2/x*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(5/2)*c+1/4/d^5*e^6*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2)*x+1/8/d^6*e^7*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/4/d^5
*e^6*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/8/d^6*e^7*a^2/c*((
x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/16/d^4*e^7*a^2*ln((1/2*a*e^2-
1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(1/2))/(c*d*e)^(1/2)+1/4/d^3*e^4*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(1/2)*x+1/32/e*d^2/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-5/96
/e^2*d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^3-19/192/e/a^3*c^3*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/32*d^3/a/(a*d*e)^(1/2)*ln((2*a*d*e
+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x
)*c^3+5/64*d/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^5\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{x^5\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**5/(e*x+d),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**5*(d + e*x)), x)
```

3.289 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx$

Optimal. Leaf size=395

$$\frac{\left(-35a^2e^4+12acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{240a^2d^3e^2x^3}+\frac{\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\right.}{256a^{7/2}d^{9/2}e^{7/2}}$$

Rubi [A] time = 0.51, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128a^3d^4e^3x^2}+\frac{\left(-35a^2e^4+12acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{240a^2d^3e^2x^3}+\frac{\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256a^{7/2}d^{9/2}e^{7/2}}-\frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{40x^4}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

```
[Out] -((c*d^2 - a*e^2)*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*a^3*d^4*e^3*
x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(5*d*x^5) - (((3*c)/(a
*e) - (7*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(40*x^4) +
((15*c^2*d^4 + 12*a*c*d^2*e^2 - 35*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(240*a^2*d^3*e^2*x^3) + ((c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a
*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(7/2)*d
^(9/2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx = \int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^6}\,dx$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5} - \frac{\int \frac{\left(-\frac{1}{2}ae\left(3cd^2-7ae^2\right)+2acde^2x\right)\sqrt{ade+(cd}}{x^5}}{5ade}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{40x^4}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{40x^4}$$

$$= -\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}}{128a^3d^4e^3x^2}$$

$$= -\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}}{128a^3d^4e^3x^2}$$

$$= -\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}}{128a^3d^4e^3x^2}$$

Mathematica [A] time = 0.49, size = 310, normalized size = 0.78

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{5x^2\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^2e^2\left(-8d^2-2dex+3e^2x^2\right)-2acd^2ex(7d+4ex)-3c^2d^4x^2\right)+3x^3\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)\right)}{a^{5/2}d^{7/2}e^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}}+\frac{48x(d+ex)\left(7ae^2+5cd^2\right)(ae+cdx)^2}{ade}-384(d+ex)(ae+cdx)^2\right)}{1920adex^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-384*(a*e + c*d*x)^2*(d + e*x) + (48*(5*c*d^2 + 7*a*e^2)*x*(a*e + c*d*x)^2*(d + e*x))/(a*d*e) + (5*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*x^2*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c^2*d^4*x^2 - 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(-8*d^2 - 2*d*e*x + 3*e^2*x^2)) + 3*(c*d^2 - a*e^2)^3*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(a^(5/2)*d^(7/2)*e^(5/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(1920*a*d*e*x^5)

IntegrateAlgebraic [F] time = 180.11, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]

[Out] $Aborted

fricas [A] time = 20.24, size = 872, normalized size = 2.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm="fricas")

[Out] [-1/7680*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(384*a^5*d^5*e^5 + (45*a*c^4*d^9*e - 30*a^2*c^3*d^7*e^3 - 36*a^3*c^2*d^5*e^5 + 190*a^4*c*d^3*e^7 - 105*a^5*d*e^9)*x^4 - 2*(15*a^2*c^3*d^8*e^2 - 9*a^3*c^2*d^6*e^4 + 61*a^4*c*d^4*e^6 - 35*a^5*d^2*e^8)*x^3 + 8*(3*a^3*c^2*d^7*e^3 + 12*a^4*c*d^5*e^5 - 7*a^5*d^3*e^7)*x^2 + 48*(11*a^4*c*d^6*e^4 + a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^5), -1/3840*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(-a*d*e)*x^5*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(384*a^5*d^5*e^5 + (45*a*c^4*d^9*e - 30*a^2*c^3*d^7*e^3 - 36*a^3*c^2*d^5*e^5 + 190*a^4*c*d^3*e^7 - 105*a^5*d*e^9)*x^4 - 2*(15*a^2*c^3*d^8*e^2 - 9*a^3*c^2*d^6*e^4 + 61*a^4*c*d^4*e^6 - 35*a^5*d^2*e^8)*x^3 + 8*(3*a^3*c^2*d^7*e^3 + 12*a^4*c*d^5*e^5 - 7*a^5*d^3*e^7)*x^2 + 48*(11*a^4*c*d^6*e^4 + a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^5)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((2*exp(1)^4*a^2*exp(2)^2-4*exp(1)^6*a^2*exp(2)+2*exp(1)^8*a^2)/2/d^4/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))-(3*a^5*exp(2)^5+6*exp(1)^2*a^5*exp(2)^4+16*exp(1)^

4*a^5*exp(2)^3+96*exp(1)^6*a^5*exp(2)^2-384*exp(1)^8*a^5*exp(2)+256*exp(1)^
10*a^5+15*c*d^2*a^4*exp(2)^4+30*c^2*d^4*a^3*exp(2)^3-36*c^2*d^4*exp(1)^2*a^
3*exp(2)^2+30*c^3*d^6*a^2*exp(2)^2-48*c^3*d^6*exp(1)^2*a^2*exp(2)+16*c^3*d^
6*exp(1)^4*a^2+15*c^4*d^8*a*exp(2)-18*c^4*d^8*exp(1)^2*a+3*c^5*d^10)/128/d^
4/exp(1)^3/a^3/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x
+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-(45*(sqrt(a*d*exp(1
)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^5+90*
exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^9*a^5*exp(2)^4+240*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^3-2400*exp(1)^6*(sqrt(a*d*exp(1
)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^2+192
0*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^9*a^5*exp(2)+225*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^9*a^4*exp(2)^4+450*c^2*d^4*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^3*exp(2)^3-540*c^2*
d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^9*a^3*exp(2)^2+450*c^3*d^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2*exp(2)^2-720*c^3*d^6*exp(1)^2*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2*ex
p(2)+240*c^3*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^9*a^2+225*c^4*d^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*
x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a*exp(2)-270*c^4*d^8*exp(1)^2*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a+45*c
^5*d^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)^9-3840*d*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x
+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5*exp(2)^2+7680*d*exp(1)^7*sqrt(c*
d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)^8*a^5*exp(2)-3840*d*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5-210*d*exp(1)*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp
(2)^5-420*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^7*a^6*exp(2)^4+160*d*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)^3+8640*d*exp(1)^7
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*
a^6*exp(2)^2-7680*d*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)-1050*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)^4-3840*c
*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^7*a^5*exp(2)^2+7680*c*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)-3840*c*d^3*exp(1)^9
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*
a^5-2100*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^3+2520*c^2*d^5*exp(1)^3*(sqrt(a*d*exp(1)+
a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^2-2100*
c^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d
*exp(1))*x)^7*a^3*exp(2)^2+3360*c^3*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3*exp(2)+4000*c^3*d^7*exp
(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)^7*a^3-1050*c^4*d^9*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^7*a^2*exp(2)+1260*c^4*d^9*exp(1)^3*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2-210*c^5*d^1
1*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^7*a+3840*d^2*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)^3+7680*d^2*exp(1)^6*
sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^6*a^6*exp(2)^2-26880*d^2*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)+
15360*d^2*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6-3840*c*d^4*exp(1)^4*sqrt(c*d*exp(1)

)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6
*a^5*exp(2)^2+7680*c*d^4*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5*exp(2)-3840*c*d^4*exp
(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^6*a^5-19200*c^2*d^6*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4*exp(2)
-11520*c^3*d^8*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*
x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^3+384*d^2*exp(1)^2*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^5-12
80*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^5*a^7*exp(2)^3-11520*d^2*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^2+11520*d^2*exp(1)
^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)
^5*a^7*exp(2)+1920*c*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^4+7680*c*d^4*exp(1)^4*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)
^3-3840*c*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^2-15360*c*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*
x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)+11520*c*d
^4*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^5*a^6+3840*c^2*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^3+23040*c^2*d^6*exp(1)^4*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5
*exp(2)^2+7680*c^2*d^6*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)-3840*c^2*d^6*exp(1)^8*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5+3840*c^3
*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^5*a^4*exp(2)^2+23040*c^3*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)+10240*c^3*d^8*ex
p(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)^5*a^4+1920*c^4*d^10*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*exp(2)+7680*c^4*d^10*exp(1)^4*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3+384*c^
5*d^12*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^5*a^2-3840*d^3*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)^4-3840*d^3*
exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)^3-3840*d^3*exp(1)^7*sqrt(c*d*exp(1))*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7
*exp(2)^2+34560*d^3*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)-23040*d^3*exp(1)^11
*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^4*a^7-15360*c*d^5*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^6*exp(2)^3-1920
0*c*d^5*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^6*exp(2)^2+6400*c*d^5*exp(1)^9*sqrt(c*d*
exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)^4*a^6-23040*c^2*d^7*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)^2-26880*c^2*d^
7*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(2)-3840*c^2*d^7*exp(1)^7*sqrt(c*d*exp(1)
)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4
*a^5-15360*c^3*d^9*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4*exp(2)-11520*c^3*d^9*exp(1)
^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^4*a^4-3840*c^4*d^11*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*ex
p(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^3+210*d^3*e
xp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)^3*a^8*exp(2)^5+420*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*

d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^4+1120*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^3+6720*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^2-7680*d^3*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)+1050*c*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^4+7680*c*d^5*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^3+7680*c*d^5*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^2+7680*c*d^5*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)-11520*c*d^5*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7+2100*c^2*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^3+20520*c^2*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^2+19200*c^2*d^7*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)+2100*c^3*d^9*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)^2+19680*c^3*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)+12640*c^3*d^9*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5+1050*c^4*d^11*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4*exp(2)+6420*c^4*d^11*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4+210*c^5*d^13*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3-19200*d^4*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8*exp(2)+15360*d^4*exp(1)^12*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8-7680*c*d^6*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7*exp(2)^2-7680*c*d^6*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7*exp(2)-1280*c*d^6*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7-15360*c^2*d^8*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6*exp(2)-7680*c^2*d^8*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6-7680*c^3*d^10*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^5-45*d^4*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^5-90*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^4-240*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^3-1440*d^4*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^2+1920*d^4*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)-225*c*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^8*exp(2)^4+3840*c*d^6*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^8-450*c^2*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)^3+540*c^2*d^8*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)^2+3840*c^2*d^8*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)+3840*c^2*d^8*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7-450*c^3*d^10*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)^2+720*c^3*d^10*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)+3600*c^3*d^10*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6-225*c^4*d^12*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5*exp(2)+270*c^4*d^12*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c

```
*d*exp(1))*x)*a^5-45*c^5*d^14*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4+3840*d^5*exp(1)^11*sqrt(c*d*exp(1))
*a^9*exp(2)-3840*d^5*exp(1)^13*sqrt(c*d*exp(1))*a^9-1280*c*d^7*exp(1)^11*sq
rt(c*d*exp(1))*a^8-768*c^2*d^9*exp(1)^9*sqrt(c*d*exp(1))*a^7)/3840/d^4/exp(
1)^3/a^3/((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)^2-d*exp(1)*a)^5)
```

maple [B] time = 0.03, size = 2888, normalized size = 7.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^6/(e*x+d),x)
```

```
[Out] 7/128/d^5*e^6*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/32*d/a^2*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-3/128*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*
e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^
2-1/8/d^3*e^4*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/8/d^3/a/x^4
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+7/48/e/a^3*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(3/2)*c^3+15/128/d^3*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
*c+1/3/d^6*e^5*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+45/128/d^6*e^5
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+121/192/d^5*e^2/a/x^2*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(5/2)+19/48/d^2*e/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(3/2)*c^2+1/32/d*e^2/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+1/4/d
^6*e^7*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/8/d^7*e^8*a^2/c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/16/d^5*e^8*a^2*ln((1/2*a*e^
2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2))/(c*d*e)^(1/2)-1/4/d^4*e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2)*x+1/16/d*e^4*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)
^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/16/a^
3/e^3/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2-3/64/a^4/e^3/x*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^3-1/5/d^2/a/e/x^5*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(5/2)-3/128*d^3/a^3/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*c^4+3/128*d^2/a^4/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^4-1/128*d
^4/a^5/e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^5-3/128*d^5/a^4/e^4*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^5-3/16/d^5*e^8*a^2*ln((c*d*e*x+1/2*
a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*
d*e)^(1/2)-1/16/d*e^4*c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-263/384/d^6*e^3/a/x*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+227/384/d^4*e^3/a*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(3/2)*c+39/128/d^4*e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*x*c-7/256/d^4*e^7*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-1/32*e/a^2*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3+23/96/d/a^3*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(3/2)*x+73/192/d^3/a^2/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)
*c-25/48/d^4*e/a/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/4/d^6*e^7*a*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/8/d^7*e^8*a^2/c*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)+1/8/d/a^2/e^2/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(5/2)*c-1/4/d^2/a^2/e/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c-1/128
*d^2*e/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3-23/96/d^2/e/a^3/x*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(5/2)*c^2+1/16/d^7*e^10*a^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*
d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1
03/192/d^3*e^2/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-103/192/d^
4*e/a^2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+3/64/d^2*e^3/a*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2+263/384/d^5*e^4*c/a*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(3/2)*x+3/16/d^3*e^6*a*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c
*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c+15/256
/d^2*e^5*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-3/64/e*d^2/a^3*(c*d*e*x^2+a*d*e+(a*
```

```
e^2+c*d^2)*x)^(1/2)*x*c^4-1/16/d^7*e^10*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/
e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d
*e)^(1/2)-3/16/d^3*e^6*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(
1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/256*d^4
/a^2/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4+3/256*d^6/a^3/e^3/(a*d*e)^(1/2)*ln((2
*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2))/x)*c^5-3/128*d^4/a^4/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^5
+3/64*d/a^4/e^2*c^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-1/128*d^3/a^5
/e^4*c^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+9/64/d/a^3/e^2/x^2*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+1/64*d/a^4/e^4/x^2*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(5/2)*c^3+1/128*d^2/a^5/e^5/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(5/2)*c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^6\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**6/(e*x+d),x)
```

```
[Out] Timed out
```

3.290 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=498

$$\frac{\left(-21a^2e^4+6acd^2e^2+7c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{160a^2d^3e^2x^4}+\frac{\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(x\left(ae^2\right.\right.}{512a^4d^5}$$

Rubi [A] time = 0.72, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{\left(21a^2cd^2e^4-105a^3e^6+33ac^2d^4e^2+35c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{960a^3d^4e^3x^3}+\frac{\left(-21a^2e^4+6acd^2e^2+7c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{160a^2d^3e^2x^4}+\frac{\left(6a^2c^2d^4e^4-21a^4e^8+8ac^3d^6e^2+7c^4d^8\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512a^4d^5e^4x^2}-\frac{\left(21a^2cd^2e^4+21a^3e^6+15ac^2d^4e^2+7c^3d^6\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024a^{9/2}d^{11/2}e^{9/2}}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{20x^5}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

```
[Out] ((7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 21*a^4*e^8)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*a^4*d^
5*e^4*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(6*d*x^6) - ((c/
(a*e) - (3*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(20*x^5)
+ ((7*c^2*d^4 + 6*a*c*d^2*e^2 - 21*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(160*a^2*d^3*e^2*x^4) - ((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 2
1*a^2*c*d^2*e^4 - 105*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
))/(960*a^3*d^4*e^3*x^3) - ((c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2
 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*
Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(102
4*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
```

```
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^7}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6}-\frac{\int \frac{\left(-\frac{3}{2}ae\left(cd^2-3ae^2\right)+3acde^2x\right)\sqrt{ade+\left(cd^2 \cdots\right.}}{x^6}}{6ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\cdots\right.}{20x^5}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\cdots\right.}{20x^5}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\cdots\right.}{20x^5}$$

$$=\frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ad\cdots}}{512a^4d^5e^4x^2}$$

$$=\frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ad\cdots}}{512a^4d^5e^4x^2}$$

$$=\frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ad\cdots}}{512a^4d^5e^4x^2}$$

Mathematica [A] time = 0.77, size = 380, normalized size = 0.76

$$-\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{16x^2(d+ex)\left(63a^2e^4+54acd^2e^2+35c^2d^4\right)(ae+cdx)^2}{a^2d^2e^2}+\frac{5x^3\left(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6\right)\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^2e^2\left(-8d^2-2dex+3e^2x^2\right)-2acd^2ex(7d+4ex)-3c^2d^4x^2\right)+3x^3\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)\right)}{a^{7/2}d^{9/2}e^{7/2}\sqrt{d+ex}\sqrt{ae+cdx}}-\frac{128x(d+ex)\left(9ae^2+7cd^2\right)(ae+cdx)^2}{ade}+1280(d+ex)(ae+cdx)^2\right)}{7680adex^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

```
[Out] -1/7680*(Sqrt[(a*e + c*d*x)*(d + e*x)]*(1280*(a*e + c*d*x)^2*(d + e*x) - (1
28*(7*c*d^2 + 9*a*e^2)*x*(a*e + c*d*x)^2*(d + e*x))/(a*d*e) + (16*(35*c^2*d
^4 + 54*a*c*d^2*e^2 + 63*a^2*e^4)*x^2*(a*e + c*d*x)^2*(d + e*x))/(a^2*d^2*e
^2) + (5*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*x^3
*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c^2*d^4*x^2 -
 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(-8*d^2 - 2*d*e*x + 3*e^2*x^2)) + 3*
(c*d^2 - a*e^2)^3*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*
Sqrt[d + e*x])]))/(a^(7/2)*d^(9/2)*e^(7/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])
))/(a*d*e*x^6)
```

IntegrateAlgebraic [F] time = 180.88, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d +
e*x)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 55.73, size = 1072, normalized size = 2.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c^
3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a^6*e^12)*sqrt(a*d*
e)*x^6*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d^6*e^6 - (105*a*
c^5*d^11*e - 55*a^2*c^4*d^9*e^3 - 54*a^3*c^3*d^7*e^5 - 78*a^4*c^2*d^5*e^7 +
 525*a^5*c*d^3*e^9 - 315*a^6*d*e^11)*x^5 + 2*(35*a^2*c^4*d^10*e^2 - 16*a^3*
c^3*d^8*e^4 - 18*a^4*c^2*d^6*e^6 + 168*a^5*c*d^4*e^8 - 105*a^6*d^2*e^10)*x^
4 - 8*(7*a^3*c^3*d^9*e^3 - 3*a^4*c^2*d^7*e^5 + 33*a^5*c*d^5*e^7 - 21*a^6*d^
3*e^9)*x^3 + 16*(3*a^4*c^2*d^8*e^4 + 14*a^5*c*d^6*e^6 - 9*a^6*d^4*e^8)*x^2
+ 128*(13*a^5*c*d^7*e^5 + a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/(a^5*d^6*e^5*x^6), 1/15360*(15*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 -
 3*a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*
e^10 - 21*a^6*e^12)*sqrt(-a*d*e)*x^6*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^
2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(1280*a^6*d^6*e^6 - (105*
a*c^5*d^11*e - 55*a^2*c^4*d^9*e^3 - 54*a^3*c^3*d^7*e^5 - 78*a^4*c^2*d^5*e^7
 + 525*a^5*c*d^3*e^9 - 315*a^6*d*e^11)*x^5 + 2*(35*a^2*c^4*d^10*e^2 - 16*a^
3*c^3*d^8*e^4 - 18*a^4*c^2*d^6*e^6 + 168*a^5*c*d^4*e^8 - 105*a^6*d^2*e^10)*
x^4 - 8*(7*a^3*c^3*d^9*e^3 - 3*a^4*c^2*d^7*e^5 + 33*a^5*c*d^5*e^7 - 21*a^6*
d^3*e^9)*x^3 + 16*(3*a^4*c^2*d^8*e^4 + 14*a^5*c*d^6*e^6 - 9*a^6*d^4*e^8)*x^
2 + 128*(13*a^5*c*d^7*e^5 + a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(a^5*d^6*e^5*x^6)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-2*exp(1)^5*a^2*exp(2)^2+4*exp(1)^7*a^2*exp(2)-2*exp(1)^9*a^2)/2/d^5/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))+(7*a^6*exp(2)^6+12*exp(1)^2*a^6*exp(2)^5+24*exp(1)^4*a^6*exp(2)^4+64*exp(1)^6*a^6*exp(2)^3+384*exp(1)^8*a^6*exp(2)^2-1536*exp(1)^10*a^6*exp(2)+1024*exp(1)^12*a^6+42*c*d^2*a^5*exp(2)^5+105*c^2*d^4*a^4*exp(2)^4-120*c^2*d^4*exp(1)^2*a^4*exp(2)^3+140*c^3*d^6*a^3*exp(2)^3-240*c^3*d^6*exp(1)^2*a^3*exp(2)^2+96*c^3*d^6*exp(1)^4*a^3*exp(2)+105*c^4*d^8*a^2*exp(2)^2-180*c^4*d^8*exp(1)^2*a^2*exp(2)+72*c^4*d^8*exp(1)^4*a^2+42*c^5*d^10*a*exp(2)-48*c^5*d^10*exp(1)^2*a+7*c^6*d^12)/512/d^5/exp(1)^4/a^4/2/sqrt(-a*d*exp(1))*atan((sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-(-105*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^6*exp(2)^6-180*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^6*exp(2)^5-360*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^6*exp(2)^4-960*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^6*exp(2)^3+9600*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^6*exp(2)^2-7680*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^6*exp(2)-630*c*d^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^5*exp(2)^5-1575*c^2*d^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^4*exp(2)^4+1800*c^2*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^4*exp(2)^3-2100*c^3*d^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^3*exp(2)^3+3600*c^3*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^3*exp(2)^2-1440*c^3*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^3*exp(2)-1575*c^4*d^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^2*exp(2)^2+2700*c^4*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^2*exp(2)-1080*c^4*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a^2-630*c^5*d^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a*exp(2)+720*c^5*d^10*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11*a-105*c^6*d^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^11+15360*d*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^10*a^6*exp(2)^2-30720*d*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^10*a^6*exp(2)+15360*d*exp(1)^11*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^10*a^6+595*d*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)^6+1020*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)^5+2040*d*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)^4+320*d*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)^3-44160*d*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)^2+38400*d*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^7*exp(2)+3570*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^6*exp(2)^5+15360*c*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^6*exp(2)^2-30720*c*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^6*exp(2)+15360*c*d^3*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^6+8925*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^4-10200*c^2*d^5*exp(1)^3*

(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a
^5*exp(2)^3+11900*c^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^4*exp(2)^3-20400*c^3*d^7*exp(1)^3*(sqrt(a
*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^4*exp(
2)^2+8160*c^3*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^9*a^4*exp(2)+8925*c^4*d^9*exp(1)*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^3*exp(2)^2-15300*
c^4*d^9*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^9*a^3*exp(2)+6120*c^4*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^3+3570*c^5*d^11*exp(1)*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2
*exp(2)-4080*c^5*d^11*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^9*a^2+595*c^6*d^13*exp(1)*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a-15360*d^2*exp(1)^6*s
qrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^8*a^7*exp(2)^3-46080*d^2*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^7*exp(2)^2
+138240*d^2*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^7*exp(2)-76800*d^2*exp(1)^12*sqrt(c
*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^8*a^7+15360*c*d^4*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^6*exp(2)^2-30720*c*d^4
*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^8*a^6*exp(2)+15360*c*d^4*exp(1)^10*sqrt(c*d*exp(1))
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*
a^6-30720*c^3*d^8*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^4-1386*d^2*exp(1)^2*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^8*exp(2)^
6-2376*d^2*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^7*a^8*exp(2)^5-1680*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^8*exp(2)^4+5760*d^2*exp(
1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^7*a^8*exp(2)^3+80640*d^2*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^8*exp(2)^2-76800*d^2*exp(1)^12*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^8*exp
(2)-8316*c*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)^7*a^7*exp(2)^5-30720*c*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^7*exp(2)^3+92160*
c*d^4*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^7*a^7*exp(2)-61440*c*d^4*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^7-20790*c^2*d^6*exp(1)^2*(
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^
6*exp(2)^4+23760*c^2*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)^3+15360*c^2*d^6*exp(1)^6*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp
(2)^2-30720*c^2*d^6*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)+15360*c^2*d^6*exp(1)^10*(sqrt(a*d*ex
p(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6-27720*c^3
*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^7*a^5*exp(2)^3+47520*c^3*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)^2+116160*c^3*d^8
exp(1)^6(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)^7*a^5*exp(2)-20790*c^4*d^10*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^2+35640*c^4*d^10*exp(
1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^7*a^4*exp(2)+71760*c^4*d^10*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4-8316*c^5*d^12*exp(1)^2*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3*exp(2)+
9504*c^5*d^12*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-

sqrt(c*d*exp(1))*x)^7*a^3-1386*c^6*d^14*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2+15360*d^3*exp(1)^5*sqrt
(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^6*a^8*exp(2)^4+30720*d^3*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^8*exp(2)^3+46
080*d^3*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^8*exp(2)^2-245760*d^3*exp(1)^11*sqrt(c*d
exp(1))(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)^6*a^8*exp(2)+153600*d^3*exp(1)^13*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^8-51200*c*d^5*exp
(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^6*a^7*exp(2)^3+61440*c*d^5*exp(1)^7*sqrt(c*d*exp(1))*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^7
*exp(2)^2+30720*c*d^5*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^7*exp(2)-40960*c*d^5*exp(1
)^11*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-s
qrt(c*d*exp(1))*x)^6*a^7-245760*c^2*d^7*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)
^2-30720*c^2*d^7*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)+15360*c^2*d^7*exp(1)^9
*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^6*a^6-276480*c^3*d^9*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5*exp(2)-614
40*c^3*d^9*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5-97280*c^4*d^11*exp(1)^5*sqrt(c*d*ex
p(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)^6*a^4+1686*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^5*a^9*exp(2)^6+696*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*
x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^9*exp(2)^5-1680*d^
3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^5*a^9*exp(2)^4-9600*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*
x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^9*exp(2)^3-72960*d^3*exp(1)^11*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^9
*exp(2)^2+76800*d^3*exp(1)^13*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)^5*a^9*exp(2)+10116*c*d^5*exp(1)^3*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^8*exp(2)^5+46
080*c*d^5*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt
(c*d*exp(1))*x)^5*a^8*exp(2)^4-46080*c*d^5*exp(1)^9*(sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^8*exp(2)^2-92160*c*d^5
exp(1)^11(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^5*a^8*exp(2)+92160*c*d^5*exp(1)^13*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^8+25290*c^2*d^7*exp(1)^3*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp
(2)^4+177360*c^2*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^3+138240*c^2*d^7*exp(1)^7*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)
^2-46080*c^2*d^7*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1
))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)-15360*c^2*d^7*exp(1)^11*(sqrt(a*d*exp(1
)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7+33720*c^3*d^
9*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^5*a^6*exp(2)^3+262560*c^3*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^2+269760*c^3*d^9*e
xp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)^5*a^6*exp(2)+15360*c^3*d^9*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*
x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6+25290*c^4*d^11*exp(1)^3*(sqrt(a
*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(
2)^2+173880*c^4*d^11*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)+133200*c^4*d^11*exp(1)^7*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5+10116*c

```
^5*d^13*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^5*a^4*exp(2)+43296*c^5*d^13*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4+1686*c^6*d^15*exp(1)^
3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5
*a^3-15360*d^4*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*
x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^9*exp(2)^5-15360*d^4*exp(1)^6*sqr
t(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^4*a^9*exp(2)^4-15360*d^4*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^9*exp(2)^3-1
5360*d^4*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d
*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^9*exp(2)^2+215040*d^4*exp(1)^12*sqrt(c
*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^4*a^9*exp(2)-153600*d^4*exp(1)^14*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^9-76800*c*d^6*e
xp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)^4*a^8*exp(2)^4-122880*c*d^6*exp(1)^6*sqrt(c*d*exp(1))
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*
a^8*exp(2)^3-46080*c*d^6*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^8*exp(2)^2+30720*c*d^6*
exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^4*a^8*exp(2)+30720*c*d^6*exp(1)^12*sqrt(c*d*exp(1))
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*
a^8-153600*c^2*d^8*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)^3-276480*c^2*d^8*exp
(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^4*a^7*exp(2)^2-138240*c^2*d^8*exp(1)^8*sqrt(c*d*exp(1))
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*
a^7*exp(2)+15360*c^2*d^8*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7-153600*c^3*d^10*exp(
1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-s
qrt(c*d*exp(1))*x)^4*a^6*exp(2)^2-245760*c^3*d^10*exp(1)^6*sqrt(c*d*exp(1))
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*
a^6*exp(2)-107520*c^3*d^10*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^6-76800*c^4*d^12*exp(
1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-s
qrt(c*d*exp(1))*x)^4*a^5*exp(2)-76800*c^4*d^12*exp(1)^6*sqrt(c*d*exp(1))*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5
-15360*c^5*d^14*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^4+595*d^4*exp(1)^4*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^10*exp(2)^6+
1020*d^4*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^3*a^10*exp(2)^5+2040*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^10*exp(2)^4+5440*d^4*exp(
1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)^3*a^10*exp(2)^3+32640*d^4*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^10*exp(2)^2-38400*d^4*exp(1)^14*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^10
*exp(2)+3570*c*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)^3*a^9*exp(2)^5+30720*c*d^6*exp(1)^6*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^9*exp(2)^4+30
720*c*d^6*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt
(c*d*exp(1))*x)^3*a^9*exp(2)^3+30720*c*d^6*exp(1)^10*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^9*exp(2)^2+30720*c*d^
6*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*ex
p(1))*x)^3*a^9*exp(2)-61440*c*d^6*exp(1)^14*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^9+8925*c^2*d^8*exp(1)^4*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp
(2)^4+112680*c^2*d^8*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^3+138240*c^2*d^8*exp(1)^8*(sqrt(a*d
```

*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)
^2+61440*c^2*d^8*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)-15360*c^2*d^8*exp(1)^12*(sqrt(a*d*exp(
1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8+11900*c^3*d
^10*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^3*a^7*exp(2)^3+163920*c^3*d^10*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^2+192480*c^3*d^
10*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*ex
p(1))*x)^3*a^7*exp(2)+51200*c^3*d^10*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7+8925*c^4*d^12*exp(1)^4*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6
*exp(2)^2+107580*c^4*d^12*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*
x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)+82920*c^4*d^12*exp(1)^8*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6+357
0*c^5*d^14*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^3*a^5*exp(2)+26640*c^5*d^14*exp(1)^6*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5+595*c^6*d^16*exp(1
)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)
^3*a^4-92160*d^5*exp(1)^13*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d
^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^10*exp(2)+76800*d^5*exp(1)^15*
sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)^2*a^10-30720*c*d^7*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^9*exp(2)^3-3072
0*c*d^7*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x
^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^9*exp(2)^2-30720*c*d^7*exp(1)^11*sqrt(c*
d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)^2*a^9*exp(2)-92160*c^2*d^9*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1
)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8*exp(2)^2-737
28*c^2*d^9*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8*exp(2)-27648*c^2*d^9*exp(1)^11*sqrt
(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*e
xp(1))*x)^2*a^8-92160*c^3*d^11*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7*exp(2)-43008*c^
3*d^11*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7-30720*c^4*d^13*exp(1)^7*sqrt(c*d*exp(1)
)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2
*a^6-105*d^5*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-s
qrt(c*d*exp(1))*x)*a^11*exp(2)^6-180*d^5*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^11*exp(2)^5-360*d^5*exp(1)
^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*
a^11*exp(2)^4-960*d^5*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)*a^11*exp(2)^3-5760*d^5*exp(1)^13*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^11*exp(2)^2+76
80*d^5*exp(1)^15*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)*a^11*exp(2)-630*c*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^10*exp(2)^5+15360*c*d^7*exp(1)^
15*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*
a^10-1575*c^2*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^4+1800*c^2*d^9*exp(1)^7*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^3+15360*
c^2*d^9*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c
*d*exp(1))*x)*a^9*exp(2)^2+15360*c^2*d^9*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)+15360*c^2*d^9*ex
p(1)^13*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1)
)*x)*a^9-2100*c^3*d^11*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)*a^8*exp(2)^3+3600*c^3*d^11*exp(1)^7*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^8*exp(2)^2+
29280*c^3*d^11*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)*a^8*exp(2)+15360*c^3*d^11*exp(1)^11*(sqrt(a*d*exp(1)+a

```
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^8-1575*c^4*d^13*exp
(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)*a^7*exp(2)^2+2700*c^4*d^13*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)+14280*c^4*d^13*exp(1)^9*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7-630
*c^5*d^15*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt
(c*d*exp(1))*x)*a^6*exp(2)+720*c^5*d^15*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2
)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6-105*c^6*d^17*exp(1)^5*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^5+15
360*d^6*exp(1)^14*sqrt(c*d*exp(1))*a^11*exp(2)-15360*d^6*exp(1)^16*sqrt(c*d
*exp(1))*a^11-5120*c*d^8*exp(1)^14*sqrt(c*d*exp(1))*a^10-3072*c^2*d^10*exp(
1)^10*sqrt(c*d*exp(1))*a^9*exp(2)-3072*c^2*d^10*exp(1)^12*sqrt(c*d*exp(1))*
a^9-3072*c^3*d^12*exp(1)^10*sqrt(c*d*exp(1))*a^8)/15360/d^5/exp(1)^4/a^4/((
sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*
exp(1)*a)^6)
```

maple [B] time = 0.04, size = 3387, normalized size = 6.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/x^7/(e*x+d),x)

[Out] 1/8/d^4*e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+19/60/d^3/a/x^5
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+1/64*e/a^2*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*c^3-23/96/d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^
3-21/512/d^6*e^7*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-1/8/d^4*e^5*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c-15/512/d^2*e^3/a*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2)*c^2-703/1536/d^3*e^2/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(3/2)*c^2-491/768/d^6*e^3/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)
+257/768/d^3/a^3/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+107/192/d^5*
e^2/a/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/3/d^7*e^6*((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-533/1536/d^7*e^6*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(3/2)+1/4/d^7*e^8*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+1/8/
d^8*e^9*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/16/d^6*e^9*a^2*ln((
1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4/d^5*e^6*c*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)*x-1/16/d^2*e^5*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*
e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/
2)+7/1536*d^5/a^6/e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^6+7/512*d^6
/a^5/e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^6+7/256*d/a^3*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^4-1/6/d^2/a/e/x^6*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(5/2)-7/96/a^3/e^3/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c
^2-1/4/d^7*e^8*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/8/d^8*e^
9*a^2/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/12/e^3/a^4/x^2*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^3-109/768/e/a^4*c^4*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(3/2)*x+3/1024*d^3/a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4+65/1
92/d^3/a^2/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c-5/384/e^4*d^3/a^5*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^5-43/96*e/d^4/a/x^4*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(5/2)+13/512/e*d^2/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)*c^4+1/64/e^3*d^4/a^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^5-1
31/1536/e^2*d/a^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^4+3/16/d^6*e^9*
a^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1/16/d^2*e^5*c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c
*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+
1045/1536/d^7*e^4/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-235/384/d^5*e
^4/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-149/512/d^5*e^6*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c-91/384/e/d^2/a^3/x^2*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(5/2)*c^2+29/192/e^2/d/a^3/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
```

```
)^(5/2)*c^2+109/768/e^2/d/a^4/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^3
+41/1536/e^4*d/a^5/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^4+3/512/e^2*
d^5/a^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^5-41/1536/e^3*d^2/a^5*c^5*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+15/512/e^2*d^3/a^4*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(1/2)*x*c^5-11/48/e/d^2/a^2/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
5/2)*c-1/16/d^8*e^11*a^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-43/96/d^4*e/a^2/x^2*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+1/256*d*e^2/a/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2))/x)*c^3+3/256/d*e^2/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3-25
7/768/d^2*e/a^3*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-877/1536/d^4*
e^3/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+877/1536/d^5*e^2/a^2/
x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c-21/512/d^3*e^4/a*(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2-1045/1536/d^6*e^5*c/a*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(3/2)*x-21/512/d^3*e^6*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-3/16/d^4
*e^7*a*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*c+7/192*d/a^4/e^4/x^3*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(5/2)*c^3-7/1536*d^3/a^6/e^6/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(5/2)*c^5-7/768*d^2/a^5/e^5/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2
)*c^4-7/1024*d^7/a^4/e^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^6+7/1536*d^4/a^6/e^5*
c^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+7/60/d/a^2/e^2/x^5*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+7/512*d^5/a^5/e^4*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*x*c^6+1/16/d^8*e^11*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*
d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(
1/2)+3/16/d^4*e^7*a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+21/1024/d^5*e^
8*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+15/1024/d*e^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a
*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c
^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^7\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**7/(e*x+d),x)
```

```
[Out] Timed out
```

3.291 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=574

$$-\frac{\left(-105a^3e^6-10cdex\left(-15a^2e^4-10acd^2e^2+33c^2d^4\right)-95a^2cd^2e^4-15ac^2d^4e^2+231c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade\right.}{4480c^3d^3e^4}$$

Rubi [A] time = 0.69, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 832, 779, 612, 621, 206}

$$-\frac{3\left(35a^2cd^2e^4+15a^3e^6+45ac^2d^4e^2+33c^3d^6\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384c^5d^5e^6}+\frac{\left(35a^2cd^2e^4+15a^3e^6+45ac^2d^4e^2+33c^3d^6\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2048c^4d^4e^5}-\frac{\left(-10cdex\left(-15a^2e^4-10acd^2e^2+33c^2d^4\right)-95a^2cd^2e^4-105a^3e^6-15ac^2d^4e^2+231c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4480c^3d^3e^4}+\frac{3\left(35a^2cd^2e^4+15a^3e^6+45ac^2d^4e^2+33c^3d^6\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{32768c^{11/2}d^{11/2}e^{13/2}}+\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}+\frac{1}{112}x^2\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (-3*(c*d^2 - a*e^2)^3*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 1
5*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(16384*c^5*d^5*e^6) + ((c*d^2 - a*e^2)*(33*c^3*d^6 + 45*a*c^2*d^4*
e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*c^4*d^4*e^5) + (((5*a)/(c*d) - (
11*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/112 + (x^3*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(8*e) - ((231*c^3*d^6 - 15*a*c
^2*d^4*e^2 - 95*a^2*c*d^2*e^4 - 105*a^3*e^6 - 10*c*d*e*(33*c^2*d^4 - 10*a*c
*d^2*e^2 - 15*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4
480*c^3*d^3*e^4) + (3*(c*d^2 - a*e^2)^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35
*a^2*c*d^2*e^4 + 15*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(32768*c^(1
1/2)*d^(11/2)*e^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\int x^3(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$$

$$=\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8e}+\frac{\int x^2\left(-3acd^2e-\frac{1}{2}cd\left(11cd^2-5\right.\right.}{}$$

$$=\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{x^3\left(ade+\left(cd^2\right.\right.}{}$$

$$=\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{x^3\left(ade+\left(cd^2\right.\right.}{}$$

$$=\frac{\left(cd^2-ae^2\right)\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+\right.}{2048c^4d^4e^5}$$

$$=-\frac{3\left(cd^2-ae^2\right)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae\right.}{16384c^5d^5e^6}$$

$$=-\frac{3\left(cd^2-ae^2\right)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae\right.}{16384c^5d^5e^6}$$

$$=-\frac{3\left(cd^2-ae^2\right)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae\right.}{16384c^5d^5e^6}$$

Mathematica [A] time = 3.61, size = 681, normalized size = 1.19

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[e]*(1575*a^8*e^15 - 525*a^7*c*d*e^13*
(7*d - e*x) + 35*a^6*c^2*d^2*e^11*(29*d^2 - 37*d*e*x - 6*e^2*x^2) + 5*a^5*c
^3*d^3*e^9*(185*d^3 + 93*d^2*e*x + 100*d*e^2*x^2 + 24*e^3*x^3) + 5*a^4*c^4*
d^4*e^7*(265*d^4 + 65*d^3*e*x - 30*d^2*e^2*x^2 - 56*d*e^3*x^3 - 16*e^4*x^4)
 + a^3*c^5*d^5*e^5*(-11193*d^5 + 8359*d^4*e*x - 6088*d^3*e^2*x^2 + 5040*d^2
*e^3*x^3 + 139200*d*e^4*x^4 + 104320*e^5*x^5) + a^2*c^6*d^6*e^3*(11445*d^6
- 18669*d^5*e*x + 12962*d^4*e^2*x^2 - 10544*d^3*e^3*x^3 + 9120*d^2*e^4*x^4
+ 350080*d*e^5*x^5 + 272640*e^6*x^6) + c^8*d^8*x*(-3465*d^7 + 2310*d^6*e*x
- 1848*d^5*e^2*x^2 + 1584*d^4*e^3*x^3 - 1408*d^3*e^4*x^4 + 1280*d^2*e^5*x^5
 + 87040*d*e^6*x^6 + 71680*e^7*x^7) + a*c^7*d^7*e*(-3465*d^7 + 13755*d^6*e*
x - 9324*d^5*e^2*x^2 + 7512*d^4*e^3*x^3 - 6464*d^3*e^4*x^4 + 5760*d^2*e^5*x
^5 + 299520*d*e^6*x^6 + 240640*e^7*x^7)))/(a*e + c*d*x) + (105*Sqrt[c]*Sqrt
[d]*(c*d^2 - a*e^2)^(9/2)*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4
 + 15*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*
d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d]*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))
/(c*d^2 - a*e^2)])))/(573440*c^5*d^5*e^(13/2))
```

IntegrateAlgebraic [F] time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
 e*x),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.57, size = 1524, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/2293760*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 -
40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^2*
d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x
^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*
e^3)*x) + 4*(71680*c^8*d^8*e^8*x^7 - 3465*c^8*d^15*e + 11445*a*c^7*d^13*e^3
 - 11193*a^2*c^6*d^11*e^5 + 1325*a^3*c^5*d^9*e^7 + 925*a^4*c^4*d^7*e^9 + 10
15*a^5*c^3*d^5*e^11 - 3675*a^6*c^2*d^3*e^13 + 1575*a^7*c*d*e^15 + 5120*(17*
c^8*d^9*e^7 + 33*a*c^7*d^7*e^9)*x^6 + 1280*(c^8*d^10*e^6 + 166*a*c^7*d^8*e^
8 + 81*a^2*c^6*d^6*e^10)*x^5 - 128*(11*c^8*d^11*e^5 - 35*a*c^7*d^9*e^7 - 10
75*a^2*c^6*d^7*e^9 - 5*a^3*c^5*d^5*e^11)*x^4 + 16*(99*c^8*d^12*e^4 - 316*a*
c^7*d^10*e^6 + 290*a^2*c^6*d^8*e^8 + 100*a^3*c^5*d^6*e^10 - 45*a^4*c^4*d^4*
e^12)*x^3 - 8*(231*c^8*d^13*e^3 - 741*a*c^7*d^11*e^5 + 686*a^2*c^6*d^9*e^7
- 50*a^3*c^5*d^7*e^9 + 235*a^4*c^4*d^5*e^11 - 105*a^5*c^3*d^3*e^13)*x^2 + 2
*(1155*c^8*d^14*e^2 - 3738*a*c^7*d^12*e^4 + 3517*a^2*c^6*d^10*e^6 - 300*a^3
*c^5*d^8*e^8 - 275*a^4*c^4*d^6*e^10 + 1190*a^5*c^3*d^4*e^12 - 525*a^6*c^2*d
^2*e^14)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^7), -1/
1146880*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 - 40*
a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^2*d^4
*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/
(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(71680*c^8
*d^8*e^8*x^7 - 3465*c^8*d^15*e + 11445*a*c^7*d^13*e^3 - 11193*a^2*c^6*d^11*
e^5 + 1325*a^3*c^5*d^9*e^7 + 925*a^4*c^4*d^7*e^9 + 1015*a^5*c^3*d^5*e^11 -
3675*a^6*c^2*d^3*e^13 + 1575*a^7*c*d*e^15 + 5120*(17*c^8*d^9*e^7 + 33*a*c^7
```

```
*d^7*e^9)*x^6 + 1280*(c^8*d^10*e^6 + 166*a*c^7*d^8*e^8 + 81*a^2*c^6*d^6*e^1
0)*x^5 - 128*(11*c^8*d^11*e^5 - 35*a*c^7*d^9*e^7 - 1075*a^2*c^6*d^7*e^9 - 5
*a^3*c^5*d^5*e^11)*x^4 + 16*(99*c^8*d^12*e^4 - 316*a*c^7*d^10*e^6 + 290*a^2
*c^6*d^8*e^8 + 100*a^3*c^5*d^6*e^10 - 45*a^4*c^4*d^4*e^12)*x^3 - 8*(231*c^8
*d^13*e^3 - 741*a*c^7*d^11*e^5 + 686*a^2*c^6*d^9*e^7 - 50*a^3*c^5*d^7*e^9 +
 235*a^4*c^4*d^5*e^11 - 105*a^5*c^3*d^3*e^13)*x^2 + 2*(1155*c^8*d^14*e^2 -
3738*a*c^7*d^12*e^4 + 3517*a^2*c^6*d^10*e^6 - 300*a^3*c^5*d^8*e^8 - 275*a^4
*c^4*d^6*e^10 + 1190*a^5*c^3*d^4*e^12 - 525*a^6*c^2*d^2*e^14)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^7)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Warning, replacing 0 by ` u`, a substitution variab
le should perhaps be purged.Warning, replacing 0 by ` u`, a substitution va
riable should perhaps be purged.Warning, replacing 0 by ` u`, a substitutio
n variable should perhaps be purged.Warning, replacing 0 by ` u`, a substit
ution variable should perhaps be purged.Warning, replacing 0 by ` u`, a sub
stitution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
 substitution variable should perhaps be purged.Warning, replacing 0 by ` u
`, a substitution variable should perhaps be purged.Warning, replacing 0 by
 ` u`, a substitution variable should perhaps be purged.Evaluation time: 0.
5Error: Bad Argument Type
```

maple [B] time = 0.03, size = 3178, normalized size = 5.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d),x)
```

```
[Out] -975/16384*e^2/c*d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4+195/4096/e^4*c^2*d^9*ln
((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2))/(c*d*e)^(1/2)*a+3/64*e/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
```

^(5/2)*x*a^2+9/64*d^6/e^3*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
*x-3/256*d*e^4*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)
+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/128*d^7/e^
2*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d
*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/256*d^9/e^4*a*c^2*ln((1/2
*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64*d^2*e*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)*x+15/256*d^3*e^2*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*
c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)
^(1/2)-5/256*e^2/c^2*a^3/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-15/102
4*e^4/c^3/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a^4+45/8192*e^7/c^4
/d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^6-855/8192/e^2*c*d^7*ln((c
*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2))/(c*d*e)^(1/2)*a^2-495/4096/e^3*c*d^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)*x*a-15/4096*e^5/c^3/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x
*a^5-105/2048*e/c*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^3-45/3276
8*e^10/c^5/d^5*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^8+15/4096*e^8/c^4/d^3*ln((c*d*e
*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2))/(c*d*e)^(1/2)*a^7-15/8192*e^6/c^3/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c
*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^6+45/4
096*e^4/c^2*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^5-3/64*d^3*a^3/c*((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/16*d^6/e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(3/2)-3/128*d^9/e^6*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^
(1/2)-15/128*d^5*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-95/2048/e^5*c*d
^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+285/16384/e^6*c^2*d^9*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+45/2048/e^3*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(3/2)*a+165/16384/e^2*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+
19/64/e^3*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-15/1024*e/c^2*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^3+465/4096*d^5*ln((c*d*e*x+1/2*a*e^2+1
/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1
/2)*a^3+735/16384/c*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+13/128/
c^2/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*a^2-1/5*d^3/e^4*((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+19/128/e^4*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(5/2)-25/112/e^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+65/1024/e
/c*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^2+285/8192/e^5*c^2*d^8*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/8*d^3/e^2*a*((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(3/2)*x+1/8*d^5/e^4*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d
/e))^(3/2)*x-1/16*d^2/e*a^2/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)
-9/64*d^4/e*a^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/128*d*e^2
a^4/c^2((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/64*d^7/e^4*a*c*((x
+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/64*d^8/e^5*c^2*((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*d^11/e^6*c^3*ln((1/2*a*e^2-1/2*c*d^2
+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
)/(c*d*e)^(1/2)+45/16384*e^8/c^5/d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*a^7+15/16384*e^6/c^4/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^6-75/1
6384*e^4/c^3/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^5-465/16384*e^2/c^
2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4-285/32768/e^6*c^3*d^11*ln((
c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2))/(c*d*e)^(1/2)+3/128*e^2/c^3/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(5/2)*a^3+29/128/e^2/c*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*a-5/512/c*
d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a^2+35/256/e^2*a*d^3*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+5/32/e/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(5/2)*x*a-45/8192*e^3/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^4+115
5/8192/e*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2+1/8/e^2*x*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)/c/d-705/16384/e^4*c*d^7*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2)*a-9/112/e/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^

```
(7/2)*a-95/1024/e^4*c*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-15/2048
*e^5/c^4/d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^5-35/2048*e^3/c^3/d^
2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

3.292 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=452

$$\frac{\left(-35a^2e^4-10cdex\left(9cd^2-5ae^2\right)-20acd^2e^2+63c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{840c^2d^2e^3}-\frac{\left(5a^2e^4+10acd^2e^2+\right.}{}$$

Rubi [A] time = 0.41, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {851, 832, 779, 612, 621, 206}

$$\frac{\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024c^4d^4e^5}-\frac{\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{384c^3d^3e^4}+\frac{\left(-35a^2e^4-10cdex\left(9cd^2-5ae^2\right)-20acd^2e^2+63c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{840c^2d^2e^3}-\frac{\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2048c^{9/2}d^{9/2}e^{11/2}}+\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)^3*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2
+ 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*c^4*d^4*e^5
) - ((c*d^2 - a*e^2)*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^
2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*c^3*d^3*
e^4) + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*e) + ((63*c^2
*d^4 - 20*a*c*d^2*e^2 - 35*a^2*e^4 - 10*c*d*e*(9*c*d^2 - 5*a*e^2)*x)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(840*c^2*d^2*e^3) - ((c*d^2 - a*e^
2)^5*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(2048*c^(9/2)*d^(9/2)*e^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\int x^2(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$$

$$=\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7e}+\frac{\int x\left(-2acd^2e-\frac{1}{2}cd\left(9cd^2-5ae\right.\right.}{}$$

$$=\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7e}+\frac{\left(63c^2d^4-20acd^2e^2-35a^2e^4-\right.}{}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\right.}{384c^3d^3e^4}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+}}{1024c^4d^4e^5}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+}}{1024c^4d^4e^5}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+}}{1024c^4d^4e^5}$$

Mathematica [A] time = 5.71, size = 562, normalized size = 1.24

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(\frac{7\sqrt{cd}\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(15\sqrt{e}\sqrt{cd}\left(cd^2-ae^2\right)^{11/2}(ae+cdx)\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}-15\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^6\sqrt{ae+cdx}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-10e^{3/2}\sqrt{cd}\left(cd^2-ae^2\right)^{9/2}(ae+cdx)^2\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}+8e^{5/2}\sqrt{cd}\left(cd^2-ae^2\right)^{7/2}(ae+cdx)^3\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}+16e^{7/2}\sqrt{cd}\left(cd^2-ae^2\right)^{3/2}(ae+cdx)^4\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\left(cd(11d+8ex)-3ae^2\right)\right)}{15360c^3d^3e^{9/2}\left(cd^2-ae^2\right)^{5/2}\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/2}(ae+cdx)^2}-\frac{(d+ex)\left(7ae^2+9cd^2\right)(ae+cdx)^2}{12cde}+x(d+ex)(ae+cdx)^2\right)}{7cde}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-1/12*((9*c*d^2 + 7*a*e^2)*(a*e + c*d*x)^
2*(d + e*x))/(c*d*e) + x*(a*e + c*d*x)^2*(d + e*x) + (7*Sqrt[c*d]*(9*c^2*d^
4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(15*Sqrt[c*d]*Sqrt[e]*(c*d^2 - a*e^2)^(11/2
)*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - 10*Sqrt[c*d]*e^(3/2
)*(c*d^2 - a*e^2)^(9/2)*(a*e + c*d*x)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2
)] + 8*Sqrt[c*d]*e^(5/2)*(c*d^2 - a*e^2)^(7/2)*(a*e + c*d*x)^3*Sqrt[(c*d*(d
 + e*x))/(c*d^2 - a*e^2)] + 16*Sqrt[c*d]*e^(7/2)*(c*d^2 - a*e^2)^(3/2)*(a*e
```

```
 + c*d*x)^4*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*(-3*a*e^2 + c*d*(11*d + 8
*e*x)) - 15*Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^6*Sqrt[a*e + c*d*x]*ArcSinh[(Sq
rt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))
/(15360*c^3*d^3*e^(9/2)*(c*d^2 - a*e^2)^(5/2)*(a*e + c*d*x)^2*((c*d*(d + e*
x))/(c*d^2 - a*e^2))^(3/2))))/(7*c*d*e)
```

IntegrateAlgebraic [F] time = 180.07, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
 e*x),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.51, size = 1272, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/430080*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15*
a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e^1
2 - 5*a^7*e^14)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d
^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(15360*c^7*d^7*e
^7*x^6 + 945*c^7*d^13*e - 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5 - 600*
a^3*c^4*d^7*e^7 - 525*a^4*c^3*d^5*e^9 + 1400*a^5*c^2*d^3*e^11 - 525*a^6*c*d
*e^13 + 1280*(15*c^7*d^8*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(3*c^7*d^9*e^5 +
 380*a*c^6*d^7*e^7 + 185*a^2*c^5*d^5*e^9)*x^4 - 16*(27*c^7*d^10*e^4 - 93*a*
c^6*d^8*e^6 - 2095*a^2*c^5*d^6*e^8 - 15*a^3*c^4*d^4*e^10)*x^3 + 8*(63*c^7*d
^11*e^3 - 218*a*c^6*d^9*e^5 + 228*a^2*c^5*d^7*e^7 + 90*a^3*c^4*d^5*e^9 - 35
*a^4*c^3*d^3*e^11)*x^2 - 2*(315*c^7*d^12*e^2 - 1099*a*c^6*d^10*e^4 + 1166*a
^2*c^5*d^8*e^6 - 150*a^3*c^4*d^6*e^8 + 455*a^4*c^3*d^4*e^10 - 175*a^5*c^2*d
^2*e^12)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6), 1/2
15040*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15*a^3*c
^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e^12 - 5
*a^7*e^14)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2
 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15360*c^7*d^7*e^7*x^6 + 945*c^7*d^13*e
- 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5 - 600*a^3*c^4*d^7*e^7 - 525*a^
4*c^3*d^5*e^9 + 1400*a^5*c^2*d^3*e^11 - 525*a^6*c*d*e^13 + 1280*(15*c^7*d^8
*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(3*c^7*d^9*e^5 + 380*a*c^6*d^7*e^7 + 185
*a^2*c^5*d^5*e^9)*x^4 - 16*(27*c^7*d^10*e^4 - 93*a*c^6*d^8*e^6 - 2095*a^2*c
^5*d^6*e^8 - 15*a^3*c^4*d^4*e^10)*x^3 + 8*(63*c^7*d^11*e^3 - 218*a*c^6*d^9*
e^5 + 228*a^2*c^5*d^7*e^7 + 90*a^3*c^4*d^5*e^9 - 35*a^4*c^3*d^3*e^11)*x^2 -
 2*(315*c^7*d^12*e^2 - 1099*a*c^6*d^10*e^4 + 1166*a^2*c^5*d^8*e^6 - 150*a^3
*c^4*d^6*e^8 + 455*a^4*c^3*d^4*e^10 - 175*a^5*c^2*d^2*e^12)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Warning, replacing 0 by ` u`, a substitution variab
le should perhaps be purged.Warning, replacing 0 by ` u`, a substitution va
riable should perhaps be purged.Warning, replacing 0 by ` u`, a substitutio
n variable should perhaps be purged.Warning, replacing 0 by ` u`, a substit
ution variable should perhaps be purged.Warning, replacing 0 by ` u`, a sub
stitution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
 substitution variable should perhaps be purged.Evaluation time: 0.52Error:
 Bad Argument Type
```

maple [B] time = 0.02, size = 2731, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d),x)
```

```
[Out] 195/2048/e*c*d^6*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-5/512*e^6/c^3/d^3*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^5-3/64*d*e^2*a^3/c*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(1/2)*x-9/64*d^5/e^2*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*
(x+d/e))^(1/2)*x-15/128*d^6/e*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/
(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+
15/256*d^8/e^3*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/256*d^2*e^3*
a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+5/512*e^4/c^2/d*(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^4+15/256*e^2/c*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2)*x*a^3-85/2048/e^3*c^2*d^8*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d
*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+5/192*e^
3/c^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a^3+5/2048*e^9/c^4/d^4*
ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2))/(c*d*e)^(1/2)*a^7-15/2048*e^7/c^3/d^2*ln((c*d*e*x+1/2*a*e^2+1/
2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/
2)*a^6+55/512/e^2*c*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a+125/204
8*e^3/c*d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4-1/16*d^5/e^4*c*((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(3/2)+3/128*d^8/e^5*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^
2)*(x+d/e))^(1/2)+9/64*d^3*a^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2
)*x-3/128*e^3*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/16*d*
a^2/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-1/6/e/c*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(5/2)*a-15/1024/e*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
```

```
(1/2)*a^2-5/192/e^2*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a-15/1024/e
^5*c^2*d^8*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+5/128/e^4*c*d^5*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/7/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
7/2)/c/d+35/1024*e^3/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4-5/96/c
*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^2-35/256*d^3*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2)*x*a^2-1/4/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5
/2)*x+1/5*d^2/e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)-1/8/e^3*d^2
*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-1/12/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(5/2)*x*a-25/192/e*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a
+15/2048/e^5*c^3*d^10*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+5/384*e^4/c^3/d^3*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^4+3/256*e^5*a^5/c^2*ln((1/2*a*e^2-1/2*c*d
^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/
2))/(c*d*e)^(1/2)+1/8*d^2/e*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)
*x+3/64*d^2*e*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*d^4/e
^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-3/64*d^6/e^3*a*c*((x+d
/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+15/128*d^4*e*a^3*ln((1/2*a*e^2-1/2
*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))
^(1/2))/(c*d*e)^(1/2)+3/64*d^7/e^4*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(1/2)*x-3/256*d^10/e^5*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/51
2/e^4*c^2*d^7*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-5/1024*e^7/c^4/d^4*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^6+5/128/e^3*c*d^6*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*a-5/128*e/c*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2)*a^3+5/64/e^3*c*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-1/24*e/c
^2/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*a^2+5/192*e^2/c^2/d*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^3-15/2048*e^5/c^2*ln((c*d*e*x+1/2*a*e^2+1
/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1
/2)*a^5-225/2048*e*d^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3+5/192*e/c*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
[Out] Timed out
```

3.293 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=381

$$\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024c^{7/2}d^{7/2}e^{9/2}}-\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2ca\right.}{512c^3d^3e^4}$$

Rubi [A] time = 0.39, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {794, 664, 612, 621, 206}

$$-\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^3d^3e^4}+\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{192c^2d^2e^3}+\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024c^{7/2}d^{7/2}e^{9/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{6cde(d+ex)}-\frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] -((c*d^2 - a*e^2)^3*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*c^3*d^3*e^4) + ((c*d^2 - a*e^2)*
(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(192*c^2*d^2*e^3) - (((5*a)/(c*d) + (7*d)/e^2)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/60 + (a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(7/2)/(6*c*d*e*(d + e*x)) + ((c*d^2 - a*e^2)^5*(7*c*d^2 + 5*a*e^2)
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(1024*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
```

```
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6cde(d+ex)} + \frac{1}{12}\left(-\frac{7d}{e}-\frac{5ae}{cd}\right)\int \frac{\left(ade+\left(cd^2+a\right.\right.}{d\,\cdot}$$

$$= -\frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} + \frac{\left(ade+\left(cd^2+ae^2\right)\right.}{6cde(d+}$$

$$= \frac{\left(cd^2-ae^2\right)\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+c\right.}{192c^2d^2e^3}$$

$$= -\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{512c^3d^3e^4}$$

$$= -\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{512c^3d^3e^4}$$

$$= -\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{512c^3d^3e^4}$$

Mathematica [A] time = 2.74, size = 506, normalized size = 1.33

$$\frac{(ae+cdx)((d+ex)(ae+cdx))^{5/2}\left(7-\frac{7\sqrt{cd}\sqrt{cd^2-ae^2}\left(5ae^2+7cd^2\right)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/2}\left(15\sqrt{e}\sqrt{cd}\left(cd^2-ae^2\right)^{11/2}(ae+cdx)\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}-15\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^6\sqrt{ae+cdx}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-10e^{3/2}\sqrt{cd}\left(cd^2-ae^2\right)^{9/2}(ae+cdx)^2\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}+8e^{5/2}\sqrt{cd}\left(cd^2-ae^2\right)^{7/2}(ae+cdx)^3\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}+16e^{7/2}\sqrt{cd}\left(cd^2-ae^2\right)^{3/2}(ae+cdx)^4\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}(cd(11d+8ex)-3ae^2)\right)}{1280c^5d^5e^{7/2}(d+ex)^4(ae+cdx)^4}\right)}{42cde}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] ((a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2)*(7 - (7*Sqrt[c*d]*Sqrt[c*d^2
 - a*e^2]*(7*c*d^2 + 5*a*e^2)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/2)*(15*S
qrt[c*d]*Sqrt[e]*(c*d^2 - a*e^2)^(11/2)*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/
(c*d^2 - a*e^2)] - 10*Sqrt[c*d]*e^(3/2)*(c*d^2 - a*e^2)^(9/2)*(a*e + c*d*x)
^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + 8*Sqrt[c*d]*e^(5/2)*(c*d^2 - a*e
^2)^(7/2)*(a*e + c*d*x)^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + 16*Sqrt[c
*d]*e^(7/2)*(c*d^2 - a*e^2)^(3/2)*(a*e + c*d*x)^4*Sqrt[(c*d*(d + e*x))/(c*d
^2 - a*e^2)]*(-3*a*e^2 + c*d*(11*d + 8*e*x)) - 15*Sqrt[c]*Sqrt[d]*(c*d^2 -
a*e^2)^6*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*
x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(1280*c^5*d^5*e^(7/2)*(a*e + c*d*x)^
4*(d + e*x)^4)))/(42*c*d*e)
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e
*x),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.49, size = 1046, normalized size = 2.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/30720*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3
*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*sqrt(c*
d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e)
 + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*e
+ 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*d^5*e^7 - 245*a^4*c
^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25*a*c^5*d^5*e^7)*x^4
+ 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*d^4*e^8)*x^3 - 8*(7*c
^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^7 - 5*a^3*c^3*d^3*e^9)*x^
2 + 2*(35*c^6*d^10*e^2 - 136*a*c^5*d^8*e^4 + 174*a^2*c^4*d^6*e^6 + 80*a^3*c
^3*d^4*e^8 - 25*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(c^4*d^4*e^5), -1/15360*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2
*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10
- 5*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*
e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*
e + 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*d^5*e^7 - 245*a^4
*c^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25*a*c^5*d^5*e^7)*x^
4 + 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*d^4*e^8)*x^3 - 8*(7
*c^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^7 - 5*a^3*c^3*d^3*e^9)*
x^2 + 2*(35*c^6*d^10*e^2 - 136*a*c^5*d^8*e^4 + 174*a^2*c^4*d^6*e^6 + 80*a^3
*c^3*d^4*e^8 - 25*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(c^4*d^4*e^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
```

```
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Warning, replacing 0 by ` u`, a substitution variab
le should perhaps be purged.Warning, replacing 0 by ` u`, a substitution va
riable should perhaps be purged.Warning, replacing 0 by ` u`, a substitutio
n variable should perhaps be purged.Warning, replacing 0 by ` u`, a substit
ution variable should perhaps be purged.Evaluation time: 0.45Error: Bad Arg
ument Type
```

maple [B] time = 0.01, size = 2411, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d),x)
```

```
[Out] 9/64*d^4/e*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/256/d*e^6*
a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d
*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/256*d^7/e^2*a*c^2*ln((1/2
*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/256*d*e^4*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+
(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))
/(c*d*e)^(1/2)-75/1024*e^4/c*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/
2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4-5/1024*e^8/c^
3/d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^6+5/256*e^5/c^2/d^2*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2)*x*a^4-5/64/e*c*d^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*x*a-5/96*e^2/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a^2+15/512/e
^2*c^2*d^7*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a+15/512*e^6/c^2/d*ln((c*d*e*x+1/2*a*
e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*
e)^(1/2)*a^5+5/512/e^4*c^2*d^7*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+5/19
2*e/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^2+5/192/e*d^2*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(3/2)*a+1/12/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/
2)*a+5/48*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*a-1/16*e*a^2/c*((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-5/192/e^3*c*d^4*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(3/2)+1/16*d^4/e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))
^(3/2)-3/128*d^7/e^4*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*
d*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-5/192*e^3/c^2/d^2*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*a^3-75/1024*c*d^5*ln((c*d*e*x+1/2*a*e^2+
1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(
1/2)*a^2+1/12/e^2*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+1/6/e*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x+5/256*d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2)*a^2-1/5*d/e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)-15/512/
e^2*c*d^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a-5/96/e^2*c*d^3*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+5/256*e^2/c*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)*a^3-5/1024/e^4*c^3*d^9*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e
)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-5/64*e^3/c*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^3+15/128*e*d^2*(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2)*x*a^2+25/256*e^2*d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2
)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3+
5/512*e^6/c^3/d^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^5-15/512*e^4/c^
2/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4+5/256/e^3*c^2*d^6*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+3/64*e^3*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d
^2)*(x+d/e))^(1/2)*x+15/128*d^5*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e
)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2
)-3/64*d*e^2*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/8*d^3/e^
2*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-9/64*d^2*e*a^2*((x+d/e)
^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/128/d*e^4*a^4/c^2*((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/64*d^5/e^2*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d
```

```
^2)*(x+d/e))^(1/2)-15/128*d^3*e^2*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e
)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2
)-3/64*d^6/e^3*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*d^
9/e^4*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c
*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{5}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Integral(x*((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x), x)
```

3.294 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=274

$$-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}+\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade\cdots}}{128c^2d^2e^3}$$

Rubi [A] time = 0.19, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {664, 612, 621, 206}

$$\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^2d^2e^3}-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}+\frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(128*c^2*d^2*e^3) + ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 +
2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/16 + (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - (3*(c*d^2 - a*e^2)^5*ArcTanh[(c*d
^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])])/(256*c^(5/2)*d^(5/2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int\left(ade+\left(\right.\right.}{2e^2}$$

$$= \frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} +$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} +$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} +$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} +$$

Mathematica [A] time = 1.22, size = 384, normalized size = 1.40

$$\frac{\sqrt{cd}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(-15a^5e^9+5a^4cde^7(14d-ex)+2a^3c^2d^2e^5\left(64d^2+268dex+129e^2x^2\right)+2a^2c^3d^3e^3\left(-35d^3+87d^2ex+489de^2x^2+292e^3x^3\right)+ac^4d^4e\left(15d^4-80d^3ex+54d^2e^2x^2+688de^3x^3+464e^4x^4\right)+c^5d^5x\left(15d^4-10d^3ex+8d^2e^2x^2+176de^3x^3+128e^4x^4\right)\right)-15\left(cd^2-ae^2\right)^{11/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)\right)}{640c^{7/2}d^{7/2}e^{7/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[c*d]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(-15*a^5*e^9 + 5*a^
4*c*d*e^7*(14*d - e*x) + 2*a^3*c^2*d^2*e^5*(64*d^2 + 268*d*e*x + 129*e^2*x^
2) + 2*a^2*c^3*d^3*e^3*(-35*d^3 + 87*d^2*e*x + 489*d*e^2*x^2 + 292*e^3*x^3)
 + c^5*d^5*x*(15*d^4 - 10*d^3*e*x + 8*d^2*e^2*x^2 + 176*d*e^3*x^3 + 128*e^4
*x^4) + a*c^4*d^4*e*(15*d^4 - 80*d^3*e*x + 54*d^2*e^2*x^2 + 688*d*e^3*x^3 +
 464*e^4*x^4)) - 15*(c*d^2 - a*e^2)^(11/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d +
 e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])
/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(640*c^(7/2)*d^(7/2)*e^(7/2)*Sqrt[(a*e
+ c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [F] time = 180.04, size = 0, normalized size = 0.00

$\$Aborted$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),
x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.46, size = 844, normalized size = 3.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d
^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
```

```
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
 + 4*(128*c^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d
^5*e^5 + 70*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^
4*d^4*e^6)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^
2 - 2*(5*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d
^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/12
80*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^
6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(128*c^5*d^5*e^5*x^
4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d^5*e^5 + 70*a^3*c^2*d^3*
e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^4*d^4*e^6)*x^3 + 8*(c^5*
d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^2 - 2*(5*c^5*d^8*e^2 - 2
3*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 replacing 0 by ` u`, a substitution variable should perhaps be purged.Warn
ing, replacing 0 by ` u`, a substitution variable should perhaps be purged.
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.Warning, replacing 0 by ` u`, a substitution variable shoul
d perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable s
hould perhaps be purged.Warning, replacing 0 by ` u`, a substitution variab
le should perhaps be purged.Warning, replacing 0 by ` u`, a substitution va
riable should perhaps be purged.Evaluation time: 0.45Error: Bad Argument Ty
pe
```

maple [B] time = 0.01, size = 1123, normalized size = 4.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d),x)
```

```
[Out] 1/5/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+15/256/e*a*c^2*d^6*ln((
1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/64*e^4*a^3/c/d*((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(1/2)*x-9/64*a*c*d^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e
))^(1/2)*x+3/256*e^7*a^5/c^2/d^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*
```

```
d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/
128*e*a^2*c*d^4*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/8*e*a*((x+d/e)^2*c
*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x+3/64*e^3*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-
c*d^2)*(x+d/e))^(1/2)-1/8/e*c*d^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(
3/2)*x-1/16/e^2*c*d^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+3/128/e
^3*c^2*d^6*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/256/e^3*c^3*d^8*
ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/64/e*a*c*d^4*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(1/2)-15/256*e^5*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c
*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^
(1/2)+15/128*e^3*a^3*d^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/
2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64/e^2*c^
2*d^5*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+9/64*e^2*a^2*d*((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/128*e^5*a^4/c^2/d^2*((x+d/e)^2*
c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/16*e^2*a^2/c/d*((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(3/2)
```

maxima [B] time = 0.57, size = 915, normalized size = 3.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] -3/256*c^4*d^9*log(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e)*sqrt(c*d/e))/((c*d/e)^(3/2)*e^5) + 15/256*a*c^3*d^7*log(2*c
*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*sqrt(c
*d/e))/((c*d/e)^(3/2)*e^3) - 15/128*a^2*c^2*d^5*log(2*c*d*x + c*d^2/e + a*e
 + 2*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*sqrt(c*d/e))/((c*d/e)^(3/2
)*e) + 15/128*a^3*c*d^3*e*log(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e)*sqrt(c*d/e))/(c*d/e)^(3/2) - 15/256*a^4*d*e^3*lo
g(2*c*d*x + c*d^2/e + a*e + 2*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*s
qrt(c*d/e))/(c*d/e)^(3/2) + 3/256*a^5*e^5*log(2*c*d*x + c*d^2/e + a*e + 2*s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*sqrt(c*d/e))/(c*d*(c*d/e)^(3/2))
 - 9/64*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a*c*d^3*x + 3/64*sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c^2*d^5*x/e^2 + 9/64*sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e)*a^2*d*e^2*x - 3/64*sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e)*a^3*e^4*x/(c*d) + 3/128*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e)*c^2*d^6/e^3 - 3/64*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a*c
*d^4/e + 3/64*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a^3*e^3/c - 3/128
*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*a^4*e^5/(c^2*d^2) - 1/8*(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e)^(3/2)*c*d^2*x/e + 1/8*(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e)^(3/2)*a*e*x - 1/16*(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e)^(3/2)*c*d^3/e^2 + 1/16*(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)^(3/2)*a
^2*e^2/(c*d) + 1/5*(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)^(5/2)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)(ae+cdx))^{\frac{5}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x), x)
```

3.295 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=394

$$-a^{5/2}d^{3/2}e^{5/2}\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(-5a^3e^6-83a^2cd^2e^4-11ac^2d^4e^2+2cdex\left(c\right.\right.}{}$$

Rubi [A] time = 0.45, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 814, 843, 621, 206, 724}

$$-\frac{\left(-83a^2cd^2e^4-5a^3e^6-11ac^2d^4e^2+2cdex\left(cd^2-5ae^2\right)\left(ae^2+3cd^2\right)+3c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64cde^2}+\frac{\left(90a^2c^2d^4e^4+60a^3cd^2e^6-5a^4e^8-20ac^3d^6e^2+3c^4d^8\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128c^{3/2}d^{3/2}e^{5/2}}-a^{5/2}d^{3/2}e^{5/2}\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)+\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] -((3*c^3*d^6 - 11*a*c^2*d^4*e^2 - 83*a^2*c*d^2*e^4 - 5*a^3*e^6 + 2*c*d*e*(c
*d^2 - 5*a*e^2)*(3*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(64*c*d*e^2) + ((3*c*d^2 + 11*a*e^2 + 6*c*d*e*x)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))/(24*e) + ((3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^
2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(128*c^(3/2)*d^(3/2)*e^(5/2)) - a^(5/2)*d^(3/2)*e^(5/2)*ArcTanh[(2*a*
d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x}\,dx$$

$$=\frac{\left(3cd^2+11ae^2+6cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24e}-\int \frac{\left(-8a^2cd^2e^3+\frac{1}{2}\right.}{}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae\right.\right.}{64cde^2}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae\right.\right.}{64cde^2}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae\right.\right.}{64cde^2}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae\right.\right.}{64cde^2}$$

Mathematica [A] time = 2.01, size = 390, normalized size = 0.99

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(-\frac{384a^{5/2}cd^{5/2}e^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}+\sqrt{e}\sqrt{ae+cdx}\left(15a^3e^6+a^2cde^4(337d+118ex)+ac^2d^2e^2\left(57d^2+244dex+136e^2x^2\right)+c^3\left(-9d^6+6d^5ex+72d^4e^2x^2+48d^3e^3x^3\right)\right)+\frac{3\sqrt{c}\sqrt{d}\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right)}{192cde^{5/2}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(15*a^3*e^6 + a^2
*c*d*e^4*(337*d + 118*e*x) + a*c^2*d^2*e^2*(57*d^2 + 244*d*e*x + 136*e^2*x^
```

```
2) + c^3*(-9*d^6 + 6*d^5*e*x + 72*d^4*e^2*x^2 + 48*d^3*e^3*x^3)) + (3*Sqrt[
c]*Sqrt[d]*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^
2*e^6 - 5*a^4*e^8)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqr
t[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d +
 e*x))/(c*d^2 - a*e^2)]) - (384*a^(5/2)*c*d^(5/2)*e^5*ArcTanh[(Sqrt[d]*Sqrt
[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[d + e*x]))/(192*c*d*e
^(5/2)*Sqrt[a*e + c*d*x])
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*
x)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 45.74, size = 1873, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/768*(384*sqrt(a*d*e)*a^2*c^2*d^3*e^5*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2
) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6
 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*x^
3 - 9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 +
 8*(9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122*a*c^3*d^
4*e^4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^2*d^2*e^3), 1/384*(192*sqrt(a*d*e)*a^2*c^2*d^3*e^5*log((8*a^2*d^2*e^2 +
 (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^
2*d*e^3)*x)/x^2) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 6
0*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2
*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^3 -
9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(
9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122*a*c^3*d^4*e^
4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
2*d^2*e^3), 1/768*(768*sqrt(-a*d*e)*a^2*c^2*d^3*e^5*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/
(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 3*(3*c^4*d^8
 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sq
rt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c
*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*x^3 - 9*c^4*d^7*e
+ 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(9*c^4*d^5*e^
3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122*a*c^3*d^4*e^4 + 59*a^2*c
^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3),
1/384*(384*sqrt(-a*d*e)*a^2*c^2*d^3*e^5*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2
*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 3*(3*c^4*d^8 - 20*a*c^3*
d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*a
```

```
rctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3
)*x)) + 2*(48*c^4*d^4*e^4*x^3 - 9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^
2*d^3*e^5 + 15*a^3*c*d*e^7 + 8*(9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(
3*c^4*d^6*e^2 + 122*a*c^3*d^4*e^4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 2180, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x/(e*x+d),x)
```

```
[Out] 3/64/d^2*a^3*e^5/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+9/64*d^2
*a*e*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/256/d^3*a^5*e^8/c^
2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*
e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/128*d^3*a^2*e^2*c*ln((1/2*a*e^2
-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d
/e))^(1/2))/(c*d*e)^(1/2)+15/256/d*a^4*e^6/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e
)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*
e)^(1/2)+75/128*d^3*a^2*e^2*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2
)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-25/256/d*a^4*e^6/c
*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2))/(c*d*e)^(1/2)+19/64*d^2*a*e*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)*x+3/256/d^3*a^5*e^8/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(
1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-3/64/d^2*a^3*e^
5/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+1/8*d*c*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(3/2)*x+1/16*d^2*c/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d
/e))^(3/2)-3/128*d^5*c^2/e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-
9/64*a^2*e^3*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/64*d^3*a*c*(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/8*d^3*a*c*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2)-3/128*d^5*c^2/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)+83/64*d*a^2*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/8*d*c*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/16*d^2*c/e*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(3/2)+19/64*a^2*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-1/5/d
*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/5/d*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(5/2)-15/128*d*a^3*e^4*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(
c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3
/64*d^4*c^2/e*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*d^7*c^3
/e^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/16/d^2*a^2*e^3/c*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(3/2)-3/128/d^3*a^4*e^6/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)+75/128*d*a^3*e^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1
/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-3/64*d^4*c^2/e*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+3/256*d^7*c^3/e^2*ln((c*d*e*x+1/2*
a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*
d*e)^(1/2)-1/8/d*a*e^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-3/64
/d*a^3*e^4/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-15/256*d^5*a*c^2
*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e
```

```
^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/8/d*a^3*e^4/c*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2)+1/8/d*a*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-
d^2*a^3*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-25/256*d^5*a*c^2*ln((c*d*e*x+1/2*a*e
^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e
)^(1/2)+11/24*a*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-1/16/d^2*a^2*e^3/
c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+3/128/d^3*a^4*e^6/c^2*((x+d
/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d + ex\right) \left(ae + cdx\right)\right)^{\frac{5}{2}}}{x \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(x*(d + e*x)), x)
```

3.296 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=352

$$-\frac{1}{2}a^{3/2}\sqrt{d}\,e^{3/2}\left(3ae^2+5cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)+\frac{\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right)+\right.}{}$$

Rubi [A] time = 0.43, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {849, 812, 814, 843, 621, 206, 724}

$$\frac{\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right)+28acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e}-\frac{\left(-45a^2cd^2e^4-5a^3e^6-15ac^2d^4e^2+c^3d^6\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16\sqrt{c}\sqrt{d}\,e^{3/2}}-\frac{1}{2}a^{3/2}\sqrt{d}\,e^{3/2}\left(3ae^2+5cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] ((c^2*d^4 + 28*a*c*d^2*e^2 + 19*a^2*e^4 + 2*c*d*e*(c*d^2 + 7*a*e^2)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e) - ((3*a*e - c*d*x)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*x) - ((c^3*d^6 - 15*a*c^2*d^4*e^2
- 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt
[c]*Sqrt[d]*e^(3/2)) - (a^(3/2)*Sqrt[d]*e^(3/2)*(5*c*d^2 + 3*a*e^2)*ArcTanh
[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
```

```
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx = \int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2}\,dx$$

$$= -\frac{(3ae-cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{\left(-ae\left(5cd^2+3ae^2\right)\right.}{}$$

$$= \frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8e}$$

$$= \frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8e}$$

$$= \frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8e}$$

$$= \frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8e}$$

Mathematica [A] time = 2.07, size = 350, normalized size = 0.99

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(-\frac{24a^{3/2}\sqrt{d}\,e^3\left(3ae^2+5cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}+\frac{\sqrt{e}\sqrt{ae+cdx}\left(3a^2e^3(11ex-8d)+2acde^2x(34d+13ex)+c^2d^2x\left(3d^2+14dex+8e^2x^2\right)\right)}{x}-\frac{3\sqrt{c}\sqrt{d}\left(-5a^3e^6-45a^2cd^2e^4-15ac^2d^4e^2+c^3d^6\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right)}{24e^{3/2}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[e]*Sqrt[a*e + c*d*x]*(3*a^2*e^3*(-8*d
 + 11*e*x) + 2*a*c*d*e^2*x*(34*d + 13*e*x) + c^2*d^2*x*(3*d^2 + 14*d*e*x +
8*e^2*x^2)))/x - (3*Sqrt[c]*Sqrt[d]*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*
d^2*e^4 - 5*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(S
qrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d
 + e*x))/(c*d^2 - a*e^2)]) - (24*a^(3/2)*Sqrt[d]*e^3*(5*c*d^2 + 3*a*e^2)*Ar
cTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[d
+ e*x]))/(24*e^(3/2)*Sqrt[a*e + c*d*x])
```

IntegrateAlgebraic [F] time = 180.16, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d +
e*x)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 15.20, size = 1717, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(
c*d*e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*
d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 24*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*
sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2
- 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*c^3*d^3*e^3*x^3 -
 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e
 + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c*d*e^2*x), 1/48*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*
e^4 - 5*a^3*e^6)*sqrt(-c*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c
*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 12*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^
5)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x
^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 2*(8*c^3*d^3*e^3*x^
3 - 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^
5*e + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(c*d*e^2*x), 1/96*(48*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*sqrt(
-a*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*
e + a^2*d*e^3)*x)) - 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a
^3*e^6)*sqrt(c*d*e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a
*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(8*c^3*d^3*e^3*x^3 - 2
4*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e +
 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(c*d*e^2*x), 1/48*(24*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*sqrt(-a*d*
e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a
^2*d*e^3)*x)) + 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^
6)*sqrt(-c*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2
*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^
2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c^3*d^3*e^3*x^3 - 24*a^2*c*d^2*e^4 + 2*(7*c
^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e + 68*a*c^2*d^3*e^3 + 33*a
^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.48Error: Bad Argument Typ
e
```

maple [B] time = 0.02, size = 2364, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^2/(e*x+d),x)
```

```
[Out] -3/64*e^6/d^3*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-9/64*e^
2*d*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*e^9/d^4*a^5/c
```

```
^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/128*e^3*d^2*a^2*c*ln((1/2*a*e^
2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2))/(c*d*e)^(1/2)+15/256*e*d^4*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/
e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d
*e)^(1/2)-15/256*e^7/d^2*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64*
e^6/d^3*a^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-3/256*e^9/d^4*a^5/c
^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2))/(c*d*e)^(1/2)+121/64*d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*x*a*c*e^2+15/256/d^2/c*e^7*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/
2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4+375/128*d^2*c
*e^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2+225/256*d^4*e*ln((c*d*e*x+1/2*a*e^2+1/2
*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2
)*a*c^2-5/2*d^3*a^2*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)
^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+15/128*e^5*a^3*ln((1/2
*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64*d^3*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2)*x-1/8*e*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x+3
/128/e*d^4*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+13/64*d^3*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2+13/128*d^4/e*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2)*c^2+1/d*a*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+9
/8*e*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+25/128*e^5*ln((c*d*e*x+1/2
*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c
*d*e)^(1/2)*a^3+1/a/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+4/5/d^2*e*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+1/5*e/d^2*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(5/2)-1/16*d*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+
67/48*d*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+19/8*e^3*(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2)*a^2-13/256*d^6*c^3/e*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^
2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1/8
*e^3/d^2*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x+3/64*e^5/d^2*a^3
/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/16*e^4/d^3*a^2/c*((x+d/e
)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+9/64*e^4/d*a^2*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(1/2)*x-3/128*e^7/d^4*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)-3/64*e*d^2*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^
(1/2)-3/256/e*d^6*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/d^2/a/e/x*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)-3/2*d*a^3*e^4/(a*d*e)^(1/2)*ln((2*a*d
*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))
/x)+1/d*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-9/64/d*(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2*e^4-1/8*e^3/d^2*a*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(3/2)*x-1/16*e^4/d^3*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)
+3/128*e^7/d^4*a^4/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/64/d^2/c*e
^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+227/64*d^2*c*e*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(1/2)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{5}{2}}}{x^2\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**2/(e*x+d),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(x**2*(d + e*x)), x)
```

3.297 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=339

$$\frac{3\sqrt{c}\,\sqrt{d}\,\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{e}}-\frac{3\sqrt{a}\,\sqrt{e}\,\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\tan}{8\sqrt{}}$$

Rubi [A] time = 0.39, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 812, 843, 621, 206, 724}

$$\frac{3\sqrt{c}\,\sqrt{d}\,\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{e}}-\frac{3\sqrt{a}\,\sqrt{e}\,\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{d}}-\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}-\frac{3\left(ae\left(ae^2+3cd^2\right)-cdx\left(3ae^2+cd^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (-3*(a*e*(3*c*d^2 + a*e^2) - c*d*(c*d^2 + 3*a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(4*x) - ((a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(2*x^2) + (3*Sqrt[c]*Sqrt[d]*(c^2*d^4 + 10*a*c*d^2*e^2 +
 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*Sqrt[e]) - (3*Sqrt[a]*Sqr
t[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e
^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(8*Sqrt[d])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3}\,dx$$

$$=-\frac{(ae-cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2x^2}-\frac{3}{8}\int \frac{\left(-2ae\left(3cd^2+ae\ldots\right.\right.}{\ldots}$$

$$=-\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex\ldots}}{4x}$$

$$=-\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex\ldots}}{4x}$$

$$=-\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex\ldots}}{4x}$$

$$=-\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex\ldots}}{4x}$$

Mathematica [A] time = 2.25, size = 334, normalized size = 0.99

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{d}\sqrt{ae+cdx}\left(-a^2e^2(2d+5ex)-9acdex(d-ex)+c^2d^2x^2(5d+2ex)\right)}{x^2}+\frac{3\sqrt{c}\,d\sqrt{cd}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-\frac{3\sqrt{a}\sqrt{e}\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}\right)}{4\sqrt{d}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[d]*Sqrt[a*e + c*d*x]*(-9*a*c*d*e*x*(d
 - e*x) + c^2*d^2*x^2*(5*d + 2*e*x) - a^2*e^2*(2*d + 5*e*x)))/x^2 + (3*Sqrt
[c]*d*Sqrt[c*d]*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*ArcSinh[(Sqrt[c]*Sqr
t[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[e]*
Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (3*Sqrt[a]*Sqr
t[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d
*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[d + e*x]))/(4*Sqrt[d]*Sqrt[a*e
+ c*d*x])
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d +
e*x)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 7.42, size = 1569, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/16*(3*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(c*d/e)*x^2*log(8*c^2*d
^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e +
 a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^
3*e + a*c*d*e^3)*x) + 3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*e/d)*
x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqr
t(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*c^2*d^2*e*x^3 - 2*a^2*d
*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e^3)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, -1/16*(6*(c^2*d^4 + 10*a*c*d^2*e
^2 + 5*a^2*e^4)*sqrt(-c*d/e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*
d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4
)*sqrt(a*e/d)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*
x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a
*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*c^2*d^2*e
*x^3 - 2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e
^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, 1/16*(6*(5*c^2*d^4
 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(-a*e/d)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d
*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 3*(c^2*d^4 + 10*a*c*d^2*
e^2 + 5*a^2*e^4)*sqrt(c*d/e)*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2*c^2*
d^2*e*x^3 - 2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*
a^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, -1/8*(3*(c^2*
d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(-c*d/e)*x^2*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^
2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 3*(5*c^2*d^4 + 10*a*c
*d^2*e^2 + a^2*e^4)*sqrt(-a*e/d)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2*x^2
+ a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - 2*(2*c^2*d^2*e*x^3 - 2*a^2*d*e^2
+ (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e^3)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.53Error: Bad Argument Typ
e
```

maple [B] time = 0.02, size = 2688, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^3/(e*x+d),x)
```

```
[Out] -3/256/d^5*e^10*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2
)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/128*d*e^4
*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/256/d^3*e^8*a^4/c*ln((1/2*
a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/256*d^3*e^2*a*c^2*ln((1/2*a*e^2-1/2*c*d^2
+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
)/(c*d*e)^(1/2)+3/64/d^4*e^7*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^
(1/2)*x-3/64/d^4*e^7*a^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+3/256/
d^5*e^10*a^5/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-3/4/d/a^2/e^2/x*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(7/2)*c-15/8*d^4*a*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2-1/4*
e/d^2*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x+975/256*e^2*d^3*ln((c*d
*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2))/(c*d*e)^(1/2)*a*c^2-15/256*e^8/d^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)
/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4+2
25/128*e^4*d*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-15/4*e^3*d^2*a^2/(a*d*e)^(1/2
)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(1/2))/x)*c+3/256*d^5*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*
e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4*e
^3/d^2*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/d/a*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(5/2)*c-3/8*e^5*a^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x
+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+93/256*d^5*c^3
*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2))/(c*d*e)^(1/2)+3/4*e^4/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*a^2+1/4/d^3/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+39/64*e^3*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a*c-1/16/d^4*e^5*a^2/c*((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(3/2)-9/64/d^2*e^5*a^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^
2)*(x+d/e))^(1/2)*x+3/128/d^5*e^8*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x
+d/e))^(1/2)+3/64*d*e^2*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1
5/128/d*e^6*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/64*d^2*e*c^2*((x+d
/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/4/a^2/e*c^2*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(5/2)*x+5/4*d/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c
^2-1/2/d^2/a/e/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+3/4*d/a^2/e^2*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+5/4*d^2/a/e*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(3/2)*c^2+1/8/d^3*e^4*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2
)*x+1/16/d^4*e^5*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-3/128/d^5*e^
8*a^4/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/8*e^2/d*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*x*c+15/128*e^6/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/
(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3+3/
64*e^6/d^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+333/64*e^2*d*c*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+9/64*e^3*a*c*((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(1/2)*x-1/8/d^3*e^4*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(3/2)*x-3/64/d^3*e^6*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2
)+1/8/d*e^2*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-1/5/d^3*e^2*(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/16*e*c*((x+d/e)^2*c*d*e+(a*e
```

```
^2-c*d^2)*(x+d/e))^(3/2)-3/128*d^3*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(1/2)+31/16*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c+387/128*d^3*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2-1/20*e^2/d^3*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(5/2)+9/64*e^5/d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a
^2+147/64*e*d^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^3\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

3.298 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=371

$$-\frac{\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8dx}-\frac{\left(-a^3e^6+15a^2cd^2e^4+45ac\right.}{}$$

Rubi [A] time = 0.47, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {849, 810, 812, 843, 621, 206, 724}

$$-\frac{\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8dx}-\frac{\left(15a^2cd^2e^4-a^3e^6+45ac^2d^4e^2+5c^3d^6\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16\sqrt{a}d^{3/2}\sqrt{e}}+\frac{1}{2}c^{3/2}d^{3/2}\sqrt{e}\left(5ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] -((5*c^2*d^4 + 12*a*c*d^2*e^2 - a^2*e^4 - 2*c*d*e*(7*c*d^2 + a*e^2)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*d*x) - ((4*a*d*e + 3*(3*c*d^2 +
 a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*d*x^3) + (c^(
3/2)*d^(3/2)*Sqrt[e]*(3*c*d^2 + 5*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x
)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])
/2 - ((5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh[(
2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
```

```
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4}\,dx$$

$$=-\frac{\left(4ade+3\left(3cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12dx^3}-\int \frac{\left(-\frac{1}{2}ae(5c\right.}{}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8dx}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8dx}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8dx}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{8dx}$$

Mathematica [A] time = 3.04, size = 357, normalized size = 0.96

$$\frac{\sqrt{ae+cdx}\left(-\frac{\sqrt{d}\sqrt{e}(d+ex)\sqrt{ae+cdx}\left(a^2e^2\left(8d^2+14dex+3e^2x^2\right)+2acd^2ex(13d+34ex)+3c^2d^3x^2(11d-8ex)\right)}{x^3}-\frac{3\sqrt{d+ex}\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3d^6\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}}+\frac{24e(cd)^{5/2}\sqrt{cd^2-ae^2}\left(5ae^2+3cd^2\right)\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{c^{3/2}}\right)}{24d^{3/2}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*(-((Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(d + e*x)*(3*c^2*d
^3*x^2*(11*d - 8*e*x) + 2*a*c*d^2*e*x*(13*d + 34*e*x) + a^2*e^2*(8*d^2 + 14
*d*e*x + 3*e^2*x^2)))/x^3) + (24*(c*d)^(5/2)*e*Sqrt[c*d^2 - a*e^2]*(3*c*d^2
 + 5*a*e^2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*
Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/c^(3/2) - (3*(
5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[d + e*x]*Ar
cTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[a]
))/(24*d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d +
e*x)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 8.72, size = 1741, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/96*(24*(3*a*c^2*d^5*e + 5*a^2*c*d^3*e^3)*sqrt(c*d*e)*x^3*log(8*c^2*d^2*e
^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*
c*d*e^3)*x) - 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)
*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x
^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(24*a*c^2*d^4*e^2
*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^
2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(a*d^2*e*x^3), -1/96*(48*(3*a*c^2*d^5*e + 5*a^2*c*d^3*e^3)*sq
rt(-c*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*
d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d
^3*e + a*c*d*e^3)*x)) + 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4
- a^3*e^6)*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 +
a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(24*a*c
^2*d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3
*d*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), 1/48*(3*(5*c^3*d^6 + 45*a*c^2*d^4*e^
2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 12*(3*a*c^2*d^5
*e + 5*a^2*c*d^3*e^3)*sqrt(c*d*e)*x^3*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 2*(24*a*
```

```
c^2*d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^
3*d*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), 1/48*(3*(5*c^3*d^6 + 45*a*c^2*d^4*e
^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a
*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 24*(3*a*c^2*d^
5*e + 5*a^2*c*d^3*e^3)*sqrt(-c*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x
^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(24*a*c^2*d^4*e^2*x^3 -
8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^2 - 2*(
13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(a*d^2*e*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.11Error: Bad Argument Typ
e
```

maple [B] time = 0.03, size = 3144, normalized size = 8.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^4/(e*x+d),x)
```

```
[Out] -1/16/d*e^2*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+3/128*d^2*e*c^2
*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/64*e^3*a*c*((x+d/e)^2*c*d*
e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-5/16*d^5/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3+35/2
4*d/a*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+25/24/e/a^2*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+5/12/d^3/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(7/2)+493/128*d^2*e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2-1/8/d^
2*e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+37/48/d*e^2*c*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(3/2)-1/24/d^3*e^4*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(3/2)+107/64*e^3*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+1/5/d^4*e^3
*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+7/40/d^4*e^3*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(5/2)-1/8/d^2*e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/
e))^(3/2)*x+1/16/d^5*e^6*a^2/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2
)+9/64/d^3*e^6*a^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/128/d^
6*e^9*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+15/128/d^2*e^7*
a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(
a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64*d*e^2*c^2*((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/256*d^4*e*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d
/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*
d*e)^(1/2)-15/128*e^5*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^
(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+5/8*d^4/
a/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-1/3/d^2/a/e/x^3*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(7/2)+5/24*d^3/a^2/e^2*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(3/2)+1/8*d^2/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^3+
5/8*d^3/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3-1/8/a^3/e^3/x*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2-9/64/d^3*e^6*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*x*a^2+93/64*d*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*
```

```
x*c^2+15/128*e^5*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-3/8/d^4*e/a/x*(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(7/2)+1/16/d*e^6*a^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)-3/64
/d^4*e^7/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+1/12/d^2*e^3*c*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-15/128/d^2*e^7*ln((c*d*e*x+1/2*a*e^2+1
/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1
/2)*a^3+387/256*d^4*e*c^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+19/24/d^2*e/a*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c+5/6*e/a*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(3/2)*x+5/6/d/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-1/8/d^
4*e^5*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-1/16/d^5*e^6*a^2/c*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+3/128/d^6*e^9*a^4/c^2*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2)+3/8/d^3*e^2*c/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2
)*x-15/16*d*e^4*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/
2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+1/64/d*e^4*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*x*a*c+15/256/d^4*e^9/c*ln((c*d*e*x+1/2*a*e^2+1/2*c
*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*
a^4+625/256*d^2*e^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a*c^2+5/24*d^2/a^2/e*c^3*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x+1/8*d/a^3/e^2*c^3*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(5/2)*x-1/12/d/a^2/e^2/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(7/2)*c+3/64/d^5*e^8*a^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x-3/25
6/d^6*e^11*a^5/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-15/256/d^4*e^9*a^4/c*ln((1/2*
a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/64/d^5*e^8*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c
*d^2)*(x+d/e))^(1/2)*x-9/64/d*e^4*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e
))^(1/2)*x+3/256/d^6*e^11*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c
*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15
/256*d^2*e^3*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x
+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/8/d^4*e^5*a*((x
+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x+3/64/d^4*e^7*a^3/c*((x+d/e)^2*
c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-5/6/e/d^2/a^2/x*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(7/2)*c-45/16*e^2*d^3*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x
+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^4\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**4/(e*x+d),x)
[Out] Timed out
```

3.299 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=404

$$-\frac{\left(x\left(-3a^3e^6+11a^2cd^2e^4+83ac^2d^4e^2+5c^3d^6\right)+2ade\left(5cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64ad^2ex^2}$$

Rubi [A] time = 0.46, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 810, 843, 621, 206, 724}

$$-\frac{\left(x\left(11a^2cd^2e^4-3a^3e^6+83ac^2d^4e^2+5c^3d^6\right)+2ade\left(5cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64ad^2ex^2}+\frac{\left(-90a^2c^2d^4e^4+20a^3cd^2e^6-3a^4e^8-60ac^3d^6e^2+5c^4d^8\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128a^{3/2}d^{5/2}e^{3/2}}+c^{5/2}d^{5/2}e^{3/2}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] -((2*a*d*e*(5*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (5*c^3*d^6 + 83*a*c^2*d^4*
e^2 + 11*a^2*c*d^2*e^4 - 3*a^3*e^6)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(64*a*d^2*e*x^2) - ((6*a*d*e + (11*c*d^2 + 3*a*e^2)*x)*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*d*x^4) + c^(5/2)*d^(5/2)*e^(3/2)*Ar
cTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])] + ((5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c
^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^
2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(128*a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
```

```
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5}\,dx$$

$$=-\frac{\left(6ade+\left(11cd^2+3ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24dx^4}-\int \frac{\left(-\frac{1}{2}ae(5\ldots}{}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3\ldots\right.\right.}{64ad^2ex^2}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3\ldots\right.\right.}{64ad^2ex^2}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3\ldots\right.\right.}{64ad^2ex^2}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3\ldots\right.\right.}{64ad^2ex^2}$$

Mathematica [A] time = 3.48, size = 404, normalized size = 1.00

$$\frac{\sqrt{ae+cdx}\left(-\frac{\sqrt{d}\sqrt{e}(d+ex)\sqrt{ae+cdx}\left(3a^3e^3\left(16d^3+24d^2ex+2de^2x^2-3e^3x^3\right)+a^2cd^2e^2x\left(136d^2+244dex+57e^2x^2\right)+ac^2d^4ex^2(118d+337ex)+15c^3d^6x^3\right)}{ax^4}+\frac{3\sqrt{d+ex}\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{a^{3/2}}+384c^{3/2}d^4e^3\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)\right)}{192d^{5/2}e^{3/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*(-((Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(d + e*x)*(15*c^3*
d^6*x^3 + a*c^2*d^4*e*x^2*(118*d + 337*e*x) + a^2*c*d^2*e^2*x*(136*d^2 + 24
```

```
4*d*e*x + 57*e^2*x^2) + 3*a^3*e^3*(16*d^3 + 24*d^2*e*x + 2*d*e^2*x^2 - 3*e^
3*x^3)))/(a*x^4)) + 384*c^(3/2)*d^4*Sqrt[c*d]*e^3*Sqrt[c*d^2 - a*e^2]*Sqrt[
(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e
+ c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])] + (3*(5*c^4*d^8 - 60*a*c^3*d^6*e
^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*Sqrt[d + e*x]*ArcTa
nh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/a^(3/2)))/
(192*d^(5/2)*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [A]   time = 2.57, size = 544, normalized size = 1.35

$$-\frac{1}{2}c^2d^2e\sqrt{cde}\log\left(a^2e^4+8cdex\sqrt{cde}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2acd^2e^2-4acde^3x+c^2d^4-4c^2d^3ex-8c^2d^2e^2x^2\right)+\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\left(-48a^3d^3e^3-72a^3d^2e^4x-6a^3de^5x^2+9a^3e^6x^3-136a^2cd^4e^2x-244a^2cd^3e^3x^2-57a^2cd^2e^4x^3-118ac^2d^5ex^2-337ac^2d^4e^2x^3-15c^3d^6x^3\right)}{192ad^2ex^4}+\frac{\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8\right)\tanh^{-1}\left(\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-x\sqrt{cde}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{64a^{3/2}d^{5/2}e^{3/2}}-c^{5/2}d^{5/2}e^{3/2}\tanh^{-1}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}x\sqrt{cde}}{ae^2+cd^2}-\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{ae^2+cd^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d +
e*x)),x]
```

```
[Out] (Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2]*(-48*a^3*d^3*e^3 - 136*a^2*c*d
^4*e^2*x - 72*a^3*d^2*e^4*x - 118*a*c^2*d^5*e*x^2 - 244*a^2*c*d^3*e^3*x^2 -
 6*a^3*d*e^5*x^2 - 15*c^3*d^6*x^3 - 337*a*c^2*d^4*e^2*x^3 - 57*a^2*c*d^2*e^
4*x^3 + 9*a^3*e^6*x^3))/(192*a*d^2*e*x^4) + ((5*c^4*d^8 - 60*a*c^3*d^6*e^2
- 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*ArcTanh[(-(Sqrt[c*d*e]
*x) + Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[a]*Sqrt[d]*Sqrt[e]
)])/(64*a^(3/2)*d^(5/2)*e^(3/2)) - c^(5/2)*d^(5/2)*e^(3/2)*ArcTanh[(2*Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[c*d*e]*x)/(c*d^2 + a*e^2) - (2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d^2 + a*e^2)] - (c^2*d^
2*e*Sqrt[c*d*e]*Log[c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4 - 4*c^2*d^3*e*x - 4*a
*c*d*e^3*x - 8*c^2*d^2*e^2*x^2 + 8*c*d*e*Sqrt[c*d*e]*x*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]])/2
```

fricas [A]   time = 24.35, size = 1917, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/768*(384*sqrt(c*d*e)*a^2*c^2*d^5*e^3*x^4*log(8*c^2*d^2*e^2*x^2 + c^2*d^4
 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 3*
(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a
^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(48*a^4*d^
4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5 - 9*a^4*d*
e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2*e^6)*x^2 +
 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(a^2*d^3*e^2*x^4), -1/768*(768*sqrt(-c*d*e)*a^2*c^2*d^5*e^3*x^4*
arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
 a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^
3)*x)) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^
2*e^6 - 3*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4
*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5
 - 9*a^4*d*e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2
*e^6)*x^2 + 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^4), 1/384*(192*sqrt(c*d*e)*a^2*c^2*d^5
*e^3*x^4*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*
```

```
d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*
a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-a*d*e)*x^4*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) -
 2*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e
^5 - 9*a^4*d*e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d
^2*e^6)*x^2 + 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^4), -1/384*(384*sqrt(-c*d*e)*a^2*c^2
*d^5*e^3*x^4*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4
+ 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-a*d*e)*x^4*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(48*a^4*d^4*e
^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5 - 9*a^4*d*e^7
)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2*e^6)*x^2 + 8*
(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x))/(a^2*d^3*e^2*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.26Error: Bad Argument Typ
e
```

maple [B] time = 0.03, size = 3646, normalized size = 9.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^5/(e*x+d),x)
```

```
[Out] 1/16/d^2*e^3*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/128*d*e^2*c^
2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/64*e^3*c^2*((x+d/e)^2*c*d
*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+25/32*d^3/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(1/2)*c^3+3/8/d^3/a/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+1/96/d
/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+35/96*e/a*(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(3/2)*c^2-1/8*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
*x*c^2+127/128*d*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+3/64/d^3*e
^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+1/64/d^4*e^5*a*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(3/2)-19/96/d^2*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(3/2)*c-1/5/d^5*e^4*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)-61/320/d^
5*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)+15/256/d^5*e^10*a^4/c*ln((1/2
*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/64/d^6*e^9*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-
c*d^2)*(x+d/e))^(1/2)*x+9/64/d^2*e^5*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2)*x-3/256/d^7*e^12*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)
/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
+15/128/d*e^6*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((
x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/256*d*e^4*a*c
^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-45/64*d^2*e^3*a/(a*d*e)^(1/2)*ln(
(2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
```

```
(1/2))/x)*c^2-31/64/d^2*e/a^2*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x
-35/192/d*e^2/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^2-3/32/d^2*e^5*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a*c-15/256/d^5*e^10/c*ln((c*d*e*x
+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
)/(c*d*e)^(1/2)*a^4-15/128/d*e^6*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)
^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2+15/256*d*
e^4*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a*c^2-25/64/d^4*e^3*c/a*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(5/2)*x-5/64*d^4/a^2/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*x*c^4+5/128*d^6/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(
1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4-5/192*d^3/a^3/e^2*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^4-1/64*d^2/a^4/e^3*c^4*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(5/2)*x+1/64*d/a^4/e^4/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(7/2)*c^3+1/24/d/a^2/e^2/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*
c-3/64/d^6*e^9*a^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+3/256/d^7*e^
12*a^5/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+45/64*e*d^2/a*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*x*c^3-13/48/e/d^2/a^2/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(7/2)*c-43/192/e^2/d/a^3/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2-5/
192*d^4/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^4-1/64*d^3/a^4/e^
4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^4-5/64*d^5/a^2/e^2*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^4+1/96/a^3/e^3/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(7/2)*c^2+5/32*e^5*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*
(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+9/64/d^4*e^7*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2+25/64/d^5*e^2/a/x*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(7/2)-7/64/d^3*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
3/2)*x*c+15/128/d^3*e^8*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3-3/128/d^2*e^7*a^3/(a
*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/x)-15/32/d^3*e^2/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(5/2)*c-17/64/d*e^4*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+253/256*d^3
*e^2*c^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+3/64/d^5*e^8/c*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*a^3+1/8/d^5*e^6*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x
+1/16/d^6*e^7*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)-3/128/d^7*e^10*
a^4/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/8/d^3*e^4*c*((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-1/16/d^6*e^7*a^2/c*((x+d/e)^2*c*d*e+(a*e
^2-c*d^2)*(x+d/e))^(3/2)-9/64/d^4*e^7*a^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x
+d/e))^(1/2)*x+3/128/d^7*e^10*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e
))^(1/2)+3/64/d*e^4*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-15/12
8/d^3*e^8*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)
^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/256*d^3*e^2*c^3*ln((
1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+31/64/d^3/a^2/x*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(7/2)*c-13/32/d^4*e/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)
-15/32*e*d^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3+35/96/e*d^2/a^2*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*c^3+19/96/e^2*d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(5/2)*c^3+85/192*d/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^3+43
/192/e/a^3*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-1/4/d^2/a/e/x^4*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)-1/8/d^5*e^6*a*((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(3/2)*x-3/64/d^5*e^8*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*
(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^5\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

3.300 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx$

Optimal. Leaf size=289

$$-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}}+\frac{3\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+}}{128a^2d^3e^2x^2}$$

Rubi [A] time = 0.33, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 806, 720, 724, 206}

$$\frac{3\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128a^2d^3e^2x^2}-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5dx^5}-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (3*(c*d^2 - a*e^2)^3*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(128*a^2*d^3*e^2*x^2) - ((c/(a*e) - e/d^2)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(16*x^4)
- (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*d*x^5) - (3*(c*d^2 - a*e
^2)^5*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5dx^5}-\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\int \frac{(ade+(\cdots}{\cdots}}{2ade}\cdots$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{16x^4}-\cdots$$

$$=\frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}-\cdots$$

$$=\frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}-\cdots$$

$$=\frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}-\cdots$$

Mathematica [A] time = 0.94, size = 295, normalized size = 1.02

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(\frac{5\left(cd^2-ae^2\right)\left(\frac{x\left(cd^2-ae^2\right)\left(\frac{x\left(ae^2-cd^2\right)\left(3x^2\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(ae(2d+5ex)-3cd^2x\right)\right)}{a^{5/2}\sqrt{d}e^{5/2}}-8(d+ex)^{5/2}\sqrt{ae+cdx}\right)}{d}-16(d+ex)^{5/2}(ae+cdx)^{3/2}\right)}{64dx^4(d+ex)^{3/2}(ae+cdx)^{3/2}}-\frac{2(d+ex)(ae+cdx)}{x^5}\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*((-2*(a*e + c*d*x)*(d + e*x))/x^5 + (5*(c*
d^2 - a*e^2)*(-16*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + ((c*d^2 - a*e^2)*x*
(-8*Sqrt[a*e + c*d*x]*(d + e*x)^(5/2) + ((-(c*d^2) + a*e^2)*x*(Sqrt[a]*Sqrt
[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c*d^2*x + a*e*(2*d + 5*e*x)
) + 3*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sq
rt[e]*Sqrt[d + e*x])]))/(a^(5/2)*Sqrt[d]*e^(5/2))))/d))/(64*d*x^4*(a*e + c*
d*x)^(3/2)*(d + e*x)^(3/2))))/(10*d)
```

IntegrateAlgebraic [F] time = 180.10, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d +
e*x)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 19.50, size = 872, normalized size = 3.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d
^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e^2 + (
c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*
d*e^3)*x)/x^2) - 4*(128*a^5*d^5*e^5 - (15*a*c^4*d^9*e - 70*a^2*c^3*d^7*e^3
- 128*a^3*c^2*d^5*e^5 + 70*a^4*c*d^3*e^7 - 15*a^5*d*e^9)*x^4 + 2*(5*a^2*c^3
*d^8*e^2 + 233*a^3*c^2*d^6*e^4 + 23*a^4*c*d^4*e^6 - 5*a^5*d^2*e^8)*x^3 + 8*
(31*a^3*c^2*d^7*e^3 + 64*a^4*c*d^5*e^5 + a^5*d^3*e^7)*x^2 + 16*(21*a^4*c*d^
6*e^4 + 11*a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^
3*d^4*e^3*x^5), 1/1280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4
 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-a*d*e)*x^5*arctan
(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x
)) - 2*(128*a^5*d^5*e^5 - (15*a*c^4*d^9*e - 70*a^2*c^3*d^7*e^3 - 128*a^3*c^
2*d^5*e^5 + 70*a^4*c*d^3*e^7 - 15*a^5*d*e^9)*x^4 + 2*(5*a^2*c^3*d^8*e^2 + 2
33*a^3*c^2*d^6*e^4 + 23*a^4*c*d^4*e^6 - 5*a^5*d^2*e^8)*x^3 + 8*(31*a^3*c^2*
d^7*e^3 + 64*a^4*c*d^5*e^5 + a^5*d^3*e^7)*x^2 + 16*(21*a^4*c*d^6*e^4 + 11*a
^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^
5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-
2*exp(1)^3*a^3*exp(2)^3+6*exp(1)^5*a^3*exp(2)^2-6*exp(1)^7*a^3*exp(2)+2*exp
(1)^9*a^3)/2/d^3/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*ex
p(1))+(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*
x)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))-(-3*a^5*exp(2)^5-10*exp(1
)^2*a^5*exp(2)^4-80*exp(1)^4*a^5*exp(2)^3+480*exp(1)^6*a^5*exp(2)^2-640*exp
(1)^8*a^5*exp(2)+256*exp(1)^10*a^5-15*c*d^2*a^4*exp(2)^4-30*c^2*d^4*a^3*exp
(2)^3+60*c^2*d^4*exp(1)^2*a^3*exp(2)^2-30*c^3*d^6*a^2*exp(2)^2+80*c^3*d^6*e
xp(1)^2*a^2*exp(2)-80*c^3*d^6*exp(1)^4*a^2-15*c^4*d^8*a*exp(2)+30*c^4*d^8*e
xp(1)^2*a-3*c^5*d^10)/128/d^3/exp(1)^2/a^2/2/sqrt(-a*d*exp(1))*atan((sqrt(a
*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)/sqrt(-a*d*
exp(1)))-(-45*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^9*a^5*exp(2)^5-150*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^4+2640*exp(1)^4*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)^
3-4320*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*
d*exp(1))*x)^9*a^5*exp(2)^2+1920*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^5*exp(2)-225*c*d^2*(sqrt(a*d*exp
(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^4*exp(2)^4-4
```

50*c^2*d^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^3*exp(2)^3+900*c^2*d^4*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^3*exp(2)^2-450*c^3*d^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2*exp(2)^2+1200*c^3*d^6*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2*exp(2)+2640*c^3*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a^2-225*c^4*d^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a*exp(2)+450*c^4*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9*a-45*c^5*d^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^9+3840*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5*exp(2)^3-11520*d*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5*exp(2)^2+11520*d*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5*exp(2)-3840*d*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^5-11520*c^2*d^5*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^3*exp(2)-7680*c^3*d^7*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^8*a^2+210*d*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)^5-580*d*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)^4-8480*d*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)^3+16320*d*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)^2-7680*d*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^6*exp(2)+1050*c*d^3*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)^4+3840*c*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)^3-11520*c*d^3*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)^2+11520*c*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5*exp(2)-3840*c*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^5+2100*c^2*d^5*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^3+15000*c^2*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^4*exp(2)^2+2100*c^3*d^7*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3*exp(2)^2+16160*c^3*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3*exp(2)+3040*c^3*d^7*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^3+1050*c^4*d^9*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2*exp(2)+5580*c^4*d^9*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a^2+210*c^5*d^11*exp(1)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^7*a-3840*d^2*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)^4-3840*d^2*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)^3+34560*d^2*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)^2-42240*d^2*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6*exp(2)+15360*d^2*exp(1)^10*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^6-15360*c*d^4*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5*exp(2)^3-11520*c*d^4*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5*exp(2)^2+11520*c*d^4*exp(1)^6*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5*exp(2)-3840*c*d^4*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^

```
2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^5-23040*c^2*d^6*exp(1)^2*sqrt(c*d*exp(1))
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*
a^4*exp(2)^2-11520*c^2*d^6*exp(1)^4*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^4*exp(2)-15360*c^3*d^
8*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp
(1))-sqrt(c*d*exp(1))*x)^6*a^3*exp(2)-3840*c^3*d^8*exp(1)^4*sqrt(c*d*exp(1)
)*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6
*a^3-3840*c^4*d^10*exp(1)^2*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^6*a^2+384*d^2*exp(1)^2*(sqrt(a*d*
exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^
5+1280*d^2*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)^5*a^7*exp(2)^4+10240*d^2*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp
(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)^3-23040*d^2*ex
p(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))
*x)^5*a^7*exp(2)^2+11520*d^2*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+
c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^7*exp(2)+1920*c*d^4*exp(1)^2*(sqrt(
a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp
(2)^4+11520*c*d^4*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^3+11520*c*d^4*exp(1)^6*(sqrt(a*d*exp(1
)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6*exp(2)^2-268
80*c*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^5*a^6*exp(2)+11520*c*d^4*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(
2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^6+3840*c^2*d^6*exp(1)^2*
(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a
^5*exp(2)^3+26880*c^2*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*
x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*exp(2)^2+11520*c^2*d^6*exp(1)^6*(sqrt
(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^5*ex
p(2)-3840*c^2*d^6*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(
1))-sqrt(c*d*exp(1))*x)^5*a^5+3840*c^3*d^8*exp(1)^2*(sqrt(a*d*exp(1)+a*x*ex
p(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4*exp(2)^2+24320*c^3*d
^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*ex
p(1))*x)^5*a^4*exp(2)+10240*c^3*d^8*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^4+1920*c^4*d^10*exp(1)^2*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^3*e
xp(2)+7680*c^4*d^10*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*ex
p(1))-sqrt(c*d*exp(1))*x)^5*a^3+384*c^5*d^12*exp(1)^2*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^5*a^2-38400*d^3*exp(1)^7
*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(
c*d*exp(1))*x)^4*a^7*exp(2)^2+57600*d^3*exp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7*exp(2)
-23040*d^3*exp(1)^11*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^7-19200*c*d^5*exp(1)^5*sqrt(c*d*exp(
1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)
^4*a^6*exp(2)^2-6400*c*d^5*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*e
xp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^6*exp(2)+6400*c*d^5*e
xp(1)^9*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1)
)-sqrt(c*d*exp(1))*x)^4*a^6-38400*c^2*d^7*exp(1)^5*sqrt(c*d*exp(1))*(sqrt(a
*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5*exp(
2)-3840*c^2*d^7*exp(1)^7*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2
*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^4*a^5-19200*c^3*d^9*exp(1)^5*sqrt(c*
d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)^4*a^4-210*d^3*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*e
xp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^5-700*d^3*exp(1)^5*(sqrt(a*d*exp(1)
+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^4-5600
*d^3*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*
exp(1))*x)^3*a^8*exp(2)^3+14400*d^3*exp(1)^9*(sqrt(a*d*exp(1)+a*x*exp(2)+c*
d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^8*exp(2)^2-7680*d^3*exp(1)^11
*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*
a^8*exp(2)-1050*c*d^5*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
```

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exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)^4+19200*c*d^5*exp(1)^9*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^7*exp(2)-1
1520*c*d^5*exp(1)^11*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sq
rt(c*d*exp(1))*x)^3*a^7-2100*c^2*d^7*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c
*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)^3+4200*c^2*d^7*exp(
1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x
)^3*a^6*exp(2)^2+19200*c^2*d^7*exp(1)^7*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x
+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^6*exp(2)-2100*c^3*d^9*exp(1)^3*(sq
rt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*
exp(2)^2+5600*c^3*d^9*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*
exp(1))-sqrt(c*d*exp(1))*x)^3*a^5*exp(2)+13600*c^3*d^9*exp(1)^7*(sqrt(a*d*e
xp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^5-1050*c^4
*d^11*exp(1)^3*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d
*exp(1))*x)^3*a^4*exp(2)+2100*c^4*d^11*exp(1)^5*(sqrt(a*d*exp(1)+a*x*exp(2)
+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^4-210*c^5*d^13*exp(1)^3*(s
qrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^3*a^3
+19200*d^4*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8*exp(2)^2-34560*d^4*exp(1)^10*sqrt(c
*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp
(1))*x)^2*a^8*exp(2)+15360*d^4*exp(1)^12*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+
a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^8-6400*c*d^6*exp
(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-
sqrt(c*d*exp(1))*x)^2*a^7*exp(2)-1280*c*d^6*exp(1)^10*sqrt(c*d*exp(1))*(sqr
t(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^7-7
680*c^2*d^8*exp(1)^8*sqrt(c*d*exp(1))*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c
*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2*a^6+45*d^4*exp(1)^4*(sqrt(a*d*exp(1)+a
*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^5+150*d^4*
exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1
))*x)*a^9*exp(2)^4+1200*d^4*exp(1)^8*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*
d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^3-3360*d^4*exp(1)^10*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^9*exp(2)^2
+1920*d^4*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqr
t(c*d*exp(1))*x)*a^9*exp(2)+225*c*d^6*exp(1)^4*(sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^8*exp(2)^4-3840*c*d^6*exp(1)^
10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*
a^8*exp(2)+3840*c*d^6*exp(1)^12*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2
*exp(1))-sqrt(c*d*exp(1))*x)*a^8+450*c^2*d^8*exp(1)^4*(sqrt(a*d*exp(1)+a*x*
exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7*exp(2)^3-900*c^2*d^8
*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(
1))*x)*a^7*exp(2)^2+3840*c^2*d^8*exp(1)^10*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^
2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^7+450*c^3*d^10*exp(1)^4*(sqrt(a*d
*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6*exp(2)^2
-1200*c^3*d^10*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))
-sqrt(c*d*exp(1))*x)*a^6*exp(2)+1200*c^3*d^10*exp(1)^8*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^6+225*c^4*d^12*exp(1)
^4*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*
a^5*exp(2)-450*c^4*d^12*exp(1)^6*(sqrt(a*d*exp(1)+a*x*exp(2)+c*d^2*x+c*d*x^
2*exp(1))-sqrt(c*d*exp(1))*x)*a^5+45*c^5*d^14*exp(1)^4*(sqrt(a*d*exp(1)+a*x
*exp(2)+c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)*a^4-3840*d^5*exp(1)^9*s
qrt(c*d*exp(1))*a^9*exp(2)^2+7680*d^5*exp(1)^11*sqrt(c*d*exp(1))*a^9*exp(2)
-3840*d^5*exp(1)^13*sqrt(c*d*exp(1))*a^9+1280*c*d^7*exp(1)^9*sqrt(c*d*exp(1
))*a^8*exp(2)-1280*c*d^7*exp(1)^11*sqrt(c*d*exp(1))*a^8-768*c^2*d^9*exp(1)^
9*sqrt(c*d*exp(1))*a^7)/3840/d^3/exp(1)^2/a^2/((sqrt(a*d*exp(1)+a*x*exp(2)+
c*d^2*x+c*d*x^2*exp(1))-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a)^5)
```

maple [B] time = 0.04, size = 3991, normalized size = 13.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^6/(e*x+d),x)
```

```
[Out] -1/16/d^3*e^4*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+13/40/d^3/a/x
^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+17/160/e/a^3*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(5/2)*c^3-3/128/d^4*e^7*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*a^2+15/128/d^3*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c-1/128/d^5*
e^6*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+1/5/d^6*e^5*((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(5/2)+3/128*e^3*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(
x+d/e))^(1/2)-15/128*e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+25/128
/d^6*e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)-31/80/d^4*e/a/x^3*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+3/32*d^2*e/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)*c^3-15/128*d^3*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a
*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3+1/5/d/a^3*c^3*(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x+109/320/d^3/a^2/x^2*(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(7/2)*c+1/8/d^6*e^7*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x
+d/e))^(3/2)*x+3/64/d^6*e^9*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(
1/2)-1/8/d^4*e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x+1/16/d^7
*e^8*a^2/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)+9/64/d^5*e^8*a^2*(
(x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/128/d^8*e^11*a^4/c^2*((x+d
/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-3/64/d^2*e^5*a*c*((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(1/2)+15/128/d^4*e^9*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+
d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c
*d*e)^(1/2)+3/64/d*e^4*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-
3/256*d^2*e^3*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+
d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/256*e^5*a*c^2*l
n((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2
-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+11/320/a^4/e^3/x*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(7/2)*c^3-1/80/a^3/e^3/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(7/2)*c^2-9/128*d^4/a^2/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^4+3/128
*d^6/a^3/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^5+1/128*d^5/a^4/e^4*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c^5+3/640*d^4/a^5/e^5*(c*d*e*x^2+a*
d*e+(a*e^2+c*d^2)*x)^(5/2)*c^5-7/128*d^3/a^3/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(3/2)*c^4-17/640*d^2/a^4/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2
)*c^4+15/256*d^5/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2
)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4-3/64*d^3/a^2*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^4-1/5/d^2/a/e/x^5*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(7/2)-1/8/d^6*e^7*a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x-15/2
56*e^5*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a*c^2-9/64/d^5*e^8*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2)*x*a^2-253/640/d^6*e^3/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(7/2)+15/128/d^4*e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c-15/128/
d^4*e^9*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^3-3/64/d^6*e^9/c*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2)*a^3+15/128/d^2*e^5*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)*a+3/256*d^2*e^3*c^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+3/256/d^3*e^8*a^3/(a*d*
e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2))/x)+273/640/d^4*e^3/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
5/2)*c+129/320/d^5*e^2/a/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+47/160
/d^2*e/a^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+139/320/d^3*e^2/a^2*
c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x+15/128*d*e^4*a/(a*d*e)^(1/2)*
ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2))/x)*c^2-139/320/d^4*e/a^2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2
)*c-15/256/d*e^6*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1
/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+15/128/d^2*e^7*c*ln((c*d*
e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2))/(c*d*e)^(1/2)*a^2+15/128/d^3*e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/
2)*x*a*c+15/256/d^6*e^11/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(
c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^4+253/640/d^5*e^4*c
/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-5/64*d^2/a^3/e*(c*d*e*x^2+a*d*
```

```
e+(a*e^2+c*d^2)*x)^(3/2)*x*c^4-1/5/d^2/a^2/e/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(7/2)*c-11/320*d/a^4/e^2*c^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2
)*x+3/640*d^3/a^5/e^4*c^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-3/640*d
^2/a^5/e^5/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^4+19/320/d/a^3/e^2/x
^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2-1/320*d/a^4/e^4/x^2*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^3+1/128*d^4/a^4/e^3*(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(3/2)*x*c^5+3/128*d^5/a^3/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2)*x*c^5-3/256*d^7/a^2/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^5+3/40/d/a^2/e
^2/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c-3/64/d^7*e^10*a^3/c*((x+d/
e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-9/64/d^3*e^6*a*c*((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256/d^8*e^13*a^5/c^2*ln((1/2*a*e^2-1/2*c*
d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1
/2))/(c*d*e)^(1/2)-15/128/d^2*e^7*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d
*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1
/2)-15/256/d^6*e^11*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1
/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/256/d^8*
e^13*a^5/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+3/64/d^7*e^10*a^3/c*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*x-1/5/d^2/e/a^3/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(7/2)*c^2+5/64/d^2*e^3/a*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^2-1/
16/d^7*e^8*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+3/128/d^8*e^11*a^4
/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^6\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**6/(e*x+d),x)
```

```
[Out] Timed out
```

3.301 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=386

$$\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024a^{7/2}d^{9/2}e^{7/2}}-\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+\right.}{512a^3d^4e}$$

Rubi [A] time = 0.49, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512a^3d^4e^3x^2}+\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{192a^2d^3e^2x^4}+\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024a^{7/2}d^{9/2}e^{7/2}}-\frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60x^5}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] -((c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*a^3*d^4*e^3*x^2) + ((c*d^2 - a
*e^2)*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))/(192*a^2*d^3*e^2*x^4) - (a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(5/2)/(6*d*x^6) - (((5*c)/(a*e) - (7*e)/d^2)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(60*x^5) + ((c*d^2 - a*e^2)^5*(5*c*d^2 +
7*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(1024*a^(7/2)*d^(9/2)*e^(7/2)
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{x^7(d + ex)}\,dx = \int \frac{(ae + cdx)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{x^7}\,dx$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{6dx^6} - \frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-7ae^2\right)+acde^2x\right)\left(ade+\left(cd^2+ae^2\right)\right.}{x^6}}{6ade}$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae} - \frac{7e}{d^2}\right)\left(ade + \left(cd^2 + ae^2\right)x + cde\right.}{60x^5}$$

$$= \frac{\left(cd^2 - ae^2\right)\left(5cd^2 + 7ae^2\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\left(ade + \left(cd^2 + ae^2\right)x + \right.}{192a^2d^3e^2x^4}$$

$$= -\frac{\left(cd^2 - ae^2\right)^3\left(5cd^2 + 7ae^2\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{ade + \left(cd^2 + ae^2\right)x}}{512a^3d^4e^3x^2}$$

$$= -\frac{\left(cd^2 - ae^2\right)^3\left(5cd^2 + 7ae^2\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{ade + \left(cd^2 + ae^2\right)x}}{512a^3d^4e^3x^2}$$

$$= -\frac{\left(cd^2 - ae^2\right)^3\left(5cd^2 + 7ae^2\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{ade + \left(cd^2 + ae^2\right)x}}{512a^3d^4e^3x^2}$$

Mathematica [A] time = 0.99, size = 344, normalized size = 0.89

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(-\frac{\left(7ae^2+5cd^2\right)\left(5x\left(cd^2-ae^2\right)\left(\frac{x\left(cd^2-ae^2\right)\left(\frac{x\left(ae^2-cd^2\right)\left(3x^2\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(ae(2d+5ex)-3cd^2x\right)\right)}{a^{5/2}\sqrt{d}e^{5/2}}-8(d+ex)^{5/2}\sqrt{ae+cdx}\right)}{d}-16(d+ex)^{5/2}(ae+cdx)^{3/2}\right)-128d(d+ex)^{5/2}(ae+cdx)^{5/2}\right)}{1280d^2x^5(d+ex)^{3/2}(ae+cdx)^{3/2}}-\frac{(d+ex)(ae+cdx)^2}{x^6}\right)}{6ade}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-(((a*e + c*d*x)^2*(d + e*x))/x^6) - ((5*
c*d^2 + 7*a*e^2)*(-128*d*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2) + 5*(c*d^2 - a
*e^2)*x*(-16*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + ((c*d^2 - a*e^2)*x*(-8*S
qrt[a*e + c*d*x]*(d + e*x)^(5/2) + ((-(c*d^2) + a*e^2)*x*(Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c*d^2*x + a*e*(2*d + 5*e*x)) + 3
*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]
*Sqrt[d + e*x])]))/(a^(5/2)*Sqrt[d]*e^(5/2))))/d)))/(1280*d^2*x^5*(a*e + c*
d*x)^(3/2)*(d + e*x)^(3/2))))/(6*a*d*e)
```

IntegrateAlgebraic [F] time = 180.31, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d +
e*x)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 60.75, size = 1072, normalized size = 2.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(5*c^6*d^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3
*c^3*d^6*e^6 - 45*a^4*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(a*
d*e)*x^6*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d^6*e^6 + (75*a
*c^5*d^11*e - 245*a^2*c^4*d^9*e^3 + 150*a^3*c^3*d^7*e^5 - 546*a^4*c^2*d^5*e
^7 + 415*a^5*c*d^3*e^9 - 105*a^6*d*e^11)*x^5 - 2*(25*a^2*c^4*d^10*e^2 - 80*
a^3*c^3*d^8*e^4 - 174*a^4*c^2*d^6*e^6 + 136*a^5*c*d^4*e^8 - 35*a^6*d^2*e^10
)*x^4 + 8*(5*a^3*c^3*d^9*e^3 + 423*a^4*c^2*d^7*e^5 + 27*a^5*c*d^5*e^7 - 7*a
^6*d^3*e^9)*x^3 + 16*(135*a^4*c^2*d^8*e^4 + 278*a^5*c*d^6*e^6 + 3*a^6*d^4*e
^8)*x^2 + 128*(25*a^5*c*d^7*e^5 + 13*a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^6), -1/15360*(15*(5*c^6*d^12 - 18*a*c
^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3*c^3*d^6*e^6 - 45*a^4*c^2*d^4*e^8
+ 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(-a*d*e)*x^6*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(
a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(1280*a^6*d
^6*e^6 + (75*a*c^5*d^11*e - 245*a^2*c^4*d^9*e^3 + 150*a^3*c^3*d^7*e^5 - 546
*a^4*c^2*d^5*e^7 + 415*a^5*c*d^3*e^9 - 105*a^6*d*e^11)*x^5 - 2*(25*a^2*c^4*
d^10*e^2 - 80*a^3*c^3*d^8*e^4 - 174*a^4*c^2*d^6*e^6 + 136*a^5*c*d^4*e^8 - 3
5*a^6*d^2*e^10)*x^4 + 8*(5*a^3*c^3*d^9*e^3 + 423*a^4*c^2*d^7*e^5 + 27*a^5*c
*d^5*e^7 - 7*a^6*d^3*e^9)*x^3 + 16*(135*a^4*c^2*d^8*e^4 + 278*a^5*c*d^6*e^6
 + 3*a^6*d^4*e^8)*x^2 + 128*(25*a^5*c*d^7*e^5 + 13*a^6*d^5*e^7)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^6)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [B] time = 0.06, size = 4735, normalized size = 12.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^7/(e*x+d),x)
```

```
[Out] 1/16/d^4*e^5*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/128/d*e^4*c^
2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+7/1536/d^6*a*e^7*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(3/2)+7/512/d^5*a^2*e^8*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)-35/384/d^4*e^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*c+17/60
/d^3/a/x^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+1/512*d^3/a^2*(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^4-59/320/d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(5/2)*c^3+25/512/d*e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+9/
64/d^6*e^9*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^2+15/128/d^5*e^10*ln
((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2))/(c*d*e)^(1/2)*a^3-101/512/d^7*e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(5/2)-1/5/d^7*e^6*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(5/2)+3/64/d^
7*e^10/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3-3/256*d*e^4*c^3*ln((c*
d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/(c*d*e)^(1/2)+1017/2560/d^7/a*e^4/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(7/2)-1/512*d^5/a^6/e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^6-5/512
*d^7/a^4/e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^6-397/960/d^5/a*e^4*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c-35/1536/d^2/a*e^3*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*c^2+49/1536*d^2/a^3/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(3/2)*c^4-5/64/d^3*a*e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c-26
81/7680/d^3/a^2*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^2+1/64*d^5/a^
3/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^5-221/7680*d/a^4/e^2*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^4-57/160/d^4/a*e/x^4*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(7/2)+7/384*d^4/a^4/e^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
3/2)*c^5-1/64*d/a*e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-5/256/a*e
^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3-11/480/a^4/e^3/x^2*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^3-45/1024*a*e^5/(a*d*e)^(1/2)*ln((2*a*d*
e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/
x)*c^2-1/32/a^3/e^3/x^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2-81/1280
/a^4/e*c^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-1/6/d^2/a/e/x^6*(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)+35/768*d/a^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(3/2)*x*c^4+5/256*d^2*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3-185/1536/d^5
*e^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c+381/1280/d^3/a^3/x*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2+15/512/d^2*e^5*(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*x*c^2+89/320/d^3/a^2/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(7/2)*c-7/1024/d^4*a^3*e^9/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*
d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+1/120*d^3/a^5/e^4*(c
*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*c^5-1543/3840/d^6/a*e^3/x^2*(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)-5/1536*d^6/a^5/e^5*(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(3/2)*c^6+377/960/d^5/a*e^2/x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(7/2)+1/8/d^5*e^6*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-1/8/d^7
*e^8*a*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)*x-3/64/d^7*e^10*a^3/c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/16/d^8*e^9*a^2/c*((x+d/e)^2
*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(3/2)-9/64/d^6*e^9*a^2*((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(1/2)*x+3/128/d^9*e^12*a^4/c^2*((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)+3/64/d^3*e^6*a*c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e)
)^(1/2)-15/128/d^5*e^10*a^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^
(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-3/64/d^2
```

```
*e^5*c^2*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*d*e^4*c^3*ln
((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-
c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/8/d^7*e^8*a*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(3/2)*x+1/16/d^8*e^9*a^2/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2
)-3/128/d^9*e^12*a^4/c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+3/64/d^8*e
^11*a^3/c*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+9/64/d^4*e^7*a*c*
((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/256/d^9*e^14*a^5/c^2*ln((
1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+15/128/d^3*e^8*a^2*c*ln((1/2*a*e^2-1/2*c
*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(
1/2))/(c*d*e)^(1/2)-15/256/d*e^6*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*
e)/(c*d*e)^(1/2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/
2)+15/256/d^7*e^12*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/
2)+((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-15/128/d^3*
e^8*c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-3/64/d^8*e^11*a^3/c*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2)*x+3/256/d^9*e^14*a^5/c^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*
d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+1
5/256/d*e^6*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a*c^2-15/256/d^7*e^12/c*ln((c*d*e*x+
1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))
/(c*d*e)^(1/2)*a^4+15/512/d^2*a^2*e^7/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c+3/512*d^
4/a^3/e*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^5-5/512*d^6/a^4/e^3*(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^6-25/768/d/a^2*e^2*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(3/2)*x*c^3+89/7680*d^2/a^5/e^3*c^5*(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(5/2)*x-1/512*d^4/a^6/e^5*c^6*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(5/2)*x+1/768*d^2/a^5/e^5/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^
4+43/1536*d^3/a^4/e^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^5-5/1536*
d^5/a^5/e^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^6+29/320/d/a^3/e^2/
x^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^2+1/192*d/a^4/e^4/x^3*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^3-9/512*d^6/a^2/e/(a*d*e)^(1/2)*ln((2*a*
d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
)/x)*c^5+5/1024*d^8/a^3/e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*
d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^6+81/1280/d/a^4/e^
2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^3-89/7680*d/a^5/e^4/x*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c^4+1/512*d^3/a^6/e^6/x*(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(7/2)*c^5-11/30/d^4/a^2*e/x^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(7/2)*c+3211/7680/d^5/a^2*e^2/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2
)*c-1017/2560/d^6/a*e^5*c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-3211/76
80/d^4/a^2*e^3*c^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-381/1280/d^2/a
^3*e*c^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)*x-65/512/d^4*a*e^7*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c+7/256*d^2/a^2*e*(c*d*e*x^2+a*d*e+(a*e^
2+c*d^2)*x)^(1/2)*x*c^4+1/12/d/a^2/e^2/x^5*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(7/2)*c+15/1024*d^4/a*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*
e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^4-65/1536/d^3/a*e^4*
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)*x*c^2-43/240/d^2/a^2/e/x^4*(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2)*c-113/640/d^2/a^3/e/x^2*(c*d*e*x^2+a*d*e+(a
*e^2+c*d^2)*x)^(7/2)*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^7\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**7/(e*x+d),x)
[Out] Timed out
```

3.302 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx$

Optimal. Leaf size=500

$$\frac{\left(-63a^2e^4+20acd^2e^2+35c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{840a^2d^3e^2x^5}-\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)^5\tan}{2048a^{9/2}d^{11/2}}$$

Rubi [A] time = 0.64, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {849, 834, 806, 720, 724, 206}

$$\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024a^4d^5e^4x^2}-\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{384a^3d^4e^3x^4}+\frac{\left(-63a^2e^4+20acd^2e^2+35c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{840a^2d^3e^2x^5}-\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2048a^{9/2}d^{11/2}e^{9/2}}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{84x^6}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] ((c*d^2 - a*e^2)^3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*a^4*d^5*e
^4*x^2) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*
a^3*d^4*e^3*x^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(7*d*x^7)
- (((5*c)/(a*e) - (9*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))
/(84*x^6) + ((35*c^2*d^4 + 20*a*c*d^2*e^2 - 63*a^2*e^4)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(5/2))/(840*a^2*d^3*e^2*x^5) - ((c*d^2 - a*e^2)^5*(5*c
^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/
(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
2048*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
```

```
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{x^8(d + ex)}\,dx = \int \frac{(ae + cdx)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{x^8}\,dx$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{7dx^7} - \frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2 - 9ae^2\right) + 2acde^2x\right)\left(ade + \left(cd^2 + ae^2\right.\right.}{x^7}}{7ade}$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{7dx^7} - \frac{\left(\frac{5c}{ae} - \frac{9e}{d^2}\right)\left(ade + \left(cd^2 + ae^2\right)x + cde\right.}{84x^6}$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{7dx^7} - \frac{\left(\frac{5c}{ae} - \frac{9e}{d^2}\right)\left(ade + \left(cd^2 + ae^2\right)x + cde\right.}{84x^6}$$

$$= -\frac{\left(cd^2 - ae^2\right)\left(5c^2d^4 + 10acd^2e^2 + 9a^2e^4\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\left(ade + \left(\right.\right.}{384a^3d^4e^3x^4}$$

$$= \frac{\left(cd^2 - ae^2\right)^3\left(5c^2d^4 + 10acd^2e^2 + 9a^2e^4\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{ade +}}{1024a^4d^5e^4x^2}$$

$$= \frac{\left(cd^2 - ae^2\right)^3\left(5c^2d^4 + 10acd^2e^2 + 9a^2e^4\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{ade +}}{1024a^4d^5e^4x^2}$$

$$= \frac{\left(cd^2 - ae^2\right)^3\left(5c^2d^4 + 10acd^2e^2 + 9a^2e^4\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{ade +}}{1024a^4d^5e^4x^2}$$

Mathematica [A] time = 0.75, size = 408, normalized size = 0.82

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(\frac{7(9a^2e^4+10acd^2e^2+5c^2d^4)\left(5x(cd^2-ae^2)\left(\frac{x(cd^2-ae^2)\left(\frac{x(ae^2-cd^2)\left(3x^2(cd^2-ae^2)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}(ae(2d+5ex)-3cd^2x)\right)}{a^{5/2}\sqrt{d}e^{5/2}}-8(d+ex)^{5/2}\sqrt{ae+cdx}\right)}{d}-16(d+ex)^{5/2}(ae+cdx)^{3/2}\right)-128d(d+ex)^{5/2}(ae+cdx)^{5/2}\right)}{15360ad^3ex^5(d+ex)^{3/2}(ae+cdx)^{3/2}}+\frac{(d+ex)(9ae^2+7cd^2)(ae+cdx)^2}{12adex^6}-\frac{(d+ex)(ae+cdx)^2}{x^7}\right)}{7ade}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-(((a*e + c*d*x)^2*(d + e*x))/x^7) + ((7*
c*d^2 + 9*a*e^2)*(a*e + c*d*x)^2*(d + e*x))/(12*a*d*e*x^6) + (7*(5*c^2*d^4
+ 10*a*c*d^2*e^2 + 9*a^2*e^4)*(-128*d*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2) +
 5*(c*d^2 - a*e^2)*x*(-16*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + ((c*d^2 - a
*e^2)*x*(-8*Sqrt[a*e + c*d*x]*(d + e*x)^(5/2) + ((-(c*d^2) + a*e^2)*x*(Sqrt
[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c*d^2*x + a*e*(2*d
+ 5*e*x)) + 3*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sq
rt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(a^(5/2)*Sqrt[d]*e^(5/2))))/d)))/(15360*a*d
^3*e*x^5*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(7*a*d*e)
```

IntegrateAlgebraic [F] time = 184.30, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d +
e*x)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 124.86, size = 1300, normalized size = 2.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/430080*(105*(5*c^7*d^14 - 15*a*c^6*d^12*e^2 + 9*a^2*c^5*d^10*e^4 + 5*a^
3*c^4*d^8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*c^2*d^4*e^10 + 35*a^6*c*d^2*e^1
2 - 9*a^7*e^14)*sqrt(a*d*e)*x^7*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1
5360*a^7*d^7*e^7 - (525*a*c^6*d^13*e - 1400*a^2*c^5*d^11*e^3 + 525*a^3*c^4*
d^9*e^5 + 600*a^4*c^3*d^7*e^7 - 3689*a^5*c^2*d^5*e^9 + 3360*a^6*c*d^3*e^11
- 945*a^7*d*e^13)*x^6 + 2*(175*a^2*c^5*d^12*e^2 - 455*a^3*c^4*d^10*e^4 + 15
0*a^4*c^3*d^8*e^6 - 1166*a^5*c^2*d^6*e^8 + 1099*a^6*c*d^4*e^10 - 315*a^7*d^
2*e^12)*x^5 - 8*(35*a^3*c^4*d^11*e^3 - 90*a^4*c^3*d^9*e^5 - 228*a^5*c^2*d^7
*e^7 + 218*a^6*c*d^5*e^9 - 63*a^7*d^3*e^11)*x^4 + 16*(15*a^4*c^3*d^10*e^4 +
 2095*a^5*c^2*d^8*e^6 + 93*a^6*c*d^6*e^8 - 27*a^7*d^4*e^10)*x^3 + 128*(185*
a^5*c^2*d^9*e^5 + 380*a^6*c*d^7*e^7 + 3*a^7*d^5*e^9)*x^2 + 1280*(29*a^6*c*d
^8*e^6 + 15*a^7*d^6*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a
^5*d^6*e^5*x^7), 1/215040*(105*(5*c^7*d^14 - 15*a*c^6*d^12*e^2 + 9*a^2*c^5*
d^10*e^4 + 5*a^3*c^4*d^8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*c^2*d^4*e^10 + 3
5*a^6*c*d^2*e^12 - 9*a^7*e^14)*sqrt(-a*d*e)*x^7*arctan(1/2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c
*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(15360*a^7*d^7
*e^7 - (525*a*c^6*d^13*e - 1400*a^2*c^5*d^11*e^3 + 525*a^3*c^4*d^9*e^5 + 60
0*a^4*c^3*d^7*e^7 - 3689*a^5*c^2*d^5*e^9 + 3360*a^6*c*d^3*e^11 - 945*a^7*d*
```

```
e^13)*x^6 + 2*(175*a^2*c^5*d^12*e^2 - 455*a^3*c^4*d^10*e^4 + 150*a^4*c^3*d^
8*e^6 - 1166*a^5*c^2*d^6*e^8 + 1099*a^6*c*d^4*e^10 - 315*a^7*d^2*e^12)*x^5
- 8*(35*a^3*c^4*d^11*e^3 - 90*a^4*c^3*d^9*e^5 - 228*a^5*c^2*d^7*e^7 + 218*a
^6*c*d^5*e^9 - 63*a^7*d^3*e^11)*x^4 + 16*(15*a^4*c^3*d^10*e^4 + 2095*a^5*c^
2*d^8*e^6 + 93*a^6*c*d^6*e^8 - 27*a^7*d^4*e^10)*x^3 + 128*(185*a^5*c^2*d^9*
e^5 + 380*a^6*c*d^7*e^7 + 3*a^7*d^5*e^9)*x^2 + 1280*(29*a^6*c*d^8*e^6 + 15*
a^7*d^6*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^5*d^6*e^5*x
^7)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [B] time = 0.09, size = 5353, normalized size = 10.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^8/(e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{x^8\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**8/(e*x+d),x)
```

```
[Out] Timed out
```

3.303 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx$

Optimal. Leaf size=628

$$\frac{\left(-33a^2e^4+10acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{448a^2d^3e^2x^6}-\frac{\left(-231a^3e^6+15a^2cd^2e^4+95ac^2d^4e^2+105c^3\ldots\right.}{4480a^3d^4e^3\ldots}$$

Rubi [A] time = 0.89, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{3\left(45a^2cd^2e^4+33a^3e^6+35ac^2d^4e^2+15c^3d^6\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384a^5d^6e^5x^2}+\frac{\left(45a^2cd^2e^4+33a^3e^6+35ac^2d^4e^2+15c^3d^6\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2048a^4d^5e^4x^4}-\frac{\left(15a^2cd^2e^4-231a^3e^6+95ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4480a^3d^4e^3x^5}+\frac{\left(-33a^2e^4+10acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{448a^2d^3e^2x^6}+\frac{3\left(45a^2cd^2e^4+33a^3e^6+35ac^2d^4e^2+15c^3d^6\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{32768a^{11/2}d^{13/2}e^{11/2}}-\frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{112x^7}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (-3*(c*d^2 - a*e^2)^3*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 3
3*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(16384*a^5*d^6*e^5*x^2) + ((c*d^2 - a*e^2)*(15*c^3*d^6 + 35*a*c^
2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*a^4*d^5*e^4*x^4) - (a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(8*d*x^8) - (((5*c)/(a*e) - (11*e)
/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(112*x^7) + ((15*c^2*d
^4 + 10*a*c*d^2*e^2 - 33*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2))/(448*a^2*d^3*e^2*x^6) - ((105*c^3*d^6 + 95*a*c^2*d^4*e^2 + 15*a^2*c*d
^2*e^4 - 231*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4480*
a^3*d^4*e^3*x^5) + (3*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45
*a^2*c*d^2*e^4 + 33*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[
a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*a^
(11/2)*d^(13/2)*e^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx = \int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^9}\,dx$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-11ae^2\right)+3acde^2x\right)\left(ade+(cd\right.}{x^8}}{8ade}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{112x^7}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{112x^7}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{112x^7}$$

$$= \frac{\left(cd^2-ae^2\right)\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2\right.\right.}{2048a^4d^5e^4x^4}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\right.}{16384a^5d^6e^5x^2}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\right.}{16384a^5d^6e^5x^2}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\right.}{16384a^5d^6e^5x^2}$$

Mathematica [A] time = 1.46, size = 512, normalized size = 0.82

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(-\frac{(d+ex)\left(33a^2e^4+34acd^2e^2+21c^2d^4\right)(ae+cdx)^2}{56a^2d^2e^2x^6}+\frac{\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\left(128a^{5/2}d^{5/2}e^{5/2}(d+ex)^{5/2}(ae+cdx)^{5/2}+5x\left(cd^2-ae^2\right)\left(16a^{5/2}d^{3/2}e^{5/2}(d+ex)^{5/2}(ae+cdx)^{3/2}+x\left(cd^2-ae^2\right)\left(8a^{5/2}\sqrt{d}e^{5/2}(d+ex)^{5/2}\sqrt{ae+cdx}+x\left(cd^2-ae^2\right)\left(3x^2\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(ae(2d+5ex)-3cd^2x\right)\right)\right)\right)\right)}{10240a^{9/2}d^{11/2}e^{9/2}x^5(d+ex)^{3/2}(ae+cdx)^{3/2}}+\frac{(d+ex)\left(11ae^2+9cd^2\right)(ae+cdx)^2}{14adex^7}-\frac{(d+ex)(ae+cdx)^2}{x^8}\right)}{8ade}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-(((a*e + c*d*x)^2*(d + e*x))/x^8) + ((9*
c*d^2 + 11*a*e^2)*(a*e + c*d*x)^2*(d + e*x))/(14*a*d*e*x^7) - ((21*c^2*d^4
+ 34*a*c*d^2*e^2 + 33*a^2*e^4)*(a*e + c*d*x)^2*(d + e*x))/(56*a^2*d^2*e^2*x
^6) + ((15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(128
*a^(5/2)*d^(5/2)*e^(5/2)*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2) + 5*(c*d^2 - a
*e^2)*x*(16*a^(5/2)*d^(3/2)*e^(5/2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + (
c*d^2 - a*e^2)*x*(8*a^(5/2)*Sqrt[d]*e^(5/2)*Sqrt[a*e + c*d*x]*(d + e*x)^(5/
2) + (c*d^2 - a*e^2)*x*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d +
e*x]*(-3*c*d^2*x + a*e*(2*d + 5*e*x)) + 3*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sq
rt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])))))/(10240*a^(9/
2)*d^(11/2)*e^(9/2)*x^5*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(8*a*d*e)
```

IntegrateAlgebraic [F] time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d +
e*x)),x]
```

```
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [B] time = 0.13, size = 6030, normalized size = 9.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/x^9/(e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{x^9\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**9/(e*x+d),x)
```

```
[Out] Timed out
```

3.304 $\int \frac{x^3}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=271

$$\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}-\frac{3\left(ae^2+3cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{4c^2d^2e^3}$$

Rubi [A] time = 0.34, antiderivative size = 298, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 779, 621, 206}

$$\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}-\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cdex\left(5cd^2-ae^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4c^2d^2e^3\left(cd^2-ae^2\right)}-\frac{2dx^2\left(cdx\left(cd^2-ae^2\right)+ae\left(cd^2-ae^2\right)\right)}{e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(e*(c*d^2 - a*e^2)
^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (((5*c*d^2 - 3*a*e^2)*(3*
c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(4*c^2*d^2*e^3*(c*d^2 - a*e^2)) + (3*(5*c^2*d^4 + 2*a*c*d^2
*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*e^(7/
2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
```

```
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \int \frac{x^3(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2\int \frac{x\left(2acd^2e\left(cd^2-ae^2\right.\right.}{\sqrt{ade-}}}{cde}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(\left(5cd^2-3ae^2\right)\left(3\right.\right.}{}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(\left(5cd^2-3ae^2\right)\left(3\right.\right.}{}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(\left(5cd^2-3ae^2\right)\left(3\right.\right.}{}$$

Mathematica [A] time = 0.51, size = 331, normalized size = 1.22

$$\frac{3\sqrt{cd}\sqrt{cd^2-ae^2}\left(-a^3e^6-a^2cd^2e^4-3ac^2d^4e^2+5c^3d^6\right)\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)+c^{3/2}d^{3/2}\sqrt{e}\left(3a^3e^5(d+ex)+a^2cde^3\left(4d^2+5dex+e^2x^2\right)-ac^2d^2e\left(15d^3+d^2ex-4de^2x^2+2e^3x^3\right)+c^3d^4x\left(-15d^2-5dex+2e^2x^2\right)\right)}{4c^{7/2}d^{7/2}e^{7/2}\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (c^(3/2)*d^(3/2)*Sqrt[e]*(3*a^3*e^5*(d + e*x) + a^2*c*d*e^3*(4*d^2 + 5*d*e*
x + e^2*x^2) + c^3*d^4*x*(-15*d^2 - 5*d*e*x + 2*e^2*x^2) - a*c^2*d^2*e*(15*
d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)) + 3*Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]
*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[a*e + c*d*x]*
Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(4*c^(7/2)*d^(7/2)*e^(7/2)
*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.90, size = 758, normalized size = 2.80

$$\left[\frac{3\left(5c^3d^7 - 3ac^2d^5e^2 - a^2cd^3e^4 - a^3de^6 + \left(5c^3d^6e - 3ac^2d^4e^3 - a^2cd^2e^5 - a^3e^7\right)x\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2 + c^2d^4 + 6acd^2e^2 + a^2e^4 + 4\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(2cdex + cd^2 + ae^2\right)\sqrt{cde} + 8\left(c^2d^3e + acde^3\right)x\right) - 4\left(15c^3d^6e - 4ac^2d^4e^3 - 3a^2cd^2e^5 - 2\left(c^3d^4e^3 - ac^2d^2e^5\right)x^2 + \left(5c^3d^5e^2 - 2ac^2d^3e^4 - 3a^2cde^6\right)x\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{16\left(c^4d^6e^4 - ac^3d^4e^6 + \left(c^4d^5e^5 - ac^3d^3e^7\right)x\right)}, -\frac{3\left(5c^3d^7 - 3ac^2d^5e^2 - a^2cd^3e^4 - a^3de^6 + \left(5c^3d^6e - 3ac^2d^4e^3 - a^2cd^2e^5 - a^3e^7\right)x\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(2cdex + cd^2 + ae^2\right)\sqrt{-cde}}{2\left(c^2d^2e^2x^2 + acd^2e^2 + \left(c^2d^3e + acde^3\right)x\right)}\right) + 2\left(15c^3d^6e - 4ac^2d^4e^3 - 3a^2cd^2e^5 - 2\left(c^3d^4e^3 - ac^2d^2e^5\right)x^2 + \left(5c^3d^5e^2 - 2ac^2d^3e^4 - 3a^2cde^6\right)x\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{8\left(c^4d^6e^4 - ac^3d^4e^6 + \left(c^4d^5e^5 - ac^3d^3e^7\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/16*(3*(5*c^3*d^7 - 3*a*c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3*
d^6*e - 3*a*c^2*d^4*e^3 - a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c*d*e)*log(8*c^2
*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*
e + a*c*d*e^3)*x) - 4*(15*c^3*d^6*e - 4*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2
*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (5*c^3*d^5*e^2 - 2*a*c^2*d^3*e^4 - 3*a
^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^6*e^4 -
a*c^3*d^4*e^6 + (c^4*d^5*e^5 - a*c^3*d^3*e^7)*x), -1/8*(3*(5*c^3*d^7 - 3*a*
c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3*d^6*e - 3*a*c^2*d^4*e^3 -
a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*
x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15*c^3*d^6*e - 4*a*c^2
*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (5*c^3*d
^5*e^2 - 2*a*c^2*d^3*e^4 - 3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(c^4*d^6*e^4 - a*c^3*d^4*e^6 + (c^4*d^5*e^5 - a*c^3*d^3*e^7)
*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Evaluation time:
 0.41Error: Bad Argument Type
```

maple [A] time = 0.02, size = 391, normalized size = 1.44

$$\frac{3a^2e\ln\left(\frac{cdex+\frac{1}{2}ae^2+\frac{1}{2}cd^2}{\sqrt{cde}}+\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\right)}{8\sqrt{cde}\,c^2d^2}+\frac{3a\ln\left(\frac{cdex+\frac{1}{2}ae^2+\frac{1}{2}cd^2}{\sqrt{cde}}+\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\right)}{4\sqrt{cde}\,ce}+\frac{15d^2\ln\left(\frac{cdex+\frac{1}{2}ae^2+\frac{1}{2}cd^2}{\sqrt{cde}}+\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\right)}{8\sqrt{cde}\,e^3}+\frac{2\sqrt{\left(x+\frac{d}{e}\right)^2cde+\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}\,d^3}{\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)e^4}+\frac{\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\,x}{2cde^2}-\frac{3\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}\,a}{4c^2d^2e}-\frac{7\sqrt{cdex^2+ade+\left(ae^2+cd^2\right)x}}{4ce^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/2/e^2*x/c/d*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/4/e/c^2/d^2*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a-7/4/e^3/c*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
```

```
*x)^(1/2)+3/8*e/c^2/d^2*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2+3/4/e/c*ln((c*d*e*x+
1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))
/(c*d*e)^(1/2)*a+15/8*d^2/e^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2
)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+2*d^3/e^4/(a*e^2-c
*d^2)/(x+d/e)*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.305 $\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=195

$$-\frac{(ae^2+3cd^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}+\frac{2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+a}}{cde^2}$$

Rubi [A] time = 0.35, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1638, 792, 621, 206}

$$-\frac{(ae^2+3cd^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}+\frac{2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cde^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d*e^2) + (2*d^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(c*d^2 - a*e^2)*(d + e*x)) - ((3*c*d^
2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 1638

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
```

```
0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{\int \frac{-\frac{1}{2}de\left(cd^2+ae^2\right)-\frac{1}{2}e^2\left(3cd^2+ae^2\right)x}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cde^3}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\left(cd^2-ae^2\right)(d+ex)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\left(cd^2-ae^2\right)(d+ex)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\left(cd^2-ae^2\right)(d+ex)}$$

Mathematica [A] time = 0.36, size = 255, normalized size = 1.31

$$\frac{c^{3/2}d^{3/2}\sqrt{e}\left(-a^2e^3(d+ex)+acde\left(3d^2-e^2x^2\right)+c^2d^3x(3d+ex)\right)-\sqrt{cd}\sqrt{cd^2-ae^2}\left(-a^2e^4-2acd^2e^2+3c^2d^4\right)\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{c^{5/2}d^{5/2}e^{5/2}\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (c^(3/2)*d^(3/2)*Sqrt[e]*(-(a^2*e^3*(d + e*x)) + c^2*d^3*x*(3*d + e*x) + a*
c*d*e*(3*d^2 - e^2*x^2)) - Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*(3*c^2*d^4 - 2*a*c
*d^2*e^2 - a^2*e^4)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]
*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2
- a*e^2])])/(c^(5/2)*d^(5/2)*e^(5/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d
+ e*x)])
```

IntegrateAlgebraic [A] time = 1.80, size = 328, normalized size = 1.68

$$\frac{\sqrt{cde}\left(ae^2+3cd^2\right)\log\left(a^2e^4+8cdex\sqrt{cde}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2acd^2e^2-4acde^3x+c^2d^4-4c^2d^3ex-8c^2d^2e^2x^2\right)}{4c^2d^2e^3}+\frac{\left(-ae^2-3cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\left(2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2x\sqrt{cde}\right)}{ae^2+cd^2}\right)}{2c^{3/2}d^{3/2}e^{5/2}}-\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\left(ade^2+ae^3x-3cd^3-cd^2ex\right)}{cde^2(d+ex)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

```
[Out] -(((-3*c*d^3 + a*d*e^2 - c*d^2*e*x + a*e^3*x)*Sqrt[a*d*e + c*d^2*x + a*e^2*
x + c*d*e*x^2])/(c*d*e^2*(c*d^2 - a*e^2)*(d + e*x))) + ((-3*c*d^2 - a*e^2)*
ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-2*Sqrt[c*d*e]*x + 2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]))/(c*d^2 + a*e^2)])/(2*c^(3/2)*d^(3/2)*e^(5/2)) + (
Sqrt[c*d*e]*(3*c*d^2 + a*e^2)*Log[c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4 - 4*c^2
*d^3*e*x - 4*a*c*d*e^3*x - 8*c^2*d^2*e^2*x^2 + 8*c*d*e*Sqrt[c*d*e]*x*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(4*c^2*d^2*e^3)
```

fricas [A] time = 0.57, size = 586, normalized size = 3.01

$$\left[\frac{\left(3c^2d^5-2acd^3e^2-a^2de^4+\left(3c^2d^4e-2acd^2e^3-a^2e^5\right)x\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4-4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdex+cd^2+ae^2\right)\sqrt{cde}+8\left(c^2d^3e+acde^3\right)x\right)+4\left(3c^2d^4e-acd^2e^3+\left(c^2d^3e^2-acde^4\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{4\left(c^3d^5e^3-ac^2d^3e^5+\left(c^3d^4e^4-ac^2d^2e^6\right)x\right)}, \frac{\left(3c^2d^5-2acd^3e^2-a^2de^4+\left(3c^2d^4e-2acd^2e^3-a^2e^5\right)x\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdex+cd^2+ae^2\right)\sqrt{-cde}}{2\left(c^2d^2e^2x^2+acd^2e^2+\left(c^2d^3e+acde^3\right)x\right)}\right)+2\left(3c^2d^4e-acd^2e^3+\left(c^2d^3e^2-acde^4\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{2\left(c^3d^5e^3-ac^2d^3e^5+\left(c^3d^4e^4-ac^2d^2e^6\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/4*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^3
 - a^2*e^5)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(3*c^2*d^4*e - a*
c*d^2*e^3 + (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c^3*d^5*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x),
 1/2*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^3
 - a^2*e^5)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2
*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3*c^2*d^4*e - a*c*d^2*e^3 + (c^2*d^
3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^5
*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Evaluation time: 0.41Err
or: Bad Argument Type
```

maple [A] time = 0.01, size = 241, normalized size = 1.24

$$-\frac{a \ln \left(\frac{c d e x+\frac{1}{2} a e^{2}+\frac{1}{2} c d^{2}}{\sqrt{c d e}}+\sqrt{c d e x^{2}+a d e+\left(a e^{2}+c d^{2}\right) x}\right)}{2 \sqrt{c d e}\, c d}-\frac{3 d \ln \left(\frac{c d e x+\frac{1}{2} a e^{2}+\frac{1}{2} c d^{2}}{\sqrt{c d e}}+\sqrt{c d e x^{2}+a d e+\left(a e^{2}+c d^{2}\right) x}\right)}{2 \sqrt{c d e}\, e^{2}}-\frac{2 \sqrt{\left(x+\frac{d}{e}\right)^{2} c d e+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)}\, d^{2}}{\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right) e^{3}}+\frac{\sqrt{c d e x^{2}+a d e+\left(a e^{2}+c d^{2}\right) x}}{c d e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] (c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/c/d/e^2-1/2/c/d*ln((c*d*e*x+1/2*a*e
^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e
)^(1/2)*a-3/2/e^2*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)-2*d^2/e^3/(a*e^2-c*d^2)/(x+d
/e)*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
[Out] Integral(x**2/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.306 $\int \frac{x}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=139

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\,e^{3/2}}-\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)\left(cd^2-ae^2\right)}$$

Rubi [A] time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {792, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\,e^{3/2}}-\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(c*d^2 - a*e^2)*(d +
e*x)) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{2d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\left(cd^2-ae^2\right)(d+ex)} + \frac{\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{e}$$

$$= -\frac{2d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\left(cd^2-ae^2\right)(d+ex)} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{\quad}{\sqrt{ade\ldots}}\right)}{e}$$

$$= -\frac{2d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\left(cd^2-ae^2\right)(d+ex)} + \frac{\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+a\ldots\right.}}\right)}{\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Mathematica [A] time = 0.42, size = 189, normalized size = 1.36

$$\frac{2\sqrt{cd}\left(cd^2-ae^2\right)^{3/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-2c^{3/2}d^{5/2}\sqrt{e}(ae+cdx)}{c^{3/2}d^{3/2}e^{3/2}\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*c^(3/2)*d^(5/2)*Sqrt[e]*(a*e + c*d*x) + 2*Sqrt[c*d]*(c*d^2 - a*e^2)^(3/
2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(c^(3
/2)*d^(3/2)*e^(3/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [B] time = 0.74, size = 296, normalized size = 2.13

$$-\frac{\sqrt{cde}\log\left(a^2e^4+8cdex\sqrt{cde}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-2acd^2e^2-4acde^3x+c^2d^4-4c^2d^3ex-8c^2d^2e^2x^2\right)}{2cde^2}+\frac{2d\sqrt{ade+ae^2x+cd^2x+cdex^2}}{e(d+ex)\left(ae^2-cd^2\right)}-\frac{\tanh^{-1}\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{e}x\sqrt{cde}}{ae^2+cd^2}-\frac{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{ae^2+cd^2}\right)}{\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
),x]
```

```
[Out] (2*d*Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2])/(e*(-(c*d^2) + a*e^2)*(d
+ e*x)) - ArcTanh[(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[c*d*e]*x)/(c*d^2 + a*e^2)
 - (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(c*d^2 + a*e^2)]/(Sqrt[c]*Sqrt[d]*e^(3/2)) - (Sqrt[c*d*e]*Log[c^2*d^4 - 2*a
*c*d^2*e^2 + a^2*e^4 - 4*c^2*d^3*e*x - 4*a*c*d*e^3*x - 8*c^2*d^2*e^2*x^2 +
8*c*d*e*Sqrt[c*d*e]*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2*c*d*
e^2)
```

fricas [A] time = 0.55, size = 443, normalized size = 3.19

$$\left[-\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,cd^2e-\left(cd^3-ade^2+\left(cd^2e-ae^3\right)x\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdex+cd^2+ae^2\right)\sqrt{cde}+8\left(c^2d^3e+acde^3\right)x\right)}{2\left(c^2d^4e^2-acd^2e^4+\left(c^2d^3e^3-acde^5\right)x\right)}, -\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,cd^2e+\left(cd^3-ade^2+\left(cd^2e-ae^3\right)x\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdex+cd^2+ae^2\right)\sqrt{-cde}}{2\left(c^2d^2e^2x^2+acd^2e^2+\left(c^2d^3e+acde^3\right)x\right)}\right)}{c^2d^4e^2-acd^2e^4+\left(c^2d^3e^3-acde^5\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d^2*e - (c*d^3 - a*d
*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6
*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c
```

```
*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c^2*d^
4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)*x), -(2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*c*d^2*e + (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*
sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)))/(c^2*d^4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)
*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.01, size = 131, normalized size = 0.94

$$\frac{\ln\left(\frac{cdex+\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2}{\sqrt{cde}}+\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}\right)}{\sqrt{cde}\;e}+\frac{2\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}\;d}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/e*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+2*d/e^2/(a*e^2-c*d^2)/(x+d/e)*((x+d/e)^2*c*d
*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(d+ex)\,(ae+cdx)}\;(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.307 $\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)\left(cd^2-ae^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {650}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\left(cd^2-ae^2\right)(d+ex)}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.81

$$\frac{2(ae+cdx)}{\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*(a*e + c*d*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [A] time = 0.00, size = 52, normalized size = 1.00

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x))
```

fricas [A] time = 0.48, size = 59, normalized size = 1.13

$$\frac{2\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{cd^3 - ade^2 + \left(cd^2e - ae^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^3 - a*d*e^2 + (c*d^2*e -
 a*e^3)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2/sqr
t(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(1))+(sqrt(c*d*exp(
1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)*exp(1))/sqrt(-a*d
*exp(1)^3+a*d*exp(1)*exp(2)))
```

maple [A] time = 0.01, size = 51, normalized size = 0.98

$$-\frac{2\,(cdx + ae)}{\left(a\,e^2 - c\,d^2\right)\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2*(c*d*x+a*e)/(a*e^2-c*d^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [B] time = 2.64, size = 50, normalized size = 0.96

$$-\frac{2\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{\left(a\,e^2 - c\,d^2\right)\,(d + e\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] -(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e^2 - c*d^2)*(d + e*
x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(d+ex)(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.308 $\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=143

$$-\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{a}\,d^{3/2}\sqrt{e}}$$

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 822, 12, 724, 206}

$$-\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{a}\,d^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\int \frac{ae+cdx}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$=-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2\int -\frac{ae(\cdots}{2x\sqrt{ade+(\cdots}}}{ade\left(c\cdots\right.}$$

$$=-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\int \frac{1}{x\sqrt{ade+\left(cd^2+\cdots\right.}}}{\cdots}$$

$$=-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2\,\mathrm{Subst}\left(\int \frac{\cdots}{4a\cdots}\right.}{\cdots}$$

$$=-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\tanh^{-1}\left(\frac{\cdots}{2\sqrt{a}\,\sqrt{\cdots}}\right.}{\cdots}$$

Mathematica [A] time = 0.13, size = 131, normalized size = 0.92

$$\frac{2\left(-\frac{\sqrt{d}\,e^{3/2}(ae+cdx)}{cd^2-ae^2}-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{ae+cdx}}{\sqrt{a}\,\sqrt{e}\,\sqrt{d+ex}}\right)}{\sqrt{a}}\right)}{d^{3/2}\sqrt{e}\,\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*(-((Sqrt[d]*e^(3/2)*(a*e + c*d*x))/(c*d^2 - a*e^2)) - (Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d
+ e*x])])/Sqrt[a]))/(d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [A] time = 0.55, size = 146, normalized size = 1.02

$$\frac{2\tanh^{-1}\left(\frac{x\sqrt{cde}}{\sqrt{a}\,\sqrt{d}\,\sqrt{e}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\,\sqrt{d}\,\sqrt{e}}\right)}{\sqrt{a}\,d^{3/2}\sqrt{e}}-\frac{2e\sqrt{ade+ae^2x+cd^2x+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

```
[Out] (-2*e*Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2])/(d*(c*d^2 - a*e^2)*(d +
e*x)) + (2*ArcTanh[(Sqrt[c*d*e]*x)/(Sqrt[a]*Sqrt[d]*Sqrt[e]) - Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[a]*Sqrt[d]*Sqrt[e])])/(Sqrt[a]*d^(3/2
)*Sqrt[e])
```

fricas [A] time = 0.77, size = 454, normalized size = 3.17

$$\left[-\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,ade^2-\left(cd^3-ade^2+\left(cd^2e-ae^3\right)x\right)\sqrt{ade}\,\log\left(\frac{8a^2d^2e^2+\left(c^2d^4+6acd^2e^2+a^2e^4\right)x^2-4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}+8\left(acd^3e+a^2de^3\right)x}{x^2}\right)}{2\left(acd^5e-a^2d^3e^3+\left(acd^4e^2-a^2d^2e^4\right)x\right)},-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,ade^2-\left(cd^3-ade^2+\left(cd^2e-ae^3\right)x\right)\sqrt{-ade}\,\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{-ade}}{2\left(acd^2e^2x^2+a^2d^2e^2+\left(acd^3e+a^2de^3\right)x\right)}\right)}{acd^5e-a^2d^3e^3+\left(acd^4e^2-a^2d^2e^4\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*d
*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2
))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d^4*e^2 - a^2*d^2*e^4)*x), -(2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*d*e^2 + (c*d^2*e -
a*e^3)*x)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^
2 + (a*c*d^3*e + a^2*d*e^3)*x)))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d^4*e^2 -
a^2*d^2*e^4)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.02, size = 136, normalized size = 0.95

$$-\frac{\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}}{x}\right)}{\sqrt{ade}\;d}+\frac{2\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+2/d/(a*e^2-c*d^2)/(x+d/e)*((x+d/e)^2*c*d*e+
(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{(d+ex)\,(ae+cdx)}\,\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
[Out] Integral(1/(x*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.309 $\int \frac{1}{x^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=229

$$\frac{\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(cd^2-3ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{ad^2ex\left(cd^2-ae^2\right)}-\frac{}{dx\left(cd^2-ae^2\right)}$$

Rubi [A] time = 0.29, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 822, 806, 724, 206}

$$\frac{\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(cd^2-3ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{ad^2ex\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{dx\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]) - ((c*d^2 - 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(a*d^2*e*(c*d^2 - a*e^2)*x) + ((c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d
^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])])/(2*a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
```

```
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \int \frac{ae+cdx}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\int \frac{-\frac{1}{2}ae\left(cd^2\right.}{x^2}}{}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-3ae^2\right.}{}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-3ae^2\right.}{}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-3ae^2\right.}{}$$

Mathematica [A] time = 0.12, size = 201, normalized size = 0.88

$$\frac{x\sqrt{d+ex}\left(-3a^2e^4+2acd^2e^2+c^2d^4\right)\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\left(a^2e^3(d+3ex)-acde\left(d^2-3e^2x^2\right)-c^2d^3x(d+ex)\right)}{a^{3/2}d^{5/2}e^{3/2}x\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c^2*d^3*x*(d + e*x)) + a^2*e^3*(d + 3*e*x) - a*
c*d*e*(d^2 - 3*e^2*x^2)) + (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*x*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])])/(a^(3/2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)*x*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

IntegrateAlgebraic [A] time = 0.97, size = 178, normalized size = 0.78

$$\frac{\left(-3ae^2-cd^2\right)\tanh^{-1}\left(\frac{x\sqrt{cde}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{a^{3/2}d^{5/2}e^{3/2}}+\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\left(-ade^2-3ae^3x+cd^3+cd^2ex\right)}{ad^2ex(d+ex)\left(ae^2-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

```
[Out] ((c*d^3 - a*d*e^2 + c*d^2*e*x - 3*a*e^3*x)*Sqrt[a*d*e + c*d^2*x + a*e^2*x +
 c*d*e*x^2])/(a*d^2*e*(-(c*d^2) + a*e^2)*x*(d + e*x)) + ((-(c*d^2) - 3*a*e^
2)*ArcTanh[(Sqrt[c*d*e]*x - Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(S
qrt[a]*Sqrt[d]*Sqrt[e])])/(a^(3/2)*d^(5/2)*e^(3/2))
```

fricas [A] time = 1.54, size = 610, normalized size = 2.66

$$\left[\frac{\sqrt{ade}\left(\left(c^2d^4e+2\,acd^2e^3-3\,a^2e^5\right)x^2+\left(c^2d^5+2\,acd^3e^2-3\,a^2de^4\right)x\right)\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2+4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}+8\left(acd^3e+a^2de^3\right)x}{x^2}\right)-4\left(acd^4e-a^2d^2e^3+\left(acd^3e^2-3\,a^2de^4\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{4\left(\left(a^2cd^5e^3-a^3d^3e^5\right)x^2+\left(a^2cd^6e^2-a^3d^4e^4\right)x\right)},-\frac{\sqrt{-ade}\left(\left(c^2d^4e+2\,acd^2e^3-3\,a^2e^5\right)x^2+\left(c^2d^5+2\,acd^3e^2-3\,a^2de^4\right)x\right)\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,ade+\left(cd^2+ae^2\right)x\right)\sqrt{-ade}}{2\left(acd^2e^2x^2+a^2d^2e^2+\left(acd^3e+a^2de^3\right)x\right)}\right)+2\left(acd^4e-a^2d^2e^3+\left(acd^3e^2-3\,a^2de^4\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{2\left(\left(a^2cd^5e^3-a^3d^3e^5\right)x^2+\left(a^2cd^6e^2-a^3d^4e^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/4*(sqrt(a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + (c^2*d^5 +
 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(
a*c*d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^2*c*d^6*
e^2 - a^3*d^4*e^4)*x), -1/2*(sqrt(-a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a
^2*e^5)*x^2 + (c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*
d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(a*c*
d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^2*c*d^6*e^2
- a^3*d^4*e^4)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(2*
exp(1)^2/2/d^2/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp(
1))+(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)
*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))-(a*exp(2)+2*exp(1)^2*a+c*d^
2)/d^2/exp(1)/a/2/sqrt(-a*d*exp(1))*atan((sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c
*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-((sqrt(c*d*exp(1)*
x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)*a*exp(2)+c*d^2*(sqrt
(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)-2*d*exp(
1)*sqrt(c*d*exp(1))*a)/2/d^2/exp(1)/a/((sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d
^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)^2-d*exp(1)*a))
```

maple [A] time = 0.02, size = 253, normalized size = 1.10

$$\frac{c\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}}{x}\right)}{2\sqrt{ade}\,ae}+\frac{3e\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}}{x}\right)}{2\sqrt{ade}\,d^2}-\frac{2\sqrt{\left(x+\frac{d}{e}\right)^2cde+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)e}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)d^2}-\frac{\sqrt{cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x}}{a\,d^2ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/d^2/a/e/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+3/2*e/d^2/(a*d*e)^(1/2
)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
```

```
)*x)^(1/2))/x)+1/2/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-2/d^2*e/(a*e^2-c*d^2)/(
x+d/e)*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,(ex + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{(d + ex)\,(ae + cdx)}\,\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.310 $\int \frac{1}{x^3(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=329

$$\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4a^2d^3e^2x\left(cd^2-ae^2\right)}-\frac{3\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right.}}\right.}{8a^{5/2}d^{7/2}e^{5/2}}$$

Rubi [A] time = 0.51, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {851, 822, 834, 806, 724, 206}

$$-\frac{3\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}}+\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4a^2d^3e^2x\left(cd^2-ae^2\right)}-\frac{\left(cd^2-5ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2ad^2ex^2\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{dx^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) - ((c*d^2 - 5*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(2*a*d^2*e*(c*d^2 - a*e^2)*x^2) + ((3*c*d^2 - 5*a*e^2)*(c*d^2 + 3*a*e^
2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*a^2*d^3*e^2*(c*d^2 - a*e
^2)*x) - (3*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(8*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
```

```
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \int \frac{ae+cdx}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\int \frac{-\frac{1}{2}ae(cd}{\cdots}}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-5ae\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-5ae\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-5ae\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(cd^2-5ae\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.19, size = 283, normalized size = 0.86

$$\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^3e^4\left(2d^2-5dex-15e^2x^2\right)-a^2cde^2\left(2d^3-4d^2ex+de^2x^2+15e^3x^3\right)+ac^2d^3ex\left(d^2+5dex+4e^2x^2\right)+3c^3d^5x^2(d+ex)\right)-3x^2\sqrt{d+ex}\left(-5a^3e^6+3a^2cd^2e^4+ac^2d^4e^2+c^3d^6\right)\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{4a^{5/2}d^{7/2}e^{5/2}x^2\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^3*d^5*x^2*(d + e*x) + a^3*e^4*(2*d^2 - 5*d*e*
x - 15*e^2*x^2) + a*c^2*d^3*e*x*(d^2 + 5*d*e*x + 4*e^2*x^2) - a^2*c*d*e^2*(
2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 15*e^3*x^3)) - 3*(c^3*d^6 + a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*x^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(
Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(4*a^(5/2)*d^(
7/2)*e^(5/2)*(c*d^2 - a*e^2)*x^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [A] time = 1.45, size = 256, normalized size = 0.78

$$\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\left(-2a^2d^2e^3+5a^2de^4x+15a^2e^5x^2+2acd^4e-2acd^3e^2x-4acd^2e^3x^2-3c^2d^5x-3c^2d^4ex^2\right)}{4a^2d^3e^2x^2(d+ex)\left(ae^2-cd^2\right)}+\frac{3\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{x\sqrt{cde}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{4a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

```
[Out] (Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2]*(2*a*c*d^4*e - 2*a^2*d^2*e^3 -
 3*c^2*d^5*x - 2*a*c*d^3*e^2*x + 5*a^2*d*e^4*x - 3*c^2*d^4*e*x^2 - 4*a*c*d^
2*e^3*x^2 + 15*a^2*e^5*x^2))/(4*a^2*d^3*e^2*(-(c*d^2) + a*e^2)*x^2*(d + e*x
)) + (3*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(Sqrt[c*d*e]*x - Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[a]*Sqrt[d]*Sqrt[e])])/(4*a^(
5/2)*d^(7/2)*e^(5/2))
```

fricas [A] time = 3.72, size = 792, normalized size = 2.41

$$\left[\frac{3\left(\left(c^3d^6e+ac^2d^4e^3+3a^2cd^2e^5-5a^3e^7\right)x^3+\left(c^3d^7+ac^2d^5e^2+3a^2cd^3e^4-5a^3de^6\right)x^2\right)\sqrt{ade}\log\left(\frac{8a^2d^2e^2+\left(c^2d^4+6acd^2e^2+a^2e^4\right)x^2-4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}+8\left(acd^3e+a^2de^3\right)x}{x^2}\right)-4\left(2a^2cd^5e^2-2a^3d^3e^4-\left(3ac^2d^5e^2+4a^2cd^3e^4-15a^3de^6\right)x^2-\left(3ac^2d^6e+2a^2cd^4e^3-5a^3d^2e^5\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{16\left(\left(a^3cd^6e^4-a^4d^4e^6\right)x^3+\left(a^3cd^7e^3-a^4d^5e^5\right)x^2\right)}, \frac{3\left(\left(c^3d^6e+ac^2d^4e^3+3a^2cd^2e^5-5a^3e^7\right)x^3+\left(c^3d^7+ac^2d^5e^2+3a^2cd^3e^4-5a^3de^6\right)x^2\right)\sqrt{-ade}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{-ade}}{2\left(acd^2e^2x^2+a^2d^2e^2+\left(acd^3e+a^2de^3\right)x\right)}\right)-2\left(2a^2cd^5e^2-2a^3d^3e^4-\left(3ac^2d^5e^2+4a^2cd^3e^4-15a^3de^6\right)x^2-\left(3ac^2d^6e+2a^2cd^4e^3-5a^3d^2e^5\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{8\left(\left(a^3cd^6e^4-a^4d^4e^6\right)x^3+\left(a^3cd^7e^3-a^4d^5e^5\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/16*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^7)*x^3 + (
c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sqrt(a*d*e)*l
og((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e)
+ 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e^4 -
(3*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6*e + 2
*a^2*c*d^4*e^3 - 5*a^3*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x))/((a^3*c*d^6*e^4 - a^4*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*e^5)*x^2
), 1/8*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^7)*x^3 +
(c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sqrt(-a*d*e)
*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d
*e^3)*x)) - 2*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e^4 - (3*a*c^2*d^5*e^2 + 4*a^2*c
*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6*e + 2*a^2*c*d^4*e^3 - 5*a^3*d^2
*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c*d^6*e^4 - a^4
*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*e^5)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(-2
*exp(1)^3/2/d^3/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2))*atan((-d*sqrt(c*d*exp
(1))+(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x
)*exp(1))/sqrt(-a*d*exp(1)^3+a*d*exp(1)*exp(2)))+(3*a^2*exp(2)^2+4*exp(1)^2
*a^2*exp(2)+8*exp(1)^4*a^2+6*c*d^2*a*exp(2)+3*c^2*d^4)/4/d^3/exp(1)^2/a^2/2
/sqrt(-a*d*exp(1))*atan((sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)
-sqrt(c*d*exp(1))*x)/sqrt(-a*d*exp(1)))-(-3*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)
+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)^3*a^2*exp(2)^2-4*exp(1)^2*(sqrt(c*
d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)^3*a^2*exp(2
)-6*c*d^2*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(
1))*x)^3*a*exp(2)-3*c^2*d^4*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2)
)*x)-sqrt(c*d*exp(1))*x)^3+8*d*exp(1)^3*sqrt(c*d*exp(1))*(sqrt(c*d*exp(1)*x
^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)^2*a^2+5*d*exp(1)*(sqr
t(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)*a^3*exp
(2)^2+4*d*exp(1)^3*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt
(c*d*exp(1))*x)*a^3*exp(2)+10*c*d^3*exp(1)*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+
(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)*a^2*exp(2)+8*c*d^3*exp(1)^3*(sqrt(c
*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)*a^2+5*c^2*
d^5*exp(1)*(sqrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp
(1))*x)*a-8*d^2*exp(1)^2*sqrt(c*d*exp(1))*a^3*exp(2)-8*d^2*exp(1)^4*sqrt(c*
d*exp(1))*a^3-8*c*d^4*exp(1)^2*sqrt(c*d*exp(1))*a^2)/8/d^3/exp(1)^2/a^2/((s
qrt(c*d*exp(1)*x^2+a*d*exp(1)+(c*d^2+a*exp(2))*x)-sqrt(c*d*exp(1))*x)^2-d*e
xp(1)*a)^2)
```

maple [A] time = 0.02, size = 414, normalized size = 1.26

$$-\frac{3c\ln\left(\frac{2ade+\left(a e^2+c d^2\right)x+2\sqrt{ade}\,\sqrt{cde\,x^2+ade+\left(a e^2+c d^2\right)x}}{x}\right)}{4\sqrt{ade}\,ad}-\frac{3c^2d\ln\left(\frac{2ade+\left(a e^2+c d^2\right)x+2\sqrt{ade}\,\sqrt{cde\,x^2+ade+\left(a e^2+c d^2\right)x}}{x}\right)}{8\sqrt{ade}\,a^2e^2}-\frac{15e^2\ln\left(\frac{2ade+\left(a e^2+c d^2\right)x+2\sqrt{ade}\,\sqrt{cde\,x^2+ade+\left(a e^2+c d^2\right)x}}{x}\right)}{8\sqrt{ade}\,d^3}+\frac{2\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}\,e^2}{\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)d^3}+\frac{7\sqrt{cde\,x^2+ade+\left(a e^2+c d^2\right)x}}{4a\,d^3x}+\frac{3\sqrt{cde\,x^2+ade+\left(a e^2+c d^2\right)x}\,c}{4a^2d\,e^2x}-\frac{\sqrt{cde\,x^2+ade+\left(a e^2+c d^2\right)x}}{2a\,d^2e\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 7/4/d^3/a/x*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-15/8/d^3*e^2/(a*d*e)^(1
/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2))/x)-3/4/d/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-1/2/d^2/a/e/x^2*(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+3/4/d/a^2/e^2/x*(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*c-3/8*d/a^2/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2+2/d^3*e^2/(a
*e^2-c*d^2)/(x+d/e)*((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{(d+ex)(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
[Out] Integral(1/(x**3*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

3.311 $\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=515

$$\frac{5\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{7/2}d^{7/2}e^{9/2}}-\frac{2x^2\left(ade\left(cd^2-ae^2\right)\left(-3a^2e^4-12acd^2e^2+\cdots\right.\right.}{3cde^2\left(cd^2-\cdots\right.}$$

Rubi [A] time = 0.62, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 779, 621, 206}

$$-\frac{2x^2\left(x\left(cd^2-ae^2\right)\left(-a^2cd^2e^4-3a^3e^6-11ac^2d^4e^2+7c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(-3a^2e^4-12acd^2e^2+7c^2d^4\right)\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(-2cdex\left(9a^2cd^2e^4-15a^3e^6-61ac^2d^4e^2+35c^3d^6\right)+36a^2c^2d^4e^4+30a^3cd^2e^6-45a^4e^8-190ac^3d^6e^2+105c^4d^8\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12c^3d^3e^4\left(cd^2-ae^2\right)^3}+\frac{5\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{7/2}d^{7/2}e^{9/2}}-\frac{2dx^4\left(cdx\left(cd^2-ae^2\right)+ae\left(cd^2-ae^2\right)\right)}{3e\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^4*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x^2*(a*d*e*(c*d^2
- a*e^2)*(7*c^2*d^4 - 12*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(7*c^3*
d^6 - 11*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 -
 a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((105*c^4*d^8 - 19
0*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 30*a^3*c*d^2*e^6 - 45*a^4*e^8 - 2*c*
d*e*(35*c^3*d^6 - 61*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 15*a^3*e^6)*x)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^3*d^3*e^4*(c*d^2 - a*e^2)^3)
+ (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d
*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(8*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
```

```
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^5(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$=-\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\int \frac{x^3\left(4acd^2e\cdots\right.}{\cdots}}{\cdots}$$

$$=-\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x^2\left(ade\left(c\cdots\right.\right.}{\cdots}$$

$$=-\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x^2\left(ade\left(c\cdots\right.\right.}{\cdots}$$

$$=-\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x^2\left(ade\left(c\cdots\right.\right.}{\cdots}$$

$$=-\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x^2\left(ade\left(c\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 5.63, size = 296, normalized size = 0.57

$$\frac{\frac{2(d+ex)^2(ae+cdx)^2\left(\frac{24a^5e^9}{c^3\left(cd^2-ae^2\right)^3(ae+cdx)}-\frac{3\left(7ae^2+11cd^2\right)}{c^3}+\frac{8d^8}{(d+ex)^2\left(cd^2-ae^2\right)^2}+\frac{40\left(3ad^7e^2-2cd^9\right)}{(d+ex)\left(cd^2-ae^2\right)^3}+\frac{6dex}{c^2}\right)}{3d^3e^4}+\frac{5(d+ex)^{3/2}\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)(ae+cdx)^{3/2}\log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}+ae^2+cd(d+2ex)\right)}{c^{7/2}d^{7/2}e^{9/2}}}{8((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] ((2*(a*e + c*d*x)^2*(d + e*x)^2*((-3*(11*c*d^2 + 7*a*e^2))/c^3 + (6*d*e*x)/
c^2 + (24*a^5*e^9)/(c^3*(c*d^2 - a*e^2)^3*(a*e + c*d*x)) + (8*d^8)/((c*d^2
- a*e^2)^2*(d + e*x)^2) + (40*(-2*c*d^9 + 3*a*d^7*e^2))/((c*d^2 - a*e^2)^3*
```

```
(d + e*x))))/(3*d^3*e^4) + (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(a*e
+ c*d*x)^(3/2)*(d + e*x)^(3/2)*Log[a*e^2 + 2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a
*e + c*d*x]*Sqrt[d + e*x] + c*d*(d + 2*e*x)])/(c^(7/2)*d^(7/2)*e^(9/2)))/(8
*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [F] time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 6.65, size = 2120, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/48*(15*(7*a*c^5*d^12*e - 15*a^2*c^4*d^10*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^4
*c^2*d^6*e^7 + 3*a^5*c*d^4*e^9 - 3*a^6*d^2*e^11 + (7*c^6*d^11*e^2 - 15*a*c^
5*d^9*e^4 + 6*a^2*c^4*d^7*e^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^10 - 3*
a^5*c*d*e^12)*x^3 + (14*c^6*d^12*e - 23*a*c^5*d^10*e^3 - 3*a^2*c^4*d^8*e^5
+ 10*a^3*c^3*d^6*e^7 + 8*a^4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^11 - 3*a^6*e^13)*x
^2 + (7*c^6*d^13 - a*c^5*d^11*e^2 - 24*a^2*c^4*d^9*e^4 + 14*a^3*c^3*d^7*e^6
 + 7*a^4*c^2*d^5*e^8 + 3*a^5*c*d^3*e^10 - 6*a^6*d*e^12)*x)*sqrt(c*d*e)*log(
8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2
*d^3*e + a*c*d*e^3)*x) - 4*(105*a*c^5*d^11*e^2 - 190*a^2*c^4*d^9*e^4 + 36*a
^3*c^3*d^7*e^6 + 30*a^4*c^2*d^5*e^8 - 45*a^5*c*d^3*e^10 - 6*(c^6*d^9*e^4 -
3*a*c^5*d^7*e^6 + 3*a^2*c^4*d^5*e^8 - a^3*c^3*d^3*e^10)*x^4 + 3*(7*c^6*d^10
*e^3 - 16*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 + 8*a^3*c^3*d^4*e^9 - 5*a^4*c^2
*d^2*e^11)*x^3 + (140*c^6*d^11*e^2 - 237*a*c^5*d^9*e^4 + 12*a^2*c^4*d^7*e^6
 + 66*a^3*c^3*d^5*e^8 - 45*a^5*c*d*e^12)*x^2 + (105*c^6*d^12*e - 50*a*c^5*d
^10*e^3 - 222*a^2*c^4*d^8*e^5 + 84*a^3*c^3*d^6*e^7 + 45*a^4*c^2*d^4*e^9 - 9
0*a^5*c*d^2*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^7*d^
12*e^6 - 3*a^2*c^6*d^10*e^8 + 3*a^3*c^5*d^8*e^10 - a^4*c^4*d^6*e^12 + (c^8*
d^11*e^7 - 3*a*c^7*d^9*e^9 + 3*a^2*c^6*d^7*e^11 - a^3*c^5*d^5*e^13)*x^3 + (
2*c^8*d^12*e^6 - 5*a*c^7*d^10*e^8 + 3*a^2*c^6*d^8*e^10 + a^3*c^5*d^6*e^12 -
 a^4*c^4*d^4*e^14)*x^2 + (c^8*d^13*e^5 - a*c^7*d^11*e^7 - 3*a^2*c^6*d^9*e^9
 + 5*a^3*c^5*d^7*e^11 - 2*a^4*c^4*d^5*e^13)*x), -1/24*(15*(7*a*c^5*d^12*e -
 15*a^2*c^4*d^10*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^4*c^2*d^6*e^7 + 3*a^5*c*d^4*
e^9 - 3*a^6*d^2*e^11 + (7*c^6*d^11*e^2 - 15*a*c^5*d^9*e^4 + 6*a^2*c^4*d^7*e
^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^10 - 3*a^5*c*d*e^12)*x^3 + (14*c^6
*d^12*e - 23*a*c^5*d^10*e^3 - 3*a^2*c^4*d^8*e^5 + 10*a^3*c^3*d^6*e^7 + 8*a^
4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^11 - 3*a^6*e^13)*x^2 + (7*c^6*d^13 - a*c^5*d^
11*e^2 - 24*a^2*c^4*d^9*e^4 + 14*a^3*c^3*d^7*e^6 + 7*a^4*c^2*d^5*e^8 + 3*a^
5*c*d^3*e^10 - 6*a^6*d*e^12)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(105*a*c^5*d^11*e^2
 - 190*a^2*c^4*d^9*e^4 + 36*a^3*c^3*d^7*e^6 + 30*a^4*c^2*d^5*e^8 - 45*a^5*c
*d^3*e^10 - 6*(c^6*d^9*e^4 - 3*a*c^5*d^7*e^6 + 3*a^2*c^4*d^5*e^8 - a^3*c^3*
d^3*e^10)*x^4 + 3*(7*c^6*d^10*e^3 - 16*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 +
8*a^3*c^3*d^4*e^9 - 5*a^4*c^2*d^2*e^11)*x^3 + (140*c^6*d^11*e^2 - 237*a*c^5
*d^9*e^4 + 12*a^2*c^4*d^7*e^6 + 66*a^3*c^3*d^5*e^8 - 45*a^5*c*d*e^12)*x^2 +
 (105*c^6*d^12*e - 50*a*c^5*d^10*e^3 - 222*a^2*c^4*d^8*e^5 + 84*a^3*c^3*d^6
```

```
*e^7 + 45*a^4*c^2*d^4*e^9 - 90*a^5*c*d^2*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a*c^7*d^12*e^6 - 3*a^2*c^6*d^10*e^8 + 3*a^3*c^5*d^8*e^
10 - a^4*c^4*d^6*e^12 + (c^8*d^11*e^7 - 3*a*c^7*d^9*e^9 + 3*a^2*c^6*d^7*e^1
1 - a^3*c^5*d^5*e^13)*x^3 + (2*c^8*d^12*e^6 - 5*a*c^7*d^10*e^8 + 3*a^2*c^6*
d^8*e^10 + a^3*c^5*d^6*e^12 - a^4*c^4*d^4*e^14)*x^2 + (c^8*d^13*e^5 - a*c^7
*d^11*e^7 - 3*a^2*c^6*d^9*e^9 + 5*a^3*c^5*d^7*e^11 - 2*a^4*c^4*d^5*e^13)*x)
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.03, size = 1680, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 15/8*e^4/c^3/d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*x*a^4+5/2*e^2/c^2/d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*a^3-7/16/e^3/c^2/(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*a+51/16/e^5/c*d^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-
9/4/e^3/c*x^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+51/8/e^4*c*d^5/(-a^2*
e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+15/16*
e^5/c^4/d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)*a^5+35/16*e^3/c^3/d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4-15/4/e^2/c^2/d*x/(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*a+15/4/e^2/c^2/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(
1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a-8/3*d^6/e^3*c
/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*a-16/3*d^7/e
^4*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/4/
c*d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2
)*x*a^2-5/4/e/c^2/d^2*x^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+21/8/e/
c*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1
/2)*a^2+11/2/e^2*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2)*x*a+9/8*e/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+95/16/e^3*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^
2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+2*d^4/e^5*(2*c*d*e*x+a*e^2
+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)+15/8/c^3/d^3*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2
+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)*a^2-15/8/c^3/d^3*x/(c*d*e*x^2+
a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2-35/8/e^4/c*d*x/(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2)+15/16*e/c^4/d^4/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+51
/16/e^5*c*d^6/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2)+35/8/e^4/c*d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+2/3*d^5/e^6/(a*e^2-c*d^
2)/(x+d/e)/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-8/3*d^8/e^5*c^2/(a
*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+5/16/e/c^3/d^2/
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+1/2/e^2*x^3/c/d/(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.312 $\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=438

$$-\frac{2x\left(ade\left(cd^2-ae^2\right)\left(-3a^2e^4-10acd^2e^2+5c^2d^4\right)+x\left(cd^2-ae^2\right)\left(-3a^3e^6-a^2cd^2e^4-9ac^2d^4e^2+5c^3d^6\right)\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\left(-9a^3\right.}{}$$

Rubi [A] time = 0.54, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 640, 621, 206}

$$-\frac{2x\left(x\left(cd^2-ae^2\right)\left(-a^2cd^2e^4-3a^3e^6-9ac^2d^4e^2+5c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(-3a^2e^4-10acd^2e^2+5c^2d^4\right)\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\left(9a^2cd^2e^4-9a^3e^6-31ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2e^3\left(cd^2-ae^2\right)^3}-\frac{\left(3ae^2+5cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{5/2}d^{5/2}e^{7/2}}-\frac{2dx^3\left(cdx\left(cd^2-ae^2\right)+ae\left(cd^2-ae^2\right)\right)}{3e\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^3*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x*(a*d*e*(c*d^2 -
a*e^2)*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(5*c^3*d^
6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*
e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((15*c^3*d^6 - 31*a*c
^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(3*c^2*d^2*e^3*(c*d^2 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*ArcTan
h[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(5/2)*d^(5/2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
```

```
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^4(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x^2(3a}{}}{}$$

$$= -\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x\left(ade\right.}{}$$

$$= -\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x\left(ade\right.}{}$$

$$= -\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x\left(ade\right.}{}$$

$$= -\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x\left(ade\right.}{}$$

Mathematica [A] time = 1.34, size = 387, normalized size = 0.88

$$\frac{(ae+cdx)\left(-\frac{ae\left(3ae^2-cd^2\right)\left(a^2e^2\left(8d^2+12dex+3e^2x^2\right)+2acd^2ex(2d+3ex)-c^2d^4x^2\right)}{cd\left(cd^2-ae^2\right)^3}-\frac{\left(3ae^2+5cd^2\right)\sqrt{ae+cdx}\left(c^{3/2}d^{7/2}\sqrt{e}\left(cd^2-ae^2\right)\sqrt{ae+cdx}-(d+ex)\left(2c^{3/2}d^{5/2}\sqrt{e}\left(2cd^2-3ae^2\right)\sqrt{ae+cdx}-3\sqrt{cd}\left(cd^2-ae^2\right)^{5/2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)\right)\right)}{c^{5/2}d^{5/2}e^{5/2}\left(cd^2-ae^2\right)^2}+3x^3\right)}{3cde((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] ((a*e + c*d*x)*(3*x^3 - (a*e*(-(c*d^2) + 3*a*e^2)*(-(c^2*d^4*x^2) + 2*a*c*d
^2*e*x*(2*d + 3*e*x) + a^2*e^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2)))/(c*d*(c*d^2
 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*Sqrt[a*e + c*d*x]*(c^(3/2)*d^(7/2)*Sqrt
[e]*(c*d^2 - a*e^2)*Sqrt[a*e + c*d*x] - (d + e*x)*(2*c^(3/2)*d^(5/2)*Sqrt[e
]*(2*c*d^2 - 3*a*e^2)*Sqrt[a*e + c*d*x] - 3*Sqrt[c*d]*(c*d^2 - a*e^2)^(5/2)
*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
```

```
t[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])))/(c^(5/2)*d^(5/2)*e^(5/2
)*(c*d^2 - a*e^2)^2)))/(3*c*d*e*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [F] time = 180.17, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 2.66, size = 1782, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/12*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^6*e^5 + 4*a^4*c
*d^4*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 + 6*a^2*c^3*d^
5*e^6 + 4*a^3*c^2*d^3*e^8 - 3*a^4*c*d*e^10)*x^3 + (10*c^5*d^10*e - 19*a*c^4
*d^8*e^3 + 14*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 - 3*a^5*e^11)*x^2 + (5*c^5*
d^11 - 2*a*c^4*d^9*e^2 - 18*a^2*c^3*d^7*e^4 + 16*a^3*c^2*d^5*e^6 + 5*a^4*c*
d^3*e^8 - 6*a^5*d*e^10)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*
d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(15*a
*c^4*d^9*e^2 - 31*a^2*c^3*d^7*e^4 + 9*a^3*c^2*d^5*e^6 - 9*a^4*c*d^3*e^8 + 3
*(c^5*d^8*e^3 - 3*a*c^4*d^6*e^5 + 3*a^2*c^3*d^4*e^7 - a^3*c^2*d^2*e^9)*x^3
+ (20*c^5*d^9*e^2 - 39*a*c^4*d^7*e^4 + 9*a^2*c^3*d^5*e^6 + 3*a^3*c^2*d^3*e^
8 - 9*a^4*c*d*e^10)*x^2 + (15*c^5*d^10*e - 11*a*c^4*d^8*e^3 - 33*a^2*c^3*d^
6*e^5 + 15*a^3*c^2*d^4*e^7 - 18*a^4*c*d^2*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a*c^6*d^11*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a^3*c^4*d^7*e^9
 - a^4*c^3*d^5*e^11 + (c^7*d^10*e^6 - 3*a*c^6*d^8*e^8 + 3*a^2*c^5*d^6*e^10
- a^3*c^4*d^4*e^12)*x^3 + (2*c^7*d^11*e^5 - 5*a*c^6*d^9*e^7 + 3*a^2*c^5*d^7
*e^9 + a^3*c^4*d^5*e^11 - a^4*c^3*d^3*e^13)*x^2 + (c^7*d^12*e^4 - a*c^6*d^1
0*e^6 - 3*a^2*c^5*d^8*e^8 + 5*a^3*c^4*d^6*e^10 - 2*a^4*c^3*d^4*e^12)*x), 1/
6*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^6*e^5 + 4*a^4*c*d^4
*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 + 6*a^2*c^3*d^5*e^
6 + 4*a^3*c^2*d^3*e^8 - 3*a^4*c*d*e^10)*x^3 + (10*c^5*d^10*e - 19*a*c^4*d^8
*e^3 + 14*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 - 3*a^5*e^11)*x^2 + (5*c^5*d^11
 - 2*a*c^4*d^9*e^2 - 18*a^2*c^3*d^7*e^4 + 16*a^3*c^2*d^5*e^6 + 5*a^4*c*d^3*
e^8 - 6*a^5*d*e^10)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 +
 a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15*a*c^4*d^9*e^2 - 31*a^2*c
^3*d^7*e^4 + 9*a^3*c^2*d^5*e^6 - 9*a^4*c*d^3*e^8 + 3*(c^5*d^8*e^3 - 3*a*c^4
*d^6*e^5 + 3*a^2*c^3*d^4*e^7 - a^3*c^2*d^2*e^9)*x^3 + (20*c^5*d^9*e^2 - 39*
a*c^4*d^7*e^4 + 9*a^2*c^3*d^5*e^6 + 3*a^3*c^2*d^3*e^8 - 9*a^4*c*d*e^10)*x^2
 + (15*c^5*d^10*e - 11*a*c^4*d^8*e^3 - 33*a^2*c^3*d^6*e^5 + 15*a^3*c^2*d^4*
e^7 - 18*a^4*c*d^2*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*
c^6*d^11*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a^3*c^4*d^7*e^9 - a^4*c^3*d^5*e^11 + (
c^7*d^10*e^6 - 3*a*c^6*d^8*e^8 + 3*a^2*c^5*d^6*e^10 - a^3*c^4*d^4*e^12)*x^3
 + (2*c^7*d^11*e^5 - 5*a*c^6*d^9*e^7 + 3*a^2*c^5*d^7*e^9 + a^3*c^4*d^5*e^11
 - a^4*c^3*d^3*e^13)*x^2 + (c^7*d^12*e^4 - a*c^6*d^10*e^6 - 3*a^2*c^5*d^8*e
^8 + 5*a^3*c^4*d^6*e^10 - 2*a^4*c^3*d^4*e^12)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.01, size = 1266, normalized size = 2.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -3/2*e^3/c^2/d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*x*a^3+5/2/e^3/c*x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/
4/c^3/d^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2-9/4/e^4/c*d/(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-5/2/e^3/c*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(
c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+16/3*d^
6/e^3*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+8
/3*d^5/e^2*c/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*
a-9/2/e*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)
*x)^(1/2)*x*a-9/2/e^3*c*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d
*e+(a*e^2+c*d^2)*x)^(1/2)*x-3/4*e^4/c^3/d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4
)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^4-3/2*e^2/c^2/d/(-a^2*e^4+2*a*c
*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3-3/2/e/c^2/d^2
*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d*e*x^2+a*d*e+(a*e^2+c*d
^2)*x)^(1/2))/(c*d*e)^(1/2)*a+3/2/e/c^2/d^2*x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(1/2)*a-3/2*e/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e
^2+c*d^2)*x)^(1/2)*x*a^2-2/3*d^4/e^5/(a*e^2-c*d^2)/(x+d/e)/((x+d/e)^2*c*d*e
+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d^7/e^4*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*
d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-2*d^3/e^4*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d
^2*e^2-(a*e^2+c*d^2)^2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/e^2*x^2/c
/d/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/c*d/(-a^2*e^4+2*a*c*d^2*e^2-c^
2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2-9/2/e^2*d^3/(-a^2*e^4+2*
a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a-9/4/e^4*c*d^
5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(x**4/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.313 $\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=297

$$-\frac{2\left(x\left(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+3c^3d^6\right)+ade\left(cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\right)}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ad}}\right)}{c^{3/2}d^{3/2}e^{5/2}}$$

Rubi [A] time = 0.29, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 777, 621, 206}

$$-\frac{2\left(x\left(-a^2cd^2e^4-3a^3e^6-7ac^2d^4e^2+3c^3d^6\right)+ade\left(cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\right)}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}}-\frac{2dx^2\left(cdx\left(cd^2-ae^2\right)+ae\left(cd^2-ae^2\right)\right)}{3e\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*(a*d*e*(c*d^2 - 3*
a*e^2)*(3*c*d^2 + a*e^2) + (3*c^3*d^6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3
*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(c^(3/2)*d^(3/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
```

```
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^3(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x\left(2acd^2e(\cdots\right.}{\left(ade\cdots\right.}}{3c\cdots}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(ade\left(cd^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(ade\left(cd^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(ade\left(cd^2\cdots\right.\right.}{\cdots}$$

Mathematica [C] time = 4.64, size = 1443, normalized size = 4.86

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (a^3*e^3*(a*e + c*d*x)^2*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(5/2)*((2520*c*d
*(d + e*x))/(a*e^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (1330*c*d*(d +
e*x))/(e*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (1050*c*d*(
a*e + c*d*x)*(d + e*x))/(a^2*e^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) + (
196*c*d*(a*e + c*d*x)^2*(d + e*x))/(a^3*e^4*Sqrt[(c*d*(d + e*x))/(c*d^2 - a
*e^2)]) + 1568*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + (1575*(c*d^2 - a*e^2
)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(a^2*e^4) + (1995*(c*d^2 - a*e^2
)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(e^2*(a*e + c*d*x)^2) - (3780*(c
*d^2 - a*e^2)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(a*e^3*(a*e + c*d*x)
) - (294*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
2)])/(a^3*e^5) - 504*(1 + (c*d*x)/(a*e))*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
```

```
2)] + 336*(1 + (c*d*x)/(a*e))^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - 56*
(1 + (c*d*x)/(a*e))^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - (1995*ArcSin[
Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]])/((e*(a*e + c*d*x))/(-(c*d^2) +
 a*e^2))^(5/2) + (3780*(a*e + c*d*x)*ArcSin[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2
) + a*e^2)]])/(a*e*((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^(5/2)) - (1575*(a
*e + c*d*x)^2*ArcSin[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]])/(a^2*e^2*
((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^(5/2)) + (294*(a*e + c*d*x)^3*ArcSin
[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]])/(a^3*e^3*((e*(a*e + c*d*x))/(
-(c*d^2) + a*e^2))^(5/2)) - (168*e^2*(a*e + c*d*x)^2*Hypergeometric2F1[3/2,
 9/2, 11/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c*d^2 - a*e^2)^2 + (392
*e*(a*e + c*d*x)^3*Hypergeometric2F1[3/2, 9/2, 11/2, (e*(a*e + c*d*x))/(-(c
*d^2) + a*e^2)])/(a*(c*d^2 - a*e^2)^2) - (280*(a*e + c*d*x)^4*Hypergeometri
c2F1[3/2, 9/2, 11/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a^2*(c*d^2 - a
*e^2)^2) + (56*(a*e + c*d*x)^5*Hypergeometric2F1[3/2, 9/2, 11/2, (e*(a*e +
c*d*x))/(-(c*d^2) + a*e^2)])/(a^3*e*(c*d^2 - a*e^2)^2) - (96*e*(a*e + c*d*x
)*HypergeometricPFQ[{1/2, 2, 2, 7/2}, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d
^2) + a*e^2)])/(-(c*d^2) + a*e^2) + (288*(a*e + c*d*x)^2*HypergeometricPFQ[
{1/2, 2, 2, 7/2}, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a*(-
(c*d^2) + a*e^2)) - (288*(a*e + c*d*x)^3*HypergeometricPFQ[{1/2, 2, 2, 7/2}
, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a^2*(-(c*d^2*e) + a*
e^3)) + (96*(a*e + c*d*x)^4*HypergeometricPFQ[{1/2, 2, 2, 7/2}, {1, 1, 9/2}
, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a^3*e^2*(-(c*d^2) + a*e^2))))/(25
2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2))
```

IntegrateAlgebraic [B] time = 10.11, size = 10635, normalized size = 35.81

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 3.24, size = 1466, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
(c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*
c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*
x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^
4*d*e^8)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a
^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
 a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(3*a*c^3*d^7*e^2 - 8
*a^2*c^2*d^5*e^4 - 3*a^3*c*d^3*e^6 + (4*c^4*d^7*e^2 - 9*a*c^3*d^5*e^4 - 3*a
^3*c*d*e^8)*x^2 + (3*c^4*d^8*e - 4*a*c^3*d^6*e^3 - 9*a^2*c^2*d^4*e^5 - 6*a^
3*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^5*d^10*e^
4 - 3*a^2*c^4*d^8*e^6 + 3*a^3*c^3*d^6*e^8 - a^4*c^2*d^4*e^10 + (c^6*d^9*e^5
 - 3*a*c^5*d^7*e^7 + 3*a^2*c^4*d^5*e^9 - a^3*c^3*d^3*e^11)*x^3 + (2*c^6*d^1
0*e^4 - 5*a*c^5*d^8*e^6 + 3*a^2*c^4*d^6*e^8 + a^3*c^3*d^4*e^10 - a^4*c^2*d^
2*e^12)*x^2 + (c^6*d^11*e^3 - a*c^5*d^9*e^5 - 3*a^2*c^4*d^7*e^7 + 5*a^3*c^3
*d^5*e^9 - 2*a^4*c^2*d^3*e^11)*x), -1/3*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3
 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c
^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*
d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c
```

```
^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c
*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3*a
*c^3*d^7*e^2 - 8*a^2*c^2*d^5*e^4 - 3*a^3*c*d^3*e^6 + (4*c^4*d^7*e^2 - 9*a*c
^3*d^5*e^4 - 3*a^3*c*d*e^8)*x^2 + (3*c^4*d^8*e - 4*a*c^3*d^6*e^3 - 9*a^2*c^
2*d^4*e^5 - 6*a^3*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(a*c^5*d^10*e^4 - 3*a^2*c^4*d^8*e^6 + 3*a^3*c^3*d^6*e^8 - a^4*c^2*d^4*e^1
0 + (c^6*d^9*e^5 - 3*a*c^5*d^7*e^7 + 3*a^2*c^4*d^5*e^9 - a^3*c^3*d^3*e^11)*
x^3 + (2*c^6*d^10*e^4 - 5*a*c^5*d^8*e^6 + 3*a^2*c^4*d^6*e^8 + a^3*c^3*d^4*e
^10 - a^4*c^2*d^2*e^12)*x^2 + (c^6*d^11*e^3 - a*c^5*d^9*e^5 - 3*a^2*c^4*d^7
*e^7 + 5*a^3*c^3*d^5*e^9 - 2*a^4*c^2*d^3*e^11)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.01, size = 977, normalized size = 3.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -1/e^2*x/c/d/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+1/2/e/c^2/d^2/(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a+3/2/e^3/c/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*
x)^(1/2)+e^2/c/d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*x*a^2+4*d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+
(a*e^2+c*d^2)*x)^(1/2)*x*a+3/e^2*c*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x+1/2*e^3/c^2/d^2/(-a^2*e^4+2*a*c*d^2*
e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^3+5/2*e/c/(-a^2*e^4+
2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*a^2+7/2/e*d^
2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*
a+3/2/e^3*c*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(1/2)+1/e^2/c/d*ln((c*d*e*x+1/2*a*e^2+1/2*c*d^2)/(c*d*e)^(1/2)+(c*d
*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/(c*d*e)^(1/2)+2*d^2/e^3*(2*c*d*e*x+a*e
^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2)+2/3*d^3/e^4/(a*e^2-c*d^2)/(x+d/e)/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+
d/e))^(1/2)-16/3*d^5/e^2*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)
*(x+d/e))^(1/2)*x-8/3*d^4/e*c/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2
)*(x+d/e))^(1/2)*a-8/3*d^6/e^3*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-
c*d^2)*(x+d/e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(x**3/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.314 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8ae\left(x\left(ae^2+cd^2\right)+2ade\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Rubi [A] time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {854, 12, 636}

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8ae\left(x\left(ae^2+cd^2\right)+2ade\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) - (8*a*e*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 854

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((2*c*d - b*e)*(f + g*x)^n*(a + b*x +
 c*x^2)^(p + 1))/(e*p*(b^2 - 4*a*c)*(d + e*x)), x] - Dist[1/(d*e*p*(b^2 - 4
*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f*(
2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p + 1)
*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && IGt
Q[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2\int -\cdots}{(\cdots}$$

$$= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(4ae)\cdots}{\cdots}$$

$$= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\cdots}{3\left(cd^2\cdots\right.}$$

Mathematica [A] time = 0.07, size = 99, normalized size = 0.79

$$\frac{-2a^2e^2\left(8d^2+12dex+3e^2x^2\right)-4acd^2ex(2d+3ex)+2c^2d^4x^2}{3(d+ex)\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*c^2*d^4*x^2 - 4*a*c*d^2*e*x*(2*d + 3*e*x) - 2*a^2*e^2*(8*d^2 + 12*d*e*x
+ 3*e^2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [B] time = 152.02, size = 20752, normalized size = 164.70

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 2.83, size = 308, normalized size = 2.44

$$-\frac{2\left(8a^2d^2e^2-\left(c^2d^4-6acd^2e^2-3a^2e^4\right)x^2+4\left(acd^3e+3a^2de^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{3\left(ac^3d^8e-3a^2c^2d^6e^3+3a^3cd^4e^5-a^4d^2e^7+\left(c^4d^7e^2-3ac^3d^5e^4+3a^2c^2d^3e^6-a^3cde^8\right)x^3+\left(2c^4d^8e-5ac^3d^6e^3+3a^2c^2d^4e^5+a^3cd^2e^7-a^4e^9\right)x^2+\left(c^4d^9-ac^3d^7e^2-3a^2c^2d^5e^4+5a^3cd^3e^6-2a^4de^8\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2/3*(8*a^2*d^2*e^2 - (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*x^2 + 4*(a*c*d^
3*e + 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^
8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*
a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c
^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 -
a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 145, normalized size = 1.15

$$\frac{2\,(cdx+ae)\left(3a^2e^4x^2+6ac\,d^2e^2x^2-c^2d^4x^2+12a^2d\,e^3x+4ac\,d^3ex+8a^2d^2e^2\right)}{3\left(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(3*a^2*e^4*x^2+6*a*c*d^2*e^2*x^2-c^2*d^4*x^2+12*a^2*d*e^3*x
+4*a*c*d^3*e*x+8*a^2*d^2*e^2)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*
d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [B] time = 3.60, size = 1071, normalized size = 8.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (4*c*d^3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(a^3*d*e^7 - c^3
*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3*a^2*c*d^3*e^5 - c^3*d^6*e^2*x + 3*
a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) - (2*d^2*(a*d*e + a*e^2*x + c*d^2*x +
 c*d*e*x^2)^(1/2))/(3*c^2*d^6*e + 3*a^2*d^2*e^5 + 3*a^2*e^7*x^2 + 6*c^2*d^5
*e^2*x + 3*c^2*d^4*e^3*x^2 - 6*a*c*d^4*e^3 + 6*a^2*d*e^6*x - 12*a*c*d^3*e^4
*x - 6*a*c*d^2*e^5*x^2) - (4*a*d*e^2*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2
)^(1/2))/(3*(a^3*d*e^7 - c^3*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3*a^2*c*
d^3*e^5 - c^3*d^6*e^2*x + 3*a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) + (2*c^4*
d^7*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a*c^4*
d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)) + (22*a^3*c
*d^2*e^5)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a
*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)) - (28*
a^2*c^2*d^4*e^3)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*
e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9))
 + (2*a*c^3*d^6*e)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*
e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9))
 + (10*a^2*c^2*d^3*e^4*x)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*
(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*
c*d*e^9)) + (2*a^3*c*d*e^6*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2
```

```
)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^
4*c*d*e^9)) - (22*a*c^3*d^5*e^2*x)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^
2)^(1/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e
^7 + a^4*c*d*e^9))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(x**2/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.315 $\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{2\left(3ae^2+cd^2\right)\left(ae^2+cd^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {792, 613}

$$\frac{2\left(3ae^2+cd^2\right)\left(ae^2+cd^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*(c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*e*(c*d^2 - a
*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(cd^2+\cdots\right.}{\cdots}$$

$$=-\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\cdots}{3e\left(cd^2\cdots\right.}$$

Mathematica [A] time = 0.04, size = 100, normalized size = 0.72

$$\frac{2\left(a^2e^3(2d+3ex)+2acde\left(3d^2+5dex+3e^2x^2\right)+c^2d^3x(3d+2ex)\right)}{3(d+ex)\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(c^2*d^3*x*(3*d + 2*e*x) + a^2*e^3*(2*d + 3*e*x) + 2*a*c*d*e*(3*d^2 + 5*
d*e*x + 3*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d +
 e*x)])
```

IntegrateAlgebraic [B] time = 153.71, size = 25359, normalized size = 183.76

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 2.72, size = 314, normalized size = 2.28

$$\frac{2\left(6\,acd^3e+2\,a^2de^3+2\left(c^2d^3e+3\,acde^3\right)x^2+\left(3\,c^2d^4+10\,acd^2e^2+3\,a^2e^4\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+\left(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8\right)x^3+\left(2\,c^4d^8e-5\,ac^3d^6e^3+3\,a^2c^2d^4e^5+a^3cd^2e^7-a^4e^9\right)x^2+\left(c^4d^9-ac^3d^7e^2-3\,a^2c^2d^5e^4+5\,a^3cd^3e^6-2\,a^4de^8\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(6*a*c*d^3*e + 2*a^2*d*e^3 + 2*(c^2*d^3*e + 3*a*c*d*e^3)*x^2 + (3*c^2*d
^4 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)/(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^
4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4
*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2
 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d
*e^8)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 149, normalized size = 1.08

$$-\frac{2\left(cdx+ae\right)\left(6acd\,e^3x^2+2c^2d^3e\,x^2+3a^2e^4x+10ac\,d^2e^2x+3c^2d^4x+2a^2d\,e^3+6ac\,d^3e\right)}{3\left(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(6*a*c*d*e^3*x^2+2*c^2*d^3*e*x^2+3*a^2*e^4*x+10*a*c*d^2*e^
2*x+3*c^2*d^4*x+2*a^2*d*e^3+6*a*c*d^3*e)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d
^4*e^2-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [B] time = 3.32, size = 499, normalized size = 3.62

$$\frac{4\,a^2\,d\,e^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+6\,a^2\,e^4\,x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+6\,c^2\,d^4\,x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+4\,c^2\,d^3\,e\,x^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+12\,a\,c\,d^3\,e\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+20\,a\,c\,d^2\,e^2\,x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+12\,a\,c\,d\,e^3\,x^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{-3\,a^4\,d^2\,e^7-6\,a^4\,d\,e^8\,x-3\,a^4\,e^9\,x^2+9\,a^3\,c\,d^4\,e^5+15\,a^3\,c\,d^3\,e^6\,x+3\,a^3\,c\,d^2\,e^7\,x^2-3\,a^3\,c\,d\,e^8\,x^3-9\,a^2\,c^2\,d^6\,e^3-9\,a^2\,c^2\,d^5\,e^4\,x+9\,a^2\,c^2\,d^4\,e^5\,x^2+9\,a^2\,c^2\,d^3\,e^6\,x^3+3\,a\,c^3\,d^8\,e-3\,a\,c^3\,d^7\,e^2\,x-15\,a\,c^3\,d^6\,e^3\,x^2-9\,a\,c^3\,d^5\,e^4\,x^3+3\,c^4\,d^9\,x+6\,c^4\,d^8\,e\,x^2+3\,c^4\,d^7\,e^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (4*a^2*d*e^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*a^2*e^4*x*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*c^2*d^4*x*(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)^(1/2) + 4*c^2*d^3*e*x^2*(x*(a*e^2 + c*d^2) + a*d*e +
 c*d*e*x^2)^(1/2) + 12*a*c*d^3*e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2) + 20*a*c*d^2*e^2*x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 12*a
*c*d*e^3*x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(3*c^4*d^9*x -
3*a^4*d^2*e^7 - 3*a^4*e^9*x^2 + 9*a^3*c*d^4*e^5 + 6*c^4*d^8*e*x^2 - 9*a^2*c
^2*d^6*e^3 + 3*c^4*d^7*e^2*x^3 + 3*a*c^3*d^8*e - 6*a^4*d*e^8*x + 9*a^2*c^2*
d^4*e^5*x^2 + 9*a^2*c^2*d^3*e^6*x^3 - 3*a*c^3*d^7*e^2*x + 15*a^3*c*d^3*e^6*
x - 3*a^3*c*d*e^8*x^3 - 9*a^2*c^2*d^5*e^4*x - 15*a*c^3*d^6*e^3*x^2 + 3*a^3*
c*d^2*e^7*x^2 - 9*a*c^3*d^5*e^4*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\left(d + ex\right)\left(ae + cdx\right)\right)^{\frac{3}{2}}\left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.316 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 613}

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] 2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
 - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{(4cd)}{\cdots}$$

$$=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\cdots}{3\left(cd^2\cdots\right.}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 0.79

$$\frac{2a^2e^4-4acde^2(3d+2ex)-2c^2d^2\left(3d^2+12dex+8e^2x^2\right)}{3(d+ex)\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*a^2*e^4 - 4*a*c*d*e^2*(3*d + 2*e*x) - 2*c^2*d^2*(3*d^2 + 12*d*e*x + 8*e^
2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [B] time = 0.02, size = 27688, normalized size = 228.83

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 2.30, size = 306, normalized size = 2.53

$$-\frac{2\left(8\,c^2d^2e^2x^2+3\,c^2d^4+6\,acd^2e^2-a^2e^4+4\left(3\,c^2d^3e+acde^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+\left(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8\right)x^3+\left(2\,c^4d^8e-5\,ac^3d^6e^3+3\,a^2c^2d^4e^5+a^3cd^2e^7-a^4e^9\right)x^2+\left(c^4d^9-ac^3d^7e^2-3\,a^2c^2d^5e^4+5\,a^3cd^3e^6-2\,a^4de^8\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2/3*(8*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 4*(3*c^2*d^
3*e + a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^8*
e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*
c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3
*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*
c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.01, size = 138, normalized size = 1.14

$$-\frac{2\left(cdx+ae\right)\left(-8c^2d^2e^2x^2-4acd\,e^3x-12c^2d^3ex+a^2e^4-6ac\,d^2e^2-3c^2d^4\right)}{3\left(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-8*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x-12*c^2*d^3*e*x+a^2*e^4-6
*a*c*d^2*e^2-3*c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)/(
c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [B] time = 2.88, size = 120, normalized size = 0.99

$$\frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2+4\,a\,c\,d\,e^3\,x+3\,c^2\,d^4+12\,c^2\,d^3\,e\,x+8\,c^2\,d^2\,e^2\,x^2\right)}{3\,\left(a\,e+c\,d\,x\right)\,{\left(a\,e^2-c\,d^2\right)}^3\,{\left(d+e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*c^2*d^4 - a^2*e^4 + 8*c
^2*d^2*e^2*x^2 + 6*a*c*d^2*e^2 + 12*c^2*d^3*e*x + 4*a*c*d*e^3*x))/(3*(a*e +
 c*d*x)*(a*e^2 - c*d^2)^3*(d + e*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.317 $\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}+\frac{2\left(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3c^3\right.}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Rubi [A] time = 0.34, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {851, 822, 12, 724, 206}

$$\frac{2\left(7a^2cd^2e^4-3a^3e^6+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3c^3d^6\right)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*e^6
 + c*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)
^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])]/(a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{ae+cdx}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2\int \frac{-\frac{3}{2}ae}{x(}}{}$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6\right.}{}$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6\right.}{}$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6\right.}{}$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6\right.}{}$$

Mathematica [A] time = 0.41, size = 262, normalized size = 0.97

$$\frac{2\left(-\frac{(d+ex)(ae+cdx)^{3/2}\left(\sqrt{a}\sqrt{d}\sqrt{e}\left(3a^2e^5-8acd^2e^3-3c^2d^4e\right)\sqrt{ae+cdx}+3\sqrt{d+ex}\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)\right)}{3\sqrt{a}\,d^{5/2}\sqrt{e}\left(cd^2-ae^2\right)^2}+\frac{\left(ae^3+3cd^2e\right)(ae+cdx)^2}{3cd^3-3ade^2}+cd(ae+cdx)\right)}{ae\left(cd^2-ae^2\right)((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(c*d*(a*e + c*d*x) + ((3*c*d^2*e + a*e^3)*(a*e + c*d*x)^2)/(3*c*d^3 - 3*
a*d*e^2) - ((a*e + c*d*x)^(3/2)*(d + e*x)*(Sqrt[a]*Sqrt[d]*Sqrt[e]*(-3*c^2*
d^4*e - 8*a*c*d^2*e^3 + 3*a^2*e^5)*Sqrt[a*e + c*d*x] + 3*(c*d^2 - a*e^2)^3*
Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d +
 e*x])]))/(3*Sqrt[a]*d^(5/2)*Sqrt[e]*(c*d^2 - a*e^2)^2)))/(a*e*(c*d^2 - a*e
^2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [F] time = 180.57, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 6.52, size = 1476, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
(c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*
c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*
x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^
4*d*e^8)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(3*a*c^3*d
^8*e + 9*a^3*c*d^4*e^5 - 4*a^4*d^2*e^7 + (3*a*c^3*d^6*e^3 + 8*a^2*c^2*d^4*e
^5 - 3*a^3*c*d^2*e^7)*x^2 + (6*a*c^3*d^7*e^2 + 9*a^2*c^2*d^5*e^4 + 4*a^3*c*
d^3*e^6 - 3*a^4*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3
*c^3*d^11*e^3 - 3*a^4*c^2*d^9*e^5 + 3*a^5*c*d^7*e^7 - a^6*d^5*e^9 + (a^2*c^
4*d^10*e^4 - 3*a^3*c^3*d^8*e^6 + 3*a^4*c^2*d^6*e^8 - a^5*c*d^4*e^10)*x^3 +
(2*a^2*c^4*d^11*e^3 - 5*a^3*c^3*d^9*e^5 + 3*a^4*c^2*d^7*e^7 + a^5*c*d^5*e^9
 - a^6*d^3*e^11)*x^2 + (a^2*c^4*d^12*e^2 - a^3*c^3*d^10*e^4 - 3*a^4*c^2*d^8
*e^6 + 5*a^5*c*d^6*e^8 - 2*a^6*d^4*e^10)*x), 1/3*(3*(a*c^3*d^8*e - 3*a^2*c^
2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4
+ 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3
*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2
- 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*sqrt(-a*d*e)*arctan
(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x
)) + 2*(3*a*c^3*d^8*e + 9*a^3*c*d^4*e^5 - 4*a^4*d^2*e^7 + (3*a*c^3*d^6*e^3
+ 8*a^2*c^2*d^4*e^5 - 3*a^3*c*d^2*e^7)*x^2 + (6*a*c^3*d^7*e^2 + 9*a^2*c^2*d
^5*e^4 + 4*a^3*c*d^3*e^6 - 3*a^4*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(a^3*c^3*d^11*e^3 - 3*a^4*c^2*d^9*e^5 + 3*a^5*c*d^7*e^7 - a^6*
d^5*e^9 + (a^2*c^4*d^10*e^4 - 3*a^3*c^3*d^8*e^6 + 3*a^4*c^2*d^6*e^8 - a^5*c
*d^4*e^10)*x^3 + (2*a^2*c^4*d^11*e^3 - 5*a^3*c^3*d^9*e^5 + 3*a^4*c^2*d^7*e^
7 + a^5*c*d^5*e^9 - a^6*d^3*e^11)*x^2 + (a^2*c^4*d^12*e^2 - a^3*c^3*d^10*e^
4 - 3*a^4*c^2*d^8*e^6 + 5*a^5*c*d^6*e^8 - 2*a^6*d^4*e^10)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.02, size = 682, normalized size = 2.52

$$-\frac{16c^2 d e^2 x}{3\left(a e^2-c d^2\right)^3 \sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{8ace^3}{3\left(a e^2-c d^2\right)^3 \sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{8c^2 d^2 e}{3\left(a e^2-c d^2\right)^3 \sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{2c^2 dx}{\left(-a^2e^4+2acd^2e^2-c^2d^4\right)\sqrt{cdex^2+ade+\left(a e^2+c d^2\right)x}\, a}-\frac{2ce^2x}{\left(-a^2e^4+2acd^2e^2-c^2d^4\right)\sqrt{cdex^2+ade+\left(a e^2+c d^2\right)x}\, d}-\frac{a e^3}{\left(-a^2e^4+2acd^2e^2-c^2d^4\right)\sqrt{cdex^2+ade+\left(a e^2+c d^2\right)x}\, d^2}-\frac{c^2d^2}{\left(-a^2e^4+2acd^2e^2-c^2d^4\right)\sqrt{cdex^2+ade+\left(a e^2+c d^2\right)x}\, ae}-\frac{2ce}{\left(-a^2e^4+2acd^2e^2-c^2d^4\right)\sqrt{cdex^2+ade+\left(a e^2+c d^2\right)x}}+\frac{2}{3\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\sqrt{\left(x+\frac{d}{e}\right)^2 cde+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}\, d}-\frac{\ln\left(\frac{2ade+\left(a e^2+c d^2\right)x+2\sqrt{ade}\,\sqrt{cdex^2+ade+\left(a e^2+c d^2\right)x}}{x}\right)}{\sqrt{ade}\, a d^2 e}+\frac{1}{\sqrt{cdex^2+ade+\left(a e^2+c d^2\right)x}\, a d^2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 1/d^2/a/e/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-2/d*e^2/(-a^2*e^4+2*a*c*d
^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c-2*d/a/(-a^2*e^4
+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2-1/d^2
*a*e^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)-2*e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2)*c-d^2/a/e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*c^2-1/d^2/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a
*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+2/3/d/(a*e^2-c*d^2)
/(x+d/e)/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)-16/3*d*c^2*e^2/(a*e^
2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-8/3*c*e^3/(a*e^2
-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*a-8/3*d^2*c^2*e/(a*
e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}(ex + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,(d + e\,x)\,\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(d + ex\right)\left(ae + cdx\right)\right)^{\frac{3}{2}}\left(d + ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.318 $\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=394

$$\frac{\left(5ae^2+3cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{5/2}d^{7/2}e^{5/2}}+\frac{2\left(-5a^3e^6+cdex\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+9a^2cd^2e^4\right.}{3ad^2ex\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+c}}$$

Rubi [A] time = 0.59, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {851, 822, 806, 724, 206}

$$-\frac{\left(31a^2cd^2e^4-15a^3e^6-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x\left(cd^2-ae^2\right)^3}+\frac{2\left(cdex\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+9a^2cd^2e^4-5a^3e^6+ac^2d^4e^2+3c^3d^6\right)}{3ad^2ex\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\left(5ae^2+3cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{5/2}d^{7/2}e^{5/2}}-\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 5*a^3*e
^6 + c*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 - 5*a^2*e^4)*x))/(3*a*d^2*e*(c*d^2 -
 a*e^2)^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((9*c^3*d^6 - 9*
a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - 15*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x) + ((3*c*d^2 + 5*a*e^2)
*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
```

```
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int -$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c\right.}{}$$

Mathematica [A] time = 0.57, size = 370, normalized size = 0.94

$$\frac{(ae+cdx)\left(3a^{3/2}d^{5/2}e^{3/2}\left(ae^2-cd^2\right)^3+\sqrt{a}\,d^{3/2}\sqrt{e}\,x\left(ae^2-cd^2\right)\left(5a^2e^5-6acd^2e^3+9c^2d^4e\right)(ae+cdx)+x(d+ex)\sqrt{ae+cdx}\left(\sqrt{a}\sqrt{d}\sqrt{e}\left(15a^3e^7-31a^2cd^2e^5+9ac^2d^4e^3-9c^3d^6e\right)\sqrt{ae+cdx}+3\sqrt{d+ex}\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)\right)+3\sqrt{a}\,cd^{7/2}\sqrt{e}\,x\left(cd^2-ae^2\right)^2\left(ae^2-3cd^2\right)\right)}{3a^{5/2}d^{7/2}e^{5/2}x\left(cd^2-ae^2\right)^3((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((a*e + c*d*x)*(3*a^(3/2)*d^(5/2)*e^(3/2)*(-(c*d^2) + a*e^2)^3 + 3*Sqrt[a]*
c*d^(7/2)*Sqrt[e]*(c*d^2 - a*e^2)^2*(-3*c*d^2 + a*e^2)*x + Sqrt[a]*d^(3/2)*
Sqrt[e]*(-(c*d^2) + a*e^2)*(9*c^2*d^4*e - 6*a*c*d^2*e^3 + 5*a^2*e^5)*x*(a*e
 + c*d*x) + x*Sqrt[a*e + c*d*x]*(d + e*x)*(Sqrt[a]*Sqrt[d]*Sqrt[e]*(-9*c^3*
d^6*e + 9*a*c^2*d^4*e^3 - 31*a^2*c*d^2*e^5 + 15*a^3*e^7)*Sqrt[a*e + c*d*x]
```

```
+ 3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sq
rt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])))/(3*a^(5/2)*d^(7/2)*e^(
5/2)*(c*d^2 - a*e^2)^3*x*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [F] time = 180.04, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 19.59, size = 1812, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/12*(3*((3*c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 - 6*a^2*c^3*d^5*e^6 + 12*a^3*c^2
*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 + (6*c^5*d^10*e - 5*a*c^4*d^8*e^3 - 16*a^2*c
^3*d^6*e^5 + 18*a^3*c^2*d^4*e^7 + 2*a^4*c*d^2*e^9 - 5*a^5*e^11)*x^3 + (3*c^
5*d^11 + 2*a*c^4*d^9*e^2 - 14*a^2*c^3*d^7*e^4 + 19*a^4*c*d^3*e^8 - 10*a^5*d
*e^10)*x^2 + (3*a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 - 6*a^3*c^2*d^6*e^5 + 12*a
^4*c*d^4*e^7 - 5*a^5*d^2*e^9)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4
+ 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x
)/x^2) - 4*(3*a^2*c^3*d^9*e^2 - 9*a^3*c^2*d^7*e^4 + 9*a^4*c*d^5*e^6 - 3*a^5
*d^3*e^8 + (9*a*c^4*d^8*e^3 - 9*a^2*c^3*d^6*e^5 + 31*a^3*c^2*d^4*e^7 - 15*a
^4*c*d^2*e^9)*x^3 + (18*a*c^4*d^9*e^2 - 15*a^2*c^3*d^7*e^4 + 33*a^3*c^2*d^5
*e^6 + 11*a^4*c*d^3*e^8 - 15*a^5*d*e^10)*x^2 + (9*a*c^4*d^10*e - 3*a^2*c^3*
d^8*e^3 - 9*a^3*c^2*d^6*e^5 + 39*a^4*c*d^4*e^7 - 20*a^5*d^2*e^9)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^4*d^11*e^5 - 3*a^4*c^3*d^9*e^
7 + 3*a^5*c^2*d^7*e^9 - a^6*c*d^5*e^11)*x^4 + (2*a^3*c^4*d^12*e^4 - 5*a^4*c
^3*d^10*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^10 - a^7*d^4*e^12)*x^3 + (a^3
*c^4*d^13*e^3 - a^4*c^3*d^11*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a^6*c*d^7*e^9 - 2*
a^7*d^5*e^11)*x^2 + (a^4*c^3*d^12*e^4 - 3*a^5*c^2*d^10*e^6 + 3*a^6*c*d^8*e^
8 - a^7*d^6*e^10)*x), -1/6*(3*((3*c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 - 6*a^2*c^3
*d^5*e^6 + 12*a^3*c^2*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 + (6*c^5*d^10*e - 5*a*c
^4*d^8*e^3 - 16*a^2*c^3*d^6*e^5 + 18*a^3*c^2*d^4*e^7 + 2*a^4*c*d^2*e^9 - 5*
a^5*e^11)*x^3 + (3*c^5*d^11 + 2*a*c^4*d^9*e^2 - 14*a^2*c^3*d^7*e^4 + 19*a^4
*c*d^3*e^8 - 10*a^5*d*e^10)*x^2 + (3*a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 - 6*a
^3*c^2*d^6*e^5 + 12*a^4*c*d^4*e^7 - 5*a^5*d^2*e^9)*x)*sqrt(-a*d*e)*arctan(1
/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x))
 + 2*(3*a^2*c^3*d^9*e^2 - 9*a^3*c^2*d^7*e^4 + 9*a^4*c*d^5*e^6 - 3*a^5*d^3*e
^8 + (9*a*c^4*d^8*e^3 - 9*a^2*c^3*d^6*e^5 + 31*a^3*c^2*d^4*e^7 - 15*a^4*c*d
^2*e^9)*x^3 + (18*a*c^4*d^9*e^2 - 15*a^2*c^3*d^7*e^4 + 33*a^3*c^2*d^5*e^6 +
 11*a^4*c*d^3*e^8 - 15*a^5*d*e^10)*x^2 + (9*a*c^4*d^10*e - 3*a^2*c^3*d^8*e^
3 - 9*a^3*c^2*d^6*e^5 + 39*a^4*c*d^4*e^7 - 20*a^5*d^2*e^9)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^4*d^11*e^5 - 3*a^4*c^3*d^9*e^7 + 3*
a^5*c^2*d^7*e^9 - a^6*c*d^5*e^11)*x^4 + (2*a^3*c^4*d^12*e^4 - 5*a^4*c^3*d^1
0*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^10 - a^7*d^4*e^12)*x^3 + (a^3*c^4*d
^13*e^3 - a^4*c^3*d^11*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a^6*c*d^7*e^9 - 2*a^7*d^
5*e^11)*x^2 + (a^4*c^3*d^12*e^4 - 3*a^5*c^2*d^10*e^6 + 3*a^6*c*d^8*e^8 - a^
7*d^6*e^10)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.41Unable to transpose Err
or: Bad Argument Value
```

maple [B] time = 0.02, size = 912, normalized size = 2.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -1/d^2/a/e/x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-5/2/d^3/a/(c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^(1/2)-3/2/d/a^2/e^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2)*c+5/d^2*e^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2
+c*d^2)*x)^(1/2)*x*c+3*d^2/a^2/e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^
2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3+5/2/d^3*a*e^4/(-a^2*e^4+2*a*c*d^2*e^2-
c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+5/2/d*e^2/(-a^2*e^4+2*a*c*
d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c+3/2*d/a/(-a^2*e^
4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+3/2*d^
3/a^2/e^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x
)^(1/2)*c^3+5/2/d^3/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(
1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)+3/2/d/a^2/e^2/(a*d*e)^(1/2
)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2
)*x)^(1/2))/x)*c-2/3*e/d^2/(a*e^2-c*d^2)/(x+d/e)/((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)+16/3*e^3*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*
d^2)*(x+d/e))^(1/2)*x+8/3*e^4/d*c/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c
*d^2)*(x+d/e))^(1/2)*a+8/3*e^2*d*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^
2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}} (ex + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}} (d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(1/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.319 $\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=522

$$-\frac{5\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{7/2}d^{9/2}e^{7/2}}+\frac{2\left(-7a^3e^6+cdex\left(-7a^2e^4+12acd^2e^2+\right.\right.}{3ad^2ex^2\left(cd^2-ae^2\right)^3\sqrt{x\left(\right.}}$$

Rubi [A] time = 0.80, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {851, 822, 834, 806, 724, 206}

$$-\frac{\left(61a^2cd^2e^4-35a^3e^6-9ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{6a^2d^3e^2x^2\left(cd^2-ae^2\right)^3}+\frac{\left(-36a^2c^2d^4e^4+190a^3cd^2e^6-105a^4e^8-30ac^3d^6e^2+45c^4d^8\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12a^3d^4e^3x\left(cd^2-ae^2\right)^3}+\frac{2\left(cdex\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+11a^2cd^2e^4-7a^3e^6+ac^2d^4e^2+3c^3d^6\right)}{3ad^2ex^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{7/2}d^{9/2}e^{7/2}}-\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 7*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((15*c^3*d^
6 - 9*a*c^2*d^4*e^2 + 61*a^2*c*d^2*e^4 - 35*a^3*e^6)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(6*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^2) + ((45*c^4*d^8
 - 30*a*c^3*d^6*e^2 - 36*a^2*c^2*d^4*e^4 + 190*a^3*c*d^2*e^6 - 105*a^4*e^8)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^
2)^3*x) - (5*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])])/(8*a^(7/2)*d^(9/2)*e^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
```

```
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \cdots}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.93, size = 467, normalized size = 0.89

$$\frac{(ae+cdx)\left(6a^{5/2}d^{7/2}e^{5/2}\left(ae^2-cd^2\right)^3+x\left(3a^{3/2}d^{5/2}e^{3/2}\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3-3\sqrt{a}d^{5/2}\sqrt{e}x\left(7a^2cde^4-15c^3d^5\right)\left(cd^2-ae^2\right)^2-\sqrt{a}d^{3/2}\sqrt{e}x\left(ae^2-cd^2\right)\left(35a^3e^7-33a^2cd^2e^5-15ac^2d^4e^3+45c^3d^6e\right)(ae+cdx)-x(d+ex)\sqrt{ae+cdx}\left(15\sqrt{d+ex}\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\left(105a^4e^9-190a^3cd^2e^7+36a^2c^2d^4e^5+30ac^3d^6e^3-45c^4d^8e\right)\sqrt{ae+cdx}\right)\right)\right)}{12a^{7/2}d^{9/2}e^{7/2}x^2\left(cd^2-ae^2\right)^3((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((a*e + c*d*x)*(6*a^(5/2)*d^(7/2)*e^(5/2)*(-(c*d^2) + a*e^2)^3 + x*(3*a^(3/
2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2) - 3*Sqrt[a]*d^(5/2
)*Sqrt[e]*(c*d^2 - a*e^2)^2*(-15*c^3*d^5 + 7*a^2*c*d*e^4)*x - Sqrt[a]*d^(3/
2)*Sqrt[e]*(-(c*d^2) + a*e^2)*(45*c^3*d^6*e - 15*a*c^2*d^4*e^3 - 33*a^2*c*d
^2*e^5 + 35*a^3*e^7)*x*(a*e + c*d*x) - x*Sqrt[a*e + c*d*x]*(d + e*x)*(Sqrt[
a]*Sqrt[d]*Sqrt[e]*(-45*c^4*d^8*e + 30*a*c^3*d^6*e^3 + 36*a^2*c^2*d^4*e^5 -
 190*a^3*c*d^2*e^7 + 105*a^4*e^9)*Sqrt[a*e + c*d*x] + 15*(c*d^2 - a*e^2)^3*
(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt
[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))))/(12*a^(7/2)*d^(9/2)*e^(
7/2)*(c*d^2 - a*e^2)^3*x^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [F] time = 180.17, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 42.29, size = 2162, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/48*(15*((3*c^6*d^11*e^2 - 3*a*c^5*d^9*e^4 - 2*a^2*c^4*d^7*e^6 - 6*a^3*c^
3*d^5*e^8 + 15*a^4*c^2*d^3*e^10 - 7*a^5*c*d*e^12)*x^5 + (6*c^6*d^12*e - 3*a
*c^5*d^10*e^3 - 7*a^2*c^4*d^8*e^5 - 14*a^3*c^3*d^6*e^7 + 24*a^4*c^2*d^4*e^9
 + a^5*c*d^2*e^11 - 7*a^6*e^13)*x^4 + (3*c^6*d^13 + 3*a*c^5*d^11*e^2 - 8*a^
2*c^4*d^9*e^4 - 10*a^3*c^3*d^7*e^6 + 3*a^4*c^2*d^5*e^8 + 23*a^5*c*d^3*e^10
- 14*a^6*d*e^12)*x^3 + (3*a*c^5*d^12*e - 3*a^2*c^4*d^10*e^3 - 2*a^3*c^3*d^8
*e^5 - 6*a^4*c^2*d^6*e^7 + 15*a^5*c*d^4*e^9 - 7*a^6*d^2*e^11)*x^2)*sqrt(a*d
*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*
d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(6*a^3*c^3*d^10*e^3 - 18*a^4*c
^2*d^8*e^5 + 18*a^5*c*d^6*e^7 - 6*a^6*d^4*e^9 - (45*a*c^5*d^10*e^3 - 30*a^2
*c^4*d^8*e^5 - 36*a^3*c^3*d^6*e^7 + 190*a^4*c^2*d^4*e^9 - 105*a^5*c*d^2*e^1
1)*x^4 - (90*a*c^5*d^11*e^2 - 45*a^2*c^4*d^9*e^4 - 84*a^3*c^3*d^7*e^6 + 222
*a^4*c^2*d^5*e^8 + 50*a^5*c*d^3*e^10 - 105*a^6*d*e^12)*x^3 - (45*a*c^5*d^12
*e - 66*a^3*c^3*d^8*e^5 - 12*a^4*c^2*d^6*e^7 + 237*a^5*c*d^4*e^9 - 140*a^6*
d^2*e^11)*x^2 - 3*(5*a^2*c^4*d^11*e^2 - 8*a^3*c^3*d^9*e^4 - 6*a^4*c^2*d^7*e
^6 + 16*a^5*c*d^5*e^8 - 7*a^6*d^3*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/((a^4*c^4*d^12*e^6 - 3*a^5*c^3*d^10*e^8 + 3*a^6*c^2*d^8*e^10 -
 a^7*c*d^6*e^12)*x^5 + (2*a^4*c^4*d^13*e^5 - 5*a^5*c^3*d^11*e^7 + 3*a^6*c^2
*d^9*e^9 + a^7*c*d^7*e^11 - a^8*d^5*e^13)*x^4 + (a^4*c^4*d^14*e^4 - a^5*c^3
*d^12*e^6 - 3*a^6*c^2*d^10*e^8 + 5*a^7*c*d^8*e^10 - 2*a^8*d^6*e^12)*x^3 + (
a^5*c^3*d^13*e^5 - 3*a^6*c^2*d^11*e^7 + 3*a^7*c*d^9*e^9 - a^8*d^7*e^11)*x^2
), 1/24*(15*((3*c^6*d^11*e^2 - 3*a*c^5*d^9*e^4 - 2*a^2*c^4*d^7*e^6 - 6*a^3*
c^3*d^5*e^8 + 15*a^4*c^2*d^3*e^10 - 7*a^5*c*d*e^12)*x^5 + (6*c^6*d^12*e - 3
*a*c^5*d^10*e^3 - 7*a^2*c^4*d^8*e^5 - 14*a^3*c^3*d^6*e^7 + 24*a^4*c^2*d^4*e
^9 + a^5*c*d^2*e^11 - 7*a^6*e^13)*x^4 + (3*c^6*d^13 + 3*a*c^5*d^11*e^2 - 8*
a^2*c^4*d^9*e^4 - 10*a^3*c^3*d^7*e^6 + 3*a^4*c^2*d^5*e^8 + 23*a^5*c*d^3*e^1
0 - 14*a^6*d*e^12)*x^3 + (3*a*c^5*d^12*e - 3*a^2*c^4*d^10*e^3 - 2*a^3*c^3*d
^8*e^5 - 6*a^4*c^2*d^6*e^7 + 15*a^5*c*d^4*e^9 - 7*a^6*d^2*e^11)*x^2)*sqrt(-
a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e +
 a^2*d*e^3)*x)) - 2*(6*a^3*c^3*d^10*e^3 - 18*a^4*c^2*d^8*e^5 + 18*a^5*c*d^6
*e^7 - 6*a^6*d^4*e^9 - (45*a*c^5*d^10*e^3 - 30*a^2*c^4*d^8*e^5 - 36*a^3*c^3
*d^6*e^7 + 190*a^4*c^2*d^4*e^9 - 105*a^5*c*d^2*e^11)*x^4 - (90*a*c^5*d^11*e
^2 - 45*a^2*c^4*d^9*e^4 - 84*a^3*c^3*d^7*e^6 + 222*a^4*c^2*d^5*e^8 + 50*a^5
*c*d^3*e^10 - 105*a^6*d*e^12)*x^3 - (45*a*c^5*d^12*e - 66*a^3*c^3*d^8*e^5 -
 12*a^4*c^2*d^6*e^7 + 237*a^5*c*d^4*e^9 - 140*a^6*d^2*e^11)*x^2 - 3*(5*a^2*
c^4*d^11*e^2 - 8*a^3*c^3*d^9*e^4 - 6*a^4*c^2*d^7*e^6 + 16*a^5*c*d^5*e^8 - 7
*a^6*d^3*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^4*c^4*d^
12*e^6 - 3*a^5*c^3*d^10*e^8 + 3*a^6*c^2*d^8*e^10 - a^7*c*d^6*e^12)*x^5 + (2
*a^4*c^4*d^13*e^5 - 5*a^5*c^3*d^11*e^7 + 3*a^6*c^2*d^9*e^9 + a^7*c*d^7*e^11
 - a^8*d^5*e^13)*x^4 + (a^4*c^4*d^14*e^4 - a^5*c^3*d^12*e^6 - 3*a^6*c^2*d^1
0*e^8 + 5*a^7*c*d^8*e^10 - 2*a^8*d^6*e^12)*x^3 + (a^5*c^3*d^13*e^5 - 3*a^6*
c^2*d^11*e^7 + 3*a^7*c*d^9*e^9 - a^8*d^7*e^11)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [B] time = 0.02, size = 1319, normalized size = 2.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -15/4*d^3/a^3/e^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+
c*d^2)*x)^(1/2)*x*c^4+7/4*e^2/d/a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2-8/3*e^3*c^2/(a*e^2-c*d^2)^3/((x+d/e)^
2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+15/8/a^3/e^3/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*c^2+9/4/d^3/a/x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+35/8
*e/d^4/a/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-16/3/d*e^4*c^2/(a*e^2-c*d^
2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-8/3/d^2*e^5*c/(a*e^2-c
*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*a-15/8*d^4/a^3/e^3/(-
a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^4+
5/4/d/a^2/e^2/x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c-35/4*e^4/d^3/(-a^
2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c-5/
4*d/a^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2)*x*c^3-5/2/e*d^2/a^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e
+(a*e^2+c*d^2)*x)^(1/2)*c^3-15/4/e/d^2/a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2
+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-15/
8/a^3/e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*
x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2-1/2/d^2/a/e/x^2/(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2)+2/3/d^3*e^2/(a*e^2-c*d^2)/(x+d/e)/((x+d/e)^2*c*d*e+(a
*e^2-c*d^2)*(x+d/e))^(1/2)+15/4/e/d^2/a^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)
^(1/2)*c-35/8*e^5/d^4*a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(
a*e^2+c*d^2)*x)^(1/2)-7/2*e^3/d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x
^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c+1/4*e/a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/
(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2-35/8*e/d^4/a/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}} (ex + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^3),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.320 $\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=664

$$\frac{2\left(-9a^3e^6+cdex\left(-9a^2e^4+14acd^2e^2+3c^2d^4\right)+13a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{3ad^2ex^3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(-21a^3e^6+33a^2cd^2e^4-3ac^2\cdots\right.}{3a^2\cdots}$$

Rubi [A] time = 1.17, antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {851, 822, 834, 806, 724, 206}

$$-\frac{\left(33a^2cd^2e^4-21a^3e^6-3ac^2d^4e^2+7c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x^3\left(cd^2-ae^2\right)^3}-\frac{\left(-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd^2e^8-315a^5e^{10}-55ac^4d^8e^2+105c^5d^{10}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{24a^4d^5e^4x\left(cd^2-ae^2\right)^3}+\frac{\left(-18a^2c^2d^4e^4+168a^3cd^2e^6-105a^4e^8-16ac^3d^6e^2+35c^4d^8\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12a^3d^4e^3x^2\left(cd^2-ae^2\right)^3}+\frac{2\left(cdex\left(-9a^2e^4+14acd^2e^2+3c^2d^4\right)+13a^2cd^2e^4-9a^3e^6+ac^2d^4e^2+3c^3d^6\right)}{3ad^2ex^3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5\left(21a^2cd^2e^4+21a^3e^6+15ac^2d^4e^2+7c^3d^6\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{9/2}d^{11/2}e^{9/2}}-\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 13*a^2*c*d^2*e^4 - 9*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 14*a*c*d^2*e^2 - 9*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((7*c^3*d^6
 - 3*a*c^2*d^4*e^2 + 33*a^2*c*d^2*e^4 - 21*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^3) + ((35*c^4*d^8
- 16*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 168*a^3*c*d^2*e^6 - 105*a^4*e^8)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^2
)^3*x^2) - ((105*c^5*d^10 - 55*a*c^4*d^8*e^2 - 54*a^2*c^3*d^6*e^4 - 78*a^3*
c^2*d^4*e^6 + 525*a^4*c*d^2*e^8 - 315*a^5*e^10)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(24*a^4*d^5*e^4*(c*d^2 - a*e^2)^3*x) + (5*(7*c^3*d^6 + 15
*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 +
 a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(16*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \cdots}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.39, size = 593, normalized size = 0.89

$$\frac{(ae+cdx)\left(24a^{7/2}d^{9/2}e^{7/2}\left(ae^2-cd^2\right)^3+x\left(6a^{5/2}d^{7/2}e^{5/2}\left(cd^2-ae^2\right)^3\left(9ae^2+7cd^2\right)+3a^{3/2}d^{5/2}e^{3/2}x\left(ae^2-cd^2\right)^3\left(63a^2e^4+54acd^2e^2+35c^2d^4\right)+x^2\left(9\sqrt{a}cd^{7/2}\sqrt{e}\left(cd^2-ae^2\right)^2\left(21a^3e^6+3a^2cd^2e^4-5ac^2d^4e^2-35c^3d^6\right)+3\sqrt{a}d^{3/2}\sqrt{e}\left(ae^2-cd^2\right)\left(105a^4e^9-84a^3cd^2e^7-42a^2c^2d^4e^5-20ac^3d^6e^3+105c^4d^8e\right)(ae+cdx)+(d+ex)\sqrt{ae+cdx}\left(45\sqrt{d+ex}\left(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+3\sqrt{a}\sqrt{d}\sqrt{e}\left(315a^5e^{11}-525a^4cd^2e^9+78a^3c^2d^4e^7+54a^2c^3d^6e^5+55ac^4d^8e^3-105c^5d^{10}e\right)\sqrt{ae+cdx}\right)\right)\right)\right)}{72a^{9/2}d^{11/2}e^{9/2}x^3\left(cd^2-ae^2\right)^3((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((a*e + c*d*x)*(24*a^(7/2)*d^(9/2)*e^(7/2)*(-(c*d^2) + a*e^2)^3 + x*(6*a^(5
/2)*d^(7/2)*e^(5/2)*(c*d^2 - a*e^2)^3*(7*c*d^2 + 9*a*e^2) + 3*a^(3/2)*d^(5/
2)*e^(3/2)*(-(c*d^2) + a*e^2)^3*(35*c^2*d^4 + 54*a*c*d^2*e^2 + 63*a^2*e^4)*
x + x^2*(9*Sqrt[a]*c*d^(7/2)*Sqrt[e]*(c*d^2 - a*e^2)^2*(-35*c^3*d^6 - 5*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 21*a^3*e^6) + 3*Sqrt[a]*d^(3/2)*Sqrt[e]*(-(c
*d^2) + a*e^2)*(105*c^4*d^8*e - 20*a*c^3*d^6*e^3 - 42*a^2*c^2*d^4*e^5 - 84*
a^3*c*d^2*e^7 + 105*a^4*e^9)*(a*e + c*d*x) + Sqrt[a*e + c*d*x]*(d + e*x)*(3
*Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^5*d^10*e + 55*a*c^4*d^8*e^3 + 54*a^2*c^3*d
^6*e^5 + 78*a^3*c^2*d^4*e^7 - 525*a^4*c*d^2*e^9 + 315*a^5*e^11)*Sqrt[a*e +
c*d*x] + 45*(c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*
e^4 + 21*a^3*e^6)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a
]*Sqrt[e]*Sqrt[d + e*x])])))))/(72*a^(9/2)*d^(11/2)*e^(9/2)*(c*d^2 - a*e^2)
^3*x^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [A] time = 9.13, size = 711, normalized size = 1.07

$$\frac{\sqrt{ade+ae^2x+cd^2x+cdex^2}\left(-8a^6d^4e^9+18a^6d^3e^{10}x-63a^6d^2e^{11}x^2-420a^6de^{12}x^3-315a^6e^{13}x^4+24a^5cd^6e^7-40a^5cd^5e^8x+135a^5cd^4e^9x^2+651a^5cd^3e^{10}x^3+105a^5cd^2e^{11}x^4-315a^5cde^{12}x^5-24a^4c^2d^8e^5+12a^4c^2d^7e^6x-62a^4c^2d^6e^7x^2-3a^4c^2d^5e^8x^3+636a^4c^2d^4e^9x^4+525a^4c^2d^3e^{10}x^5+8a^3c^3d^{10}e^3+24a^3c^3d^9e^4x+6a^3c^3d^8e^5x^2-106a^3c^3d^7e^6x^3-174a^3c^3d^6e^7x^4-78a^3c^3d^5e^8x^5-14a^2c^4d^{11}e^2x-51a^2c^4d^{10}e^3x^2-114a^2c^4d^9e^4x^3-131a^2c^4d^8e^5x^4-54a^2c^4d^7e^6x^5+35ac^5d^{12}ex^2+15ac^5d^{11}e^2x^3-75ac^5d^{10}e^3x^4-55ac^5d^9e^4x^5+105c^6d^{13}x^3+210c^6d^{12}ex^4+105c^6d^{11}e^2x^5\right)}{24a^4d^5e^4x^3(d+ex)^2\left(ae^2-cd^2\right)^3(ae+cdx)}-\frac{5\left(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6\right)\tanh^{-1}\left(\frac{x\sqrt{cde}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right)}{8a^{9/2}d^{11/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2)),x]
```

```
[Out] (Sqrt[a*d*e + c*d^2*x + a*e^2*x + c*d*e*x^2]*(8*a^3*c^3*d^10*e^3 - 24*a^4*c
^2*d^8*e^5 + 24*a^5*c*d^6*e^7 - 8*a^6*d^4*e^9 - 14*a^2*c^4*d^11*e^2*x + 24*
a^3*c^3*d^9*e^4*x + 12*a^4*c^2*d^7*e^6*x - 40*a^5*c*d^5*e^8*x + 18*a^6*d^3*
e^10*x + 35*a*c^5*d^12*e*x^2 - 51*a^2*c^4*d^10*e^3*x^2 + 6*a^3*c^3*d^8*e^5*
x^2 - 62*a^4*c^2*d^6*e^7*x^2 + 135*a^5*c*d^4*e^9*x^2 - 63*a^6*d^2*e^11*x^2
+ 105*c^6*d^13*x^3 + 15*a*c^5*d^11*e^2*x^3 - 114*a^2*c^4*d^9*e^4*x^3 - 106*
a^3*c^3*d^7*e^6*x^3 - 3*a^4*c^2*d^5*e^8*x^3 + 651*a^5*c*d^3*e^10*x^3 - 420*
a^6*d*e^12*x^3 + 210*c^6*d^12*e*x^4 - 75*a*c^5*d^10*e^3*x^4 - 131*a^2*c^4*d
^8*e^5*x^4 - 174*a^3*c^3*d^6*e^7*x^4 + 636*a^4*c^2*d^4*e^9*x^4 + 105*a^5*c*
d^2*e^11*x^4 - 315*a^6*e^13*x^4 + 105*c^6*d^11*e^2*x^5 - 55*a*c^5*d^9*e^4*x
^5 - 54*a^2*c^4*d^7*e^6*x^5 - 78*a^3*c^3*d^5*e^8*x^5 + 525*a^4*c^2*d^3*e^10
*x^5 - 315*a^5*c*d*e^12*x^5))/(24*a^4*d^5*e^4*(-(c*d^2) + a*e^2)^3*x^3*(a*e
 + c*d*x)*(d + e*x)^2) - (5*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^
4 + 21*a^3*e^6)*ArcTanh[(Sqrt[c*d*e]*x - Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(Sqrt[a]*Sqrt[d]*Sqrt[e])])/(8*a^(9/2)*d^(11/2)*e^(9/2))
```

fricas [B] time = 93.16, size = 2526, normalized size = 3.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/96*(15*((7*c^7*d^13*e^2 - 6*a*c^6*d^11*e^4 - 3*a^2*c^5*d^9*e^6 - 4*a^3*c
^4*d^7*e^8 - 15*a^4*c^3*d^5*e^10 + 42*a^5*c^2*d^3*e^12 - 21*a^6*c*d*e^14)*x
^6 + (14*c^7*d^14*e - 5*a*c^6*d^12*e^3 - 12*a^2*c^5*d^10*e^5 - 11*a^3*c^4*d
^8*e^7 - 34*a^4*c^3*d^6*e^9 + 69*a^5*c^2*d^4*e^11 - 21*a^7*e^15)*x^5 + (7*c
^7*d^15 + 8*a*c^6*d^13*e^2 - 15*a^2*c^5*d^11*e^4 - 10*a^3*c^4*d^9*e^6 - 23*
a^4*c^3*d^7*e^8 + 12*a^5*c^2*d^5*e^10 + 63*a^6*c*d^3*e^12 - 42*a^7*d*e^14)*
x^4 + (7*a*c^6*d^14*e - 6*a^2*c^5*d^12*e^3 - 3*a^3*c^4*d^10*e^5 - 4*a^4*c^3
*d^8*e^7 - 15*a^5*c^2*d^6*e^9 + 42*a^6*c*d^4*e^11 - 21*a^7*d^2*e^13)*x^3)*s
qrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*a^4*c^3*d^11*e^4 - 2
4*a^5*c^2*d^9*e^6 + 24*a^6*c*d^7*e^8 - 8*a^7*d^5*e^10 + (105*a*c^6*d^12*e^3
 - 55*a^2*c^5*d^10*e^5 - 54*a^3*c^4*d^8*e^7 - 78*a^4*c^3*d^6*e^9 + 525*a^5*
c^2*d^4*e^11 - 315*a^6*c*d^2*e^13)*x^5 + (210*a*c^6*d^13*e^2 - 75*a^2*c^5*d
^11*e^4 - 131*a^3*c^4*d^9*e^6 - 174*a^4*c^3*d^7*e^8 + 636*a^5*c^2*d^5*e^10
+ 105*a^6*c*d^3*e^12 - 315*a^7*d*e^14)*x^4 + (105*a*c^6*d^14*e + 15*a^2*c^5
*d^12*e^3 - 114*a^3*c^4*d^10*e^5 - 106*a^4*c^3*d^8*e^7 - 3*a^5*c^2*d^6*e^9
+ 651*a^6*c*d^4*e^11 - 420*a^7*d^2*e^13)*x^3 + (35*a^2*c^5*d^13*e^2 - 51*a^
3*c^4*d^11*e^4 + 6*a^4*c^3*d^9*e^6 - 62*a^5*c^2*d^7*e^8 + 135*a^6*c*d^5*e^1
0 - 63*a^7*d^3*e^12)*x^2 - 2*(7*a^3*c^4*d^12*e^3 - 12*a^4*c^3*d^10*e^5 - 6*
a^5*c^2*d^8*e^7 + 20*a^6*c*d^6*e^9 - 9*a^7*d^4*e^11)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x))/((a^5*c^4*d^13*e^7 - 3*a^6*c^3*d^11*e^9 + 3*a^7*c
^2*d^9*e^11 - a^8*c*d^7*e^13)*x^6 + (2*a^5*c^4*d^14*e^6 - 5*a^6*c^3*d^12*e^
8 + 3*a^7*c^2*d^10*e^10 + a^8*c*d^8*e^12 - a^9*d^6*e^14)*x^5 + (a^5*c^4*d^1
5*e^5 - a^6*c^3*d^13*e^7 - 3*a^7*c^2*d^11*e^9 + 5*a^8*c*d^9*e^11 - 2*a^9*d^
7*e^13)*x^4 + (a^6*c^3*d^14*e^6 - 3*a^7*c^2*d^12*e^8 + 3*a^8*c*d^10*e^10 -
a^9*d^8*e^12)*x^3), -1/48*(15*((7*c^7*d^13*e^2 - 6*a*c^6*d^11*e^4 - 3*a^2*c
^5*d^9*e^6 - 4*a^3*c^4*d^7*e^8 - 15*a^4*c^3*d^5*e^10 + 42*a^5*c^2*d^3*e^12
```

```
- 21*a^6*c*d*e^14)*x^6 + (14*c^7*d^14*e - 5*a*c^6*d^12*e^3 - 12*a^2*c^5*d^1
0*e^5 - 11*a^3*c^4*d^8*e^7 - 34*a^4*c^3*d^6*e^9 + 69*a^5*c^2*d^4*e^11 - 21*
a^7*e^15)*x^5 + (7*c^7*d^15 + 8*a*c^6*d^13*e^2 - 15*a^2*c^5*d^11*e^4 - 10*a
^3*c^4*d^9*e^6 - 23*a^4*c^3*d^7*e^8 + 12*a^5*c^2*d^5*e^10 + 63*a^6*c*d^3*e^
12 - 42*a^7*d*e^14)*x^4 + (7*a*c^6*d^14*e - 6*a^2*c^5*d^12*e^3 - 3*a^3*c^4*
d^10*e^5 - 4*a^4*c^3*d^8*e^7 - 15*a^5*c^2*d^6*e^9 + 42*a^6*c*d^4*e^11 - 21*
a^7*d^2*e^13)*x^3)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a
^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(8*a^4*c^3*d^11*e^4 - 24*a^5*c
^2*d^9*e^6 + 24*a^6*c*d^7*e^8 - 8*a^7*d^5*e^10 + (105*a*c^6*d^12*e^3 - 55*a
^2*c^5*d^10*e^5 - 54*a^3*c^4*d^8*e^7 - 78*a^4*c^3*d^6*e^9 + 525*a^5*c^2*d^4
*e^11 - 315*a^6*c*d^2*e^13)*x^5 + (210*a*c^6*d^13*e^2 - 75*a^2*c^5*d^11*e^4
 - 131*a^3*c^4*d^9*e^6 - 174*a^4*c^3*d^7*e^8 + 636*a^5*c^2*d^5*e^10 + 105*a
^6*c*d^3*e^12 - 315*a^7*d*e^14)*x^4 + (105*a*c^6*d^14*e + 15*a^2*c^5*d^12*e
^3 - 114*a^3*c^4*d^10*e^5 - 106*a^4*c^3*d^8*e^7 - 3*a^5*c^2*d^6*e^9 + 651*a
^6*c*d^4*e^11 - 420*a^7*d^2*e^13)*x^3 + (35*a^2*c^5*d^13*e^2 - 51*a^3*c^4*d
^11*e^4 + 6*a^4*c^3*d^9*e^6 - 62*a^5*c^2*d^7*e^8 + 135*a^6*c*d^5*e^10 - 63*
a^7*d^3*e^12)*x^2 - 2*(7*a^3*c^4*d^12*e^3 - 12*a^4*c^3*d^10*e^5 - 6*a^5*c^2
*d^8*e^7 + 20*a^6*c*d^6*e^9 - 9*a^7*d^4*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/((a^5*c^4*d^13*e^7 - 3*a^6*c^3*d^11*e^9 + 3*a^7*c^2*d^9*
e^11 - a^8*c*d^7*e^13)*x^6 + (2*a^5*c^4*d^14*e^6 - 5*a^6*c^3*d^12*e^8 + 3*a
^7*c^2*d^10*e^10 + a^8*c*d^8*e^12 - a^9*d^6*e^14)*x^5 + (a^5*c^4*d^15*e^5 -
 a^6*c^3*d^13*e^7 - 3*a^7*c^2*d^11*e^9 + 5*a^8*c*d^9*e^11 - 2*a^9*d^7*e^13)
*x^4 + (a^6*c^3*d^14*e^6 - 3*a^7*c^2*d^12*e^8 + 3*a^8*c*d^10*e^10 - a^9*d^8
*e^12)*x^3)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.42Unable to transpose Err
or: Bad Argument Value
```

maple [**B**] time = 0.02, size = 1705, normalized size = 2.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 35/8*d^4/a^4/e^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c
*d^2)*x)^(1/2)*x*c^5+25/12/e*d^2/a^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^4-41/12/d^2*e^3/a/(-a^2*e^4+2*a*c*d^
2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^2+75/16/e^2/d/a^
3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^2-1/6*e/a^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)
/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*x*c^3-43/24/d*e^2/a/(-a^2*e^4+2*a*
c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2+105/8/d^4*e^
5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*
x*c+35/16*d^5/a^4/e^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2)*c^5+35/16*d/a^4/e^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c^3+16/3
/d^2*e^5*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)*
x+8/3/d^3*e^6*c/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/
2)*a-17/6/e/d^2/a^2/x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c+155/48/e^2*
```

```
d^3/a^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2)*c^4+7/12/d/a^2/e^2/x^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c+13/1
2/d^3/a/x^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-105/16/d^5*e^2/a/(c*d*e
*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)-105/16/d^3/a^2/(c*d*e*x^2+a*d*e+(a*e^2+c*
d^2)*x)^(1/2)*c-89/24/d^4*e/a/x/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)+105
/16/d^5*e^6*a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^
2)*x)^(1/2)+233/48/d^3*e^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*d*e*x^2+a*d*
e+(a*e^2+c*d^2)*x)^(1/2)*c+23/24*d/a^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(c*
d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3+105/16/d^5*e^2/a/(a*d*e)^(1/2)*ln(
(2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^
(1/2))/x)+105/16/d^3/a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2))/x)*c-75/16/e^2/d/a^3/(c*d*
e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^2-35/24/a^3/e^3/x/(c*d*e*x^2+a*d*e+(a*
e^2+c*d^2)*x)^(1/2)*c^2-1/3/d^2/a/e/x^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(
1/2)-35/16*d/a^4/e^4/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)*c^3-2/3/d^4*e^
3/(a*e^2-c*d^2)/(x+d/e)/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3/d
*e^4*c^2/(a*e^2-c*d^2)^3/((x+d/e)^2*c*d*e+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}} (ex + d)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^4),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(\left(d + ex\right) \left(ae + cdx\right)\right)^{\frac{3}{2}} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

3.321 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=259

$$\frac{8\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8\left(x\left(-2a^3e^6+a^2cd^2e^4+c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(3ae^2+cd^2\right.\right.}{15e\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Rubi [A] time = 0.24, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {854, 777, 613}

$$\frac{8\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8\left(x\left(a^2cd^2e^4-2a^3e^6+c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)}{15e\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{2x^2}{5(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*x^2)/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) - (8*(a*d*e*(c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (c^3*d^6 + a^2*c*d
^2*e^4 - 2*a^3*e^6)*x))/(15*e*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2)) + (8*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*
e^2 + 2*c*d*e*x))/(15*e*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 854

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((2*c*d - b*e)*(f + g*x)^n*(a + b*x +
 c*x^2)^(p + 1))/(e*p*(b^2 - 4*a*c)*(d + e*x)), x] - Dist[1/(d*e*p*(b^2 - 4
*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f*(
2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p + 1)
*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && IGt
Q[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x(-}{}}{}$$

$$= \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{8\left(ade\right.}{1}$$

$$= \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{8\left(ade\right.}{1}$$

Mathematica [A] time = 0.12, size = 235, normalized size = 0.91

$$\frac{2\left(a^4e^6\left(8d^2+20dex+15e^2x^2\right)+4a^3cde^4\left(20d^3+53d^2ex+45de^2x^2+15e^3x^3\right)+2a^2c^2d^2e^2\left(20d^4+110d^3ex+189d^2e^2x^2+110de^3x^3+20e^4x^4\right)+4ac^3d^4ex\left(15d^3+45d^2ex+53de^2x^2+20e^3x^3\right)+c^4d^6x^2\left(15d^2+20dex+8e^2x^2\right)\right)}{15(d+ex)\left(cd^2-ae^2\right)^5((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*(c^4*d^6*x^2*(15*d^2 + 20*d*e*x + 8*e^2*x^2) + a^4*e^6*(8*d^2 + 20*d*e*x
 + 15*e^2*x^2) + 4*a^3*c*d*e^4*(20*d^3 + 53*d^2*e*x + 45*d*e^2*x^2 + 15*e^3
*x^3) + 4*a*c^3*d^4*e*x*(15*d^3 + 45*d^2*e*x + 53*d*e^2*x^2 + 20*e^3*x^3) +
 2*a^2*c^2*d^2*e^2*(20*d^4 + 110*d^3*e*x + 189*d^2*e^2*x^2 + 110*d*e^3*x^3
+ 20*e^4*x^4)))/(15*(c*d^2 - a*e^2)^5*(d + e*x)*((a*e + c*d*x)*(d + e*x))^(
3/2))
```

IntegrateAlgebraic [F] time = 180.07, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 27.13, size = 820, normalized size = 3.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"fricas")
```

```
[Out] 2/15*(40*a^2*c^2*d^6*e^2 + 80*a^3*c*d^4*e^4 + 8*a^4*d^2*e^6 + 8*(c^4*d^6*e^
2 + 10*a*c^3*d^4*e^4 + 5*a^2*c^2*d^2*e^6)*x^4 + 4*(5*c^4*d^7*e + 53*a*c^3*d
^5*e^3 + 55*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7)*x^3 + 3*(5*c^4*d^8 + 60*a*c^3
*d^6*e^2 + 126*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 + 5*a^4*e^8)*x^2 + 4*(15*
a*c^3*d^7*e + 55*a^2*c^2*d^5*e^3 + 53*a^3*c*d^3*e^5 + 5*a^4*d*e^7)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*d^11*e
^4 + 10*a^4*c^3*d^9*e^6 - 10*a^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^7*d^3*e
^12 + (c^7*d^12*e^3 - 5*a*c^6*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^3*c^4*d^
6*e^9 + 5*a^4*c^3*d^4*e^11 - a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e^2 - 13*a
*c^6*d^11*e^4 + 20*a^2*c^5*d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c^3*d^5*e^1
0 + 7*a^5*c^2*d^3*e^12 - 2*a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9*a*c^6*d^12
```

```
*e^3 + a^2*c^5*d^10*e^5 + 25*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e^9 + 17*a^5*
c^2*d^4*e^11 - a^6*c*d^2*e^13 - a^7*e^15)*x^3 + (c^7*d^15 + a*c^6*d^13*e^2
- 17*a^2*c^5*d^11*e^4 + 35*a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e^8 - a^5*c^2*d
^5*e^10 + 9*a^6*c*d^3*e^12 - 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^14*e - 7*a^2*c^
5*d^12*e^3 + 5*a^3*c^4*d^10*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a^5*c^2*d^6*e^9 +
 13*a^6*c*d^4*e^11 - 3*a^7*d^2*e^13)*x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.5Unable to transpose Erro
r: Bad Argument Value
```

maple [**A**] time = 0.01, size = 366, normalized size = 1.41

$$-\frac{2 \left(c d x+a e\right) \left(40 a^{2} c^{2} d^{2} e^{6} x^{4}+80 a \,c^{3} d^{4} e^{4} x^{4}+8 c^{4} d^{6} e^{2} x^{4}+60 a^{3} c d \,e^{7} x^{3}+220 a^{2} c^{2} d^{3} e^{5} x^{3}+212 a \,c^{3} d^{5} e^{3} x^{3}+20 c^{4} d^{7} e \,x^{3}+15 a^{4} e^{8} x^{2}+180 a^{3} c \,d^{2} e^{6} x^{2}+378 a^{2} c^{2} d^{4} e^{4} x^{2}+180 a \,c^{3} d^{6} e^{2} x^{2}+15 c^{4} d^{8} x^{2}+20 a^{4} d \,e^{7} x+212 a^{3} c \,d^{3} e^{5} x+220 a^{2} c^{2} d^{5} e^{3} x+60 a \,c^{3} d^{7} e x+8 a^{4} d^{2} e^{6}+80 a^{3} c \,d^{4} e^{4}+40 a^{2} c^{2} d^{6} e^{2}\right)}{15 \left(a^{5} e^{10}-5 a^{4} c \,d^{2} e^{8}+10 a^{3} c^{2} d^{4} e^{6}-10 a^{2} c^{3} d^{6} e^{4}+5 a \,c^{4} d^{8} e^{2}-c^{5} d^{10}\right) \left(c d e \,x^{2}+a \,e^{2} x+c \,d^{2} x+a d e \right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(40*a^2*c^2*d^2*e^6*x^4+80*a*c^3*d^4*e^4*x^4+8*c^4*d^6*e^
2*x^4+60*a^3*c*d*e^7*x^3+220*a^2*c^2*d^3*e^5*x^3+212*a*c^3*d^5*e^3*x^3+20*c
^4*d^7*e*x^3+15*a^4*e^8*x^2+180*a^3*c*d^2*e^6*x^2+378*a^2*c^2*d^4*e^4*x^2+1
80*a*c^3*d^6*e^2*x^2+15*c^4*d^8*x^2+20*a^4*d*e^7*x+212*a^3*c*d^3*e^5*x+220*
a^2*c^2*d^5*e^3*x+60*a*c^3*d^7*e*x+8*a^4*d^2*e^6+80*a^3*c*d^4*e^4+40*a^2*c^
2*d^6*e^2)/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+
5*a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [**B**] time = 4.33, size = 3099, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

```
[Out] (((6*a*e^2 - 10*c*d^2)/(15*(a*e^2 - c*d^2)^4) - (4*c*d^2)/(5*(a*e^2 - c*d^2
)^4))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d*((e*(
2*a*e^3 - 2*c*d^2*e))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (4*c*d^
2*e^2)/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e))))/e + (e*(2*c*d^3 + 2*a*
d*e^2))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*
```

```
x^2)^(1/2)*(x*((((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2
*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^
2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(a*e^2 + c*d^2))/(c*d*e)
 - (6*c^2*d^2*e*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2*
d^3*e^3 + a^2*c*d*e^5)) + (8*a*c^3*d^4*e^3)/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e
 - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^2*d^2*e*(46*a^2*e^4 + 4*c^2*d^4 +
 66*a*c*d^2*e^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*
c*d*e^5))) + (a*((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2
*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^
2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - (c*d*(a*e^2 + c*d^2
)*(46*a^2*e^4 + 4*c^2*d^4 + 66*a*c*d^2*e^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)*(d + e*x)) + ((x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*(((a*e^2 + c*d^2)*((4*c^4*d^4
*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
a^2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(10*c^
3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e
 - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*(a*e^2 - c*d^2)^3
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3*e^2*(a*e^2 + c*d
^2)*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/
(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^4
*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*
e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*((4*c^4*d^4*e^
3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2
*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(10*c^3*d
^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*(a*e^2 - c*d^2)^3*(c
^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3*e^2*(a*e^2 + c*d^2)
*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^
2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(12*a^3*e^5 - 36*a^2*c*d^2*e^3))/(15
*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(a
*e^2 + c*d^2)*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 -
c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*a^3*c^
2*d^3*e^6)/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)
) - (c*d*e*(12*a^3*e^5 - 36*a^2*c*d^2*e^3)*(a*e^2 + c*d^2))/(15*(a*e^2 - c*
d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) + (a*((a*((4*c^4*d^4*e
^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^
2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c
*d^2)*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e
^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c
^2*d^2*e^2*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d
^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*
(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3
*e^2*(a*e^2 + c*d^2)*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e -
2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(12*a^3*e^5 - 36
*a^2*c*d^2*e^3))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c
*d*e^5)) - (c*d*e*(a*e^2 + c*d^2)*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d
*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))
/c + (4*a^3*c*d^2*e^5*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)^2*(d + e*x)^2) - (2*d^2*e*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((d + e*x)^3*(5*a^3*e^7 - 5*c^
3*d^6*e + 15*a*c^2*d^4*e^3 - 15*a^2*c*d^2*e^5))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

3.322 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx$

Optimal. Leaf size=341

$$-\frac{128cd\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105e\left(cd^2-ae^2\right)^5\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{8(\ldots}{35\ldots}$$

Rubi [A] time = 0.29, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {854, 777, 614, 613}

$$-\frac{128cd\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105e\left(cd^2-ae^2\right)^5\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{8\left(x\left(3a^2e^4+acd^2e^2+2c^2d^4\right)+2ade\left(2ae^2+cd^2\right)\right)}{35e\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}+\frac{2x^2}{7(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

```
[Out] (2*x^2)/(7*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)) - (8*(2*a*d*e*(c*d^2 + 2*a*e^2) + (2*c^2*d^4 + a*c*d^2*e^2 + 3*a^2
*e^4)*x))/(35*e*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)) + (16*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d
*e*x))/(105*e*(c*d^2 - a*e^2)^5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)) - (128*c*d*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*
c*d*e*x))/(105*(c*d^2 - a*e^2)^7*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 854

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((2*c*d - b*e)*(f + g*x)^n*(a + b*x +
 c*x^2)^(p + 1))/(e*p*(b^2 - 4*a*c)*(d + e*x)), x] - Dist[1/(d*e*p*(b^2 - 4
*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f*(
2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p + 1)
*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && IGt
Q[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx = \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} + \frac{2\int}{}$$

$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} - \frac{8(2}{35}$$

$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} - \frac{8(2}{35}$$

$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} - \frac{8(2}{35}$$

Mathematica [A] time = 0.19, size = 433, normalized size = 1.27

$$-\frac{2\sqrt{(d+ex)(ae+cdx)}\left(-a^6e^{10}\left(8d^2+28dex+35e^2x^2\right)+2a^5cde^8\left(112d^3+382d^2ex+455de^2x^2+140e^3x^3\right)+5a^4c^2d^2e^6\left(336d^4+1288d^3ex+1859d^2e^2x^2+1288de^3x^3+336e^4x^4\right)+20a^3c^3d^3e^4\left(56d^5+406d^4ex+1001d^3e^2x^2+1084d^2e^3x^3+560de^4x^4+112e^5x^5\right)+a^2c^4d^4e^2\left(56d^6+2996d^5ex+13195d^4e^2x^2+24080d^3e^3x^3+20320d^2e^4x^4+7616de^5x^5+896e^6x^6\right)+2ac^5d^6ex\left(70d^5+1295d^4ex+4060d^3e^2x^2+5600d^2e^3x^3+3616de^4x^4+896e^5x^5\right)+3c^6d^8x^2\left(35d^4+280d^3ex+560d^2e^2x^2+448de^3x^3+128e^4x^4\right)\right)}{105(d+ex)^4\left(cd^2-ae^2\right)^7(ae+cdx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

```
[Out] (-2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a^6*e^10*(8*d^2 + 28*d*e*x + 35*e^2*x^
2)) + 2*a^5*c*d*e^8*(112*d^3 + 382*d^2*e*x + 455*d*e^2*x^2 + 140*e^3*x^3) +
 3*c^6*d^8*x^2*(35*d^4 + 280*d^3*e*x + 560*d^2*e^2*x^2 + 448*d*e^3*x^3 + 12
8*e^4*x^4) + 5*a^4*c^2*d^2*e^6*(336*d^4 + 1288*d^3*e*x + 1859*d^2*e^2*x^2 +
 1288*d*e^3*x^3 + 336*e^4*x^4) + 20*a^3*c^3*d^3*e^4*(56*d^5 + 406*d^4*e*x +
 1001*d^3*e^2*x^2 + 1084*d^2*e^3*x^3 + 560*d*e^4*x^4 + 112*e^5*x^5) + 2*a*c
^5*d^6*e*x*(70*d^5 + 1295*d^4*e*x + 4060*d^3*e^2*x^2 + 5600*d^2*e^3*x^3 + 3
616*d*e^4*x^4 + 896*e^5*x^5) + a^2*c^4*d^4*e^2*(56*d^6 + 2996*d^5*e*x + 131
95*d^4*e^2*x^2 + 24080*d^3*e^3*x^3 + 20320*d^2*e^4*x^4 + 7616*d*e^5*x^5 + 8
96*e^6*x^6)))/(105*(c*d^2 - a*e^2)^7*(a*e + c*d*x)^3*(d + e*x)^4)
```

IntegrateAlgebraic [F] time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
7/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 167.97, size = 1540, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"fricas")
```

```
[Out] -2/105*(56*a^2*c^4*d^10*e^2 + 1120*a^3*c^3*d^8*e^4 + 1680*a^4*c^2*d^6*e^6 +
 224*a^5*c*d^4*e^8 - 8*a^6*d^2*e^10 + 128*(3*c^6*d^8*e^4 + 14*a*c^5*d^6*e^6
 + 7*a^2*c^4*d^4*e^8)*x^6 + 64*(21*c^6*d^9*e^3 + 113*a*c^5*d^7*e^5 + 119*a^
2*c^4*d^5*e^7 + 35*a^3*c^3*d^3*e^9)*x^5 + 80*(21*c^6*d^10*e^2 + 140*a*c^5*d
^8*e^4 + 254*a^2*c^4*d^6*e^6 + 140*a^3*c^3*d^4*e^8 + 21*a^4*c^2*d^2*e^10)*x
^4 + 40*(21*c^6*d^11*e + 203*a*c^5*d^9*e^3 + 602*a^2*c^4*d^7*e^5 + 542*a^3*
c^3*d^5*e^7 + 161*a^4*c^2*d^3*e^9 + 7*a^5*c*d*e^11)*x^3 + 5*(21*c^6*d^12 +
518*a*c^5*d^10*e^2 + 2639*a^2*c^4*d^8*e^4 + 4004*a^3*c^3*d^6*e^6 + 1859*a^4
*c^2*d^4*e^8 + 182*a^5*c*d^2*e^10 - 7*a^6*e^12)*x^2 + 4*(35*a*c^5*d^11*e +
749*a^2*c^4*d^9*e^3 + 2030*a^3*c^3*d^7*e^5 + 1610*a^4*c^2*d^5*e^7 + 191*a^5
*c*d^3*e^9 - 7*a^6*d*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(
a^3*c^7*d^18*e^3 - 7*a^4*c^6*d^16*e^5 + 21*a^5*c^5*d^14*e^7 - 35*a^6*c^4*d^
12*e^9 + 35*a^7*c^3*d^10*e^11 - 21*a^8*c^2*d^8*e^13 + 7*a^9*c*d^6*e^15 - a^
10*d^4*e^17 + (c^10*d^17*e^4 - 7*a*c^9*d^15*e^6 + 21*a^2*c^8*d^13*e^8 - 35*
a^3*c^7*d^11*e^10 + 35*a^4*c^6*d^9*e^12 - 21*a^5*c^5*d^7*e^14 + 7*a^6*c^4*d
^5*e^16 - a^7*c^3*d^3*e^18)*x^7 + (4*c^10*d^18*e^3 - 25*a*c^9*d^16*e^5 + 63
*a^2*c^8*d^14*e^7 - 77*a^3*c^7*d^12*e^9 + 35*a^4*c^6*d^10*e^11 + 21*a^5*c^5
*d^8*e^13 - 35*a^6*c^4*d^6*e^15 + 17*a^7*c^3*d^4*e^17 - 3*a^8*c^2*d^2*e^19)
*x^6 + 3*(2*c^10*d^19*e^2 - 10*a*c^9*d^17*e^4 + 15*a^2*c^8*d^15*e^6 + 7*a^3
*c^7*d^13*e^8 - 49*a^4*c^6*d^11*e^10 + 63*a^5*c^5*d^9*e^12 - 35*a^6*c^4*d^7
*e^14 + 5*a^7*c^3*d^5*e^16 + 3*a^8*c^2*d^3*e^18 - a^9*c*d*e^20)*x^5 + (4*c^
10*d^20*e - 10*a*c^9*d^18*e^3 - 30*a^2*c^8*d^16*e^5 + 155*a^3*c^7*d^14*e^7
- 245*a^4*c^6*d^12*e^9 + 147*a^5*c^5*d^10*e^11 + 35*a^6*c^4*d^8*e^13 - 95*a
^7*c^3*d^6*e^15 + 45*a^8*c^2*d^4*e^17 - 5*a^9*c*d^2*e^19 - a^10*e^21)*x^4 +
 (c^10*d^21 + 5*a*c^9*d^19*e^2 - 45*a^2*c^8*d^17*e^4 + 95*a^3*c^7*d^15*e^6
- 35*a^4*c^6*d^13*e^8 - 147*a^5*c^5*d^11*e^10 + 245*a^6*c^4*d^9*e^12 - 155*
a^7*c^3*d^7*e^14 + 30*a^8*c^2*d^5*e^16 + 10*a^9*c*d^3*e^18 - 4*a^10*d*e^20)
*x^3 + 3*(a*c^9*d^20*e - 3*a^2*c^8*d^18*e^3 - 5*a^3*c^7*d^16*e^5 + 35*a^4*c
^6*d^14*e^7 - 63*a^5*c^5*d^12*e^9 + 49*a^6*c^4*d^10*e^11 - 7*a^7*c^3*d^8*e^
13 - 15*a^8*c^2*d^6*e^15 + 10*a^9*c*d^4*e^17 - 2*a^10*d^2*e^19)*x^2 + (3*a^
2*c^8*d^19*e^2 - 17*a^3*c^7*d^17*e^4 + 35*a^4*c^6*d^15*e^6 - 21*a^5*c^5*d^1
3*e^8 - 35*a^6*c^4*d^11*e^10 + 77*a^7*c^3*d^9*e^12 - 63*a^8*c^2*d^7*e^14 +
25*a^9*c*d^5*e^16 - 4*a^10*d^3*e^18)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.6Unable to transpose Erro
r: Bad Argument Value
```

maple [B] time = 0.02, size = 663, normalized size = 1.94

$$-\frac{2\left(cdx+ae\right)\left(-896a^{2}c^{4}d^{4}e^{8}x^{6}-1792a\,c^{5}d^{6}e^{6}x^{6}-384c^{6}d^{8}e^{4}x^{6}-2240a^{3}c^{3}d^{3}e^{9}x^{5}-7616a^{2}c^{4}d^{5}e^{7}x^{5}-7232a\,c^{5}d^{7}e^{5}x^{5}-1344c^{6}d^{9}e^{3}x^{5}-1680a^{4}c^{2}d^{2}e^{10}x^{4}-11200a^{3}c^{3}d^{4}e^{8}x^{4}-20320a^{2}c^{4}d^{6}e^{6}x^{4}-11200a\,c^{5}d^{8}e^{4}x^{4}-1680c^{6}d^{10}e^{2}x^{4}-280a^{5}cd\,e^{11}x^{3}-6440a^{4}c^{2}d^{3}e^{9}x^{3}-21680a^{3}c^{3}d^{5}e^{7}x^{3}-24080a^{2}c^{4}d^{7}e^{5}x^{3}-8120a\,c^{5}d^{9}e^{3}x^{3}-840c^{6}d^{11}e\,x^{3}+35a^{6}e^{12}x^{2}-910a^{5}c\,d^{2}e^{10}x^{2}-9295a^{4}c^{2}d^{4}e^{8}x^{2}-20020a^{3}c^{3}d^{6}e^{6}x^{2}-13195a^{2}c^{4}d^{8}e^{4}x^{2}-2590a\,c^{5}d^{10}e^{2}x^{2}-105c^{6}d^{12}x^{2}+28a^{6}d\,e^{11}x-764a^{5}c\,d^{3}e^{9}x-6440a^{4}c^{2}d^{5}e^{7}x-8120a^{3}c^{3}d^{7}e^{5}x-2996a^{2}c^{4}d^{9}e^{3}x-140a\,c^{5}d^{11}ex+8a^{6}d^{2}e^{10}-224a^{5}c\,d^{4}e^{8}-1680a^{4}c^{2}d^{6}e^{6}-1120a^{3}c^{3}d^{8}e^{4}-56a^{2}c^{4}d^{10}e^{2}\right)}{105\left(a^{7}e^{14}-7a^{6}c\,d^{2}e^{12}+21a^{5}c^{2}d^{4}e^{10}-35a^{4}c^{3}d^{6}e^{8}+35a^{3}c^{4}d^{8}e^{6}-21a^{2}c^{5}d^{10}e^{4}+7a\,c^{6}d^{12}e^{2}-c^{7}d^{14}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(7/2),x)
```

```
[Out] -2/105*(c*d*x+a*e)*(-896*a^2*c^4*d^4*e^8*x^6-1792*a*c^5*d^6*e^6*x^6-384*c^6
*d^8*e^4*x^6-2240*a^3*c^3*d^3*e^9*x^5-7616*a^2*c^4*d^5*e^7*x^5-7232*a*c^5*d
^7*e^5*x^5-1344*c^6*d^9*e^3*x^5-1680*a^4*c^2*d^2*e^10*x^4-11200*a^3*c^3*d^4
*e^8*x^4-20320*a^2*c^4*d^6*e^6*x^4-11200*a*c^5*d^8*e^4*x^4-1680*c^6*d^10*e^
2*x^4-280*a^5*c*d*e^11*x^3-6440*a^4*c^2*d^3*e^9*x^3-21680*a^3*c^3*d^5*e^7*x
^3-24080*a^2*c^4*d^7*e^5*x^3-8120*a*c^5*d^9*e^3*x^3-840*c^6*d^11*e*x^3+35*a
```

```
^6*e^12*x^2-910*a^5*c*d^2*e^10*x^2-9295*a^4*c^2*d^4*e^8*x^2-20020*a^3*c^3*d
^6*e^6*x^2-13195*a^2*c^4*d^8*e^4*x^2-2590*a*c^5*d^10*e^2*x^2-105*c^6*d^12*x
^2+28*a^6*d*e^11*x-764*a^5*c*d^3*e^9*x-6440*a^4*c^2*d^5*e^7*x-8120*a^3*c^3*
d^7*e^5*x-2996*a^2*c^4*d^9*e^3*x-140*a*c^5*d^11*e*x+8*a^6*d^2*e^10-224*a^5*
c*d^4*e^8-1680*a^4*c^2*d^6*e^6-1120*a^3*c^3*d^8*e^4-56*a^2*c^4*d^10*e^2)/(a
^7*e^14-7*a^6*c*d^2*e^12+21*a^5*c^2*d^4*e^10-35*a^4*c^3*d^6*e^8+35*a^3*c^4*
d^8*e^6-21*a^2*c^5*d^10*e^4+7*a*c^6*d^12*e^2-c^7*d^14)/(c*d*e*x^2+a*e^2*x+c
*d^2*x+a*d*e)^(7/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` for
more details)Is a*e^2-c*d^2 zero or nonzero?
```

mupad [**B**] time = 7.72, size = 11469, normalized size = 33.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)),x)
```

```
[Out] ((6*c^3*d^5 + 36*a*c^2*d^3*e^2 - 10*a^2*c*d*e^4)/(105*(a*e^2 - c*d^2)^6) -
x*((16*c^2*d^2*e)/(105*(a*e^2 - c*d^2)^5) - (8*c^2*d^2*e*(a*e^2 + c*d^2))/(
105*(a*e^2 - c*d^2)^6)) + (8*a*c^2*d^3*e^2)/(105*(a*e^2 - c*d^2)^6))/(x*(a*
e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + (x*((a*((64*c^5*d^5*e^4*(a*e^2 +
c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))
 - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e -
 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*(
(64*c^5*d^5*e^4*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^
2*d^3*e^3 + a^2*c*d*e^5)) - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^
2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c
^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (128*a*c^5*d^6*e^5)/(105*(a
*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (32*c^4*d^4*
e^3*(a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(60*c^4*d^7 -
204*a*c^3*d^5*e^2 - 156*a^2*c^2*d^3*e^4 + 44*a^3*c*d*e^6))/(105*(a*e^2 - c*
d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^3*d^3*e^2*(a*e^
2 + c*d^2)*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*(((a*e^2 + c*d^2)*((64*
c^5*d^5*e^4*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^
3*e^3 + a^2*c*d*e^5)) - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 -
c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^4*d
^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (128*a*c^5*d^6*e^5)/(105*(a*e^2
 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (32*c^4*d^4*e^3*
(a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (c*d*e*(a*e^2 + c*d^2)*(60*c^4*d^7 - 20
4*a*c^3*d^5*e^2 - 156*a^2*c^2*d^3*e^4 + 44*a^3*c*d*e^6))/(105*(a*e^2 - c*d^
2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))/(x*(a*e^2 + c*d^2) + a*d
*e + c*d*e*x^2)^(1/2) + (x*((a*((8*c^3*d^3*e^2*(a*e^2 + c*d^2))/(105*(a*e^2
 - c*d^2)^6) - (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6)))/
c + (36*c^4*d^7*e - 76*a*c^3*d^5*e^3 - 36*a^2*c^2*d^3*e^5 + 12*a^3*c*d*e^7)
```

```
/(105*e*(a*e^2 - c*d^2)^6) + ((a*e^2 + c*d^2)*((8*a*c^3*d^4*e^3)/(105*(a*e^
2 - c*d^2)^6) - (((8*c^3*d^3*e^2*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6) -
 (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6))*(a*e^2 + c*d^2)
)/(c*d*e) + (2*c^2*d^2*e*(11*c^2*d^4 - 13*a^2*e^4 + 14*a*c*d^2*e^2))/(105*(
a*e^2 - c*d^2)^6)))/(c*d*e)) + (30*a^4*e^8 - 22*c^4*d^8 + 20*a*c^3*d^6*e^2
 - 132*a^3*c*d^2*e^6 + 72*a^2*c^2*d^4*e^4)/(105*e*(a*e^2 - c*d^2)^6) + (a*((
8*a*c^3*d^4*e^3)/(105*(a*e^2 - c*d^2)^6) - (((8*c^3*d^3*e^2*(a*e^2 + c*d^2)
)/(105*(a*e^2 - c*d^2)^6) - (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 -
 c*d^2)^6))*(a*e^2 + c*d^2))/(c*d*e) + (2*c^2*d^2*e*(11*c^2*d^4 - 13*a^2*e^
4 + 14*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6)))/c)/(x*(a*e^2 + c*d^2) + a*d*
e + c*d*e*x^2)^(3/2) - (((d*((e*(2*a*e^4 - 2*c*d^2*e^2))/(7*(a*e^2 - c*d^2)
^4*(5*a*e^3 - 5*c*d^2*e)) - (4*c*d^2*e^3)/(7*(a*e^2 - c*d^2)^4*(5*a*e^3 - 5
*c*d^2*e))))/e + (e*(2*a*d*e^3 + 2*c*d^3*e))/(7*(a*e^2 - c*d^2)^4*(5*a*e^3
 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 +
 (((e*(10*a*e^3 - 14*c*d^2*e))/(35*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e))
 - (4*c*d^2*e^2)/(7*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c
*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*((a*((a*((4*c^5*d^5*e^4
*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*
c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^
5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d
^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a
*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2
 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4
*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 -
c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e
^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(14*c^4*d^7 - 56*a*c^3*d^
5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^
5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^3*d^3*e^2*(a*e^2 + c*d^2)*(7*c
^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a
*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*((a*(((a*e^2 + c*d^2)*(
(4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*
d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*
d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4*d^4*
e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2
)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e^2 +
c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^
3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((4*c^5*d^5*e^4*(a*e^2 + c*d^2
))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*
c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*((4*c^5*d^5
*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4*d^4*e^3*(7*c^2
*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c
^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*
a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*
d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(14*c^4*d^7 - 56*a*c^3*d^5*e^2 - 12*a^2*
c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5)) - (2*c^3*d^3*e^2*(a*e^2 + c*d^2)*(7*c^2*d^4 - 9*a^2*
e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(16*a^4*e^7 - 64*a^3*c*d^2*e^5))/(
35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*
(a*e^2 + c*d^2)*(14*c^4*d^7 - 56*a*c^3*d^5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^
3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^
5))))/(c*d*e) + (32*a^4*c^2*d^3*e^8)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a
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*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(16*a^4*e^7 - 64*a^3*c*d^2*e^5)*(a*e^
2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^
5))) - (a*((a*(((a*e^2 + c*d^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2
 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(
7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*
c*d*e^5))))/(c*d*e) - (4*c^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2
))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*
a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d
*e^5)) + (2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d
^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*(
(a*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2
- c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d
^2)*(((a*e^2 + c*d^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^
4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 -
c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))
)/(c*d*e) - (4*c^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*
e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*
e^5)/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (
2*c^4*d^4*e^3*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3
*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(14*c^4
*d^7 - 56*a*c^3*d^5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2
- c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^3*d^3*e^2*(a
*e^2 + c*d^2)*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^
4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(
16*a^4*e^7 - 64*a^3*c*d^2*e^5))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*
d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(a*e^2 + c*d^2)*(14*c^4*d^7 - 56*a*c^3*d^5
*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (16*a^4*c*d^2*e^7*(a*e^2 + c*d^2
))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(x*(
a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e + c*d*x)^3*(d + e*x)^3) -
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*((16*c^5*d^5*e^4*(a*e
^2 + c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e
^5)) - (16*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*
e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2
)*((16*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(35*(a*e
^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (16*
c^4*d^4*e^3*(c^2*d^4 - 7*a^2*e^4 + 14*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^5*d^6*e^5)/(35*(a*e^2
 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^4*d^4*e^3*(
a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(484*c^4*d^7 + 1228*
a*c^3*d^5*e^2 - 1092*a^2*c^2*d^3*e^4 - 812*a^3*c*d*e^6))/(105*(a*e^2 - c*d^
2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*c^3*d^3*e^2*(a*e^2 +
 c*d^2)*(c^2*d^4 - 7*a^2*e^4 + 14*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^6*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*(((a*e^2 + c*d^2)*((16*c^5*d^
5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5)) - (16*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (16*c^4*d^4*e^3*(
c^2*d^4 - 7*a^2*e^4 + 14*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^5*d^6*e^5)/(35*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^4*d^4*e^3*(a*e^2 + c*d^2
)*(5*a*e^2 - 3*c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5))))/c + (c*d*e*(a*e^2 + c*d^2)*(484*c^4*d^7 + 1228*a*c^3*d^5*e
^2 - 1092*a^2*c^2*d^3*e^4 - 812*a^3*c*d*e^6))/(105*(a*e^2 - c*d^2)^6*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)*(d + e*x)) + ((x*(a
*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*((a*((16*c^6*d^6*e^5*(a*e^2
 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^
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5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*
((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c
^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2
- c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^
5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(105*(a*e^
2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e^4*
(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*c^4*d^4*e^3*(25*a^3*e^6 + 5*c^3*
d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^4*d^4*e^3*(a*e^2 + c*d^2)*(2*c
^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*((a*(((a*e^2 + c*d^2)*
((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c
^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2
- c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^
5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(105*(a*e^
2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e^4*
(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*
c^2*d^3*e^3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((16*c^6*d^6*e^5*(a*
e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d
*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^
2)*((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*
e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32
*c^5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^
6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(105*(a
*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e
^4*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*c^4*d^4*e^3*(25*a^3*e^6 + 5*c
^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e^2 - c*d^2)^6*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^4*d^4*e^3*(a*e^2 + c*d^2)*(
2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e -
2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^3*d^3*e^2*(7*a^4*e^8 - 3
*c^4*d^8 - 28*a^3*c*d^2*e^6 + 12*a^2*c^2*d^4*e^4))/(105*(a*e^2 - c*d^2)^6*(
c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2
)*(25*a^3*e^6 + 5*c^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e
^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c
^2*d^2*e*(88*c^5*d^10 - 152*a^5*e^10 + 80*a*c^4*d^8*e^2 + 272*a^4*c*d^2*e^8
- 520*a^2*c^3*d^6*e^4 + 296*a^3*c^2*d^4*e^6))/(105*(a*e^2 - c*d^2)^6*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^2*d^2*e*(a*e^2 + c*d^2)*(7*
a^4*e^8 - 3*c^4*d^8 - 28*a^3*c*d^2*e^6 + 12*a^2*c^2*d^4*e^4))/(105*(a*e^2 -
c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*((a*(((a*e^2 +
c*d^2)*((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(10
5*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e)
- (32*c^5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*
d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)/(1
05*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*
d^5*e^4*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*
e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((16*c^6*d^6
e^5(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*
(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^
2 + c*d^2)*((16*c^6*d^6*e^5*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^
5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^6*d^6*e^5*(7*a*e^2 - c*d^2))/

```
(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*
e) - (32*c^5*d^5*e^4*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 -
 c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (32*a*c^6*d^7*e^6)
/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c
^5*d^5*e^4*(a*e^2 + c*d^2)*(7*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*c^4*d^4*e^3*(25*a^3*e
^6 + 5*c^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(105*(a*e^2 - c*d^2)
^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^4*d^4*e^3*(a*e^2 +
c*d^2)*(2*c^2*d^4 - 6*a^2*e^4 + 5*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^3
*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^3*d^3*e^2*(7*a^4
*e^8 - 3*c^4*d^8 - 28*a^3*c*d^2*e^6 + 12*a^2*c^2*d^4*e^4))/(105*(a*e^2 - c*
d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2
 + c*d^2)*(25*a^3*e^6 + 5*c^3*d^6 - 23*a*c^2*d^4*e^2 - 51*a^2*c*d^2*e^4))/(
105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (c
*d*(a*e^2 + c*d^2)*(88*c^5*d^10 - 152*a^5*e^10 + 80*a*c^4*d^8*e^2 + 272*a^4
*c*d^2*e^8 - 520*a^2*c^3*d^6*e^4 + 296*a^3*c^2*d^4*e^6))/(105*(a*e^2 - c*d^
2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)^2*(d + e
*x)^2) - (2*d^2*e^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((d + e*
x)^4*(7*a^4*e^9 + 7*c^4*d^8*e - 28*a*c^3*d^6*e^3 - 28*a^3*c*d^2*e^7 + 42*a^
2*c^2*d^4*e^5)) + (8*c*d*e*(5*a*e^2 + c*d^2)*(x*(a*e^2 + c*d^2) + a*d*e + c
*d*e*x^2)^(1/2))/(105*(a*e^2 - c*d^2)^6*(d + e*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2),x)
```

```
[Out] Timed out
```

3.323 $\int x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=23

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx=\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.00

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

IntegrateAlgebraic [F] time = 33.85, size = 0, normalized size = 0.00

$$\int x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
[Out] Defer[IntegrateAlgebraic][x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2], x]
```

fricas [A] time = 0.38, size = 22, normalized size = 0.96

$$\frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

giac [B] time = 0.24, size = 67, normalized size = 2.91

$$\frac{2}{315}\left(\left(5\left(7x-23\right)\left(x+1\right)+258\right)\left(x+1\right)-213\right)\sqrt{\left(x+1\right)^{2}-3x}\sqrt{x+1}+\frac{2}{105}\left(3\left(5x-12\right)\left(x+1\right)+71\right)\sqrt{\left(x+1\right)^{2}-3x}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] 2/315*((5*(7*x - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*sq
rt(x + 1) + 2/105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(x
+ 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{2\left(x+1\right)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x+1)^(1/2)*(x^2-x+1)^(1/2),x)
[Out] 2/9*(x+1)^(3/2)*(x^2-x+1)^(3/2)
```

maxima [A] time = 0.95, size = 22, normalized size = 0.96

$$\frac{2}{9}\left(x^{3}+1\right)\sqrt{x^{2}-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] 2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

mupad [B] time = 2.62, size = 22, normalized size = 0.96

$$\frac{2\left(x^{3}+1\right)\sqrt{x+1}\sqrt{x^{2}-x+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
[Out] (2*(x^3 + 1)*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/9
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{x+1}\sqrt{x^{2}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
[Out] Integral(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

3.324 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx$

Optimal. Leaf size=66

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\sqrt{x^2-x+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 50, 63, 207}

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\sqrt{x^2-x+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcT
anh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x^3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x}\,dx}{\sqrt{1+x^3}}\\
&= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(2\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}-\frac{2\sqrt{1+x}\sqrt{1-x+x^2}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] time = 0.41, size = 197, normalized size = 2.98

$$\frac{\sqrt{x+1}\left(2\left(x^2-x+1\right)+\frac{3i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\,\Pi\left(\frac{3}{2}-\frac{i\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{3\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

```
[Out] (Sqrt[1 + x]*(2*(1 - x + x^2) + ((3*I)*Sqrt[2]*Sqrt[(I + Sqrt[3] - (2*I)*x)
/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticP
i[3/2 - (I/2)*Sqrt[3], I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3]
)]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]
))/(3*Sqrt[1 - x + x^2])
```

IntegrateAlgebraic [F] time = 22.48, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

```
[Out] Defer[IntegrateAlgebraic][(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x, x]
```

fricas [A] time = 0.40, size = 60, normalized size = 0.91

$$\frac{2}{3}\sqrt{x^2-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) +
 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

maple [A] time = 0.03, size = 43, normalized size = 0.65

$$-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(\operatorname{arctanh}\left(\sqrt{x^3+1}\right)-\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)*(x^2-x+1)^(1/2)/x,x)
```

```
[Out] -2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(-(x^3+1)^(1/2)+arctanh((x^3+1)^(1/2)))/(x
^3+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x,x)
```

```
[Out] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x, x)
```

3.325 $\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 1.00

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

IntegrateAlgebraic [F] time = 71.30, size = 0, normalized size = 0.00

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2), x]
```

fricas [A] time = 0.39, size = 27, normalized size = 1.17

$$\frac{2}{15}\left(x^6+2\,x^3+1\right)\sqrt{x^2-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

giac [B] time = 0.61, size = 173, normalized size = 7.52

$$\frac{2}{45045}\left(\left(\left(7\left(3\left(11\left(13x-80\right)(x+1)+3165\right)(x+1)-16442\right)(x+1)+121227\right)(x+1)-80187\right)(x+1)+34077\right)\sqrt{(x+1)^2-3x}\sqrt{x+1}+\frac{2}{45045}\left(\left(5\left(7\left(9\left(11x-57\right)(x+1)+1601\right)(x+1)-15837\right)(x+1)+65172\right)(x+1)-34077\right)\sqrt{(x+1)^2-3x}\sqrt{x+1}+\frac{2}{315}\left(\left(5\left(7x-23\right)(x+1)+258\right)(x+1)-213\right)\sqrt{(x+1)^2-3x}\sqrt{x+1}+\frac{2}{105}\left(3\left(5x-12\right)(x+1)+71\right)\sqrt{(x+1)^2-3x}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(((7*(3*(11*(13*x - 80)*(x + 1) + 3165)*(x + 1) - 16442)*(x + 1) +
121227)*(x + 1) - 80187)*(x + 1) + 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
 + 2/45045*((5*(7*(9*(11*x - 57)*(x + 1) + 1601)*(x + 1) - 15837)*(x + 1) +
 65172)*(x + 1) - 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/315*((5*(7*x
 - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/
105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{2\left(x+1\right)^{\frac{5}{2}}\left(x^{2}-x+1\right)^{\frac{5}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x+1)^(3/2)*(x^2-x+1)^(3/2),x)
```

```
[Out] 2/15*(x+1)^(5/2)*(x^2-x+1)^(5/2)
```

maxima [A] time = 0.97, size = 27, normalized size = 1.17

$$\frac{2}{15}\left(x^{6}+2x^{3}+1\right)\sqrt{x^{2}-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

mupad [B] time = 0.12, size = 25, normalized size = 1.09

$$\frac{2\sqrt{x+1}\left(x^{2}-x+1\right)^{5/2}\left(x^{2}+2x+1\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

```
[Out] (2*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2)*(2*x + x^2 + 1))/15
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(x+1\right)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

3.326 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}+\frac{2}{9}\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 50, 63, 207}

$$\frac{2}{9}\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)+\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 + (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 +
 x^3))/9 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt
[1 + x^3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
```

```
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x}\,dx}{\sqrt{1+x^3}}$$
$$= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\operatorname{Subst}\left(\int \frac{(1+x)^{3/2}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}$$
$$= \frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}$$
$$= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)}{3\sqrt{}}$$
$$= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(2\sqrt{1+x}\sqrt{1-x+x^2}\right.}{}$$
$$= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{2\sqrt{1+x}\sqrt{1-x+x^2}\ \mathrm{t}}{3\sqrt{1+}}$$

Mathematica [C] time = 0.30, size = 201, normalized size = 2.14

$$\frac{\sqrt{x+1}\left(\frac{2}{9}\left(x^2-x+1\right)\left(x^3+4\right)+\frac{i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\,\Pi\left(\frac{3}{2}-\frac{i\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[1 + x]*((2*(1 - x + x^2)*(4 + x^3))/9 + (I*Sqrt[2]*Sqrt[(I + Sqrt[3]
- (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]
*EllipticPi[3/2 - (I/2)*Sqrt[3], I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I
 + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I +
 Sqrt[3])]))/Sqrt[1 - x + x^2]
```

IntegrateAlgebraic [F] time = 48.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

```
[Out] Defer[IntegrateAlgebraic][((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x, x]
```

fricas [A] time = 0.40, size = 65, normalized size = 0.69

$$\frac{2}{9}\left(x^3+4\right)\sqrt{x^2-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqr
t(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

maple [A] time = 0.01, size = 57, normalized size = 0.61

$$-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(-\sqrt{x^3+1}\,x^3+3\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)-4\sqrt{x^3+1}\right)}{9\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)*(x^2-x+1)^(3/2)/x,x)
```

```
[Out] -2/9*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(-x^3*(x^3+1)^(1/2)+3*arctanh((x^3+1)^(1/2
))-4*(x^3+1)^(1/2))/(x^3+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x,x)
```

```
[Out] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}\,\left(x^2-x+1\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x,x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x, x)
```

3.327 $\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}} dx$

Optimal. Leaf size=23

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}} dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

IntegrateAlgebraic [F] time = 150.83, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]), x]
```

fricas [A] time = 0.38, size = 17, normalized size = 0.74

$$\frac{2}{3}\sqrt{x^2-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

giac [A] time = 0.18, size = 18, normalized size = 0.78

$$\frac{2}{3}\sqrt{(x+1)^2-3x}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+1)^(1/2)/(x^2-x+1)^(1/2),x)
[Out] 2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)
```

maxima [A] time = 0.97, size = 22, normalized size = 0.96

$$\frac{2\left(x^3+1\right)}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x^3 + 1)/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

mupad [B] time = 0.15, size = 9, normalized size = 0.39

$$\frac{2\sqrt{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] (2*(x^3 + 1)^(1/2))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.328 $\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {915, 266, 63, 207}

$$-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (-2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{\left(2\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= -\frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 12.80, size = 2463, normalized size = 58.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] 2*(((-I)*(1 + x)*Sqrt[1 - 6/((3 - I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*S
qrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]]
, (3 - I*Sqrt[3])/(3 + I*Sqrt[3])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sq
rt[3 - 3*(1 + x) + (1 + x)^2]) + (Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sq
rt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x
])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt
[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1
/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(S
qrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sq
rt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqr
t[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6
/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*
Sqrt[3])] - 6/Sqrt[1 + x]))]*((1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*EllipticF[Arc
Sin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3
])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*
(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqr
t[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sq
rt[3]))/3]*EllipticPi[((-1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/S
qrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-S
qrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[
3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 +
 x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])]
 - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3
 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1
/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt
[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) - (Sqrt[
3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt
[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*
(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] +
 Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*
Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x
]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (
I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sq
rt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] +
Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((-1 + Sqr
t[1/2 - (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[
```

```
3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqr
t[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]],
(Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3
 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))/3]*EllipticPi[((1 + Sqrt[1/2 -
 (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/(
(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I
/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(S
qrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I
*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3
]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2])
)/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/
2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3
 - 3*(1 + x) + (1 + x)^2]))
```

IntegrateAlgebraic [A] time = 8.25, size = 61, normalized size = 1.45

$$-\frac{1}{3}\log\left(\frac{\sqrt{x+1}\,\sqrt{(x+1)^2-3(x+1)+3}\,+1}{1-\sqrt{x+1}\,\sqrt{(x+1)^2-3(x+1)+3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -1/3*Log[(1 + Sqrt[1 + x]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])/(1 - Sqrt[1 + x]
*Sqrt[3 - 3*(1 + x) + (1 + x)^2])]
```

fricas [A] time = 0.39, size = 43, normalized size = 1.02

$$-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}\,+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqr
t(x + 1) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

maple [A] time = 0.03, size = 33, normalized size = 0.79

$$-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+1)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(x+1)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^2-x+1}\,\sqrt{x+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
[Out] Integral(1/(x*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

3.329 $\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.00

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

IntegrateAlgebraic [F] time = 173.28, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)), x]
```

fricas [A] time = 0.38, size = 24, normalized size = 1.04

$$-\frac{2\sqrt{x^2-x+1}\sqrt{x+1}}{3\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] -2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$-\frac{2}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+1)^(3/2)/(x^2-x+1)^(3/2),x)
[Out] -2/3/(x+1)^(1/2)/(x^2-x+1)^(1/2)
```

maxima [A] time = 0.98, size = 17, normalized size = 0.74

$$-\frac{2}{3\,\sqrt{x^2-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] -2/3/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

mupad [B] time = 2.69, size = 17, normalized size = 0.74

$$-\frac{2}{3\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] -2/(3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(x**2/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.330 $\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}-\frac{2\sqrt{x^3+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 51, 63, 207}

$$\frac{2}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}-\frac{2\sqrt{x^3+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3
]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx &= \frac{\sqrt{1+x^3}\int \frac{1}{x\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx,x,x^3\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{\left(2\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{2\sqrt{1+x^3}\,\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}
\end{aligned}
$$

Mathematica [C] time = 6.08, size = 2511, normalized size = 38.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] Sqrt[1 + x]*Sqrt[1 - x + x^2]*(2/(9*(1 + x)) - (2*(-2 + x))/(9*(1 - x + x^2
))) + 2*(((-I)*(1 + x)*Sqrt[1 - 6/((3 - I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3
 + I*Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1
 + x]], (3 - I*Sqrt[3])/(3 + I*Sqrt[3])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[3])^(-
1)]*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) + (Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]
] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt
[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] +
1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-
Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))
/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]
] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])
)]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3]
)] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(
3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*Ellipti
cF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*
Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(S
qrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 +
 I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3
- I*Sqrt[3]))/3]*EllipticPi[((-1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (
I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((-1 - Sqrt[1/2 - (I/2)/Sqrt[3]
])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[(
(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sq
rt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqr
t[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(
Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 -
Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2
)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) -
(Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*
(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]
))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt
```

```
[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 +
x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt
[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1
/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3
+ I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[
3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((-1
 + Sqrt[1/2 - (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] -
 Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 -
 I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]
))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] -
Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))/3]*EllipticPi[((1 + Sqrt
[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3
]]))/((1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/
2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3
]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt
[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*
Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]
])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2
 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*
Sqrt[3 - 3*(1 + x) + (1 + x)^2]))
```

IntegrateAlgebraic [F] time = 122.67, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)), x]
```

fricas [A] time = 0.39, size = 78, normalized size = 1.18

$$-\frac{\left(x^3+1\right)\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}\,+1\right)-\left(x^3+1\right)\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}\,-1\right)-2\,\sqrt{x^2-x+1}\,\sqrt{x+1}}{3\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*((x^3 + 1)*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) - (x^3 + 1)*log(sqrt
(x^2 - x + 1)*sqrt(x + 1) - 1) - 2*sqrt(x^2 - x + 1)*sqrt(x + 1))/(x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

maple [A] time = 0.04, size = 43, normalized size = 0.65

$$-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(\sqrt{x^3+1}\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)-1\right)}{3\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+1)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] -2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)-1)/(
x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

```
[Out] int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(x+1)^{\frac{3}{2}}\,\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(1/(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

3.331 $\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{9(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$-\frac{2}{9(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=-\frac{2}{9(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 1.00

$$-\frac{2}{9(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

IntegrateAlgebraic [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
[Out] $Aborted
```

fricas [A] time = 0.40, size = 29, normalized size = 1.26

$$-\frac{2\sqrt{x^2-x+1}\,\sqrt{x+1}}{9\left(x^6+2\,x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
[Out] -2/9*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + 2*x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
[Out] integrate(x^2/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{2}{9\,(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+1)^(5/2)/(x^2-x+1)^(5/2),x)
[Out] -2/9/(x+1)^(3/2)/(x^2-x+1)^(3/2)
```

maxima [A] time = 0.97, size = 24, normalized size = 1.04

$$-\frac{2}{9\left(x^3+1\right)\sqrt{x^2-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
[Out] -2/9/((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1))
```

mupad [B] time = 2.88, size = 82, normalized size = 3.57

$$\frac{18\,\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}-18\,x\,\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}}{(x+1)\,\left(81\,x\,\left(x^2-x+1\right)^4-162\,\left(x^2-x+1\right)^4+81\,\left(x^2-x+1\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] (18*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2) - 18*x*(x + 1)^(1/2)*(x^2 - x + 1)^(5
/2))/((x + 1)*(81*x*(x^2 - x + 1)^4 - 162*(x^2 - x + 1)^4 + 81*(x^2 - x + 1
)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
[Out] Integral(x**2/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.332 $\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Rubi [A] time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 51, 63, 207}

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
 + x^3)) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 -
 x + x^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
```

```
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{5/2}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int\right.}{3\sqrt{1+x}\sqrt{1}}$$

$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(2\sqrt{1+x^3}\right)\text{Subs}}{3\sqrt{1+}}$$

$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{2\sqrt{1+x^3}\tanh^{-1}(}{3\sqrt{1+x}\sqrt{1-}}$$

Mathematica [C] time = 6.09, size = 2539, normalized size = 26.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] Sqrt[1 + x]*Sqrt[1 - x + x^2]*(2/(81*(1 + x)^2) + 22/(81*(1 + x)) - (2*(-1
+ x))/(27*(1 - x + x^2)^2) - (2*(-21 + 11*x))/(81*(1 - x + x^2))) + 2*(((-I
)*(1 + x)*Sqrt[1 - 6/((3 - I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*Sqrt[3])
*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]], (3 -
I*Sqrt[3])/(3 + I*Sqrt[3])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sqrt[3 -
3*(1 + x) + (1 + x)^2]) + (Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2
+ (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sq
rt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]
))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I
/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2
 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2
+ (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqr
t[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1
 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3]
)] - 6/Sqrt[1 + x]))]*((1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqr
t[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6
/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*
Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^
2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))
/3]*EllipticPi[((-1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]]
 + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2
 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[3 - I*S
qrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/(
```

```
(Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sq
rt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sq
rt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I
/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] -
Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) - (Sqrt[3/2]*(S
qrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 -
(I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[
1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1
/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(S
qrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((S
qrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sq
rt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])
*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3
+ I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((-1 + Sqrt[1/2 -
 (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*S
qrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] +
 Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3
 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sq
rt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))/3]*EllipticPi[((1 + Sqrt[1/2 - (I/2)/
Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((1 - Sq
rt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqr
t[3]])), ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(
3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3
]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sq
rt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt
[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt
[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1
 + x) + (1 + x)^2]))
```

IntegrateAlgebraic [F] time = 169.09, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)), x]
```

fricas [A] time = 0.38, size = 101, normalized size = 1.05

$$\frac{2\left(3x^3+4\right)\sqrt{x^2-x+1}\,\sqrt{x+1}-3\left(x^6+2x^3+1\right)\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}+1\right)+3\left(x^6+2x^3+1\right)\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}-1\right)}{9\left(x^6+2x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(3*x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 3*(x^6 + 2*x^3 + 1)*log(
sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 3*(x^6 + 2*x^3 + 1)*log(sqrt(x^2 - x +
 1)*sqrt(x + 1) - 1))/(x^6 + 2*x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

maple [A] time = 0.06, size = 69, normalized size = 0.72

$$-\frac{2\left(3\sqrt{x^3+1}\,x^3\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)-3x^3+3\sqrt{x^3+1}\,\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)-4\right)}{9\left(x^3+1\right)\sqrt{x^2-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+1)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] -2/9*(3*(x^3+1)^(1/2)*arctanh((x^3+1)^(1/2))*x^3-3*x^3+3*arctanh((x^3+1)^(1
/2))*(x^3+1)^(1/2)-4)/(x^3+1)/(x^2-x+1)^(1/2)/(x+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

```
[Out] int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(1/(x*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

3.333 $\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{44x+39}{276(1-x)^2\left(4x^2+5x+3\right)}-\frac{11\log\left(4x^2+5x+3\right)}{4608}-\frac{97}{4416(1-x)}-\frac{21}{736(1-x)^2}+\frac{11\log(1-x)}{2304}+\frac{6023\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}$$

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {822, 800, 634, 618, 204, 628}

$$\frac{44x+39}{276(1-x)^2\left(4x^2+5x+3\right)}-\frac{11\log\left(4x^2+5x+3\right)}{4608}-\frac{97}{4416(1-x)}-\frac{21}{736(1-x)^2}+\frac{11\log(1-x)}{2304}+\frac{6023\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] -21/(736*(1 - x)^2) - 97/(4416*(1 - x)) + (39 + 44*x)/(276*(1 - x)^2*(3 + 5
*x + 4*x^2)) + (6023*ArcTan[(5 + 8*x)/Sqrt[23]])/(52992*Sqrt[23]) + (11*Log
[1 - x])/2304 - (11*Log[3 + 5*x + 4*x^2])/4608
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
```

```
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx = \frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{1}{276}\int \frac{57+132x}{(-1+x)^3\left(3+5x+4x^2\right)}\,dx$$

$$= \frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{1}{276}\int \left(\frac{63}{4(-1+x)^3}-\frac{97}{16(-1+x)^2}+\frac{253}{192(-1$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{11\log(1-x)}{2304}$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{11\log(1-x)}{2304}$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{11\log(1-x)}{2304}$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{6023\tan^{-1}\left(\frac{5}{\sqrt{}}\right.}{52992\sqrt{23}}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.80

$$\frac{\frac{184(2204x+975)}{4x^2+5x+3}-17457\log\left(4x^2+5x+3\right)+\frac{59248}{x-1}-\frac{25392}{(x-1)^2}+34914\log(1-x)+36138\sqrt{23}\,\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{7312896}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] (-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3 + 5*x + 4*x^2
) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 34914*Log[1 - x] - 17457*Lo
g[3 + 5*x + 4*x^2])/7312896
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2), x]
```

fricas [A] time = 0.39, size = 134, normalized size = 1.38

$$\frac{214176\,x^3+12046\,\sqrt{23}\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(8\,x+5)\right)-224664\,x^2-5819\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)\log\left(4\,x^2+5\,x+3\right)+11638\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)\log\left(x-1\right)-66240\,x-24840}{2437632\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/2437632*(214176*x^3 + 12046*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arct
an(1/23*sqrt(23)*(8*x + 5)) - 224664*x^2 - 5819*(4*x^4 - 3*x^3 - 3*x^2 - x
+ 3)*log(4*x^2 + 5*x + 3) + 11638*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*log(x - 1
) - 66240*x - 24840)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

giac [A] time = 0.18, size = 71, normalized size = 0.73

$$\frac{6023}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(8\,x+5)\right)+\frac{388\,x^3-407\,x^2-120\,x-45}{4416\left(4\,x^2+5\,x+3\right)(x-1)^2}-\frac{11}{4608}\log\left(4\,x^2+5\,x+3\right)+\frac{11}{2304}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="giac")
```

```
[Out] 6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 - 4
07*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) - 11/4608*log(4*x^2 + 5*
x + 3) + 11/2304*log(abs(x - 1))
```

maple [A] time = 0.01, size = 68, normalized size = 0.70

$$\frac{6023\sqrt{23}\ \arctan\left(\frac{(8x+5)\sqrt{23}}{23}\right)}{1218816}+\frac{11\ln(x-1)}{2304}-\frac{11\ln\left(4x^2+5x+3\right)}{4608}-\frac{-\frac{2204x}{23}-\frac{975}{23}}{6912\left(x^2+\frac{5}{4}x+\frac{3}{4}\right)}-\frac{1}{288\,(x-1)^2}+\frac{7}{864\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-1+x)^3/(4*x^2+5*x+3)^2,x)
```

```
[Out] -1/6912*(-2204/23*x-975/23)/(x^2+5/4*x+3/4)-11/4608*ln(4*x^2+5*x+3)+6023/12
18816*arctan(1/23*(5+8*x)*23^(1/2))*23^(1/2)-1/288/(-1+x)^2+7/864/(-1+x)+11
/2304*ln(-1+x)
```

maxima [A] time = 0.96, size = 75, normalized size = 0.77

$$\frac{6023}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(8\,x+5)\right)+\frac{388\,x^3-407\,x^2-120\,x-45}{4416\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)}-\frac{11}{4608}\log\left(4\,x^2+5\,x+3\right)+\frac{11}{2304}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="maxima")
```

```
[Out] 6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 - 4
07*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3) - 11/4608*log(4*x^2 +
5*x + 3) + 11/2304*log(x - 1)
```

mupad [B] time = 0.13, size = 84, normalized size = 0.87

$$\frac{11\ \ln(x-1)}{2304}+\frac{-\frac{97\,x^3}{4416}+\frac{407\,x^2}{17664}+\frac{5\,x}{736}+\frac{15}{5888}}{-x^4+\frac{3\,x^3}{4}+\frac{3\,x^2}{4}+\frac{x}{4}-\frac{3}{4}}-\ln\left(x+\frac{5}{8}-\frac{\sqrt{23}\ 1\mathrm{i}}{8}\right)\left(\frac{11}{4608}+\frac{\sqrt{23}\ 6023\mathrm{i}}{2437632}\right)+\ln\left(x+\frac{5}{8}+\frac{\sqrt{23}\ 1\mathrm{i}}{8}\right)\left(-\frac{11}{4608}+\frac{\sqrt{23}\ 6023\mathrm{i}}{2437632}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x - 1)^3*(5*x + 4*x^2 + 3)^2),x)
```

```
[Out] (11*log(x - 1))/2304 + ((5*x)/736 + (407*x^2)/17664 - (97*x^3)/4416 + 15/58
88)/(x/4 + (3*x^2)/4 + (3*x^3)/4 - x^4 - 3/4) - log(x - (23^(1/2)*1i)/8 + 5
/8)*((23^(1/2)*6023i)/2437632 + 11/4608) + log(x + (23^(1/2)*1i)/8 + 5/8)*(
(23^(1/2)*6023i)/2437632 - 11/4608)
```

sympy [A] time = 0.24, size = 88, normalized size = 0.91

$$\frac{388x^3 - 407x^2 - 120x - 45}{17664x^4 - 13248x^3 - 13248x^2 - 4416x + 13248} + \frac{11\log(x-1)}{2304} - \frac{11\log\left(x^2 + \frac{5x}{4} + \frac{3}{4}\right)}{4608} + \frac{6023\sqrt{23}\operatorname{atan}\left(\frac{8\sqrt{23}\,x}{23} + \frac{5\sqrt{23}}{23}\right)}{1218816}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

```
[Out] (388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 - 13248*x**2 -
4416*x + 13248) + 11*log(x - 1)/2304 - 11*log(x**2 + 5*x/4 + 3/4)/4608 + 60
23*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*sqrt(23)/23)/1218816
```

3.334 $\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=490

$$\frac{\sqrt{2}\left(-\frac{-5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right.}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Rubi [A] time = 14.85, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(-\frac{-5a^2bc^2e+2a^2c^3d-4ab^2c^2d+5ab^3ce+b^4cd+b^5(-e)}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^3cd+b^4(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{-5a^2bc^2e+2a^2c^3d-4ab^2c^2d+5ab^3ce+b^4cd+b^5(-e)}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^3cd+b^4(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{9/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2(d+ex)^{3/2}\left(ce(bd-ae)+b^2e^2+c^2d^2\right)}{3c^3e^3}-\frac{2b\left(b^2-2ac\right)\sqrt{d+ex}}{c^4}-\frac{2(d+ex)^{5/2}(be+2cd)}{5c^2e^3}+\frac{2(d+ex)^{7/2}}{7ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (-2*b*(b^2 - 2*a*c)*Sqrt[d + e*x])/c^4 + (2*(c^2*d^2 + b^2*e^2 + c*e*(b*d -
 a*e))*(d + e*x)^(3/2))/(3*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*x)^(5/2))/(5*
c^2*e^3) + (2*(d + e*x)^(7/2))/(7*c*e^3) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d
- b^4*e + 3*a*b^2*c*e - a^2*c^2*e - (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d
- b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d -
 b^4*e + 3*a*b^2*c*e - a^2*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d -
 b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
```

```
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^4}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{\left(b^3-2abc\right)e}{c^4}+\frac{\left(c^2d^2+b^2e^2+ce(bd-ae)\right)x^2}{c^3e^2}-\frac{(2cd+be)x^4}{c^2e^2}+\frac{x^6}{ce^2}+\frac{b\left(b^2-2ac\right)\left(cd^2-bde+ae^2\right)-\left(b^3c\right.}{c^4e\left(\frac{cd^2-bde+ae^2}{e^2}\right.}\right.\right.}{e}$$

$$= -\frac{2b\left(b^2-2ac\right)\sqrt{d+ex}}{c^4}+\frac{2\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex}{5c^2e^3}$$

$$= -\frac{2b\left(b^2-2ac\right)\sqrt{d+ex}}{c^4}+\frac{2\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex}{5c^2e^3}$$

$$= -\frac{2b\left(b^2-2ac\right)\sqrt{d+ex}}{c^4}+\frac{2\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex}{5c^2e^3}$$

Mathematica [A] time = 0.75, size = 568, normalized size = 1.16

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-105*b^3*e^3 - 7*c^2*e*(d + e*x)*(-2*b*d + 5*a*e + 3*b*e*
x) + c^3*(8*d^3 - 4*d^2*e*x + 3*d*e^2*x^2 + 15*e^3*x^3) + 35*b*c*e^2*(6*a*e
 + b*(d + e*x))))/(105*c^4*e^3) - (Sqrt[2]*(-(b^5*e) + a*b*c^2*(2*Sqrt[b^2
- 4*a*c]*d - 5*a*e) + b^3*c*(-(Sqrt[b^2 - 4*a*c]*d) + 5*a*e) + b^4*(c*d + S
qrt[b^2 - 4*a*c]*e) + a^2*c^2*(2*c*d + Sqrt[b^2 - 4*a*c]*e) - a*b^2*c*(4*c*
d + 3*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*
c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(9/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b^5*e - b^3*c*(Sqrt[b^2 - 4*a*c]*d
 + 5*a*e) + a*b*c^2*(2*Sqrt[b^2 - 4*a*c]*d + 5*a*e) + a*b^2*c*(4*c*d - 3*Sq
rt[b^2 - 4*a*c]*e) + a^2*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e) + b^4*(-(c*d) +
 Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e])
```

IntegrateAlgebraic [C] time = 2.22, size = 784, normalized size = 1.60

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-105*b^3*e^3 + 210*a*b*c*e^3 + 35*c^3*d^2*(d + e*x) + 35*
b*c^2*d*e*(d + e*x) + 35*b^2*c*e^2*(d + e*x) - 35*a*c^2*e^2*(d + e*x) - 42*
c^3*d*(d + e*x)^2 - 21*b*c^2*e*(d + e*x)^2 + 15*c^3*(d + e*x)^3))/(105*c^4*
e^3) + (((-I)*Sqrt[2]*b^4*c*d + (4*I)*Sqrt[2]*a*b^2*c^2*d - (2*I)*Sqrt[2]*a
^2*c^3*d - Sqrt[2]*b^3*c*Sqrt[-b^2 + 4*a*c]*d + 2*Sqrt[2]*a*b*c^2*Sqrt[-b^2
 + 4*a*c]*d + I*Sqrt[2]*b^5*e - (5*I)*Sqrt[2]*a*b^3*c*e + (5*I)*Sqrt[2]*a^2
*b*c^2*e + Sqrt[2]*b^4*Sqrt[-b^2 + 4*a*c]*e - 3*Sqrt[2]*a*b^2*c*Sqrt[-b^2 +
 4*a*c]*e + Sqrt[2]*a^2*c^2*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*S
qrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-
b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + ((I*Sqrt[2]*b^4
*c*d - (4*I)*Sqrt[2]*a*b^2*c^2*d + (2*I)*Sqrt[2]*a^2*c^3*d - Sqrt[2]*b^3*c*
Sqrt[-b^2 + 4*a*c]*d + 2*Sqrt[2]*a*b*c^2*Sqrt[-b^2 + 4*a*c]*d - I*Sqrt[2]*b
^5*e + (5*I)*Sqrt[2]*a*b^3*c*e - (5*I)*Sqrt[2]*a^2*b*c^2*e + Sqrt[2]*b^4*Sq
rt[-b^2 + 4*a*c]*e - 3*Sqrt[2]*a*b^2*c*Sqrt[-b^2 + 4*a*c]*e + Sqrt[2]*a^2*c
^2*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d
 + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d
+ b*e + I*Sqrt[-b^2 + 4*a*c]*e])
```

fricas [B] time = 1.00, size = 5507, normalized size = 11.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/210*(105*sqrt(2)*c^4*e^3*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16
*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^
3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c
^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 +
 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^
3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^
8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4
*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^
2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(sqrt(2)*((b^12*c - 12*
a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 - 32*a^5*b^
2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4
*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e - (b^6*c^9 - 8*a*b^4*c^10 + 18*a
^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4
 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2
 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*
b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*
a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c
^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4
*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)
*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4
 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2
 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*
b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*
a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c
^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 10*a^6*b^3*c
^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*
c^3 + a^8*c^4)*e)*sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^3*sqrt(((b^8*c - 8*a*b
^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c
+ 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*s
qrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^
4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^
2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 +
```

```
 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2
 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 -
20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)
)*log(-sqrt(2)*((b^12*c - 12*a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4
+ 104*a^4*b^4*c^5 - 32*a^5*b^2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^
2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e - (
b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12
*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*
b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3
 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4
*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
+ 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^
8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4
*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 -
30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12
*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*
b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3
 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4
*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
+ 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^
8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c -
 6*a^5*b^5*c^2 + 10*a^6*b^3*c^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c +
 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e)*sqrt(e*x + d)) + 105*sqrt(2)
*c^4*e^3*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a
^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^
4)*e - (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c
^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d
^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^
4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 1
4*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5
*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a
*c^19)))/(b^2*c^9 - 4*a*c^10))*log(sqrt(2)*((b^12*c - 12*a*b^10*c^2 + 54*a^
2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 - 32*a^5*b^2*c^6)*d - (b^13 -
 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*
b^3*c^5 + 8*a^6*b*c^6)*e + (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^
3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5
 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*
a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*
b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*
b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b
^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^8*
c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9
*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e - (b^2*c^9 - 4*
a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5
 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*
a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*
b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*
b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b
^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 -
4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 10*a^6*b^3*c^3 - 4*a^7*b*c^4)*
d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e)*
sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^3*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^
4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 -
 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e - (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 1
2*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5
*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^
3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 -
4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a
```

```
^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(-sqrt(2)*((b
^12*c - 12*a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5
- 32*a^5*b^2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^
3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e + (b^6*c^9 - 8*a*b^4*
c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a
^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b
^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4
 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e +
(b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4
- 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c
^18 - 4*a*c^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*
c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9
*a^4*b*c^4)*e - (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a
^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b
^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4
 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e +
(b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4
- 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c
^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 1
0*a^6*b^3*c^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 -
10*a^7*b^2*c^3 + a^8*c^4)*e)*sqrt(e*x + d)) + 4*(15*c^3*e^3*x^3 + 8*c^3*d^3
 + 14*b*c^2*d^2*e + 35*(b^2*c - a*c^2)*d*e^2 - 105*(b^3 - 2*a*b*c)*e^3 + 3*
(c^3*d*e^2 - 7*b*c^2*e^3)*x^2 - (4*c^3*d^2*e + 7*b*c^2*d*e^2 - 35*(b^2*c -
a*c^2)*e^3)*x)*sqrt(e*x + d))/(c^4*e^3)
```

giac [B] time = 0.83, size = 1171, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*d*e - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*
c^2 - 2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^4*c^2 - 2*a*b^2*c
^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^2
)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d +
 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^
2 - (3*b^5*c^3 - 14*a*b^3*c^4 + 12*a^2*b*c^5)*d*e + (b^6*c^2 - 5*a*b^4*c^3
+ 5*a^2*b^2*c^4)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^24
 - b*c^7*e^25 + sqrt(-4*(c^8*d^2*e^24 - b*c^7*d*e^25 + a*c^7*e^26)*c^8*e^24
 + (2*c^8*d*e^24 - b*c^7*e^25)^2))*e^(-24)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^
2 - sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) + 1/4*(
sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c - 6*a*b^3*c^2 + 8*
a^2*b*c^3)*d*e - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*c^2 +
2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^4*c^2 - 2*a*b^2*c^3)*sq
rt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt
(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*
c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^2 - (3
*b^5*c^3 - 14*a*b^3*c^4 + 12*a^2*b*c^5)*d*e + (b^6*c^2 - 5*a*b^4*c^3 + 5*a^
2*b^2*c^4)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^24 - b*c
^7*e^25 - sqrt(-4*(c^8*d^2*e^24 - b*c^7*d*e^25 + a*c^7*e^26)*c^8*e^24 + (2*
c^8*d*e^24 - b*c^7*e^25)^2))*e^(-24)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^2 - sq
rt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) + 2/105*(15*(
x*e + d)^(7/2)*c^6*e^18 - 42*(x*e + d)^(5/2)*c^6*d*e^18 + 35*(x*e + d)^(3/2
)*c^6*d^2*e^18 - 21*(x*e + d)^(5/2)*b*c^5*e^19 + 35*(x*e + d)^(3/2)*b*c^5*d
*e^19 + 35*(x*e + d)^(3/2)*b^2*c^4*e^20 - 35*(x*e + d)^(3/2)*a*c^5*e^20 - 1
05*sqrt(x*e + d)*b^3*c^3*e^21 + 210*sqrt(x*e + d)*a*b*c^4*e^21)*e^(-21)/c^7
```

maple [B] time = 0.11, size = 2218, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/7*(e*x+d)^(7/2)/c/e^3-2/5/e^2/c^2*(e*x+d)^(5/2)*b-2/3/e/c^2*(e*x+d)^(3/2)
*a+2/3/e/c^3*(e*x+d)^(3/2)*b^2-4/5/e^3/c*(e*x+d)^(5/2)*d+2/3/e^3/c*(e*x+d)^
(3/2)*d^2+4/c^3*a*b*(e*x+d)^(1/2)+4*e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/
((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/
2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2*d+4*e/c^2/(-e^2*(4
*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2))*a*b^2*d+5*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*b-2*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^
(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*d-e/c^3/(-e^2*(4
*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2))*b^4*d-2/c^4*b^3*(e*x+d)^(1/2)+2/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2))*a*b*d-2/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2))*a*b*d+e^2/c^4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*
e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^5+3*e/c^3*2^(1/2)/((-b*e
+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/(
(-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2+e^2/c^4/(-e^2*(4*a*c-
b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c
*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^5-
3*e/c^3*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*
x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2-5*
e^2/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))*a*b^3+e/c^4*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2))*b^4+1/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2))*b^3*d-1/c^3*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b^3*d-e/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2))*a^2-e/c^4*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2))*b^4+e/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arcta
n(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a
^2+5*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2))*a^2*b-2*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e
-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((
b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*d-5*e^2/c^3/(-e^2*(4*a*c-
b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c
*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^
3-e/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2))*b^4*d+2/3/e^2/c^2*(e*x+d)^(3/2)*b*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,x^4}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x^4/(c*x^2 + b*x + a), x)
```

mupad [B] time = 4.86, size = 13879, normalized size = 28.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (d + e*x)^(3/2)*((4*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(3*c^2
*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d
*e^3))/(3*c*e^3)) - atan(((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d
*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^
7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^
(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d -
 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e
- 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2
) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c
- b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d
*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*
a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2
*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*
d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^1
0*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7
*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^
3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^
3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 -
8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a
^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^
2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e
^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a
*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^
11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6
*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b
^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^
9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2)
- 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c -
 b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3
*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1
/2)*1i - (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*
c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2
*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e +
 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d
 + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3
*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e
+ 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*
b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +
```

4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^
3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(
4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b
^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b
^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^
6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*
b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b
^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2)
 - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c
 - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^
3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(
1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 -
50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e
^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c
^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2
 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^11*e + 8*a^5*c^6*
d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b
^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^
4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*
c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(
4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^
4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) +
15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2
)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((((8*(a*
b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6
*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 -
8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^
8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4
*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*
c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d
 - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c
 - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-
(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2
*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(
1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^
2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6
*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*
b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a
^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8
*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-
(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c
^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) +
 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^
2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*
x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^
4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e
^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a
*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*
d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*
c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4
*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4
*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*
c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^
2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(1
6*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (16*(a^5*b^4*e^5 + a^7*c^2*e
^5 - 3*a^6*b^2*c*e^5 - a^4*b^5*d*e^4 + a^6*c^3*d^2*e^3 - a^4*b^3*c^2*d^3*e^

2 - 5*a^5*b^2*c^2*d^2*e^3 + 2*a^5*b^3*c*d*e^4 + a^6*b*c^2*d*e^4 + 2*a^4*b^4*c*d^2*e^3 + 2*a^5*b*c^3*d^3*e^2))/c^7 + (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)))*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i - atan(((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a

*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2
) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11
 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c
^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a
^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*
b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^
5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3
))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d
+ 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e
 + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/
2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^
2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c
 - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*
d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10
*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^
2*c^10)))^(1/2)*1i - (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3
- 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2
*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 + (8*(d + e*x)^(1/2)
*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^
2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*
a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13
*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(
1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)
^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*
a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^
2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)
))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2)
)/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d +
 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e
+ 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2
) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c
- b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d
*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*
a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2
c^10)))^(1/2) + (8(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*
c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8
*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16
*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^
3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*((b^8*e*(-(4
*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d -
 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e
 - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e -
12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^
6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4
*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)
^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*
a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/
((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4
+ b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d
^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c -
b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3
*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*
a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^
8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(
-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*
c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2)
+ 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b

^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^
3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*
a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d -
96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e
- 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 1
2*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6
*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*
a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^
(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a
c - b^2)^3)^(1/2))/(2(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*
(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4
*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*
b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2
 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b
^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^5*c^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d +
66*a^4*b^2*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e
 + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*
a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2
)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*
c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*
c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))
/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (16*(a^5*b^4*e^5 + a^7
*c^2*e^5 - 3*a^6*b^2*c*e^5 - a^4*b^5*d*e^4 + a^6*c^3*d^2*e^3 - a^4*b^3*c^2*
d^3*e^2 - 5*a^5*b^2*c^2*d^2*e^3 + 2*a^5*b^3*c*d*e^4 + a^6*b*c^2*d*e^4 + 2*a
^4*b^4*c*d^2*e^3 + 2*a^5*b*c^3*d^3*e^2))/c^7 + (((8*(a*b^5*c^5*e^4 + 8*a^3*
b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c
^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3)
)/c^7 + (8*(d + e*x)^(1/2)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d -
 b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d
 - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(
4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^
7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a
*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2
) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11
 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*
c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c
^6*d - b^11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2
*c^5*d - 63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4
e(-(4*a*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*
e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3
)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^
2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2
*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4
 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 2
0*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^
4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4
*d*e^3))/c^7)*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^11*e + b^1
0*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d - 63*a^2*b^7
*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^
3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^
3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 -
8*a*b^2*c^10)))^(1/2)))*((b^8*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^5*c^6*d - b^

```
11*e + b^10*c*d + 52*a^2*b^6*c^3*d - 96*a^3*b^4*c^4*d + 66*a^4*b^2*c^5*d -
63*a^2*b^7*c^2*e + 138*a^3*b^5*c^3*e - 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a
*c - b^2)^3)^(1/2) + 13*a*b^9*c*e - 12*a*b^8*c^2*d + 36*a^5*b*c^5*e - b^7*c
*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^
5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) -
 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 +
b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i - ((8*d)/(5*c*e^3) + (2*(b*e^4 - 2*c*d*e
^3))/(5*c^2*e^6))*(d + e*x)^(5/2) - (d + e*x)^(1/2)*((8*d^3)/(c*e^3) - (((8
*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e
^4))/(c*e^3) + ((b*e^4 - 2*c*d*e^3)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e
^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e
^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(c*e^3)) + (2*(d + e*x)^(7/2))/(7*c*e^3
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.335 $\int \frac{x^3 \sqrt{d+ex}}{a+bx+cx^2} \, dx$

Optimal. Leaf size=326

$$\frac{2\left(b^2-ac\right)\sqrt{d+ex}}{c^3}+\frac{\left(-\sqrt{b^2-4ac}\left(b^2-ac\right)-3abc+b^3\right)\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}}$$

Rubi [A] time = 7.47, antiderivative size = 397, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {897, 1287, 1166, 208}

$$-\frac{\sqrt{2}\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2\left(b^2-ac\right)\sqrt{d+ex}}{c^3}-\frac{2(d+ex)^{3/2}(be+cd)}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*(b^2 - a*c)*Sqrt[d + e*x])/c^3 - (2*(c*d + b*e)*(d + e*x)^(3/2))/(3*c^2*
e^2) + (2*(d + e*x)^(5/2))/(5*c*e^2) - (Sqrt[2]*(b^2*c*d - a*c^2*d - b^3*e
+ 2*a*b*c*e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/S
qrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- (Sqrt[2]*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d
- b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(b^2-ac)e}{c^3}-\frac{(cd+be)x^2}{c^2e}+\frac{x^4}{ce}-\frac{(b^2-ac)(cd^2-bde+ae^2)-(b^2cd-ac^2d-b^3e+2abce)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d-\right.}{e}$$

$$= \frac{2(b^2-ac)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}-\frac{2\,\text{Subst}\left(\int \frac{(b^2-ac)(cd^2-bde+ac}{\frac{cd^2-bde+}{e^2}}\right.}{}$$

$$= \frac{2(b^2-ac)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}+\frac{\left(b^2cd-ac^2d-b^3e+2abce\right.}{}$$

$$= \frac{2(b^2-ac)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}-\frac{\sqrt{2}\left(b^2cd-ac^2d-b^3e+2\right.}{}$$

Mathematica [A] time = 0.53, size = 466, normalized size = 1.43

$$\frac{2\sqrt{d+ex}\left(-5ce(3ae+b(d+ex))+15b^2e^2+c^2\left(-2d^2+dex+3e^2x^2\right)\right)}{15c^3e^2}+\frac{\sqrt{2}\left(ac^2\left(d\sqrt{b^2-4ac}-2ae\right)+b^2c\left(4ae-d\sqrt{b^2-4ac}\right)-abc\left(2e\sqrt{b^2-4ac}+3cd\right)+b^3\left(e\sqrt{b^2-4ac}+cd\right)+b^4(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{\sqrt{b^2-4ac}\,e-be+2cd}}\right)}{c^{7/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\left(ac^2\left(d\sqrt{b^2-4ac}+2ae\right)-b^2c\left(d\sqrt{b^2-4ac}+4ae\right)+abc\left(3cd-2e\sqrt{b^2-4ac}\right)+b^3\left(e\sqrt{b^2-4ac}-cd\right)+b^4e\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{7/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x + 3*e^2*x^2) - 5*c*e*(3*
a*e + b*(d + e*x))))/(15*c^3*e^2) + (Sqrt[2]*(-(b^4*e) + a*c^2*(Sqrt[b^2 -
4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^3*(c*d + Sqr
t[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(7/2)*
Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^4
*e + a*c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a
*e) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*
c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e])
```

IntegrateAlgebraic [A] time = 1.50, size = 584, normalized size = 1.79

$$\frac{\left(2\sqrt{2}a^2c^2e-\sqrt{2}ac^2d\sqrt{b^2-4ac}+\sqrt{2}b^2cd\sqrt{b^2-4ac}-4\sqrt{2}ab^2ce+2\sqrt{2}abce\sqrt{b^2-4ac}-\sqrt{2}b^3e\sqrt{b^2-4ac}+3\sqrt{2}abc^2d+\sqrt{2}b^4e-\sqrt{2}b^3cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{c^{7/2}\sqrt{b^2-4ac}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}+\frac{\left(-2\sqrt{2}a^2c^2e-\sqrt{2}ac^2d\sqrt{b^2-4ac}+\sqrt{2}b^2cd\sqrt{b^2-4ac}+4\sqrt{2}ab^2ce+2\sqrt{2}abce\sqrt{b^2-4ac}-\sqrt{2}b^3e\sqrt{b^2-4ac}-3\sqrt{2}abc^2d-\sqrt{2}b^4e+\sqrt{2}b^3cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right)}{c^{7/2}\sqrt{b^2-4ac}\sqrt{e\sqrt{b^2-4ac}+be-2cd}}+\frac{2\left(-15ace^2\sqrt{d+ex}+15b^2e^2\sqrt{d+ex}-5bce(d+ex)^{3/2}+3c^2(d+ex)^{5/2}-5c^2d(d+ex)^{3/2}\right)}{15c^3e^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*(15*b^2*e^2*Sqrt[d + e*x] - 15*a*c*e^2*Sqrt[d + e*x] - 5*c^2*d*(d + e*x)
^(3/2) - 5*b*c*e*(d + e*x)^(3/2) + 3*c^2*(d + e*x)^(5/2)))/(15*c^3*e^2) + (
(-(Sqrt[2]*b^3*c*d) + 3*Sqrt[2]*a*b*c^2*d + Sqrt[2]*b^2*c*Sqrt[b^2 - 4*a*c]
*d - Sqrt[2]*a*c^2*Sqrt[b^2 - 4*a*c]*d + Sqrt[2]*b^4*e - 4*Sqrt[2]*a*b^2*c*
```

```
e + 2*Sqrt[2]*a^2*c^2*e - Sqrt[2]*b^3*Sqrt[b^2 - 4*a*c]*e + 2*Sqrt[2]*a*b*c
*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d +
b*e - Sqrt[b^2 - 4*a*c]*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e -
 Sqrt[b^2 - 4*a*c]*e]) + ((Sqrt[2]*b^3*c*d - 3*Sqrt[2]*a*b*c^2*d + Sqrt[2]*
b^2*c*Sqrt[b^2 - 4*a*c]*d - Sqrt[2]*a*c^2*Sqrt[b^2 - 4*a*c]*d - Sqrt[2]*b^4
*e + 4*Sqrt[2]*a*b^2*c*e - 2*Sqrt[2]*a^2*c^2*e - Sqrt[2]*b^3*Sqrt[b^2 - 4*a
*c]*e + 2*Sqrt[2]*a*b*c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(7/2)*Sqrt[b^2 - 4*a
*c]*Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e])
```

fricas [B] time = 0.72, size = 4245, normalized size = 13.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3
*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 - 4
*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9
*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c
^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c
^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*
c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(sqrt(2)*((b^9*c - 9*a*b^7*c^2 +
 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e - (b^5*c^7 -
 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b
^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*
b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e
 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*
a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3
 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c
 + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*
c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*
a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6
*c^3)*e)*sqrt(e*x + d)) - 15*sqrt(2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9
*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3)*e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
- 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^
7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((
b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (
b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^
5*c^5)*e - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8
*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9
*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6
)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c -
 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 +
 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9
*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e
+ (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))
 + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c
+ 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt(e*x + d)) + 15*sqrt(2)*c^3*e^2*sqrt(((b^
```

```
6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^
2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*
c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*
a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)
*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c
^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*
c^8))*log(sqrt(2)*((b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 +
 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 2
9*a^4*b^2*c^4 - 4*a^5*c^5)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(
((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)
*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b
^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b
^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^
15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*
a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^10
*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 -
 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5
 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/
(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^
3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt(e*x + d)) - 15*sqrt(
2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7
 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt((
(b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*
d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^
3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^1
5)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c
^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e + (b^5*c^7 - 7*a*b^3*c^8
+ 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4
*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a
^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^
2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a
^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 -
 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 +
 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5
*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^
2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 +
 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt
(e*x + d)) + 4*(3*c^2*e^2*x^2 - 2*c^2*d^2 - 5*b*c*d*e + 15*(b^2 - a*c)*e^2
+ (c^2*d*e - 5*b*c*e^2)*x)*sqrt(e*x + d))/(c^3*e^2)
```

giac [B] time = 0.56, size = 1045, normalized size = 3.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c - 5*a*b^2*c^2
 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*c^2 - 2*((b^2*c^3
- a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d*e
+ (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqr
t(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)
*e)*(2*(b^3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d
*e + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*
e + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e
```

```
^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))*e^(-12)/c^6))/
((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d*e + sqrt(b^2 - 4*a*
c)*a*c^5*e^2)*c^2) - 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*
((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^
2)*c^2 + 2*((b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*s
qrt(b^2 - 4*a*c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4
*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*c -
 sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b
^2*c^4 + 4*a^2*c^5)*d*e + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^2))*arcta
n(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 - sqrt(-4*(c^6
*d^2*e^12 - b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^1
3)^2))*e^(-12)/c^6))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*
d*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2) + 2/15*(3*(x*e + d)^(5/2)*c^4*e^8 -
 5*(x*e + d)^(3/2)*c^4*d*e^8 - 5*(x*e + d)^(3/2)*b*c^3*e^9 + 15*sqrt(x*e +
d)*b^2*c^2*e^10 - 15*sqrt(x*e + d)*a*c^3*e^10)*e^(-10)/c^5
```

maple [B] time = 0.06, size = 1764, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/5*(e*x+d)^(5/2)/c/e^2-2/3/e/c^2*(e*x+d)^(3/2)*b-2/3/e^2/c*(e*x+d)^(3/2)*d
-2/c^2*a*(e*x+d)^(1/2)+2/c^3*b^2*(e*x+d)^(1/2)-2*e^2/c/(-(4*a*c-b^2)*e^2)^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d
)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2+4*e^
2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2
))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*c)*a*b^2-3*e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*
c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*d-e^2/c^3/(-(4*a*c-b^2)*e^2)^
(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+
d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4+e/c
^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*
c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2
))*c)^(1/2)*c)*b^3*d-2*e/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*c)*a*b+1/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(
1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c
)^(1/2)*c)*a*d+e/c^3*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(
1/2)*c)*b^3-1/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*c)*b^2*d-2*e^2/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2+4*e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)
/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2-3*e/c/(-(4*a*c-b^2
)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan(
(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b
*d-e^2/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*c)*b^4+e/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d+2*e/c^2*2^(1/2)/((b*e-2*c*d+(-
(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b-1/c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*d-e/c^3*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
```

```
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*b^3+1/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1
/2))*c)^(1/2)*c)*b^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,x^3}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x^3/(c*x^2 + b*x + a), x)
```

mupad [B] time = 4.37, size = 11143, normalized size = 34.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e
^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*
d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5
*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b
^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*
c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2
*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^
(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)
*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2
*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c
^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^
4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/
2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*
c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 +
20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e
^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d
*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^
5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a
^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3
*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*
c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(
1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^
2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^
4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5
*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*
b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d + e*x
)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d -
 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e
+ a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a
^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)
^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c
 - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9
 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^
8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)
^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*
```

```
e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e +
10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^
4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3
*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^
(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)
*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3
*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*
c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2
 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^
2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^
2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e
 + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*
b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c
^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*
e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6
*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c
 - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b
^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7
*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) +
 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1
/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b
^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3
 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c
^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3
*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2
)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*
a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b
^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))
)^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4
 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4
 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4
*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4
*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a
^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b
^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(
4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*
d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2
*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8
)))^(1/2) - (16*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4 + a^4*c^3*d^3*
e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 + 2*a^3*b^3*c*d
^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5 + (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 -
b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5
*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d +
e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*
d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3
*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 2
8*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b
^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*
a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*
c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b
*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b
^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c
^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e
 + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a
*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^
```

3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)))*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i - ((2*d)/(c*e^2) + (2*(b*e^3 - 2*c*d*e^2))/(3*c^2*e^4))*(d + e*x)^(3/2) + atan(((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)

^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (16*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4 + a^4*c^3*d^3*e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 + 2*a^3*b^3*c*d^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5 + (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d + e*x)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a

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^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*
d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*
c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*
b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^
7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2)
+ 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c -
b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)))*((8*a^4*c^
5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d -
 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*
c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*
d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) -
6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*2i + (d + e*x)^(1/2)*((6*d^2)/(c*e^2) - (2*(a*e^4 + c*d^2*e^
2 - b*d*e^3))/(c^2*e^4) + (((6*d)/(c*e^2) + (2*(b*e^3 - 2*c*d*e^2))/(c^2*e^
4))*(b*e^3 - 2*c*d*e^2))/(c*e^2)) + (2*(d + e*x)^(5/2))/(5*c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.336 $\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=316

$$\frac{\sqrt{2}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+\right.}{c^{5/2}\sqrt{2cd-}}$$

Rubi [A] time = 3.16, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (-2*b*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c*e) + (Sqrt[2]*(b*c*d -
b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b*c*
d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{be}{c^2}+\frac{x^2}{c}+\frac{b\left(cd^2-bde+ae^2\right)-\left(bcd-b^2e+ace\right)x^2}{c^2e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{2\,\text{Subst}\left(\int \frac{b\left(cd^2-bde+ae^2\right)+\left(-bcd+b^2e-ace\right)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{c^2e^2}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}-\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt{b^2-4ac}}{2e}-\ldots}\right.}{c^2e^2}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{\sqrt{2}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{2cd-(\ldots}}\right.}{c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.44, size = 375, normalized size = 1.19

$$\frac{\sqrt{2}\left(-b^2\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(d\sqrt{b^2-4ac}-3ae\right)+ac\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\sqrt{2}\left(b^2\left(e\sqrt{b^2-4ac}-cd\right)-bc\left(d\sqrt{b^2-4ac}+3ae\right)+ac\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2\sqrt{d+ex}\,(c(d+ex)-3be)}{3c^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-3*b*e + c*(d + e*x)))/(3*c^2*e) + (Sqrt[2]*(b^3*e + b*c*
(Sqrt[b^2 - 4*a*c]*d - 3*a*e) - b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) + a*c*(2*c*
d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2 - 4*a*c]*d + 3
*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2 - 4*a*c]
*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e])
```

IntegrateAlgebraic [A] time = 1.16, size = 445, normalized size = 1.41

$$\frac{\left(-\sqrt{2}bcd\sqrt{b^2-4ac}+\sqrt{2}b^2e\sqrt{b^2-4ac}-\sqrt{2}ace\sqrt{b^2-4ac}+3\sqrt{2}abce-2\sqrt{2}ac^2d-\sqrt{2}b^3e+\sqrt{2}b^2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}+\frac{\left(-\sqrt{2}bcd\sqrt{b^2-4ac}+\sqrt{2}b^2e\sqrt{b^2-4ac}-\sqrt{2}ace\sqrt{b^2-4ac}-3\sqrt{2}abce+2\sqrt{2}ac^2d+\sqrt{2}b^3e-\sqrt{2}b^2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{e\sqrt{b^2-4ac}+be-2cd}}-\frac{2\left(3be\sqrt{d+ex}-c(d+ex)^{3/2}\right)}{3c^2e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(3*b*e*Sqrt[d + e*x] - c*(d + e*x)^(3/2)))/(3*c^2*e) + ((Sqrt[2]*b^2*c*
d - 2*Sqrt[2]*a*c^2*d - Sqrt[2]*b*c*Sqrt[b^2 - 4*a*c]*d - Sqrt[2]*b^3*e + 3
*Sqrt[2]*a*b*c*e + Sqrt[2]*b^2*Sqrt[b^2 - 4*a*c]*e - Sqrt[2]*a*c*Sqrt[b^2 -
 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[
b^2 - 4*a*c]*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2 -
```

```
4*a*c]*e]) + ((-(Sqrt[2]*b^2*c*d) + 2*Sqrt[2]*a*c^2*d - Sqrt[2]*b*c*Sqrt[b
^2 - 4*a*c]*d + Sqrt[2]*b^3*e - 3*Sqrt[2]*a*b*c*e + Sqrt[2]*b^2*Sqrt[b^2 -
4*a*c]*e - Sqrt[2]*a*c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e])
```

fricas [**B**] time = 0.52, size = 2966, normalized size = 9.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a
*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3
+ 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4
)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(
b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)*((b^6*c - 6*a*b^4*c
^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e
- (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*
b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e +
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^
2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a
^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) - 3*sqrt(2)*c^2*e*sqrt(((
b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b
^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^
7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*
c^5 - 4*a*c^6))*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e - (b^4*c^5 - 6*a*b^2*c^6 +
8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5
*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c
 - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^
5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c -
 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 -
 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*e)*sqrt(e*x + d)) + 3*sqrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2
*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^
6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)
*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c
^2 - 4*a^3*b*c^3)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 -
4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*
d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2
- 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3
 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c -
 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) - 3*s
qrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*
b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e +
```

```
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 + 8*
a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e + (b^4*
c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4
)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*
c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^
2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c
^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*
a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4
- 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) + 4*(c*e*x + c*d - 3*b*e)*sqrt(e
*x + d))/(c^2*e)
```

giac [**B**] time = 0.45, size = 868, normalized size = 2.75

$$-\frac{\left(\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\left(\left(b^3c-4abc^2\right)de-\left(b^4-5ab^2c+4a^2c^2\right)e^2\right)c^2-2\left(\sqrt{b^2-4ac}\,bc^3d^2-\sqrt{b^2-4ac}\,b^2c^2de+\sqrt{b^2-4ac}\,abc^2e^2\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,|c|+\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\left(2\left(b^2c^4-2ac^5\right)d^2-\left(3b^3c^3-8abc^4\right)de+\left(b^4c^2-3ab^2c^3\right)e^2\right)\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{\left(2c^4de^4-bc^3e^5+\sqrt{-4\left(c^4d^2e^4-bc^3de^5+ac^3e^6\right)c^4e^4+\left(2c^4de^4-bc^3e^5\right)^2}\right)e^{(-4)}}{c^4}}}\right)}{4\left(\sqrt{b^2-4ac}\,c^5d^2-\sqrt{b^2-4ac}\,bc^4de+\sqrt{b^2-4ac}\,ac^4e^2\right)c^2}+\frac{\left(\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\left(\left(b^3c-4abc^2\right)de-\left(b^4-5ab^2c+4a^2c^2\right)e^2\right)c^2+2\left(\sqrt{b^2-4ac}\,bc^3d^2-\sqrt{b^2-4ac}\,b^2c^2de+\sqrt{b^2-4ac}\,abc^2e^2\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,|c|+\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\left(2\left(b^2c^4-2ac^5\right)d^2-\left(3b^3c^3-8abc^4\right)de+\left(b^4c^2-3ab^2c^3\right)e^2\right)\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{\left(2c^4de^4-bc^3e^5-\sqrt{-4\left(c^4d^2e^4-bc^3de^5+ac^3e^6\right)c^4e^4+\left(2c^4de^4-bc^3e^5\right)^2}\right)e^{(-4)}}{c^4}}}\right)}{4\left(\sqrt{b^2-4ac}\,c^5d^2-\sqrt{b^2-4ac}\,bc^4de+\sqrt{b^2-4ac}\,ac^4e^2\right)c^2}+\frac{2\left(\left(xe+d\right)^{\frac{3}{2}}c^2e^2-3\sqrt{xe+d}\,bce^3\right)e^{(-3)}}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c - 4*a*b*c^2)
*d*e - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^3*
d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*sqrt(-
4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*a*b*c
^4)*d*e + (b^4*c^2 - 3*a*b^2*c^3)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sq
rt(-(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e
^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))*e^(-4)/c^4))/((sqrt(b^2 - 4*a*c
)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2)
 + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c - 4*a*b*c^
2)*d*e - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2 + 2*(sqrt(b^2 - 4*a*c)*b*c^
3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*sqrt
(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*
c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*a*b
*c^4)*d*e + (b^4*c^2 - 3*a*b^2*c^3)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/
sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 - sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3
*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))*e^(-4)/c^4))/((sqrt(b^2 - 4*a
*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^
2) + 2/3*((x*e + d)^(3/2)*c^2*e^2 - 3*sqrt(x*e + d)*b*c*e^3)*e^(-3)/c^3
```

maple [**B**] time = 0.04, size = 1329, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/3*(e*x+d)^(3/2)/c/e-2*b*(e*x+d)^(1/2)/c^2-3*e^2/c/(-(4*a*c-b^2)*e^2)^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b+2*e/(-(
4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/
2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^
(1/2)*c)*a*d+e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c
-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3-e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/(
(-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d+e/c*2^(1/2)/((-b
*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/(
(-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a-e/c^2*2^(1/2)/((-b*e+2*
```

```
c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e
+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2+1/c*2^(1/2)/((-b*e+2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*
d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d-3*e^2/c/(-(4*a*c-b^2)*e^2)^(1/2
)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/
2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b+2*e/(-(4*a
*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*a
rctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*
c)*a*d+e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b^3-e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*
d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c
*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d-e/c*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(
4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a+e/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*b^2-1/c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x^2/(c*x^2 + b*x + a), x)
```

mupad [B] time = 3.91, size = 8171, normalized size = 25.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(3/2))/(3*c*e) - atan(((((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 -
 b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/
c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^
(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-
(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e
*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*
(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^
7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c
^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
+ 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*
(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4
- 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 -
 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 -
4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3
)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*
(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*1i - (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*
e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*(-(b
^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*
```

```
c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
 + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*
b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2
*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^
2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(
4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2
*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^
3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/
2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c
^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^
3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-
(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^
2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c
*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*
a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/
(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^5 - a^3*b
^2*e^5 + a^2*b^3*d*e^4 + a^3*c^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*
d^2*e^3))/c^3 + (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3
*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 - (8*(d + e*x)^(
1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18
*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/
2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^
(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2
*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d +
b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^
2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20
*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*
a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b
^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2)
)/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d -
18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3
)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*
e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2
 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^
4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*
e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a
^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a
*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) -
 b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c
- b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a
*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d
 + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*
e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3
- 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d +
b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^
2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20
*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2)))*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3
)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*
```

```
c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*
(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*2i - atan(((((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^
4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 - (8*(d + e*x)^(1/2
)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2
*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^
5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) +
 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2
))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6
*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1
/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e +
 a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*
b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)
^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8
*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b
^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*
d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3
)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2
*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^
5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) +
 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2
))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((8*(a*b^3*c^3*e^4
 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 +
4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e +
a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b
*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7
*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*
c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2
)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c -
b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*
x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 +
 b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*
a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e +
a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b
*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^5 - a^3*b^2*e^5 + a^2*b^3*d*e^4 + a^3*
c^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*d^2*e^3))/c^3 + (((8*(a*b^3*c
^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*
e^2 + 4*a*b^2*c^4*d*e^3))/c^3 - (8*(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^
3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^
2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20
*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8
*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e
+ b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c
 - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*
a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(
d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2
*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3
 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^
```

```
3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^
2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20
*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*
d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*
(d + e*x)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^
6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b
^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c
- b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e
^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*
c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a
^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c
^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7
 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3
*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*
e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3
*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^
6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b
^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c
- b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*((b^4*e*
(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d
 - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*
a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2
*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*
a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - ((4*d)/(c*e) + (2*(b*e^2 - 2*
c*d*e))/(c^2*e^2))*(d + e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.337 $\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=287

$$\frac{\sqrt{2}\left(-\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}\,(cd-be)+2ace+\right.}{c^{3/2}\sqrt{b^2-4ac}\,\sqrt{}}$$

Rubi [A] time = 3.20, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {824, 826, 1166, 208}

$$\frac{\sqrt{2}\left(-\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2\sqrt{d+ex}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x])/c + (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]
*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d
- b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx &= \frac{2\sqrt{d+ex}}{c} + \frac{\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx}{c}\\
&= \frac{2\sqrt{d+ex}}{c} + \frac{2\,\text{Subst}\left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c}\\
&= \frac{2\sqrt{d+ex}}{c} - \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\right.}{c\sqrt{b^2-4ac}}\\
&= \frac{2\sqrt{d+ex}}{c} + \frac{\sqrt{2}\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} -
\end{aligned}
$$

Mathematica [A] time = 0.41, size = 301, normalized size = 1.05

$$
\frac{\frac{\sqrt{2}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\sqrt{2}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}-2ace+b^2e-bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + 2\sqrt{c}\sqrt{d+ex}}{c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x] + (Sqrt[2]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e
+ 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(-(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d +
b^2*e - 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]))/c^(3/2)
```

IntegrateAlgebraic [C] time = 1.18, size = 387, normalized size = 1.35

$$
\frac{\left(\sqrt{2}\,cd\sqrt{4ac-b^2}-\sqrt{2}\,be\sqrt{4ac-b^2}+2i\sqrt{2}\,ace-i\sqrt{2}\,b^2e+i\sqrt{2}\,bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{c^{3/2}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}} + \frac{\left(\sqrt{2}\,cd\sqrt{4ac-b^2}-\sqrt{2}\,be\sqrt{4ac-b^2}-2i\sqrt{2}\,ace+i\sqrt{2}\,b^2e-i\sqrt{2}\,bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{c^{3/2}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}} + \frac{2\sqrt{d+ex}}{c}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x])/c + ((I*Sqrt[2]*b*c*d + Sqrt[2]*c*Sqrt[-b^2 + 4*a*c]*d -
I*Sqrt[2]*b^2*e + (2*I)*Sqrt[2]*a*c*e - Sqrt[2]*b*Sqrt[-b^2 + 4*a*c]*e)*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c
]*e]])/(c^(3/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]
*e]) + (((-I)*Sqrt[2]*b*c*d + Sqrt[2]*c*Sqrt[-b^2 + 4*a*c]*d + I*Sqrt[2]*b^
2*e - (2*I)*Sqrt[2]*a*c*e - Sqrt[2]*b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(3
/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])
```

fricas [B] time = 0.46, size = 1721, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4
*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3*c
- 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sq
rt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2
)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^
2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b
^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*
d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) - sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d
 - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a
*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d
*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c -
2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*
(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7
)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) +
 sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c
^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^
2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3*c - 4*
a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e + (b^3*c^3 - 4*a*b*c^4)*sqrt((
b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b
^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^
3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c
 + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d -
(a*b^2 - a^2*c)*e)*sqrt(e*x + d)) - sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (
b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c
^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*e + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e +
 (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*
c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3
*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/
(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) + 4*s
qrt(e*x + d))/c
```

giac [B] time = 0.41, size = 753, normalized size = 2.62

$$\frac{2\sqrt{xe+d}}{c}+\frac{\left(\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\left(\left(b^2c-4ac^2\right)de-\left(b^3-4abc\right)e^2\right)c^2-2\left(\sqrt{b^2-4ac}\,c^3d^2-\sqrt{b^2-4ac}\,bc^2de+\sqrt{b^2-4ac}\,ac^2e^2\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,|c|+\left(2bc^4d^2-\left(3b^2c^3-4ac^4\right)de+\left(b^3c^2-2abc^3\right)e^2\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2c^2d-bce+\sqrt{-4\left(c^2d^2-bcde+ace^2\right)c^2+\left(2c^2d-bce\right)^2}}{c^2}}}\right)}{4\left(\sqrt{b^2-4ac}\,c^4d^2-\sqrt{b^2-4ac}\,bc^3de+\sqrt{b^2-4ac}\,ac^3e^2\right)c^2}-\frac{\left(\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\left(\left(b^2c-4ac^2\right)de-\left(b^3-4abc\right)e^2\right)c^2+2\left(\sqrt{b^2-4ac}\,c^3d^2-\sqrt{b^2-4ac}\,bc^2de+\sqrt{b^2-4ac}\,ac^2e^2\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,|c|+\left(2bc^4d^2-\left(3b^2c^3-4ac^4\right)de+\left(b^3c^2-2abc^3\right)e^2\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2c^2d-bce-\sqrt{-4\left(c^2d^2-bcde+ace^2\right)c^2+\left(2c^2d-bce\right)^2}}{c^2}}}\right)}{4\left(\sqrt{b^2-4ac}\,c^4d^2-\sqrt{b^2-4ac}\,bc^3de+\sqrt{b^2-4ac}\,ac^3e^2\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(
(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 - 2*(sqrt(b^2 - 4*a*c)*c^3
*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*sqrt(-4*c
^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^4*d^2 - (3*b^2*c^3
- 4*a*c^4)*d*e + (b^3*c^2 - 2*a*b*c^3)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b
^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^2*d - b*c*e
+ sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/(
(sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c
)*a*c^3*e^2)*c^2) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(
(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 + 2*(sqrt(b^2 - 4*a*c)*c^3
*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*sqrt(-4*c
^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^4*d^2 - (3*b^2*c^3
- 4*a*c^4)*d*e + (b^3*c^2 - 2*a*b*c^3)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b
^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^2*d - b*c*e
- sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/(
```

```
(sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c
)*a*c^3*e^2)*c^2)
```

maple [B] time = 0.04, size = 926, normalized size = 3.23

$$\frac{2\sqrt{2}\,ae^2\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}+\frac{2\sqrt{2}\,ae^2\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}-\frac{\sqrt{2}\,b^2e^2\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\,c}-\frac{\sqrt{2}\,b^2e^2\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\,c}+\frac{\sqrt{2}\,bde\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}+\frac{\sqrt{2}\,bde\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}+\frac{\sqrt{2}\,be\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\,c}-\frac{\sqrt{2}\,be\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\,c}-\frac{\sqrt{2}\,d\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}+\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}+\frac{2\sqrt{ex+d}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2*(e*x+d)^(1/2)/c+2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*e^2-1/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/
2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*e^2+1/(-(4*a*c-b^
2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arcta
nh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)
*b*d*e+1/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh(
(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*
e-2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d+2/(-(4*a*
c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*ar
ctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*a*e^2-1/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*b^2*e^2+1/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*
d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d*e-1/c*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*e+2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*c)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,x}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x/(c*x^2 + b*x + a), x)
```

mupad [B] time = 3.82, size = 5664, normalized size = 19.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4
*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d +
e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d
 + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^
(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2
))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b
```

^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a
^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e +
 a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*
c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i
- (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b
^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*((8*a^2*c^3*d - b
^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*
a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a
^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 -
2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e
- b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c -
 b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*
c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)
*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e
^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(
4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1
/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(
16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^4 - a*b^
2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d
e^3))/c - (8(d + e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)
^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*
e^3 + 8*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(
1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*
a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 -
2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c
^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*
c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^
3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) - (16*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^
4 - 2*a*b*c*d^2*e^3))/c + (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2
*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)
^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7
*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2
) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c
)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^
3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6
*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2) + (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c
^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c
^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c
e(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*
d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*((8*a
^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e +
 a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*
c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
- atan(((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e
^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d + e*x)^(1/2)*(-(b^5*e -
8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*
e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d
 + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3
*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^
2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-
(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 1
2*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*
x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a

```
*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d -
b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b
^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^
2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((8*(4*a^2*c^3*e
^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*
a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*
a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2
) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 -
4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c
- b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) +
b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2
*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^
3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^
(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4
*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*
c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d
 + e*x)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4
*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)
^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d
*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d
 - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^
(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^
2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*
(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3
*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*
a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/
2) - (16*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 - 2*a*b*c*d
^2*e^3))/c + (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^
2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*(-(b^5
*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e +
 a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*
c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^
3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e -
 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c
*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*
d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d
 + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2
- 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3
*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*
c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2
*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5*e - 8*a^
2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-
(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 1
2*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
[Out] Timed out
```

3.338 $\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=198

$$\frac{\sqrt{2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.27, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {699, 1130, 208}

$$\frac{\sqrt{2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] -((Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2
 - 4*a*c])) + (Sqrt[2]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = (2e)\,\text{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)$$
$$= -\left(\left(-e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)\right)+$$
$$= -\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{2}\sqrt{2cd-\left(b+\sqrt{b^2-4\right.}}}{\sqrt{c}}$$

Mathematica [A] time = 0.40, size = 175, normalized size = 0.88

$$\frac{\sqrt{2}\left(\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)-\sqrt{e\sqrt{b^2-4ac}-be+2cd}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (Sqrt[2]*(-(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]) + Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]]))/(Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [C] time = 0.00, size = 275, normalized size = 1.39

$$\frac{\sqrt{2}\left(e\sqrt{4ac-b^2}+ibe-2icd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{c}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\sqrt{2}\left(e\sqrt{4ac-b^2}-ibe+2icd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{c}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (Sqrt[2]*((-2*I)*c*d + I*b*e + Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[c]*Sqr
t[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (Sqrt[2]*((2
*I)*c*d - I*b*e + Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*
c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])
```

fricas [B] time = 0.42, size = 715, normalized size = 3.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*
c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 -
 4*a*c^3))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^
3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*
e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log
(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e
```

```
 + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*
sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e
^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*s
qrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2
/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) - 1/2*sqrt(2
)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2
*c - 4*a*c^2))*log(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))
*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*
c - 4*a*c^2)) + 2*sqrt(e*x + d)*e)
```

giac [A] time = 0.27, size = 223, normalized size = 1.13

$$-\frac{\sqrt{-4\,c^{2}d+2\left(bc-\sqrt{b^{2}-4\,ac}\,c\right)e}\,\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}\,\sqrt{xe+d}}{\sqrt{-\frac{2\,cd-be+\sqrt{(2\,cd-be)^{2}-4\left(cd^{2}-bde+ae^{2}\right)c}}{c}}}\right)}{\sqrt{b^{2}-4\,ac}\,|c|}+\frac{\sqrt{-4\,c^{2}d+2\left(bc+\sqrt{b^{2}-4\,ac}\,c\right)e}\,\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}\,\sqrt{xe+d}}{\sqrt{-\frac{2\,cd-be-\sqrt{(2\,cd-be)^{2}-4\left(cd^{2}-bde+ae^{2}\right)c}}{c}}}\right)}{\sqrt{b^{2}-4\,ac}\,|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sqrt(x
*e + d)/sqrt(-(2*c*d - b*e + sqrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^
2)*c))/c))/(sqrt(b^2 - 4*a*c)*abs(c)) + sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 -
 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c*d - b*e - sqrt((2
*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/(sqrt(b^2 - 4*a*c)*abs(c)
)
```

maple [B] time = 0.03, size = 545, normalized size = 2.75

$$\frac{\sqrt{2}\,b\,e^{2}\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}+\frac{\sqrt{2}\,b\,e^{2}\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}-\frac{2\sqrt{2}\,cde\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}-\frac{2\sqrt{2}\,cde\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}-\frac{\sqrt{2}\,e\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}+\frac{\sqrt{2}\,e\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] e^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*c)*b-2*c*e/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-
(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d-e*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)+e^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*
d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b-2*c*e/(-(4*a*c-b^2)*e^2)^(1/2)*2^
(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2
^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d+e*2^(1/2)/((b*e-
2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e
-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{cx^{2}+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x + a), x)
```

mupad [B] time = 2.99, size = 709, normalized size = 3.58

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,\left(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)+\frac{\sqrt{d+e\,x}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3\,e^3+64\,a\,d\,c^4\,e^2\right)\left(b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e\right)}{2\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\sqrt{-\frac{b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e}{2\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{16\,c^2\,d^2\,e^3-16\,b\,c\,d\,e^4+16\,a\,c\,e^5}\right)\sqrt{-\frac{b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e}{2\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,\left(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)-\frac{\sqrt{d+e\,x}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3\,e^3+64\,a\,d\,c^4\,e^2\right)\left(e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e\right)}{2\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\sqrt{\frac{e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e}{2\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{16\,c^2\,d^2\,e^3-16\,b\,c\,d\,e^4+16\,a\,c\,e^5}\right)\sqrt{\frac{e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e}{2\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(a + b*x + c*x^2),x)
```

```
[Out] - 2*atanh((2*((d + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2
+ 16*b*c^2*d*e^3) + ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32
*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^
2*d - 2*b^2*c*d - 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^
3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5 - 16
*b*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d
- 4*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh((2*((d
 + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2 + 16*b*c^2*d*e^3
) - ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c^3*e^3 + 6
4*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d
+ 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*a*c - b^2)^3
)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3
 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5 - 16*b*c*d*e^4))*((e*
(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

sympy [A] time = 50.10, size = 155, normalized size = 0.78

$$2 e \operatorname{RootSum}{\left(t^{4} \left(256 a^{2} c^{3} e^{4} - 128 a b^{2} c^{2} e^{4} + 16 b^{4} c e^{4}\right) + t^{2} \left(- 16 a b c e^{3} + 32 a c^{2} d e^{2} + 4 b^{3} e^{3} - 8 b^{2} c d e^{2}\right) + a e^{2} - b d e + c d^{2}, \left(t \mapsto t \log{\left(64 t^{3} a c^{2} e^{2} - 16 t^{3} b^{2} c e^{2} - 2 t b e + 4 t c d + \sqrt{d + e x} \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] 2*e*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e*
*4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e
**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 - 16
*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x))))
```

3.339 $\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=275

$$\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\cdots}{\sqrt{2cd-e(\cdots}}\cdots\right.}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\cdots$$

Rubi [A] time = 1.13, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {897, 1287, 206, 1166, 208}

$$\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a + (Sqrt[2]*Sqrt[c]*(b*d + Sqr
t[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{de}{a\left(d-x^2\right)}+\frac{e\left(cd^2-bde+ae^2-cdx^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+\cdots}\right.}{a}$$

$$= -\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\left(c\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)\cdots}\right.}{a\sqrt{b^2-4ac}}$$

$$= -\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e\cdots}}\right.}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.94, size = 267, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b
^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^
2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a
```

IntegrateAlgebraic [A] time = 0.99, size = 274, normalized size = 1.00

$$-\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{a\sqrt{b^2-4ac}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}-\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right)}{a\sqrt{b^2-4ac}\sqrt{e\sqrt{b^2-4ac}+be-2cd}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] -((Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(a*Sqrt[b^2 -
 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e])) - (Sqrt[2]*Sqrt[c]*(-(b*
d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sq
rt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d +
 b*e + Sqrt[b^2 - 4*a*c]*e]) - (2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a
```

fricas [B] time = 0.79, size = 2446, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((
b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*l
og(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e -
(b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d
)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*lo
g(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e -
(b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d
)) + sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*lo
g(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)
) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^
2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log
(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)
) + 2*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x))/a, 1/2*(sqrt(2)
*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a
*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((
b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)
*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*
a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) - sqrt(2)*
a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((
b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)
*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*
a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) + sqrt(2)*
a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) - sqrt(2)*a
*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2
```

```
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) + 4*sqrt(-d
)*arctan(sqrt(e*x + d)*sqrt(-d)/d))/a]
```

giac [B] time = 0.39, size = 712, normalized size = 2.59

$$\frac{2\,d\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}} - \frac{\left(\sqrt{-4\,c^2d + 2\left(bc - \sqrt{b^2-4\,ac}\,c\right)e}\,\left(b^2-4\,ac\right)a^2de - 2\left(\sqrt{b^2-4\,ac}\,acd^2 - \sqrt{b^2-4\,ac}\,abde + \sqrt{b^2-4\,ac}\,a^2e^2\right)\sqrt{-4\,c^2d + 2\left(bc - \sqrt{b^2-4\,ac}\,c\right)e}\,|a| - \left(2\,a^2bcd^2 + 2\,a^3be^2 - \left(a^2b^2 + 4\,a^3c\right)de\right)\sqrt{-4\,c^2d + 2\left(bc - \sqrt{b^2-4\,ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2\,acd - abe + \sqrt{-4\left(acd^2 - abde + a^2e^2\right)ac + \left(2\,acd - abe\right)^2}}{ac}}}\right)}{4\left(\sqrt{b^2-4\,ac}\,a^2cd^2 - \sqrt{b^2-4\,ac}\,a^2bde + \sqrt{b^2-4\,ac}\,a^3e^2\right)|a||c|} + \frac{\left(\sqrt{-4\,c^2d + 2\left(bc + \sqrt{b^2-4\,ac}\,c\right)e}\,\left(b^2-4\,ac\right)a^2de + 2\left(\sqrt{b^2-4\,ac}\,acd^2 - \sqrt{b^2-4\,ac}\,abde + \sqrt{b^2-4\,ac}\,a^2e^2\right)\sqrt{-4\,c^2d + 2\left(bc + \sqrt{b^2-4\,ac}\,c\right)e}\,|a| - \left(2\,a^2bcd^2 + 2\,a^3be^2 - \left(a^2b^2 + 4\,a^3c\right)de\right)\sqrt{-4\,c^2d + 2\left(bc + \sqrt{b^2-4\,ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2\,acd - abe - \sqrt{-4\left(acd^2 - abde + a^2e^2\right)ac + \left(2\,acd - abe\right)^2}}{ac}}}\right)}{4\left(\sqrt{b^2-4\,ac}\,a^2cd^2 - \sqrt{b^2-4\,ac}\,a^2bde + \sqrt{b^2-4\,ac}\,a^3e^2\right)|a||c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*d*arctan(sqrt(x*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e - 2*(sqrt(b^2 - 4*a*c)*a
*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2
*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2
 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c*d
^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*
a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*a
bs(a)*abs(c)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^2
- 4*a*c)*a^2*d*e + 2*(sqrt(b^2 - 4*a*c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e
 + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)
*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt(-
4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d
)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a
*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)
*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c))
```

maple [B] time = 0.06, size = 581, normalized size = 2.11

$$\frac{\sqrt{2}\,bcde\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\,a} + \frac{\sqrt{2}\,bcde\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\,a} - \frac{2\sqrt{2}\,c\,e^2\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}} - \frac{2\sqrt{2}\,c\,e^2\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{-\left(4ac-b^2\right)e^2}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}} + \frac{\sqrt{2}\,cd\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\,a} - \frac{\sqrt{2}\,cd\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\,c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^2\right)e^2}\right)c}\,a} - \frac{2\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x)
```

```
[Out] -2*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a-2*e^2*c/(-(4*a*c-b^2)*e^2)^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)+e/a*c/(-(4*
a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)
*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1
/2)*c)*b*d+1/a*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*ar
ctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)
*c)*d-2*e^2*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)+e/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d-1/a*c*2^(1/2)/((b*e-2*c*d+(-(4*a*
c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x), x)
```

mupad [B] time = 7.41, size = 10894, normalized size = 39.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x*(a + b*x + c*x^2)),x)

[Out] - atan(((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b
*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a
^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*
a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*
c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4
*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c -
b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3
*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e
^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^
8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 3
84*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b
^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)
^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576
*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d
*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e
^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*
c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*
e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*
a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^
3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10
) + (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 +
 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e
+ a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d
 + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i - (((b^4
*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c -
b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3
*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*
(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a
^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*
((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a
*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*
c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d
^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9
) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96*a^2*b^2*c^4*d^3*e^8 + 96*
a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*a^3*b^2*c^3*d*e^10) - (d +
 e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11
+ 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*
c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*
d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-
(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2
*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a^2*c^4*d^2*e^
10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b
^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) - (d + e*x)^
(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^
2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c
 - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e
)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i)/((((b^4*d + 8*a^2*c^2
*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2
)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-
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(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) + (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96*a^2*b^2*c^4*d^3*e^8 + 96*a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) - 64*c^4*d^3*e^10 + 64*b*c^3*d^2*e^11 - 64*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i - atan(((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b

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^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 -
 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^
2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2
*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2
*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2
) + 96*a*c^5*d^4*e^8 + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^
2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^1
1 - 192*a*b^2*c^3*d^2*e^10) + (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4
*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*
d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b
^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*
b^2*c)))^(1/2)*1i - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^
3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^
2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*
a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)
^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c -
b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(
2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b
^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d
^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e
^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96
*a^2*b^2*c^4*d^3*e^8 + 96*a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*
a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e
^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b
^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^
3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^
2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(
1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))
^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 6
4*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*
c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c
^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*
d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) -
 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)
*1i)/((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d
*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4
*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*
c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*
e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d
 + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^
2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b
^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^1
0 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8
- 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384
*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b^3
*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(
1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a
^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e
^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9
 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c
- b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)
/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*a^
2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*
e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10)
+ (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 6
4*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e -
a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d +
```

4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96*a^2*b^2*c^4*d^3*e^8 + 96*a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) - 64*c^4*d^3*e^10 + 64*b*c^3*d^2*e^11 - 64*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i - (2*d^(1/2)*atanh((640*c^4*d^(5/2)*e^10*(d + e*x)^(1/2))/(640*c^4*d^3*e^10 - 384*b*c^3*d^2*e^11 + (576*c^5*d^5*e^8)/a + 64*a*c^3*d*e^12 + (192*b^2*c^3*d^3*e^10)/a + (64*b^3*c^2*d^2*e^11)/a - (128*b^2*c^4*d^5*e^8)/a^2 + (192*b^3*c^3*d^4*e^9)/a^2 - (64*b^4*c^2*d^3*e^10)/a^2 - (896*b*c^4*d^4*e^9)/a) + (576*c^5*d^(9/2)*e^8*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 384*a*b*c^3*d^2*e^11) + (64*b^3*c^2*d^(3/2)*e^11*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 384*a*b*c^3*d^2*e^11) + (192*b^2*c^3*d^(5/2)*e^10*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 384*a*b*c^3*d^2*e^11) - (64*b^4*c^2*d^(5/2)*e^10*(d + e*x)^(1/2))/(576*a*c^5*d^5*e^8 + 64*a^3*c^3*d*e^12 + 640*a^2*c^4*d^3*e^10 - 128*b^2*c^4*d^5*e^8 + 192*b^3*c^3*d^4*e^9 - 64*b^4*c^2*d^3*e^10 - 896*a*b*c^4*d^4*e^9 + 192*a*b^2*c^3*d^3*e^10 + 64*a*b^3*c^2*d^2*e^11 - 384*a^2*b*c^3*d^2*e^11) + (192*b^3*c^3*d^(7/2)*e^9*(d + e*x)^(1/2))/(576*a*c^5*d^5*e^8 + 64*a^3*c^3*d*e^12 + 640*a^2*c^4*d^3*e^10 - 128*b^2*c^4*d^5*e^8 + 192*b^3*c^3*d^4*e^9 - 64*b^4*c^2*d^3*e^10 - 896*a*b*c^4*d^4*e^9 + 192*a*b^2*c^3*d^3*e^10 + 64*a*b^3*c^2*d^2*e^11 - 384*a^2*b*c^3*d^2*e^11) - (128*b^2*c^4*d^(9/2)*e^8*(d + e*x)^(1/2))/(576*a*c^5*d^5*e^8 + 64*a^3*c^3*d*e^12 + 640*a^2*c^4*d^3*e^10 - 128*b^2*c^4*d^5*e^8 + 192*b^3*c^3*d^4*e^9 - 64*b^4*c^2*d^3*e^10 - 896*a*b*c^4*d^4*e^9 + 192*a*b^2*c^3*d^3*e^10 + 64*a*b^3*c^2*d^2*e^11 - 384*a^2*b*c^3*d^2*e^11) + (64*a*c^3*d^(1/2)*e^12*(d + e*x)^(1/2))/(640*c^4*d^3*e^10 - 384*b*c^3*d^2*e^11 + (576*c^5*d^5*e^8)/a + 64*a*c^3*d*e^12 + (192*b^2*c^3*d^3*e^10)/a + (64*b^3*c^2*d^2*e^11)/a - (128*b^2*c^4*d^5*e^8)/a^2 + (192*b^3*c^3*d^4*e^9)/a^2 - (64*b^4*c^2*d^3*e^10)/a^2 - (896*b*c^4*d^4*e^9)/a) - (384*b*c^3*d^(3/2)*e^11*(d + e*x)^(1/2))/(640*c^4*d^3*e^10 - 384*b*c^3*d^2*e^11 + (576*c^5*d^5*e^8)/a + 64*a*c^3*d*e^12 + (192*b^2*c^3*d^3*e^10)/a + (64*

```
b^3*c^2*d^2*e^11)/a - (128*b^2*c^4*d^5*e^8)/a^2 + (192*b^3*c^3*d^4*e^9)/a^2
 - (64*b^4*c^2*d^3*e^10)/a^2 - (896*b*c^4*d^4*e^9)/a) - (896*b*c^4*d^(7/2)*
e^9*(d + e*x)^(1/2))/(576*c^5*d^5*e^8 + 640*a*c^4*d^3*e^10 + 64*a^2*c^3*d*e
^12 - 896*b*c^4*d^4*e^9 + 192*b^2*c^3*d^3*e^10 + 64*b^3*c^2*d^2*e^11 - (128
*b^2*c^4*d^5*e^8)/a + (192*b^3*c^3*d^4*e^9)/a - (64*b^4*c^2*d^3*e^10)/a - 3
84*a*b*c^3*d^2*e^11)))/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.340 $\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx$

Optimal. Leaf size=368

$$-\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\sqrt{c}\left(-b\left(d\sqrt{b^2-4ac}+ae\right)\right.}{a^2\sqrt{b}}$$

Rubi [A] time = 3.66, antiderivative size = 356, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\sqrt{c}\left(-\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{d+ex}}{ax}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -(Sqrt[d + e*x]/(a*x)) + (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a*Sqrt[d]) + (
2*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sqrt[2]*Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c
]*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqr
t[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{de^2}{a\left(d-x^2\right)^2}-\frac{e(-bd+ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^2} + \frac{(2de)\,\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d}\right.}{a}$$

$$= -\frac{\sqrt{d+ex}}{ax} + \frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} + \frac{e\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{\left(c\left(b^2d-\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex}}{ax} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}} + \frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd-a\right.}{a^2\sqrt{b}}$$

Mathematica [A] time = 1.59, size = 364, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(-bd\sqrt{b^2-4ac}+ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{a\sqrt{d+ex}}{x}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] (-((a*Sqrt[d + e*x])/x) + (a*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (2
*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (Sqrt[2]*Sqrt[c]*(-(
b^2*d) + 2*a*c*d - b*Sqrt[b^2 - 4*a*c]*d + a*b*e + a*Sqrt[b^2 - 4*a*c]*e)*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]
*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[
2]*Sqrt[c]*(-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2
 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt
```

```
[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]))/a^2
```

<u>IntegrateAlgebraic</u> [A] time = 1.71, size = 424, normalized size = 1.15

$$\frac{\left(\sqrt{2}\,b\sqrt{c}\,d\sqrt{b^2-4ac}-\sqrt{2}\,a\sqrt{c}\,e\sqrt{b^2-4ac}-\sqrt{2}\,ab\sqrt{c}\,e-2\sqrt{2}\,ac^{3/2}d+\sqrt{2}\,b^2\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}+\frac{\left(\sqrt{2}\,b\sqrt{c}\,d\sqrt{b^2-4ac}-\sqrt{2}\,a\sqrt{c}\,e\sqrt{b^2-4ac}+\sqrt{2}\,ab\sqrt{c}\,e+2\sqrt{2}\,ac^{3/2}d-\sqrt{2}\,b^2\sqrt{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{e\sqrt{b^2-4ac}+be-2cd}}+\frac{(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{d+ex}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -(Sqrt[d + e*x]/(a*x)) + ((Sqrt[2]*b^2*Sqrt[c]*d - 2*Sqrt[2]*a*c^(3/2)*d +
Sqrt[2]*b*Sqrt[c]*Sqrt[b^2 - 4*a*c]*d - Sqrt[2]*a*b*Sqrt[c]*e - Sqrt[2]*a*S
qrt[c]*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*
c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(a^2*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e
 - Sqrt[b^2 - 4*a*c]*e]) + ((-(Sqrt[2]*b^2*Sqrt[c]*d) + 2*Sqrt[2]*a*c^(3/2)
*d + Sqrt[2]*b*Sqrt[c]*Sqrt[b^2 - 4*a*c]*d + Sqrt[2]*a*b*Sqrt[c]*e - Sqrt[2
]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqr
t[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])/(a^2*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d
+ b*e + Sqrt[b^2 - 4*a*c]*e]) + ((2*b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d
]])/(a^2*Sqrt[d])
```

<u>fricas</u> [B] time = 9.99, size = 4860, normalized size = 13.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^
2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2
- 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*
c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*
a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b
^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 -
 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c
^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b
^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2
 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x
*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 -
 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^
3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 -
4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*
c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8
*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^
3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 -
4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e +
(a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5
- 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(
a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a
*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) + sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^
2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6
 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d
*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2
- 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a
^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6
- 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*
e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4
 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*s
```

```
qrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3
*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/
(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e
)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d -
(a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*
b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt
(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c
^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*
b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2
*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*
c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b
^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))
 + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - (
2*b*d - a*e)*sqrt(d)*x*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*sqr
t(e*x + d)*a*d)/(a^2*d*x), 1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b
^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^
2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*
c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c
 + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^
4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2
*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b
^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^
6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*
d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2
 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e
*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 -
 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c +
 a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^
6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e -
(a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c +
 a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d
 - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^
2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a
^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b
^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) + sqrt(2)*a
^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4
*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*
a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*
b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2
*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*
c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a
^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b
^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)
*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a
*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e
^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d
 - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4
*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(
((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c
^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4
*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5
 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(
((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c
^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt
```

```
(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^
5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c +
 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9
*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*
c^3)*e)*sqrt(e*x + d)) - 2*(2*b*d - a*e)*sqrt(-d)*x*arctan(sqrt(e*x + d)*sq
rt(-d)/d) - 2*sqrt(e*x + d)*a*d)/(a^2*d*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 1.17Done
```

maple [B] time = 0.05, size = 999, normalized size = 2.71

$$\frac{\sqrt{2}\, b c e^{2} \operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\, c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}\, a}+\frac{\sqrt{2}\, b c e^{2} \arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\, c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}\, a}+\frac{2\sqrt{2}\, c^{2} d e \operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\, c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}\, a}+\frac{2\sqrt{2}\, c^{2} d e \arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\, c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}\, a}-\frac{\sqrt{2}\, b^{2} c d e \operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\, c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}\, a^{2}}-\frac{\sqrt{2}\, b^{2} c d e \arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\, c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{-\left(4ac-b^{2}\right)e^{2}}\,\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}\, a^{2}}+\frac{\sqrt{2}\, c e \operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\, c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}\, a}-\frac{\sqrt{2}\, c e \arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\, c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}\, a}-\frac{\sqrt{2}\, b c d \operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,\sqrt{2}\, c}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}\, a^{2}}+\frac{\sqrt{2}\, b c d \arctan\left(\frac{\sqrt{ex+d}\,\sqrt{2}\, c}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-\left(4ac-b^{2}\right)e^{2}}\right)c}\, a^{2}}-\frac{e \operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{a\sqrt{d}}+\frac{2b\sqrt{d}\, \operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{a^{2}}-\frac{\sqrt{ex+d}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x)
```

```
[Out] -(e*x+d)^(1/2)/a/x-e*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)+2/a^2*d^(1/2)
*arctanh((e*x+d)^(1/2)/d^(1/2))*b+e^2/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/
2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b+2*e/a*c^2/(-(4*a*c-
b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arc
tanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*
c)*d-e/a^2*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*b^2*d+e/a*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)-1/a^2*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b*d+e^2/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*
d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b+2*e/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2
)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/
2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d-e/a^2*c/(-(4
*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)
*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*c)*b^2*d-e/a*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arc
tan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)
+1/a^2*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x
+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\left(cx^{2}+bx+a\right)x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^2), x)
```

mupad [B] time = 6.81, size = 19887, normalized size = 54.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x^2*(a + b*x + c*x^2)),x)
```

```
[Out] (atan((((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e
^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*
a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 - ((a*e
 - 2*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8
*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^
4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b
^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d
^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^
11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 - ((a*e - 2*b*d)*(
(8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e
^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*
a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108
*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*
a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 -
(((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c
^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4
*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^
8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*
c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 +
8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^
6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a
^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)))*1i)/(2*a^2*d^(1/2)) + ((a*
e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*
c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d
*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 + ((a*e - 2*b*d)*(
(8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^
3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e
^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*
e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3
*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4
*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + ((a*e - 2*b*d)*((8*(d + e*x
)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^
5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2
*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^
5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3
*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 + (((8*(32*a^
8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9
+ 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 -
64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*
b^3*c^3*d*e^10))/a^4 + (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 +
4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c
^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d
*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a^2*d^(1/2))
))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)))*1i)/(2*a^2*d^(1/2)))/((16*(a^3*c^5*e^
13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4
*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11))/a^4
 - ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 +
 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*
b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 - ((a*e - 2
*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4
*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^
```

4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 - ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 - (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)) + ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 + ((a*e - 2*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2))))*(a*e - 2*b*d)*1i)/(a^2*d^(1/2)) - atan(((((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) -

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(8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e
^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*
a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108
*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*
a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(
-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b
^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e +
 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b
^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3
*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d +
8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c
^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2
))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(6*
a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 +
 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a
*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(
1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2)
 + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2
*c)))^(1/2)*1i - (((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*
e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 2
0*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 +
84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b
^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*
c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32
*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e
^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9
 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a
^6*b^3*c^3*d*e^10))/a^4 + (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d
*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(
4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 1
6*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32
*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2
*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d
*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5
*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*
a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d
*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
+ (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2
*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 +
8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 1
08*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 8
7*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)
*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2
*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e
 + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c -
 b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c
^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d
+ 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b
*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1
/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(
6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8
 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12
*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)
^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/
2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b
^2*c)))^(1/2)*1i)/((((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^
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2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a *b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^ 4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2 *e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e ^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24 *a^6*b^3*c^3*d*e^10))/a^4 - (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3 *d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a* b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b ^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3 *d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b ^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c *d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c ^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^ 4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a ^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c *e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3 *c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2* d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3 *b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^ (1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2) *(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e ^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^ 3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^ 2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5 *b^2*c)))^(1/2) + (((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2 *e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a* b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4 *c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(3 2*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2* e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^ 9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24* a^6*b^3*c^3*d*e^10))/a^4 + (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3* d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b ^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(- (4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 3 2*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^ 2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3* d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^ 5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7 *a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c* d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^

```
2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 +
 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 -
108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 +
87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4
)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^
2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*
e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c
 - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*
c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d
 + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*
b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(
1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*
(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^
8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 1
2*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3
)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1
/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*
b^2*c)))^(1/2) + (16*(a^3*c^5*e^13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*
a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e
^12 + 2*a*b^2*c^5*d^2*e^11))/a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^
2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2
 - 8*a^5*b^2*c)))^(1/2)*2i - (d + e*x)^(1/2)/(a*x) - atan(((((8*(16*a^5*b*c^
4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4
*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5
*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*
c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*
e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10
 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11
 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5
*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5
*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (8*(d +
e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*
e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a
^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*
(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(
64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2
*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9
 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d
 + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*
d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2
*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4
*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(60*a^6*b*c^4
*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a
^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4
*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^
3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^
5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-
(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a
*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a
^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^
2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8
 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^
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3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3
*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*
d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3
*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^
(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((8*(16*a^5*b*
c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a
^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b
^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^
5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*
d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^
10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^
11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a
^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b
^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (8*(d
+ e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^
5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7
*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*
d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d
^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e
^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6
*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*
c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a
^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a
^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(60*a^6*b*c
^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40
*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b
^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*
b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*
c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*
(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2
e(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4
*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16
*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c -
b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)
^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 -
8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e
^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*
a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a
^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^
2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a
^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3
)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((((8*(16*a^5*
b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20
*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2
*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*
b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^
2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*
e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*
e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6
*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5
*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (8*(
d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*
b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) -
 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*
c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/
2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5
*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d

*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 + (8*(d + e*x)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4)*(-(8*

```
a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c
^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*
a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^
3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (16*(a^3*c^5*e^
13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4
*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11))/a^4
))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a
^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c
*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c
 - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.341 $\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)} \, dx$

Optimal. Leaf size=531

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-\right.}}}$$

Rubi [A] time = 3.57, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)-ab\left(3cd-e\sqrt{b^2-4ac}\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}}+\frac{\sqrt{d+ex}(bd-ae)}{a^2dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}}-\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -Sqrt[d + e*x]/(2*a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*d*x) + ((b*d - a*e)*Sqr
t[d + e*x])/(a^2*d*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*d^(3/2)
) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*d^(3/2)) - (2*(b^2*
d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[2]
*Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a*c
]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d - b^2
*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c
*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{de^3}{a\left(d-x^2\right)^3}+\frac{e^2(-bd+ae)}{a^2\left(d-x^2\right)^2}+\frac{e\left(-b^2d+acd+abe\right)}{a^3\left(d-x^2\right)}+\frac{e\left(\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\cdots\right.\right.}{a^3\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\cdots\right.\right.}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^3} - \frac{\left(2de^2\right)\text{Subst}\left(\int \frac{1}{(d-x\cdots}\cdots\right.}{a\cdots}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{2\left(b^2d-acd-abe\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}-\frac{\left(3e^2\right)\text{Subs}\cdots}{\cdots}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}}-\frac{2\left(b^2d\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}}-\frac{e(bd-ae)\tan\cdots}{a^2\cdots}$$

Mathematica [A] time = 2.34, size = 516, normalized size = 0.97

$$-\frac{\frac{3a^2e\left(ex\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)-\sqrt{d}\sqrt{d+ex}\right)}{d^{3/2}x}+\frac{2a^2\sqrt{d+ex}}{x^2}+\frac{8\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{\sqrt{d}}+\frac{4\sqrt{2}\sqrt{c}\left(b^2\left(ae-d\sqrt{b^2-4ac}\right)+ab\left(e\sqrt{b^2-4ac}+3cd\right)+ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{4\sqrt{2}\sqrt{c}\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)+ab\left(e\sqrt{b^2-4ac}-3cd\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{4ae(ae-bd)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{4a\sqrt{d+ex}(bd-ae)}{dx}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/4*((2*a^2*Sqrt[d + e*x])/x^2 - (4*a*(b*d - a*e)*Sqrt[d + e*x])/(d*x) - (
4*a*e*(-(b*d) + a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(3/2) + (8*(b^2*d -
a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (3*a^2*e*(-(Sqrt[d
]*Sqrt[d + e*x]) + e*x*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]))/(d^(3/2)*x) + (4*Sq
rt[2]*Sqrt[c]*(-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b
^2 - 4*a*c]*d) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (4*Sqrt[2]*Sqrt[c]*(b
^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)
+ a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]))/a^3
```

IntegrateAlgebraic [A] time = 2.46, size = 577, normalized size = 1.09

$$-\frac{\sqrt{d+ex}\left(ae(d+ex)+ade+4bd^2-4bd(d+ex)\right)}{4a^2dex^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(a^2e^2+4abde+8acd^2-8b^2d^2\right)}{4a^3d^{3/2}}+\frac{\left(-2\sqrt{2}a^2c^{3/2}e+\sqrt{2}ac^{3/2}d\sqrt{b^2-4ac}-\sqrt{2}b^2\sqrt{c}d\sqrt{b^2-4ac}+\sqrt{2}ab^2\sqrt{c}e+\sqrt{2}ab\sqrt{c}e\sqrt{b^2-4ac}+3\sqrt{2}abc^{3/2}d-\sqrt{2}b^3\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{a^3\sqrt{b^2-4ac}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}+\frac{\left(2\sqrt{2}a^2c^{3/2}e+\sqrt{2}ac^{3/2}d\sqrt{b^2-4ac}-\sqrt{2}b^2\sqrt{c}d\sqrt{b^2-4ac}-\sqrt{2}ab^2\sqrt{c}e+\sqrt{2}ab\sqrt{c}e\sqrt{b^2-4ac}-3\sqrt{2}abc^{3/2}d+\sqrt{2}b^3\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right)}{a^3\sqrt{b^2-4ac}\sqrt{e\sqrt{b^2-4ac}+be-2cd}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/4*(Sqrt[d + e*x]*(4*b*d^2 + a*d*e - 4*b*d*(d + e*x) + a*e*(d + e*x)))/(a
^2*d*e*x^2) + ((-(Sqrt[2]*b^3*Sqrt[c]*d) + 3*Sqrt[2]*a*b*c^(3/2)*d - Sqrt[2
]*b^2*Sqrt[c]*Sqrt[b^2 - 4*a*c]*d + Sqrt[2]*a*c^(3/2)*Sqrt[b^2 - 4*a*c]*d +
 Sqrt[2]*a*b^2*Sqrt[c]*e - 2*Sqrt[2]*a^2*c^(3/2)*e + Sqrt[2]*a*b*Sqrt[c]*Sq
rt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e
 - Sqrt[b^2 - 4*a*c]*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b
^2 - 4*a*c]*e]) + ((Sqrt[2]*b^3*Sqrt[c]*d - 3*Sqrt[2]*a*b*c^(3/2)*d - Sqrt[
2]*b^2*Sqrt[c]*Sqrt[b^2 - 4*a*c]*d + Sqrt[2]*a*c^(3/2)*Sqrt[b^2 - 4*a*c]*d
- Sqrt[2]*a*b^2*Sqrt[c]*e + 2*Sqrt[2]*a^2*c^(3/2)*e + Sqrt[2]*a*b*Sqrt[c]*S
qrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*
e + Sqrt[b^2 - 4*a*c]*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e + Sqrt[
b^2 - 4*a*c]*e]) + ((-8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e + a^2*e^2)*ArcTanh[
Sqrt[d + e*x]/Sqrt[d]])/(4*a^3*d^(3/2))
```

fricas [B] time = 149.30, size = 7425, normalized size = 13.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*
c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2 - 4*a^7*c)*sqrt((
(b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 -
2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e
 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/
(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c))*log(sqrt(2)*((b^9 - 9*a*b^7*c
+ 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 - 8*a^2*b^6*c
+ 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e - (a^6*b^5 - 7*a^7*b^3*c +
 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 +
9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c
^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2
*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))*sqrt(((b^6 - 6*a*b^4*c + 9*a^2
*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2
- 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^
4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 +
 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3
 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c)) - 4*((b^5*c^3
 - 4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^2*c^4 + a^3*c^5)*e)*
```

```
sqrt(e*x + d)) - 4*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c
^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2 - 4*a^
7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*
c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5
*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6
*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c))*log(-sqrt(2)*((b^9
- 9*a*b^7*c + 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 -
8*a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e - (a^6*b^5 - 7
*a^7*b^3*c + 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^
3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 -
13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2
 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))*sqrt(((b^6 - 6*a*b
^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e
 + (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^
4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a
^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6
*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c)) -
 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^2*c^4 +
a^3*c^5)*e)*sqrt(e*x + d)) + 4*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a*b^4*c +
 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e - (a^
6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3
+ 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3
*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b
^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c))*log(sqr
t(2)*((b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d
- (a*b^8 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e + (
a^6*b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*
c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3
*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*
a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))*sqrt(((
b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a
^3*b*c^2)*e - (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2
- 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5
*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*
b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 -
4*a^7*c)) - 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2
*b^2*c^4 + a^3*c^5)*e)*sqrt(e*x + d)) - 4*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 -
6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c
^2)*e - (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a
^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 -
 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^
2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*
c))*log(-sqrt(2)*((b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a
^4*b*c^4)*d - (a*b^8 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^
5*c^4)*e + (a^6*b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c +
 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^
7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3
*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*
c)))*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2
*b^3*c + 5*a^3*b*c^2)*e - (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*
a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c
+ 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6
*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))
/(a^6*b^2 - 4*a^7*c)) - 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4
*c^3 - 3*a^2*b^2*c^4 + a^3*c^5)*e)*sqrt(e*x + d)) + (4*a*b*d*e + a^2*e^2 -
8*(b^2 - a*c)*d^2)*sqrt(d)*x^2*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x)
 - 2*(2*a^2*d^2 - (4*a*b*d^2 - a^2*d*e)*x)*sqrt(e*x + d))/(a^3*d^2*x^2), 1/
4*(2*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3
)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2 - 4*a^7*c)*sqrt(((b^
```

10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(
a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e +
(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^
12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c))*log(sqrt(2)*((b^9 - 9*a*b^7*c + 2
7*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 - 8*a^2*b^6*c + 2
0*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e - (a^6*b^5 - 7*a^7*b^3*c + 12
*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a
^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3
+ 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^
3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^
2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2 - 4
*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b
^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*
a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 +
a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c)) - 4*((b^5*c^3 -
4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^2*c^4 + a^3*c^5)*e)*sqr
t(e*x + d)) - 2*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2
- 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2 - 4*a^7*c
)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4
)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*
c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^
4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c))*log(-sqrt(2)*((b^9 - 9
*a*b^7*c + 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 - 8*a
^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e - (a^6*b^5 - 7*a^
7*b^3*c + 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b
^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*
a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 -
6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))*sqrt(((b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e +
(a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c
^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*
b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^
5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c)) - 4*
((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^2*c^4 + a^3
*c^5)*e)*sqrt(e*x + d)) + 2*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a*b^4*c + 9*
a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e - (a^6*b
^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9
*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^
3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*
c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c))*log(sqrt(2
)*((b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (
a*b^8 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e + (a^6
*b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2
 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^
5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4
*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))*sqrt(((b^6
 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*
b*c^2)*e - (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 2
4*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^
2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4
*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a
^7*c)) - 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^
2*c^4 + a^3*c^5)*e)*sqrt(e*x + d)) - 2*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a
*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)
*e - (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*
b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13
*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 -
 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c))
log(-sqrt(2)((b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*

```
b*c^4)*d - (a*b^8 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c
^4)*e + (a^6*b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c + 22
*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c
 + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^
6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c))
)*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^
3*c + 5*a^3*b*c^2)*e - (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2
*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 1
6*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c
+ 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a
^6*b^2 - 4*a^7*c)) - 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^
3 - 3*a^2*b^2*c^4 + a^3*c^5)*e)*sqrt(e*x + d)) - (4*a*b*d*e + a^2*e^2 - 8*(
b^2 - a*c)*d^2)*sqrt(-d)*x^2*arctan(sqrt(e*x + d)*sqrt(-d)/d) - (2*a^2*d^2
- (4*a*b*d^2 - a^2*d*e)*x)*sqrt(e*x + d))/(a^3*d^2*x^2)]
```

giac [B] time = 0.59, size = 1041, normalized size = 1.96

$$-\frac{\left(\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\left((b^4-5ab^2c+4a^2c^2)de-(ab^3-4a^2bc)e^2\right)a^2-2\left((ab^2c-a^2c^2)\sqrt{b^2-4ac}\,d^2-(ab^3-a^2bc)\sqrt{b^2-4ac}\,de+(a^2b^2-a^3c)\sqrt{b^2-4ac}\,e^2\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,|a|-\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\left(2(a^2b^3c-3a^3bc^2)d^2-(a^2b^4-a^3b^2c-4a^4c^2)de+(a^3b^3-2a^4bc)e^2\right)\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{ex+d}}{\sqrt{-\frac{2a^3cd-a^3be+\sqrt{-4(a^3cd^2-a^3bde+a^4e^2)a^3c+(2a^3cd-a^3be)^2}}{a^3c}}}\right)}{4\left(\sqrt{b^2-4ac}\,a^4cd^2-\sqrt{b^2-4ac}\,a^4bde+\sqrt{b^2-4ac}\,a^5e^2\right)|a||c|}+\frac{\left(\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\left((b^4-5ab^2c+4a^2c^2)de-(ab^3-4a^2bc)e^2\right)a^2+2\left((ab^2c-a^2c^2)\sqrt{b^2-4ac}\,d^2-(ab^3-a^2bc)\sqrt{b^2-4ac}\,de+(a^2b^2-a^3c)\sqrt{b^2-4ac}\,e^2\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,|a|-\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\left(2(a^2b^3c-3a^3bc^2)d^2-(a^2b^4-a^3b^2c-4a^4c^2)de+(a^3b^3-2a^4bc)e^2\right)\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{ex+d}}{\sqrt{-\frac{2a^3cd-a^3be-\sqrt{-4(a^3cd^2-a^3bde+a^4e^2)a^3c+(2a^3cd-a^3be)^2}}{a^3c}}}\right)}{4\left(\sqrt{b^2-4ac}\,a^4cd^2-\sqrt{b^2-4ac}\,a^4bde+\sqrt{b^2-4ac}\,a^5e^2\right)|a||c|}+\frac{(8b^2d^2-8acd^2-4abde-a^2e^2)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{4a^3\sqrt{-d}\,d}+\frac{4(ex+d)^{\frac{3}{2}}bde-4\sqrt{ex+d}\,bd^2e-(ex+d)^{\frac{3}{2}}ae^2-\sqrt{ex+d}\,ade^2}{4a^2e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*a^2 - 2*((a*b^2*c - a^2*c^2)*sqrt
(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^
3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*abs(a) - sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c -
 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^
4*b*c)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e +
sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))
/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sq
rt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqr
t(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*
c)*e^2)*a^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b
*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-
4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3
*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^4*b*c)*e^2))*arctan(2*sqrt(1/2)*sq
rt(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a
^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4
*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*ab
s(c)) + 1/4*(8*b^2*d^2 - 8*a*c*d^2 - 4*a*b*d*e - a^2*e^2)*arctan(sqrt(x*e +
 d)/sqrt(-d))/(a^3*sqrt(-d)*d) + 1/4*(4*(x*e + d)^(3/2)*b*d*e - 4*sqrt(x*e
+ d)*b*d^2*e - (x*e + d)^(3/2)*a*e^2 - sqrt(x*e + d)*a*d*e^2)*e^(-2)/(a^2*d
*x^2)
```

maple [B] time = 0.05, size = 1486, normalized size = 2.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x)
```

```
[Out] -1/4/a/x^2/d*(e*x+d)^(3/2)+1/e/a^2/x^2*(e*x+d)^(3/2)*b-1/e/a^2/x^2*(e*x+d)^
(1/2)*b*d-1/4*(e*x+d)^(1/2)/a/x^2+1/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/
d^(3/2)+e/a^2/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*b+2/a^2*d^(1/2)*arctan
h((e*x+d)^(1/2)/d^(1/2))*c-2/a^3*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*b^2
+2*e^2/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)-e^2/a^2*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+
```

```
2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b
*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2-3*e/a^2*c^2/(-(4*a*c-b^2
)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan
h((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*
b*d+e/a^3*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b^3*d-e/a^2*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*b-1/a^2*c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*d+1/a^3*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*b^2*d+2*e^2/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b
*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((
b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)-e^2/a^2*c/(-(4*a*c-b^2)*e^2
)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+
d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2-3*e/
a^2*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1
/2))*c)^(1/2)*c)*b*d+e/a^3*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d+e/a^2*c*2^(1/2)/((b*e-2*c*d+(-(
4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-
(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b+1/a^2*c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*
c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d-1/a^3*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^3), x)
```

mupad [B] time = 8.09, size = 33838, normalized size = 63.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x^3*(a + b*x + c*x^2)),x)
```

```
[Out] atan(((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^
10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*
e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*
d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*
b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^1
1*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*(
(b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^
4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4
+ 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*
d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b
^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*
```

a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*
a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 3
8*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c
^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^
2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*
c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 5
12*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1
152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^
10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3
*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b
^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 451
2*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 +
896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 -
102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*
c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*
b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(
4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e
*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)
+ (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*
e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 12
8*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 +
960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^
10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d
^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*
c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a
^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12
+ 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d
^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e
^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e
^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-
(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25
*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*
e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(
-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^
2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(
1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^
6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 70
4*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10
- 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*
e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6
*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^
6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^
5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*
d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2
*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/
2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a
^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*
a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6
*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3
*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*
c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^1
0*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8
*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2
*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^
3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e

+ a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b
^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c
 - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^1
2*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^1
0*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 6
4*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e
^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) -
a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2
) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) -
 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^
(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(
8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e
^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8
 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^
4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*
c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^
7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 +
568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2
*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^1
2 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d
 + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*
d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^
4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a
^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*
b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*
e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^
9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^
5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c
^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*
b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 140
8*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^
11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^
5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*
b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3
*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((
b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4
*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*
a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 +
 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2
*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*
e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e
^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*
d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*
c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*
b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*
c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^
5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1
/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^
3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3
)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^
2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((((((128*
a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^

```
4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*
b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a
^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^1
1 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^1
1 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*((b^8*d + 8*a^4*
c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^
3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*
b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e
 + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 -
8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*
a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8
+ 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^
2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5
*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d
 - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a
*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*
c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a
^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c
)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*
b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*
e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4
*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b
^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*
a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9
+ 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d
^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*
d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4
*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/
2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3
*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)
^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2
)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^
14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*
c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d
^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^
5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9
*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a
^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 +
1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*
e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*
c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a
^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*
b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b
^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3
)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e
+ a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^
2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^
3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c
- b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*
x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*
c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d
^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c
^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b
^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*
a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*
a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5
*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d
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- b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*

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d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c
^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*
b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128
*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 -
 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*
a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*
d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33
*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^
5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2
*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a
^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7*d*e^14 + 56*a^3*c^9
*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 +
 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 112*a^3*b^2*c^7*d^3*e
^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^10
- 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*b*c^7*d^2*e^13)/(a^
8*d^2)))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e +
 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2
*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*
c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2
*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i - (((a*e^2 + 4*b*d*e)*(d +
 e*x)^(1/2))/(4*a^2) + ((a*e^2 - 4*b*d*e)*(d + e*x)^(3/2))/(4*a^2*d))/((d +
 e*x)^2 - 2*d*(d + e*x) + d^2) + atan((((((128*a^12*c^4*d*e^12 + 768*a^10*
c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*
c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b
^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*
a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9
+ 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)
/(2*a^8*d^2) - ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^
2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^
2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c
- b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b
^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*
a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*
a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*
a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9
+ 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^
3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2)
 - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(
1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2
) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^
3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2
)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^
3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*
e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3
*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b
^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096
*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10
 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*
d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*
e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^
8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c
^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20
*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*
c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 1
6*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a
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^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d
^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5
*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4
*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^
4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a
^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 +
1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3
*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3
*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a
^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*
c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))
*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*
b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e
 + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-
(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^
4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14
- 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d
^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^
5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c
^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b
^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a
^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b
^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3
*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)
^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e -
 a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2
)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3
)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c -
 b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((1
28*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8
*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a
^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 2
4*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*
e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*
e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a
^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38
*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10
*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^
3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2
 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 1
28*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e
^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3
*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d +
b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^
3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d
- a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b
^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +
3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^
2*c)))^(1/2) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a
^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d
^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*
c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^
6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 -
64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e
^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^
5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c
^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*

c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b

^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c

^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 112*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*b*c^7*d^2*e^13)/(a^8*d^2)))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1

/2)*2i + (atan((((((4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(8*a^3*(d^3)^(1/2)) - (((((4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 +

384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) + ((d + e*x)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(8*a^3*(d^3)^(1/2)))/((7*a^5*c^7*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 112*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*b*c^7*d^2*e^13)/(a^8*d^2) + (((((4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b^3*c^5*d^4*e^9 + 4944*a

^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 4512*a^8*b^2*c^5*d^3*e^10
- 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 + 896*a^8*b*c^6*d^4*e^9
+ 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 - 102*a^9*b^2*c^4*d*e^12)
)/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^
(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2))
- ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8
 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2
*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512
*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10
- 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*
e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e
^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9
 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*(a^2*e^2 -
8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1/2)) + (((((4*a^9*c^5*e^
14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*
c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d
^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^
5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9
*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a
^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 +
1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*
e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*
c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a
^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*
b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b
^3*c^4*d*e^13)/(2*a^8*d^2) + (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^
8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^
9 - 32*a^8*b^6*c^2*d^3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4
*e^9 + 392*a^9*b^4*c^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c
^4*d^3*e^10 - 192*a^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b
^4*c^2*d*e^12 + 384*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^
2) + ((d + e*x)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(1536*a
^12*c^5*d^4*e^8 + 1024*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a
^10*b^5*c^2*d^3*e^9 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 +
 64*a^11*b^4*c^2*d^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3
*e^9))/(16*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*
d*e))/(8*a^3*(d^3)^(1/2)) - ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b
*c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 +
512*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 -
1152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e
^10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^
3*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*
b^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 45
12*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12
+ 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 -
 102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4
*a*b*d*e))/(8*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*
e))/(8*a^3*(d^3)^(1/2)) + ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^
14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b
^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^
2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 7
52*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^1
0 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2
*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9
 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/
(2*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(8*a^3*(d^3)^(1
/2))))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(4*a^3*(d^3)^(1/2)
)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x**3/(c*x**2+b*x+a),x)
[Out] Timed out
```

3.342 $\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=650

$$-\frac{2\sqrt{d+ex}\left(-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)}{c^5}+\frac{\sqrt{2}\left(\frac{10a^2bc^3de-2a^2c^3\left(cd^2-ae^2\right)-b^4c\left(cd^2-6ae^2\right)-10ab^3c^2de+ab^2c^2\left(4cd\right.}{\sqrt{b^2-4ac}}\right.}{\ldots}$$

Rubi [A] time = 2.68, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(\frac{10a^2bc^3de-2a^2c^3\left(cd^2-ae^2\right)+ab^2c^2\left(4cd^2-9ae^2\right)-10ab^3c^2de-b^4c\left(cd^2-6ae^2\right)+2b^5cde+b^6\left(-e^2\right)}{\sqrt{b^2-4ac}}+\left(ace+b^2(-e)+bcd\right)\left(3abce-2ac^2d+b^2cd+b^3(-e)\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{11/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\left(ace+b^2(-e)+bcd\right)\left(3abce-2ac^2d+b^2cd+b^3(-e)\right)-\frac{10a^2bc^3de-2a^2c^3\left(cd^2-ae^2\right)+ab^2c^2\left(4cd^2-9ae^2\right)-10ab^3c^2de-b^4c\left(cd^2-6ae^2\right)+2b^5cde+b^6\left(-e^2\right)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{11/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{d+ex}\left(-a^2c^2e+3ab^2ce-2abc^2d+b^3cd+b^4(-e)\right)}{c^5}+\frac{2(d+ex)^{5/2}\left(ce(bd-ae)+b^2e^2+c^2d^2\right)}{5c^3e^3}-\frac{2b\left(b^2-2ac\right)(d+ex)^{3/2}}{3c^4}-\frac{2(d+ex)^{7/2}(be+2cd)}{7c^2e^3}+\frac{2(d+ex)^{9/2}}{9ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*Sqrt[d + e*x]
)/c^5 - (2*b*(b^2 - 2*a*c)*(d + e*x)^(3/2))/(3*c^4) + (2*(c^2*d^2 + b^2*e^2
 + c*e*(b*d - a*e))*(d + e*x)^(5/2))/(5*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*
x)^(7/2))/(7*c^2*e^3) + (2*(d + e*x)^(9/2))/(9*c*e^3) + (Sqrt[2]*((b*c*d -
b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) + (2*b^5*c*d*e - 1
0*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2
) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]])/(c^(11/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((b*c
*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) - (2*b^5*c*d*
e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*
a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(11/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^4}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(-\frac{e\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)}{c^5}-\frac{b\left(b^2-2ac\right)ex^2}{c^4}+\frac{\left(c^2d^2+b^2e^2+ce(bd-ae)\right)x^4}{c^3e^2}-\frac{(2cd+be)x^6}{c^2e^2}\right.\right.}{}$$

$$= -\frac{2\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)\sqrt{d+ex}}{c^5}-\frac{2b\left(b^2-2ac\right)(d+ex)^{3/2}}{3c^4}+\frac{2\left(c^2d^2\right.}{}$$

$$= -\frac{2\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)\sqrt{d+ex}}{c^5}-\frac{2b\left(b^2-2ac\right)(d+ex)^{3/2}}{3c^4}+\frac{2\left(c^2d^2\right.}{}$$

$$= -\frac{2\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)\sqrt{d+ex}}{c^5}-\frac{2b\left(b^2-2ac\right)(d+ex)^{3/2}}{3c^4}+\frac{2\left(c^2d^2\right.}{}$$

Mathematica [A] time = 1.12, size = 808, normalized size = 1.24

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(315*b^4*e^4 - 105*b^2*c*e^3*(4*b*d + 9*a*e + b*e*x) - 9*c
^3*e*(d + e*x)^2*(-2*b*d + 7*a*e + 5*b*e*x) + c^4*(d + e*x)^2*(8*d^2 - 20*d
*e*x + 35*e^2*x^2) + 21*c^2*e^2*(15*a^2*e^2 + 3*b^2*(d + e*x)^2 + 10*a*b*e*
(4*d + e*x))))/(315*c^5*e^3) + (Sqrt[2]*(-(b^6*e^2) + b^5*e*(2*c*d + Sqrt[b
^2 - 4*a*c]*e) + a*b^2*c^2*(4*c*d^2 + 6*Sqrt[b^2 - 4*a*c]*d*e - 9*a*e^2) +
b^3*c*(-4*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 10*a*e)) + a
*b*c^2*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - 2*c*d*(Sqrt[b^2 - 4*a*c]*d - 5*a*e)) -
b^4*c*(c*d^2 + 2*e*(Sqrt[b^2 - 4*a*c]*d - 3*a*e)) + 2*a^2*c^3*(-(c*d^2) + e
*(-(Sqrt[b^2 - 4*a*c]*d) + a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(11/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^6*e^2 + b^5*e*(-2*c*d + S
qrt[b^2 - 4*a*c]*e) + a*b^2*c^2*(-4*c*d^2 + 6*Sqrt[b^2 - 4*a*c]*d*e + 9*a*e
^2) - 2*a^2*c^3*(-(c*d^2) + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b^4*c*(c*d^2 -
 2*e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + a*b*c^2*(3*a*Sqrt[b^2 - 4*a*c]*e^2 -
2*c*d*(Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + b^3*c*(-4*a*Sqrt[b^2 - 4*a*c]*e^2 +
c*d*(Sqrt[b^2 - 4*a*c]*d + 10*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(11/2)*Sqrt[b^2 - 4*a*c]*Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

IntegrateAlgebraic [C] time = 3.48, size = 1229, normalized size = 1.89

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-315*b^3*c*d*e^3 + 630*a*b*c^2*d*e^3 + 315*b^4*e^4 - 945*
a*b^2*c*e^4 + 315*a^2*c^2*e^4 - 105*b^3*c*e^3*(d + e*x) + 210*a*b*c^2*e^3*(
d + e*x) + 63*c^4*d^2*(d + e*x)^2 + 63*b*c^3*d*e*(d + e*x)^2 + 63*b^2*c^2*e
^2*(d + e*x)^2 - 63*a*c^3*e^2*(d + e*x)^2 - 90*c^4*d*(d + e*x)^3 - 45*b*c^3
*e*(d + e*x)^3 + 35*c^4*(d + e*x)^4))/(315*c^5*e^3) + (((-I)*Sqrt[2]*b^4*c^
2*d^2 + (4*I)*Sqrt[2]*a*b^2*c^3*d^2 - (2*I)*Sqrt[2]*a^2*c^4*d^2 - Sqrt[2]*b
^3*c^2*Sqrt[-b^2 + 4*a*c]*d^2 + 2*Sqrt[2]*a*b*c^3*Sqrt[-b^2 + 4*a*c]*d^2 +
(2*I)*Sqrt[2]*b^5*c*d*e - (10*I)*Sqrt[2]*a*b^3*c^2*d*e + (10*I)*Sqrt[2]*a^2
*b*c^3*d*e + 2*Sqrt[2]*b^4*c*Sqrt[-b^2 + 4*a*c]*d*e - 6*Sqrt[2]*a*b^2*c^2*S
qrt[-b^2 + 4*a*c]*d*e + 2*Sqrt[2]*a^2*c^3*Sqrt[-b^2 + 4*a*c]*d*e - I*Sqrt[2
]*b^6*e^2 + (6*I)*Sqrt[2]*a*b^4*c*e^2 - (9*I)*Sqrt[2]*a^2*b^2*c^2*e^2 + (2*
I)*Sqrt[2]*a^3*c^3*e^2 - Sqrt[2]*b^5*Sqrt[-b^2 + 4*a*c]*e^2 + 4*Sqrt[2]*a*b
^3*c*Sqrt[-b^2 + 4*a*c]*e^2 - 3*Sqrt[2]*a^2*b*c^2*Sqrt[-b^2 + 4*a*c]*e^2)*A
rcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a
*c]*e]])/(c^(11/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a
*c]*e]) + ((I*Sqrt[2]*b^4*c^2*d^2 - (4*I)*Sqrt[2]*a*b^2*c^3*d^2 + (2*I)*Sqr
t[2]*a^2*c^4*d^2 - Sqrt[2]*b^3*c^2*Sqrt[-b^2 + 4*a*c]*d^2 + 2*Sqrt[2]*a*b*c
^3*Sqrt[-b^2 + 4*a*c]*d^2 - (2*I)*Sqrt[2]*b^5*c*d*e + (10*I)*Sqrt[2]*a*b^3*
c^2*d*e - (10*I)*Sqrt[2]*a^2*b*c^3*d*e + 2*Sqrt[2]*b^4*c*Sqrt[-b^2 + 4*a*c]
*d*e - 6*Sqrt[2]*a*b^2*c^2*Sqrt[-b^2 + 4*a*c]*d*e + 2*Sqrt[2]*a^2*c^3*Sqrt[
-b^2 + 4*a*c]*d*e + I*Sqrt[2]*b^6*e^2 - (6*I)*Sqrt[2]*a*b^4*c*e^2 + (9*I)*S
qrt[2]*a^2*b^2*c^2*e^2 - (2*I)*Sqrt[2]*a^3*c^3*e^2 - Sqrt[2]*b^5*Sqrt[-b^2
+ 4*a*c]*e^2 + 4*Sqrt[2]*a*b^3*c*Sqrt[-b^2 + 4*a*c]*e^2 - 3*Sqrt[2]*a^2*b*c
^2*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c
*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(11/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c
*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])
```

fricas [B] time = 11.22, size = 14340, normalized size = 22.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/630*(315*sqrt(2)*c^5*e^3*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 -
 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^
4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2
*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*
a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*
e^3 + (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c
^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12
)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 2
39*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e +
 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 183
5*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*
a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 29
70*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 -
700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 +
 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^
7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^
2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1
068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 +
95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c
```

^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12))*log(sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5 + 54*a^2*b^8*c^6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4 - (4*b^13*c^3 - 52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^12*c^3 + 154*a^2*b^10*c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a^6*b^2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360*a^2*b^11*c^3 - 1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 561*a^6*b^3*c^7 - 68*a^7*b*c^8)*d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^2 - 352*a^3*b^10*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8)*e^4 - ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*c^14 - 8*a^3*c^15)*d - (b^7*c^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*b*c^14)*e)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b^3*c^6 - 4*a^7*b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2*c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c - 15*a^5*b^8*c^2 + 35*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^11 - 6*a^5*b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e^4 - (a^5*b^10 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 15*a^9*b^2*c^4 - a^10*c^5)*e^5)*sqrt(e*x + d)) - 315*sqrt(2)*c^5*e^3*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6

+ 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12))*log(-sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5 + 54*a^2*b^8*c^6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4 - (4*b^13*c^3 - 52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^12*c^3 + 154*a^2*b^10*c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a^6*b^2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360*a^2*b^11*c^3 - 1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 561*a^6*b^3*c^7 - 68*a^7*b*c^8)*d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^2 - 352*a^3*b^10*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8)*e^4 - ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*c^14 - 8*a^3*c^15)*d - (b^7*c^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*b*c^14)*e)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 +

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5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 +
 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4
- 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c
^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c
- 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 -
 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5
*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*
c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b
^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a
*c^23)))/(b^2*c^11 - 4*a*c^12)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*
b^3*c^6 - 4*a^7*b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c
^5 - 34*a^7*b^2*c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3
+ 51*a^6*b^5*c^4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c
- 15*a^5*b^8*c^2 + 35*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^
4*b^11 - 6*a^5*b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14
*a^9*b*c^5)*d*e^4 - (a^5*b^10 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c
^3 + 15*a^9*b^2*c^4 - a^10*c^5)*e^5)*sqrt(e*x + d)) + 315*sqrt(2)*c^5*e^3*s
qrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*
d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*
c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 2
5*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77
*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*s
qrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^
4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^
13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*
c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^
5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*
c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^1
7*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^
9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c
^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*
b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a
^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17
*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b
^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10
)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*
a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 29
5*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b
^2*c^11 - 4*a*c^12))*log(sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5 + 54*a^2*b^8*c^
6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4 - (4*b^13*c^3 -
 52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4*b^5*c^7 - 350*
a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^12*c^3 + 154*a^2
*b^10*c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a^6*b^
2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360*a^2*b^11*c^3 -
 1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 561*a^6*b^3*c^7 -
 68*a^7*b*c^8)*d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^2 - 352*a^3*b^1
0*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6 - 73*a^7*b^2*c^
7 + 4*a^8*c^8)*e^4 + ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*c^14 - 8*a^3*c^
15)*d - (b^7*c^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*b*c^14)*e)*sqrt
(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b
^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*
c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^1
0 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 +
 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9
 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c
^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c
^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11
)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^1
2*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*
```

b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^
2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*
c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d
*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4
*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a
^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))*sqrt(
((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3
- 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)
*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^
4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3
*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*sqrt(
((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^
6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c
^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10
 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 +
395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9
+ 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^
3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^
7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)
*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12
*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b
^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2
 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c
^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*
e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*
b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^
8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c
^11 - 4*a*c^12)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b^3*c^6 - 4*a^7
*b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2
*c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^
4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c - 15*a^5*b^8*c^
2 + 35*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^11 - 6*a^5*
b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e
^4 - (a^5*b^10 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 15*a^9*b^2
*c^4 - a^10*c^5)*e^5)*sqrt(e*x + d)) - 315*sqrt(2)*c^5*e^3*sqrt(((b^8*c^3 -
 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^
2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*
(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 -
 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 +
55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6
- 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80
*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2
*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b
^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^1
2*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b
^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b
^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^
5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 +
3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855
*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 12
5*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*
b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a
^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20
 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 -
2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 -
 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c
^12))*log(-sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5 + 54*a^2*b^8*c^6 - 112*a^3*b^
6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4 - (4*b^13*c^3 - 52*a*b^11*c^4
 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 +

```
40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^12*c^3 + 154*a^2*b^10*c^4 - 42
0*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a^6*b^2*c^8 - 4*a^7*
c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360*a^2*b^11*c^3 - 1100*a^3*b^9*
c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 561*a^6*b^3*c^7 - 68*a^7*b*c^8)
*d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^2 - 352*a^3*b^10*c^3 + 660*a^
4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8)
*e^4 + ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*c^14 - 8*a^3*c^15)*d - (b^7*c
^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*b*c^14)*e)*sqrt(((b^14*c^6 -
12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a
^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b
^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3
*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*
c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4
*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^1
5*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*
b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*
(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a
^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*
a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^
15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6
*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 -
 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 22
32*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 3
0*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))*sqrt(((b^8*c^3 - 8*
a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 -
 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^
10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*
a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*
a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 1
2*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^
5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^
11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*
c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c
^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*
c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15
*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b
^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(
5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^
4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a
^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^1
5*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*
b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 -
18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 223
2*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30
*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12
)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b^3*c^6 - 4*a^7*b*c^7)*d^5 -
(4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2*c^6 + 3*a^8*c
^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^4 - 40*a^7*b^3
*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c - 15*a^5*b^8*c^2 + 35*a^6*b^6
*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^11 - 6*a^5*b^9*c + 4*a^6*
b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e^4 - (a^5*b^10
 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 15*a^9*b^2*c^4 - a^10*c^
5)*e^5)*sqrt(e*x + d)) - 4*(35*c^4*e^4*x^4 + 8*c^4*d^4 + 18*b*c^3*d^3*e + 6
3*(b^2*c^2 - a*c^3)*d^2*e^2 - 420*(b^3*c - 2*a*b*c^2)*d*e^3 + 315*(b^4 - 3*
a*b^2*c + a^2*c^2)*e^4 + 5*(10*c^4*d*e^3 - 9*b*c^3*e^4)*x^3 + 3*(c^4*d^2*e^
2 - 24*b*c^3*d*e^3 + 21*(b^2*c^2 - a*c^3)*e^4)*x^2 - (4*c^4*d^3*e + 9*b*c^3
*d^2*e^2 - 126*(b^2*c^2 - a*c^3)*d*e^3 + 105*(b^3*c - 2*a*b*c^2)*e^4)*x)*sq
rt(e*x + d))/(c^5*e^3)
```

giac [B] time = 0.69, size = 1577, normalized size = 2.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^2*e - 2*(b^6*c - 7*a*b^4*c^2
 + 13*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^2 + (b^7 - 8*a*b^5*c + 19*a^2*b^3*c^2 -
12*a^3*b*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2 - 2
*((b^3*c^4 - 2*a*b*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*a*b^2*c^4 +
a^2*c^5)*sqrt(b^2 - 4*a*c)*d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2*b*c^4)*sqrt
(b^2 - 4*a*c)*d*e^2 - (a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*sqrt(b^2 - 4*a*
c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*(b^4*c
^5 - 4*a*b^2*c^6 + 2*a^2*c^7)*d^3 - (5*b^5*c^4 - 24*a*b^3*c^5 + 22*a^2*b*c^
6)*d^2*e + 2*(2*b^6*c^3 - 11*a*b^4*c^4 + 14*a^2*b^2*c^5 - 2*a^3*c^6)*d*e^2
- (b^7*c^2 - 6*a*b^5*c^3 + 9*a^2*b^3*c^4 - 2*a^3*b*c^5)*e^3)*sqrt(-4*c^2*d
+ 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-
(2*c^10*d*e^30 - b*c^9*e^31 + sqrt(-4*(c^10*d^2*e^30 - b*c^9*d*e^31 + a*c^9
*e^32)*c^10*e^30 + (2*c^10*d*e^30 - b*c^9*e^31)^2))*e^(-30)/c^10))/((sqrt(b
^2 - 4*a*c)*c^8*d^2 - sqrt(b^2 - 4*a*c)*b*c^7*d*e + sqrt(b^2 - 4*a*c)*a*c^7
*e^2)*c^2) + 1/4*(((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^2*e - 2*(b^6*c -
 7*a*b^4*c^2 + 13*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^2 + (b^7 - 8*a*b^5*c + 19*a^
2*b^3*c^2 - 12*a^3*b*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c
)*e)*c^2 + 2*((b^3*c^4 - 2*a*b*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*
a*b^2*c^4 + a^2*c^5)*sqrt(b^2 - 4*a*c)*d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2
*b*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*sqr
t(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)
 + (2*(b^4*c^5 - 4*a*b^2*c^6 + 2*a^2*c^7)*d^3 - (5*b^5*c^4 - 24*a*b^3*c^5 +
 22*a^2*b*c^6)*d^2*e + 2*(2*b^6*c^3 - 11*a*b^4*c^4 + 14*a^2*b^2*c^5 - 2*a^3
*c^6)*d*e^2 - (b^7*c^2 - 6*a*b^5*c^3 + 9*a^2*b^3*c^4 - 2*a^3*b*c^5)*e^3)*sq
rt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e
 + d)/sqrt(-(2*c^10*d*e^30 - b*c^9*e^31 - sqrt(-4*(c^10*d^2*e^30 - b*c^9*d*
e^31 + a*c^9*e^32)*c^10*e^30 + (2*c^10*d*e^30 - b*c^9*e^31)^2))*e^(-30)/c^1
0))/((sqrt(b^2 - 4*a*c)*c^8*d^2 - sqrt(b^2 - 4*a*c)*b*c^7*d*e + sqrt(b^2 -
4*a*c)*a*c^7*e^2)*c^2) + 2/315*(35*(x*e + d)^(9/2)*c^8*e^24 - 90*(x*e + d)^
(7/2)*c^8*d*e^24 + 63*(x*e + d)^(5/2)*c^8*d^2*e^24 - 45*(x*e + d)^(7/2)*b*c
^7*e^25 + 63*(x*e + d)^(5/2)*b*c^7*d*e^25 + 63*(x*e + d)^(5/2)*b^2*c^6*e^26
 - 63*(x*e + d)^(5/2)*a*c^7*e^26 - 105*(x*e + d)^(3/2)*b^3*c^5*e^27 + 210*(
x*e + d)^(3/2)*a*b*c^6*e^27 - 315*sqrt(x*e + d)*b^3*c^5*d*e^27 + 630*sqrt(x
*e + d)*a*b*c^6*d*e^27 + 315*sqrt(x*e + d)*b^4*c^4*e^28 - 945*sqrt(x*e + d)
*a*b^2*c^5*e^28 + 315*sqrt(x*e + d)*a^2*c^6*e^28)*e^(-27)/c^9
```

maple [B] time = 0.08, size = 3685, normalized size = 5.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/9*(e*x+d)^(9/2)/c/e^3+2*e/c^3*a^2*(e*x+d)^(1/2)+2*e/c^5*b^4*(e*x+d)^(1/2)
-4/7/e^3/c*(e*x+d)^(7/2)*d+2/5/e^3/c*(e*x+d)^(5/2)*d^2+4/3/c^3*(e*x+d)^(3/2
)*a*b-2/c^4*b^3*d*(e*x+d)^(1/2)-2/7/e^2/c^2*(e*x+d)^(7/2)*b-2/5/e/c^2*(e*x+
d)^(5/2)*a+2/5/e/c^3*(e*x+d)^(5/2)*b^2-10*e^2/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*
2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/
2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^3*d+4*e/c
^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*
c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2
))*c)^(1/2)*c)*a*b^2*d^2+10*e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-
2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e
-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*b*d-10*e^2/c^3/(-(4*a*c-b^
2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan
```

((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*
b^3*d+4*e/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*a*b^2*d^2+10*e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*b*d+4/c^3*a*b*d
*(e*x+d)^(1/2)+2*e^3/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^3-e^3/c^5/(-(4*a*c-b^2)*e^2)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^6+2*e^3/c^2/
(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(
1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^
(1/2)*c)*a^3-e^3/c^5/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c
-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^6-3*e^2/c^3*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*b+2*e/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*a^2*d+4*e^2/c^4*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*a*b^3+2*e/c^4*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b^4*d+3*e^2/c^3*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*b-2*e/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*d-4*e^2/c^4*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*
c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^3-2*e/c^4*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4*d+2/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*d^2-2/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*d^2+2/5/e^2/c^2*(e*x+d)^(5/2)*b*d-6*e/c^4*
a*b^2*(e*x+d)^(1/2)+1/c^3*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)
^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*c)*b^3*d^2-1/c^3*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)
^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c
)^(1/2)*c)*b^3*d^2+e^2/c^5*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c
)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2)
)*c)^(1/2)*c)*b^5-e^2/c^5*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^
(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)
^(1/2)*c)*b^5-2/3/c^4*(e*x+d)^(3/2)*b^3+6*e/c^3*2^(1/2)/((-b*e+2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(
4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2*d-6*e/c^3*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(
4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2*d-2*e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*d^2-9*e^3/
c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*
c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*c)*a^2*b^2-2*e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(
4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-
(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*d^2+6*e^3/c^4/(-(4*a*c-b^2)*e^2)^(1
/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(
1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^4+2*e^2/
c^4/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*

```
c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*c)*b^5*d-e/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4*d^2+6*e^3/c^4/(-(4*a*c-b^2)*e^2)^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^4+2*e^2
/c^4/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2)
)*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1
/2))*c)^(1/2)*c)*b^5*d-e/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c
*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4*d^2-9*e^3/c^3/(-(4*a*c-b^2)*e^
2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e
*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*
b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}x^4}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x^4/(c*x^2 + b*x + a), x)
```

mupad [B] time = 7.97, size = 31485, normalized size = 48.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] (d + e*x)^(1/2)*((2*d^4)/(c*e^3) - ((a*e^5 + c*d^2*e^3 - b*d*e^4)*((12*d^2)
/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3) +
(2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(c*e^3) +
 ((b*e^4 - 2*c*d*e^3)*((8*d^3)/(c*e^3) - (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*
d*e^3))/(c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c*e^3) + ((b*e^4 - 2*c*d
*e^3)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((
8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e
^3)))/(c*e^3)))/(c*e^3)) - atan(((((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*
c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b
^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d
*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a
^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*
e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12
*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 5
2*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^
2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a
^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*
b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e
^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5
*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e +
 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 30
6*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a
```

*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2
*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^
13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*
a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3
*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*
e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4
*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 +
363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^
(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d
*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^
4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b
^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 22
5*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a
^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^
6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*
c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^
4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*
d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12))
)^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6
 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c
^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b
^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2
 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e
^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^
2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*
a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*
b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13
*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e -
52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c
^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 +
a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a
*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*
e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e
+ 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 3
06*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^
2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c
^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*1i - (((8*(4*a^4*c^9*e^5 - a*b^6*c^6
*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*
d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*
a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^
2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/
2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^

2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88
*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*
c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4
*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b
^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b
^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*
c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5
*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) -
 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d
e^2(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2
*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12
*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3
 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44
*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 +
 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7
*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 -
 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*
c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*
d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4
*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*
e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2
 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a
^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a
*b^2*c^12)))^(1/2) + (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*
b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^
6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^
3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*
c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^
2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4
*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^
4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5
 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c
^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c
^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 8
8*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3
*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^
4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*
b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5

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*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^
5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(
2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*1i)/((16*(a^6*b^5*e^8 - 4
*a^7*b^3*c*e^8 + 3*a^8*b*c^2*e^8 - 2*a^5*b^6*d*e^7 - 2*a^8*c^3*d*e^7 + a^4*
b^7*d^2*e^6 - 2*a^6*c^5*d^5*e^3 - 4*a^7*c^4*d^3*e^5 + a^4*b^3*c^4*d^6*e^2 -
 4*a^4*b^4*c^3*d^5*e^3 + 6*a^4*b^5*c^2*d^4*e^4 + 10*a^5*b^2*c^4*d^5*e^3 - 1
6*a^5*b^3*c^3*d^4*e^4 + 8*a^5*b^4*c^2*d^3*e^5 + 8*a^6*b^2*c^3*d^3*e^5 - 16*
a^6*b^3*c^2*d^2*e^6 + 6*a^6*b^4*c*d*e^7 - 4*a^4*b^6*c*d^3*e^5 - 2*a^5*b*c^5
*d^6*e^2 + 2*a^5*b^5*c*d^2*e^6 + 3*a^6*b*c^4*d^4*e^4 + 8*a^7*b*c^3*d^2*e^6)
)/c^9 + (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7
*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^
7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4
- 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2
*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^1
0*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b
*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^
3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e
^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2
)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^1
2*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) -
15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e
+ 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e
 - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 +
387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a
^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a
^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3
*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c
^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d
*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 +
3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7
*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231
*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3
*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3
5*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e
 + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(
4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414
*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^
4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^
4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^
5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^
(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1
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/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d ^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^ 10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^ 3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3* e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^ 5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a* b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b* c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^ 10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6* b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a ^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3* e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^ 2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^ 12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e ^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2) ^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2* e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3* a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3) ^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45* a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^ 3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2* c^12)))^(1/2) + (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2 *b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^ 9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d ^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b ^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a *c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^ 6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^ (1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a* c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^ 7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c ^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4* d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e ^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2* d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(- (4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60 *a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8 *a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8* a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7* d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4 *b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e ^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d ^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3

```
*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c
^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6
*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e
^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e
- 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^
3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c -
b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (8*(d +
 e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^
3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a
^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 -
 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^
5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*
c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*
c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3
 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 3
6*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*
d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 +
 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^
3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*
b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*
a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^
3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b
^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c
^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*
c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4
*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*
e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6
0*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 -
8*a*b^2*c^12)))^(1/2)))*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*
c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66
*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c
^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2
*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b
^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b
*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3
*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^
2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c
^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(
```

-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3
0*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*
c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*2i -
atan(((((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e
^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*
d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 -
6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b
^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*
c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c
^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*
b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3
 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^
3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*
c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15
*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e -
3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e -
 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 38
7*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4
*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2
*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c
^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^1
2)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e
^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*
b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d
^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a
^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*
a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e +
 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*
a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a
^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*
b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*
b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1
/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1/2
)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 10
5*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2
*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10
*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3
+ 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^
3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5
+ 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^
8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^
7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10
*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*
c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3

*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*1i - (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4

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*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a
^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (8*(d
+ e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c
^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*
a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4
- 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c
^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3
*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9
*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^
3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 -
36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8
*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3
+ 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d
^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3
*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4
*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e
^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*
b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*
c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7
*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^
4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d
*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^
2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 -
 8*a*b^2*c^12)))^(1/2)*1i)/((16*(a^6*b^5*e^8 - 4*a^7*b^3*c*e^8 + 3*a^8*b*c^
2*e^8 - 2*a^5*b^6*d*e^7 - 2*a^8*c^3*d*e^7 + a^4*b^7*d^2*e^6 - 2*a^6*c^5*d^5
*e^3 - 4*a^7*c^4*d^3*e^5 + a^4*b^3*c^4*d^6*e^2 - 4*a^4*b^4*c^3*d^5*e^3 + 6*
a^4*b^5*c^2*d^4*e^4 + 10*a^5*b^2*c^4*d^5*e^3 - 16*a^5*b^3*c^3*d^4*e^4 + 8*a
^5*b^4*c^2*d^3*e^5 + 8*a^6*b^2*c^3*d^3*e^5 - 16*a^6*b^3*c^2*d^2*e^6 + 6*a^6
*b^4*c*d*e^7 - 4*a^4*b^6*c*d^3*e^5 - 2*a^5*b*c^5*d^6*e^2 + 2*a^5*b^5*c*d^2*
e^6 + 3*a^6*b*c^4*d^4*e^4 + 8*a^7*b*c^3*d^2*e^6))/c^9 + (((8*(4*a^4*c^9*e^5
 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 +
 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*
d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*
a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(
d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2
 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*
c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 -
231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*
d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^
2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 -
414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675
*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15
*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c -
```

b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 + (8*(d + e*x)^(1/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*

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a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^
4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*
b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^
9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*
c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*
d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2
*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^
3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30
*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c
^11*e^3 - 2*b^2*c^12*d*e^2 - 4*a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13
*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e -
52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c
^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 -
a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*
e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^
5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e
+ 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 3
06*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 63*a^
2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c
^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) + (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a
^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6
 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^
4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c
*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4
*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c
^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*
e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 1
76*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a
^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*
c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66
*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c
^4*e^3 - 231*a^5*b^3*c^5*e^3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2
*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b
^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b
*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3
*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^
```

```
2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c
^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3
0*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*
c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2))*(-(
b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 + b^10*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*
e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b
^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^
3 - a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 - 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 35*a^3*b^4*c^3*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^
2*d*e^2 - 108*a^5*b*c^7*d^2*e - 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 6*
a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^
2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2
 + 306*a^5*b^2*c^6*d*e^2 + 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^8
*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b*c^5*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6
3*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 30*a^3*b^2*c^5*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a
^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*2i - ((8*d)/(7*c*e^3) + (2*(b*e^
4 - 2*c*d*e^3))/(7*c^2*e^6))*(d + e*x)^(7/2) + (d + e*x)^(5/2)*((12*d^2)/(5
*c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(5*c^2*e^6) + (((8*d)/(c*e^3) +
 (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(5*c*e^3)) - (d +
e*x)^(3/2)*((8*d^3)/(3*c*e^3) - (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(
c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(3*c*e^3) + ((b*e^4 - 2*c*d*e^3)*(
(12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c
*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(
3*c*e^3)) + (2*(d + e*x)^(9/2))/(9*c*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.343 $\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=581

$$\frac{\sqrt{2}\left(-\frac{4a^2c^3de-b^3c\left(cd^2-5ae^2\right)-8ab^2c^2de+abc^2\left(3cd^2-5ae^2\right)+b^5\left(-e^2\right)+2b^4cde}{\sqrt{b^2-4ac}}-b^2c\left(cd^2-3ae^2\right)-4abc^2de+ac^2\left(cd^2-ae^2\right)+b^4\left(\cdots\right.\right.}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Rubi [A] time = 15.25, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(-\frac{4a^2c^3de-8ab^2c^2de-b^3c\left(cd^2-5ae^2\right)+abc^2\left(3cd^2-5ae^2\right)+2b^4cde+b^5\left(-e^2\right)}{\sqrt{b^2-4ac}}-b^2c\left(cd^2-3ae^2\right)-4abc^2de+ac^2\left(cd^2-ae^2\right)+2b^3cde+b^4\left(-e^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{4a^2c^3de-8ab^2c^2de-b^3c\left(cd^2-5ae^2\right)+abc^2\left(3cd^2-5ae^2\right)+2b^4cde+b^5\left(-e^2\right)}{\sqrt{b^2-4ac}}-b^2c\left(cd^2-3ae^2\right)-4abc^2de+ac^2\left(cd^2-ae^2\right)+2b^3cde+b^4\left(-e^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{9/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2\left(b^2-ac\right)(d+ex)^{3/2}}{3c^3}+\frac{2\sqrt{d+ex}\left(2abce-ac^2d+b^2cd+b^3(-e)\right)}{c^4}-\frac{2(d+ex)^{5/2}(be+cd)}{5c^2e^2}+\frac{2(d+ex)^{7/2}}{7ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Sqrt[d + e*x])/c^4 + (2*(b^2 - a
*c)*(d + e*x)^(3/2))/(3*c^3) - (2*(c*d + b*e)*(d + e*x)^(5/2))/(5*c^2*e^2)
+ (2*(d + e*x)^(7/2))/(7*c*e^2) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b
^4*e^2 - b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) - (2*b^4*c*d*e - 8
*a*b^2*c^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^
2*(3*c*d^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b^4*e^2 -
 b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) + (2*b^4*c*d*e - 8*a*b^2*c
^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^2*(3*c*d
^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{e\left(b^2cd-ac^2d-b^3e+2abce\right)}{c^4}+\frac{\left(b^2-ac\right)ex^2}{c^3}-\frac{(cd+be)x^4}{c^2e}+\frac{x^6}{ce}-\frac{\left(b^2cd-ac^2d-b^3e+2abce\right)\left(cd^2-bde+\right.}{c^4e}\right.\right.}{e}$$

$$= \frac{2\left(b^2cd-ac^2d-b^3e+2abce\right)\sqrt{d+ex}}{c^4}+\frac{2\left(b^2-ac\right)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}$$

$$= \frac{2\left(b^2cd-ac^2d-b^3e+2abce\right)\sqrt{d+ex}}{c^4}+\frac{2\left(b^2-ac\right)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}$$

$$= \frac{2\left(b^2cd-ac^2d-b^3e+2abce\right)\sqrt{d+ex}}{c^4}+\frac{2\left(b^2-ac\right)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}$$

Mathematica [A] time = 0.90, size = 680, normalized size = 1.17

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(105*b^3*e^3 + 3*c^3*(2*d - 5*e*x)*(d + e*x)^2 - 35*b*c*e
^2*(4*b*d + 6*a*e + b*e*x) + 7*c^2*e*(3*b*(d + e*x)^2 + 5*a*e*(4*d + e*x)))
)/(105*c^4*e^2) - (Sqrt[2]*(-(b^5*e^2) + b^4*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e
) + b^2*c*(-3*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e))
- b^3*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 5*a*e)) + a*b*c^2*(3*c*d^2 + e*
(4*Sqrt[b^2 - 4*a*c]*d - 5*a*e)) + a*c^2*(a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(-(
Sqrt[b^2 - 4*a*c]*d) + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqr
t[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(9/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b^5*e^2 + b^4*e*(-2*c*d + Sqrt
[b^2 - 4*a*c]*e) + a*c^2*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*
d + 4*a*e)) + b^3*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + a*b*c^2*(
-3*c*d^2 + e*(4*Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + b^2*c*(-3*a*Sqrt[b^2 - 4*a*
c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 8*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[b^2 - 4*a
*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

IntegrateAlgebraic [C] time = 2.97, size = 996, normalized size = 1.71

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(105*b^2*c*d*e^2 - 105*a*c^2*d*e^2 - 105*b^3*e^3 + 210*a*b
*c*e^3 + 35*b^2*c*e^2*(d + e*x) - 35*a*c^2*e^2*(d + e*x) - 21*c^3*d*(d + e*
x)^2 - 21*b*c^2*e*(d + e*x)^2 + 15*c^3*(d + e*x)^3))/(105*c^4*e^2) + ((I*Sq
rt[2]*b^3*c^2*d^2 - (3*I)*Sqrt[2]*a*b*c^3*d^2 + Sqrt[2]*b^2*c^2*Sqrt[-b^2 +
 4*a*c]*d^2 - Sqrt[2]*a*c^3*Sqrt[-b^2 + 4*a*c]*d^2 - (2*I)*Sqrt[2]*b^4*c*d*
e + (8*I)*Sqrt[2]*a*b^2*c^2*d*e - (4*I)*Sqrt[2]*a^2*c^3*d*e - 2*Sqrt[2]*b^3
*c*Sqrt[-b^2 + 4*a*c]*d*e + 4*Sqrt[2]*a*b*c^2*Sqrt[-b^2 + 4*a*c]*d*e + I*Sq
rt[2]*b^5*e^2 - (5*I)*Sqrt[2]*a*b^3*c*e^2 + (5*I)*Sqrt[2]*a^2*b*c^2*e^2 + S
qrt[2]*b^4*Sqrt[-b^2 + 4*a*c]*e^2 - 3*Sqrt[2]*a*b^2*c*Sqrt[-b^2 + 4*a*c]*e^
2 + Sqrt[2]*a^2*c^2*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-b^2 +
4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((-I)*Sqrt[2]*b^3*c^
2*d^2 + (3*I)*Sqrt[2]*a*b*c^3*d^2 + Sqrt[2]*b^2*c^2*Sqrt[-b^2 + 4*a*c]*d^2
- Sqrt[2]*a*c^3*Sqrt[-b^2 + 4*a*c]*d^2 + (2*I)*Sqrt[2]*b^4*c*d*e - (8*I)*Sq
rt[2]*a*b^2*c^2*d*e + (4*I)*Sqrt[2]*a^2*c^3*d*e - 2*Sqrt[2]*b^3*c*Sqrt[-b^2
 + 4*a*c]*d*e + 4*Sqrt[2]*a*b*c^2*Sqrt[-b^2 + 4*a*c]*d*e - I*Sqrt[2]*b^5*e^
2 + (5*I)*Sqrt[2]*a*b^3*c*e^2 - (5*I)*Sqrt[2]*a^2*b*c^2*e^2 + Sqrt[2]*b^4*S
qrt[-b^2 + 4*a*c]*e^2 - 3*Sqrt[2]*a*b^2*c*Sqrt[-b^2 + 4*a*c]*e^2 + Sqrt[2]*
a^2*c^2*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt
[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[
-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])
```

fricas [B] time = 6.06, size = 11459, normalized size = 19.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/210*(105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 -
2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d
^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5
)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)
*e^3 + (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8
- 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2
*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^1
2*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8
 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 +
 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39
*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 64
0*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*
c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^
4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5
 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c
^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^
2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(sqrt(2)*((b^9*c^4 - 9*a*b^7*
c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^10*c^3 - 4
0*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2*c^7 - 12*a^5*
c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^
5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48*a*b^10*c^2 + 2
16*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^
6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181
*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 - ((b^5*c^10 - 7*a*b^3*c^1
1 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^
12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*
a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5
*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5
```

+ 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3
*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 91
0*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 +
3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a
^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^1
5*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 -
166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*
c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5
+ 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))*
sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2
- 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^
2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c +
27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 + (b^2*c^9 - 4*a*c^10)*
sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2
*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 +
22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*
a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^
10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b
^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b
^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*
c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 1
3*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^
5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*
a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a
^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^
9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^
3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b
^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*
b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3
+ (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d
*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*
e^5)*sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*
a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 -
7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c
^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3
+ 9*a^4*b*c^4)*e^3 + (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 2
2*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^
9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^
5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 2
30*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 11
0*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^
5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2
*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^
2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 -
174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^
7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 +
367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8
*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(-sqrt(2)*((b^9
*c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 -
(4*b^10*c^3 - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^
2*c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 -
155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48
*a*b^10*c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5
*b^2*c^6 + 4*a^6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^
3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 - ((b^5*c^1
0 - 7*a*b^3*c^11 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*
c^11 - 8*a^3*c^12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a
^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c
^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4

- 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 + (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) + 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(sqrt(2)*((b^9*c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^10*c^3 - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2*c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48*a*b^10*c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 + ((b^5*c^10 - 7*a*b^3*c^11 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e +

3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(-sqrt(2)*((b^9*c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^10*c^3 - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2*c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48*a*b^10*c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 + ((b^5*c^10 - 7*a*b^3*c^11 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b

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*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^
6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13
*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7
 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3
+ 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 +
80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^
11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c
^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b
^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*
c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^6*c^3 - 6*a*b^4*c^4 +
9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4
- 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2
*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^
3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 +
 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*
b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*
d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 +
 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 -
110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*
a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a
^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*
b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3
- 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*
a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a
^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^
4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21
*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^
5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5
*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^
5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c +
 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) - 4*(15*c^3
*e^3*x^3 - 6*c^3*d^3 - 21*b*c^2*d^2*e + 140*(b^2*c - a*c^2)*d*e^2 - 105*(b^
3 - 2*a*b*c)*e^3 + 3*(8*c^3*d*e^2 - 7*b*c^2*e^3)*x^2 + (3*c^3*d^2*e - 42*b*
c^2*d*e^2 + 35*(b^2*c - a*c^2)*e^3)*x)*sqrt(e*x + d))/(c^4*e^2)
```

giac [B] time = 0.58, size = 1362, normalized size = 2.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e - 2*(b^5*c - 6*a*b^3*c^2 +
8*a^2*b*c^3)*d*e^2 + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3)*sq
rt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2 - 2*((b^2*c^4 - a*c^5)*s
qrt(b^2 - 4*a*c)*d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2*e + (b
^4*c^2 - a*b^2*c^3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^3*c^2 - 2*a^2*
b*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)
*e)*abs(c) + (2*(b^3*c^5 - 3*a*b*c^6)*d^3 - (5*b^4*c^4 - 19*a*b^2*c^5 + 8*a
^2*c^6)*d^2*e + 2*(2*b^5*c^3 - 9*a*b^3*c^4 + 7*a^2*b*c^5)*d*e^2 - (b^6*c^2
- 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a
*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^16 - b*c^7*e^1
7 + sqrt(-4*(c^8*d^2*e^16 - b*c^7*d*e^17 + a*c^7*e^18)*c^8*e^16 + (2*c^8*d*
e^16 - b*c^7*e^17)^2))*e^(-16)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^2 - sqrt(b^2
 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) - 1/4*(((b^4*c^2 -
5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e^
2 + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3)*sqrt(-4*c^2*d + 2*(
b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2 + 2*((b^2*c^4 - a*c^5)*sqrt(b^2 - 4*a*c)*
d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2*e + (b^4*c^2 - a*b^2*c^
```

```
3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 -
 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*(
b^3*c^5 - 3*a*b*c^6)*d^3 - (5*b^4*c^4 - 19*a*b^2*c^5 + 8*a^2*c^6)*d^2*e + 2
*(2*b^5*c^3 - 9*a*b^3*c^4 + 7*a^2*b*c^5)*d*e^2 - (b^6*c^2 - 5*a*b^4*c^3 + 5
*a^2*b^2*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan
(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^16 - b*c^7*e^17 - sqrt(-4*(c^8*
d^2*e^16 - b*c^7*d*e^17 + a*c^7*e^18)*c^8*e^16 + (2*c^8*d*e^16 - b*c^7*e^17
)^2))*e^(-16)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^2 - sqrt(b^2 - 4*a*c)*b*c^6*d
*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) + 2/105*(15*(x*e + d)^(7/2)*c^6*e^12
 - 21*(x*e + d)^(5/2)*c^6*d*e^12 - 21*(x*e + d)^(5/2)*b*c^5*e^13 + 35*(x*e
+ d)^(3/2)*b^2*c^4*e^14 - 35*(x*e + d)^(3/2)*a*c^5*e^14 + 105*sqrt(x*e + d)
*b^2*c^4*d*e^14 - 105*sqrt(x*e + d)*a*c^5*d*e^14 - 105*sqrt(x*e + d)*b^3*c^
3*e^15 + 210*sqrt(x*e + d)*a*b*c^4*e^15)*e^(-14)/c^7
```

maple [B] time = 0.06, size = 2988, normalized size = 5.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)

[Out] 2/7*(e*x+d)^(7/2)/c/e^2-2/5/e^2/c*(e*x+d)^(5/2)*d-2/c^2*a*d*(e*x+d)^(1/2)+2
/c^3*b^2*d*(e*x+d)^(1/2)-2/5/e/c^2*(e*x+d)^(5/2)*b-2*e/c^4*b^3*(e*x+d)^(1/2
)+8*e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*a*b^2*d-3*e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2
*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-
2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*d^2+8*e^2/c^2/(-(4*a*c-b^2)
*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh
((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a
*b^2*d-3*e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*a*b*d^2+e^3/c^4/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/(
(b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/
((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^5+3*e^2/c^3*2^(1/2)/((-
b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2+2*e/c^3*2^(1/2)/((
-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d-3*e^2/c^3*2^(1/2)
/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2
)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2-2*e/c^3*2^(1/2)/(
(b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/
((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d+e^3/c^4/(-(4*a*c-b^
2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arcta
nh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)
*b^5+4*e/c^3*a*b*(e*x+d)^(1/2)+5*e^3/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/(
(b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/
((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*b-4*e^2/c/(-(4*a*c-b^
2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan
((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^
2*d-5*e^3/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*a*b^3-2*e^2/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b
*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((
b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4*d+e/c^2/(-(4*a*c-b^2)*e
^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*
x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d^
2+5*e^3/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*a^2*b-4*e/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
```

```
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*a*b*d+4*e/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*a*b*d-4*e^2/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*
e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2*d-5*e^3/c^3/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*a*b^3-2*e^2/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c
-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4*d+e/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/
2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d^2-2/3/c^2*(e*x+
d)^(3/2)*a+2/3/c^3*(e*x+d)^(3/2)*b^2+1/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d^2+1/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*a*d^2-1/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*b^2*d^2-e^2/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2-e^2/c^4*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*b^4+e^2/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*a^2+e^2/c^4*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b^4-1/c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)
^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c
)^(1/2)*c)*a*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}x^3}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x^3/(c*x^2 + b*x + a), x)
```

mupad [B] time = 7.14, size = 25497, normalized size = 43.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e
^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^
2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12
*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11
*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*
a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*
a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3
*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^
4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*
b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
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+ 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i - (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2

*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((16*(a^5*b^4*e^8 + a^7*c^2*e^8 - 3*a^6*b^2*c*e^8 - 2*a^4*b^5*d*e^7 + a^3*b^6*d^2*e^6 - a^4*c^5*d^6*e^2 - a^5*c^4*d^4*e^4 + a^6*c^3*d^2*e^6 + a^3*b^2*c^4*d^6*e^2 - 4*a^3*b^3*c^3*d^5*e^3 + 6*a^3*b^4*c^2*d^4*e^4 - 10*a^4*b^2*c^3*d^4*e^4 + 4*a^4*b^3*c^2*d^3*e^5 - 12*a^5*b^2*c^2*d^2*e^6 + 4*a^5*b^3*c*d*e^7 + 2*a^6*b*c^2*d*e^7 - 4*a^3*b^5*c*d^3*e^5 + 6*a^4*b*c^4*d^5*e^3 + 3*a^4*b^4*c*d^2*e^6 + 8*a^5*b*c^3*d^3*e^5))/c^7 + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*

b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 +

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24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d
^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c
^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13
*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e
 - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*
e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 -
 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*
a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a
*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c
^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2
 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c
^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c -
 b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*
b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33
*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2
*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c
^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*
d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2
*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2
) + (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*
a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e
^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e
^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a
^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b
*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*
e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5)
)/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c
^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 1
38*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(
1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e
^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3
*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 -
 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*
a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*
a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)))*(-(b^1
1*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10
*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33
*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^
3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b
```

^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a
*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b
^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*
c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a
*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/
2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i - ((6*d)/(5*c*e^2)
 + (2*(b*e^3 - 2*c*d*e^2))/(5*c^2*e^4))*(d + e*x)^(5/2) + atan(((((8*(4*a^3
*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^
6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c
^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^
3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*
d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 3
6*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 +
 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^
3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c
e^3(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 8
4*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^
3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126
*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^
3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^
3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^1
1 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b
*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3
 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 +
 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d
^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c
^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13
*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e
 + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*
e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 -
 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*
a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a
*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*
b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 +
 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e
^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^
5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2
*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 -
48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a
^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^
6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^
2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(

4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e
^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*
b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^
2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b
^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*
a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^
3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)
))^(1/2)*1i - (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6
*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^
5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*
e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8*(d + e*x)^(1/2)
*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2
*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3
*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) +
b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 1
5*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e
+ 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*
a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a
^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^
5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^
2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 -
2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*
a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4
*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c
^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 12
9*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4
*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7
*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d
*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^
3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e
+ 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16
*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^6
- 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*
e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*
d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^
6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c
^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2
+ 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*
a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^
3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*
a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 3
8*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*

c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*
a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a
^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*
b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*
b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11
+ b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i)/((16*(a^5*b^4*e^8 + a^7*c^2*e^8 - 3*a
^6*b^2*c*e^8 - 2*a^4*b^5*d*e^7 + a^3*b^6*d^2*e^6 - a^4*c^5*d^6*e^2 - a^5*c^
4*d^4*e^4 + a^6*c^3*d^2*e^6 + a^3*b^2*c^4*d^6*e^2 - 4*a^3*b^3*c^3*d^5*e^3 +
 6*a^3*b^4*c^2*d^4*e^4 - 10*a^4*b^2*c^3*d^4*e^4 + 4*a^4*b^3*c^2*d^3*e^5 - 1
2*a^5*b^2*c^2*d^2*e^6 + 4*a^5*b^3*c*d*e^7 + 2*a^6*b*c^2*d*e^7 - 4*a^3*b^5*c
*d^3*e^5 + 6*a^4*b*c^4*d^5*e^3 + 3*a^4*b^4*c*d^2*e^6 + 8*a^5*b*c^3*d^3*e^5)
)/c^7 + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d
*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*
d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 -
12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^
11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 1
0*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 3
3*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c
^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*
b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*
a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3
*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*
a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1
/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2
*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^
7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 -
 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3
 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*
b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6
*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 +
 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*
d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 1
26*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*
a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*
a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c
^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^
5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 -
2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^
3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*
e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2

*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) + (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^

```
4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*
b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5
*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^
3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b
^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2
*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^
3 + 129*a^4*b^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c
^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8
*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*
d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4
*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)))*(-(b^11*e^3 - 8*a^4*c^7
*d^3 - b^8*c^3*d^3 - b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 -
36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3
+ 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b
^3*c^4*e^3 - a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 - 15*a^2*b^4*c^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a*b^6*
c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 +
84*a^4*b*c^6*d^2*e + 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^4*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 12
6*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a
^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 + 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^4*c^3*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) - 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 12*a
^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^
11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*2i + (d + e*x)^(3/2)*((2*d^2)/(c*e^2)
- (2*(a*e^4 + c*d^2*e^2 - b*d*e^3))/(3*c^2*e^4) + (((6*d)/(c*e^2) + (2*(b*e
^3 - 2*c*d*e^2))/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))/(3*c*e^2)) - (d + e*x)^(1/
2)*((2*d^3)/(c*e^2) - (((6*d)/(c*e^2) + (2*(b*e^3 - 2*c*d*e^2))/(c^2*e^4))*
(a*e^4 + c*d^2*e^2 - b*d*e^3))/(c*e^2) + ((b*e^3 - 2*c*d*e^2)*((6*d^2)/(c*e
^2) - (2*(a*e^4 + c*d^2*e^2 - b*d*e^3))/(c^2*e^4) + (((6*d)/(c*e^2) + (2*(b
*e^3 - 2*c*d*e^2))/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))/(c*e^2)))/(c*e^2)) + (2*
(d + e*x)^(7/2))/(7*c*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.344 $\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2} \, dx$

Optimal. Leaf size=441

$$-\frac{2\sqrt{d+ex}\left(ace+b^2(-e)+bcd\right)}{c^3}+\frac{\sqrt{2}\left((cd-be)\left(2ace+b^2(-e)+bcd\right)+\frac{-b^2c\left(cd^2-4ae^2\right)-6abc^2de+2ac^2\left(cd^2-ae^2\right)+b^4\left(-e^2\right)+2}{\sqrt{b^2-4ac}}\right.}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Rubi [A] time = 2.15, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(\frac{-b^2c\left(cd^2-4ae^2\right)-6abc^2de+2ac^2\left(cd^2-ae^2\right)+2b^3cde+b^4\left(-e^2\right)}{\sqrt{b^2-4ac}}+(cd-be)\left(2ace+b^2(-e)+bcd\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left((cd-be)\left(2ace+b^2(-e)+bcd\right)-\frac{-b^2c\left(cd^2-4ae^2\right)-6abc^2de+2ac^2\left(cd^2-ae^2\right)+2b^3cde+b^4\left(-e^2\right)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{d+ex}\left(ace+b^2(-e)+bcd\right)}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x])/c^3 - (2*b*(d + e*x)^(3/2))/(3*c
^2) + (2*(d + e*x)^(5/2))/(5*c*e) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e +
2*a*c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2)
 + 2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c
*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*
a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
```

```
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{e\left(bcd-b^2e+ace\right)}{c^3}-\frac{bex^2}{c^2}+\frac{x^4}{c}+\frac{\left(bcd-b^2e+ace\right)\left(cd^2-bde+ae^2\right)-(cd-be)\left(bcd-b^2e+2ace\right)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, \ldots}{e}$$

$$= -\frac{2\left(bcd-b^2e+ace\right)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}+\frac{2\,\text{Subst}\left(\int \frac{\left(bcd-b^2e+ace\right)(c\ldots}{cd^2\ldots}\ldots\right)}{\ldots}$$

$$= -\frac{2\left(bcd-b^2e+ace\right)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}-\frac{\left((cd-be)\left(bcd-b^2e+2\ldots\right.\right.}{\ldots}$$

$$= -\frac{2\left(bcd-b^2e+ace\right)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}+\frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e\ldots\right.\right.}{\ldots}$$

Mathematica [A] time = 0.67, size = 538, normalized size = 1.22

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(15*b^2*e^2 + 3*c^2*(d + e*x)^2 - 5*c*e*(4*b*d + 3*a*e + b
*e*x)))/(15*c^3*e) + (Sqrt[2]*(-(b^4*e^2) + b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c
]*e) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 6*a*e))
 - b^2*c*(c*d^2 + 2*e*(Sqrt[b^2 - 4*a*c]*d - 2*a*e)) + 2*a*c^2*(c*d^2 + e*(
Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2
*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^4*e^2 + b^3*e*(-2*c*d + Sqrt[b^
2 - 4*a*c]*e) + 2*a*c^2*(-(c*d^2) + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b^2*c*
(c*d^2 - 2*e*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e
^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 6*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

IntegrateAlgebraic [A] time = 1.93, size = 702, normalized size = 1.59

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-15*b*c*d*e + 15*b^2*e^2 - 15*a*c*e^2 - 5*b*c*e*(d + e*x)
 + 3*c^2*(d + e*x)^2))/(15*c^3*e) + ((Sqrt[2]*b^2*c^2*d^2 - 2*Sqrt[2]*a*c^3
*d^2 - Sqrt[2]*b*c^2*Sqrt[b^2 - 4*a*c]*d^2 - 2*Sqrt[2]*b^3*c*d*e + 6*Sqrt[2
]*a*b*c^2*d*e + 2*Sqrt[2]*b^2*c*Sqrt[b^2 - 4*a*c]*d*e - 2*Sqrt[2]*a*c^2*Sqr
t[b^2 - 4*a*c]*d*e + Sqrt[2]*b^4*e^2 - 4*Sqrt[2]*a*b^2*c*e^2 + 2*Sqrt[2]*a^
2*c^2*e^2 - Sqrt[2]*b^3*Sqrt[b^2 - 4*a*c]*e^2 + 2*Sqrt[2]*a*b*c*Sqrt[b^2 -
4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt
[b^2 - 4*a*c]*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2
- 4*a*c]*e]) + ((-(Sqrt[2]*b^2*c^2*d^2) + 2*Sqrt[2]*a*c^3*d^2 - Sqrt[2]*b*c
^2*Sqrt[b^2 - 4*a*c]*d^2 + 2*Sqrt[2]*b^3*c*d*e - 6*Sqrt[2]*a*b*c^2*d*e + 2*
Sqrt[2]*b^2*c*Sqrt[b^2 - 4*a*c]*d*e - 2*Sqrt[2]*a*c^2*Sqrt[b^2 - 4*a*c]*d*e
 - Sqrt[2]*b^4*e^2 + 4*Sqrt[2]*a*b^2*c*e^2 - 2*Sqrt[2]*a^2*c^2*e^2 - Sqrt[2
]*b^3*Sqrt[b^2 - 4*a*c]*e^2 + 2*Sqrt[2]*a*b*c*Sqrt[b^2 - 4*a*c]*e^2)*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])
/(c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e])
```

<u>fricas</u> [B] time = 2.40, size = 8530, normalized size = 19.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*c^3*e*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(
b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2
*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)
*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5
*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*
e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29
*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*
a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2
 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 +
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*l
og(sqrt(2)*((b^6*c^4 - 6*a*b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a
*b^5*c^4 + 53*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c
^3 + 39*a^2*b^4*c^4 - 29*a^3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a
*b^7*c^2 + 107*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10
- 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5
)*e^4 - ((b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 +
12*a^2*b*c^9)*e)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7
*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30
*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b
^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d
^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 +
 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7
*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^
2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a
*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7
+ 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c
^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 +
 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*
a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^
2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11
*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5
*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*
```

```
a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c
^7 - 4*a*c^8)) + 4*((a^2*b^3*c^4 - 2*a^3*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a
^3*b^2*c^4 + 3*a^4*c^5)*d^4*e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b
*c^4)*d^3*e^2 - 2*(2*a^2*b^6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d
^2*e^3 + (a^2*b^7 - 2*a^3*b^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3
*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e^5)*sqrt(e*x + d)) - 15*sqrt
(2)*c^3*e*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*
b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a
^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 + (b^2
*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*
c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*
a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^
9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^
3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 +
45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*
c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((
b^6*c^4 - 6*a*b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53
*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b
^4*c^4 - 29*a^3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 10
7*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10 - 10*a*b^8*c
+ 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 - ((b^4
*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)
*e)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^
5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 +
55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a
*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5
*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c
^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3
*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^
6)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 -
3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*
a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*
b*c^3)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c
^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3
*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d
^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 +
 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 1
20*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*
c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5
 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)
) + 4*((a^2*b^3*c^4 - 2*a^3*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a^3*b^2*c^4 +
3*a^4*c^5)*d^4*e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b*c^4)*d^3*e^2
 - 2*(2*a^2*b^6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d^2*e^3 + (a^2
*b^7 - 2*a^3*b^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3*b^6 - 5*a^4*
b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e^5)*sqrt(e*x + d)) + 15*sqrt(2)*c^3*e*sqr
t(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a
^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2
 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a*c^
8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5
*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 5
5*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*
b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*
b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^
6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*
b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^
```

2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(sqrt(2)*((b^6*c^4 - 6*a* b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b^4*c^4 - 29*a^ 3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 107*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c ^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 + ((b^4*c^8 - 6*a*b^2 *c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*e)*sqrt(((b^6 *c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2* b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160 *a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40 *a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22* a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6 2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5 *a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b ^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10 *b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7) *d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b ^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10* a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^ 5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^2*b^ 3*c^4 - 2*a^3*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a^3*b^2*c^4 + 3*a^4*c^5)*d^4 *e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b*c^4)*d^3*e^2 - 2*(2*a^2*b^ 6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d^2*e^3 + (a^2*b^7 - 2*a^3*b ^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5* b^2*c^2 - a^6*c^3)*e^5)*sqrt(e*x + d)) - 15*sqrt(2)*c^3*e*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2* e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a* b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a*c^8)*sqrt(((b^6* c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b ^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160* a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40* a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7) *d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a ^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62 *a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((b^6*c^4 - 6*a*b^4*c^5 + 8*a ^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b^4*c^4 - 29*a^3*b^2*c^5 + 4 *a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 107*a^2*b^5*c^3 - 118*a^3*b^3 *c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b ^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 + ((b^4*c^8 - 6*a*b^2*c^9 + 8*a^2* c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*e)*sqrt(((b^6*c^6 - 4*a*b^ 4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a ^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2 *c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6 *(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))*

```
sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 +
5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*
e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a
*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*
b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5
+ 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70
*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*
(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2
*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a
^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37
*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*
e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^2*b^3*c^4 - 2*a^3
*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a^3*b^2*c^4 + 3*a^4*c^5)*d^4*e + 2*(3*a^2
*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b*c^4)*d^3*e^2 - 2*(2*a^2*b^6*c - 7*a^3*b
^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d^2*e^3 + (a^2*b^7 - 2*a^3*b^5*c - 6*a^4*
b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6
*c^3)*e^5)*sqrt(e*x + d)) - 4*(3*c^2*e^2*x^2 + 3*c^2*d^2 - 20*b*c*d*e + 15*
(b^2 - a*c)*e^2 + (6*c^2*d*e - 5*b*c*e^2)*x)*sqrt(e*x + d))/(c^3*e)
```

giac [B] time = 0.53, size = 1160, normalized size = 2.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(((b^3*c^2 - 4*a*b*c^3)*d^2*e - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*
e^2 + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^
2 - 4*a*c)*c)*e)*c^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^4*d^3 + sqrt(b^2 - 4*a*c)*b
^3*c^2*d*e^2 - (2*b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2*e - (a*b^2*c^2 - a
^2*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c
)*e)*abs(c) + (2*(b^2*c^5 - 2*a*c^6)*d^3 - (5*b^3*c^4 - 14*a*b*c^5)*d^2*e +
 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^2 - (b^5*c^2 - 4*a*b^3*c^3 + 2
*a^2*b*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2
*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^6*d*e^6 - b*c^5*e^7 + sqrt(-4*(c^6*d^2*
e^6 - b*c^5*d*e^7 + a*c^5*e^8)*c^6*e^6 + (2*c^6*d*e^6 - b*c^5*e^7)^2))*e^(-
6)/c^6))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d*e + sqrt(b
^2 - 4*a*c)*a*c^5*e^2)*c^2) + 1/4*(((b^3*c^2 - 4*a*b*c^3)*d^2*e - 2*(b^4*c
- 5*a*b^2*c^2 + 4*a^2*c^3)*d*e^2 + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^3)*sqr
t(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2 + 2*(sqrt(b^2 - 4*a*c)*b*
c^4*d^3 + sqrt(b^2 - 4*a*c)*b^3*c^2*d*e^2 - (2*b^2*c^3 - a*c^4)*sqrt(b^2 -
4*a*c)*d^2*e - (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d +
 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*(b^2*c^5 - 2*a*c^6)*d^3 - (5*
b^3*c^4 - 14*a*b*c^5)*d^2*e + 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^2
 - (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt
(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^6*d*e^6 -
b*c^5*e^7 - sqrt(-4*(c^6*d^2*e^6 - b*c^5*d*e^7 + a*c^5*e^8)*c^6*e^6 + (2*c^
6*d*e^6 - b*c^5*e^7)^2))*e^(-6)/c^6))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^
2 - 4*a*c)*b*c^5*d*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2) + 2/15*(3*(x*e + d
)^(5/2)*c^4*e^4 - 5*(x*e + d)^(3/2)*b*c^3*e^5 - 15*sqrt(x*e + d)*b*c^3*d*e^
5 + 15*sqrt(x*e + d)*b^2*c^2*e^6 - 15*sqrt(x*e + d)*a*c^3*e^6)*e^(-5)/c^5
```

maple [B] time = 0.06, size = 2358, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] -e^2/c^3*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x
+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3-1/c
```

```
*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2
)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2+1/c*2^(1/
2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^
(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2+e^2/c^3*2^(1
/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3-2/3*b*(e*x+d)
^(3/2)/c^2+2/5*(e*x+d)^(5/2)/c/e-2*e/c^2*a*(e*x+d)^(1/2)+2*e/c^3*b^2*(e*x+d
)^(1/2)-2/c^2*b*d*(e*x+d)^(1/2)-6*e^2/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*d-6*e^2/c/(-(4*a*c-
b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*
a*b*d-e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b^2*d^2+4*e^3/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*
e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2+2*e^2/c^2/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*b^3*d-e/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b^2*d^2+4*e^3/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)
/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b^2-2*e^3/c/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*a^2-e^3/c^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*b^4-2*e^2/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*a*b+2*e/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*a*d-2*e/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*b^2*d-2*e^3/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e
-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*
e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a^2-e^3/c^3/(-(4*a*c-b^2)*e^2
)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+
d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^4+2*e^
2/c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)
^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b-2*e/c*
2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)
*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*d+2*e/c^2*2^(1
/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(
1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d+2*e/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*a*d^2+2*e/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*a*d^2+2*e^2/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e
-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*
e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x^2/(c*x^2 + b*x + a), x)
```

mupad [B] time = 5.72, size = 19465, normalized size = 44.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e
^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^
3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/2)*
(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e
- 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^
3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^
7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d
*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2
 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4
*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(
4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3
*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^
3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^
4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*
b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*
c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 -
 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b
^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^
3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 1
2*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b
^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x
)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6
+ 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*
e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^
2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*
a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^
3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2
 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c
^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^
5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 9
9*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*
a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5
*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3
+ 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d
 + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*
```

b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^
3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c -
 b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2
*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2
*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b
*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8
)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/
c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d
^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c
^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^
5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*
d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) -
3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^
2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)
 + (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^
3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 -
4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*
a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5
*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e
^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 -
 b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24
*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3
- 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b
^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2
 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3
*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*
b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^
4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5
*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^
4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3
))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*
e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*
c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*
a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60
*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*
d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c
^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7
 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*
a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2
 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c

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^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^
5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 9
9*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*
a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c
^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*
c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7
*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5
 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a
^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 -
 b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*
b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^
2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*
a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a
*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3
*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(1
6*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((8*(4*a^3*c^6*e^5 + a*b^4*c^
4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3
*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*
c^6*d^2*e^3))/c^5 + (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3
*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^
3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^
2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*
c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) +
 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2
*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a
^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^
9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c
^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^
6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^
4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 6
3*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4
*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*
c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 -
60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^
4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2
*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c
^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 2
0*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2
*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*
e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*
b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2
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*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (16*(a^4*b^3*e^8 - 2*a^3*b^4*d*e^7 + 2*a^5*c^2*d*e^7 + a^2*b^5*d^2*e^6 + 2*a^3*c^4*d^5*e^3 + 4*a^4*c^3*d^3*e^5 - 2*a^5*b*c*e^8 - 4*a^2*b^2*c^3*d^5*e^3 + 6*a^2*b^3*c^2*d^4*e^4 + 2*a^4*b^2*c*d*e^7 + a^2*b*c^4*d^6*e^2 - 4*a^2*b^4*c*d^3*e^5 - 4*a^3*b*c^3*d^4*e^4 + 4*a^3*b^3*c*d^2*e^6 - 7*a^4*b*c^2*d^2*e^6))/c^5))*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i + atan((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6

```
*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 +
 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c
^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d
*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^
7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3
- a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e
 - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b
^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c
^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))
)^(1/2)*1i - (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^
2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b
*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d + e*x)
^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2
*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3
*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) -
11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*
b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^
3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*
e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/
2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-
(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e -
18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*
(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*
c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*
c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 +
 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*
a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(
d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c
^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c
^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2
*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^
2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56
*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^
5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^
3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a
^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^
2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/
```

2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b

```
^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4
*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3
+ 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d
*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b
^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*
b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e
 - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b
^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 +
b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4
*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d
*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 -
20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d
^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*
a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 4
2*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/
2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2
+ 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e +
30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2
 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^3*c^2*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(
2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (16*(a^4*b^3*e^8 - 2*a^3*b
^4*d*e^7 + 2*a^5*c^2*d*e^7 + a^2*b^5*d^2*e^6 + 2*a^3*c^4*d^5*e^3 + 4*a^4*c^
3*d^3*e^5 - 2*a^5*b*c*e^8 - 4*a^2*b^2*c^3*d^5*e^3 + 6*a^2*b^3*c^2*d^4*e^4 +
 2*a^4*b^2*c*d*e^7 + a^2*b*c^4*d^6*e^2 - 4*a^2*b^4*c*d^3*e^5 - 4*a^3*b*c^3*
d^4*e^4 + 4*a^3*b^3*c*d^2*e^6 - 7*a^4*b*c^2*d^2*e^6))/c^5))*(-(b^9*e^3 + 8*
a^3*c^6*d^3 - b^6*c^3*d^3 + b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*
d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^
5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 - a^3*c^3*e^3*(-(4*a*c - b^
2)^3)^(1/2) - b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8
*c*d*e^2 + 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^4*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3
*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e - 3*b^5*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*
c^4*d*e^2 + 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^4*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^
3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)
^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i + (d + e*x)^(1/2
)*((2*d^2)/(c*e) - (2*(a*e^3 - b*d*e^2 + c*d^2*e))/(c^2*e^2) + (((4*d)/(c*e
) + (2*(b*e^2 - 2*c*d*e))/(c^2*e^2))*(b*e^2 - 2*c*d*e))/(c*e)) - ((4*d)/(3*
c*e) + (2*(b*e^2 - 2*c*d*e))/(3*c^2*e^2))*(d + e*x)^(3/2) + (2*(d + e*x)^(5
/2))/(5*c*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.345 $\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2} dx$

Optimal. Leaf size=453

$$\frac{\sqrt{2}\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Rubi [A] time = 4.53, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {824, 826, 1166, 208}

$$\frac{\sqrt{2}\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(bc\left(cd^2-e\left(2d\sqrt{b^2-4ac}+3ae\right)\right)-c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}+4ae\right)\right)-b^2e\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2\sqrt{d+ex}(cd-be)}{c^2}+\frac{2(d+ex)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(c*d - b*e)*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c) + (Sqrt[2]*(b
^3*e^2 - b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d
 - 3*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - (Sqrt[2]*(b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c
*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2(d+ex)^{3/2}}{3c} + \frac{\int \frac{\sqrt{d+ex}\,(-ae+(cd-be)x)}{a+bx+cx^2}\,dx}{c}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{\int \frac{-ae(2cd-be)+\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx}{c^2}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{2\,\mathrm{Subst}\left(\int \frac{-ae^2(2cd-be)-d\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)+\left(c^2d^2+b^2e^2-ce \cdots\right.}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} \cdots\right.}{c^2}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} - \frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-4ac}\,e^2-c \cdots\right.\right.}{\cdots}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{\sqrt{2}\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-4ac}\,e^2 \cdots\right.\right.}{c^5 \cdots}$$

Mathematica [A] time = 1.49, size = 779, normalized size = 1.72

$$\frac{2\left(\frac{3\sqrt{c}\,d\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{3\sqrt{c}\,d\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{3\left(3c^2de\left(d\sqrt{b^2-4ac}-2ae-bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)+b^2e^3\left(\sqrt{b^2-4ac}-b\right)+2c^3d^3\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{3\left(-3c^2de\left(d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(3b\left(d\sqrt{b^2-4ac}+ae\right)+ae\sqrt{b^2-4ac}+3b^2d\right)-b^2e^3\left(\sqrt{b^2-4ac}+b\right)+2c^3d^3\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-3e\sqrt{d+ex}\,(be-2cd)+ce(d+ex)^{3/2}-3cde\sqrt{d+ex}\right)}{3c^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(-3*c*d*e*Sqrt[d + e*x] - 3*e*(-2*c*d + b*e)*Sqrt[d + e*x] + c*e*(d + e*
x)^(3/2) + (3*Sqrt[c]*d*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*
(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (3*(2*c^3*d^3 + b^2*(-b + Sqrt[b^2 -
4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*d - 2*a*e) + c*e^2*(3*b
^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])
 - (3*Sqrt[c]*d*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + S
qrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]) + (3*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^
3 - 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a*Sqrt
[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]
*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/(3*c^2*e)
```

IntegrateAlgebraic [C] time = 2.02, size = 599, normalized size = 1.32

$$\frac{\left(\sqrt{2}c^2d^2\sqrt{4ac-b^2}-2\sqrt{2}bcde\sqrt{4ac-b^2}+\sqrt{2}b^2e^2\sqrt{4ac-b^2}-\sqrt{2}ace^2\sqrt{4ac-b^2}-3i\sqrt{2}abce^2+4i\sqrt{2}ac^2de+i\sqrt{2}b^3e^2-2i\sqrt{2}b^2cde+i\sqrt{2}bc^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{c^{5/2}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(\sqrt{2}c^2d^2\sqrt{4ac-b^2}-2\sqrt{2}bcde\sqrt{4ac-b^2}+\sqrt{2}b^2e^2\sqrt{4ac-b^2}-\sqrt{2}ace^2\sqrt{4ac-b^2}+3i\sqrt{2}abce^2-4i\sqrt{2}ac^2de-i\sqrt{2}b^3e^2+2i\sqrt{2}b^2cde-i\sqrt{2}bc^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{c^{5/2}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}+\frac{2\sqrt{d+ex}\,(-3be+c(d+ex)+3cd)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(3*c*d - 3*b*e + c*(d + e*x)))/(3*c^2) + ((I*Sqrt[2]*b*c^2
*d^2 + Sqrt[2]*c^2*Sqrt[-b^2 + 4*a*c]*d^2 - (2*I)*Sqrt[2]*b^2*c*d*e + (4*I)
*Sqrt[2]*a*c^2*d*e - 2*Sqrt[2]*b*c*Sqrt[-b^2 + 4*a*c]*d*e + I*Sqrt[2]*b^3*e
^2 - (3*I)*Sqrt[2]*a*b*c*e^2 + Sqrt[2]*b^2*Sqrt[-b^2 + 4*a*c]*e^2 - Sqrt[2]
*a*c*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2
*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(5/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*
c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((-I)*Sqrt[2]*b*c^2*d^2 + Sqrt[2]*c
^2*Sqrt[-b^2 + 4*a*c]*d^2 + (2*I)*Sqrt[2]*b^2*c*d*e - (4*I)*Sqrt[2]*a*c^2*d
*e - 2*Sqrt[2]*b*c*Sqrt[-b^2 + 4*a*c]*d*e - I*Sqrt[2]*b^3*e^2 + (3*I)*Sqrt[
2]*a*b*c*e^2 + Sqrt[2]*b^2*Sqrt[-b^2 + 4*a*c]*e^2 - Sqrt[2]*a*c*Sqrt[-b^2 +
 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*S
qrt[-b^2 + 4*a*c]*e]])/(c^(5/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sq
rt[-b^2 + 4*a*c]*e])
```

fricas [B] time = 1.13, size = 5572, normalized size = 12.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)
*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*e^3 + (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^
6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3
 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*
a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)*((b^3
*c^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(
2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2
+ 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4
*a^3*b*c^3)*e^4 - ((b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2
*c^7)*e)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 1
0*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*
c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^
2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*
a*c^11)))*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3
*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*
e^3 + (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e +
 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^
3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^
4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c
^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^
6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2
*c^3 - 3*a^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^
4*c - 3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2
*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^5)*sqrt(e*x + d)) - 3*sqrt(2)*c^2*sqrt(((b^
2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c
^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^5 - 4*
a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*
a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^
5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*
e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^3*c^4 - 4*a*b*c^5)*d^4 - (4*b
^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 1
2*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4
)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 - ((b^3*c^6
```

```
- 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*sqrt((b^2*c^6*d^6 -
 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4
*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2
- 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c
^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^2*c^3 - 2
*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^
2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^5 - 4*a*c^6)*sq
rt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5
 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^
3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(
b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(
b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a^2*c^4)*d^4*e + 2*
(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^4*c - 3*a^2*b^2*c^2 - a^3*c^
3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2
)*e^5)*sqrt(e*x + d)) + 3*sqrt(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^
3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5
 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 - (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6
*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e
^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 -
20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2
 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
*log(sqrt(2)*((b^3*c^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^
2*c^5)*d^3*e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6
*c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c +
13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 + ((b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6
*a*b^2*c^6 + 8*a^2*c^7)*e)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e
+ 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b
^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c
^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*
c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e
^6)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*
a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3
*c + 5*a^2*b*c^2)*e^3 - (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5
- a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10
*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c
^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*(a*b*
c^4*d^5 - (4*a*b^2*c^3 - 3*a^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d
^3*e^2 - 2*(2*a*b^4*c - 3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b
*c^2)*d*e^4 - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^5)*sqrt(e*x + d)) - 3*sqr
t(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*
(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e
^3 - (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e +
3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3
*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4
 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^
4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6
)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^3*c^4 - 4*a
*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(2*b^5*c^2
- 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2 + 37*a^2*b
^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3
)*e^4 + ((b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*s
qrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^
5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e
^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*
```

```
(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))*
sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c -
4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 - (b^2
*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*
c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19
*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*
c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a
^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^4*c - 3*a^
2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2*b^4 - 3*a
^3*b^2*c + a^4*c^2)*e^5)*sqrt(e*x + d)) - 4*(c*e*x + 4*c*d - 3*b*e)*sqrt(e*
x + d))/c^2
```

giac [B] time = 0.46, size = 978, normalized size = 2.16

$$\frac{\left(\left((b^2c^2-4ac^3)d^2e-2(b^3c-4abc^2)de^2+(b^4-5ab^2c+4a^2c^2)e^3\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,c^2-2\left(\sqrt{b^2-4ac}\,c^4d^3-2\sqrt{b^2-4ac}\,bc^3d^2e-\sqrt{b^2-4ac}\,abc^2e^3+(b^2c^2+ac^3)\sqrt{b^2-4ac}\,de^2\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,|c|+\left(2bc^5d^3-(5b^2c^4-8ac^5)d^2e+2(2b^3c^3-5abc^4)de^2-(b^4c^2-3ab^2c^3)e^3\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2c^4d-bc^3e+\sqrt{-4(c^4d^2-bc^3de+ac^3e^2)c^4+(2c^4d-bc^3e)^2}}{c^4}}}\right)}{4\left(\sqrt{b^2-4ac}\,c^5d^2-\sqrt{b^2-4ac}\,bc^4de+\sqrt{b^2-4ac}\,ac^4e^2\right)c^2}-\frac{\left(\left((b^2c^2-4ac^3)d^2e-2(b^3c-4abc^2)de^2+(b^4-5ab^2c+4a^2c^2)e^3\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,c^2+2\left(\sqrt{b^2-4ac}\,c^4d^3-2\sqrt{b^2-4ac}\,bc^3d^2e-\sqrt{b^2-4ac}\,abc^2e^3+(b^2c^2+ac^3)\sqrt{b^2-4ac}\,de^2\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,|c|+\left(2bc^5d^3-(5b^2c^4-8ac^5)d^2e+2(2b^3c^3-5abc^4)de^2-(b^4c^2-3ab^2c^3)e^3\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2c^4d-bc^3e-\sqrt{-4(c^4d^2-bc^3de+ac^3e^2)c^4+(2c^4d-bc^3e)^2}}{c^4}}}\right)}{4\left(\sqrt{b^2-4ac}\,c^5d^2-\sqrt{b^2-4ac}\,bc^4de+\sqrt{b^2-4ac}\,ac^4e^2\right)c^2}+\frac{2\left((xe+d)^{\frac{3}{2}}c^2+3\sqrt{xe+d}\,c^2d-3\sqrt{xe+d}\,bce\right)}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(((b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*
b^2*c + 4*a^2*c^2)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^
2 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b
^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^5*d^3 - (5*b^2*c
^4 - 8*a*c^5)*d^2*e + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^2 - (b^4*c^2 - 3*a*b^2*
c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1
/2)*sqrt(x*e + d)/sqrt(-(2*c^4*d - b*c^3*e + sqrt(-4*(c^4*d^2 - b*c^3*d*e +
 a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((sqrt(b^2 - 4*a*c)*c^5*d^2
 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2) - 1/4*((
(b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^2*c
+ 4*a^2*c^2)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2 + 2*
(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4
*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*
d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^5*d^3 - (5*b^2*c^4 - 8
*a*c^5)*d^2*e + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^2 - (b^4*c^2 - 3*a*b^2*c^3)*e
^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sq
rt(x*e + d)/sqrt(-(2*c^4*d - b*c^3*e - sqrt(-4*(c^4*d^2 - b*c^3*d*e + a*c^3
*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((sqrt(b^2 - 4*a*c)*c^5*d^2 - sqr
t(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2) + 2/3*((x*e +
d)^(3/2)*c^2 + 3*sqrt(x*e + d)*c^2*d - 3*sqrt(x*e + d)*b*c*e)/c^3
```

maple [B] time = 0.05, size = 1714, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/3*(e*x+d)^(3/2)/c-2/c^2*b*e*(e*x+d)^(1/2)+2/c*d*(e*x+d)^(1/2)-3/c/(-(4*a*
c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*c)*a*b*e^3+4/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*a*d*e^2+1/c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/(
(-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*e^3-2/c/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
```

```
anh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c
)*b^2*d*e^2+1/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2*e+1/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*c)*a*e^2-1/c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b^2*e^2+2/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*c)*b*d*e-2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))
*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*c)*d^2-3/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a
*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*b*e^3+4/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)
/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2
)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*d*e^2+1/c^2/(-(4*a*c-
b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*
b^3*e^3-2/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*b^2*d*e^2+1/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c
*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*
c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2*e-1/c*2^(1/2)/((b*e-2*c*d+(
-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*e^2+1/c^2*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*e^2-2/c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c
-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d*e+2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*c)*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}x}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x/(c*x^2 + b*x + a), x)
```

mupad [B] time = 4.72, size = 13841, normalized size = 30.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(3/2))/(3*c) - ((2*d)/c + (2*(b*e - 2*c*d))/c^2)*(d + e*x)^(1/
2) - atan(((((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*
c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5
*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*
c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3
- 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2
+ 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54
*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d
```

e^2(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1
/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(
-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) +
24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4
*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 -
8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c
^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b
^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b
*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e
^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c
^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c
e^3(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 3
6*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5
+ 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*
b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x
)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3
)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2
e^3(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*
a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 +
b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e
^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3
*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d
*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b
^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*
e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*
e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e
^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3
*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d
^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3
- b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e +
25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3
- 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2
*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2
+ 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2
)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c
*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*d*e^7 - a*c^4*d^6*e^2 - a^2*
c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c^3*d^5*e^3 + 4*a*b^3*c*d^3*e^5 - 6*a
*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5 - 5*a^2*b^2*c*d^2*e^6))/c^3 + (((8*(
a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c

^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*

```
a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*
e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a
^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*
c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3
*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^
2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c
*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^
3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d
*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c
- b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - atan(
((((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4
- b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 +
 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 -
b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b
*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^
2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b
^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^
3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c
^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*
a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c
^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3
- 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*
c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^
4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3
*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c
^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2
 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3
*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e
^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*
(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
+ 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^
4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))^(1/2)*1i - (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6
*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^
2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x)^(1/2)*(-
(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) +
24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*
c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*
d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 -
8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c
^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c
```

- b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*d*e^7 - a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c^3*d^5*e^3 + 4*a*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5 - 5*a^2*b^2*c*d^2*e^6))/c^3 + (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3

```
*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^
2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b
^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6
*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24
*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^
2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*
d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*
c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^
4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4)
)/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 - a^2*c^2*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 3*a*b^2*c*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a
^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b
^4*c^5 - 8*a*b^2*c^6)))^(1/2)))*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 -
b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 + b*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a
^2*b^3*c^2*e^3 - a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b
^6*c*d*e^2 + 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e +
24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e + 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 - 3*b
^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^
(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
[Out] Timed out
```

3.346 $\int \frac{(d+ex)^{3/2}}{a+bx+cx^2} \, dx$

Optimal. Leaf size=322

$$-\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(-2ce\left(d \cdots\right.\right.}{}$$

Rubi [A] time = 1.24, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {703, 826, 1166, 208}

$$-\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2e\sqrt{d+ex}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*e*Sqrt[d + e*x])/c - (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c
^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}}{c} + \frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{2\,\text{Subst}\left(\int \frac{-de(2cd-be)+e\left(cd^2-ae^2\right)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\text{Subst}\left(\int \frac{}{-\frac{1}{2}}\right.}{c\sqrt{b^2-4ac}}$$

$$= \frac{2e\sqrt{d+ex}}{c} - \frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\right.}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.70, size = 317, normalized size = 0.98

$$\frac{\frac{\sqrt{2}\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+2\sqrt{c}\,e\sqrt{d+ex}}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[d + e*x] + (Sqrt[2]*(-2*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*
c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/c^(3/2)
```

IntegrateAlgebraic [C] time = 0.00, size = 436, normalized size = 1.35

$$\frac{\left(2\sqrt{2}cde\sqrt{4ac-b^2}-\sqrt{2}be^2\sqrt{4ac-b^2}+2i\sqrt{2}ace^2-i\sqrt{2}b^2e^2+2i\sqrt{2}bcde-2i\sqrt{2}c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{c^{3/2}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(2\sqrt{2}cde\sqrt{4ac-b^2}-\sqrt{2}be^2\sqrt{4ac-b^2}-2i\sqrt{2}ace^2+i\sqrt{2}b^2e^2-2i\sqrt{2}bcde+2i\sqrt{2}c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{c^{3/2}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}+\frac{2e\sqrt{d+ex}}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*e*Sqrt[d + e*x])/c + (((-2*I)*Sqrt[2]*c^2*d^2 + (2*I)*Sqrt[2]*b*c*d*e +
2*Sqrt[2]*c*Sqrt[-b^2 + 4*a*c]*d*e - I*Sqrt[2]*b^2*e^2 + (2*I)*Sqrt[2]*a*c*
e^2 - Sqrt[2]*b*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(3/2)*Sqrt[-b^2 + 4*a*
c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((2*I)*Sqrt[2]*c^2*d^2 -
 (2*I)*Sqrt[2]*b*c*d*e + 2*Sqrt[2]*c*Sqrt[-b^2 + 4*a*c]*d*e + I*Sqrt[2]*b^2
*e^2 - (2*I)*Sqrt[2]*a*c*e^2 - Sqrt[2]*b*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/
(c^(3/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])
```

fricas [B] time = 0.54, size = 2770, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2
 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3
*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b
^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l
og(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (
b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*
a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3
)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b
^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3
+ 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - sqrt(2)*c*sqrt((2*
c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 +
 (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2
- 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2
)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-sqrt(2)*(3*(b^2*c^2
- 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2
*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4
*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c -
a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqr
t((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*
e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2
*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6
*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b
^2 - a^2*c)*e^5)*sqrt(e*x + d)) + sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e
 + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sq
rt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*
(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(
b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4
- 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*
e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 -
2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d
^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4
)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4
- 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^
2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*
x + d)) - sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log(-sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^
3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3
 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*
a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^
6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c
^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 -
 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d
*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*
a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 -
(b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - 4*sqrt(e*x +
d)*e)/c
```

giac [B] time = 0.42, size = 783, normalized size = 2.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*e/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)
*(2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*c^2 - 2*(sqrt(b^2 - 4*a*
c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)
*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) - (4*c^5*d^3 - 6*b
*c^4*d^2*e + 4*(b^2*c^3 - a*c^4)*d*e^2 - (b^3*c^2 - 2*a*b*c^3)*e^3)*sqrt(-4
*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)
/sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^
2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^
3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*c^
2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b
^2 - 4*a*c)*a*c^2*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs
(c) - (4*c^5*d^3 - 6*b*c^4*d^2*e + 4*(b^2*c^3 - a*c^4)*d*e^2 - (b^3*c^2 - 2
*a*b*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*s
qrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e
+ a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 -
sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2)
```

maple [B] time = 0.04, size = 1138, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2*e*(e*x+d)^(1/2)/c+2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*
c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*e^3-1/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*e^3+2/(-(4*a*c-
b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arc
tanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*
c)*b*d*e^2-2*e*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-
b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^2+1/c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)*b*e^2-2*e*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*c)*d+2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*a*e^3-1/c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2
)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/
2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*e^3+2/(-(4*a*c-b^2
)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan(
(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d
*e^2-2*e*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)
^(1/2))*c)^(1/2)*c)*d^2-1/c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c
)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*
c)^(1/2)*c)*b*e^2+2*e*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/
2)*c)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a), x)
```

mupad [B] time = 4.44, size = 8334, normalized size = 25.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(a + b*x + c*x^2),x)
```

```
[Out] (2*e*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c
^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(d
+ e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(
b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*
b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*
x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*
c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^
2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*1i - (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*
e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*(-(b^5*e^3
 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e
^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2
*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^
2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 -
 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a
^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a
*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*
d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a
^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2
*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3
 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e
^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2
*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^
3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 -
 4*a*b*c^3*d*e^4))/c - (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*
c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*
```

```
d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*
e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e +
 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4
*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^
2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*
e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^
6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*
c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*
d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*
e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e +
 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*
a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*
(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e -
3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*
(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18
*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d +
 e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4
*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b
*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e -
3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) - (16*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 - 5
*b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b*c*
d^2*e^6))/c))*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*
c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i - atan(((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3
*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(d + e*x)^(1/2)*(-(
b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*
b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3
 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^
4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3
 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a
*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(
2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^
6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*
b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(
b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
```

12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)

```
 + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18
*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (16*(2*
c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 - 5*b*c^2*d^4*e^4 +
 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b*c*d^2*e^6))/c))*(-
(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 + b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e + 3*c^2*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 - a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b
^4*c*d*e^2 - 12*a*b*c^3*d^2*e - 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a
*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.347 $\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx$

Optimal. Leaf size=340

$$-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4\right.\right.}{}$$

Rubi [A] time = 1.58, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {897, 1287, 206, 1166, 208}

$$-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a - (Sqrt[2]*(a*Sqrt[b^2 - 4*a*
c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*S
qrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2
]*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 +
 a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]])/(a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{d^2e}{a\left(d-x^2\right)}+\frac{e\left(d\left(cd^2-bde+ae^2\right)-\left(cd^2-ae^2\right)x^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{d\left(cd^2-bde+ae^2\right)+\left(-cd^2+ae^2\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\ldots}\right.}{a}$$

$$= -\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\right)\right)\text{S}\ldots}{a\sqrt{b^2-4a\ldots}}$$

$$= -\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\ldots\right.\right.}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-\ldots}\right.}}$$

Mathematica [A] time = 1.16, size = 331, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\left(cd\left(d\sqrt{b^2-4ac}+4ae\right)-ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[2]*(-(a*Sqrt[b^2 - 4*a*c
]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqr
t[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]
*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) - b*(c*d^2
 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]))/a
```

IntegrateAlgebraic [C] time = 1.40, size = 383, normalized size = 1.13

$$-\frac{\sqrt{2}\left(cd^2\sqrt{4ac-b^2}-ae^2\sqrt{4ac-b^2}-iabe^2+4iacde-ibcd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{a\sqrt{c}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}-\frac{\sqrt{2}\left(cd^2\sqrt{4ac-b^2}-ae^2\sqrt{4ac-b^2}+iabe^2-4iacde+ibcd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{a\sqrt{c}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

[Out] -((Sqrt[2]*((-I)*b*c*d^2 + c*Sqrt[-b^2 + 4*a*c]*d^2 + (4*I)*a*c*d*e - I*a*b*e^2 - a*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e])) - (Sqrt[2]*(I*b*c*d^2 + c*Sqrt[-b^2 + 4*a*c]*d^2 - (4*I)*a*c*d*e + I*a*b*e^2 - a*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]) - (2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a

fricas [B] time = 7.61, size = 5167, normalized size = 15.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")

[Out] [-1/2*(sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) + sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*

```
a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sq
rt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a
^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e
^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^
3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^
2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - 2
*d^(3/2)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x))/a, -1/2*(sqrt(2)*a*s
qrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (
a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*
e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c
^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*
b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*
b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e
 + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*
b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (
b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^
2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6
)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a
*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*
d^4*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^
2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6
 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^
4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(-sqrt
(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4
*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)
*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^
3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d
^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a
^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c
*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*
c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*
d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) + sqrt(2)*a*sqrt(
-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*
b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2
+ 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3))
)/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*
c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^
2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9
*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*
c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*
c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^
5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a
^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c
*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*
e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*
e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6
*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 +
a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(-sqrt(2)*
((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3
*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqr
t((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 -
6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e
 - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c
^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3
*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c -
4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^
4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - 4*sqrt(-d)*d*arctan
(sqrt(e*x + d)*sqrt(-d)/d))/a]
```

giac [B] time = 0.41, size = 822, normalized size = 2.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*d^2*arctan(sqrt(x*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(((b^2*c - 4*a*c^2)
*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*
c)*e)*a^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e
+ sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)
*c)*e)*abs(a) - (2*a^2*b*c^2*d^3 + 6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2
*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*ar
ctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c*d^2 -
a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*
a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*
abs(a)*abs(c)) + 1/4*(((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*sqr
t(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a^2 + 2*(sqrt(b^2 - 4*a*c)*a*
c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sq
rt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c^2*d^3 +
6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^2*
d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt
(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d -
 a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2
*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*abs(a)*abs(c))
```

maple [B] time = 0.04, size = 944, normalized size = 2.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x)
```

```
[Out] -2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/a+e^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^
(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b-4*e^2*c/(-(4*
a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)
*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1
/2)*c)*d+e/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^
2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2-e^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*
e^2)^(1/2))*c)^(1/2)*c)+1/a*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2)
)*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1
/2))*c)^(1/2)*c)*d^2+e^3/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*
a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4
*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b-4*e^2*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2
)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/
2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d+e/a*c/(-(4*a*c-b^2)*
e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e
*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2
+e^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^
(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)-1/a*c*2^(1/
2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1
/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x), x)
```

mupad [B] time = 8.16, size = 20897, normalized size = 61.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x)
```

```
[Out] atan(((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c -
8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c
*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*
c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2
*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d
^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e
 + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(
512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5
*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4
*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^
8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9
 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11
 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3
*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8
 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2
*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*
b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*
b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d
^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b
^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))
^(1/2) + 96*a*c^5*d^7*e^8 + 32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*
a^3*c^3*d^3*e^12 - 32*b^2*c^4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5
*e^10 - 96*a^2*b^2*c^2*d^3*e^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14
 - 288*a*b^2*c^3*d^5*e^10 - 160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^1
1 + 32*a^2*b^3*c*d^2*e^13 + 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a
^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^
4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^
6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 +
 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))
*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*
b^2*c^2)))^(1/2)*1i + (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e
^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^
3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*
c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d
*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e -
```

3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c^4*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*b^3*c^3*d^6*e^9 - 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c^4*d^6*e^9 + 32*a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d^4*e^11 - 1280*a^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i)/((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*

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(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e
^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^
3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^
14 + 672*a^2*c^4*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*
b^3*c^3*d^6*e^9 - 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c
^4*d^6*e^9 + 32*a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d
^4*e^11 - 1280*a^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d
^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*
e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a
^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c
*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3
*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3
+ a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4
*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*
(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - (((b^4*c*d^3 - a^2*b^3*e
^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 2
4*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a
^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4
*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6
*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^
2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*
b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a
^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*
c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*
e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9
+ 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^8 - 384*a^4*c^4*d^2*e^10 + 96*
a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9 + 32*a^2*b^4*c^2*d^2*e^10 - 3
2*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11 + 512*a^3*b*c^4*d^3*e^9 - 32*
a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^
13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b
^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^
3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^
3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3
*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d
^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2
+ 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*
c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/
(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 96*a*c^5*d^7*e^8 + 32
*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*a^3*c^3*d^3*e^12 - 32*b^2*c^4*
d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5*e^10 - 96*a^2*b^2*c^2*d^3*e^1
2 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14 - 288*a*b^2*c^3*d^5*e^10 - 16
0*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^11 + 32*a^2*b^3*c*d^2*e^13 + 12
8*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 2
56*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4
*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11
- 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*
a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8
*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*
c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^
2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2
*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 192*c^4*d^8
*e^10 + 448*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 512*b*c^3*d^7*e^11 - 128*b
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^3*c*d^5*e^13 + 320*a^2*c^2*d^4*e^14 + 448*b^2*c^2*d^6*e^12 - 768*a*b*c^2*d
^5*e^13 + 320*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15))*((b^4*c*d^3 - a^2*b
^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3
 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e +
 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2i
+ atan(((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c
 - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b
*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^
4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a
^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2
*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2
*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)
*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c
^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c
^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*
e^8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e
^9 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^
11 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a
^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e
^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a
^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*
a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^
2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c
*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3
)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)
))^(1/2) + 96*a*c^5*d^7*e^8 + 32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 73
6*a^3*c^3*d^3*e^12 - 32*b^2*c^4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d
^5*e^10 - 96*a^2*b^2*c^2*d^3*e^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^
14 - 288*a*b^2*c^3*d^5*e^10 - 160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e
^11 + 32*a^2*b^3*c*d^2*e^13 + 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32
*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*
b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*
d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13
 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14
))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^
3*b^2*c^2)))^(1/2)*1i + (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c
*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*
c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^
2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2
*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e
 + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*
d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)
*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
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12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c^4*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 32*b^3*c^3*d^6*e^9 - 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b*c^4*d^6*e^9 + 32*a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2*d^4*e^11 - 1280*a^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i)/((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 96*a*c^5*d^7*e^8 - 32*a^4*c^2*d*e^14 + 672*a^2*c^4*d^5*e^10 + 736*a^3*c^3*d^3*e^12 + 32*b^2*c^4*d^7*e^8 + 3

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2*b^3*c^3*d^6*e^9 - 64*b^4*c^2*d^5*e^10 + 96*a^2*b^2*c^2*d^3*e^12 - 256*a*b
*c^4*d^6*e^9 + 32*a^3*b^2*c*d*e^14 + 288*a*b^2*c^3*d^5*e^10 + 160*a*b^3*c^2
*d^4*e^11 - 1280*a^2*b*c^3*d^4*e^11 - 32*a^2*b^3*c*d^2*e^13 - 128*a^3*b*c^2
*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^
6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128
*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b
*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d
^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^
3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 +
 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(
2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - (((b^4*c*d^3 - a^2*b^3
*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 -
 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*
b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6
*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(((b
^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*
c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 - a
^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^
3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*a*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16
*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^
4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^
2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^
9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^8 - 384*a^4*c^4*d^2*e^10 + 9
6*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9 + 32*a^2*b^4*c^2*d^2*e^10 -
 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11 + 512*a^3*b*c^4*d^3*e^9 - 3
2*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*
e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64
*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*
b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*
c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c
^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3
*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^
2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e + 3*
a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2
)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 96*a*c^5*d^7*e^8 +
32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*a^3*c^3*d^3*e^12 - 32*b^2*c^
4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5*e^10 - 96*a^2*b^2*c^2*d^3*e
^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14 - 288*a*b^2*c^3*d^5*e^10 -
160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^11 + 32*a^2*b^3*c*d^2*e^13 +
128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a^4*c*e^16 + 96*c^5*d^8*e^8 -
 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^4*c*d^4*e^12 + 256*a^2*c^3*d
^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^6*e^10 - 256*b^3*c^2*d^5*e^1
1 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 + 256*a*b^2*c^2*d^4*e^12 - 38
4*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))*((b^4*c*d^3 - a^2*b^3*e^3 +
 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^
3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*
d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b
^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 192*c^4*d
^8*e^10 + 448*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 512*b*c^3*d^7*e^11 - 128
*b^3*c*d^5*e^13 + 320*a^2*c^2*d^4*e^14 + 448*b^2*c^2*d^6*e^12 - 768*a*b*c^2
*d^5*e^13 + 320*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15))*((b^4*c*d^3 - a^2
*b^3*e^3 + 8*a^2*c^3*d^3 - a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d
^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 - b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
3*a*b^3*c*d^2*e + 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e
```

+ 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2
i - (2*atanh((64*a^3*c*e^16*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10
+ 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*
c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*
e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d
^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 102
4*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*
c^4*d^9*e^9)/a) + (576*c^5*d^8*e^8*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*c^5*d^
10*e^8 + 2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 1536*b*c^4*d^9*e^9 + 192
0*a^2*c^3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c^3*d^8*e^10 + 384*b^3*
c^2*d^7*e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^3*d^9*e^9)/a - (192*b^
4*c^2*d^8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3*c*d^5*e^13 - 256*a^3*
b*c*d^3*e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^2*d^5*e^13 + 384*a^2*b
^2*c*d^4*e^14) + (2304*c^4*d^6*e^10*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*
d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 -
192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*
c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b
^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a
^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 -
(1536*b*c^4*d^9*e^9)/a) - (128*b^2*c^4*d^8*e^8*(d^3)^(1/2)*(d + e*x)^(1/2))
/(576*a*c^5*d^10*e^8 + 64*a^5*c*d^2*e^16 + 2304*a^2*c^4*d^8*e^10 + 1920*a^3
*c^3*d^6*e^12 + 256*a^4*c^2*d^4*e^14 - 128*b^2*c^4*d^10*e^8 + 320*b^3*c^3*d
^9*e^9 - 192*b^4*c^2*d^8*e^10 + 640*a^2*b^2*c^2*d^6*e^12 - 1536*a*b*c^4*d^9
*e^9 - 256*a^4*b*c*d^3*e^15 + 640*a*b^2*c^3*d^8*e^10 + 384*a*b^3*c^2*d^7*e^
11 - 3328*a^2*b*c^3*d^7*e^11 - 192*a^2*b^3*c*d^5*e^13 - 1024*a^3*b*c^2*d^5*
e^13 + 384*a^3*b^2*c*d^4*e^14) + (320*b^3*c^3*d^7*e^9*(d^3)^(1/2)*(d + e*x)
^(1/2))/(576*a*c^5*d^10*e^8 + 64*a^5*c*d^2*e^16 + 2304*a^2*c^4*d^8*e^10 + 1
920*a^3*c^3*d^6*e^12 + 256*a^4*c^2*d^4*e^14 - 128*b^2*c^4*d^10*e^8 + 320*b^
3*c^3*d^9*e^9 - 192*b^4*c^2*d^8*e^10 + 640*a^2*b^2*c^2*d^6*e^12 - 1536*a*b*
c^4*d^9*e^9 - 256*a^4*b*c*d^3*e^15 + 640*a*b^2*c^3*d^8*e^10 + 384*a*b^3*c^2
*d^7*e^11 - 3328*a^2*b*c^3*d^7*e^11 - 192*a^2*b^3*c*d^5*e^13 - 1024*a^3*b*c
^2*d^5*e^13 + 384*a^3*b^2*c*d^4*e^14) - (192*b^4*c^2*d^6*e^10*(d^3)^(1/2)*(
d + e*x)^(1/2))/(576*a*c^5*d^10*e^8 + 64*a^5*c*d^2*e^16 + 2304*a^2*c^4*d^8*
e^10 + 1920*a^3*c^3*d^6*e^12 + 256*a^4*c^2*d^4*e^14 - 128*b^2*c^4*d^10*e^8
+ 320*b^3*c^3*d^9*e^9 - 192*b^4*c^2*d^8*e^10 + 640*a^2*b^2*c^2*d^6*e^12 - 1
536*a*b*c^4*d^9*e^9 - 256*a^4*b*c*d^3*e^15 + 640*a*b^2*c^3*d^8*e^10 + 384*a
*b^3*c^2*d^7*e^11 - 3328*a^2*b*c^3*d^7*e^11 - 192*a^2*b^3*c*d^5*e^13 - 1024
*a^3*b*c^2*d^5*e^13 + 384*a^3*b^2*c*d^4*e^14) + (1920*a*c^3*d^4*e^12*(d^3)^
(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*
d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14
+ (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a +
(384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^
9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d
^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (3328*b*c^3*d^5*
e^11*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12
+ 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^
2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8
*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3
*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 38
4*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (192*
b^3*c*d^3*e^13*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3
*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 +
256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b
^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2
+ (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5
*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/
a) + (640*b^2*c^3*d^6*e^10*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*c^5*d^10*e^8 +
2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 1536*b*c^4*d^9*e^9 + 1920*a^2*c^
3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c^3*d^8*e^10 + 384*b^3*c^2*d^7*

```
e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^3*d^9*e^9)/a - (192*b^4*c^2*d^
8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3*c*d^5*e^13 - 256*a^3*b*c*d^3*
e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^2*d^5*e^13 + 384*a^2*b^2*c*d^4
*e^14) + (384*b^3*c^2*d^5*e^11*(d^3)^(1/2)*(d + e*x)^(1/2))/(576*c^5*d^10*e
^8 + 2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 1536*b*c^4*d^9*e^9 + 1920*a^
2*c^3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c^3*d^8*e^10 + 384*b^3*c^2*
d^7*e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^3*d^9*e^9)/a - (192*b^4*c^
2*d^8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3*c*d^5*e^13 - 256*a^3*b*c*
d^3*e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^2*d^5*e^13 + 384*a^2*b^2*c
*d^4*e^14) + (256*a^2*c^2*d^2*e^14*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d
^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 1
92*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c
^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^
2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^
2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (
1536*b*c^4*d^9*e^9)/a) + (640*b^2*c^2*d^4*e^12*(d^3)^(1/2)*(d + e*x)^(1/2))
/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*
d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a
 + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)
/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*
d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*
d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (1536*b*c^4*d^7*e^9*(d^3)^(1/2)*(d + e
*x)^(1/2))/(576*c^5*d^10*e^8 + 2304*a*c^4*d^8*e^10 + 64*a^4*c*d^2*e^16 - 15
36*b*c^4*d^9*e^9 + 1920*a^2*c^3*d^6*e^12 + 256*a^3*c^2*d^4*e^14 + 640*b^2*c
^3*d^8*e^10 + 384*b^3*c^2*d^7*e^11 - (128*b^2*c^4*d^10*e^8)/a + (320*b^3*c^
3*d^9*e^9)/a - (192*b^4*c^2*d^8*e^10)/a - 3328*a*b*c^3*d^7*e^11 - 192*a*b^3
*c*d^5*e^13 - 256*a^3*b*c*d^3*e^15 + 640*a*b^2*c^2*d^6*e^12 - 1024*a^2*b*c^
2*d^5*e^13 + 384*a^2*b^2*c*d^4*e^14) - (256*a^2*b*c*d*e^15*(d^3)^(1/2)*(d +
 e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3*c*d^2*e^16 -
 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^14 + (576*c^5
*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a + (384*b^3*c
^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9*e^9)/a^2 - (
192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*c*d^4*e^14 -
256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) - (1024*a*b*c^2*d^3*e^13*(d^
3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e^12 + 64*a^3
*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^2*c^2*d^4*e^
14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3*d^8*e^10)/a
 + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320*b^3*c^3*d^9
*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13 + 384*a*b^2*
c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a) + (384*a*b^2*c*
d^2*e^14*(d^3)^(1/2)*(d + e*x)^(1/2))/(2304*c^4*d^8*e^10 + 1920*a*c^3*d^6*e
^12 + 64*a^3*c*d^2*e^16 - 3328*b*c^3*d^7*e^11 - 192*b^3*c*d^5*e^13 + 256*a^
2*c^2*d^4*e^14 + (576*c^5*d^10*e^8)/a + 640*b^2*c^2*d^6*e^12 + (640*b^2*c^3
*d^8*e^10)/a + (384*b^3*c^2*d^7*e^11)/a - (128*b^2*c^4*d^10*e^8)/a^2 + (320
*b^3*c^3*d^9*e^9)/a^2 - (192*b^4*c^2*d^8*e^10)/a^2 - 1024*a*b*c^2*d^5*e^13
+ 384*a*b^2*c*d^4*e^14 - 256*a^2*b*c*d^3*e^15 - (1536*b*c^4*d^9*e^9)/a))*(d
^3)^(1/2))/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.348 $\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx$

Optimal. Leaf size=403

$$-\frac{\sqrt{2}\sqrt{c}\left(-2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(d\sqrt{b^2-4ac}-2ae\right)+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}}{}$$

Rubi [A] time = 3.07, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{2}\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\sqrt{c}\left(-bd\left(d\sqrt{b^2-4ac}+2ae\right)+2ae\left(d\sqrt{b^2-4ac}+ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2}-\frac{d\sqrt{d+ex}}{ax}+\frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((d*Sqrt[d + e*x])/(a*x)) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a +
 (2*Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a^2 - (Sqrt[2]*Sq
rt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*e*(Sqr
t[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*(Sqrt[
b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt
[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x^2\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{d^2e^2}{a\left(d-x^2\right)^2}-\frac{de(-bd+2ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-\left((bd-ae)\left(cd^2-bde+ae^2\right)\right)+cd(bd-2ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{-\left((bd-ae)\left(cd^2-bde+ae^2\right)\right)+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^2} + \frac{\left(2d^2e\right)\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\,d\right.}{a}$$

$$= -\frac{d\sqrt{d+ex}}{ax} + \frac{2\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a} +$$

$$= -\frac{d\sqrt{d+ex}}{ax} + \frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{2\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} - \frac{\sqrt{2}\sqrt{c}\left(b^2d^2\right.}{}$$

Mathematica [A] time = 1.54, size = 393, normalized size = 0.98

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(2ae-d\sqrt{b^2-4ac}\right)-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\sqrt{2}\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}+2ae\right)-2ae\left(d\sqrt{b^2-4ac}+ae\right)+2acd^2-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+2\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)-\frac{ad\sqrt{d+ex}}{x}+a\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x])/x) + a*Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + 2*
Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[2]*Sqrt[c]*(-(
b^2*d^2) + b*d*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) + 2*a*(c*d^2 + e*(Sqrt[b^2
- 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*
e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 -
 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(-(b^2*d^2) + 2*a*c*d^2 - 2*a*e*(Sqrt[b^2 -
```

```
 4*a*c]*d + a*e) + b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqrt[2]*Sqrt
[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a^2
```

IntegrateAlgebraic [C] time = 2.00, size = 529, normalized size = 1.31

$$\frac{\left(-2i\sqrt{2}a^2\sqrt{c}e^2+\sqrt{2}b\sqrt{c}d^2\sqrt{4ac-b^2}-2\sqrt{2}a\sqrt{c}de\sqrt{4ac-b^2}+2i\sqrt{2}ab\sqrt{c}de+2i\sqrt{2}ac^{3/2}d^2-i\sqrt{2}b^2\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{a^2\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(2i\sqrt{2}a^2\sqrt{c}e^2+\sqrt{2}b\sqrt{c}d^2\sqrt{4ac-b^2}-2\sqrt{2}a\sqrt{c}de\sqrt{4ac-b^2}-2i\sqrt{2}ab\sqrt{c}de-2i\sqrt{2}ac^{3/2}d^2+i\sqrt{2}b^2\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{a^2\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(2bd^{3/2}-3a\sqrt{d}e\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2}-\frac{d\sqrt{d+ex}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((d*Sqrt[d + e*x])/(a*x)) + (((-I)*Sqrt[2]*b^2*Sqrt[c]*d^2 + (2*I)*Sqrt[2]
*a*c^(3/2)*d^2 + Sqrt[2]*b*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d^2 + (2*I)*Sqrt[2]*a
*b*Sqrt[c]*d*e - 2*Sqrt[2]*a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d*e - (2*I)*Sqrt[2]
*a^2*Sqrt[c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e
- I*Sqrt[-b^2 + 4*a*c]*e]])/(a^2*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*S
qrt[-b^2 + 4*a*c]*e]) + ((I*Sqrt[2]*b^2*Sqrt[c]*d^2 - (2*I)*Sqrt[2]*a*c^(3/
2)*d^2 + Sqrt[2]*b*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d^2 - (2*I)*Sqrt[2]*a*b*Sqrt[
c]*d*e - 2*Sqrt[2]*a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d*e + (2*I)*Sqrt[2]*a^2*Sqr
t[c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt
[-b^2 + 4*a*c]*e]])/(a^2*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2
 + 4*a*c]*e]) + ((2*b*d^(3/2) - 3*a*Sqrt[d]*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d
]])/a^2
```

fricas [B] time = 34.64, size = 8653, normalized size = 21.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 +
 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a
^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^
6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2
*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^
2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2
)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3
*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^
3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b^2*c +
8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b
^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)
*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 -
 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)
))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*
b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5
*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a
^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^
4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e
^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c
*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e +
2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt
(e*x + d)) - sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)
*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2
 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c
^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a
^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*
a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(
-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c +
 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*
```

(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt(e*x + d)) + sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ((a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt(e*x + d)) - sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ((a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt(e*x + d)) + (2*b*d - 3*a*e)*sqrt(d)*x*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*sqrt(e*x + d)*a*d)/(a^2*x), -1/2*(sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*

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a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 -
3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)
*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^
2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b
^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3*b*c^2)*d^3*e +
 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*
d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d -
(a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c
+ 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a
^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^
3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))*sqrt(-(a^3*b
*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*
(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*
e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^
3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a
^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 -
4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2
 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c -
a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt(e*x + d)) - sq
rt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3
 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqr
t(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*
b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^
4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5
*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6
- 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3*b*c^2)*
d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^3 - 4*a^
4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^
2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a
*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e -
 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*
b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))*sqrt(
-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2
*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5
 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c
 + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 +
2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8
*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 + (
b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b
^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt(e*x + d
)) + sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3
*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5
*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6
+ 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c
 + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2
- 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*
((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3*b
*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^3
- 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ((a^4*b^4 - 6*a^5*b^2*c + 8*
a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6
 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d
^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 1
1*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))
*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*
c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b
*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2
*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*
e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4
)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e
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^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*
(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt(e
*x + d)) - sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d
^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 -
 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2
)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3
*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^
4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-s
qrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 2
0*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a
^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ((a^4*b^4 - 6*a^5*b^2
*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - a^6*e^6
 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b
*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*
b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a
^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3
*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(
6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5
- 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^
2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*
d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 -
a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4
*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)
*sqrt(e*x + d)) + 2*(2*b*d - 3*a*e)*sqrt(-d)*x*arctan(sqrt(e*x + d)*sqrt(-d
)/d) + 2*sqrt(e*x + d)*a*d)/(a^2*x)]
```

giac [A] time = 0.55, size = 425, normalized size = 1.05

$$-\frac{\sqrt{xe+d}\,d}{ax}-\frac{\left(2\,bd^{2}-3\,ade\right)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{a^{2}\sqrt{-d}}-\frac{\sqrt{-4\,c^{2}d+2\left(bc-\sqrt{b^{2}-4\,ac}\,c\right)e}\left(\left(b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,b\right)d-\left(ab+\sqrt{b^{2}-4\,ac}\,a\right)e\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2\,a^{2}cd-a^{2}be+\sqrt{-4\left(a^{2}cd^{2}-a^{2}bde+a^{3}e^{2}\right)a^{2}c+\left(2\,a^{2}cd-a^{2}be\right)^{2}}}{a^{2}c}}}\right)}{2\sqrt{b^{2}-4\,ac}\,a^{2}|c|}+\frac{\sqrt{-4\,c^{2}d+2\left(bc+\sqrt{b^{2}-4\,ac}\,c\right)e}\left(\left(b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}\,b\right)d-\left(ab-\sqrt{b^{2}-4\,ac}\,a\right)e\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2\,a^{2}cd-a^{2}be-\sqrt{-4\left(a^{2}cd^{2}-a^{2}bde+a^{3}e^{2}\right)a^{2}c+\left(2\,a^{2}cd-a^{2}be\right)^{2}}}{a^{2}c}}}\right)}{2\sqrt{b^{2}-4\,ac}\,a^{2}|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(x*e + d)*d/(a*x) - (2*b*d^2 - 3*a*d*e)*arctan(sqrt(x*e + d)/sqrt(-d))
/(a^2*sqrt(-d)) - 1/2*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^
2 - 2*a*c + sqrt(b^2 - 4*a*c)*b)*d - (a*b + sqrt(b^2 - 4*a*c)*a)*e)*arctan(
2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 -
 a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sqrt(b^2
 - 4*a*c)*a^2*abs(c)) + 1/2*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e
)*((b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*b)*d - (a*b - sqrt(b^2 - 4*a*c)*a)*e)*a
rctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c
*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sq
rt(b^2 - 4*a*c)*a^2*abs(c))
```

maple [B] time = 0.05, size = 1215, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x)
```

```
[Out] -d*(e*x+d)^(1/2)/a/x-3*e*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+2*d^(3/2)
/a^2*arctanh((e*x+d)^(1/2)/d^(1/2))*b-2*e^3*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1
/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)+2*e^2/a*c/(-(4*a*
c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*c)*b*d+2*e/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*
c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^2-e/a^2*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)
```

```
/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1
/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d^2+2*e/a*c*2^(1
/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d-1/a^2*c*2^(1/2)
/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1
/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2-2*e^3*c/(-(4*a
*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*a
rctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*
c)+2*e^2/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*c)*b*d+2*e/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-
2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e
-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^2-e/a^2*c/(-(4*a*c-b^2)*e^2)
^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d
)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*d^2-2
*e/a*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d
)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d+1/a^2*c
*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2
)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+bx+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^2), x)
```

mupad [B] time = 7.36, size = 29890, normalized size = 74.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x)
```

```
[Out] (d^(1/2)*atan(((d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8
*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b
^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2
*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*
a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3
*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*
d^3*e^13))/a^4 - (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 1
00*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11
*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 +
 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e
^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*
d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9
+ 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252
*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 + (d^(1/2)*((8*(d + e*x)^(
1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c
^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d
^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d
^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^
3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b
^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b
*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*((8*
(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4
*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*
```

c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d)*1i)/(2*a^2) + (d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4 + (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 - (d^(1/2)*((8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d)*1i)/(2*a^2))/((16*(6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16))/a^4 - (d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4 - (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 + (d^(1/2)*((8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c

^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2) + (d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4 + (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 - (d^(1/2)*((8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2)))*(3*a*e - 2*b*d)*1i)/a^2 - atan((((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c

^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) -

(8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6

*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^
7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^
10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8
 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e
^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2
*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5
*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*
d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c -
b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4
*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d
^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 3
6*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(
1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*
e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 -
 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^
9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3
*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^
2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d
^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8
+ 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a
^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1
/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2
 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a
*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^
2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(
a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(4*a^6*c^3
*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a
^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3
*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b
^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c
^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5
*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^
3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3
*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*
a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2
e(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2
) + (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8
- 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 +
36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112
*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2)*((b^
6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^
2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3
*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*
b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e
- 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4
+ 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 -
 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*
b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^
3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^
4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*
d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e -
36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^

(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)
*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d
^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e
^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e
^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^
3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c
^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4
*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^
3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
 + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3
+ 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c
*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^
(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13
 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^
11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^
4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c
^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b
*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e
^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12
 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^
3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b
^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*
c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^
2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(
4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8
*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 +
 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^
2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*
b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a
^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*
d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 -
 a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c
^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2
*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^
3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^
6*c^2 - 8*a^5*b^2*c)))^(1/2) + (16*(6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*
b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^
9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 +
 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a
*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b
*c^3*d^2*e^16))/a^4))*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 - a^3*e^3*(-(
4*a*c - b^2)^3)^(1/2) + b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2
+ 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3
*a*b^5*d^2*e - 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d
^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 + 3*a^2*c*d^2*e*(-(4*a*c - b
^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - atan((((
(((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^
5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6
*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b
*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3
- a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*
c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*

a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a

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*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2
*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2
)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^1
0 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b
^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c
^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*
d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 +
4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*
b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d
^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1
/2) - (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3
*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e
^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^
9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*
e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*
d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^
2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3
- a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*
c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a
^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a
^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^1
1 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10
 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e
^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*
d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2
*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6
*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10
+ 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*
e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c -
 b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 -
8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d
*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8
*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^
8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*
c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^
4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4
*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^
5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3
*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b
^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*
c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^
2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36
*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((((((8*(80*a^8*c^
4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e
^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^
10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 -
 28*a^7*b^2*c^3*d*e^11))/a^4 - (8*(d + e*x)^(1/2)*((b^6*d^3 - a^3*b^3*e^3 -
 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b
```

^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(
1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2
- 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*
b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10
+ 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112
*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*
d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*
d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*
b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e -
18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 +
16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 +
88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*
d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d
^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5
*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c
^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5
*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^
2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*
a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 +
24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a
*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2
*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2
)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^
6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7
*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*
d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c
^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*
b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5
*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^
7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^
14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)
^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2
*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e
+ 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^
3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
)/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(4*a
^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12
- 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a
^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28
*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a
*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c
^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*
c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^
3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) +
3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))
)^(1/2) + (((((8*(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^
4*e^8 - 6*a^5*b^4*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*
e^9 + 36*a^6*b^3*c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11
- 112*a^7*b*c^4*d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 + (8*(d + e*x)^(1/2
)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2
) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 +

18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - (8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/a^4)*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (16*(6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16))/a^4))*((b^6*d^3 - a^3*b^3*e^3 - 8*a^3*c^3*d^3 + a^3*e

```
^3*(-(4*a*c - b^2)^3)^(1/2) - b^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^4*
d*e^2 + 24*a^4*c^2*d*e^2 + 18*a^2*b^2*c^2*d^3 - 8*a*b^4*c*d^3 + 4*a^4*b*c*e
^3 - 3*a*b^5*d^2*e + 2*a*b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 21*a^2*b
^3*c*d^2*e - 36*a^3*b*c^2*d^2*e - 18*a^3*b^2*c*d*e^2 - 3*a^2*c*d^2*e*(-(4*a
*c - b^2)^3)^(1/2))/(2*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - (d
*(d + e*x)^(1/2))/(a*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.349 $\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx$

Optimal. Leaf size=607

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-2abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(-ab\left(e\left(2d\sqrt{b^2-4ac}-ae\right)+3cd^2\right)+a\left(ae^2\sqrt{b^2}\right.\right.}{}$$

Rubi [A] time = 3.93, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {897, 1287, 199, 206, 1166, 208}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -(d*Sqrt[d + e*x])/(2*a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*x) + ((b*d - 2*a*e)
*Sqrt[d + e*x])/(a^2*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*Sqrt[
d]) - (e*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (2*(
b^2*d^2 - 2*a*b*d*e - a*(c*d^2 - a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a
^3*Sqrt[d]) + (Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*
e) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - a*b*
(3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^2*
d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d +
 a*e)) - a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx,x,\sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{d^2e^3}{a\left(d-x^2\right)^3}+\frac{de^2(-bd+2ae)}{a^2\left(d-x^2\right)^2}+\frac{e\left(-b^2d^2+2abde+a\left(cd^2-ae^2\right)\right)}{a^3\left(d-x^2\right)}+\frac{e\left(\left(b^2d-acd-abe\right)\left(cd^2-bde+ae^2\right)\ldots}{a^3\left(cd^2-bde+ae^2\ldots\right.}\right.\right.}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\left(b^2d-acd-abe\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d^2-2abde-a\left(cd^2-ae^2\right)\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx,x,\sqrt{d+ex}\right)}{a^3} - \frac{\left(2d^2e^2\right)\text{Su}\ldots}{\ldots}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{2\left(b^2d^2-2abde-a\left(cd^2-ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}-\ldots$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{e(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{2\left(b^2\ldots\right.}{\ldots}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}}-\frac{e(bd-2ae)\,\text{ta}\ldots}{a^2\ldots}$$

Mathematica [A] time = 2.86, size = 587, normalized size = 0.97

$$\frac{-\frac{2a^2d\sqrt{d+ex}}{x^2}+3a^2e\left(\frac{\sqrt{d+ex}}{x}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}\right)-\frac{8\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-2abde+a\left(ae^2-cd^2\right)+b^2d^2\right)}{\sqrt{d}}+\frac{4\sqrt{2}\sqrt{c}\left(ab\left(e\left(ae-2d\sqrt{b^2-4ac}\right)-3cd^2\right)+a\left(cd\left(4ae-d\sqrt{b^2-4ac}\right)+ae^2\sqrt{b^2-4ac}\right)+b^2d\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{4\sqrt{2}\sqrt{c}\left(ab\left(e\left(2d\sqrt{b^2-4ac}+ae\right)-3cd^2\right)+a\left(cd\left(d\sqrt{b^2-4ac}+4ae\right)-ae^2\sqrt{b^2-4ac}\right)-b^2d\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{4a\sqrt{d+ex}(bd-2ae)}{x}+\frac{4ae(2ae-bd)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] ((-2*a^2*d*Sqrt[d + e*x])/x^2 + (4*a*(b*d - 2*a*e)*Sqrt[d + e*x])/x + (4*a*
e*(-(b*d) + 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] - (8*(b^2*d^2 -
2*a*b*d*e + a*(-(c*d^2) + a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] +
 3*a^2*e*(Sqrt[d + e*x]/x - (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d]) + (
4*Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*b*(-3*
c*d^2 + e*(-2*Sqrt[b^2 - 4*a*c]*d + a*e)) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 + c*
d*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e]) - (4*Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^2*d*(Sqrt[b
^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d + a*e)) +
 a*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(4*a^3)
```

IntegrateAlgebraic [C] time = 3.20, size = 759, normalized size = 1.25

$$-\frac{\sqrt{d+ex}\left(5ae(d+ex)-3ade+4bd^2-4bd(d+ex)\right)}{4a^2ex^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-3a^2e^2+12abde+8acd^2-8b^2d^2\right)}{4a^3\sqrt{d}}+\frac{\left(-\sqrt{2}a^2\sqrt{c}e^2\sqrt{4ac-b^2}+i\sqrt{2}a^2b\sqrt{c}e^2+4i\sqrt{2}a^2c^{3/2}de+\sqrt{2}ac^{3/2}d^2\sqrt{4ac-b^2}-\sqrt{2}b^2\sqrt{c}d^2\sqrt{4ac-b^2}-2i\sqrt{2}ab^2\sqrt{c}de+2\sqrt{2}ab\sqrt{c}de\sqrt{4ac-b^2}-3i\sqrt{2}abc^{3/2}d^2+i\sqrt{2}b^3\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{a^3\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}+\frac{\left(-\sqrt{2}a^2\sqrt{c}e^2\sqrt{4ac-b^2}-i\sqrt{2}a^2b\sqrt{c}e^2-4i\sqrt{2}a^2c^{3/2}de+\sqrt{2}ac^{3/2}d^2\sqrt{4ac-b^2}-\sqrt{2}b^2\sqrt{c}d^2\sqrt{4ac-b^2}+2i\sqrt{2}ab^2\sqrt{c}de+2\sqrt{2}ab\sqrt{c}de\sqrt{4ac-b^2}+3i\sqrt{2}abc^{3/2}d^2-i\sqrt{2}b^3\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{a^3\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/4*(Sqrt[d + e*x]*(4*b*d^2 - 3*a*d*e - 4*b*d*(d + e*x) + 5*a*e*(d + e*x))
)/(a^2*e*x^2) + ((I*Sqrt[2]*b^3*Sqrt[c]*d^2 - (3*I)*Sqrt[2]*a*b*c^(3/2)*d^2
 - Sqrt[2]*b^2*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d^2 + Sqrt[2]*a*c^(3/2)*Sqrt[-b^2
 + 4*a*c]*d^2 - (2*I)*Sqrt[2]*a*b^2*Sqrt[c]*d*e + (4*I)*Sqrt[2]*a^2*c^(3/2)
*d*e + 2*Sqrt[2]*a*b*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d*e + I*Sqrt[2]*a^2*b*Sqrt[
c]*e^2 - Sqrt[2]*a^2*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(a^3*Sqrt[-b
^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((-I)*Sqrt[2]*b
^3*Sqrt[c]*d^2 + (3*I)*Sqrt[2]*a*b*c^(3/2)*d^2 - Sqrt[2]*b^2*Sqrt[c]*Sqrt[-
b^2 + 4*a*c]*d^2 + Sqrt[2]*a*c^(3/2)*Sqrt[-b^2 + 4*a*c]*d^2 + (2*I)*Sqrt[2]
*a*b^2*Sqrt[c]*d*e - (4*I)*Sqrt[2]*a^2*c^(3/2)*d*e + 2*Sqrt[2]*a*b*Sqrt[c]*
Sqrt[-b^2 + 4*a*c]*d*e - I*Sqrt[2]*a^2*b*Sqrt[c]*e^2 - Sqrt[2]*a^2*Sqrt[c]*
Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d
+ b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(a^3*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e
 + I*Sqrt[-b^2 + 4*a*c]*e]) + ((-8*b^2*d^2 + 8*a*c*d^2 + 12*a*b*d*e - 3*a^2
*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a^3*Sqrt[d])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.61, size = 1121, normalized size = 1.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*e - 2*(a*b^3 - 4*a^2*b*c)*d*e^2 +
(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*a
^2 + 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2*e + sqrt(b^2 - 4*a*c)*a^3*b*e^3 - (a*b^
2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^3 - (2*a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c
)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a) + (a^4*b^2
```

```
*e^3 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*d^3 + (a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*
d^2*e - 2*(a^3*b^3 - a^4*b*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*
a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e + s
qrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/
(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqr
t(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)) + 1/4*(((b^4 - 5*a*b^2*c + 4*a^2*c^2
)*d^2*e - 2*(a*b^3 - 4*a^2*b*c)*d*e^2 + (a^2*b^2 - 4*a^3*c)*e^3)*sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a^2 - 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2
*e + sqrt(b^2 - 4*a*c)*a^3*b*e^3 - (a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^
3 - (2*a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*abs(a) + (a^4*b^2*e^3 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*
d^3 + (a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*d^2*e - 2*(a^3*b^3 - a^4*b*c)*d*e^2
)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt
(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4
*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c
*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(
c)) + 1/4*(8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e + 3*a^2*e^2)*arctan(sqrt(x*e
+ d)/sqrt(-d))/(a^3*sqrt(-d)) + 1/4*(4*(x*e + d)^(3/2)*b*d*e - 4*sqrt(x*e +
 d)*b*d^2*e - 5*(x*e + d)^(3/2)*a*e^2 + 3*sqrt(x*e + d)*a*d*e^2)*e^(-2)/(a^
2*x^2)
```

maple [B] time = 0.06, size = 1880, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x)
```

```
[Out] -1/a^2*c^2*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh(
(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^
2+1/a^2*c^2*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((
e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d^2+
e^2/a*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*
x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)-e^2/a
*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1
/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)-1/e/a^2/x^2*(
e*x+d)^(1/2)*b*d^2+1/e/a^2/x^2*(e*x+d)^(3/2)*b*d-2/a^3*d^(3/2)*arctanh((e*x
+d)^(1/2)/d^(1/2))*b^2+2/a^2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c-3*e/a
^2*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2
))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*c)*b*d^2+e/a^3*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+
(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d
+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d^2-2*e^2/a^2*c/(-(4*a*c-b^2)*e^
2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e
*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^2*
d-3*e/a^2*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e
^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2
)*e^2)^(1/2))*c)^(1/2)*c)*b*d^2+e/a^3*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b^3*d^2-2*e^2/a^2*c/(-(
4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2
)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/
2)*c)*b^2*d-3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)+3/4*d*(e*x+d)^
(1/2)/a/x^2-5/4/a/x^2*(e*x+d)^(3/2)+3*e/a^2*d^(1/2)*arctanh((e*x+d)^(1/2)/d
^(1/2))*b+2*e/a^2*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*
arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)
*c)*b*d+e^3/a*c/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)
*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b
^2)*e^2)^(1/2))*c)^(1/2)*c)*b+4*e^2/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/
```

```
2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*d-2*e/a^2*c*2^(1/2)/(
(-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b*d+e^3/a*c/(-(4*a*c-b
^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*arcta
n((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(1/2))*c)^(1/2)*c)*b
+4*e^2/a*c^2/(-(4*a*c-b^2)*e^2)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^
2)^(1/2))*c)^(1/2)*c)*d+1/a^3*c*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b^2*d^2-1/a^3*c*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-(4*a*c-b^2)*e^2)^
(1/2))*c)^(1/2)*c)*b^2*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^3), x)
```

mupad [B] time = 8.19, size = 44649, normalized size = 73.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x)
```

```
[Out] (((3*a*d*e^2 - 4*b*d^2*e)*(d + e*x)^(1/2))/(4*a^2) - ((5*a*e^2 - 4*b*d*e)*(
d + e*x)^(3/2))/(4*a^2))/((d + e*x)^2 - 2*d*(d + e*x) + d^2) + atan((((((1
92*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*
c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*
c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b
^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 14
08*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576
*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 +
8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5
*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*
c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*
e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2
*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^
2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)
^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10
 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 1
28*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9
- 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^
3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*
e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e
^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^
2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3
*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a
```

c - b^2)^3)^(1/2))/(2(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d +
e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^
13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 -
128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^
8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d
^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*
c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a
^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10
+ 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4
*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4
*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3
- 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d
^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^
4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*
a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c
^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a
^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7
*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3
*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7
*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a
^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 -
2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^
7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^
5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 374
4*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3
*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b
^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^
7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*
a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*
d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b
^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10
*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a
*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e
^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*
b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*
d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4
+ 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 1
92*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*
c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*
c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^
3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 +
5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d
^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*
b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7
*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*
c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^
2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^
6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 +
60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d

e^2(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9

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+ 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3
*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^
6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13
 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*
d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^
3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^
8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*
a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a
^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 -
 a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*
a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b
*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*
e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/
2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*
d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8
 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^
6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*
c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496
*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^
12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*
d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*
e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3
 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b
^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^
6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*
b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3
*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*
d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7
5*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i
)/((216*a^3*c^9*d^8*e^10 - 15*a^7*c^5*e^18 + 391*a^4*c^8*d^6*e^12 + 119*a^5
*c^7*d^4*e^14 - 71*a^6*c^6*d^2*e^16 - 64*b^4*c^8*d^10*e^8 + 128*b^5*c^7*d^9
*e^9 - 64*b^6*c^6*d^8*e^10 + 1472*a^2*b^3*c^7*d^7*e^11 - 1344*a^2*b^4*c^6*d
^6*e^12 + 32*a^2*b^5*c^5*d^5*e^13 - 1264*a^3*b^2*c^7*d^6*e^12 + 2088*a^3*b^
3*c^6*d^5*e^13 - 152*a^3*b^4*c^5*d^4*e^14 - 1689*a^4*b^2*c^6*d^4*e^14 + 280
*a^4*b^3*c^5*d^3*e^15 - 247*a^5*b^2*c^5*d^2*e^16 + 102*a^6*b*c^5*d*e^17 + 6
4*a*b^2*c^9*d^10*e^8 + 192*a*b^3*c^8*d^9*e^9 - 704*a*b^4*c^7*d^8*e^10 + 448
*a*b^5*c^6*d^7*e^11 - 224*a^2*b*c^9*d^9*e^9 - 504*a^3*b*c^8*d^7*e^11 + 250*
a^4*b*c^7*d^5*e^13 + 632*a^5*b*c^6*d^3*e^15)/a^8 + (((((192*a^11*b^2*c^3*e^
12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*
a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a
^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 4
88*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^
11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^
11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^
5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2
*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3
*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d
*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^
2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^
2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^
```

4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2
*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^
2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2
*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*
c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2
*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*
e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24
*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 5
4*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*
c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
)/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(876*a
^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c
^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^
5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^
3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b
^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 409
6*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^1
0 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c
^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^
4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))
*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2)
 + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*
d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e
 - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*
a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c))
)^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 -
114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^
8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*
b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840
*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8
 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*
d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b
^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3
497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*
e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b
^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 +
3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 +
 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14
)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^
3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a
*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*
d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a
^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8
 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*
b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344

*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^

(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^
3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a
^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(
2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*
a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d
^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7
*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*
d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*
c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a
^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12
- 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^
5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b
^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 -
12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*
d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6
*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b
*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 +
8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^
5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5
*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2
*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b
^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^
2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a
^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3
)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)
*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c
^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^
8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*
c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a
^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11
+ 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5
*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^
6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*
d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^
4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c
^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*
d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e -
24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3
)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e +
54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^
2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)))*((b^8*d^3 - a^3*b^5*e
^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 1
2*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24
*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c
*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c
- b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^
2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^
2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i + atan(((((
((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^

10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024

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*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 19
71*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b
^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*
c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^
2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) +
3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*
d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a
^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c
^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b
^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16
+ 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a
^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b
^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944
*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^1
1 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^
5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a
^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*
b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6
*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^
3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4
*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a
*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b
^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2
 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^
3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*
e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 +
16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((192*a^11*b^2*c^3*e^12 - 24*a^10
*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2
*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d
^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c
^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*
b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8)
+ ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3
 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d
^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^
4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*
a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c
^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*
a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*
a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960
*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^
5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2
*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3
*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d
*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^
2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^
2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^
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4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2)

- 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((216*a^3*c^9*d^8*e^10 - 15*a^7*c^5*e^18 + 391*a^4*c^8*d^6*e^12 + 119*a^5*c^7*d^4*e^14 - 71*a^6*c^6*d^2*e^16 - 64*b^4*c^8*d^10*e^8 + 128*b^5*c^7*d^9*e^9 - 64*b^6*c^6*d^8*e^10 + 1472*a^2*b^3*c^7*d^7*e^11 - 1344*a^2*b^4*c^6*d^6*e^12 + 32*a^2*b^5*c^5*d^5*e^13 - 1264*a^3*b^2*c^7*d^6*e^12 + 2088*a^3*b^3*c^6*d^5*e^13 - 152*a^3*b^4*c^5*d^4*e^14 - 1689*a^4*b^2*c^6*d^4*e^14 + 280*a^4*b^3*c^5*d^3*e^15 - 247*a^5*b^2*c^5*d^2*e^16 + 102*a^6*b*c^5*d*e^17 + 64*a*b^2*c^9*d^10*e^8 + 192*a*b^3*c^8*d^9*e^9 - 704*a*b^4*c^7*d^8*e^10 + 448*a*b^5*c^6*d^7*e^11 - 224*a^2*b*c^9*d^9*e^9 - 504*a^3*b*c^8*d^7*e^11 + 250*a^4*b*c^7*d^5*e^13 + 632*a^5*b*c^6*d^3*e^15)/a^8 + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^

3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) + ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(

1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9)) /(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b

^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 224
8*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e
^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*
c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6
*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c
^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8
*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*
b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c
^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e
- 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*
e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2
*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)))*((b^8*d^3 - a^3*b^
5*e^3 + 8*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3
- 12*a^5*b*c^2*e^3 - a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 -
24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 4*a*b^3*c*d^3*(-
(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^
5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e - 3*a^2*b*c^2*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3
*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 + 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c*d*e^2*(-(4*a*c -
b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i - (atan(
(((((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^
6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 +
704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*
e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^
6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a
^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12
- 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*
e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^
9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8) + (((108*a^9
*b*c^4*e^15 + 302*a^9*c^5*d*e^14 + (15*a^7*b^5*c^2*e^15)/2 - 57*a^8*b^3*c^3
*e^15 + 96*a^6*c^8*d^7*e^8 - 672*a^7*c^7*d^5*e^10 - 466*a^8*c^6*d^3*e^12 +
64*a^2*b^8*c^4*d^7*e^8 - 48*a^2*b^9*c^3*d^6*e^9 - 16*a^2*b^10*c^2*d^5*e^10
- 480*a^3*b^6*c^5*d^7*e^8 + 64*a^3*b^7*c^4*d^6*e^9 + 420*a^3*b^8*c^3*d^5*e^
10 + 76*a^3*b^9*c^2*d^4*e^11 + 1088*a^4*b^4*c^6*d^7*e^8 + 1168*a^4*b^5*c^5*
d^6*e^9 - 1824*a^4*b^6*c^4*d^5*e^10 - 1248*a^4*b^7*c^3*d^4*e^11 - 140*a^4*b
^8*c^2*d^3*e^12 - 800*a^5*b^2*c^7*d^7*e^8 - 3008*a^5*b^3*c^6*d^6*e^9 + 1164
*a^5*b^4*c^5*d^5*e^10 + 5108*a^5*b^5*c^4*d^4*e^11 + (3497*a^5*b^6*c^3*d^3*e
^12)/2 + (247*a^5*b^7*c^2*d^2*e^13)/2 + 1872*a^6*b^2*c^6*d^5*e^10 - 5456*a^
6*b^3*c^5*d^4*e^11 - (12151*a^6*b^4*c^4*d^3*e^12)/2 - 1249*a^6*b^5*c^3*d^2*
e^13 + (10885*a^7*b^2*c^5*d^3*e^12)/2 + (7081*a^7*b^3*c^4*d^2*e^13)/2 + 160
0*a^6*b*c^7*d^6*e^9 - 51*a^6*b^6*c^2*d*e^14 + 512*a^7*b*c^6*d^4*e^11 + (867
*a^7*b^4*c^3*d*e^14)/2 - 2146*a^8*b*c^5*d^2*e^13 - (1971*a^8*b^2*c^4*d*e^14
)/2)/a^8 + ((((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 +
73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a
^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*
a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 -
3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e
^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^
5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^
8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 -
4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11
- 5034*a^9*b^2*c^4*d*e^12))/(2*a^8) - (((96*a^11*b^2*c^3*e^12 - 12*a^10*b^4
*c^2*e^12 - 192*a^12*c^4*e^12 + 384*a^10*c^6*d^4*e^8 + 192*a^11*c^5*d^2*e^1

0 + 64*a^8*b^4*c^4*d^4*e^8 - 48*a^8*b^5*c^3*d^3*e^9 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2*c^5*d^4*e^8 + 160*a^9*b^3*c^4*d^3*e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^10*b^2*c^4*d^2*e^10 + 704*a^11*b*c^4*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^10*b*c^5*d^3*e^9 - 288*a^10*b^3*c^3*d*e^11)/a^8 - ((d + e*x)^(1/2)*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*1i)/(8*a^3*d^(1/2)) + ((((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8) - (((108*a^9*b*c^4*e^15 + 302*a^9*c^5*d*e^14 + (15*a^7*b^5*c^2*e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c^8*d^7*e^8 - 672*a^7*c^7*d^5*e^10 - 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d^7*e^8 - 48*a^2*b^9*c^3*d^6*e^9 - 16*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^5*d^7*e^8 + 64*a^3*b^7*c^4*d^6*e^9 + 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*c^2*d^4*e^11 + 1088*a^4*b^4*c^6*d^7*e^8 + 1168*a^4*b^5*c^5*d^6*e^9 - 1824*a^4*b^6*c^4*d^5*e^10 - 1248*a^4*b^7*c^3*d^4*e^11 - 140*a^4*b^8*c^2*d^3*e^12 - 800*a^5*b^2*c^7*d^7*e^8 - 3008*a^5*b^3*c^6*d^6*e^9 + 1164*a^5*b^4*c^5*d^5*e^10 + 5108*a^5*b^5*c^4*d^4*e^11 + (3497*a^5*b^6*c^3*d^3*e^12)/2 + (247*a^5*b^7*c^2*d^2*e^13)/2 + 1872*a^6*b^2*c^6*d^5*e^10 - 5456*a^6*b^3*c^5*d^4*e^11 - (12151*a^6*b^4*c^4*d^3*e^12)/2 - 1249*a^6*b^5*c^3*d^2*e^13 + (10885*a^7*b^2*c^5*d^3*e^12)/2 + (7081*a^7*b^3*c^4*d^2*e^13)/2 + 1600*a^6*b*c^7*d^6*e^9 - 51*a^6*b^6*c^2*d*e^14 + 512*a^7*b*c^6*d^4*e^11 + (867*a^7*b^4*c^3*d*e^14)/2 - 2146*a^8*b*c^5*d^2*e^13 - (1971*a^8*b^2*c^4*d*e^14)/2)/a^8 - ((((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8) + (((96*a^11*b^2*c^3*e^12 - 12*a^10*b^4*c^2*e^12 - 192*a^12*c^4*e^12 + 384*a^10*c^6*d^4*e^8 + 192*a^11*c^5*d^2*e^10 + 64*a^8*b^4*c^4*d^4*e^8 - 48*a^8*b^5*c^3*d^3*e^9 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2*c^5*d^4*e^8 + 160*a^9*b^3*c^4*d^3*e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^10*b^2*c^4*d^2*e^10 + 704*a^11*b*c^4*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^10*b*c^5*d^3*e^9 - 288*a^10*b^3*c^3*d*e^11)/a^8 + ((d + e*x)^(1/2)*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*1i)/(8*a^3*d^(1/2)))/((216*a^3*c^9*d^8*e^10 - 15*a^7*c^5*e^18 + 391*a^4*c^8*d^6*e^12 + 119*a^5*c^7*d^4*e^14 - 71*a^6*c^6*d^2*e^16 - 64*b^4*c^8*d^10*e^8 + 128*b^5*c^7*d^9*e^9 - 64*b^6*c^6*d^8*e^10 + 1472*a^2*b^3*c^7*d^7*e^11 - 1344*a^2*

b^4*c^6*d^6*e^12 + 32*a^2*b^5*c^5*d^5*e^13 - 1264*a^3*b^2*c^7*d^6*e^12 + 20
88*a^3*b^3*c^6*d^5*e^13 - 152*a^3*b^4*c^5*d^4*e^14 - 1689*a^4*b^2*c^6*d^4*e
^14 + 280*a^4*b^3*c^5*d^3*e^15 - 247*a^5*b^2*c^5*d^2*e^16 + 102*a^6*b*c^5*d
*e^17 + 64*a*b^2*c^9*d^10*e^8 + 192*a*b^3*c^8*d^9*e^9 - 704*a*b^4*c^7*d^8*e
^10 + 448*a*b^5*c^6*d^7*e^11 - 224*a^2*b*c^9*d^9*e^9 - 504*a^3*b*c^8*d^7*e^
11 + 250*a^4*b*c^7*d^5*e^13 + 632*a^5*b*c^6*d^3*e^15)/a^8 - ((((d + e*x)^(1
/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^
6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^14 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7
*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b
^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 224
8*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e
^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*
c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6
*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c
^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^13))/(2*a^8) + (((108*a^9*b*c^4*e^15 + 30
2*a^9*c^5*d*e^14 + (15*a^7*b^5*c^2*e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c
^8*d^7*e^8 - 672*a^7*c^7*d^5*e^10 - 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d
^7*e^8 - 48*a^2*b^9*c^3*d^6*e^9 - 16*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^
5*d^7*e^8 + 64*a^3*b^7*c^4*d^6*e^9 + 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*
c^2*d^4*e^11 + 1088*a^4*b^4*c^6*d^7*e^8 + 1168*a^4*b^5*c^5*d^6*e^9 - 1824*a
^4*b^6*c^4*d^5*e^10 - 1248*a^4*b^7*c^3*d^4*e^11 - 140*a^4*b^8*c^2*d^3*e^12
- 800*a^5*b^2*c^7*d^7*e^8 - 3008*a^5*b^3*c^6*d^6*e^9 + 1164*a^5*b^4*c^5*d^5
*e^10 + 5108*a^5*b^5*c^4*d^4*e^11 + (3497*a^5*b^6*c^3*d^3*e^12)/2 + (247*a^
5*b^7*c^2*d^2*e^13)/2 + 1872*a^6*b^2*c^6*d^5*e^10 - 5456*a^6*b^3*c^5*d^4*e^
11 - (12151*a^6*b^4*c^4*d^3*e^12)/2 - 1249*a^6*b^5*c^3*d^2*e^13 + (10885*a^
7*b^2*c^5*d^3*e^12)/2 + (7081*a^7*b^3*c^4*d^2*e^13)/2 + 1600*a^6*b*c^7*d^6*
e^9 - 51*a^6*b^6*c^2*d*e^14 + 512*a^7*b*c^6*d^4*e^11 + (867*a^7*b^4*c^3*d*e
^14)/2 - 2146*a^8*b*c^5*d^2*e^13 - (1971*a^8*b^2*c^4*d*e^14)/2)/a^8 + ((((d
 + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*
e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10
- 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*
e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5
*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^
7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824
*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^
10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d
^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c
^4*d*e^12))/(2*a^8) - (((96*a^11*b^2*c^3*e^12 - 12*a^10*b^4*c^2*e^12 - 192*
a^12*c^4*e^12 + 384*a^10*c^6*d^4*e^8 + 192*a^11*c^5*d^2*e^10 + 64*a^8*b^4*c
^4*d^4*e^8 - 48*a^8*b^5*c^3*d^3*e^9 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2
*c^5*d^4*e^8 + 160*a^9*b^3*c^4*d^3*e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^1
0*b^2*c^4*d^2*e^10 + 704*a^11*b*c^4*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^
10*b*c^5*d^3*e^9 - 288*a^10*b^3*c^3*d*e^11)/a^8 - ((d + e*x)^(1/2)*(3*a^2*e
^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*
c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3
*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*
c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*
d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*
a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2
- 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b
*d*e))/(8*a^3*d^(1/2)) + ((((d + e*x)^(1/2)*(82*a^8*c^5*e^16 + 192*a^4*c^9*
d^8*e^8 - 608*a^5*c^8*d^6*e^10 + 1106*a^6*c^7*d^4*e^12 + 52*a^7*c^6*d^2*e^1
4 + 64*b^8*c^5*d^8*e^8 + 704*a^2*b^4*c^7*d^8*e^8 + 2240*a^2*b^5*c^6*d^7*e^9
 + 1344*a^2*b^6*c^5*d^6*e^10 - 512*a^3*b^2*c^8*d^8*e^8 - 2944*a^3*b^3*c^7*d
^7*e^9 - 5424*a^3*b^4*c^6*d^6*e^10 - 2248*a^3*b^5*c^5*d^5*e^11 + 5184*a^4*b
^2*c^7*d^6*e^10 + 6496*a^4*b^3*c^6*d^5*e^11 + 2409*a^4*b^4*c^5*d^4*e^12 - 3
748*a^5*b^2*c^6*d^4*e^12 - 1876*a^5*b^3*c^5*d^3*e^13 + 1110*a^6*b^2*c^5*d^2
*e^14 - 436*a^7*b*c^5*d*e^15 - 384*a*b^6*c^6*d^8*e^8 - 448*a*b^7*c^5*d^7*e^
9 + 896*a^4*b*c^8*d^7*e^9 - 4048*a^5*b*c^7*d^5*e^11 + 780*a^6*b*c^6*d^3*e^1

```
3))/(2*a^8) - (((108*a^9*b*c^4*e^15 + 302*a^9*c^5*d*e^14 + (15*a^7*b^5*c^2*
e^15)/2 - 57*a^8*b^3*c^3*e^15 + 96*a^6*c^8*d^7*e^8 - 672*a^7*c^7*d^5*e^10 -
 466*a^8*c^6*d^3*e^12 + 64*a^2*b^8*c^4*d^7*e^8 - 48*a^2*b^9*c^3*d^6*e^9 - 1
6*a^2*b^10*c^2*d^5*e^10 - 480*a^3*b^6*c^5*d^7*e^8 + 64*a^3*b^7*c^4*d^6*e^9
+ 420*a^3*b^8*c^3*d^5*e^10 + 76*a^3*b^9*c^2*d^4*e^11 + 1088*a^4*b^4*c^6*d^7
*e^8 + 1168*a^4*b^5*c^5*d^6*e^9 - 1824*a^4*b^6*c^4*d^5*e^10 - 1248*a^4*b^7*
c^3*d^4*e^11 - 140*a^4*b^8*c^2*d^3*e^12 - 800*a^5*b^2*c^7*d^7*e^8 - 3008*a^
5*b^3*c^6*d^6*e^9 + 1164*a^5*b^4*c^5*d^5*e^10 + 5108*a^5*b^5*c^4*d^4*e^11 +
 (3497*a^5*b^6*c^3*d^3*e^12)/2 + (247*a^5*b^7*c^2*d^2*e^13)/2 + 1872*a^6*b^
2*c^6*d^5*e^10 - 5456*a^6*b^3*c^5*d^4*e^11 - (12151*a^6*b^4*c^4*d^3*e^12)/2
 - 1249*a^6*b^5*c^3*d^2*e^13 + (10885*a^7*b^2*c^5*d^3*e^12)/2 + (7081*a^7*b
^3*c^4*d^2*e^13)/2 + 1600*a^6*b*c^7*d^6*e^9 - 51*a^6*b^6*c^2*d*e^14 + 512*a
^7*b*c^6*d^4*e^11 + (867*a^7*b^4*c^3*d*e^14)/2 - 2146*a^8*b*c^5*d^2*e^13 -
(1971*a^8*b^2*c^4*d*e^14)/2)/a^8 - ((((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13
+ 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*
a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4
*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448
*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9
+ 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d
^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^
5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 3
28*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 -
4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8) + (((96*a^11*b^
2*c^3*e^12 - 12*a^10*b^4*c^2*e^12 - 192*a^12*c^4*e^12 + 384*a^10*c^6*d^4*e^
8 + 192*a^11*c^5*d^2*e^10 + 64*a^8*b^4*c^4*d^4*e^8 - 48*a^8*b^5*c^3*d^3*e^9
 - 16*a^8*b^6*c^2*d^2*e^10 - 352*a^9*b^2*c^5*d^4*e^8 + 160*a^9*b^3*c^4*d^3*
e^9 + 244*a^9*b^4*c^3*d^2*e^10 - 768*a^10*b^2*c^4*d^2*e^10 + 704*a^11*b*c^4
*d*e^11 + 28*a^9*b^5*c^2*d*e^11 + 128*a^10*b*c^5*d^3*e^9 - 288*a^10*b^3*c^3
*d*e^11)/a^8 + ((d + e*x)^(1/2)*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b
*d*e)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 +
1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8
 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9)
)/(16*a^11*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a
^3*d^(1/2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/
2)))*(3*a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2)))*(3*
a^2*e^2 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e))/(8*a^3*d^(1/2))))*(3*a^2*e^2
 + 8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e)*1i)/(4*a^3*d^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.350 $\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=141

$$-\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3}-\frac{dx^2\left(4d^2g^2+7defg+2e^2f^2\right)}{e}-\frac{d^2x\left(8d^2g^2+16defg+7e^2f^2\right)}{e^2}-\frac{1}{2}egx^4(2dg+ef)-\frac{1}{3}x^3$$

Rubi [A] time = 0.18, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{dx^2\left(4d^2g^2+7defg+2e^2f^2\right)}{e}-\frac{d^2x\left(8d^2g^2+16defg+7e^2f^2\right)}{e^2}-\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3}-\frac{1}{2}egx^4(2dg+ef)-\frac{1}{3}x^3(dg+ef)(7dg+ef)-\frac{1}{5}e^2g^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((d^2*(7*e^2*f^2 + 16*d*e*f*g + 8*d^2*g^2)*x)/e^2) - (d*(2*e^2*f^2 + 7*d*e
*f*g + 4*d^2*g^2)*x^2)/e - ((e*f + d*g)*(e*f + 7*d*g)*x^3)/3 - (e*g*(e*f +
2*d*g)*x^4)/2 - (e^2*g^2*x^5)/5 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(d+ex)^3(f+gx)^2}{d-ex}\,dx\\ &= \int \left(-\frac{d^2\left(7e^2f^2+16defg+8d^2g^2\right)}{e^2}-\frac{2d\left(2e^2f^2+7defg+4d^2g^2\right)x}{e}+(-ef-7dg)(ef\right.\\ &= -\frac{d^2\left(7e^2f^2+16defg+8d^2g^2\right)x}{e^2}-\frac{d\left(2e^2f^2+7defg+4d^2g^2\right)x^2}{e}-\frac{1}{3}(ef+dg)(ef+\end{aligned}$$

Mathematica [A] time = 0.08, size = 134, normalized size = 0.95

$$-\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3}-\frac{x\left(240d^4g^2+120d^3eg(4f+gx)+70d^2e^2\left(3f^2+3fgx+g^2x^2\right)+10de^3x\left(6f^2+8fgx+3g^2x^2\right)+e^4x^2\left(10f^2+15fgx+6g^2x^2\right)\right)}{30e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/30*(x*(240*d^4*g^2 + 120*d^3*e*g*(4*f + g*x) + 70*d^2*e^2*(3*f^2 + 3*f*g
*x + g^2*x^2) + 10*d*e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2) + e^4*x^2*(10*f^2
+ 15*f*g*x + 6*g^2*x^2)))/e^2 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2), x]
```

fricas [A] time = 0.39, size = 176, normalized size = 1.25

$$-\frac{6\,e^5g^2x^5+15\left(e^5fg+2\,de^4g^2\right)x^4+10\left(e^5f^2+8\,de^4fg+7\,d^2e^3g^2\right)x^3+30\left(2\,de^4f^2+7\,d^2e^3fg+4\,d^3e^2g^2\right)x^2+30\left(7\,d^2e^3f^2+16\,d^3e^2fg+8\,d^4eg^2\right)x+240\left(d^3e^2f^2+2\,d^4efg+d^5g^2\right)\log\left(ex-d\right)}{30\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/30*(6*e^5*g^2*x^5 + 15*(e^5*f*g + 2*d*e^4*g^2)*x^4 + 10*(e^5*f^2 + 8*d*e
^4*f*g + 7*d^2*e^3*g^2)*x^3 + 30*(2*d*e^4*f^2 + 7*d^2*e^3*f*g + 4*d^3*e^2*g
^2)*x^2 + 30*(7*d^2*e^3*f^2 + 16*d^3*e^2*f*g + 8*d^4*e*g^2)*x + 240*(d^3*e^
2*f^2 + 2*d^4*e*f*g + d^5*g^2)*log(e*x - d))/e^3
```

giac [A] time = 0.16, size = 249, normalized size = 1.77

$$-4\left(d^5g^2e^3+2\,d^4fge^4+d^3f^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{30}\left(6\,g^2x^5e^{12}+30\,dg^2x^4e^{11}+70\,d^2g^2x^3e^{10}+120\,d^3g^2x^2e^9+240\,d^4g^2xe^8+15\,fgx^4e^{12}+80\,dfgx^3e^{11}+210\,d^2fgx^2e^{10}+480\,d^3fgxe^9+10\,f^2x^3e^{12}+60\,df^2x^2e^{11}+210\,d^2f^2xe^{10}\right)e^{(-10)}-\frac{4\left(d^6g^2e^4+2\,d^5fge^5+d^4f^2e^6\right)e^{(-7)}\log\left(\frac{\left|2\,xe^2-2\,|d||e|\right|}{\left|2\,xe^2+2\,|d||e|\right|}\right)}{|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -4*(d^5*g^2*e^3 + 2*d^4*f*g*e^4 + d^3*f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2
)) - 1/30*(6*g^2*x^5*e^12 + 30*d*g^2*x^4*e^11 + 70*d^2*g^2*x^3*e^10 + 120*d
^3*g^2*x^2*e^9 + 240*d^4*g^2*x*e^8 + 15*f*g*x^4*e^12 + 80*d*f*g*x^3*e^11 +
210*d^2*f*g*x^2*e^10 + 480*d^3*f*g*x*e^9 + 10*f^2*x^3*e^12 + 60*d*f^2*x^2*e
^11 + 210*d^2*f^2*x*e^10)*e^(-10) - 4*(d^6*g^2*e^4 + 2*d^5*f*g*e^5 + d^4*f^
2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(
d)
```

maple [A] time = 0.01, size = 186, normalized size = 1.32

$$-\frac{e^2g^2x^5}{5}-deg^2x^4-\frac{e^2fgx^4}{2}-\frac{7d^2g^2x^3}{3}-\frac{8defgx^3}{3}-\frac{e^2f^2x^3}{3}-\frac{4d^3g^2x^2}{e}-7d^2fgx^2-2def^2x^2-\frac{8d^5g^2\ln\left(ex-d\right)}{e^3}-\frac{16d^4fg\ln\left(ex-d\right)}{e^2}-\frac{8d^4g^2x}{e^2}-\frac{8d^3f^2\ln\left(ex-d\right)}{e}-\frac{16d^3fgx}{e}-7d^2f^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/5*e^2*g^2*x^5-e*x^4*d*g^2-1/2*e^2*x^4*f*g-7/3*x^3*d^2*g^2-8/3*e*x^3*d*f*
g-1/3*e^2*x^3*f^2-4/e*x^2*d^3*g^2-7*x^2*d^2*f*g-2*e*x^2*d*f^2-8/e^2*x*d^4*g
^2-16/e*x*d^3*f*g-7*x*d^2*f^2-8*d^5/e^3*ln(e*x-d)*g^2-16*d^4/e^2*ln(e*x-d)*
f*g-8*d^3/e*ln(e*x-d)*f^2
```

maxima [A] time = 0.47, size = 175, normalized size = 1.24

$$-\frac{6\,e^4g^2x^5+15\left(e^4fg+2\,de^3g^2\right)x^4+10\left(e^4f^2+8\,de^3fg+7\,d^2e^2g^2\right)x^3+30\left(2\,de^3f^2+7\,d^2e^2fg+4\,d^3eg^2\right)x^2+30\left(7\,d^2e^2f^2+16\,d^3efg+8\,d^4g^2\right)x}{30\,e^2}-\frac{8\left(d^3e^2f^2+2\,d^4efg+d^5g^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/30*(6*e^4*g^2*x^5 + 15*(e^4*f*g + 2*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 8*d*e
^3*f*g + 7*d^2*e^2*g^2)*x^3 + 30*(2*d*e^3*f^2 + 7*d^2*e^2*f*g + 4*d^3*e*g^2
```

```
)*x^2 + 30*(7*d^2*e^2*f^2 + 16*d^3*e*f*g + 8*d^4*g^2)*x)/e^2 - 8*(d^3*e^2*f
^2 + 2*d^4*e*f*g + d^5*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.11, size = 351, normalized size = 2.49

$$-x^2\left(\frac{d^3g^2+6d^2efg+3de^2f^2}{2e}+\frac{d\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{e}+\frac{d\left(eg\left(3dg+2ef\right)+deg^2\right)}{e}\right)}{2e}\right)-x^3\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{3e}+\frac{d\left(eg\left(3dg+2ef\right)+deg^2\right)}{3e}\right)-x^4\left(\frac{eg\left(3dg+2ef\right)}{4}+\frac{deg^2}{4}\right)-x\left(\frac{d\left(\frac{d^3g^2+6d^2efg+3de^2f^2}{e}+\frac{d\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{e}+\frac{d\left(eg\left(3dg+2ef\right)+deg^2\right)}{e}\right)}{e}\right)}{e}+\frac{d^2f\left(2dg+3ef\right)}{e}\right)-\frac{\ln\left(ex-d\right)\left(8d^5g^2+16d^4efg+8d^3e^2f^2\right)}{e^3}-\frac{e^2g^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^2*((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/(2*e) + (d*((e^3*f^2 + 3*d^2*e
*g^2 + 6*d*e^2*f*g)/e + (d*(e*g*(3*d*g + 2*e*f) + d*e*g^2))/e))/(2*e)) - x^
3*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2*f*g)/(3*e) + (d*(e*g*(3*d*g + 2*e*f) +
d*e*g^2))/(3*e)) - x^4*((e*g*(3*d*g + 2*e*f))/4 + (d*e*g^2)/4) - x*((d*((d^
3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/e + (d*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2
*f*g)/e + (d*(e*g*(3*d*g + 2*e*f) + d*e*g^2))/e))/e))/e + (d^2*f*(2*d*g + 3
*e*f))/e) - (log(e*x - d)*(8*d^5*g^2 + 8*d^3*e^2*f^2 + 16*d^4*e*f*g))/e^3 -
 (e^2*g^2*x^5)/5
```

sympy [A] time = 0.60, size = 150, normalized size = 1.06

$$-\frac{8d^3\left(dg+ef\right)^2\log\left(-d+ex\right)}{e^3}-\frac{e^2g^2x^5}{5}-x^4\left(deg^2+\frac{e^2fg}{2}\right)-x^3\left(\frac{7d^2g^2}{3}+\frac{8defg}{3}+\frac{e^2f^2}{3}\right)-x^2\left(\frac{4d^3g^2}{e}+7d^2fg+2def^2\right)-x\left(\frac{8d^4g^2}{e^2}+\frac{16d^3fg}{e}+7d^2f^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -8*d**3*(d*g + e*f)**2*log(-d + e*x)/e**3 - e**2*g**2*x**5/5 - x**4*(d*e*g*
*2 + e**2*f*g/2) - x**3*(7*d**2*g**2/3 + 8*d*e*f*g/3 + e**2*f**2/3) - x**2*
(4*d**3*g**2/e + 7*d**2*f*g + 2*d*e*f**2) - x*(8*d**4*g**2/e**2 + 16*d**3*f
*g/e + 7*d**2*f**2)
```

3.351 $\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{4d^2(dg+ef)^2\log(d-ex)}{e^3}-\frac{x^2\left(4d^2g^2+6defg+e^2f^2\right)}{2e}-\frac{dx(2dg+ef)(2dg+3ef)}{e^2}-\frac{1}{3}gx^3(3dg+2ef)-\frac{1}{4}eg^2x^4$$

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{x^2\left(4d^2g^2+6defg+e^2f^2\right)}{2e}-\frac{4d^2(dg+ef)^2\log(d-ex)}{e^3}-\frac{dx(2dg+ef)(2dg+3ef)}{e^2}-\frac{1}{3}gx^3(3dg+2ef)-\frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((d*(e*f + 2*d*g)*(3*e*f + 2*d*g)*x)/e^2) - ((e^2*f^2 + 6*d*e*f*g + 4*d^2*
g^2)*x^2)/(2*e) - (g*(2*e*f + 3*d*g)*x^3)/3 - (e*g^2*x^4)/4 - (4*d^2*(e*f +
 d*g)^2*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(d+ex)^2(f+gx)^2}{d-ex}\,dx\\ &= \int \left(\frac{d(-3ef-2dg)(ef+2dg)}{e^2}-\frac{\left(e^2f^2+6defg+4d^2g^2\right)x}{e}-g(2ef+3dg)x^2-eg^2\right.\\ &= -\frac{d(ef+2dg)(3ef+2dg)x}{e^2}-\frac{\left(e^2f^2+6defg+4d^2g^2\right)x^2}{2e}-\frac{1}{3}g(2ef+3dg)x^3-\frac{1}{4}eg\end{aligned}$$

Mathematica [A] time = 0.05, size = 103, normalized size = 0.94

$$-\frac{48d^2(dg+ef)^2\log(d-ex)+ex\left(48d^3g^2+24d^2eg(4f+gx)+12de^2\left(3f^2+3fgx+g^2x^2\right)+e^3x\left(6f^2+8fgx+3g^2x^2\right)\right)}{12e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/12*(e*x*(48*d^3*g^2 + 24*d^2*e*g*(4*f + g*x) + 12*d*e^2*(3*f^2 + 3*f*g*x
 + g^2*x^2) + e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2)) + 48*d^2*(e*f + d*g)^2*L
og[d - e*x])/e^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2), x]
```

fricas [A] time = 0.41, size = 139, normalized size = 1.28

$$-\frac{3\,e^4g^2x^4+4\left(2\,e^4fg+3\,de^3g^2\right)x^3+6\left(e^4f^2+6\,de^3fg+4\,d^2e^2g^2\right)x^2+12\left(3\,de^3f^2+8\,d^2e^2fg+4\,d^3eg^2\right)x+48\left(d^2e^2f^2+2\,d^3efg+d^4g^2\right)\log\left(ex-d\right)}{12\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*e^4*g^2*x^4 + 4*(2*e^4*f*g + 3*d*e^3*g^2)*x^3 + 6*(e^4*f^2 + 6*d*e
^3*f*g + 4*d^2*e^2*g^2)*x^2 + 12*(3*d*e^3*f^2 + 8*d^2*e^2*f*g + 4*d^3*e*g^2
)*x + 48*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(e*x - d))/e^3
```

giac [B] time = 0.21, size = 211, normalized size = 1.94

$$-2\left(d^4g^2e^3+2\,d^3fge^4+d^2f^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{12}\left(3\,g^2x^4e^9+12\,dg^2x^3e^8+24\,d^2g^2x^2e^7+48\,d^3g^2xe^6+8\,fgx^3e^9+36\,dfgx^2e^8+96\,d^2fgxe^7+6\,f^2x^2e^9+36\,df^2xe^8\right)e^{(-8)}-\frac{2\left(d^5g^2e^2+2\,d^4fge^3+d^3f^2e^4\right)e^{(-5)}\log\left(\frac{\left|2\,xe^2-2\,\left|d\right|e\right|}{\left|2\,xe^2+2\,\left|d\right|e\right|}\right)}{\left|d\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -2*(d^4*g^2*e^3 + 2*d^3*f*g*e^4 + d^2*f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2
)) - 1/12*(3*g^2*x^4*e^9 + 12*d*g^2*x^3*e^8 + 24*d^2*g^2*x^2*e^7 + 48*d^3*g
^2*x*e^6 + 8*f*g*x^3*e^9 + 36*d*f*g*x^2*e^8 + 96*d^2*f*g*x*e^7 + 6*f^2*x^2*
e^9 + 36*d*f^2*x*e^8)*e^(-8) - 2*(d^5*g^2*e^2 + 2*d^4*f*g*e^3 + d^3*f^2*e^4
)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

maple [A] time = 0.00, size = 145, normalized size = 1.33

$$-\frac{eg^2x^4}{4}-dg^2x^3-\frac{2efgx^3}{3}-\frac{2d^2g^2x^2}{e}-3dfgx^2-\frac{ef^2x^2}{2}-\frac{4d^4g^2\ln\left(ex-d\right)}{e^3}-\frac{8d^3fg\ln\left(ex-d\right)}{e^2}-\frac{4d^3g^2x}{e^2}-\frac{4d^2f^2\ln\left(ex-d\right)}{e}-\frac{8d^2fgx}{e}-3df^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/4*e*g^2*x^4-x^3*d*g^2-2/3*e*x^3*f*g-2/e*x^2*d^2*g^2-3*x^2*d*f*g-1/2*e*x^
2*f^2-4/e^2*x*d^3*g^2-8/e*x*d^2*f*g-3*x*d*f^2-4*d^4/e^3*ln(e*x-d)*g^2-8*d^3
/e^2*ln(e*x-d)*f*g-4*d^2/e*ln(e*x-d)*f^2
```

maxima [A] time = 0.45, size = 138, normalized size = 1.27

$$-\frac{3\,e^3g^2x^4+4\left(2\,e^3fg+3\,de^2g^2\right)x^3+6\left(e^3f^2+6\,de^2fg+4\,d^2eg^2\right)x^2+12\left(3\,de^2f^2+8\,d^2efg+4\,d^3g^2\right)x}{12\,e^2}-\frac{4\left(d^2e^2f^2+2\,d^3efg+d^4g^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*e^3*g^2*x^4 + 4*(2*e^3*f*g + 3*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 6*d*e
^2*f*g + 4*d^2*e*g^2)*x^2 + 12*(3*d*e^2*f^2 + 8*d^2*e*f*g + 4*d^3*g^2)*x)/e
^2 - 4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.59, size = 197, normalized size = 1.81

$$-x^3\left(\frac{2g\left(dg+ef\right)}{3}+\frac{dg^2}{3}\right)-x^2\left(\frac{d^2g^2+4defg+e^2f^2}{2e}+\frac{d\left(2g\left(dg+ef\right)+dg^2\right)}{2e}\right)-x\left(\frac{d\left(\frac{d^2g^2+4defg+e^2f^2}{e}+\frac{d\left(2g\left(dg+ef\right)+dg^2\right)}{e}\right)}{e}+\frac{2df\left(dg+ef\right)}{e}\right)-\frac{\ln\left(ex-d\right)\left(4d^4g^2+8d^3efg+4d^2e^2f^2\right)}{e^3}-\frac{eg^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^3*((2*g*(d*g + e*f))/3 + (d*g^2)/3) - x^2*((d^2*g^2 + e^2*f^2 + 4*d*e*f
*g)/(2*e) + (d*(2*g*(d*g + e*f) + d*g^2))/(2*e)) - x*((d*((d^2*g^2 + e^2*f^
2 + 4*d*e*f*g)/e + (d*(2*g*(d*g + e*f) + d*g^2))/e))/e + (2*d*f*(d*g + e*f)
)/e) - (log(e*x - d)*(4*d^4*g^2 + 4*d^2*e^2*f^2 + 8*d^3*e*f*g))/e^3 - (e*g^
2*x^4)/4
```

sympy [A] time = 0.48, size = 109, normalized size = 1.00

$$-\frac{4d^2\left(dg+ef\right)^2\log\left(-d+ex\right)}{e^3}-\frac{eg^2x^4}{4}-x^3\left(dg^2+\frac{2efg}{3}\right)-x^2\left(\frac{2d^2g^2}{e}+3dfg+\frac{ef^2}{2}\right)-x\left(\frac{4d^3g^2}{e^2}+\frac{8d^2fg}{e}+3df^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -4*d**2*(d*g + e*f)**2*log(-d + e*x)/e**3 - e*g**2*x**4/4 - x**3*(d*g**2 +
2*e*f*g/3) - x**2*(2*d**2*g**2/e + 3*d*f*g + e*f**2/2) - x*(4*d**3*g**2/e**
2 + 8*d**2*f*g/e + 3*d*f**2)
```

3.352 $\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{2d(dg+ef)^2\log(d-ex)}{e^3}-\frac{2dgx(dg+ef)}{e^2}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}$$

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {848, 77}

$$-\frac{2dgx(dg+ef)}{e^2}-\frac{2d(dg+ef)^2\log(d-ex)}{e^3}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] (-2*d*g*(e*f + d*g)*x)/e^2 - (d*(f + g*x)^2)/e - (f + g*x)^3/(3*g) - (2*d*(
e*f + d*g)^2*Log[d - e*x])/e^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(d+ex)(f+gx)^2}{d-ex}\,dx\\ &= \int \left(-\frac{2dg(ef+dg)}{e^2}-\frac{2d(ef+dg)^2}{e^2(-d+ex)}-\frac{2dg(f+gx)}{e}-(f+gx)^2\right)dx\\ &= -\frac{2dg(ef+dg)x}{e^2}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}-\frac{2d(ef+dg)^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.12

$$-\frac{ex\left(6d^2g^2+3deg(4f+gx)+e^2\left(3f^2+3fgx+g^2x^2\right)\right)+6d(dg+ef)^2\log(d-ex)}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/3*(e*x*(6*d^2*g^2 + 3*d*e*g*(4*f + g*x) + e^2*(3*f^2 + 3*f*g*x + g^2*x^2
)) + 6*d*(e*f + d*g)^2*Log[d - e*x])/e^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2), x]
```

fricas [A] time = 0.39, size = 98, normalized size = 1.51

$$-\frac{e^3g^2x^3+3\left(e^3fg+de^2g^2\right)x^2+3\left(e^3f^2+4\,de^2fg+2\,d^2eg^2\right)x+6\left(de^2f^2+2\,d^2efg+d^3g^2\right)\log\left(ex-d\right)}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/3*(e^3*g^2*x^3 + 3*(e^3*f*g + d*e^2*g^2)*x^2 + 3*(e^3*f^2 + 4*d*e^2*f*g
+ 2*d^2*e*g^2)*x + 6*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(e*x - d))/e^3
```

giac [B] time = 0.16, size = 172, normalized size = 2.65

$$-\left(d^3g^2e+2\,d^2fge^2+df^2e^3\right)e^{(-4)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{3}\left(g^2x^3e^6+3\,dg^2x^2e^5+6\,d^2g^2xe^4+3\,fgx^2e^6+12\,dfgxe^5+3\,f^2xe^6\right)e^{(-6)}-\frac{\left(d^4g^2e^2+2\,d^3fge^3+d^2f^2e^4\right)e^{(-5)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -(d^3*g^2*e + 2*d^2*f*g*e^2 + d*f^2*e^3)*e^(-4)*log(abs(x^2*e^2 - d^2)) - 1
/3*(g^2*x^3*e^6 + 3*d*g^2*x^2*e^5 + 6*d^2*g^2*x*e^4 + 3*f*g*x^2*e^6 + 12*d*
f*g*x*e^5 + 3*f^2*x*e^6)*e^(-6) - (d^4*g^2*e^2 + 2*d^3*f*g*e^3 + d^2*f^2*e^
4)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

maple [A] time = 0.00, size = 110, normalized size = 1.69

$$-\frac{g^2x^3}{3}-\frac{d\,g^2x^2}{e}-f\,g\,x^2-\frac{2d^3g^2\ln\left(ex-d\right)}{e^3}-\frac{4d^2fg\ln\left(ex-d\right)}{e^2}-\frac{2d^2g^2x}{e^2}-\frac{2d\,f^2\ln\left(ex-d\right)}{e}-\frac{4dfgx}{e}-f^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/3*g^2*x^3-1/e*x^2*d*g^2-x^2*f*g-2/e^2*x*d^2*g^2-4/e*x*d*f*g-x*f^2-2*d^3/
e^3*ln(e*x-d)*g^2-4*d^2/e^2*ln(e*x-d)*f*g-2*d/e*ln(e*x-d)*f^2
```

maxima [A] time = 0.45, size = 97, normalized size = 1.49

$$-\frac{e^2g^2x^3+3\left(e^2fg+deg^2\right)x^2+3\left(e^2f^2+4\,defg+2\,d^2g^2\right)x}{3\,e^2}-\frac{2\left(de^2f^2+2\,d^2efg+d^3g^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/3*(e^2*g^2*x^3 + 3*(e^2*f*g + d*e*g^2)*x^2 + 3*(e^2*f^2 + 4*d*e*f*g + 2*
d^2*g^2)*x)/e^2 - 2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.07, size = 127, normalized size = 1.95

$$-x^2\left(\frac{d\,g^2+2\,e\,f\,g}{2\,e}+\frac{d\,g^2}{2\,e}\right)-x\left(\frac{e\,f^2+2\,d\,g\,f}{e}+\frac{d\,\left(\frac{d\,g^2+2\,e\,f\,g}{e}+\frac{d\,g^2}{e}\right)}{e}\right)-\frac{g^2\,x^3}{3}-\frac{\ln\left(e\,x-d\right)\,\left(2\,d^3\,g^2+4\,d^2\,e\,f\,g+2\,d\,e^2\,f^2\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^2*((d*g^2 + 2*e*f*g)/(2*e) + (d*g^2)/(2*e)) - x*((e*f^2 + 2*d*f*g)/e +
(d*((d*g^2 + 2*e*f*g)/e + (d*g^2)/e))/e) - (g^2*x^3)/3 - (log(e*x - d)*(2*d
^3*g^2 + 2*d*e^2*f^2 + 4*d^2*e*f*g))/e^3
```

sympy [A] time = 0.38, size = 70, normalized size = 1.08

$$-\frac{2d\left(dg+ef\right)^2\log\left(-d+ex\right)}{e^3}-\frac{g^2x^3}{3}-x^2\left(\frac{dg^2}{e}+fg\right)-x\left(\frac{2d^2g^2}{e^2}+\frac{4dfg}{e}+f^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -2*d*(d*g + e*f)**2*log(-d + e*x)/e**3 - g**2*x**3/3 - x**2*(d*g**2/e + f*g
) - x*(2*d**2*g**2/e**2 + 4*d*f*g/e + f**2)
```

3.353 $\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2} \, dx$

Optimal. Leaf size=50

$$-\frac{(dg+ef)^2\log(d-ex)}{e^3}-\frac{gx(dg+ef)}{e^2}-\frac{(f+gx)^2}{2e}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {799, 43}

$$-\frac{gx(dg+ef)}{e^2}-\frac{(dg+ef)^2\log(d-ex)}{e^3}-\frac{(f+gx)^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((g*(e*f + d*g)*x)/e^2) - (f + g*x)^2/(2*e) - ((e*f + d*g)^2*Log[d - e*x])
/e^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 799

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c*x)/g)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(f+gx)^2}{d-ex}\,dx\\ &= \int \left(-\frac{g(ef+dg)}{e^2}+\frac{(ef+dg)^2}{e^2(d-ex)}-\frac{g(f+gx)}{e}\right)dx\\ &= -\frac{g(ef+dg)x}{e^2}-\frac{(f+gx)^2}{2e}-\frac{(ef+dg)^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.86

$$-\frac{egx(2dg+4ef+egx)+2(dg+ef)^2\log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/2*(e*g*x*(4*e*f + 2*d*g + e*g*x) + 2*(e*f + d*g)^2*Log[d - e*x])/e^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2), x]
```

fricas [A] time = 0.39, size = 64, normalized size = 1.28

$$-\frac{e^{2} g^{2} x^{2} + 2 \left(2 e^{2} f g + d e g^{2}\right) x + 2 \left(e^{2} f^{2} + 2 d e f g + d^{2} g^{2}\right) \log\left(e x - d\right)}{2 e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/2*(e^2*g^2*x^2 + 2*(2*e^2*f*g + d*e*g^2)*x + 2*(e^2*f^2 + 2*d*e*f*g + d^
2*g^2)*log(e*x - d))/e^3
```

giac [B] time = 0.17, size = 134, normalized size = 2.68

$$-\frac{1}{2} \left(d^{2} g^{2} e + 2 d f g e^{2} + f^{2} e^{3}\right) e^{\left(-4\right)} \log\left(\left|x^{2} e^{2} - d^{2}\right|\right) - \frac{1}{2} \left(g^{2} x^{2} e^{3} + 2 d g^{2} x e^{2} + 4 f g x e^{3}\right) e^{\left(-4\right)} - \frac{\left(d^{3} g^{2} + 2 d^{2} f g e + d f^{2} e^{2}\right) e^{\left(-3\right)} \log\left(\frac{\left|2 x e^{2} - 2 \left|d\right| e\right|}{\left|2 x e^{2} + 2 \left|d\right| e\right|}\right)}{2 \left|d\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -1/2*(d^2*g^2*e + 2*d*f*g*e^2 + f^2*e^3)*e^(-4)*log(abs(x^2*e^2 - d^2)) - 1
/2*(g^2*x^2*e^3 + 2*d*g^2*x*e^2 + 4*f*g*x*e^3)*e^(-4) - 1/2*(d^3*g^2 + 2*d^
2*f*g*e + d*f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*a
bs(d)*e))/abs(d)
```

maple [A] time = 0.00, size = 82, normalized size = 1.64

$$-\frac{g^{2} x^{2}}{2 e} - \frac{d^{2} g^{2} \ln\left(e x - d\right)}{e^{3}} - \frac{2 d f g \ln\left(e x - d\right)}{e^{2}} - \frac{d g^{2} x}{e^{2}} - \frac{f^{2} \ln\left(e x - d\right)}{e} - \frac{2 f g x}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/2*g^2*x^2/e-g^2/e^2*d*x-2*g/e*f*x-1/e^3*ln(e*x-d)*d^2*g^2-2/e^2*ln(e*x-d
)*d*f*g-1/e*ln(e*x-d)*f^2
```

maxima [A] time = 0.44, size = 63, normalized size = 1.26

$$-\frac{e g^{2} x^{2} + 2 \left(2 e f g + d g^{2}\right) x}{2 e^{2}} - \frac{\left(e^{2} f^{2} + 2 d e f g + d^{2} g^{2}\right) \log\left(e x - d\right)}{e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/2*(e*g^2*x^2 + 2*(2*e*f*g + d*g^2)*x)/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g
^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.61, size = 65, normalized size = 1.30

$$-x \left(\frac{d g^{2}}{e^{2}} + \frac{2 f g}{e}\right) - \frac{\ln\left(e x - d\right) \left(d^{2} g^{2} + 2 d e f g + e^{2} f^{2}\right)}{e^{3}} - \frac{g^{2} x^{2}}{2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2),x)
```

```
[Out] - x*((d*g^2)/e^2 + (2*f*g)/e) - (log(e*x - d)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*
g))/e^3 - (g^2*x^2)/(2*e)
```

sympy [A] time = 0.29, size = 46, normalized size = 0.92

$$-x\left(\frac{dg^2}{e^2}+\frac{2fg}{e}\right)-\frac{g^2x^2}{2e}-\frac{\left(dg+ef\right)^2\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -x*(d*g**2/e**2 + 2*f*g/e) - g**2*x**2/(2*e) - (d*g + e*f)**2*log(-d + e*x)
/e**3
```

3.354 $\int \frac{(f+gx)^2}{d^2-e^2x^2} dx$

Optimal. Leaf size=62

$$-\frac{(dg+ef)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}-\frac{g^2x}{e^2}$$

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {702, 633, 31}

$$-\frac{(dg+ef)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}-\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

```
[Out] -((g^2*x)/e^2) - ((e*f + d*g)^2*Log[d - e*x])/(2*d*e^3) + ((e*f - d*g)^2*Lo
g[d + e*x])/(2*d*e^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \left(-\frac{g^2}{e^2}+\frac{e^2f^2+d^2g^2+2e^2fgx}{e^2\left(d^2-e^2x^2\right)}\right)dx\\
&= -\frac{g^2x}{e^2}+\frac{\int \frac{e^2f^2+d^2g^2+2e^2fgx}{d^2-e^2x^2}\,dx}{e^2}\\
&= -\frac{g^2x}{e^2}-\frac{(ef-dg)^2\int \frac{1}{-de-e^2x}\,dx}{2de}+\frac{(ef+dg)^2\int \frac{1}{de-e^2x}\,dx}{2de}\\
&= -\frac{g^2x}{e^2}-\frac{(ef+dg)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.89

$$\frac{\left(d^2g^2+e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)-deg\left(f\log\left(d^2-e^2x^2\right)+gx\right)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

```
[Out] ((e^2*f^2 + d^2*g^2)*ArcTanh[(e*x)/d] - d*e*g*(g*x + f*Log[d^2 - e^2*x^2]))
/(d*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/(d^2 - e^2*x^2), x]
```

fricas [A] time = 0.41, size = 76, normalized size = 1.23

$$-\frac{2\,deg^2x-\left(e^2f^2-2\,defg+d^2g^2\right)\log\left(ex+d\right)+\left(e^2f^2+2\,defg+d^2g^2\right)\log\left(ex-d\right)}{2\,de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*e*g^2*x - (e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d) + (e^2*f^2
 + 2*d*e*f*g + d^2*g^2)*log(e*x - d))/(d*e^3)
```

giac [A] time = 0.15, size = 81, normalized size = 1.31

$$-g^2xe^{(-2)}-fge^{(-2)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{\left(d^2g^2+f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{2\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -g^2*x*e^(-2) - f*g*e^(-2)*log(abs(x^2*e^2 - d^2)) - 1/2*(d^2*g^2 + f^2*e^2
)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

maple [A] time = 0.01, size = 107, normalized size = 1.73

$$-\frac{d\,g^2\ln\left(ex-d\right)}{2e^3}+\frac{d\,g^2\ln\left(ex+d\right)}{2e^3}-\frac{f^2\ln\left(ex-d\right)}{2de}+\frac{f^2\ln\left(ex+d\right)}{2de}-\frac{fg\ln\left(ex-d\right)}{e^2}-\frac{fg\ln\left(ex+d\right)}{e^2}-\frac{g^2x}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -g^2*x/e^2-1/2/e^3*d*ln(e*x-d)*g^2-1/e^2*ln(e*x-d)*f*g-1/2/e/d*ln(e*x-d)*f^
2+1/2/e^3*d*ln(e*x+d)*g^2-1/e^2*ln(e*x+d)*f*g+1/2/e/d*ln(e*x+d)*f^2
```

maxima [A] time = 0.44, size = 82, normalized size = 1.32

$$-\frac{g^2x}{e^2}+\frac{\left(e^2f^2-2\,defg+d^2g^2\right)\log\left(ex+d\right)}{2\,de^3}-\frac{\left(e^2f^2+2\,defg+d^2g^2\right)\log\left(ex-d\right)}{2\,de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -g^2*x/e^2 + 1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d*e^3) - 1/2
*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d*e^3)
```

mupad [B] time = 0.15, size = 81, normalized size = 1.31

$$\frac{\ln(d+e\,x)\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}{2\,d\,e^3}-\frac{g^2\,x}{e^2}-\frac{\ln(d-e\,x)\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{2\,d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2),x)
```

```
[Out] (log(d + e*x)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g))/(2*d*e^3) - (g^2*x)/e^2 - (l
og(d - e*x)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(2*d*e^3)
```

sympy [B] time = 0.64, size = 112, normalized size = 1.81

$$-\frac{g^2x}{e^2}+\frac{\left(dg-ef\right)^2\log\left(x+\frac{2d^2fg+\frac{d\left(dg-ef\right)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3}-\frac{\left(dg+ef\right)^2\log\left(x+\frac{2d^2fg-\frac{d\left(dg+ef\right)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -g**2*x/e**2 + (d*g - e*f)**2*log(x + (2*d**2*f*g + d*(d*g - e*f)**2/e)/(d*
*2*g**2 + e**2*f**2))/(2*d*e**3) - (d*g + e*f)**2*log(x + (2*d**2*f*g - d*(
d*g + e*f)**2/e)/(d**2*g**2 + e**2*f**2))/(2*d*e**3)
```

3.355 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx$

Optimal. Leaf size=86

$$\frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3}-\frac{(dg+ef)^2\log(d-ex)}{4d^2e^3}-\frac{(ef-dg)^2}{2de^3(d+ex)}$$

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3}-\frac{(dg+ef)^2\log(d-ex)}{4d^2e^3}-\frac{(ef-dg)^2}{2de^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(2*d*e^3*(d + e*x)) - ((e*f + d*g)^2*Log[d - e*x])/(4*d^2*e^
3) + ((e*f - d*g)*(e*f + 3*d*g)*Log[d + e*x])/(4*d^2*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^2}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{4d^2e^2(d-ex)}+\frac{(-ef+dg)^2}{2de^2(d+ex)^2}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)}\right)dx\\
&= -\frac{(ef-dg)^2}{2de^3(d+ex)}-\frac{(ef+dg)^2\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)(ef+3dg)\log(d+ex)}{4d^2e^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.95

$$\frac{(ef-dg)((d+ex)(3dg+ef)\log(d+ex)+2d(dg-ef))-(d+ex)(dg+ef)^2\log(d-ex)}{4d^2e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

```
[Out] (-((e*f + d*g)^2*(d + e*x)*Log[d - e*x]) + (e*f - d*g)*(2*d*(-(e*f) + d*g)
+ (e*f + 3*d*g)*(d + e*x)*Log[d + e*x]))/(4*d^2*e^3*(d + e*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)), x]
```

fricas [B] time = 0.40, size = 165, normalized size = 1.92

$$-\frac{2\,de^2f^2-4\,d^2efg+2\,d^3g^2-\left(de^2f^2+2\,d^2efg-3\,d^3g^2+\left(e^3f^2+2\,de^2fg-3\,d^2eg^2\right)x\right)\log\left(ex+d\right)+\left(de^2f^2+2\,d^2efg+d^3g^2+\left(e^3f^2+2\,de^2fg+d^2eg^2\right)x\right)\log\left(ex-d\right)}{4\left(d^2e^4x+d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*e^2*f^2 - 4*d^2*e*f*g + 2*d^3*g^2 - (d*e^2*f^2 + 2*d^2*e*f*g - 3*
d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e*x + d) + (d*e^2*f^
2 + 2*d^2*e*f*g + d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e*x
- d))/(d^2*e^4*x + d^3*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(d^2
*g^2-2*d*exp(1)*g*f+exp(1)^2*f^2)/(exp(2)*d^2*exp(1)-d^2*exp(1)^3)*ln(abs(x
*exp(1)+d))-(2*exp(2)*d*g*f-exp(2)*exp(1)*f^2-d^2*exp(1)*g^2)/(2*exp(2)^2*d
^2-2*exp(2)*d^2*exp(1)^2)*ln(abs(x^2*exp(2)-d^2))-(exp(2)*f^2+d^2*g^2-2*d*e
xp(1)*g*f)*1/2/(exp(2)*d-d*exp(1)^2)/exp(1)/abs(d)*ln(abs(2*x*exp(2)-2*exp(
1)*abs(d))/abs(2*x*exp(2)+2*exp(1)*abs(d)))
```

maple [A] time = 0.01, size = 149, normalized size = 1.73

$$-\frac{d\,g^2}{2\left(ex+d\right)e^3}-\frac{f^2}{2\left(ex+d\right)de}-\frac{fg\ln\left(ex-d\right)}{2d\,e^2}+\frac{fg\ln\left(ex+d\right)}{2d\,e^2}-\frac{f^2\ln\left(ex-d\right)}{4d^2e}+\frac{f^2\ln\left(ex+d\right)}{4d^2e}+\frac{fg}{\left(ex+d\right)e^2}-\frac{g^2\ln\left(ex-d\right)}{4e^3}-\frac{3g^2\ln\left(ex+d\right)}{4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x)
```

```
[Out] -1/4/e^3*ln(e*x-d)*g^2-1/2/e^2/d*ln(e*x-d)*f*g-1/4/e/d^2*ln(e*x-d)*f^2-3/4/
e^3*ln(e*x+d)*g^2+1/2/e^2/d*ln(e*x+d)*f*g+1/4/e/d^2*ln(e*x+d)*f^2-1/2/e^3*d
/(e*x+d)*g^2+1/e^2/(e*x+d)*f*g-1/2/e/d/(e*x+d)*f^2
```

maxima [A] time = 0.46, size = 113, normalized size = 1.31

$$-\frac{e^2f^2-2\,defg+d^2g^2}{2\left(de^4x+d^2e^3\right)}+\frac{\left(e^2f^2+2\,defg-3\,d^2g^2\right)\log\left(ex+d\right)}{4\,d^2e^3}-\frac{\left(e^2f^2+2\,defg+d^2g^2\right)\log\left(ex-d\right)}{4\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(d*e^4*x + d^2*e^3) + 1/4*(e^2*f^2 + 2
*d*e*f*g - 3*d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 + 2*d*e*f*g + d
^2*g^2)*log(e*x - d)/(d^2*e^3)
```

mupad [B] time = 2.70, size = 109, normalized size = 1.27

$$\frac{\ln(d+e\,x)\,\left(-3\,d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3}-\frac{\ln(d-e\,x)\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3}-\frac{d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2}{2\,d\,e^3\,(d+e\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(e^2*f^2 - 3*d^2*g^2 + 2*d*e*f*g))/(4*d^2*e^3) - (log(d - e*x
)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(4*d^2*e^3) - (d^2*g^2 + e^2*f^2 - 2*d*e
*f*g)/(2*d*e^3*(d + e*x))
```

sympy [B] time = 1.00, size = 182, normalized size = 2.12

$$-\frac{d^2g^2-2defg+e^2f^2}{2d^2e^3+2de^4x}-\frac{(dg-ef)\,(3dg+ef)\log\left(x+\frac{-2d^3g^2+d(dg-ef)(3dg+ef)}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3}-\frac{(dg+ef)^2\log\left(x+\frac{-2d^3g^2+d(dg+ef)^2}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2),x)
```

```
[Out] -(d**2*g**2 - 2*d*e*f*g + e**2*f**2)/(2*d**2*e**3 + 2*d*e**4*x) - (d*g - e*
f)*(3*d*g + e*f)*log(x + (-2*d**3*g**2 + d*(d*g - e*f)*(3*d*g + e*f))/(d**2
*e*g**2 - 2*d*e**2*f*g - e**3*f**2))/(4*d**2*e**3) - (d*g + e*f)**2*log(x +
 (-2*d**3*g**2 + d*(d*g + e*f)**2)/(d**2*e*g**2 - 2*d*e**2*f*g - e**3*f**2)
)/(4*d**2*e**3)
```

3.356 $\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx$

Optimal. Leaf size=87

$$\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}-\frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)}-\frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}-\frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(4*d*e^3*(d + e*x)^2) - ((e*f - d*g)*(e*f + 3*d*g))/(4*d^2*e
^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)}\,dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^3}\,dx\\
&= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^3}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^2}+\frac{(ef+dg)^2}{4d^2e^2\left(d^2-e^2x^2\right)}\right)dx\\
&= -\frac{(ef-dg)^2}{4de^3(d+ex)^2}-\frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)}+\frac{(ef+dg)^2\int \frac{1}{d^2-e^2x^2}\,dx}{4d^2e^2}\\
&= -\frac{(ef-dg)^2}{4de^3(d+ex)^2}-\frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)}+\frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 1.00

$$\frac{\frac{2d(dg-ef)\left(2d^2g+de(2f+3gx)+e^2fx\right)}{(d+ex)^2}+(dg+ef)^2(-\log(d-ex))+(dg+ef)^2\log(d+ex)}{8d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

```
[Out] ((2*d*(-(e*f) + d*g)*(2*d^2*g + e^2*f*x + d*e*(2*f + 3*g*x)))/(d + e*x)^2 -
 (e*f + d*g)^2*Log[d - e*x] + (e*f + d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)), x]
```

fricas [B] time = 0.41, size = 271, normalized size = 3.11

$$-\frac{4\,d^2e^2f^2-4\,d^4g^2+2\left(de^3f^2+2\,d^2e^2fg-3\,d^3eg^2\right)x-\left(d^2e^2f^2+2\,d^3efg+d^4g^2+\left(e^4f^2+2\,de^3fg+d^2e^2g^2\right)x^2+2\left(de^3f^2+2\,d^2e^2fg+d^3eg^2\right)x\right)\log\left(ex+d\right)+\left(d^2e^2f^2+2\,d^3efg+d^4g^2+\left(e^4f^2+2\,de^3fg+d^2e^2g^2\right)x^2+2\left(de^3f^2+2\,d^2e^2fg+d^3eg^2\right)x\right)\log\left(ex-d\right)}{8\left(d^3e^5x^2+2\,d^4e^4x+d^5e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*d^2*e^2*f^2 - 4*d^4*g^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g - 3*d^3*e*g^
2)*x - (d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 + 2*d*e^3*f*g + d^2*
e^2*g^2)*x^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x + d) +
(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 + 2*d*e^3*f*g + d^2*e^2*g^2
)*x^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x - d))/(d^3*e^5
*x^2 + 2*d^4*e^4*x + d^5*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(-g^
2*d^2*exp(1)+g*d*exp(1)^2*f+g*d*f*exp(2)-exp(1)*f^2*exp(2))/(d^3*exp(1)^4-2
*d^3*exp(1)^2*exp(2)+d^3*exp(2)^2)*ln(abs(-(-(exp(1)*x+d)^-1/exp(1))^2*d^2*
exp(1)^4+(-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^2*exp(2)-2*(exp(1)*x+d)^-1/
exp(1)*d*exp(1)*exp(2)+exp(2)))-(g^2*d^2*exp(1)^4+g^2*d^2*exp(1)^2*exp(2)-4
*g*d*exp(1)^3*f*exp(2)+exp(1)^4*f^2*exp(2)+exp(1)^2*f^2*exp(2)^2)/2/(d^2*ex
p(1)^4-2*d^2*exp(1)^2*exp(2)+d^2*exp(2)^2)/exp(1)/abs(d)/exp(1)^2*ln(abs(2*
(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^4-2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^2*e
xp(2)+2*d*exp(1)*exp(2)-2*exp(1)*abs(d)*exp(1)^2)/abs(2*(exp(1)*x+d)^-1/exp
(1)*d^2*exp(1)^4-2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^2*exp(2)+2*d*exp(1)*ex
p(2)+2*exp(1)*abs(d)*exp(1)^2))-((exp(1)*x+d)^-1/exp(1)*g^2*d^2*exp(1)^2-2*
(exp(1)*x+d)^-1/exp(1)*g*d*exp(1)^3*f+(exp(1)*x+d)^-1/exp(1)*exp(1)^4*f^2)/
(d^2*exp(1)^4-d^2*exp(1)^2*exp(2))
```

maple [B] time = 0.01, size = 206, normalized size = 2.37

$$-\frac{d\,g^2}{4\left(ex+d\right)^2e^3}-\frac{f^2}{4\left(ex+d\right)^2de}+\frac{fg}{2\left(ex+d\right)^2e^2}-\frac{fg}{2\left(ex+d\right)d\,e^2}-\frac{g^2\ln\left(ex-d\right)}{8d\,e^3}+\frac{g^2\ln\left(ex+d\right)}{8d\,e^3}-\frac{f^2}{4\left(ex+d\right)d^2e}-\frac{fg\ln\left(ex-d\right)}{4d^2e^2}+\frac{fg\ln\left(ex+d\right)}{4d^2e^2}-\frac{f^2\ln\left(ex-d\right)}{8d^3e}+\frac{f^2\ln\left(ex+d\right)}{8d^3e}+\frac{3g^2}{4\left(ex+d\right)e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/8/e^3/d*ln(e*x-d)*g^2-1/4/e^2/d^2*ln(e*x-d)*f*g-1/8/e/d^3*ln(e*x-d)*f^2+
3/4/e^3/(e*x+d)*g^2-1/2/d/e^2/(e*x+d)*f*g-1/4/d^2/e/(e*x+d)*f^2-1/4/e^3*d/(
e*x+d)^2*g^2+1/2/e^2/(e*x+d)^2*f*g-1/4/e/d/(e*x+d)^2*f^2+1/8/e^3/d*ln(e*x+d
)*g^2+1/4/e^2/d^2*ln(e*x+d)*f*g+1/8/e/d^3*ln(e*x+d)*f^2
```

maxima [A] time = 0.45, size = 149, normalized size = 1.71

$$-\frac{2\,de^2f^2-2\,d^3g^2+\left(e^3f^2+2\,de^2fg-3\,d^2eg^2\right)x}{4\left(d^2e^5x^2+2\,d^3e^4x+d^4e^3\right)}+\frac{\left(e^2f^2+2\,defg+d^2g^2\right)\log\left(ex+d\right)}{8\,d^3e^3}-\frac{\left(e^2f^2+2\,defg+d^2g^2\right)\log\left(ex-d\right)}{8\,d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*d*e^2*f^2 - 2*d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g - 3*d^2*e*g^2)*x)/(d
^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3) + 1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*l
og(e*x + d)/(d^3*e^3) - 1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d
^3*e^3)
```

mupad [B] time = 0.13, size = 100, normalized size = 1.15

$$\frac{\frac{d^2\,g^2-e^2\,f^2}{2\,d\,e^3}-\frac{x\,\left(-3\,d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^2}}{d^2+2\,d\,e\,x+e^2\,x^2}+\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g+e\,f\right)}^2}{4\,d^3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - e^2*f^2)/(2*d*e^3) - (x*(e^2*f^2 - 3*d^2*g^2 + 2*d*e*f*g))/(4*d
^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*x) + (atanh((e*x)/d)*(d*g + e*f)^2)/(4*d^3*
e^3)
```

sympy [B] time = 1.03, size = 185, normalized size = 2.13

$$-\frac{-2d^3g^2+2de^2f^2+x\left(-3d^2eg^2+2de^2fg+e^3f^2\right)}{4d^4e^3+8d^3e^4x+4d^2e^5x^2}-\frac{\left(dg+ef\right)^2\log\left(-\frac{d\left(dg+ef\right)^2}{e\left(d^2g^2+2defg+e^2f^2\right)}+x\right)}{8d^3e^3}+\frac{\left(dg+ef\right)^2\log\left(\frac{d\left(dg+ef\right)^2}{e\left(d^2g^2+2defg+e^2f^2\right)}+x\right)}{8d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -(-2*d**3*g**2 + 2*d*e**2*f**2 + x*(-3*d**2*e*g**2 + 2*d*e**2*f*g + e**3*f*
*2))/(4*d**4*e**3 + 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g + e*f)**2*log(
-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3
) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f*
*2)) + x)/(8*d**3*e**3)
```

3.357 $\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx$

Optimal. Leaf size=113

$$\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}-\frac{(dg+ef)^2}{8d^3e^3(d+ex)}-\frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2}-\frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

Rubi [A] time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2}-\frac{(dg+ef)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}-\frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(6*d*e^3*(d + e*x)^3) - ((e*f - d*g)*(e*f + 3*d*g))/(8*d^2*e
^3*(d + e*x)^2) - (e*f + d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcT
anh[(e*x)/d])/(8*d^4*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^4}\,dx\\
&= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^4}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^3}+\frac{(ef+dg)^2}{8d^3e^2(d+ex)^2}+\frac{(ef+dg)^2}{8d^3e^2\left(d^2-e^2x^2\right)}\right)\\
&= -\frac{(ef-dg)^2}{6de^3(d+ex)^3}-\frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2}-\frac{(ef+dg)^2}{8d^3e^3(d+ex)}+\frac{(ef+dg)^2\int \frac{1}{d^2-e^2x^2}\,dx}{8d^3e^2}\\
&= -\frac{(ef-dg)^2}{6de^3(d+ex)^3}-\frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2}-\frac{(ef+dg)^2}{8d^3e^3(d+ex)}+\frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 122, normalized size = 1.08

$$\frac{-\frac{8d^3(ef-dg)^2}{(d+ex)^3}+\frac{6d^2\left(3d^2g^2-2defg-e^2f^2\right)}{(d+ex)^2}-\frac{6d(dg+ef)^2}{d+ex}-3(dg+ef)^2\log(d-ex)+3(dg+ef)^2\log(d+ex)}{48d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] ((-8*d^3*(e*f - d*g)^2)/(d + e*x)^3 + (6*d^2*(-(e^2*f^2) - 2*d*e*f*g + 3*d^
2*g^2))/(d + e*x)^2 - (6*d*(e*f + d*g)^2)/(d + e*x) - 3*(e*f + d*g)^2*Log[d
 - e*x] + 3*(e*f + d*g)^2*Log[d + e*x])/(48*d^4*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)), x]
```

fricas [B] time = 0.39, size = 400, normalized size = 3.54

$$-\frac{20\,d^3e^2f^2+8\,d^4efg-4\,d^5g^2+6\left(de^4f^2+2\,d^2e^3fg+d^3e^2g^2\right)x^2+6\left(3\,d^2e^3f^2+6\,d^3e^2fg-d^4eg^2\right)x-3\left(d^3e^2f^2+2\,d^4efg+d^5g^2+\left(e^5f^2+2\,de^4fg+d^2e^3g^2\right)x^3+3\left(de^4f^2+2\,d^2e^3fg+d^3e^2g^2\right)x^2+3\left(d^2e^3f^2+2\,d^3e^2fg+d^4eg^2\right)x\right)\log\left(ex+d\right)+3\left(d^3e^2f^2+2\,d^4efg+d^5g^2+\left(e^5f^2+2\,de^4fg+d^2e^3g^2\right)x^3+3\left(de^4f^2+2\,d^2e^3fg+d^3e^2g^2\right)x^2+3\left(d^2e^3f^2+2\,d^3e^2fg+d^4eg^2\right)x\right)\log\left(ex-d\right)}{48\left(d^4e^6x^3+3\,d^5e^5x^2+3\,d^6e^4x+d^7e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/48*(20*d^3*e^2*f^2 + 8*d^4*e*f*g - 4*d^5*g^2 + 6*(d*e^4*f^2 + 2*d^2*e^3*
f*g + d^3*e^2*g^2)*x^2 + 6*(3*d^2*e^3*f^2 + 6*d^3*e^2*f*g - d^4*e*g^2)*x -
3*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2 + (e^5*f^2 + 2*d*e^4*f*g + d^2*e^3*g
^2)*x^3 + 3*(d*e^4*f^2 + 2*d^2*e^3*f*g + d^3*e^2*g^2)*x^2 + 3*(d^2*e^3*f^2
+ 2*d^3*e^2*f*g + d^4*e*g^2)*x)*log(e*x + d) + 3*(d^3*e^2*f^2 + 2*d^4*e*f*g
 + d^5*g^2 + (e^5*f^2 + 2*d*e^4*f*g + d^2*e^3*g^2)*x^3 + 3*(d*e^4*f^2 + 2*d
^2*e^3*f*g + d^3*e^2*g^2)*x^2 + 3*(d^2*e^3*f^2 + 2*d^3*e^2*f*g + d^4*e*g^2)
*x)*log(e*x - d))/(d^4*e^6*x^3 + 3*d^5*e^5*x^2 + 3*d^6*e^4*x + d^7*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(2*e
xp(2)^2*d*g*f-3*exp(2)^2*exp(1)*f^2-3*exp(2)*d^2*exp(1)*g^2+6*exp(2)*d*exp(
1)^2*g*f-exp(2)*exp(1)^3*f^2-d^2*exp(1)^3*g^2)/(2*exp(2)^3*d^4-6*exp(2)^2*d
^4*exp(1)^2+6*exp(2)*d^4*exp(1)^4-2*d^4*exp(1)^6)*ln(abs(-x^2*exp(2)+d^2))-
(-exp(2)^3*f^2-exp(2)^2*d^2*g^2+6*exp(2)^2*d*exp(1)*g*f-3*exp(2)^2*exp(1)^2
*f^2-3*exp(2)*d^2*exp(1)^2*g^2+2*exp(2)*d*exp(1)^3*g*f)*1/2/(exp(2)^3*d^3-3
*exp(2)^2*d^3*exp(1)^2+3*exp(2)*d^3*exp(1)^4-d^3*exp(1)^6)/exp(1)/abs(d)*ln
(abs(-2*x*exp(2)-2*exp(1)*abs(d))/abs(-2*x*exp(2)+2*exp(1)*abs(d)))-(-2*exp
(2)^2*d*exp(1)*g*f+3*exp(2)^2*exp(1)^2*f^2+3*exp(2)*d^2*exp(1)^2*g^2-6*exp(
2)*d*exp(1)^3*g*f+exp(2)*exp(1)^4*f^2+d^2*exp(1)^4*g^2)/(exp(2)^3*d^4*exp(1
)-3*exp(2)^2*d^4*exp(1)^3+3*exp(2)*d^4*exp(1)^5-d^4*exp(1)^7)*ln(abs(x*exp(
1)+d))-(-exp(2)^2*d^4*g^2+6*exp(2)^2*d^3*exp(1)*g*f-5*exp(2)^2*d^2*exp(1)^2
```

```
*f^2-2*exp(2)*d^4*exp(1)^2*g^2-4*exp(2)*d^3*exp(1)^3*g*f+6*exp(2)*d^2*exp(1
)^4*f^2+3*d^4*exp(1)^4*g^2-2*d^3*exp(1)^5*g*f-d^2*exp(1)^6*f^2+(4*exp(2)^2*
d^2*exp(1)^2*g*f-4*exp(2)^2*d*exp(1)^3*f^2-4*exp(2)*d^3*exp(1)^3*g^2+4*exp(
2)*d*exp(1)^5*f^2+4*d^3*exp(1)^5*g^2-4*d^2*exp(1)^6*g*f)*x)/2/d^4/exp(1)/(e
xp(2)-exp(1)^2)^3/(x*exp(1)+d)^2
```

maple [B] time = 0.01, size = 259, normalized size = 2.29

$$-\frac{d g^2}{6(ex+d)^3 e^3}-\frac{f^2}{6(ex+d)^3 de}+\frac{fg}{3(ex+d)^3 e^2}-\frac{fg}{4(ex+d)^2 d e^2}-\frac{f^2}{8(ex+d)^2 d^2 e}+\frac{3g^2}{8(ex+d)^2 e^3}-\frac{g^2}{8(ex+d) d e^3}-\frac{fg}{4(ex+d) d^2 e^2}-\frac{g^2 \ln(ex-d)}{16 d^2 e^3}+\frac{g^2 \ln(ex+d)}{16 d^2 e^3}-\frac{f^2}{8(ex+d) d^3 e}-\frac{fg \ln(ex-d)}{8 d^3 e^2}+\frac{fg \ln(ex+d)}{8 d^3 e^2}-\frac{f^2 \ln(ex-d)}{16 d^4 e}+\frac{f^2 \ln(ex+d)}{16 d^4 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x)
```

```
[Out] -1/16/e^3/d^2*ln(e*x-d)*g^2-1/8/e^2/d^3*ln(e*x-d)*f*g-1/16/e/d^4*ln(e*x-d)*
f^2+3/8/e^3/(e*x+d)^2*g^2-1/4/d/e^2/(e*x+d)^2*f*g-1/8/d^2/e/(e*x+d)^2*f^2-1
/6/e^3*d/(e*x+d)^3*g^2+1/3/e^2/(e*x+d)^3*f*g-1/6/e/d/(e*x+d)^3*f^2+1/16/e^3
/d^2*ln(e*x+d)*g^2+1/8/e^2/d^3*ln(e*x+d)*f*g+1/16/e/d^4*ln(e*x+d)*f^2-1/8/d
/e^3/(e*x+d)*g^2-1/4/d^2/e^2/(e*x+d)*f*g-1/8/d^3/e/(e*x+d)*f^2
```

maxima [A] time = 0.48, size = 206, normalized size = 1.82

$$-\frac{10 d^2 e^2 f^2 + 4 d^3 e f g - 2 d^4 g^2 + 3 \left(e^4 f^2 + 2 d e^3 f g + d^2 e^2 g^2\right) x^2 + 3 \left(3 d e^3 f^2 + 6 d^2 e^2 f g - d^3 e g^2\right) x}{24 \left(d^3 e^6 x^3 + 3 d^4 e^5 x^2 + 3 d^5 e^4 x + d^6 e^3\right)} + \frac{\left(e^2 f^2 + 2 d e f g + d^2 g^2\right) \log(ex+d)}{16 d^4 e^3} - \frac{\left(e^2 f^2 + 2 d e f g + d^2 g^2\right) \log(ex-d)}{16 d^4 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/24*(10*d^2*e^2*f^2 + 4*d^3*e*f*g - 2*d^4*g^2 + 3*(e^4*f^2 + 2*d*e^3*f*g
+ d^2*e^2*g^2)*x^2 + 3*(3*d*e^3*f^2 + 6*d^2*e^2*f*g - d^3*e*g^2)*x)/(d^3*e^
6*x^3 + 3*d^4*e^5*x^2 + 3*d^5*e^4*x + d^6*e^3) + 1/16*(e^2*f^2 + 2*d*e*f*g
+ d^2*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*lo
g(e*x - d)/(d^4*e^3)
```

mupad [B] time = 2.65, size = 152, normalized size = 1.35

$$\frac{\operatorname{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g+e\,f\right)}^2}{8\,d^4\,e^3}-\frac{\frac{-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2}{12\,d\,e^3}+\frac{x\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{8\,d^3\,e}}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^3),x)
```

```
[Out] (atanh((e*x)/d)*(d*g + e*f)^2)/(8*d^4*e^3) - ((5*e^2*f^2 - d^2*g^2 + 2*d*e*
f*g)/(12*d*e^3) + (x*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(8*d^2*e^2) + (x^2*
(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(8*d^3*e))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 +
3*d^2*e*x)
```

sympy [B] time = 1.42, size = 248, normalized size = 2.19

$$-\frac{-2d^4g^2 + 4d^3efg + 10d^2e^2f^2 + x^2\left(3d^2e^2g^2 + 6de^3fg + 3e^4f^2\right) + x\left(-3d^3eg^2 + 18d^2e^2fg + 9de^3f^2\right)}{24d^6e^3 + 72d^5e^4x + 72d^4e^5x^2 + 24d^3e^6x^3} - \frac{\left(dg+ef\right)^2 \log\left(-\frac{d\left(dg+ef\right)^2}{e\left(d^2g^2+2defg+e^2f^2\right)} + x\right)}{16d^4e^3} + \frac{\left(dg+ef\right)^2 \log\left(\frac{d\left(dg+ef\right)^2}{e\left(d^2g^2+2defg+e^2f^2\right)} + x\right)}{16d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2),x)
```

```
[Out] -(-2*d**4*g**2 + 4*d**3*e*f*g + 10*d**2*e**2*f**2 + x**2*(3*d**2*e**2*g**2
+ 6*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 18*d**2*e**2*f*g + 9*d*
e**3*f**2))/(24*d**6*e**3 + 72*d**5*e**4*x + 72*d**4*e**5*x**2 + 24*d**3*e
*6*x**3) - (d*g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g +
 e**2*f**2)) + x)/(16*d**4*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(
d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(16*d**4*e**3)
```

3.358 $\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx$

Optimal. Leaf size=139

$$\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3}-\frac{(dg+ef)^2}{16d^4e^3(d+ex)}-\frac{(dg+ef)^2}{16d^3e^3(d+ex)^2}-\frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3}-\frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3}-\frac{(dg+ef)^2}{16d^4e^3(d+ex)}-\frac{(dg+ef)^2}{16d^3e^3(d+ex)^2}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3}-\frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(8*d*e^3*(d + e*x)^4) - ((e*f - d*g)*(e*f + 3*d*g))/(12*d^2*
e^3*(d + e*x)^3) - (e*f + d*g)^2/(16*d^3*e^3*(d + e*x)^2) - (e*f + d*g)^2/(
16*d^4*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(16*d^5*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)}\,dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^5}\,dx\\ &= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^5}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^4}+\frac{(ef+dg)^2}{8d^3e^2(d+ex)^3}+\frac{(ef+dg)^2}{16d^4e^2(d+ex)^2}+\frac{}{16\ldots}\right.\\ &= -\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(ef+\ldots}{}\\ &= -\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(ef+\ldots}{}\end{aligned}$$

Mathematica [A] time = 0.09, size = 142, normalized size = 1.02

$$-\frac{\frac{12d^4(ef-dg)^2}{(d+ex)^4}+\frac{6d^2(dg+ef)^2}{(d+ex)^2}+\frac{8d^3\left(-3d^2g^2+2defg+e^2f^2\right)}{(d+ex)^3}+\frac{6d(dg+ef)^2}{d+ex}+3(dg+ef)^2\log(d-ex)-3(dg+ef)^2\log(d+ex)}{96d^5e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/96*((12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(e^2*f^2 + 2*d*e*f*g - 3
*d^2*g^2))/(d + e*x)^3 + (6*d^2*(e*f + d*g)^2)/(d + e*x)^2 + (6*d*(e*f + d*
g)^2)/(d + e*x) + 3*(e*f + d*g)^2*Log[d - e*x] - 3*(e*f + d*g)^2*Log[d + e*
x])/(d^5*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)), x]
```

fricas [B] time = 0.42, size = 511, normalized size = 3.68

$$-\frac{32d^4e^2f^2+16d^5efg+6\left(de^5f^2+2d^2e^4fg+d^3e^3g^2\right)x^3+24\left(d^2e^4f^2+2d^3e^3fg+d^4e^2g^2\right)x^2+2\left(19d^3e^3f^2+38d^4e^2fg+3d^5eg^2\right)x-3\left(d^4e^2f^2+2d^5efg+d^6g^2+\left(e^6f^2+2de^5fg+d^2e^4g^2\right)x^4+4\left(de^5f^2+2d^2e^4fg+d^3e^3g^2\right)x^3+6\left(d^2e^4f^2+2d^3e^3fg+d^4e^2g^2\right)x^2+4\left(d^3e^3f^2+2d^4e^2fg+d^5eg^2\right)x\right)\log(ex+d)+3\left(d^4e^2f^2+2d^5efg+d^6g^2+\left(e^6f^2+2de^5fg+d^2e^4g^2\right)x^4+4\left(de^5f^2+2d^2e^4fg+d^3e^3g^2\right)x^3+6\left(d^2e^4f^2+2d^3e^3fg+d^4e^2g^2\right)x^2+4\left(d^3e^3f^2+2d^4e^2fg+d^5eg^2\right)x\right)\log(ex-d)}{96\left(d^5e^7x^4+4d^6e^6x^3+6d^7e^5x^2+4d^8e^4x+d^9e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/96*(32*d^4*e^2*f^2 + 16*d^5*e*f*g + 6*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e
^3*g^2)*x^3 + 24*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 2*(19*d^
3*e^3*f^2 + 38*d^4*e^2*f*g + 3*d^5*e*g^2)*x - 3*(d^4*e^2*f^2 + 2*d^5*e*f*g
+ d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x^4 + 4*(d*e^5*f^2 + 2*d^
2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2
)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)*x)*log(e*x + d) + 3*(d^
4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x
^4 + 4*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d
^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)
*x)*log(e*x - d))/(d^5*e^7*x^4 + 4*d^6*e^6*x^3 + 6*d^7*e^5*x^2 + 4*d^8*e^4*
x + d^9*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(exp
(2)^3*d*g*f-2*exp(2)^3*exp(1)*f^2-2*exp(2)^2*d^2*exp(1)*g^2+6*exp(2)^2*d*ex
p(1)^2*g*f-2*exp(2)^2*exp(1)^3*f^2-2*exp(2)*d^2*exp(1)^3*g^2+exp(2)*d*exp(1
)^4*g*f)/(exp(2)^4*d^5-4*exp(2)^3*d^5*exp(1)^2+6*exp(2)^2*d^5*exp(1)^4-4*ex
p(2)*d^5*exp(1)^6+d^5*exp(1)^8)*ln(abs(-x^2*exp(2)+d^2))-(-exp(2)^4*f^2-exp
(2)^3*d^2*g^2+8*exp(2)^3*d*exp(1)*g*f-6*exp(2)^3*exp(1)^2*f^2-6*exp(2)^2*d^
2*exp(1)^2*g^2+8*exp(2)^2*d*exp(1)^3*g*f-exp(2)^2*exp(1)^4*f^2-exp(2)*d^2*e
xp(1)^4*g^2)*1/2/(exp(2)^4*d^4-4*exp(2)^3*d^4*exp(1)^2+6*exp(2)^2*d^4*exp(1
)^4-4*exp(2)*d^4*exp(1)^6+d^4*exp(1)^8)/exp(1)/abs(d)*ln(abs(-2*x*exp(2)-2*
```

```
exp(1)*abs(d))/abs(-2*x*exp(2)+2*exp(1)*abs(d)))-(-2*exp(2)^3*d*exp(1)*g*f+
4*exp(2)^3*exp(1)^2*f^2+4*exp(2)^2*d^2*exp(1)^2*g^2-12*exp(2)^2*d*exp(1)^3*
g*f+4*exp(2)^2*exp(1)^4*f^2+4*exp(2)*d^2*exp(1)^4*g^2-2*exp(2)*d*exp(1)^5*g
*f)/(exp(2)^4*d^5*exp(1)-4*exp(2)^3*d^5*exp(1)^3+6*exp(2)^2*d^5*exp(1)^5-4*
exp(2)*d^5*exp(1)^7+d^5*exp(1)^9)*ln(abs(x*exp(1)+d))-((6*exp(2)^3*d^2*exp(
1)^3*g*f-9*exp(2)^3*d*exp(1)^4*f^2-9*exp(2)^2*d^3*exp(1)^4*g^2+12*exp(2)^2*
d^2*exp(1)^5*g*f+6*exp(2)^2*d*exp(1)^6*f^2+6*exp(2)*d^3*exp(1)^6*g^2-18*exp
(2)*d^2*exp(1)^7*g*f+3*exp(2)*d*exp(1)^8*f^2+3*d^3*exp(1)^8*g^2)*x^2+(15*ex
p(2)^3*d^3*exp(1)^2*g*f-21*exp(2)^3*d^2*exp(1)^3*f^2-21*exp(2)^2*d^4*exp(1)
^3*g^2+21*exp(2)^2*d^3*exp(1)^4*g*f+18*exp(2)^2*d^2*exp(1)^5*f^2+18*exp(2)*
d^4*exp(1)^5*g^2-39*exp(2)*d^3*exp(1)^6*g*f+3*exp(2)*d^2*exp(1)^7*f^2+3*d^4
*exp(1)^7*g^2+3*d^3*exp(1)^8*g*f)*x-exp(2)^3*d^5*g^2+11*exp(2)^3*d^4*exp(1)
*g*f-13*exp(2)^3*d^3*exp(1)^2*f^2-9*exp(2)^2*d^5*exp(1)^2*g^2+3*exp(2)^2*d^
4*exp(1)^3*g*f+15*exp(2)^2*d^3*exp(1)^4*f^2+9*exp(2)*d^5*exp(1)^4*g^2-15*ex
p(2)*d^4*exp(1)^5*g*f-3*exp(2)*d^3*exp(1)^6*f^2+d^5*exp(1)^6*g^2+d^4*exp(1)
^7*g*f+d^3*exp(1)^8*f^2)/3/d^5/exp(1)/(exp(2)-exp(1)^2)^4/(x*exp(1)+d)^3
```

maple [B] time = 0.01, size = 312, normalized size = 2.24

$$-\frac{d g^2}{8(ex+d)^4 e^3}-\frac{f^2}{8(ex+d)^4 de}+\frac{fg}{4(ex+d)^4 e^2}-\frac{fg}{6(ex+d)^3 d e^2}-\frac{f^2}{12(ex+d)^3 d^2 e}+\frac{g^2}{4(ex+d)^3 e^3}-\frac{g^2}{16(ex+d)^2 d e^3}-\frac{fg}{8(ex+d)^2 d^2 e^2}-\frac{f^2}{16(ex+d)^2 d^3 e}-\frac{g^2}{16(ex+d) d^2 e^3}-\frac{fg}{8(ex+d) d^3 e^2}-\frac{g^2 \ln(ex-d)}{32 d^3 e^3}+\frac{g^2 \ln(ex+d)}{32 d^3 e^3}-\frac{f^2}{16(ex+d) d^4 e}-\frac{fg \ln(ex-d)}{16 d^4 e^2}+\frac{fg \ln(ex+d)}{16 d^4 e^2}-\frac{f^2 \ln(ex-d)}{32 d^5 e}+\frac{f^2 \ln(ex+d)}{32 d^5 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x)
```

```
[Out] -1/32/e^3/d^3*ln(e*x-d)*g^2-1/16/e^2/d^4*ln(e*x-d)*f*g-1/32/e/d^5*ln(e*x-d)
*f^2+1/4/e^3/(e*x+d)^3*g^2-1/6/d/e^2/(e*x+d)^3*f*g-1/12/d^2/e/(e*x+d)^3*f^2
-1/8/e^3*d/(e*x+d)^4*g^2+1/4/e^2/(e*x+d)^4*f*g-1/8/e/d/(e*x+d)^4*f^2+1/32/e
^3/d^3*ln(e*x+d)*g^2+1/16/e^2/d^4*ln(e*x+d)*f*g+1/32/e/d^5*ln(e*x+d)*f^2-1/
16/d^2/e^3/(e*x+d)*g^2-1/8/d^3/e^2/(e*x+d)*f*g-1/16/d^4/e/(e*x+d)*f^2-1/16/
d/e^3/(e*x+d)^2*g^2-1/8/d^2/e^2/(e*x+d)^2*f*g-1/16/d^3/e/(e*x+d)^2*f^2
```

maxima [A] time = 0.49, size = 236, normalized size = 1.70

$$-\frac{16 d^3 e f^2 + 8 d^4 f g + 3\left(e^4 f^2 + 2 d e^3 f g + d^2 e^2 g^2\right) x^3 + 12\left(d e^3 f^2 + 2 d^2 e^2 f g + d^3 e g^2\right) x^2 + \left(19 d^2 e^2 f^2 + 38 d^3 e f g + 3 d^4 g^2\right) x}{48\left(d^4 e^6 x^4 + 4 d^5 e^5 x^3 + 6 d^6 e^4 x^2 + 4 d^7 e^3 x + d^8 e^2\right)} + \frac{\left(e^2 f^2 + 2 d e f g + d^2 g^2\right)\log(ex+d)}{32 d^5 e^3} - \frac{\left(e^2 f^2 + 2 d e f g + d^2 g^2\right)\log(ex-d)}{32 d^5 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/48*(16*d^3*e*f^2 + 8*d^4*f*g + 3*(e^4*f^2 + 2*d*e^3*f*g + d^2*e^2*g^2)*x
^3 + 12*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x^2 + (19*d^2*e^2*f^2 + 38*
d^3*e*f*g + 3*d^4*g^2)*x)/(d^4*e^6*x^4 + 4*d^5*e^5*x^3 + 6*d^6*e^4*x^2 + 4*
d^7*e^3*x + d^8*e^2) + 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d
^5*e^3) - 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^5*e^3)
```

mupad [B] time = 0.15, size = 180, normalized size = 1.29

$$\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g+e\,f\right)}^2}{16\,d^5\,e^3}-\frac{\frac{x^3\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{16\,d^4}+\frac{2\,e\,f^2+d\,g\,f}{6\,d\,e^2}+\frac{x\,\left(3\,d^2\,g^2+38\,d\,e\,f\,g+19\,e^2\,f^2\right)}{48\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^3\,e}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^4),x)
```

```
[Out] (atanh((e*x)/d)*(d*g + e*f)^2)/(16*d^5*e^3) - ((x^3*(d^2*g^2 + e^2*f^2 + 2*
d*e*f*g))/(16*d^4) + (2*e*f^2 + d*f*g)/(6*d*e^2) + (x*(3*d^2*g^2 + 19*e^2*f
^2 + 38*d*e*f*g))/(48*d^2*e^2) + (x^2*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(4*d
^3*e))/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

sympy [B] time = 1.93, size = 282, normalized size = 2.03

$$-\frac{8d^4fg + 16d^3ef^2 + x^3\left(3d^2e^2g^2 + 6de^3fg + 3e^4f^2\right) + x^2\left(12d^3eg^2 + 24d^2e^2fg + 12de^3f^2\right) + x\left(3d^4g^2 + 38d^3efg + 19d^2e^2f^2\right)}{48d^8e^2 + 192d^7e^3x + 288d^6e^4x^2 + 192d^5e^5x^3 + 48d^4e^6x^4} - \frac{\left(dg+ef\right)^2 \log\left(-\frac{d\left(dg+ef\right)^2}{e\left(d^2g^2+2defg+e^2f^2\right)} + x\right)}{32d^5e^3} + \frac{\left(dg+ef\right)^2 \log\left(\frac{d\left(dg+ef\right)^2}{e\left(d^2g^2+2defg+e^2f^2\right)} + x\right)}{32d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2),x)
```

```
[Out] -(8*d**4*f*g + 16*d**3*e*f**2 + x**3*(3*d**2*e**2*g**2 + 6*d*e**3*f*g + 3*e
**4*f**2) + x**2*(12*d**3*e*g**2 + 24*d**2*e**2*f*g + 12*d*e**3*f**2) + x*(
3*d**4*g**2 + 38*d**3*e*f*g + 19*d**2*e**2*f**2))/(48*d**8*e**2 + 192*d**7*
e**3*x + 288*d**6*e**4*x**2 + 192*d**5*e**5*x**3 + 48*d**4*e**6*x**4) - (d*
g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) +
 x)/(32*d**5*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*
d*e*f*g + e**2*f**2)) + x)/(32*d**5*e**3)
```

3.359 $\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=218

$$\frac{32d^5(dg+ef)^2}{e^3(d-ex)}+\frac{16d^4(dg+ef)(9dg+5ef)\log(d-ex)}{e^3}+\frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right)+\frac{1}{3}dx^3\left(49d^2g^2+46def\right.$$

Rubi [A] time = 0.28, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right)+\frac{1}{3}dx^3\left(49d^2g^2+46defg+7e^2f^2\right)+\frac{d^2x^2\left(80d^2g^2+98defg+23e^2f^2\right)}{2e}+\frac{d^3x\left(112d^2g^2+160defg+49e^2f^2\right)}{e^2}+\frac{32d^5(dg+ef)^2}{e^3(d-ex)}+\frac{16d^4(dg+ef)(9dg+5ef)\log(d-ex)}{e^3}+\frac{1}{5}e^2gx^5(7dg+2ef)+\frac{1}{6}e^3g^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 + (32*d^5*(e*f + d*g)^2)/(e^3*(d - e*x)) +
 (16*d^4*(e*f + d*g)*(5*e*f + 9*d*g)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^5(f+gx)^2}{(d-ex)^2}\,dx\\
&= \int \left(\frac{d^3\left(49e^2f^2+160defg+112d^2g^2\right)}{e^2}+\frac{d^2\left(23e^2f^2+98defg+80d^2g^2\right)x}{e}+d\left(7e^2f^2\right.\right.\\
&= \frac{d^3\left(49e^2f^2+160defg+112d^2g^2\right)x}{e^2}+\frac{d^2\left(23e^2f^2+98defg+80d^2g^2\right)x^2}{2e}+\frac{1}{3}d\left(7e^2f\right.
\end{aligned}$$

Mathematica [A] time = 0.12, size = 226, normalized size = 1.04

$$-\frac{32d^5(dg+ef)^2}{e^3(ex-d)}+\frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right)+\frac{1}{3}dx^3\left(49d^2g^2+46defg+7e^2f^2\right)+\frac{d^2x^2\left(80d^2g^2+98defg+23e^2f^2\right)}{2e}+\frac{16d^4\left(9d^2g^2+14defg+5e^2f^2\right)\log(d-ex)}{e^3}+\frac{d^3x\left(112d^2g^2+160defg+49e^2f^2\right)}{e^2}+\frac{1}{5}e^2gx^5(7dg+2ef)+\frac{1}{6}e^3g^2x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 - (32*d^5*(e*f + d*g)^2)/(e^3*(-d + e*x))
+ (16*d^4*(5*e^2*f^2 + 14*d*e*f*g + 9*d^2*g^2)*Log[d - e*x])/e^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2, x]
```

fricas [A] time = 0.42, size = 328, normalized size = 1.50

$$\frac{10\,e^7g^2x^7-1920\,d^5e^2f^2-3840\,d^6efg-1920\,d^7g^2+2\left(12\,e^7fg+37\,de^6g^2\right)x^6+3\left(5\,e^7f^2+62\,de^6fg+87\,d^2e^5g^2\right)x^5+5\left(25\,de^6f^2+142\,d^2e^5fg+127\,d^3e^4g^2\right)x^4+10\left(55\,d^2e^5f^2+202\,d^3e^4fg+142\,d^4e^3g^2\right)x^3+90\left(25\,d^3e^4f^2+74\,d^4e^3fg+48\,d^5e^2g^2\right)x^2-60\left(49\,d^4e^3f^2+160\,d^5e^2fg+112\,d^6eg^2\right)x-960\left(5\,d^5e^2f^2+14\,d^6efg+9\,d^7g^2-\left(5\,d^4e^3f^2+14\,d^5e^2fg+9\,d^6eg^2\right)x\right)\log\left(ex-d\right)}{60\left(e^4x-de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*e^7*g^2*x^7 - 1920*d^5*e^2*f^2 - 3840*d^6*e*f*g - 1920*d^7*g^2 + 2
*(12*e^7*f*g + 37*d*e^6*g^2)*x^6 + 3*(5*e^7*f^2 + 62*d*e^6*f*g + 87*d^2*e^5
*g^2)*x^5 + 5*(25*d*e^6*f^2 + 142*d^2*e^5*f*g + 127*d^3*e^4*g^2)*x^4 + 10*(
55*d^2*e^5*f^2 + 202*d^3*e^4*f*g + 142*d^4*e^3*g^2)*x^3 + 90*(25*d^3*e^4*f^
2 + 74*d^4*e^3*f*g + 48*d^5*e^2*g^2)*x^2 - 60*(49*d^4*e^3*f^2 + 160*d^5*e^2
*f*g + 112*d^6*e*g^2)*x - 960*(5*d^5*e^2*f^2 + 14*d^6*e*f*g + 9*d^7*g^2 - (
5*d^4*e^3*f^2 + 14*d^5*e^2*f*g + 9*d^6*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e
^3)
```

giac [A] time = 0.18, size = 367, normalized size = 1.68

$$8\left(9\,d^6g^2e^7+14\,d^5fge^8+5\,d^4f^2e^9\right)e^{(-10)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{1}{60}\left(10\,g^2x^6e^{27}+84\,dg^2x^5e^{26}+345\,d^2g^2x^4e^{25}+980\,d^3g^2x^3e^{24}+2400\,d^4g^2x^2e^{23}+6720\,d^5g^2xe^{22}+24\,fgx^5e^{27}+210\,dfgx^4e^{26}+920\,d^2fgx^3e^{25}+2940\,d^3fgx^2e^{24}+9600\,d^4fgxe^{23}+15\,f^2x^4e^{27}+140\,df^2x^3e^{26}+690\,d^2f^2x^2e^{25}+2940\,d^3f^2xe^{24}\right)e^{(-24)}+\frac{8\left(9\,d^7g^2e^6+14\,d^6fge^7+5\,d^5f^2e^8\right)e^{(-9)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{|d|}-\frac{32\left(d^8g^2e^7+2\,d^7fge^8+d^6f^2e^9+\left(d^7g^2e^8+2\,d^6fge^9+d^5f^2e^{10}\right)x\right)e^{(-10)}}{x^2e^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 8*(9*d^6*g^2*e^7 + 14*d^5*f*g*e^8 + 5*d^4*f^2*e^9)*e^(-10)*log(abs(x^2*e^2
- d^2)) + 1/60*(10*g^2*x^6*e^27 + 84*d*g^2*x^5*e^26 + 345*d^2*g^2*x^4*e^25
+ 980*d^3*g^2*x^3*e^24 + 2400*d^4*g^2*x^2*e^23 + 6720*d^5*g^2*x*e^22 + 24*f
*g*x^5*e^27 + 210*d*f*g*x^4*e^26 + 920*d^2*f*g*x^3*e^25 + 2940*d^3*f*g*x^2*
e^24 + 9600*d^4*f*g*x*e^23 + 15*f^2*x^4*e^27 + 140*d*f^2*x^3*e^26 + 690*d^2
*f^2*x^2*e^25 + 2940*d^3*f^2*x*e^24)*e^(-24) + 8*(9*d^7*g^2*e^6 + 14*d^6*f*
g*e^7 + 5*d^5*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2
*abs(d)*e))/abs(d) - 32*(d^8*g^2*e^7 + 2*d^7*f*g*e^8 + d^6*f^2*e^9 + (d^7*g
^2*e^8 + 2*d^6*f*g*e^9 + d^5*f^2*e^10)*x)*e^(-10)/(x^2*e^2 - d^2)
```

maple [A] time = 0.01, size = 286, normalized size = 1.31

$$\frac{e^3g^2x^6}{6}+\frac{7d\,e^2g^2x^5}{5}+\frac{2e^3fg\,x^5}{5}+\frac{23d^2e\,g^2x^4}{4}+\frac{7d\,e^2fg\,x^4}{2}+\frac{e^3f^2x^4}{4}+\frac{49d^3g^2x^3}{3}+\frac{46d^2efg\,x^3}{3}+\frac{7d\,e^2f^2x^3}{3}+\frac{40d^4g^2x^2}{e}+49d^3fg\,x^2+\frac{23d^2e\,f^2x^2}{2}-\frac{32d^7g^2}{(ex-d)\,e^3}-\frac{64d^6fg}{(ex-d)\,e^2}+\frac{144d^6g^2\ln(ex-d)}{e^3}-\frac{32d^5f^2}{(ex-d)\,e}+\frac{224d^5fg\ln(ex-d)}{e^2}+\frac{112d^5g^2x}{e^2}+\frac{80d^4f^2\ln(ex-d)}{e}+\frac{160d^4fgx}{e}+49d^3f^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/6*e^3*g^2*x^6+7/5*e^2*x^5*d*g^2+2/5*e^3*x^5*f*g+23/4*e*x^4*d^2*g^2+7/2*e^
2*x^4*d*f*g+1/4*e^3*x^4*f^2+49/3*x^3*d^3*g^2+46/3*e*x^3*d^2*f*g+7/3*e^2*x^3
```

```
*d*f^2+40/e*x^2*d^4*g^2+49*x^2*d^3*f*g+23/2*e*x^2*d^2*f^2+112/e^2*d^5*g^2*x
+160/e*d^4*f*g*x+49*d^3*f^2*x+144*d^6/e^3*ln(e*x-d)*g^2+224*d^5/e^2*ln(e*x-
d)*f*g+80*d^4/e*ln(e*x-d)*f^2-32*d^7/e^3/(e*x-d)*g^2-64*d^6/e^2/(e*x-d)*f*g
-32*d^5/e/(e*x-d)*f^2
```

maxima [A] time = 0.45, size = 258, normalized size = 1.18

$$-\frac{32\left(d^5e^2f^2+2d^6efg+d^7g^2\right)}{e^4x-de^3}+\frac{10e^5g^2x^6+12\left(2e^5fg+7de^4g^2\right)x^5+15\left(e^5f^2+14de^4fg+23d^2e^3g^2\right)x^4+20\left(7de^4f^2+46d^2e^3fg+49d^3e^2g^2\right)x^3+30\left(23d^2e^3f^2+98d^3e^2fg+80d^4eg^2\right)x^2+60\left(49d^3e^2f^2+160d^4efg+112d^5g^2\right)x}{60e^2}+\frac{16\left(5d^4e^2f^2+14d^5efg+9d^6g^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -32*(d^5*e^2*f^2 + 2*d^6*e*f*g + d^7*g^2)/(e^4*x - d*e^3) + 1/60*(10*e^5*g^
2*x^6 + 12*(2*e^5*f*g + 7*d*e^4*g^2)*x^5 + 15*(e^5*f^2 + 14*d*e^4*f*g + 23*
d^2*e^3*g^2)*x^4 + 20*(7*d*e^4*f^2 + 46*d^2*e^3*f*g + 49*d^3*e^2*g^2)*x^3 +
 30*(23*d^2*e^3*f^2 + 98*d^3*e^2*f*g + 80*d^4*e*g^2)*x^2 + 60*(49*d^3*e^2*f
^2 + 160*d^4*e*f*g + 112*d^5*g^2)*x)/e^2 + 16*(5*d^4*e^2*f^2 + 14*d^5*e*f*g
 + 9*d^6*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.64, size = 1029, normalized size = 4.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^7)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x^5*((e^2*g*(5*d*g + 2*e*f))/5 + (2*d*e^2*g^2)/5) + x^3*((5*d*(2*d^2*g^2 +
e^2*f^2 + 4*d*e*f*g))/3 + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e
^2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/(3*e) - (d
^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/(3*e^2)) + x^4*((e^5*f^2 + 10*d^2
*e^3*g^2 + 10*d*e^4*f*g)/(4*e^2) - (d^2*e*g^2)/4 + (d*(e^2*g*(5*d*g + 2*e*f
) + 2*d*e^2*g^2))/(2*e)) + x^2*((5*d^2*(d^2*g^2 + 2*e^2*f^2 + 4*d*e*f*g))/(
2*e) - (d^2*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2
*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/(2*e^2) + (d*(5*d*(2*d^2*g^2
+ e^2*f^2 + 4*d*e*f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^
2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e
^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e^2))/e) + x*((d^5*g^2 + 10*d^3*e^2*f^
2 + 10*d^4*e*f*g)/e^2 - (d^2*(5*d*(2*d^2*g^2 + e^2*f^2 + 4*d*e*f*g) + (2*d*
((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(5
*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2
*g^2))/e^2))/e^2 + (2*d*((5*d^2*(d^2*g^2 + 2*e^2*f^2 + 4*d*e*f*g))/e - (d^2
*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(
5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e^2 + (2*d*(5*d*(2*d^2*g^2 + e^2*f^2 + 4
*d*e*f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2
 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e^2*g*(5*d*g +
 2*e*f) + 2*d*e^2*g^2))/e^2))/e))/e) + (log(e*x - d)*(144*d^6*g^2 + 80*d^4*
e^2*f^2 + 224*d^5*e*f*g))/e^3 + (32*(d^7*g^2 + d^5*e^2*f^2 + 2*d^6*e*f*g))/
(e*(d*e^2 - e^3*x)) + (e^3*g^2*x^6)/6
```

sympy [A] time = 1.20, size = 250, normalized size = 1.15

$$\frac{16d^4\left(dg+ef\right)\left(9dg+5ef\right)\log\left(-d+ex\right)}{e^3}+\frac{e^3g^2x^6}{6}+x^5\left(\frac{7de^2g^2}{5}+\frac{2e^3fg}{5}\right)+x^4\left(\frac{23d^2eg^2}{4}+\frac{7de^2fg}{2}+\frac{e^3f^2}{4}\right)+x^3\left(\frac{49d^3g^2}{3}+\frac{46d^2efg}{3}+\frac{7de^2f^2}{3}\right)+x^2\left(\frac{40d^4g^2}{e}+49d^3fg+\frac{23d^2ef^2}{2}\right)+x\left(\frac{112d^5g^2}{e^2}+\frac{160d^4fg}{e}+49d^3f^2\right)+\frac{-32d^7g^2-64d^6efg-32d^5e^2f^2}{-de^3+e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 16*d**4*(d*g + e*f)*(9*d*g + 5*e*f)*log(-d + e*x)/e**3 + e**3*g**2*x**6/6 +
 x**5*(7*d*e**2*g**2/5 + 2*e**3*f*g/5) + x**4*(23*d**2*e*g**2/4 + 7*d*e**2*
f*g/2 + e**3*f**2/4) + x**3*(49*d**3*g**2/3 + 46*d**2*e*f*g/3 + 7*d*e**2*f*
```

```
*2/3) + x**2*(40*d**4*g**2/e + 49*d**3*f*g + 23*d**2*e*f**2/2) + x*(112*d**
5*g**2/e**2 + 160*d**4*f*g/e + 49*d**3*f**2) + (-32*d**7*g**2 - 64*d**6*e*f
*g - 32*d**5*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.360 $\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=177

$$\frac{16d^4(dg+ef)^2}{e^3(d-ex)}+\frac{32d^3(dg+ef)(2dg+ef)\log(d-ex)}{e^3}+\frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right)+\frac{dx^2\left(16d^2g^2+17defg+3\right.}{e}$$

Rubi [A] time = 0.23, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right)+\frac{dx^2\left(16d^2g^2+17defg+3e^2f^2\right)}{e}+\frac{d^2x\left(48d^2g^2+64defg+17e^2f^2\right)}{e^2}+\frac{16d^4(dg+ef)^2}{e^3(d-ex)}+\frac{32d^3(dg+ef)(2dg+ef)\log(d-ex)}{e^3}+\frac{1}{2}egx^4(3dg+ef)+\frac{1}{5}e^2g^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 + (16*d^4*(e*f + d*g)^2)/(e^3*(d
 - e*x)) + (32*d^3*(e*f + d*g)*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^2}\,dx\\
&= \int \left(\frac{d^2\left(17e^2f^2+64defg+48d^2g^2\right)}{e^2}+\frac{2d\left(3e^2f^2+17defg+16d^2g^2\right)x}{e}+\left(e^2f^2+12d\right.\right.\\
&= \frac{d^2\left(17e^2f^2+64defg+48d^2g^2\right)x}{e^2}+\frac{d\left(3e^2f^2+17defg+16d^2g^2\right)x^2}{e}+\frac{1}{3}\left(e^2f^2+12d\right.
\end{aligned}$$

Mathematica [A] time = 0.12, size = 185, normalized size = 1.05

$$-\frac{16d^4(dg+ef)^2}{e^3(ex-d)}+\frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right)+\frac{dx^2\left(16d^2g^2+17defg+3e^2f^2\right)}{e}+\frac{d^2x\left(48d^2g^2+64defg+17e^2f^2\right)}{e^2}+\frac{32d^3\left(2d^2g^2+3defg+e^2f^2\right)\log(d-ex)}{e^3}+\frac{1}{2}egx^4(3dg+ef)+\frac{1}{5}e^2g^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
```

```
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 - (16*d^4*(e*f + d*g)^2)/(e^3*(-
d + e*x)) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2*g^2)*Log[d - e*x])/e^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2, x]
```

fricas [A] time = 0.39, size = 288, normalized size = 1.63

$$\frac{6\,e^{6}g^{2}x^{6}-480\,d^{4}e^{2}f^{2}-960\,d^{5}efg-480\,d^{6}g^{2}+3\left(5\,e^{6}fg+13\,de^{5}g^{2}\right)x^{5}+5\left(2\,e^{6}f^{2}+21\,de^{5}fg+25\,d^{2}e^{4}g^{2}\right)x^{4}+10\left(8\,de^{5}f^{2}+39\,d^{2}e^{4}fg+31\,d^{3}e^{3}g^{2}\right)x^{3}+30\left(14\,d^{2}e^{4}f^{2}+47\,d^{3}e^{3}fg+32\,d^{4}e^{2}g^{2}\right)x^{2}-30\left(17\,d^{3}e^{3}f^{2}+64\,d^{4}e^{2}fg+48\,d^{5}eg^{2}\right)x-960\left(d^{4}e^{2}f^{2}+3\,d^{5}efg+2\,d^{6}g^{2}-\left(d^{3}e^{3}f^{2}+3\,d^{4}e^{2}fg+2\,d^{5}eg^{2}\right)x\right)\log\left(ex-d\right)}{30\left(e^{4}x-de^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/30*(6*e^6*g^2*x^6 - 480*d^4*e^2*f^2 - 960*d^5*e*f*g - 480*d^6*g^2 + 3*(5*
e^6*f*g + 13*d*e^5*g^2)*x^5 + 5*(2*e^6*f^2 + 21*d*e^5*f*g + 25*d^2*e^4*g^2)
*x^4 + 10*(8*d*e^5*f^2 + 39*d^2*e^4*f*g + 31*d^3*e^3*g^2)*x^3 + 30*(14*d^2*
e^4*f^2 + 47*d^3*e^3*f*g + 32*d^4*e^2*g^2)*x^2 - 30*(17*d^3*e^3*f^2 + 64*d^
4*e^2*f*g + 48*d^5*e*g^2)*x - 960*(d^4*e^2*f^2 + 3*d^5*e*f*g + 2*d^6*g^2 -
(d^3*e^3*f^2 + 3*d^4*e^2*f*g + 2*d^5*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3
)
```

giac [A] time = 0.18, size = 327, normalized size = 1.85

$$16\left(2\,d^{5}g^{2}e^{5}+3\,d^{4}fge^{6}+d^{3}f^{2}e^{7}\right)e^{(-8)}\log\left(\left|x^{2}e^{2}-d^{2}\right|\right)+\frac{1}{30}\left(6\,g^{2}x^{5}e^{22}+45\,dg^{2}x^{4}e^{21}+170\,d^{2}g^{2}x^{3}e^{20}+480\,d^{3}g^{2}x^{2}e^{19}+1440\,d^{4}g^{2}xe^{18}+15\,fgx^{4}e^{22}+120\,dfgx^{3}e^{21}+510\,d^{2}fgx^{2}e^{20}+1920\,d^{3}fgxe^{19}+10\,f^{2}x^{3}e^{22}+90\,df^{2}x^{2}e^{21}+510\,d^{2}f^{2}xe^{20}\right)e^{(-20)}+\frac{16\left(2\,d^{6}g^{2}e^{6}+3\,d^{5}fge^{7}+d^{4}f^{2}e^{8}\right)e^{(-9)}\log\left(\frac{\left|2\,xe^{2}-2\,\left|d\right|\left|e\right|\right|}{\left|2\,xe^{2}+2\,\left|d\right|\left|e\right|\right|}\right)}{\left|d\right|}-\frac{16\left(d^{7}g^{2}e^{5}+2\,d^{6}fge^{6}+d^{5}f^{2}e^{7}+\left(d^{6}g^{2}e^{6}+2\,d^{5}fge^{7}+d^{4}f^{2}e^{8}\right)x\right)e^{(-8)}}{x^{2}e^{2}-d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 16*(2*d^5*g^2*e^5 + 3*d^4*f*g*e^6 + d^3*f^2*e^7)*e^(-8)*log(abs(x^2*e^2 - d
^2)) + 1/30*(6*g^2*x^5*e^22 + 45*d*g^2*x^4*e^21 + 170*d^2*g^2*x^3*e^20 + 48
0*d^3*g^2*x^2*e^19 + 1440*d^4*g^2*x*e^18 + 15*f*g*x^4*e^22 + 120*d*f*g*x^3*
e^21 + 510*d^2*f*g*x^2*e^20 + 1920*d^3*f*g*x*e^19 + 10*f^2*x^3*e^22 + 90*d*
f^2*x^2*e^21 + 510*d^2*f^2*x*e^20)*e^(-20) + 16*(2*d^6*g^2*e^6 + 3*d^5*f*g*
e^7 + d^4*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs
(d)*e))/abs(d) - 16*(d^7*g^2*e^5 + 2*d^6*f*g*e^6 + d^5*f^2*e^7 + (d^6*g^2*e
^6 + 2*d^5*f*g*e^7 + d^4*f^2*e^8)*x)*e^(-8)/(x^2*e^2 - d^2)
```

maple [A] time = 0.01, size = 245, normalized size = 1.38

$$\frac{e^{2}g^{2}x^{5}}{5}+\frac{3deg^{2}x^{4}}{2}+\frac{e^{2}fgx^{4}}{2}+\frac{17d^{2}g^{2}x^{3}}{3}+4defgx^{3}+\frac{e^{2}f^{2}x^{3}}{3}+\frac{16d^{3}g^{2}x^{2}}{e}+17d^{2}fgx^{2}+3def^{2}x^{2}-\frac{16d^{6}g^{2}}{\left(ex-d\right)e^{3}}-\frac{32d^{5}fg}{\left(ex-d\right)e^{2}}+\frac{64d^{5}g^{2}\ln\left(ex-d\right)}{e^{3}}-\frac{16d^{4}f^{2}}{\left(ex-d\right)e}+\frac{96d^{4}fg\ln\left(ex-d\right)}{e^{2}}+\frac{48d^{4}g^{2}x}{e^{2}}+\frac{32d^{3}f^{2}\ln\left(ex-d\right)}{e}+\frac{64d^{3}fgx}{e}+17d^{2}f^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/5*e^2*g^2*x^5+3/2*d*e*g^2*x^4+1/2*e^2*f*g*x^4+17/3*d^2*g^2*x^3+4*d*e*f*g*
x^3+1/3*e^2*f^2*x^3+16*d^3/e*g^2*x^2+17*d^2*f*g*x^2+3*d*e*f^2*x^2+48*d^4/e^
2*g^2*x+64*d^3/e*f*g*x+17*d^2*f^2*x+64*d^5/e^3*g^2*ln(e*x-d)+96*d^4/e^2*f*g
*ln(e*x-d)+32*d^3/e*f^2*ln(e*x-d)-16*d^6/e^3/(e*x-d)*g^2-32*d^5/e^2/(e*x-d)
*f*g-16*d^4/e/(e*x-d)*f^2
```

maxima [A] time = 0.45, size = 218, normalized size = 1.23

$$-\frac{16\left(d^{4}e^{2}f^{2}+2\,d^{5}efg+d^{6}g^{2}\right)}{e^{4}x-de^{3}}+\frac{6\,e^{4}g^{2}x^{5}+15\left(e^{4}fg+3\,de^{3}g^{2}\right)x^{4}+10\left(e^{4}f^{2}+12\,de^{3}fg+17\,d^{2}e^{2}g^{2}\right)x^{3}+30\left(3\,de^{3}f^{2}+17\,d^{2}e^{2}fg+16\,d^{3}eg^{2}\right)x^{2}+30\left(17\,d^{2}e^{2}f^{2}+64\,d^{3}efg+48\,d^{4}g^{2}\right)x}{30\,e^{2}}+\frac{32\left(d^{3}e^{2}f^{2}+3\,d^{4}efg+2\,d^{5}g^{2}\right)\log\left(ex-d\right)}{e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")

```
[Out] -16*(d^4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2)/(e^4*x - d*e^3) + 1/30*(6*e^4*g^2
*x^5 + 15*(e^4*f*g + 3*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 12*d*e^3*f*g + 17*d^2
*e^2*g^2)*x^3 + 30*(3*d*e^3*f^2 + 17*d^2*e^2*f*g + 16*d^3*e*g^2)*x^2 + 30*(
17*d^2*e^2*f^2 + 64*d^3*e*f*g + 48*d^4*g^2)*x)/e^2 + 32*(d^3*e^2*f^2 + 3*d^
4*e*f*g + 2*d^5*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.61, size = 565, normalized size = 3.19

$$x^2\left(\frac{2d\left(d^2g^2+3defg+e^2f^2\right)}{e}-\frac{d^2\left(2eg\left(2dg+ef\right)+2deg^2\right)}{2e^2}+\frac{d\left(\frac{6d^2e^2g^2+8de^3fg+e^4f^2}{e^2}-d^2g^2+\frac{2d\left(2eg\left(2dg+ef\right)+2deg^2\right)}{e}\right)}{e}\right)+x^4\left(\frac{eg\left(2dg+ef\right)}{2}+\frac{deg^2}{2}\right)+x\left(\frac{d^4g^2+8d^3efg+6d^2e^2f^2}{e^2}-\frac{d^2\left(\frac{6d^2e^2g^2+8de^3fg+e^4f^2}{e^2}-d^2g^2+\frac{2d\left(2eg\left(2dg+ef\right)+2deg^2\right)}{e}\right)}{e^2}+\frac{2d\left(\frac{4d\left(d^2g^2+3defg+e^2f^2\right)}{e}-\frac{d^2\left(2eg\left(2dg+ef\right)+2deg^2\right)}{e^2}+\frac{2d\left(\frac{6d^2e^2g^2+8de^3fg+e^4f^2}{e^2}-d^2g^2+\frac{2d\left(2eg\left(2dg+ef\right)+2deg^2\right)}{e}\right)}{e}\right)}{e}\right)+x^3\left(\frac{6d^2e^2g^2+8de^3fg+e^4f^2}{3e^2}-\frac{d^2g^2}{3}+\frac{2d\left(2eg\left(2dg+ef\right)+2deg^2\right)}{3e}\right)+\frac{\ln\left(ex-d\right)\left(64d^5g^2+96d^4efg+32d^3e^2f^2\right)}{e^3}+\frac{16\left(d^6g^2+2d^5efg+d^4e^2f^2\right)}{e\left(de^2-e^3x\right)}+\frac{e^2g^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f + g*x)^2*(d + e*x)^6)/(d^2 - e^2*x^2)^2,x)

```
[Out] x^2*((2*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f*g))/e - (d^2*(2*e*g*(2*d*g + e*f) +
2*d*e*g^2))/(2*e^2) + (d*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2
*g^2 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/e))/e) + x^4*((e*g*(2*d*g +
e*f))/2 + (d*e*g^2)/2) + x*((d^4*g^2 + 6*d^2*e^2*f^2 + 8*d^3*e*f*g)/e^2 - (
d^2*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2*g^2 + (2*d*(2*e*g*(2
*d*g + e*f) + 2*d*e*g^2))/e))/e^2 + (2*d*((4*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f
*g))/e - (d^2*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/e^2 + (2*d*((e^4*f^2 + 6*d
^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2*g^2 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e
*g^2))/e))/e))/e) + x^3*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/(3*e^2) -
(d^2*g^2)/3 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/(3*e)) + (log(e*x - d
)*(64*d^5*g^2 + 32*d^3*e^2*f^2 + 96*d^4*e*f*g))/e^3 + (16*(d^6*g^2 + d^4*e^
2*f^2 + 2*d^5*e*f*g))/(e*(d*e^2 - e^3*x)) + (e^2*g^2*x^5)/5
```

sympy [A] time = 1.01, size = 199, normalized size = 1.12

$$\frac{32d^3\left(dg+ef\right)\left(2dg+ef\right)\log\left(-d+ex\right)}{e^3}+\frac{e^2g^2x^5}{5}+x^4\left(\frac{3deg^2}{2}+\frac{e^2fg}{2}\right)+x^3\left(\frac{17d^2g^2}{3}+4defg+\frac{e^2f^2}{3}\right)+x^2\left(\frac{16d^3g^2}{e}+17d^2fg+3def^2\right)+x\left(\frac{48d^4g^2}{e^2}+\frac{64d^3fg}{e}+17d^2f^2\right)+\frac{-16d^6g^2-32d^5efg-16d^4e^2f^2}{-de^3+e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)

```
[Out] 32*d**3*(d*g + e*f)*(2*d*g + e*f)*log(-d + e*x)/e**3 + e**2*g**2*x**5/5 + x
**4*(3*d*e*g**2/2 + e**2*f*g/2) + x**3*(17*d**2*g**2/3 + 4*d*e*f*g + e**2*f
**2/3) + x**2*(16*d**3*g**2/e + 17*d**2*f*g + 3*d*e*f**2) + x*(48*d**4*g**2
/e**2 + 64*d**3*f*g/e + 17*d**2*f**2) + (-16*d**6*g**2 - 32*d**5*e*f*g - 16
*d**4*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.361 $\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=146

$$\frac{8d^3(dg+ef)^2}{e^3(d-ex)}+\frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3}+\frac{x^2\left(12d^2g^2+10defg+e^2f^2\right)}{2e}+\frac{dx\left(20d^2g^2+24defg+5\right.}{e^2}$$

Rubi [A] time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{x^2\left(12d^2g^2+10defg+e^2f^2\right)}{2e}+\frac{dx\left(20d^2g^2+24defg+5e^2f^2\right)}{e^2}+\frac{8d^3(dg+ef)^2}{e^3(d-ex)}+\frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3}+\frac{1}{3}gx^3(5dg+2ef)+\frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g +
12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 + (8*d^3
*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d^2*(e*f + d*g)*(3*e*f + 7*d*g)*Log[d
- e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^2}\,dx\\ &= \int \left(\frac{d\left(5e^2f^2+24defg+20d^2g^2\right)}{e^2}+\frac{\left(e^2f^2+10defg+12d^2g^2\right)x}{e}+g(2ef+5dg)x\right.\\ &= \frac{d\left(5e^2f^2+24defg+20d^2g^2\right)x}{e^2}+\frac{\left(e^2f^2+10defg+12d^2g^2\right)x^2}{2e}+\frac{1}{3}g(2ef+5dg)x\end{aligned}$$

Mathematica [A] time = 0.09, size = 154, normalized size = 1.05

$$-\frac{8d^3(dg+ef)^2}{e^3(ex-d)}+\frac{x^2\left(12d^2g^2+10defg+e^2f^2\right)}{2e}+\frac{dx\left(20d^2g^2+24defg+5e^2f^2\right)}{e^2}+\frac{4d^2\left(7d^2g^2+10defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{1}{3}gx^3(5dg+2ef)+\frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g + 12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 - (8*d^3*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d^2*(3*e^2*f^2 + 10*d*e*f*g + 7*d^2*g^2)*Log[d - e*x])/e^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]

[Out] IntegrateAlgebraic[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2, x]

fricas [A] time = 0.40, size = 251, normalized size = 1.72

$$\frac{3\,e^5g^2x^5-96\,d^3e^2f^2-192\,d^4efg-96\,d^5g^2+\left(8\,e^5fg+17\,de^4g^2\right)x^4+2\left(3\,e^5f^2+26\,de^4fg+26\,d^2e^3g^2\right)x^3+6\left(9\,de^4f^2+38\,d^2e^3fg+28\,d^3e^2g^2\right)x^2-12\left(5\,d^2e^3f^2+24\,d^3e^2fg+20\,d^4eg^2\right)x-48\left(3\,d^3e^2f^2+10\,d^4efg+7\,d^5g^2-\left(3\,d^2e^3f^2+10\,d^3e^2fg+7\,d^4eg^2\right)x\right)\log\left(ex-d\right)}{12\left(e^4x-de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")

[Out] 1/12*(3*e^5*g^2*x^5 - 96*d^3*e^2*f^2 - 192*d^4*e*f*g - 96*d^5*g^2 + (8*e^5*f*g + 17*d*e^4*g^2)*x^4 + 2*(3*e^5*f^2 + 26*d*e^4*f*g + 26*d^2*e^3*g^2)*x^3 + 6*(9*d*e^4*f^2 + 38*d^2*e^3*f*g + 28*d^3*e^2*g^2)*x^2 - 12*(5*d^2*e^3*f^2 + 24*d^3*e^2*f*g + 20*d^4*e*g^2)*x - 48*(3*d^3*e^2*f^2 + 10*d^4*e*f*g + 7*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f*g + 7*d^4*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3)

giac [B] time = 0.19, size = 291, normalized size = 1.99

$$2\left(7\,d^4g^2e^5+10\,d^3fge^6+3\,d^2f^2e^7\right)e^{(-8)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{1}{12}\left(3\,g^2x^4e^{17}+20\,dg^2x^3e^{16}+72\,d^2g^2x^2e^{15}+240\,d^3g^2xe^{14}+8\,fgx^3e^{17}+60\,dfgx^2e^{16}+288\,d^2fgxe^{15}+6\,f^2x^2e^{17}+60\,df^2xe^{16}\right)e^{(-16)}+\frac{2\left(7\,d^5g^2e^4+10\,d^4fge^5+3\,d^3f^2e^6\right)e^{(-7)}\log\left(\frac{\left|2\,xe^2-2\,\left|d\right|e\right|}{\left|2\,xe^2+2\,\left|d\right|e\right|}\right)}{\left|d\right|}-\frac{8\left(d^6g^2e^5+2\,d^5fge^6+d^4f^2e^7+\left(d^5g^2e^6+2\,d^4fge^7+d^3f^2e^8\right)x\right)e^{(-8)}}{x^2e^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")

[Out] 2*(7*d^4*g^2*e^5 + 10*d^3*f*g*e^6 + 3*d^2*f^2*e^7)*e^(-8)*log(abs(x^2*e^2 - d^2)) + 1/12*(3*g^2*x^4*e^17 + 20*d*g^2*x^3*e^16 + 72*d^2*g^2*x^2*e^15 + 240*d^3*g^2*x*e^14 + 8*f*g*x^3*e^17 + 60*d*f*g*x^2*e^16 + 288*d^2*f*g*x*e^15 + 6*f^2*x^2*e^17 + 60*d*f^2*x*e^16)*e^(-16) + 2*(7*d^5*g^2*e^4 + 10*d^4*f*g*e^5 + 3*d^3*f^2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d) - 8*(d^6*g^2*e^5 + 2*d^5*f*g*e^6 + d^4*f^2*e^7 + (d^5*g^2*e^6 + 2*d^4*f*g*e^7 + d^3*f^2*e^8)*x)*e^(-8)/(x^2*e^2 - d^2)

maple [A] time = 0.01, size = 204, normalized size = 1.40

$$\frac{e\,g^2x^4}{4}+\frac{5d\,g^2x^3}{3}+\frac{2efg\,x^3}{3}+\frac{6d^2g^2x^2}{e}+5dfg\,x^2+\frac{e\,f^2x^2}{2}-\frac{8d^5g^2}{(ex-d)\,e^3}-\frac{16d^4fg}{(ex-d)\,e^2}+\frac{28d^4g^2\ln(ex-d)}{e^3}-\frac{8d^3f^2}{(ex-d)\,e}+\frac{40d^3fg\ln(ex-d)}{e^2}+\frac{20d^3g^2x}{e^2}+\frac{12d^2f^2\ln(ex-d)}{e}+\frac{24d^2fgx}{e}+5d\,f^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)

[Out] 1/4*e*g^2*x^4+5/3*d*g^2*x^3+2/3*e*f*g*x^3+6*d^2/e*g^2*x^2+5*d*f*g*x^2+1/2*e*f^2*x^2+20*d^3/e^2*g^2*x+24*d^2/e*f*g*x+5*d*f^2*x+28*d^4/e^3*g^2*ln(e*x-d)+40*d^3/e^2*f*g*ln(e*x-d)+12*d^2/e*f^2*ln(e*x-d)-8*d^5/e^3/(e*x-d)*g^2-16*d^4/e^2/(e*x-d)*f*g-8*d^3/e/(e*x-d)*f^2

maxima [A] time = 0.46, size = 182, normalized size = 1.25

$$-\frac{8\left(d^3e^2f^2+2\,d^4efg+d^5g^2\right)}{e^4x-de^3}+\frac{3\,e^3g^2x^4+4\left(2\,e^3fg+5\,de^2g^2\right)x^3+6\left(e^3f^2+10\,de^2fg+12\,d^2eg^2\right)x^2+12\left(5\,de^2f^2+24\,d^2efg+20\,d^3g^2\right)x}{12\,e^2}+\frac{4\left(3\,d^2e^2f^2+10\,d^3efg+7\,d^4g^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -8*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2)/(e^4*x - d*e^3) + 1/12*(3*e^3*g^2*
x^4 + 4*(2*e^3*f*g + 5*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 10*d*e^2*f*g + 12*d^2*
e*g^2)*x^2 + 12*(5*d*e^2*f^2 + 24*d^2*e*f*g + 20*d^3*g^2)*x)/e^2 + 4*(3*d^2
*e^2*f^2 + 10*d^3*e*f*g + 7*d^4*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.09, size = 316, normalized size = 2.16

$$x\left(\frac{d^3g^2+6d^2efg+3de^2f^2}{e^2}-\frac{d^2\left(g\left(3dg+2ef\right)+2dg^2\right)}{e^2}+\frac{2d\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{e^2}-\frac{d^2g^2}{e}+\frac{2d\left(g\left(3dg+2ef\right)+2dg^2\right)}{e}\right)}{e}\right)+x^2\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{2e^2}-\frac{d^2g^2}{2e}+\frac{d\left(g\left(3dg+2ef\right)+2dg^2\right)}{e}\right)+x^3\left(\frac{g\left(3dg+2ef\right)}{3}+\frac{2dg^2}{3}\right)+\frac{\ln\left(ex-d\right)\left(28d^4g^2+40d^3efg+12d^2e^2f^2\right)}{e^3}+\frac{8\left(d^5g^2+2d^4efg+d^3e^2f^2\right)}{e\left(de^2-e^3x\right)}+\frac{eg^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^5)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x*((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/e^2 - (d^2*(g*(3*d*g + 2*e*f) + 2*
d*g^2))/e^2 + (2*d*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2*f*g)/e^2 - (d^2*g^2)/e
 + (2*d*(g*(3*d*g + 2*e*f) + 2*d*g^2))/e))/e) + x^2*((e^3*f^2 + 3*d^2*e*g^2
 + 6*d*e^2*f*g)/(2*e^2) - (d^2*g^2)/(2*e) + (d*(g*(3*d*g + 2*e*f) + 2*d*g^2
))/e) + x^3*((g*(3*d*g + 2*e*f))/3 + (2*d*g^2)/3) + (log(e*x - d)*(28*d^4*g
^2 + 12*d^2*e^2*f^2 + 40*d^3*e*f*g))/e^3 + (8*(d^5*g^2 + d^3*e^2*f^2 + 2*d^
4*e*f*g))/(e*(d*e^2 - e^3*x)) + (e*g^2*x^4)/4
```

sympy [A] time = 0.85, size = 162, normalized size = 1.11

$$\frac{4d^2\left(dg+ef\right)\left(7dg+3ef\right)\log\left(-d+ex\right)}{e^3}+\frac{eg^2x^4}{4}+x^3\left(\frac{5dg^2}{3}+\frac{2efg}{3}\right)+x^2\left(\frac{6d^2g^2}{e}+5dfg+\frac{ef^2}{2}\right)+x\left(\frac{20d^3g^2}{e^2}+\frac{24d^2fg}{e}+5df^2\right)+\frac{-8d^5g^2-16d^4efg-8d^3e^2f^2}{-de^3+e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 4*d**2*(d*g + e*f)*(7*d*g + 3*e*f)*log(-d + e*x)/e**3 + e*g**2*x**4/4 + x**
3*(5*d*g**2/3 + 2*e*f*g/3) + x**2*(6*d**2*g**2/e + 5*d*f*g + e*f**2/2) + x*
(20*d**3*g**2/e**2 + 24*d**2*f*g/e + 5*d*f**2) + (-8*d**5*g**2 - 16*d**4*e*
f*g - 8*d**3*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.362 $\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=107

$$\frac{4d^2(dg+ef)^2}{e^3(d-ex)}+\frac{x\left(8d^2g^2+8defg+e^2f^2\right)}{e^2}+\frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{x\left(8d^2g^2+8defg+e^2f^2\right)}{e^2}+\frac{4d^2(dg+ef)^2}{e^3(d-ex)}+\frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 + (4*d^2*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d*(e*f + d*g)*(e*f + 3*
d*g)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^2}\,dx$$
$$=\int \left(\frac{e^2f^2+8defg+8d^2g^2}{e^2}+\frac{2g(ef+2dg)x}{e}+g^2x^2+\frac{4d(-ef-3dg)(ef+dg)}{e^2(d-ex)}+\frac{4d^2(\cdots}{e^2(\cdots}\right.$$
$$=\frac{\left(e^2f^2+8defg+8d^2g^2\right)x}{e^2}+\frac{g(ef+2dg)x^2}{e}+\frac{g^2x^3}{3}+\frac{4d^2(ef+dg)^2}{e^3(d-ex)}+\frac{4d(ef+dg)(e\cdots}{\cdots}$$

Mathematica [A] time = 0.09, size = 115, normalized size = 1.07

$$-\frac{4d^2(dg+ef)^2}{e^3(ex-d)}+\frac{x\left(8d^2g^2+8defg+e^2f^2\right)}{e^2}+\frac{4d\left(3d^2g^2+4defg+e^2f^2\right)\log(d-ex)}{e^3}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 - (4*d^2*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d*(e^2*f^2 + 4*d*e*f*g
 + 3*d^2*g^2)*Log[d - e*x])/e^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2, x]
```

fricas [A] time = 0.39, size = 206, normalized size = 1.93

$$\frac{e^4g^2x^4-12\,d^2e^2f^2-24\,d^3efg-12\,d^4g^2+\left(3\,e^4fg+5\,de^3g^2\right)x^3+3\left(e^4f^2+7\,de^3fg+6\,d^2e^2g^2\right)x^2-3\left(de^3f^2+8\,d^2e^2fg+8\,d^3eg^2\right)x-12\left(d^2e^2f^2+4\,d^3efg+3\,d^4g^2-\left(de^3f^2+4\,d^2e^2fg+3\,d^3eg^2\right)x\right)\log\left(ex-d\right)}{3\left(e^4x-de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(e^4*g^2*x^4 - 12*d^2*e^2*f^2 - 24*d^3*e*f*g - 12*d^4*g^2 + (3*e^4*f*g
+ 5*d*e^3*g^2)*x^3 + 3*(e^4*f^2 + 7*d*e^3*f*g + 6*d^2*e^2*g^2)*x^2 - 3*(d*e
^3*f^2 + 8*d^2*e^2*f*g + 8*d^3*e*g^2)*x - 12*(d^2*e^2*f^2 + 4*d^3*e*f*g + 3
*d^4*g^2 - (d*e^3*f^2 + 4*d^2*e^2*f*g + 3*d^3*e*g^2)*x)*log(e*x - d))/(e^4*
x - d*e^3)
```

giac [B] time = 0.17, size = 250, normalized size = 2.34

$$2\left(3\,d^3g^2e^3+4\,d^2fge^4+df^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{1}{3}\left(g^2x^3e^{12}+6\,dg^2x^2e^{11}+24\,d^2g^2xe^{10}+3\,fgx^2e^{12}+24\,dfgxe^{11}+3\,f^2xe^{12}\right)e^{(-12)}+\frac{2\left(3\,d^4g^2e^4+4\,d^3fge^5+d^2f^2e^6\right)e^{(-7)}\log\left(\frac{\left|2\,xe^2-2\,\left|d\right|e\right|}{\left|2\,xe^2+2\,\left|d\right|e\right|}\right)}{\left|d\right|}-\frac{4\left(d^5g^2e^3+2\,d^4fge^4+d^3f^2e^5+\left(d^4g^2e^4+2\,d^3fge^5+d^2f^2e^6\right)x\right)e^{(-6)}}{x^2e^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 2*(3*d^3*g^2*e^3 + 4*d^2*f*g*e^4 + d*f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2)
) + 1/3*(g^2*x^3*e^12 + 6*d*g^2*x^2*e^11 + 24*d^2*g^2*x*e^10 + 3*f*g*x^2*e^
12 + 24*d*f*g*x*e^11 + 3*f^2*x*e^12)*e^(-12) + 2*(3*d^4*g^2*e^4 + 4*d^3*f*g
*e^5 + d^2*f^2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*ab
s(d)*e))/abs(d) - 4*(d^5*g^2*e^3 + 2*d^4*f*g*e^4 + d^3*f^2*e^5 + (d^4*g^2*e
^4 + 2*d^3*f*g*e^5 + d^2*f^2*e^6)*x)*e^(-6)/(x^2*e^2 - d^2)
```

maple [A] time = 0.01, size = 167, normalized size = 1.56

$$\frac{g^2x^3}{3}+\frac{2d\,g^2x^2}{e}+f\,g\,x^2-\frac{4d^4g^2}{(ex-d)\,e^3}-\frac{8d^3fg}{(ex-d)\,e^2}+\frac{12d^3g^2\ln(ex-d)}{e^3}-\frac{4d^2f^2}{(ex-d)\,e}+\frac{16d^2fg\ln(ex-d)}{e^2}+\frac{8d^2g^2x}{e^2}+\frac{4d\,f^2\ln(ex-d)}{e}+\frac{8dfgx}{e}+f^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/3*g^2*x^3+2*d/e*g^2*x^2+f*g*x^2+8*d^2/e^2*g^2*x+8*d/e*f*g*x+f^2*x+12*d^3/
e^3*g^2*ln(e*x-d)+16*d^2/e^2*f*g*ln(e*x-d)+4*d/e*f^2*ln(e*x-d)-4*d^4/e^3/(e
*x-d)*g^2-8*d^3/e^2/(e*x-d)*f*g-4*d^2/e/(e*x-d)*f^2
```

maxima [A] time = 0.45, size = 141, normalized size = 1.32

$$-\frac{4\left(d^2e^2f^2+2\,d^3efg+d^4g^2\right)}{e^4x-de^3}+\frac{e^2g^2x^3+3\left(e^2fg+2\,deg^2\right)x^2+3\left(e^2f^2+8\,defg+8\,d^2g^2\right)x}{3\,e^2}+\frac{4\left(de^2f^2+4\,d^2efg+3\,d^3g^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)/(e^4*x - d*e^3) + 1/3*(e^2*g^2*x^3
 + 3*(e^2*f*g + 2*d*e*g^2)*x^2 + 3*(e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2
 + 4*(d*e^2*f^2 + 4*d^2*e*f*g + 3*d^3*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.07, size = 185, normalized size = 1.73

$$x^2\left(\frac{g\left(dg+ef\right)}{e}+\frac{dg^2}{e}\right)+x\left(\frac{d^2g^2+4defg+e^2f^2}{e^2}+\frac{2d\left(\frac{2g\left(dg+ef\right)}{e}+\frac{2dg^2}{e}\right)}{e}-\frac{d^2g^2}{e^2}\right)+\frac{g^2x^3}{3}+\frac{4\left(d^4g^2+2d^3efg+d^2e^2f^2\right)}{e\left(de^2-e^3x\right)}+\frac{\ln\left(ex-d\right)\left(12d^3g^2+16d^2efg+4de^2f^2\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x^2*((g*(d*g + e*f))/e + (d*g^2)/e) + x*((d^2*g^2 + e^2*f^2 + 4*d*e*f*g)/e^
2 + (2*d*((2*g*(d*g + e*f))/e + (2*d*g^2)/e))/e - (d^2*g^2)/e^2) + (g^2*x^3
)/3 + (4*(d^4*g^2 + d^2*e^2*f^2 + 2*d^3*e*f*g))/(e*(d*e^2 - e^3*x)) + (log(
e*x - d)*(12*d^3*g^2 + 4*d*e^2*f^2 + 16*d^2*e*f*g))/e^3
```

sympy [A] time = 0.74, size = 119, normalized size = 1.11

$$\frac{4d\left(dg+ef\right)\left(3dg+ef\right)\log\left(-d+ex\right)}{e^3}+\frac{g^2x^3}{3}+x^2\left(\frac{2dg^2}{e}+fg\right)+x\left(\frac{8d^2g^2}{e^2}+\frac{8dfg}{e}+f^2\right)+\frac{-4d^4g^2-8d^3efg-4d^2e^2f^2}{-de^3+e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 4*d*(d*g + e*f)*(3*d*g + e*f)*log(-d + e*x)/e**3 + g**2*x**3/3 + x**2*(2*d*
g**2/e + f*g) + x*(8*d**2*g**2/e**2 + 8*d*f*g/e + f**2) + (-4*d**4*g**2 - 8
*d**3*e*f*g - 4*d**2*e**2*f**2)/(-d*e**3 + e**4*x)
```

3.363 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

Optimal. Leaf size=78

$$\frac{2d(dg+ef)^2}{e^3(d-ex)}+\frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3}+\frac{gx(3dg+2ef)}{e^2}+\frac{g^2x^2}{2e}$$

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 77}

$$\frac{2d(dg+ef)^2}{e^3(d-ex)}+\frac{gx(3dg+2ef)}{e^2}+\frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (g*(2*e*f + 3*d*g)*x)/e^2 + (g^2*x^2)/(2*e) + (2*d*(e*f + d*g)^2)/(e^3*(d -
 e*x)) + ((e*f + d*g)*(e*f + 5*d*g)*Log[d - e*x])/e^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(d+ex)(f+gx)^2}{(d-ex)^2}\,dx\\
&= \int \left(\frac{g(2ef+3dg)}{e^2}+\frac{g^2x}{e}+\frac{(-ef-5dg)(ef+dg)}{e^2(d-ex)}+\frac{2d(ef+dg)^2}{e^2(-d+ex)^2}\right)dx\\
&= \frac{g(2ef+3dg)x}{e^2}+\frac{g^2x^2}{2e}+\frac{2d(ef+dg)^2}{e^3(d-ex)}+\frac{(ef+dg)(ef+5dg)\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 1.06

$$\frac{2\left(5d^2g^2+6defg+e^2f^2\right)\log(d-ex)+\frac{4d(dg+ef)^2}{d-ex}+2egx(3dg+2ef)+e^2g^2x^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (2*e*g*(2*e*f + 3*d*g)*x + e^2*g^2*x^2 + (4*d*(e*f + d*g)^2)/(d - e*x) + 2*
(e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*Log[d - e*x])/(2*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2, x]
```

fricas [B] time = 0.39, size = 157, normalized size = 2.01

$$\frac{e^3g^2x^3-4\,de^2f^2-8\,d^2efg-4\,d^3g^2+\left(4\,e^3fg+5\,de^2g^2\right)x^2-2\left(2\,de^2fg+3\,d^2eg^2\right)x-2\left(de^2f^2+6\,d^2efg+5\,d^3g^2-\left(e^3f^2+6\,de^2fg+5\,d^2eg^2\right)x\right)\log\left(ex-d\right)}{2\left(e^4x-de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(e^3*g^2*x^3 - 4*d*e^2*f^2 - 8*d^2*e*f*g - 4*d^3*g^2 + (4*e^3*f*g + 5*d
*e^2*g^2)*x^2 - 2*(2*d*e^2*f*g + 3*d^2*e*g^2)*x - 2*(d*e^2*f^2 + 6*d^2*e*f*
g + 5*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^2)*x)*log(e*x - d))/(e^4
*x - d*e^3)
```

giac [B] time = 0.18, size = 212, normalized size = 2.72

$$\frac{1}{2}\left(5\,d^2g^2e^3+6\,dfge^4+f^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{1}{2}\left(g^2x^2e^7+6\,dg^2xe^6+4\,fgxe^7\right)e^{(-8)}+\frac{\left(5\,d^3g^2e^2+6\,d^2fge^3+df^2e^4\right)e^{(-5)}\log\left(\frac{\left|2\,xe^2-2\,\left|d\right|\left|e\right|\right|}{\left|2\,xe^2+2\,\left|d\right|\left|e\right|\right|}\right)}{2\left|d\right|}-\frac{2\left(d^4g^2e^3+2\,d^3fge^4+d^2f^2e^5+\left(d^3g^2e^4+2\,d^2fge^5+df^2e^6\right)x\right)e^{(-6)}}{x^2e^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*d^2*g^2*e^3 + 6*d*f*g*e^4 + f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2))
+ 1/2*(g^2*x^2*e^7 + 6*d*g^2*x*e^6 + 4*f*g*x*e^7)*e^(-8) + 1/2*(5*d^3*g^2*e
^2 + 6*d^2*f*g*e^3 + d*f^2*e^4)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*
x*e^2 + 2*abs(d)*e))/abs(d) - 2*(d^4*g^2*e^3 + 2*d^3*f*g*e^4 + d^2*f^2*e^5
+ (d^3*g^2*e^4 + 2*d^2*f*g*e^5 + d*f^2*e^6)*x)*e^(-6)/(x^2*e^2 - d^2)
```

maple [A] time = 0.01, size = 138, normalized size = 1.77

$$\frac{g^2x^2}{2e}-\frac{2d^3g^2}{\left(ex-d\right)e^3}-\frac{4d^2fg}{\left(ex-d\right)e^2}+\frac{5d^2g^2\ln\left(ex-d\right)}{e^3}-\frac{2d\,f^2}{\left(ex-d\right)e}+\frac{6dfg\ln\left(ex-d\right)}{e^2}+\frac{3d\,g^2x}{e^2}+\frac{f^2\ln\left(ex-d\right)}{e}+\frac{2fgx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/2/e*g^2*x^2+3*d/e^2*g^2*x+2/e*f*g*x+5*d^2/e^3*g^2*ln(e*x-d)+6*d/e^2*f*g*l
n(e*x-d)+1/e*f^2*ln(e*x-d)-2*d^3/e^3/(e*x-d)*g^2-4*d^2/e^2/(e*x-d)*f*g-2*d/
e/(e*x-d)*f^2
```

maxima [A] time = 0.44, size = 104, normalized size = 1.33

$$-\frac{2\left(de^2f^2+2\,d^2efg+d^3g^2\right)}{e^4x-de^3}+\frac{eg^2x^2+2\left(2\,efg+3\,dg^2\right)x}{2\,e^2}+\frac{\left(e^2f^2+6\,defg+5\,d^2g^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)/(e^4*x - d*e^3) + 1/2*(e*g^2*x^2 + 2
*(2*e*f*g + 3*d*g^2)*x)/e^2 + (e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*log(e*x - d
)/e^3
```

mupad [B] time = 2.53, size = 116, normalized size = 1.49

$$x\left(\frac{d\,g^2+2\,e\,f\,g}{e^2}+\frac{2\,d\,g^2}{e^2}\right)+\frac{\ln\left(e\,x-d\right)\,\left(5\,d^2\,g^2+6\,d\,e\,f\,g+e^2\,f^2\right)}{e^3}+\frac{g^2\,x^2}{2\,e}+\frac{2\,\left(d^3\,g^2+2\,d^2\,e\,f\,g+d\,e^2\,f^2\right)}{e\,\left(d\,e^2-e^3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x*((d*g^2 + 2*e*f*g)/e^2 + (2*d*g^2)/e^2) + (log(e*x - d)*(5*d^2*g^2 + e^2*
f^2 + 6*d*e*f*g))/e^3 + (g^2*x^2)/(2*e) + (2*(d^3*g^2 + d*e^2*f^2 + 2*d^2*e
*f*g))/(e*(d*e^2 - e^3*x))
```

sympy [A] time = 0.59, size = 94, normalized size = 1.21

$$x\left(\frac{3dg^2}{e^2}+\frac{2fg}{e}\right)+\frac{-2d^3g^2-4d^2efg-2de^2f^2}{-de^3+e^4x}+\frac{g^2x^2}{2e}+\frac{\left(dg+ef\right)\left(5dg+ef\right)\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] x*(3*d*g**2/e**2 + 2*f*g/e) + (-2*d**3*g**2 - 4*d**2*e*f*g - 2*d*e**2*f**2)
/(-d*e**3 + e**4*x) + g**2*x**2/(2*e) + (d*g + e*f)*(5*d*g + e*f)*log(-d +
e*x)/e**3
```

3.364 $\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=50

$$\frac{(dg+ef)^2}{e^3(d-ex)}+\frac{2g(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x}{e^2}$$

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 43}

$$\frac{(dg+ef)^2}{e^3(d-ex)}+\frac{2g(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (g^2*x)/e^2 + (e*f + d*g)^2/(e^3*(d - e*x)) + (2*g*(e*f + d*g)*Log[d - e*x]
)/e^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2}\,dx\\ &= \int \left(\frac{g^2}{e^2}+\frac{(ef+dg)^2}{e^2(-d+ex)^2}+\frac{2g(ef+dg)}{e^2(-d+ex)}\right)dx\\ &= \frac{g^2x}{e^2}+\frac{(ef+dg)^2}{e^3(d-ex)}+\frac{2g(ef+dg)\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 0.92

$$\frac{\frac{(dg+ef)^2}{d-ex}+2g(dg+ef)\log(d-ex)+eg^2x}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (e*g^2*x + (e*f + d*g)^2/(d - e*x) + 2*g*(e*f + d*g)*Log[d - e*x])/e^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2, x]
```

fricas [A] time = 0.38, size = 95, normalized size = 1.90

$$\frac{e^2g^2x^2-deg^2x-e^2f^2-2\,defg-d^2g^2-2\left(defg+d^2g^2-\left(e^2fg+deg^2\right)x\right)\log\left(ex-d\right)}{e^4x-de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] (e^2*g^2*x^2 - d*e*g^2*x - e^2*f^2 - 2*d*e*f*g - d^2*g^2 - 2*(d*e*f*g + d^2
*g^2 - (e^2*f*g + d*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3)
```

giac [B] time = 0.17, size = 160, normalized size = 3.20

$$g^2xe^{(-2)}+\left(dg^2e+fge^2\right)e^{(-4)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{\left(d^2g^2e^2+dfge^3\right)e^{(-5)}\log\left(\frac{\left|2\,xe^2-2\,|d||e|\right|}{\left|2\,xe^2+2\,|d||e|\right|}\right)}{|d|}-\frac{\left(d^3g^2e+2\,d^2fge^2+df^2e^3+\left(d^2g^2e^2+2\,dfge^3+f^2e^4\right)x\right)e^{(-4)}}{x^2e^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] g^2*x*e^(-2) + (d*g^2*e + f*g*e^2)*e^(-4)*log(abs(x^2*e^2 - d^2)) + (d^2*g^
2*e^2 + d*f*g*e^3)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs
(d)*e))/abs(d) - (d^3*g^2*e + 2*d^2*f*g*e^2 + d*f^2*e^3 + (d^2*g^2*e^2 + 2*
d*f*g*e^3 + f^2*e^4)*x)*e^(-4)/(x^2*e^2 - d^2)
```

maple [A] time = 0.01, size = 96, normalized size = 1.92

$$-\frac{d^2g^2}{(ex-d)\,e^3}-\frac{2dfg}{(ex-d)\,e^2}+\frac{2d\,g^2\ln\left(ex-d\right)}{e^3}-\frac{f^2}{(ex-d)\,e}+\frac{2fg\ln\left(ex-d\right)}{e^2}+\frac{g^2x}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/e^2*g^2*x+2*d/e^3*g^2*ln(e*x-d)+2/e^2*f*g*ln(e*x-d)-1/e^3/(e*x-d)*d^2*g^2
-2/e^2/(e*x-d)*d*f*g-1/e/(e*x-d)*f^2
```

maxima [A] time = 0.44, size = 69, normalized size = 1.38

$$\frac{g^2x}{e^2}-\frac{e^2f^2+2\,defg+d^2g^2}{e^4x-de^3}+\frac{2\left(efg+dg^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] g^2*x/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(e^4*x - d*e^3) + 2*(e*f*g + d*
g^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.56, size = 72, normalized size = 1.44

$$\frac{d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{e\,\left(d\,e^2-e^3\,x\right)}+\frac{g^2\,x}{e^2}+\frac{\ln\left(e\,x-d\right)\,\left(2\,d\,g^2+2\,e\,f\,g\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] (d^2*g^2 + e^2*f^2 + 2*d*e*f*g)/(e*(d*e^2 - e^3*x)) + (g^2*x)/e^2 + (log(e*
x - d)*(2*d*g^2 + 2*e*f*g))/e^3
```

sympy [A] time = 0.40, size = 61, normalized size = 1.22

$$\frac{-d^2g^2-2defg-e^2f^2}{-de^3+e^4x}+\frac{g^2x}{e^2}+\frac{2g\left(dg+ef\right)\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-d**2*g**2 - 2*d*e*f*g - e**2*f**2)/(-d*e**3 + e**4*x) + g**2*x/e**2 + 2*g
*(d*g + e*f)*log(-d + e*x)/e**3
```

3.365 $\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^2} \, dx$

Optimal. Leaf size=86

$$\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}-\frac{(ef-3dg)(dg+ef)\log(d-ex)}{4d^2e^3}+\frac{(dg+ef)^2}{2de^3(d-ex)}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {799, 88}

$$\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}-\frac{(ef-3dg)(dg+ef)\log(d-ex)}{4d^2e^3}+\frac{(dg+ef)^2}{2de^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (e*f + d*g)^2/(2*d*e^3*(d - e*x)) - ((e*f - 3*d*g)*(e*f + d*g)*Log[d - e*x]
)/(4*d^2*e^3) + ((e*f - d*g)^2*Log[d + e*x])/(4*d^2*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 799

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c*x)/g)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{2de^2(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^2(d-ex)}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)}\right)dx\\
&= \frac{(ef+dg)^2}{2de^3(d-ex)}-\frac{(ef-3dg)(ef+dg)\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 1.06

$$\frac{(d-ex)\left(3d^2g^2+2defg-e^2f^2\right)\log(d-ex)+(d-ex)(ef-dg)^2\log(d+ex)+2d(dg+ef)^2}{4d^2e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (2*d*(e*f + d*g)^2 + (-(e^2*f^2) + 2*d*e*f*g + 3*d^2*g^2)*(d - e*x)*Log[d -
 e*x] + (e*f - d*g)^2*(d - e*x)*Log[d + e*x])/(4*d^2*e^3*(d - e*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2, x]
```

fricas [B] time = 0.39, size = 168, normalized size = 1.95

$$-\frac{2\,de^2f^2+4\,d^2efg+2\,d^3g^2+\left(de^2f^2-2\,d^2efg+d^3g^2-\left(e^3f^2-2\,de^2fg+d^2eg^2\right)x\right)\log\left(ex+d\right)-\left(de^2f^2-2\,d^2efg-3\,d^3g^2-\left(e^3f^2-2\,de^2fg-3\,d^2eg^2\right)x\right)\log\left(ex-d\right)}{4\left(d^2e^4x-d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*e^2*f^2 + 4*d^2*e*f*g + 2*d^3*g^2 + (d*e^2*f^2 - 2*d^2*e*f*g + d^
3*g^2 - (e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e*x + d) - (d*e^2*f^2 -
2*d^2*e*f*g - 3*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e*x
- d))/(d^2*e^4*x - d^3*e^3)
```

giac [A] time = 0.19, size = 159, normalized size = 1.85

$$\frac{1}{2}\,g^2e^{(-3)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{\left(d^2g^2+2\,dfge-f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{4\,d|d|}-\frac{\left(\left(d^2g^2+2\,dfge+f^2e^2\right)x+\left(d^3g^2e+2\,d^2fge^2+df^2e^3\right)e^{(-2)}\right)e^{(-2)}}{2\left(x^2e^2-d^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*g^2*e^(-3)*log(abs(x^2*e^2 - d^2)) + 1/4*(d^2*g^2 + 2*d*f*g*e - f^2*e^2
)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/(d*abs(d)
) - 1/2*((d^2*g^2 + 2*d*f*g*e + f^2*e^2)*x + (d^3*g^2*e + 2*d^2*f*g*e^2 + d
*f^2*e^3)*e^(-2))*e^(-2)/((x^2*e^2 - d^2)*d)
```

maple [A] time = 0.01, size = 156, normalized size = 1.81

$$-\frac{d\,g^2}{2\left(ex-d\right)e^3}-\frac{f^2}{2\left(ex-d\right)de}+\frac{fg\ln\left(ex-d\right)}{2d\,e^2}-\frac{fg\ln\left(ex+d\right)}{2d\,e^2}-\frac{f^2\ln\left(ex-d\right)}{4d^2e}+\frac{f^2\ln\left(ex+d\right)}{4d^2e}-\frac{fg}{\left(ex-d\right)e^2}+\frac{3g^2\ln\left(ex-d\right)}{4e^3}+\frac{g^2\ln\left(ex+d\right)}{4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/2/e^3*d/(e*x-d)*g^2-1/e^2/(e*x-d)*f*g-1/2/e/d/(e*x-d)*f^2+3/4/e^3*g^2*ln
(e*x-d)+1/2/d/e^2*f*g*ln(e*x-d)-1/4/d^2/e*f^2*ln(e*x-d)+1/4/e^3*g^2*ln(e*x+
d)-1/2/d/e^2*f*g*ln(e*x+d)+1/4/d^2/e*f^2*ln(e*x+d)
```

maxima [A] time = 0.45, size = 114, normalized size = 1.33

$$-\frac{e^2f^2+2\,defg+d^2g^2}{2\left(de^4x-d^2e^3\right)}+\frac{\left(e^2f^2-2\,defg+d^2g^2\right)\log\left(ex+d\right)}{4\,d^2e^3}-\frac{\left(e^2f^2-2\,defg-3\,d^2g^2\right)\log\left(ex-d\right)}{4\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(d*e^4*x - d^2*e^3) + 1/4*(e^2*f^2 - 2
*d*e*f*g + d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 - 2*d*e*f*g - 3*d
^2*g^2)*log(e*x - d)/(d^2*e^3)
```

mupad [B] time = 2.64, size = 111, normalized size = 1.29

$$\frac{d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{2\,d\,e^3\,\left(d-e\,x\right)}+\frac{\ln\left(d+e\,x\right)\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3}+\frac{\ln\left(d-e\,x\right)\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2\right)}{4\,d^2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2)^2,x)
```

```
[Out] (d^2*g^2 + e^2*f^2 + 2*d*e*f*g)/(2*d*e^3*(d - e*x)) + (log(d + e*x)*(d^2*g^
2 + e^2*f^2 - 2*d*e*f*g))/(4*d^2*e^3) + (log(d - e*x)*(3*d^2*g^2 - e^2*f^2
+ 2*d*e*f*g))/(4*d^2*e^3)
```

sympy [B] time = 1.04, size = 182, normalized size = 2.12

$$\frac{-d^2g^2-2defg-e^2f^2}{-2d^2e^3+2de^4x}+\frac{\left(dg-ef\right)^2\log\left(x+\frac{2d^3g^2-d\left(dg-ef\right)^2}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}+\frac{\left(dg+ef\right)\left(3dg-ef\right)\log\left(x+\frac{2d^3g^2-d\left(dg+ef\right)\left(3dg-ef\right)}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-d**2*g**2 - 2*d*e*f*g - e**2*f**2)/(-2*d**2*e**3 + 2*d*e**4*x) + (d*g - e
*f)**2*log(x + (2*d**3*g**2 - d*(d*g - e*f)**2)/(d**2*e*g**2 + 2*d*e**2*f*g
 - e**3*f**2))/(4*d**2*e**3) + (d*g + e*f)*(3*d*g - e*f)*log(x + (2*d**3*g*
*2 - d*(d*g + e*f)*(3*d*g - e*f))/(d**2*e*g**2 + 2*d*e**2*f*g - e**3*f**2))
/(4*d**2*e**3)
```

3.366 $\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{(ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}+\frac{(f+gx)\left(d^2g+e^2fx\right)}{2d^2e^2\left(d^2-e^2x^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {723, 208}

$$\frac{(f+gx)\left(d^2g+e^2fx\right)}{2d^2e^2\left(d^2-e^2x^2\right)}+\frac{(ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(2*d^2*e^2*(d^2 - e^2*x^2)) + ((e*f - d*g)*(e
*f + d*g)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \frac{\left(d^2g+e^2fx\right)(f+gx)}{2d^2e^2\left(d^2-e^2x^2\right)}-\frac{1}{2}\left(-\frac{f^2}{d^2}+\frac{g^2}{e^2}\right)\int \frac{1}{d^2-e^2x^2}\,dx\\ &= \frac{\left(d^2g+e^2fx\right)(f+gx)}{2d^2e^2\left(d^2-e^2x^2\right)}+\frac{(ef-dg)(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.15

$$\frac{-2d^2fg-d^2g^2x-e^2f^2x}{2d^2e^2\left(e^2x^2-d^2\right)}-\frac{\left(d^2g^2-e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (-2*d^2*f*g - e^2*f^2*x - d^2*g^2*x)/(2*d^2*e^2*(-d^2 + e^2*x^2)) - ((-(e^2
*f^2) + d^2*g^2)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/(d^2 - e^2*x^2)^2, x]
```

fricas [B] time = 0.41, size = 155, normalized size = 2.09

$$-\frac{4\,d^3efg+2\left(de^3f^2+d^3eg^2\right)x+\left(d^2e^2f^2-d^4g^2-\left(e^4f^2-d^2e^2g^2\right)x^2\right)\log\left(ex+d\right)-\left(d^2e^2f^2-d^4g^2-\left(e^4f^2-d^2e^2g^2\right)x^2\right)\log\left(ex-d\right)}{4\left(d^3e^5x^2-d^5e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d^3*e*f*g + 2*(d*e^3*f^2 + d^3*e*g^2)*x + (d^2*e^2*f^2 - d^4*g^2 -
(e^4*f^2 - d^2*e^2*g^2)*x^2)*log(e*x + d) - (d^2*e^2*f^2 - d^4*g^2 - (e^4*f
^2 - d^2*e^2*g^2)*x^2)*log(e*x - d))/(d^3*e^5*x^2 - d^5*e^3)
```

giac [A] time = 0.16, size = 101, normalized size = 1.36

$$\frac{\left(d^2g^2-f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d||e|\right|}{\left|2\,xe^2+2\,|d||e|\right|}\right)}{4\,d^2|d|}-\frac{\left(d^2g^2x+2\,d^2fg+f^2xe^2\right)e^{(-2)}}{2\left(x^2e^2-d^2\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(d^2*g^2 - f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 +
2*abs(d)*e))/(d^2*abs(d)) - 1/2*(d^2*g^2*x + 2*d^2*f*g + f^2*x*e^2)*e^(-2)/
((x^2*e^2 - d^2)*d^2)
```

maple [B] time = 0.01, size = 180, normalized size = 2.43

$$-\frac{fg}{2\left(ex-d\right)d\,e^2}+\frac{fg}{2\left(ex+d\right)d\,e^2}+\frac{g^2\ln\left(ex-d\right)}{4d\,e^3}-\frac{g^2\ln\left(ex+d\right)}{4d\,e^3}-\frac{f^2}{4\left(ex-d\right)d^2e}-\frac{f^2}{4\left(ex+d\right)d^2e}-\frac{f^2\ln\left(ex-d\right)}{4d^3e}+\frac{f^2\ln\left(ex+d\right)}{4d^3e}-\frac{g^2}{4\left(ex-d\right)e^3}-\frac{g^2}{4\left(ex+d\right)e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/4/d/e^3*g^2*ln(e*x-d)-1/4/d^3/e*f^2*ln(e*x-d)-1/4/e^3/(e*x-d)*g^2-1/2/e^2
/d/(e*x-d)*f*g-1/4/e/d^2/(e*x-d)*f^2-1/4/d/e^3*g^2*ln(e*x+d)+1/4/d^3/e*f^2*
ln(e*x+d)-1/4/(e*x+d)/e^3*g^2+1/2/(e*x+d)/d/e^2*f*g-1/4/(e*x+d)/d^2/e*f^2
```

maxima [A] time = 0.44, size = 111, normalized size = 1.50

$$-\frac{2\,d^2fg+\left(e^2f^2+d^2g^2\right)x}{2\left(d^2e^4x^2-d^4e^2\right)}+\frac{\left(e^2f^2-d^2g^2\right)\log\left(ex+d\right)}{4\,d^3e^3}-\frac{\left(e^2f^2-d^2g^2\right)\log\left(ex-d\right)}{4\,d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*d^2*f*g + (e^2*f^2 + d^2*g^2)*x)/(d^2*e^4*x^2 - d^4*e^2) + 1/4*(e^2
*f^2 - d^2*g^2)*log(e*x + d)/(d^3*e^3) - 1/4*(e^2*f^2 - d^2*g^2)*log(e*x -
d)/(d^3*e^3)
```

mupad [B] time = 2.61, size = 115, normalized size = 1.55

$$\frac{\frac{f\,g}{e^2}+\frac{x\,\left(d^2\,g^2+e^2\,f^2\right)}{2\,d^2\,e^2}}{d^2-e^2\,x^2}-\frac{2\,\mathrm{atanh}\left(\frac{4\,e\,x\,\left(\frac{d^2\,g^2}{4}-\frac{e^2\,f^2}{4}\right)}{d\,\left(d^2\,g^2-e^2\,f^2\right)}\right)\,\left(\frac{d^2\,g^2}{4}-\frac{e^2\,f^2}{4}\right)}{d^3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2)^2,x)
```

```
[Out] ((f*g)/e^2 + (x*(d^2*g^2 + e^2*f^2))/(2*d^2*e^2))/(d^2 - e^2*x^2) - (2*atan
h((4*e*x*((d^2*g^2)/4 - (e^2*f^2)/4))/(d*(d^2*g^2 - e^2*f^2)))*((d^2*g^2)/4
 - (e^2*f^2)/4))/(d^3*e^3)
```

sympy [B] time = 0.71, size = 156, normalized size = 2.11

$$\frac{-2d^2fg+x\left(-d^2g^2-e^2f^2\right)}{-2d^4e^2+2d^2e^4x^2}+\frac{\left(dg-ef\right)\left(dg+ef\right)\log\left(-\frac{d\left(dg-ef\right)\left(dg+ef\right)}{e\left(d^2g^2-e^2f^2\right)}+x\right)}{4d^3e^3}-\frac{\left(dg-ef\right)\left(dg+ef\right)\log\left(\frac{d\left(dg-ef\right)\left(dg+ef\right)}{e\left(d^2g^2-e^2f^2\right)}+x\right)}{4d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**2*f*g + x*(-d**2*g**2 - e**2*f**2))/(-2*d**4*e**2 + 2*d**2*e**4*x**2
) + (d*g - e*f)*(d*g + e*f)*log(-d*(d*g - e*f)*(d*g + e*f)/(e*(d**2*g**2 -
e**2*f**2)) + x)/(4*d**3*e**3) - (d*g - e*f)*(d*g + e*f)*log(d*(d*g - e*f)*
(d*g + e*f)/(e*(d**2*g**2 - e**2*f**2)) + x)/(4*d**3*e**3)
```

3.367 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=121

$$\frac{(3ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}+\frac{(dg+ef)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}$$

Rubi [A] time = 0.14, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}+\frac{(dg+ef)^2}{8d^3e^3(d-ex)}+\frac{(3ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(8*d^3*e^3*(d - e*x)) - (e*f - d*g)^2/(8*d^2*e^3*(d + e*x)^2)
 - (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d + e*x)) + ((3*e*f - d*g)*(e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^3}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{8d^3e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^3}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^2}+\frac{(3ef-dg)(ef+dg)}{8d^3e^2\left(d^2-e^2x^2\right)}\right) d\\
&= \frac{(ef+dg)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}+\frac{((3ef-dg)(ef+dg))\int \frac{1}{d^2-e^2x}}{8d^3e^2}\\
&= \frac{(ef+dg)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}+\frac{(3ef-dg)(ef+dg)\tanh^{-1}(}{8d^4e^3}\end{aligned}$$

Mathematica [A] time = 0.11, size = 139, normalized size = 1.15

$$\frac{\frac{4d\left(d^2g^2-e^2f^2\right)}{d+ex}+\left(d^2g^2-2defg-3e^2f^2\right)\log(d-ex)+\left(-d^2g^2+2defg+3e^2f^2\right)\log(d+ex)-\frac{2d^2(ef-dg)^2}{(d+ex)^2}+\frac{2d(dg+ef)^2}{d-ex}}{16d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((2*d*(e*f + d*g)^2)/(d - e*x) - (2*d^2*(e*f - d*g)^2)/(d + e*x)^2 + (4*d*(
-(e^2*f^2) + d^2*g^2))/(d + e*x) + (-3*e^2*f^2 - 2*d*e*f*g + d^2*g^2)*Log[d
 - e*x] + (3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2), x]
```

fricas [B] time = 0.38, size = 417, normalized size = 3.45

$$\frac{4d^3e^2f^2-8d^4efg-4d^5g^2-2\left(3de^4f^2+2d^2e^3fg-d^3e^2g^2\right)x^2-2\left(3d^2e^3f^2+2d^3e^2fg+3d^4eg^2\right)x-\left(3d^3e^2f^2+2d^4efg-d^5g^2-\left(3e^5f^2+2de^4fg-d^2e^3g^2\right)x^3-\left(3de^4f^2+2d^2e^3fg-d^3e^2g^2\right)x^2+\left(3d^2e^3f^2+2d^3e^2fg-d^4eg^2\right)x\right)\log(ex+d)+\left(3d^3e^2f^2+2d^4efg-d^5g^2-\left(3e^5f^2+2de^4fg-d^2e^3g^2\right)x^3-\left(3de^4f^2+2d^2e^3fg-d^3e^2g^2\right)x^2+\left(3d^2e^3f^2+2d^3e^2fg-d^4eg^2\right)x\right)\log(ex-d)}{16\left(d^4e^6x^3+d^5e^5x^2-d^6e^4x-d^7e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d^3*e^2*f^2 - 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 + 2*d^2*e^3*
f*g - d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^3*f^2 + 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x
- (3*d^3*e^2*f^2 + 2*d^4*e*f*g - d^5*g^2 - (3*e^5*f^2 + 2*d*e^4*f*g - d^2*e
^3*g^2)*x^3 - (3*d*e^4*f^2 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 + (3*d^2*e^3*
f^2 + 2*d^3*e^2*f*g - d^4*e*g^2)*x)*log(e*x + d) + (3*d^3*e^2*f^2 + 2*d^4*e
*f*g - d^5*g^2 - (3*e^5*f^2 + 2*d*e^4*f*g - d^2*e^3*g^2)*x^3 - (3*d*e^4*f^2
 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 + (3*d^2*e^3*f^2 + 2*d^3*e^2*f*g - d^4*
e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 + d^5*e^5*x^2 - d^6*e^4*x - d^7*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (d^2*
exp(1)^2*g^2-2*d*exp(1)^3*g*f+exp(1)^4*f^2)/(exp(2)^2*d^4*exp(1)-2*exp(2)*d
^4*exp(1)^3+d^4*exp(1)^5)*ln(abs(x*exp(1)+d))+(-d^2*exp(1)*g^2+2*d*exp(1)^2
*g*f-exp(1)^3*f^2)/(2*exp(2)^2*d^4-4*exp(2)*d^4*exp(1)^2+2*d^4*exp(1)^4)*ln
(abs(-x^2*exp(2)+d^2))+(exp(2)^2*f^2-exp(2)*d^2*g^2+2*exp(2)*d*exp(1)*g*f-3
*exp(2)*exp(1)^2*f^2-d^2*exp(1)^2*g^2+2*d*exp(1)^3*g*f)*1/2/(2*exp(2)^2*d^3
-4*exp(2)*d^3*exp(1)^2+2*d^3*exp(1)^4)/exp(1)/abs(d)*ln(abs(-2*x*exp(2)-2*e
xp(1)*abs(d))/abs(-2*x*exp(2)+2*exp(1)*abs(d)))-(-2*exp(2)^2*d^3*g*f+exp(2)
^2*d^2*exp(1)*f^2+exp(2)*d^4*exp(1)*g^2+2*exp(2)*d^3*exp(1)^2*g*f-exp(2)*d^
2*exp(1)^3*f^2-d^4*exp(1)^3*g^2+(-exp(2)^3*d*f^2-exp(2)^2*d^3*g^2+2*exp(2)^
2*d^2*exp(1)*g*f+exp(2)^2*d*exp(1)^2*f^2+exp(2)*d^3*exp(1)^2*g^2-2*exp(2)*d
^2*exp(1)^3*g*f)*x)/2/d^4/exp(2)/(exp(2)-exp(1)^2)^2/(-x^2*exp(2)+d^2)
```

maple [B] time = 0.01, size = 253, normalized size = 2.09

$$\frac{fg}{4\left(ex+d\right)^{2}de^{2}}-\frac{f^{2}}{8\left(ex+d\right)^{2}d^{2}e}-\frac{g^{2}}{8\left(ex+d\right)^{2}e^{3}}-\frac{g^{2}}{8\left(ex-d\right)de^{3}}+\frac{g^{2}}{4\left(ex+d\right)de^{3}}-\frac{fg}{4\left(ex-d\right)d^{2}e^{2}}+\frac{g^{2}\ln\left(ex-d\right)}{16d^{2}e^{3}}-\frac{g^{2}\ln\left(ex+d\right)}{16d^{2}e^{3}}-\frac{f^{2}}{8\left(ex-d\right)d^{3}e}-\frac{f^{2}}{4\left(ex+d\right)d^{3}e}-\frac{fg\ln\left(ex-d\right)}{8d^{3}e^{2}}+\frac{fg\ln\left(ex+d\right)}{8d^{3}e^{2}}-\frac{3f^{2}\ln\left(ex-d\right)}{16d^{4}e}+\frac{3f^{2}\ln\left(ex+d\right)}{16d^{4}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/16/d^2/e^3*g^2*ln(e*x-d)-1/8/d^3/e^2*f*g*ln(e*x-d)-3/16/d^4/e*f^2*ln(e*x-
d)-1/8/e^3/d/(e*x-d)*g^2-1/4/e^2/d^2/(e*x-d)*f*g-1/8/e/d^3/(e*x-d)*f^2+1/4/
(e*x+d)/d/e^3*g^2-1/4/(e*x+d)/d^3/e*f^2-1/16/d^2/e^3*g^2*ln(e*x+d)+1/8/d^3/
e^2*f*g*ln(e*x+d)+3/16/d^4/e*f^2*ln(e*x+d)-1/8/(e*x+d)^2/e^3*g^2+1/4/(e*x+d
)^2/d/e^2*f*g-1/8/(e*x+d)^2/d^2/e*f^2
```

maxima [A] time = 0.47, size = 212, normalized size = 1.75

$$\frac{2\,d^{2}e^{2}f^{2}-4\,d^{3}efg-2\,d^{4}g^{2}-\left(3\,e^{4}f^{2}+2\,de^{3}fg-d^{2}e^{2}g^{2}\right)x^{2}-\left(3\,de^{3}f^{2}+2\,d^{2}e^{2}fg+3\,d^{3}eg^{2}\right)x}{8\left(d^{3}e^{6}x^{3}+d^{4}e^{5}x^{2}-d^{5}e^{4}x-d^{6}e^{3}\right)}+\frac{\left(3\,e^{2}f^{2}+2\,defg-d^{2}g^{2}\right)\log\left(ex+d\right)}{16\,d^{4}e^{3}}-\frac{\left(3\,e^{2}f^{2}+2\,defg-d^{2}g^{2}\right)\log\left(ex-d\right)}{16\,d^{4}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*d^2*e^2*f^2 - 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 + 2*d*e^3*f*g - d
^2*e^2*g^2)*x^2 - (3*d*e^3*f^2 + 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6*x
^3 + d^4*e^5*x^2 - d^5*e^4*x - d^6*e^3) + 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2
*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(e
*x - d)/(d^4*e^3)
```

mupad [B] time = 0.15, size = 198, normalized size = 1.64

$$\frac{\frac{d^{2}\,g^{2}+2\,d\,e\,f\,g-e^{2}\,f^{2}}{4\,d\,e^{3}}+\frac{x\,\left(3\,d^{2}\,g^{2}+2\,d\,e\,f\,g+3\,e^{2}\,f^{2}\right)}{8\,d^{2}\,e^{2}}+\frac{x^{2}\,\left(-d^{2}\,g^{2}+2\,d\,e\,f\,g+3\,e^{2}\,f^{2}\right)}{8\,d^{3}\,e}}{d^{3}+d^{2}\,e\,x-d\,e^{2}\,x^{2}-e^{3}\,x^{3}}+\frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g+e\,f\right)\,\left(d\,g-3\,e\,f\right)}{d\,\left(-d^{2}\,g^{2}+2\,d\,e\,f\,g+3\,e^{2}\,f^{2}\right)}\right)\,\left(d\,g+e\,f\right)\,\left(d\,g-3\,e\,f\right)}{8\,d^{4}\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)),x)
```

```
[Out] ((d^2*g^2 - e^2*f^2 + 2*d*e*f*g)/(4*d*e^3) + (x*(3*d^2*g^2 + 3*e^2*f^2 + 2*
d*e*f*g))/(8*d^2*e^2) + (x^2*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g))/(8*d^3*e))/
(d^3 - e^3*x^3 - d*e^2*x^2 + d^2*e*x) + (atanh((e*x*(d*g + e*f)*(d*g - 3*e*
f))/(d*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g)))*(d*g + e*f)*(d*g - 3*e*f))/(8*d^
4*e^3)
```

sympy [B] time = 1.26, size = 279, normalized size = 2.31

$$\frac{-2d^{4}g^{2}-4d^{3}efg+2d^{2}e^{2}f^{2}+x^{2}\left(d^{2}e^{2}g^{2}-2de^{3}fg-3e^{4}f^{2}\right)+x\left(-3d^{3}eg^{2}-2d^{2}e^{2}fg-3de^{3}f^{2}\right)}{-8d^{6}e^{3}-8d^{5}e^{4}x+8d^{4}e^{5}x^{2}+8d^{3}e^{6}x^{3}}+\frac{\left(dg-3ef\right)\left(dg+ef\right)\log\left(-\frac{d\left(dg-3ef\right)\left(dg+ef\right)}{e\left(d^{2}g^{2}-2defg-3e^{2}f^{2}\right)}+x\right)}{16d^{4}e^{3}}-\frac{\left(dg-3ef\right)\left(dg+ef\right)\log\left(\frac{d\left(dg-3ef\right)\left(dg+ef\right)}{e\left(d^{2}g^{2}-2defg-3e^{2}f^{2}\right)}+x\right)}{16d^{4}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**4*g**2 - 4*d**3*e*f*g + 2*d**2*e**2*f**2 + x**2*(d**2*e**2*g**2 - 2*
d*e**3*f*g - 3*e**4*f**2) + x*(-3*d**3*e*g**2 - 2*d**2*e**2*f*g - 3*d*e**3*
f**2))/(-8*d**6*e**3 - 8*d**5*e**4*x + 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3)
 + (d*g - 3*e*f)*(d*g + e*f)*log(-d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2
 - 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - 3*e*f)*(d*g + e*f
)*log(d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2 - 2*d*e*f*g - 3*e**2*f**2))
 + x)/(16*d**4*e**3)
```

3.368 $\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=146

$$\frac{f(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2}+\frac{(dg+ef)^2}{16d^4e^3(d-ex)}-\frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}$$

Rubi [A] time = 0.16, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}+\frac{(dg+ef)^2}{16d^4e^3(d-ex)}-\frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)}+\frac{f(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(16*d^4*e^3*(d - e*x)) - (e*f - d*g)^2/(12*d^2*e^3*(d + e*x)^
3) - (e^2*f^2 - d^2*g^2)/(8*d^3*e^3*(d + e*x)^2) - ((3*e*f - d*g)*(e*f + d*
g))/(16*d^4*e^3*(d + e*x)) + (f*(e*f + d*g)*ArcTanh[(e*x)/d])/(4*d^5*e^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^4}\,dx$$

$$=\int \left(\frac{(ef+dg)^2}{16d^4e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^4}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^3}+\frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^2}+ \cdots\right)$$

$$=\frac{(ef+dg)^2}{16d^4e^3(d-ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)}+\frac{(f(ef \cdots}{}$$

$$=\frac{(ef+dg)^2}{16d^4e^3(d-ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)}+\frac{f(ef \cdots}{}$$

Mathematica [**A**] time = 0.09, size = 171, normalized size = 1.17

$$\frac{2d\left(2d^5g^2+2d^4eg(f+2gx)+d^3e^2f(gx-4f)+d^2e^3fx(f+6gx)+3de^4fx^2(2f+gx)+3e^5f^2x^3\right)+3ef(ex-d)(d+ex)^3(dg+ef)\log(d-ex)+3ef(d-ex)(d+ex)^3(dg+ef)\log(d+ex)}{24d^5e^3(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (2*d*(2*d^5*g^2 + 3*e^5*f^2*x^3 + d^3*e^2*f*(-4*f + g*x) + 3*d*e^4*f*x^2*(2
*f + g*x) + 2*d^4*e*g*(f + 2*g*x) + d^2*e^3*f*x*(f + 6*g*x)) + 3*e*f*(e*f +
 d*g)*(-d + e*x)*(d + e*x)^3*Log[d - e*x] + 3*e*f*(e*f + d*g)*(d - e*x)*(d
+ e*x)^3*Log[d + e*x])/(24*d^5*e^3*(d - e*x)*(d + e*x)^3)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2), x]
```

fricas [**B**] time = 0.38, size = 337, normalized size = 2.31

$$\frac{8\,d^4e^2f^2-4\,d^5efg-4\,d^6g^2-6\left(de^5f^2+d^2e^4fg\right)x^3-12\left(d^2e^4f^2+d^3e^3fg\right)x^2-2\left(d^3e^3f^2+d^4e^2fg+4\,d^5eg^2\right)x-3\left(d^4e^2f^2+d^5efg-\left(e^6f^2+de^5fg\right)x^4-2\left(de^5f^2+d^2e^4fg\right)x^3+2\left(d^3e^3f^2+d^4e^2fg\right)x\right)\log\left(ex+d\right)+3\left(d^4e^2f^2+d^5efg-\left(e^6f^2+de^5fg\right)x^4-2\left(de^5f^2+d^2e^4fg\right)x^3+2\left(d^3e^3f^2+d^4e^2fg\right)x\right)\log\left(ex-d\right)}{24\left(d^5e^7x^4+2\,d^6e^6x^3-2\,d^8e^4x-d^9e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(8*d^4*e^2*f^2 - 4*d^5*e*f*g - 4*d^6*g^2 - 6*(d*e^5*f^2 + d^2*e^4*f*g)
*x^3 - 12*(d^2*e^4*f^2 + d^3*e^3*f*g)*x^2 - 2*(d^3*e^3*f^2 + d^4*e^2*f*g +
4*d^5*e*g^2)*x - 3*(d^4*e^2*f^2 + d^5*e*f*g - (e^6*f^2 + d*e^5*f*g)*x^4 - 2
*(d*e^5*f^2 + d^2*e^4*f*g)*x^3 + 2*(d^3*e^3*f^2 + d^4*e^2*f*g)*x)*log(e*x +
 d) + 3*(d^4*e^2*f^2 + d^5*e*f*g - (e^6*f^2 + d*e^5*f*g)*x^4 - 2*(d*e^5*f^2
 + d^2*e^4*f*g)*x^3 + 2*(d^3*e^3*f^2 + d^4*e^2*f*g)*x)*log(e*x - d))/(d^5*e
^7*x^4 + 2*d^6*e^6*x^3 - 2*d^8*e^4*x - d^9*e^3)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-(ex
p(1)*x+d)^-1/exp(1)*g^2*d^2*exp(1)^6+2*(exp(1)*x+d)^-1/exp(1)*g*d*exp(1)^7*
f-(exp(1)*x+d)^-1/exp(1)*exp(1)^8*f^2)/(d^4*exp(1)^8-2*d^4*exp(1)^6*exp(2)+
d^4*exp(1)^4*exp(2)^2)-(-(-g^2*d^3*exp(1)^6-6*g^2*d^3*exp(1)^4*exp(2)-g^2*d
^3*exp(1)^2*exp(2)^2+8*g*d^2*exp(1)^5*exp(2)*f+8*g*d^2*exp(1)^3*exp(2)^2*f-
d*exp(1)^6*exp(2)*f^2-6*d*exp(1)^4*exp(2)^2*f^2-d*exp(1)^2*exp(2)^3*f^2)/(e
xp(1)^2-exp(2))*(exp(1)*x+d)^-1/exp(1)+(-3*g^2*d^2*exp(1)^3*exp(2)-g^2*d^2*
exp(1)*exp(2)^2+2*g*d*exp(1)^4*exp(2)*f+6*g*d*exp(1)^2*exp(2)^2*f-3*exp(1)^
3*exp(2)^2*f^2-exp(1)*exp(2)^3*f^2)/(exp(1)^2-exp(2)))/2/d^5/(exp(2)-exp(1)
^2)^2/(-(-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^4+(-(exp(1)*x+d)^-1/exp(1))^
2*d^2*exp(1)^2*exp(2)-2*(exp(1)*x+d)^-1/exp(1)*d*exp(1)*exp(2)+exp(2))+(g^2
*d^2*exp(1)^3+g^2*d^2*exp(1)*exp(2)-g*d*exp(1)^4*f-3*g*d*exp(1)^2*f*exp(2)+
2*exp(1)^3*f^2*exp(2))/(d^5*exp(1)^6-3*d^5*exp(1)^4*exp(2)+3*d^5*exp(1)^2*e
xp(2)^2-d^5*exp(2)^3)*ln(abs(-(-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^4+(-(e
```

```
xp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^2*exp(2)-2*(exp(1)*x+d)^-1/exp(1)*d*exp(
1)*exp(2)+exp(2)))+(-g^2*d^2*exp(1)^6-6*g^2*d^2*exp(1)^4*exp(2)-g^2*d^2*exp
(1)^2*exp(2)^2+12*g*d*exp(1)^5*f*exp(2)+4*g*d*exp(1)^3*f*exp(2)^2-3*exp(1)^
6*f^2*exp(2)-6*exp(1)^4*f^2*exp(2)^2+exp(1)^2*f^2*exp(2)^3)/2/(2*d^4*exp(1)
^6-6*d^4*exp(1)^4*exp(2)+6*d^4*exp(1)^2*exp(2)^2-2*d^4*exp(2)^3)/exp(1)/abs
(d)/exp(1)^2*ln(abs(2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^4-2*(exp(1)*x+d)^-1
/exp(1)*d^2*exp(1)^2*exp(2)+2*d*exp(1)*exp(2)-2*exp(1)*abs(d)*exp(1)^2)/abs
(2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^4-2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^
2*exp(2)+2*d*exp(1)*exp(2)+2*exp(1)*abs(d)*exp(1)^2))
```

maple [A] time = 0.02, size = 270, normalized size = 1.85

$$\frac{fg}{6\left(ex+d\right)^{3}de^{2}}-\frac{f^{2}}{12\left(ex+d\right)^{3}d^{2}e}-\frac{g^{2}}{12\left(ex+d\right)^{3}e^{3}}+\frac{g^{2}}{8\left(ex+d\right)^{2}de^{3}}-\frac{f^{2}}{8\left(ex+d\right)^{2}d^{3}e}-\frac{g^{2}}{16\left(ex-d\right)d^{2}e^{3}}+\frac{g^{2}}{16\left(ex+d\right)d^{2}e^{3}}-\frac{fg}{8\left(ex-d\right)d^{3}e^{2}}-\frac{fg}{8\left(ex+d\right)d^{3}e^{2}}-\frac{f^{2}}{16\left(ex-d\right)d^{4}e}-\frac{3f^{2}}{16\left(ex+d\right)d^{4}e}-\frac{fg\ln\left(ex-d\right)}{8d^{4}e^{2}}+\frac{fg\ln\left(ex+d\right)}{8d^{4}e^{2}}-\frac{f^{2}\ln\left(ex-d\right)}{8d^{5}e}+\frac{f^{2}\ln\left(ex+d\right)}{8d^{5}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/16/e^3/d^2/(e*x-d)*g^2-1/8/e^2/d^3/(e*x-d)*f*g-1/16/e/d^4/(e*x-d)*f^2-1/
8/d^4/e^2*f*g*ln(e*x-d)-1/8/d^5/e*f^2*ln(e*x-d)+1/8/(e*x+d)^2/d/e^3*g^2-1/8
/(e*x+d)^2/d^3/e*f^2+1/16/(e*x+d)/d^2/e^3*g^2-1/8/(e*x+d)/d^3/e^2*f*g-3/16/
(e*x+d)/d^4/e*f^2-1/12/(e*x+d)^3/e^3*g^2+1/6/(e*x+d)^3/d/e^2*f*g-1/12/(e*x+
d)^3/d^2/e*f^2+1/8/d^4/e^2*f*g*ln(e*x+d)+1/8/d^5/e*f^2*ln(e*x+d)
```

maxima [A] time = 0.48, size = 197, normalized size = 1.35

$$\frac{4d^{3}e^{2}f^{2}-2d^{4}efg-2d^{5}g^{2}-3\left(e^{5}f^{2}+de^{4}fg\right)x^{3}-6\left(de^{4}f^{2}+d^{2}e^{3}fg\right)x^{2}-\left(d^{2}e^{3}f^{2}+d^{3}e^{2}fg+4d^{4}eg^{2}\right)x}{12\left(d^{4}e^{7}x^{4}+2d^{5}e^{6}x^{3}-2d^{7}e^{4}x-d^{8}e^{3}\right)}+\frac{\left(ef^{2}+dfg\right)\log\left(ex+d\right)}{8d^{5}e^{2}}-\frac{\left(ef^{2}+dfg\right)\log\left(ex-d\right)}{8d^{5}e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(4*d^3*e^2*f^2 - 2*d^4*e*f*g - 2*d^5*g^2 - 3*(e^5*f^2 + d*e^4*f*g)*x^3
 - 6*(d*e^4*f^2 + d^2*e^3*f*g)*x^2 - (d^2*e^3*f^2 + d^3*e^2*f*g + 4*d^4*e*g
^2)*x)/(d^4*e^7*x^4 + 2*d^5*e^6*x^3 - 2*d^7*e^4*x - d^8*e^3) + 1/8*(e*f^2 +
 d*f*g)*log(e*x + d)/(d^5*e^2) - 1/8*(e*f^2 + d*f*g)*log(e*x - d)/(d^5*e^2)
```

mupad [B] time = 2.63, size = 148, normalized size = 1.01

$$\frac{\frac{d^2\,g^2+d\,e\,f\,g-2\,e^2\,f^2}{6\,d\,e^3}+\frac{f\,x^2\,\left(d\,g+e\,f\right)}{2\,d^3}+\frac{x\,\left(4\,d^2\,g^2+d\,e\,f\,g+e^2\,f^2\right)}{12\,d^2\,e^2}+\frac{e\,f\,x^3\,\left(d\,g+e\,f\right)}{4\,d^4}}{d^4+2\,d^3\,e\,x-2\,d\,e^3\,x^3-e^4\,x^4}+\frac{f\,\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(d\,g+e\,f\right)}{4\,d^5\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - 2*e^2*f^2 + d*e*f*g)/(6*d*e^3) + (f*x^2*(d*g + e*f))/(2*d^3) +
(x*(4*d^2*g^2 + e^2*f^2 + d*e*f*g))/(12*d^2*e^2) + (e*f*x^3*(d*g + e*f))/(4
*d^4))/(d^4 - e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x) + (f*atanh((e*x)/d)*(d*g +
 e*f))/(4*d^5*e^2)
```

sympy [A] time = 1.36, size = 241, normalized size = 1.65

$$\frac{-2d^{5}g^{2}-2d^{4}efg+4d^{3}e^{2}f^{2}+x^{3}\left(-3de^{4}fg-3e^{5}f^{2}\right)+x^{2}\left(-6d^{2}e^{3}fg-6de^{4}f^{2}\right)+x\left(-4d^{4}eg^{2}-d^{3}e^{2}fg-d^{2}e^{3}f^{2}\right)}{-12d^{8}e^{3}-24d^{7}e^{4}x+24d^{5}e^{6}x^{3}+12d^{4}e^{7}x^{4}}-\frac{f\left(dg+ef\right)\log\left(-\frac{df\left(dg+ef\right)}{e\left(dfg+ef^{2}\right)}+x\right)}{8d^{5}e^{2}}+\frac{f\left(dg+ef\right)\log\left(\frac{df\left(dg+ef\right)}{e\left(dfg+ef^{2}\right)}+x\right)}{8d^{5}e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**5*g**2 - 2*d**4*e*f*g + 4*d**3*e**2*f**2 + x**3*(-3*d*e**4*f*g - 3*e
**5*f**2) + x**2*(-6*d**2*e**3*f*g - 6*d*e**4*f**2) + x*(-4*d**4*e*g**2 - d
```

```
**3*e**2*f*g - d**2*e**3*f**2))/(-12*d**8*e**3 - 24*d**7*e**4*x + 24*d**5*e
**6*x**3 + 12*d**4*e**7*x**4) - f*(d*g + e*f)*log(-d*f*(d*g + e*f)/(e*(d*f*
g + e*f**2)) + x)/(8*d**5*e**2) + f*(d*g + e*f)*log(d*f*(d*g + e*f)/(e*(d*f
*g + e*f**2)) + x)/(8*d**5*e**2)
```

3.369 $\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=178

$$\frac{(dg+ef)(dg+5ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^6e^3}+\frac{(dg+ef)^2}{32d^5e^3(d-ex)}-\frac{f(dg+ef)}{8d^5e^2(d+ex)}-\frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2 \cdots}{12d^3e^3(\cdots}$$

Rubi [A] time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}+\frac{(dg+ef)^2}{32d^5e^3(d-ex)}-\frac{f(dg+ef)}{8d^5e^2(d+ex)}-\frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2}+\frac{(dg+ef)(dg+5ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^6e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(32*d^5*e^3*(d - e*x)) - (e*f - d*g)^2/(16*d^2*e^3*(d + e*x)^
4) - (e^2*f^2 - d^2*g^2)/(12*d^3*e^3*(d + e*x)^3) - ((3*e*f - d*g)*(e*f + d
*g))/(32*d^4*e^3*(d + e*x)^2) - (f*(e*f + d*g))/(8*d^5*e^2*(d + e*x)) + ((e
*f + d*g)*(5*e*f + d*g)*ArcTanh[(e*x)/d])/(32*d^6*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^5}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{32d^5e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^5}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^4}+\frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^3}+\cdots\right.\\
&= \frac{(ef+dg)^2}{32d^5e^3(d-ex)}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2}-\frac{f(\cdots}{8d^5\cdots}\\
&= \frac{(ef+dg)^2}{32d^5e^3(d-ex)}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2}-\frac{f(\cdots}{8d^5\cdots}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 195, normalized size = 1.10

$$\frac{-\frac{12d^4(ef-dg)^2}{(d+ex)^4}+\frac{6d^2\left(d^2g^2-2defg-3e^2f^2\right)}{(d+ex)^2}-3\left(d^2g^2+6defg+5e^2f^2\right)\log(d-ex)+3\left(d^2g^2+6defg+5e^2f^2\right)\log(d+ex)+\frac{16d^3\left(d^2g^2-e^2f^2\right)}{(d+ex)^3}+\frac{6d(dg+ef)^2}{d-ex}-\frac{24def(dg+ef)}{d+ex}}{192d^6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((6*d*(e*f + d*g)^2)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (16*d
^3*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^3 + (6*d^2*(-3*e^2*f^2 - 2*d*e*f*g + d
^2*g^2))/(d + e*x)^2 - (24*d*e*f*(e*f + d*g))/(d + e*x) - 3*(5*e^2*f^2 + 6*
d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*Log[d
 + e*x])/(192*d^6*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2), x]
```

fricas [B] time = 0.40, size = 648, normalized size = 3.64

$$\frac{64d^5e^2f^2-16d^7g^2-6\left(5de^6f^2+6d^2e^5fg+d^3e^4g^2\right)x^4-18\left(5d^2e^5f^2+6d^3e^4fg+d^4e^3g^2\right)x^3-14\left(5d^3e^4f^2+6d^4e^3fg+d^5e^2g^2\right)x^2+6\left(5d^4e^3f^2+6d^5e^2fg-7d^6eg^2\right)x-3\left(5d^5e^2f^2+6d^6efg+d^7g^2-\left(5e^7f^2+6de^6fg+d^2e^5g^2\right)x^5-3\left(5de^6f^2+6d^2e^5fg+d^3e^4g^2\right)x^4-2\left(5d^2e^5f^2+6d^3e^4fg+d^4e^3g^2\right)x^3+2\left(5d^3e^4f^2+6d^4e^3fg+d^5e^2g^2\right)x^2+3\left(5d^4e^3f^2+6d^5e^2fg+d^6eg^2\right)x\right)\log(ex+d)+3\left(5d^5e^2f^2+6d^6efg+d^7g^2-\left(5e^7f^2+6de^6fg+d^2e^5g^2\right)x^5-3\left(5de^6f^2+6d^2e^5fg+d^3e^4g^2\right)x^4-2\left(5d^2e^5f^2+6d^3e^4fg+d^4e^3g^2\right)x^3+2\left(5d^3e^4f^2+6d^4e^3fg+d^5e^2g^2\right)x^2+3\left(5d^4e^3f^2+6d^5e^2fg+d^6eg^2\right)x\right)\log(ex-d)}{192\left(d^6e^8x^5+3d^7e^7x^4+2d^8e^6x^3-2d^9e^5x^2-3d^{10}e^4x-d^{11}e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(64*d^5*e^2*f^2 - 16*d^7*g^2 - 6*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^3*e
^4*g^2)*x^4 - 18*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 14*(5*
d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 6*(5*d^4*e^3*f^2 + 6*d^5*e
^2*f*g - 7*d^6*e*g^2)*x - 3*(5*d^5*e^2*f^2 + 6*d^6*e*f*g + d^7*g^2 - (5*e^7
*f^2 + 6*d*e^6*f*g + d^2*e^5*g^2)*x^5 - 3*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^
3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 + 2*(5
*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3*f^2 + 6*d^5*
e^2*f*g + d^6*e*g^2)*x)*log(e*x + d) + 3*(5*d^5*e^2*f^2 + 6*d^6*e*f*g + d^7
*g^2 - (5*e^7*f^2 + 6*d*e^6*f*g + d^2*e^5*g^2)*x^5 - 3*(5*d*e^6*f^2 + 6*d^2
*e^5*f*g + d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^
2)*x^3 + 2*(5*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3
*f^2 + 6*d^5*e^2*f*g + d^6*e*g^2)*x)*log(e*x - d))/(d^6*e^8*x^5 + 3*d^7*e^7
*x^4 + 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 - 3*d^10*e^4*x - d^11*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-3*e
xp(2)^2*d^2*exp(1)*g^2+12*exp(2)^2*d*exp(1)^2*g*f-10*exp(2)^2*exp(1)^3*f^2-
8*exp(2)*d^2*exp(1)^3*g^2+12*exp(2)*d*exp(1)^4*g*f-2*exp(2)*exp(1)^5*f^2-d^
2*exp(1)^5*g^2)/(2*exp(2)^4*d^6-8*exp(2)^3*d^6*exp(1)^2+12*exp(2)^2*d^6*exp
(1)^4-8*exp(2)*d^6*exp(1)^6+2*d^6*exp(1)^8)*ln(abs(-x^2*exp(2)+d^2))+(exp(2
)^4*f^2-exp(2)^3*d^2*g^2+6*exp(2)^3*d*exp(1)*g*f-10*exp(2)^3*exp(1)^2*f^2-1
```

```
4*exp(2)^2*d^2*exp(1)^2*g^2+36*exp(2)^2*d*exp(1)^3*g*f-15*exp(2)^2*exp(1)^4
*f^2-9*exp(2)*d^2*exp(1)^4*g^2+6*exp(2)*d*exp(1)^5*g*f)*1/2/(2*exp(2)^4*d^5
-8*exp(2)^3*d^5*exp(1)^2+12*exp(2)^2*d^5*exp(1)^4-8*exp(2)*d^5*exp(1)^6+2*d
^5*exp(1)^8)/exp(1)/abs(d)*ln(abs(-2*x*exp(2)-2*exp(1)*abs(d))/abs(-2*x*exp
(2)+2*exp(1)*abs(d)))+(3*exp(2)^2*d^2*exp(1)^2*g^2-12*exp(2)^2*d*exp(1)^3*g
*f+10*exp(2)^2*exp(1)^4*f^2+8*exp(2)*d^2*exp(1)^4*g^2-12*exp(2)*d*exp(1)^5*
g*f+2*exp(2)*exp(1)^6*f^2+d^2*exp(1)^6*g^2)/(exp(2)^4*d^6*exp(1)-4*exp(2)^3
*d^6*exp(1)^3+6*exp(2)^2*d^6*exp(1)^5-4*exp(2)*d^6*exp(1)^7+d^6*exp(1)^9)*l
n(abs(x*exp(1)+d))-((-exp(2)^4*d*exp(1)^2*f^2-5*exp(2)^3*d^3*exp(1)^2*g^2+1
8*exp(2)^3*d^2*exp(1)^3*g*f-10*exp(2)^3*d*exp(1)^4*f^2-2*exp(2)^2*d^3*exp(1
)^4*g^2-12*exp(2)^2*d^2*exp(1)^5*g*f+11*exp(2)^2*d*exp(1)^6*f^2+7*exp(2)*d^
3*exp(1)^6*g^2-6*exp(2)*d^2*exp(1)^7*g*f)*x^3+(-2*exp(2)^4*d^2*exp(1)*f^2-7
*exp(2)^3*d^4*exp(1)*g^2+24*exp(2)^3*d^3*exp(1)^2*g*f-10*exp(2)^3*d^2*exp(1
)^3*f^2+exp(2)^2*d^4*exp(1)^3*g^2-24*exp(2)^2*d^3*exp(1)^4*g*f+14*exp(2)^2*
d^2*exp(1)^5*f^2+7*exp(2)*d^4*exp(1)^5*g^2-2*exp(2)*d^2*exp(1)^7*f^2-d^4*ex
p(1)^7*g^2)*x^2+(-exp(2)^4*d^3*f^2-exp(2)^3*d^5*g^2+2*exp(2)^3*d^4*exp(1)*g
*f+4*exp(2)^3*d^3*exp(1)^2*f^2+8*exp(2)^2*d^5*exp(1)^2*g^2-24*exp(2)^2*d^4*
exp(1)^3*g*f+7*exp(2)^2*d^3*exp(1)^4*f^2-exp(2)*d^5*exp(1)^4*g^2+18*exp(2)*
d^4*exp(1)^5*g*f-10*exp(2)*d^3*exp(1)^6*f^2-6*d^5*exp(1)^6*g^2+4*d^4*exp(1)
^7*g*f)*x-2*exp(2)^3*d^5*g*f+3*exp(2)^3*d^4*exp(1)*f^2+8*exp(2)^2*d^6*exp(1
)*g^2-18*exp(2)^2*d^5*exp(1)^2*g*f+7*exp(2)^2*d^4*exp(1)^3*f^2-4*exp(2)*d^6
*exp(1)^3*g^2+18*exp(2)*d^5*exp(1)^4*g*f-11*exp(2)*d^4*exp(1)^5*f^2-4*d^6*e
xp(1)^5*g^2+2*d^5*exp(1)^6*g*f+d^4*exp(1)^7*f^2)/2/d^6/(exp(2)-exp(1)^2)^4/
(-x*exp(1)-d)^2/(-x^2*exp(2)+d^2)
```

maple [B] time = 0.02, size = 341, normalized size = 1.92

$$\frac{fg}{8\left(ex+d\right)^{4}de^{2}}-\frac{f^{2}}{16\left(ex+d\right)^{4}d^{2}e}-\frac{g^{2}}{16\left(ex+d\right)^{4}e^{3}}+\frac{g^{2}}{12\left(ex+d\right)^{3}de^{3}}-\frac{f^{2}}{12\left(ex+d\right)^{3}d^{3}e}+\frac{g^{2}}{32\left(ex+d\right)^{2}d^{2}e^{3}}-\frac{fg}{16\left(ex+d\right)^{2}d^{3}e^{2}}-\frac{3f^{2}}{32\left(ex+d\right)^{2}d^{4}e}-\frac{g^{2}}{32\left(ex-d\right)d^{3}e^{3}}-\frac{fg}{16\left(ex-d\right)d^{4}e^{2}}-\frac{fg}{8\left(ex+d\right)d^{4}e^{2}}-\frac{g^{2}\ln\left(ex-d\right)}{64d^{4}e^{3}}+\frac{g^{2}\ln\left(ex+d\right)}{64d^{4}e^{3}}-\frac{f^{2}}{32\left(ex-d\right)d^{5}e}-\frac{f^{2}}{8\left(ex+d\right)d^{5}e}-\frac{3fg\ln\left(ex-d\right)}{32d^{5}e^{2}}+\frac{3fg\ln\left(ex+d\right)}{32d^{5}e^{2}}-\frac{5f^{2}\ln\left(ex-d\right)}{64d^{6}e}+\frac{5f^{2}\ln\left(ex+d\right)}{64d^{6}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/64/e^3/d^4*ln(e*x-d)*g^2-3/32/e^2/d^5*ln(e*x-d)*f*g-5/64/e/d^6*ln(e*x-d)
*f^2-1/32/e^3/d^3/(e*x-d)*g^2-1/16/e^2/d^4/(e*x-d)*f*g-1/32/e/d^5/(e*x-d)*f
^2+1/64/e^3/d^4*ln(e*x+d)*g^2+3/32/e^2/d^5*ln(e*x+d)*f*g+5/64/e/d^6*ln(e*x+
d)*f^2+1/12/e^3/d/(e*x+d)^3*g^2-1/12/e/d^3/(e*x+d)^3*f^2+1/32/e^3/d^2/(e*x+
d)^2*g^2-1/16/e^2/d^3/(e*x+d)^2*f*g-3/32/e/d^4/(e*x+d)^2*f^2-1/16/e^3/(e*x+
d)^4*g^2+1/8/e^2/d/(e*x+d)^4*f*g-1/16/e/d^2/(e*x+d)^4*f^2-1/8/e^2*f/d^4/(e*
x+d)*g-1/8/e*f^2/d^5/(e*x+d)
```

maxima [A] time = 0.50, size = 298, normalized size = 1.67

$$\frac{32\,d^{4}e^{2}f^{2}-8\,d^{6}g^{2}-3\left(5\,e^{6}f^{2}+6\,de^{5}fg+d^{2}e^{4}g^{2}\right)x^{4}-9\left(5\,de^{5}f^{2}+6\,d^{2}e^{4}fg+d^{3}e^{3}g^{2}\right)x^{3}-7\left(5\,d^{2}e^{4}f^{2}+6\,d^{3}e^{3}fg+d^{4}e^{2}g^{2}\right)x^{2}+3\left(5\,d^{3}e^{3}f^{2}+6\,d^{4}e^{2}fg-7\,d^{5}eg^{2}\right)x}{96\left(d^{5}e^{8}x^{5}+3\,d^{6}e^{7}x^{4}+2\,d^{7}e^{6}x^{3}-2\,d^{8}e^{5}x^{2}-3\,d^{9}e^{4}x-d^{10}e^{3}\right)}+\frac{\left(5\,e^{2}f^{2}+6\,defg+d^{2}g^{2}\right)\log\left(ex+d\right)}{64\,d^{6}e^{3}}-\frac{\left(5\,e^{2}f^{2}+6\,defg+d^{2}g^{2}\right)\log\left(ex-d\right)}{64\,d^{6}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(32*d^4*e^2*f^2 - 8*d^6*g^2 - 3*(5*e^6*f^2 + 6*d*e^5*f*g + d^2*e^4*g^2
)*x^4 - 9*(5*d*e^5*f^2 + 6*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 - 7*(5*d^2*e^4*f^
2 + 6*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 3*(5*d^3*e^3*f^2 + 6*d^4*e^2*f*g - 7
*d^5*e*g^2)*x)/(d^5*e^8*x^5 + 3*d^6*e^7*x^4 + 2*d^7*e^6*x^3 - 2*d^8*e^5*x^2
 - 3*d^9*e^4*x - d^10*e^3) + 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(e*x
 + d)/(d^6*e^3) - 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^6*
e^3)
```

mupad [B] time = 2.70, size = 274, normalized size = 1.54

$$\frac{\frac{d^2\,g^2-4\,e^2\,f^2}{12\,d\,e^3}+\frac{3\,x^3\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^4}+\frac{e\,x^4\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^5}-\frac{x\,\left(-7\,d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^2\,e^2}+\frac{7\,x^2\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{96\,d^3\,e}}{d^5+3\,d^4\,e\,x+2\,d^3\,e^2\,x^2-2\,d^2\,e^3\,x^3-3\,d\,e^4\,x^4-e^5\,x^5}+\frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g+e\,f\right)\,\left(d\,g+5\,e\,f\right)}{d\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}\right)\,\left(d\,g+e\,f\right)\,\left(d\,g+5\,e\,f\right)}{32\,d^6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^3),x)
```

```
[Out] ((d^2*g^2 - 4*e^2*f^2)/(12*d*e^3) + (3*x^3*(d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*g
))/(32*d^4) + (e*x^4*(d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*g))/(32*d^5) - (x*(5*e^
2*f^2 - 7*d^2*g^2 + 6*d*e*f*g))/(32*d^2*e^2) + (7*x^2*(d^2*g^2 + 5*e^2*f^2
+ 6*d*e*f*g))/(96*d^3*e))/(d^5 - e^5*x^5 - 3*d*e^4*x^4 + 2*d^3*e^2*x^2 - 2*
d^2*e^3*x^3 + 3*d^4*e*x) + (atanh((e*x*(d*g + e*f)*(d*g + 5*e*f))/(d*(d^2*g
^2 + 5*e^2*f^2 + 6*d*e*f*g)))*(d*g + e*f)*(d*g + 5*e*f))/(32*d^6*e^3)
```

sympy [B] time = 1.93, size = 376, normalized size = 2.11

$$\frac{-8d^6g^2 + 32d^4e^2f^2 + x^4\left(-3d^2e^4g^2 - 18de^5fg - 15e^6f^2\right) + x^3\left(-9d^3e^3g^2 - 54d^2e^4fg - 45de^5f^2\right) + x^2\left(-7d^4e^2g^2 - 42d^3e^3fg - 35d^2e^4f^2\right) + x\left(-21d^5eg^2 + 18d^4e^2fg + 15d^3e^3f^2\right)}{-96d^{10}e^3 - 288d^9e^4x - 192d^8e^5x^2 + 192d^7e^6x^3 + 288d^6e^7x^4 + 96d^5e^8x^5} - \frac{\left(dg + ef\right)\left(dg + 5ef\right)\log\left(-\frac{d\left(dg+ef\right)\left(dg+5ef\right)}{e\left(d^2g^2+6defg+5e^2f^2\right)} + x\right)}{64d^6e^3} + \frac{\left(dg + ef\right)\left(dg + 5ef\right)\log\left(\frac{d\left(dg+ef\right)\left(dg+5ef\right)}{e\left(d^2g^2+6defg+5e^2f^2\right)} + x\right)}{64d^6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-8*d**6*g**2 + 32*d**4*e**2*f**2 + x**4*(-3*d**2*e**4*g**2 - 18*d*e**5*f*g
 - 15*e**6*f**2) + x**3*(-9*d**3*e**3*g**2 - 54*d**2*e**4*f*g - 45*d*e**5*f
**2) + x**2*(-7*d**4*e**2*g**2 - 42*d**3*e**3*f*g - 35*d**2*e**4*f**2) + x*
(-21*d**5*e*g**2 + 18*d**4*e**2*f*g + 15*d**3*e**3*f**2))/(-96*d**10*e**3 -
 288*d**9*e**4*x - 192*d**8*e**5*x**2 + 192*d**7*e**6*x**3 + 288*d**6*e**7*
x**4 + 96*d**5*e**8*x**5) - (d*g + e*f)*(d*g + 5*e*f)*log(-d*(d*g + e*f)*(d
*g + 5*e*f)/(e*(d**2*g**2 + 6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3) +
 (d*g + e*f)*(d*g + 5*e*f)*log(d*(d*g + e*f)*(d*g + 5*e*f)/(e*(d**2*g**2 +
6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3)
```

3.370 $\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{(dg+ef)(dg+3ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^7e^3}+\frac{(dg+ef)^2}{64d^6e^3(d-ex)}-\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)}-\frac{f(dg+ef)}{16d^5e^2(d+ex)^2}-\frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3}-$$

Rubi [A] time = 0.24, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}+\frac{(dg+ef)^2}{64d^6e^3(d-ex)}-\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)}-\frac{f(dg+ef)}{16d^5e^2(d+ex)^2}-\frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3}+\frac{(dg+ef)(dg+3ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^7e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(64*d^6*e^3*(d - e*x)) - (e*f - d*g)^2/(20*d^2*e^3*(d + e*x)^
5) - (e^2*f^2 - d^2*g^2)/(16*d^3*e^3*(d + e*x)^4) - ((3*e*f - d*g)*(e*f + d
*g))/(48*d^4*e^3*(d + e*x)^3) - (f*(e*f + d*g))/(16*d^5*e^2*(d + e*x)^2) -
((e*f + d*g)*(5*e*f + d*g))/(64*d^6*e^3*(d + e*x)) + ((e*f + d*g)*(3*e*f +
d*g)*ArcTanh[(e*x)/d])/(32*d^7*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^6}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{64d^6e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^6}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^5}+\frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^4}+\right.\\
&= \frac{(ef+dg)^2}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3}-\frac{f}{16d^5}\\
&= \frac{(ef+dg)^2}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3}-\frac{f}{16d^5}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 229, normalized size = 1.09

$$\frac{-\frac{48d^5(ef-dg)^2}{(d+ex)^5}-\frac{15d\left(d^2g^2+6defg+5e^2f^2\right)}{d+ex}-15\left(d^2g^2+4defg+3e^2f^2\right)\log(d-ex)+15\left(d^2g^2+4defg+3e^2f^2\right)\log(d+ex)-\frac{60d^2ef(dg+ef)}{(d+ex)^2}+\frac{60d^4\left(d^2g^2-e^2f^2\right)}{(d+ex)^4}+\frac{20d^3\left(d^2g^2-2defg-3e^2f^2\right)}{(d+ex)^3}+\frac{15d(dg+ef)^2}{d-ex}}{960d^7e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((15*d*(e*f + d*g)^2)/(d - e*x) - (48*d^5*(e*f - d*g)^2)/(d + e*x)^5 + (60*
d^4*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^4 + (20*d^3*(-3*e^2*f^2 - 2*d*e*f*g +
 d^2*g^2))/(d + e*x)^3 - (60*d^2*e*f*(e*f + d*g))/(d + e*x)^2 - (15*d*(5*e^
2*f^2 + 6*d*e*f*g + d^2*g^2))/(d + e*x) - 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g
^2)*Log[d - e*x] + 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*Log[d + e*x])/(960*
d^7*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2), x]
```

fricas [B] time = 0.40, size = 693, normalized size = 3.30

$$\frac{288d^6e^2f^2+64d^7efg-32d^8g^2-30\left(3de^7f^2+4d^2e^6fg+d^3e^5g^2\right)x^5-120\left(3d^2e^6f^2+4d^3e^5fg+d^4e^4g^2\right)x^4-160\left(3d^3e^5f^2+4d^4e^4fg+d^5e^3g^2\right)x^3-40\left(3d^4e^4f^2+4d^5e^3fg+d^6e^2g^2\right)x^2+2\left(141d^5e^3f^2+188d^6e^2fg-49d^7eg^2\right)x-15\left(3d^6e^2f^2+4d^7efg+d^8g^2-\left(3e^8f^2+4de^7fg+d^2e^6g^2\right)x^6-4\left(3de^7f^2+4d^2e^6fg+d^3e^5g^2\right)x^5-5\left(3d^2e^6f^2+4d^3e^5fg+d^4e^4g^2\right)x^4+5\left(3d^4e^4f^2+4d^5e^3fg+d^6e^2g^2\right)x^2+4\left(3d^5e^3f^2+4d^6e^2fg+d^7eg^2\right)x\right)\log(ex+d)+15\left(3d^6e^2f^2+4d^7efg+d^8g^2-\left(3e^8f^2+4de^7fg+d^2e^6g^2\right)x^6-4\left(3de^7f^2+4d^2e^6fg+d^3e^5g^2\right)x^5-5\left(3d^2e^6f^2+4d^3e^5fg+d^4e^4g^2\right)x^4+5\left(3d^4e^4f^2+4d^5e^3fg+d^6e^2g^2\right)x^2+4\left(3d^5e^3f^2+4d^6e^2fg+d^7eg^2\right)x\right)\log(ex-d)}{960\left(d^7e^9x^6+4d^8e^8x^5+5d^9e^7x^4-5d^{11}e^5x^2-4d^{12}e^4x-d^{13}e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/960*(288*d^6*e^2*f^2 + 64*d^7*e*f*g - 32*d^8*g^2 - 30*(3*d*e^7*f^2 + 4*d^
2*e^6*f*g + d^3*e^5*g^2)*x^5 - 120*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4
*g^2)*x^4 - 160*(3*d^3*e^5*f^2 + 4*d^4*e^4*f*g + d^5*e^3*g^2)*x^3 - 40*(3*d
^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 2*(141*d^5*e^3*f^2 + 188*d^
6*e^2*f*g - 49*d^7*e*g^2)*x - 15*(3*d^6*e^2*f^2 + 4*d^7*e*f*g + d^8*g^2 - (
3*e^8*f^2 + 4*d*e^7*f*g + d^2*e^6*g^2)*x^6 - 4*(3*d*e^7*f^2 + 4*d^2*e^6*f*g
 + d^3*e^5*g^2)*x^5 - 5*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4*g^2)*x^4 +
 5*(3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 4*(3*d^5*e^3*f^2 + 4
*d^6*e^2*f*g + d^7*e*g^2)*x)*log(e*x + d) + 15*(3*d^6*e^2*f^2 + 4*d^7*e*f*g
 + d^8*g^2 - (3*e^8*f^2 + 4*d*e^7*f*g + d^2*e^6*g^2)*x^6 - 4*(3*d*e^7*f^2 +
 4*d^2*e^6*f*g + d^3*e^5*g^2)*x^5 - 5*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*
e^4*g^2)*x^4 + 5*(3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 4*(3*d
^5*e^3*f^2 + 4*d^6*e^2*f*g + d^7*e*g^2)*x)*log(e*x - d))/(d^7*e^9*x^6 + 4*d
^8*e^8*x^5 + 5*d^9*e^7*x^4 - 5*d^11*e^5*x^2 - 4*d^12*e^4*x - d^13*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-2*e
xp(2)^3*d^2*exp(1)*g^2+10*exp(2)^3*d*exp(1)^2*g*f-10*exp(2)^3*exp(1)^3*f^2-
10*exp(2)^2*d^2*exp(1)^3*g^2+20*exp(2)^2*d*exp(1)^4*g*f-6*exp(2)^2*exp(1)^5
*f^2-4*exp(2)*d^2*exp(1)^5*g^2+2*exp(2)*d*exp(1)^6*g*f)/(exp(2)^5*d^7-5*exp
```

```
(2)^4*d^7*exp(1)^2+10*exp(2)^3*d^7*exp(1)^4-10*exp(2)^2*d^7*exp(1)^6+5*exp(
2)*d^7*exp(1)^8-d^7*exp(1)^10)*ln(abs(-x^2*exp(2)+d^2))+(exp(2)^5*f^2-exp(2
)^4*d^2*g^2+8*exp(2)^4*d*exp(1)*g*f-15*exp(2)^4*exp(1)^2*f^2-25*exp(2)^3*d^
2*exp(1)^2*g^2+80*exp(2)^3*d*exp(1)^3*g*f-45*exp(2)^3*exp(1)^4*f^2-35*exp(2
)^2*d^2*exp(1)^4*g^2+40*exp(2)^2*d*exp(1)^5*g*f-5*exp(2)^2*exp(1)^6*f^2-3*e
xp(2)*d^2*exp(1)^6*g^2)*1/2/(2*exp(2)^5*d^6-10*exp(2)^4*d^6*exp(1)^2+20*exp
(2)^3*d^6*exp(1)^4-20*exp(2)^2*d^6*exp(1)^6+10*exp(2)*d^6*exp(1)^8-2*d^6*ex
p(1)^10)/exp(1)/abs(d)*ln(abs(-2*x*exp(2)-2*exp(1)*abs(d))/abs(-2*x*exp(2)+
2*exp(1)*abs(d)))+(4*exp(2)^3*d^2*exp(1)^2*g^2-20*exp(2)^3*d*exp(1)^3*g*f+2
0*exp(2)^3*exp(1)^4*f^2+20*exp(2)^2*d^2*exp(1)^4*g^2-40*exp(2)^2*d*exp(1)^5
*g*f+12*exp(2)^2*exp(1)^6*f^2+8*exp(2)*d^2*exp(1)^6*g^2-4*exp(2)*d*exp(1)^7
*g*f)/(exp(2)^5*d^7*exp(1)-5*exp(2)^4*d^7*exp(1)^3+10*exp(2)^3*d^7*exp(1)^5
-10*exp(2)^2*d^7*exp(1)^7+5*exp(2)*d^7*exp(1)^9-d^7*exp(1)^11)*ln(abs(x*exp
(1)+d))-((-3*exp(2)^5*d*exp(1)^3*f^2-21*exp(2)^4*d^3*exp(1)^3*g^2+96*exp(2)
^4*d^2*exp(1)^4*g*f-75*exp(2)^4*d*exp(1)^5*f^2-45*exp(2)^3*d^3*exp(1)^5*g^2
+63*exp(2)^3*d*exp(1)^7*f^2+57*exp(2)^2*d^3*exp(1)^7*g^2-96*exp(2)^2*d^2*ex
p(1)^8*g*f+15*exp(2)^2*d*exp(1)^9*f^2+9*exp(2)*d^3*exp(1)^9*g^2)*x^4+(-9*ex
p(2)^5*d^2*exp(1)^2*f^2-51*exp(2)^4*d^4*exp(1)^2*g^2+228*exp(2)^4*d^3*exp(1
)^3*g*f-165*exp(2)^4*d^2*exp(1)^4*f^2-87*exp(2)^3*d^4*exp(1)^4*g^2-60*exp(2
)^3*d^3*exp(1)^5*g*f+165*exp(2)^3*d^2*exp(1)^6*f^2+135*exp(2)^2*d^4*exp(1)^
6*g^2-180*exp(2)^2*d^3*exp(1)^7*g*f+9*exp(2)^2*d^2*exp(1)^8*f^2+3*exp(2)*d^
4*exp(1)^8*g^2+12*exp(2)*d^3*exp(1)^9*g*f)*x^3+(-9*exp(2)^5*d^3*exp(1)*f^2-
35*exp(2)^4*d^5*exp(1)*g^2+148*exp(2)^4*d^4*exp(1)^2*g*f-83*exp(2)^4*d^3*ex
p(1)^3*f^2-9*exp(2)^3*d^5*exp(1)^3*g^2-204*exp(2)^3*d^4*exp(1)^4*g*f+183*ex
p(2)^3*d^3*exp(1)^5*f^2+117*exp(2)^2*d^5*exp(1)^5*g^2-36*exp(2)^2*d^4*exp(1
)^6*g*f-81*exp(2)^2*d^3*exp(1)^7*f^2-67*exp(2)*d^5*exp(1)^7*g^2+92*exp(2)*d
^4*exp(1)^8*g*f-10*exp(2)*d^3*exp(1)^9*f^2-6*d^5*exp(1)^9*g^2)*x^2+(-3*exp(
2)^5*d^4*f^2-3*exp(2)^4*d^6*g^2+6*exp(2)^4*d^5*exp(1)*g*f+21*exp(2)^4*d^4*e
xp(1)^2*f^2+63*exp(2)^3*d^6*exp(1)^2*g^2-252*exp(2)^3*d^5*exp(1)^3*g*f+147*
exp(2)^3*d^4*exp(1)^4*f^2+69*exp(2)^2*d^6*exp(1)^4*g^2+96*exp(2)^2*d^5*exp(
1)^5*g*f-153*exp(2)^2*d^4*exp(1)^6*f^2-123*exp(2)*d^6*exp(1)^6*g^2+156*exp(
2)*d^5*exp(1)^7*g*f-12*exp(2)*d^4*exp(1)^8*f^2-6*d^6*exp(1)^8*g^2-6*d^5*exp
(1)^9*g*f)*x-6*exp(2)^4*d^6*g*f+12*exp(2)^4*d^5*exp(1)*f^2+38*exp(2)^3*d^7*
exp(1)*g^2-124*exp(2)^3*d^6*exp(1)^2*g*f+74*exp(2)^3*d^5*exp(1)^3*f^2+18*ex
p(2)^2*d^7*exp(1)^3*g^2+72*exp(2)^2*d^6*exp(1)^4*g*f-90*exp(2)^2*d^5*exp(1)
^5*f^2-54*exp(2)*d^7*exp(1)^5*g^2+60*exp(2)*d^6*exp(1)^6*g*f+6*exp(2)*d^5*e
xp(1)^7*f^2-2*d^7*exp(1)^7*g^2-2*d^6*exp(1)^8*g*f-2*d^5*exp(1)^9*f^2)/6/d^7
/(exp(2)-exp(1)^2)^5/(-x*exp(1)-d)^3/(x^2*exp(2)-d^2)
```

maple [B] time = 0.02, size = 394, normalized size = 1.88

$$\frac{fg}{10\,(ex+d)^5\,d\,e^2}-\frac{f^2}{20\,(ex+d)^5\,d^2e}-\frac{g^2}{20\,(ex+d)^5\,e^3}+\frac{g^2}{16\,(ex+d)^4\,d\,e^3}-\frac{f^2}{16\,(ex+d)^4\,d^3e}+\frac{g^2}{48\,(ex+d)^3\,d^2e^3}-\frac{fg}{24\,(ex+d)^3\,d^3e^2}-\frac{f^2}{16\,(ex+d)^3\,d^4e}-\frac{fg}{16\,(ex+d)^2\,d^4e^2}-\frac{f^2}{16\,(ex+d)^2\,d^5e}-\frac{g^2}{64\,(ex-d)\,d^4e^3}-\frac{g^2}{64\,(ex+d)\,d^4e^3}-\frac{fg}{32\,(ex-d)\,d^5e^2}-\frac{3fg}{32\,(ex+d)\,d^5e^2}-\frac{g^2\ln(ex-d)}{64d^5e^3}+\frac{g^2\ln(ex+d)}{64d^5e^3}-\frac{f^2}{64\,(ex-d)\,d^6e}-\frac{5f^2}{64\,(ex+d)\,d^6e}-\frac{fg\ln(ex-d)}{16d^6e^2}+\frac{fg\ln(ex+d)}{16d^6e^2}-\frac{3f^2\ln(ex-d)}{64d^7e}+\frac{3f^2\ln(ex+d)}{64d^7e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/64/e^3/d^5*ln(e*x-d)*g^2-1/16/e^2/d^6*ln(e*x-d)*f*g-3/64/e/d^7*ln(e*x-d)
*f^2-1/64/e^3/d^4/(e*x-d)*g^2-1/32/e^2/d^5/(e*x-d)*f*g-1/64/e/d^6/(e*x-d)*f
^2+1/64/e^3/d^5*ln(e*x+d)*g^2+1/16/e^2/d^6*ln(e*x+d)*f*g+3/64/e/d^7*ln(e*x+
d)*f^2-1/64/e^3/d^4/(e*x+d)*g^2-3/32/e^2/d^5/(e*x+d)*f*g-5/64/e/d^6/(e*x+d)
*f^2+1/16/e^3/d/(e*x+d)^4*g^2-1/16/e/d^3/(e*x+d)^4*f^2+1/48/e^3/d^2/(e*x+d)
^3*g^2-1/24/e^2/d^3/(e*x+d)^3*f*g-1/16/e/d^4/(e*x+d)^3*f^2-1/20/e^3/(e*x+d)
^5*g^2+1/10/e^2/d/(e*x+d)^5*f*g-1/20/e/d^2/(e*x+d)^5*f^2-1/16/e^2*f/d^4/(e*
x+d)^2*g-1/16/e*f^2/d^5/(e*x+d)^2
```

maxima [A] time = 0.53, size = 342, normalized size = 1.63

$$\frac{144\,d^5e^2f^2+32\,d^6efg-16\,d^7g^2-15\left(3\,e^7f^2+4\,de^6fg+d^2e^5g^2\right)x^5-60\left(3\,de^6f^2+4\,d^2e^5fg+d^3e^4g^2\right)x^4-80\left(3\,d^2e^5f^2+4\,d^3e^4fg+d^4e^3g^2\right)x^3-20\left(3\,d^3e^4f^2+4\,d^4e^3fg+d^5e^2g^2\right)x^2+\left(141\,d^4e^3f^2+188\,d^5e^2fg-49\,d^6eg^2\right)x}{480\left(d^6e^9x^6+4\,d^7e^8x^5+5\,d^8e^7x^4-5\,d^{10}e^5x^2-4\,d^{11}e^4x-d^{12}e^3\right)}+\frac{\left(3\,e^2f^2+4\,defg+d^2g^2\right)\log\left(ex+d\right)}{64\,d^7e^3}-\frac{\left(3\,e^2f^2+4\,defg+d^2g^2\right)\log\left(ex-d\right)}{64\,d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/480*(144*d^5*e^2*f^2 + 32*d^6*e*f*g - 16*d^7*g^2 - 15*(3*e^7*f^2 + 4*d*e^
6*f*g + d^2*e^5*g^2)*x^5 - 60*(3*d*e^6*f^2 + 4*d^2*e^5*f*g + d^3*e^4*g^2)*x
^4 - 80*(3*d^2*e^5*f^2 + 4*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 20*(3*d^3*e^4*f
^2 + 4*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + (141*d^4*e^3*f^2 + 188*d^5*e^2*f*g
- 49*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 4*d^7*e^8*x^5 + 5*d^8*e^7*x^4 - 5*d^10*e^
5*x^2 - 4*d^11*e^4*x - d^12*e^3) + 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*l
og(e*x + d)/(d^7*e^3) - 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*log(e*x - d)
/(d^7*e^3)
```

mupad [B] time = 2.72, size = 314, normalized size = 1.50

$$\frac{\frac{x^3\left(d^2g^2+4defg+3e^2f^2\right)}{6d^4}-\frac{-d^2g^2+2defg+9e^2f^2}{30de^3}+\frac{ex^4\left(d^2g^2+4defg+3e^2f^2\right)}{8d^5}-\frac{x\left(-49d^2g^2+188defg+141e^2f^2\right)}{480d^2e^2}+\frac{x^2\left(d^2g^2+4defg+3e^2f^2\right)}{24d^3e}+\frac{e^2x^5\left(d^2g^2+4defg+3e^2f^2\right)}{32d^6}}{d^6+4d^5ex+5d^4e^2x^2-5d^2e^4x^4-4de^5x^5-e^6x^6}+\frac{\mathrm{atanh}\left(\frac{ex\left(dg+ef\right)\left(dg+3ef\right)}{d\left(d^2g^2+4defg+3e^2f^2\right)}\right)\left(dg+ef\right)\left(dg+3ef\right)}{32d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^4),x)
```

```
[Out] ((x^3*(d^2*g^2 + 3*e^2*f^2 + 4*d*e*f*g))/(6*d^4) - (9*e^2*f^2 - d^2*g^2 + 2
*d*e*f*g)/(30*d*e^3) + (e*x^4*(d^2*g^2 + 3*e^2*f^2 + 4*d*e*f*g))/(8*d^5) -
(x*(141*e^2*f^2 - 49*d^2*g^2 + 188*d*e*f*g))/(480*d^2*e^2) + (x^2*(d^2*g^2
+ 3*e^2*f^2 + 4*d*e*f*g))/(24*d^3*e) + (e^2*x^5*(d^2*g^2 + 3*e^2*f^2 + 4*d*
e*f*g))/(32*d^6))/(d^6 - e^6*x^6 - 4*d*e^5*x^5 + 5*d^4*e^2*x^2 - 5*d^2*e^4*
x^4 + 4*d^5*e*x) + (atanh((e*x*(d*g + e*f)*(d*g + 3*e*f))/(d*(d^2*g^2 + 3*e
^2*f^2 + 4*d*e*f*g)))*(d*g + e*f)*(d*g + 3*e*f))/(32*d^7*e^3)
```

sympy [B] time = 2.15, size = 427, normalized size = 2.03

$$\frac{-16d^7g^2+32d^6efg+144d^5e^2f^2+x^5\left(-15d^2e^5g^2-60de^6fg-45e^7f^2\right)+x^4\left(-60d^3e^4g^2-240d^2e^5fg-180de^6f^2\right)+x^3\left(-80d^4e^3g^2-320d^3e^4fg-240d^2e^5f^2\right)+x^2\left(-20d^5e^2g^2-80d^4e^3fg-60d^3e^4f^2\right)+x\left(-49d^6eg^2+188d^5e^2fg+141d^4e^3f^2\right)}{-480d^{12}e^3-1920d^{11}e^4x-2400d^{10}e^5x^2+2400d^8e^7x^4+1920d^7e^8x^5+480d^6e^9x^6}-\frac{\left(dg+ef\right)\left(dg+3ef\right)\log\left(-\frac{d\left(dg+ef\right)\left(dg+3ef\right)}{e\left(d^2g^2+4defg+3e^2f^2\right)}+x\right)}{64d^7e^3}+\frac{\left(dg+ef\right)\left(dg+3ef\right)\log\left(\frac{d\left(dg+ef\right)\left(dg+3ef\right)}{e\left(d^2g^2+4defg+3e^2f^2\right)}+x\right)}{64d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-16*d**7*g**2 + 32*d**6*e*f*g + 144*d**5*e**2*f**2 + x**5*(-15*d**2*e**5*g
**2 - 60*d*e**6*f*g - 45*e**7*f**2) + x**4*(-60*d**3*e**4*g**2 - 240*d**2*e
**5*f*g - 180*d*e**6*f**2) + x**3*(-80*d**4*e**3*g**2 - 320*d**3*e**4*f*g -
 240*d**2*e**5*f**2) + x**2*(-20*d**5*e**2*g**2 - 80*d**4*e**3*f*g - 60*d**
3*e**4*f**2) + x*(-49*d**6*e*g**2 + 188*d**5*e**2*f*g + 141*d**4*e**3*f**2)
)/(-480*d**12*e**3 - 1920*d**11*e**4*x - 2400*d**10*e**5*x**2 + 2400*d**8*e
**7*x**4 + 1920*d**7*e**8*x**5 + 480*d**6*e**9*x**6) - (d*g + e*f)*(d*g + 3
*e*f)*log(-d*(d*g + e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f
**2)) + x)/(64*d**7*e**3) + (d*g + e*f)*(d*g + 3*e*f)*log(d*(d*g + e*f)*(d*
g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f**2)) + x)/(64*d**7*e**3)
```

3.371 $\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=179

$$\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2}-\frac{32d^3(dg+ef)(2dg+ef)}{e^3(d-ex)}-\frac{dx\left(56d^2g^2+48defg+7e^2f^2\right)}{e^2}-\frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)\log(d$$

Rubi [A] time = 0.24, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{dx\left(56d^2g^2+48defg+7e^2f^2\right)}{e^2}-\frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2}-\frac{32d^3(dg+ef)(2dg+ef)}{e^3(d-ex)}-\frac{1}{3}gx^3(7dg+2ef)-\frac{x^2(2dg+ef)(12dg+ef)}{2e}-\frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e*f + 2*d*g)*(e*f +
12*d*g)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*d^4*(e*
f + d*g)^2)/(e^3*(d - e*x)^2) - (32*d^3*(e*f + d*g)*(e*f + 2*d*g))/(e^3*(d
- e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx=\int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^3}\,dx$$

$$=\int \left(-\frac{d\left(7e^2f^2+48defg+56d^2g^2\right)}{e^2}+\frac{(-ef-12dg)(ef+2dg)x}{e}-g(2ef+7dg)x^2-e$$

$$=-\frac{d\left(7e^2f^2+48defg+56d^2g^2\right)x}{e^2}-\frac{(ef+2dg)(ef+12dg)x^2}{2e}-\frac{1}{3}g(2ef+7dg)x^3-\frac{1}{4}e$$

Mathematica [A] time = 0.10, size = 193, normalized size = 1.08

$$\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2}-\frac{x^2\left(24d^2g^2+14defg+e^2f^2\right)}{2e}-\frac{dx\left(56d^2g^2+48defg+7e^2f^2\right)}{e^2}-\frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{32d^3\left(2d^2g^2+3defg+e^2f^2\right)}{e^3(ex-d)}-\frac{1}{3}gx^3(7dg+2ef)-\frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e^2*f^2 + 14*d*e*f*g
 + 24*d^2*g^2)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*
d^4*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2
*g^2))/(e^3*(-d + e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[
d - e*x])/e^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3, x]
```

fricas [A] time = 0.38, size = 336, normalized size = 1.88

$$-\frac{3\,e^6g^2x^6+288\,d^4e^2f^2+960\,d^5efg+672\,d^6g^2+2\left(4\,e^6fg+11\,de^5g^2\right)x^5+\left(6\,e^6f^2+68\,de^5fg+91\,d^2e^4g^2\right)x^4+4\left(18\,de^5f^2+104\,d^2e^4fg+103\,d^3e^3g^2\right)x^3-6\left(27\,d^2e^4f^2+178\,d^3e^3fg+200\,d^4e^2g^2\right)x^2-12\left(25\,d^3e^3f^2+48\,d^4e^2fg+8\,d^5eg^2\right)x+96\left(3\,d^4e^2f^2+14\,d^5efg+13\,d^6g^2+\left(3\,d^2e^4f^2+14\,d^3e^3fg+13\,d^4e^2g^2\right)x^2-2\left(3\,d^3e^3f^2+14\,d^4e^2fg+13\,d^5eg^2\right)x\right)\log\left(ex-d\right)}{12\left(e^5x^2-2\,de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*e^6*g^2*x^6 + 288*d^4*e^2*f^2 + 960*d^5*e*f*g + 672*d^6*g^2 + 2*(4
*e^6*f*g + 11*d*e^5*g^2)*x^5 + (6*e^6*f^2 + 68*d*e^5*f*g + 91*d^2*e^4*g^2)*
x^4 + 4*(18*d*e^5*f^2 + 104*d^2*e^4*f*g + 103*d^3*e^3*g^2)*x^3 - 6*(27*d^2*
e^4*f^2 + 178*d^3*e^3*f*g + 200*d^4*e^2*g^2)*x^2 - 12*(25*d^3*e^3*f^2 + 48*
d^4*e^2*f*g + 8*d^5*e*g^2)*x + 96*(3*d^4*e^2*f^2 + 14*d^5*e*f*g + 13*d^6*g^
2 + (3*d^2*e^4*f^2 + 14*d^3*e^3*f*g + 13*d^4*e^2*g^2)*x^2 - 2*(3*d^3*e^3*f^
2 + 14*d^4*e^2*f*g + 13*d^5*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x +
d^2*e^3)
```

giac [B] time = 0.19, size = 364, normalized size = 2.03

$$-4\left(13\,d^4g^2e^7+14\,d^3fge^8+3\,d^2f^2e^9\right)e^{(-10)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{12}\left(3\,g^2x^4e^{25}+28\,dg^2x^3e^{24}+144\,d^2g^2x^2e^{23}+672\,d^3g^2xe^{22}+8\,fgx^3e^{25}+84\,dfgx^2e^{24}+576\,d^2fgxe^{23}+6\,f^2x^2e^{25}+84\,df^2xe^{24}\right)e^{(-24)}-\frac{4\left(13\,d^5g^2e^6+14\,d^4fge^7+3\,d^3f^2e^8\right)e^{(-9)}\log\left(\frac{\left|2\,xe^2-2\,\left|d\right|e\right|}{\left|2\,xe^2+2\,\left|d\right|e\right|}\right)}{\left|d\right|}-\frac{8\left(7\,d^8g^2e^7+10\,d^7fge^8+3\,d^6f^2e^9-4\left(2\,d^5g^2e^{10}+3\,d^4fge^{11}+d^3f^2e^{12}\right)x^3-\left(9\,d^6g^2e^9+14\,d^5fge^{10}+5\,d^4f^2e^{11}\right)x^2+2\left(3\,d^7g^2e^8+4\,d^6fge^9+d^5f^2e^{10}\right)x\right)e^{(-10)}}{\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -4*(13*d^4*g^2*e^7 + 14*d^3*f*g*e^8 + 3*d^2*f^2*e^9)*e^(-10)*log(abs(x^2*e^
2 - d^2)) - 1/12*(3*g^2*x^4*e^25 + 28*d*g^2*x^3*e^24 + 144*d^2*g^2*x^2*e^23
 + 672*d^3*g^2*x*e^22 + 8*f*g*x^3*e^25 + 84*d*f*g*x^2*e^24 + 576*d^2*f*g*x*
e^23 + 6*f^2*x^2*e^25 + 84*d*f^2*x*e^24)*e^(-24) - 4*(13*d^5*g^2*e^6 + 14*d
^4*f*g*e^7 + 3*d^3*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^
2 + 2*abs(d)*e))/abs(d) - 8*(7*d^8*g^2*e^7 + 10*d^7*f*g*e^8 + 3*d^6*f^2*e^9
 - 4*(2*d^5*g^2*e^10 + 3*d^4*f*g*e^11 + d^3*f^2*e^12)*x^3 - (9*d^6*g^2*e^9
+ 14*d^5*f*g*e^10 + 5*d^4*f^2*e^11)*x^2 + 2*(3*d^7*g^2*e^8 + 4*d^6*f*g*e^9
+ d^5*f^2*e^10)*x)*e^(-10)/(x^2*e^2 - d^2)^2
```

maple [A] time = 0.01, size = 263, normalized size = 1.47

$$-\frac{e\,g^2x^4}{4}-\frac{7d\,g^2x^3}{3}-\frac{2ef\,g\,x^3}{3}+\frac{8d^6g^2}{\left(ex-d\right)^2e^3}+\frac{16d^5fg}{\left(ex-d\right)^2e^2}+\frac{8d^4f^2}{\left(ex-d\right)^2e}-\frac{12d^2g^2x^2}{e}-7dfg\,x^2-\frac{e\,f^2x^2}{2}+\frac{64d^5g^2}{\left(ex-d\right)e^3}+\frac{96d^4fg}{\left(ex-d\right)e^2}-\frac{104d^4g^2\ln\left(ex-d\right)}{e^3}+\frac{32d^3f^2}{\left(ex-d\right)e}-\frac{112d^3fg\ln\left(ex-d\right)}{e^2}-\frac{56d^3g^2x}{e^2}-\frac{24d^2f^2\ln\left(ex-d\right)}{e}-\frac{48d^2fgx}{e}-7d\,f^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/4*e*g^2*x^4-7/3*d*g^2*x^3-2/3*e*f*g*x^3-12*d^2/e*g^2*x^2-7*d*f*g*x^2-1/2
*e*f^2*x^2-56*d^3/e^2*g^2*x-48*d^2/e*f*g*x-7*d*f^2*x-104*d^4/e^3*g^2*ln(e*x
```

```
-d)-112*d^3/e^2*f*g*ln(e*x-d)-24*d^2/e*f^2*ln(e*x-d)+8*d^6/e^3/(e*x-d)^2*g^
2+16*d^5/e^2/(e*x-d)^2*f*g+8*d^4/e/(e*x-d)^2*f^2+64/(e*x-d)*d^5/e^3*g^2+96/
(e*x-d)*d^4/e^2*f*g+32/(e*x-d)*d^3/e*f^2
```

maxima [A] time = 0.47, size = 227, normalized size = 1.27

$$-\frac{8\left(3 d^{4} e^{2} f^{2}+10 d^{5} e f g+7 d^{6} g^{2}-4\left(d^{3} e^{3} f^{2}+3 d^{4} e^{2} f g+2 d^{5} e g^{2}\right) x\right)}{e^{5} x^{2}-2 d e^{4} x+d^{2} e^{3}}-\frac{3 e^{3} g^{2} x^{4}+4\left(2 e^{3} f g+7 d e^{2} g^{2}\right) x^{3}+6\left(e^{3} f^{2}+14 d e^{2} f g+24 d^{2} e g^{2}\right) x^{2}+12\left(7 d e^{2} f^{2}+48 d^{2} e f g+56 d^{3} g^{2}\right) x}{12 e^{2}}-\frac{8\left(3 d^{2} e^{2} f^{2}+14 d^{3} e f g+13 d^{4} g^{2}\right) \log\left(e x-d\right)}{e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -8*(3*d^4*e^2*f^2 + 10*d^5*e*f*g + 7*d^6*g^2 - 4*(d^3*e^3*f^2 + 3*d^4*e^2*f
*g + 2*d^5*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/12*(3*e^3*g^2*x^4
+ 4*(2*e^3*f*g + 7*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 14*d*e^2*f*g + 24*d^2*e*g^
2)*x^2 + 12*(7*d*e^2*f^2 + 48*d^2*e*f*g + 56*d^3*g^2)*x)/e^2 - 8*(3*d^2*e^2
*f^2 + 14*d^3*e*f*g + 13*d^4*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.14, size = 375, normalized size = 2.09

$$\frac{x\left(64 d^{5} g^{2}+96 d^{4} e f g+32 d^{3} e^{2} f^{2}\right)-\frac{8\left(7 d^{6} g^{2}+10 d^{5} e f g+3 d^{4} e^{2} f^{2}\right)}{e}}{d^{2} e^{2}-2 d e^{3} x+e^{4} x^{2}}-x^{2}\left(\frac{6 d^{2} e^{2} g^{2}+8 d e^{3} f g+e^{4} f^{2}}{2 e^{3}}-\frac{3 d^{2} g^{2}}{2 e}+\frac{3 d\left(2 g\left(2 d g+e f\right)+3 d g^{2}\right)}{2 e}\right)-x\left(\frac{d^{3} g^{2}}{e^{2}}-\frac{3 d^{2}\left(2 g\left(2 d g+e f\right)+3 d g^{2}\right)}{e^{2}}+\frac{4 d\left(d^{2} g^{2}+3 d e f g+e^{2} f^{2}\right)}{e^{2}}+\frac{3 d\left(\frac{6 d^{2} e^{2} g^{2}+8 d e^{3} f g+e^{4} f^{2}}{e^{3}}-\frac{3 d^{2} g^{2}}{e}+\frac{3 d\left(2 g\left(2 d g+e f\right)+3 d g^{2}\right)}{e}\right)}{e}\right)-x^{3}\left(\frac{2 g\left(2 d g+e f\right)}{3}+d g^{2}\right)-\frac{\ln\left(e x-d\right)\left(104 d^{4} g^{2}+112 d^{3} e f g+24 d^{2} e^{2} f^{2}\right)}{e^{3}}-\frac{e g^{2} x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^7)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (x*(64*d^5*g^2 + 32*d^3*e^2*f^2 + 96*d^4*e*f*g) - (8*(7*d^6*g^2 + 3*d^4*e^2
*f^2 + 10*d^5*e*f*g))/e)/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x^2*((e^4*f^2 +
6*d^2*e^2*g^2 + 8*d*e^3*f*g)/(2*e^3) - (3*d^2*g^2)/(2*e) + (3*d*(2*g*(2*d*g
 + e*f) + 3*d*g^2))/(2*e)) - x*((d^3*g^2)/e^2 - (3*d^2*(2*g*(2*d*g + e*f) +
 3*d*g^2))/e^2 + (4*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f*g))/e^2 + (3*d*((e^4*f^2
 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^3 - (3*d^2*g^2)/e + (3*d*(2*g*(2*d*g + e*
f) + 3*d*g^2))/e))/e) - x^3*((2*g*(2*d*g + e*f))/3 + d*g^2) - (log(e*x - d)
*(104*d^4*g^2 + 24*d^2*e^2*f^2 + 112*d^3*e*f*g))/e^3 - (e*g^2*x^4)/4
```

sympy [A] time = 1.55, size = 219, normalized size = 1.22

$$-\frac{8 d^{2}\left(13 d^{2} g^{2}+14 d e f g+3 e^{2} f^{2}\right) \log\left(-d+e x\right)}{e^{3}}-\frac{e g^{2} x^{4}}{4}-x^{3}\left(\frac{7 d g^{2}}{3}+\frac{2 e f g}{3}\right)-x^{2}\left(\frac{12 d^{2} g^{2}}{e}+7 d f g+\frac{e f^{2}}{2}\right)-x\left(\frac{56 d^{3} g^{2}}{e^{2}}+\frac{48 d^{2} f g}{e}+7 d f^{2}\right)-\frac{56 d^{6} g^{2}+80 d^{5} e f g+24 d^{4} e^{2} f^{2}+x\left(-64 d^{5} e g^{2}-96 d^{4} e^{2} f g-32 d^{3} e^{3} f^{2}\right)}{d^{2} e^{3}-2 d e^{4} x+e^{5} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -8*d**2*(13*d**2*g**2 + 14*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - e*g*
*2*x**4/4 - x**3*(7*d*g**2/3 + 2*e*f*g/3) - x**2*(12*d**2*g**2/e + 7*d*f*g
+ e*f**2/2) - x*(56*d**3*g**2/e**2 + 48*d**2*f*g/e + 7*d*f**2) - (56*d**6*g
**2 + 80*d**5*e*f*g + 24*d**4*e**2*f**2 + x*(-64*d**5*e*g**2 - 96*d**4*e**2
*f*g - 32*d**3*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2)
```

3.372 $\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=149

$$\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d^2(dg+ef)(7dg+3ef)}{e^3(d-ex)}-\frac{x\left(18d^2g^2+12defg+e^2f^2\right)}{e^2}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log(d-}{e^3}$$

Rubi [A] time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{x\left(18d^2g^2+12defg+e^2f^2\right)}{e^2}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d^2(dg+ef)(7dg+3ef)}{e^3(d-ex)}-\frac{gx^2(3dg+ef)}{e}-\frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
(g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - (4*d^2*(e*f + d*g)*
(3*e*f + 7*d*g))/(e^3*(d - e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g + 19*d^2*g^
2)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^3}\,dx\\ &= \int \left(\frac{-e^2f^2-12defg-18d^2g^2}{e^2}-\frac{2g(ef+3dg)x}{e}-g^2x^2+\frac{4d^2(-3ef-7dg)(ef+dg}{e^2(d-ex)^2}\right.\\ &= -\frac{\left(e^2f^2+12defg+18d^2g^2\right)x}{e^2}-\frac{g(ef+3dg)x^2}{e}-\frac{g^2x^3}{3}+\frac{4d^3(ef+dg)^2}{e^3(d-ex)^2}-\frac{4d^2(ef}{}\end{aligned}$$

Mathematica [A] time = 0.08, size = 157, normalized size = 1.05

$$\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{x\left(18d^2g^2+12defg+e^2f^2\right)}{e^2}+\frac{4d^2\left(7d^2g^2+10defg+3e^2f^2\right)}{e^3(ex-d)}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log(d-ex)}{e^3}-\frac{gx^2(3dg+ef)}{e}-\frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
(g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (4*d^2*(3*e^2*f^2 +
 10*d*e*f*g + 7*d^2*g^2))/(e^3*(-d + e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g +
 19*d^2*g^2)*Log[d - e*x])/e^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3, x]
```

fricas [A] time = 0.38, size = 294, normalized size = 1.97

$$-\frac{e^5g^2x^5+24\,d^3e^2f^2+96\,d^4efg+72\,d^5g^2+\left(3\,e^5fg+7\,de^4g^2\right)x^4+\left(3\,e^5f^2+30\,de^4fg+37\,d^2e^3g^2\right)x^3-3\left(2\,de^4f^2+23\,d^2e^3fg+33\,d^3e^2g^2\right)x^2-3\left(11\,d^2e^3f^2+28\,d^3e^2fg+10\,d^4eg^2\right)x+6\left(3\,d^3e^2f^2+18\,d^4efg+19\,d^5g^2+\left(3\,de^4f^2+18\,d^2e^3fg+19\,d^3e^2g^2\right)x^2-2\left(3\,d^2e^3f^2+18\,d^3e^2fg+19\,d^4eg^2\right)x\right)\log\left(ex-d\right)}{3\left(e^5x^2-2\,de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(e^5*g^2*x^5 + 24*d^3*e^2*f^2 + 96*d^4*e*f*g + 72*d^5*g^2 + (3*e^5*f*g
 + 7*d*e^4*g^2)*x^4 + (3*e^5*f^2 + 30*d*e^4*f*g + 37*d^2*e^3*g^2)*x^3 - 3*(
2*d*e^4*f^2 + 23*d^2*e^3*f*g + 33*d^3*e^2*g^2)*x^2 - 3*(11*d^2*e^3*f^2 + 28
*d^3*e^2*f*g + 10*d^4*e*g^2)*x + 6*(3*d^3*e^2*f^2 + 18*d^4*e*f*g + 19*d^5*g
^2 + (3*d*e^4*f^2 + 18*d^2*e^3*f*g + 19*d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^3*f^2
 + 18*d^3*e^2*f*g + 19*d^4*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d
^2*e^3)
```

giac [B] time = 0.20, size = 324, normalized size = 2.17

$$-\left(19\,d^3g^2e^5+18\,d^2fge^6+3\,df^2e^7\right)e^{(-8)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{3}\left(g^2x^3e^{18}+9\,dg^2x^2e^{17}+54\,d^2g^2xe^{16}+3\,fgx^2e^{18}+36\,dfgxe^{17}+3\,f^2xe^{18}\right)e^{(-18)}-\frac{\left(19\,d^4g^2e^6+18\,d^3fge^7+3\,d^2f^2e^8\right)e^{(-9)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{|d|}-\frac{4\left(6\,d^7g^2e^5+8\,d^6fge^6+2\,d^5f^2e^7-\left(7\,d^4g^2e^8+10\,d^3fge^9+3\,d^2f^2e^{10}\right)x^3-4\left(2\,d^5g^2e^7+3\,d^4fge^8+d^3f^2e^9\right)x^2+\left(5\,d^6g^2e^6+6\,d^5fge^7+d^4f^2e^8\right)x\right)e^{(-8)}}{\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -(19*d^3*g^2*e^5 + 18*d^2*f*g*e^6 + 3*d*f^2*e^7)*e^(-8)*log(abs(x^2*e^2 - d
^2)) - 1/3*(g^2*x^3*e^18 + 9*d*g^2*x^2*e^17 + 54*d^2*g^2*x*e^16 + 3*f*g*x^2
*e^18 + 36*d*f*g*x*e^17 + 3*f^2*x*e^18)*e^(-18) - (19*d^4*g^2*e^6 + 18*d^3*
f*g*e^7 + 3*d^2*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 +
 2*abs(d)*e))/abs(d) - 4*(6*d^7*g^2*e^5 + 8*d^6*f*g*e^6 + 2*d^5*f^2*e^7 - (
7*d^4*g^2*e^8 + 10*d^3*f*g*e^9 + 3*d^2*f^2*e^10)*x^3 - 4*(2*d^5*g^2*e^7 + 3
*d^4*f*g*e^8 + d^3*f^2*e^9)*x^2 + (5*d^6*g^2*e^6 + 6*d^5*f*g*e^7 + d^4*f^2*
e^8)*x)*e^(-8)/(x^2*e^2 - d^2)^2
```

maple [A] time = 0.01, size = 228, normalized size = 1.53

$$-\frac{g^2x^3}{3}+\frac{4d^5g^2}{(ex-d)^2e^3}+\frac{8d^4fg}{(ex-d)^2e^2}+\frac{4d^3f^2}{(ex-d)^2e}-\frac{3dg^2x^2}{e}-fgx^2+\frac{28d^4g^2}{(ex-d)e^3}+\frac{40d^3fg}{(ex-d)e^2}-\frac{38d^3g^2\ln(ex-d)}{e^3}+\frac{12d^2f^2}{(ex-d)e}-\frac{36d^2fg\ln(ex-d)}{e^2}-\frac{18d^2g^2x}{e^2}-\frac{6df^2\ln(ex-d)}{e}-\frac{12dfgx}{e}-f^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/3*g^2*x^3-3*d/e*g^2*x^2-f*g*x^2-18*d^2/e^2*g^2*x-12*d/e*f*g*x-f^2*x-38*d
^3/e^3*g^2*ln(e*x-d)-36*d^2/e^2*f*g*ln(e*x-d)-6*d/e*f^2*ln(e*x-d)+4*d^5/e^3
/(e*x-d)^2*g^2+8*d^4/e^2/(e*x-d)^2*f*g+4*d^3/e/(e*x-d)^2*f^2+28/(e*x-d)*d^4
/e^3*g^2+40/(e*x-d)*d^3/e^2*f*g+12/(e*x-d)*d^2/e*f^2
```

maxima [A] time = 0.46, size = 188, normalized size = 1.26

$$-\frac{4\left(2d^3e^2f^2+8d^4efg+6d^5g^2-\left(3d^2e^3f^2+10d^3e^2fg+7d^4eg^2\right)x\right)}{e^5x^2-2de^4x+d^2e^3}-\frac{e^2g^2x^3+3\left(e^2fg+3deg^2\right)x^2+3\left(e^2f^2+12defg+18d^2g^2\right)x}{3e^2}-\frac{2\left(3de^2f^2+18d^2efg+19d^3g^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -4*(2*d^3*e^2*f^2 + 8*d^4*e*f*g + 6*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f
*g + 7*d^4*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/3*(e^2*g^2*x^3 + 3
*(e^2*f*g + 3*d*e*g^2)*x^2 + 3*(e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2 -
 2*(3*d*e^2*f^2 + 18*d^2*e*f*g + 19*d^3*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 0.10, size = 240, normalized size = 1.61

$$\frac{x\left(28d^4g^2+40d^3efg+12d^2e^2f^2\right)-\frac{8\left(3d^5g^2+4d^4efg+d^3e^2f^2\right)}{e}}{d^2e^2-2de^3x+e^4x^2}-x\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{e^3}+\frac{3d\left(\frac{g\left(3dg+2ef\right)}{e}+\frac{3dg^2}{e}\right)}{e}-\frac{3d^2g^2}{e^2}\right)-x^2\left(\frac{g\left(3dg+2ef\right)}{2e}+\frac{3dg^2}{2e}\right)-\frac{g^2x^3}{3}-\frac{\ln\left(ex-d\right)\left(38d^3g^2+36d^2efg+6de^2f^2\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^6)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (x*(28*d^4*g^2 + 12*d^2*e^2*f^2 + 40*d^3*e*f*g) - (8*(3*d^5*g^2 + d^3*e^2*f
^2 + 4*d^4*e*f*g))/e)/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x*((e^3*f^2 + 3*d^2
*e*g^2 + 6*d*e^2*f*g)/e^3 + (3*d*((g*(3*d*g + 2*e*f))/e + (3*d*g^2)/e))/e -
 (3*d^2*g^2)/e^2) - x^2*((g*(3*d*g + 2*e*f))/(2*e) + (3*d*g^2)/(2*e)) - (g^
2*x^3)/3 - (log(e*x - d)*(38*d^3*g^2 + 6*d*e^2*f^2 + 36*d^2*e*f*g))/e^3
```

sympy [A] time = 1.37, size = 178, normalized size = 1.19

$$-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log\left(-d+ex\right)}{e^3}-\frac{g^2x^3}{3}-x^2\left(\frac{3dg^2}{e}+fg\right)-x\left(\frac{18d^2g^2}{e^2}+\frac{12dfg}{e}+f^2\right)-\frac{24d^5g^2+32d^4efg+8d^3e^2f^2+x\left(-28d^4eg^2-40d^3e^2fg-12d^2e^3f^2\right)}{d^2e^3-2de^4x+e^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -2*d*(19*d**2*g**2 + 18*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - g**2*x*
*3/3 - x**2*(3*d*g**2/e + f*g) - x*(18*d**2*g**2/e**2 + 12*d*f*g/e + f**2)
- (24*d**5*g**2 + 32*d**4*e*f*g + 8*d**3*e**2*f**2 + x*(-28*d**4*e*g**2 - 4
0*d**3*e**2*f*g - 12*d**2*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2)
```

3.373 $\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=118

$$\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2}-\frac{\left(13d^2g^2+10defg+e^2f^2\right)\log(d-ex)}{e^3}-\frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)}-\frac{gx(5dg+2ef)}{e^2}-\frac{g^2x^2}{2e}$$

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{\left(13d^2g^2+10defg+e^2f^2\right)\log(d-ex)}{e^3}+\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)}-\frac{gx(5dg+2ef)}{e^2}-\frac{g^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((g*(2*e*f + 5*d*g)*x)/e^2) - (g^2*x^2)/(2*e) + (2*d^2*(e*f + d*g)^2)/(e^3
*(d - e*x)^2) - (4*d*(e*f + d*g)*(e*f + 3*d*g))/(e^3*(d - e*x)) - ((e^2*f^2
 + 10*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^3}\,dx\\ &= \int \left(-\frac{g(2ef+5dg)}{e^2}-\frac{g^2x}{e}+\frac{4d(-ef-3dg)(ef+dg)}{e^2(d-ex)^2}-\frac{4d^2(ef+dg)^2}{e^2(-d+ex)^3}+\frac{-e^2f^2-10d\ldots}{e^2(-d\ldots}\right.\\ &= -\frac{g(2ef+5dg)x}{e^2}-\frac{g^2x^2}{2e}+\frac{2d^2(ef+dg)^2}{e^3(d-ex)^2}-\frac{4d(ef+dg)(ef+3dg)}{e^3(d-ex)}-\frac{\left(e^2f^2+10defg\ldots\right.}{}\end{aligned}$$

Mathematica [A] time = 0.09, size = 118, normalized size = 1.00

$$-\frac{\frac{8d\left(3d^2g^2+4defg+e^2f^2\right)}{d-ex}+2\left(13d^2g^2+10defg+e^2f^2\right)\log(d-ex)-\frac{4d^2(dg+ef)^2}{(d-ex)^2}+2egx(5dg+2ef)+e^2g^2x^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

[Out] -1/2*(2*e*g*(2*e*f + 5*d*g)*x + e^2*g^2*x^2 - (4*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (8*d*(e^2*f^2 + 4*d*e*f*g + 3*d^2*g^2))/(d - e*x) + 2*(e^2*f^2 + 10*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]

[Out] IntegrateAlgebraic[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3, x]

fricas [B] time = 0.39, size = 241, normalized size = 2.04

$$-\frac{e^4g^2x^4+4d^2e^2f^2+24d^3efg+20d^4g^2+4\left(e^4fg+2de^3g^2\right)x^3-\left(8de^3fg+19d^2e^2g^2\right)x^2-2\left(4de^3f^2+14d^2e^2fg+7d^3eg^2\right)x+2\left(d^2e^2f^2+10d^3efg+13d^4g^2+\left(e^4f^2+10de^3fg+13d^2e^2g^2\right)x^2-2\left(de^3f^2+10d^2e^2fg+13d^3eg^2\right)x\right)\log\left(ex-d\right)}{2\left(e^5x^2-2de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")

[Out] -1/2*(e^4*g^2*x^4 + 4*d^2*e^2*f^2 + 24*d^3*e*f*g + 20*d^4*g^2 + 4*(e^4*f*g + 2*d*e^3*g^2)*x^3 - (8*d*e^3*f*g + 19*d^2*e^2*g^2)*x^2 - 2*(4*d*e^3*f^2 + 14*d^2*e^2*f*g + 7*d^3*e*g^2)*x + 2*(d^2*e^2*f^2 + 10*d^3*e*f*g + 13*d^4*g^2 + (e^4*f^2 + 10*d*e^3*f*g + 13*d^2*e^2*g^2)*x^2 - 2*(d*e^3*f^2 + 10*d^2*e^2*f*g + 13*d^3*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)

giac [B] time = 0.38, size = 273, normalized size = 2.31

$$-\frac{1}{2}\left(13d^2g^2e^5+10dfge^6+f^2e^7\right)e^{(-8)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{2}\left(g^2x^2e^{11}+10dg^2xe^{10}+4fgxe^{11}\right)e^{(-12)}-\frac{\left(13d^3g^2e^4+10d^2fge^5+df^2e^6\right)e^{(-7)}\log\left(\frac{\left|2xe^2-2|d|e\right|}{\left|2xe^2+2|d|e\right|}\right)}{2|d|}-\frac{2\left(5d^6g^2e^5+6d^5fge^6+d^4f^2e^7-2\left(3d^3g^2e^8+4d^2fge^9+df^2e^{10}\right)x^3-\left(7d^4g^2e^7+10d^3fge^8+3d^2f^2e^9\right)x^2+4\left(d^5g^2e^6+d^4fge^7\right)x\right)e^{(-8)}}{\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")

[Out] -1/2*(13*d^2*g^2*e^5 + 10*d*f*g*e^6 + f^2*e^7)*e^(-8)*log(abs(x^2*e^2 - d^2)) - 1/2*(g^2*x^2*e^11 + 10*d*g^2*x*e^10 + 4*f*g*x*e^11)*e^(-12) - 1/2*(13*d^3*g^2*e^4 + 10*d^2*f*g*e^5 + d*f^2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d) - 2*(5*d^6*g^2*e^5 + 6*d^5*f*g*e^6 + d^4*f^2*e^7 - 2*(3*d^3*g^2*e^8 + 4*d^2*f*g*e^9 + d*f^2*e^10)*x^3 - (7*d^4*g^2*e^7 + 10*d^3*f*g*e^8 + 3*d^2*f^2*e^9)*x^2 + 4*(d^5*g^2*e^6 + d^4*f*g*e^7)*x)*e^(-8)/(x^2*e^2 - d^2)^2

maple [A] time = 0.01, size = 198, normalized size = 1.68

$$\frac{2d^4g^2}{\left(ex-d\right)^2e^3}+\frac{4d^3fg}{\left(ex-d\right)^2e^2}+\frac{2d^2f^2}{\left(ex-d\right)^2e}-\frac{g^2x^2}{2e}+\frac{12d^3g^2}{\left(ex-d\right)e^3}+\frac{16d^2fg}{\left(ex-d\right)e^2}-\frac{13d^2g^2\ln\left(ex-d\right)}{e^3}+\frac{4df^2}{\left(ex-d\right)e}-\frac{10dfg\ln\left(ex-d\right)}{e^2}-\frac{5dg^2x}{e^2}-\frac{f^2\ln\left(ex-d\right)}{e}-\frac{2fgx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)

[Out] -1/2/e*g^2*x^2-5*d/e^2*g^2*x-2/e*f*g*x-13*d^2/e^3*g^2*ln(e*x-d)-10*d/e^2*f*g*ln(e*x-d)-1/e*f^2*ln(e*x-d)+2*d^4/e^3/(e*x-d)^2*g^2+4*d^3/e^2/(e*x-d)^2*f*g+2*d^2/e/(e*x-d)^2*f^2+12/(e*x-d)*d^3/e^3*g^2+16/(e*x-d)*d^2/e^2*f*g+4/(e*x-d)*d/e*f^2

maxima [A] time = 0.46, size = 149, normalized size = 1.26

$$-\frac{2\left(d^2e^2f^2+6d^3efg+5d^4g^2-2\left(de^3f^2+4d^2e^2fg+3d^3eg^2\right)x\right)}{e^5x^2-2de^4x+d^2e^3}-\frac{eg^2x^2+2\left(2efg+5dg^2\right)x}{2e^2}-\frac{\left(e^2f^2+10defg+13d^2g^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -2*(d^2*e^2*f^2 + 6*d^3*e*f*g + 5*d^4*g^2 - 2*(d*e^3*f^2 + 4*d^2*e^2*f*g +
3*d^3*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/2*(e*g^2*x^2 + 2*(2*e*f
*g + 5*d*g^2)*x)/e^2 - (e^2*f^2 + 10*d*e*f*g + 13*d^2*g^2)*log(e*x - d)/e^3
```

mupad [B] time = 2.60, size = 161, normalized size = 1.36

$$-\frac{\frac{2\left(5\,d^4\,g^2+6\,d^3\,e\,f\,g+d^2\,e^2\,f^2\right)}{e}-x\,\left(12\,d^3\,g^2+16\,d^2\,e\,f\,g+4\,d\,e^2\,f^2\right)}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2}-x\,\left(\frac{2\,g\,\left(d\,g+e\,f\right)}{e^2}+\frac{3\,d\,g^2}{e^2}\right)-\frac{\ln\left(e\,x-d\right)\,\left(13\,d^2\,g^2+10\,d\,e\,f\,g+e^2\,f^2\right)}{e^3}-\frac{g^2\,x^2}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^5)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((2*(5*d^4*g^2 + d^2*e^2*f^2 + 6*d^3*e*f*g))/e - x*(12*d^3*g^2 + 4*d*e^2*
f^2 + 16*d^2*e*f*g))/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x*((2*g*(d*g + e*f))
/e^2 + (3*d*g^2)/e^2) - (log(e*x - d)*(13*d^2*g^2 + e^2*f^2 + 10*d*e*f*g))/
e^3 - (g^2*x^2)/(2*e)
```

sympy [A] time = 1.21, size = 151, normalized size = 1.28

$$-x\left(\frac{5dg^2}{e^2}+\frac{2fg}{e}\right)-\frac{10d^4g^2+12d^3efg+2d^2e^2f^2+x\left(-12d^3eg^2-16d^2e^2fg-4de^3f^2\right)}{d^2e^3-2de^4x+e^5x^2}-\frac{g^2x^2}{2e}-\frac{\left(13d^2g^2+10defg+e^2f^2\right)\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -x*(5*d*g**2/e**2 + 2*f*g/e) - (10*d**4*g**2 + 12*d**3*e*f*g + 2*d**2*e**2*
f**2 + x*(-12*d**3*e*g**2 - 16*d**2*e**2*f*g - 4*d*e**3*f**2))/(d**2*e**3 -
 2*d*e**4*x + e**5*x**2) - g**2*x**2/(2*e) - (13*d**2*g**2 + 10*d*e*f*g + e
**2*f**2)*log(-d + e*x)/e**3
```

3.374 $\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)}+\frac{d(dg+ef)^2}{e^3(d-ex)^2}-\frac{2g(2dg+ef)\log(d-ex)}{e^3}-\frac{g^2x}{e^2}$$

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 77}

$$-\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)}+\frac{d(dg+ef)^2}{e^3(d-ex)^2}-\frac{2g(2dg+ef)\log(d-ex)}{e^3}-\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((g^2*x)/e^2) + (d*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - ((e*f + d*g)*(e*f +
5*d*g))/(e^3*(d - e*x)) - (2*g*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(d+ex)(f+gx)^2}{(d-ex)^3}\,dx\\
&= \int \left(-\frac{g^2}{e^2}+\frac{(-ef-5dg)(ef+dg)}{e^2(d-ex)^2}-\frac{2d(ef+dg)^2}{e^2(-d+ex)^3}-\frac{2g(ef+2dg)}{e^2(-d+ex)}\right)dx\\
&= -\frac{g^2x}{e^2}+\frac{d(ef+dg)^2}{e^3(d-ex)^2}-\frac{(ef+dg)(ef+5dg)}{e^3(d-ex)}-\frac{2g(ef+2dg)\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 93, normalized size = 1.15

$$\frac{-4d^3g^2+4d^2eg(gx-f)+2de^2gx(3f+gx)-2g(d-ex)^2(2dg+ef)\log(d-ex)+e^3x\left(f^2-g^2x^2\right)}{e^3(d-ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

[Out] (-4*d^3*g^2 + 4*d^2*e*g*(-f + g*x) + 2*d*e^2*g*x*(3*f + g*x) + e^3*x*(f^2 - g^2*x^2) - 2*g*(e*f + 2*d*g)*(d - e*x)^2*Log[d - e*x])/(e^3*(d - e*x)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]

[Out] IntegrateAlgebraic[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3, x]

fricas [A] time = 0.40, size = 159, normalized size = 1.96

$$-\frac{e^3g^2x^3-2\,de^2g^2x^2+4\,d^2efg+4\,d^3g^2-\left(e^3f^2+6\,de^2fg+4\,d^2eg^2\right)x+2\left(d^2efg+2\,d^3g^2+\left(e^3fg+2\,de^2g^2\right)x^2-2\left(de^2fg+2\,d^2eg^2\right)x\right)\log\left(ex-d\right)}{e^5x^2-2\,de^4x+d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")

[Out] -(e^3*g^2*x^3 - 2*d*e^2*g^2*x^2 + 4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 4*d^2*e*g^2)*x + 2*(d^2*e*f*g + 2*d^3*g^2 + (e^3*f*g + 2*d*e^2*g^2)*x^2 - 2*(d*e^2*f*g + 2*d^2*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)

giac [B] time = 0.21, size = 227, normalized size = 2.80

$$-g^2xe^{(-2)}-\left(2\,dg^2e^3+fge^4\right)e^{(-6)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{\left(2\,d^2g^2e^4+dfge^5\right)e^{(-7)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{|d|}-\frac{\left(4\,d^5g^2e^3+4\,d^4fge^4-\left(5\,d^2g^2e^6+6\,dfge^7+f^2e^8\right)x^3-2\left(3\,d^3g^2e^5+4\,d^2fge^6+df^2e^7\right)x^2+\left(3\,d^4g^2e^4+2\,d^3fge^5-d^2f^2e^6\right)x\right)e^{(-6)}}{\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")

[Out] -g^2*x*e^(-2) - (2*d*g^2*e^3 + f*g*e^4)*e^(-6)*log(abs(x^2*e^2 - d^2)) - (2*d^2*g^2*e^4 + d*f*g*e^5)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d) - (4*d^5*g^2*e^3 + 4*d^4*f*g*e^4 - (5*d^2*g^2*e^6 + 6*d*f*g*e^7 + f^2*e^8)*x^3 - 2*(3*d^3*g^2*e^5 + 4*d^2*f*g*e^6 + d*f^2*e^7)*x^2 + (3*d^4*g^2*e^4 + 2*d^3*f*g*e^5 - d^2*f^2*e^6)*x)*e^(-6)/(x^2*e^2 - d^2)^2

maple [A] time = 0.01, size = 151, normalized size = 1.86

$$\frac{d^3g^2}{\left(ex-d\right)^2e^3}+\frac{2d^2fg}{\left(ex-d\right)^2e^2}+\frac{d\,f^2}{\left(ex-d\right)^2e}+\frac{5d^2g^2}{\left(ex-d\right)e^3}+\frac{6dfg}{\left(ex-d\right)e^2}-\frac{4d\,g^2\ln\left(ex-d\right)}{e^3}+\frac{f^2}{\left(ex-d\right)e}-\frac{2fg\ln\left(ex-d\right)}{e^2}-\frac{g^2x}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)

[Out] -1/e^2*g^2*x-4*d/e^3*g^2*ln(e*x-d)-2/e^2*f*g*ln(e*x-d)+d^3/e^3/(e*x-d)^2*g^2+2*d^2/e^2/(e*x-d)^2*f*g+d/e/(e*x-d)^2*f^2+5/(e*x-d)*d^2/e^3*g^2+6/(e*x-d)*d/e^2*f*g+1/(e*x-d)/e*f^2

maxima [A] time = 0.45, size = 105, normalized size = 1.30

$$-\frac{g^2x}{e^2}-\frac{4\,d^2efg+4\,d^3g^2-\left(e^3f^2+6\,de^2fg+5\,d^2eg^2\right)x}{e^5x^2-2\,de^4x+d^2e^3}-\frac{2\left(efg+2\,dg^2\right)\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -g^2*x/e^2 - (4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^
2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 2*(e*f*g + 2*d*g^2)*log(e*x - d)/e^
3
```

mupad [B] time = 2.60, size = 107, normalized size = 1.32

$$-\frac{\frac{4\left(d^3\,g^2+e\,f\,d^2\,g\right)}{e}-x\,\left(5\,d^2\,g^2+6\,d\,e\,f\,g+e^2\,f^2\right)}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2}-\frac{g^2\,x}{e^2}-\frac{\ln\left(e\,x-d\right)\,\left(4\,d\,g^2+2\,e\,f\,g\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((4*(d^3*g^2 + d^2*e*f*g))/e - x*(5*d^2*g^2 + e^2*f^2 + 6*d*e*f*g))/(d^2*
e^2 + e^4*x^2 - 2*d*e^3*x) - (g^2*x)/e^2 - (log(e*x - d)*(4*d*g^2 + 2*e*f*g
))/e^3
```

sympy [A] time = 0.87, size = 102, normalized size = 1.26

$$-\frac{4d^3g^2+4d^2efg+x\left(-5d^2eg^2-6de^2fg-e^3f^2\right)}{d^2e^3-2de^4x+e^5x^2}-\frac{g^2x}{e^2}-\frac{2g\left(2dg+ef\right)\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(4*d**3*g**2 + 4*d**2*e*f*g + x*(-5*d**2*e*g**2 - 6*d*e**2*f*g - e**3*f**2
))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2) - g**2*x/e**2 - 2*g*(2*d*g + e*f)*l
og(-d + e*x)/e**3
```

3.375 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=61

$$-\frac{2g(dg+ef)}{e^3(d-ex)}+\frac{(dg+ef)^2}{2e^3(d-ex)^2}-\frac{g^2\log(d-ex)}{e^3}$$

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 43}

$$-\frac{2g(dg+ef)}{e^3(d-ex)}+\frac{(dg+ef)^2}{2e^3(d-ex)^2}-\frac{g^2\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(2*e^3*(d - e*x)^2) - (2*g*(e*f + d*g))/(e^3*(d - e*x)) - (g^
2*Log[d - e*x])/e^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3}\,dx\\ &= \int \left(\frac{(ef+dg)^2}{e^2(d-ex)^3}-\frac{2g(ef+dg)}{e^2(d-ex)^2}+\frac{g^2}{e^2(d-ex)}\right)dx\\ &= \frac{(ef+dg)^2}{2e^3(d-ex)^2}-\frac{2g(ef+dg)}{e^3(d-ex)}-\frac{g^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.80

$$\frac{\frac{(dg+ef)(e(f+4gx)-3dg)}{(d-ex)^2}-2g^2\log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (((e*f + d*g)*(-3*d*g + e*(f + 4*g*x)))/(d - e*x)^2 - 2*g^2*Log[d - e*x])/(
2*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3, x]
```

fricas [A] time = 0.38, size = 100, normalized size = 1.64

$$\frac{e^2f^2-2\,defg-3\,d^2g^2+4\left(e^2fg+deg^2\right)x-2\left(e^2g^2x^2-2\,deg^2x+d^2g^2\right)\log\left(ex-d\right)}{2\left(e^5x^2-2\,de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x - 2*(e^2*g^2
*x^2 - 2*d*e*g^2*x + d^2*g^2)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)
```

giac [B] time = 0.20, size = 195, normalized size = 3.20

$$-\frac{dg^2e^{(-3)}\log\left(\frac{|2\,xe^2-2\,|d|e|}{|2\,xe^2+2\,|d|e|}\right)}{2\,|d|}-\frac{1}{2}\,g^2e^{(-3)}\log\left(|x^2e^2-d^2|\right)+\frac{\left(4\left(d^2g^2e^4+dfge^5\right)x^3+\left(5\,d^3g^2e^3+6\,d^2fge^4+df^2e^5\right)x^2-2\left(d^4g^2e^2-d^2f^2e^4\right)x-\left(3\,d^5g^2e^3+2\,d^4fge^4-d^3f^2e^5\right)e^{(-2)}\right)e^{(-4)}}{2\left(x^2e^2-d^2\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*d*g^2*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/
abs(d) - 1/2*g^2*e^(-3)*log(abs(x^2*e^2 - d^2)) + 1/2*(4*(d^2*g^2*e^4 + d*f
*g*e^5)*x^3 + (5*d^3*g^2*e^3 + 6*d^2*f*g*e^4 + d*f^2*e^5)*x^2 - 2*(d^4*g^2*
e^2 - d^2*f^2*e^4)*x - (3*d^5*g^2*e^3 + 2*d^4*f*g*e^4 - d^3*f^2*e^5)*e^(-2)
)*e^(-4)/((x^2*e^2 - d^2)^2*d)
```

maple [A] time = 0.01, size = 105, normalized size = 1.72

$$\frac{d^2g^2}{2\left(ex-d\right)^2e^3}+\frac{dfg}{\left(ex-d\right)^2e^2}+\frac{f^2}{2\left(ex-d\right)^2e}+\frac{2dg^2}{\left(ex-d\right)e^3}+\frac{2fg}{\left(ex-d\right)e^2}-\frac{g^2\ln\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/e^3*g^2*ln(e*x-d)+1/2/e^3/(e*x-d)^2*d^2*g^2+1/e^2/(e*x-d)^2*d*f*g+1/2/e/
(e*x-d)^2*f^2+2/(e*x-d)*d/e^3*g^2+2/(e*x-d)/e^2*f*g
```

maxima [A] time = 0.44, size = 81, normalized size = 1.33

$$\frac{e^2f^2-2\,defg-3\,d^2g^2+4\left(e^2fg+deg^2\right)x}{2\left(e^5x^2-2\,de^4x+d^2e^3\right)}-\frac{g^2\log\left(ex-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x)/(e^5*x^2 -
2*d*e^4*x + d^2*e^3) - g^2*log(e*x - d)/e^3
```

mupad [B] time = 0.07, size = 80, normalized size = 1.31

$$-\frac{\frac{3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2}{2\,e^3}-\frac{2\,g\,x\,\left(d\,g+e\,f\right)}{e^2}}{d^2-2\,d\,e\,x+e^2\,x^2}-\frac{g^2\,\ln\left(e\,x-d\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((3*d^2*g^2 - e^2*f^2 + 2*d*e*f*g)/(2*e^3) - (2*g*x*(d*g + e*f))/e^2)/(d^
2 + e^2*x^2 - 2*d*e*x) - (g^2*log(e*x - d))/e^3
```

sympy [A] time = 0.54, size = 83, normalized size = 1.36

$$-\frac{3d^2g^2+2defg-e^2f^2+x\left(-4deg^2-4e^2fg\right)}{2d^2e^3-4de^4x+2e^5x^2}-\frac{g^2\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(3*d**2*g**2 + 2*d*e*f*g - e**2*f**2 + x*(-4*d*e*g**2 - 4*e**2*f*g))/(2*d*
*2*e**3 - 4*d*e**4*x + 2*e**5*x**2) - g**2*log(-d + e*x)/e**3
```

3.376 $\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=88

$$\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}+\frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)}+\frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$\frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}+\frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(4*d*e^3*(d - e*x)^2) + ((e*f - 3*d*g)*(e*f + d*g))/(4*d^2*e^
3*(d - e*x)) + ((e*f - d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{2de^2(d-ex)^3}+\frac{(ef-3dg)(ef+dg)}{4d^2e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2\left(d^2-e^2x^2\right)}\right)dx\\
&= \frac{(ef+dg)^2}{4de^3(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\int \frac{1}{d^2-e^2x^2}\,dx}{4d^2e^2}\\
&= \frac{(ef+dg)^2}{4de^3(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 1.02

$$\frac{-\frac{2d(dg+ef)\left(2d^2g-de(2f+3gx)+e^2fx\right)}{(d-ex)^2}+(ef-dg)^2(-\log(d-ex))+(ef-dg)^2\log(d+ex)}{8d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((-2*d*(e*f + d*g)*(2*d^2*g + e^2*f*x - d*e*(2*f + 3*g*x)))/(d - e*x)^2 - (
e*f - d*g)^2*Log[d - e*x] + (e*f - d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3, x]
```

fricas [B] time = 0.41, size = 271, normalized size = 3.08

$$\frac{4\,d^2e^2f^2-4\,d^4g^2-2\left(de^3f^2-2\,d^2e^2fg-3\,d^3eg^2\right)x+\left(d^2e^2f^2-2\,d^3efg+d^4g^2+\left(e^4f^2-2\,de^3fg+d^2e^2g^2\right)x^2-2\left(de^3f^2-2\,d^2e^2fg+d^3eg^2\right)x\right)\log\left(ex+d\right)-\left(d^2e^2f^2-2\,d^3efg+d^4g^2+\left(e^4f^2-2\,de^3fg+d^2e^2g^2\right)x^2-2\left(de^3f^2-2\,d^2e^2fg+d^3eg^2\right)x\right)\log\left(ex-d\right)}{8\left(d^3e^5x^2-2\,d^4e^4x+d^5e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*d^2*e^2*f^2 - 4*d^4*g^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g - 3*d^3*e*g^2
)*x + (d^2*e^2*f^2 - 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 - 2*d*e^3*f*g + d^2*e
^2*g^2)*x^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x + d) - (
d^2*e^2*f^2 - 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 - 2*d*e^3*f*g + d^2*e^2*g^2)
*x^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x - d))/(d^3*e^5*
x^2 - 2*d^4*e^4*x + d^5*e^3)
```

giac [B] time = 0.17, size = 197, normalized size = 2.24

$$-\frac{\left(d^2g^2e^2-2\,dfge^3+f^2e^4\right)e^{(-5)}\log\left(\frac{|2\,xe^2-2\,|d|e|}{|2\,xe^2+2\,|d|e|}\right)}{8\,d^2|d|}+\frac{\left(3\,d^2g^2x^3e^4+4\,d^3g^2x^2e^3-d^4g^2xe^2-2\,d^5g^2e+2\,dfgx^3e^5+4\,d^2fgx^2e^4+2\,d^3fgxe^3-f^2x^3e^6+3\,d^2f^2xe^4+2\,d^3f^2e^3\right)e^{(-4)}}{4\left(x^2e^2-d^2\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d
)*e)/abs(2*x*e^2 + 2*abs(d)*e))/(d^2*abs(d)) + 1/4*(3*d^2*g^2*x^3*e^4 + 4*d
^3*g^2*x^2*e^3 - d^4*g^2*x*e^2 - 2*d^5*g^2*e + 2*d*f*g*x^3*e^5 + 4*d^2*f*g*
x^2*e^4 + 2*d^3*f*g*x*e^3 - f^2*x^3*e^6 + 3*d^2*f^2*x*e^4 + 2*d^3*f^2*e^3)*
e^(-4)/((x^2*e^2 - d^2)^2*d^2)
```

maple [B] time = 0.01, size = 218, normalized size = 2.48

$$\frac{dg^2}{4\left(ex-d\right)^2e^3}+\frac{f^2}{4\left(ex-d\right)^2de}+\frac{fg}{2\left(ex-d\right)^2e^2}+\frac{fg}{2\left(ex-d\right)de^2}-\frac{g^2\ln\left(ex-d\right)}{8de^3}+\frac{g^2\ln\left(ex+d\right)}{8de^3}-\frac{f^2}{4\left(ex-d\right)d^2e}+\frac{fg\ln\left(ex-d\right)}{4d^2e^2}-\frac{fg\ln\left(ex+d\right)}{4d^2e^2}-\frac{f^2\ln\left(ex-d\right)}{8d^3e}+\frac{f^2\ln\left(ex+d\right)}{8d^3e}+\frac{3g^2}{4\left(ex-d\right)e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 3/4/(e*x-d)/e^3*g^2+1/2/(e*x-d)/d/e^2*f*g-1/4/(e*x-d)/d^2/e*f^2+1/4/e^3*d/(
e*x-d)^2*g^2+1/2/e^2/(e*x-d)^2*f*g+1/4/e/d/(e*x-d)^2*f^2-1/8/d/e^3*g^2*ln(e
*x-d)+1/4/d^2/e^2*f*g*ln(e*x-d)-1/8/d^3/e*f^2*ln(e*x-d)+1/8/d/e^3*g^2*ln(e*
x+d)-1/4/d^2/e^2*f*g*ln(e*x+d)+1/8/d^3/e*f^2*ln(e*x+d)
```

maxima [A] time = 0.46, size = 150, normalized size = 1.70

$$\frac{2\,de^2f^2-2\,d^3g^2-\left(e^3f^2-2\,de^2fg-3\,d^2eg^2\right)x}{4\left(d^2e^5x^2-2\,d^3e^4x+d^4e^3\right)}+\frac{\left(e^2f^2-2\,defg+d^2g^2\right)\log\left(ex+d\right)}{8\,d^3e^3}-\frac{\left(e^2f^2-2\,defg+d^2g^2\right)\log\left(ex-d\right)}{8\,d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*e^2*f^2 - 2*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)/(d^
2*e^5*x^2 - 2*d^3*e^4*x + d^4*e^3) + 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*lo
g(e*x + d)/(d^3*e^3) - 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^
3*e^3)
```

mupad [B] time = 0.13, size = 103, normalized size = 1.17

$$\frac{\operatorname{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g-e\,f\right)}^2}{4\,d^3\,e^3}-\frac{\frac{d^2\,g^2-e^2\,f^2}{2\,d\,e^3}-\frac{x\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2\right)}{4\,d^2\,e^2}}{d^2-2\,d\,e\,x+e^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (atanh((e*x)/d)*(d*g - e*f)^2)/(4*d^3*e^3) - ((d^2*g^2 - e^2*f^2)/(2*d*e^3)
 - (x*(3*d^2*g^2 - e^2*f^2 + 2*d*e*f*g))/(4*d^2*e^2))/(d^2 + e^2*x^2 - 2*d*
e*x)
```

sympy [B] time = 1.01, size = 185, normalized size = 2.10

$$-\frac{2d^3g^2-2de^2f^2+x\left(-3d^2eg^2-2de^2fg+e^3f^2\right)}{4d^4e^3-8d^3e^4x+4d^2e^5x^2}-\frac{\left(dg-ef\right)^2\log\left(-\frac{d\left(dg-ef\right)^2}{e\left(d^2g^2-2defg+e^2f^2\right)}+x\right)}{8d^3e^3}+\frac{\left(dg-ef\right)^2\log\left(\frac{d\left(dg-ef\right)^2}{e\left(d^2g^2-2defg+e^2f^2\right)}+x\right)}{8d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(2*d**3*g**2 - 2*d*e**2*f**2 + x*(-3*d**2*e*g**2 - 2*d*e**2*f*g + e**3*f**
2))/(4*d**4*e**3 - 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g - e*f)**2*log(-
d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3)
 + (d*g - e*f)**2*log(d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**
2)) + x)/(8*d**3*e**3)
```

3.377 $\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=122

$$\frac{(dg+3ef)(ef-dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}$$

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {799, 88, 208}

$$\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2}{8d^2e^3(d-ex)^2}+\frac{(dg+3ef)(ef-dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(8*d^2*e^3*(d - e*x)^2) + (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d -
 e*x)) - (e*f - d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f - d*g)*(3*e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 799

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c*x)/g)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^2}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{4d^2e^2(d-ex)^3}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d-ex)^2}+\frac{(-ef+dg)^2}{8d^3e^2(d+ex)^2}+\frac{(ef-dg)(3ef+dg)}{8d^3e^2\left(d^2-e^2x^2\right)}\right)dx\\
&= \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{((ef-dg)(3ef+dg))\int \frac{1}{d^2-e^2x^2}\,dx}{8d^3e^2}\\
&= \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(ef-dg)(3ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 140, normalized size = 1.15

$$\frac{\frac{4de^2f^2-4d^3g^2}{d-ex}+\left(d^2g^2+2defg-3e^2f^2\right)\log(d-ex)+\left(-d^2g^2-2defg+3e^2f^2\right)\log(d+ex)+\frac{2d^2(dg+ef)^2}{(d-ex)^2}-\frac{2d(ef-dg)^2}{d+ex}}{16d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((2*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (4*d*e^2*f^2 - 4*d^3*g^2)/(d - e*x) -
(2*d*(e*f - d*g)^2)/(d + e*x) + (-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2)*Log[d -
e*x] + (3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3, x]
```

fricas [B] time = 0.40, size = 417, normalized size = 3.42

$$\frac{4d^3e^2f^2+8d^4efg-4d^5g^2-2\left(3de^4f^2-2d^2e^3fg-d^3e^2g^2\right)x^2+2\left(3d^2e^3f^2-2d^3e^2fg+3d^4eg^2\right)x+\left(3d^3e^2f^2-2d^4efg-d^5g^2+\left(3e^5f^2-2de^4fg-d^2e^3g^2\right)x^3-\left(3de^4f^2-2d^2e^3fg-d^3e^2g^2\right)x^2-\left(3d^2e^3f^2-2d^3e^2fg-d^4eg^2\right)x\right)\log(ex+d)-\left(3d^3e^2f^2-2d^4efg-d^5g^2+\left(3e^5f^2-2de^4fg-d^2e^3g^2\right)x^3-\left(3de^4f^2-2d^2e^3fg-d^3e^2g^2\right)x^2-\left(3d^2e^3f^2-2d^3e^2fg-d^4eg^2\right)x\right)\log(ex-d)}{16\left(d^4e^6x^3-d^5e^5x^2-d^6e^4x+d^7e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d^3*e^2*f^2 + 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 - 2*d^2*e^3*
f*g - d^3*e^2*g^2)*x^2 + 2*(3*d^2*e^3*f^2 - 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x
+ (3*d^3*e^2*f^2 - 2*d^4*e*f*g - d^5*g^2 + (3*e^5*f^2 - 2*d*e^4*f*g - d^2*e
^3*g^2)*x^3 - (3*d*e^4*f^2 - 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - (3*d^2*e^3*
f^2 - 2*d^3*e^2*f*g - d^4*e*g^2)*x)*log(e*x + d) - (3*d^3*e^2*f^2 - 2*d^4*e
*f*g - d^5*g^2 + (3*e^5*f^2 - 2*d*e^4*f*g - d^2*e^3*g^2)*x^3 - (3*d*e^4*f^2
 - 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - (3*d^2*e^3*f^2 - 2*d^3*e^2*f*g - d^4*
e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 - d^5*e^5*x^2 - d^6*e^4*x + d^7*e^3)
```

giac [A] time = 0.17, size = 191, normalized size = 1.57

$$\frac{\left(d^2g^2+2dfge-3f^2e^2\right)e^{(-3)}\log\left(\frac{|2xe^2-2|d||e||}{|2xe^2+2|d||e||}\right)}{16d^3|d|}+\frac{\left(d^2g^2x^3e^4+4d^3g^2x^2e^3+d^4g^2xe^2-2d^5g^2e+2dfgx^3e^5+2d^3fgxe^3+4d^4fge^2-3f^2x^3e^6+5d^2f^2xe^4+2d^3f^2e^3\right)e^{(-4)}}{8\left(x^2e^2-d^2\right)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(d^2*g^2 + 2*d*f*g*e - 3*f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)
/abs(2*x*e^2 + 2*abs(d)*e))/(d^3*abs(d)) + 1/8*(d^2*g^2*x^3*e^4 + 4*d^3*g^2
*x^2*e^3 + d^4*g^2*x*e^2 - 2*d^5*g^2*e + 2*d*f*g*x^3*e^5 + 2*d^3*f*g*x*e^3
+ 4*d^4*f*g*e^2 - 3*f^2*x^3*e^6 + 5*d^2*f^2*x*e^4 + 2*d^3*f^2*e^3)*e^(-4)/(
(x^2*e^2 - d^2)^2*d^3)
```

maple [B] time = 0.03, size = 257, normalized size = 2.11

$$\frac{fg}{4(ex-d)^2de^2}+\frac{f^2}{8(ex-d)^2d^2e}+\frac{g^2}{8(ex-d)^2e^3}+\frac{g^2}{4(ex-d)de^3}-\frac{g^2}{8(ex+d)de^3}+\frac{fg}{4(ex+d)d^2e^2}+\frac{g^2\ln(ex-d)}{16d^2e^3}-\frac{g^2\ln(ex+d)}{16d^2e^3}-\frac{f^2}{4(ex-d)d^3e}-\frac{f^2}{8(ex+d)d^3e}+\frac{fg\ln(ex-d)}{8d^3e^2}-\frac{fg\ln(ex+d)}{8d^3e^2}-\frac{3f^2\ln(ex-d)}{16d^4e}+\frac{3f^2\ln(ex+d)}{16d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 1/4/(e*x-d)/d/e^3*g^2-1/4/(e*x-d)/d^3/e*f^2+1/8/e^3/(e*x-d)^2*g^2+1/4/e^2/d
/(e*x-d)^2*f*g+1/8/e/d^2/(e*x-d)^2*f^2+1/16/d^2/e^3*g^2*ln(e*x-d)+1/8/d^3/e
^2*f*g*ln(e*x-d)-3/16/d^4/e*f^2*ln(e*x-d)-1/16/d^2/e^3*g^2*ln(e*x+d)-1/8/d^
3/e^2*f*g*ln(e*x+d)+3/16/d^4/e*f^2*ln(e*x+d)-1/8/(e*x+d)/d/e^3*g^2+1/4/(e*x
+d)/d^2/e^2*f*g-1/8/(e*x+d)/d^3/e*f^2
```

maxima [A] time = 0.47, size = 211, normalized size = 1.73

$$\frac{2 d^2e^2f^2 + 4 d^3efg - 2 d^4g^2 - \left(3 e^4f^2 - 2 de^3fg - d^2e^2g^2\right)x^2 + \left(3 de^3f^2 - 2 d^2e^2fg + 3 d^3eg^2\right)x}{8 \left(d^3e^6x^3 - d^4e^5x^2 - d^5e^4x + d^6e^3\right)} + \frac{\left(3 e^2f^2 - 2 defg - d^2g^2\right)\log\left(ex + d\right)}{16 d^4e^3} - \frac{\left(3 e^2f^2 - 2 defg - d^2g^2\right)\log\left(ex - d\right)}{16 d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*d^2*e^2*f^2 + 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 - 2*d*e^3*f*g - d
^2*e^2*g^2)*x^2 + (3*d*e^3*f^2 - 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6*x
^3 - d^4*e^5*x^2 - d^5*e^4*x + d^6*e^3) + 1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2
*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*log(e
*x - d)/(d^4*e^3)
```

mupad [B] time = 2.64, size = 198, normalized size = 1.62

$$\frac{\frac{-d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{4\,d\,e^3}+\frac{x\,\left(3\,d^2\,g^2-2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2\right)}{8\,d^3\,e}}{d^3-d^2\,e\,x-d\,e^2\,x^2+e^3\,x^3}-\frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g-e\,f\right)\,\left(d\,g+3\,e\,f\right)}{d\,\left(d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2\right)}\right)\,\left(d\,g-e\,f\right)\,\left(d\,g+3\,e\,f\right)}{8\,d^4\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2)^3,x)
```

```
[Out] ((e^2*f^2 - d^2*g^2 + 2*d*e*f*g)/(4*d*e^3) + (x*(3*d^2*g^2 + 3*e^2*f^2 - 2*
d*e*f*g))/(8*d^2*e^2) + (x^2*(d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g))/(8*d^3*e))/
(d^3 + e^3*x^3 - d*e^2*x^2 - d^2*e*x) - (atanh((e*x*(d*g - e*f)*(d*g + 3*e*
f))/(d*(d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g)))*(d*g - e*f)*(d*g + 3*e*f))/(8*d^
4*e^3)
```

sympy [B] time = 1.32, size = 277, normalized size = 2.27

$$-\frac{2d^4g^2 - 4d^3efg - 2d^2e^2f^2 + x^2\left(-d^2e^2g^2 - 2de^3fg + 3e^4f^2\right) + x\left(-3d^3eg^2 + 2d^2e^2fg - 3de^3f^2\right)}{8d^6e^3 - 8d^5e^4x - 8d^4e^5x^2 + 8d^3e^6x^3} + \frac{\left(dg - ef\right)\left(dg + 3ef\right)\log\left(-\frac{d(dg-ef)(dg+3ef)}{e(d^2g^2+2defg-3e^2f^2)} + x\right)}{16d^4e^3} - \frac{\left(dg - ef\right)\left(dg + 3ef\right)\log\left(\frac{d(dg-ef)(dg+3ef)}{e(d^2g^2+2defg-3e^2f^2)} + x\right)}{16d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(2*d**4*g**2 - 4*d**3*e*f*g - 2*d**2*e**2*f**2 + x**2*(-d**2*e**2*g**2 - 2
*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 2*d**2*e**2*f*g - 3*d*e**3
*f**2))/(8*d**6*e**3 - 8*d**5*e**4*x - 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3)
 + (d*g - e*f)*(d*g + 3*e*f)*log(-d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2
 + 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - e*f)*(d*g + 3*e*f
)*log(d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 2*d*e*f*g - 3*e**2*f**2))
 + x)/(16*d**4*e**3)
```

3.378 $\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^3} \, dx$

Optimal. Leaf size=127

$$\frac{(f+gx)\left(d^2g+e^2fx\right)}{4d^2e^2\left(d^2-e^2x^2\right)^2}+\frac{\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}+\frac{x\left(3e^2f^2-d^2g^2\right)+2d^2fg}{8d^4e^2\left(d^2-e^2x^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {739, 639, 208}

$$\frac{x\left(3e^2f^2-d^2g^2\right)+2d^2fg}{8d^4e^2\left(d^2-e^2x^2\right)}+\frac{\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}+\frac{(f+gx)\left(d^2g+e^2fx\right)}{4d^2e^2\left(d^2-e^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(4*d^2*e^2*(d^2 - e^2*x^2)^2) + (2*d^2*f*g +
(3*e^2*f^2 - d^2*g^2)*x)/(8*d^4*e^2*(d^2 - e^2*x^2)) + ((3*e^2*f^2 - d^2*g^
2)*ArcTanh[(e*x)/d])/(8*d^5*e^3)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^3} \, dx &= \frac{\left(d^2g+e^2fx\right)(f+gx)}{4d^2e^2\left(d^2-e^2x^2\right)^2}-\frac{\int \frac{-3e^2f^2+d^2g^2-2e^2fgx}{\left(d^2-e^2x^2\right)^2} \, dx}{4d^2e^2}\\
&= \frac{\left(d^2g+e^2fx\right)(f+gx)}{4d^2e^2\left(d^2-e^2x^2\right)^2}+\frac{2d^2fg+\left(3e^2f^2-d^2g^2\right)x}{8d^4e^2\left(d^2-e^2x^2\right)}-\frac{\left(-\frac{3e^2f^2}{d^2}+g^2\right)\int \frac{1}{d^2-e^2x^2} \, dx}{8d^2e^2}\\
&= \frac{\left(d^2g+e^2fx\right)(f+gx)}{4d^2e^2\left(d^2-e^2x^2\right)^2}+\frac{2d^2fg+\left(3e^2f^2-d^2g^2\right)x}{8d^4e^2\left(d^2-e^2x^2\right)}+\frac{\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 110, normalized size = 0.87

$$\frac{d^5eg(4f+gx)+d^3e^3x\left(5f^2+g^2x^2\right)+\left(d^2-e^2x^2\right)^2\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)-3de^5f^2x^3}{8d^5e^3\left(d^2-e^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (-3*d*e^5*f^2*x^3 + d^5*e*g*(4*f + g*x) + d^3*e^3*x*(5*f^2 + g^2*x^2) + (3*
e^2*f^2 - d^2*g^2)*(d^2 - e^2*x^2)^2*ArcTanh[(e*x)/d])/(8*d^5*e^3*(d^2 - e^
2*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/(d^2 - e^2*x^2)^3, x]
```

fricas [B] time = 0.40, size = 252, normalized size = 1.98

$$\frac{8\,d^5efg-2\left(3\,de^5f^2-d^3e^3g^2\right)x^3+2\left(5\,d^3e^3f^2+d^5eg^2\right)x+\left(3\,d^4e^2f^2-d^6g^2+\left(3\,e^6f^2-d^2e^4g^2\right)x^4-2\left(3\,d^2e^4f^2-d^4e^2g^2\right)x^2\right)\log\left(ex+d\right)-\left(3\,d^4e^2f^2-d^6g^2+\left(3\,e^6f^2-d^2e^4g^2\right)x^4-2\left(3\,d^2e^4f^2-d^4e^2g^2\right)x^2\right)\log\left(ex-d\right)}{16\left(d^5e^7x^4-2\,d^7e^5x^2+d^9e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(8*d^5*e*f*g - 2*(3*d*e^5*f^2 - d^3*e^3*g^2)*x^3 + 2*(5*d^3*e^3*f^2 +
d^5*e*g^2)*x + (3*d^4*e^2*f^2 - d^6*g^2 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2
*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)*log(e*x + d) - (3*d^4*e^2*f^2 - d^6*g^2
 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)*log
(e*x - d))/(d^5*e^7*x^4 - 2*d^7*e^5*x^2 + d^9*e^3)
```

giac [A] time = 0.17, size = 127, normalized size = 1.00

$$\frac{\left(d^2g^2-3\,f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d||e|\right|}{\left|2\,xe^2+2\,|d||e|\right|}\right)}{16\,d^4|d|}+\frac{\left(d^2g^2x^3e^2+d^4g^2x+4\,d^4fg-3\,f^2x^3e^4+5\,d^2f^2xe^2\right)e^{(-2)}}{8\left(x^2e^2-d^2\right)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(d^2*g^2 - 3*f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2
 + 2*abs(d)*e))/(d^4*abs(d)) + 1/8*(d^2*g^2*x^3*e^2 + d^4*g^2*x + 4*d^4*f*g
 - 3*f^2*x^3*e^4 + 5*d^2*f^2*x*e^2)*e^(-2)/((x^2*e^2 - d^2)^2*d^4)
```

maple [B] time = 0.02, size = 298, normalized size = 2.35

$$\frac{g^2}{16\left(ex-d\right)^2de^3}-\frac{g^2}{16\left(ex+d\right)^2de^3}+\frac{fg}{8\left(ex-d\right)^2d^2e^2}+\frac{fg}{8\left(ex+d\right)^2d^2e^2}+\frac{f^2}{16\left(ex-d\right)^2d^3e}-\frac{f^2}{16\left(ex+d\right)^2d^3e}+\frac{g^2}{16\left(ex-d\right)d^2e^3}+\frac{g^2}{16\left(ex+d\right)d^2e^3}-\frac{fg}{8\left(ex-d\right)d^3e^2}+\frac{fg}{8\left(ex+d\right)d^3e^2}+\frac{g^2\ln\left(ex-d\right)}{16d^3e^3}-\frac{g^2\ln\left(ex+d\right)}{16d^3e^3}-\frac{3f^2}{16\left(ex-d\right)d^4e}-\frac{3f^2}{16\left(ex+d\right)d^4e}-\frac{3f^2\ln\left(ex-d\right)}{16d^5e}+\frac{3f^2\ln\left(ex+d\right)}{16d^5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 1/16/d^3/e^3*g^2*ln(e*x-d)-3/16/d^5/e*f^2*ln(e*x-d)+1/16/e^3/d/(e*x-d)^2*g^
2+1/8/e^2/d^2/(e*x-d)^2*f*g+1/16/e/d^3/(e*x-d)^2*f^2+1/16/(e*x-d)/d^2/e^3*g
^2-1/8/(e*x-d)/d^3/e^2*f*g-3/16/(e*x-d)/d^4/e*f^2-1/16/d^3/e^3*g^2*ln(e*x+d
)+3/16/d^5/e*f^2*ln(e*x+d)+1/16/(e*x+d)/d^2/e^3*g^2+1/8/(e*x+d)/d^3/e^2*f*g
-3/16/(e*x+d)/d^4/e*f^2-1/16/(e*x+d)^2/d/e^3*g^2+1/8/(e*x+d)^2/d^2/e^2*f*g-
1/16/(e*x+d)^2/d^3/e*f^2
```

maxima [A] time = 0.45, size = 152, normalized size = 1.20

$$\frac{4\,d^4fg-\left(3\,e^4f^2-d^2e^2g^2\right)x^3+\left(5\,d^2e^2f^2+d^4g^2\right)x}{8\left(d^4e^6x^4-2\,d^6e^4x^2+d^8e^2\right)}+\frac{\left(3\,e^2f^2-d^2g^2\right)\log\left(ex+d\right)}{16\,d^5e^3}-\frac{\left(3\,e^2f^2-d^2g^2\right)\log\left(ex-d\right)}{16\,d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*d^4*f*g - (3*e^4*f^2 - d^2*e^2*g^2)*x^3 + (5*d^2*e^2*f^2 + d^4*g^2)*
x)/(d^4*e^6*x^4 - 2*d^6*e^4*x^2 + d^8*e^2) + 1/16*(3*e^2*f^2 - d^2*g^2)*log
(e*x + d)/(d^5*e^3) - 1/16*(3*e^2*f^2 - d^2*g^2)*log(e*x - d)/(d^5*e^3)
```

mupad [B] time = 0.10, size = 114, normalized size = 0.90

$$\frac{\frac{x^3\left(d^2\,g^2-3\,e^2\,f^2\right)}{8\,d^4}+\frac{f\,g}{2\,e^2}+\frac{x\left(d^2\,g^2+5\,e^2\,f^2\right)}{8\,d^2\,e^2}}{d^4-2\,d^2\,e^2\,x^2+e^4\,x^4}-\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\left(d^2\,g^2-3\,e^2\,f^2\right)}{8\,d^5\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2)^3,x)
```

```
[Out] ((x^3*(d^2*g^2 - 3*e^2*f^2))/(8*d^4) + (f*g)/(2*e^2) + (x*(d^2*g^2 + 5*e^2*
f^2))/(8*d^2*e^2))/(d^4 + e^4*x^4 - 2*d^2*e^2*x^2) - (atanh((e*x)/d)*(d^2*g
^2 - 3*e^2*f^2))/(8*d^5*e^3)
```

sympy [A] time = 1.00, size = 144, normalized size = 1.13

$$-\frac{-4d^4fg+x^3\left(-d^2e^2g^2+3e^4f^2\right)+x\left(-d^4g^2-5d^2e^2f^2\right)}{8d^8e^2-16d^6e^4x^2+8d^4e^6x^4}+\frac{\left(d^2g^2-3e^2f^2\right)\log\left(-\frac{d}{e}+x\right)}{16d^5e^3}-\frac{\left(d^2g^2-3e^2f^2\right)\log\left(\frac{d}{e}+x\right)}{16d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-4*d**4*f*g + x**3*(-d**2*e**2*g**2 + 3*e**4*f**2) + x*(-d**4*g**2 - 5*d*
*2*e**2*f**2))/(8*d**8*e**2 - 16*d**6*e**4*x**2 + 8*d**4*e**6*x**4) + (d**2
*g**2 - 3*e**2*f**2)*log(-d/e + x)/(16*d**5*e**3) - (d**2*g**2 - 3*e**2*f**
2)*log(d/e + x)/(16*d**5*e**3)
```

3.379 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=188

$$\frac{f(dg+ef)}{8d^5e^2(d-ex)}-\frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2}+\frac{(dg+ef)^2}{32d^4e^3(d-ex)^2}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}+\frac{\left(-d^2g^2+2defg+5e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^6e^3}-$$

Rubi [A] time = 0.21, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}+\frac{\left(-d^2g^2+2defg+5e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^6e^3}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2}+\frac{f(dg+ef)}{8d^5e^2(d-ex)}+\frac{(dg+ef)^2}{32d^4e^3(d-ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] (e*f + d*g)^2/(32*d^4*e^3*(d - e*x)^2) + (f*(e*f + d*g))/(8*d^5*e^2*(d - e*
x)) - (e*f - d*g)^2/(24*d^3*e^3*(d + e*x)^3) - ((e*f - d*g)*(3*e*f + d*g))/
(32*d^4*e^3*(d + e*x)^2) - (3*e^2*f^2 - d^2*g^2)/(16*d^5*e^3*(d + e*x)) + (
(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(16*d^6*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^4}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{16d^4e^2(d-ex)^3}+\frac{f(ef+dg)}{8d^5e(d-ex)^2}+\frac{(-ef+dg)^2}{8d^3e^2(d+ex)^4}+\frac{(ef-dg)(3ef+dg)}{16d^4e^2(d+ex)^3}+\right.\\
&= \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2}+\frac{f(ef+dg)}{8d^5e^2(d-ex)}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2}-\\
&= \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2}+\frac{f(ef+dg)}{8d^5e^2(d-ex)}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2}-
\end{aligned}$$

Mathematica [A] time = 0.15, size = 197, normalized size = 1.05

$$\frac{-\frac{4d^3(ef-dg)^2}{(d+ex)^3}+\frac{3d^2\left(d^2g^2+2defg-3e^2f^2\right)}{(d+ex)^2}+\frac{6d\left(d^2g^2-3e^2f^2\right)}{d+ex}+3\left(d^2g^2-2defg-5e^2f^2\right)\log(d-ex)+3\left(-d^2g^2+2defg+5e^2f^2\right)\log(d+ex)+\frac{3d^2(dg+ef)^2}{(d-ex)^2}+\frac{12def(dg+ef)}{d-ex}}{96d^6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] ((3*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (12*d*e*f*(e*f + d*g))/(d - e*x) - (4*
d^3*(e*f - d*g)^2)/(d + e*x)^3 + (3*d^2*(-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2))
/(d + e*x)^2 + (6*d*(-3*e^2*f^2 + d^2*g^2))/(d + e*x) + 3*(-5*e^2*f^2 - 2*d
*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d
+ e*x])/(96*d^6*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3), x]
```

fricas [B] time = 0.40, size = 662, normalized size = 3.52

$$-\frac{16d^5e^2f^2-32d^6efg-8d^7g^2+6\left(5de^6f^2+2d^2e^5fg-d^3e^4g^2\right)x^4+6\left(5d^2e^5f^2+2d^3e^4fg-d^4e^3g^2\right)x^3-10\left(5d^3e^4f^2+2d^4e^3fg-d^5e^2g^2\right)x^2-2\left(25d^4e^3f^2+10d^5e^2fg+7d^6eg^2\right)x-3\left(5d^5e^2f^2+2d^6efg-d^7g^2+\left(5e^7f^2+2de^6fg-d^2e^5g^2\right)x^5+\left(5de^6f^2+2d^2e^5fg-d^3e^4g^2\right)x^4-2\left(5d^2e^5f^2+2d^3e^4fg-d^4e^3g^2\right)x^3-2\left(5d^3e^4f^2+2d^4e^3fg-d^5e^2g^2\right)x^2+\left(5d^4e^3f^2+2d^5e^2fg-d^6eg^2\right)x\right)\log(ex+d)+3\left(5d^5e^2f^2+2d^6efg-d^7g^2+\left(5e^7f^2+2de^6fg-d^2e^5g^2\right)x^5+\left(5de^6f^2+2d^2e^5fg-d^3e^4g^2\right)x^4-2\left(5d^2e^5f^2+2d^3e^4fg-d^4e^3g^2\right)x^3-2\left(5d^3e^4f^2+2d^4e^3fg-d^5e^2g^2\right)x^2+\left(5d^4e^3f^2+2d^5e^2fg-d^6eg^2\right)x\right)\log(ex-d)}{96\left(d^6e^8x^5+d^7e^7x^4-2d^8e^6x^3-2d^9e^5x^2+d^{10}e^4x+d^{11}e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(16*d^5*e^2*f^2 - 32*d^6*e*f*g - 8*d^7*g^2 + 6*(5*d*e^6*f^2 + 2*d^2*e
^5*f*g - d^3*e^4*g^2)*x^4 + 6*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^2)
*x^3 - 10*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - 2*(25*d^4*e^3
*f^2 + 10*d^5*e^2*f*g + 7*d^6*e*g^2)*x - 3*(5*d^5*e^2*f^2 + 2*d^6*e*f*g - d
^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^2 + 2*d^2
*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^
2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*d^4*e^3*f
^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x)*log(e*x + d) + 3*(5*d^5*e^2*f^2 + 2*d^6*
e*f*g - d^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^
2 + 2*d^2*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d
^4*e^3*g^2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*
d^4*e^3*f^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x)*log(e*x - d))/(d^6*e^8*x^5 + d^
7*e^7*x^4 - 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 + d^10*e^4*x + d^11*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -(d^2
*exp(1)^4*g^2-2*d*exp(1)^5*g*f+exp(1)^6*f^2)/(exp(2)^3*d^6*exp(1)-3*exp(2)^
2*d^6*exp(1)^3+3*exp(2)*d^6*exp(1)^5-d^6*exp(1)^7)*ln(abs(x*exp(1)+d))-(-d^
2*exp(1)^3*g^2+2*d*exp(1)^4*g*f-exp(1)^5*f^2)/(2*exp(2)^3*d^6-6*exp(2)^2*d^
6*exp(1)^2+6*exp(2)*d^6*exp(1)^4-2*d^6*exp(1)^6)*ln(abs(-x^2*exp(2)+d^2))-(
-3*exp(2)^3*f^2+exp(2)^2*d^2*g^2-2*exp(2)^2*d*exp(1)*g*f+10*exp(2)^2*exp(1)
```

```
^2*f^2-6*exp(2)*d^2*exp(1)^2*g^2+12*exp(2)*d*exp(1)^3*g*f-15*exp(2)*exp(1)^
4*f^2-3*d^2*exp(1)^4*g^2+6*d*exp(1)^5*g*f)*1/2/(8*exp(2)^3*d^5-24*exp(2)^2*
d^5*exp(1)^2+24*exp(2)*d^5*exp(1)^4-8*d^5*exp(1)^6)/exp(1)/abs(d)*ln(abs(-2
*x*exp(2)-2*exp(1)*abs(d))/abs(-2*x*exp(2)+2*exp(1)*abs(d)))-((3*exp(2)^5*d
*f^2-exp(2)^4*d^3*g^2+2*exp(2)^4*d^2*exp(1)*g*f-10*exp(2)^4*d*exp(1)^2*f^2-
2*exp(2)^3*d^3*exp(1)^2*g^2+4*exp(2)^3*d^2*exp(1)^3*g*f+7*exp(2)^3*d*exp(1)
^4*f^2+3*exp(2)^2*d^3*exp(1)^4*g^2-6*exp(2)^2*d^2*exp(1)^5*g*f)*x^3+(4*exp(
2)^3*d^4*exp(1)*g^2-8*exp(2)^3*d^3*exp(1)^2*g*f+4*exp(2)^3*d^2*exp(1)^3*f^2
-4*exp(2)^2*d^4*exp(1)^3*g^2+8*exp(2)^2*d^3*exp(1)^4*g*f-4*exp(2)^2*d^2*exp
(1)^5*f^2)*x^2+(-5*exp(2)^4*d^3*f^2-exp(2)^3*d^5*g^2+2*exp(2)^3*d^4*exp(1)*
g*f+14*exp(2)^3*d^3*exp(1)^2*f^2+6*exp(2)^2*d^5*exp(1)^2*g^2-12*exp(2)^2*d^
4*exp(1)^3*g*f-9*exp(2)^2*d^3*exp(1)^4*f^2-5*exp(2)*d^5*exp(1)^4*g^2+10*exp
(2)*d^4*exp(1)^5*g*f)*x-4*exp(2)^3*d^5*g*f+2*exp(2)^3*d^4*exp(1)*f^2-2*exp(
2)^2*d^6*exp(1)*g^2+16*exp(2)^2*d^5*exp(1)^2*g*f-8*exp(2)^2*d^4*exp(1)^3*f^
2-12*exp(2)*d^5*exp(1)^4*g*f+6*exp(2)*d^4*exp(1)^5*f^2+2*d^6*exp(1)^5*g^2)/
8/d^6/exp(2)/(exp(2)-exp(1)^2)^3/(-x^2*exp(2)+d^2)^2
```

maple [A] time = 0.02, size = 348, normalized size = 1.85

$$-\frac{g^2}{24(ex+d)^3 d e^3}+\frac{fg}{12(ex+d)^3 d^2e^2}-\frac{f^2}{24(ex+d)^3 d^3e}+\frac{g^2}{32(ex-d)^2 d^2e^3}+\frac{g^2}{32(ex+d)^2 d^2e^3}+\frac{fg}{16(ex-d)^2 d^3e^2}+\frac{fg}{16(ex+d)^2 d^3e^2}+\frac{f^2}{32(ex-d)^2 d^4e}-\frac{3f^2}{32(ex+d)^2 d^4e}+\frac{g^2}{16(ex+d) d^3e^3}-\frac{fg}{8(ex-d) d^4e^2}+\frac{g^2\ln(ex-d)}{32d^4e^3}-\frac{g^2\ln(ex+d)}{32d^4e^3}-\frac{f^2}{8(ex-d) d^5e}-\frac{3f^2}{16(ex+d) d^5e}-\frac{fg\ln(ex-d)}{16d^5e^2}+\frac{fg\ln(ex+d)}{16d^5e^2}-\frac{5f^2\ln(ex-d)}{32d^6e}+\frac{5f^2\ln(ex+d)}{32d^6e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 1/32/e^3/d^2/(e*x-d)^2*g^2+1/16/e^2/d^3/(e*x-d)^2*f*g+1/32/e/d^4/(e*x-d)^2*
f^2+1/32/d^4/e^3*g^2*ln(e*x-d)-1/16/d^5/e^2*f*g*ln(e*x-d)-5/32/d^6/e*f^2*ln
(e*x-d)-1/8/(e*x-d)/d^4/e^2*f*g-1/8/(e*x-d)/d^5/e*f^2+1/16/e^3/d^3/(e*x+d)*
g^2-3/16/(e*x+d)/d^5/e*f^2+1/32/(e*x+d)^2/d^2/e^3*g^2+1/16/(e*x+d)^2/d^3/e^
2*f*g-3/32/(e*x+d)^2/d^4/e*f^2-1/32/d^4/e^3*g^2*ln(e*x+d)+1/16/d^5/e^2*f*g*
ln(e*x+d)+5/32/d^6/e*f^2*ln(e*x+d)-1/24/(e*x+d)^3/d/e^3*g^2+1/12/e^2/d^2/(e
*x+d)^3*f*g-1/24/(e*x+d)^3/d^3/e*f^2
```

maxima [A] time = 0.50, size = 308, normalized size = 1.64

$$-\frac{8\,d^4e^2f^2-16\,d^5efg-4\,d^6g^2+3\left(5\,e^6f^2+2\,de^5fg-d^2e^4g^2\right)x^4+3\left(5\,de^5f^2+2\,d^2e^4fg-d^3e^3g^2\right)x^3-5\left(5\,d^2e^4f^2+2\,d^3e^3fg-d^4e^2g^2\right)x^2-\left(25\,d^3e^3f^2+10\,d^4e^2fg+7\,d^5eg^2\right)x}{48\left(d^5e^8x^5+d^6e^7x^4-2\,d^7e^6x^3-2\,d^8e^5x^2+d^9e^4x+d^{10}e^3\right)}+\frac{\left(5\,e^2f^2+2\,defg-d^2g^2\right)\log\left(ex+d\right)}{32\,d^6e^3}-\frac{\left(5\,e^2f^2+2\,defg-d^2g^2\right)\log\left(ex-d\right)}{32\,d^6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(8*d^4*e^2*f^2 - 16*d^5*e*f*g - 4*d^6*g^2 + 3*(5*e^6*f^2 + 2*d*e^5*f*
g - d^2*e^4*g^2)*x^4 + 3*(5*d*e^5*f^2 + 2*d^2*e^4*f*g - d^3*e^3*g^2)*x^3 -
5*(5*d^2*e^4*f^2 + 2*d^3*e^3*f*g - d^4*e^2*g^2)*x^2 - (25*d^3*e^3*f^2 + 10*
d^4*e^2*f*g + 7*d^5*e*g^2)*x)/(d^5*e^8*x^5 + d^6*e^7*x^4 - 2*d^7*e^6*x^3 -
2*d^8*e^5*x^2 + d^9*e^4*x + d^10*e^3) + 1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g
^2)*log(e*x + d)/(d^6*e^3) - 1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(e*x
 - d)/(d^6*e^3)
```

mupad [B] time = 2.68, size = 249, normalized size = 1.32

$$\frac{\frac{d^2\,g^2+4\,d\,e\,f\,g-2\,e^2\,f^2}{12\,d\,e^3}-\frac{x^3\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^4}-\frac{e\,x^4\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^5}+\frac{x\,\left(7\,d^2\,g^2+10\,d\,e\,f\,g+25\,e^2\,f^2\right)}{48\,d^2\,e^2}+\frac{5\,x^2\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{48\,d^3\,e}}{d^5+d^4\,e\,x-2\,d^3\,e^2\,x^2-2\,d^2\,e^3\,x^3+d\,e^4\,x^4+e^5\,x^5}+\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^3*(d + e*x)),x)
```

```
[Out] ((d^2*g^2 - 2*e^2*f^2 + 4*d*e*f*g)/(12*d*e^3) - (x^3*(5*e^2*f^2 - d^2*g^2 +
 2*d*e*f*g))/(16*d^4) - (e*x^4*(5*e^2*f^2 - d^2*g^2 + 2*d*e*f*g))/(16*d^5)
+ (x*(7*d^2*g^2 + 25*e^2*f^2 + 10*d*e*f*g))/(48*d^2*e^2) + (5*x^2*(5*e^2*f^
2 - d^2*g^2 + 2*d*e*f*g))/(48*d^3*e))/(d^5 + e^5*x^5 + d*e^4*x^4 - 2*d^3*e^
```

```
2*x^2 - 2*d^2*e^3*x^3 + d^4*e*x) + (atanh((e*x)/d)*(5*e^2*f^2 - d^2*g^2 + 2
*d*e*f*g))/(16*d^6*e^3)
```

sympy [A] time = 1.83, size = 321, normalized size = 1.71

$$-\frac{-4d^6g^2-16d^5efg+8d^4e^2f^2+x^4\left(-3d^2e^4g^2+6de^5fg+15e^6f^2\right)+x^3\left(-3d^3e^3g^2+6d^2e^4fg+15de^5f^2\right)+x^2\left(5d^4e^2g^2-10d^3e^3fg-25d^2e^4f^2\right)+x\left(-7d^5eg^2-10d^4e^2fg-25d^3e^3f^2\right)}{48d^{10}e^3+48d^9e^4x-96d^8e^5x^2-96d^7e^6x^3+48d^6e^7x^4+48d^5e^8x^5}+\frac{\left(d^2g^2-2defg-5e^2f^2\right)\log\left(-\frac{d}{e}+x\right)}{32d^6e^3}-\frac{\left(d^2g^2-2defg-5e^2f^2\right)\log\left(\frac{d}{e}+x\right)}{32d^6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-4*d**6*g**2 - 16*d**5*e*f*g + 8*d**4*e**2*f**2 + x**4*(-3*d**2*e**4*g**2
 + 6*d*e**5*f*g + 15*e**6*f**2) + x**3*(-3*d**3*e**3*g**2 + 6*d**2*e**4*f*g
 + 15*d*e**5*f**2) + x**2*(5*d**4*e**2*g**2 - 10*d**3*e**3*f*g - 25*d**2*e*
*4*f**2) + x*(-7*d**5*e*g**2 - 10*d**4*e**2*f*g - 25*d**3*e**3*f**2))/(48*d
**10*e**3 + 48*d**9*e**4*x - 96*d**8*e**5*x**2 - 96*d**7*e**6*x**3 + 48*d**
6*e**7*x**4 + 48*d**5*e**8*x**5) + (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*lo
g(-d/e + x)/(32*d**6*e**3) - (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*log(d/e
+ x)/(32*d**6*e**3)
```

3.380 $\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=235

$$\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d-ex)}+\frac{(dg+ef)^2}{64d^5e^3(d-ex)^2}-\frac{(dg+3ef)(ef-dg)}{48d^4e^3(d+ex)^3}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}+\frac{\left(-d^2g^2+10defg+15e^2f^2\right)\tanh^{-1}}{64d^7e^3}$$

Rubi [A] time = 0.27, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {848, 88, 208}

$$-\frac{-d^2g^2+2defg+5e^2f^2}{32d^6e^3(d+ex)}-\frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2}+\frac{\left(-d^2g^2+10defg+15e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{64d^7e^3}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}-\frac{(dg+3ef)(ef-dg)}{48d^4e^3(d+ex)^3}+\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d-ex)}+\frac{(dg+ef)^2}{64d^5e^3(d-ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] (e*f + d*g)^2/(64*d^5*e^3*(d - e*x)^2) + ((e*f + d*g)*(5*e*f + d*g))/(64*d^
6*e^3*(d - e*x)) - (e*f - d*g)^2/(32*d^3*e^3*(d + e*x)^4) - ((e*f - d*g)*(3
*e*f + d*g))/(48*d^4*e^3*(d + e*x)^3) - (3*e^2*f^2 - d^2*g^2)/(32*d^5*e^3*(
d + e*x)^2) - (5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)/(32*d^6*e^3*(d + e*x)) + ((
15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(64*d^7*e^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^5}\,dx\\
&= \int \left(\frac{(ef+dg)^2}{32d^5e^2(d-ex)^3}+\frac{(ef+dg)(5ef+dg)}{64d^6e^2(d-ex)^2}+\frac{(-ef+dg)^2}{8d^3e^2(d+ex)^5}+\frac{(ef-dg)(3ef+dg)}{16d^4e^2(d+ex)^4}\right.\\
&= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2}+\frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}-\frac{(ef-dg)(3ef+dg)}{48d^4e^3(d+ex)^3}-\\
&= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2}+\frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}-\frac{(ef-dg)(3ef+dg)}{48d^4e^3(d+ex)^3}-
\end{aligned}$$

Mathematica [A] time = 0.17, size = 244, normalized size = 1.04

$$\frac{-\frac{12d^4(ef-dg)^2}{(d+ex)^4}+\frac{12d^2\left(d^2g^2-3e^2f^2\right)}{(d+ex)^2}+\frac{6d\left(d^2g^2+6defg+5e^2f^2\right)}{d-ex}+\frac{12d\left(d^2g^2-2defg-5e^2f^2\right)}{d+ex}+3\left(d^2g^2-10defg-15e^2f^2\right)\log(d-ex)+3\left(-d^2g^2+10defg+15e^2f^2\right)\log(d+ex)+\frac{6d^2(dg+ef)^2}{(d-ex)^2}+\frac{8d^3\left(d^2g^2+2defg-3e^2f^2\right)}{(d+ex)^3}}{384d^7e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] ((6*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (6*d*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)
)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(-3*e^2*f^2 + 2*d
*e*f*g + d^2*g^2))/(d + e*x)^3 + (12*d^2*(-3*e^2*f^2 + d^2*g^2))/(d + e*x)^
2 + (12*d*(-5*e^2*f^2 - 2*d*e*f*g + d^2*g^2))/(d + e*x) + 3*(-15*e^2*f^2 -
10*d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*
Log[d + e*x])/(384*d^7*e^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3), x]
```

fricas [B] time = 0.41, size = 793, normalized size = 3.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/384*(96*d^6*e^2*f^2 - 64*d^7*e*f*g - 32*d^8*g^2 + 6*(15*d*e^7*f^2 + 10*d
^2*e^6*f*g - d^3*e^5*g^2)*x^5 + 12*(15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e
^4*g^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - 20*(1
5*d^4*e^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 - 2*(51*d^5*e^3*f^2 + 34*
d^6*e^2*f*g + 35*d^7*e*g^2)*x - 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8*g^2
+ (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 10*d^2*
e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g^2
)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15*d^4*e^4
*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*d^6*e^2*f
*g - d^7*e*g^2)*x)*log(e*x + d) + 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8*g^
2 + (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 10*d^
2*e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g
^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15*d^4*e
^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*d^6*e^2
*f*g - d^7*e*g^2)*x)*log(e*x - d))/(d^7*e^9*x^6 + 2*d^8*e^8*x^5 - d^9*e^7*x
^4 - 4*d^10*e^6*x^3 - d^11*e^5*x^2 + 2*d^12*e^4*x + d^13*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -((ex
p(1)*x+d)^-1/exp(1)*g^2*d^2*exp(1)^10-2*(exp(1)*x+d)^-1/exp(1)*g*d*exp(1)^1
1*f+(exp(1)*x+d)^-1/exp(1)*exp(1)^12*f^2)/(d^6*exp(1)^12-3*d^6*exp(1)^10*ex
```

```
p(2)+3*d^6*exp(1)^8*exp(2)^2-d^6*exp(1)^6*exp(2)^3)-((5*g^2*d^5*exp(1)^12+5
0*g^2*d^5*exp(1)^10*exp(2)-20*g^2*d^5*exp(1)^8*exp(2)^2-34*g^2*d^5*exp(1)^6
*exp(2)^3-g^2*d^5*exp(1)^4*exp(2)^4-68*g*d^4*exp(1)^11*exp(2)*f-52*g*d^4*ex
p(1)^9*exp(2)^2*f+116*g*d^4*exp(1)^7*exp(2)^3*f+4*g*d^4*exp(1)^5*exp(2)^4*f
+9*d^3*exp(1)^12*exp(2)*f^2+66*d^3*exp(1)^10*exp(2)^2*f^2-60*d^3*exp(1)^8*e
xp(2)^3*f^2-18*d^3*exp(1)^6*exp(2)^4*f^2+3*d^3*exp(1)^4*exp(2)^5*f^2)*(-(ex
p(1)*x+d)^-1/exp(1))^3+(17*g^2*d^4*exp(1)^9*exp(2)-85*g^2*d^4*exp(1)^7*exp(
2)^2-89*g^2*d^4*exp(1)^5*exp(2)^3-3*g^2*d^4*exp(1)^3*exp(2)^4-16*g*d^3*exp(
1)^10*exp(2)*f+44*g*d^3*exp(1)^8*exp(2)^2*f+280*g*d^3*exp(1)^6*exp(2)^3*f+1
2*g*d^3*exp(1)^4*exp(2)^4*f+21*d^2*exp(1)^9*exp(2)^2*f^2-145*d^2*exp(1)^7*e
xp(2)^3*f^2-45*d^2*exp(1)^5*exp(2)^4*f^2+9*d^2*exp(1)^3*exp(2)^5*f^2)*(-(ex
p(1)*x+d)^-1/exp(1))^2-(-3*g^2*d^3*exp(1)^8*exp(2)-77*g^2*d^3*exp(1)^6*exp(
2)^2-77*g^2*d^3*exp(1)^4*exp(2)^3-3*g^2*d^3*exp(1)^2*exp(2)^4+76*g*d^2*exp(
1)^7*exp(2)^2*f+232*g*d^2*exp(1)^5*exp(2)^3*f+12*g*d^2*exp(1)^3*exp(2)^4*f-
7*d*exp(1)^8*exp(2)^2*f^2-121*d*exp(1)^6*exp(2)^3*f^2-41*d*exp(1)^4*exp(2)^
4*f^2+9*d*exp(1)^2*exp(2)^5*f^2)*(exp(1)*x+d)^-1/exp(1)-17*g^2*d^2*exp(1)^5
*exp(2)^2-22*g^2*d^2*exp(1)^3*exp(2)^3-g^2*d^2*exp(1)*exp(2)^4+12*g*d*exp(1
)^6*exp(2)^2*f+64*g*d*exp(1)^4*exp(2)^3*f+4*g*d*exp(1)^2*exp(2)^4*f-29*exp(
1)^5*exp(2)^3*f^2-14*exp(1)^3*exp(2)^4*f^2+3*exp(1)*exp(2)^5*f^2)/8/d^7/(ex
p(2)-exp(1)^2)^4/((-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^4-(-(exp(1)*x+d)^-
1/exp(1))^2*d^2*exp(1)^2*exp(2)+2*(exp(1)*x+d)^-1/exp(1)*d*exp(1)*exp(2)-ex
p(2))^2-(g^2*d^2*exp(1)^5+2*g^2*d^2*exp(1)^3*exp(2)-g*d*exp(1)^6*f-5*g*d*ex
p(1)^4*f*exp(2)+3*exp(1)^5*f^2*exp(2))/(-d^7*exp(1)^8+4*d^7*exp(1)^6*exp(2)
-6*d^7*exp(1)^4*exp(2)^2+4*d^7*exp(1)^2*exp(2)^3-d^7*exp(2)^4)*ln(abs((-(ex
p(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^4-(-(exp(1)*x+d)^-1/exp(1))^2*d^2*exp(1)^
2*exp(2)+2*(exp(1)*x+d)^-1/exp(1)*d*exp(1)*exp(2)-exp(2)))-(3*g^2*d^2*exp(1
)^8+33*g^2*d^2*exp(1)^6*exp(2)+13*g^2*d^2*exp(1)^4*exp(2)^2-g^2*d^2*exp(1)^
2*exp(2)^3-60*g*d*exp(1)^7*f*exp(2)-40*g*d*exp(1)^5*f*exp(2)^2+4*g*d*exp(1)
^3*f*exp(2)^3+15*exp(1)^8*f^2*exp(2)+45*exp(1)^6*f^2*exp(2)^2-15*exp(1)^4*f
^2*exp(2)^3+3*exp(1)^2*f^2*exp(2)^4)/2/(-8*d^6*exp(1)^8+32*d^6*exp(1)^6*exp
(2)-48*d^6*exp(1)^4*exp(2)^2+32*d^6*exp(1)^2*exp(2)^3-8*d^6*exp(2)^4)/exp(1
)/abs(d)/exp(1)^2*ln(abs(-2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^4+2*(exp(1)*x
+d)^-1/exp(1)*d^2*exp(1)^2*exp(2)-2*d*exp(1)*exp(2)-2*exp(1)*abs(d)*exp(1)^
2)/abs(-2*(exp(1)*x+d)^-1/exp(1)*d^2*exp(1)^4+2*(exp(1)*x+d)^-1/exp(1)*d^2*
exp(1)^2*exp(2)-2*d*exp(1)*exp(2)+2*exp(1)*abs(d)*exp(1)^2))
```

maple [A] time = 0.02, size = 421, normalized size = 1.79

$$-\frac{g^2}{32(ex+d)^4de^3}+\frac{fg}{16(ex+d)^4d^2e^2}-\frac{f^2}{32(ex+d)^4d^3e}+\frac{g^2}{48(ex+d)^3d^2e^3}+\frac{fg}{24(ex+d)^3d^3e^2}-\frac{f^2}{16(ex+d)^3d^4e}+\frac{g^2}{64(ex-d)^2d^3e^3}+\frac{g^2}{32(ex+d)^2d^3e^3}+\frac{fg}{32(ex-d)^2d^4e^2}+\frac{f^2}{64(ex-d)^2d^5e}-\frac{3f^2}{32(ex+d)^2d^5e}-\frac{g^2}{64(ex-d)d^4e^3}+\frac{g^2}{32(ex+d)d^4e^3}-\frac{3fg}{32(ex-d)d^5e^2}-\frac{fg}{16(ex+d)d^5e^2}+\frac{g^2\ln(ex-d)}{128d^5e^3}-\frac{g^2\ln(ex+d)}{128d^5e^3}-\frac{5f^2}{64(ex-d)d^6e}-\frac{5f^2}{32(ex+d)d^6e}-\frac{5fg\ln(ex-d)}{64d^6e^2}+\frac{5fg\ln(ex+d)}{64d^6e^2}-\frac{15f^2\ln(ex-d)}{128d^7e}+\frac{15f^2\ln(ex+d)}{128d^7e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/64/(e*x-d)/d^4/e^3*g^2-3/32/(e*x-d)/d^5/e^2*f*g-5/64/(e*x-d)/d^6/e*f^2+1
/64/e^3/d^3/(e*x-d)^2*g^2+1/32/e^2/d^4/(e*x-d)^2*f*g+1/64/e/d^5/(e*x-d)^2*f
^2+1/128/d^5/e^3*g^2*ln(e*x-d)-5/64/d^6/e^2*f*g*ln(e*x-d)-15/128/d^7/e*f^2*
ln(e*x-d)+1/32/e^3/d^3/(e*x+d)^2*g^2-3/32/(e*x+d)^2/d^5/e*f^2+1/48/(e*x+d)^
3/d^2/e^3*g^2+1/24/(e*x+d)^3/d^3/e^2*f*g-1/16/(e*x+d)^3/d^4/e*f^2+1/32/(e*x
+d)/d^4/e^3*g^2-1/16/(e*x+d)/d^5/e^2*f*g-5/32/(e*x+d)/d^6/e*f^2-1/128/d^5/e
^3*g^2*ln(e*x+d)+5/64/d^6/e^2*f*g*ln(e*x+d)+15/128/d^7/e*f^2*ln(e*x+d)-1/32
/(e*x+d)^4/d/e^3*g^2+1/16/e^2/d^2/(e*x+d)^4*f*g-1/32/(e*x+d)^4/d^3/e*f^2
```

maxima [A] time = 0.51, size = 359, normalized size = 1.53

$$-\frac{48d^5e^2f^2-32d^6efg-16d^7g^2+3\left(15e^7f^2+10de^6fg-d^2e^5g^2\right)x^5+6\left(15de^6f^2+10d^2e^5fg-d^3e^4g^2\right)x^4-2\left(15d^2e^5f^2+10d^3e^4fg-d^4e^3g^2\right)x^3-10\left(15d^3e^4f^2+10d^4e^3fg-d^5e^2g^2\right)x^2-\left(51d^4e^3f^2+34d^5e^2fg+35d^6eg^2\right)x}{192\left(d^6e^9x^6+2d^7e^8x^5-d^8e^7x^4-4d^9e^6x^3-d^{10}e^5x^2+2d^{11}e^4x+d^{12}e^3\right)}+\frac{\left(15e^2f^2+10defg-d^2g^2\right)\log(ex+d)}{128d^7e^3}-\frac{\left(15e^2f^2+10defg-d^2g^2\right)\log(ex-d)}{128d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(48*d^5*e^2*f^2 - 32*d^6*e*f*g - 16*d^7*g^2 + 3*(15*e^7*f^2 + 10*d*e
^6*f*g - d^2*e^5*g^2)*x^5 + 6*(15*d*e^6*f^2 + 10*d^2*e^5*f*g - d^3*e^4*g^2)
*x^4 - 2*(15*d^2*e^5*f^2 + 10*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 10*(15*d^3*e
^4*f^2 + 10*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - (51*d^4*e^3*f^2 + 34*d^5*e^2*f
*g + 35*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 2*d^7*e^8*x^5 - d^8*e^7*x^4 - 4*d^9*e^
6*x^3 - d^10*e^5*x^2 + 2*d^11*e^4*x + d^12*e^3) + 1/128*(15*e^2*f^2 + 10*d*
e*f*g - d^2*g^2)*log(e*x + d)/(d^7*e^3) - 1/128*(15*e^2*f^2 + 10*d*e*f*g -
d^2*g^2)*log(e*x - d)/(d^7*e^3)
```

mupad [B] time = 2.64, size = 296, normalized size = 1.26

$$\frac{\frac{d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2}{12\,d\,e^3}+\frac{x^3\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{96\,d^4}-\frac{e\,x^4\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{32\,d^5}+\frac{x\,\left(35\,d^2\,g^2+34\,d\,e\,f\,g+51\,e^2\,f^2\right)}{192\,d^2\,e^2}+\frac{5\,x^2\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{96\,d^3\,e}-\frac{e^2\,x^5\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{64\,d^6}}{d^6+2\,d^5\,e\,x-d^4\,e^2\,x^2-4\,d^3\,e^3\,x^3-d^2\,e^4\,x^4+2\,d\,e^5\,x^5+e^6\,x^6}+\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{64\,d^7\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^3*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g)/(12*d*e^3) + (x^3*(15*e^2*f^2 - d^2*g^2
+ 10*d*e*f*g))/(96*d^4) - (e*x^4*(15*e^2*f^2 - d^2*g^2 + 10*d*e*f*g))/(32*d
^5) + (x*(35*d^2*g^2 + 51*e^2*f^2 + 34*d*e*f*g))/(192*d^2*e^2) + (5*x^2*(15
*e^2*f^2 - d^2*g^2 + 10*d*e*f*g))/(96*d^3*e) - (e^2*x^5*(15*e^2*f^2 - d^2*g
^2 + 10*d*e*f*g))/(64*d^6))/(d^6 + e^6*x^6 + 2*d*e^5*x^5 - d^4*e^2*x^2 - 4*
d^3*e^3*x^3 - d^2*e^4*x^4 + 2*d^5*e*x) + (atanh((e*x)/d)*(15*e^2*f^2 - d^2*
g^2 + 10*d*e*f*g))/(64*d^7*e^3)
```

sympy [A] time = 2.15, size = 372, normalized size = 1.58

$$-\frac{-16d^7g^2-32d^6efg+48d^5e^2f^2+x^5\left(-3d^2e^5g^2+30de^6fg+45e^7f^2\right)+x^4\left(-6d^3e^4g^2+60d^2e^5fg+90de^6f^2\right)+x^3\left(2d^4e^3g^2-20d^3e^4fg-30d^2e^5f^2\right)+x^2\left(10d^5e^2g^2-100d^4e^3fg-150d^3e^4f^2\right)+x\left(-35d^6eg^2-34d^5e^2fg-51d^4e^3f^2\right)}{192d^{12}e^3+384d^{11}e^4x-192d^{10}e^5x^2-768d^9e^6x^3-192d^8e^7x^4+384d^7e^8x^5+192d^6e^9x^6}+\frac{\left(d^2g^2-10defg-15e^2f^2\right)\log\left(-\frac{d}{e}+x\right)}{128d^7e^3}-\frac{\left(d^2g^2-10defg-15e^2f^2\right)\log\left(\frac{d}{e}+x\right)}{128d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-16*d**7*g**2 - 32*d**6*e*f*g + 48*d**5*e**2*f**2 + x**5*(-3*d**2*e**5*g*
*2 + 30*d*e**6*f*g + 45*e**7*f**2) + x**4*(-6*d**3*e**4*g**2 + 60*d**2*e**5
*f*g + 90*d*e**6*f**2) + x**3*(2*d**4*e**3*g**2 - 20*d**3*e**4*f*g - 30*d**
2*e**5*f**2) + x**2*(10*d**5*e**2*g**2 - 100*d**4*e**3*f*g - 150*d**3*e**4*
f**2) + x*(-35*d**6*e*g**2 - 34*d**5*e**2*f*g - 51*d**4*e**3*f**2))/(192*d*
*12*e**3 + 384*d**11*e**4*x - 192*d**10*e**5*x**2 - 768*d**9*e**6*x**3 - 19
2*d**8*e**7*x**4 + 384*d**7*e**8*x**5 + 192*d**6*e**9*x**6) + (d**2*g**2 -
10*d*e*f*g - 15*e**2*f**2)*log(-d/e + x)/(128*d**7*e**3) - (d**2*g**2 - 10*
d*e*f*g - 15*e**2*f**2)*log(d/e + x)/(128*d**7*e**3)
```

3.381 $\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{g^3\left(13d^2g^2+30defg+20e^2f^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{g^4\sqrt{d^2-e^2x^2}\,(3dg+5ef)}{e^6}+\frac{(d+ex)^2(2ef-23dg)(dg+ef)^4}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}}\cdot$$

Rubi [A] time = 0.97, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1635, 1815, 641, 217, 203}

$$\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{15d^3e^6\sqrt{d^2-e^2x^2}}-\frac{g^3\left(13d^2g^2+30defg+20e^2f^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{g^4\sqrt{d^2-e^2x^2}\,(3dg+5ef)}{e^6}+\frac{(d+ex)^2(2ef-23dg)(dg+ef)^4}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^5}{5de^6\left(d^2-e^2x^2\right)^{5/2}}+\frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^5*(d + e*x)^3)/(5*d*e^6*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 23*
d*g)*(e*f + d*g)^4*(d + e*x)^2)/(15*d^2*e^6*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2)*(d + e*x))/(15*d^3*e^6*Sqrt
[d^2 - e^2*x^2]) + (g^4*(5*e*f + 3*d*g)*Sqrt[d^2 - e^2*x^2])/e^6 + (g^5*x*S
qrt[d^2 - e^2*x^2])/(2*e^5) - (g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
```

```
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^5}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^5f^5-15de^4f^4g-30d^2e^3f^3g^2-30d^3e^2f^2g^3-15d^4efg^4-3d^5g^5}{e^5}+\frac{5dg^2(10e^3f^3 \cdots}{\cdots}\right)}{(d^2- \cdots}}{\cdots}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^5f^5-15de^4f^4g+70d^2e \cdots}{\cdots}\right)}{\cdots}}{\cdots}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21d \cdots\right.}{15d^3e^6\sqrt{d \cdots}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21d \cdots\right.}{15d^3e^6\sqrt{d \cdots}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21d \cdots\right.}{15d^3e^6\sqrt{d \cdots}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21d \cdots\right.}{15d^3e^6\sqrt{d \cdots}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21d \cdots\right.}{15d^3e^6\sqrt{d \cdots}}$$

Mathematica [A] time = 0.97, size = 193, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{2(2ef-23dg)(dg+ef)^4}{d^2(d-ex)^2}+\frac{2(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{d^3(d-ex)}+30g^4(3dg+5ef)+\frac{6(dg+ef)^5}{d(d-ex)^3}+15eg^5x\right)-15g^3\left(13d^2g^2+30defg+20e^2f^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{30e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(30*g^4*(5*e*f + 3*d*g) + 15*e*g^5*x + (6*(e*f + d*g)^
5)/(d*(d - e*x)^3) + (2*(2*e*f - 23*d*g)*(e*f + d*g)^4)/(d^2*(d - e*x)^2) +
 (2*(e*f + d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2))/(d^3*(d - e*x)))
- 15*g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/(30*e^6)
```

IntegrateAlgebraic [A] time = 1.98, size = 385, normalized size = 1.43

$$\frac{\sqrt{d^2-e^2x^2}\left(304d^7g^5+720d^6efg^4-717d^6eg^5x+440d^5e^2f^2g^3-1710d^5e^2fg^4x+479d^5e^2g^5x^2+40d^4e^3f^3g^2-1020d^4e^3f^2g^3x+1170d^4e^3fg^4x^2-45d^4e^3g^5x^3-30d^3e^4f^4g-120d^3e^4f^3g^2x+640d^3e^4f^2g^3x^2-150d^3e^4fg^4x^3-15d^3e^4g^5x^4+14d^2e^5f^5+90d^2e^5f^4gx+140d^2e^5f^3g^2x^2-12de^6f^5x-30de^6f^4gx^2+4e^7f^5x^2\right)}{30d^3e^6(d-ex)^3}-\frac{\sqrt{-e^2}\left(13d^2g^5+30defg^4+20e^2f^2g^3\right)\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{2e^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(14*d^2*e^5*f^5 - 30*d^3*e^4*f^4*g + 40*d^4*e^3*f^3*g^
2 + 440*d^5*e^2*f^2*g^3 + 720*d^6*e*f*g^4 + 304*d^7*g^5 - 12*d*e^6*f^5*x +
```

```
90*d^2*e^5*f^4*g*x - 120*d^3*e^4*f^3*g^2*x - 1020*d^4*e^3*f^2*g^3*x - 1710*
d^5*e^2*f*g^4*x - 717*d^6*e*g^5*x + 4*e^7*f^5*x^2 - 30*d*e^6*f^4*g*x^2 + 14
0*d^2*e^5*f^3*g^2*x^2 + 640*d^3*e^4*f^2*g^3*x^2 + 1170*d^4*e^3*f*g^4*x^2 +
479*d^5*e^2*g^5*x^2 - 150*d^3*e^4*f*g^4*x^3 - 45*d^4*e^3*g^5*x^3 - 15*d^3*e
^4*g^5*x^4))/(30*d^3*e^6*(d - e*x)^3) - (Sqrt[-e^2]*(20*e^2*f^2*g^3 + 30*d*
e*f*g^4 + 13*d^2*g^5)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^7)
```

fricas [B] time = 0.49, size = 807, normalized size = 3.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(14*d^3*e^5*f^5 - 30*d^4*e^4*f^4*g + 40*d^5*e^3*f^3*g^2 + 440*d^6*e^2
*f^2*g^3 + 720*d^7*e*f*g^4 + 304*d^8*g^5 - 2*(7*e^8*f^5 - 15*d*e^7*f^4*g +
20*d^2*e^6*f^3*g^2 + 220*d^3*e^5*f^2*g^3 + 360*d^4*e^4*f*g^4 + 152*d^5*e^3*
g^5)*x^3 + 6*(7*d*e^7*f^5 - 15*d^2*e^6*f^4*g + 20*d^3*e^5*f^3*g^2 + 220*d^4
*e^4*f^2*g^3 + 360*d^5*e^3*f*g^4 + 152*d^6*e^2*g^5)*x^2 - 6*(7*d^2*e^6*f^5
- 15*d^3*e^5*f^4*g + 20*d^4*e^4*f^3*g^2 + 220*d^5*e^3*f^2*g^3 + 360*d^6*e^2
*f*g^4 + 152*d^7*e*g^5)*x + 30*(20*d^6*e^2*f^2*g^3 + 30*d^7*e*f*g^4 + 13*d^
8*g^5 - (20*d^3*e^5*f^2*g^3 + 30*d^4*e^4*f*g^4 + 13*d^5*e^3*g^5)*x^3 + 3*(2
0*d^4*e^4*f^2*g^3 + 30*d^5*e^3*f*g^4 + 13*d^6*e^2*g^5)*x^2 - 3*(20*d^5*e^3*
f^2*g^3 + 30*d^6*e^2*f*g^4 + 13*d^7*e*g^5)*x)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) - (15*d^3*e^4*g^5*x^4 - 14*d^2*e^5*f^5 + 30*d^3*e^4*f^4*g - 40
*d^4*e^3*f^3*g^2 - 440*d^5*e^2*f^2*g^3 - 720*d^6*e*f*g^4 - 304*d^7*g^5 + 15
*(10*d^3*e^4*f*g^4 + 3*d^4*e^3*g^5)*x^3 - (4*e^7*f^5 - 30*d*e^6*f^4*g + 140
*d^2*e^5*f^3*g^2 + 640*d^3*e^4*f^2*g^3 + 1170*d^4*e^3*f*g^4 + 479*d^5*e^2*g
^5)*x^2 + 3*(4*d*e^6*f^5 - 30*d^2*e^5*f^4*g + 40*d^3*e^4*f^3*g^2 + 340*d^4*
e^3*f^2*g^3 + 570*d^5*e^2*f*g^4 + 239*d^6*e*g^5)*x)*sqrt(-e^2*x^2 + d^2))/(
d^3*e^9*x^3 - 3*d^4*e^8*x^2 + 3*d^5*e^7*x - d^6*e^6)
```

giac [B] time = 0.66, size = 537, normalized size = 2.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/2*(13*d^2*g^5 + 30*d*f*g^4*e + 20*f^2*g^3*e^2)*arcsin(x*e/d)*e^(-6)*sgn(
d) + 1/30*sqrt(-x^2*e^2 + d^2)*((((((15*(g^5*x*e + 2*(3*d^5*g^5*e^12 + 5*d^
4*f*g^4*e^13)*e^(-12)/d^4)*x - (299*d^6*g^5*e^11 + 720*d^5*f*g^4*e^12 + 640
*d^4*f^2*g^3*e^13 + 140*d^3*f^3*g^2*e^14 - 30*d^2*f^4*g*e^15 + 4*d*f^5*e^16
)*e^(-12)/d^4)*x - 30*(19*d^7*g^5*e^10 + 45*d^6*f*g^4*e^11 + 30*d^5*f^2*g^3
*e^12 + 10*d^4*f^3*g^2*e^13)*e^(-12)/d^4)*x + 5*(91*d^8*g^5*e^9 + 210*d^7*f
*g^4*e^10 + 140*d^6*f^2*g^3*e^11 - 20*d^5*f^3*g^2*e^12 - 30*d^4*f^4*g*e^13
+ 2*d^3*f^5*e^14)*e^(-12)/d^4)*x + 10*(76*d^9*g^5*e^8 + 180*d^8*f*g^4*e^9 +
 110*d^7*f^2*g^3*e^10 + 10*d^6*f^3*g^2*e^11 - 15*d^5*f^4*g*e^12 - d^4*f^5*e
^13)*e^(-12)/d^4)*x - 15*(13*d^10*g^5*e^7 + 30*d^9*f*g^4*e^8 + 20*d^8*f^2*g
^3*e^9 + 2*d^5*f^5*e^12)*e^(-12)/d^4)*x - 2*(152*d^11*g^5*e^6 + 360*d^10*f*
g^4*e^7 + 220*d^9*f^2*g^3*e^8 + 20*d^8*f^3*g^2*e^9 - 15*d^7*f^4*g*e^10 + 7*
d^6*f^5*e^11)*e^(-12)/d^4)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.06, size = 1308, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 3/2*d^2/e*x/(-e^2*x^2+d^2)^(5/2)*f^4*g+1/2/e^4*x/(-e^2*x^2+d^2)^(3/2)*d^3*f
*g^4+3/e^3*x/(-e^2*x^2+d^2)^(3/2)*d^2*f^2*g^3+7/3/e^2*x/(-e^2*x^2+d^2)^(3/2
)*d*f^3*g^2-5/e^2*x^3/(-e^2*x^2+d^2)^(3/2)*d*f*g^4+16/e^4*x/(-e^2*x^2+d^2)^
(1/2)*d*f*g^4-15/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
*d*f*g^4-110/3*d^3/e^2*x^2/(-e^2*x^2+d^2)^(5/2)*f^2*g^3-10/3*d^2/e*x^2/(-e^
2*x^2+d^2)^(5/2)*f^3*g^2+5/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)*d^3*f*g^4+15*x^3/
e/(-e^2*x^2+d^2)^(5/2)*d^2*f^2*g^3-3/2*d^5/e^4*x/(-e^2*x^2+d^2)^(5/2)*f*g^4
+45*d^2/e*x^4/(-e^2*x^2+d^2)^(5/2)*f*g^4-60*d^4/e^3*x^2/(-e^2*x^2+d^2)^(5/2
)*f*g^4+14/3/d/e^2*x/(-e^2*x^2+d^2)^(1/2)*f^3*g^2-1/d^2/e*x/(-e^2*x^2+d^2)^
(1/2)*f^4*g-9*d^4/e^3*x/(-e^2*x^2+d^2)^(5/2)*f^2*g^3-7*d^3/e^2*x/(-e^2*x^2+
d^2)^(5/2)*f^3*g^2-1/2*e*g^5*x^7/(-e^2*x^2+d^2)^(5/2)+7/15*d^2/e/(-e^2*x^2+
d^2)^(5/2)*f^5+4/5*x/(-e^2*x^2+d^2)^(5/2)*d*f^5+1/15/d*x/(-e^2*x^2+d^2)^(3/
2)*f^5+2/15/d^3*x/(-e^2*x^2+d^2)^(1/2)*f^5+1/3*x^2*e/(-e^2*x^2+d^2)^(5/2)*f
^5-3*x^6/(-e^2*x^2+d^2)^(5/2)*d*g^5+152/15*d^7/e^6/(-e^2*x^2+d^2)^(5/2)*g^5
+5/2*x^3*e/(-e^2*x^2+d^2)^(5/2)*f^4*g-1/2/e*x/(-e^2*x^2+d^2)^(3/2)*f^4*g+5*
x^2/(-e^2*x^2+d^2)^(5/2)*d*f^4*g-d^3/e^2/(-e^2*x^2+d^2)^(5/2)*f^4*g-5*x^6*e
/(-e^2*x^2+d^2)^(5/2)*f*g^4+19*d^3/e^2*x^4/(-e^2*x^2+d^2)^(5/2)*g^5-76/3*d^
5/e^4*x^2/(-e^2*x^2+d^2)^(5/2)*g^5+24*d^6/e^5/(-e^2*x^2+d^2)^(5/2)*f*g^4+3*
x^5/(-e^2*x^2+d^2)^(5/2)*d*f*g^4+2*x^5*e/(-e^2*x^2+d^2)^(5/2)*f^2*g^3-10/3/
e*x^3/(-e^2*x^2+d^2)^(3/2)*f^2*g^3+16/e^3*x/(-e^2*x^2+d^2)^(1/2)*f^2*g^3-10
/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)*f^2*g^3+30*x^4/
(-e^2*x^2+d^2)^(5/2)*d*f^2*g^3+10*x^4*e/(-e^2*x^2+d^2)^(5/2)*f^3*g^2+13/10/
e*g^5*d^2*x^5/(-e^2*x^2+d^2)^(5/2)-13/6/e^3*g^5*d^2*x^3/(-e^2*x^2+d^2)^(3/2
)+13/2/e^5*g^5*d^2*x/(-e^2*x^2+d^2)^(1/2)-13/2/e^5*g^5*d^2/(e^2)^(1/2)*arct
an((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+44/3*d^5/e^4/(-e^2*x^2+d^2)^(5/2)*f^
2*g^3+4/3*d^4/e^3/(-e^2*x^2+d^2)^(5/2)*f^3*g^2+15*x^3/(-e^2*x^2+d^2)^(5/2)*
d*f^3*g^2
```

maxima [B] time = 1.05, size = 1579, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*e*g^5*x^7/(-e^2*x^2 + d^2)^(5/2) + 7/30*d^2*e*g^5*x*(15*x^4/((-e^2*x^2
 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2
*x^2 + d^2)^(5/2)*e^6)) - 7/6*d^2*g^5*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2)
 - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + 1/5*d*f^5*x/(-e^2*x^2 + d^2)^(5/
2) + 3/5*d^2*f^5/((-e^2*x^2 + d^2)^(5/2)*e) + d^3*f^4*g/((-e^2*x^2 + d^2)^(
5/2)*e^2) + 4/15*f^5*x/((-e^2*x^2 + d^2)^(3/2)*d) + 14/15*d^4*g^5*x/((-e^2*
x^2 + d^2)^(3/2)*e^5) + 1/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5
)*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) + 8/15*f^5*x/(sqrt(-e^2*x^2 +
 d^2)*d^3) - 49/30*d^2*g^5*x/(sqrt(-e^2*x^2 + d^2)*e^5) - (5*e^3*f*g^4 + 3*
d*e^2*g^5)*x^6/((-e^2*x^2 + d^2)^(5/2)*e^2) - 7/2*d^2*g^5*arcsin(e*x/d)/e^6
 - 1/3*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5)*x*(3*x^2/((-e^2*x^2
+ d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^2 + 6*(5*e^3*f*g^
4 + 3*d*e^2*g^5)*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^4) + (10*e^3*f^3*g^2 + 3
0*d*e^2*f^2*g^3 + 15*d^2*e*f*g^4 + d^3*g^5)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2
) + 5/2*(e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^2*g^3 + d^3*f*g^4)*x^3/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 8*(5*e^3*f*g^4 + 3*d*e^2*g^5)*d^4*x^2/((-e^2*x^
2 + d^2)^(5/2)*e^6) - 4/3*(10*e^3*f^3*g^2 + 30*d*e^2*f^2*g^3 + 15*d^2*e*f*g
^4 + d^3*g^5)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/3*(e^3*f^5 + 15*d*e^
2*f^4*g + 30*d^2*e*f^3*g^2 + 10*d^3*f^2*g^3)*x^2/((-e^2*x^2 + d^2)^(5/2)*e^
2) - 3/2*(e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^2*g^3 + d^3*f*g^4)*d^2*x/
((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/5*(3*d*e^2*f^5 + 15*d^2*e*f^4*g + 10*d^3*f
^3*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 16/5*(5*e^3*f*g^4 + 3*d*e^2*g^5)*d
^6/((-e^2*x^2 + d^2)^(5/2)*e^8) + 8/15*(10*e^3*f^3*g^2 + 30*d*e^2*f^2*g^3 +
 15*d^2*e*f*g^4 + d^3*g^5)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(e^3*f^5
```

```
 + 15*d*e^2*f^4*g + 30*d^2*e*f^3*g^2 + 10*d^3*f^2*g^3)*d^2/((-e^2*x^2 + d^2
)^(5/2)*e^4) + 4/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5)*d^2*x/(
(-e^2*x^2 + d^2)^(3/2)*e^6) + 1/2*(e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^
2*g^3 + d^3*f*g^4)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/15*(3*d*e^2*f^5 + 15*
d^2*e*f^4*g + 10*d^3*f^3*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 7/15*(10
*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5)*x/(sqrt(-e^2*x^2 + d^2)*e^6) +
 (e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^2*g^3 + d^3*f*g^4)*x/(sqrt(-e^2*x
^2 + d^2)*d^2*e^4) - 2/15*(3*d*e^2*f^5 + 15*d^2*e*f^4*g + 10*d^3*f^3*g^2)*x
/(sqrt(-e^2*x^2 + d^2)*d^4*e^2) - (10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*
e*g^5)*arcsin(e*x/d)/e^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^5\,{\left(d+e\,x\right)}^3}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((f + g*x)^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**5/(-e**2*x**2+d**2)**(7/2),x)
[Out] Timed out
```

3.382 $\int \frac{(d+ex)^3(f+gx)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{g^3(3dg+4ef)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{e^5}+\frac{2(d\ \cdots}{}$$

Rubi [A] time = 0.67, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1635, 641, 217, 203}

$$\frac{2(d+ex)(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)}{15d^3e^5\sqrt{d^2-e^2x^2}}-\frac{g^3(3dg+4ef)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^4*(d + e*x)^3)/(5*d*e^5*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 9*d
*g)*(e*f + d*g)^3*(d + e*x)^2)/(15*d^2*e^5*(d^2 - e^2*x^2)^(3/2)) + (2*(e*f
 + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d + e*x))/(15*d^3*e^5*Sqrt[d^
2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/e^5 - (g^3*(4*e*f + 3*d*g)*ArcTan
[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^4f^4-12de^3f^3g-18d^2e^2f^2g^2-12d^3efg^3-3d^4g^4}{e^4}+\frac{5dg^2\left(6e^2f^2+4defg+d^2g^2\right)x}{e^3}+\cdots\right)}{\left(d^2-e^2x^2\right)^{5/2}}}{5d}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^4f^4-12de^3f^3g+42d^2e^2f^2g^2\cdots}{e^4}\cdots\right)}{(\cdots}}{\cdots}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+\cdots\right.}{15d^3e^5\sqrt{d^2-e^2\cdots}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+\cdots\right.}{15d^3e^5\sqrt{d^2-e^2\cdots}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+\cdots\right.}{15d^3e^5\sqrt{d^2-e^2\cdots}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+\cdots\right.}{15d^3e^5\sqrt{d^2-e^2\cdots}}$$

Mathematica [A] time = 0.74, size = 168, normalized size = 0.78

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(15d^3g^4(d-ex)^3+2(d-ex)^2(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)+3d^2(dg+ef)^4+2d(d-ex)(ef-9dg)(dg+ef)^3\right)}{d^3(d-ex)^3}-15g^3(3dg+4ef)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^2*(e*f + d*g)^4 + 2*d*(e*f - 9*d*g)*(e*f + d*g)^
3*(d - e*x) + 2*(e*f + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d - e*x)^
2 + 15*d^3*g^4*(d - e*x)^3))/(d^3*(d - e*x)^3) - 15*g^3*(4*e*f + 3*d*g)*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^5)
```

IntegrateAlgebraic [A] time = 1.57, size = 294, normalized size = 1.37

$$\frac{\sqrt{d^2-e^2x^2}\left(72d^6g^4+88d^5efg^3-171d^5eg^4x+12d^4e^2f^2g^2-204d^4e^2fg^3x+117d^4e^2g^4x^2-12d^3e^3f^3g-36d^3e^3f^2g^2x+128d^3e^3fg^3x^2-15d^3e^3g^4x^3+7d^2e^4f^4+36d^2e^4f^3gx+42d^2e^4f^2g^2x^2-6de^5f^4x-12de^5f^3gx^2+2e^6f^4x^2\right)}{15d^3e^5(d-ex)^3}-\frac{\sqrt{-e^2}\left(3dg^4+4efg^3\right)\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}\,x\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(7*d^2*e^4*f^4 - 12*d^3*e^3*f^3*g + 12*d^4*e^2*f^2*g^2
 + 88*d^5*e*f*g^3 + 72*d^6*g^4 - 6*d*e^5*f^4*x + 36*d^2*e^4*f^3*g*x - 36*d^
3*e^3*f^2*g^2*x - 204*d^4*e^2*f*g^3*x - 171*d^5*e*g^4*x + 2*e^6*f^4*x^2 - 1
2*d*e^5*f^3*g*x^2 + 42*d^2*e^4*f^2*g^2*x^2 + 128*d^3*e^3*f*g^3*x^2 + 117*d^
4*e^2*g^4*x^2 - 15*d^3*e^3*g^4*x^3))/(15*d^3*e^5*(d - e*x)^3) - (Sqrt[-e^2]
*(4*e*f*g^3 + 3*d*g^4)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^6
```

fricas [B] time = 0.57, size = 624, normalized size = 2.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^4*f^4 - 12*d^4*e^3*f^3*g + 12*d^5*e^2*f^2*g^2 + 88*d^6*e*f*g
^3 + 72*d^7*g^4 - (7*e^7*f^4 - 12*d*e^6*f^3*g + 12*d^2*e^5*f^2*g^2 + 88*d^3
*e^4*f*g^3 + 72*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 - 12*d^2*e^5*f^3*g + 12*d
^3*e^4*f^2*g^2 + 88*d^4*e^3*f*g^3 + 72*d^5*e^2*g^4)*x^2 - 3*(7*d^2*e^5*f^4
- 12*d^3*e^4*f^3*g + 12*d^4*e^3*f^2*g^2 + 88*d^5*e^2*f*g^3 + 72*d^6*e*g^4)*
x + 30*(4*d^6*e*f*g^3 + 3*d^7*g^4 - (4*d^3*e^4*f*g^3 + 3*d^4*e^3*g^4)*x^3 +
 3*(4*d^4*e^3*f*g^3 + 3*d^5*e^2*g^4)*x^2 - 3*(4*d^5*e^2*f*g^3 + 3*d^6*e*g^4
)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (15*d^3*e^3*g^4*x^3 - 7*d^
2*e^4*f^4 + 12*d^3*e^3*f^3*g - 12*d^4*e^2*f^2*g^2 - 88*d^5*e*f*g^3 - 72*d^6
*g^4 - (2*e^6*f^4 - 12*d*e^5*f^3*g + 42*d^2*e^4*f^2*g^2 + 128*d^3*e^3*f*g^3
 + 117*d^4*e^2*g^4)*x^2 + 3*(2*d*e^5*f^4 - 12*d^2*e^4*f^3*g + 12*d^3*e^3*f^
2*g^2 + 68*d^4*e^2*f*g^3 + 57*d^5*e*g^4)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^8*
x^3 - 3*d^4*e^7*x^2 + 3*d^5*e^6*x - d^6*e^5)
```

giac [B] time = 0.40, size = 411, normalized size = 1.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -(3*d*g^4 + 4*f*g^3*e)*arcsin(x*e/d)*e^(-5)*sgn(d) + 1/15*sqrt(-x^2*e^2 + d
^2)*((((((15*g^4*x*e - 2*(36*d^5*g^4*e^10 + 64*d^4*f*g^3*e^11 + 21*d^3*f^2*
g^2*e^12 - 6*d^2*f^3*g*e^13 + d*f^4*e^14)*e^(-10)/d^4)*x - 45*(3*d^6*g^4*e^
9 + 4*d^5*f*g^3*e^10 + 2*d^4*f^2*g^2*e^11)*e^(-10)/d^4)*x + 5*(21*d^7*g^4*e
^8 + 28*d^6*f*g^3*e^9 - 6*d^5*f^2*g^2*e^10 - 12*d^4*f^3*g*e^11 + d^3*f^4*e^
12)*e^(-10)/d^4)*x + 5*(36*d^8*g^4*e^7 + 44*d^7*f*g^3*e^8 + 6*d^6*f^2*g^2*e
^9 - 12*d^5*f^3*g*e^10 - d^4*f^4*e^11)*e^(-10)/d^4)*x - 15*(3*d^9*g^4*e^6 +
 4*d^8*f*g^3*e^7 + d^5*f^4*e^10)*e^(-10)/d^4)*x - (72*d^10*g^4*e^5 + 88*d^9
*f*g^3*e^6 + 12*d^8*f^2*g^2*e^7 - 12*d^7*f^3*g*e^8 + 7*d^6*f^4*e^9)*e^(-10)
/d^4)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.01, size = 1030, normalized size = 4.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*x/(-e^2*x^2+d^2)^(5/2)*d*f^4+1/15/d*x/(-e^2*x^2+d^2)^(3/2)*f^4+2/15/d^3
*x/(-e^2*x^2+d^2)^(1/2)*f^4+1/3*x^2*e/(-e^2*x^2+d^2)^(5/2)*f^4+7/15*d^2/e/(
-e^2*x^2+d^2)^(5/2)*f^4+3/5*x^5/(-e^2*x^2+d^2)^(5/2)*d*g^4+24/5/e^5*g^4*d^6
/(-e^2*x^2+d^2)^(5/2)-e*g^4*x^6/(-e^2*x^2+d^2)^(5/2)+6*x^3/e/(-e^2*x^2+d^2)
^(5/2)*d^2*f*g^3-18/5*d^4/e^3*x/(-e^2*x^2+d^2)^(5/2)*f*g^3-44/3*d^3/e^2*x^2
/(-e^2*x^2+d^2)^(5/2)*f*g^3-2*d^2/e*x^2/(-e^2*x^2+d^2)^(5/2)*f^2*g^2+14/5/d
/e^2*x/(-e^2*x^2+d^2)^(1/2)*f^2*g^2-4/5/d^2/e*x/(-e^2*x^2+d^2)^(1/2)*f^3*g-
21/5*d^3/e^2*x/(-e^2*x^2+d^2)^(5/2)*f^2*g^2+6/5*d^2/e*x/(-e^2*x^2+d^2)^(5/2
)*f^3*g+6/5/e^3*x/(-e^2*x^2+d^2)^(3/2)*d^2*f*g^3+7/5/e^2*x/(-e^2*x^2+d^2)^(
3/2)*d*f^2*g^2+6*x^4*e/(-e^2*x^2+d^2)^(5/2)*f^2*g^2+88/15*d^5/e^4/(-e^2*x^2
+d^2)^(5/2)*f*g^3+4/5*d^4/e^3/(-e^2*x^2+d^2)^(5/2)*f^2*g^2-1/e^2*x^3/(-e^2*
x^2+d^2)^(3/2)*d*g^4-4/3/e*x^3/(-e^2*x^2+d^2)^(3/2)*f*g^3+16/5/e^4*x/(-e^2*
x^2+d^2)^(1/2)*d*g^4+32/5/e^3*x/(-e^2*x^2+d^2)^(1/2)*f*g^3-3/e^4/(e^2)^(1/2
)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)*d*g^4-4/e^3/(e^2)^(1/2)*arctan
((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)*f*g^3+4/5*x^5*e/(-e^2*x^2+d^2)^(5/2)*f
*g^3+2*x^3*e/(-e^2*x^2+d^2)^(5/2)*f^3*g-3/10*d^5/e^4*x/(-e^2*x^2+d^2)^(5/2)
*g^4+12*x^4/(-e^2*x^2+d^2)^(5/2)*d*f*g^3+9*x^3/(-e^2*x^2+d^2)^(5/2)*d*f^2*g
```

```
^2+1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)*d^3*g^4-12/e^3*g^4*d^4*x^2/(-e^2*x^2+d^
2)^(5/2)+9/e*g^4*d^2*x^4/(-e^2*x^2+d^2)^(5/2)+1/10/e^4*x/(-e^2*x^2+d^2)^(3/
2)*d^3*g^4-2/5/e*x/(-e^2*x^2+d^2)^(3/2)*f^3*g+4*x^2/(-e^2*x^2+d^2)^(5/2)*d*
f^3*g-4/5*d^3/e^2/(-e^2*x^2+d^2)^(5/2)*f^3*g
```

maxima [B] time = 1.04, size = 1178, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -e*g^4*x^6/(-e^2*x^2 + d^2)^(5/2) + 6*d^2*g^4*x^4/((-e^2*x^2 + d^2)^(5/2)*e
) - 8*d^4*g^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) + 1/5*d*f^4*x/(-e^2*x^2 + d^
2)^(5/2) + 1/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(15*x^4/((-e^2*x^2 + d^2)^(5/
2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)
^(5/2)*e^6)) + 3/5*d^2*f^4/((-e^2*x^2 + d^2)^(5/2)*e) + 4/5*d^3*f^3*g/((-e^
2*x^2 + d^2)^(5/2)*e^2) + 16/5*d^6*g^4/((-e^2*x^2 + d^2)^(5/2)*e^5) + 4/15*
f^4*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^4*x/(sqrt(-e^2*x^2 + d^2)*d^3) -
1/3*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d
^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^2 + 3*(2*e^3*f^2*g^2 + 4*d*e^2*f*g^3 + d
^2*e*g^4)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) + 1/2*(4*e^3*f^3*g + 18*d*e^2*f^
2*g^2 + 12*d^2*e*f*g^3 + d^3*g^4)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4*(2*e
^3*f^2*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4
) + 1/3*(e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3)*x^2/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 3/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e
*f*g^3 + d^3*g^4)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/5*(d*e^2*f^4 + 4*d
^2*e*f^3*g + 2*d^3*f^2*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/5*(2*e^3*f^2
*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(
e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3)*d^2/((-e^2*x^2 +
 d^2)^(5/2)*e^4) + 4/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*d^2*x/((-e^2*x^2 + d^2)
^(3/2)*e^6) + 1/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e*f*g^3 + d^3*g
^4)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^2*f^4 + 4*d^2*e*f^3*g + 2*d^3
*f^2*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 7/15*(4*e^3*f*g^3 + 3*d*e^2*
g^4)*x/(sqrt(-e^2*x^2 + d^2)*e^6) + 1/5*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 1
2*d^2*e*f*g^3 + d^3*g^4)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/5*(d*e^2*f^4
+ 4*d^2*e*f^3*g + 2*d^3*f^2*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2) - (4*e^3*
f*g^3 + 3*d*e^2*g^4)*arcsin(e*x/d)/e^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^4\,{\left(d+e\,x\right)}^3}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int(((f + g*x)^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

3.383 $\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=183

$$\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{g^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}+\frac{(d+ex)(dg+ef)\left(32d^2g^2-11de\right.}{15d^3e^4\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.40, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1635, 778, 217, 203}

$$\frac{(d+ex)(dg+ef)\left(32d^2g^2-11defg+2e^2f^2\right)}{15d^3e^4\sqrt{d^2-e^2x^2}}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{g^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^3*(d + e*x)^3)/(5*d*e^4*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 13*
d*g)*(e*f + d*g)^2*(d + e*x)^2)/(15*d^2*e^4*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)*(2*e^2*f^2 - 11*d*e*f*g + 32*d^2*g^2)*(d + e*x))/(15*d^3*e^4*Sqrt[d^
2 - e^2*x^2]) - (g^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^3f^3-9de^2f^2g-9d^2efg^2-3d^3g^3}{e^3}+\frac{5dg^2(3ef+dg)x}{e^2}+\frac{5dg^3x^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^3f^3-9de^2f^2g+21d^2efg^2+1}{e^3}\right.}{\left(d^2-e^2x^2\right)^{3/2}}}{15d^2}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-11defg\right.}{15d^3e^4\sqrt{d^2-}}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-11defg\right.}{15d^3e^4\sqrt{d^2-}}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-11defg\right.}{15d^3e^4\sqrt{d^2-}}$$

Mathematica [A] time = 0.81, size = 182, normalized size = 0.99

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}(dg+ef)\left(22d^4g^2-d^3eg(16f+51gx)+d^2e^2\left(7f^2+33fgx+32g^2x^2\right)-de^3fx(6f+11gx)+2e^4f^2x^2\right)-15d^2g^3(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{15d^3e^4(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*((e*f + d*g)*Sqrt[1 - (e^2*x^2)/d^2]*(22*d^4*g^2 + 2*e^4*f^2*x^2
 - d*e^3*f*x*(6*f + 11*g*x) - d^3*e*g*(16*f + 51*g*x) + d^2*e^2*(7*f^2 + 33
*f*g*x + 32*g^2*x^2)) - 15*d^2*g^3*(d - e*x)^3*ArcSin[(e*x)/d]))/(15*d^3*e^
4*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

IntegrateAlgebraic [A] time = 1.12, size = 223, normalized size = 1.22

$$\frac{\sqrt{d^2-e^2x^2}\left(22d^5g^3+6d^4efg^2-51d^4eg^3x-9d^3e^2f^2g-18d^3e^2fg^2x+32d^3e^2g^3x^2+7d^2e^3f^3+27d^2e^3f^2gx+21d^2e^3fg^2x^2-6de^4f^3x-9de^4f^2gx^2+2e^5f^3x^2\right)}{15d^3e^4(d-ex)^3}-\frac{\sqrt{-e^2}g^3\log\left(\sqrt{d^2-e^2x^2}-\sqrt{-e^2}x\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(7*d^2*e^3*f^3 - 9*d^3*e^2*f^2*g + 6*d^4*e*f*g^2 + 22*
d^5*g^3 - 6*d*e^4*f^3*x + 27*d^2*e^3*f^2*g*x - 18*d^3*e^2*f*g^2*x - 51*d^4*
e*g^3*x + 2*e^5*f^3*x^2 - 9*d*e^4*f^2*g*x^2 + 21*d^2*e^3*f*g^2*x^2 + 32*d^3
*e^2*g^3*x^2))/(15*d^3*e^4*(d - e*x)^3) - (Sqrt[-e^2]*g^3*Log[-(Sqrt[-e^2]*
x) + Sqrt[d^2 - e^2*x^2]])/e^5
```

fricas [B] time = 0.44, size = 454, normalized size = 2.48

$$-\frac{7d^3e^3f^3-9d^4e^2f^2g+6d^5efg^2+22d^6g^3-\left(7e^6f^3-9de^5f^2g+6d^2e^4fg^2+22d^3e^3g^3\right)x^3+3\left(7de^5f^3-9d^2e^4f^2g+6d^3e^3fg^2+22d^4e^2g^3\right)x^2-3\left(7d^2e^4f^3-9d^3e^3f^2g+6d^4e^2fg^2+22d^5eg^3\right)x-30\left(d^3e^3g^3x^3-3d^4e^2g^3x^2+3d^5eg^3x-d^6g^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(7d^2e^3f^3-9d^3e^2f^2g+6d^4efg^2+22d^5g^3+\left(2e^5f^3-9de^4f^2g+21d^2e^3fg^2+32d^3e^2g^3\right)x^2-3\left(2de^4f^3-9d^2e^3f^2g+6d^3e^2fg^2+17d^4eg^3\right)x\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^7x^3-3d^4e^6x^2+3d^5e^5x-d^6e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^3*f^3 - 9*d^4*e^2*f^2*g + 6*d^5*e*f*g^2 + 22*d^6*g^3 - (7*e^
6*f^3 - 9*d*e^5*f^2*g + 6*d^2*e^4*f*g^2 + 22*d^3*e^3*g^3)*x^3 + 3*(7*d*e^5*
f^3 - 9*d^2*e^4*f^2*g + 6*d^3*e^3*f*g^2 + 22*d^4*e^2*g^3)*x^2 - 3*(7*d^2*e^
4*f^3 - 9*d^3*e^3*f^2*g + 6*d^4*e^2*f*g^2 + 22*d^5*e*g^3)*x - 30*(d^3*e^3*g
^3*x^3 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*arctan(-(d - sqrt(-e^
2*x^2 + d^2))/(e*x)) + (7*d^2*e^3*f^3 - 9*d^3*e^2*f^2*g + 6*d^4*e*f*g^2 + 2
2*d^5*g^3 + (2*e^5*f^3 - 9*d*e^4*f^2*g + 21*d^2*e^3*f*g^2 + 32*d^3*e^2*g^3)
*x^2 - 3*(2*d*e^4*f^3 - 9*d^2*e^3*f^2*g + 6*d^3*e^2*f*g^2 + 17*d^4*e*g^3)*x
)*sqrt(-e^2*x^2 + d^2))/(d^3*e^7*x^3 - 3*d^4*e^6*x^2 + 3*d^5*e^5*x - d^6*e^
4)
```

giac [A] time = 0.38, size = 309, normalized size = 1.69

$$-g^3\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d) - \frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{(32d^4g^3e^8+21d^3fg^2e^9-9d^2f^2ge^{10}+2df^3e^{11})xe^{(-7)}}{d^4}+\frac{45(d^5g^3e^7+d^4fg^2e^8)e^{(-7)}}{d^4}\right)-\frac{5(7d^6g^3e^6-3d^5fg^2e^7-9d^4f^2ge^8+d^3f^3e^9)e^{(-7)}}{d^4}\right)x-\frac{5(11d^7g^3e^5+3d^6fg^2e^6-9d^5f^2ge^7-d^4f^3e^8)e^{(-7)}}{d^4}\right)x+\frac{15(d^8g^3e^4+d^5f^3e^7)e^{(-7)}}{d^4}\right)x+\frac{(22d^9g^3e^3+6d^8fg^2e^4-9d^7f^2ge^5+7d^6f^3e^6)e^{(-7)}}{d^4}\right)}{15(x^2e^2-d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -g^3*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/15*sqrt(-x^2*e^2 + d^2)*((((x*((32*d^4
*g^3*e^8 + 21*d^3*f*g^2*e^9 - 9*d^2*f^2*g*e^10 + 2*d*f^3*e^11)*x*e^(-7)/d^4
 + 45*(d^5*g^3*e^7 + d^4*f*g^2*e^8)*e^(-7)/d^4) - 5*(7*d^6*g^3*e^6 - 3*d^5*
f*g^2*e^7 - 9*d^4*f^2*g*e^8 + d^3*f^3*e^9)*e^(-7)/d^4)*x - 5*(11*d^7*g^3*e^
5 + 3*d^6*f*g^2*e^6 - 9*d^5*f^2*g*e^7 - d^4*f^3*e^8)*e^(-7)/d^4)*x + 15*(d^
8*g^3*e^4 + d^5*f^3*e^7)*e^(-7)/d^4)*x + (22*d^9*g^3*e^3 + 6*d^8*f*g^2*e^4
- 9*d^7*f^2*g*e^5 + 7*d^6*f^3*e^6)*e^(-7)/d^4)/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.01, size = 713, normalized size = 3.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*x/(-e^2*x^2+d^2)^(5/2)*d*f^3+1/15/d*x/(-e^2*x^2+d^2)^(3/2)*f^3+2/15/d^3
*x/(-e^2*x^2+d^2)^(1/2)*f^3+7/15*d^2/e/(-e^2*x^2+d^2)^(5/2)*f^3+1/3*x^2*e/(
-e^2*x^2+d^2)^(5/2)*f^3+3*x^4/(-e^2*x^2+d^2)^(5/2)*d*g^3+22/15*d^5/e^4/(-e^
2*x^2+d^2)^(5/2)*g^3+8/5/e^3*g^3*x/(-e^2*x^2+d^2)^(1/2)-1/e^3*g^3/(e^2)^(1/
2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/5*e*g^3*x^5/(-e^2*x^2+d^2)^
(5/2)-1/3/e*g^3*x^3/(-e^2*x^2+d^2)^(3/2)-21/10*d^3/e^2*x/(-e^2*x^2+d^2)^(5/
2)*f*g^2-d^2/e*x^2/(-e^2*x^2+d^2)^(5/2)*f*g^2+9/10*d^2/e*x/(-e^2*x^2+d^2)^(
5/2)*f^2*g+7/10/e^2*x/(-e^2*x^2+d^2)^(3/2)*d*f*g^2+7/5/d/e^2*x/(-e^2*x^2+d^
2)^(1/2)*f*g^2-3/5/d^2/e*x/(-e^2*x^2+d^2)^(1/2)*f^2*g-9/10*d^4/e^3*x/(-e^2*
x^2+d^2)^(5/2)*g^3+3/10/e^3*x/(-e^2*x^2+d^2)^(3/2)*d^2*g^3-3/10/e*x/(-e^2*x
^2+d^2)^(3/2)*f^2*g+3/2*x^3*e/(-e^2*x^2+d^2)^(5/2)*f^2*g+3*x^2/(-e^2*x^2+d^
2)^(5/2)*d*f^2*g-3/5*d^3/e^2/(-e^2*x^2+d^2)^(5/2)*f^2*g-11/3*d^3/e^2*x^2/(-
e^2*x^2+d^2)^(5/2)*g^3+2/5*d^4/e^3/(-e^2*x^2+d^2)^(5/2)*f*g^2+3/2*x^3/e/(-e
^2*x^2+d^2)^(5/2)*d^2*g^3+9/2*x^3/(-e^2*x^2+d^2)^(5/2)*d*f*g^2+3*x^4*e/(-e^
2*x^2+d^2)^(5/2)*f*g^2
```

maxima [B] time = 1.03, size = 891, normalized size = 4.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^3*g^3*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2
 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e*g^3*x*(3*x
^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 1/5
```

```
*d*f^3*x/(-e^2*x^2 + d^2)^(5/2) + 3/5*d^2*f^3/((-e^2*x^2 + d^2)^(5/2)*e) +
3/5*d^3*f^2*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 4/15*f^3*x/((-e^2*x^2 + d^2)^(
3/2)*d) + 4/15*d^2*g^3*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 8/15*f^3*x/(sqrt(-e
^2*x^2 + d^2)*d^3) - 7/15*g^3*x/(sqrt(-e^2*x^2 + d^2)*e^3) + 3*(e^3*f*g^2 +
 d*e^2*g^3)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - g^3*arcsin(e*x/d)/e^4 + 3/2*
(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) -
4*(e^3*f*g^2 + d*e^2*g^3)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/3*(e^3*f
^3 + 9*d*e^2*f^2*g + 9*d^2*e*f*g^2 + d^3*g^3)*x^2/((-e^2*x^2 + d^2)^(5/2)*e
^2) - 9/10*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*d^2*x/((-e^2*x^2 + d^2)^
(5/2)*e^4) + 3/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2)*x/((-e^2*x^2 + d^2
)^(5/2)*e^2) + 8/5*(e^3*f*g^2 + d*e^2*g^3)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)
 - 2/15*(e^3*f^3 + 9*d*e^2*f^2*g + 9*d^2*e*f*g^2 + d^3*g^3)*d^2/((-e^2*x^2
+ d^2)^(5/2)*e^4) + 3/10*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x/((-e^2*x
^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2)*x/((-e^2
*x^2 + d^2)^(3/2)*d^2*e^2) + 3/5*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x/
(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2
)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f+g\,x\right)^3\,\left(d+e\,x\right)^3}{\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int(((f + g*x)^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\left(f+gx\right)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.384 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{(d+ex)^3(dg+ef)^2}{5de^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)^2(ef-4dg)(dg+ef)}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)\left(7d^2g^2-6defg+2e^2f^2\right)}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.22, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1635, 789, 637}

$$\frac{(d+ex)\left(7d^2g^2-6defg+2e^2f^2\right)}{15d^3e^3\sqrt{d^2-e^2x^2}}+\frac{(d+ex)^3(dg+ef)^2}{5de^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)^2(ef-4dg)(dg+ef)}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^2*(d + e*x)^3)/(5*d*e^3*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 4*d
*g)*(e*f + d*g)*(d + e*x)^2)/(15*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) + ((2*e^2*f
^2 - 6*d*e*f*g + 7*d^2*g^2)*(d + e*x))/(15*d^3*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-2f^2+\frac{6dfg}{e}+\frac{3d^2g^2}{e^2}+\frac{5dg^2x}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$

$$= \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\left(2e^2f^2-6defg+7d^2g^2\right)\int -\ldots}{15d^2e^2}$$

$$= \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\left(2e^2f^2-6defg+7d^2g^2\right)(d+\ldots}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.39, size = 110, normalized size = 0.76

$$\frac{(d+ex)\left(2d^4g^2-6d^3eg(f+gx)+d^2e^2\left(7f^2+18fgx+7g^2x^2\right)-6de^3fx(f+gx)+2e^4f^2x^2\right)}{15d^3e^3(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(2*d^4*g^2 + 2*e^4*f^2*x^2 - 6*d^3*e*g*(f + g*x) - 6*d*e^3*f*x*(
f + g*x) + d^2*e^2*(7*f^2 + 18*f*g*x + 7*g^2*x^2)))/(15*d^3*e^3*(d - e*x)^2
*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.77, size = 129, normalized size = 0.89

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^4g^2-6d^3efg-6d^3eg^2x+7d^2e^2f^2+18d^2e^2fgx+7d^2e^2g^2x^2-6de^3f^2x-6de^3fgx^2+2e^4f^2x^2\right)}{15d^3e^3(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(7*d^2*e^2*f^2 - 6*d^3*e*f*g + 2*d^4*g^2 - 6*d*e^3*f^2
*x + 18*d^2*e^2*f*g*x - 6*d^3*e*g^2*x + 2*e^4*f^2*x^2 - 6*d*e^3*f*g*x^2 + 7
*d^2*e^2*g^2*x^2))/(15*d^3*e^3*(d - e*x)^3)
```

fricas [B] time = 0.42, size = 279, normalized size = 1.92

$$-\frac{7d^3e^2f^2-6d^4efg+2d^5g^2-\left(7e^5f^2-6de^4fg+2d^2e^3g^2\right)x^3+3\left(7de^4f^2-6d^2e^3fg+2d^3e^2g^2\right)x^2-3\left(7d^2e^3f^2-6d^3e^2fg+2d^4eg^2\right)x+\left(7d^2e^2f^2-6d^3efg+2d^4g^2+\left(2e^4f^2-6de^3fg+7d^2e^2g^2\right)x^2-6\left(de^3f^2-3d^2e^2fg+d^3eg^2\right)x\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^6x^3-3d^4e^5x^2+3d^5e^4x-d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^2*f^2 - 6*d^4*e*f*g + 2*d^5*g^2 - (7*e^5*f^2 - 6*d*e^4*f*g +
 2*d^2*e^3*g^2)*x^3 + 3*(7*d*e^4*f^2 - 6*d^2*e^3*f*g + 2*d^3*e^2*g^2)*x^2 -
 3*(7*d^2*e^3*f^2 - 6*d^3*e^2*f*g + 2*d^4*e*g^2)*x + (7*d^2*e^2*f^2 - 6*d^3
*e*f*g + 2*d^4*g^2 + (2*e^4*f^2 - 6*d*e^3*f*g + 7*d^2*e^2*g^2)*x^2 - 6*(d*e
^3*f^2 - 3*d^2*e^2*f*g + d^3*e*g^2)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^3 -
 3*d^4*e^5*x^2 + 3*d^5*e^4*x - d^6*e^3)
```

giac [A] time = 0.34, size = 198, normalized size = 1.37

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(15\,df^2+\left(\left(\left(15\,g^2e+\frac{\left(7\,d^3g^2e^6-6\,d^2fge^7+2\,df^2e^8\right)xe^{(-4)}}{d^4}\right)x+\frac{5\left(d^5g^2e^4+6\,d^4fge^5-d^3f^2e^6\right)e^{(-4)}}{d^4}\right)x-\frac{5\left(d^6g^2e^3-6\,d^5fge^4-d^4f^2e^5\right)e^{(-4)}}{d^4}\right)x\right)x+\frac{\left(2\,d^8g^2e-6\,d^7fge^2+7\,d^6f^2e^3\right)e^{(-4)}}{d^4}\right)}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((15*d*f^2 + (((15*g^2*e + (7*d^3*g^2*e^6 - 6*d^
2*f*g*e^7 + 2*d*f^2*e^8)*x*e^(-4)/d^4)*x + 5*(d^5*g^2*e^4 + 6*d^4*f*g*e^5 -
 d^3*f^2*e^6)*e^(-4)/d^4)*x - 5*(d^6*g^2*e^3 - 6*d^5*f*g*e^4 - d^4*f^2*e^5)
*e^(-4)/d^4)*x)*x + (2*d^8*g^2*e - 6*d^7*f*g*e^2 + 7*d^6*f^2*e^3)*e^(-4)/d^
4)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 131, normalized size = 0.90

$$\frac{(-ex+d)(ex+d)^4\left(7d^2e^2g^2x^2-6d\,e^3fg\,x^2+2e^4f^2x^2-6d^3e\,g^2x+18d^2e^2fgx-6d\,e^3f^2x+2d^4g^2-6d^3efg+7d^2e^2f^2\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(7*d^2*e^2*g^2*x^2-6*d*e^3*f*g*x^2+2*e^4*f^2*x^2-6*
d^3*e*g^2*x+18*d^2*e^2*f*g*x-6*d*e^3*f^2*x+2*d^4*g^2-6*d^3*e*f*g+7*d^2*e^2*
f^2)/d^3/e^3/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.47, size = 583, normalized size = 4.02

$$\frac{eg^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{4\,d^2g^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d\,f^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{3\,d^2f^2}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{2\,d^3fg}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}+\frac{8\,d^4g^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}+\frac{4\,f^2x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}+\frac{8\,f^2x}{15\sqrt{-e^2x^2+d^2}\,d^3}+\frac{\left(2\,e^3fg+3\,de^2g^2\right)x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}+\frac{\left(e^3f^2+6\,de^2fg+3\,d^2eg^2\right)x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{3\left(2\,e^3fg+3\,de^2g^2\right)d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{\left(3\,de^2f^2+6\,d^2efg+d^3g^2\right)x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{2\left(e^3f^2+6\,de^2fg+3\,d^2eg^2\right)d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{\left(2\,e^3fg+3\,de^2g^2\right)x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}-\frac{\left(3\,de^2f^2+6\,d^2efg+d^3g^2\right)x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^2}+\frac{\left(2\,e^3fg+3\,de^2g^2\right)x}{5\sqrt{-e^2x^2+d^2}\,d^2e^4}-\frac{2\left(3\,de^2f^2+6\,d^2efg+d^3g^2\right)x}{15\sqrt{-e^2x^2+d^2}\,d^4e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] e*g^2*x^4/(-e^2*x^2 + d^2)^(5/2) - 4/3*d^2*g^2*x^2/((-e^2*x^2 + d^2)^(5/2)*
e) + 1/5*d*f^2*x/(-e^2*x^2 + d^2)^(5/2) + 3/5*d^2*f^2/((-e^2*x^2 + d^2)^(5/
2)*e) + 2/5*d^3*f*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/15*d^4*g^2/((-e^2*x^2
+ d^2)^(5/2)*e^3) + 4/15*f^2*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^2*x/(sqr
t(-e^2*x^2 + d^2)*d^3) + 1/2*(2*e^3*f*g + 3*d*e^2*g^2)*x^3/((-e^2*x^2 + d^2
)^(5/2)*e^2) + 1/3*(e^3*f^2 + 6*d*e^2*f*g + 3*d^2*e*g^2)*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^2) - 3/10*(2*e^3*f*g + 3*d*e^2*g^2)*d^2*x/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 1/5*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/((-e^2*x^2 + d^2)^(5/
2)*e^2) - 2/15*(e^3*f^2 + 6*d*e^2*f*g + 3*d^2*e*g^2)*d^2/((-e^2*x^2 + d^2)^
(5/2)*e^4) + 1/10*(2*e^3*f*g + 3*d*e^2*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*e^4)
- 1/15*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*
e^2) + 1/5*(2*e^3*f*g + 3*d*e^2*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/1
5*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

mupad [B] time = 2.87, size = 125, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^4\,g^2-6\,d^3\,e\,f\,g-6\,d^3\,e\,g^2\,x+7\,d^2\,e^2\,f^2+18\,d^2\,e^2\,f\,g\,x+7\,d^2\,e^2\,g^2\,x^2-6\,d\,e^3\,f^2\,x-6\,d\,e^3\,f\,g\,x^2+2\,e^4\,f^2\,x^2\right)}{15\,d^3\,e^3\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^4*g^2 + 7*d^2*e^2*f^2 + 2*e^4*f^2*x^2 - 6*d^3*e
*f*g + 7*d^2*e^2*g^2*x^2 - 6*d*e^3*f^2*x - 6*d^3*e*g^2*x + 18*d^2*e^2*f*g*x
 - 6*d*e^3*f*g*x^2))/(15*d^3*e^3*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\left(f+gx\right)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.385 $\int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=117

$$\frac{(d+ex)^3(dg+ef)}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)(2ef-3dg)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(2ef-3dg)}{15d^3e\sqrt{d^2-e^2x^2}}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {789, 653, 191}

$$\frac{(d+ex)^3(dg+ef)}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)(2ef-3dg)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(2ef-3dg)}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)*(d + e*x)^3)/(5*d*e^2*(d^2 - e^2*x^2)^(5/2)) + (2*(2*e*f - 3*d
*g)*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^(3/2)) + ((2*e*f - 3*d*g)*x)/(15*d
^3*e*Sqrt[d^2 - e^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{(-5ef+3(ef+dg))\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5de}\\ &= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{(-5ef+3(ef+dg))\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}\\ &= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{(2ef-3dg)x}{15d^3e\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.24, size = 83, normalized size = 0.71

$$-\frac{(d+ex)\left(3d^3g-d^2e(7f+9gx)+3de^2x(2f+gx)-2e^3fx^2\right)}{15d^3e^2(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -1/15*((d + e*x)*(3*d^3*g - 2*e^3*f*x^2 + 3*d*e^2*x*(2*f + g*x) - d^2*e*(7*
f + 9*g*x)))/(d^3*e^2*(d - e*x)^2*Sqrt[d^2 - e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.55, size = 83, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(-3d^3g+7d^2ef+9d^2egx-6de^2fx-3de^2gx^2+2e^3fx^2\right)}{15d^3e^2(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(7*d^2*e*f - 3*d^3*g - 6*d*e^2*f*x + 9*d^2*e*g*x + 2*e
^3*f*x^2 - 3*d*e^2*g*x^2))/(15*d^3*e^2*(d - e*x)^3)
```

fricas [A] time = 0.39, size = 183, normalized size = 1.56

$$-\frac{7\,d^3ef-3\,d^4g-\left(7\,e^4f-3\,de^3g\right)x^3+3\left(7\,de^3f-3\,d^2e^2g\right)x^2-3\left(7\,d^2e^2f-3\,d^3eg\right)x+\left(7\,d^2ef-3\,d^3g+\left(2\,e^3f-3\,de^2g\right)x^2-3\left(2\,de^2f-3\,d^2eg\right)x\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^5x^3-3\,d^4e^4x^2+3\,d^5e^3x-d^6e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e*f - 3*d^4*g - (7*e^4*f - 3*d*e^3*g)*x^3 + 3*(7*d*e^3*f - 3*d
^2*e^2*g)*x^2 - 3*(7*d^2*e^2*f - 3*d^3*e*g)*x + (7*d^2*e*f - 3*d^3*g + (2*e
^3*f - 3*d*e^2*g)*x^2 - 3*(2*d*e^2*f - 3*d^2*e*g)*x)*sqrt(-e^2*x^2 + d^2))/
(d^3*e^5*x^3 - 3*d^4*e^4*x^2 + 3*d^5*e^3*x - d^6*e^2)
```

giac [A] time = 0.33, size = 139, normalized size = 1.19

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(15\,df-\left(x\left(\frac{\left(3\,d^2ge^7-2\,dfe^8\right)x^2e^{(-4)}}{d^4}-\frac{5\left(3\,d^4ge^5-d^3fe^6\right)e^{(-4)}}{d^4}\right)-\frac{5\left(3\,d^5ge^4+d^4fe^5\right)e^{(-4)}}{d^4}\right)x\right)x-\frac{\left(3\,d^7ge^2-7\,d^6fe^3\right)e^{(-4)}}{d^4}\right)}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((15*d*f - (x*((3*d^2*g*e^7 - 2*d*f*e^8)*x^2*e^(
-4)/d^4 - 5*(3*d^4*g*e^5 - d^3*f*e^6)*e^(-4)/d^4) - 5*(3*d^5*g*e^4 + d^4*f*
e^5)*e^(-4)/d^4)*x)*x - (3*d^7*g*e^2 - 7*d^6*f*e^3)*e^(-4)/d^4)/(x^2*e^2 -
d^2)^3
```

maple [A] time = 0.01, size = 85, normalized size = 0.73

$$-\frac{(-ex+d)\,(ex+d)^4\left(3d\,e^2g\,x^2-2e^3f\,x^2-9d^2egx+6d\,e^2fx+3d^3g-7d^2ef\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^4*(3*d*e^2*g*x^2-2*e^3*f*x^2-9*d^2*e*g*x+6*d*e^2*f*x
+3*d^3*g-7*d^2*e*f)/d^3/e^2/(-e^2*x^2+d^2)^(7/2)
```

maxima [B] time = 0.46, size = 373, normalized size = 3.19

$$\frac{egx^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{dfx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{3\,d^2gx}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{3\,d^2f}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d^3g}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}+\frac{4\,fx}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}+\frac{gx}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e}+\frac{8\,fx}{15\sqrt{-e^2x^2+d^2}\,d^3}+\frac{gx}{5\sqrt{-e^2x^2+d^2}\,d^2e}+\frac{\left(e^3f+3\,de^2g\right)x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}+\frac{3\left(de^2f+d^2eg\right)x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{2\left(e^3f+3\,de^2g\right)d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}-\frac{\left(de^2f+d^2eg\right)x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e^2}-\frac{2\left(de^2f+d^2eg\right)x}{5\sqrt{-e^2x^2+d^2}\,d^4e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*g*x^3/(-e^2*x^2 + d^2)^(5/2) + 1/5*d*f*x/(-e^2*x^2 + d^2)^(5/2) - 3/1
0*d^2*g*x/((-e^2*x^2 + d^2)^(5/2)*e) + 3/5*d^2*f/((-e^2*x^2 + d^2)^(5/2)*e)
 + 1/5*d^3*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 4/15*f*x/((-e^2*x^2 + d^2)^(3/2
)*d) + 1/10*g*x/((-e^2*x^2 + d^2)^(3/2)*e) + 8/15*f*x/(sqrt(-e^2*x^2 + d^2)
*d^3) + 1/5*g*x/(sqrt(-e^2*x^2 + d^2)*d^2*e) + 1/3*(e^3*f + 3*d*e^2*g)*x^2/
((-e^2*x^2 + d^2)^(5/2)*e^2) + 3/5*(d*e^2*f + d^2*e*g)*x/((-e^2*x^2 + d^2)^
(5/2)*e^2) - 2/15*(e^3*f + 3*d*e^2*g)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) - 1/
5*(d*e^2*f + d^2*e*g)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 2/5*(d*e^2*f + d
^2*e*g)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

mupad [B] time = 2.79, size = 79, normalized size = 0.68

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,g\,d^3-9\,g\,d^2\,e\,x-7\,f\,d^2\,e+3\,g\,d\,e^2\,x^2+6\,f\,d\,e^2\,x-2\,f\,e^3\,x^2\right)}{15\,d^3\,e^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(3*d^3*g - 2*e^3*f*x^2 - 7*d^2*e*f + 6*d*e^2*f*x -
9*d^2*e*g*x + 3*d*e^2*g*x^2))/(15*d^3*e^2*(d - e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^3\left(f+gx\right)}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.386 $\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {655, 659, 651}

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx &= \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}\\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}\\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.56

$$\frac{(d+ex)\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

IntegrateAlgebraic [A] time = 0.00, size = 53, normalized size = 0.51

$$\frac{\sqrt{d^2-e^2x^2}\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^3)
```

fricas [A] time = 0.40, size = 106, normalized size = 1.03

$$\frac{7\,e^3x^3-21\,de^2x^2+21\,d^2ex-7\,d^3-\left(2\,e^2x^2-6\,dex+7\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^4x^3-3\,d^4e^3x^2+3\,d^5e^2x-d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
+ 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x -
 d^6*e)
```

giac [A] time = 0.30, size = 70, normalized size = 0.68

$$-\frac{\sqrt{-x^2e^2+d^2}\left(7\,d^2e^{(-1)}+\left(\left(x\left(\frac{2\,x^2e^4}{d^3}-\frac{5\,e^2}{d}\right)+5\,e\right)x+15\,d\right)x\right)}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(7*d^2*e^(-1) + ((x*(2*x^2*e^4/d^3 - 5*e^2/d) +
5*e)*x + 15*d)*x)/(x^2*e^2 - d^2)^3
```

maple [A] time = 0.01, size = 55, normalized size = 0.53

$$\frac{(-ex+d)\,(ex+d)^4\left(2e^2x^2-6dex+7d^2\right)}{15\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/e/(-e^2*x^2+d^2)^(7/2
)
```

maxima [A] time = 0.44, size = 101, normalized size = 0.98

$$\frac{ex^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{4\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{7\,d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}+\frac{2\,x}{15\sqrt{-e^2x^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^
2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

mupad [B] time = 2.70, size = 49, normalized size = 0.48

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^3
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

3.387 $\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=242

$$\frac{g^3\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(dg+ef)^3\sqrt{e^2f^2-d^2g^2}}-\frac{5d(ef-dg)-ex(11dg+ef)}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^2}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}+\frac{15d^3g^2+ex\left(22d^2g^2\right.}{15d^3\sqrt{d^2-e^2x}}$$

Rubi [A] time = 0.62, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1647, 823, 12, 725, 204}

$$\frac{ex\left(22d^2g^2+9defg+2e^2f^2\right)+15d^3g^2}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^3}+\frac{g^3\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(dg+ef)^3\sqrt{e^2f^2-d^2g^2}}-\frac{5d(ef-dg)-ex(11dg+ef)}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^2}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*(d + e*x))/(5*(e*f + d*g)*(d^2 - e^2*x^2)^(5/2)) - (5*d*(e*f - d*g) -
e*(e*f + 11*d*g)*x)/(15*d*(e*f + d*g)^2*(d^2 - e^2*x^2)^(3/2)) + (15*d^3*g^
2 + e*(2*e^2*f^2 + 9*d*e*f*g + 22*d^2*g^2)*x)/(15*d^3*(e*f + d*g)^3*Sqrt[d^
2 - e^2*x^2]) + (g^3*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/((e*f + d*g)^3*Sqrt[e^2*f^2 - d^2*g^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
```

```
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2(ef+5dg)}{ef+dg}-\frac{d^2e^3(5ef-11dg)x}{ef+dg}}{(f+gx)\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-\frac{d^3e^4(ef-dg)\left(2e^2f^2+7defg+1\cdots\right.}{ef+dg}\cdots}{(f+gx)\left(\cdots\right.}}{15d^4e^4\left(\cdots\right.}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2+9\cdots\right.}{15d^3(ef+dg)^3\cdots}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2+9\cdots\right.}{15d^3(ef+dg)^3\cdots}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2+9\cdots\right.}{15d^3(ef+dg)^3\cdots}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2+9\cdots\right.}{15d^3(ef+dg)^3\cdots}$$

Mathematica [A] time = 0.39, size = 225, normalized size = 0.93

$$\frac{\frac{(d+ex)\left(d^2g^2-e^2f^2\right)\left(32d^4g^2+3d^3eg(8f-17gx)+d^2e^2\left(7f^2-27fgx+22g^2x^2\right)+3de^3fx(3gx-2f)+2e^4f^2x^2\right)}{d^3(d-ex)^2\sqrt{d^2-e^2x^2}}-15g^3\sqrt{e^2f^2-d^2g^2}\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{15(dg-ef)(dg+ef)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (((-(e^2*f^2) + d^2*g^2)*(d + e*x)*(32*d^4*g^2 + 2*e^4*f^2*x^2 + 3*d^3*e*g*
(8*f - 17*g*x) + 3*d*e^3*f*x*(-2*f + 3*g*x) + d^2*e^2*(7*f^2 - 27*f*g*x + 2
2*g^2*x^2)))/(d^3*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]) - 15*g^3*Sqrt[e^2*f^2 -
d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x
^2])])/(15*(-(e*f) + d*g)*(e*f + d*g)^4)
```

IntegrateAlgebraic [F] time = 180.98, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.48, size = 1767, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(7*d^3*e^4*f^4 + 24*d^4*e^3*f^3*g + 25*d^5*e^2*f^2*g^2 - 24*d^6*e*f*g
^3 - 32*d^7*g^4 - (7*e^7*f^4 + 24*d*e^6*f^3*g + 25*d^2*e^5*f^2*g^2 - 24*d^3
*e^4*f*g^3 - 32*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 + 24*d^2*e^5*f^3*g + 25*d
^3*e^4*f^2*g^2 - 24*d^4*e^3*f*g^3 - 32*d^5*e^2*g^4)*x^2 + 15*(d^3*e^3*g^3*x
^3 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*sqrt(-e^2*f^2 + d^2*g^2)*
log((d*e^2*f*g*x + d^3*g^2 - sqrt(-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sq
rt(-e^2*x^2 + d^2)*d*g) - (e^2*f^2 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x +
f)) - 3*(7*d^2*e^5*f^4 + 24*d^3*e^4*f^3*g + 25*d^4*e^3*f^2*g^2 - 24*d^5*e^2
*f*g^3 - 32*d^6*e*g^4)*x + (7*d^2*e^4*f^4 + 24*d^3*e^3*f^3*g + 25*d^4*e^2*f
^2*g^2 - 24*d^5*e*f*g^3 - 32*d^6*g^4 + (2*e^6*f^4 + 9*d*e^5*f^3*g + 20*d^2*
e^4*f^2*g^2 - 9*d^3*e^3*f*g^3 - 22*d^4*e^2*g^4)*x^2 - 3*(2*d*e^5*f^4 + 9*d^
2*e^4*f^3*g + 15*d^3*e^3*f^2*g^2 - 9*d^4*e^2*f*g^3 - 17*d^5*e*g^4)*x)*sqrt(
-e^2*x^2 + d^2))/(d^6*e^5*f^5 + 3*d^7*e^4*f^4*g + 2*d^8*e^3*f^3*g^2 - 2*d^9
*e^2*f^2*g^3 - 3*d^10*e*f*g^4 - d^11*g^5 - (d^3*e^8*f^5 + 3*d^4*e^7*f^4*g +
 2*d^5*e^6*f^3*g^2 - 2*d^6*e^5*f^2*g^3 - 3*d^7*e^4*f*g^4 - d^8*e^3*g^5)*x^3
 + 3*(d^4*e^7*f^5 + 3*d^5*e^6*f^4*g + 2*d^6*e^5*f^3*g^2 - 2*d^7*e^4*f^2*g^3
 - 3*d^8*e^3*f*g^4 - d^9*e^2*g^5)*x^2 - 3*(d^5*e^6*f^5 + 3*d^6*e^5*f^4*g +
2*d^7*e^4*f^3*g^2 - 2*d^8*e^3*f^2*g^3 - 3*d^9*e^2*f*g^4 - d^10*e*g^5)*x), 1
/15*(7*d^3*e^4*f^4 + 24*d^4*e^3*f^3*g + 25*d^5*e^2*f^2*g^2 - 24*d^6*e*f*g^3
 - 32*d^7*g^4 - (7*e^7*f^4 + 24*d*e^6*f^3*g + 25*d^2*e^5*f^2*g^2 - 24*d^3*e
^4*f*g^3 - 32*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 + 24*d^2*e^5*f^3*g + 25*d^3
*e^4*f^2*g^2 - 24*d^4*e^3*f*g^3 - 32*d^5*e^2*g^4)*x^2 - 30*(d^3*e^3*g^3*x^3
 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*sqrt(e^2*f^2 - d^2*g^2)*arc
tan((d*g*x + d*f - sqrt(-e^2*x^2 + d^2)*f)/(sqrt(e^2*f^2 - d^2*g^2)*x)) - 3
*(7*d^2*e^5*f^4 + 24*d^3*e^4*f^3*g + 25*d^4*e^3*f^2*g^2 - 24*d^5*e^2*f*g^3
- 32*d^6*e*g^4)*x + (7*d^2*e^4*f^4 + 24*d^3*e^3*f^3*g + 25*d^4*e^2*f^2*g^2
- 24*d^5*e*f*g^3 - 32*d^6*g^4 + (2*e^6*f^4 + 9*d*e^5*f^3*g + 20*d^2*e^4*f^2
*g^2 - 9*d^3*e^3*f*g^3 - 22*d^4*e^2*g^4)*x^2 - 3*(2*d*e^5*f^4 + 9*d^2*e^4*f
^3*g + 15*d^3*e^3*f^2*g^2 - 9*d^4*e^2*f*g^3 - 17*d^5*e*g^4)*x)*sqrt(-e^2*x^
2 + d^2))/(d^6*e^5*f^5 + 3*d^7*e^4*f^4*g + 2*d^8*e^3*f^3*g^2 - 2*d^9*e^2*f^
2*g^3 - 3*d^10*e*f*g^4 - d^11*g^5 - (d^3*e^8*f^5 + 3*d^4*e^7*f^4*g + 2*d^5*
e^6*f^3*g^2 - 2*d^6*e^5*f^2*g^3 - 3*d^7*e^4*f*g^4 - d^8*e^3*g^5)*x^3 + 3*(d
^4*e^7*f^5 + 3*d^5*e^6*f^4*g + 2*d^6*e^5*f^3*g^2 - 2*d^7*e^4*f^2*g^3 - 3*d^
8*e^3*f*g^4 - d^9*e^2*g^5)*x^2 - 3*(d^5*e^6*f^5 + 3*d^6*e^5*f^4*g + 2*d^7*e
^4*f^3*g^2 - 2*d^8*e^3*f^2*g^3 - 3*d^9*e^2*f*g^4 - d^10*e*g^5)*x)]
```

giac [B] time = 0.46, size = 2966, normalized size = 12.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2*(d^3*g^6*e^2 - 3*d^2*f*g^5*e^3 + 3*d*f^2*g^4*e^4 - f^3*g^3*e^5)*arctan((
d*g*e + (d*e + sqrt(-x^2*e^2 + d^2)*e)*f/x)/sqrt(-d^2*g^2*e^2 + f^2*e^4))/(
(d^6*g^6*e - 3*d^4*f^2*g^4*e^3 + 3*d^2*f^4*g^2*e^5 - f^6*e^7)*sqrt(-d^2*g^2
*e^2 + f^2*e^4)) - 1/15*sqrt(-x^2*e^2 + d^2)*((((((22*d^18*g^17*e^9 + 339*d
^17*f*g^16*e^10 + 2447*d^16*f^2*g^15*e^11 + 10985*d^15*f^3*g^14*e^12 + 3433
5*d^14*f^4*g^13*e^13 + 79261*d^13*f^5*g^12*e^14 + 139867*d^12*f^6*g^11*e^15
 + 192621*d^11*f^7*g^10*e^16 + 209495*d^10*f^8*g^9*e^17 + 180895*d^9*f^9*g^
8*e^18 + 123981*d^8*f^10*g^7*e^19 + 67067*d^7*f^11*g^6*e^20 + 28301*d^6*f^1
2*g^5*e^21 + 9135*d^5*f^13*g^4*e^22 + 2185*d^4*f^14*g^3*e^23 + 367*d^3*f^15
*g^2*e^24 + 39*d^2*f^16*g*e^25 + 2*d*f^17*e^26)*x/(d^22*g^18*e^4 + 18*d^21*
f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19*f^3*g^15*e^7 + 3060*d^18*f^4*
```

```
g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^16*f^6*g^12*e^10 + 31824*d^15*f
^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 48620*d^13*f^9*g^9*e^13 + 43758*d
^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15 + 18564*d^10*f^12*g^6*e^16 + 8
568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^18 + 816*d^7*f^15*g^3*e^19 + 15
3*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^4*f^18*e^22) + 15*(d^19*g^17*e
^8 + 15*d^18*f*g^16*e^9 + 105*d^17*f^2*g^15*e^10 + 455*d^16*f^3*g^14*e^11 +
 1365*d^15*f^4*g^13*e^12 + 3003*d^14*f^5*g^12*e^13 + 5005*d^13*f^6*g^11*e^1
4 + 6435*d^12*f^7*g^10*e^15 + 6435*d^11*f^8*g^9*e^16 + 5005*d^10*f^9*g^8*e^
17 + 3003*d^9*f^10*g^7*e^18 + 1365*d^8*f^11*g^6*e^19 + 455*d^7*f^12*g^5*e^2
0 + 105*d^6*f^13*g^4*e^21 + 15*d^5*f^14*g^3*e^22 + d^4*f^15*g^2*e^23)/(d^22
*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19*f^3*g^15*
e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^16*f^6*g^12
*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 48620*d^13*f^
9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15 + 18564*d^
10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^18 + 816*d^
7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^4*f^18*e^2
2))*x - 5*(11*d^20*g^17*e^7 + 171*d^19*f*g^16*e^8 + 1246*d^18*f^2*g^15*e^9
+ 5650*d^17*f^3*g^14*e^10 + 17850*d^16*f^4*g^13*e^11 + 41678*d^15*f^5*g^12*
e^12 + 74438*d^14*f^6*g^11*e^13 + 103818*d^13*f^7*g^10*e^14 + 114400*d^12*f
^8*g^9*e^15 + 100100*d^11*f^9*g^8*e^16 + 69498*d^10*f^10*g^7*e^17 + 38038*d
^9*f^11*g^6*e^18 + 16198*d^8*f^12*g^5*e^19 + 5250*d^7*f^13*g^4*e^20 + 1250*
d^6*f^14*g^3*e^21 + 206*d^5*f^15*g^2*e^22 + 21*d^4*f^16*g*e^23 + d^3*f^17*e
^24)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19
*f^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^1
6*f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 486
20*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15
+ 18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^1
8 + 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^
4*f^18*e^22))*x - 5*(7*d^21*g^17*e^6 + 105*d^20*f*g^16*e^7 + 734*d^19*f^2*g
^15*e^8 + 3170*d^18*f^3*g^14*e^9 + 9450*d^17*f^4*g^13*e^10 + 20566*d^16*f^5
*g^12*e^11 + 33670*d^15*f^6*g^11*e^12 + 42042*d^14*f^7*g^10*e^13 + 40040*d^
13*f^8*g^9*e^14 + 28600*d^12*f^9*g^8*e^15 + 14586*d^11*f^10*g^7*e^16 + 4550
*d^10*f^11*g^6*e^17 + 182*d^9*f^12*g^5*e^18 - 630*d^8*f^13*g^4*e^19 - 350*d
^7*f^14*g^3*e^20 - 98*d^6*f^15*g^2*e^21 - 15*d^5*f^16*g*e^22 - d^4*f^17*e^2
3)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19*f
^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^16*
f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 48620
*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15 +
18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^18
+ 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^4*
f^18*e^22))*x + 15*(3*d^22*g^17*e^5 + 48*d^21*f*g^16*e^6 + 361*d^20*f^2*g^1
5*e^7 + 1695*d^19*f^3*g^14*e^8 + 5565*d^18*f^4*g^13*e^9 + 13559*d^17*f^5*g^
12*e^10 + 25389*d^16*f^6*g^11*e^11 + 37323*d^15*f^7*g^10*e^12 + 43615*d^14*
f^8*g^9*e^13 + 40755*d^13*f^9*g^8*e^14 + 30459*d^12*f^10*g^7*e^15 + 18109*d
^11*f^11*g^6*e^16 + 8463*d^10*f^12*g^5*e^17 + 3045*d^9*f^13*g^4*e^18 + 815*
d^8*f^14*g^3*e^19 + 153*d^7*f^15*g^2*e^20 + 18*d^6*f^16*g*e^21 + d^5*f^17*e
^22)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19
*f^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^1
6*f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 486
20*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15
+ 18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^1
8 + 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^
4*f^18*e^22))*x + (32*d^23*g^17*e^4 + 504*d^22*f*g^16*e^5 + 3727*d^21*f^2*g
^15*e^6 + 17185*d^20*f^3*g^14*e^7 + 55335*d^19*f^4*g^13*e^8 + 132041*d^18*f
^5*g^12*e^9 + 241787*d^17*f^6*g^11*e^10 + 347061*d^16*f^7*g^10*e^11 + 39539
5*d^15*f^8*g^9*e^12 + 359645*d^14*f^9*g^8*e^13 + 261261*d^13*f^10*g^7*e^14
+ 150787*d^12*f^11*g^6*e^15 + 68341*d^11*f^12*g^5*e^16 + 23835*d^10*f^13*g^
4*e^17 + 6185*d^9*f^14*g^3*e^18 + 1127*d^8*f^15*g^2*e^19 + 129*d^7*f^16*g*e
^20 + 7*d^6*f^17*e^21)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g
```

```
^16*e^6 + 816*d^19*f^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^
13*e^9 + 18564*d^16*f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f
^8*g^10*e^12 + 48620*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d
^11*f^11*g^7*e^15 + 18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 306
0*d^8*f^14*g^4*e^18 + 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^
5*f^17*g*e^21 + d^4*f^18*e^22))/(x^2*e^2 - d^2)^3
```

maple [B] time = 0.05, size = 3961, normalized size = 16.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] g/(d^2*g^2-e^2*f^2)^3*f^4/d^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^
2*f^2)/g^2)^(1/2)*e^5*x+3*g^4/(d^2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(
1/2)*ln((2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2
)^(1/2)*(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+
f/g))*d^2*e*f+1/3/g/(d^2*g^2-e^2*f^2)^2*e^5*f^4/d^2/(-(x+f/g)^2*e^2+2*e^2*f
/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x+2/3/g/(d^2*g^2-e^2*f^2)^2*e^5*f^4
/d^4/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x-g^4/(
d^2*g^2-e^2*f^2)^3*f*d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/
g^2)^(1/2)*e^2*x-3*g^2/(d^2*g^2-e^2*f^2)^3*f^3/d/(-(x+f/g)^2*e^2+2*e^2*f/g*
(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*e^4*x-4/5*e^2/g^2/d^3*f*x/(-e^2*x^2+d^
2)^(3/2)+3/5/g*e^3*f^2/(d^2*g^2-e^2*f^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+
(d^2*g^2-e^2*f^2)/g^2)^(5/2)*x+8/15*e^3/g^3*f^2/d^6*x/(-e^2*x^2+d^2)^(1/2)-
3/5*e^2/g^2/d*f*x/(-e^2*x^2+d^2)^(5/2)-8/5*e^2/g^2/d^5*f*x/(-e^2*x^2+d^2)^(
1/2)+1/5*e^3/g^3*f^2*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/15*e^3/g^3*f^2/d^4*x/(-e^
2*x^2+d^2)^(3/2)+3/5/g/(d^2*g^2-e^2*f^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+
(d^2*g^2-e^2*f^2)/g^2)^(5/2)*d*e^2*f^2-g^2/(d^2*g^2-e^2*f^2)^2/(-(x+f/g)^2*
e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*d^2*e*f-1/5*e^2*f/(d^2*g
^2-e^2*f^2)*d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2
)*x-8/15*e^2*f/(d^2*g^2-e^2*f^2)/d^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2
*g^2-e^2*f^2)/g^2)^(1/2)*x-1/(d^2*g^2-e^2*f^2)^2*e^4*f^3/d/(-(x+f/g)^2*e^2+
2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x-2/(d^2*g^2-e^2*f^2)^2*e^4*
f^3/d^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x-4/
15*e^2*f/(d^2*g^2-e^2*f^2)/d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2
*f^2)/g^2)^(3/2)*x+g/(d^2*g^2-e^2*f^2)^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+
(d^2*g^2-e^2*f^2)/g^2)^(3/2)*d*e^2*f^2-3*g^4/(d^2*g^2-e^2*f^2)^3/(-(x+f/g)^
2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*d^2*e*f+3*g^3/(d^2*g^2
-e^2*f^2)^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*
d*e^2*f^2+g^2/(d^2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln((2*(d^2*
g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*(-(x+f/g
)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g))*e^3*f^3+3*
g^3/(d^2*g^2-e^2*f^2)^3*f^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*
f^2)/g^2)^(1/2)*e^3*x+g/(d^2*g^2-e^2*f^2)^2*e^3*f^2/(-(x+f/g)^2*e^2+2*e^2*f
/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x+1/5*g/(d^2*g^2-e^2*f^2)/(-(x+f/g)
^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*d^3+1/3*g^3/(d^2*g^2-
e^2*f^2)^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*d
^3+g^5/(d^2*g^2-e^2*f^2)^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f
^2)/g^2)^(1/2)*d^3-1/3/(d^2*g^2-e^2*f^2)^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g
)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*e^3*f^3+4/5*e/g*x/(-e^2*x^2+d^2)^(5/2)-1/5*e
/g^2/(-e^2*x^2+d^2)^(5/2)*f+3/5/g/(-e^2*x^2+d^2)^(5/2)*d+8/15/g^3*e^5*f^4/(
d^2*g^2-e^2*f^2)/d^6/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^
2)^(1/2)*x-1/3*g^2/(d^2*g^2-e^2*f^2)^2*e^2*f*d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x
+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x+11/15*e/g/d^2*x/(-e^2*x^2+d^2)^(3/2)+2
2/15*e/g/d^4*x/(-e^2*x^2+d^2)^(1/2)-3/5/(d^2*g^2-e^2*f^2)/(-(x+f/g)^2*e^2+2
*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*d^2*e*f-1/5/g^2/(d^2*g^2-e^2*
f^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*e^3*f^3
-g^2/(d^2*g^2-e^2*f^2)^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2
```

```
)/g^2)^(1/2)*e^3*f^3-g^5/(d^2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*
ln((2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/
2)*(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g))
*d^3-3*g^3/(d^2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln((2*(d^2*g^2
-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^2
*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g))*d*e^2*f^2-3/5
/g^2*e^4*f^3/(d^2*g^2-e^2*f^2)/d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2
-e^2*f^2)/g^2)^(5/2)*x+1/5/g^3*e^5*f^4/(d^2*g^2-e^2*f^2)/d^2/(-(x+f/g)^2*e^
2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*x+4/5/g*e^3*f^2/(d^2*g^2-e
^2*f^2)/d^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*
x-2/3*g^2/(d^2*g^2-e^2*f^2)^2*e^2*f/d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^
2*g^2-e^2*f^2)/g^2)^(1/2)*x+2*g/(d^2*g^2-e^2*f^2)^2*e^3*f^2/d^2/(-(x+f/g)^2
*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x-4/5/g^2*e^4*f^3/(d^2*
g^2-e^2*f^2)/d^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(
3/2)*x+4/15/g^3*e^5*f^4/(d^2*g^2-e^2*f^2)/d^4/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+
f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x+8/5/g*e^3*f^2/(d^2*g^2-e^2*f^2)/d^4/(-(
x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x-8/5/g^2*e^4*f
^3/(d^2*g^2-e^2*f^2)/d^5/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2
)/g^2)^(1/2)*x
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d*g-e*f)>0)', see `assume?` for mo
re details)Is (d*g-e*f)    *(d*g+e*f)    positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\left(f+g\,x\right)\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)), x)
```

3.388 $\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=311

$$\frac{eg^3(4ef-3dg)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)(dg+ef)^4\sqrt{e^2f^2-d^2g^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)(dg+ef)^4}-\frac{e(5d(ef-3dg)-ex(21dg+ef))}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^3}+\frac{}{5}$$

Rubi [A] time = 1.26, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1647, 807, 725, 204}

$$\frac{e\left(ex\left(57d^2g^2+14defg+2e^2f^2\right)+45d^3g^2\right)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^4}+\frac{eg^3(4ef-3dg)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)(dg+ef)^4\sqrt{e^2f^2-d^2g^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)(dg+ef)^4}-\frac{e(5d(ef-3dg)-ex(21dg+ef))}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^3}+\frac{4de(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*e*(d + e*x))/(5*(e*f + d*g)^2*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d*(e*f -
3*d*g) - e*(e*f + 21*d*g)*x))/(15*d*(e*f + d*g)^3*(d^2 - e^2*x^2)^(3/2)) +
(e*(45*d^3*g^2 + e*(2*e^2*f^2 + 14*d*e*f*g + 57*d^2*g^2)*x))/(15*d^3*(e*f +
 d*g)^4*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/((e*f - d*g)*(e*f
+ d*g)^4*(f + g*x)) + (e*g^3*(4*e*f - 3*d*g)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt
[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/((e*f - d*g)*(e*f + d*g)^4*Sqrt[
e^2*f^2 - d^2*g^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2\left(e^2f^2+10defg+5d^2g^2\right)}{(ef+dg)^2}-\frac{d^2e^3(ef-5dg)(5ef+3dg)x}{(ef+dg)^2}+\frac{16d^3e^4g^2x^2}{(ef+dg)^2}}{(f+gx)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{d^3e^4\left(2e^3f^3+12de^2f^2\cdots\right.}{(ef\cdots}}{\cdots}}{\cdots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\left(2\cdots\right.\right.}{15d^3(e\cdots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\left(2\cdots\right.\right.}{15d^3(e\cdots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\left(2\cdots\right.\right.}{15d^3(e\cdots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\left(2\cdots\right.\right.}{15d^3(e\cdots}$$

Mathematica [A] time = 0.61, size = 341, normalized size = 1.10

$$\frac{15eg^3(4ef-3dg)\sqrt{e^2f^2-d^2g^2}\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)+\frac{(d+ex)\left(e^2f^2-d^2g^2\right)\left(15d^6g^4-9d^5eg^3(8f+13gx)+d^4e^2g^2\left(38f^2+164fgx+171g^2x^2\right)-3d^3e^3g\left(-9f^3+19f^2gx+47fg^2x^2+24g^3x^3\right)+d^2e^4f\left(7f^3-29f^2gx+7fg^2x^2+43g^3x^3\right)+6de^5f^2x\left(-f^2+fgx+2g^2x^2\right)+2e^6f^3x^2(f+gx)\right)}{d^3(d-ex)^2\sqrt{d^2-e^2x^2}(f+gx)}}{15(ef-dg)^2(dg+ef)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (((e^2*f^2 - d^2*g^2)*(d + e*x)*(15*d^6*g^4 + 2*e^6*f^3*x^2*(f + g*x) - 9*d
^5*e*g^3*(8*f + 13*g*x) + 6*d*e^5*f^2*x*(-f^2 + f*g*x + 2*g^2*x^2) + d^4*e^
2*g^2*(38*f^2 + 164*f*g*x + 171*g^2*x^2) - 3*d^3*e^3*g*(-9*f^3 + 19*f^2*g*x
 + 47*f*g^2*x^2 + 24*g^3*x^3) + d^2*e^4*f*(7*f^3 - 29*f^2*g*x + 7*f*g^2*x^2
 + 43*g^3*x^3)))/(d^3*(d - e*x)^2*(f + g*x)*Sqrt[d^2 - e^2*x^2]) + 15*e*g^3
*(4*e*f - 3*d*g)*Sqrt[e^2*f^2 - d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2
*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/(15*(e*f - d*g)^2*(e*f + d*g)^5)
```

IntegrateAlgebraic [F] time = 180.05, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 1.02, size = 3305, normalized size = 10.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] [1/15*(7*d^3*e^6*f^7 + 27*d^4*e^5*f^6*g + 31*d^5*e^4*f^5*g^2 - 99*d^6*e^3*f
^4*g^3 - 23*d^7*e^2*f^3*g^4 + 72*d^8*e*f^2*g^5 - 15*d^9*f*g^6 - (7*e^9*f^6*
g + 27*d*e^8*f^5*g^2 + 31*d^2*e^7*f^4*g^3 - 99*d^3*e^6*f^3*g^4 - 23*d^4*e^5
*f^2*g^5 + 72*d^5*e^4*f*g^6 - 15*d^6*e^3*g^7)*x^4 - (7*e^9*f^7 + 6*d*e^8*f^
6*g - 50*d^2*e^7*f^5*g^2 - 192*d^3*e^6*f^4*g^3 + 274*d^4*e^5*f^3*g^4 + 141*
d^5*e^4*f^2*g^5 - 231*d^6*e^3*f*g^6 + 45*d^7*e^2*g^7)*x^3 + 3*(7*d*e^8*f^7
+ 20*d^2*e^7*f^6*g + 4*d^3*e^6*f^5*g^2 - 130*d^4*e^5*f^4*g^3 + 76*d^5*e^4*f
^3*g^4 + 95*d^6*e^3*f^2*g^5 - 87*d^7*e^2*f*g^6 + 15*d^8*e*g^7)*x^2 - 15*(4*
d^6*e^2*f^3*g^3 - 3*d^7*e*f^2*g^4 - (4*d^3*e^5*f^2*g^4 - 3*d^4*e^4*f*g^5)*x
^4 - (4*d^3*e^5*f^3*g^3 - 15*d^4*e^4*f^2*g^4 + 9*d^5*e^3*f*g^5)*x^3 + 3*(4*
d^4*e^4*f^3*g^3 - 7*d^5*e^3*f^2*g^4 + 3*d^6*e^2*f*g^5)*x^2 - (12*d^5*e^3*f^
3*g^3 - 13*d^6*e^2*f^2*g^4 + 3*d^7*e*f*g^5)*x)*sqrt(-e^2*f^2 + d^2*g^2)*log
((d*e^2*f*g*x + d^3*g^2 - sqrt(-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sqrt(
-e^2*x^2 + d^2)*d*g) - (e^2*f^2 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x + f))
 - (21*d^2*e^7*f^7 + 74*d^3*e^6*f^6*g + 66*d^4*e^5*f^5*g^2 - 328*d^5*e^4*f^
4*g^3 + 30*d^6*e^3*f^3*g^4 + 239*d^7*e^2*f^2*g^5 - 117*d^8*e*f*g^6 + 15*d^9
*g^7)*x + (7*d^2*e^6*f^7 + 27*d^3*e^5*f^6*g + 31*d^4*e^4*f^5*g^2 - 99*d^5*e
^3*f^4*g^3 - 23*d^6*e^2*f^3*g^4 + 72*d^7*e*f^2*g^5 - 15*d^8*f*g^6 + (2*e^8*
f^6*g + 12*d*e^7*f^5*g^2 + 41*d^2*e^6*f^4*g^3 - 84*d^3*e^5*f^3*g^4 - 43*d^4
*e^4*f^2*g^5 + 72*d^5*e^3*f*g^6)*x^3 + (2*e^8*f^7 + 6*d*e^7*f^6*g + 5*d^2*e
^6*f^5*g^2 - 147*d^3*e^5*f^4*g^3 + 164*d^4*e^4*f^3*g^4 + 141*d^5*e^3*f^2*g^
5 - 171*d^6*e^2*f*g^6)*x^2 - (6*d*e^7*f^7 + 29*d^2*e^6*f^6*g + 51*d^3*e^5*f
^5*g^2 - 193*d^4*e^4*f^4*g^3 + 60*d^5*e^3*f^3*g^4 + 164*d^6*e^2*f^2*g^5 - 1
17*d^7*e*f*g^6)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*f^9 + 3*d^7*e^6*f^8*g + d
^8*e^5*f^7*g^2 - 5*d^9*e^4*f^6*g^3 - 5*d^10*e^3*f^5*g^4 + d^11*e^2*f^4*g^5
+ 3*d^12*e*f^3*g^6 + d^13*f^2*g^7 - (d^3*e^10*f^8*g + 3*d^4*e^9*f^7*g^2 + d
^5*e^8*f^6*g^3 - 5*d^6*e^7*f^5*g^4 - 5*d^7*e^6*f^4*g^5 + d^8*e^5*f^3*g^6 +
3*d^9*e^4*f^2*g^7 + d^10*e^3*f*g^8)*x^4 - (d^3*e^10*f^9 - 8*d^5*e^8*f^7*g^2
 - 8*d^6*e^7*f^6*g^3 + 10*d^7*e^6*f^5*g^4 + 16*d^8*e^5*f^4*g^5 - 8*d^10*e^3
*f^2*g^7 - 3*d^11*e^2*f*g^8)*x^3 + 3*(d^4*e^9*f^9 + 2*d^5*e^8*f^8*g - 2*d^6
*e^7*f^7*g^2 - 6*d^7*e^6*f^6*g^3 + 6*d^9*e^4*f^4*g^5 + 2*d^10*e^3*f^3*g^6 -
 2*d^11*e^2*f^2*g^7 - d^12*e*f*g^8)*x^2 - (3*d^5*e^8*f^9 + 8*d^6*e^7*f^8*g
- 16*d^8*e^5*f^6*g^3 - 10*d^9*e^4*f^5*g^4 + 8*d^10*e^3*f^4*g^5 + 8*d^11*e^2
*f^3*g^6 - d^13*f*g^8)*x), 1/15*(7*d^3*e^6*f^7 + 27*d^4*e^5*f^6*g + 31*d^5*
e^4*f^5*g^2 - 99*d^6*e^3*f^4*g^3 - 23*d^7*e^2*f^3*g^4 + 72*d^8*e*f^2*g^5 -
15*d^9*f*g^6 - (7*e^9*f^6*g + 27*d*e^8*f^5*g^2 + 31*d^2*e^7*f^4*g^3 - 99*d^
3*e^6*f^3*g^4 - 23*d^4*e^5*f^2*g^5 + 72*d^5*e^4*f*g^6 - 15*d^6*e^3*g^7)*x^4
 - (7*e^9*f^7 + 6*d*e^8*f^6*g - 50*d^2*e^7*f^5*g^2 - 192*d^3*e^6*f^4*g^3 +
274*d^4*e^5*f^3*g^4 + 141*d^5*e^4*f^2*g^5 - 231*d^6*e^3*f*g^6 + 45*d^7*e^2*
g^7)*x^3 + 3*(7*d*e^8*f^7 + 20*d^2*e^7*f^6*g + 4*d^3*e^6*f^5*g^2 - 130*d^4*
e^5*f^4*g^3 + 76*d^5*e^4*f^3*g^4 + 95*d^6*e^3*f^2*g^5 - 87*d^7*e^2*f*g^6 +
15*d^8*e*g^7)*x^2 + 30*(4*d^6*e^2*f^3*g^3 - 3*d^7*e*f^2*g^4 - (4*d^3*e^5*f^
2*g^4 - 3*d^4*e^4*f*g^5)*x^4 - (4*d^3*e^5*f^3*g^3 - 15*d^4*e^4*f^2*g^4 + 9*
d^5*e^3*f*g^5)*x^3 + 3*(4*d^4*e^4*f^3*g^3 - 7*d^5*e^3*f^2*g^4 + 3*d^6*e^2*f
*g^5)*x^2 - (12*d^5*e^3*f^3*g^3 - 13*d^6*e^2*f^2*g^4 + 3*d^7*e*f*g^5)*x)*sq
rt(e^2*f^2 - d^2*g^2)*arctan((d*g*x + d*f - sqrt(-e^2*x^2 + d^2)*f)/(sqrt(e
^2*f^2 - d^2*g^2)*x)) - (21*d^2*e^7*f^7 + 74*d^3*e^6*f^6*g + 66*d^4*e^5*f^5
*g^2 - 328*d^5*e^4*f^4*g^3 + 30*d^6*e^3*f^3*g^4 + 239*d^7*e^2*f^2*g^5 - 117
*d^8*e*f*g^6 + 15*d^9*g^7)*x + (7*d^2*e^6*f^7 + 27*d^3*e^5*f^6*g + 31*d^4*e
^4*f^5*g^2 - 99*d^5*e^3*f^4*g^3 - 23*d^6*e^2*f^3*g^4 + 72*d^7*e*f^2*g^5 - 1
5*d^8*f*g^6 + (2*e^8*f^6*g + 12*d*e^7*f^5*g^2 + 41*d^2*e^6*f^4*g^3 - 84*d^3
*e^5*f^3*g^4 - 43*d^4*e^4*f^2*g^5 + 72*d^5*e^3*f*g^6)*x^3 + (2*e^8*f^7 + 6*
d*e^7*f^6*g + 5*d^2*e^6*f^5*g^2 - 147*d^3*e^5*f^4*g^3 + 164*d^4*e^4*f^3*g^4
 + 141*d^5*e^3*f^2*g^5 - 171*d^6*e^2*f*g^6)*x^2 - (6*d*e^7*f^7 + 29*d^2*e^6
*f^6*g + 51*d^3*e^5*f^5*g^2 - 193*d^4*e^4*f^4*g^3 + 60*d^5*e^3*f^3*g^4 + 16
4*d^6*e^2*f^2*g^5 - 117*d^7*e*f*g^6)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*f^9
+ 3*d^7*e^6*f^8*g + d^8*e^5*f^7*g^2 - 5*d^9*e^4*f^6*g^3 - 5*d^10*e^3*f^5*g^
4 + d^11*e^2*f^4*g^5 + 3*d^12*e*f^3*g^6 + d^13*f^2*g^7 - (d^3*e^10*f^8*g +
```

```
3*d^4*e^9*f^7*g^2 + d^5*e^8*f^6*g^3 - 5*d^6*e^7*f^5*g^4 - 5*d^7*e^6*f^4*g^5
 + d^8*e^5*f^3*g^6 + 3*d^9*e^4*f^2*g^7 + d^10*e^3*f*g^8)*x^4 - (d^3*e^10*f^
9 - 8*d^5*e^8*f^7*g^2 - 8*d^6*e^7*f^6*g^3 + 10*d^7*e^6*f^5*g^4 + 16*d^8*e^5
*f^4*g^5 - 8*d^10*e^3*f^2*g^7 - 3*d^11*e^2*f*g^8)*x^3 + 3*(d^4*e^9*f^9 + 2*
d^5*e^8*f^8*g - 2*d^6*e^7*f^7*g^2 - 6*d^7*e^6*f^6*g^3 + 6*d^9*e^4*f^4*g^5 +
 2*d^10*e^3*f^3*g^6 - 2*d^11*e^2*f^2*g^7 - d^12*e*f*g^8)*x^2 - (3*d^5*e^8*f
^9 + 8*d^6*e^7*f^8*g - 16*d^8*e^5*f^6*g^3 - 10*d^9*e^4*f^5*g^4 + 8*d^10*e^3
*f^4*g^5 + 8*d^11*e^2*f^3*g^6 - d^13*f*g^8)*x)]
```

giac [C] time = 2.97, size = 4343, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(-45*I*d^9*g^12*e^6*log(d^2*g^4*e^2) - 75*I*d^8*f*g^11*e^7*log(d^
2*g^4*e^2) + 90*I*d^7*f^2*g^10*e^8*log(d^2*g^4*e^2) + 144*sqrt(d^2*g^2 - f^
2*e^2)*d^8*g^10*abs(g)*e^6 + 210*I*d^6*f^3*g^9*e^9*log(d^2*g^4*e^2) + 346*s
qrt(d^2*g^2 - f^2*e^2)*d^7*f*g^9*abs(g)*e^7 + 15*I*d^5*f^4*g^8*e^10*log(d^2
*g^4*e^2) + 6*sqrt(d^2*g^2 - f^2*e^2)*d^6*f^2*g^8*abs(g)*e^8 - 135*I*d^4*f^
5*g^7*e^11*log(d^2*g^4*e^2) - 536*sqrt(d^2*g^2 - f^2*e^2)*d^5*f^3*g^7*abs(g
)*e^9 - 60*I*d^3*f^6*g^6*e^12*log(d^2*g^4*e^2) - 320*sqrt(d^2*g^2 - f^2*e^2
)*d^4*f^4*g^6*abs(g)*e^10 + 154*sqrt(d^2*g^2 - f^2*e^2)*d^3*f^5*g^5*abs(g)*
e^11 + 166*sqrt(d^2*g^2 - f^2*e^2)*d^2*f^6*g^4*abs(g)*e^12 + 36*sqrt(d^2*g^
2 - f^2*e^2)*d*f^7*g^3*abs(g)*e^13 + 4*sqrt(d^2*g^2 - f^2*e^2)*f^8*g^2*abs(
g)*e^14)*sgn(1/(g*x + f))*sgn(g)/(30*I*sqrt(d^2*g^2 - f^2*e^2)*d^13*g^10*ab
s(g)*e^5 + 180*I*sqrt(d^2*g^2 - f^2*e^2)*d^12*f*g^9*abs(g)*e^6 + 390*I*sqrt
(d^2*g^2 - f^2*e^2)*d^11*f^2*g^8*abs(g)*e^7 + 240*I*sqrt(d^2*g^2 - f^2*e^2)
*d^10*f^3*g^7*abs(g)*e^8 - 420*I*sqrt(d^2*g^2 - f^2*e^2)*d^9*f^4*g^6*abs(g)
*e^9 - 840*I*sqrt(d^2*g^2 - f^2*e^2)*d^8*f^5*g^5*abs(g)*e^10 - 420*I*sqrt(d
^2*g^2 - f^2*e^2)*d^7*f^6*g^4*abs(g)*e^11 + 240*I*sqrt(d^2*g^2 - f^2*e^2)*d
^6*f^7*g^3*abs(g)*e^12 + 390*I*sqrt(d^2*g^2 - f^2*e^2)*d^5*f^8*g^2*abs(g)*e
^13 + 180*I*sqrt(d^2*g^2 - f^2*e^2)*d^4*f^9*g*abs(g)*e^14 + 30*I*sqrt(d^2*g
^2 - f^2*e^2)*d^3*f^10*abs(g)*e^15) + 15*(3*d*g^7*e - 4*f*g^6*e^2)*log(abs(
f*g*e^2 + sqrt(d^2*g^2 - f^2*e^2)*(sqrt(d^2*g^2/(g*x + f)^2 + 2*f*e^2/(g*x
+ f) - f^2*e^2/(g*x + f)^2 - e^2) + sqrt(d^2*g^4 - f^2*g^2*e^2)/((g*x + f)*
g))*abs(g)))/(sqrt(d^2*g^2 - f^2*e^2)*d^5*g^5*abs(g)*sgn(1/(g*x + f))*sgn(g
) + 3*sqrt(d^2*g^2 - f^2*e^2)*d^4*f*g^4*abs(g)*e*sgn(1/(g*x + f))*sgn(g) +
2*sqrt(d^2*g^2 - f^2*e^2)*d^3*f^2*g^3*abs(g)*e^2*sgn(1/(g*x + f))*sgn(g) -
2*sqrt(d^2*g^2 - f^2*e^2)*d^2*f^3*g^2*abs(g)*e^3*sgn(1/(g*x + f))*sgn(g) -
3*sqrt(d^2*g^2 - f^2*e^2)*d*f^4*g*abs(g)*e^4*sgn(1/(g*x + f))*sgn(g) - sqrt
(d^2*g^2 - f^2*e^2)*f^5*abs(g)*e^5*sgn(1/(g*x + f))*sgn(g)) - ((72*d^8*g^24
*e^10*sgn(1/(g*x + f))^3*sgn(g)^3 - 187*d^7*f*g^23*e^11*sgn(1/(g*x + f))^3*
sgn(g)^3 + 146*d^6*f^2*g^22*e^12*sgn(1/(g*x + f))^3*sgn(g)^3 - 21*d^5*f^3*g
^21*e^13*sgn(1/(g*x + f))^3*sgn(g)^3 - 8*d^4*f^4*g^20*e^14*sgn(1/(g*x + f))
^3*sgn(g)^3 - 2*d^3*f^5*g^19*e^15*sgn(1/(g*x + f))^3*sgn(g)^3)/(d^13*g^24*e
^4*sgn(1/(g*x + f))^4*sgn(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^
4 - 3*d^11*f^2*g^22*e^6*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*s
gn(1/(g*x + f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4
 + 3*d^8*f^5*g^19*e^9*sgn(1/(g*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1
/(g*x + f))^4*sgn(g)^4 - d^6*f^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4) + (
5*(9*d^9*g^26*e^9*sgn(1/(g*x + f))^3*sgn(g)^3 - 102*d^8*f*g^25*e^10*sgn(1/(
g*x + f))^3*sgn(g)^3 + 220*d^7*f^2*g^24*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 -
158*d^6*f^3*g^23*e^12*sgn(1/(g*x + f))^3*sgn(g)^3 + 21*d^5*f^4*g^22*e^13*sg
n(1/(g*x + f))^3*sgn(g)^3 + 8*d^4*f^5*g^21*e^14*sgn(1/(g*x + f))^3*sgn(g)^3
 + 2*d^3*f^6*g^20*e^15*sgn(1/(g*x + f))^3*sgn(g)^3)/(d^13*g^24*e^4*sgn(1/(g
*x + f))^4*sgn(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*
f^2*g^22*e^6*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x +
 f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^
```

5*g^19*e^9*sgn(1/(g*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))
^4*sgn(g)^4 - d^6*f^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4) - (5*(36*d^10*
g^28*e^8*sgn(1/(g*x + f))^3*sgn(g)^3 - 53*d^9*f*g^27*e^9*sgn(1/(g*x + f))^3
*sgn(g)^3 - 206*d^8*f^2*g^26*e^10*sgn(1/(g*x + f))^3*sgn(g)^3 + 512*d^7*f^3
*g^25*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 - 350*d^6*f^4*g^24*e^12*sgn(1/(g*x +
 f))^3*sgn(g)^3 + 41*d^5*f^5*g^23*e^13*sgn(1/(g*x + f))^3*sgn(g)^3 + 16*d^4
*f^6*g^22*e^14*sgn(1/(g*x + f))^3*sgn(g)^3 + 4*d^3*f^7*g^21*e^15*sgn(1/(g*x
 + f))^3*sgn(g)^3)/(d^13*g^24*e^4*sgn(1/(g*x + f))^4*sgn(g)^4 + d^12*f*g^23
*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*f^2*g^22*e^6*sgn(1/(g*x + f))^4*s
gn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*
e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sgn(1/(g*x + f))^4*sgn
(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 - d^6*f^7*g^17*e^11*s
gn(1/(g*x + f))^4*sgn(g)^4) + (5*(21*d^11*g^30*e^7*sgn(1/(g*x + f))^3*sgn(g
)^3 - 178*d^10*f*g^29*e^8*sgn(1/(g*x + f))^3*sgn(g)^3 + 287*d^9*f^2*g^28*e^
9*sgn(1/(g*x + f))^3*sgn(g)^3 + 132*d^8*f^3*g^27*e^10*sgn(1/(g*x + f))^3*sg
n(g)^3 - 601*d^7*f^4*g^26*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 + 398*d^6*f^5*g^
25*e^12*sgn(1/(g*x + f))^3*sgn(g)^3 - 39*d^5*f^6*g^24*e^13*sgn(1/(g*x + f))
^3*sgn(g)^3 - 16*d^4*f^7*g^23*e^14*sgn(1/(g*x + f))^3*sgn(g)^3 - 4*d^3*f^8*
g^22*e^15*sgn(1/(g*x + f))^3*sgn(g)^3)/(d^13*g^24*e^4*sgn(1/(g*x + f))^4*sg
n(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*f^2*g^22*e^6*
sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)
^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sg
n(1/(g*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 -
 d^6*f^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4) - (5*(27*d^12*g^32*e^6*sgn(
1/(g*x + f))^3*sgn(g)^3 - 18*d^11*f*g^31*e^7*sgn(1/(g*x + f))^3*sgn(g)^3 -
227*d^10*f^2*g^30*e^8*sgn(1/(g*x + f))^3*sgn(g)^3 + 406*d^9*f^3*g^29*e^9*sg
n(1/(g*x + f))^3*sgn(g)^3 - 27*d^8*f^4*g^28*e^10*sgn(1/(g*x + f))^3*sgn(g)^
3 - 368*d^7*f^5*g^27*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 + 235*d^6*f^6*g^26*e^
12*sgn(1/(g*x + f))^3*sgn(g)^3 - 18*d^5*f^7*g^25*e^13*sgn(1/(g*x + f))^3*sg
n(g)^3 - 8*d^4*f^8*g^24*e^14*sgn(1/(g*x + f))^3*sgn(g)^3 - 2*d^3*f^9*g^23*e
^15*sgn(1/(g*x + f))^3*sgn(g)^3)/(d^13*g^24*e^4*sgn(1/(g*x + f))^4*sgn(g)^4
 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*f^2*g^22*e^6*sgn(1/
(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)^4 + 3
*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sgn(1/(g
*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 - d^6*f
^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4) + (2*(36*d^13*g^34*e^5*sgn(1/(g*x
 + f))^3*sgn(g)^3 - 181*d^12*f*g^33*e^6*sgn(1/(g*x + f))^3*sgn(g)^3 + 203*d
^11*f^2*g^32*e^7*sgn(1/(g*x + f))^3*sgn(g)^3 + 217*d^10*f^3*g^31*e^8*sgn(1/
(g*x + f))^3*sgn(g)^3 - 504*d^9*f^4*g^30*e^9*sgn(1/(g*x + f))^3*sgn(g)^3 +
113*d^8*f^5*g^29*e^10*sgn(1/(g*x + f))^3*sgn(g)^3 + 256*d^7*f^6*g^28*e^11*s
gn(1/(g*x + f))^3*sgn(g)^3 - 153*d^6*f^7*g^27*e^12*sgn(1/(g*x + f))^3*sgn(g
)^3 + 8*d^5*f^8*g^26*e^13*sgn(1/(g*x + f))^3*sgn(g)^3 + 4*d^4*f^9*g^25*e^14
*sgn(1/(g*x + f))^3*sgn(g)^3 + d^3*f^10*g^24*e^15*sgn(1/(g*x + f))^3*sgn(g)
^3)/(d^13*g^24*e^4*sgn(1/(g*x + f))^4*sgn(g)^4 + d^12*f*g^23*e^5*sgn(1/(g*x
 + f))^4*sgn(g)^4 - 3*d^11*f^2*g^22*e^6*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^1
0*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^9*f^4*g^20*e^8*sgn(1/(g*x
+ f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sgn(1/(g*x + f))^4*sgn(g)^4 - d^7*f^6
*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 - d^6*f^7*g^17*e^11*sgn(1/(g*x + f))
^4*sgn(g)^4) - 15*(d^14*g^36*e^4*sgn(1/(g*x + f))^3*sgn(g)^3 - 2*d^13*f*g^3
5*e^5*sgn(1/(g*x + f))^3*sgn(g)^3 - 2*d^12*f^2*g^34*e^6*sgn(1/(g*x + f))^3*
sgn(g)^3 + 6*d^11*f^3*g^33*e^7*sgn(1/(g*x + f))^3*sgn(g)^3 - 6*d^9*f^5*g^31
*e^9*sgn(1/(g*x + f))^3*sgn(g)^3 + 2*d^8*f^6*g^30*e^10*sgn(1/(g*x + f))^3*s
gn(g)^3 + 2*d^7*f^7*g^29*e^11*sgn(1/(g*x + f))^3*sgn(g)^3 - d^6*f^8*g^28*e^
12*sgn(1/(g*x + f))^3*sgn(g)^3)/((d^13*g^24*e^4*sgn(1/(g*x + f))^4*sgn(g)^4
 + d^12*f*g^23*e^5*sgn(1/(g*x + f))^4*sgn(g)^4 - 3*d^11*f^2*g^22*e^6*sgn(1/
(g*x + f))^4*sgn(g)^4 - 3*d^10*f^3*g^21*e^7*sgn(1/(g*x + f))^4*sgn(g)^4 + 3
*d^9*f^4*g^20*e^8*sgn(1/(g*x + f))^4*sgn(g)^4 + 3*d^8*f^5*g^19*e^9*sgn(1/(g
*x + f))^4*sgn(g)^4 - d^7*f^6*g^18*e^10*sgn(1/(g*x + f))^4*sgn(g)^4 - d^6*f
^7*g^17*e^11*sgn(1/(g*x + f))^4*sgn(g)^4)*(g*x + f)*g))/((g*x + f)*g))/((g*

```
x + f)*g))/((g*x + f)*g))/((g*x + f)*g))/((g*x + f)*g))/(d^2*g^2/(g*x + f)^
2 + 2*f*e^2/(g*x + f) - f^2*e^2/(g*x + f)^2 - e^2)^(5/2))/g^2
```

maple [B] time = 0.03, size = 6760, normalized size = 21.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d*g-e*f)>0)', see `assume?` for mo
re details)Is (d*g-e*f)    *(d*g+e*f)    positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\left(f+g\,x\right)^2\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)**2), x)
```

3.389 $\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=398

$$\frac{e^2g^3\left(13d^2g^2-30defg+20e^2f^2\right)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{2(ef-dg)^2(dg+ef)^5\sqrt{e^2f^2-d^2g^2}}+\frac{3eg^4\sqrt{d^2-e^2x^2}\,(3ef-2dg)}{2(f+gx)(ef-dg)^2(dg+ef)^5}+\frac{g^4\sqrt{d^2}}{2(f+gx)^2(ef-}$$

Rubi [A] time = 2.57, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1647, 1651, 807, 725, 204}

$$\frac{e^2\left(ex\left(107d^2g^2+19defg+2e^2f^2\right)+90d^3g^2\right)}{15d^3\sqrt{d^2-e^2x^2}\,(dg+ef)^5}+\frac{e^2g^3\left(13d^2g^2-30defg+20e^2f^2\right)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{2(ef-dg)^2(dg+ef)^5\sqrt{e^2f^2-d^2g^2}}+\frac{3eg^4\sqrt{d^2-e^2x^2}\,(3ef-2dg)}{2(f+gx)(ef-dg)^2(dg+ef)^5}+\frac{g^4\sqrt{d^2-e^2x^2}}{2(f+gx)^2(ef-dg)(dg+ef)^4}-\frac{e^2(5d(ef-5dg)-ex(31dg+ef))}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^4}+\frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*e^2*(d + e*x))/(5*(e*f + d*g)^3*(d^2 - e^2*x^2)^(5/2)) - (e^2*(5*d*(e*
f - 5*d*g) - e*(e*f + 31*d*g)*x))/(15*d*(e*f + d*g)^4*(d^2 - e^2*x^2)^(3/2)
) + (e^2*(90*d^3*g^2 + e*(2*e^2*f^2 + 19*d*e*f*g + 107*d^2*g^2)*x))/(15*d^3
*(e*f + d*g)^5*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/(2*(e*f - d
*g)*(e*f + d*g)^4*(f + g*x)^2) + (3*e*g^4*(3*e*f - 2*d*g)*Sqrt[d^2 - e^2*x^
2])/(2*(e*f - d*g)^2*(e*f + d*g)^5*(f + g*x)) + (e^2*g^3*(20*e^2*f^2 - 30*d
*e*f*g + 13*d^2*g^2)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/(2*(e*f - d*g)^2*(e*f + d*g)^5*Sqrt[e^2*f^2 - d^2*g^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2\left(e^3f^3+15de^2f^2g+15d^2efg^2+5d^3g^3\right)}{(ef+dg)^3} - \frac{d^2e^3\left(5e^3f^3-33de^2f^2g-45d^2efg\right.}{(ef+dg)^3}}{(f+gx)^3\left(d^2-e^2x^2\right)^{5/2}}}{5d^2e^2}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{d^3e^4\left(2e^4f^4+17de^3\right.}{}}{}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\right.}{15d^3(}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\right.}{15d^3(}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\right.}{15d^3(}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\right.}{15d^3(}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\right.}{15d^3(}$$

Mathematica [C] time = 1.14, size = 387, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{2e^2(dg+ef)(17dg+2ef)}{d^2(d-ex)^2}+\frac{2e^2\left(107d^2g^2+19defg+2e^2f^2\right)}{d^3(d-ex)}+\frac{6e^2(dg+ef)^2}{d(d-ex)^3}+\frac{45eg^4(3ef-2dg)}{(f+gx)(ef-dg)^2}+\frac{15g^4(dg+ef)}{(f+gx)^2(ef-dg)}\right)-\frac{15ie^2g^3\left(13d^2g^2-30defg+20e^2f^2\right)\log\left(\frac{4(ef-dg)^2(dg+ef)^5\left(\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}+id^2g+ie^2fx\right)}{e^2g^2(f+gx)\sqrt{e^2f^2-d^2g^2}\left(13d^2g^2-30defg+20e^2f^2\right)}\right)}{(ef-dg)^2\sqrt{e^2f^2-d^2g^2}}}{30(dg+ef)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((6*e^2*(e*f + d*g)^2)/(d*(d - e*x)^3) + (2*e^2*(e*f +
 d*g)*(2*e*f + 17*d*g))/(d^2*(d - e*x)^2) + (2*e^2*(2*e^2*f^2 + 19*d*e*f*g
+ 107*d^2*g^2))/(d^3*(d - e*x)) + (15*g^4*(e*f + d*g))/((e*f - d*g)*(f + g*
x)^2) + (45*e*g^4*(3*e*f - 2*d*g))/((e*f - d*g)^2*(f + g*x))) - ((15*I)*e^2
*g^3*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*Log[(4*(e*f - d*g)^2*(e*f + d*g
```

```
)^5*(I*d^2*g + I*e^2*f*x + Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2]))/(e
^2*g^2*Sqrt[e^2*f^2 - d^2*g^2]*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*(f +
g*x))])/((e*f - d*g)^2*Sqrt[e^2*f^2 - d^2*g^2]))/(30*(e*f + d*g)^5)
```

IntegrateAlgebraic [F] time = 180.61, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 3.55, size = 5361, normalized size = 13.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/30*(14*d^3*e^8*f^10 + 60*d^4*e^7*f^9*g + 78*d^5*e^6*f^8*g^2 - 480*d^6*e^
5*f^7*g^3 + 312*d^7*e^4*f^6*g^4 + 330*d^8*e^3*f^5*g^5 - 419*d^9*e^2*f^4*g^6
 + 90*d^10*e*f^3*g^7 + 15*d^11*f^2*g^8 - (14*e^11*f^8*g^2 + 60*d*e^10*f^7*g
^3 + 78*d^2*e^9*f^6*g^4 - 480*d^3*e^8*f^5*g^5 + 312*d^4*e^7*f^4*g^6 + 330*d
^5*e^6*f^3*g^7 - 419*d^6*e^5*f^2*g^8 + 90*d^7*e^4*f*g^9 + 15*d^8*e^3*g^10)*
x^5 - (28*e^11*f^9*g + 78*d*e^10*f^8*g^2 - 24*d^2*e^9*f^7*g^3 - 1194*d^3*e^
8*f^6*g^4 + 2064*d^4*e^7*f^5*g^5 - 276*d^5*e^6*f^4*g^6 - 1828*d^6*e^5*f^3*g
^7 + 1437*d^7*e^4*f^2*g^8 - 240*d^8*e^3*f*g^9 - 45*d^9*e^2*g^10)*x^4 - (14*
e^11*f^10 - 24*d*e^10*f^9*g - 240*d^2*e^9*f^8*g^2 - 768*d^3*e^8*f^7*g^3 + 3
426*d^4*e^7*f^6*g^4 - 2982*d^5*e^6*f^5*g^5 - 1463*d^6*e^5*f^4*g^6 + 3594*d^
7*e^4*f^3*g^7 - 1782*d^8*e^3*f^2*g^8 + 180*d^9*e^2*f*g^9 + 45*d^10*e*g^10)*
x^3 + (42*d*e^10*f^10 + 96*d^2*e^9*f^9*g - 112*d^3*e^8*f^8*g^2 - 1848*d^4*e
^7*f^7*g^3 + 3894*d^5*e^6*f^6*g^4 - 1362*d^6*e^5*f^5*g^5 - 2925*d^7*e^4*f^4
*g^6 + 3114*d^8*e^3*f^3*g^7 - 914*d^9*e^2*f^2*g^8 + 15*d^11*g^10)*x^2 - 15*
(20*d^6*e^4*f^6*g^3 - 30*d^7*e^3*f^5*g^4 + 13*d^8*e^2*f^4*g^5 - (20*d^3*e^7
*f^4*g^5 - 30*d^4*e^6*f^3*g^6 + 13*d^5*e^5*f^2*g^7)*x^5 - (40*d^3*e^7*f^5*g
^4 - 120*d^4*e^6*f^4*g^5 + 116*d^5*e^5*f^3*g^6 - 39*d^6*e^4*f^2*g^7)*x^4 -
(20*d^3*e^7*f^6*g^3 - 150*d^4*e^6*f^5*g^4 + 253*d^5*e^5*f^4*g^5 - 168*d^6*e
^4*f^3*g^6 + 39*d^7*e^3*f^2*g^7)*x^3 + (60*d^4*e^6*f^6*g^3 - 210*d^5*e^5*f^
5*g^4 + 239*d^6*e^4*f^4*g^5 - 108*d^7*e^3*f^3*g^6 + 13*d^8*e^2*f^2*g^7)*x^2
 - (60*d^5*e^5*f^6*g^3 - 130*d^6*e^4*f^5*g^4 + 99*d^7*e^3*f^4*g^5 - 26*d^8*
e^2*f^3*g^6)*x)*sqrt(-e^2*f^2 + d^2*g^2)*log((d*e^2*f*g*x + d^3*g^2 - sqrt(
-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sqrt(-e^2*x^2 + d^2)*d*g) - (e^2*f^2
 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x + f)) - (42*d^2*e^9*f^10 + 152*d^3*e
^8*f^9*g + 114*d^4*e^7*f^8*g^2 - 1596*d^5*e^6*f^7*g^3 + 1896*d^6*e^5*f^6*g^
4 + 366*d^7*e^4*f^5*g^5 - 1917*d^8*e^3*f^4*g^6 + 1108*d^9*e^2*f^3*g^7 - 135
*d^10*e*f^2*g^8 - 30*d^11*f*g^9)*x + (14*d^2*e^8*f^10 + 60*d^3*e^7*f^9*g +
78*d^4*e^6*f^8*g^2 - 480*d^5*e^5*f^7*g^3 + 312*d^6*e^4*f^6*g^4 + 330*d^7*e^
3*f^5*g^5 - 419*d^8*e^2*f^4*g^6 + 90*d^9*e*f^3*g^7 + 15*d^10*f^2*g^8 + (4*e
^10*f^8*g^2 + 30*d*e^9*f^7*g^3 + 138*d^2*e^8*f^6*g^4 - 555*d^3*e^7*f^5*g^5
 + 162*d^4*e^6*f^4*g^6 + 525*d^5*e^5*f^3*g^7 - 304*d^6*e^4*f^2*g^8)*x^4 + (8
*e^10*f^9*g + 48*d*e^9*f^8*g^2 + 186*d^2*e^8*f^7*g^3 - 1224*d^3*e^7*f^6*g^4
 + 1539*d^4*e^6*f^5*g^5 + 459*d^5*e^5*f^4*g^6 - 1733*d^6*e^4*f^3*g^7 + 717*
d^7*e^3*f^2*g^8)*x^3 + (4*e^10*f^10 + 6*d*e^9*f^9*g - 28*d^2*e^8*f^8*g^2 -
828*d^3*e^7*f^7*g^3 + 2400*d^4*e^6*f^6*g^4 - 1197*d^5*e^5*f^5*g^5 - 1897*d^
6*e^4*f^4*g^6 + 2019*d^7*e^3*f^3*g^7 - 479*d^8*e^2*f^2*g^8)*x^2 - (12*d*e^9
*f^10 + 62*d^2*e^8*f^9*g + 114*d^3*e^7*f^8*g^2 - 1056*d^4*e^6*f^7*g^3 + 162
6*d^5*e^5*f^6*g^4 + 81*d^6*e^4*f^5*g^5 - 1707*d^7*e^3*f^4*g^6 + 913*d^8*e^2
*f^3*g^7 - 45*d^9*e*f^2*g^8)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^9*f^13 + 3*d^7
*e^8*f^12*g - 8*d^9*e^6*f^10*g^3 - 6*d^10*e^5*f^9*g^4 + 6*d^11*e^4*f^8*g^5
```

+ 8*d^12*e^3*f^7*g^6 - 3*d^14*e*f^5*g^8 - d^15*f^4*g^9 - (d^3*e^12*f^11*g^2 + 3*d^4*e^11*f^10*g^3 - 8*d^6*e^9*f^8*g^5 - 6*d^7*e^8*f^7*g^6 + 6*d^8*e^7*f^6*g^7 + 8*d^9*e^6*f^5*g^8 - 3*d^11*e^4*f^3*g^10 - d^12*e^3*f^2*g^11)*x^5 - (2*d^3*e^12*f^12*g + 3*d^4*e^11*f^11*g^2 - 9*d^5*e^10*f^10*g^3 - 16*d^6*e^9*f^9*g^4 + 12*d^7*e^8*f^8*g^5 + 30*d^8*e^7*f^7*g^6 - 2*d^9*e^6*f^6*g^7 - 24*d^10*e^5*f^5*g^8 - 6*d^11*e^4*f^4*g^9 + 7*d^12*e^3*f^3*g^10 + 3*d^13*e^2*f^2*g^11)*x^4 - (d^3*e^12*f^13 - 3*d^4*e^11*f^12*g - 15*d^5*e^10*f^11*g^2 + d^6*e^9*f^10*g^3 + 42*d^7*e^8*f^9*g^4 + 18*d^8*e^7*f^8*g^5 - 46*d^9*e^6*f^7*g^6 - 30*d^10*e^5*f^6*g^7 + 21*d^11*e^4*f^5*g^8 + 17*d^12*e^3*f^4*g^9 - 3*d^13*e^2*f^3*g^10 - 3*d^14*e*f^2*g^11)*x^3 + (3*d^4*e^11*f^13 + 3*d^5*e^10*f^12*g - 17*d^6*e^9*f^11*g^2 - 21*d^7*e^8*f^10*g^3 + 30*d^8*e^7*f^9*g^4 + 46*d^9*e^6*f^8*g^5 - 18*d^10*e^5*f^7*g^6 - 42*d^11*e^4*f^6*g^7 - d^12*e^3*f^5*g^8 + 15*d^13*e^2*f^4*g^9 + 3*d^14*e*f^3*g^10 - d^15*f^2*g^11)*x^2 - (3*d^5*e^10*f^13 + 7*d^6*e^9*f^12*g - 6*d^7*e^8*f^11*g^2 - 24*d^8*e^7*f^10*g^3 - 2*d^9*e^6*f^9*g^4 + 30*d^10*e^5*f^8*g^5 + 12*d^11*e^4*f^7*g^6 - 16*d^12*e^3*f^6*g^7 - 9*d^13*e^2*f^5*g^8 + 3*d^14*e*f^4*g^9 + 2*d^15*f^3*g^10)*x)$,

1/30*(14*d^3*e^8*f^10 + 60*d^4*e^7*f^9*g + 78*d^5*e^6*f^8*g^2 - 480*d^6*e^5*f^7*g^3 + 312*d^7*e^4*f^6*g^4 + 330*d^8*e^3*f^5*g^5 - 419*d^9*e^2*f^4*g^6 + 90*d^10*e*f^3*g^7 + 15*d^11*f^2*g^8 - (14*e^11*f^8*g^2 + 60*d*e^10*f^7*g^3 + 78*d^2*e^9*f^6*g^4 - 480*d^3*e^8*f^5*g^5 + 312*d^4*e^7*f^4*g^6 + 330*d^5*e^6*f^3*g^7 - 419*d^6*e^5*f^2*g^8 + 90*d^7*e^4*f*g^9 + 15*d^8*e^3*g^10)*x^5 - (28*e^11*f^9*g + 78*d*e^10*f^8*g^2 - 24*d^2*e^9*f^7*g^3 - 1194*d^3*e^8*f^6*g^4 + 2064*d^4*e^7*f^5*g^5 - 276*d^5*e^6*f^4*g^6 - 1828*d^6*e^5*f^3*g^7 + 1437*d^7*e^4*f^2*g^8 - 240*d^8*e^3*f*g^9 - 45*d^9*e^2*g^10)*x^4 - (14*e^11*f^10 - 24*d*e^10*f^9*g - 240*d^2*e^9*f^8*g^2 - 768*d^3*e^8*f^7*g^3 + 3426*d^4*e^7*f^6*g^4 - 2982*d^5*e^6*f^5*g^5 - 1463*d^6*e^5*f^4*g^6 + 3594*d^7*e^4*f^3*g^7 - 1782*d^8*e^3*f^2*g^8 + 180*d^9*e^2*f*g^9 + 45*d^10*e*g^10)*x^3 + (42*d*e^10*f^10 + 96*d^2*e^9*f^9*g - 112*d^3*e^8*f^8*g^2 - 1848*d^4*e^7*f^7*g^3 + 3894*d^5*e^6*f^6*g^4 - 1362*d^6*e^5*f^5*g^5 - 2925*d^7*e^4*f^4*g^6 + 3114*d^8*e^3*f^3*g^7 - 914*d^9*e^2*f^2*g^8 + 15*d^11*g^10)*x^2 + 30*(20*d^6*e^4*f^6*g^3 - 30*d^7*e^3*f^5*g^4 + 13*d^8*e^2*f^4*g^5 - (20*d^3*e^7*f^4*g^5 - 30*d^4*e^6*f^3*g^6 + 13*d^5*e^5*f^2*g^7)*x^5 - (40*d^3*e^7*f^5*g^4 - 120*d^4*e^6*f^4*g^5 + 116*d^5*e^5*f^3*g^6 - 39*d^6*e^4*f^2*g^7)*x^4 - (20*d^3*e^7*f^6*g^3 - 150*d^4*e^6*f^5*g^4 + 253*d^5*e^5*f^4*g^5 - 168*d^6*e^4*f^3*g^6 + 39*d^7*e^3*f^2*g^7)*x^3 + (60*d^4*e^6*f^6*g^3 - 210*d^5*e^5*f^5*g^4 + 239*d^6*e^4*f^4*g^5 - 108*d^7*e^3*f^3*g^6 + 13*d^8*e^2*f^2*g^7)*x^2 - (60*d^5*e^5*f^6*g^3 - 130*d^6*e^4*f^5*g^4 + 99*d^7*e^3*f^4*g^5 - 26*d^8*e^2*f^3*g^6)*x)*sqrt(e^2*f^2 - d^2*g^2)*arctan((d*g*x + d*f - sqrt(-e^2*x^2 + d^2)*f)/(sqrt(e^2*f^2 - d^2*g^2)*x)) - (42*d^2*e^9*f^10 + 152*d^3*e^8*f^9*g + 114*d^4*e^7*f^8*g^2 - 1596*d^5*e^6*f^7*g^3 + 1896*d^6*e^5*f^6*g^4 + 366*d^7*e^4*f^5*g^5 - 1917*d^8*e^3*f^4*g^6 + 1108*d^9*e^2*f^3*g^7 - 135*d^10*e*f^2*g^8 - 30*d^11*f*g^9)*x + (14*d^2*e^8*f^10 + 60*d^3*e^7*f^9*g + 78*d^4*e^6*f^8*g^2 - 480*d^5*e^5*f^7*g^3 + 312*d^6*e^4*f^6*g^4 + 330*d^7*e^3*f^5*g^5 - 419*d^8*e^2*f^4*g^6 + 90*d^9*e*f^3*g^7 + 15*d^10*f^2*g^8 + (4*e^10*f^8*g^2 + 30*d*e^9*f^7*g^3 + 138*d^2*e^8*f^6*g^4 - 555*d^3*e^7*f^5*g^5 + 162*d^4*e^6*f^4*g^6 + 525*d^5*e^5*f^3*g^7 - 304*d^6*e^4*f^2*g^8)*x^4 + (8*e^10*f^9*g + 48*d*e^9*f^8*g^2 + 186*d^2*e^8*f^7*g^3 - 1224*d^3*e^7*f^6*g^4 + 1539*d^4*e^6*f^5*g^5 + 459*d^5*e^5*f^4*g^6 - 1733*d^6*e^4*f^3*g^7 + 717*d^7*e^3*f^2*g^8)*x^3 + (4*e^10*f^10 + 6*d*e^9*f^9*g - 28*d^2*e^8*f^8*g^2 - 828*d^3*e^7*f^7*g^3 + 2400*d^4*e^6*f^6*g^4 - 1197*d^5*e^5*f^5*g^5 - 1897*d^6*e^4*f^4*g^6 + 2019*d^7*e^3*f^3*g^7 - 479*d^8*e^2*f^2*g^8)*x^2 - (12*d*e^9*f^10 + 62*d^2*e^8*f^9*g + 114*d^3*e^7*f^8*g^2 - 1056*d^4*e^6*f^7*g^3 + 1626*d^5*e^5*f^6*g^4 + 81*d^6*e^4*f^5*g^5 - 1707*d^7*e^3*f^4*g^6 + 913*d^8*e^2*f^3*g^7 - 45*d^9*e*f^2*g^8)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^9*f^13 + 3*d^7*e^8*f^12*g - 8*d^9*e^6*f^10*g^3 - 6*d^10*e^5*f^9*g^4 + 6*d^11*e^4*f^8*g^5 + 8*d^12*e^3*f^7*g^6 - 3*d^14*e*f^5*g^8 - d^15*f^4*g^9 - (d^3*e^12*f^11*g^2 + 3*d^4*e^11*f^10*g^3 - 8*d^6*e^9*f^8*g^5 - 6*d^7*e^8*f^7*g^6 + 6*d^8*e^7*f^6*g^7 + 8*d^9*e^6*f^5*g^8 - 3*d^11*e^4*f^3*g^10 - d^12*e^3*f^2*g^11)*x^5 - (2*d^3*e^12*f^12*g + 3*d^4*e^11*f^11*g^2 - 9*d^5*e^10*f^10*g^3 - 16*d^6*e^9*f^

```
9*g^4 + 12*d^7*e^8*f^8*g^5 + 30*d^8*e^7*f^7*g^6 - 2*d^9*e^6*f^6*g^7 - 24*d^
10*e^5*f^5*g^8 - 6*d^11*e^4*f^4*g^9 + 7*d^12*e^3*f^3*g^10 + 3*d^13*e^2*f^2*
g^11)*x^4 - (d^3*e^12*f^13 - 3*d^4*e^11*f^12*g - 15*d^5*e^10*f^11*g^2 + d^6
*e^9*f^10*g^3 + 42*d^7*e^8*f^9*g^4 + 18*d^8*e^7*f^8*g^5 - 46*d^9*e^6*f^7*g^
6 - 30*d^10*e^5*f^6*g^7 + 21*d^11*e^4*f^5*g^8 + 17*d^12*e^3*f^4*g^9 - 3*d^1
3*e^2*f^3*g^10 - 3*d^14*e*f^2*g^11)*x^3 + (3*d^4*e^11*f^13 + 3*d^5*e^10*f^1
2*g - 17*d^6*e^9*f^11*g^2 - 21*d^7*e^8*f^10*g^3 + 30*d^8*e^7*f^9*g^4 + 46*d
^9*e^6*f^8*g^5 - 18*d^10*e^5*f^7*g^6 - 42*d^11*e^4*f^6*g^7 - d^12*e^3*f^5*g
^8 + 15*d^13*e^2*f^4*g^9 + 3*d^14*e*f^3*g^10 - d^15*f^2*g^11)*x^2 - (3*d^5*
e^10*f^13 + 7*d^6*e^9*f^12*g - 6*d^7*e^8*f^11*g^2 - 24*d^8*e^7*f^10*g^3 - 2
*d^9*e^6*f^9*g^4 + 30*d^10*e^5*f^8*g^5 + 12*d^11*e^4*f^7*g^6 - 16*d^12*e^3*
f^6*g^7 - 9*d^13*e^2*f^5*g^8 + 3*d^14*e*f^4*g^9 + 2*d^15*f^3*g^10)*x)]
```

giac [B] time = 2.09, size = 6017, normalized size = 15.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -(13*d^9*g^12*e^8 - 69*d^8*f*g^11*e^9 + 123*d^7*f^2*g^10*e^10 - 25*d^6*f^3*
g^9*e^11 - 195*d^5*f^4*g^8*e^12 + 237*d^4*f^5*g^7*e^13 - 31*d^3*f^6*g^6*e^1
4 - 123*d^2*f^7*g^5*e^15 + 90*d*f^8*g^4*e^16 - 20*f^9*g^3*e^17)*arctan((d*g
*e + (d*e + sqrt(-x^2*e^2 + d^2)*e)*f/x)/sqrt(-d^2*g^2*e^2 + f^2*e^4))/((d^
14*g^14*e^5 - 7*d^12*f^2*g^12*e^7 + 21*d^10*f^4*g^10*e^9 - 35*d^8*f^6*g^8*e
^11 + 35*d^6*f^8*g^6*e^13 - 21*d^4*f^10*g^4*e^15 + 7*d^2*f^12*g^2*e^17 - f^
14*e^19)*sqrt(-d^2*g^2*e^2 + f^2*e^4)) - 1/15*sqrt(-x^2*e^2 + d^2)*((((((10
7*d^28*g^27*e^11 + 2694*d^27*f*g^26*e^12 + 32577*d^26*f^2*g^25*e^13 + 25185
0*d^25*f^3*g^24*e^14 + 1397850*d^24*f^4*g^23*e^15 + 5929860*d^23*f^5*g^22*e
^16 + 19984470*d^22*f^6*g^21*e^17 + 54906060*d^21*f^7*g^20*e^18 + 125216025
*d^20*f^8*g^19*e^19 + 240109650*d^19*f^9*g^18*e^20 + 390736995*d^18*f^10*g^
17*e^21 + 543134190*d^17*f^11*g^16*e^22 + 647660220*d^16*f^12*g^15*e^23 + 6
64152600*d^15*f^13*g^14*e^24 + 586148100*d^14*f^14*g^13*e^25 + 444848520*d^
13*f^15*g^12*e^26 + 289619565*d^12*f^16*g^11*e^27 + 161082570*d^11*f^17*g^1
0*e^28 + 76070775*d^10*f^18*g^9*e^29 + 30246150*d^9*f^19*g^8*e^30 + 1001121
0*d^8*f^20*g^7*e^31 + 2717220*d^7*f^21*g^6*e^32 + 592710*d^6*f^22*g^5*e^33
+ 101100*d^5*f^23*g^4*e^34 + 12975*d^4*f^24*g^3*e^35 + 1182*d^3*f^25*g^2*e^
36 + 69*d^2*f^26*g*e^37 + 2*d*f^27*e^38)*x/(d^34*g^30*e^4 + 30*d^33*f*g^29*
e^5 + 435*d^32*f^2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*
e^8 + 142506*d^29*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f
^7*g^23*e^11 + 5852925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 3
0045015*d^24*f^10*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*
f^12*g^18*e^16 + 119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e
^18 + 155117520*d^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 11975
9850*d^17*f^17*g^13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^1
9*g^11*e^23 + 30045015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 +
5852925*d^12*f^22*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*
g^6*e^28 + 142506*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^
27*g^3*e^31 + 435*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34) +
 90*(d^29*g^27*e^10 + 25*d^28*f*g^26*e^11 + 300*d^27*f^2*g^25*e^12 + 2300*d
^26*f^3*g^24*e^13 + 12650*d^25*f^4*g^23*e^14 + 53130*d^24*f^5*g^22*e^15 + 1
77100*d^23*f^6*g^21*e^16 + 480700*d^22*f^7*g^20*e^17 + 1081575*d^21*f^8*g^1
9*e^18 + 2042975*d^20*f^9*g^18*e^19 + 3268760*d^19*f^10*g^17*e^20 + 4457400
*d^18*f^11*g^16*e^21 + 5200300*d^17*f^12*g^15*e^22 + 5200300*d^16*f^13*g^14
*e^23 + 4457400*d^15*f^14*g^13*e^24 + 3268760*d^14*f^15*g^12*e^25 + 2042975
*d^13*f^16*g^11*e^26 + 1081575*d^12*f^17*g^10*e^27 + 480700*d^11*f^18*g^9*e
^28 + 177100*d^10*f^19*g^8*e^29 + 53130*d^9*f^20*g^7*e^30 + 12650*d^8*f^21*
g^6*e^31 + 2300*d^7*f^22*g^5*e^32 + 300*d^6*f^23*g^4*e^33 + 25*d^5*f^24*g^3
*e^34 + d^4*f^25*g^2*e^35)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f
^2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^2
```

9*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5
852925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^1
0*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 +
119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*
d^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g
^13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30
045015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^2
2*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 14250
6*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 43
5*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x - 5*(49*d^30*g
^27*e^9 + 1239*d^29*f*g^26*e^10 + 15051*d^28*f^2*g^25*e^11 + 116925*d^27*f^
3*g^24*e^12 + 652350*d^26*f^4*g^23*e^13 + 2782770*d^25*f^5*g^22*e^14 + 9434
370*d^24*f^6*g^21*e^15 + 26086830*d^23*f^7*g^20*e^16 + 59904075*d^22*f^8*g^
19*e^17 + 115728525*d^21*f^9*g^18*e^18 + 189852465*d^20*f^10*g^17*e^19 + 26
6218215*d^19*f^11*g^16*e^20 + 320487060*d^18*f^12*g^15*e^21 + 332076300*d^1
7*f^13*g^14*e^22 + 296417100*d^16*f^14*g^13*e^23 + 227773140*d^15*f^15*g^12
*e^24 + 150325815*d^14*f^16*g^11*e^25 + 84867585*d^13*f^17*g^10*e^26 + 4073
9325*d^12*f^18*g^9*e^27 + 16489275*d^11*f^19*g^8*e^28 + 5563470*d^10*f^20*g
^7*e^29 + 1540770*d^9*f^21*g^6*e^30 + 342930*d^8*f^22*g^5*e^31 + 59550*d^7*
f^23*g^4*e^32 + 7725*d^6*f^24*g^3*e^33 + 699*d^5*f^25*g^2*e^34 + 39*d^4*f^2
6*g*e^35 + d^3*f^27*e^36)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^
2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29
*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 58
52925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10
*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 +
119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d
^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^
13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 300
45015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22
*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506
*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435
*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x - 5*(41*d^31*g^
27*e^8 + 1029*d^30*f*g^26*e^9 + 12399*d^29*f^2*g^25*e^10 + 95475*d^28*f^3*g
^24*e^11 + 527550*d^27*f^4*g^23*e^12 + 2226630*d^26*f^5*g^22*e^13 + 7460970
*d^25*f^6*g^21*e^14 + 20363970*d^24*f^7*g^20*e^15 + 46090275*d^23*f^8*g^19*
e^16 + 87607575*d^22*f^9*g^18*e^17 + 141109485*d^21*f^10*g^17*e^18 + 193785
465*d^20*f^11*g^16*e^19 + 227773140*d^19*f^12*g^15*e^20 + 229556100*d^18*f^
13*g^14*e^21 + 198354300*d^17*f^14*g^13*e^22 + 146648460*d^16*f^15*g^12*e^2
3 + 92379615*d^15*f^16*g^11*e^24 + 49247715*d^14*f^17*g^10*e^25 + 21992025*
d^13*f^18*g^9*e^26 + 8102325*d^12*f^19*g^8*e^27 + 2406030*d^11*f^20*g^7*e^2
8 + 554070*d^10*f^21*g^6*e^29 + 91770*d^9*f^22*g^5*e^30 + 8850*d^8*f^23*g^4
*e^31 - 75*d^7*f^24*g^3*e^32 - 159*d^6*f^25*g^2*e^33 - 21*d^5*f^26*g*e^34 -
d^4*f^27*e^35)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^2*g^28*e^6
+ 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29*f^5*g^25*
e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5852925*d^26
*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10*g^20*e^14
+ 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 + 119759850*
d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d^19*f^15*g
^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^13*e^21 +
86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30045015*d^14
*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22*g^8*e^26
+ 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506*d^9*f^25*
g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435*d^6*f^28*
g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x + 15*(10*d^32*g^27*e^7 +
255*d^31*f*g^26*e^8 + 3126*d^30*f^2*g^25*e^9 + 24525*d^29*f^3*g^24*e^10 + 1
38300*d^28*f^4*g^23*e^11 + 596850*d^27*f^5*g^22*e^12 + 2049300*d^26*f^6*g^2
1*e^13 + 5745630*d^25*f^7*g^20*e^14 + 13396350*d^24*f^8*g^19*e^15 + 2631832
5*d^23*f^9*g^18*e^16 + 43984050*d^22*f^10*g^17*e^17 + 62960775*d^21*f^11*g^
16*e^18 + 77558760*d^20*f^12*g^15*e^19 + 82461900*d^19*f^13*g^14*e^20 + 757

75800*d^18*f^14*g^13*e^21 + 60174900*d^17*f^15*g^12*e^22 + 41230950*d^16*f^16*g^11*e^23 + 24299385*d^15*f^17*g^10*e^24 + 12257850*d^14*f^18*g^9*e^25 + 5256075*d^13*f^19*g^8*e^26 + 1897500*d^12*f^20*g^7*e^27 + 569250*d^11*f^21*g^6*e^28 + 139380*d^10*f^22*g^5*e^29 + 27150*d^9*f^23*g^4*e^30 + 4050*d^8*f^24*g^3*e^31 + 435*d^7*f^25*g^2*e^32 + 30*d^6*f^26*g*e^33 + d^5*f^27*e^34)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5852925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 + 119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30045015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x + (127*d^33*g^27*e^6 + 3219*d^32*f*g^26*e^7 + 39207*d^31*f^2*g^25*e^8 + 305475*d^30*f^3*g^24*e^9 + 1709850*d^29*f^4*g^23*e^10 + 7320210*d^28*f^5*g^22*e^11 + 24917970*d^27*f^6*g^21*e^12 + 69213210*d^26*f^7*g^20*e^13 + 159750525*d^25*f^8*g^19*e^14 + 310412025*d^24*f^9*g^18*e^15 + 512594445*d^23*f^10*g^17*e^16 + 724216065*d^22*f^11*g^16*e^17 + 879445020*d^21*f^12*g^15*e^18 + 920453100*d^20*f^13*g^14*e^19 + 831305100*d^19*f^14*g^13*e^20 + 647660220*d^18*f^15*g^12*e^21 + 434485065*d^17*f^16*g^11*e^22 + 250132245*d^16*f^17*g^10*e^23 + 122939025*d^15*f^18*g^9*e^24 + 51213525*d^14*f^19*g^8*e^25 + 17904810*d^13*f^20*g^7*e^26 + 5183970*d^12*f^21*g^6*e^27 + 1220610*d^11*f^22*g^5*e^28 + 227850*d^10*f^23*g^4*e^29 + 32475*d^9*f^24*g^3*e^30 + 3327*d^8*f^25*g^2*e^31 + 219*d^7*f^26*g*e^32 + 7*d^6*f^27*e^33)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5852925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 + 119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30045015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))/(x^2*e^2 - d^2)^3 + (2*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^10*g^13*e^3/x^2 + 2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^9*f*g^12*e^6/x + 6*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^9*f*g^12*e^4/x^2 + 2*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^9*f*g^12*e^2/x^3 + d^8*f^2*g^11*e^9 + 12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^8*f^2*g^11*e^7/x - 51*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^8*f^2*g^11*e^5/x^2 + 3*d^7*f^3*g^10*e^10 - 79*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^7*f^3*g^10*e^8/x + 91*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^7*f^3*g^10*e^6/x^2 - 25*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^7*f^3*g^10*e^4/x^3 - 26*d^6*f^4*g^9*e^11 + 127*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6*f^4*g^9*e^9/x - 48*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*f^4*g^9*e^7/x^2 + 49*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^6*f^4*g^9*e^5/x^3 + 44*d^5*f^5*g^8*e^12 - 28*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^5*f^5*g^8*e^10/x - 30*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*f^5*g^8*e^8/x^2 - 16*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^5*f^5*g^8*e^6/x^3 - 11*d^4*f^6*g^7*e^13 - 110*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*f^6*g^7*e^11/x + 61*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*f^6*g^7*e^9/x^2 - 38*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*f^6*g^7*e^7/x^3 - 37*d^3*f^7*g^6*e^14 + 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*f^7*g^6*e^12/x - 57*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*f^7*g^6*e^10/x^2 + 39*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*f^7*g^6*e^8/x^3 + 36*d^2*f^8*g^5*e^15 - 29*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2*f^8*g^5*e^13/x + 36*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*f^8*g^5*e^11/x^2 - 11*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*f^8*g^5*e^9/

```
x^3 - 10*d*f^9*g^4*e^16 - 10*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*f^9*g^4*e^1
2/x^2)/((d^12*f^2*g^12*e^5 - 6*d^10*f^4*g^10*e^7 + 15*d^8*f^6*g^8*e^9 - 20*
d^6*f^8*g^6*e^11 + 15*d^4*f^10*g^4*e^13 - 6*d^2*f^12*g^2*e^15 + f^14*e^17)*
(2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*g*e^(-1)/x + f*e^2 + (d*e + sqrt(-x^2*e
^2 + d^2)*e)^2*f*e^(-2)/x^2)^2)
```

maple [B] time = 0.03, size = 9593, normalized size = 24.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d*g-e*f)>0)', see `assume?` for mo
re details)Is (d*g-e*f)    *(d*g+e*f)    positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\left(f+g\,x\right)^3\,\left(d^2-e^2\,x^2\right)^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}\left(f+gx\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)**3), x)
```

3.390 $\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal. Leaf size=112

$$-\frac{2\left(ae^2+cd^2\right)}{e^2\sqrt{d+ex}\,(ef-dg)}-\frac{2\left(ag^2+cf^2\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^2g}$$

Rubi [A] time = 0.22, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {898, 1261, 205}

$$-\frac{2\left(ae^2+cd^2\right)}{e^2\sqrt{d+ex}\,(ef-dg)}-\frac{2\left(ag^2+cf^2\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*(c*d^2 + a*e^2))/(e^2*(e*f - d*g)*Sqrt[d + e*x]) + (2*c*Sqrt[d + e*x])/
(e^2*g) - (2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]
])/(g^(3/2)*(e*f - d*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cd^2+ae^2}{e^2}-\frac{2cdx^2}{e^2}+\frac{cx^4}{e^2}}{x^2\left(\frac{ef-dg}{e}+\frac{gx^2}{e}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cd^2+ae^2}{e(ef-dg)x^2}-\frac{e\left(cf^2+ag^2\right)}{g(-ef+dg)\left(-ef+dg-gx^2\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= -\frac{2\left(cd^2+ae^2\right)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}+\frac{\left(2\left(cf^2+ag^2\right)\right)\text{Subst}\left(\int \frac{1}{-ef+dg-gx^2}\,dx, x, \sqrt{d+ex}\right)}{g(ef-dg)}$$

$$= -\frac{2\left(cd^2+ae^2\right)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}-\frac{2\left(cf^2+ag^2\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

Mathematica [C] time = 0.09, size = 91, normalized size = 0.81

$$\frac{2c(ef-dg)(2dg+e(f+gx))-2e^2\left(ag^2+cf^2\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{g(d+ex)}{dg-ef}\right)}{e^2g^2\sqrt{d+ex}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (2*c*(e*f - d*g)*(2*d*g + e*(f + g*x)) - 2*e^2*(c*f^2 + a*g^2)*Hypergeometr
ic2F1[-1/2, 1, 1/2, (g*(d + e*x))/(-(e*f) + d*g)])/(e^2*g^2*(e*f - d*g)*Sqr
t[d + e*x])
```

IntegrateAlgebraic [A] time = 0.22, size = 118, normalized size = 1.05

$$-\frac{2\left(ae^2g+cd^2g-cef(d+ex)+cdg(d+ex)\right)}{e^2g\sqrt{d+ex}\,(ef-dg)}-\frac{2\left(ag^2+cf^2\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*(c*d^2*g + a*e^2*g - c*e*f*(d + e*x) + c*d*g*(d + e*x)))/(e^2*g*(e*f -
d*g)*Sqrt[d + e*x]) - (2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqr
t[e*f - d*g]])/(g^(3/2)*(e*f - d*g)^(3/2))
```

fricas [B] time = 0.43, size = 499, normalized size = 4.46

$$\left[\frac{\left(cde^2f^2+ade^2g^2+\left(ce^3f^2+ae^3g^2\right)x\right)\sqrt{-efg+dg^2}\log\left(\frac{egx-ef+2dg-2\sqrt{-efg+dg^2}\sqrt{ex+d}}{gx+f}\right)+2\left(cde^2f^2g-\left(3cd^2e+ae^3\right)fg^2+\left(2cd^3+ade^2\right)g^3+\left(ce^3f^2g-2cde^2fg^2+cd^2eg^3\right)x\right)\sqrt{ex+d}}{de^4f^2g^2-2d^2e^3fg^3+d^3e^2g^4+\left(e^5f^2g^2-2de^4fg^3+d^2e^3g^4\right)x}, \frac{2\left(\left(cde^2f^2+ade^2g^2+\left(ce^3f^2+ae^3g^2\right)x\right)\sqrt{efg-dg^2}\arctan\left(\frac{\sqrt{efg-dg^2}\sqrt{ex+d}}{egx+dg}\right)+\left(cde^2f^2g-\left(3cd^2e+ae^3\right)fg^2+\left(2cd^3+ade^2\right)g^3+\left(ce^3f^2g-2cde^2fg^2+cd^2eg^3\right)x\right)\sqrt{ex+d}\right)}{de^4f^2g^2-2d^2e^3fg^3+d^3e^2g^4+\left(e^5f^2g^2-2de^4fg^3+d^2e^3g^4\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="fricas")
```

```
[Out] [((c*d*e^2*f^2 + a*d*e^2*g^2 + (c*e^3*f^2 + a*e^3*g^2)*x)*sqrt(-e*f*g + d*g
^2)*log((e*g*x - e*f + 2*d*g - 2*sqrt(-e*f*g + d*g^2)*sqrt(e*x + d))/(g*x +
 f)) + 2*(c*d*e^2*f^2*g - (3*c*d^2*e + a*e^3)*f*g^2 + (2*c*d^3 + a*d*e^2)*g
^3 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(e*x + d))/(d*e^4
*f^2*g^2 - 2*d^2*e^3*f*g^3 + d^3*e^2*g^4 + (e^5*f^2*g^2 - 2*d*e^4*f*g^3 + d
^2*e^3*g^4)*x), 2*((c*d*e^2*f^2 + a*d*e^2*g^2 + (c*e^3*f^2 + a*e^3*g^2)*x)*
```

```
sqrt(e*f*g - d*g^2)*arctan(sqrt(e*f*g - d*g^2)*sqrt(e*x + d)/(e*g*x + d*g))
 + (c*d*e^2*f^2*g - (3*c*d^2*e + a*e^3)*f*g^2 + (2*c*d^3 + a*d*e^2)*g^3 + (
c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(e*x + d))/(d*e^4*f^2*g
^2 - 2*d^2*e^3*f*g^3 + d^3*e^2*g^4 + (e^5*f^2*g^2 - 2*d*e^4*f*g^3 + d^2*e^3
*g^4)*x)]
```

giac [A] time = 0.19, size = 116, normalized size = 1.04

$$\frac{2\sqrt{xe+d}\,ce^{(-2)}}{g}+\frac{2\left(cf^2+ag^2\right)\arctan\left(\frac{\sqrt{xe+d}\,g}{\sqrt{-dg^2+fge}}\right)}{\left(dg^2-fge\right)\sqrt{-dg^2+fge}}+\frac{2\left(cd^2+ae^2\right)}{\left(dge^2-fe^3\right)\sqrt{xe+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*c*e^(-2)/g + 2*(c*f^2 + a*g^2)*arctan(sqrt(x*e + d)*g/sqrt(
-d*g^2 + f*g*e))/((d*g^2 - f*g*e)*sqrt(-d*g^2 + f*g*e)) + 2*(c*d^2 + a*e^2)
/((d*g*e^2 - f*e^3)*sqrt(x*e + d))
```

maple [A] time = 0.02, size = 114, normalized size = 1.02

$$\frac{-\frac{2\left(a\,g^2+c\,f^2\right)e^2\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}\,g}{\sqrt{\left(dg-ef\right)g}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)g}\,g}+\frac{2\sqrt{ex+d}\,c}{g}-\frac{2\left(-a\,e^2-c\,d^2\right)}{\left(dg-ef\right)\sqrt{ex+d}}}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x)
```

```
[Out] 2/e^2*(c/g*(e*x+d)^(1/2)-e^2*(a*g^2+c*f^2)/(d*g-e*f)/g/((d*g-e*f)*g)^(1/2)*
arctanh(g*(e*x+d)^(1/2)/((d*g-e*f)*g)^(1/2))-(-a*e^2-c*d^2)/(d*g-e*f)/(e*x+
d)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g*(d*g-e*f)>0)', see `assume?` for
more details)Is g*(d*g-e*f)    positive or negative?
```

mupad [B] time = 0.23, size = 124, normalized size = 1.11

$$\frac{2\,c\,\sqrt{d+e\,x}}{e^2\,g}+\frac{2\,\left(c\,g\,d^2+a\,g\,e^2\right)}{e^2\,g\,\left(d\,g-e\,f\right)\,\sqrt{d+e\,x}}+\frac{\mathrm{atan}\left(\frac{d\,g^{3/2}\,\sqrt{d+e\,x}\,1{}\mathrm{i}-e\,f\,\sqrt{g}\,\sqrt{d+e\,x}\,1{}\mathrm{i}}{{\left(d\,g-e\,f\right)}^{3/2}}\right)\,\left(c\,f^2+a\,g^2\right)\,2{}\mathrm{i}}{g^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)*(d + e*x)^(3/2)),x)
```

```
[Out] (atan((d*g^(3/2)*(d + e*x)^(1/2)*1i - e*f*g^(1/2)*(d + e*x)^(1/2)*1i)/(d*g
- e*f)^(3/2))*(a*g^2 + c*f^2)*2i)/(g^(3/2)*(d*g - e*f)^(3/2)) + (2*c*(d + e
```

```
*x)^(1/2))/(e^2*g) + (2*(a*e^2*g + c*d^2*g))/(e^2*g*(d*g - e*f)*(d + e*x)^(
1/2))
```

sympy [A] time = 87.88, size = 107, normalized size = 0.96

$$\frac{2c\sqrt{d+ex}}{e^2g}+\frac{2\left(ag^2+cf^2\right)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-\frac{dg-ef}{g}}}\right)}{g^2\sqrt{-\frac{dg-ef}{g}}\left(dg-ef\right)}+\frac{2\left(ae^2+cd^2\right)}{e^2\sqrt{d+ex}\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(3/2)/(g*x+f),x)
```

```
[Out] 2*c*sqrt(d + e*x)/(e**2*g) + 2*(a*g**2 + c*f**2)*atan(sqrt(d + e*x)/sqrt(-(
d*g - e*f)/g))/(g**2*sqrt(-(d*g - e*f)/g)*(d*g - e*f)) + 2*(a*e**2 + c*d**2
)/(e**2*sqrt(d + e*x)*(d*g - e*f))
```

3.391 $\int \frac{(d+ex)^3\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=240

$$\frac{2e(f+gx)^{7/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}-\frac{2(f+gx)^{5/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2\right.\right.}{5g^6}$$

Rubi [A] time = 0.34, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {898, 1153}

$$\frac{2e(f+gx)^{7/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}-\frac{2(f+gx)^{5/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^3}{g^6}+\frac{2(f+gx)^{3/2}(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{3g^6}-\frac{2ce^2(f+gx)^{9/2}(5ef-3dg)}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*g)^2*(3*
a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^6) - (2*(e*f - d*g)*(3
*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6)
 + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/
2))/(7*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c*e^3*
(f + g*x)^(11/2))/(11*g^6)
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx &= \frac{2\,\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)dx,x,\sqrt{f+gx}\right)}{g}\\ &= \frac{2\,\text{Subst}\left(\int \left(\frac{(-ef+dg)^3\left(cf^2+ag^2\right)}{g^5}+\frac{(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)x^2}{g^5}+\frac{(ef-dg)\left(-3ae^2g^2-c\left(10e^2f^2-\right.\right.}{g^5}\right.\right.}{}\\ &= -\frac{2(ef-dg)^3\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)(f+gx}{3g^6}\end{aligned}$$

Mathematica [A] time = 0.24, size = 207, normalized size = 0.86

$$\frac{2\sqrt{f+gx}\left(495e(f+gx)^3\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)-693(f+gx)^2(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)-3465\left(ag^2+cf^2\right)(ef-dg)^3+1155(f+gx)(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)-385ce^2(f+gx)^4(5ef-3dg)+315ce^3(f+gx)^5\right)}{3465g^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-3465*(e*f - d*g)^3*(c*f^2 + a*g^2) + 1155*(e*f - d*g)^2*
(3*a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x) - 693*(e*f - d*g)*(3*a*e^2*g^2
+ c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^2 + 495*e*(a*e^2*g^2 + c*
(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^3 - 385*c*e^2*(5*e*f - 3*d
*g)*(f + g*x)^4 + 315*c*e^3*(f + g*x)^5))/(3465*g^6)
```

IntegrateAlgebraic [A] time = 0.17, size = 427, normalized size = 1.78

$$\frac{2\sqrt{f+gx}\left(3465ad^3g^5+3465ad^2eg^4(f+gx)-10395ad^2efg^4+10395ade^2f^2g^3-6930ade^2fg^3(f+gx)+2079ade^2g^3(f+gx)^2-3465ae^3f^3g^2+3465ae^3f^2g^2(f+gx)-2079ae^3fg^2(f+gx)^2+495ae^3g^2(f+gx)^3+3465cd^3f^2g^3-2310cd^3fg^3(f+gx)+693cd^3g^3(f+gx)^2-10395cd^2ef^3g^2+10395cd^2ef^2g^2(f+gx)-6237cd^2efg^2(f+gx)^2+1485cd^2eg^2(f+gx)^3+10395cde^2f^4g-13860cde^2f^3g(f+gx)+12474cde^2f^2g(f+gx)^2-5940cde^2fg(f+gx)^3+1155cde^2g(f+gx)^4-3465ce^3f^5+5775ce^3f^4(f+gx)-6930ce^3f^3(f+gx)^2+4950ce^3f^2(f+gx)^3-1925ce^3f(f+gx)^4+315ce^3(f+gx)^5\right)}{3465g^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-3465*c*e^3*f^5 + 10395*c*d*e^2*f^4*g - 10395*c*d^2*e*f^3
*g^2 - 3465*a*e^3*f^3*g^2 + 3465*c*d^3*f^2*g^3 + 10395*a*d*e^2*f^2*g^3 - 10
395*a*d^2*e*f*g^4 + 3465*a*d^3*g^5 + 5775*c*e^3*f^4*(f + g*x) - 13860*c*d*e
^2*f^3*g*(f + g*x) + 10395*c*d^2*e*f^2*g^2*(f + g*x) + 3465*a*e^3*f^2*g^2*(
f + g*x) - 2310*c*d^3*f*g^3*(f + g*x) - 6930*a*d*e^2*f*g^3*(f + g*x) + 3465
*a*d^2*e*g^4*(f + g*x) - 6930*c*e^3*f^3*(f + g*x)^2 + 12474*c*d*e^2*f^2*g*(
f + g*x)^2 - 6237*c*d^2*e*f*g^2*(f + g*x)^2 - 2079*a*e^3*f*g^2*(f + g*x)^2
+ 693*c*d^3*g^3*(f + g*x)^2 + 2079*a*d*e^2*g^3*(f + g*x)^2 + 4950*c*e^3*f^2
*(f + g*x)^3 - 5940*c*d*e^2*f*g*(f + g*x)^3 + 1485*c*d^2*e*g^2*(f + g*x)^3
+ 495*a*e^3*g^2*(f + g*x)^3 - 1925*c*e^3*f*(f + g*x)^4 + 1155*c*d*e^2*g*(f
+ g*x)^4 + 315*c*e^3*(f + g*x)^5))/(3465*g^6)
```

fricas [A] time = 0.40, size = 324, normalized size = 1.35

$$\frac{2\left(315ce^3g^5x^5-1280ce^3f^5+4224cde^2f^4g-6930ad^2efg^4+3465ad^3g^5-1584\left(3cd^2e+ae^3\right)f^3g^2+1848\left(cd^3+3ade^2\right)f^2g^3-35\left(10ce^3fg^4-33cde^2g^5\right)x^4+5\left(80ce^3f^2g^3-264cde^2fg^4+99\left(3cd^2e+ae^3\right)g^5\right)x^3-3\left(160ce^3f^3g^2-528cde^2f^2g^3+198\left(3cd^2e+ae^3\right)fg^4-231\left(cd^3+3ade^2\right)g^5\right)x^2+\left(640ce^3f^4g-2112cde^2f^3g^2+3465ad^2eg^5+792\left(3cd^2e+ae^3\right)f^2g^3-924\left(cd^3+3ade^2\right)fg^4\right)x\right)\sqrt{gx+f}}{3465g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 4224*c*d*e^2*f^4*g - 6930*a*d^
2*e*f*g^4 + 3465*a*d^3*g^5 - 1584*(3*c*d^2*e + a*e^3)*f^3*g^2 + 1848*(c*d^3
 + 3*a*d*e^2)*f^2*g^3 - 35*(10*c*e^3*f*g^4 - 33*c*d*e^2*g^5)*x^4 + 5*(80*c*
e^3*f^2*g^3 - 264*c*d*e^2*f*g^4 + 99*(3*c*d^2*e + a*e^3)*g^5)*x^3 - 3*(160*
c*e^3*f^3*g^2 - 528*c*d*e^2*f^2*g^3 + 198*(3*c*d^2*e + a*e^3)*f*g^4 - 231*(
c*d^3 + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 2112*c*d*e^2*f^3*g^2 + 346
5*a*d^2*e*g^5 + 792*(3*c*d^2*e + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*a*d*e^2)*f
*g^4)*x)*sqrt(g*x + f)/g^6
```

giac [A] time = 0.18, size = 378, normalized size = 1.58

$$\frac{2\left(3465\sqrt{gx+f}\,ad^3+\frac{3465\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)ad^2e}{g}+\frac{231\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)cd^3}{g^2}+\frac{693\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)ade^2}{g^2}+\frac{297\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^2-35\sqrt{gx+f}\,f^3\right)cd^2e}{g^3}+\frac{99\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^2-35\sqrt{gx+f}\,f^3\right)ae^3}{g^3}+\frac{33\left(35(gx+f)^{\frac{9}{2}}-180(gx+f)^{\frac{7}{2}}f+378(gx+f)^{\frac{5}{2}}f^2-420(gx+f)^{\frac{3}{2}}f^3+315\sqrt{gx+f}\,f^4\right)cde^2}{g^4}+\frac{5\left(63(gx+f)^{\frac{11}{2}}-385(gx+f)^{\frac{9}{2}}f+990(gx+f)^{\frac{7}{2}}f^2-1386(gx+f)^{\frac{5}{2}}f^3+1155(gx+f)^{\frac{3}{2}}f^4-693\sqrt{gx+f}\,f^5\right)ce^3}{g^5}\right)}{3465g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(g*x + f)*a*d^3 + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*
f)*a*d^2*e/g + 231*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
+ f)*f^2)*c*d^3/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sq
rt(g*x + f)*f^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*
f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 99*(5*(g*x
 + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x +
 f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(
```

```
g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*d*e
^2/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/
2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x
 + f)*f^5)*c*e^3/g^5)/g
```

maple [A] time = 0.01, size = 365, normalized size = 1.52

$$\frac{2\sqrt{gx+f}\,\left(315e^3c\,x^5g^5+1155cd\,e^2g^5x^4-350c\,e^3f\,g^4x^4+495a\,e^3g^5x^3+1485c\,d^2e\,g^5x^3-1320cd\,e^2f\,g^4x^3+400c\,e^3f^2g^3x^3+2079ad\,e^2g^5x^2-594a\,e^3f\,g^4x^2+693c\,d^3g^5x^2-1782c\,d^2ef\,g^4x^2+1584cd\,e^2f^2g^3x^2-480c\,e^3f^3g^2x^2+3465a\,d^2e\,g^5x-2772ad\,e^2f\,g^4x+792a\,e^3f^2g^3x-924c\,d^3f\,g^4x+2376c\,d^2e\,f^2g^3x-2112cd\,e^2f^3g^2x+640c\,e^3f^4gx+3465d^3a\,g^5-6930a\,d^2ef\,g^4+5544ad\,e^2f^2g^3-1584a\,e^3f^3g^2+1848c\,d^3f^2g^3-4752c\,d^2e\,f^3g^2+4224cd\,e^2f^4g-1280c\,e^3f^5\right)}{3465g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3465*(g*x+f)^(1/2)*(315*c*e^3*g^5*x^5+1155*c*d*e^2*g^5*x^4-350*c*e^3*f*g^
4*x^4+495*a*e^3*g^5*x^3+1485*c*d^2*e*g^5*x^3-1320*c*d*e^2*f*g^4*x^3+400*c*e
^3*f^2*g^3*x^3+2079*a*d*e^2*g^5*x^2-594*a*e^3*f*g^4*x^2+693*c*d^3*g^5*x^2-1
782*c*d^2*e*f*g^4*x^2+1584*c*d*e^2*f^2*g^3*x^2-480*c*e^3*f^3*g^2*x^2+3465*a
*d^2*e*g^5*x-2772*a*d*e^2*f*g^4*x+792*a*e^3*f^2*g^3*x-924*c*d^3*f*g^4*x+237
6*c*d^2*e*f^2*g^3*x-2112*c*d*e^2*f^3*g^2*x+640*c*e^3*f^4*g*x+3465*a*d^3*g^5
-6930*a*d^2*e*f*g^4+5544*a*d*e^2*f^2*g^3-1584*a*e^3*f^3*g^2+1848*c*d^3*f^2*
g^3-4752*c*d^2*e*f^3*g^2+4224*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6
```

maxima [A] time = 0.45, size = 326, normalized size = 1.36

$$\frac{2\left(315\left(gx+f\right)^{\frac{11}{2}}ce^3-385\left(5ce^3f-3cde^2g\right)\left(gx+f\right)^{\frac{9}{2}}+495\left(10ce^3f^2-12cde^2fg+\left(3cd^2e+ae^3\right)g^2\right)\left(gx+f\right)^{\frac{7}{2}}-693\left(10ce^3f^3-18cde^2f^2g+3\left(3cd^2e+ae^3\right)fg^2-\left(cd^3+3ade^2\right)g^3\right)\left(gx+f\right)^{\frac{5}{2}}+1155\left(5ce^3f^4-12cde^2f^3g+3ad^2eg^4+3\left(3cd^2e+ae^3\right)f^2g^2-2\left(cd^3+3ade^2\right)fg^3\right)\left(gx+f\right)^{\frac{3}{2}}-3465\left(ce^3f^5-3cde^2f^4g+3ad^2efg^4-ad^3g^5+\left(3cd^2e+ae^3\right)f^3g^2-\left(cd^3+3ade^2\right)f^2g^3\right)\sqrt{gx+f}\right)}{3465\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f
)^(9/2) + 495*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*
x + f)^(7/2) - 693*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e^3)
*f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 12*
c*d*e^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*a*d*e^2)*f*g^3)*(g*x + f)^(3/2) - 3465*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*a
*d^2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*d*e^2
)*f^2*g^3)*sqrt(g*x + f))/g^6
```

mupad [B] time = 0.12, size = 222, normalized size = 0.92

$$\frac{{\left(f+g\,x\right)}^{7/2}\,\left(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2\right)}{7\,g^6}+\frac{2\,\sqrt{f+g\,x}\,\left(c\,f^2+a\,g^2\right)\,{\left(d\,g-e\,f\right)}^3}{g^6}+\frac{2\,c\,e^3\,{\left(f+g\,x\right)}^{11/2}}{11\,g^6}+\frac{2\,{\left(f+g\,x\right)}^{3/2}\,{\left(d\,g-e\,f\right)}^2\,\left(5\,c\,e\,f^2-2\,c\,d\,f\,g+3\,a\,e\,g^2\right)}{3\,g^6}+\frac{2\,{\left(f+g\,x\right)}^{5/2}\,\left(d\,g-e\,f\right)\,\left(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+10\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{5\,g^6}+\frac{2\,c\,e^2\,{\left(f+g\,x\right)}^{9/2}\,\left(3\,d\,g-5\,e\,f\right)}{9\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f
*g))/(7*g^6) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^3)/g^6 + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6) + (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*e*g
^2 + 5*c*e*f^2 - 2*c*d*f*g))/(3*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(3*a*
e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g))/(5*g^6) + (2*c*e^2*(f +
g*x)^(9/2)*(3*d*g - 5*e*f))/(9*g^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.392 $\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=175

$$\frac{2(f+gx)^{5/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^2}{g^5}-\frac{4(f+gx)^{3/2}(ef-dg)\left(aeg^2\right.}{3g^5}$$

Rubi [A] time = 0.24, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {898, 1153}

$$\frac{2(f+gx)^{5/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^2}{g^5}-\frac{4(f+gx)^{3/2}(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{3g^5}-\frac{4ce(f+gx)^{7/2}(2ef-dg)}{7g^5}+\frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^5 - (4*(e*f - d*g)*(a*e*g
^2 + c*f*(2*e*f - d*g))*(f + g*x)^(3/2))/(3*g^5) + (2*(a*e^2*g^2 + c*(6*e^2
*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (4*c*e*(2*e*f - d*g
)*(f + g*x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx &= \frac{2\,\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)dx,x,\sqrt{f+gx}\right)}{g}\\ &= \frac{2\,\text{Subst}\left(\int \left(\frac{(-ef+dg)^2\left(cf^2+ag^2\right)}{g^4}+\frac{2(ef-dg)\left(-aeg^2-cf(2ef-dg)\right)x^2}{g^4}+\frac{\left(ae^2g^2+c\left(6e^2f^2-6defg+d^2g^2\right)\right)x^4}{g^4}\right.\right.}{g}\\ &= \frac{2(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^5}-\frac{4(ef-dg)\left(aeg^2+cf(2ef-dg)\right)(f+gx)^{3/2}}{3g^5}+\frac{2}{}\end{aligned}$$

Mathematica [A] time = 0.15, size = 149, normalized size = 0.85

$$\frac{2\sqrt{f+gx}\left(63(f+gx)^2\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)+315\left(ag^2+cf^2\right)(ef-dg)^2-210(f+gx)(ef-dg)\left(aeg^2+cf(2ef-dg)\right)-90ce(f+gx)^3(2ef-dg)+35ce^2(f+gx)^4\right)}{315g^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(315*(e*f - d*g)^2*(c*f^2 + a*g^2) - 210*(e*f - d*g)*(a*e*
g^2 + c*f*(2*e*f - d*g))*(f + g*x) + 63*(a*e^2*g^2 + c*(6*e^2*f^2 - 6*d*e*f
*g + d^2*g^2))*(f + g*x)^2 - 90*c*e*(2*e*f - d*g)*(f + g*x)^3 + 35*c*e^2*(f
 + g*x)^4))/(315*g^5)
```

IntegrateAlgebraic [A] time = 0.11, size = 250, normalized size = 1.43

$$\frac{2\sqrt{f+gx}\left(315ad^2g^4+210adeg^3(f+gx)-630adefg^3+315ae^2f^2g^2-210ae^2fg^2(f+gx)+63ae^2g^2(f+gx)^2+315cd^2f^2g^2-210cd^2fg^2(f+gx)+63cd^2g^2(f+gx)^2-630cdef^3g+630cdef^2g(f+gx)-378cdefg(f+gx)^2+90cdeg(f+gx)^3+315ce^2f^4-420ce^2f^3(f+gx)+378ce^2f^2(f+gx)^2-180ce^2f(f+gx)^3+35ce^2(f+gx)^4\right)}{315g^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(315*c*e^2*f^4 - 630*c*d*e*f^3*g + 315*c*d^2*f^2*g^2 + 315
*a*e^2*f^2*g^2 - 630*a*d*e*f*g^3 + 315*a*d^2*g^4 - 420*c*e^2*f^3*(f + g*x)
+ 630*c*d*e*f^2*g*(f + g*x) - 210*c*d^2*f*g^2*(f + g*x) - 210*a*e^2*f*g^2*(
f + g*x) + 210*a*d*e*g^3*(f + g*x) + 378*c*e^2*f^2*(f + g*x)^2 - 378*c*d*e*
f*g*(f + g*x)^2 + 63*c*d^2*g^2*(f + g*x)^2 + 63*a*e^2*g^2*(f + g*x)^2 - 180
*c*e^2*f*(f + g*x)^3 + 90*c*d*e*g*(f + g*x)^3 + 35*c*e^2*(f + g*x)^4))/(315
*g^5)
```

fricas [A] time = 0.39, size = 197, normalized size = 1.13

$$\frac{2\left(35ce^2g^4x^4+128ce^2f^4-288cdef^3g-420adefg^3+315ad^2g^4+168\left(cd^2+ae^2\right)f^2g^2-10\left(4ce^2fg^3-9cdeg^4\right)x^3+3\left(16ce^2f^2g^2-36cdefg^3+21\left(cd^2+ae^2\right)g^4\right)x^2-2\left(32ce^2f^3g-72cdef^2g^2-105adeg^4+42\left(cd^2+ae^2\right)fg^3\right)x\right)\sqrt{gx+f}}{315g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 - 288*c*d*e*f^3*g - 420*a*d*e*f*g^3
 + 315*a*d^2*g^4 + 168*(c*d^2 + a*e^2)*f^2*g^2 - 10*(4*c*e^2*f*g^3 - 9*c*d*
e*g^4)*x^3 + 3*(16*c*e^2*f^2*g^2 - 36*c*d*e*f*g^3 + 21*(c*d^2 + a*e^2)*g^4)
*x^2 - 2*(32*c*e^2*f^3*g - 72*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 42*(c*d^2 + a
*e^2)*f*g^3)*x)*sqrt(g*x + f)/g^5
```

giac [A] time = 0.17, size = 243, normalized size = 1.39

$$\frac{2\left(315\sqrt{gx+f}\,ad^2+\frac{210\left(\left(gx+f\right)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)ade}{g}+\frac{21\left(3\left(gx+f\right)^{\frac{5}{2}}-10\left(gx+f\right)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)cd^2}{g^2}+\frac{21\left(3\left(gx+f\right)^{\frac{5}{2}}-10\left(gx+f\right)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)ae^2}{g^2}+\frac{18\left(5\left(gx+f\right)^{\frac{7}{2}}-21\left(gx+f\right)^{\frac{5}{2}}f+35\left(gx+f\right)^{\frac{3}{2}}f^2-35\sqrt{gx+f}\,f^3\right)cde}{g^3}+\frac{\left(35\left(gx+f\right)^{\frac{9}{2}}-180\left(gx+f\right)^{\frac{7}{2}}f+378\left(gx+f\right)^{\frac{5}{2}}f^2-420\left(gx+f\right)^{\frac{3}{2}}f^3+315\sqrt{gx+f}\,f^4\right)ce^2}{g^4}\right)}{315g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(g*x + f)*a*d^2 + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*
a*d*e/g + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*c*d^2/g^2 + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
+ f)*f^2)*a*e^2/g^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*
x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + (35*(g*x + f)^(9/2) -
180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 +
 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

maple [A] time = 0.01, size = 215, normalized size = 1.23

$$\frac{2\sqrt{gx+f}\,\left(35e^2cx^4g^4+90cdeg^4x^3-40ce^2fg^3x^3+63ae^2g^4x^2+63cd^2g^4x^2-108cdefg^3x^2+48ce^2f^2g^2x^2+210adeg^4x-84ae^2fg^3x-84cd^2fg^3x+144cdef^2g^2x-64ce^2f^3gx+315d^2ag^4-420adefg^3+168ae^2f^2g^2+168cd^2f^2g^2-288cdef^3g+128ce^2f^4\right)}{315g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/315*(g*x+f)^(1/2)*(35*c*e^2*g^4*x^4+90*c*d*e*g^4*x^3-40*c*e^2*f*g^3*x^3+6
3*a*e^2*g^4*x^2+63*c*d^2*g^4*x^2-108*c*d*e*f*g^3*x^2+48*c*e^2*f^2*g^2*x^2+2
```

```
10*a*d*e*g^4*x-84*a*e^2*f*g^3*x-84*c*d^2*f*g^3*x+144*c*d*e*f^2*g^2*x-64*c*e
^2*f^3*g*x+315*a*d^2*g^4-420*a*d*e*f*g^3+168*a*e^2*f^2*g^2+168*c*d^2*f^2*g^
2-288*c*d*e*f^3*g+128*c*e^2*f^4)/g^5
```

maxima [A] time = 0.45, size = 197, normalized size = 1.13

$$\frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^2-90\left(2\,ce^2f-cdeg\right)(gx+f)^{\frac{7}{2}}+63\left(6\,ce^2f^2-6\,cdefg+\left(cd^2+ae^2\right)g^2\right)(gx+f)^{\frac{5}{2}}-210\left(2\,ce^2f^3-3\,cdef^2g-adeg^3+\left(cd^2+ae^2\right)fg^2\right)(gx+f)^{\frac{3}{2}}+315\left(ce^2f^4-2\,cdef^3g-2\,adefg^3+ad^2g^4+\left(cd^2+ae^2\right)f^2g^2\right)\sqrt{gx+f}\right)}{315\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(g*x + f)^(9/2)*c*e^2 - 90*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(7/2)
+ 63*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(5/2) - 21
0*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*(g*x +
f)^(3/2) + 315*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 + (c*
d^2 + a*e^2)*f^2*g^2)*sqrt(g*x + f))/g^5
```

mupad [B] time = 2.58, size = 159, normalized size = 0.91

$$\frac{{\left(f+g\,x\right)}^{5/2}\,\left(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2\right)}{5\,g^5}+\frac{2\,\sqrt{f+g\,x}\,\left(c\,f^2+a\,g^2\right)\,{\left(d\,g-e\,f\right)}^2}{g^5}+\frac{4\,{\left(f+g\,x\right)}^{3/2}\,\left(d\,g-e\,f\right)\,\left(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2\right)}{3\,g^5}+\frac{2\,c\,e^2\,{\left(f+g\,x\right)}^{9/2}}{9\,g^5}+\frac{4\,c\,e\,{\left(f+g\,x\right)}^{7/2}\,\left(d\,g-2\,e\,f\right)}{7\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g))
/(5*g^5) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2)/g^5 + (4*(f +
g*x)^(3/2)*(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/(3*g^5) + (2*c*e^2*
(f + g*x)^(9/2))/(9*g^5) + (4*c*e*(f + g*x)^(7/2)*(d*g - 2*e*f))/(7*g^5)
```

sympy [A] time = 108.50, size = 673, normalized size = 3.85

$$\begin{cases} \frac{-\frac{2ad^2f}{\sqrt{f+gx}}-2ad^2\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)-\frac{4adef\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)}{g}-\frac{4ade\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}-\frac{2ae^2f\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2}-\frac{2ae^2\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}-\frac{2cd^2f\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2}-\frac{2cd^2\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}-\frac{4cdef\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^3}-\frac{4cde\left(\frac{f^4}{\sqrt{f+gx}}+4f^3\sqrt{f+gx}-2f^2(f+gx)^{\frac{3}{2}}+\frac{4f(f+gx)^{\frac{5}{2}}}{5}-\frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^3}-\frac{2ce^2f\left(\frac{f^4}{\sqrt{f+gx}}+4f^3\sqrt{f+gx}-2f^2(f+gx)^{\frac{3}{2}}+\frac{4f(f+gx)^{\frac{5}{2}}}{5}-\frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^4}-\frac{2ce^2\left(-\frac{f^5}{\sqrt{f+gx}}-5f^4\sqrt{f+gx}+\frac{10f^3(f+gx)^{\frac{3}{2}}}{3}-2f^2(f+gx)^{\frac{5}{2}}+\frac{5f(f+gx)^{\frac{7}{2}}}{7}-\frac{(f+gx)^{\frac{9}{2}}}{9}\right)}{g^4}}{g} & \text{for}\: g \neq 0 \\ \frac{ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3\left(ae^2+cd^2\right)}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d**2*f/sqrt(f + g*x) - 2*a*d**2*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) - 4*a*d*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 4*a*d*e*(f**2/s
qrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*a*e**2*f*(f**2
/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*a*e**2*(-
f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)*
*(5/2)/5)/g**2 - 2*c*d**2*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f +
g*x)**(3/2)/3)/g**2 - 2*c*d**2*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x)
+ f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 4*c*d*e*f*(-f**3/sqrt(f +
 g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**
3 - 4*c*d*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**
(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*c*e**2*f*(f**
4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f +
 g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 - 2*c*e**2*(-f**5/sqrt(f + g*x) -
 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2
) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4)/g, Ne(g, 0)), ((a*d*
*2*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2 + c*d**2)/3
)/sqrt(f), True))
```

3.393 $\int \frac{(d+ex)\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=113

$$-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}{g^4}+\frac{2(f+gx)^{3/2}\left(aeg^2+cf(3ef-2dg)\right)}{3g^4}-\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4}+\frac{2ce(f+gx}{7g^4}$$

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {772}

$$-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}{g^4}+\frac{2(f+gx)^{3/2}\left(aeg^2+cf(3ef-2dg)\right)}{3g^4}-\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(a*e*g^2 + c*f*(3*e
*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^4) - (2*c*(3*e*f - d*g)*(f + g*x)^(5/2))
/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7*g^4)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx &= \int \left(\frac{(-ef+dg)\left(cf^2+ag^2\right)}{g^3\sqrt{f+gx}}+\frac{\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^3}+\frac{c(-3ef+dg)(f}{g^3}\right.\\ &= -\frac{2(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^4}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)(f+gx)^{3/2}}{3g^4}-\frac{2c(3ef}{}\end{aligned}$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.83

$$\frac{2\sqrt{f+gx}\left(35ag^2(3dg-2ef+egx)+7cdg\left(8f^2-4fgx+3g^2x^2\right)-3ce\left(16f^3-8f^2gx+6fg^2x^2-5g^3x^3\right)\right)}{105g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(35*a*g^2*(-2*e*f + 3*d*g + e*g*x) + 7*c*d*g*(8*f^2 - 4*f*
g*x + 3*g^2*x^2) - 3*c*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))/(
105*g^4)
```

IntegrateAlgebraic [A] time = 0.07, size = 117, normalized size = 1.04

$$\frac{2\sqrt{f+gx}\left(105adg^3+35aeg^2(f+gx)-105aefg^2+105cdf^2g-70cdfg(f+gx)+21cdg(f+gx)^2-105cef^3+105cef^2(f+gx)-63cef(f+gx)^2+15ce(f+gx)^3\right)}{105g^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-105*c*e*f^3 + 105*c*d*f^2*g - 105*a*e*f*g^2 + 105*a*d*g^
3 + 105*c*e*f^2*(f + g*x) - 70*c*d*f*g*(f + g*x) + 35*a*e*g^2*(f + g*x) - 6
3*c*e*f*(f + g*x)^2 + 21*c*d*g*(f + g*x)^2 + 15*c*e*(f + g*x)^3))/(105*g^4)
```

fricas [A] time = 0.39, size = 100, normalized size = 0.88

$$\frac{2\left(15\,ceg^3x^3-48\,cef^3+56\,cdf^2g-70\,aefg^2+105\,adg^3-3\left(6\,cefg^2-7\,cdg^3\right)x^2+\left(24\,cef^2g-28\,cdfg^2+35\,aeg^3\right)x\right)\sqrt{gx+f}}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 56*c*d*f^2*g - 70*a*e*f*g^2 + 105*a*d*
g^3 - 3*(6*c*e*f*g^2 - 7*c*d*g^3)*x^2 + (24*c*e*f^2*g - 28*c*d*f*g^2 + 35*a
*e*g^3)*x)*sqrt(g*x + f)/g^4
```

giac [A] time = 0.18, size = 134, normalized size = 1.19

$$\frac{2\left(105\,\sqrt{gx+f}\,ad+\frac{35\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}\,f\right)ae}{g}+\frac{7\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}\,f^2\right)cd}{g^2}+\frac{3\left(5\,(gx+f)^{\frac{7}{2}}-21\,(gx+f)^{\frac{5}{2}}f+35\,(gx+f)^{\frac{3}{2}}f^2-35\,\sqrt{gx+f}\,f^3\right)ce}{g^3}\right)}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e
/g + 7*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*
d/g^2 + 3*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^
2 - 35*sqrt(g*x + f)*f^3)*c*e/g^3)/g
```

maple [A] time = 0.00, size = 101, normalized size = 0.89

$$\frac{2\sqrt{gx+f}\,\left(15ce\,x^3g^3+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x-28cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-70aef\,g^2+56cd\,f^2g-48ce\,f^3\right)}{105g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(15*c*e*g^3*x^3+21*c*d*g^3*x^2-18*c*e*f*g^2*x^2+35*a*e*
g^3*x-28*c*d*f*g^2*x+24*c*e*f^2*g*x+105*a*d*g^3-70*a*e*f*g^2+56*c*d*f^2*g-4
8*c*e*f^3)/g^4
```

maxima [A] time = 0.44, size = 104, normalized size = 0.92

$$\frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce-21\,\left(3\,cef-cdg\right)(gx+f)^{\frac{5}{2}}+35\,\left(3\,cef^2-2\,cdfg+aeg^2\right)(gx+f)^{\frac{3}{2}}-105\,\left(cef^3-cdf^2g+aefg^2-adg^3\right)\sqrt{gx+f}\right)}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - c*d*g)*(g*x + f)^(5/2) + 35*(
3*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*(g*x + f)^(3/2) - 105*(c*e*f^3 - c*d*f^2*g
 + a*e*f*g^2 - a*d*g^3)*sqrt(g*x + f))/g^4
```

mupad [B] time = 0.07, size = 100, normalized size = 0.88

$$\frac{{\left(f+g\,x\right)}^{3/2}\,\left(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2\right)}{3\,g^4}+\frac{2\,c\,e\,{\left(f+g\,x\right)}^{7/2}}{7\,g^4}+\frac{2\,c\,{\left(f+g\,x\right)}^{5/2}\,\left(d\,g-3\,e\,f\right)}{5\,g^4}+\frac{2\,\sqrt{f+g\,x}\,\left(c\,f^2+a\,g^2\right)\,\left(d\,g-e\,f\right)}{g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/(3*g^4) + (2*c*e*(f +
 g*x)^(7/2))/(7*g^4) + (2*c*(f + g*x)^(5/2)*(d*g - 3*e*f))/(5*g^4) + (2*(f
+ g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f))/g^4
```

sympy [A] time = 61.13, size = 374, normalized size = 3.31

$$\begin{cases} \frac{-\frac{2adf}{\sqrt{f+gx}} - 2ad\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) - \frac{2aef\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} - \frac{2ae\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} - \frac{2cdf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2cd\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2} - \frac{2cef\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^3} - \frac{2ce\left(\frac{f^4}{\sqrt{f+gx}} + 4f^3\sqrt{f+gx} - 2f^2(f+gx)^{\frac{3}{2}} + \frac{4f(f+gx)^{\frac{5}{2}}}{5} - \frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^3}}{g} & \text{for}\ g \neq 0 \\ \frac{adx + \frac{aex^2}{2} + \frac{cdx^3}{3} + \frac{cex^4}{4}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d*f/sqrt(f + g*x) - 2*a*d*(-f/sqrt(f + g*x) - sqrt(f + g*x
)) - 2*a*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*a*e*(f**2/sqrt(f + g*
x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*c*d*f*(f**2/sqrt(f + g*x
) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d*(-f**3/sqrt(f + g*
x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 -
 2*c*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) -
 (f + g*x)**(5/2)/5)/g**3 - 2*c*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x
) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/
g**3)/g, Ne(g, 0)), ((a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/sqrt(f)
, True))
```

3.394 $\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)}{g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)}{g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^3 - (4*c*f*(f + g*x)^(3/2))/(3*g^3) + (
2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx &= \int \left(\frac{cf^2+ag^2}{g^2\sqrt{f+gx}}-\frac{2cf\sqrt{f+gx}}{g^2}+\frac{c(f+gx)^{3/2}}{g^2}\right)dx\\ &= \frac{2\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.72

$$\frac{2\sqrt{f+gx}\left(15ag^2+c\left(8f^2-4fgx+3g^2x^2\right)\right)}{15g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(15*a*g^2 + c*(8*f^2 - 4*f*g*x + 3*g^2*x^2)))/(15*g^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 48, normalized size = 0.79

$$\frac{2\sqrt{f+gx}\left(15ag^2+15cf^2-10cf(f+gx)+3c(f+gx)^2\right)}{15g^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(15*c*f^2 + 15*a*g^2 - 10*c*f*(f + g*x) + 3*c*(f + g*x)^2)
)/(15*g^3)
```

fricas [**A**] time = 0.40, size = 40, normalized size = 0.66

$$\frac{2\left(3\,cg^2x^2-4\,cfgx+8\,cf^2+15\,ag^2\right)\sqrt{gx+f}}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*c*g^2*x^2 - 4*c*f*g*x + 8*c*f^2 + 15*a*g^2)*sqrt(g*x + f)/g^3
```

giac [**A**] time = 0.15, size = 53, normalized size = 0.87

$$\frac{2\left(15\,\sqrt{gx+f}\,a+\frac{\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*c/g^2)/g
```

maple [**A**] time = 0.00, size = 41, normalized size = 0.67

$$\frac{2\sqrt{gx+f}\,\left(3c\,x^2g^2-4cfxg+15a\,g^2+8c\,f^2\right)}{15g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(g*x+f)^(1/2),x)
[Out] 2/15*(g*x+f)^(1/2)*(3*c*g^2*x^2-4*c*f*g*x+15*a*g^2+8*c*f^2)/g^3
```

maxima [**A**] time = 0.43, size = 53, normalized size = 0.87

$$\frac{2\left(15\,\sqrt{gx+f}\,a+\frac{\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*c/g^2)/g
```

mupad [**B**] time = 2.56, size = 44, normalized size = 0.72

$$\frac{2\,\sqrt{f+g\,x}\,\left(3\,c\,{\left(f+g\,x\right)}^2+15\,a\,g^2+15\,c\,f^2-10\,c\,f\,\left(f+g\,x\right)\right)}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/(f + g*x)^(1/2),x)
```

```
[Out] (2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 - 10*c*f*(f + g*x
)))/(15*g^3)
```

sympy [A] time = 13.10, size = 150, normalized size = 2.46

$$\begin{cases} \frac{-\frac{2af}{\sqrt{f+gx}} - 2a\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) - \frac{2cf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2c\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for } g \neq 0 \\ \frac{ax + \frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*f/sqrt(f + g*x) - 2*a*(-f/sqrt(f + g*x) - sqrt(f + g*x)) -
 2*c*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 -
 2*c*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f
+ g*x)**(5/2)/5)/g**2)/g, Ne(g, 0)), ((a*x + c*x**3/3)/sqrt(f), True))
```

3.395 $\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=104

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{2c\sqrt{f+gx}\,(dg+ef)}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

Rubi [A] time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {898, 1153, 208}

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{2c\sqrt{f+gx}\,(dg+ef)}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*c*(e*f + d*g)*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))/(3*e*g^2
) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e
^(5/2)*Sqrt[e*f - d*g])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{c(ef+dg)}{e^2g}+\frac{cx^2}{eg}+\frac{cd^2+ae^2}{e^2\left(d-\frac{ef}{g}+\frac{ex^2}{g}\right)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}+\frac{\left(2\left(a+\frac{cd^2}{e^2}\right)\right)\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+g$$

$$= -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2\left(cd^2+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

Mathematica [A] time = 0.16, size = 92, normalized size = 0.88

$$\frac{2c\sqrt{f+gx}\,(-3dg-2ef+egx)}{3e^2g^2}-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x]*(-2*e*f - 3*d*g + e*g*x))/(3*e^2*g^2) - (2*(c*d^2 + a*e^
2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*
g])
```

IntegrateAlgebraic [A] time = 0.16, size = 105, normalized size = 1.01

$$\frac{2c\sqrt{f+gx}\,(-3dg+e(f+gx)-3ef)}{3e^2g^2}-\frac{2\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)}{e^{5/2}\sqrt{dg-ef}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x]*(-3*e*f - 3*d*g + e*(f + g*x)))/(3*e^2*g^2) - (2*(c*d^2
+ a*e^2)*ArcTan[(Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[f + g*x])/(e*f - d*g)])/(e
^(5/2)*Sqrt[-(e*f) + d*g])
```

fricas [A] time = 0.41, size = 297, normalized size = 2.86

$$\left[\frac{3\left(cd^2+ae^2\right)\sqrt{e^2f-deg}\,g^2\log\left(\frac{egx+2ef-dg-2\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2ce^3f^2+cde^2fg-3cd^2eg^2-\left(ce^3fg-cde^2g^2\right)x\right)\sqrt{gx+f}}{3\left(e^4fg^2-de^3g^3\right)}, \frac{2\left(3\left(cd^2+ae^2\right)\sqrt{-e^2f+deg}\,g^2\arctan\left(\frac{\sqrt{-e^2f+deg}\sqrt{gx+f}}{egx+ef}\right)-\left(2ce^3f^2+cde^2fg-3cd^2eg^2-\left(ce^3fg-cde^2g^2\right)x\right)\sqrt{gx+f}\right)}{3\left(e^4fg^2-de^3g^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c*d^2 + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f - d*g -
2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2 + c*d*e^2*
f*g - 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^
2 - d*e^3*g^3), 2/3*(3*(c*d^2 + a*e^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(sqrt
```

```
(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*e^3*f^2 + c*d*e^2*f*g
- 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 -
d*e^3*g^3)]
```

giac [A] time = 0.20, size = 107, normalized size = 1.03

$$\frac{2\left(cd^2+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)e^{(-2)}}{\sqrt{dge-fe^2}}-\frac{2\left(3\sqrt{gx+f}\,cdg^5e-\left(gx+f\right)^{\frac{3}{2}}cg^4e^2+3\sqrt{gx+f}\,cfg^4e^2\right)e^{(-3)}}{3g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))*e^(-2)/sqrt(d
*g*e - f*e^2) - 2/3*(3*sqrt(g*x + f)*c*d*g^5*e - (g*x + f)^(3/2)*c*g^4*e^2
+ 3*sqrt(g*x + f)*c*f*g^4*e^2)*e^(-3)/g^6
```

maple [A] time = 0.02, size = 132, normalized size = 1.27

$$\frac{2a\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\sqrt{\left(dg-ef\right)e}}+\frac{2cd^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\sqrt{\left(dg-ef\right)e}\,e^2}-\frac{2\sqrt{gx+f}\,cd}{e^2g}-\frac{2\sqrt{gx+f}\,cf}{e\,g^2}+\frac{2\left(gx+f\right)^{\frac{3}{2}}c}{3e\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3*c*(g*x+f)^(3/2)/e/g^2-2/g*c/e^2*d*(g*x+f)^(1/2)-2/g^2*c/e*f*(g*x+f)^(1/
2)+2/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)*e/((d*g-e*f)*e)^(1/2))*a+2/e^
2/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)*e/((d*g-e*f)*e)^(1/2))*c*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.11, size = 107, normalized size = 1.03

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(c\,d^2+a\,e^2\right)}{e^{5/2}\,\sqrt{d\,g-e\,f}}-\sqrt{f+g\,x}\,\left(\frac{2\,c\,\left(d\,g^3-e\,f\,g^2\right)}{e^2\,g^4}+\frac{4\,c\,f}{e\,g^2}\right)+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

```
[Out] (2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2))/(e^(5
/2)*(d*g - e*f)^(1/2)) - (f + g*x)^(1/2)*((2*c*(d*g^3 - e*f*g^2))/(e^2*g^4)
 + (4*c*f)/(e*g^2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

sympy [A] time = 48.66, size = 100, normalized size = 0.96

$$\frac{2c(f+gx)^{\frac{3}{2}}}{3eg^2}-\frac{2c\sqrt{f+gx}\,(dg+ef)}{e^2g^2}-\frac{2(ae^2+cd^2)\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\,\sqrt{f+gx}}\right)}{e^2\sqrt{\frac{e}{dg-ef}}\,(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*e*g**2) - 2*c*sqrt(f + g*x)*(d*g + e*f)/(e**2*g**2)
 - 2*(a*e**2 + c*d**2)*atan(1/(sqrt(e/(d*g - e*f))*sqrt(f + g*x)))/(e**2*sq
rt(e/(d*g - e*f))*(d*g - e*f))
```

3.396 $\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{\left(ae^2g+cd(4ef-3dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

Rubi [A] time = 0.20, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {898, 1157, 388, 208}

$$-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{\left(ae^2g+cd(4ef-3dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((a + (c*d^2)/e^2)*Sqrt[f + g*x])/((e*f - d*g
)*(d + e*x)) + ((a*e^2*g + c*d*(4*e*f - 3*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g
*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} + \frac{\mathrm{Subst}\left(\int \frac{-a+\frac{cd^2}{e^2}-\frac{2cf^2}{g^2}+\frac{2c(ef-dg)x^2}{eg^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} - \frac{\left(a+\frac{cd(4ef-3dg)}{e^2g}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+\ldots}\right.}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} + \frac{\left(ae^2g+cd(4ef-3dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

Mathematica [A] time = 0.29, size = 171, normalized size = 1.40

$$\frac{\frac{\left(ae^2+cd^2\right)\left(\sqrt{e}\sqrt{f+gx}\,(dg-ef)+g(d+ex)\sqrt{dg-ef}\,\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)\right)}{(d+ex)(ef-dg)^2}+\frac{4cd\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}+\frac{2c\sqrt{e}\sqrt{f+gx}}{g}}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] ((2*c*Sqrt[e]*Sqrt[f + g*x])/g + ((c*d^2 + a*e^2)*(Sqrt[e]*(-(e*f) + d*g)*S
qrt[f + g*x] + g*Sqrt[-(e*f) + d*g]*(d + e*x)*ArcTan[(Sqrt[e]*Sqrt[f + g*x]
)/Sqrt[-(e*f) + d*g]]))/((e*f - d*g)^2*(d + e*x)) + (4*c*d*ArcTanh[(Sqrt[e]
*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g])/e^(5/2)
```

IntegrateAlgebraic [A] time = 0.51, size = 179, normalized size = 1.47

$$\frac{\sqrt{f+gx}\left(ae^2g^2+3cd^2g^2+2cdeg(f+gx)-4cdefg+2ce^2f^2-2ce^2f(f+gx)\right)}{e^2g(ef-dg)(-dg-e(f+gx)+ef)}+\frac{\left(-ae^2g+3cd^2g-4cdef\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)}{e^{5/2}(dg-ef)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*(2*c*e^2*f^2 - 4*c*d*e*f*g + 3*c*d^2*g^2 + a*e^2*g^2 - 2*c*e
^2*f*(f + g*x) + 2*c*d*e*g*(f + g*x)))/(e^2*g*(e*f - d*g)*(e*f - d*g - e*(f
 + g*x))) + ((-4*c*d*e*f + 3*c*d^2*g - a*e^2*g)*ArcTan[(Sqrt[e]*Sqrt[-(e*f)
 + d*g]*Sqrt[f + g*x])/(e*f - d*g)])/(e^(5/2)*(-(e*f) + d*g)^(3/2))
```

fricas [B] time = 0.41, size = 539, normalized size = 4.42

$$\left[-\frac{\left(4cd^2efg-\left(3cd^3-ade^2\right)g^2+\left(4cde^2fg-\left(3cd^2e-ae^3\right)g^2\right)x\right)\sqrt{e^2f-deg}\,\log\left(\frac{egx+2ef-dg-2\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2cde^3f^2-\left(5cd^2e^2+ae^4\right)fg+\left(3cd^3e+ade^3\right)g^2+2\left(ce^4f^2-2cde^3fg+cd^2e^2g^2\right)x\right)\sqrt{gx+f}}{2\left(de^5f^2g-2d^2e^4fg^2+d^3e^3g^3+\left(e^6f^2g-2de^5fg^2+d^2e^4g^3\right)x\right)}, -\frac{\left(4cd^2efg-\left(3cd^3-ade^2\right)g^2+\left(4cde^2fg-\left(3cd^2e-ae^3\right)g^2\right)x\right)\sqrt{-e^2f+deg}\,\arctan\left(\frac{\sqrt{-e^2f+deg}\sqrt{gx+f}}{egx+ef}\right)-\left(2cde^3f^2-\left(5cd^2e^2+ae^4\right)fg+\left(3cd^3e+ade^3\right)g^2+2\left(ce^4f^2-2cde^3fg+cd^2e^2g^2\right)x\right)\sqrt{gx+f}}{de^5f^2g-2d^2e^4fg^2+d^3e^3g^3+\left(e^6f^2g-2de^5fg^2+d^2e^4g^3\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((4*c*d^2*e*f*g - (3*c*d^3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2
*e - a*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e
^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^2 +
```

```
a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*
d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3
 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -((4*c*d^2*e*f*g - (3*c*d^
3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2*e - a*e^3)*g^2)*x)*sqrt(-e^2*f
 + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*d
*e^3*f^2 - (5*c*d^2*e^2 + a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4
*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^
2*e^4*f*g^2 + d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x)]
```

giac [A] time = 0.17, size = 148, normalized size = 1.21

$$\frac{2\sqrt{gx+f}\,ce^{(-2)}}{g}-\frac{\left(3\,cd^2g-4\,cdfe-age^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(dge^2-fe^3\right)\sqrt{dge-fe^2}}+\frac{\sqrt{gx+f}\,cd^2g+\sqrt{gx+f}\,age^2}{\left(dge^2-fe^3\right)\left(dg+\left(gx+f\right)e-fe\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(g*x + f)*c*e^(-2)/g - (3*c*d^2*g - 4*c*d*f*e - a*g*e^2)*arctan(sqrt(
g*x + f)*e/sqrt(d*g*e - f*e^2))/((d*g*e^2 - f*e^3)*sqrt(d*g*e - f*e^2)) + (
sqrt(g*x + f)*c*d^2*g + sqrt(g*x + f)*a*g*e^2)/((d*g*e^2 - f*e^3)*(d*g + (g
*x + f)*e - f*e))
```

maple [B] time = 0.02, size = 237, normalized size = 1.94

$$\frac{ag\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}}-\frac{3c\,d^2g\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e^2}+\frac{4cdf\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e}+\frac{\sqrt{gx+f}\,ag}{\left(dg-ef\right)\left(egx+dg\right)}+\frac{\sqrt{gx+f}\,c\,d^2g}{\left(dg-ef\right)\left(egx+dg\right)e^2}+\frac{2\sqrt{gx+f}\,c}{e^2g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

```
[Out] 2*c*(g*x+f)^(1/2)/e^2/g+g/(d*g-e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*a+g/e^2/(d*g-
e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*c*d^2+g/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan
((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*a-3*g/e^2/(d*g-e*f)/((d*g-e*f)*e)^(1/
2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d^2+4/e/(d*g-e*f)/((d*g-e*
f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 2.68, size = 128, normalized size = 1.05

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(-3\,c\,g\,d^2+4\,c\,f\,d\,e+a\,g\,e^2\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}}+\frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)}+\frac{2\,c\,\sqrt{f+g\,x}}{e^2\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 3*c*d^2*g + 4
*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)^(1/2)*(a*e^2*g + c*d^2*
g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f + d*e^2*g)) + (2*c*(f + g*x)^(1/2))
/(e^2*g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.397 $\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=178

$$-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(d+ex)^2(ef-dg)}-\frac{\left(3ae^2g^2+c\left(3d^2g^2-8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}+\frac{\sqrt{f+gx}\left(3ae^2g+cd(8ef-\right.}{4e^2(d+ex)(ef-dg)^2}$$

Rubi [A] time = 0.30, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {898, 1157, 385, 208}

$$-\frac{\left(3ae^2g^2+c\left(3d^2g^2-8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}\left(3ae^2g+cd(8ef-5dg)\right)}{4e^2(d+ex)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -((a + (c*d^2)/e^2)*Sqrt[f + g*x])/(2*(e*f - d*g)*(d + e*x)^2) + ((3*a*e^2*
g + c*d*(8*e*f - 5*d*g))*Sqrt[f + g*x])/(4*e^2*(e*f - d*g)^2*(d + e*x)) - (
(3*a*e^2*g^2 + c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt
[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(5/2)*(e*f - d*g)^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-3a+\frac{cd^2}{e^2}-\frac{4cf^2}{g^2}+\frac{4c(ef-dg)x^2}{eg^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}dx, x, \sqrt{f+gx}\right)}{2(ef-dg)}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(8ef-5dg)\right)\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} + \frac{\left(3ae^2g^2+c\left(8e^2f^2-8\right.\right.}{}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(8ef-5dg)\right)\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} - \frac{\left(3ae^2g^2+c\left(8e^2f^2-8\right.\right.}{4e}$$

Mathematica [C] time = 0.82, size = 207, normalized size = 1.16

$$\frac{2\left(\frac{\sqrt{e}g^2\sqrt{f+gx}\left(ae^2+cd^2\right){}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{e(f+gx)}{ef-dg}\right)}{(dg-ef)^3}-\frac{cd\left(\sqrt{e}\sqrt{f+gx}(dg-ef)+g(d+ex)\sqrt{dg-ef}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)\right)}{(d+ex)(ef-dg)^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] (2*(-((c*d*(Sqrt[e]*(-(e*f) + d*g)*Sqrt[f + g*x] + g*Sqrt[-(e*f) + d*g]*(d
+ e*x)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]]))/((e*f - d*g)^2*
(d + e*x))) - (c*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e*f
 - d*g] + (Sqrt[e]*(c*d^2 + a*e^2)*g^2*Sqrt[f + g*x]*Hypergeometric2F1[1/2,
 3, 3/2, (e*(f + g*x))/(e*f - d*g)])/(-(e*f) + d*g)^3))/e^(5/2)
```

IntegrateAlgebraic [A] time = 0.86, size = 225, normalized size = 1.26

$$\frac{\left(-3ae^2g^2-3cd^2g^2+8cdefg-8ce^2f^2\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)}{4e^{5/2}(dg-ef)^{5/2}}-\frac{g\sqrt{f+gx}\left(-5ade^2g^2-3ae^3g(f+gx)+5ae^3fg+3cd^3g^2+5cd^2eg(f+gx)-11cd^2efg+8cde^2f^2-8cde^2f(f+gx)\right)}{4e^2(ef-dg)^2(-dg-e(f+gx)+ef)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -1/4*(g*Sqrt[f + g*x]*(8*c*d*e^2*f^2 - 11*c*d^2*e*f*g + 5*a*e^3*f*g + 3*c*d
^3*g^2 - 5*a*d*e^2*g^2 - 8*c*d*e^2*f*(f + g*x) + 5*c*d^2*e*g*(f + g*x) - 3*
a*e^3*g*(f + g*x)))/(e^2*(e*f - d*g)^2*(e*f - d*g - e*(f + g*x))^2) + ((-8*
c*e^2*f^2 + 8*c*d*e*f*g - 3*c*d^2*g^2 - 3*a*e^2*g^2)*ArcTan[(Sqrt[e]*Sqrt[-
(e*f) + d*g]*Sqrt[f + g*x])/(e*f - d*g)])/(4*e^(5/2)*(-(e*f) + d*g)^(5/2))
```

fricas [B] time = 0.43, size = 896, normalized size = 5.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a*d^2*e^2)*g^2 + (8*c*e
^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2
```

```
- 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((
e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(
2*(3*c*d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d^4*e -
5*a*d^2*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g + (5*c*d^3
*e^2 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3
*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 -
 d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*
e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a*d^2*e^2)*
g^2 + (8*c*e^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*x^2 + 2*(8*
c*d*e^3*f^2 - 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*sqrt(-e^2*f +
 d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*(3*c*
d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d^4*e - 5*a*d^2
*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g + (5*c*d^3*e^2 -
3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3*d^4*e^
4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 - d^3*e^
5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*e^4*g^3
)*x)]
```

giac [A] time = 0.21, size = 278, normalized size = 1.56

$$\frac{\left(3\,cd^2g^2-8\,cdfge+8\,cf^2e^2+3\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{4\,\left(d^2g^2e^2-2\,dfge^3+f^2e^4\right)\sqrt{dge-fe^2}}-\frac{3\,\sqrt{gx+f}\,cd^3g^3+5\,\left(gx+f\right)^{\frac{3}{2}}cd^2g^2e-11\,\sqrt{gx+f}\,cd^2fg^2e-8\,\left(gx+f\right)^{\frac{3}{2}}cdfge^2+8\,\sqrt{gx+f}\,cdf^2ge^2-5\,\sqrt{gx+f}\,adg^3e^2-3\,\left(gx+f\right)^{\frac{3}{2}}ag^2e^3+5\,\sqrt{gx+f}\,afg^2e^3}{4\,\left(d^2g^2e^2-2\,dfge^3+f^2e^4\right)\left(dg+\left(gx+f\right)e-fe\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*g^2 - 8*c*d*f*g*e + 8*c*f^2*e^2 + 3*a*g^2*e^2)*arctan(sqrt(g*x
 + f)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*sqrt(d*
g*e - f*e^2)) - 1/4*(3*sqrt(g*x + f)*c*d^3*g^3 + 5*(g*x + f)^(3/2)*c*d^2*g^
2*e - 11*sqrt(g*x + f)*c*d^2*f*g^2*e - 8*(g*x + f)^(3/2)*c*d*f*g*e^2 + 8*sq
rt(g*x + f)*c*d*f^2*g*e^2 - 5*sqrt(g*x + f)*a*d*g^3*e^2 - 3*(g*x + f)^(3/2)
*a*g^2*e^3 + 5*sqrt(g*x + f)*a*f*g^2*e^3)/((d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2
*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

maple [B] time = 0.02, size = 384, normalized size = 2.16

$$\frac{3a\,g^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{4\left(d^2g^2-2defg+e^2f^2\right)\sqrt{\left(dg-ef\right)e}}+\frac{3c\,d^2g^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{4\left(d^2g^2-2defg+e^2f^2\right)\sqrt{\left(dg-ef\right)e}\,e^2}-\frac{2cdfg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{\left(d^2g^2-2defg+e^2f^2\right)\sqrt{\left(dg-ef\right)e}\,e}+\frac{2c\,f^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{\left(d^2g^2-2defg+e^2f^2\right)\sqrt{\left(dg-ef\right)e}}+\frac{\frac{\left(3a\,e^2g-5c\,d^2g+8cdef\right)\left(gx+f\right)^{\frac{3}{2}}g}{4\left(d^2g^2-2defg+e^2f^2\right)e}+\frac{\left(5a\,e^2g-3c\,d^2g+8cdef\right)\sqrt{gx+f}\,g}{4\left(dg-ef\right)e^2}}{\left(dg-ef+\left(gx+f\right)e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x)
```

```
[Out] 2*(1/8*g*(3*a*e^2*g-5*c*d^2*g+8*c*d*e*f)/e/(d^2*g^2-2*d*e*f*g+e^2*f^2)*(g*x
+f)^(3/2)+1/8*(5*a*e^2*g-3*c*d^2*g+8*c*d*e*f)/e^2*g/(d*g-e*f)*(g*x+f)^(1/2)
)/(e*(g*x+f)+d*g-e*f)^2+3/4/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)
*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*a*g^2+3/4/(d^2*g^2-2*d*e*f*g+e
^2*f^2)/e^2/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)
*c*d^2*g^2-2/(d^2*g^2-2*d*e*f*g+e^2*f^2)/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+
f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d*f*g+2/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((d*g
-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 2.91, size = 224, normalized size = 1.26

$$\frac{\frac{\sqrt{f+g\,x}\,\left(-3\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+5\,a\,e^2\,g^2\right)}{4\,e^2\,\left(d\,g-e\,f\right)}+\frac{{\left(f+g\,x\right)}^{3/2}\,\left(-5\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+3\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^2\,f-2\,d\,e\,g\right)+d^2\,g^2+e^2\,f^2-2\,d\,e\,f\,g}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{4\,e^{5/2}\,{\left(d\,g-e\,f\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

```
[Out] (((f + g*x)^(1/2)*(5*a*e^2*g^2 - 3*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e^2*(d*g -
e*f)) + ((f + g*x)^(3/2)*(3*a*e^2*g^2 - 5*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e*(d
*g - e*f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2 +
e^2*f^2 - 2*d*e*f*g) + (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(
3*a*e^2*g^2 + 3*c*d^2*g^2 + 8*c*e^2*f^2 - 8*c*d*e*f*g))/(4*e^(5/2)*(d*g - e
*f)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.398 $\int \frac{(d+ex)^3\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=238

$$\frac{2e(f+gx)^{5/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}-\frac{2(f+gx)^{3/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2\right.\right.}{3g^6}$$

Rubi [A] time = 0.27, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {898, 1261}

$$\frac{2e(f+gx)^{5/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}-\frac{2(f+gx)^{3/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{3g^6}+\frac{2\left(ag^2+cf^2\right)(ef-dg)^3}{g^6\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{g^6}-\frac{2ce^2(f+gx)^{7/2}(5ef-3dg)}{7g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^3*(c*f^2 + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d*g)^2*(3
*a*e*g^2 + c*f*(5*e*f - 2*d*g))*Sqrt[f + g*x])/g^6 - (2*(e*f - d*g)*(3*a*e^
2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) + (2
*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(
5*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*(f +
g*x)^(9/2))/(9*g^6)
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx,x,\sqrt{f+gx}\right)}{g}\\ &= \frac{2\,\text{Subst}\left(\int \left(\frac{(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{g^5}+\frac{(-ef+dg)^3\left(cf^2+ag^2\right)}{g^5x^2}+\frac{(ef-dg)\left(-3ae^2g^2-c\left(10e^2f^2-8d\right.\right.}{g^5}\right.\right.}{}\\ &= \frac{2(ef-dg)^3\left(cf^2+ag^2\right)}{g^6\sqrt{f+gx}}+\frac{2(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)\sqrt{f+gx}}{g^6}-\frac{2(ef-}{}\end{aligned}$$

Mathematica [A] time = 0.24, size = 207, normalized size = 0.87

$$\frac{2\left(63e(f+gx)^3\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)-105(f+gx)^2(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)+315\left(ag^2+cf^2\right)(ef-dg)^3+315(f+gx)(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)-45ce^2(f+gx)^4(5ef-3dg)+35ce^3(f+gx)^5\right)}{315g^6\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(315*(e*f - d*g)^3*(c*f^2 + a*g^2) + 315*(e*f - d*g)^2*(3*a*e*g^2 + c*f*
(5*e*f - 2*d*g))*(f + g*x) - 105*(e*f - d*g)*(3*a*e^2*g^2 + c*(10*e^2*f^2 -
 8*d*e*f*g + d^2*g^2))*(f + g*x)^2 + 63*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d
*e*f*g + 3*d^2*g^2))*(f + g*x)^3 - 45*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^4 + 3
5*c*e^3*(f + g*x)^5))/(315*g^6*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.18, size = 427, normalized size = 1.79

$$\frac{2\left(-315ad^3g^5+945ad^2eg^4(f+gx)+945ad^2efg^4-945ade^2f^2g^3-1890ade^2fg^3(f+gx)+315ade^2g^3(f+gx)^2+315ae^3f^3g^2+945ae^3f^2g^2(f+gx)-315ae^3fg^2(f+gx)^2+63ae^3g^2(f+gx)^3-315cd^3f^2g^3-630cd^3fg^3(f+gx)+105cd^3g^3(f+gx)^2+945cd^2ef^3g^2+2835cd^2ef^2g^2(f+gx)-945cd^2efg^2(f+gx)^2+189cd^2eg^2(f+gx)^3-945cde^2f^4g-3780cde^2f^3g(f+gx)+1890cde^2f^2g(f+gx)^2-756cde^2fg(f+gx)^3+135cde^2g(f+gx)^4+315ce^3f^5+1575ce^3f^4(f+gx)-1050ce^3f^3(f+gx)^2+630ce^3f^2(f+gx)^3-225ce^3f(f+gx)^4+35ce^3(f+gx)^5\right)}{315g^6\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(315*c*e^3*f^5 - 945*c*d*e^2*f^4*g + 945*c*d^2*e*f^3*g^2 + 315*a*e^3*f^3
*g^2 - 315*c*d^3*f^2*g^3 - 945*a*d*e^2*f^2*g^3 + 945*a*d^2*e*f*g^4 - 315*a*
d^3*g^5 + 1575*c*e^3*f^4*(f + g*x) - 3780*c*d*e^2*f^3*g*(f + g*x) + 2835*c*
d^2*e*f^2*g^2*(f + g*x) + 945*a*e^3*f^2*g^2*(f + g*x) - 630*c*d^3*f*g^3*(f
+ g*x) - 1890*a*d*e^2*f*g^3*(f + g*x) + 945*a*d^2*e*g^4*(f + g*x) - 1050*c*
e^3*f^3*(f + g*x)^2 + 1890*c*d*e^2*f^2*g*(f + g*x)^2 - 945*c*d^2*e*f*g^2*(f
 + g*x)^2 - 315*a*e^3*f*g^2*(f + g*x)^2 + 105*c*d^3*g^3*(f + g*x)^2 + 315*a
*d*e^2*g^3*(f + g*x)^2 + 630*c*e^3*f^2*(f + g*x)^3 - 756*c*d*e^2*f*g*(f + g
*x)^3 + 189*c*d^2*e*g^2*(f + g*x)^3 + 63*a*e^3*g^2*(f + g*x)^3 - 225*c*e^3*
f*(f + g*x)^4 + 135*c*d*e^2*g*(f + g*x)^4 + 35*c*e^3*(f + g*x)^5))/(315*g^6
*Sqrt[f + g*x])
```

fricas [A] time = 0.39, size = 333, normalized size = 1.40

$$\frac{2\left(35ce^3g^5x^5+1280ce^3f^5-3456cde^2f^4g+1890ad^2efg^4-315ad^3g^5+1008\left(3cd^2e+ae^3\right)f^3g^2-840\left(cd^3+3ade^2\right)f^2g^3-5\left(10ce^3fg^4-27cde^2g^5\right)x^4+\left(80ce^3f^2g^3-216cde^2fg^4+63\left(3cd^2e+ae^3\right)g^5\right)x^3-\left(160ce^3f^3g^2-432cde^2f^2g^3+126\left(3cd^2e+ae^3\right)fg^4-105\left(cd^3+3ade^2\right)g^5\right)x^2+\left(640ce^3f^4g-1728cde^2f^3g^2+945ad^2eg^5+504\left(3cd^2e+ae^3\right)f^2g^3-420\left(cd^3+3ade^2\right)fg^4\right)x\right)\sqrt{gx+f}}{315\left(g^7x+fg^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 3456*c*d*e^2*f^4*g + 1890*a*d^2*
e*f*g^4 - 315*a*d^3*g^5 + 1008*(3*c*d^2*e + a*e^3)*f^3*g^2 - 840*(c*d^3 + 3
*a*d*e^2)*f^2*g^3 - 5*(10*c*e^3*f*g^4 - 27*c*d*e^2*g^5)*x^4 + (80*c*e^3*f^2
*g^3 - 216*c*d*e^2*f*g^4 + 63*(3*c*d^2*e + a*e^3)*g^5)*x^3 - (160*c*e^3*f^3
*g^2 - 432*c*d*e^2*f^2*g^3 + 126*(3*c*d^2*e + a*e^3)*f*g^4 - 105*(c*d^3 + 3
*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 1728*c*d*e^2*f^3*g^2 + 945*a*d^2*e*
g^5 + 504*(3*c*d^2*e + a*e^3)*f^2*g^3 - 420*(c*d^3 + 3*a*d*e^2)*f*g^4)*x)*s
qrt(g*x + f)/(g^7*x + f*g^6)
```

giac [B] time = 0.21, size = 453, normalized size = 1.90

$$-\frac{2\left(cd^3f^2g^3+ad^3g^5-3cd^2f^3g^2e-3ad^2fg^4e+3cdf^4ge^2+3adf^2g^3e^2-cf^5e^3-af^3g^2e^3\right)}{\sqrt{gx+f}\,g^6}+\frac{2\left(105(gx+f)^{\frac{3}{2}}cd^3g^{51}-630\sqrt{gx+f}\,cd^3fg^{51}+189(gx+f)^{\frac{5}{2}}cd^2g^{50}e-945(gx+f)^{\frac{3}{2}}cd^2fg^{50}e+2835\sqrt{gx+f}\,cd^2f^2g^{50}e+945\sqrt{gx+f}\,ad^2g^{52}e+135(gx+f)^{\frac{7}{2}}cdg^{49}e^2-756(gx+f)^{\frac{5}{2}}cdfg^{49}e^2+1890(gx+f)^{\frac{3}{2}}cdf^2g^{49}e^2-3780\sqrt{gx+f}\,cdf^3g^{49}e^2+315(gx+f)^{\frac{3}{2}}adg^{51}e^2-1890\sqrt{gx+f}\,adfg^{51}e^2+35(gx+f)^{\frac{9}{2}}cg^{48}e^3-225(gx+f)^{\frac{7}{2}}cfg^{48}e^3+630(gx+f)^{\frac{5}{2}}cf^2g^{48}e^3-1050(gx+f)^{\frac{3}{2}}cf^3g^{48}e^3+1575\sqrt{gx+f}\,cf^4g^{48}e^3+63(gx+f)^{\frac{5}{2}}ag^{50}e^3-315(gx+f)^{\frac{3}{2}}afg^{50}e^3+945\sqrt{gx+f}\,af^2g^{50}e^3\right)}{315g^{54}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^3*f^2*g^3 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e - 3*a*d^2*f*g^4*e + 3*c*d
*f^4*g*e^2 + 3*a*d*f^2*g^3*e^2 - c*f^5*e^3 - a*f^3*g^2*e^3)/(sqrt(g*x + f)*
g^6) + 2/315*(105*(g*x + f)^(3/2)*c*d^3*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^
51 + 189*(g*x + f)^(5/2)*c*d^2*g^50*e - 945*(g*x + f)^(3/2)*c*d^2*f*g^50*e
+ 2835*sqrt(g*x + f)*c*d^2*f^2*g^50*e + 945*sqrt(g*x + f)*a*d^2*g^52*e + 13
5*(g*x + f)^(7/2)*c*d*g^49*e^2 - 756*(g*x + f)^(5/2)*c*d*f*g^49*e^2 + 1890*
(g*x + f)^(3/2)*c*d*f^2*g^49*e^2 - 3780*sqrt(g*x + f)*c*d*f^3*g^49*e^2 + 31
5*(g*x + f)^(3/2)*a*d*g^51*e^2 - 1890*sqrt(g*x + f)*a*d*f*g^51*e^2 + 35*(g*
```

```
x + f)^(9/2)*c*g^48*e^3 - 225*(g*x + f)^(7/2)*c*f*g^48*e^3 + 630*(g*x + f)^
(5/2)*c*f^2*g^48*e^3 - 1050*(g*x + f)^(3/2)*c*f^3*g^48*e^3 + 1575*sqrt(g*x
+ f)*c*f^4*g^48*e^3 + 63*(g*x + f)^(5/2)*a*g^50*e^3 - 315*(g*x + f)^(3/2)*a
*f*g^50*e^3 + 945*sqrt(g*x + f)*a*f^2*g^50*e^3)/g^54
```

maple [A] time = 0.01, size = 365, normalized size = 1.53

$$-\frac{2\left(-35c e^{3} g^{5} x^{5}-135c d e^{2} g^{5} x^{4}+50c e^{3} f g^{4} x^{4}-63a e^{3} g^{5} x^{3}-189c d^{2} e g^{5} x^{3}+216c d e^{2} f g^{4} x^{3}-80c e^{3} f^{2} g^{3} x^{3}-315a d e^{2} g^{5} x^{2}+126a e^{3} f g^{4} x^{2}-105c d^{3} g^{5} x^{2}+378c d^{2} e f g^{4} x^{2}-432c d e^{2} f^{2} g^{3} x^{2}+160c e^{3} f^{3} g^{2} x^{2}-945a d^{2} e g^{5} x+1260a d e^{2} f g^{4} x-504a e^{3} f^{2} g^{3} x+420c d^{3} f g^{4} x-1512c d^{2} e f^{2} g^{3} x+1728c d e^{2} f^{3} g^{2} x-640c e^{3} f^{4} g x+315a d^{3} g^{5}-1890a d^{2} e f g^{4}+2520a d e^{2} f^{2} g^{3}-1008a e^{3} f^{3} g^{2}+840c d^{3} f^{2} g^{3}-3024c d^{2} e f^{3} g^{2}+3456c d e^{2} f^{4} g-1280c e^{3} f^{5}\right)}{315\sqrt{g x+f}\, g^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/315/(g*x+f)^(1/2)*(-35*c*e^3*g^5*x^5-135*c*d*e^2*g^5*x^4+50*c*e^3*f*g^4*
x^4-63*a*e^3*g^5*x^3-189*c*d^2*e*g^5*x^3+216*c*d*e^2*f*g^4*x^3-80*c*e^3*f^2
*g^3*x^3-315*a*d*e^2*g^5*x^2+126*a*e^3*f*g^4*x^2-105*c*d^3*g^5*x^2+378*c*d^
2*e*f*g^4*x^2-432*c*d*e^2*f^2*g^3*x^2+160*c*e^3*f^3*g^2*x^2-945*a*d^2*e*g^5
*x+1260*a*d*e^2*f*g^4*x-504*a*e^3*f^2*g^3*x+420*c*d^3*f*g^4*x-1512*c*d^2*e*
f^2*g^3*x+1728*c*d*e^2*f^3*g^2*x-640*c*e^3*f^4*g*x+315*a*d^3*g^5-1890*a*d^2
*e*f*g^4+2520*a*d*e^2*f^2*g^3-1008*a*e^3*f^3*g^2+840*c*d^3*f^2*g^3-3024*c*d
^2*e*f^3*g^2+3456*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6
```

maxima [A] time = 0.46, size = 334, normalized size = 1.40

$$\frac{2\left(\frac{35\left(gx+f\right)^{\frac{9}{2}}ce^{3}-45\left(5\,ce^{3}f-3\,cde^{2}g\right)\left(gx+f\right)^{\frac{7}{2}}+63\left(10\,ce^{3}f^{2}-12\,cde^{2}fg+\left(3\,cd^{2}e+ae^{3}\right)g^{2}\right)\left(gx+f\right)^{\frac{5}{2}}-105\left(10\,ce^{3}f^{3}-18\,cde^{2}f^{2}g+3\left(3\,cd^{2}e+ae^{3}\right)fg^{2}-\left(cd^{3}+3\,ade^{2}\right)g^{3}\right)\left(gx+f\right)^{\frac{3}{2}}+315\left(5\,ce^{3}f^{4}-12\,cde^{2}f^{3}g+3\,ad^{2}eg^{4}+3\left(3\,cd^{2}e+ae^{3}\right)f^{2}g^{2}-2\left(cd^{3}+3\,ade^{2}\right)fg^{3}\right)\sqrt{gx+f}}{g^{5}}+\frac{315\left(ce^{3}f^{5}-3\,cde^{2}f^{4}g+3\,ad^{2}efg^{4}-ad^{3}g^{5}+\left(3\,cd^{2}e+ae^{3}\right)f^{3}g^{2}-\left(cd^{3}+3\,ade^{2}\right)f^{2}g^{3}\right)}{\sqrt{gx+f}\,g^{5}}\right)}{315\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f)^(
7/2) + 63*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*x +
f)^(5/2) - 105*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e^3)*f*g
^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 12*c*d*e
^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*a*d
*e^2)*f*g^3)*sqrt(g*x + f))/g^5 + 315*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*a*d^
2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*d*e^2)*f
^2*g^3)/(sqrt(g*x + f)*g^5))/g
```

mupad [B] time = 0.09, size = 292, normalized size = 1.23

$$\frac{{\left(f+g\,x\right)}^{5/2}\,\left(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2\right)}{5\,g^6}-\frac{2\,c\,d^3\,f^2\,g^3+2\,a\,d^3\,g^5-6\,c\,d^2\,e\,f^3\,g^2-6\,a\,d^2\,e\,f\,g^4+6\,c\,d\,e^2\,f^4\,g+6\,a\,d\,e^2\,f^2\,g^3-2\,c\,e^3\,f^5-2\,a\,e^3\,f^3\,g^2}{g^6\,\sqrt{f+g\,x}}+\frac{2\,c\,e^3\,{\left(f+g\,x\right)}^{9/2}}{9\,g^6}+\frac{2\,\sqrt{f+g\,x}\,{\left(d\,g-e\,f\right)}^2\,\left(5\,c\,e\,f^2-2\,c\,d\,f\,g+3\,a\,e\,g^2\right)}{g^6}+\frac{2\,{\left(f+g\,x\right)}^{3/2}\,\left(d\,g-e\,f\right)\,\left(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+10\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{3\,g^6}+\frac{2\,c\,e^2\,{\left(f+g\,x\right)}^{7/2}\,\left(3\,d\,g-5\,e\,f\right)}{7\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f
*g))/(5*g^6) - (2*a*d^3*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2*g
^3 - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*c*d^2*e*f^3*
g^2)/(g^6*(f + g*x)^(1/2)) + (2*c*e^3*(f + g*x)^(9/2))/(9*g^6) + (2*(f + g*
x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2 + 5*c*e*f^2 - 2*c*d*f*g))/g^6 + (2*(f + g
*x)^(3/2)*(d*g - e*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g
))/(3*g^6) + (2*c*e^2*(f + g*x)^(7/2)*(3*d*g - 5*e*f))/(7*g^6)
```

sympy [A] time = 110.87, size = 328, normalized size = 1.38

$$\frac{2ce^{3}\left(f+gx\right)^{\frac{9}{2}}}{9g^{6}}+\frac{\left(f+gx\right)^{\frac{7}{2}}\left(6cde^{2}g-10ce^{3}f\right)}{7g^{6}}+\frac{\left(f+gx\right)^{\frac{5}{2}}\left(2ae^{3}g^{2}+6cd^{2}eg^{2}-24cde^{2}fg+20ce^{3}f^{2}\right)}{5g^{6}}+\frac{\left(f+gx\right)^{\frac{3}{2}}\left(6ade^{2}g^{3}-6ae^{3}fg^{2}+2cd^{3}g^{3}-18cd^{2}efg^{2}+36cde^{2}f^{2}g-20ce^{3}f^{3}\right)}{3g^{6}}+\frac{\sqrt{f+gx}\left(6ad^{2}eg^{4}-12ade^{2}fg^{3}+6ae^{3}f^{2}g^{2}-4cd^{3}fg^{3}+18cd^{2}ef^{2}g^{2}-24cde^{2}f^{3}g+10ce^{3}f^{4}\right)}{g^{6}}-\frac{2\left(ag^{2}+cf^{2}\right)\left(dg-ef\right)^{3}}{g^{6}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**3*(f + g*x)**(9/2)/(9*g**6) + (f + g*x)**(7/2)*(6*c*d*e**2*g - 10*c*
e**3*f)/(7*g**6) + (f + g*x)**(5/2)*(2*a*e**3*g**2 + 6*c*d**2*e*g**2 - 24*c
*d*e**2*f*g + 20*c*e**3*f**2)/(5*g**6) + (f + g*x)**(3/2)*(6*a*d*e**2*g**3
- 6*a*e**3*f*g**2 + 2*c*d**3*g**3 - 18*c*d**2*e*f*g**2 + 36*c*d*e**2*f**2*g
 - 20*c*e**3*f**3)/(3*g**6) + sqrt(f + g*x)*(6*a*d**2*e*g**4 - 12*a*d*e**2*
f*g**3 + 6*a*e**3*f**2*g**2 - 4*c*d**3*f*g**3 + 18*c*d**2*e*f**2*g**2 - 24*
c*d*e**2*f**3*g + 10*c*e**3*f**4)/g**6 - 2*(a*g**2 + c*f**2)*(d*g - e*f)**3
/(g**6*sqrt(f + g*x))
```

3.399 $\int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{2(f+gx)^{3/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2\left(ag^2+cf^2\right)(ef-dg)^2}{g^5\sqrt{f+gx}}-\frac{4\sqrt{f+gx}\,(ef-dg)\left(aeg^2+cf(2e\right.}{g^5}$$

Rubi [A] time = 0.20, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {898, 1261}

$$\frac{2(f+gx)^{3/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2\left(ag^2+cf^2\right)(ef-dg)^2}{g^5\sqrt{f+gx}}-\frac{4\sqrt{f+gx}\,(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{g^5}-\frac{4ce(f+gx)^{5/2}(2ef-dg)}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 + a*g^2))/(g^5*Sqrt[f + g*x]) - (4*(e*f - d*g)*(a*
e*g^2 + c*f*(2*e*f - d*g))*Sqrt[f + g*x])/g^5 + (2*(a*e^2*g^2 + c*(6*e^2*f^
2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^5) - (4*c*e*(2*e*f - d*g)*(
f + g*x)^(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx=\frac{2\,\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx,x,\sqrt{f+gx}\right)}{g}$$

$$=\frac{2\,\text{Subst}\left(\int \left(\frac{2(ef-dg)\left(-aeg^2-cf(2ef-dg)\right)}{g^4}+\frac{(-ef+dg)^2\left(cf^2+ag^2\right)}{g^4x^2}+\frac{\left(ae^2g^2+c\left(6e^2f^2-6defg+d^2g^2\right)\right)x}{g^4}\right.\right.}{g}$$

$$=-\frac{2(ef-dg)^2\left(cf^2+ag^2\right)}{g^5\sqrt{f+gx}}-\frac{4(ef-dg)\left(aeg^2+cf(2ef-dg)\right)\sqrt{f+gx}}{g^5}+\frac{2\left(ae^2g^2\right.}{}$$

Mathematica [A] time = 0.15, size = 149, normalized size = 0.86

$$\frac{2\left(35(f+gx)^2\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)-105\left(ag^2+cf^2\right)(ef-dg)^2-210(f+gx)(ef-dg)\left(aeg^2+cf(2ef-dg)\right)-42ce(f+gx)^3(2ef-dg)+15ce^2(f+gx)^4\right)}{105g^5\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-105*(e*f - d*g)^2*(c*f^2 + a*g^2) - 210*(e*f - d*g)*(a*e*g^2 + c*f*(2*
e*f - d*g))*(f + g*x) + 35*(a*e^2*g^2 + c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2)
)*(f + g*x)^2 - 42*c*e*(2*e*f - d*g)*(f + g*x)^3 + 15*c*e^2*(f + g*x)^4))/(
105*g^5*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.12, size = 250, normalized size = 1.45

$$\frac{2\left(-105ad^2g^4+210adeg^3(f+gx)+210adefg^3-105ae^2f^2g^2-210ae^2fg^2(f+gx)+35ae^2g^2(f+gx)^2-105cd^2f^2g^2-210cd^2fg^2(f+gx)+35cd^2g^2(f+gx)^2+210cdef^3g+630cdef^2g(f+gx)-210cdefg(f+gx)^2+42cdeg(f+gx)^3-105ce^2f^4-420ce^2f^3(f+gx)+210ce^2f^2(f+gx)^2-84ce^2f(f+gx)^3+15ce^2(f+gx)^4\right)}{105g^5\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-105*c*e^2*f^4 + 210*c*d*e*f^3*g - 105*c*d^2*f^2*g^2 - 105*a*e^2*f^2*g^
2 + 210*a*d*e*f*g^3 - 105*a*d^2*g^4 - 420*c*e^2*f^3*(f + g*x) + 630*c*d*e*f
^2*g*(f + g*x) - 210*c*d^2*f*g^2*(f + g*x) - 210*a*e^2*f*g^2*(f + g*x) + 21
0*a*d*e*g^3*(f + g*x) + 210*c*e^2*f^2*(f + g*x)^2 - 210*c*d*e*f*g*(f + g*x)
^2 + 35*c*d^2*g^2*(f + g*x)^2 + 35*a*e^2*g^2*(f + g*x)^2 - 84*c*e^2*f*(f +
g*x)^3 + 42*c*d*e*g*(f + g*x)^3 + 15*c*e^2*(f + g*x)^4))/(105*g^5*Sqrt[f +
g*x])
```

fricas [A] time = 0.38, size = 206, normalized size = 1.19

$$\frac{2\left(15ce^2g^4x^4-384ce^2f^4+672cdef^3g+420adefg^3-105ad^2g^4-280\left(cd^2+ae^2\right)f^2g^2-6\left(4ce^2fg^3-7cdeg^4\right)x^3+\left(48ce^2f^2g^2-84cdefg^3+35\left(cd^2+ae^2\right)g^4\right)x^2-2\left(96ce^2f^3g-168cdef^2g^2-105adeg^4+70\left(cd^2+ae^2\right)fg^3\right)x\right)\sqrt{gx+f}}{105\left(g^6x+fg^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 + 672*c*d*e*f^3*g + 420*a*d*e*f*g^3
 - 105*a*d^2*g^4 - 280*(c*d^2 + a*e^2)*f^2*g^2 - 6*(4*c*e^2*f*g^3 - 7*c*d*e
*g^4)*x^3 + (48*c*e^2*f^2*g^2 - 84*c*d*e*f*g^3 + 35*(c*d^2 + a*e^2)*g^4)*x^
2 - 2*(96*c*e^2*f^3*g - 168*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 70*(c*d^2 + a*e
^2)*f*g^3)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

giac [A] time = 0.33, size = 275, normalized size = 1.59

$$-\frac{2\left(cd^2f^2g^2+ad^2g^4-2cdf^3ge-2adfg^3e+cf^4e^2+af^2g^2e^2\right)}{\sqrt{gx+f}\,g^5}+\frac{2\left(35(gx+f)^{\frac{3}{2}}cd^2g^{32}-210\sqrt{gx+f}\,cd^2fg^{32}+42(gx+f)^{\frac{5}{2}}cdg^{31}e-210(gx+f)^{\frac{3}{2}}cdfg^{31}e+630\sqrt{gx+f}\,cdf^2g^{31}e+210\sqrt{gx+f}\,adg^{33}e+15(gx+f)^{\frac{7}{2}}cg^{30}e^2-84(gx+f)^{\frac{5}{2}}cfg^{30}e^2+210(gx+f)^{\frac{3}{2}}cf^2g^{30}e^2-420\sqrt{gx+f}\,cf^3g^{30}e^2+35(gx+f)^{\frac{3}{2}}ag^{32}e^2-210\sqrt{gx+f}\,afg^{32}e^2\right)}{105g^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*f^2*g^2 + a*d^2*g^4 - 2*c*d*f^3*g*e - 2*a*d*f*g^3*e + c*f^4*e^2 +
 a*f^2*g^2*e^2)/(sqrt(g*x + f)*g^5) + 2/105*(35*(g*x + f)^(3/2)*c*d^2*g^32
- 210*sqrt(g*x + f)*c*d^2*f*g^32 + 42*(g*x + f)^(5/2)*c*d*g^31*e - 210*(g*x
 + f)^(3/2)*c*d*f*g^31*e + 630*sqrt(g*x + f)*c*d*f^2*g^31*e + 210*sqrt(g*x
+ f)*a*d*g^33*e + 15*(g*x + f)^(7/2)*c*g^30*e^2 - 84*(g*x + f)^(5/2)*c*f*g^
30*e^2 + 210*(g*x + f)^(3/2)*c*f^2*g^30*e^2 - 420*sqrt(g*x + f)*c*f^3*g^30*
e^2 + 35*(g*x + f)^(3/2)*a*g^32*e^2 - 210*sqrt(g*x + f)*a*f*g^32*e^2)/g^35
```

maple [A] time = 0.01, size = 215, normalized size = 1.24

$$-\frac{2\left(-15e^2cx^4g^4-42cdeg^4x^3+24ce^2fg^3x^3-35ae^2g^4x^2-35cd^2g^4x^2+84cdefg^3x^2-48ce^2f^2g^2x^2-210adeg^4x+140ae^2fg^3x+140cd^2fg^3x-336cdef^2g^2x+192ce^2f^3gx+105d^2ag^4-420adefg^3+280ae^2f^2g^2+280cd^2f^2g^2-672cdef^3g+384ce^2f^4\right)}{105\sqrt{gx+f}\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/105/(g*x+f)^(1/2)*(-15*c*e^2*g^4*x^4-42*c*d*e*g^4*x^3+24*c*e^2*f*g^3*x^3
-35*a*e^2*g^4*x^2-35*c*d^2*g^4*x^2+84*c*d*e*f*g^3*x^2-48*c*e^2*f^2*g^2*x^2-
210*a*d*e*g^4*x+140*a*e^2*f*g^3*x+140*c*d^2*f*g^3*x-336*c*d*e*f^2*g^2*x+192
*c*e^2*f^3*g*x+105*a*d^2*g^4-420*a*d*e*f*g^3+280*a*e^2*f^2*g^2+280*c*d^2*f^
2*g^2-672*c*d*e*f^3*g+384*c*e^2*f^4)/g^5
```

maxima [A] time = 0.45, size = 205, normalized size = 1.18

$$\frac{2\left(\frac{15\,(gx+f)^{\frac{7}{2}}ce^2-42\,(2\,ce^2f-cdeg)(gx+f)^{\frac{5}{2}}+35\,(6\,ce^2f^2-6\,cdefg+(cd^2+ae^2)g^2)(gx+f)^{\frac{3}{2}}-210\,(2\,ce^2f^3-3\,cdef^2g-adeg^3+(cd^2+ae^2)fg^2)\sqrt{gx+f}}{g^4}-\frac{105\,(ce^2f^4-2\,cdef^3g-2\,adefg^3+ad^2g^4+(cd^2+ae^2)f^2g^2)}{\sqrt{gx+f}\,g^4}\right)}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 - 42*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(5/2)
 + 35*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(3/2) - 2
10*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*sqrt(g
*x + f))/g^4 - 105*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 +
 (c*d^2 + a*e^2)*f^2*g^2)/(sqrt(g*x + f)*g^4))/g
```

mupad [B] time = 2.66, size = 199, normalized size = 1.15

$$\frac{(f+g\,x)^{3/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2)}{3\,g^5}-\frac{2\,c\,d^2\,f^2\,g^2+2\,a\,d^2\,g^4-4\,c\,d\,e\,f^3\,g-4\,a\,d\,e\,f\,g^3+2\,c\,e^2\,f^4+2\,a\,e^2\,f^2\,g^2}{g^5\,\sqrt{f+g\,x}}+\frac{4\,\sqrt{f+g\,x}\,(d\,g-e\,f)\,(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2)}{g^5}+\frac{2\,c\,e^2\,(f+g\,x)^{7/2}}{7\,g^5}+\frac{4\,c\,e\,(f+g\,x)^{5/2}\,(d\,g-2\,e\,f)}{5\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g))
/(3*g^5) - (2*a*d^2*g^4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2 -
 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*x)^(1/2)) + (4*(f + g*x)^(1/2)*
(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/g^5 + (2*c*e^2*(f + g*x)^(7/2)
)/(7*g^5) + (4*c*e*(f + g*x)^(5/2)*(d*g - 2*e*f))/(5*g^5)
```

sympy [A] time = 50.85, size = 204, normalized size = 1.18

$$\frac{2ce^2\left(f+gx\right)^{\frac{7}{2}}}{7g^5}+\frac{\left(f+gx\right)^{\frac{5}{2}}\left(4cdeg-8ce^2f\right)}{5g^5}+\frac{\left(f+gx\right)^{\frac{3}{2}}\left(2ae^2g^2+2cd^2g^2-12cdefg+12ce^2f^2\right)}{3g^5}+\frac{\sqrt{f+gx}\left(4adeg^3-4ae^2fg^2-4cd^2fg^2+12cdef^2g-8ce^2f^3\right)}{g^5}-\frac{2\left(ag^2+cf^2\right)\left(dg-ef\right)^2}{g^5\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**2*(f + g*x)**(7/2)/(7*g**5) + (f + g*x)**(5/2)*(4*c*d*e*g - 8*c*e**2
*f)/(5*g**5) + (f + g*x)**(3/2)*(2*a*e**2*g**2 + 2*c*d**2*g**2 - 12*c*d*e*f
*g + 12*c*e**2*f**2)/(3*g**5) + sqrt(f + g*x)*(4*a*d*e*g**3 - 4*a*e**2*f*g*
*2 - 4*c*d**2*f*g**2 + 12*c*d*e*f**2*g - 8*c*e**2*f**3)/g**5 - 2*(a*g**2 +
c*f**2)*(d*g - e*f)**2/(g**5*sqrt(f + g*x))
```

3.400 $\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\left(ag^2+cf^2\right)(ef-dg)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\left(aeg^2+cf(3ef-2dg)\right)}{g^4}-\frac{2c(f+gx)^{3/2}(3ef-dg)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {772}

$$\frac{2\left(ag^2+cf^2\right)(ef-dg)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\left(aeg^2+cf(3ef-2dg)\right)}{g^4}-\frac{2c(f+gx)^{3/2}(3ef-dg)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)*(c*f^2 + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(a*e*g^2 + c*f*(3*
e*f - 2*d*g))*Sqrt[f + g*x])/g^4 - (2*c*(3*e*f - d*g)*(f + g*x)^(3/2))/(3*g
^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{(-ef+dg)\left(cf^2+ag^2\right)}{g^3(f+gx)^{3/2}}+\frac{aeg^2+cf(3ef-2dg)}{g^3\sqrt{f+gx}}+\frac{c(-3ef+dg)\sqrt{f+gx}}{g^3}+\frac{ce(f\ -\right.\\ &= \frac{2(ef-dg)\left(cf^2+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^4}-\frac{2c(3ef-dg)(f+gx)^{3/}}{3g^4}\end{aligned}$$

Mathematica [A] time = 0.08, size = 92, normalized size = 0.83

$$\frac{30ag^2(-dg+2ef+egx)+10cdg\left(-8f^2-4fgx+g^2x^2\right)+6ce\left(16f^3+8f^2gx-2fg^2x^2+g^3x^3\right)}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (30*a*g^2*(2*e*f - d*g + e*g*x) + 10*c*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 6
*c*e*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))/(15*g^4*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.07, size = 117, normalized size = 1.05

$$\frac{2\left(-15adg^3+15aeg^2(f+gx)+15aefg^2-15cdf^2g-30cdfg(f+gx)+5cdg(f+gx)^2+15cef^3+45cef^2(f+gx)-15cef(f+gx)^2+3ce(f+gx)^3\right)}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(15*c*e*f^3 - 15*c*d*f^2*g + 15*a*e*f*g^2 - 15*a*d*g^3 + 45*c*e*f^2*(f +
 g*x) - 30*c*d*f*g*(f + g*x) + 15*a*e*g^2*(f + g*x) - 15*c*e*f*(f + g*x)^2
+ 5*c*d*g*(f + g*x)^2 + 3*c*e*(f + g*x)^3))/(15*g^4*Sqrt[f + g*x])
```

fricas [A] time = 0.40, size = 110, normalized size = 0.99

$$\frac{2\left(3\,ceg^{3}x^{3}+48\,cef^{3}-40\,cdf^{2}g+30\,aefg^{2}-15\,adg^{3}-\left(6\,cefg^{2}-5\,cdg^{3}\right)x^{2}+\left(24\,cef^{2}g-20\,cdfg^{2}+15\,aeg^{3}\right)x\right)\sqrt{gx+f}}{15\left(g^{5}x+fg^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 40*c*d*f^2*g + 30*a*e*f*g^2 - 15*a*d*g^3
 - (6*c*e*f*g^2 - 5*c*d*g^3)*x^2 + (24*c*e*f^2*g - 20*c*d*f*g^2 + 15*a*e*g^
3)*x)*sqrt(g*x + f)/(g^5*x + f*g^4)
```

giac [A] time = 0.21, size = 143, normalized size = 1.29

$$-\frac{2\left(cdf^{2}g+adg^{3}-cf^{3}e-afg^{2}e\right)}{\sqrt{gx+f}\,g^{4}}+\frac{2\left(5\left(gx+f\right)^{\frac{3}{2}}cdg^{17}-30\,\sqrt{gx+f}\,cdfg^{17}+3\left(gx+f\right)^{\frac{5}{2}}cg^{16}e-15\left(gx+f\right)^{\frac{3}{2}}cfg^{16}e+45\,\sqrt{gx+f}\,cf^{2}g^{16}e+15\,\sqrt{gx+f}\,ag^{18}e\right)}{15\,g^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d*f^2*g + a*d*g^3 - c*f^3*e - a*f*g^2*e)/(sqrt(g*x + f)*g^4) + 2/15*(
5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f*g^17 + 3*(g*x + f)^(5/2
)*c*g^16*e - 15*(g*x + f)^(3/2)*c*f*g^16*e + 45*sqrt(g*x + f)*c*f^2*g^16*e
+ 15*sqrt(g*x + f)*a*g^18*e)/g^20
```

maple [A] time = 0.00, size = 101, normalized size = 0.91

$$-\frac{2\left(-3ce\,x^{3}g^{3}-5cd\,g^{3}x^{2}+6cef\,g^{2}x^{2}-15ae\,g^{3}x+20cdf\,g^{2}x-24ce\,f^{2}gx+15ad\,g^{3}-30aef\,g^{2}+40cd\,f^{2}g-48ce\,f^{3}\right)}{15\sqrt{gx+f}\,g^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/15/(g*x+f)^(1/2)*(-3*c*e*g^3*x^3-5*c*d*g^3*x^2+6*c*e*f*g^2*x^2-15*a*e*g^
3*x+20*c*d*f*g^2*x-24*c*e*f^2*g*x+15*a*d*g^3-30*a*e*f*g^2+40*c*d*f^2*g-48*c
*e*f^3)/g^4
```

maxima [A] time = 0.45, size = 112, normalized size = 1.01

$$\frac{2\left(\frac{3\left(gx+f\right)^{\frac{5}{2}}ce-5\left(3\,cef-cdg\right)\left(gx+f\right)^{\frac{3}{2}}+15\left(3\,cef^{2}-2\,cdfg+aeg^{2}\right)\sqrt{gx+f}}{g^{3}}+\frac{15\left(cef^{3}-cdf^{2}g+aefg^{2}-adg^{3}\right)}{\sqrt{gx+f}\,g^{3}}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - c*d*g)*(g*x + f)^(3/2) + 15*(3*
c*e*f^2 - 2*c*d*f*g + a*e*g^2)*sqrt(g*x + f))/g^3 + 15*(c*e*f^3 - c*d*f^2*g
 + a*e*f*g^2 - a*d*g^3)/(sqrt(g*x + f)*g^3))/g
```

mupad [B] time = 0.07, size = 111, normalized size = 1.00

$$\frac{\sqrt{f+g\,x}\,\left(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2\right)}{g^4}-\frac{-2\,c\,e\,f^3+2\,c\,d\,f^2\,g-2\,a\,e\,f\,g^2+2\,a\,d\,g^3}{g^4\,\sqrt{f+g\,x}}+\frac{2\,c\,e\,{\left(f+g\,x\right)}^{5/2}}{5\,g^4}+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}\,\left(d\,g-3\,e\,f\right)}{3\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(1/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/g^4 - (2*a*d*g^3 - 2*
c*e*f^3 - 2*a*e*f*g^2 + 2*c*d*f^2*g)/(g^4*(f + g*x)^(1/2)) + (2*c*e*(f + g*
x)^(5/2))/(5*g^4) + (2*c*(f + g*x)^(3/2)*(d*g - 3*e*f))/(3*g^4)
```

sympy [A] time = 25.28, size = 112, normalized size = 1.01

$$\frac{2ce\left(f+gx\right)^{\frac{5}{2}}}{5g^4}+\frac{\left(f+gx\right)^{\frac{3}{2}}\left(2cdg-6cef\right)}{3g^4}+\frac{\sqrt{f+gx}\left(2aeg^2-4cdfg+6cef^2\right)}{g^4}-\frac{2\left(ag^2+cf^2\right)\left(dg-ef\right)}{g^4\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e*(f + g*x)**(5/2)/(5*g**4) + (f + g*x)**(3/2)*(2*c*d*g - 6*c*e*f)/(3*g
**4) + sqrt(f + g*x)*(2*a*e*g**2 - 4*c*d*f*g + 6*c*e*f**2)/g**4 - 2*(a*g**2
 + c*f**2)*(d*g - e*f)/(g**4*sqrt(f + g*x))
```

3.401 $\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{2\left(ag^2+cf^2\right)}{g^3\sqrt{f+gx}}+\frac{2c(f+gx)^{3/2}}{3g^3}-\frac{4cf\sqrt{f+gx}}{g^3}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{2\left(ag^2+cf^2\right)}{g^3\sqrt{f+gx}}+\frac{2c(f+gx)^{3/2}}{3g^3}-\frac{4cf\sqrt{f+gx}}{g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(c*f^2 + a*g^2))/(g^3*Sqrt[f + g*x]) - (4*c*f*Sqrt[f + g*x])/g^3 + (2*c
*(f + g*x)^(3/2))/(3*g^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{cf^2+ag^2}{g^2(f+gx)^{3/2}}-\frac{2cf}{g^2\sqrt{f+gx}}+\frac{c\sqrt{f+gx}}{g^2}\right)dx\\ &= -\frac{2\left(cf^2+ag^2\right)}{g^3\sqrt{f+gx}}-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.73

$$\frac{2\left(c\left(-8f^2-4fgx+g^2x^2\right)-3ag^2\right)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-3*a*g^2 + c*(-8*f^2 - 4*f*g*x + g^2*x^2)))/(3*g^3*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.03, size = 47, normalized size = 0.80

$$\frac{2\left(-3ag^2-3cf^2-6cf(f+gx)+c(f+gx)^2\right)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)/(f + g*x)^(3/2),x]
```

[Out] (2*(-3*c*f^2 - 3*a*g^2 - 6*c*f*(f + g*x) + c*(f + g*x)^2))/(3*g^3*Sqrt[f + g*x])

fricas [A] time = 0.38, size = 49, normalized size = 0.83

$$\frac{2\left(cg^{2}x^{2}-4\,cfgx-8\,cf^{2}-3\,ag^{2}\right)\sqrt{gx+f}}{3\left(g^{4}x+fg^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")

[Out] 2/3*(c*g^2*x^2 - 4*c*f*g*x - 8*c*f^2 - 3*a*g^2)*sqrt(g*x + f)/(g^4*x + f*g^3)

giac [A] time = 0.18, size = 56, normalized size = 0.95

$$-\frac{2\left(cf^{2}+ag^{2}\right)}{\sqrt{gx+f}\,g^{3}}+\frac{2\left(\left(gx+f\right)^{\frac{3}{2}}cg^{6}-6\,\sqrt{gx+f}\,cfg^{6}\right)}{3\,g^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")

[Out] -2*(c*f^2 + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6 - 6*sqrt(g*x + f)*c*f*g^6)/g^9

maple [A] time = 0.00, size = 41, normalized size = 0.69

$$-\frac{2\left(-c\,x^{2}g^{2}+4cfxg+3a\,g^{2}+8c\,f^{2}\right)}{3\sqrt{gx+f}\,\,g^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)/(g*x+f)^(3/2),x)

[Out] -2/3/(g*x+f)^(1/2)*(-c*g^2*x^2+4*c*f*g*x+3*a*g^2+8*c*f^2)/g^3

maxima [A] time = 0.44, size = 54, normalized size = 0.92

$$\frac{2\left(\frac{\left(gx+f\right)^{\frac{3}{2}}c-6\,\sqrt{gx+f}\,cf}{g^{2}}-\frac{3\left(cf^{2}+ag^{2}\right)}{\sqrt{gx+f}\,g^{2}}\right)}{3\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")

[Out] 2/3*(((g*x + f)^(3/2)*c - 6*sqrt(g*x + f)*c*f)/g^2 - 3*(c*f^2 + a*g^2)/(sqrt(g*x + f)*g^2))/g

mupad [B] time = 0.05, size = 44, normalized size = 0.75

$$-\frac{6\,a\,g^{2}-2\,c\,\left(f+g\,x\right)^{2}+6\,c\,f^{2}+12\,c\,f\,\left(f+g\,x\right)}{3\,g^{3}\,\sqrt{f+g\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)/(f + g*x)^(3/2),x)

```
[Out] -(6*a*g^2 - 2*c*(f + g*x)^2 + 6*c*f^2 + 12*c*f*(f + g*x))/(3*g^3*(f + g*x)^
(1/2))
```

sympy [A] time = 10.14, size = 58, normalized size = 0.98

$$-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3g^3}-\frac{2\left(ag^2+cf^2\right)}{g^3\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] -4*c*f*sqrt(f + g*x)/g**3 + 2*c*(f + g*x)**(3/2)/(3*g**3) - 2*(a*g**2 + c*f
**2)/(g**3*sqrt(f + g*x))
```

3.402 $\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2+cf^2\right)}{g^2\sqrt{f+gx}\,(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

Rubi [A] time = 0.17, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {898, 1261, 208}

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2+cf^2\right)}{g^2\sqrt{f+gx}\,(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f + g*x])/(
e*g^2) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]
])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cf^2+ag^2}{g(-ef+dg)x^2}-\frac{(cd^2+ae^2)g}{e(ef-dg)(ef-dg-ex^2)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{(2(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{ef-dg-ex^2}\,dx, x, \sqrt{f\right.}}{e(ef-dg)}$$

$$= \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

Mathematica [C] time = 0.07, size = 90, normalized size = 0.80

$$-\frac{2\left(g^2\left(ae^2+cd^2\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)(dg+2ef+egx)\right)}{e^2g^2\sqrt{f+gx}\,(dg-ef)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*(e*f - d*g)*(2*e*f + d*g + e*g*x) + (c*d^2 + a*e^2)*g^2*Hypergeometr
ic2F1[-1/2, 1, 1/2, (e*(f + g*x))/(e*f - d*g)]))/(e^2*g^2*(-(e*f) + d*g)*Sq
rt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.22, size = 128, normalized size = 1.14

$$\frac{2\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)}{e^{3/2}(dg-ef)^{3/2}}+\frac{2\left(aeg^2-cdg(f+gx)+cef^2+cef(f+gx)\right)}{eg^2\sqrt{f+gx}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*e*f^2 + a*e*g^2 + c*e*f*(f + g*x) - c*d*g*(f + g*x)))/(e*g^2*(e*f - d
*g)*Sqrt[f + g*x]) + (2*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*Sqrt[-(e*f) + d*g]*
Sqrt[f + g*x])/(e*f - d*g)])/(e^(3/2)*(-(e*f) + d*g)^(3/2))
```

fricas [B] time = 0.42, size = 492, normalized size = 4.39

$$\left[-\frac{\left((cd^2+ae^2)g^3x+(cd^2+ae^2)fg^2\right)\sqrt{e^2f-deg}\log\left(\frac{egx+2ef-dg+2\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2ce^3f^3-3cde^2f^2g-ade^2g^3+(cd^2e+ae^3)fg^2+(ce^3f^2g-2cde^2fg^2+cd^2eg^3)x\right)\sqrt{gx+f}}{e^4f^3g^2-2de^3f^2g^3+d^2e^2fg^4+(e^4f^2g^3-2de^3fg^4+d^2e^2g^5)x}, \frac{2\left(\left((cd^2+ae^2)g^3x+(cd^2+ae^2)fg^2\right)\sqrt{-e^2f+deg}\arctan\left(\frac{\sqrt{-e^2f+deg}\sqrt{gx+f}}{egx+ef}\right)+\left(2ce^3f^3-3cde^2f^2g-ade^2g^3+(cd^2e+ae^3)fg^2+(ce^3f^2g-2cde^2fg^2+cd^2eg^3)x\right)\sqrt{gx+f}\right)}{e^4f^3g^2-2de^3f^2g^3+d^2e^2fg^4+(e^4f^2g^3-2de^3fg^4+d^2e^2g^5)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(e^2*f - d*e*g)*log(
(e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*
(2*c*e^3*f^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 + (c
*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2
- 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*
g^5)*x), 2*(((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(-e^2*f + d
*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*e^3*f
```

```
^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 + (c*e^3*f^2*g
 - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*
f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x)]
```

giac [A] time = 0.20, size = 101, normalized size = 0.90

$$-\frac{2\left(cd^2+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(dge-fe^2\right)^{\frac{3}{2}}}+\frac{2\sqrt{gx+f}\,ce^{(-1)}}{g^2}-\frac{2\left(cf^2+ag^2\right)}{\left(dg^3-fg^2e\right)\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/(d*g*e - f*e
^2)^(3/2) + 2*sqrt(g*x + f)*c*e^(-1)/g^2 - 2*(c*f^2 + a*g^2)/((d*g^3 - f*g^
2*e)*sqrt(g*x + f))
```

maple [A] time = 0.01, size = 165, normalized size = 1.47

$$-\frac{2ae\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}}-\frac{2c\,d^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e}-\frac{2a}{\left(dg-ef\right)\sqrt{gx+f}}-\frac{2c\,f^2}{\left(dg-ef\right)\sqrt{gx+f}\,g^2}+\frac{2\sqrt{gx+f}\,c}{e\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x)
```

```
[Out] 2*c*(g*x+f)^(1/2)/e/g^2-2/(d*g-e*f)*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1
/2)/((d*g-e*f)*e)^(1/2)*e)*a-2/(d*g-e*f)/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+
f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d^2-2/(d*g-e*f)/(g*x+f)^(1/2)*a-2/g^2/(d*
g-e*f)/(g*x+f)^(1/2)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.14, size = 141, normalized size = 1.26

$$\frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)\,\left(e^2\,f-d\,e\,g\right)}{\sqrt{e}\,\left(2\,c\,d^2+2\,a\,e^2\right)\,{\left(d\,g-e\,f\right)}^{3/2}}\right)\,\left(c\,d^2+a\,e^2\right)}{e^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}}+\frac{2\,c\,\sqrt{f+g\,x}}{e\,g^2}-\frac{2\,\left(c\,e\,f^2+a\,e\,g^2\right)}{e\,g^2\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

```
[Out] (2*atan((2*(f + g*x)^(1/2)*(a*e^2 + c*d^2)*(e^2*f - d*e*g))/(e^(1/2)*(2*a*e
^2 + 2*c*d^2)*(d*g - e*f)^(3/2)))*(a*e^2 + c*d^2))/(e^(3/2)*(d*g - e*f)^(3/
2)) + (2*c*(f + g*x)^(1/2))/(e*g^2) - (2*(a*e*g^2 + c*e*f^2))/(e*g^2*(f + g
*x)^(1/2)*(d*g - e*f))
```

sympy [A] time = 41.28, size = 104, normalized size = 0.93

$$\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2\left(ag^2+cf^2\right)}{g^2\sqrt{f+gx}\left(dg-ef\right)}-\frac{2\left(ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*sqrt(f + g*x)/(e*g**2) - 2*(a*g**2 + c*f**2)/(g**2*sqrt(f + g*x)*(d*g -
 e*f)) - 2*(a*e**2 + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e**2*
sqrt((d*g - e*f)/e)*(d*g - e*f))
```

3.403 $\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\sqrt{f+gx}\left(ae^2+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\left(3ae^2g+cd(4ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2+cf^2\right)}{g\sqrt{f+gx}\,(ef-dg)^2}$$

Rubi [A] time = 0.27, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {898, 1259, 453, 208}

$$-\frac{\sqrt{f+gx}\left(ae^2+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\left(3ae^2g+cd(4ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2+cf^2\right)}{g\sqrt{f+gx}\,(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*f^2 + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqr
t[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((3*a*e^2*g + c*d*(4*e*f - d*g))*
ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2
))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{g^3\,\text{Subst}\left(\int \frac{\frac{2e^2(ef-dg)\left(cf^2+ag^2\right)}{g^5}+\frac{e\left(ae^2g^2-c\left(2e^2f^2-4defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx, \ldots\right.}{e^2(ef-dg)^2}$$

$$= -\frac{2\left(cf^2+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{\left(3ae^2g+cd(4ef-dg)\right)\text{Subst}\left(\ldots\right.}{eg(ef-\ldots}$$

$$= -\frac{2\left(cf^2+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{\left(3ae^2g+cd(4ef-dg)\right)\tanh^{-1}\left(\ldots\right.}{e^{3/2}(ef-dg)^{5/2}}$$

Mathematica [C] time = 0.08, size = 118, normalized size = 0.82

$$-\frac{2\left(g^2\left(ae^2+cd^2\right){}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+2cdg(ef-dg)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2\right)}{e^2g\sqrt{f+gx}\,(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*(e*f - d*g)^2 + 2*c*d*g*(e*f - d*g)*Hypergeometric2F1[-1/2, 1, 1/2,
(e*(f + g*x))/(e*f - d*g)] + (c*d^2 + a*e^2)*g^2*Hypergeometric2F1[-1/2, 2,
 1/2, (e*(f + g*x))/(e*f - d*g)]))/(e^2*g*(e*f - d*g)^2*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.57, size = 210, normalized size = 1.46

$$\frac{2adeg^3+3ae^2g^2(f+gx)-2ae^2fg^2+cd^2g^2(f+gx)+2cdef^2g-2ce^2f^3+2ce^2f^2(f+gx)}{eg\sqrt{f+gx}\,(ef-dg)^2(-dg-e(f+gx)+ef)}+\frac{\left(3ae^2g-cd^2g+4cdef\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)}{e^{3/2}(ef-dg)^2\sqrt{dg-ef}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*c*e^2*f^3 + 2*c*d*e*f^2*g - 2*a*e^2*f*g^2 + 2*a*d*e*g^3 + 2*c*e^2*f^2*(
f + g*x) + c*d^2*g^2*(f + g*x) + 3*a*e^2*g^2*(f + g*x))/(e*g*(e*f - d*g)^2*
Sqrt[f + g*x]*(e*f - d*g - e*(f + g*x))) + ((4*c*d*e*f - c*d^2*g + 3*a*e^2*
g)*ArcTan[(Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[f + g*x])/(e*f - d*g)])/(e^(3/2)
*(e*f - d*g)^2*Sqrt[-(e*f) + d*g])
```

fricas [B] time = 0.43, size = 906, normalized size = 6.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e^2*f*g^2 - (c*
d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e + a*e^3)*f*g^2 -
```

```
(c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g +
2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3 - 2*a*d^
2*e^2*g^3 - (c*d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2*c*e^
4*f^3 - 2*c*d*e^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*a*d*e^
3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*
g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^
3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g
^5)*x), -((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e^2*f*g^2 -
 (c*d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e + a*e^3)*f*g^
2 - (c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d
*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*e^2*g^3 - (c*
d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2*c*e^4*f^3 - 2*c*d*e
^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*a*d*e^3)*g^3)*x)*sqrt
(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*f
*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2
+ (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x)]
```

giac [A] time = 0.19, size = 225, normalized size = 1.56

$$\frac{\left(cd^2g - 4\,cdfe - 3\,age^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(d^2g^2e - 2\,dfge^2 + f^2e^3\right)\sqrt{dge - fe^2}} - \frac{\left(gx+f\right)cd^2g^2 + 2\,cdf^2ge + 2\,adg^3e + 2\left(gx+f\right)cf^2e^2 - 2\,cf^3e^2 + 3\left(gx+f\right)ag^2e^2 - 2\,afg^2e^2}{\left(d^2g^3e - 2\,dfg^2e^2 + f^2ge^3\right)\left(\sqrt{gx+f}\,dg + \left(gx+f\right)^{\frac{3}{2}}e - \sqrt{gx+f}\,fe\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] (c*d^2*g - 4*c*d*f*e - 3*a*g*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2
))/((d^2*g^2*e - 2*d*f*g*e^2 + f^2*e^3)*sqrt(d*g*e - f*e^2)) - ((g*x + f)*c
*d^2*g^2 + 2*c*d*f^2*g*e + 2*a*d*g^3*e + 2*(g*x + f)*c*f^2*e^2 - 2*c*f^3*e^
2 + 3*(g*x + f)*a*g^2*e^2 - 2*a*f*g^2*e^2)/((d^2*g^3*e - 2*d*f*g^2*e^2 + f^
2*g*e^3)*(sqrt(g*x + f)*d*g + (g*x + f)^(3/2)*e - sqrt(g*x + f)*f*e))
```

maple [B] time = 0.02, size = 269, normalized size = 1.87

$$-\frac{3aeg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^2\sqrt{\left(dg-ef\right)e}} + \frac{c\,d^2g\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^2\sqrt{\left(dg-ef\right)e}\,e} - \frac{4cdf\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^2\sqrt{\left(dg-ef\right)e}} - \frac{\sqrt{gx+f}\,aeg}{\left(dg-ef\right)^2\left(egx+dg\right)} - \frac{\sqrt{gx+f}\,c\,d^2g}{\left(dg-ef\right)^2\left(egx+dg\right)e} - \frac{2ag}{\left(dg-ef\right)^2\sqrt{gx+f}} - \frac{2c\,f^2}{\left(dg-ef\right)^2\sqrt{gx+f}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x)
```

```
[Out] -g/(d*g-e*f)^2*e*(g*x+f)^(1/2)/(e*g*x+d*g)*a-g/(d*g-e*f)^2/e*(g*x+f)^(1/2)/
(e*g*x+d*g)*c*d^2-3*g/(d*g-e*f)^2*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2
)/((d*g-e*f)*e)^(1/2)*e)*a+g/(d*g-e*f)^2/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+
f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d^2-4/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)*arc
tan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d*f-2*g/(d*g-e*f)^2/(g*x+f)^(1/2
)*a-2/g/(d*g-e*f)^2/(g*x+f)^(1/2)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 3.29, size = 187, normalized size = 1.30

$$-\frac{\frac{2\left(cf^2+ag^2\right)}{dg-ef}+\frac{\left(f+gx\right)\left(cd^2g^2+2ce^2f^2+3ae^2g^2\right)}{e\left(dg-ef\right)^2}}{\sqrt{f+gx}\,\left(dg^2-efg\right)+eg\left(f+gx\right)^{3/2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{f+gx}\,\left(d^2eg^2-2de^2fg+e^3f^2\right)}{\sqrt{e}\,\left(dg-ef\right)^{5/2}}\right)\left(-cgd^2+4cfde+3age^2\right)}{e^{3/2}\left(dg-ef\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

```
[Out] - ((2*(a*g^2 + c*f^2))/(d*g - e*f) + ((f + g*x)*(3*a*e^2*g^2 + c*d^2*g^2 +
2*c*e^2*f^2))/(e*(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(f
+ g*x)^(3/2)) - (atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*g))
/(e^(1/2)*(d*g - e*f)^(5/2)))*(3*a*e^2*g - c*d^2*g + 4*c*d*e*f))/(e^(3/2)*(
d*g - e*f)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

3.404 $\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{\sqrt{f+gx}\left(ae^2+cd^2\right)}{2e(d+ex)^2(ef-dg)^2}-\frac{\left(15ae^2g^2+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}+\frac{\sqrt{f+gx}\left(7ae^2g+cd(8ef-\right.}{4e(d+ex)(ef-dg)^3}$$

Rubi [A] time = 0.50, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {898, 1259, 456, 453, 208}

$$-\frac{\left(15ae^2g^2+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}-\frac{\sqrt{f+gx}\left(ae^2+cd^2\right)}{2e(d+ex)^2(ef-dg)^2}+\frac{\sqrt{f+gx}\left(7ae^2g+cd(8ef-dg)\right)}{4e(d+ex)(ef-dg)^3}+\frac{2\left(ag^2+cf^2\right)}{\sqrt{f+gx}(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqrt[f
 + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((7*a*e^2*g + c*d*(8*e*f - d*g))
*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d + e*x)) - ((15*a*e^2*g^2 + c*(8*e^2*f
^2 + 8*d*e*f*g - d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(4*e^(3/2)*(e*f - d*g)^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} - \frac{g^3\,\text{Subst}\left(\int \frac{\frac{4e^2(ef-dg)\left(cf^2+ag^2\right)}{g^5}+\frac{e\left(3ae^2g^2-c\left(4e^2f^2-8defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx \ldots\right.}{2e^2(ef-dg)^2}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{\left(7ae^2g+cd(8ef-dg)\right)\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} + \frac{g^3\,\text{Subst}\left(\int \frac{\frac{8e^2(cf^2+\ldots}{g^4}\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{2\left(cf^2+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{\left(7ae^2g+cd(8ef-dg)\right)\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} \ldots$$

$$= \frac{2\left(cf^2+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{\left(7ae^2g+cd(8ef-dg)\right)\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} \ldots$$

Mathematica [C] time = 0.10, size = 140, normalized size = 0.65

$$\frac{2\left(g\left(g\left(ae^2+cd^2\right){}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+2cd(ef-dg)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)\right)+c(ef-dg)^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)\right)}{e^2\sqrt{f+gx}\,(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*(e*f - d*g)^2*Hypergeometric2F1[-1/2, 1, 1/2, (e*(f + g*x))/(e*f - d*
g)] + g*(2*c*d*(e*f - d*g)*Hypergeometric2F1[-1/2, 2, 1/2, (e*(f + g*x))/(e
*f - d*g)] + (c*d^2 + a*e^2)*g*Hypergeometric2F1[-1/2, 3, 1/2, (e*(f + g*x)
)/(e*f - d*g)])))/(e^2*(e*f - d*g)^3*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 1.06, size = 369, normalized size = 1.72

$$\frac{\left(-15ae^2g^2+cd^2g^2-8cdefg-8ce^2f^2\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)}{4e^{3/2}(ef-dg)^3\sqrt{dg-ef}}+\frac{8ad^2eg^4+25ade^2g^3(f+gx)-16ade^2fg^3+8ae^3f^2g^2-25ae^3fg^2(f+gx)+15ae^3g^2(f+gx)^2+cd^3g^3(f+gx)+8cd^2ef^2g^2+7cd^2efg^2(f+gx)-cd^2eg^2(f+gx)^2-16cde^2f^3g+8cde^2f^2g(f+gx)+8cde^2fg(f+gx)^2+8ce^3f^4-16ce^3f^3(f+gx)+8ce^3f^2(f+gx)^2}{4e\sqrt{f+gx}\,(ef-dg)^3(-dg-e(f+gx)+ef)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (8*c*e^3*f^4 - 16*c*d*e^2*f^3*g + 8*c*d^2*e*f^2*g^2 + 8*a*e^3*f^2*g^2 - 16*
a*d*e^2*f*g^3 + 8*a*d^2*e*g^4 - 16*c*e^3*f^3*(f + g*x) + 8*c*d*e^2*f^2*g*(f
```

```
 + g*x) + 7*c*d^2*e*f*g^2*(f + g*x) - 25*a*e^3*f*g^2*(f + g*x) + c*d^3*g^3*
(f + g*x) + 25*a*d*e^2*g^3*(f + g*x) + 8*c*e^3*f^2*(f + g*x)^2 + 8*c*d*e^2*
f*g*(f + g*x)^2 - c*d^2*e*g^2*(f + g*x)^2 + 15*a*e^3*g^2*(f + g*x)^2)/(4*e*
(e*f - d*g)^3*Sqrt[f + g*x]*(e*f - d*g - e*(f + g*x))^2) + ((-8*c*e^2*f^2 -
 8*c*d*e*f*g + c*d^2*g^2 - 15*a*e^2*g^2)*ArcTan[(Sqrt[e]*Sqrt[-(e*f) + d*g]
*Sqrt[f + g*x])/(e*f - d*g)])/(4*e^(3/2)*(e*f - d*g)^3*Sqrt[-(e*f) + d*g])
```

fricas [B] time = 0.45, size = 1539, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((8*c*d^2*e^2*f^3 + 8*c*d^3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 +
(8*c*e^4*f^2*g + 8*c*d*e^3*f*g^2 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c*e
^4*f^3 + 24*c*d*e^3*f^2*g + 15*(c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e - 15*
a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e + 5*a
*d*e^3)*f*g^2 - (c*d^4 - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g
*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(8*a
*d^3*e^2*g^3 - 2*(7*c*d^2*e^3 - a*e^5)*f^3 + (13*c*d^3*e^2 - 11*a*d*e^4)*f^
2*g + (c*d^4*e + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 - 3*(3*c*d^2*e^3 - 5*a*e^5
)*f*g^2 + (c*d^3*e^2 - 15*a*d*e^4)*g^3)*x^2 - (24*c*d*e^4*f^3 - (19*c*d^2*e
^3 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 5*a*d*e^4)*f*g^2 - (c*d^4*e + 25*a*d^2
*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3
*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6
*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*
f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3
*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*
f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x), 1/4*((8*c*d^2*e^2*f^3 + 8*c*d^
3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 8*c*d*e^3*f*g^2
 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 24*c*d*e^3*f^2*g + 15*(
c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*
f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e + 5*a*d*e^3)*f*g^2 - (c*d^4 - 15*a*d^
2*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x +
f)/(e*g*x + e*f)) - (8*a*d^3*e^2*g^3 - 2*(7*c*d^2*e^3 - a*e^5)*f^3 + (13*c*
d^3*e^2 - 11*a*d*e^4)*f^2*g + (c*d^4*e + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 -
3*(3*c*d^2*e^3 - 5*a*e^5)*f*g^2 + (c*d^3*e^2 - 15*a*d*e^4)*g^3)*x^2 - (24*c
*d*e^4*f^3 - (19*c*d^2*e^3 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 5*a*d*e^4)*f*g
^2 - (c*d^4*e + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e
^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4
*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x
^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d
^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*
e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x)]
```

giac [A] time = 0.22, size = 361, normalized size = 1.69

$$\frac{\left(cd^2g^2 - 8\,cdfge - 8\,cf^2e^2 - 15\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{4\left(d^3g^3e - 3\,d^2fg^2e^2 + 3\,df^2ge^3 - f^3e^4\right)\sqrt{dge - fe^2}} - \frac{2\left(cf^2 + ag^2\right)}{\left(d^3g^3 - 3\,d^2fg^2e + 3\,df^2ge^2 - f^3e^3\right)\sqrt{gx + f}} - \frac{\sqrt{gx + f}\,cd^3g^3 - \left(gx + f\right)^{\frac{3}{2}}cd^2g^2e + 7\,\sqrt{gx + f}\,cd^2fg^2e + 8\left(gx + f\right)^{\frac{3}{2}}cdfge^2 - 8\,\sqrt{gx + f}\,cdf^2ge^2 + 9\,\sqrt{gx + f}\,adg^3e^2 + 7\left(gx + f\right)^{\frac{3}{2}}ag^2e^3 - 9\,\sqrt{gx + f}\,afg^2e^3}{4\left(d^3g^3e - 3\,d^2fg^2e^2 + 3\,df^2ge^3 - f^3e^4\right)\left(dg + \left(gx + f\right)e - fe\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(c*d^2*g^2 - 8*c*d*f*g*e - 8*c*f^2*e^2 - 15*a*g^2*e^2)*arctan(sqrt(g*x
+ f)*e/sqrt(d*g*e - f*e^2))/((d^3*g^3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^3 -
 f^3*e^4)*sqrt(d*g*e - f*e^2)) - 2*(c*f^2 + a*g^2)/((d^3*g^3 - 3*d^2*f*g^2*
e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(g*x + f)) - 1/4*(sqrt(g*x + f)*c*d^3*g^3
- (g*x + f)^(3/2)*c*d^2*g^2*e + 7*sqrt(g*x + f)*c*d^2*f*g^2*e + 8*(g*x + f)
^(3/2)*c*d*f*g*e^2 - 8*sqrt(g*x + f)*c*d*f^2*g*e^2 + 9*sqrt(g*x + f)*a*d*g^
3*e^2 + 7*(g*x + f)^(3/2)*a*g^2*e^3 - 9*sqrt(g*x + f)*a*f*g^2*e^3)/((d^3*g^
```

```
3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^3 - f^3*e^4)*(d*g + (g*x + f)*e - f*e)^
2)
```

maple [B] time = 0.02, size = 546, normalized size = 2.55

$$-\frac{9\sqrt{gx+f}\,ade\,g^{3}}{4\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}+\frac{9\sqrt{gx+f}\,a\,e^{2}f\,g^{2}}{4\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}-\frac{\sqrt{gx+f}\,c\,d^{3}g^{3}}{4\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}e}-\frac{7\sqrt{gx+f}\,c\,d^{2}f\,g^{2}}{4\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}+\frac{2\sqrt{gx+f}\,cde\,f^{2}g}{\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}-\frac{7\left(gx+f\right)^{\frac{3}{2}}a\,e^{2}g^{2}}{4\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}-\frac{15ae\,g^{2}\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{4\left(dg-ef\right)^{3}\sqrt{\left(dg-ef\right)e}}+\frac{c\,d^{2}g^{2}\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{4\left(dg-ef\right)^{3}\sqrt{\left(dg-ef\right)e}\,e}+\frac{\left(gx+f\right)^{\frac{3}{2}}c\,d^{2}g^{2}}{4\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}-\frac{2\left(gx+f\right)^{\frac{3}{2}}cdefg}{\left(dg-ef\right)^{3}\left(egx+dg\right)^{2}}-\frac{2cdfg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^{3}\sqrt{\left(dg-ef\right)e}}-\frac{2ce\,f^{2}\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^{3}\sqrt{\left(dg-ef\right)e}}-\frac{2a\,g^{2}}{\left(dg-ef\right)^{3}\sqrt{gx+f}}-\frac{2c\,f^{2}}{\left(dg-ef\right)^{3}\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x)
```

```
[Out] -7/4/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*a*e^2*g^2+1/4/(d*g-e*f)^3/(e*g
*x+d*g)^2*(g*x+f)^(3/2)*c*d^2*g^2-2/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)
*c*d*e*f*g-9/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3*e*(g*x+f)^(1/2)*a*d+9/4/(d*g-e
*f)^3/(e*g*x+d*g)^2*g^2*e^2*(g*x+f)^(1/2)*a*f-1/4/(d*g-e*f)^3/(e*g*x+d*g)^2
*g^3/e*(g*x+f)^(1/2)*c*d^3-7/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^2*(g*x+f)^(1/2)*
f*c*d^2+2/(d*g-e*f)^3/(e*g*x+d*g)^2*g*e*(g*x+f)^(1/2)*c*d*f^2-15/4/(d*g-e*f
)^3*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*a*g^2
+1/4/(d*g-e*f)^3/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(
1/2)*e)*c*d^2*g^2-2/(d*g-e*f)^3/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((
d*g-e*f)*e)^(1/2)*e)*c*d*f*g-2/(d*g-e*f)^3*e/((d*g-e*f)*e)^(1/2)*arctan((g*
x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*f^2-2/(d*g-e*f)^3/(g*x+f)^(1/2)*a*g^2-2
/(d*g-e*f)^3/(g*x+f)^(1/2)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 3.37, size = 310, normalized size = 1.45

$$\frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+15\,a\,e^2\,g^2\right)}{4\,e^{3/2}\,{\left(d\,g-e\,f\right)}^{7/2}}-\frac{\frac{2\,\left(c\,f^2+a\,g^2\right)}{d\,g-e\,f}+\frac{{\left(f+g\,x\right)}^2\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+15\,a\,e^2\,g^2\right)}{4\,{\left(d\,g-e\,f\right)}^3}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2+8\,c\,d\,e\,f\,g+16\,c\,e^2\,f^2+25\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^{5/2}-{\left(f+g\,x\right)}^{3/2}\,\left(2\,e^2\,f-2\,d\,e\,g\right)+\sqrt{f+g\,x}\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^2
*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 +
 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)) - ((2*(a*g^2 + c*f^2))/(d*g -
e*f) + ((f + g*x)^2*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 + 8*c*d*e*f*g))
/(4*(d*g - e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f^2 +
8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x)^(5/2) - (f + g*x)^(3/2)*(
2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

3.405 $\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=147

$$\frac{\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}-\frac{c\sqrt{d+ex}\sqrt{f+gx}\,(5dg+3ef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {952, 80, 63, 217, 206}

$$\frac{\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}-\frac{c\sqrt{d+ex}\sqrt{f+gx}\,(5dg+3ef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -(c*(3*e*f + 5*d*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) + (c*(d + e*x)
^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g
+ 3*d^2*g^2))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(4*
e^(5/2)*g^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
```

```
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rubi steps

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}ce(3ef+5dg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{1}{8}\left(8a + \frac{c\left(3e^2f^2+2def\right.}{e^2g^2}\right.$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(8a + \frac{c\left(3e^2f^2+2defg+3d^2g^2\right.}{e^2g^2}\right.}{}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(8a + \frac{c\left(3e^2f^2+2defg+3d^2g^2\right.}{e^2g^2}\right.}{}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(8ae^2g^2 + c\left(3e^2f^2 + 2d\right.\right.}{}$$

Mathematica [A] time = 0.56, size = 155, normalized size = 1.05

$$\frac{\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)+ce\sqrt{g}\sqrt{d+ex}(f+gx)(-3dg-3ef+2egx)}{4e^3g^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (c*e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(-3*e*f - 3*d*g + 2*e*g*x) + Sqrt[e*f
- d*g]*(8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*Sqrt[(e*(f + g
*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(4*e^3*
g^(5/2)*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.40, size = 216, normalized size = 1.47

$$\frac{\left(8ae^2g^2+3cd^2g^2+2cdefg+3ce^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}} - \frac{c\sqrt{f+gx}\left(-\frac{5d^2eg^2(f+gx)}{d+ex}+3d^2g^3+\frac{3e^3f^2(f+gx)}{d+ex}+\frac{2de^2fg(f+gx)}{d+ex}+2defg^2-5e^2f^2g\right)}{4e^2g^2\sqrt{d+ex}\left(\frac{e(f+gx)}{d+ex}-g\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -1/4*(c*Sqrt[f + g*x]*(-5*e^2*f^2*g + 2*d*e*f*g^2 + 3*d^2*g^3 + (3*e^3*f^2*
(f + g*x))/(d + e*x) + (2*d*e^2*f*g*(f + g*x))/(d + e*x) - (5*d^2*e*g^2*(f
+ g*x))/(d + e*x)))/(e^2*g^2*Sqrt[d + e*x]*(-g + (e*(f + g*x))/(d + e*x))^2
) + ((3*c*e^2*f^2 + 2*c*d*e*f*g + 3*c*d^2*g^2 + 8*a*e^2*g^2)*ArcTanh[(Sqrt[
e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^(5/2)*g^(5/2))
```

fricas [A] time = 0.47, size = 336, normalized size = 2.29

$$\left[\frac{\left(3ce^2f^2+2cdefg+\left(3cd^2+8ae^2\right)g^2\right)\sqrt{eg}\log\left(8e^2g^2x^2+e^2f^2+6defg+d^2g^2+4\left(2egx+ef+dg\right)\sqrt{eg}\sqrt{ex+d}\sqrt{gx+f}+8\left(e^2fg+deg^2\right)x\right)+4\left(2ce^2g^2x-3ce^2fg-3cdeg^2\right)\sqrt{ex+d}\sqrt{gx+f}}{16e^3g^3}, -\frac{\left(3ce^2f^2+2cdefg+\left(3cd^2+8ae^2\right)g^2\right)\sqrt{-eg}\arctan\left(\frac{\left(2egx+ef+dg\right)\sqrt{-eg}\sqrt{ex+d}\sqrt{gx+f}}{2\left(e^2g^2x^2+defg+\left(e^2fg+deg^2\right)x\right)}\right)-2\left(2ce^2g^2x-3ce^2fg-3cdeg^2\right)\sqrt{ex+d}\sqrt{gx+f}}{8e^3g^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(e*g)*log(
8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqr
t(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(2*c*e^2*
g^2*x - 3*c*e^2*f*g - 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3),
-1/8*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(-e*g)*arct
an(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^
2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g
- 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

giac [A] time = 0.27, size = 155, normalized size = 1.05

$$\frac{1}{4}\sqrt{(xe+d)ge-dge+fe^2}\,\sqrt{xe+d}\left(\frac{2\,(xe+d)ce^{(-3)}}{g}-\frac{\left(5\,cdg^2e^5+3\,cfge^6\right)e^{(-8)}}{g^3}\right)-\frac{\left(3\,cd^2g^2+2\,cdfge+3\,cf^2e^2+8\,ag^2e^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-\sqrt{xe+d}\,\sqrt{g}\,e^{\frac{1}{2}}+\sqrt{(xe+d)ge-dge+fe^2}\right|\right)}{4\,g^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*(2*(x*e + d)*c*e^(-3)
/g - (5*c*d*g^2*e^5 + 3*c*f*g*e^6)*e^(-8)/g^3) - 1/4*(3*c*d^2*g^2 + 2*c*d*f
*g*e + 3*c*f^2*e^2 + 8*a*g^2*e^2)*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e
^(1/2) + sqrt((x*e + d)*g*e - d*g*e + f*e^2)))/g^(5/2)
```

maple [B] time = 0.04, size = 306, normalized size = 2.08

$$\frac{\left(8a\,e^2g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+3c\,d^2g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+2cdefg\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+3c\,e^2f^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+4\sqrt{eg}\,\sqrt{(ex+d)\left(gx+f\right)}\,cegx-6\sqrt{(ex+d)\left(gx+f\right)}\,\sqrt{eg}\,cdg-6\sqrt{(ex+d)\left(gx+f\right)}\,\sqrt{eg}\,cef\right)\sqrt{ex+d}\,\sqrt{gx+f}}{8\sqrt{eg}\,\sqrt{(ex+d)\left(gx+f\right)}\,e^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(8*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)
^(1/2))*a*e^2*g^2+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*
(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^2*f^2+2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g
*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*g+4*(e*g)^(1/2)*((e*
x+d)*(g*x+f))^(1/2)*x*c*e*g-6*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*d*g-6*(
(e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)
^(1/2)/g^2/e^2/((e*x+d)*(g*x+f))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [B] time = 20.13, size = 569, normalized size = 3.87

$$\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,e^{5/2}\,g^{5/2}}-\frac{4\,a\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}{\sqrt{-e\,g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{-e\,g}}-\frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}-\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{g^5\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3}+\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7\left(\frac{3\,c\,d^2\,g^2}{2}+c\,d\,e\,f\,g+\frac{3\,c\,e^2\,f^2}{2}\right)}{e^2\,g^3\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^7}-\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{e\,g^4\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^5}+\frac{\sqrt{d}\,\sqrt{f}\,\left(32\,c\,d\,g+32\,c\,e\,f\right)\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{g^4\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^4}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4\,e\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{g\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^6}-\frac{4\,e^3\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{g^3\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^2}+\frac{6\,e^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{g^2\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) -
f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*g))/(2*e^(5/2)*g^(5/2)) - (4*a*
atan((e*((f + g*x)^(1/2) - f^(1/2)))/((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/
2)))))/(-e*g)^(1/2) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*e^3*f^2)/2 + (3*c
*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1/2))) - (((d + e*
x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))
/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) - d^(1/2))^7*((3*c
*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f + g*x)^(1/2) - f^(
1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2
)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5) + (d^(1/2)*f^(1/
2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/
2) - f^(1/2))^4))/((((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x)^(1/2) - f^(1/2)
)^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f + g*x)^(1/2) - f
^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((f + g*x)^(1/2) -
f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*((f + g*x)^(1/2) -
 f^(1/2))^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + cx^2}{\sqrt{d + ex}\sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.406 $\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx$

Optimal. Leaf size=16

$$\sqrt{x-1}\,x\sqrt{x+1}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {384}

$$\sqrt{x-1}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[-1 + x]*x*Sqrt[1 + x]
```

Rule 384

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2), x] /; FreeQ[{a1, b1, a2, b2, c, d, n
, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d - b1*b2*
c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{-1+x}\,x\sqrt{1+x}$$

Mathematica [C] time = 0.09, size = 66, normalized size = 4.12

$$\frac{\sqrt{x-1}\left(x\sqrt{1-x^2}-2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)}{\sqrt{1-x}}+2\tanh^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[-1 + x]*(x*Sqrt[1 - x^2] - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/Sqrt[1 - x
] + 2*ArcTanh[Sqrt[(-1 + x)/(1 + x)]]
```

IntegrateAlgebraic [B] time = 0.05, size = 46, normalized size = 2.88

$$\frac{2\left(\frac{(x-1)^{3/2}}{(x+1)^{3/2}}+\frac{\sqrt{x-1}}{\sqrt{x+1}}\right)}{\left(\frac{x-1}{x+1}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

```
[Out] (2*((-1 + x)^(3/2)/(1 + x)^(3/2) + Sqrt[-1 + x]/Sqrt[1 + x]))/(-1 + (-1 + x
)/(1 + x))^2
```

fricas [A] time = 0.39, size = 12, normalized size = 0.75

$$\sqrt{x+1}\,\sqrt{x-1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(x - 1)*x
```

giac [A] time = 0.17, size = 12, normalized size = 0.75

$$\sqrt{x+1}\,\sqrt{x-1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*sqrt(x - 1)*x
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\sqrt{x-1}\,\sqrt{x+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-1)/(x-1)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] x*(x-1)^(1/2)*(x+1)^(1/2)
```

maxima [C] time = 0.43, size = 9, normalized size = 0.56

$$\sqrt{x^2-1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*x
```

mupad [B] time = 2.80, size = 16, normalized size = 1.00

$$\frac{\left(x^2+x\right)\,\sqrt{x-1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 1)/((x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] ((x + x^2)*(x - 1)^(1/2))/(x + 1)^(1/2)
```

sympy [C] time = 43.48, size = 129, normalized size = 8.06

$$-\begin{cases} 2\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) & \text{for}\ \frac{|x+1|}{2}>1 \\ -2i\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) & \text{otherwise} \end{cases} + \frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1 \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\,\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}} - \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0 \end{matrix}\middle|\,\frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-1)/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] -Piecewise((2*acosh(sqrt(2)*sqrt(x + 1)/2), Abs(x + 1)/2 > 1), (-2*I*asin(s
qrt(2)*sqrt(x + 1)/2), True)) + meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)),
 ((-1, -3/4, -1/2, -1/4, 0, 0), ()), x**(-2))/(2*pi**(3/2)) - I*meijerg(((-
3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_
polar(2*I*pi)/x**2)/(2*pi**(3/2))
```

3.407 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal. Leaf size=411

$$\frac{\left(\frac{a\left(ae^2g-cd(dg+2ef)\right)}{\sqrt{c}}-\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{ac\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}+\frac{\left(\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)+\frac{a(ae\cdots}{}\right.}{ac\sqrt{\sqrt{-a}e+\cdots}}$$

Rubi [A] time = 2.51, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {904, 80, 63, 217, 206, 6725, 93, 208}

$$\frac{\left(\frac{a\left(ae^2g-cd(dg+2ef)\right)}{\sqrt{c}}-\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{ac\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}+\frac{\left(\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)+\frac{a\left(ae^2g-cd(dg+2ef)\right)}{\sqrt{c}}\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{ac\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}+\frac{e\sqrt{d+ex}\sqrt{f+gx}}{c}+\frac{\sqrt{e}(3dg+ef)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (e*Sqrt[d + e*x]*Sqrt[f + g*x])/c + (Sqrt[e]*(e*f + 3*d*g)*ArcTanh[(Sqrt[g]
*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqrt[g]) + (((a*(a*e^2*g - c*d
*(2*e*f + d*g)))/Sqrt[c] - Sqrt[-a]*(c*d^2*f - a*e*(e*f + 2*d*g)))*ArcTanh[
(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*
Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a
]*g]) + (((a*(a*e^2*g - c*d*(2*e*f + d*g)))/Sqrt[c] + Sqrt[-a]*(c*d^2*f - a
*e*(e*f + 2*d*g)))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*
e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 904

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[g/c, Int[Simp[2*e*f + d*g + e*g*x, x]*(d + e*x)^(m
- 1)*(f + g*x)^(n - 2), x], x] + Dist[1/c, Int[(Simp[c*d*f^2 - 2*a*e*f*g -
a*d*g^2 + (c*e*f^2 + 2*c*d*f*g - a*e*g^2)*x, x]*(d + e*x)^(m - 1)*(f + g*x)
^(n - 2))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2
+ a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\int \frac{cd^2f-ae(ef+2dg)-\left(ae^2g-cd(2ef+dg)\right)x}{\sqrt{d+ex}\,\sqrt{f+gx}\,\left(a+cx^2\right)}\,dx}{c}+\frac{e\int \frac{ef+2dg+egx}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{c}$$

$$=\frac{e\sqrt{d+ex}\,\sqrt{f+gx}}{c}+\frac{\int \left(\frac{-\frac{a\left(-ae^2g+cd(2ef+dg)\right)}{\sqrt{c}}+\sqrt{-a}\left(cd^2f-ae(ef+2dg)\right)}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\frac{a\left(-ae^2g+cd(2ef+dg)\right)}{\sqrt{c}}+\sqrt{\ldots}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\ldots}\ldots\right.}{c}$$

$$=\frac{e\sqrt{d+ex}\,\sqrt{f+gx}}{c}+\frac{(ef+3dg)\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx,x,\sqrt{d+ex}\right)}{c}+\frac{\left(\frac{a\left(ae^2g-c\ldots\right.}{\ldots}\right.}{\ldots}$$

$$=\frac{e\sqrt{d+ex}\,\sqrt{f+gx}}{c}+\frac{(ef+3dg)\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c}+\frac{\left(\frac{a\left(ae^2g-cd(2ef+dg)\right)}{\sqrt{c}}\ldots\right.}{\ldots}$$

$$=\frac{e\sqrt{d+ex}\,\sqrt{f+gx}}{c}+\frac{\sqrt{e}\,(ef+3dg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}}+\frac{\left(\frac{a\left(ae^2g-cd(2ef+dg)\right)}{\sqrt{c}}-\sqrt{-\ldots}\right.}{ac\ldots}$$

Mathematica [A] time = 2.44, size = 410, normalized size = 1.00

$$\frac{-\frac{\left(\sqrt{-a}\,cd^2+2a\sqrt{c}\,de+(-a)^{3/2}e^2\right)\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{a\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}+\frac{\left(\sqrt{-a}\,cd^2-2a\sqrt{c}\,de+(-a)^{3/2}e^2\right)\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{a\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}+\sqrt{c}\,e\sqrt{d+ex}\,\sqrt{f+gx}+\frac{\sqrt{c}\,\sqrt{ef-dg}\,(3dg+ef)\sqrt{\frac{e(f+gx)}{ef-dg}}\,\sinh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{\sqrt{g}\,\sqrt{f+gx}}}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[f + g*x] + (Sqrt[c]*Sqrt[e*f - d*g]*(e*f + 3*
d*g)*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e
*f - d*g]])/(Sqrt[g]*Sqrt[f + g*x]) - ((Sqrt[-a]*c*d^2 + 2*a*Sqrt[c]*d*e +
(-a)^(3/2)*e^2)*Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f)
+ Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x]
)])/(a*Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]) + ((Sqrt[-a]*c*d^2 - 2*a*Sqrt[c]*d*
e + (-a)^(3/2)*e^2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f +
Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(
a*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]))/c^(3/2)
```

IntegrateAlgebraic [C] time = 1.44, size = 580, normalized size = 1.41

$$\frac{\left(-icdf\sqrt{ae^2+cd^2}+\sqrt{a}\sqrt{c}ef\sqrt{ae^2+cd^2}+\sqrt{a}\sqrt{c}dg\sqrt{ae^2+cd^2}+iaeg\sqrt{ae^2+cd^2}\right)\tan^{-1}\left(\frac{\sqrt{f+gx}\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\sqrt{-i\sqrt{a}\sqrt{c}dg+i\sqrt{a}\sqrt{c}ef-aeg-cdf}}\right)}{\sqrt{a}c^{3/2}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}}+\frac{\left(icdf\sqrt{ae^2+cd^2}+\sqrt{a}\sqrt{c}ef\sqrt{ae^2+cd^2}+\sqrt{a}\sqrt{c}dg\sqrt{ae^2+cd^2}-iaeg\sqrt{ae^2+cd^2}\right)\tan^{-1}\left(\frac{\sqrt{f+gx}\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\sqrt{i\sqrt{a}\sqrt{c}dg-i\sqrt{a}\sqrt{c}ef-aeg-cdf}}\right)}{\sqrt{a}c^{3/2}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}}+\frac{\left(3d\sqrt{e}g+e^{3/2}f\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{c\sqrt{g}}-\frac{e\sqrt{f+gx}(ef-dg)}{c\sqrt{d+ex}\left(g-\frac{e(f+gx)}{d+ex}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] -((e*(e*f - d*g)*Sqrt[f + g*x])/(c*Sqrt[d + e*x]*(g - (e*(f + g*x))/(d + e*
x)))) + (((-I)*c*d*Sqrt[c*d^2 + a*e^2]*f + Sqrt[a]*Sqrt[c]*e*Sqrt[c*d^2 + a
*e^2]*f + Sqrt[a]*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*g + I*a*e*Sqrt[c*d^2 + a*e^
2]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-(c*d*f) + I*Sqrt[a]
*Sqrt[c]*e*f - I*Sqrt[a]*Sqrt[c]*d*g - a*e*g]*Sqrt[d + e*x])])/(Sqrt[a]*c^(
3/2)*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + ((I*c*
d*Sqrt[c*d^2 + a*e^2]*f + Sqrt[a]*Sqrt[c]*e*Sqrt[c*d^2 + a*e^2]*f + Sqrt[a]
*Sqrt[c]*d*Sqrt[c*d^2 + a*e^2]*g - I*a*e*Sqrt[c*d^2 + a*e^2]*g)*ArcTan[(Sqr
t[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-(c*d*f) - I*Sqrt[a]*Sqrt[c]*e*f + I*
Sqrt[a]*Sqrt[c]*d*g - a*e*g]*Sqrt[d + e*x])])/(Sqrt[a]*c^(3/2)*Sqrt[-((Sqrt
[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]) + ((e^(3/2)*f + 3*d*Sqrt[
e]*g)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(c*Sqrt[g])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.11, size = 2497, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(3*(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c
*d*f)/c)^(1/2)*ln(1/2*(2*e*g*x+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(e*g)^(1/2
)+d*g+e*f)/(e*g)^(1/2))*c*d*e*g+(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*
```

```
f)/c)^(1/2)*ln(1/2*(2*e*g*x+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*c*e^2*f+(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*
e*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*
c)^(1/2)))*a*e^2*g-(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-
c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))
*c*d^2*g-2*(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c
)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+(-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d*e*f
+(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*
g)^(1/2)*ln((x*c*d*g+x*c*e*f-2*(-a*c)^(1/2)*x*e*g+2*c*d*f+2*(e*g*x^2+d*g*x+
e*f*x+d*f)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*e^2*g-(-a*c)^(1
/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)*l
n((x*c*d*g+x*c*e*f-2*(-a*c)^(1/2)*x*e*g+2*c*d*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)
^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^
(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*d^2*g-2*(-a*c)^(1/2)*(((-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((x*c*d
*g+x*c*e*f-2*(-a*c)^(1/2)*x*e*g+2*c*d*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(
-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*
g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*d*e*f+2*a*c*(-((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+
x*c*d*g+x*c*e*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*
f)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*
f)/(c*x-(-a*c)^(1/2)))*d*e*g+a*c*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g
-c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/
2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2))
)*e^2*f-(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1
/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a
*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*d^2*f-2*a*c*(((-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((x*c*d*g+
x*c*e*f-2*(-a*c)^(1/2)*x*e*g+2*c*d*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(
-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*d*e*g-a*c*(((-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((x*c*d*g+x*c*e*f-2*(-a*c)^(1/2
)*x*e*g+2*c*d*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x
+(-a*c)^(1/2)))*e^2*f+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(
1/2)*(e*g)^(1/2)*ln((x*c*d*g+x*c*e*f-2*(-a*c)^(1/2)*x*e*g+2*c*d*f+2*(e*g*x^
2+d*g*x+e*f*x+d*f)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/
c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c^2*d^2*f
+2*(-a*c)^(1/2)*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c
*d*f)/c)^(1/2)*(e*g)^(1/2)*c*e)/(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)/(-a*c)^(1/2
)/c^2/(e*g)^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2
)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
```

```
[Out] Integral((d + e*x)**(3/2)*sqrt(f + g*x)/(a + c*x**2), x)
```

3.408 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal. Leaf size=342

$$\frac{\left(-\sqrt{-a}\sqrt{c}\,(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(\sqrt{-a}\sqrt{c}\,(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\cdots}{\sqrt{f+gx}\cdots}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\sqrt{\sqrt{-a}\,g+\sqrt{c}\cdots}}$$

Rubi [A] time = 2.09, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {906, 63, 217, 206, 6725, 93, 208}

$$\frac{\left(-\sqrt{-a}\sqrt{c}\,(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(\sqrt{-a}\sqrt{c}\,(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}+\frac{2\sqrt{e}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])]
)/c + ((c*d*f - a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x]
)])/(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g])
- ((c*d*f - a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]*f +
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 906

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(e*g)/c, Int[(d + e*x)^(m - 1)*(f + g*x)^(n - 1), x
], x] + Dist[1/c, Int[(Simp[c*d*f - a*e*g + (c*e*f + c*d*g)*x, x]*(d + e*x)
^(m - 1)*(f + g*x)^(n - 1))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0
] && GtQ[n, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{c} + \frac{(eg)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{-a\sqrt{c}(ef+dg)+\sqrt{-a}(cdf-aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{a\sqrt{c}(ef+dg)+\sqrt{-a}(cdf-aeg)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c} + \frac{(2g)\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\right.}{c}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} - \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+e}}}{2\sqrt{-a}\,c}$$

$$= \frac{2\sqrt{e}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} - \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-(}\right.}{\sqrt{-a}\,c}$$

$$= \frac{2\sqrt{e}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} + \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f}}\right.}{\sqrt{-a}\,c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}$$

Mathematica [A] time = 1.27, size = 339, normalized size = 0.99

$$\frac{\frac{\frac{\left(\sqrt{-a}\sqrt{c}d-ae\right)\sqrt{\sqrt{-a}g+\sqrt{c}f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e+\sqrt{c}d}} - \frac{\left(\sqrt{-a}\sqrt{c}d+ae\right)\sqrt{\sqrt{-a}g-\sqrt{c}f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e-\sqrt{c}d}}}{a} + \frac{2\sqrt{g}\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{\sqrt{f+gx}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] ((2*Sqrt[g]*Sqrt[e*f - d*g]*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g
]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/Sqrt[f + g*x] + (-(((Sqrt[-a]*Sqrt[c]*d
+ a*e)*Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a
]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/Sqrt[
-(Sqrt[c]*d) + Sqrt[-a]*e]) + ((Sqrt[-a]*Sqrt[c]*d - a*e)*Sqrt[Sqrt[c]*f +
Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt
[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e])/a)/c
```

IntegrateAlgebraic [C] time = 1.08, size = 378, normalized size = 1.11

$$\frac{\sqrt{ae^2+cd^2}\left(\sqrt{a}\,g-i\sqrt{c}\,f\right)\tan^{-1}\left(\frac{\sqrt{f+gx}\,\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\,\sqrt{-i\sqrt{a}\,\sqrt{c}\,dg+i\sqrt{a}\,\sqrt{c}\,ef-aeg-cdf}}\right)}{\sqrt{a}\,c\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}}+\frac{\sqrt{ae^2+cd^2}\left(\sqrt{a}\,g+i\sqrt{c}\,f\right)\tan^{-1}\left(\frac{\sqrt{f+gx}\,\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\,\sqrt{i\sqrt{a}\,\sqrt{c}\,dg-i\sqrt{a}\,\sqrt{c}\,ef-aeg-cdf}}\right)}{\sqrt{a}\,c\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}}+\frac{2\sqrt{e}\,\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{d+ex}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (Sqrt[c*d^2 + a*e^2]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^
2]*Sqrt[f + g*x])/(Sqrt[-(c*d*f) + I*Sqrt[a]*Sqrt[c]*e*f - I*Sqrt[a]*Sqrt[c
]*d*g - a*e*g]*Sqrt[d + e*x])])/(Sqrt[a]*c*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)
*(Sqrt[c]*f + I*Sqrt[a]*g))]) + (Sqrt[c*d^2 + a*e^2]*(I*Sqrt[c]*f + Sqrt[a]
*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-(c*d*f) - I*Sqrt[a]*S
qrt[c]*e*f + I*Sqrt[a]*Sqrt[c]*d*g - a*e*g]*Sqrt[d + e*x])])/(Sqrt[a]*c*Sqr
t[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]) + (2*Sqrt[e]*Sqr
t[g]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/c
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.03, size = 1569, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+d*g+e*f+2*(e*g*x^2+d*g*x
+e*f*x+d*f)^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f)/c)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f)/c)^(1/2)*(-a*c)^(1/2)*e*g-(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*((-a*e*g
+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*(
-a*c)^(1/2)*d*g-(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*((-a*e*g+c*d*f+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*(-a*c)^(1/2)*e
*f+(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*l
n((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a*e*g-(e*g)^(1/2)*(-(a*e*g-
c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d
*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+
```

```
e*f*x+d*f)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)
*c)/(c*x-(-a*c)^(1/2)))*c*d*f-(e*g)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a
*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*
f)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x
+(-a*c)^(1/2)))*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*
(-a*c)^(1/2)*d*g-(e*g)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g
*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-(a
*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)
))*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2)*
e*f-(e*g)^(1/2)*ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/
2)*d*g-(-a*c)^(1/2)*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-(a*e*g-c*d*f+(-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*((-a*e*g+c
*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*a*e*g+(e*g)^(1/2)*ln((c*d*
g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+
2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)*c*d*f)/(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)/(-a*c)^(1/2
)/c/(e*g)^(1/2)/(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)/
((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,\sqrt{gx+f}}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(g*x + f)/(c*x^2 + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(a + c*x^2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
[Out] Integral(sqrt(d + e*x)*sqrt(f + g*x)/(a + c*x**2), x)
```

3.409 $\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}$$

Rubi [A] time = 0.34, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {910, 93, 208}

$$\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] (Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*d - Sqrt[-a]*e]) - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}\,f-\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\sqrt{-a}\,f+\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\right)dx$$

$$= \frac{1}{2}\left(\frac{af}{(-a)^{3/2}}-\frac{g}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx+\frac{1}{2}\left(\frac{af}{(-a)^{3/2}}+\frac{g}{\sqrt{c}}\right)\int \frac{}{\left(\sqrt{-\right.}}$$

$$= \left(\frac{af}{(-a)^{3/2}}-\frac{g}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}\,d+\sqrt{-a}\,e-\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)+\left(\frac{}{(-}\right.$$

$$= \frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{}}\right.}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}}$$

Mathematica [A] time = 0.35, size = 229, normalized size = 0.95

$$\frac{\frac{\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}-\frac{\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}}{\sqrt{-a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] ((Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*
Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/Sqrt[-(Sqr
t[c]*d) + Sqrt[-a]*e] - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]
*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x]
)])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e])/(Sqrt[-a]*Sqrt[c])
```

IntegrateAlgebraic [C] time = 172.25, size = 1248, normalized size = 5.20

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] (e^2*f^2*Sqrt[e/g]*g^2 - 2*d*e*f*Sqrt[e/g]*g^3 + d^2*Sqrt[e/g]*g^4)*RootSum
[c*e^4*f^4 - 4*c*d*e^3*f^3*g + 6*c*d^2*e^2*f^2*g^2 - 4*c*d^3*e*f*g^3 + c*d^
4*g^4 - 4*c*e^3*f^3*g*#1^2 + 4*c*d*e^2*f^2*g^2*#1^2 + 4*c*d^2*e*f*g^3*#1^2
- 4*c*d^3*g^4*#1^2 + 6*c*e^2*f^2*g^2*#1^4 + 4*c*d*e*f*g^3*#1^4 + 6*c*d^2*g^
4*#1^4 + 16*a*e^2*g^4*#1^4 - 4*c*e*f*g^3*#1^6 - 4*c*d*g^4*#1^6 + c*g^4*#1^8
 & , Log[-(Sqrt[e/g]*Sqrt[f + g*x]) + Sqrt[d - (e*f)/g + (e*(f + g*x))/g] -
 #1]/(c*e^3*f^3 - c*d*e^2*f^2*g - c*d^2*e*f*g^2 + c*d^3*g^3 - 3*c*e^2*f^2*g
*#1^2 - 2*c*d*e*f*g^2*#1^2 - 3*c*d^2*g^3*#1^2 - 8*a*e^2*g^3*#1^2 + 3*c*e*f*
g^2*#1^4 + 3*c*d*g^3*#1^4 - c*g^3*#1^6) & ] - 2*(e*f*Sqrt[e/g]*g^3 - d*Sqrt
[e/g]*g^4)*RootSum[c*e^4*f^4 - 4*c*d*e^3*f^3*g + 6*c*d^2*e^2*f^2*g^2 - 4*c*
d^3*e*f*g^3 + c*d^4*g^4 - 4*c*e^3*f^3*g*#1^2 + 4*c*d*e^2*f^2*g^2*#1^2 + 4*c
*d^2*e*f*g^3*#1^2 - 4*c*d^3*g^4*#1^2 + 6*c*e^2*f^2*g^2*#1^4 + 4*c*d*e*f*g^3
*#1^4 + 6*c*d^2*g^4*#1^4 + 16*a*e^2*g^4*#1^4 - 4*c*e*f*g^3*#1^6 - 4*c*d*g^4
*#1^6 + c*g^4*#1^8 & , (Log[-(Sqrt[e/g]*Sqrt[f + g*x]) + Sqrt[d - (e*f)/g +
 (e*(f + g*x))/g] - #1]*#1^2)/(-(c*e^3*f^3) + c*d*e^2*f^2*g + c*d^2*e*f*g^2
 - c*d^3*g^3 + 3*c*e^2*f^2*g*#1^2 + 2*c*d*e*f*g^2*#1^2 + 3*c*d^2*g^3*#1^2 +
 8*a*e^2*g^3*#1^2 - 3*c*e*f*g^2*#1^4 - 3*c*d*g^3*#1^4 + c*g^3*#1^6) & ] - S
qrt[e/g]*g^4*RootSum[c*e^4*f^4 - 4*c*d*e^3*f^3*g + 6*c*d^2*e^2*f^2*g^2 - 4*
```

```
c*d^3*e*f*g^3 + c*d^4*g^4 - 4*c*e^3*f^3*g*#1^2 + 4*c*d*e^2*f^2*g^2*#1^2 + 4
*c*d^2*e*f*g^3*#1^2 - 4*c*d^3*g^4*#1^2 + 6*c*e^2*f^2*g^2*#1^4 + 4*c*d*e*f*g
^3*#1^4 + 6*c*d^2*g^4*#1^4 + 16*a*e^2*g^4*#1^4 - 4*c*e*f*g^3*#1^6 - 4*c*d*g
^4*#1^6 + c*g^4*#1^8 & , (Log[-(Sqrt[e/g]*Sqrt[f + g*x]) + Sqrt[d - (e*f)/g
 + (e*(f + g*x))/g] - #1]*#1^4)/(-(c*e^3*f^3) + c*d*e^2*f^2*g + c*d^2*e*f*g
^2 - c*d^3*g^3 + 3*c*e^2*f^2*g*#1^2 + 2*c*d*e*f*g^2*#1^2 + 3*c*d^2*g^3*#1^2
 + 8*a*e^2*g^3*#1^2 - 3*c*e*f*g^2*#1^4 - 3*c*d*g^3*#1^4 + c*g^3*#1^6) & ]
```

fricas [B] time = 10.39, size = 1921, normalized size = 8.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*
f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a
^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g - (a*c^2*d^2*e +
a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2
*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c
^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^
2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^
2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a
*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^
2*d^2*e^2 + a^3*c*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c
*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2
+ a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*d*e*
f - c*d^2*g - (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^
2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f
)*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g
 + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*
c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*
e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d
^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x) - 1/4*sqrt(-(c*d*f
 + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a
*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(
e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g + (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-
(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^
2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a
^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*(c^2*d^
3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x)*s
qrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c
*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*
f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a
*c^2*d^2 + a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*d*e*f - c*d^2*g + (a*
c^2*d^2*e + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2
*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f +
a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^
3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*
f*g - d*e*g^2)*x - (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f +
(c^2*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^
4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.Non regular value [0
,0] was discarded and replaced randomly by 0=[62,91]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.Non regular value [0,0] was discarded and replaced randomly by 0=[44,-43]W
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.Non regular value [0,0] was discarded and replaced r
andomly by 0=[-18,-31]Precision problem choosing root in common_EXT, curren
t precision 14Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{-2,[0,1]%%
%},0,%%%{1,[2,2]%%%}+%%%{2,[1,2]%%%}+%%%{1,[0,2]%%%}] at parameters values
[-27,26]Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{-2,[0,1]%%%},0,%
%%{1,[2,2]%%%}+%%%{2,[1,2]%%%}+%%%{1,[0,2]%%%}] at parameters values [-89,6
3]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.Non regular value [0,0] was discarded and replace
d randomly by 0=[-59,-77]Precision problem choosing root in common_EXT, cur
rent precision 14Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.Non regular value [0,0] was discar
ded and replaced randomly by 0=[-37,-94]Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regular
 value [0,0] was discarded and replaced randomly by 0=[-32,97]Warning, choo
sing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%{-2,
[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-82.3579015951,0]Warning,
choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%
{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-29.292030761,22]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.Non regular value [0,0] was discarded and replaced rando
mly by 0=[2,-99]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.Non regular value [0,0] was discard
ed and replaced randomly by 0=[-13,69]Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [0,0] was discarded and replaced randomly by 0=[-55,-78]Warning, choos
ing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%{-2,[
2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-57.0371161718,0]Warning, c
hoosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%{
-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-53.6704242053,49]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.Non regular value [0,0] was discarded and replaced rando
mly by 0=[-20,-31]Precision problem choosing root in common_EXT, current pr
ecision 14Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.Non regular value [0,0] was discarded and
 replaced randomly by 0=[-67,8]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.Non regular value [0
,0] was discarded and replaced randomly by 0=[-69,98]Warning, choosing root
 of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%{-2,[2,1]%%%}
+%%%{1,[2,0]%%%}] at parameters values [-41.1343540126,0]Warning, choosing
root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%%%{-2,[2,1]
%%%}+%%%{1,[2,0]%%%}] at parameters values [-46.2420096635,-70]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.Non regular value [0,0] was discarded and replaced randomly by
0=[-53,73]Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.Non regular value [0,0] was discarded and
 replaced randomly by 0=[-78,50]Warning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.Non regular value [
0,0] was discarded and replaced randomly by 0=[-61,27]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.Non regular value [0,0] was discarded and replaced randomly by 0=[-18,-4
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.Non regular value [0,0] was discarded and replaced
```

```
 randomly by 0=[15,-93]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.Non regular value [0,0] was
discarded and replaced randomly by 0=[97,57]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.Non reg
ular value [0,0] was discarded and replaced randomly by 0=[70,-37]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.Non regular value [0,0] was discarded and replaced randomly
by 0=[8,40]Warning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.Non regular value [0,0] was discarded an
d replaced randomly by 0=[10,9]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.Non regular value [0
,0] was discarded and replaced randomly by 0=[85,-92]Warning, need to choos
e a branch for the root of a polynomial with parameters. This might be wron
g.Non regular value [0,0] was discarded and replaced randomly by 0=[-83,95]
Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2
]%%%}+%%%{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-49.355685115
3,0]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.Non regular value [0,0] was discarded and repla
ced randomly by 0=[66,42]Warning, need to choose a branch for the root of a
 polynomial with parameters. This might be wrong.Non regular value [0,0] wa
s discarded and replaced randomly by 0=[20,-21]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.Non
regular value [0,0] was discarded and replaced randomly by 0=[13,-34]Warnin
g, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+
%%%{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-90.5690937298,0]Wa
rning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%
%%}+%%%{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-36.6004387327,
-85]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.Non regular value [0,0] was discarded and repla
ced randomly by 0=[99,-89]Warning, need to choose a branch for the root of
a polynomial with parameters. This might be wrong.Non regular value [0,0] w
as discarded and replaced randomly by 0=[2,-9]Warning, need to choose a bra
nch for the root of a polynomial with parameters. This might be wrong.Non r
egular value [0,0] was discarded and replaced randomly by 0=[-74,46]Warning
, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%%}+%
%%{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-4.22288109735,0]War
ning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[1,0]%%%},0,%%%{1,[2,2]%%
%}+%%%{-2,[2,1]%%%}+%%%{1,[2,0]%%%}] at parameters values [-6.87379696826,-
21]Warning, need to choose a branch for the root of a polynomial with param
eters. This might be wrong.Non regular value [0,0] was discarded and replac
ed randomly by 0=[-35,-95]Warning, need to choose a branch for the root of
a polynomial with parameters. This might be wrong.Non regular value [0,0] w
as discarded and replaced randomly by 0=[-9,27]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.Non
regular value [0,0] was discarded and replaced randomly by 0=[-19,90]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.Non regular value [0,0] was discarded and replaced random
ly by 0=[2,-39]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.Non regular value [0,0] was discarde
d and replaced randomly by 0=[55,-73]Warning, need to choose a branch for t
he root of a polynomial with parameters. This might be wrong.Non regular va
lue [0,0] was discarded and replaced randomly by 0=[61,1]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.Non regular value [0,0] was discarded and replaced randomly by 0=[10,
40]Warning, need to choose a branch for the root of a polynomial with param
eters. This might be wrong.Non regular value [0,0] was discarded and replac
ed randomly by 0=[83,49]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.Non regular value [0,0] was
 discarded and replaced randomly by 0=[1,-81]Warning, need to choose a bran
```

```
ch for the root of a polynomial with parameters. This might be wrong.Non re
gular value [0,0] was discarded and replaced randomly by 0=[76,-13]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.Non regular value [0,0] was discarded and replaced randomly
 by 0=[45,19]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.Non regular value [0,0] was discarded
and replaced randomly by 0=[37,-12]Evaluation time: 4.14index.cc index_m op
erator + Error: Bad Argument Value
```

maple [B] time = 0.04, size = 1383, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x)
```

```
[Out] -1/2*(g*x+f)^(1/2)*(e*x+d)^(1/2)*(ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*
a*c*e^2*f*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+ln((2*
(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*
x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1
/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*e^2*g*(-a*c)^(1/2)*(-(a*e*g-c*d*f+(-a*
c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*
e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+
c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1
/2)))*c^2*d^2*f*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+
ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+
2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)
/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d^2*g*(-a*c)^(1/2)*(-(a*e*g-c*d*
f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d*g*x+c*e*f*x-2*(-a*c)^
(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a
*c)^(1/2)))*a*c*e^2*f*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^
(1/2)+ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^
(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c-(-a*c)^(
1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*e^2*g*((-a*e*g+c*d*f+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2)-ln((c*d*g*x+c*e*f*x-2*(
-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(
1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c
*x+(-a*c)^(1/2)))*c^2*d^2*f*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*
x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c-(-
a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*d^2*g*((-a*e*g+c*d*f
+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2))/((e*x+d)*(g*x+f)
)^(1/2)/(c*d-(-a*c)^(1/2)*e)/(-a*c)^(1/2)/(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f)/c)^(1/2)/((-a*c)^(1/2)*e+c*d)/((-a*e*g+c*d*f+(-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\left(cx^2+a\right)\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*sqrt(e*x + d)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(1/2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f + gx}}{\left(a + cx^{2}\right)\sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(1/2)/(c*x**2+a),x)
[Out] Integral(sqrt(f + g*x)/((a + c*x**2)*sqrt(d + e*x)), x)
```

3.410 $\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\left(a+cx^2\right)} \, dx$

Optimal. Leaf size=351

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(ae^2+cd^2\right)\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg\right.}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}}$$

Rubi [A] time = 2.15, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {908, 37, 6725, 93, 208}

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(ae^2+cd^2\right)\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(ae^2+cd^2\right)\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + ((c*d*f + a*e*g + Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((c*d*f +
a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\left(a+cx^2\right)}\,dx=\frac{\int \frac{cdf+aeg-c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{cd^2+ae^2}+\frac{(e(ef-dg))\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{cd^2+ae^2}$$

$$=-\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{\int \left(\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)}{cd^2+ae^2}$$

$$=-\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}}-\frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}}{2\sqrt{-a}\left(cd^2+ae^2\right)}$$

$$=-\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}}-\frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-(\sqrt{c}}\right.}{\sqrt{-a}\left(cd^2+ae^2\right)}$$

$$=-\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}}+\frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f}}\right.}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(cd^2+ae^2\right)\sqrt{\sqrt{c}f-\sqrt{-a}g}}$$

Mathematica [A] time = 0.70, size = 265, normalized size = 0.75

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{a\sqrt{\sqrt{-a}g-\sqrt{c}f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{(-a)^{3/2}\left(\sqrt{-a}e-\sqrt{c}d\right)^{3/2}}+\frac{a\sqrt{\sqrt{-a}g+\sqrt{c}f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{(-a)^{3/2}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (a*Sqrt[-(Sqrt[c]*f)
 + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqr
t[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/((-a)^(3/2)*(-(Sqrt[c]*d) + S
qrt[-a]*e)^(3/2)) + (a*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f
 + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]
)/((-a)^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2))
```

IntegrateAlgebraic [C] time = 1.25, size = 401, normalized size = 1.14

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{c}d-i\sqrt{a}e\right)^2\left(\sqrt{a}g-i\sqrt{c}f\right)\tan^{-1}\left(\frac{\sqrt{f+gx}\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\sqrt{-i\sqrt{a}\sqrt{c}dg+i\sqrt{a}\sqrt{c}ef-aeg-cdf}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^{3/2}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}}+\frac{\left(\sqrt{c}d+i\sqrt{a}e\right)^2\left(\sqrt{a}g+i\sqrt{c}f\right)\tan^{-1}\left(\frac{\sqrt{f+gx}\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\sqrt{i\sqrt{a}\sqrt{c}dg-i\sqrt{a}\sqrt{c}ef-aeg-cdf}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^{3/2}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + ((Sqrt[c]*d - I*Sqrt
[a]*e)^2*((-I)*Sqrt[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f +
g*x])/(Sqrt[-(c*d*f) + I*Sqrt[a]*Sqrt[c]*e*f - I*Sqrt[a]*Sqrt[c]*d*g - a*e*
g]*Sqrt[d + e*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*Sqrt[-((Sqrt[c]*d - I*Sq
rt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + ((Sqrt[c]*d + I*Sqrt[a]*e)^2*(I*Sqr
```

```
t[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-(c*d*
f) - I*Sqrt[a]*Sqrt[c]*e*f + I*Sqrt[a]*Sqrt[c]*d*g - a*e*g]*Sqrt[d + e*x])]
)/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*
f - I*Sqrt[a]*g))])
```

fricas [B] time = 52.33, size = 5816, normalized size = 16.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2
)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*
d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2
 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*
c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a
^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10
 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))
*log(((3*c*d^2*e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g - (c*d^4 - 3*a*
d^2*e^2)*g^2 + 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^5 - 4*
a*c*d^3*e^2 + 3*a^2*d*e^4)*g - 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + 3*a^3*c
*d^3*e^5 + a^4*d*e^7)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*
f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6
*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 1
5*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e
^10 + a^7*e^12)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2
)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*
d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2
 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*
c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a
^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10
 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))
 + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a*d*e^3)*g^2)*x + (2*(c^3*d^
7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*f + ((c^3*d^6*e + 3*a*c^
2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a
^2*c*d^3*e^4 + a^3*d*e^6)*g)*x)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a
^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c
^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^
10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a
^6*c*d^2*e^10 + a^7*e^12)))/x) - (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sq
rt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3
*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c
^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d
*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12
+ 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2
*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 +
3*a^3*c*d^2*e^4 + a^4*e^6))*log(((3*c*d^2*e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a
*d*e^3)*f*g - (c*d^4 - 3*a*d^2*e^2)*g^2 - 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 +
 a^2*e^5)*f - (c^2*d^5 - 4*a*c*d^3*e^2 + 3*a^2*d*e^4)*g - 2*(a*c^3*d^7*e +
3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 + a^4*d*e^7)*sqrt(-((9*c^3*d^4*e^2 - 6*
a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*
c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^
12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*
c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))*sqrt(e*x + d)*sqrt(g*x + f)*sq
rt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3
*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c
^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d
*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12
+ 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2
```

```
*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 +
3*a^3*c*d^2*e^4 + a^4*e^6)) + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a
*d*e^3)*g^2)*x + (2*(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^
6)*f + ((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3*
d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*g)*x)*sqrt(-((9*c^3*d^
4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^
3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)
/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6
 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/x) + (c*d^3 + a*d*e^
2 + (c*d^2*e + a*e^3)*x)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e -
a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sq
rt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 1
0*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c
*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a
^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3
*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))*log(((3*c*d^2*e^2 -
a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g - (c*d^4 - 3*a*d^2*e^2)*g^2 + 2*((3*
c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^5 - 4*a*c*d^3*e^2 + 3*a^2*d
*e^4)*g + 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 + a^4*d*e^7)
*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e
- 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^
2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 2
0*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))*sqr
t(e*x + d)*sqrt(g*x + f)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e -
a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sq
rt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 1
0*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c
*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a
^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3
*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)) + 2*((3*c*d^2*e^2 -
a*e^4)*f*g - (c*d^3*e - 3*a*d*e^3)*g^2)*x - (2*(c^3*d^7 + 3*a*c^2*d^5*e^2 +
 3*a^2*c*d^3*e^4 + a^3*d*e^6)*f + ((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d
^2*e^5 + a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*
e^6)*g)*x)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*
c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*
e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d
^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e
^12)))/x) - (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-((c^2*d^3 - 3*a*c
*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*
a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^
6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6
- 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2
+ 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^
2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4
*e^6))*log(((3*c*d^2*e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g - (c*d^4
- 3*a*d^2*e^2)*g^2 - 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^
5 - 4*a*c*d^3*e^2 + 3*a^2*d*e^4)*g + 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + 3
*a^3*c*d^3*e^5 + a^4*d*e^7)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c
*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d
^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e
^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c
*d^2*e^10 + a^7*e^12)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-((c^2*d^3 - 3*a*c
*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*
a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^
6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6
- 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2
+ 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^
2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4
*e^6)) + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a*d*e^3)*g^2)*x - (2*(
```

```
c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*f + ((c^3*d^6*e +
3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2
 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*g)*x)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e
^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*
g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*
c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8
 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/x) + 8*sqrt(e*x + d)*sqrt(g*x + f)*e)/(c*
d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.06, size = 5383, normalized size = 15.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\left(cx^2+a\right)(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(3/2)/(c*x**2+a),x)
[Out] Timed out
```

3.411 $\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=613

$$\frac{e\sqrt{f+gx}\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\,\sqrt{d+ex}\,\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)(ef-dg)}-\frac{e\sqrt{f+gx}\left(\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\,\sqrt{d+ex}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)(ef-dg)}+\frac{}{3\sqrt{d}}$$

Rubi [A] time = 3.16, antiderivative size = 613, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {908, 45, 37, 6725, 96, 93, 208}

$$\frac{e\sqrt{f+gx}\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\,\sqrt{d+ex}\,\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)(ef-dg)}-\frac{e\sqrt{f+gx}\left(\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\,\sqrt{d+ex}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)(ef-dg)}+\frac{4eg\sqrt{f+gx}}{3\sqrt{d+ex}\,\left(ae^2+cd^2\right)(ef-dg)}-\frac{2e\sqrt{f+gx}}{3(d+ex)^{3/2}\left(ae^2+cd^2\right)}+\frac{\sqrt{c}\left(\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)^{3/2}\left(ae^2+cd^2\right)\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}+\frac{\sqrt{c}\left(a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,cdf+\sqrt{-a}\,aeg\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{a\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)^{3/2}\left(ae^2+cd^2\right)\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) + (4*e*g*Sqrt[f +
g*x])/(3*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (e*(c*d*f + a*e*g - S
qrt[-a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]
*e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) - (e*(c*d*f + a*e*g + Sqrt[-
a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(
c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (Sqrt[c]*(c*d*f + a*e*g + Sqrt[
-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x
])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sq
rt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) + (Sqrt[c]*(S
qrt[-a]*c*d*f + Sqrt[-a]*a*e*g + a*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[
c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*
x])]])/(a*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sq
rt[-a]*g])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\left(a+cx^2\right)}\,dx = \frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{cd^2+ae^2} + \frac{(e(ef-dg))\int \frac{1}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{\int \left(\frac{a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}} + \frac{-a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf+aeg)}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+g}\right.}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{\right.}{}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{\right.}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{\right.}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{\right.}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)}$$

Mathematica [A] time = 2.86, size = 353, normalized size = 0.58

$$-\frac{2e\sqrt{f+gx}\left(ae^3(f+gx)+cd\left(-6d^2g+7def-5degx+6e^2fx\right)\right)}{3(d+ex)^{3/2}\left(ae^2+cd^2\right)^2(ef-dg)}-\frac{\sqrt{c}\sqrt{\sqrt{-a}g-\sqrt{c}f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\left(\sqrt{-a}e-\sqrt{c}d\right)^{3/2}\left(\sqrt{-a}\sqrt{c}d+ae\right)}-\frac{\sqrt{c}\sqrt{\sqrt{-a}g+\sqrt{c}f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\left(\sqrt{-a}\sqrt{c}d-ae\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x]*(a*e^3*(f + g*x) + c*d*(7*d*e*f - 6*d^2*g + 6*e^2*f*x -
 5*d*e*g*x)))/(3*(c*d^2 + a*e^2)^2*(e*f - d*g)*(d + e*x)^(3/2)) - (Sqrt[c]*
Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sq
rt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/((-(Sqrt[c]*
d) + Sqrt[-a]*e)^(3/2)*(Sqrt[-a]*Sqrt[c]*d + a*e)) - (Sqrt[c]*Sqrt[Sqrt[c]*
f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[
Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/((Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(S
qrt[-a]*Sqrt[c]*d - a*e))
```

IntegrateAlgebraic [C] time = 1.78, size = 498, normalized size = 0.81

$$\frac{\left(\sqrt{c}d-i\sqrt{a}e\right)^3\left(\sqrt{a}\sqrt{c}g-icf\right)\tan^{-1}\left(\frac{\sqrt{f+gx}\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\sqrt{-i\sqrt{a}\sqrt{c}dg+i\sqrt{a}\sqrt{c}ef-aeg-cdf}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^{5/2}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}}+\frac{\left(\sqrt{c}d+i\sqrt{a}e\right)^3\left(\sqrt{a}\sqrt{c}g+icf\right)\tan^{-1}\left(\frac{\sqrt{f+gx}\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\sqrt{i\sqrt{a}\sqrt{c}dg-i\sqrt{a}\sqrt{c}ef-aeg-cdf}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^{5/2}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}}+\frac{2\left(\frac{ae^4(f+gx)^{3/2}}{(d+ex)^{3/2}}+\frac{cd^2e^2(f+gx)^{3/2}}{(d+ex)^{3/2}}-\frac{6cd^2eg\sqrt{f+gx}}{\sqrt{d+ex}}+\frac{6cde^2f\sqrt{f+gx}}{\sqrt{d+ex}}\right)}{3\left(ae^2+cd^2\right)^2(dg-ef)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (2*((6*c*d*e^2*f*Sqrt[f + g*x])/Sqrt[d + e*x] - (6*c*d^2*e*g*Sqrt[f + g*x])
/Sqrt[d + e*x] + (c*d^2*e^2*(f + g*x)^(3/2))/(d + e*x)^(3/2) + (a*e^4*(f +
g*x)^(3/2))/(d + e*x)^(3/2)))/(3*(c*d^2 + a*e^2)^2*(-(e*f) + d*g)) + ((Sqrt
[c]*d - I*Sqrt[a]*e)^3*((-I)*c*f + Sqrt[a]*Sqrt[c]*g)*ArcTan[(Sqrt[c*d^2 +
a*e^2]*Sqrt[f + g*x])/(Sqrt[-(c*d*f) + I*Sqrt[a]*Sqrt[c]*e*f - I*Sqrt[a]*Sq
rt[c]*d*g - a*e*g]*Sqrt[d + e*x])]])/(Sqrt[a]*(c*d^2 + a*e^2)^(5/2)*Sqrt[-((
Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + ((Sqrt[c]*d + I*Sqr
t[a]*e)^3*(I*c*f + Sqrt[a]*Sqrt[c]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f +
g*x])/(Sqrt[-(c*d*f) - I*Sqrt[a]*Sqrt[c]*e*f + I*Sqrt[a]*Sqrt[c]*d*g - a*e*
g]*Sqrt[d + e*x])]])/(Sqrt[a]*(c*d^2 + a*e^2)^(5/2)*Sqrt[-((Sqrt[c]*d + I*Sq
rt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.09, size = 14861, normalized size = 24.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{gx+f}}{\left(cx^2+a\right)(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)),x)
```

```
[Out] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(5/2)/(c*x**2+a),x)
```

```
[Out] Timed out
```

3.412 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx$

Optimal. Leaf size=337

$$\frac{\left(-2\sqrt{-a}\sqrt{c}\,de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(2\sqrt{-a}\sqrt{c}\,de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Rubi [A] time = 2.46, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {910, 63, 217, 206, 6725, 93, 208}

$$\frac{\left(-2\sqrt{-a}\sqrt{c}\,de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(2\sqrt{-a}\sqrt{c}\,de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}+\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) + ((c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f -
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((
c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*
g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*
c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(a+cx^2\right)}\,dx = \int \left(\frac{e^2}{c\sqrt{d+ex}\sqrt{f+gx}} + \frac{cd^2-ae^2+2cdex}{c\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\right)dx$$

$$= \frac{\int \frac{cd^2-ae^2+2cdex}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx}{c} + \frac{e^2\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{-2a\sqrt{c}de+\sqrt{-a}\left(cd^2-ae^2\right)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{2a\sqrt{c}de+\sqrt{-a}\left(cd^2-ae^2\right)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c} + \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}} \cdots\right)}{c}$$

$$= \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} - \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+g \cdots}}}{2\sqrt{-a}\,c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\mathrm{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a} \cdots\right.}\right.}{\sqrt{-a}\,c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \cdots$$

Mathematica [A] time = 1.09, size = 339, normalized size = 1.01

$$\frac{\frac{\frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(\sqrt{-a}\sqrt{c}d-ae\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}g+\sqrt{c}f}} - \frac{\sqrt{\sqrt{-a}e-\sqrt{c}d}\left(\sqrt{-a}\sqrt{c}d+ae\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}g-\sqrt{c}f}}}{a} + \frac{2(ef-dg)^{3/2}\left(\frac{e(f+gx)}{ef-dg}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{\sqrt{g}(f+gx)^{3/2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((2*(e*f - d*g)^(3/2)*((e*(f + g*x))/(e*f - d*g))^(3/2)*ArcSinh[(Sqrt[g]*Sq
rt[d + e*x])/Sqrt[e*f - d*g]])/(Sqrt[g]*(f + g*x)^(3/2)) + (-((Sqrt[-(Sqrt[
c]*d) + Sqrt[-a]*e]*(Sqrt[-a]*Sqrt[c]*d + a*e)*ArcTanh[(Sqrt[-(Sqrt[c]*f) +
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])
```

```
])/Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]) + (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[-
a]*Sqrt[c]*d - a*e)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(S
qrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/Sqrt[Sqrt[c]*f + Sqrt[-a]*g])/
a)/c
```

IntegrateAlgebraic [C] time = 1.16, size = 378, normalized size = 1.12

$$\frac{\left(\sqrt{a}\,e-i\sqrt{c}\,d\right)\sqrt{ae^2+cd^2}\,\tan^{-1}\left(\frac{\sqrt{f+gx}\,\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\,\sqrt{-i\sqrt{a}\,\sqrt{c}\,dg+i\sqrt{a}\,\sqrt{c}\,ef-aeg-cdf}}\right)}{\sqrt{a}\,c\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}}+\frac{\left(\sqrt{a}\,e+i\sqrt{c}\,d\right)\sqrt{ae^2+cd^2}\,\tan^{-1}\left(\frac{\sqrt{f+gx}\,\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\,\sqrt{i\sqrt{a}\,\sqrt{c}\,dg-i\sqrt{a}\,\sqrt{c}\,ef-aeg-cdf}}\right)}{\sqrt{a}\,c\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}}+\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{d+ex}}\right)}{c\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*ArcTan[(Sqrt[c*d^2 + a*e^
2]*Sqrt[f + g*x])/(Sqrt[-(c*d*f) + I*Sqrt[a]*Sqrt[c]*e*f - I*Sqrt[a]*Sqrt[c
]*d*g - a*e*g]*Sqrt[d + e*x])])/(Sqrt[a]*c*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)
*(Sqrt[c]*f + I*Sqrt[a]*g))]) + ((I*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e
^2]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-(c*d*f) - I*Sqrt[a]*S
qrt[c]*e*f + I*Sqrt[a]*Sqrt[c]*d*g - a*e*g]*Sqrt[d + e*x])])/(Sqrt[a]*c*Sqr
t[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]) + (2*e^(3/2)*Arc
Tanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(c*Sqrt[g])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 2336, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+
f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*a*e^2*g^2*(-a*c)^(1/2)*(-(a*e*g-c*d*f+(
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f)/c)^(1/2)+2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1
/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*e^2*f^2*(-a*c)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)
^(1/2)*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a^2*e^2*g^2*(
e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c
*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
```

```
*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a*c*d^2*g^2*(e*g)^(1/2)*(-(a*e*g-c*d*f+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(
-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/
2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c
)^(1/2)))*a*c*e^2*f^2*(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f)/c)^(1/2)-2*ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a*d*e*g^2*(-a*
c)^(1/2)*(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(
1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*c^2*d^2*f^2*(e*g)^(1/2)*(-(a*
e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-2*ln((c*d*g*x+c*e*f*x
+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*
(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c
)/(c*x-(-a*c)^(1/2)))*c*d*e*f^2*(-a*c)^(1/2)*(e*g)^(1/2)*(-(a*e*g-c*d*f+(-a
*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*
c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*
(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(
1/2)))*a^2*e^2*g^2*(e*g)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)
*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2
)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a*c*d^2*g^2*(e*g)^(
1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d*g*x
+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*(
(e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)
^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*(e*g)^(1/2)*((-a*e*g+c*d*f+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-2*ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c
)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(
-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1
/2)))*a*d*e*g^2*(-a*c)^(1/2)*(e*g)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(
-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*c^2*d^2*f
^2*(e*g)^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-2
*ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(
1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*c*d*e*f^2*(-a*c)^(1/2)*(e*g)^(1/2
)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2))/((e*x+d)*(g*x
+f))^(1/2)/(c*f-g*(-a*c)^(1/2))/(-a*c)^(1/2)/(e*g)^(1/2)/(-(a*e*g-c*d*f+(-a
*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)/(g*(-a*c)^(1/2)+c*f)/((-a*e*g+c*d*
f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+a\right)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
[Out] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral((d + e*x)**(3/2)/((a + c*x**2)*sqrt(f + g*x)), x)
```

3.413 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Rubi **[A]** time = 0.33, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {910, 93, 208}

$$\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}\,d-\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\sqrt{-a}\,d+\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\right) dx$$

$$= \frac{1}{2}\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx+\frac{1}{2}\left(\frac{ad}{(-a)^{3/2}}+\frac{e}{\sqrt{c}}\right)\int$$

$$= \left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}\,d+\sqrt{-a}\,e-\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)+$$

$$= \frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\ldots}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\ldots}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

Mathematica [A] time = 0.33, size = 229, normalized size = 0.95

$$\frac{\frac{\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}-\frac{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}}{\sqrt{-a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*
Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/Sqrt[-(Sqr
t[c]*f) + Sqrt[-a]*g] - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]
*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x]
)])/Sqrt[Sqrt[c]*f + Sqrt[-a]*g])/(Sqrt[-a]*Sqrt[c])
```

IntegrateAlgebraic [C] time = 116.51, size = 1253, normalized size = 5.22

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (e^3*f^2*Sqrt[e/g]*g - 2*d*e^2*f*Sqrt[e/g]*g^2 + d^2*e*Sqrt[e/g]*g^3)*RootS
um[c*e^4*f^4 - 4*c*d*e^3*f^3*g + 6*c*d^2*e^2*f^2*g^2 - 4*c*d^3*e*f*g^3 + c*
d^4*g^4 - 4*c*e^3*f^3*g*#1^2 + 4*c*d*e^2*f^2*g^2*#1^2 + 4*c*d^2*e*f*g^3*#1^
2 - 4*c*d^3*g^4*#1^2 + 6*c*e^2*f^2*g^2*#1^4 + 4*c*d*e*f*g^3*#1^4 + 6*c*d^2*
g^4*#1^4 + 16*a*e^2*g^4*#1^4 - 4*c*e*f*g^3*#1^6 - 4*c*d*g^4*#1^6 + c*g^4*#1
^8 & , Log[-(Sqrt[e/g]*Sqrt[f + g*x]) + Sqrt[d - (e*f)/g + (e*(f + g*x))/g]
 - #1]/(c*e^3*f^3 - c*d*e^2*f^2*g - c*d^2*e*f*g^2 + c*d^3*g^3 - 3*c*e^2*f^2
*g*#1^2 - 2*c*d*e*f*g^2*#1^2 - 3*c*d^2*g^3*#1^2 - 8*a*e^2*g^3*#1^2 + 3*c*e*
f*g^2*#1^4 + 3*c*d*g^3*#1^4 - c*g^3*#1^6) & ] + 2*(e^2*f*Sqrt[e/g]*g^2 - d*
e*Sqrt[e/g]*g^3)*RootSum[c*e^4*f^4 - 4*c*d*e^3*f^3*g + 6*c*d^2*e^2*f^2*g^2
- 4*c*d^3*e*f*g^3 + c*d^4*g^4 - 4*c*e^3*f^3*g*#1^2 + 4*c*d*e^2*f^2*g^2*#1^2
 + 4*c*d^2*e*f*g^3*#1^2 - 4*c*d^3*g^4*#1^2 + 6*c*e^2*f^2*g^2*#1^4 + 4*c*d*e
*f*g^3*#1^4 + 6*c*d^2*g^4*#1^4 + 16*a*e^2*g^4*#1^4 - 4*c*e*f*g^3*#1^6 - 4*c
*d*g^4*#1^6 + c*g^4*#1^8 & , (Log[-(Sqrt[e/g]*Sqrt[f + g*x]) + Sqrt[d - (e*
f)/g + (e*(f + g*x))/g] - #1]*#1^2)/(-(c*e^3*f^3) + c*d*e^2*f^2*g + c*d^2*e
*f*g^2 - c*d^3*g^3 + 3*c*e^2*f^2*g*#1^2 + 2*c*d*e*f*g^2*#1^2 + 3*c*d^2*g^3*
#1^2 + 8*a*e^2*g^3*#1^2 - 3*c*e*f*g^2*#1^4 - 3*c*d*g^3*#1^4 + c*g^3*#1^6) &
 ] - e*Sqrt[e/g]*g^3*RootSum[c*e^4*f^4 - 4*c*d*e^3*f^3*g + 6*c*d^2*e^2*f^2*
```

```
g^2 - 4*c*d^3*e*f*g^3 + c*d^4*g^4 - 4*c*e^3*f^3*g*#1^2 + 4*c*d*e^2*f^2*g^2*
#1^2 + 4*c*d^2*e*f*g^3*#1^2 - 4*c*d^3*g^4*#1^2 + 6*c*e^2*f^2*g^2*#1^4 + 4*c
*d*e*f*g^3*#1^4 + 6*c*d^2*g^4*#1^4 + 16*a*e^2*g^4*#1^4 - 4*c*e*f*g^3*#1^6 -
 4*c*d*g^4*#1^6 + c*g^4*#1^8 & , (Log[-(Sqrt[e/g]*Sqrt[f + g*x]) + Sqrt[d -
 (e*f)/g + (e*(f + g*x))/g] - #1]*#1^4)/(-(c*e^3*f^3) + c*d*e^2*f^2*g + c*d
^2*e*f*g^2 - c*d^3*g^3 + 3*c*e^2*f^2*g*#1^2 + 2*c*d*e*f*g^2*#1^2 + 3*c*d^2*
g^3*#1^2 + 8*a*e^2*g^3*#1^2 - 3*c*e*f*g^2*#1^4 - 3*c*d*g^3*#1^4 + c*g^3*#1^
6) & ]
```

fricas [B] time = 9.63, size = 1913, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*
f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a
^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g + (a*c^2*f^2*g +
a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2
*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c
^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^
2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g - d*e*g^
2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^
2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*
c^2*f^2*g^2 + a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2
*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^
2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*e*
f^2 - c*d*f*g + (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*
g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f
*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^
2*c*g^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e
*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g
 + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x) - 1/4*sqrt(-(c
*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2
)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*lo
g(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g - (a*c^2*f^2*g + a^2*c*g^3)*sq
rt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*
g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*
c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2
 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*c^2
*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3
)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 +
 a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(
-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4
)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*e*f^2 - c*d*f*g
 - (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f
^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*
d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)
/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2
*(e^2*f*g - d*e*g^2)*x + (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*
f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(
a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 1387, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)
*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g
-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a
*c*d*g^2*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d
*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f
+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a*e*g^2*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)
^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-
(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/
2)))*c^2*d*f^2*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-l
n((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/
2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*c*e*f^2*((-a*e*g+c*d*f+(-a*c)^(1/2)
*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f+2*
(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1
/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*
c)^(1/2)))*a*c*d*g^2*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(
1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a*e*g^2*(-(a*e*g-c*d*f+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*
d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+
f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*
x-(-a*c)^(1/2)))*c^2*d*f^2*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
)/c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*c*e*f^2*(-(a*e*g-c*d*f+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*(-a*c)^(1/2))/((e*x+d)*(g*x+f))
^(1/2)/(c*f-(-a*c)^(1/2)*g)/(-a*c)^(1/2)/(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)/(c*f+(-a*c)^(1/2)*g)/((-a*e*g+c*d*f+(-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f)/c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+a\right)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/((a + c*x**2)*sqrt(f + g*x)), x)
```

3.414 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)} dx$

Optimal. Leaf size=230

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

Rubi [A] time = 0.20, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {912, 93, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt
[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]) - ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(S
qrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt
[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\,dx = \int\left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\sqrt{f+g}\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\sqrt{c}\,d+\sqrt{-a}\,e-\left(-\sqrt{c}\,f+\sqrt{-a}\,g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}\,d+\sqrt{-a}\,e-\left(\sqrt{}\right.}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\sqrt{\sqrt{c}\,f+\sqrt{-a}\,}}$$

Mathematica [A] time = 0.19, size = 225, normalized size = 0.98

$$\frac{-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (-(ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*
d) + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[-(Sq
rt[c]*f) + Sqrt[-a]*g])) - ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e
*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[Sqrt[c]*d + Sqrt[-
a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/Sqrt[-a]
```

IntegrateAlgebraic [C] time = 1.20, size = 310, normalized size = 1.35

$$\frac{\sqrt[4]{-1}\sqrt{\sqrt{a}\,e+i\sqrt{c}\,d}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt{f+gx}\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\sqrt{\sqrt{a}\,e+i\sqrt{c}\,d}\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}}\right)}{\sqrt{a}\sqrt{ae^2+cd^2}\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}}-\frac{\sqrt[4]{-1}\sqrt{\sqrt{a}\,e-i\sqrt{c}\,d}\tan^{-1}\left(\frac{\sqrt[4]{-1}\sqrt{f+gx}\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\sqrt{\sqrt{a}\,e-i\sqrt{c}\,d}\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}}\right)}{\sqrt{a}\sqrt{ae^2+cd^2}\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] -(((-1)^(1/4)*Sqrt[(-I)*Sqrt[c]*d + Sqrt[a]*e]*ArcTan[((-1)^(1/4)*Sqrt[c*d^
2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[(-I)*Sqrt[c]*d + Sqrt[a]*e]*Sqrt[Sqrt[c]*f
- I*Sqrt[a]*g]*Sqrt[d + e*x])]])/(Sqrt[a]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*f
 - I*Sqrt[a]*g])) + ((-1)^(1/4)*Sqrt[I*Sqrt[c]*d + Sqrt[a]*e]*ArcTanh[((-1)
^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[I*Sqrt[c]*d + Sqrt[a]*e]*Sq
rt[Sqrt[c]*f + I*Sqrt[a]*g]*Sqrt[d + e*x])]])/(Sqrt[a]*Sqrt[c*d^2 + a*e^2]*S
qrt[Sqrt[c]*f + I*Sqrt[a]*g])
```

fricas [B] time = 23.24, size = 4325, normalized size = 18.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3
*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*
c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c
*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^
2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g +
 d^2*g^2 + 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d^2*e +
 a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)
*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^
2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4
+ 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
 a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f - a*e*g + ((a*c^2
*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d
*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*
(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a
^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*
(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d
*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*s
qrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2
+ a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^
2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x) + 1/4*sqrt(-(c*d*f
- a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(
c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*
c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2
 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g + d^2*g^2 - 2*(c*d*
e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d^2*e + a^2*c*e^3)*f^3 +
(a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^
3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4
 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^
2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*s
qrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*
f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2
)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a
^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*
(e^2*f*g + d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e
^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c
*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 +
2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4
 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 +
 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x) - 1/4*sqrt(-(c*d*f - a*e*g - ((a*c^2*
d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*
e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(
a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4
*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^
3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g + d^2*g^2 + 2*(c*d*e*f^2 - a*d*e*g^2
+ (c*d^2 - a*e^2)*f*g + ((a*c^2*d^2*e + a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c
*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3
)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e
^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2
*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 +
 a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*
a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a
^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^
2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2
)*x - (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*
d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*
```

```
f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^
2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4
+ 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
 a^5*e^4)*g^4)))/x) + 1/4*sqrt(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f
^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)
/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^
3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e
^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((
e^2*f^2 + 2*d*e*f*g + d^2*g^2 - 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*
f*g + ((a*c^2*d^2*e + a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a
^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2
 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*
f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*
d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqr
t(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 +
a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2
+ (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)
*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x - (2*(c^2*d^3
+ a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f
^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 +
 a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4
 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^
2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 1415, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] -1/2*c^2*(ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a^2*e^2*g^2*(-(a*e*g-c*d
*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f+
2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^
(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-
a*c)^(1/2)))*a*c*d^2*g^2*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/
c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f+2*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(-(a*e*g-c*d*
f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x+2*c*d*f+2
*(-a*c)^(1/2)*e*g*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(
1/2)*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x-(-a
*c)^(1/2)))*c^2*d^2*f^2*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c
)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(
-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a^2*e^2*g^2*((-a*e*g+c*d*f
+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f-2*
```

```
(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1
/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*
c)^(1/2)))*a*c*d^2*g^2*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)
^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-
a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*((-a*e*g+c*d*f+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)-ln((c*d*g*x+c*e*f*x+2*c*d*f-2*(
-a*c)^(1/2)*e*g*x-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/
2)*(-(a*e*g-c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)*c)/(c*x+(-a*c
)^(1/2)))*c^2*d^2*f^2*((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^
(1/2))*(g*x+f)^(1/2)*(e*x+d)^(1/2)/((-a*e*g+c*d*f+(-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f)/c)^(1/2)/(c*f+(-a*c)^(1/2)*g)/(c*d+(-a*c)^(1/2)*e)/(-(a*e*g-c*d*f
+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f)/c)^(1/2)/(-a*c)^(1/2)/(c*f-(-a*c)^(1/2)
*g)/(c*d-(-a*c)^(1/2)*e)/((e*x+d)*(g*x+f))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+a\right)\sqrt{ex+d}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^2\right)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.415 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=354

$$-\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)}+\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-}}\right.}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)^{3/2}\sqrt{\sqrt{c}}}$$

Rubi [A] time = 0.61, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {912, 96, 93, 208}

$$-\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)}+\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)^{3/2}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)^{3/2}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] -((e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d +
 e*x])) + (e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*
Sqrt[d + e*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*
x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - S
qrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[c]*ArcTanh[(Sqrt[Sqr
t[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f +
g*x])]])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*
g])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+ex)^{3/}}\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(ef-d}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(ef-d}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(ef-d}$$

Mathematica [A] time = 0.70, size = 287, normalized size = 0.81

$$\frac{\frac{2\sqrt{-a}\,e^2\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)(dg-ef)} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\left(\sqrt{-a}\,e-\sqrt{c}\,d\right)^{3/2}\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)^{3/2}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((2*Sqrt[-a]*e^2*Sqrt[f + g*x])/((c*d^2 + a*e^2)*(-(e*f) + d*g)*Sqrt[d + e*
x]) + (Sqrt[c]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqr
t[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/((-(Sqrt[c]*d) + Sqrt[-a]*e)^
(3/2)*Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]) - (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f +
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/
((Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/Sqrt[-a]
```

IntegrateAlgebraic [C] time = 1.08, size = 393, normalized size = 1.11

$$\frac{2e^2\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)(dg-ef)} - \frac{i\sqrt{c}\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)^2\tan^{-1}\left(\frac{\sqrt{f+gx}\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\sqrt{-i\sqrt{a}\sqrt{c}\,dg+i\sqrt{a}\sqrt{c}\,ef-aeg-cdf}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^{3/2}\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}} + \frac{i\sqrt{c}\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)^2\tan^{-1}\left(\frac{\sqrt{f+gx}\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\sqrt{i\sqrt{a}\sqrt{c}\,dg-i\sqrt{a}\sqrt{c}\,ef-aeg-cdf}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^{3/2}\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x])/((c*d^2 + a*e^2)*(-(e*f) + d*g)*Sqrt[d + e*x]) - (I*S
qrt[c]*(Sqrt[c]*d - I*Sqrt[a]*e)^2*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x
])/(Sqrt[-(c*d*f) + I*Sqrt[a]*Sqrt[c]*e*f - I*Sqrt[a]*Sqrt[c]*d*g - a*e*g]*
Sqrt[d + e*x])]])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*Sqrt[-((Sqrt[c]*d - I*Sqrt[
a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + (I*Sqrt[c]*(Sqrt[c]*d + I*Sqrt[a]*e)^2
*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-(c*d*f) - I*Sqrt[a]*Sqrt
[c]*e*f + I*Sqrt[a]*Sqrt[c]*d*g - a*e*g]*Sqrt[d + e*x])]])/(Sqrt[a]*(c*d^2 +
 a*e^2)^(3/2)*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))])
```

fricas [B] time = 80.36, size = 11846, normalized size = 33.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(8*sqrt(e*x + d)*sqrt(g*x + f)*e^2 + ((c*d^3*e + a*d*e^3)*f - (c*d^4 +
 a*d^2*e^2)*g + ((c*d^2*e^2 + a*e^4)*f - (c*d^3*e + a*d*e^3)*g)*x)*sqrt(-((
c^3*d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*g + ((a*c^4*d^6 +
3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a
^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((9*c^5*d^4*e^2 - 6*
a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^
2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a
*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 +
15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^
12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*
c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*
a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4
*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2
+ 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3
*a^4*c*d^2*e^4 + a^5*e^6)*g^2))*log(-((3*c^3*d^2*e^2 - a*c^2*e^4)*f^2 + 4*(
c^3*d^3*e - a*c^2*d*e^3)*f*g + (c^3*d^4 - 3*a*c^2*d^2*e^2)*g^2 + 2*((3*c^4*
d^4*e - 4*a*c^3*d^2*e^3 + a^2*c^2*e^5)*f^2 + (c^4*d^5 - 10*a*c^3*d^3*e^2 +
5*a^2*c^2*d*e^4)*f*g - 2*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*g^2 - (2*(a*c^5*
d^7*e + 3*a^2*c^4*d^5*e^3 + 3*a^3*c^3*d^3*e^5 + a^4*c^2*d*e^7)*f^3 + (a*c^5
*d^8 + 2*a^2*c^4*d^6*e^2 - 2*a^4*c^2*d^2*e^6 - a^5*c*e^8)*f^2*g + 2*(a^2*c^
4*d^7*e + 3*a^3*c^3*d^5*e^3 + 3*a^4*c^2*d^3*e^5 + a^5*c*d*e^7)*f*g^2 + (a^2
*c^4*d^8 + 2*a^3*c^3*d^6*e^2 - 2*a^5*c*d^2*e^6 - a^6*e^8)*g^3)*sqrt(-((9*c^
5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*
d^3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2
*e^4)*g^2)/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*
c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 +
 2*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6
*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3
*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 +
15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))*sqrt(e*x + d)*sqrt
(g*x + f)*sqrt(-((c^3*d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*
g + ((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 +
(a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((
9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*
c^4*d^3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3
*d^2*e^4)*g^2)/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*
a^4*c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f
^4 + 2*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4
*d^6*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 +
(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^
6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/((a*c^4*d^6 +
3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a
^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)) + 2*((3*c^3*d^2*e^2 - a*c
^2*e^4)*f*g + (c^3*d^3*e - 3*a*c^2*d*e^3)*g^2)*x + (2*(c^5*d^7 + 3*a*c^4*d^
5*e^2 + 3*a^2*c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^3 + 2*(a*c^4*d^7 + 3*a^2*c^3*d
^5*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*f*g^2 + ((c^5*d^6*e + 3*a*c^4*d^4
*e^3 + 3*a^2*c^3*d^2*e^5 + a^3*c^2*e^7)*f^3 + (c^5*d^7 + 3*a*c^4*d^5*e^2 +
3*a^2*c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^2*g + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3
 + 3*a^3*c^2*d^2*e^5 + a^4*c*e^7)*f*g^2 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 +
3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*g^3)*x)*sqrt(-((9*c^5*d^4*e^2 - 6*a*c^4*d^
2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d*
e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c^8*d^1
2 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c
^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12 + 6*a
```

^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*d^4*
e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^4*c^5*
d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6
*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/x) - ((c*d^3*e + a*d*e^3)*f - (c*d^4 + a
*d^2*e^2)*g + ((c*d^2*e^2 + a*e^4)*f - (c*d^3*e + a*d*e^3)*g)*x)*sqrt(-((c^
3*d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*g + ((a*c^4*d^6 + 3*
a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3
*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((9*c^5*d^4*e^2 - 6*a*
c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*
c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c
^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15
*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12
 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^
3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^
4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e
^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 +
3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a
^4*c*d^2*e^4 + a^5*e^6)*g^2))*log(-((3*c^3*d^2*e^2 - a*c^2*e^4)*f^2 + 4*(c^
3*d^3*e - a*c^2*d*e^3)*f*g + (c^3*d^4 - 3*a*c^2*d^2*e^2)*g^2 - 2*((3*c^4*d^
4*e - 4*a*c^3*d^2*e^3 + a^2*c^2*e^5)*f^2 + (c^4*d^5 - 10*a*c^3*d^3*e^2 + 5*
a^2*c^2*d*e^4)*f*g - 2*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*g^2 - (2*(a*c^5*d^
7*e + 3*a^2*c^4*d^5*e^3 + 3*a^3*c^3*d^3*e^5 + a^4*c^2*d*e^7)*f^3 + (a*c^5*d
^8 + 2*a^2*c^4*d^6*e^2 - 2*a^4*c^2*d^2*e^6 - a^5*c*e^8)*f^2*g + 2*(a^2*c^4*
d^7*e + 3*a^3*c^3*d^5*e^3 + 3*a^4*c^2*d^3*e^5 + a^5*c*d*e^7)*f*g^2 + (a^2*c
^4*d^8 + 2*a^3*c^3*d^6*e^2 - 2*a^5*c*d^2*e^6 - a^6*e^8)*g^3)*sqrt(-((9*c^5*
d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^
3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e
^4)*g^2)/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^
5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2
*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e
^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c
^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15
*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))*sqrt(e*x + d)*sqrt(g
*x + f)*sqrt(-((c^3*d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*g
 + ((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a
^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((9*
c^5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^
4*d^3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d
^2*e^4)*g^2)/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^
4*c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4
 + 2*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d
^6*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a
^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6
 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/((a*c^4*d^6 + 3*
a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3
*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)) + 2*((3*c^3*d^2*e^2 - a*c^2
*e^4)*f*g + (c^3*d^3*e - 3*a*c^2*d*e^3)*g^2)*x + (2*(c^5*d^7 + 3*a*c^4*d^5*
e^2 + 3*a^2*c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^3 + 2*(a*c^4*d^7 + 3*a^2*c^3*d^5
*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*f*g^2 + ((c^5*d^6*e + 3*a*c^4*d^4*e
^3 + 3*a^2*c^3*d^2*e^5 + a^3*c^2*e^7)*f^3 + (c^5*d^7 + 3*a*c^4*d^5*e^2 + 3*
a^2*c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^2*g + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3 +
 3*a^3*c^2*d^2*e^5 + a^4*c*e^7)*f*g^2 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*
a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*g^3)*x)*sqrt(-((9*c^5*d^4*e^2 - 6*a*c^4*d^2*
e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d*e^
5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c^8*d^12
 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c^4
*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12 + 6*a^3
*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*d^4*e^
8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^4*c^5*d^

10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a
^8*c*d^2*e^10 + a^9*e^12)*g^4)))/x) + ((c*d^3*e + a*d*e^3)*f - (c*d^4 + a*d
^2*e^2)*g + ((c*d^2*e^2 + a*e^4)*f - (c*d^3*e + a*d*e^3)*g)*x)*sqrt(-((c^3*
d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*g - ((a*c^4*d^6 + 3*a^
2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c
^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((9*c^5*d^4*e^2 - 6*a*c^
4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^
3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c^8
*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a
^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12 +
 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*
d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^4*
c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8
 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*
a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4
*c*d^2*e^4 + a^5*e^6)*g^2))*log(-((3*c^3*d^2*e^2 - a*c^2*e^4)*f^2 + 4*(c^3*
d^3*e - a*c^2*d*e^3)*f*g + (c^3*d^4 - 3*a*c^2*d^2*e^2)*g^2 + 2*((3*c^4*d^4*
e - 4*a*c^3*d^2*e^3 + a^2*c^2*e^5)*f^2 + (c^4*d^5 - 10*a*c^3*d^3*e^2 + 5*a^
2*c^2*d*e^4)*f*g - 2*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*g^2 + (2*(a*c^5*d^7*
e + 3*a^2*c^4*d^5*e^3 + 3*a^3*c^3*d^3*e^5 + a^4*c^2*d*e^7)*f^3 + (a*c^5*d^8
 + 2*a^2*c^4*d^6*e^2 - 2*a^4*c^2*d^2*e^6 - a^5*c*e^8)*f^2*g + 2*(a^2*c^4*d^
7*e + 3*a^3*c^3*d^5*e^3 + 3*a^4*c^2*d^3*e^5 + a^5*c*d*e^7)*f*g^2 + (a^2*c^4
*d^8 + 2*a^3*c^3*d^6*e^2 - 2*a^5*c*d^2*e^6 - a^6*e^8)*g^3)*sqrt(-((9*c^5*d^
4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*
e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4
)*g^2)/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*
d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(
a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6
 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6
*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a
^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))*sqrt(e*x + d)*sqrt(g*x
 + f)*sqrt(-((c^3*d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*g -
((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2
*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((9*c^
5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*
d^3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2
*e^4)*g^2)/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*
c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 +
 2*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6
*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3
*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 +
15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/((a*c^4*d^6 + 3*a^
2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c
^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)) + 2*((3*c^3*d^2*e^2 - a*c^2*e
^4)*f*g + (c^3*d^3*e - 3*a*c^2*d*e^3)*g^2)*x - (2*(c^5*d^7 + 3*a*c^4*d^5*e^
2 + 3*a^2*c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^3 + 2*(a*c^4*d^7 + 3*a^2*c^3*d^5*e
^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*f*g^2 + ((c^5*d^6*e + 3*a*c^4*d^4*e^3
 + 3*a^2*c^3*d^2*e^5 + a^3*c^2*e^7)*f^3 + (c^5*d^7 + 3*a*c^4*d^5*e^2 + 3*a^
2*c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^2*g + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3 + 3
*a^3*c^2*d^2*e^5 + a^4*c*e^7)*f*g^2 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*a^
3*c^2*d^3*e^4 + a^4*c*d*e^6)*g^3)*x)*sqrt(-((9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^
4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d*e^5)
*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c^8*d^12 +
6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c^4*d
^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12 + 6*a^3*c
^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*d^4*e^8
+ 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10
*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8
*c*d^2*e^10 + a^9*e^12)*g^4)))/x) - ((c*d^3*e + a*d*e^3)*f - (c*d^4 + a*d^2

```
*e^2)*g + ((c*d^2*e^2 + a*e^4)*f - (c*d^3*e + a*d*e^3)*g)*x)*sqrt(-((c^3*d^
3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*g - ((a*c^4*d^6 + 3*a^2*
c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2
*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((9*c^5*d^4*e^2 - 6*a*c^4*
d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*
d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c^8*d
^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5
*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12 + 6
*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*d^
4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^4*c^
5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 +
 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^
3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c
*d^2*e^4 + a^5*e^6)*g^2))*log(-((3*c^3*d^2*e^2 - a*c^2*e^4)*f^2 + 4*(c^3*d^
3*e - a*c^2*d*e^3)*f*g + (c^3*d^4 - 3*a*c^2*d^2*e^2)*g^2 - 2*((3*c^4*d^4*e
 - 4*a*c^3*d^2*e^3 + a^2*c^2*e^5)*f^2 + (c^4*d^5 - 10*a*c^3*d^3*e^2 + 5*a^2*
c^2*d*e^4)*f*g - 2*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*g^2 + (2*(a*c^5*d^7*e
 + 3*a^2*c^4*d^5*e^3 + 3*a^3*c^3*d^3*e^5 + a^4*c^2*d*e^7)*f^3 + (a*c^5*d^8 +
 2*a^2*c^4*d^6*e^2 - 2*a^4*c^2*d^2*e^6 - a^5*c*e^8)*f^2*g + 2*(a^2*c^4*d^7*
e + 3*a^3*c^3*d^5*e^3 + 3*a^4*c^2*d^3*e^5 + a^5*c*d*e^7)*f*g^2 + (a^2*c^4*d
^8 + 2*a^3*c^3*d^6*e^2 - 2*a^5*c*d^2*e^6 - a^6*e^8)*g^3)*sqrt(-((9*c^5*d^4*
e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^
3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*
g^2)/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^
6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^
2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 +
 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d
^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7
*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(-((c^3*d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*g - ((
a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c
^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((9*c^5*
d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^
3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e
^4)*g^2)/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^
5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2
*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e
^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c
^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15
*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/((a*c^4*d^6 + 3*a^2*
c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2
*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)) + 2*((3*c^3*d^2*e^2 - a*c^2*e^4
)*f*g + (c^3*d^3*e - 3*a*c^2*d*e^3)*g^2)*x - (2*(c^5*d^7 + 3*a*c^4*d^5*e^2
+ 3*a^2*c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^3 + 2*(a*c^4*d^7 + 3*a^2*c^3*d^5*e^2
 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*f*g^2 + ((c^5*d^6*e + 3*a*c^4*d^4*e^3 +
 3*a^2*c^3*d^2*e^5 + a^3*c^2*e^7)*f^3 + (c^5*d^7 + 3*a*c^4*d^5*e^2 + 3*a^2*
c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^2*g + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3 + 3*a
^3*c^2*d^2*e^5 + a^4*c*e^7)*f*g^2 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*a^3*
c^2*d^3*e^4 + a^4*c*d*e^6)*g^3)*x)*sqrt(-((9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^4
+ a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d*e^5)*f
*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c^8*d^12 + 6*
a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c^4*d^4
*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12 + 6*a^3*c^6
*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*d^4*e^8 +
6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e
^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c
*d^2*e^10 + a^9*e^12)*g^4)))/x))/((c*d^3*e + a*d*e^3)*f - (c*d^4 + a*d^2*e^
2)*g + ((c*d^2*e^2 + a*e^4)*f - (c*d^3*e + a*d*e^3)*g)*x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [**B**] time = 0.09, size = 10977, normalized size = 31.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+a\right)(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^2\right)(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

3.416 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=625

$$\frac{2\sqrt{d+ex}\,(ef-dg)}{\sqrt{f+gx}\,\left(ag^2+cf^2\right)}-\frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{\sqrt{g}\,\left(ag^2+cf^2\right)}-\frac{\sqrt{e}\,\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d}}{\sqrt{e}\,\sqrt{f}}\right.}{\sqrt{-a}\,\sqrt{c}\,\sqrt{g}\,\left(ag^2+cf^2\right)}$$

Rubi [A] time = 2.53, antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {908, 47, 63, 217, 206, 6725, 105, 93, 208}

$$\frac{2\sqrt{d+ex}\,(ef-dg)}{\sqrt{f+gx}\,\left(ag^2+cf^2\right)}-\frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{\sqrt{g}\,\left(ag^2+cf^2\right)}-\frac{\sqrt{e}\,\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{g}\,\left(ag^2+cf^2\right)}+\frac{\sqrt{e}\,\left(\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{g}\,\left(ag^2+cf^2\right)}+\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\left(ag^2+cf^2\right)}-\frac{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\left(\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,\left(ag^2+cf^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (2*(e*f - d*g)*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[e]*
(e*f - d*g)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt
[g]*(c*f^2 + a*g^2)) - (Sqrt[e]*(c*d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*
g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqr
t[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*
(e*f - d*g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqr
t[-a]*Sqrt[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(c*d
*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-
a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-
a]*Sqrt[c]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - (Sqrt[Sqrt[c]*d
+ Sqrt[-a]*e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[
Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
 + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \frac{\int \frac{\sqrt{d+ex}\,(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{cf^2+ag^2} - \frac{(g(ef-dg))\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}}\,dx}{cf^2+ag^2}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{\int \left(\frac{\left(-a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf+aeg)\right)\sqrt{d+ex}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{f+gx}} + \frac{\left(a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf \cdots}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{\cdots}}\right.}{cf^2+ag^2}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{(2(ef-dg))\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d+ex}\right)}{cf^2+ag^2} - \frac{\left(cd \cdots\right.}{\cdots}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{(2(ef-dg))\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{cf^2+ag^2} - \frac{\left(e\left(cdf+a \cdots\right.\right.}{\cdots}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{\sqrt{g}\left(cf^2+ag^2\right)} - \frac{\left(cdf+aeg-\sqrt{-a}\,\cdots\right.}{\cdots}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{\sqrt{g}\left(cf^2+ag^2\right)} + \frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\left(cd \cdots\right.}{\cdots}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{\sqrt{g}\left(cf^2+ag^2\right)} - \frac{\sqrt{e}\,\left(cdf+aeg-\sqrt{\cdots}\right.}{\sqrt{-a \cdots}}$$

Mathematica [A] time = 0.73, size = 336, normalized size = 0.54

$$-\left(\left(\frac{d}{\sqrt{-a}}-\frac{e}{\sqrt{c}}\right)\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)}+\frac{\sqrt{\sqrt{-a}e-\sqrt{c}d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\left(\sqrt{-a}g-\sqrt{c}f\right)^{3/2}}\right)\right)-\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)}-\frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] -((d/Sqrt[-a] - e/Sqrt[c])*(Sqrt[d + e*x]/((Sqrt[c]*f - Sqrt[-a]*g)*Sqrt[f
+ g*x]) + (Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqr
t[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(
-(Sqrt[c]*f) + Sqrt[-a]*g)^(3/2))) - ((a*d)/(-a)^(3/2) - e/Sqrt[c])*(Sqrt[d
 + e*x]/((Sqrt[c]*f + Sqrt[-a]*g)*Sqrt[f + g*x]) - (Sqrt[Sqrt[c]*d + Sqrt[-
a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d
+ Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

IntegrateAlgebraic [C] time = 92.44, size = 1541, normalized size = 2.47

result too large to display

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (2*(e*f - d*g)*Sqrt[d - (e*f)/g + (e*(f + g*x))/g])/((c*f^2 + a*g^2)*Sqrt[f
 + g*x]) + ((2*c*d*e^3*f^3*Sqrt[e/g]*g - 5*c*d^2*e^2*f^2*Sqrt[e/g]*g^2 + a*
```

```
e^4*f^2*Sqrt[e/g]*g^2 + 4*c*d^3*e*f*Sqrt[e/g]*g^3 - 2*a*d*e^3*f*Sqrt[e/g]*g
^3 - c*d^4*Sqrt[e/g]*g^4 + a*d^2*e^2*Sqrt[e/g]*g^4)*RootSum[c*e^4*f^4 - 4*c
*d*e^3*f^3*g + 6*c*d^2*e^2*f^2*g^2 - 4*c*d^3*e*f*g^3 + c*d^4*g^4 - 4*c*e^3*
f^3*g*#1^2 + 4*c*d*e^2*f^2*g^2*#1^2 + 4*c*d^2*e*f*g^3*#1^2 - 4*c*d^3*g^4*#1
^2 + 6*c*e^2*f^2*g^2*#1^4 + 4*c*d*e*f*g^3*#1^4 + 6*c*d^2*g^4*#1^4 + 16*a*e^
2*g^4*#1^4 - 4*c*e*f*g^3*#1^6 - 4*c*d*g^4*#1^6 + c*g^4*#1^8 & , Log[-(Sqrt[
e/g]*Sqrt[f + g*x]) + Sqrt[d - (e*f)/g + (e*(f + g*x))/g] - #1]/(c*e^3*f^3
- c*d*e^2*f^2*g - c*d^2*e*f*g^2 + c*d^3*g^3 - 3*c*e^2*f^2*g*#1^2 - 2*c*d*e*
f*g^2*#1^2 - 3*c*d^2*g^3*#1^2 - 8*a*e^2*g^3*#1^2 + 3*c*e*f*g^2*#1^4 + 3*c*d
*g^3*#1^4 - c*g^3*#1^6) & ])/(c*f^2 + a*g^2) + (2*(2*c*d*e^2*f^2*Sqrt[e/g]*
g^2 - c*d^2*e*f*Sqrt[e/g]*g^3 + 3*a*e^3*f*Sqrt[e/g]*g^3 - c*d^3*Sqrt[e/g]*g
^4 - 3*a*d*e^2*Sqrt[e/g]*g^4)*RootSum[c*e^4*f^4 - 4*c*d*e^3*f^3*g + 6*c*d^2
*e^2*f^2*g^2 - 4*c*d^3*e*f*g^3 + c*d^4*g^4 - 4*c*e^3*f^3*g*#1^2 + 4*c*d*e^2
*f^2*g^2*#1^2 + 4*c*d^2*e*f*g^3*#1^2 - 4*c*d^3*g^4*#1^2 + 6*c*e^2*f^2*g^2*#
1^4 + 4*c*d*e*f*g^3*#1^4 + 6*c*d^2*g^4*#1^4 + 16*a*e^2*g^4*#1^4 - 4*c*e*f*g
^3*#1^6 - 4*c*d*g^4*#1^6 + c*g^4*#1^8 & , (Log[-(Sqrt[e/g]*Sqrt[f + g*x]) +
 Sqrt[d - (e*f)/g + (e*(f + g*x))/g] - #1]*#1^2)/(-(c*e^3*f^3) + c*d*e^2*f^
2*g + c*d^2*e*f*g^2 - c*d^3*g^3 + 3*c*e^2*f^2*g*#1^2 + 2*c*d*e*f*g^2*#1^2 +
 3*c*d^2*g^3*#1^2 + 8*a*e^2*g^3*#1^2 - 3*c*e*f*g^2*#1^4 - 3*c*d*g^3*#1^4 +
c*g^3*#1^6) & ])/(c*f^2 + a*g^2) + ((-2*c*d*e*f*Sqrt[e/g]*g^3 + c*d^2*Sqrt[
e/g]*g^4 - a*e^2*Sqrt[e/g]*g^4)*RootSum[c*e^4*f^4 - 4*c*d*e^3*f^3*g + 6*c*d
^2*e^2*f^2*g^2 - 4*c*d^3*e*f*g^3 + c*d^4*g^4 - 4*c*e^3*f^3*g*#1^2 + 4*c*d*e
^2*f^2*g^2*#1^2 + 4*c*d^2*e*f*g^3*#1^2 - 4*c*d^3*g^4*#1^2 + 6*c*e^2*f^2*g^2
*#1^4 + 4*c*d*e*f*g^3*#1^4 + 6*c*d^2*g^4*#1^4 + 16*a*e^2*g^4*#1^4 - 4*c*e*f
*g^3*#1^6 - 4*c*d*g^4*#1^6 + c*g^4*#1^8 & , (Log[-(Sqrt[e/g]*Sqrt[f + g*x])
 + Sqrt[d - (e*f)/g + (e*(f + g*x))/g] - #1]*#1^4)/(-(c*e^3*f^3) + c*d*e^2*
f^2*g + c*d^2*e*f*g^2 - c*d^3*g^3 + 3*c*e^2*f^2*g*#1^2 + 2*c*d*e*f*g^2*#1^2
 + 3*c*d^2*g^3*#1^2 + 8*a*e^2*g^3*#1^2 - 3*c*e*f*g^2*#1^4 - 3*c*d*g^3*#1^4
+ c*g^3*#1^6) & ])/(c*f^2 + a*g^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.07, size = 8264, normalized size = 13.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\left(f+g\,x\right)^{3/2}\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

```
[Out] Timed out
```

3.417 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal. Leaf size=351

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)}+\frac{\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\left(ag^2+cf^2\right)}-\frac{\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg\right.}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}}}$$

Rubi [A] time = 1.81, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {908, 37, 6725, 93, 208}

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)}+\frac{\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\left(ag^2+cf^2\right)}-\frac{\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}\left(ag^2+cf^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + ((c*d*f + a*e*g - Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - ((c*d*f +
a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\left(a+cx^2\right)}dx = \frac{\int \frac{cdf+aeg+c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}dx}{cf^2+ag^2} - \frac{(g(ef-dg))\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}}dx}{cf^2+ag^2}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{\int \left(\frac{-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)}{cf^2+ag^2}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+}}}{2\sqrt{-a}\left(cf^2+ag^2\right)}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{}{-\sqrt{c}d+\sqrt{-a}e-}\right.}{\sqrt{-a}\left(cf^2+ag^2\right)}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right.}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\left(cf^2+ag^2\right)}$$

Mathematica [A] time = 0.64, size = 265, normalized size = 0.75

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)} + \frac{a\sqrt{\sqrt{-a}e-\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{(-a)^{3/2}\left(\sqrt{-a}g-\sqrt{c}f\right)^{3/2}} + \frac{a\sqrt{\sqrt{-a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{(-a)^{3/2}\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + (a*Sqrt[-(Sqrt[c]*d)
 + Sqrt[-a]*e]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqr
t[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/((-a)^(3/2)*(-(Sqrt[c]*f) + S
qrt[-a]*g)^(3/2)) + (a*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f
 + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]
)/((-a)^(3/2)*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

IntegrateAlgebraic [C] time = 1.28, size = 401, normalized size = 1.14

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)} + \frac{\left(\sqrt{a}e-i\sqrt{c}d\right)\left(\sqrt{c}f-i\sqrt{a}g\right)^2\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{ag^2+cf^2}}{\sqrt{f+gx}\sqrt{i\sqrt{a}\sqrt{c}dg-i\sqrt{a}\sqrt{c}ef-aeg-cdf}}\right)}{\sqrt{a}\left(ag^2+cf^2\right)^{3/2}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}} + \frac{\left(\sqrt{a}e+i\sqrt{c}d\right)\left(\sqrt{c}f+i\sqrt{a}g\right)^2\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{ag^2+cf^2}}{\sqrt{f+gx}\sqrt{-i\sqrt{a}\sqrt{c}dg+i\sqrt{a}\sqrt{c}ef-aeg-cdf}}\right)}{\sqrt{a}\left(ag^2+cf^2\right)^{3/2}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + ((I*Sqrt[c]*d + Sqrt
[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)^2*ArcTan[(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x
])/(Sqrt[-(c*d*f) + I*Sqrt[a]*Sqrt[c]*e*f - I*Sqrt[a]*Sqrt[c]*d*g - a*e*g]*
Sqrt[f + g*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*S
qrt[a]*g))]*(c*f^2 + a*g^2)^(3/2)) + (((-I)*Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[c]
```

```
*f - I*Sqrt[a]*g)^2*ArcTan[(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x])/(Sqrt[-(c*d*
f) - I*Sqrt[a]*Sqrt[c]*e*f + I*Sqrt[a]*Sqrt[c]*d*g - a*e*g]*Sqrt[f + g*x])]
)/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*(c*
f^2 + a*g^2)^(3/2))
```

fricas [B] time = 66.39, size = 5844, normalized size = 16.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((c*f^3 + a*f*g^2 + (c*f^2*g + a*g^3)*x)*sqrt(-(c^2*d*f^3 + 3*a*c*e*f^
2*g - 3*a*c*d*f*g^2 - a^2*e*g^3 + (a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*
f^2*g^4 + a^4*g^6)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*
g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g
^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^
2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*
f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a
^4*g^6))*log((c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 - 3*(c*
d^2 + a*e^2)*f^2*g^2 + 2*(c^2*e*f^5 - 3*c^2*d*f^4*g - 4*a*c*e*f^3*g^2 + 4*a
*c*d*f^2*g^3 + 3*a^2*e*f*g^4 - a^2*d*g^5 + 2*(a*c^3*f^7*g + 3*a^2*c^2*f^5*g
^3 + 3*a^3*c*f^3*g^5 + a^4*f*g^7)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20
*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a
*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*
a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4
*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c^
2*d*f^3 + 3*a*c*e*f^2*g - 3*a*c*d*f*g^2 - a^2*e*g^3 + (a*c^3*f^6 + 3*a^2*c^
2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g
 + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2
- 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12
 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^
2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 +
 3*a^3*c*f^2*g^4 + a^4*g^6)) + 2*(c*e^2*f^3*g - 3*c*d*e*f^2*g^2 - 3*a*e^2*f
*g^3 + a*d*e*g^4)*x - (2*c^3*d*f^7 + 6*a*c^2*d*f^5*g^2 + 6*a^2*c*d*f^3*g^4
+ 2*a^3*d*f*g^6 + (c^3*e*f^7 + c^3*d*f^6*g + 3*a*c^2*e*f^5*g^2 + 3*a*c^2*d*
f^4*g^3 + 3*a^2*c*e*f^3*g^4 + 3*a^2*c*d*f^2*g^5 + a^3*e*f*g^6 + a^3*d*g^7)*
x)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*
e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*
d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f
^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g
^12)))/x) - (c*f^3 + a*f*g^2 + (c*f^2*g + a*g^3)*x)*sqrt(-(c^2*d*f^3 + 3*a*
c*e*f^2*g - 3*a*c*d*f*g^2 - a^2*e*g^3 + (a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*
a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*
e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)
*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f
^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*
a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g
^4 + a^4*g^6))*log((c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 -
 3*(c*d^2 + a*e^2)*f^2*g^2 - 2*(c^2*e*f^5 - 3*c^2*d*f^4*g - 4*a*c*e*f^3*g^2
 + 4*a*c*d*f^2*g^3 + 3*a^2*e*f*g^4 - a^2*d*g^5 + 2*(a*c^3*f^7*g + 3*a^2*c^2
*f^5*g^3 + 3*a^3*c*f^3*g^5 + a^4*f*g^7)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*
g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2
 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^1
2 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c
^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))*sqrt(e*x + d)*sqrt(g*x + f)*sqr
t(-(c^2*d*f^3 + 3*a*c*e*f^2*g - 3*a*c*d*f*g^2 - a^2*e*g^3 + (a*c^3*f^6 + 3*
a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e
*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^
3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^
```

6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*
a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4
*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)) + 2*(c*e^2*f^3*g - 3*c*d*e*f^2*g^2 - 3*a
*e^2*f*g^3 + a*d*e*g^4)*x - (2*c^3*d*f^7 + 6*a*c^2*d*f^5*g^2 + 6*a^2*c*d*f^
3*g^4 + 2*a^3*d*f*g^6 + (c^3*e*f^7 + c^3*d*f^6*g + 3*a*c^2*e*f^5*g^2 + 3*a*
c^2*d*f^4*g^3 + 3*a^2*c*e*f^3*g^4 + 3*a^2*c*d*f^2*g^5 + a^3*e*f*g^6 + a^3*d
*g^7)*x)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^
2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*
a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3
*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 +
 a^7*g^12)))/x) + (c*f^3 + a*f*g^2 + (c*f^2*g + a*g^3)*x)*sqrt(-(c^2*d*f^3
+ 3*a*c*e*f^2*g - 3*a*c*d*f*g^2 - a^2*e*g^3 - (a*c^3*f^6 + 3*a^2*c^2*f^4*g^
2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*
c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^
2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2
*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^
8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c
*f^2*g^4 + a^4*g^6))*log((c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2
g^4 - 3(c*d^2 + a*e^2)*f^2*g^2 + 2*(c^2*e*f^5 - 3*c^2*d*f^4*g - 4*a*c*e*f
^3*g^2 + 4*a*c*d*f^2*g^3 + 3*a^2*e*f*g^4 - a^2*d*g^5 - 2*(a*c^3*f^7*g + 3*a
^2*c^2*f^5*g^3 + 3*a^3*c*f^3*g^5 + a^4*f*g^7)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*
e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c
^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c
^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15
*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))*sqrt(e*x + d)*sqrt(g*x +
f)*sqrt(-(c^2*d*f^3 + 3*a*c*e*f^2*g - 3*a*c*d*f*g^2 - a^2*e*g^3 - (a*c^3*f^
6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(c^3*e^2*f^6 - 6*c
^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3
*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)
/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6
 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c
^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)) + 2*(c*e^2*f^3*g - 3*c*d*e*f^2*g^2
 - 3*a*e^2*f*g^3 + a*d*e*g^4)*x + (2*c^3*d*f^7 + 6*a*c^2*d*f^5*g^2 + 6*a^2*
c*d*f^3*g^4 + 2*a^3*d*f*g^6 + (c^3*e*f^7 + c^3*d*f^6*g + 3*a*c^2*e*f^5*g^2
+ 3*a*c^2*d*f^4*g^3 + 3*a^2*c*e*f^3*g^4 + 3*a^2*c*d*f^2*g^5 + a^3*e*f*g^6 +
 a^3*d*g^7)*x)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3
- 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 -
 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 +
15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*
g^10 + a^7*g^12)))/x) - (c*f^3 + a*f*g^2 + (c*f^2*g + a*g^3)*x)*sqrt(-(c^2*
d*f^3 + 3*a*c*e*f^2*g - 3*a*c*d*f*g^2 - a^2*e*g^3 - (a*c^3*f^6 + 3*a^2*c^2*
f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g +
 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 -
2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 +
 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*
f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3
*a^3*c*f^2*g^4 + a^4*g^6))*log((c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 +
 a*d^2*g^4 - 3*(c*d^2 + a*e^2)*f^2*g^2 - 2*(c^2*e*f^5 - 3*c^2*d*f^4*g - 4*a
*c*e*f^3*g^2 + 4*a*c*d*f^2*g^3 + 3*a^2*e*f*g^4 - a^2*d*g^5 - 2*(a*c^3*f^7*g
 + 3*a^2*c^2*f^5*g^3 + 3*a^3*c*f^3*g^5 + a^4*f*g^7)*sqrt(-(c^3*e^2*f^6 - 6*
c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 +
3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4
)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^
6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(-(c^2*d*f^3 + 3*a*c*e*f^2*g - 3*a*c*d*f*g^2 - a^2*e*g^3 - (a*
c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(c^3*e^2*f^6
 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g
^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^
2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f

```
^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3
*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)) + 2*(c*e^2*f^3*g - 3*c*d*e*f
^2*g^2 - 3*a*e^2*f*g^3 + a*d*e*g^4)*x + (2*c^3*d*f^7 + 6*a*c^2*d*f^5*g^2 +
6*a^2*c*d*f^3*g^4 + 2*a^3*d*f*g^6 + (c^3*e*f^7 + c^3*d*f^6*g + 3*a*c^2*e*f^
5*g^2 + 3*a*c^2*d*f^4*g^3 + 3*a^2*c*e*f^3*g^4 + 3*a^2*c*d*f^2*g^5 + a^3*e*f
*g^6 + a^3*d*g^7)*x)*sqrt(-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^
3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4
*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*
g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*
c*f^2*g^10 + a^7*g^12)))/x) + 8*sqrt(e*x + d)*sqrt(g*x + f)*g)/(c*f^3 + a*f
*g^2 + (c*f^2*g + a*g^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.06, size = 5383, normalized size = 15.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+a\right)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^{3/2}\,\left(c\,x^2+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
[Out] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Timed out
```

3.418 $\int \frac{1}{\sqrt{d+ex}\,(f+gx)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=354

$$\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)}-\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\cdots}}{\sqrt{f+gx}\sqrt{\sqrt{\cdots}}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}\cdots\right.}$$

Rubi [A] time = 0.76, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {912, 96, 93, 208}

$$\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)}-\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)*Sqrt[f + g
*x]) - (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)*Sqr
t[f + g*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])
/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d -
Sqrt[-a]*e]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c
]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x
])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,(f+gx)^{3/2}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,(f+\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx}{2\sqrt{-a}}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)(ef-dg)\cdot}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)(ef-dg)\cdot}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)(ef-dg)\cdot}$$

Mathematica [A] time = 0.78, size = 287, normalized size = 0.81

$$\frac{\frac{2\sqrt{-a}\,g^2\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}\left(\sqrt{-a}\,g-\sqrt{c}\,f\right)^{3/2}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\left(\sqrt{-a}\,g+\sqrt{c}\,f\right)^{3/2}}}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] ((2*Sqrt[-a]*g^2*Sqrt[d + e*x])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])
 + (Sqrt[c]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-
(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]
*(-(Sqrt[c]*f) + Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f + Sq
rt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2)))/Sqrt[-a]
```

IntegrateAlgebraic [C] time = 1.05, size = 393, normalized size = 1.11

$$\frac{2g^2\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)} + \frac{i\sqrt{c}\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)^2\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{ag^2+cf^2}}{\sqrt{f+gx}\sqrt{-i\sqrt{a}\sqrt{c}\,dg+i\sqrt{a}\sqrt{c}\,ef-aeg-cdf}}\right)}{\sqrt{a}\left(ag^2+cf^2\right)^{3/2}\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}} - \frac{i\sqrt{c}\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)^2\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{ag^2+cf^2}}{\sqrt{f+gx}\sqrt{i\sqrt{a}\sqrt{c}\,dg-i\sqrt{a}\sqrt{c}\,ef-aeg-cdf}}\right)}{\sqrt{a}\left(ag^2+cf^2\right)^{3/2}\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (2*g^2*Sqrt[d + e*x])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) + (I*Sqrt
[c]*(Sqrt[c]*f + I*Sqrt[a]*g)^2*ArcTan[(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x])/
(Sqrt[-(c*d*f) + I*Sqrt[a]*Sqrt[c]*e*f - I*Sqrt[a]*Sqrt[c]*d*g - a*e*g]*Sqr
t[f + g*x])]])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt
[a]*g))]*(c*f^2 + a*g^2)^(3/2)) - (I*Sqrt[c]*(Sqrt[c]*f - I*Sqrt[a]*g)^2*Ar
cTan[(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x])/(Sqrt[-(c*d*f) - I*Sqrt[a]*Sqrt[c]
*e*f + I*Sqrt[a]*Sqrt[c]*d*g - a*e*g]*Sqrt[f + g*x])]])/(Sqrt[a]*Sqrt[-((Sqr
t[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*(c*f^2 + a*g^2)^(3/2))
```

fricas [B] time = 126.49, size = 12028, normalized size = 33.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(8*sqrt(e*x + d)*sqrt(g*x + f)*g^2 - (c*e*f^4 - c*d*f^3*g + a*e*f^2*g^2
 - a*d*f*g^3 + (c*e*f^3*g - c*d*f^2*g^2 + a*e*f*g^3 - a*d*g^4)*x)*sqrt(-(c^
3*d*f^3 - 3*a*c^2*e*f^2*g - 3*a*c^2*d*f*g^2 + a^2*c*e*g^3 + ((a*c^4*d^2 + a
^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^2 +
a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)*sqrt(-(c^5*e^2*f^6 + 6*c^5*
d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*g^5 + a^2*c^3*d^2*g^6 +
3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^2*c^3*e^2)*f^2*g
^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^12 + 6*(a^2*c^7*d^4 +
2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3*c^6*d^4 + 2*a^4*c^5*d^2
*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5*c^4*d^2*e^2 + a^6*c^3
*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 + a^7*c^2*e^4)*f^4*g^8
+ 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g^10 + (a^7*c^2*d^4 +
 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/((a*c^4*d^2 + a^2*c^3*e^2)*f^6 + 3*(a^2
*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^
4*c*d^2 + a^5*e^2)*g^6))*log(-(c^3*e^2*f^4 + 4*c^3*d*e*f^3*g - 4*a*c^2*d*e*
f*g^3 - a*c^2*d^2*g^4 + 3*(c^3*d^2 - a*c^2*e^2)*f^2*g^2 + 2*(c^4*d*e*f^5 -
10*a*c^3*d*e*f^3*g^2 + 5*a^2*c^2*d*e*f*g^4 + a^2*c^2*d^2*g^5 + (3*c^4*d^2 -
 2*a*c^3*e^2)*f^4*g - 2*(2*a*c^3*d^2 - 3*a^2*c^2*e^2)*f^2*g^3 - ((a*c^5*d^2
*e + a^2*c^4*e^3)*f^8 + 2*(a*c^5*d^3 + a^2*c^4*d*e^2)*f^7*g + 2*(a^2*c^4*d^
2*e + a^3*c^3*e^3)*f^6*g^2 + 6*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f^5*g^3 + 6*(a
^3*c^3*d^3 + a^4*c^2*d*e^2)*f^3*g^5 - 2*(a^4*c^2*d^2*e + a^5*c*e^3)*f^2*g^6
 + 2*(a^4*c^2*d^3 + a^5*c*d*e^2)*f*g^7 - (a^5*c*d^2*e + a^6*e^3)*g^8)*sqrt(
-(c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*g^
5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2
- 3*a^2*c^3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^
12 + 6*(a^2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3*c
^6*d^4 + 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5
*c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 +
 a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2
*g^10 + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))*sqrt(e*x + d)*sqr
t(g*x + f)*sqrt(-(c^3*d*f^3 - 3*a*c^2*e*f^2*g - 3*a*c^2*d*f*g^2 + a^2*c*e*g
^3 + ((a*c^4*d^2 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2
 + 3*(a^3*c^2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)*sqrt(-(
c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*g^5
+ a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 -
3*a^2*c^3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^12
 + 6*(a^2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3*c^6
*d^4 + 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5*c
^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 + a
^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g
^10 + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/((a*c^4*d^2 + a^2*c
^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^2 + a^4*
c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)) + 2*(c^3*e^2*f^3*g + 3*c^3*d*e
*f^2*g^2 - 3*a*c^2*e^2*f*g^3 - a*c^2*d*e*g^4)*x + (2*(c^5*d^3 + a*c^4*d*e^2
)*f^7 + 6*(a*c^4*d^3 + a^2*c^3*d*e^2)*f^5*g^2 + 6*(a^2*c^3*d^3 + a^3*c^2*d*
e^2)*f^3*g^4 + 2*(a^3*c^2*d^3 + a^4*c*d*e^2)*f*g^6 + ((c^5*d^2*e + a*c^4*e^
3)*f^7 + (c^5*d^3 + a*c^4*d*e^2)*f^6*g + 3*(a*c^4*d^2*e + a^2*c^3*e^3)*f^5*
g^2 + 3*(a*c^4*d^3 + a^2*c^3*d*e^2)*f^4*g^3 + 3*(a^2*c^3*d^2*e + a^3*c^2*e^
3)*f^3*g^4 + 3*(a^2*c^3*d^3 + a^3*c^2*d*e^2)*f^2*g^5 + (a^3*c^2*d^2*e + a^4
*c*e^3)*f*g^6 + (a^3*c^2*d^3 + a^4*c*d*e^2)*g^7)*x)*sqrt(-(c^5*e^2*f^6 + 6*
c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*g^5 + a^2*c^3*d^2*g^
6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^2*c^3*e^2)*f
^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^12 + 6*(a^2*c^7*d^
4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3*c^6*d^4 + 2*a^4*c^5
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*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5*c^4*d^2*e^2 + a^6
*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 + a^7*c^2*e^4)*f^4*
g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g^10 + (a^7*c^2*d
^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/x) + (c*e*f^4 - c*d*f^3*g + a*e*f^2
*g^2 - a*d*f*g^3 + (c*e*f^3*g - c*d*f^2*g^2 + a*e*f*g^3 - a*d*g^4)*x)*sqrt(
-(c^3*d*f^3 - 3*a*c^2*e*f^2*g - 3*a*c^2*d*f*g^2 + a^2*c*e*g^3 + ((a*c^4*d^2
 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^
2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)*sqrt(-(c^5*e^2*f^6 + 6*
c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*g^5 + a^2*c^3*d^2*g^
6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^2*c^3*e^2)*f
^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^12 + 6*(a^2*c^7*d^
4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3*c^6*d^4 + 2*a^4*c^5
*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5*c^4*d^2*e^2 + a^6
*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 + a^7*c^2*e^4)*f^4*
g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g^10 + (a^7*c^2*d
^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/((a*c^4*d^2 + a^2*c^3*e^2)*f^6 + 3*
(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^2 + a^4*c*e^2)*f^2*g^4 +
 (a^4*c*d^2 + a^5*e^2)*g^6))*log(-(c^3*e^2*f^4 + 4*c^3*d*e*f^3*g - 4*a*c^2*
d*e*f*g^3 - a*c^2*d^2*g^4 + 3*(c^3*d^2 - a*c^2*e^2)*f^2*g^2 - 2*(c^4*d*e*f^
5 - 10*a*c^3*d*e*f^3*g^2 + 5*a^2*c^2*d*e*f*g^4 + a^2*c^2*d^2*g^5 + (3*c^4*d
^2 - 2*a*c^3*e^2)*f^4*g - 2*(2*a*c^3*d^2 - 3*a^2*c^2*e^2)*f^2*g^3 - ((a*c^5
*d^2*e + a^2*c^4*e^3)*f^8 + 2*(a*c^5*d^3 + a^2*c^4*d*e^2)*f^7*g + 2*(a^2*c^
4*d^2*e + a^3*c^3*e^3)*f^6*g^2 + 6*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f^5*g^3 +
6*(a^3*c^3*d^3 + a^4*c^2*d*e^2)*f^3*g^5 - 2*(a^4*c^2*d^2*e + a^5*c*e^3)*f^2
*g^6 + 2*(a^4*c^2*d^3 + a^5*c*d*e^2)*f*g^7 - (a^5*c*d^2*e + a^6*e^3)*g^8)*s
qrt(-(c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*
f*g^5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*
d^2 - 3*a^2*c^3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4
)*f^12 + 6*(a^2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a
^3*c^6*d^4 + 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2
*a^5*c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e
^2 + a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)
*f^2*g^10 + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))*sqrt(e*x + d)
*sqrt(g*x + f)*sqrt(-(c^3*d*f^3 - 3*a*c^2*e*f^2*g - 3*a*c^2*d*f*g^2 + a^2*c
*e*g^3 + ((a*c^4*d^2 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4
*g^2 + 3*(a^3*c^2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)*sqr
t(-(c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*
g^5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^
2 - 3*a^2*c^3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*
f^12 + 6*(a^2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3
*c^6*d^4 + 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a
^5*c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2
 + a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f
^2*g^10 + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/((a*c^4*d^2 + a
^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^2 +
a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)) + 2*(c^3*e^2*f^3*g + 3*c^3
*d*e*f^2*g^2 - 3*a*c^2*e^2*f*g^3 - a*c^2*d*e*g^4)*x + (2*(c^5*d^3 + a*c^4*d
*e^2)*f^7 + 6*(a*c^4*d^3 + a^2*c^3*d*e^2)*f^5*g^2 + 6*(a^2*c^3*d^3 + a^3*c^
2*d*e^2)*f^3*g^4 + 2*(a^3*c^2*d^3 + a^4*c*d*e^2)*f*g^6 + ((c^5*d^2*e + a*c^
4*e^3)*f^7 + (c^5*d^3 + a*c^4*d*e^2)*f^6*g + 3*(a*c^4*d^2*e + a^2*c^3*e^3)*
f^5*g^2 + 3*(a*c^4*d^3 + a^2*c^3*d*e^2)*f^4*g^3 + 3*(a^2*c^3*d^2*e + a^3*c^
2*e^3)*f^3*g^4 + 3*(a^2*c^3*d^3 + a^3*c^2*d*e^2)*f^2*g^5 + (a^3*c^2*d^2*e +
 a^4*c*e^3)*f*g^6 + (a^3*c^2*d^3 + a^4*c*d*e^2)*g^7)*x)*sqrt(-(c^5*e^2*f^6
+ 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*g^5 + a^2*c^3*d^
2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^2*c^3*e^
2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^12 + 6*(a^2*c^
7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3*c^6*d^4 + 2*a^4
*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5*c^4*d^2*e^2 +
 a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 + a^7*c^2*e^4)*
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f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g^10 + (a^7*c
^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/x) - (c*e*f^4 - c*d*f^3*g + a*e
*f^2*g^2 - a*d*f*g^3 + (c*e*f^3*g - c*d*f^2*g^2 + a*e*f*g^3 - a*d*g^4)*x)*s
qrt(-(c^3*d*f^3 - 3*a*c^2*e*f^2*g - 3*a*c^2*d*f*g^2 + a^2*c*e*g^3 - ((a*c^4
*d^2 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^
2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)*sqrt(-(c^5*e^2*f^6
+ 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*g^5 + a^2*c^3*d^
2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^2*c^3*e^
2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^12 + 6*(a^2*c^
7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3*c^6*d^4 + 2*a^4
*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5*c^4*d^2*e^2 +
 a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 + a^7*c^2*e^4)*
f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g^10 + (a^7*c
^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/((a*c^4*d^2 + a^2*c^3*e^2)*f^6
+ 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^2 + a^4*c*e^2)*f^2*g
^4 + (a^4*c*d^2 + a^5*e^2)*g^6))*log(-(c^3*e^2*f^4 + 4*c^3*d*e*f^3*g - 4*a*
c^2*d*e*f*g^3 - a*c^2*d^2*g^4 + 3*(c^3*d^2 - a*c^2*e^2)*f^2*g^2 + 2*(c^4*d*
e*f^5 - 10*a*c^3*d*e*f^3*g^2 + 5*a^2*c^2*d*e*f*g^4 + a^2*c^2*d^2*g^5 + (3*c
^4*d^2 - 2*a*c^3*e^2)*f^4*g - 2*(2*a*c^3*d^2 - 3*a^2*c^2*e^2)*f^2*g^3 + ((a
*c^5*d^2*e + a^2*c^4*e^3)*f^8 + 2*(a*c^5*d^3 + a^2*c^4*d*e^2)*f^7*g + 2*(a^
2*c^4*d^2*e + a^3*c^3*e^3)*f^6*g^2 + 6*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f^5*g^
3 + 6*(a^3*c^3*d^3 + a^4*c^2*d*e^2)*f^3*g^5 - 2*(a^4*c^2*d^2*e + a^5*c*e^3)
*f^2*g^6 + 2*(a^4*c^2*d^3 + a^5*c*d*e^2)*f*g^7 - (a^5*c*d^2*e + a^6*e^3)*g^
8)*sqrt(-(c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*
d*e*f*g^5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*
c^4*d^2 - 3*a^2*c^3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6
*e^4)*f^12 + 6*(a^2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 1
5*(a^3*c^6*d^4 + 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4
 + 2*a^5*c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d
^2*e^2 + a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*
e^4)*f^2*g^10 + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))*sqrt(e*x
+ d)*sqrt(g*x + f)*sqrt(-(c^3*d*f^3 - 3*a*c^2*e*f^2*g - 3*a*c^2*d*f*g^2 + a
^2*c*e*g^3 - ((a*c^4*d^2 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)
*f^4*g^2 + 3*(a^3*c^2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)
*sqrt(-(c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*
e*f*g^5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^
4*d^2 - 3*a^2*c^3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e
^4)*f^12 + 6*(a^2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*
(a^3*c^6*d^4 + 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 +
 2*a^5*c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2
*e^2 + a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^
4)*f^2*g^10 + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/((a*c^4*d^2
 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^
2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)) + 2*(c^3*e^2*f^3*g + 3
*c^3*d*e*f^2*g^2 - 3*a*c^2*e^2*f*g^3 - a*c^2*d*e*g^4)*x - (2*(c^5*d^3 + a*c
^4*d*e^2)*f^7 + 6*(a*c^4*d^3 + a^2*c^3*d*e^2)*f^5*g^2 + 6*(a^2*c^3*d^3 + a^
3*c^2*d*e^2)*f^3*g^4 + 2*(a^3*c^2*d^3 + a^4*c*d*e^2)*f*g^6 + ((c^5*d^2*e +
a*c^4*e^3)*f^7 + (c^5*d^3 + a*c^4*d*e^2)*f^6*g + 3*(a*c^4*d^2*e + a^2*c^3*e
^3)*f^5*g^2 + 3*(a*c^4*d^3 + a^2*c^3*d*e^2)*f^4*g^3 + 3*(a^2*c^3*d^2*e + a^
3*c^2*e^3)*f^3*g^4 + 3*(a^2*c^3*d^3 + a^3*c^2*d*e^2)*f^2*g^5 + (a^3*c^2*d^2
*e + a^4*c*e^3)*f*g^6 + (a^3*c^2*d^3 + a^4*c*d*e^2)*g^7)*x)*sqrt(-(c^5*e^2*
f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*g^5 + a^2*c^
3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^2*c^
3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^12 + 6*(a^
2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3*c^6*d^4 + 2
*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5*c^4*d^2*e
^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 + a^7*c^2*e
^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g^10 + (a
^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/x) + (c*e*f^4 - c*d*f^3*g +
```

a*e*f^2*g^2 - a*d*f*g^3 + (c*e*f^3*g - c*d*f^2*g^2 + a*e*f*g^3 - a*d*g^4)*
x)*sqrt(-(c^3*d*f^3 - 3*a*c^2*e*f^2*g - 3*a*c^2*d*f*g^2 + a^2*c*e*g^3 - ((a
*c^4*d^2 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^
3*c^2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)*sqrt(-(c^5*e^2*
f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*g^5 + a^2*c^
3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^2*c^
3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^12 + 6*(a^
2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3*c^6*d^4 + 2
*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5*c^4*d^2*e
^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 + a^7*c^2*e
^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g^10 + (a
^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/((a*c^4*d^2 + a^2*c^3*e^2)*
f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^2 + a^4*c*e^2)*f
^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6))*log(-(c^3*e^2*f^4 + 4*c^3*d*e*f^3*g -
4*a*c^2*d*e*f*g^3 - a*c^2*d^2*g^4 + 3*(c^3*d^2 - a*c^2*e^2)*f^2*g^2 - 2*(c^
4*d*e*f^5 - 10*a*c^3*d*e*f^3*g^2 + 5*a^2*c^2*d*e*f*g^4 + a^2*c^2*d^2*g^5 +
(3*c^4*d^2 - 2*a*c^3*e^2)*f^4*g - 2*(2*a*c^3*d^2 - 3*a^2*c^2*e^2)*f^2*g^3 +
((a*c^5*d^2*e + a^2*c^4*e^3)*f^8 + 2*(a*c^5*d^3 + a^2*c^4*d*e^2)*f^7*g + 2
*(a^2*c^4*d^2*e + a^3*c^3*e^3)*f^6*g^2 + 6*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f^
5*g^3 + 6*(a^3*c^3*d^3 + a^4*c^2*d*e^2)*f^3*g^5 - 2*(a^4*c^2*d^2*e + a^5*c*
e^3)*f^2*g^6 + 2*(a^4*c^2*d^3 + a^5*c*d*e^2)*f*g^7 - (a^5*c*d^2*e + a^6*e^3
)*g^8)*sqrt(-(c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*
c^3*d*e*f*g^5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(
2*a*c^4*d^2 - 3*a^2*c^3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3
*c^6*e^4)*f^12 + 6*(a^2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2
+ 15*(a^3*c^6*d^4 + 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5
*d^4 + 2*a^5*c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c
^3*d^2*e^2 + a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^
8*c*e^4)*f^2*g^10 + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))*sqrt(
e*x + d)*sqrt(g*x + f)*sqrt(-(c^3*d*f^3 - 3*a*c^2*e*f^2*g - 3*a*c^2*d*f*g^2
+ a^2*c*e*g^3 - ((a*c^4*d^2 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*
e^2)*f^4*g^2 + 3*(a^3*c^2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*
g^6)*sqrt(-(c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^
3*d*e*f*g^5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*
a*c^4*d^2 - 3*a^2*c^3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c
^6*e^4)*f^12 + 6*(a^2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 +
15*(a^3*c^6*d^4 + 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d
^4 + 2*a^5*c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3
*d^2*e^2 + a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*
c*e^4)*f^2*g^10 + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/((a*c^4
*d^2 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^
2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)) + 2*(c^3*e^2*f^3*g
+ 3*c^3*d*e*f^2*g^2 - 3*a*c^2*e^2*f*g^3 - a*c^2*d*e*g^4)*x - (2*(c^5*d^3 +
a*c^4*d*e^2)*f^7 + 6*(a*c^4*d^3 + a^2*c^3*d*e^2)*f^5*g^2 + 6*(a^2*c^3*d^3
+ a^3*c^2*d*e^2)*f^3*g^4 + 2*(a^3*c^2*d^3 + a^4*c*d*e^2)*f*g^6 + ((c^5*d^2*
e + a*c^4*e^3)*f^7 + (c^5*d^3 + a*c^4*d*e^2)*f^6*g + 3*(a*c^4*d^2*e + a^2*c
^3*e^3)*f^5*g^2 + 3*(a*c^4*d^3 + a^2*c^3*d*e^2)*f^4*g^3 + 3*(a^2*c^3*d^2*e
+ a^3*c^2*e^3)*f^3*g^4 + 3*(a^2*c^3*d^3 + a^3*c^2*d*e^2)*f^2*g^5 + (a^3*c^2
*d^2*e + a^4*c*e^3)*f*g^6 + (a^3*c^2*d^3 + a^4*c*d*e^2)*g^7)*x)*sqrt(-(c^5*
e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f*g^5 + a^
2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^
2*c^3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^12 + 6
*(a^2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^10*g^2 + 15*(a^3*c^6*d^4
+ 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5*c^4*d
^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 + a^7*c
^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g^10
+ (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^12)))/x))/(c*e*f^4 - c*d*f^3*
g + a*e*f^2*g^2 - a*d*f*g^3 + (c*e*f^3*g - c*d*f^2*g^2 + a*e*f*g^3 - a*d*g^
4)*x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.09, size = 10977, normalized size = 31.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2+a)\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*sqrt(e*x + d)*(g*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(f+g\,x)^{3/2}\,(c\,x^2+a)\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

3.419 $\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=549

$$-\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)}+\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(ef-dg)}+\frac{g}{\sqrt{-a}\sqrt{f+\cdots}}$$

Rubi [A] time = 1.32, antiderivative size = 543, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {912, 104, 152, 12, 93, 208}

$$-\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}+\frac{g\sqrt{d+ex}\left(2aeg-\sqrt{-a}\sqrt{c}(dg+ef)\right)}{a\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)^2}+\frac{g\sqrt{d+ex}\left(\sqrt{-a}\sqrt{c}(dg+ef)+2aeg\right)}{a\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)^2}+\frac{c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] -(e/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])) + e/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt
[f + g*x]) + (g*(2*a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*
(Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*
x]) + (g*(2*a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*(Sqrt[c
]*d - Sqrt[-a]*e)*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*x]) + (
c*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sq
rt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)^(3/2)*(Sqrt[c
]*f - Sqrt[-a]*g)^(3/2)) - (c*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*(Sqrt[c]*d
 + Sqrt[-a]*e)^(3/2)*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}(f+gx)^{3/2}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+e \cdots}\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}(f+gx)^{3/2}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+ex)^{3/2}(f+gx)^{3/2}}\,dx}{2\sqrt{-a}}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{\cdots}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-\cdots}\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{\cdots}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-\cdots}\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{\cdots}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-\cdots}\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{\cdots}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-\cdots}\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{\cdots}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-\cdots}\right.}$$

Mathematica [A] time = 2.07, size = 521, normalized size = 0.95

$$\frac{\frac{e}{\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}\,e-\sqrt{c}\,d\right)} + \frac{e}{\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)} + \frac{g\sqrt{d+ex}\left(2\sqrt{-a}\,eg+\sqrt{c}\,(dg+ef)\right)}{\sqrt{f+gx}\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(\sqrt{-a}\,g+\sqrt{c}\,f\right)(ef-dg)} + \frac{\frac{g\sqrt{d+ex}\left(2\sqrt{-a}\,eg-\sqrt{c}\,(dg+ef)\right)}{\sqrt{f+gx}\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)} + \frac{c(ef-dg)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g-\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{-a}\,e-\sqrt{c}\,d}\left(\sqrt{-a}\,g-\sqrt{c}\,f\right)^{3/2}}}{\sqrt{c}\,d-\sqrt{-a}\,e} + \frac{c(dg-ef)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)^{3/2}\left(\sqrt{-a}\,g+\sqrt{c}\,f\right)^{3/2}}}{\sqrt{-a}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (e/((-(Sqrt[c]*d) + Sqrt[-a]*e)*Sqrt[d + e*x]*Sqrt[f + g*x]) + e/((Sqrt[c]*
d + Sqrt[-a]*e)*Sqrt[d + e*x]*Sqrt[f + g*x]) + (g*(2*Sqrt[-a]*e*g + Sqrt[c]
*(e*f + d*g))*Sqrt[d + e*x])/((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a
]*g)*(e*f - d*g)*Sqrt[f + g*x]) + ((g*(2*Sqrt[-a]*e*g - Sqrt[c]*(e*f + d*g)
)*Sqrt[d + e*x])/((Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)*Sqrt[f + g*x]) + (c*
(e*f - d*g)*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-
(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]
*(-(Sqrt[c]*f) + Sqrt[-a]*g)^(3/2)))/(Sqrt[c]*d - Sqrt[-a]*e) + (c*(-(e*f)
+ d*g)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d
 + Sqrt[-a]*e]*Sqrt[f + g*x])])/((Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(Sqrt[c]*f
+ Sqrt[-a]*g)^(3/2)))/(Sqrt[-a]*(e*f - d*g))
```

IntegrateAlgebraic [C] time = 1.69, size = 492, normalized size = 0.90

$$-\frac{ic\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)^2\tan^{-1}\left(\frac{\sqrt{f+gx}\,\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\,\sqrt{-i\sqrt{a}\,\sqrt{c}\,dg+i\sqrt{a}\,\sqrt{c}\,ef-aeg-cdf}}\right)}{\sqrt{a}\,\left(ae^2+cd^2\right)^{3/2}\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}}+\frac{ic\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)^2\tan^{-1}\left(\frac{\sqrt{f+gx}\,\sqrt{ae^2+cd^2}}{\sqrt{d+ex}\,\sqrt{i\sqrt{a}\,\sqrt{c}\,dg-i\sqrt{a}\,\sqrt{c}\,ef-aeg-cdf}}\right)}{\sqrt{a}\,\left(ae^2+cd^2\right)^{3/2}\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}}-\frac{2\sqrt{d+ex}\left(\frac{ae^3g^2(f+gx)}{d+ex}+ae^2g^3+cd^2g^3+\frac{ce^3f^2(f+gx)}{d+ex}\right)}{\sqrt{f+gx}\left(ae^2+cd^2\right)\left(ag^2+cf^2\right)(dg-ef)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(c*d^2*g^3 + a*e^2*g^3 + (c*e^3*f^2*(f + g*x))/(d + e*x)
+ (a*e^3*g^2*(f + g*x))/(d + e*x)))/((c*d^2 + a*e^2)*(-(e*f) + d*g)^2*(c*f^
2 + a*g^2)*Sqrt[f + g*x]) - (I*c*(Sqrt[c]*d - I*Sqrt[a]*e)^2*ArcTan[(Sqrt[c
*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-(c*d*f) + I*Sqrt[a]*Sqrt[c]*e*f - I*Sqr
t[a]*Sqrt[c]*d*g - a*e*g]*Sqrt[d + e*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*(
Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sq
rt[a]*g))]) + (I*c*(Sqrt[c]*d + I*Sqrt[a]*e)^2*ArcTan[(Sqrt[c*d^2 + a*e^2]*
Sqrt[f + g*x])/(Sqrt[-(c*d*f) - I*Sqrt[a]*Sqrt[c]*e*f + I*Sqrt[a]*Sqrt[c]*d
*g - a*e*g]*Sqrt[d + e*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*(Sqrt[c]*f - I*
Sqrt[a]*g)*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.23, size = 30656, normalized size = 55.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+a\right)(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*(g*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f+g\,x\right)^{3/2}\,\left(c\,x^2+a\right)\,\left(d+e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^2\right)\left(d+ex\right)^{\frac{3}{2}}\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*(f + g*x)**(3/2)), x)
```

3.420 $\int \frac{\sqrt{x}}{\sqrt{1+x}\left(1+x^2\right)}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{x}}{\sqrt{x+1}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\,\sqrt{x}}{\sqrt{x+1}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {910, 93, 208}

$$-\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{x}}{\sqrt{x+1}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\,\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] -((1 - I)^(3/2)*ArcTanh[(Sqrt[1 - I]*Sqrt[x])/Sqrt[1 + x]])/2 - ((1 + I)^(3
/2)*ArcTanh[(Sqrt[1 + I]*Sqrt[x])/Sqrt[1 + x]])/2
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{1+x}\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{2(i-x)\sqrt{x}\,\sqrt{1+x}}+\frac{1}{2\sqrt{x}\,(i+x)\sqrt{1+x}}\right)dx\\
&= -\left(\frac{1}{2}\int \frac{1}{(i-x)\sqrt{x}\,\sqrt{1+x}}\,dx\right)+\frac{1}{2}\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{1+x}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{i-(1+i)x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{1+x}}\right)+\text{Subst}\left(\int \frac{1}{i+(1-i)x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{1+x}}\right)\\
&= -\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{x}}{\sqrt{1+x}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\,\sqrt{x}}{\sqrt{1+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.97

$$\frac{1}{2}\left(-(-1+i)^{3/2}\tan^{-1}\left(\sqrt{-1+i}\sqrt{\frac{x}{x+1}}\right)-(1+i)^{3/2}\tanh^{-1}\left(\sqrt{1+i}\sqrt{\frac{x}{x+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] (-((-1 + I)^(3/2)*ArcTan[Sqrt[-1 + I]*Sqrt[x/(1 + x)]]) - (1 + I)^(3/2)*Arc
Tanh[Sqrt[1 + I]*Sqrt[x/(1 + x)]])/2
```

IntegrateAlgebraic [C] time = 0.09, size = 59, normalized size = 0.91

$$-\text{RootSum}\left[\#1^4+16\#1^2+32\#1+16\&,\frac{\#1^2\log\left(\#1-2x+2\sqrt{x+1}\sqrt{x}\right)}{\#1^3+8\#1+8}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] -RootSum[16 + 32*#1 + 16*#1^2 + #1^4 & , (Log[-2*x + 2*Sqrt[x]*Sqrt[1 + x]
+ #1]*#1^2)/(8 + 8*#1 + #1^3) & ]
```

fricas [B] time = 0.48, size = 744, normalized size = 11.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-8*sqrt(x + 1)*x^(3/2) +
8*x^2 + 2*(2^(1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x
+ 1) - 2*x))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) - 1/8*2^(1/4)*sqrt(
2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-8*sqrt(x + 1)*x^(3/2) + 8*x^2 - 2*(2^(1/4
)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x + 1) - 2*x))*sqrt
(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) - 1/2*2^(1/4)*sqrt(2*sqrt(2) + 4)*ar
ctan(1/7*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*sqrt(x + 1)*sqrt(x) - 1/
7*sqrt(2)*(sqrt(2)*(5*x + 1) + 6*x + 4) - 1/28*sqrt(-8*sqrt(x + 1)*x^(3/2)
+ 8*x^2 - 2*(2^(1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(
x + 1) - 2*x))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*(2*sqrt(2)*(5*sqr
t(2) + 6) - (2^(3/4)*(3*sqrt(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(2*sqrt
(2) + 4) + 16*sqrt(2) + 8) - 1/7*sqrt(2)*(8*x + 3) - 1/14*((2^(3/4)*(3*sqrt
(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(x + 1)*sqrt(x) - 2^(3/4)*(sqrt(2)*
(3*x + 2) + 5*x + 1) - 2*2^(1/4)*(sqrt(2)*(x + 3) + 4*x - 2))*sqrt(2*sqrt(2
) + 4) - 4/7*x - 5/7) - 1/2*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*(sqrt(2
)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*sqrt(x + 1)*sqrt(x) + 1/7*sqrt(2)*(sqrt(
2)*(5*x + 1) + 6*x + 4) + 1/28*sqrt(-8*sqrt(x + 1)*x^(3/2) + 8*x^2 + 2*(2^(
1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x + 1) - 2*x))*s
qrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*(2*sqrt(2)*(5*sqrt(2) + 6) + (2^(
3/4)*(3*sqrt(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(2*sqrt(2) + 4) + 16*sq
rt(2) + 8) + 1/7*sqrt(2)*(8*x + 3) - 1/14*((2^(3/4)*(3*sqrt(2) + 5) + 2*2^(
1/4)*(sqrt(2) + 4))*sqrt(x + 1)*sqrt(x) - 2^(3/4)*(sqrt(2)*(3*x + 2) + 5*x
+ 1) - 2*2^(1/4)*(sqrt(2)*(x + 3) + 4*x - 2))*sqrt(2*sqrt(2) + 4) + 4/7*x +
 5/7)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.17, size = 305, normalized size = 4.69

$$\frac{\sqrt{\frac{(x+1)x}{\left(x+\sqrt{2}-1\right)^{2}}}\,\left(x+\sqrt{2}-1\right)\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{\frac{(x+1)x}{\left(x+\sqrt{2}-1\right)^{2}}}}{\sqrt{1+\sqrt{2}}}\right)-6\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{\frac{(x+1)x}{\left(x+\sqrt{2}-1\right)^{2}}}}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{-2+2\sqrt{2}}\,\sqrt{1+\sqrt{2}}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{\left(3\sqrt{2}-4\right)(x+1)\left(4+3\sqrt{2}\right)x}{\left(x+\sqrt{2}-1\right)^{2}}}\,\sqrt{-2+2\sqrt{2}}\,\left(3+2\sqrt{2}\right)\left(-x+\sqrt{2}+1\right)\left(3\sqrt{2}-4\right)\left(x+\sqrt{2}-1\right)}{4(x+1)x}\right)-2\sqrt{-2+2\sqrt{2}}\,\sqrt{1+\sqrt{2}}\,\arctan\left(\frac{\sqrt{\frac{\left(3\sqrt{2}-4\right)(x+1)\left(4+3\sqrt{2}\right)x}{\left(x+\sqrt{2}-1\right)^{2}}}\,\sqrt{-2+2\sqrt{2}}\,\left(3+2\sqrt{2}\right)\left(-x+\sqrt{2}+1\right)\left(3\sqrt{2}-4\right)\left(x+\sqrt{2}-1\right)}{4(x+1)x}\right)\right)\sqrt{2}}{4\sqrt{x+1}\,\left(3\sqrt{2}-4\right)\sqrt{1+\sqrt{2}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)/(x+1)^(1/2),x)
```

```
[Out] 1/4/x^(1/2)/(x+1)^(1/2)*(x*(x+1)/(2^(1/2)-1+x)^2)^(1/2)*(2^(1/2)-1+x)*((-2+
2*2^(1/2))^(1/2)*arctan(1/4*((3*2^(1/2)-4)*x*(x+1)*(4+3*2^(1/2))/(2^(1/2)-1
+x)^2)^(1/2)*(-2+2*2^(1/2))^(1/2)*(3+2*2^(1/2))*(2^(1/2)+1-x)*(3*2^(1/2)-4)
*(2^(1/2)-1+x)/x/(x+1))*(1+2^(1/2))^(1/2)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2)*ar
ctan(1/4*((3*2^(1/2)-4)*x*(x+1)*(4+3*2^(1/2))/(2^(1/2)-1+x)^2)^(1/2)*(-2+2*
2^(1/2))^(1/2)*(3+2*2^(1/2))*(2^(1/2)+1-x)*(3*2^(1/2)-4)*(2^(1/2)-1+x)/x/(x
+1))*(1+2^(1/2))^(1/2)+4*arctanh(2^(1/2)*(x*(x+1)/(2^(1/2)-1+x)^2)^(1/2)/(1
+2^(1/2))^(1/2))*2^(1/2)-6*arctanh(2^(1/2)*(x*(x+1)/(2^(1/2)-1+x)^2)^(1/2)/
(1+2^(1/2))^(1/2)))*2^(1/2)/(3*2^(1/2)-4)/(1+2^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(x^2+1\right)\sqrt{x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/((x^2 + 1)*sqrt(x + 1)), x)
```

mupad [B] time = 8.49, size = 1610, normalized size = 24.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((x^2 + 1)*(x + 1)^(1/2)),x)
```

```
[Out] - atan(((((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((28454
158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/
2)/16 - 1/16)^(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) - ((53150
2202880*x)/((x + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/
2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16
 - 1/16)^(1/2)) - (12079595520*x)/((x + 1)^(1/2) - 1)^2 + 68451041280))*((-
 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (13555990528*x)/((
x + 1)^(1/2) - 1)^2 + 9529458688) + (3556769792*x^(1/2))/((x + 1)^(1/2) - 1
))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*1i - (((- 2^(1
/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*((13555990528*x)/((x + 1)
^(1/2) - 1)^2 - ((28454158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16
 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((112742891520*x^(1/2))/((x +
1)^(1/2) - 1) + ((531502202880*x)/((x + 1)^(1/2) - 1)^2 - 241591910400)*((-
 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/
16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (12079595520*x)/((x + 1)^(1/2) - 1
)^2 - 68451041280))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2
)) + 9529458688) - (3556769792*x^(1/2))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16
 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*1i)/((((- 2^(1/2)/16 - 1/16)^(1
/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((28454158336*x^(1/2))/((x + 1)^(1/2) - 1
) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((1127428915
20*x^(1/2))/((x + 1)^(1/2) - 1) - ((531502202880*x)/((x + 1)^(1/2) - 1)^2 -
```

```
 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*(
(- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) - (12079595520*x)/
((x + 1)^(1/2) - 1)^2 + 68451041280))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/
2)/16 - 1/16)^(1/2)) + (13555990528*x)/((x + 1)^(1/2) - 1)^2 + 9529458688)
+ (3556769792*x^(1/2))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) -
(2^(1/2)/16 - 1/16)^(1/2)) + (((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 -
1/16)^(1/2))*((13555990528*x)/((x + 1)^(1/2) - 1)^2 - ((28454158336*x^(1/2)
)/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^
(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) + ((531502202880*x)/((x
 + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/
16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)
) + (12079595520*x)/((x + 1)^(1/2) - 1)^2 - 68451041280))*((- 2^(1/2)/16 -
1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + 9529458688) - (3556769792*x^(1/2
))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^
(1/2)) + (7549747200*x)/((x + 1)^(1/2) - 1)^2 + 503316480))*((- 2^(1/2)/16
- 1/16)^(1/2)*2i - (2^(1/2)/16 - 1/16)^(1/2)*2i) - atan(((x^(1/2)*(- 2^(1/2
)/16 - 1/16)^(1/2)*848i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)
^(1/2)*848i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(- 2^(1/2)/16 - 1/16)^(3/2)*678
4i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(3/2)*6784i)/((x + 1
)^(1/2) - 1) + (x^(1/2)*(- 2^(1/2)/16 - 1/16)^(5/2)*26880i)/((x + 1)^(1/2)
- 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(5/2)*26880i)/((x + 1)^(1/2) - 1) + (x^
(1/2)*(2^(1/2)/16 - 1/16)^2*(- 2^(1/2)/16 - 1/16)^(1/2)*134400i)/((x + 1)^(
1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(2^(1/2)/16 + 1/16)^2*134400
i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)*(- 2^(1/2)/16 - 1/16)
^(1/2)*20352i)/((x + 1)^(1/2) - 1) - (x^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(2^
(1/2)/16 + 1/16)*20352i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)
*(- 2^(1/2)/16 - 1/16)^(3/2)*268800i)/((x + 1)^(1/2) - 1) - (x^(1/2)*(2^(1/
2)/16 - 1/16)^(3/2)*(2^(1/2)/16 + 1/16)*268800i)/((x + 1)^(1/2) - 1))/(4544
*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + 65280*(2^(1/2)/16
- 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(3/2) + 65280*(2^(1/2)/16 - 1/16)^(3/2)
*(- 2^(1/2)/16 - 1/16)^(1/2) + 345600*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/
16 - 1/16)^(5/2) + 1152000*(2^(1/2)/16 - 1/16)^(3/2)*(- 2^(1/2)/16 - 1/16)^
(3/2) + 345600*(2^(1/2)/16 - 1/16)^(5/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + x/((
x + 1)^(1/2) - 1)^2 + (6464*x*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/1
6)^(1/2))/((x + 1)^(1/2) - 1)^2 - (11520*x*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(
1/2)/16 - 1/16)^(3/2))/((x + 1)^(1/2) - 1)^2 - (11520*x*(2^(1/2)/16 - 1/16)
^(3/2)*(- 2^(1/2)/16 - 1/16)^(1/2))/((x + 1)^(1/2) - 1)^2 - (760320*x*(2^(1
/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(5/2))/((x + 1)^(1/2) - 1)^2 - (
2534400*x*(2^(1/2)/16 - 1/16)^(3/2)*(- 2^(1/2)/16 - 1/16)^(3/2))/((x + 1)^(
1/2) - 1)^2 - (760320*x*(2^(1/2)/16 - 1/16)^(5/2)*(- 2^(1/2)/16 - 1/16)^(1/
2))/((x + 1)^(1/2) - 1)^2 + 1))*((- 2^(1/2)/16 - 1/16)^(1/2)*2i + (2^(1/2)/
16 - 1/16)^(1/2)*2i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x+1}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/(sqrt(x + 1)*(x**2 + 1)), x)
```

3.421 $\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx$

Optimal. Leaf size=80

$$\frac{(x+1)^4(f+g)^2}{5\left(1-x^2\right)^{5/2}}+\frac{(x+1)^3(f-9g)(f+g)}{15\left(1-x^2\right)^{3/2}}+\frac{2g^2(x+1)}{\sqrt{1-x^2}}-g^2\sin^{-1}(x)$$

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {853, 1635, 789, 653, 216}

$$\frac{(x+1)^4(f+g)^2}{5\left(1-x^2\right)^{5/2}}+\frac{(x+1)^3(f-9g)(f+g)}{15\left(1-x^2\right)^{3/2}}+\frac{2g^2(x+1)}{\sqrt{1-x^2}}-g^2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

```
[Out] ((f + g)^2*(1 + x)^4)/(5*(1 - x^2)^(5/2)) + ((f - 9*g)*(f + g)*(1 + x)^3)/(
15*(1 - x^2)^(3/2)) + (2*g^2*(1 + x))/Sqrt[1 - x^2] - g^2*ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx &= \int \frac{(1+x)^4(f+gx)^2}{\left(1-x^2\right)^{7/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} - \frac{1}{5}\int \frac{(1+x)^3\left(-f^2+8fg+4g^2+5g^2x\right)}{\left(1-x^2\right)^{5/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}} + g^2\int \frac{(1+x)^2}{\left(1-x^2\right)^{3/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}} + \frac{2g^2(1+x)}{\sqrt{1-x^2}} - g^2\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}} + \frac{2g^2(1+x)}{\sqrt{1-x^2}} - g^2\sin^{-1}(x)
\end{aligned}$$

Mathematica [C] time = 0.14, size = 91, normalized size = 1.14

$$\frac{\sqrt{1-x^2}\left((x+1)^{3/2}\left(f^2(x-4)+fg(2-8x)+g^2(x-4)\right)-20\sqrt{2}g^2(x-1)\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1-x}{2}\right)\right)}{15(x-1)^3\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

```
[Out] (Sqrt[1 - x^2]*((f*g*(2 - 8*x) + f^2*(-4 + x) + g^2*(-4 + x))*(1 + x)^(3/2)
 - 20*Sqrt[2]*g^2*(-1 + x)*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - x)/2]))
/(15*(-1 + x)^3*Sqrt[1 + x])
```

IntegrateAlgebraic [A] time = 0.35, size = 98, normalized size = 1.22

$$\frac{\sqrt{1-x^2}\left(f^2x^2-3f^2x-4f^2-8fgx^2-6fgx+2fg-39g^2x^2+57g^2x-24g^2\right)}{15(x-1)^3}+2g^2\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

```
[Out] (Sqrt[1 - x^2]*(-4*f^2 + 2*f*g - 24*g^2 - 3*f^2*x - 6*f*g*x + 57*g^2*x + f^
2*x^2 - 8*f*g*x^2 - 39*g^2*x^2))/(15*(-1 + x)^3) + 2*g^2*ArcTan[Sqrt[1 - x^
2]/(1 + x)]
```

fricas [B] time = 0.40, size = 193, normalized size = 2.41

$$\frac{2\left(2f^2-fg+12g^2\right)x^3-6\left(2f^2-fg+12g^2\right)x^2-4f^2+2fg-24g^2+6\left(2f^2-fg+12g^2\right)x+30\left(g^2x^3-3g^2x^2+3g^2x-g^2\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\left(\left(f^2-8fg-39g^2\right)x^2-4f^2+2fg-24g^2-3\left(f^2+2fg-19g^2\right)x\right)\sqrt{-x^2+1}}{15\left(x^3-3x^2+3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(2*(2*f^2 - f*g + 12*g^2)*x^3 - 6*(2*f^2 - f*g + 12*g^2)*x^2 - 4*f^2 +
 2*f*g - 24*g^2 + 6*(2*f^2 - f*g + 12*g^2)*x + 30*(g^2*x^3 - 3*g^2*x^2 + 3*
g^2*x - g^2)*arctan((sqrt(-x^2 + 1) - 1)/x) + ((f^2 - 8*f*g - 39*g^2)*x^2 -
 4*f^2 + 2*f*g - 24*g^2 - 3*(f^2 + 2*f*g - 19*g^2)*x)*sqrt(-x^2 + 1))/(x^3
- 3*x^2 + 3*x - 1)
```

giac [B] time = 0.35, size = 266, normalized size = 3.32

$$-g^2 \arcsin(x) + \frac{2\left(4 f^2 - 2 f g + 24 g^2 + \frac{5 f^2\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{10 f g\left(\sqrt{-x^2+1}-1\right)}{x} + \frac{105 g^2\left(\sqrt{-x^2+1}-1\right)}{x} + \frac{25 f^2\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + \frac{10 f g\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + \frac{165 g^2\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + \frac{15 f^2\left(\sqrt{-x^2+1}-1\right)^3}{x^3} - \frac{30 f g\left(\sqrt{-x^2+1}-1\right)^3}{x^3} + \frac{75 g^2\left(\sqrt{-x^2+1}-1\right)^3}{x^3} + \frac{15 f^2\left(\sqrt{-x^2+1}-1\right)^4}{x^4} + \frac{15 g^2\left(\sqrt{-x^2+1}-1\right)^4}{x^4}\right)}{15\left(\frac{\sqrt{-x^2+1}-1}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="giac")
```

```
[Out] -g^2*arcsin(x) + 2/15*(4*f^2 - 2*f*g + 24*g^2 + 5*f^2*(sqrt(-x^2 + 1) - 1)/
x - 10*f*g*(sqrt(-x^2 + 1) - 1)/x + 105*g^2*(sqrt(-x^2 + 1) - 1)/x + 25*f^2
*(sqrt(-x^2 + 1) - 1)^2/x^2 + 10*f*g*(sqrt(-x^2 + 1) - 1)^2/x^2 + 165*g^2*(
sqrt(-x^2 + 1) - 1)^2/x^2 + 15*f^2*(sqrt(-x^2 + 1) - 1)^3/x^3 - 30*f*g*(sqr
t(-x^2 + 1) - 1)^3/x^3 + 75*g^2*(sqrt(-x^2 + 1) - 1)^3/x^3 + 15*f^2*(sqrt(-
x^2 + 1) - 1)^4/x^4 + 15*g^2*(sqrt(-x^2 + 1) - 1)^4/x^4)/((sqrt(-x^2 + 1) -
 1)/x + 1)^5
```

maple [A] time = 0.02, size = 125, normalized size = 1.56

$$\left(-\arcsin(x) + \frac{\left(-2x-(x-1)^2+2\right)^{\frac{3}{2}}}{(x-1)^2} + \sqrt{-2x-(x-1)^2+2}\right) g^2 + \frac{2\left(f+g\right)\left(-2x-(x-1)^2+2\right)^{\frac{3}{2}} g}{3\,(x-1)^3} + \left(f^2+2fg+g^2\right)\left(\frac{\left(-2x-(x-1)^2+2\right)^{\frac{3}{2}}}{5\,(x-1)^4} - \frac{\left(-2x-(x-1)^2+2\right)^{\frac{3}{2}}}{15\,(x-1)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x)
```

```
[Out] 2/3*g*(f+g)/(x-1)^3*(-(x-1)^2-2*x+2)^(3/2)+g^2*(1/(x-1)^2*(-(x-1)^2-2*x+2)^
(3/2)+(-(x-1)^2-2*x+2)^(1/2)-arcsin(x))+(f^2+2*f*g+g^2)*(1/5/(x-1)^4*(-(x-1
)^2-2*x+2)^(3/2)-1/15/(x-1)^3*(-(x-1)^2-2*x+2)^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^2 \sqrt{-x^2+1}}{(x-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^2*sqrt(-x^2 + 1)/(x - 1)^4, x)
```

mupad [B] time = 2.96, size = 164, normalized size = 2.05

$$\sqrt{1-x^2}\left(\frac{\frac{f^2}{3}+2fg+\frac{5g^2}{3}}{x-1} - \frac{\frac{f^2}{3}+2fg+\frac{5g^2}{3}}{(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{\frac{2f^2}{5}+\frac{4fg}{5}+\frac{2g^2}{5}}{(x-1)^3} + \frac{\frac{4f^2}{15}+\frac{8fg}{15}+\frac{4g^2}{15}}{x-1} - \frac{\frac{4f^2}{15}+\frac{8fg}{15}+\frac{4g^2}{15}}{(x-1)^2}\right) - g^2\,\mathrm{asin}(x) - \frac{\sqrt{1-x^2}\,\left(4g^2+2fg\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(1 - x^2)^(1/2))/(x - 1)^4,x)
```

```
[Out] (1 - x^2)^(1/2)*((2*f*g + f^2/3 + (5*g^2)/3)/(x - 1) - (2*f*g + f^2/3 + (5*
g^2)/3)/(x - 1)^2) - (1 - x^2)^(1/2)*(((4*f*g)/5 + (2*f^2)/5 + (2*g^2)/5)/(
x - 1)^3 + ((8*f*g)/15 + (4*f^2)/15 + (4*g^2)/15)/(x - 1) - ((8*f*g)/15 + (
4*f^2)/15 + (4*g^2)/15)/(x - 1)^2) - g^2*asin(x) - ((1 - x^2)^(1/2)*(2*f*g
+ 4*g^2))/(x - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}\,\left(f+gx\right)^2}{(x-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-x**2+1)**(1/2)/(1-x)**4,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))*(f + g*x)**2/(x - 1)**4, x)
```

3.422 $\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\,\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

Rubi [A] time = 0.24, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {853, 1654, 844, 216, 725, 204}

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\,\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-a^2x^2\right)^{3/2}}{(1-ax)^2(c+dx)}\,dx &= \int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{\int \frac{-a^2d^2+a^3(ac-2d)dx}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{a^2d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(a(ac-2d))\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{d^2}+\frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}-\frac{(ac-d)^2\,\mathrm{Subst}\left(\int \frac{1}{-a^2c^2+d^2-x^2}\,dx,x,\frac{d+a^2cx}{\sqrt{1-a^2x^2}}\right)}{d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}+\frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}
\end{aligned}$$

Mathematica [C] time = 0.30, size = 148, normalized size = 1.38

$$-\frac{\frac{i(d-ac)^2\log\left(\frac{2d^3\left(\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}+ia^2cx+id\right)}{(d-ac)^2\sqrt{a^2c^2-d^2}(c+dx)}\right)}{\sqrt{a^2c^2-d^2}}+d\sqrt{1-a^2x^2}+(ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] -((d*Sqrt[1 - a^2*x^2] + (a*c - 2*d)*ArcSin[a*x] + (I*(-(a*c) + d)^2*Log[(2
*d^3*(I*d + I*a^2*c*x + Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2]))/((-(a*c) +
d)^2*Sqrt[a^2*c^2 - d^2]*(c + d*x))])/Sqrt[a^2*c^2 - d^2])/d^2)
```

IntegrateAlgebraic [B] time = 2.56, size = 971, normalized size = 9.07

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) + ((a^3*c^3*Sqrt[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^
2 - d^2]] - a^2*c^2*d*Sqrt[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]] - a
*c*d^2*Sqrt[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]] + d^3*Sqrt[2*a^2*c
^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]] + a^2*c^2*Sqrt[a^2*c^2 - d^2]*Sqrt[2*
a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]] - a*c*d*Sqrt[a^2*c^2 - d^2]*Sqrt
[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]])*ArcTan[(Sqrt[-a^2]*d*x - d*S
qrt[1 - a^2*x^2])/Sqrt[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]]])/(d^4*
(a*c + d)) + ((a^3*c^3*Sqrt[2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]] -
a^2*c^2*d*Sqrt[2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]] - a*c*d^2*Sqrt[
```

```
2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]] + d^3*Sqrt[2*a^2*c^2 - d^2 + 2
*a*c*Sqrt[a^2*c^2 - d^2]] - a^2*c^2*Sqrt[a^2*c^2 - d^2]*Sqrt[2*a^2*c^2 - d^
2 + 2*a*c*Sqrt[a^2*c^2 - d^2]] + a*c*d*Sqrt[a^2*c^2 - d^2]*Sqrt[2*a^2*c^2 -
 d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]])*ArcTan[(Sqrt[-a^2]*d*x - d*Sqrt[1 - a^2*
x^2])/Sqrt[2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]]]])/(d^4*(a*c + d)) -
 (Sqrt[-a^2]*(a*c - d)*Sqrt[-(a^2*c^2) + d^2]*ArcTan[(a^2*c^2 - a^2*d^2*x^2
 - Sqrt[-a^2]*d^2*x*Sqrt[1 - a^2*x^2])/(a*c*Sqrt[-(a^2*c^2) + d^2])])/(a*d^
2*(a*c + d)) - (Sqrt[-a^2]*(a*c - 2*d)*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x
^2]])/(a*d^2)
```

fricas [A] time = 0.54, size = 318, normalized size = 2.97

$$\left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-(a^2c^2-d^2)\sqrt{-a^2x^2+1}-\left(acd+d^2+(a^3c^2+a^2cd)x+\sqrt{-a^2x^2+1}\,(acd+d^2)\right)\sqrt{-\frac{ac-d}{ac+d}}}{dx+c}\right)-2\,(ac-2\,d)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,d}{d^2},\ \frac{2\,(ac-d)\sqrt{\frac{ac-d}{ac+d}}\arctan\left(\frac{\left(dx-\sqrt{-a^2x^2+1}\,c+c\right)\sqrt{\frac{ac-d}{ac+d}}}{(ac-d)x}\right)+2\,(ac-2\,d)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}\,d}{d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] [-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 - d
^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2*x
^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c - 2*
d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2, (2*(
a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c + c)*
sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt(-a^2*
x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

giac [B] time = 0.59, size = 208, normalized size = 1.94

$$-\left(\frac{(ax-1)\sqrt{-\frac{2}{ax-1}-1}\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)}{ad}-\frac{2\left(ac\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)-2\,d\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)\right)\arctan\left(\sqrt{-\frac{2}{ax-1}-1}\right)}{ad^2}+\frac{2\left(a^2c^2\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)-2\,acd\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)+d^2\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)\right)\arctan\left(\frac{ac\sqrt{-\frac{2}{ax-1}-1}+d\sqrt{-\frac{2}{ax-1}-1}}{\sqrt{a^2c^2-d^2}}\right)}{\sqrt{a^2c^2-d^2}\,ad^2}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -((a*x - 1)*sqrt(-2/(a*x - 1) - 1)*sgn(1/(a*x - 1))*sgn(a)/(a*d) - 2*(a*c*s
gn(1/(a*x - 1))*sgn(a) - 2*d*sgn(1/(a*x - 1))*sgn(a))*arctan(sqrt(-2/(a*x -
 1) - 1))/(a*d^2) + 2*(a^2*c^2*sgn(1/(a*x - 1))*sgn(a) - 2*a*c*d*sgn(1/(a*x
 - 1))*sgn(a) + d^2*sgn(1/(a*x - 1))*sgn(a))*arctan((a*c*sqrt(-2/(a*x - 1)
- 1) + d*sqrt(-2/(a*x - 1) - 1))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*
a*d^2))*abs(a)
```

maple [B] time = 0.04, size = 1178, normalized size = 11.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x)
```

```
[Out] -1/a^2/(a*c+d)/(x-1/a)^2*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(5/2)-1/(a*c+d)*(-(x-
1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+3/2*a/(a*c+d)*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1
/2)*x+3/2*a/(a*c+d)/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-(x-1/a)^2*a^2-2*(x-1/a
)*a)^(1/2)*x)-1/3*d/(a*c+d)^2*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+1/2*d/(a*c
+d)^2*a*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)*x+1/2*d/(a*c+d)^2*a/(a^2)^(1/2)*
arctan((a^2)^(1/2)/(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)*x)+1/3*d/(a*c+d)^2*(-
(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(3/2)+1/2/(a*c+d)^2*a^2*
c*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2)*x+3/2/(a*c+d)^
2*a^2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-
(a^2*c^2-d^2)/d^2)^(1/2))-1/d/(a*c+d)^2*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(
a^2*c^2-d^2)/d^2)^(1/2)*a^2*c^2+d/(a*c+d)^2*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/
d)-(a^2*c^2-d^2)/d^2)^(1/2)-1/d^2/(a*c+d)^2*a^4*c^3/(a^2)^(1/2)*arctan((a^2
```

```
)^(1/2)*x/(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))-1/d^3
/(a*c+d)^2/(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x
+c/d)+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c
^2-d^2)/d^2)^(1/2))/(x+c/d))*a^4*c^4+2/d/(a*c+d)^2/(-(a^2*c^2-d^2)/d^2)^(1/
2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*
(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))*a^2*c^
2-d/(a*c+d)^2/(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d
*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^
2*c^2-d^2)/d^2)^(1/2))/(x+c/d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax-1)^2(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x - 1)^2*(d*x + c)), x)
```

mupad [B] time = 0.29, size = 148, normalized size = 1.38

$$-\frac{\sqrt{1-a^2x^2}}{d}-\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)\left(2a\sqrt{-a^2}-\frac{a^2c\sqrt{-a^2}}{d}\right)}{a^2d}-\frac{\left(\ln\left(\sqrt{1-\frac{a^2c^2}{d^2}}\sqrt{1-a^2x^2}+\frac{a^2cx}{d}+1\right)-\ln\left(c+dx\right)\right)\left(a^2c^2-2acd+d^2\right)}{d^3\sqrt{1-\frac{a^2c^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((a*x - 1)^2*(c + d*x)),x)
```

```
[Out] - (1 - a^2*x^2)^(1/2)/d - (asinh(x*(-a^2)^(1/2))*(2*a*(-a^2)^(1/2) - (a^2*c
*(-a^2)^(1/2))/d))/(a^2*d) - ((log((1 - (a^2*c^2)/d^2)^(1/2)*(1 - a^2*x^2)^
(1/2) + (a^2*c*x)/d + 1) - log(c + d*x))*(d^2 + a^2*c^2 - 2*a*c*d))/(d^3*(1
 - (a^2*c^2)/d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(c+dx\right)\left(ax-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)/(-a*x+1)**2/(d*x+c),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((c + d*x)*(a*x - 1)**2), x)
```

3.423 $\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=107

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

Rubi [A] time = 0.18, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 844, 216, 725, 204}

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
```

```
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{\int \frac{-a^2d^2+a^3(ac-2d)x}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{a^2d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(a(ac-2d))\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{d^2} + \frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2} - \frac{(ac-d)^2\,\mathrm{Subst}\left(\int \frac{1}{-a^2c^2+d^2-x^2}\,dx, x, \frac{d+a^2cx}{\sqrt{1-a^2x^2}}\right)}{d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2} + \frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 120, normalized size = 1.12

$$
\frac{(ac-d)\sqrt{a^2c^2-d^2}\,\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2(ac+d)} - \frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-d)\sin^{-1}(ax)}{d^2} + \frac{\sin^{-1}(ax)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - d)*ArcSin[a*x])/d^2 + ArcSin[a*x]/d + ((a*
c - d)*Sqrt[a^2*c^2 - d^2]*ArcTan[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1
 - a^2*x^2])])/(d^2*(a*c + d))
```

IntegrateAlgebraic [B] time = 1.78, size = 971, normalized size = 9.07

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) + ((a^3*c^3*Sqrt[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^
2 - d^2]] - a^2*c^2*d*Sqrt[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]] - a
*c*d^2*Sqrt[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]] + d^3*Sqrt[2*a^2*c
^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]] + a^2*c^2*Sqrt[a^2*c^2 - d^2]*Sqrt[2*
a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]] - a*c*d*Sqrt[a^2*c^2 - d^2]*Sqrt
[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]])*ArcTan[(Sqrt[-a^2]*d*x - d*S
qrt[1 - a^2*x^2])/Sqrt[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^2*c^2 - d^2]]])/(d^4*
(a*c + d)) + ((a^3*c^3*Sqrt[2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]] -
a^2*c^2*d*Sqrt[2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]] - a*c*d^2*Sqrt[
2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]] + d^3*Sqrt[2*a^2*c^2 - d^2 + 2
*a*c*Sqrt[a^2*c^2 - d^2]] - a^2*c^2*Sqrt[a^2*c^2 - d^2]*Sqrt[2*a^2*c^2 - d^
2 + 2*a*c*Sqrt[a^2*c^2 - d^2]] + a*c*d*Sqrt[a^2*c^2 - d^2]*Sqrt[2*a^2*c^2 -
 d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]])*ArcTan[(Sqrt[-a^2]*d*x - d*Sqrt[1 - a^2*
x^2])/Sqrt[2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]]])/(d^4*(a*c + d)) -
 (Sqrt[-a^2]*(a*c - d)*Sqrt[-(a^2*c^2) + d^2]*ArcTan[(a^2*c^2 - a^2*d^2*x^2
 - Sqrt[-a^2]*d^2*x*Sqrt[1 - a^2*x^2])/(a*c*Sqrt[-(a^2*c^2) + d^2])])/(a*d^
2*(a*c + d)) - (Sqrt[-a^2]*(a*c - 2*d)*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x
^2]])/(a*d^2)
```

fricas [A] time = 0.56, size = 318, normalized size = 2.97

$$\left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-(a^2c^2-d^2)\sqrt{-a^2x^2+1}-\left(acd+d^2+(a^3c^2+a^2cd)x+\sqrt{-a^2x^2+1}\,(acd+d^2)\right)\sqrt{-\frac{ac-d}{ac+d}}}{dx+c}\right)-2\,(ac-2\,d)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,d}{d^2},\ \frac{2\,(ac-d)\sqrt{\frac{ac-d}{ac+d}}\arctan\left(\frac{\left(dx-\sqrt{-a^2x^2+1}\,c+c\right)\sqrt{\frac{ac-d}{ac+d}}}{(ac-d)x}\right)+2\,(ac-2\,d)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}\,d}{d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 - d
^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2*x
^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c - 2*
d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2, (2*(
a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c + c)*
sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt(-a^2*
x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

giac [A] time = 0.46, size = 131, normalized size = 1.22

$$-\frac{\left(a^2c-2\,ad\right)\arcsin\left(ax\right)\operatorname{sgn}(a)}{d^2|a|}-\frac{\sqrt{-a^2x^2+1}}{d}-\frac{2\left(a^3c^2-2\,a^2cd+ad^2\right)\arctan\left(\frac{d+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c}{ax}}{\sqrt{a^2c^2-d^2}}\right)}{\sqrt{a^2c^2-d^2}\,d^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(a^2*c - 2*a*d)*arcsin(a*x)*sgn(a)/(d^2*abs(a)) - sqrt(-a^2*x^2 + 1)/d - 2
*(a^3*c^2 - 2*a^2*c*d + a*d^2)*arctan((d + (sqrt(-a^2*x^2 + 1)*abs(a) + a)*
c/(a*x))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*d^2*abs(a))
```

maple [B] time = 0.02, size = 524, normalized size = 4.90

$$-\frac{a^2c^2\ln\left(\frac{\frac{2\left(x+\frac{c}{d}\right)a^2c}{d}-\frac{2(a^2c^2-d^2)}{d^2}+2\sqrt{-\frac{a^2c^2-d^2}{d^2}}\sqrt{\frac{2\left(x+\frac{c}{d}\right)a^2c}{d}-\left(x+\frac{c}{d}\right)^2a^2-\frac{a^2c^2-d^2}{d^2}}}{x+\frac{c}{d}}\right)}{\sqrt{-\frac{a^2c^2-d^2}{d^2}}\,d^3}-\frac{a^2c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}\,d^2}+\frac{2ac\ln\left(\frac{\frac{2\left(x+\frac{c}{d}\right)a^2c}{d}-\frac{2(a^2c^2-d^2)}{d^2}+2\sqrt{-\frac{a^2c^2-d^2}{d^2}}\sqrt{\frac{2\left(x+\frac{c}{d}\right)a^2c}{d}-\left(x+\frac{c}{d}\right)^2a^2-\frac{a^2c^2-d^2}{d^2}}}{x+\frac{c}{d}}\right)}{\sqrt{-\frac{a^2c^2-d^2}{d^2}}\,d^2}+\frac{2a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}\,d}-\frac{\ln\left(\frac{\frac{2\left(x+\frac{c}{d}\right)a^2c}{d}-\frac{2(a^2c^2-d^2)}{d^2}+2\sqrt{-\frac{a^2c^2-d^2}{d^2}}\sqrt{\frac{2\left(x+\frac{c}{d}\right)a^2c}{d}-\left(x+\frac{c}{d}\right)^2a^2-\frac{a^2c^2-d^2}{d^2}}}{x+\frac{c}{d}}\right)}{\sqrt{-\frac{a^2c^2-d^2}{d^2}}\,d}-\frac{\sqrt{-a^2x^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/d-a^2/d^2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*x^2+1)
^(1/2)*x)+2*a/d/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*x^2+1)^(1/2)*x)-1/d^3/
(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((2*(x+c/d)*a^2*c/d-2*(a^2*c^2-d^2)/d^2+2*(-(a
^2*c^2-d^2)/d^2)^(1/2)*(2*(x+c/d)*a^2*c/d-(x+c/d)^2*a^2-(a^2*c^2-d^2)/d^2)^
(1/2))/(x+c/d))*a^2*c^2+2/d^2/(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((2*(x+c/d)*a^2*
c/d-2*(a^2*c^2-d^2)/d^2+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*(2*(x+c/d)*a^2*c/d-(x+
c/d)^2*a^2-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))*a*c-1/d/(-(a^2*c^2-d^2)/d^2)^
(1/2)*ln((2*(x+c/d)*a^2*c/d-2*(a^2*c^2-d^2)/d^2+2*(-(a^2*c^2-d^2)/d^2)^(1/2
)*(2*(x+c/d)*a^2*c/d-(x+c/d)^2*a^2-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(d-a*c>0)', see `assume?` for more d
etails)Is d-a*c positive, negative or zero?
```

mupad [B] time = 0.12, size = 148, normalized size = 1.38

$$-\frac{\sqrt{1-a^2x^2}}{d}-\frac{\mathrm{asinh}\left(x\sqrt{-a^2}\right)\left(2a\sqrt{-a^2}-\frac{a^2c\sqrt{-a^2}}{d}\right)}{a^2d}-\frac{\left(\ln\left(\sqrt{1-\frac{a^2c^2}{d^2}}\sqrt{1-a^2x^2}+\frac{a^2cx}{d}+1\right)-\ln\left(c+dx\right)\right)\left(a^2c^2-2acd+d^2\right)}{d^3\sqrt{1-\frac{a^2c^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((1 - a^2*x^2)^(1/2)*(c + d*x)),x)
```

```
[Out] - (1 - a^2*x^2)^(1/2)/d - (asinh(x*(-a^2)^(1/2))*(2*a*(-a^2)^(1/2) - (a^2*c
*(-a^2)^(1/2))/d))/(a^2*d) - ((log((1 - (a^2*c^2)/d^2)^(1/2)*(1 - a^2*x^2)^
(1/2) + (a^2*c*x)/d + 1) - log(c + d*x))*(d^2 + a^2*c^2 - 2*a*c*d))/(d^3*(1
 - (a^2*c^2)/d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^2}{\sqrt{-(ax-1)(ax+1)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(d*x+c)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**2/(sqrt(-(a*x - 1)*(a*x + 1))*(c + d*x)), x)
```

3.424 $\int \frac{\sqrt{d+ex}\,(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=269

$$-\frac{16\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2\left(2ae^2g-cd(3ef-dg)\right)}{35c^4d^4e\sqrt{d+ex}}+\frac{16g\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cd}}{35c^3d^3e}$$

Rubi [A] time = 0.42, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{12(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{35c^2d^2\sqrt{d+ex}}+\frac{16g\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{35c^3d^3e}-\frac{16\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2\left(2ae^2g-cd(3ef-dg)\right)}{35c^4d^4e\sqrt{d+ex}}+\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(35*c^4*d^4*e*Sqrt[d + e*x]) + (16*g*(c*d*f - a*e*
g)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^3*d^3
*e) + (12*(c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(35*c^2*d^2*Sqrt[d + e*x]) + (2*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(7*c*d*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7cd\sqrt{d+ex}} + \frac{\left(6\left(cde^2f+cd^2eg-e\left(cd^2+a\right.\right.\right.}{7}$$

$$= \frac{12(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^2d^2\sqrt{d+ex}} + \frac{2(f+gx)^3\sqrt{ade\ }}{7c}$$

$$= \frac{16g(cdf-aeg)^2\sqrt{d+ex}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^3d^3e} + \frac{12(cdf-aeg)(f}{}$$

$$= -\frac{16(cdf-aeg)^2\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^4d^4e\sqrt{d+ex}} +$$

Mathematica [A] time = 0.13, size = 136, normalized size = 0.51

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(-16a^3e^3g^3+8a^2cde^2g^2(7f+gx)-2ac^2d^2eg\left(35f^2+14fgx+3g^2x^2\right)+c^3d^3\left(35f^3+35f^2gx+21fg^2x^2+5g^3x^3\right)\right)}{35c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(7*f
+ g*x) - 2*a*c^2*d^2*e*g*(35*f^2 + 14*f*g*x + 3*g^2*x^2) + c^3*d^3*(35*f^3
+ 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3)))/(35*c^4*d^4*Sqrt[d + e*x])
```

IntegrateAlgebraic [A] time = 0.85, size = 394, normalized size = 1.46

$$\frac{2\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}\left(-16a^3e^6g^3-8a^2cd^2e^4g^3+56a^2cde^5fg^2+8a^2cde^4g^3(d+ex)-6ac^2d^4e^2g^3+28ac^2d^3e^3fg^2+12ac^2d^3e^2g^3(d+ex)-70ac^2d^2e^4f^2g-28ac^2d^2e^3fg^2(d+ex)-6ac^2d^2e^2g^3(d+ex)^2-5c^3d^6g^3+21c^3d^5efg^2+15c^3d^5g^3(d+ex)-35c^3d^4e^2f^2g-42c^3d^4efg^2(d+ex)-15c^3d^4g^3(d+ex)^2+35c^3d^3e^3f^3+35c^3d^3e^2f^2g(d+ex)+21c^3d^3efg^2(d+ex)^2+5c^3d^3g^3(d+ex)^3\right)}{35c^4d^4e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*(35*c
^3*d^3*e^3*f^3 - 35*c^3*d^4*e^2*f^2*g - 70*a*c^2*d^2*e^4*f^2*g + 21*c^3*d^5
*e*f*g^2 + 28*a*c^2*d^3*e^3*f*g^2 + 56*a^2*c*d*e^5*f*g^2 - 5*c^3*d^6*g^3 -
6*a*c^2*d^4*e^2*g^3 - 8*a^2*c*d^2*e^4*g^3 - 16*a^3*e^6*g^3 + 35*c^3*d^3*e^2
*f^2*g*(d + e*x) - 42*c^3*d^4*e*f*g^2*(d + e*x) - 28*a*c^2*d^2*e^3*f*g^2*(d
 + e*x) + 15*c^3*d^5*g^3*(d + e*x) + 12*a*c^2*d^3*e^2*g^3*(d + e*x) + 8*a^2
*c*d*e^4*g^3*(d + e*x) + 21*c^3*d^3*e*f*g^2*(d + e*x)^2 - 15*c^3*d^4*g^3*(d
 + e*x)^2 - 6*a*c^2*d^2*e^2*g^3*(d + e*x)^2 + 5*c^3*d^3*g^3*(d + e*x)^3))/(
35*c^4*d^4*e^3*Sqrt[d + e*x])
```

fricas [A] time = 0.42, size = 193, normalized size = 0.72

$$\frac{2\left(5c^3d^3g^3x^3+35c^3d^3f^3-70ac^2d^2ef^2g+56a^2cde^2fg^2-16a^3e^3g^3+3\left(7c^3d^3fg^2-2ac^2d^2eg^3\right)x^2+\left(35c^3d^3f^2g-28ac^2d^2efg^2+8a^2cde^2g^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{35\left(c^4d^4ex+c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 70*a*c^2*d^2*e*f^2*g + 56*a^2*c*
d*e^2*f*g^2 - 16*a^3*e^3*g^3 + 3*(7*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2
+ (35*c^3*d^3*f^2*g - 28*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^3}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^3/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

maple [A] time = 0.01, size = 188, normalized size = 0.70

$$-\frac{2\left(cdx+ae\right)\left(-5g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a^3e^3g^3-56a^2cd\,e^2f\,g^2+70a\,c^2d^2e\,f^2g-35f^3c^3d^3\right)\sqrt{ex+d}}{35\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\;c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-5*c^3*d^3*g^3*x^3+6*a*c^2*d^2*e*g^3*x^2-21*c^3*d^3*f*g^
2*x^2-8*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*g^2*x-35*c^3*d^3*f^2*g*x+16*a^3*
e^3*g^3-56*a^2*c*d*e^2*f*g^2+70*a*c^2*d^2*e*f^2*g-35*c^3*d^3*f^3)*(e*x+d)^(
1/2)/c^4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.66, size = 218, normalized size = 0.81

$$\frac{2\sqrt{cdx+ae}\,f^3}{cd}+\frac{2\left(c^2d^2x^2-acdex-2\,a^2e^2\right)f^2g}{\sqrt{cdx+ae}\,c^2d^2}+\frac{2\left(3\,c^3d^3x^3-ac^2d^2ex^2+4\,a^2cde^2x+8\,a^3e^3\right)fg^2}{5\sqrt{cdx+ae}\,c^3d^3}+\frac{2\left(5\,c^4d^4x^4-ac^3d^3ex^3+2\,a^2c^2d^2e^2x^2-8\,a^3cde^3x-16\,a^4e^4\right)g^3}{35\sqrt{cdx+ae}\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f^3/(c*d) + 2*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f^2
*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4*a
^2*c*d*e^2*x + 8*a^3*e^3)*f*g^2/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(5*c^4*d
^4*x^4 - a*c^3*d^3*e*x^3 + 2*a^2*c^2*d^2*e^2*x^2 - 8*a^3*c*d*e^3*x - 16*a^4
*e^4)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

mupad [B] time = 3.66, size = 218, normalized size = 0.81

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(32\,a^3\,e^3\,g^3-112\,a^2\,c\,d\,e^2\,f\,g^2+140\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3\right)}{35\,c^4\,d^4\,e}-\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{7\,c\,d\,e}+\frac{6\,g^2\,x^2\,\left(2\,a\,e\,g-7\,c\,d\,f\right)\,\sqrt{d+e\,x}}{35\,c^2\,d^2\,e}-\frac{2\,g\,x\,\sqrt{d+e\,x}\,\left(8\,a^2\,e^2\,g^2-28\,a\,c\,d\,e\,f\,g+35\,c^2\,d^2\,f^2\right)}{35\,c^3\,d^3\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*e
^3*g^3 - 70*c^3*d^3*f^3 + 140*a*c^2*d^2*e*f^2*g - 112*a^2*c*d*e^2*f*g^2))/(
35*c^4*d^4*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(7*c*d*e) + (6*g^2*x^2*(2*a*e*g
 - 7*c*d*f)*(d + e*x)^(1/2))/(35*c^2*d^2*e) - (2*g*x*(d + e*x)^(1/2)*(8*a^2
*e^2*g^2 + 35*c^2*d^2*f^2 - 28*a*c*d*e*f*g))/(35*c^3*d^3*e)))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\left(f+gx\right)^{3}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.425 $\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=200

$$-\frac{8\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)}{15c^3d^3e\sqrt{d+ex}}+\frac{8g\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15c^2d^2e}$$

Rubi [A] time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{8g\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{15c^2d^2e}-\frac{8\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)}{15c^3d^3e\sqrt{d+ex}}+\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) + (8*g*(c*d*f - a*e*g)*S
qrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2*e) +
(2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d +
 e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5cd\sqrt{d+ex}} + \frac{\left(4\left(cde^2f+cd^2eg-e\left(cd^2+a\right.\right.\right.}{5}$$

$$= \frac{8g(cdf-aeg)\sqrt{d+ex}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^2d^2e} + \frac{2(f+gx)^2\sqrt{ade+}}{5cd}$$

$$= -\frac{8(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^3d^3e\sqrt{d+ex}} + \frac{8g}{}$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.44

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^2g^2-4acdeg(5f+gx)+c^2d^2\left(15f^2+10fgx+3g^2x^2\right)\right)}{15c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(5*f + g*x) +
 c^2*d^2*(15*f^2 + 10*f*g*x + 3*g^2*x^2)))/(15*c^3*d^3*Sqrt[d + e*x])
```

IntegrateAlgebraic [A] time = 0.44, size = 199, normalized size = 1.00

$$\frac{2\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}\left(8a^2e^4g^2+4acd^2e^2g^2-20acde^3fg-4acde^2g^2(d+ex)+3c^2d^4g^2-10c^2d^3efg-6c^2d^3g^2(d+ex)+15c^2d^2e^2f^2+10c^2d^2efg(d+ex)+3c^2d^2g^2(d+ex)^2\right)}{15c^3d^3e^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*(15*c
^2*d^2*e^2*f^2 - 10*c^2*d^3*e*f*g - 20*a*c*d*e^3*f*g + 3*c^2*d^4*g^2 + 4*a*
c*d^2*e^2*g^2 + 8*a^2*e^4*g^2 + 10*c^2*d^2*e*f*g*(d + e*x) - 6*c^2*d^3*g^2*
(d + e*x) - 4*a*c*d*e^2*g^2*(d + e*x) + 3*c^2*d^2*g^2*(d + e*x)^2))/(15*c^3
*d^3*e^2*Sqrt[d + e*x])
```

fricas [A] time = 0.44, size = 123, normalized size = 0.62

$$\frac{2\left(3\,c^2d^2g^2x^2+15\,c^2d^2f^2-20\,acdefg+8\,a^2e^2g^2+2\left(5\,c^2d^2fg-2\,acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{15\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g + 8*a^2*e^2*g^2 +
 2*(5*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^2}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^2/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

maple [A] time = 0.01, size = 116, normalized size = 0.58

$$\frac{2 (cdx + ae) \left(3g^{2}x^{2}c^{2}d^{2} - 4acde\,g^{2}x + 10c^{2}d^{2}fgx + 8a^{2}e^{2}g^{2} - 20acdefg + 15f^{2}c^{2}d^{2}\right) \sqrt{ex + d}}{15\sqrt{cde\,x^{2} + a\,e^{2}x + c\,d^{2}x + ade}\; c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 2/15*(c*d*x+a*e)*(3*c^2*d^2*g^2*x^2-4*a*c*d*e*g^2*x+10*c^2*d^2*f*g*x+8*a^2*
e^2*g^2-20*a*c*d*e*f*g+15*c^2*d^2*f^2)*(e*x+d)^(1/2)/c^3/d^3/(c*d*e*x^2+a*e
^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.61, size = 133, normalized size = 0.66

$$\frac{2 \sqrt{cdx + ae}\, f^{2}}{cd} + \frac{4 \left(c^{2}d^{2}x^{2} - acdex - 2\,a^{2}e^{2}\right) fg}{3 \sqrt{cdx + ae}\, c^{2}d^{2}} + \frac{2 \left(3\,c^{3}d^{3}x^{3} - ac^{2}d^{2}ex^{2} + 4\,a^{2}cde^{2}x + 8\,a^{3}e^{3}\right) g^{2}}{15 \sqrt{cdx + ae}\, c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f
*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4*
a^2*c*d*e^2*x + 8*a^3*e^3)*g^2/(sqrt(c*d*x + a*e)*c^3*d^3)
```

mupad [B] time = 3.40, size = 142, normalized size = 0.71

$$\frac{\sqrt{c\,d\,e\,x^{2} + \left(c\,d^{2} + a\,e^{2}\right)\,x + a\,d\,e}\;\left(\frac{\sqrt{d+e\,x}\,\left(16\,a^{2}\,e^{2}\,g^{2} - 40\,a\,c\,d\,e\,f\,g + 30\,c^{2}\,d^{2}\,f^{2}\right)}{15\,c^{3}\,d^{3}\,e} + \frac{2\,g^{2}\,x^{2}\,\sqrt{d+e\,x}}{5\,c\,d\,e} - \frac{4\,g\,x\,\left(2\,a\,e\,g - 5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{15\,c^{2}\,d^{2}\,e}\right)}{x + \frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e^
2*g^2 + 30*c^2*d^2*f^2 - 40*a*c*d*e*f*g))/(15*c^3*d^3*e) + (2*g^2*x^2*(d +
e*x)^(1/2))/(5*c*d*e) - (4*g*x*(2*a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*c^2
*d^2*e)))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex}\, \left(f + gx\right)^{2}}{\sqrt{\left(d + ex\right) \left(ae + cdx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.426 $\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^2d^2e\sqrt{d+ex}}$$

Rubi [A] time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^2d^2e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x
]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c^2*d^2*e*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(3*c*d*e)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\frac{2g\sqrt{d+ex}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cde}+\frac{1}{3}\left(3f-\frac{dg}{e}-\frac{2aeg}{cd}\right)\int \frac{}{\sqrt{ade}}$$

$$=-\frac{2\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^2d^2e\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{}}{}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.42

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\,(cd(3f+gx)-2aeg)}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(3*f + g*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

IntegrateAlgebraic [A] time = 0.23, size = 92, normalized size = 0.74

$$\frac{2\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}\left(-2ae^2g-cd^2g+3cdef+cdg(d+ex)\right)}{3c^2d^2e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2],x]
```

```
[Out] (2*(3*c*d*e*f - c*d^2*g - 2*a*e^2*g + c*d*g*(d + e*x))*Sqrt[-((c*d^2*(d + e
*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e])/(3*c^2*d^2*e*Sqrt[d + e*x])
```

fricas [A] time = 0.40, size = 71, normalized size = 0.57

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdgx+3\,cdf-2\,aeg\right)\sqrt{ex+d}}{3\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2*a*e*
g)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ex+d}\left(gx+f\right)}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x), x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.54

$$-\frac{2\left(cdx+ae\right)\left(-cdgx+2aeg-3cdf\right)\sqrt{ex+d}}{3\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\;c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c*d*g*x+2*a*e*g-3*c*d*f)*(e*x+d)^(1/2)/c^2/d^2/(c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.55, size = 65, normalized size = 0.52

$$\frac{2\sqrt{cdx+ae}\,f}{cd}+\frac{2\left(c^2d^2x^2-acdex-2\,a^2e^2\right)g}{3\sqrt{cdx+ae}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f/(c*d) + 2/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*g/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

mupad [B] time = 3.23, size = 88, normalized size = 0.70

$$-\frac{\left(\frac{\left(4\,a\,e\,g-6\,c\,d\,f\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}-\frac{2\,g\,x\,\sqrt{d+e\,x}}{3\,c\,d\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2),x)
```

```
[Out] -((((4*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*c^2*d^2*e) - (2*g*x*(d + e*x)^(
1/2))/(3*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex} \left(f + gx\right)}{\sqrt{\left(d + ex\right) \left(ae + cdx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.427 $\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.76

$$\frac{2\sqrt{(d+ex)(ae+cdx)}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/(c*d*Sqrt[d + e*x])
```

IntegrateAlgebraic [A] time = 0.00, size = 57, normalized size = 1.24

$$\frac{2\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
],x]
```

```
[Out] (2*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e])/(c*d
*Sqrt[d + e*x])
```

fricas [A] time = 0.41, size = 49, normalized size = 1.07

$$\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,\sqrt{ex + d}}{cdex + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^
2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex + d}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(e*x + d)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

maple [A] time = 0.00, size = 50, normalized size = 1.09

$$\frac{2\,(cdx + ae)\,\sqrt{ex + d}}{\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}\;cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 2*(c*d*x+a*e)*(e*x+d)^(1/2)/c/d/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.50, size = 18, normalized size = 0.39

$$\frac{2\sqrt{cdx + ae}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)/(c*d)
```

mupad [B] time = 3.20, size = 54, normalized size = 1.17

$$\frac{2\sqrt{d + e\,x}\,\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{c\,d\,e\,\left(x + \frac{d}{e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

```
[Out] (2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c*d*e*(x
 + d/e))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.428 $\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=80

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {874, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x
]
```

```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f
 - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*Sqrt[c*d*f - a*e*g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx &= \left(2e^2\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg)+e^2gx^2}\,dx, x, \sqrt{\right.\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 93, normalized size = 1.16

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[
c*d*f - a*e*g]])/(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)]
)
```

IntegrateAlgebraic [C] time = 5.27, size = 609, normalized size = 7.61

$$-\frac{2\left(\sqrt{e}\sqrt{cdf-aeg}-i\sqrt{c}\sqrt{d}\sqrt{dg-ef}\right)\sqrt{-2i\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}+ae^2g+cd^2g-2cdef}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{d+ex}\sqrt{-2i\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}+ae^2g+cd^2g-2cdef}}{e\sqrt{g}\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}-\sqrt{g}\sqrt{cde}(d+ex)}\right)}{g^{3/2}\left(ae^2-cd^2\right)\sqrt{cdf-aeg}}-\frac{2\left(\sqrt{e}\sqrt{cdf-aeg}+i\sqrt{c}\sqrt{d}\sqrt{dg-ef}\right)\sqrt{2i\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}+ae^2g+cd^2g-2cdef}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{d+ex}\sqrt{2i\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}+ae^2g+cd^2g-2cdef}}{e\sqrt{g}\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}-\sqrt{g}\sqrt{cde}(d+ex)}\right)}{g^{3/2}\left(ae^2-cd^2\right)\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]),x]
```

```
[Out] (-2*((-I)*Sqrt[c]*Sqrt[d]*Sqrt[-(e*f) + d*g] + Sqrt[e]*Sqrt[c*d*f - a*e*g])
*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-
(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*
g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f -
 a*e*g]]*Sqrt[d + e*x])/(-(Sqrt[c*d*e]*Sqrt[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[
-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e])]))/((-(c*d^2)
 + a*e^2)*g^(3/2)*Sqrt[c*d*f - a*e*g]) - (2*(I*Sqrt[c]*Sqrt[d]*Sqrt[-(e*f)
+ d*g] + Sqrt[e]*Sqrt[c*d*f - a*e*g])*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g +
 (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*ArcT
anh[(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sq
rt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x])/(-(Sqrt[c*d*e]
*Sqrt[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x)
 + (c*d*(d + e*x)^2)/e])]))/((-(c*d^2) + a*e^2)*g^(3/2)*Sqrt[c*d*f - a*e*g])
```

fricas [A] time = 0.42, size = 252, normalized size = 3.15

$$\left[-\frac{\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2adeg-\left(cdef-\left(cd^2+2ae^2\right)g\right)x-2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdfg+aeg^2}\sqrt{ex+d}}{egx^2+df+\left(ef+dg\right)x}\right)}{cdfg-aeg^2},-\frac{2\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{cdfg-aeg^2}\sqrt{ex+d}}{cdegx^2+adeg+\left(cd^2+ae^2\right)gx}\right)}{\sqrt{cdfg-aeg^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] [-sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e
*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x))/(
c*d*f*g - a*e*g^2), -2*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^
2)*g*x))/sqrt(c*d*f*g - a*e*g^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)), x)
```

maple [A] time = 0.03, size = 87, normalized size = 1.09

$$-\frac{2\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\;\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)}{\sqrt{ex+d}\;\sqrt{cdx+ae}\;\sqrt{\left(aeg-cdf\right)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/(e*x+d)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\,\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d+e\,x}}{\left(f+g\,x\right)\,\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

3.429 $\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\,(cdf-aeg)^{3/2}}$$

Rubi [A] time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {872, 874, 205}

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\,(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*
(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}}{2(cdf-aeg)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{\left(cde^2\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(}\right.}{c}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right.}{\sqrt{g}\,(cdf-aeg)^{3/2}}$$

Mathematica [**A**] time = 0.11, size = 136, normalized size = 0.97

$$\frac{\sqrt{d+ex}\left(\sqrt{g}\,(ae+cdx)\sqrt{cdf-aeg}+cd(f+gx)\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{\sqrt{g}\,(f+gx)\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*Sqrt[a*e +
c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/
(Sqrt[g]*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x))
```

IntegrateAlgebraic [**F**] time = 180.13, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]),x]
```

```
[Out] $Aborted
```

fricas [**B**] time = 0.43, size = 703, normalized size = 5.02

$$\left[\frac{\left(cdegx^2+cd^2f+\left(cdef+cd^2g\right)x\right)\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2adeg-\left(cdef-\left(cd^2+2ae^2\right)g\right)x+2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdfg+aeg^2}\sqrt{ex+d}}{egx^2+df+\left(ef+dg\right)x}\right)+2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdfg-aeg^2\right)\sqrt{ex+d}}{2\left(c^2d^3f^3g-2acd^2ef^2g^2+a^2de^2fg^3+\left(c^2d^2ef^2g^2-2acde^2fg^3+a^2e^3g^4\right)x^2+\left(c^2d^2ef^3g+a^2de^2g^4+\left(c^2d^3-2acde^2\right)f^2g^2-\left(2acd^2e-a^2e^3\right)fg^3\right)x\right)}, -\frac{\left(cdegx^2+cd^2f+\left(cdef+cd^2g\right)x\right)\sqrt{cdfg-aeg^2}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{cdfg-aeg^2}\sqrt{ex+d}}{cdegx^2+adeg+\left(cd^2+ae^2\right)gx}\right)-\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdfg-aeg^2\right)\sqrt{ex+d}}{c^2d^3f^3g-2acd^2ef^2g^2+a^2de^2fg^3+\left(c^2d^2ef^2g^2-2acde^2fg^3+a^2e^3g^4\right)x^2+\left(c^2d^2ef^3g+a^2de^2g^4+\left(c^2d^3-2acde^2\right)f^2g^2-\left(2acd^2e-a^2e^3\right)fg^3\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e*g
^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*
g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^
2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^3*g -
 2*a*c*d^2*e*f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f
*g^3 + a^2*e^3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 2*a
*c*d*e^2)*f^2*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x), -((c*d*e*g*x^2 + c*d
^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2
```

```
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*
g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^3*g - 2*a*c*d^2*e*
f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*g^3 + a^2*e^
3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 2*a*c*d*e^2)*f^2
*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 168, normalized size = 1.20

$$\frac{\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\left(cdgx\,\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+cdf\,\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{cdx+ae}\,\sqrt{\left(aeg-cdf\right)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(aeg-cdf\right)\left(gx+f\right)\sqrt{\left(aeg-cdf\right)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2)*g)*x*c*d*g+arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
)*g)*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+
a*e)^(1/2)/(a*e*g-c*d*f)/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2), x)
```

3.430 $\int \frac{\sqrt{d+ex}}{(f+gx)^3 \sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=213

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4\sqrt{g}\,(cdf-aeg)^{5/2}}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}$$

Rubi [A] time = 0.31, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {872, 874, 205}

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4\sqrt{g}\,(cdf-aeg)^{5/2}}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^2) + (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*
d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/(4*Sqrt[g]*(c*d*f - a*e*g)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{(3cd)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+c\cdots}}}{4(cdf-aeg)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cd\cdots}}{4(cdf-aeg)^2\sqrt{d+ex}\,(f+g\cdots}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cd\cdots}}{4(cdf-aeg)^2\sqrt{d+ex}\,(f+g\cdots}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cd\cdots}}{4(cdf-aeg)^2\sqrt{d+ex}\,(f+g\cdots}$$

Mathematica [C] time = 0.05, size = 77, normalized size = 0.36

$$\frac{2c^2d^2\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{d+ex}\,(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (2*c^2*d^2*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[1/2, 3, 3/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)^3*Sqrt[d + e*x])
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.45, size = 1283, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
+ d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(5*c^2*d^2*f^2*g - 7*a*c*d*e*f*g
^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g - 3*a*c^2*d^3*e*f^4*
g^2 + 3*a^2*c*d^2*e^2*f^3*g^3 - a^3*d*e^3*f^2*g^4 + (c^3*d^3*e*f^3*g^3 - 3*
```

```
a*c^2*d^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^3 + (2*c^3*d^3
*e*f^4*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^3*g^3 - 3*(a*c^2
*d^3*e - 2*a^2*c*d*e^3)*f^2*g^4 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^5)*x^2
+ (c^3*d^3*e*f^5*g - 2*a^3*d*e^3*f*g^5 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*
g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (6*a^2*c*d^2*e^2 - a^3*e^4)
*f^2*g^4)*x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g +
 c^2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g
^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g
^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - (5*c^2*d
^2*f^2*g - 7*a*c*d*e*f*g^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*g^2 - a*c*d*e*g^3
)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^
5*g - 3*a*c^2*d^3*e*f^4*g^2 + 3*a^2*c*d^2*e^2*f^3*g^3 - a^3*d*e^3*f^2*g^4 +
 (c^3*d^3*e*f^3*g^3 - 3*a*c^2*d^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e
^4*g^6)*x^3 + (2*c^3*d^3*e*f^4*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 6*a*c^2*d^2
*e^2)*f^3*g^3 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^4 + (3*a^2*c*d^2*e^2
- 2*a^3*e^4)*f*g^5)*x^2 + (c^3*d^3*e*f^5*g - 2*a^3*d*e^3*f*g^5 + (2*c^3*d^4
 - 3*a*c^2*d^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (6*
a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 285, normalized size = 1.34

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3c^2d^2g^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+6c^2d^2fgx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+3c^2d^2f^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-3\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,cdgx+2\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,aeg-5\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,cdf\right)}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(aeg-cdf\right)^2\left(gx+f\right)^2\sqrt{(aeg-cdf)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^2*d^2*g^2+6*arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*x*c^2*d^2*f*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2)*g)*c^2*d^2*f^2-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c
*d*g+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g-5*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f
)^2/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^3), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{\left(f+g\,x\right)^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.431 $\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=280

$$\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8\sqrt{g}\,(cdf-aeg)^{7/2}}+\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}$$

Rubi [A] time = 0.42, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {872, 874, 205}

$$\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8\sqrt{g}\,(cdf-aeg)^{7/2}}+\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^3(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^3) + (5*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(8*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (
5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqr
t[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*Sqrt[g]*(c*d*f - a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{(5cd)\int\frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+c}}}{6(cdf-aeg)}$$

$$=\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5cd\sqrt{ade+(cd^2+ae^2)x+cd}}{12(cdf-aeg)^2\sqrt{d+ex}\,(f+g}$$

$$=\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5cd\sqrt{ade+(cd^2+ae^2)x+cd}}{12(cdf-aeg)^2\sqrt{d+ex}\,(f+g}$$

$$=\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5cd\sqrt{ade+(cd^2+ae^2)x+cd}}{12(cdf-aeg)^2\sqrt{d+ex}\,(f+g}$$

$$=\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5cd\sqrt{ade+(cd^2+ae^2)x+cd}}{12(cdf-aeg)^2\sqrt{d+ex}\,(f+g}$$

Mathematica [C] time = 0.04, size = 77, normalized size = 0.28

$$\frac{2c^3d^3\sqrt{(d+ex)(ae+cdx)}\;{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{d+ex}\,(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (2*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[1/2, 4, 3/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)^4*Sqrt[d + e*x])
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.47, size = 2027, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g
^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*
d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*
e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
```

```
+ d*g)*x)) + 2*(33*c^3*d^3*f^3*g - 59*a*c^2*d^2*e*f^2*g^2 + 34*a^2*c*d*e^2*
f*g^3 - 8*a^3*e^3*g^4 + 15*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 10*(4*c^
3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g - 4*a*c^3*d^4*e*f^
6*g^2 + 6*a^2*c^2*d^3*e^2*f^5*g^3 - 4*a^3*c*d^2*e^3*f^4*g^4 + a^4*d*e^4*f^3
*g^5 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a^2*c^2*d^2*e^3*f^2
*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^4 + (3*c^4*d^4*e*f^5*g^3 + a^4*
d*e^4*g^8 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^4 - 2*(2*a*c^3*d^4*e - 9*a^2
*c^2*d^2*e^3)*f^3*g^5 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^6 - (4*a^
3*c*d^2*e^3 - 3*a^4*e^5)*f*g^7)*x^3 + 3*(c^4*d^4*e*f^6*g^2 + a^4*d*e^4*f*g^
7 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*
e^3)*f^4*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^5 - (4*a^3*c*d^2
*e^3 - a^4*e^5)*f^2*g^6)*x^2 + (c^4*d^4*e*f^7*g + 3*a^4*d*e^4*f^2*g^6 + (3*
c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^2 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^
5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^4 - (12*a^3*c*d^2*e^3 -
 a^4*e^5)*f^3*g^5)*x), -1/24*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*
d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 +
(c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/
(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - (33*c^3*d^3*f^3*g - 59*a*c
^2*d^2*e*f^2*g^2 + 34*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 15*(c^3*d^3*f*g^3
 - a*c^2*d^2*e*g^4)*x^2 + 10*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2
*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))
/(c^4*d^5*f^7*g - 4*a*c^3*d^4*e*f^6*g^2 + 6*a^2*c^2*d^3*e^2*f^5*g^3 - 4*a^3
*c*d^2*e^3*f^4*g^4 + a^4*d*e^4*f^3*g^5 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e
^2*f^3*g^5 + 6*a^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)
*x^4 + (3*c^4*d^4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*
f^4*g^4 - 2*(2*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^5 + 6*(a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^2*g^6 - (4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^7)*x^3 + 3*(
c^4*d^4*e*f^6*g^2 + a^4*d*e^4*f*g^7 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^3 -
 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a
^3*c*d*e^4)*f^3*g^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^6)*x^2 + (c^4*d^4*e
*f^7*g + 3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^2 - 6*(2
*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*
e^4)*f^4*g^4 - (12*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 450, normalized size = 1.61

$$\frac{\sqrt{c d e x^2+a e^2 x+c d^2 x+a d e}\,\left(15 c^3 d^3 g^3 x^3 \operatorname{arctanh}\left(\frac{\sqrt{c d x+a e}\, g}{\sqrt{\left(a e g-c d f\right) g}}\right)+45 c^3 d^3 f g^2 x^2 \operatorname{arctanh}\left(\frac{\sqrt{c d x+a e}\, g}{\sqrt{\left(a e g-c d f\right) g}}\right)+45 c^3 d^3 f^2 g x \operatorname{arctanh}\left(\frac{\sqrt{c d x+a e}\, g}{\sqrt{\left(a e g-c d f\right) g}}\right)+15 c^3 d^3 f^3 \operatorname{arctanh}\left(\frac{\sqrt{c d x+a e}\, g}{\sqrt{\left(a e g-c d f\right) g}}\right)-15 \sqrt{\left(a e g-c d f\right) g}\,\sqrt{c d x+a e}\, c^2 d^2 g^2 x^2+10 \sqrt{\left(a e g-c d f\right) g}\,\sqrt{c d x+a e}\, a c d e g^2 x-40 \sqrt{\left(a e g-c d f\right) g}\,\sqrt{c d x+a e}\, c^2 d^2 f g x-8 \sqrt{\left(a e g-c d f\right) g}\,\sqrt{c d x+a e}\, a^2 e^2 g^2+26 \sqrt{\left(a e g-c d f\right) g}\,\sqrt{c d x+a e}\, a c d e f g-33 \sqrt{\left(a e g-c d f\right) g}\,\sqrt{c d x+a e}\, c^2 d^2 f^2\right)}{24 \sqrt{e x+d}\,\sqrt{c d x+a e}\,\left(a e g-c d f\right)^3 \left(g x+f\right)^3 \sqrt{\left(a e g-c d f\right) g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^4/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*g^3+45*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*f*g^2+45*arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2)*g)*c^3*d^3*f^3-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2
)*x^2*c^2*d^2*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*a*c*d*e*g^
```

```
2-40*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g-8*((a*e*g-c*d*
f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+26*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x
+a*e)^(1/2)*a*c*d*e*f*g-33*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^
2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^3/(g*x+f)^3/((a*e*g-c*
d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.432 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{16g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)}{5c^4d^4e\sqrt{d+ex}}+\frac{16g^2\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+c\cdots}}{5c^3d^3e}$$

Rubi [A] time = 0.33, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {866, 870, 794, 648}

$$\frac{16g^2\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{5c^3d^3e}+\frac{12g(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5c^2d^2\sqrt{d+ex}}-\frac{16g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)}{5c^4d^4e\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^3)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (16*g*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^2*(c*d*
f - a*e*g)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^
3*d^3*e) + (12*g*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
5*c^2*d^2*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(6g)\int \frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{12g(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x}}{5c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{16g^2(cdf-aeg)\sqrt{d+ex}\,\sqrt{ade+\left(c\right.}}{5c^3d^3e}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{16g(cdf-aeg)\left(2ae^2g-cd(3ef-\right.}{5c^4d^4e}$$

Mathematica [A] time = 0.10, size = 134, normalized size = 0.52

$$\frac{2\sqrt{d+ex}\left(16a^3e^3g^3+8a^2cde^2g^2(gx-5f)-2ac^2d^2eg\left(-15f^2+10fgx+g^2x^2\right)+c^3d^3\left(-5f^3+15f^2gx+5fg^2x^2+g^3x^3\right)\right)}{5c^4d^4\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(-5*f + g*x) - 2*a*c^2
*d^2*e*g*(-15*f^2 + 10*f*g*x + g^2*x^2) + c^3*d^3*(-5*f^3 + 15*f^2*g*x + 5*
f*g^2*x^2 + g^3*x^3)))/(5*c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [A] time = 3.64, size = 202, normalized size = 0.79

$$\frac{2(d+ex)^{3/2}(ae+cdx)\left(5a^3e^3g^3-15a^2cde^2fg^2+15a^2e^2g^3(ae+cdx)+15c^2d^2f^2g(ae+cdx)+15ac^2d^2ef^2g+5cdfg^2(ae+cdx)^2-30acdefg^2(ae+cdx)+g^3(ae+cdx)^3-5aeg^3(ae+cdx)^2-5c^3d^3f^3\right)}{5c^4d^4((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(3/2),x]
```

```
[Out] (2*(a*e + c*d*x)*(d + e*x)^(3/2)*(-5*c^3*d^3*f^3 + 15*a*c^2*d^2*e*f^2*g - 1
5*a^2*c*d*e^2*f*g^2 + 5*a^3*e^3*g^3 + 15*c^2*d^2*f^2*g*(a*e + c*d*x) - 30*a
*c*d*e*f*g^2*(a*e + c*d*x) + 15*a^2*e^2*g^3*(a*e + c*d*x) + 5*c*d*f*g^2*(a*
e + c*d*x)^2 - 5*a*e*g^3*(a*e + c*d*x)^2 + g^3*(a*e + c*d*x)^3))/(5*c^4*d^4
*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [A] time = 0.42, size = 216, normalized size = 0.84

$$\frac{2\left(c^3d^3g^3x^3-5c^3d^3f^3+30ac^2d^2ef^2g-40a^2cde^2fg^2+16a^3e^3g^3+\left(5c^3d^3fg^2-2ac^2d^2eg^3\right)x^2+\left(15c^3d^3f^2g-20ac^2d^2efg^2+8a^2cde^2g^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{5\left(c^5d^5ex^2+ac^4d^5e+\left(c^5d^6+ac^4d^4e^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/5*(c^3*d^3*g^3*x^3 - 5*c^3*d^3*f^3 + 30*a*c^2*d^2*e*f^2*g - 40*a^2*c*d*e^
2*f*g^2 + 16*a^3*e^3*g^3 + (5*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 + (15*
c^3*d^3*f^2*g - 20*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x^2 + a*c^4*d^5*e +
(c^5*d^6 + a*c^4*d^4*e^2)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.71Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.01, size = 187, normalized size = 0.73

$$\frac{2\left(cdx+ae\right)\left(g^{3}x^{3}c^{3}d^{3}-2a\,c^{2}d^{2}e\,g^{3}x^{2}+5c^{3}d^{3}f\,g^{2}x^{2}+8a^{2}cd\,e^{2}g^{3}x-20a\,c^{2}d^{2}ef\,g^{2}x+15c^{3}d^{3}f^{2}gx+16a^{3}e^{3}g^{3}-40a^{2}cd\,e^{2}f\,g^{2}+30a\,c^{2}d^{2}e\,f^{2}g-5f^{3}c^{3}d^{3}\right)\left(ex+d\right)^{\frac{3}{2}}}{5\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 2/5*(c*d*x+a*e)*(c^3*d^3*g^3*x^3-2*a*c^2*d^2*e*g^3*x^2+5*c^3*d^3*f*g^2*x^2+
8*a^2*c*d*e^2*g^3*x-20*a*c^2*d^2*e*f*g^2*x+15*c^3*d^3*f^2*g*x+16*a^3*e^3*g^
3-40*a^2*c*d*e^2*f*g^2+30*a*c^2*d^2*e*f^2*g-5*c^3*d^3*f^3)*(e*x+d)^(3/2)/c^
4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [A] time = 0.70, size = 165, normalized size = 0.64

$$-\frac{2\,f^{3}}{\sqrt{cdx+ae}\,cd}+\frac{6\,(cdx+2\,ae)f^{2}g}{\sqrt{cdx+ae}\,c^{2}d^{2}}+\frac{2\left(c^{2}d^{2}x^{2}-4\,acdex-8\,a^{2}e^{2}\right)fg^{2}}{\sqrt{cdx+ae}\,c^{3}d^{3}}+\frac{2\left(c^{3}d^{3}x^{3}-2\,ac^{2}d^{2}ex^{2}+8\,a^{2}cde^{2}x+16\,a^{3}e^{3}\right)g^{3}}{5\,\sqrt{cdx+ae}\,c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] -2*f^3/(sqrt(c*d*x + a*e)*c*d) + 6*(c*d*x + 2*a*e)*f^2*g/(sqrt(c*d*x + a*e)
*c^2*d^2) + 2*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*f*g^2/(sqrt(c*d*x + a
*e)*c^3*d^3) + 2/5*(c^3*d^3*x^3 - 2*a*c^2*d^2*e*x^2 + 8*a^2*c*d*e^2*x + 16*
a^3*e^3)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

mupad [B] time = 3.61, size = 252, normalized size = 0.98

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(32\,a^3\,e^3\,g^3-80\,a^2\,c\,d\,e^2\,f\,g^2+60\,a\,c^2\,d^2\,e\,f^2\,g-10\,c^3\,d^3\,f^3\right)}{5\,c^5\,d^5\,e}+\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{5\,c^2\,d^2\,e}-\frac{2\,g^2\,x^2\,\left(2\,a\,e\,g-5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{5\,c^3\,d^3\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}\,\left(8\,a^2\,e^2\,g^2-20\,a\,c\,d\,e\,f\,g+15\,c^2\,d^2\,f^2\right)}{5\,c^4\,d^4\,e}\right)}{\frac{a}{c}+x^2+\frac{x\,\left(5\,c^5\,d^6+5\,a\,c^4\,d^4\,e^2\right)}{5\,c^5\,d^5\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*e^
3*g^3 - 10*c^3*d^3*f^3 + 60*a*c^2*d^2*e*f^2*g - 80*a^2*c*d*e^2*f*g^2))/(5*c
^5*d^5*e) + (2*g^3*x^3*(d + e*x)^(1/2))/(5*c^2*d^2*e) - (2*g^2*x^2*(2*a*e*g
 - 5*c*d*f)*(d + e*x)^(1/2))/(5*c^3*d^3*e) + (2*g*x*(d + e*x)^(1/2)*(8*a^2*
e^2*g^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(5*c^4*d^4*e)))/(a/c + x^2 + (x
*(5*c^5*d^6 + 5*a*c^4*d^4*e^2))/(5*c^5*d^5*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

3.433 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(2ae^2g-cd(3ef-dg)\right)}{3c^3d^3e\sqrt{d+ex}}+\frac{8g^2\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2e}-\frac{}{cd\sqrt{x}}$$

Rubi [A] time = 0.18, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {866, 794, 648}

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(2ae^2g-cd(3ef-dg)\right)}{3c^3d^3e\sqrt{d+ex}}+\frac{8g^2\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2e}-\frac{2\sqrt{d+ex}\,(f+gx)^2}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^2)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (8*g*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(3*c^3*d^3*e*Sqrt[d + e*x]) + (8*g^2*Sqrt[d + e*x]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,(f+gx)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(4g)\int \frac{\sqrt{d+ex}\,(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{8g^2\sqrt{d+ex}\,\sqrt{ade+\left(cd^2+ae^2\right)x\ -}}{3c^2d^2e}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8g\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+}}{3c^3d^3e\sqrt{d+ex}}$$

Mathematica [A] time = 0.07, size = 88, normalized size = 0.49

$$\frac{2\sqrt{d+ex}\left(-8a^2e^2g^2-4acdeg(gx-3f)+c^2d^2\left(-3f^2+6fgx+g^2x^2\right)\right)}{3c^3d^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-8*a^2*e^2*g^2 - 4*a*c*d*e*g*(-3*f + g*x) + c^2*d^2*(-3*f
^2 + 6*f*g*x + g^2*x^2)))/(3*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [A] time = 2.12, size = 119, normalized size = 0.66

$$\frac{2(d+ex)^{3/2}(ae+cdx)\left(-3a^2e^2g^2+6cdfg(ae+cdx)+6acdefg+g^2(ae+cdx)^2-6aeg^2(ae+cdx)-3c^2d^2f^2\right)}{3c^3d^3((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(3/2),x]
```

```
[Out] (2*(a*e + c*d*x)*(d + e*x)^(3/2)*(-3*c^2*d^2*f^2 + 6*a*c*d*e*f*g - 3*a^2*e^
2*g^2 + 6*c*d*f*g*(a*e + c*d*x) - 6*a*e*g^2*(a*e + c*d*x) + g^2*(a*e + c*d*
x)^2))/(3*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [A] time = 0.41, size = 147, normalized size = 0.81

$$\frac{2\left(c^2d^2g^2x^2-3\,c^2d^2f^2+12\,acdefg-8\,a^2e^2g^2+2\left(3\,c^2d^2fg-2\,acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{3\left(c^4d^4ex^2+ac^3d^4e+\left(c^4d^5+ac^3d^3e^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(c^2*d^2*g^2*x^2 - 3*c^2*d^2*f^2 + 12*a*c*d*e*f*g - 8*a^2*e^2*g^2 + 2*(
3*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.25Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.01, size = 116, normalized size = 0.64

$$-\frac{2\left(cdx+ae\right)\left(-g^{2}x^{2}c^{2}d^{2}+4acde\,g^{2}x-6c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-12acdefg+3f^{2}c^{2}d^{2}\right)\left(ex+d\right)^{\frac{3}{2}}}{3\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c^2*d^2*g^2*x^2+4*a*c*d*e*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^
2*g^2-12*a*c*d*e*f*g+3*c^2*d^2*f^2)*(e*x+d)^(3/2)/c^3/d^3/(c*d*e*x^2+a*e^2*
x+c*d^2*x+a*d*e)^(3/2)
```

maxima [A] time = 0.63, size = 98, normalized size = 0.54

$$-\frac{2\,f^{2}}{\sqrt{cdx+ae}\,cd}+\frac{4\left(cdx+2\,ae\right)fg}{\sqrt{cdx+ae}\,c^{2}d^{2}}+\frac{2\left(c^{2}d^{2}x^{2}-4\,acdex-8\,a^{2}e^{2}\right)g^{2}}{3\sqrt{cdx+ae}\,c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] -2*f^2/(sqrt(c*d*x + a*e)*c*d) + 4*(c*d*x + 2*a*e)*f*g/(sqrt(c*d*x + a*e)*c
^2*d^2) + 2/3*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*g^2/(sqrt(c*d*x + a*e
)*c^3*d^3)
```

mupad [B] time = 3.43, size = 178, normalized size = 0.98

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,a^2\,e^2\,g^2-24\,a\,c\,d\,e\,f\,g+6\,c^2\,d^2\,f^2\right)}{3\,c^4\,d^4\,e}-\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}+\frac{4\,g\,x\,\left(2\,a\,e\,g-3\,c\,d\,f\right)\,\sqrt{d+e\,x}}{3\,c^3\,d^3\,e}\right)}{\frac{a}{c}+x^2+\frac{x\,\left(3\,c^4\,d^5+3\,a\,c^3\,d^3\,e^2\right)}{3\,c^4\,d^4\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e
^2*g^2 + 6*c^2*d^2*f^2 - 24*a*c*d*e*f*g))/(3*c^4*d^4*e) - (2*g^2*x^2*(d + e
*x)^(1/2))/(3*c^2*d^2*e) + (4*g*x*(2*a*e*g - 3*c*d*f)*(d + e*x)^(1/2))/(3*c
^3*d^3*e)))/(a/c + x^2 + (x*(3*c^4*d^5 + 3*a*c^3*d^3*e^2))/(3*c^4*d^4*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

3.434 $\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(2ae^2g-cd(dg+ef)\right)}{c^2d^2\sqrt{d+ex}\,\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Rubi [A] time = 0.14, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {788, 648}

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(2ae^2g-cd(dg+ef)\right)}{c^2d^2\sqrt{d+ex}\,\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(c*d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e*f + d*g))*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(2\left(-\frac{1}{2}e\left(2cdef-\left(cd^2\right.\right.\right.\right.}{}$$

$$=-\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2\left(2ae^2g-cd(ef+dg)\right.}{c^2d^2\left(cd\right.}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.34

$$\frac{2\sqrt{d+ex}\,(2aeg+cd(gx-f))}{c^2d^2\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(2*a*e*g + c*d*(-f + g*x)))/(c^2*d^2*Sqrt[(a*e + c*d*x)*(d
 + e*x)])
```

IntegrateAlgebraic [A] time = 1.36, size = 63, normalized size = 0.42

$$\frac{2(d+ex)^{3/2}(ae+cdx)(g(ae+cdx)+aeg-cdf)}{c^2d^2((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2),x]
```

```
[Out] (2*(a*e + c*d*x)*(d + e*x)^(3/2)*(-(c*d*f) + a*e*g + g*(a*e + c*d*x)))/(c^2
*d^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [A] time = 0.40, size = 96, normalized size = 0.64

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdgx-cdf+2\,aeg\right)\sqrt{ex+d}}{c^3d^3ex^2+ac^2d^3e+\left(c^3d^4+ac^2d^2e^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - c*d*f + 2*a*e*g)*s
qrt(e*x + d)/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.89Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.01, size = 66, normalized size = 0.44

$$\frac{2\,(cdx+ae)\left(cdgx+2aeg-cdf\right)(ex+d)^{\frac{3}{2}}}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 2*(c*d*x+a*e)*(c*d*g*x+2*a*e*g-c*d*f)*(e*x+d)^(3/2)/c^2/d^2/(c*d*e*x^2+a*e^
2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [A] time = 0.57, size = 48, normalized size = 0.32

$$-\frac{2\,f}{\sqrt{cdx+ae}\,cd}+\frac{2\,(cdx+2\,ae)g}{\sqrt{cdx+ae}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="maxima")
```

```
[Out] -2*f/(sqrt(c*d*x + a*e)*c*d) + 2*(c*d*x + 2*a*e)*g/(sqrt(c*d*x + a*e)*c^2*d
^2)
```

mupad [B] time = 3.37, size = 118, normalized size = 0.79

$$\frac{\left(\frac{\left(4\,a\,e\,g-2\,c\,d\,f\right)\,\sqrt{d+e\,x}}{c^3\,d^3\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\frac{a}{c}+x^2+\frac{x\,\left(c^3\,d^4+a\,c^2\,d^2\,e^2\right)}{c^3\,d^3\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/
2),x)
```

```
[Out] ((((4*a*e*g - 2*c*d*f)*(d + e*x)^(1/2))/(c^3*d^3*e) + (2*g*x*(d + e*x)^(1/2
))/(c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(a/c + x^2 +
 (x*(c^3*d^4 + a*c^2*d^2*e^2))/(c^3*d^3*e))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\left(f+gx\right)}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

3.435 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.76

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [A] time = 0.00, size = 43, normalized size = 0.93

$$-\frac{2(d+ex)^{3/2}(ae+cdx)}{cd((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2),x]
```

```
[Out] (-2*(a*e + c*d*x)*(d + e*x)^(3/2))/(c*d*((a*e + c*d*x)*(d + e*x))^(3/2))
```

fricas [A] time = 0.41, size = 74, normalized size = 1.61

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{c^2d^2ex^2+acd^2e+\left(c^2d^3+acde^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x^2
 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.66Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.00, size = 50, normalized size = 1.09

$$-\frac{2\left(cdx+ae\right)\left(ex+d\right)^{\frac{3}{2}}}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2*(c*d*x+a*e)*(e*x+d)^(3/2)/c/d/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [A] time = 0.51, size = 18, normalized size = 0.39

$$-\frac{2}{\sqrt{cdx+ae}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] -2/(sqrt(c*d*x + a*e)*c*d)
```

mupad [B] time = 3.27, size = 82, normalized size = 1.78

$$-\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c^2\,d^2\,e\,\left(\frac{a}{c}+x^2+\frac{x\,\left(c^2\,d^3+a\,c\,d\,e^2\right)}{c^2\,d^2\,e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c^2*d^2
*e*(a/c + x^2 + (x*(c^2*d^3 + a*c*d*e^2))/(c^2*d^2*e)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

3.436 $\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}-\frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}$$

Rubi [A] time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {868, 874, 205}

$$-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}-\frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{g\int \frac{}{(f+gx)\sqrt{ade}}}{cd}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(2e^2g\right)\text{Subst}}{}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\sqrt{g}\tan^{-1}\left(\right.}{(c}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.53

$$-\frac{2\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 1, 1/2, (g*(a*e + c*d*x))/(-(c*d*
f) + a*e*g)])/((c*d*f - a*e*g)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.44, size = 553, normalized size = 4.16

$$\left[-\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)\sqrt{-\frac{g}{cdf-aeg}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg+2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(cdf-aeg)\sqrt{ex+d}\sqrt{-\frac{g}{cdf-aeg}}-\left(cdef-\left(cd^2+2\,ae^2\right)g\right)x}{egx^2+df+(ef+dg)x}\right)+2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{acd^2ef-a^2de^2g+\left(c^2d^2ef-acde^2g\right)x^2+\left(\left(c^2d^3+acde^2\right)f-\left(acd^2e+a^2e^3\right)g\right)x}, -\frac{2\left(\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)\sqrt{\frac{g}{cdf-aeg}}\arctan\left(-\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(cdf-aeg)\sqrt{ex+d}\sqrt{\frac{g}{cdf-aeg}}}{cdegx^2+adeg+\left(cd^2+ae^2\right)gx}\right)+\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\right)}{acd^2ef-a^2de^2g+\left(c^2d^2ef-acde^2g\right)x^2+\left(\left(c^2d^3+acde^2\right)f-\left(acd^2e+a^2e^3\right)g\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] [-((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c
*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (
c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c*d^2*e*f - a^2*d*e^2*g + (c
^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2
*e^3)*g)*x), -2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d*f - a*
e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*s
qrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^
```

```
2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c*
d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d
*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.51Unable to transpose Err
or: Bad Argument Value
```

maple [**A**] time = 0.03, size = 128, normalized size = 0.96

$$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\left(\sqrt{cdx+ae}\,g\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{\left(aeg-cdf\right)g}\right)}{\sqrt{ex+d}\,\left(cdx+ae\right)\left(aeg-cdf\right)\sqrt{\left(aeg-cdf\right)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)-((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^
(1/2)/(c*d*x+a*e)/(a*e*g-c*d*f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^{3/2}}{\left(f+g\,x\right)\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

```
[Out] Timed out
```

3.437 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{3g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}-\frac{3cd\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+c\right.}}{\sqrt{d+ex}\,\cdot}\right.}{(cdf-aeg)^{5/}}$$

Rubi [A] time = 0.26, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$-\frac{3g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}-\frac{3cd\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (3*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f
 - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) - (3*c*d*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/(c*d*f - a*e*g)^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
```

```
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(3g)\cdots}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3g\sqrt{\cdots}}{(c\cdots}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3g\sqrt{\cdots}}{(c\cdots}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{3g\sqrt{\cdots}}{(c\cdots}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.36

$$-\frac{2cd\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)),x]
```

```
[Out] (-2*c*d*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 2, 1/2, (g*(a*e + c*d*x))/(-(
c*d*f) + a*e*g)])/((c*d*f - a*e*g)^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.45, size = 1067, normalized size = 5.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] [1/2*(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f + (c^2*d^3 + a*c*d*e^
2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x)*sqrt(-g/(c*d*f - a*e
*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) -
(c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + 2*c*d*f + a*e*g)*sqrt
(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^3*d*e^3*f*g^2 + (c^
3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3)*x^3 + (c^3*d^3*e*f
^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e + a^2*c*d*e^3)*f*g^2
+ (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3 + (c^3*d^4 + a*c^2*d^
2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^2*c*d^2*e^2 - a^3*e^4
)*f*g^2)*x), -(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f + (c^2*d^3 +
 a*c*d*e^2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x)*sqrt(g/(c*d
*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a
*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2
 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x +
2*c*d*f + a*e*g)*sqrt(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g +
a^3*d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^
3)*x^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e
+ a^2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3
+ (c^3*d^4 + a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^
2*c*d^2*e^2 - a^3*e^4)*f*g^2)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.58Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.03, size = 225, normalized size = 1.11

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3\sqrt{cdx+ae}\,cd\,g^2x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3\sqrt{cdx+ae}\,cdfg\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-3\sqrt{\left(aeg-cdf\right)g}\,cdgx-\sqrt{\left(aeg-cdf\right)g}\,aeg-2\sqrt{\left(aeg-cdf\right)g}\,cdf\right)}{\sqrt{ex+d}\,\left(cdx+ae\right)\left(aeg-cdf\right)^2\left(gx+f\right)\sqrt{\left(aeg-cdf\right)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/((a*e*
g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x*c*d*g^2+3*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*c*d*f*g-3*((a*e*g-c*d*f)*g)^
(1/2)*x*c*d*g-((a*e*g-c*d*f)*g)^(1/2)*a*e*g-2*((a*e*g-c*d*f)*g)^(1/2)*c*d*f
)/(e*x+d)^(1/2)/(c*d*x+a*e)/(a*e*g-c*d*f)^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\left(f+g\,x\right)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

3.438 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{15c^2d^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{7/2}}-\frac{15cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^3}-\frac{5g\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg}$$

Rubi [A] time = 0.35, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$-\frac{15c^2d^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{7/2}}-\frac{15cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^3}-\frac{5g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) - (5*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) - (15*c*d*g*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (1
5*c^2*d^2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*(c*d*f - a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
```

```
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(5g}{}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{5g}{2(c}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{5g}{2(c}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{5g}{2(c}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{5g}{2(c}$$

Mathematica [C] time = 0.04, size = 77, normalized size = 0.28

$$-\frac{2c^2d^2\sqrt{d+ex}\;{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)),x]
```

```
[Out] (-2*c^2*d^2*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 3, 1/2, (g*(a*e + c*d*x))
/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.47, size = 1863, normalized size = 6.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(15*(c^3*d^3*e*g^2*x^4 + a*c^2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d^
4 + a*c^2*d^2*e^2)*g^2)*x^3 + (c^3*d^3*e*f^2 + a*c^2*d^3*e*g^2 + 2*(c^3*d^4
 + a*c^2*d^2*e^2)*f*g)*x^2 + (2*a*c^2*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)
*f^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g +
 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d
)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 +
d*f + (e*f + d*g)*x)) + 2*(15*c^2*d^2*g^2*x^2 + 8*c^2*d^2*f^2 + 9*a*c*d*e*f
*g - 2*a^2*e^2*g^2 + 5*(5*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^
2*f^4*g + 3*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2
- 3*a*c^3*d^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4
+ (2*c^4*d^4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e
 - a^2*c^2*d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^
3*c*d^2*e^3 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d
^5 - a*c^3*d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (
3*a^2*c^2*d^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*
g^4)*x^2 - (2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*
e + 3*a^2*c^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 -
(5*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^3)*x), -1/4*(15*(c^3*d^3*e*g^2*x^4 + a*c^
2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)*g^2)*x^3 + (c^3*
d^3*e*f^2 + a*c^2*d^3*e*g^2 + 2*(c^3*d^4 + a*c^2*d^2*e^2)*f*g)*x^2 + (2*a*c
^2*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)*f^2)*x)*sqrt(g/(c*d*f - a*e*g))*ar
ctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x
+ d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x))
 + (15*c^2*d^2*g^2*x^2 + 8*c^2*d^2*f^2 + 9*a*c*d*e*f*g - 2*a^2*e^2*g^2 + 5*
(5*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(e*x + d))/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^2*f^4*g + 3*a^3*c*d^2*e
^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2 - 3*a*c^3*d^3*e^2*f^2*g
^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4 + (2*c^4*d^4*e*f^4*g +
(c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^2
*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^3*c*d^2*e^3 + a^4*e^5)*
g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d^5 - a*c^3*d^3*e^2)*f^4
*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (3*a^2*c^2*d^3*e^2 + 5*a
^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*g^4)*x^2 - (2*a^4*d*e^4
*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*
f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 - (5*a^3*c*d^2*e^3 - a^4*
e^5)*f^2*g^3)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.85Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.04, size = 379, normalized size = 1.38

$$-\frac{\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\left(15\sqrt{cdx+ae}\,c^{2}d^{2}g^{3}x^{2}\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+30\sqrt{cdx+ae}\,c^{2}d^{2}f\,g^{2}x\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+15\sqrt{cdx+ae}\,c^{2}d^{2}f^{2}g\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-15\sqrt{\left(aeg-cdf\right)g}\,c^{2}d^{2}g^{2}x^{2}-5\sqrt{\left(aeg-cdf\right)g}\,acde\,g^{2}x-25\sqrt{\left(aeg-cdf\right)g}\,c^{2}d^{2}fgx+2\sqrt{\left(aeg-cdf\right)g}\,a^{2}e^{2}g^{2}-9\sqrt{\left(aeg-cdf\right)g}\,acdefg-8\sqrt{\left(aeg-cdf\right)g}\,c^{2}d^{2}f^{2}\right)}{4\sqrt{ex+d}\,\left(cdx+ae\right)\left(aeg-cdf\right)^{3}\left(gx+f\right)^{2}\sqrt{\left(aeg-cdf\right)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x^2*c^2*d^2*g^3+30*arctanh((c*
d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g^2
+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*
c^2*d^2*f^2*g-15*((a*e*g-c*d*f)*g)^(1/2)*x^2*c^2*d^2*g^2-5*((a*e*g-c*d*f)*g
)^(1/2)*x*a*c*d*e*g^2-25*((a*e*g-c*d*f)*g)^(1/2)*x*c^2*d^2*f*g+2*((a*e*g-c*
d*f)*g)^(1/2)*a^2*e^2*g^2-9*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g-8*((a*e*g-c
*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3/(g*x+
f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\left(f+g\,x\right)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

3.439 $\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(2ae^2g-cd(3ef-dg)\right)}{3c^4d^4e\sqrt{d+ex}}+\frac{16g^3\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^3d^3e}-\frac{\quad}{c^2d^2\sqrt{\ }}$$

Rubi [A] time = 0.28, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {866, 794, 648}

$$-\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(2ae^2g-cd(3ef-dg)\right)}{3c^4d^4e\sqrt{d+ex}}-\frac{4g\sqrt{d+ex}\,(f+gx)^2}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16g^3\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^3d^3e}-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (4*g*Sqrt[d + e*x]*(f + g*x)^2)/(c^2*d^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]) - (16*g^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^3*
Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*e)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}dx = -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(2g)\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}}{cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{4g\sqrt{d+ex}\,(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{4g\sqrt{d+ex}\,(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{4g\sqrt{d+ex}\,(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

Mathematica [A] time = 0.11, size = 131, normalized size = 0.55

$$\frac{2(d+ex)^{3/2}\left(-16a^3e^3g^3+24a^2cde^2g^2(f-gx)-6ac^2d^2eg\left(f^2-6fgx+g^2x^2\right)+c^3d^3\left(-f^3-9f^2gx+9fg^2x^2+g^3x^3\right)\right)}{3c^4d^4((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(f - g*x) - 6*a*c^
2*d^2*e*g*(f^2 - 6*f*g*x + g^2*x^2) + c^3*d^3*(-f^3 - 9*f^2*g*x + 9*f*g^2*x
^2 + g^3*x^3)))/(3*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [A] time = 4.48, size = 201, normalized size = 0.84

$$\frac{2(d+ex)^{5/2}(ae+cdx)\left(a^3e^3g^3-3a^2cde^2fg^2-9a^2e^2g^3(ae+cdx)-9c^2d^2f^2g(ae+cdx)+3ac^2d^2ef^2g+9cdfg^2(ae+cdx)^2+18acdefg^2(ae+cdx)+g^3(ae+cdx)^3-9aeg^3(ae+cdx)^2-c^3d^3f^3\right)}{3c^4d^4((d+ex)(ae+cdx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)*(d + e*x)^(5/2)*(-(c^3*d^3*f^3) + 3*a*c^2*d^2*e*f^2*g - 3*
a^2*c*d*e^2*f*g^2 + a^3*e^3*g^3 - 9*c^2*d^2*f^2*g*(a*e + c*d*x) + 18*a*c*d*
e*f*g^2*(a*e + c*d*x) - 9*a^2*e^2*g^3*(a*e + c*d*x) + 9*c*d*f*g^2*(a*e + c*
d*x)^2 - 9*a*e*g^3*(a*e + c*d*x)^2 + g^3*(a*e + c*d*x)^3))/(3*c^4*d^4*((a*e
 + c*d*x)*(d + e*x))^(5/2))
```

fricas [A] time = 0.42, size = 251, normalized size = 1.05

$$\frac{2\left(c^3d^3g^3x^3-c^3d^3f^3-6\,ac^2d^2ef^2g+24\,a^2cde^2fg^2-16\,a^3e^3g^3+3\left(3\,c^3d^3fg^2-2\,ac^2d^2eg^3\right)x^2-3\left(3\,c^3d^3f^2g-12\,ac^2d^2efg^2+8\,a^2cde^2g^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{3\left(c^6d^6ex^3+a^2c^4d^5e^2+\left(c^6d^7+2\,ac^5d^5e^2\right)x^2+\left(2\,ac^5d^6e+a^2c^4d^4e^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(c^3*d^3*g^3*x^3 - c^3*d^3*f^3 - 6*a*c^2*d^2*e*f^2*g + 24*a^2*c*d*e^2*f
*g^2 - 16*a^3*e^3*g^3 + 3*(3*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 - 3*(3*
c^3*d^3*f^2*g - 12*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
```

```
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^6*d^6*e*x^3 + a^2*c^4*d^5*e^
2 + (c^6*d^7 + 2*a*c^5*d^5*e^2)*x^2 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 6.27Unable to transpose Err
or: Bad Argument Value
```

maple [**A**] time = 0.01, size = 187, normalized size = 0.78

$$-\frac{2\left(cdx+ae\right)\left(-g^{3}x^{3}c^{3}d^{3}+6a\,c^{2}d^{2}e\,g^{3}x^{2}-9c^{3}d^{3}f\,g^{2}x^{2}+24a^{2}cd\,e^{2}g^{3}x-36a\,c^{2}d^{2}ef\,g^{2}x+9c^{3}d^{3}f^{2}gx+16a^{3}e^{3}g^{3}-24a^{2}cd\,e^{2}f\,g^{2}+6a\,c^{2}d^{2}e\,f^{2}g+f^{3}c^{3}d^{3}\right)\left(ex+d\right)^{\frac{5}{2}}}{3\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c^3*d^3*g^3*x^3+6*a*c^2*d^2*e*g^3*x^2-9*c^3*d^3*f*g^2*x^
2+24*a^2*c*d*e^2*g^3*x-36*a*c^2*d^2*e*f*g^2*x+9*c^3*d^3*f^2*g*x+16*a^3*e^3*
g^3-24*a^2*c*d*e^2*f*g^2+6*a*c^2*d^2*e*f^2*g+c^3*d^3*f^3)*(e*x+d)^(5/2)/c^4
/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [**A**] time = 0.74, size = 219, normalized size = 0.92

$$-\frac{2\,(3\,cdx+2\,ae)f^{2}g}{\left(c^{3}d^{3}x+ac^{2}d^{2}e\right)\sqrt{cdx+ae}}+\frac{2\left(3\,c^{2}d^{2}x^{2}+12\,acdex+8\,a^{2}e^{2}\right)fg^{2}}{\left(c^{4}d^{4}x+ac^{3}d^{3}e\right)\sqrt{cdx+ae}}+\frac{2\left(c^{3}d^{3}x^{3}-6\,ac^{2}d^{2}ex^{2}-24\,a^{2}cde^{2}x-16\,a^{3}e^{3}\right)g^{3}}{3\left(c^{5}d^{5}x+ac^{4}d^{4}e\right)\sqrt{cdx+ae}}-\frac{2\,f^{3}}{3\left(c^{2}d^{2}x+acde\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] -2*(3*c*d*x + 2*a*e)*f^2*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
2*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*f*g^2/((c^4*d^4*x + a*c^3*d^3*
e)*sqrt(c*d*x + a*e)) + 2/3*(c^3*d^3*x^3 - 6*a*c^2*d^2*e*x^2 - 24*a^2*c*d*e
^2*x - 16*a^3*e^3)*g^3/((c^5*d^5*x + a*c^4*d^4*e)*sqrt(c*d*x + a*e)) - 2/3*
f^3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

mupad [**B**] time = 3.77, size = 278, normalized size = 1.16

$$-\frac{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(\frac{32\,a^{3}\,e^{3}\,g^{3}}{3}-16\,a^{2}\,c\,d\,e^{2}\,f\,g^{2}+4\,a\,c^{2}\,d^{2}\,e\,f^{2}\,g+\frac{2\,c^{3}\,d^{3}\,f^{3}}{3}\right)}{c^{6}\,d^{6}\,e}-\frac{2\,g^{3}\,x^{3}\,\sqrt{d+e\,x}}{3\,c^{3}\,d^{3}\,e}+\frac{g^{2}\,x^{2}\,\left(4\,a\,e\,g-6\,c\,d\,f\right)\,\sqrt{d+e\,x}}{c^{4}\,d^{4}\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}\,\left(8\,a^{2}\,e^{2}\,g^{2}-12\,a\,c\,d\,e\,f\,g+3\,c^{2}\,d^{2}\,f^{2}\right)}{c^{5}\,d^{5}\,e}\right)}{x^{3}+\frac{a^{2}\,e}{c^{2}\,d}+\frac{a\,x\,\left(2\,c\,d^{2}+a\,e^{2}\right)}{c^{2}\,d^{2}}+\frac{x^{2}\,\left(c^{6}\,d^{7}+2\,a\,c^{5}\,d^{5}\,e^{2}\right)}{c^{6}\,d^{6}\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*((32*a^3*
e^3*g^3)/3 + (2*c^3*d^3*f^3)/3 + 4*a*c^2*d^2*e*f^2*g - 16*a^2*c*d*e^2*f*g^2
))/(c^6*d^6*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(3*c^3*d^3*e) + (g^2*x^2*(4*a*
e*g - 6*c*d*f)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d + e*x)^(1/2)*(8*a^2
*e^2*g^2 + 3*c^2*d^2*f^2 - 12*a*c*d*e*f*g))/(c^5*d^5*e)))/(x^3 + (a^2*e)/(c
```

```
^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^6*d^7 + 2*a*c^5*d^5*e^2
))/(c^6*d^6*e))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

3.440 $\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{3c^3d^3\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{8g(d+ex)^{3/2}(cdf-aeg)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\quad}{3cd\left(\right.}$$

Rubi [A] time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {866, 788, 648}

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{3c^3d^3\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{8g(d+ex)^{3/2}(cdf-aeg)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^2)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (8*g*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(3*c^2*d^2*(c*d^2 - a*e
^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*g*(2*a*e^2*g - c*d*(e
*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*(c*d^2 -
 a*e^2)*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(4g)\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}}{3cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{8g(cdf-aeg)(d}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+(c}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{8g(cdf-aeg)(d}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+(c}}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.41

$$\frac{2(d+ex)^{3/2}\left(8a^2e^2g^2-4acdeg(f-3gx)-c^2d^2\left(f^2+6fgx-3g^2x^2\right)\right)}{3c^3d^3((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(f - 3*g*x) - c^2*d^2*(f^2
+ 6*f*g*x - 3*g^2*x^2)))/(3*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [A] time = 2.99, size = 120, normalized size = 0.57

$$\frac{2(d+ex)^{5/2}(ae+cdx)\left(-a^2e^2g^2-6cdfg(ae+cdx)+2acdefg+3g^2(ae+cdx)^2+6aeg^2(ae+cdx)-c^2d^2f^2\right)}{3c^3d^3((d+ex)(ae+cdx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)*(d + e*x)^(5/2)*(-(c^2*d^2*f^2) + 2*a*c*d*e*f*g - a^2*e^2*
g^2 - 6*c*d*f*g*(a*e + c*d*x) + 6*a*e*g^2*(a*e + c*d*x) + 3*g^2*(a*e + c*d*
x)^2))/(3*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(5/2))
```

fricas [A] time = 0.41, size = 180, normalized size = 0.85

$$\frac{2\left(3\,c^2d^2g^2x^2-c^2d^2f^2-4\,acdefg+8\,a^2e^2g^2-6\left(c^2d^2fg-2\,acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{3\left(c^5d^5ex^3+a^2c^3d^4e^2+\left(c^5d^6+2\,ac^4d^4e^2\right)x^2+\left(2\,ac^4d^5e+a^2c^3d^3e^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(3*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 - 4*a*c*d*e*f*g + 8*a^2*e^2*g^2 - 6*(c
^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2
)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 4.4Unable to transpose Erro
r: Bad Argument Value
```

maple [A] time = 0.01, size = 116, normalized size = 0.55

$$\frac{2\,(cdx+ae)\left(3g^2x^2c^2d^2+12acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-4acdefg-f^2c^2d^2\right)(ex+d)^{\frac{5}{2}}}{3\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(3*c^2*d^2*g^2*x^2+12*a*c*d*e*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e
^2*g^2-4*a*c*d*e*f*g-c^2*d^2*f^2)*(e*x+d)^(5/2)/c^3/d^3/(c*d*e*x^2+a*e^2*x+
c*d^2*x+a*d*e)^(5/2)
```

maxima [A] time = 0.67, size = 138, normalized size = 0.65

$$-\frac{4\,(3\,cdx+2\,ae)fg}{3\left(c^3d^3x+ac^2d^2e\right)\sqrt{cdx+ae}}+\frac{2\left(3\,c^2d^2x^2+12\,acdex+8\,a^2e^2\right)g^2}{3\left(c^4d^4x+ac^3d^3e\right)\sqrt{cdx+ae}}-\frac{2\,f^2}{3\left(c^2d^2x+acde\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] -4/3*(3*c*d*x + 2*a*e)*f*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
2/3*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*g^2/((c^4*d^4*x + a*c^3*d^3*
e)*sqrt(c*d*x + a*e)) - 2/3*f^2/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

mupad [B] time = 3.61, size = 206, normalized size = 0.98

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{c^3\,d^3\,e}-\frac{\sqrt{d+e\,x}\,\left(-16\,a^2\,e^2\,g^2+8\,a\,c\,d\,e\,f\,g+2\,c^2\,d^2\,f^2\right)}{3\,c^5\,d^5\,e}+\frac{4\,g\,x\,\left(2\,a\,e\,g-c\,d\,f\right)\,\sqrt{d+e\,x}}{c^4\,d^4\,e}\right)}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(3\,c^5\,d^6+6\,a\,c^4\,d^4\,e^2\right)}{3\,c^5\,d^5\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^2*(d + e*x)^(1/2))
/(c^3*d^3*e) - ((d + e*x)^(1/2)*(2*c^2*d^2*f^2 - 16*a^2*e^2*g^2 + 8*a*c*d*e
*f*g))/(3*c^5*d^5*e) + (4*g*x*(2*a*e*g - c*d*f)*(d + e*x)^(1/2))/(c^4*d^4*e
)))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(3*c^
5*d^6 + 6*a*c^4*d^4*e^2))/(3*c^5*d^5*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

3.441 $\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{2\sqrt{d+ex}\left(2ae^2g+cd(ef-3dg)\right)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Rubi [A] time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {788, 648}

$$\frac{2\sqrt{d+ex}\left(2ae^2g+cd(ef-3dg)\right)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(5/2))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*(2*a*e^2*g + c*d*(e*f - 3*d*g))*Sqrt[d
 + e*x])/(3*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{\left(2ae^2g+cd(ef-\right.}{}$$

$$=-\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(2ae^2g+\right.}{3c^2d^2\left(cd^2-ae^2\right.}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.34

$$-\frac{2(d+ex)^{3/2}(2aeg+cd(f+3gx))}{3c^2d^2((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(2*a*e*g + c*d*(f + 3*g*x)))/(3*c^2*d^2*((a*e + c*d*x)*
(d + e*x))^(3/2))
```

IntegrateAlgebraic [A] time = 1.96, size = 66, normalized size = 0.43

$$-\frac{2(d+ex)^{5/2}(ae+cdx)(3g(ae+cdx)-aeg+cdf)}{3c^2d^2((d+ex)(ae+cdx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(a*e + c*d*x)*(d + e*x)^(5/2)*(c*d*f - a*e*g + 3*g*(a*e + c*d*x)))/(3*c
^2*d^2*((a*e + c*d*x)*(d + e*x))^(5/2))
```

fricas [A] time = 0.41, size = 129, normalized size = 0.84

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(3\,cdgx+cdf+2\,aeg\right)\sqrt{ex+d}}{3\left(c^4d^4ex^3+a^2c^2d^3e^2+\left(c^4d^5+2\,ac^3d^3e^2\right)x^2+\left(2\,ac^3d^4e+a^2c^2d^2e^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + c*d*f + 2*a*e
*g)*sqrt(e*x + d)/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3
*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.14Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.01, size = 66, normalized size = 0.43

$$-\frac{2\left(cdx+ae\right)\left(3cdgx+2aeg+cdf\right)\left(ex+d\right)^{\frac{5}{2}}}{3\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(3*c*d*g*x+2*a*e*g+c*d*f)*(e*x+d)^(5/2)/c^2/d^2/(c*d*e*x^2
+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [A] time = 0.60, size = 73, normalized size = 0.47

$$-\frac{2\,(3\,cdx+2\,ae)g}{3\,\left(c^3d^3x+ac^2d^2e\right)\sqrt{cdx+ae}}-\frac{2\,f}{3\,\left(c^2d^2x+acde\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="maxima")
```

```
[Out] -2/3*(3*c*d*x + 2*a*e)*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) - 2/
3*f/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

mupad [B] time = 3.50, size = 149, normalized size = 0.97

$$-\frac{\left(\frac{\left(\frac{4\,a\,e\,g}{3}+\frac{2\,c\,d\,f}{3}\right)\,\sqrt{d+e\,x}}{c^4\,d^4\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^3\,d^3\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c^4\,d^5+2\,a\,c^3\,d^3\,e^2\right)}{c^4\,d^4\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/
2),x)
```

```
[Out] -(((((4*a*e*g)/3 + (2*c*d*f)/3)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d +
e*x)^(1/2))/(c^3*d^3*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^
3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^4*d^5 + 2
*a*c^3*d^3*e^2))/(c^4*d^4*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

```
[Out] Timed out
```

3.442 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.77

$$-\frac{2(d+ex)^{3/2}}{3cd((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [A] time = 0.00, size = 45, normalized size = 0.94

$$-\frac{2(d+ex)^{5/2}(ae+cdx)}{3cd((d+ex)(ae+cdx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2),x]
```

```
[Out] (-2*(a*e + c*d*x)*(d + e*x)^(5/2))/(3*c*d*((a*e + c*d*x)*(d + e*x))^(5/2))
```

fricas [**B**] time = 0.41, size = 107, normalized size = 2.23

$$-\frac{2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{3\,\left(c^3d^3ex^3+a^2cd^2e^2+\left(c^3d^4+2\,ac^2d^2e^2\right)x^2+\left(2\,ac^2d^3e+a^2cde^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x
^3 + a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^2 + (2*a*c^2*d^3*e + a^2
*c*d*e^3)*x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.06Unable to transpose Err
or: Bad Argument Value
```

maple [**A**] time = 0.00, size = 50, normalized size = 1.04

$$-\frac{2\left(cdx+ae\right)\left(ex+d\right)^{\frac{5}{2}}}{3\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(e*x+d)^(5/2)/c/d/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [**A**] time = 0.52, size = 28, normalized size = 0.58

$$-\frac{2}{3\left(c^2d^2x+acde\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
```

```
[Out] -2/3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

mupad [**B**] time = 3.32, size = 110, normalized size = 2.29

$$-\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^2\,c\,d^2\,e^2+a^2\,c\,d\,e^3\,x+2\,a\,c^2\,d^3\,e\,x+2\,a\,c^2\,d^2\,e^2\,x^2+c^3\,d^4\,x^2+c^3\,d^3\,e\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(c^3*
d^4*x^2 + a^2*c*d^2*e^2 + c^3*d^3*e*x^3 + 2*a*c^2*d^3*e*x + a^2*c*d*e^3*x +
 2*a*c^2*d^2*e^2*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

3.443 $\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=188

$$\frac{2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}+\frac{2g\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cd\right.}$$

Rubi [A] time = 0.27, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {868, 874, 205}

$$\frac{2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}+\frac{2g\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) + (2*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)
^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{g\int \frac{\ldots}{(f+gx)\left(ade+\ldots\right.}}{c\ldots}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{(cdf-aeg)^2\ldots}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{(cdf-aeg)^2\ldots}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{(cdf-aeg)^2\ldots}$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.39

$$\frac{2(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3((d+ex)(ae+cdx))^{3/2}(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 1, -1/2, (g*(a*e + c*d*x))/(-(c*
d*f) + a*e*g)])/(3*(-(c*d*f) + a*e*g)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [F] time = 180.06, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(5/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.43, size = 1015, normalized size = 5.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2
*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d
^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a
*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)
*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^3*e^2*
```

```
f^2 - 2*a^3*c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^
2*f*g + a^2*c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*
c^3*d^4*e + 2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)
*x^2 + ((2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*
c*d*e^4)*f*g + (2*a^3*c*d^2*e^3 + a^4*e^5)*g^2)*x), 2/3*(3*(c^2*d^2*e*g*x^3
 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*
x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*
d*e*g + (c*d^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*
c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*
c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e +
2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)*x^2 + ((2*a
*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g
 + (2*a^3*c*d^2*e^3 + a^4*e^5)*g^2)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 6.14Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.03, size = 219, normalized size = 1.16

$$-\frac{2\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\left(3\sqrt{cdx+ae}\;cd\,g^{2}x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3\sqrt{cdx+ae}\;ae\,g^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-3\sqrt{\left(aeg-cdf\right)g}\;cdgx-4\sqrt{\left(aeg-cdf\right)g}\;aeg+\sqrt{\left(aeg-cdf\right)g}\;cdf\right)}{3\sqrt{ex+d}\;\left(cdx+ae\right)^{2}\left(aeg-cdf\right)^{2}\sqrt{\left(aeg-cdf\right)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x*c*d*g^2+3*arctanh((c*d*x+a*e)
^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*a*e*g^2*(c*d*x+a*e)^(1/2)-3*((a*e*g-c*d*f
)*g)^(1/2)*c*d*g*x-4*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*
c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+e\,x\right)}^{5/2}}{\left(f+g\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

```
[Out] Timed out
```

3.444 $\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$\frac{5cdg^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}}+\frac{5g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{10g\sqrt{d+ex}}{3(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+\cdots}}$$

Rubi [A] time = 0.34, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$\frac{5g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5cdg^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}}+\frac{10g\sqrt{d+ex}}{3(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2)) + (10*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*x)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (5*c
*d*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sq
rt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
```

```
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(5g)}{}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(c}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(c}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(c}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(c}$$

Mathematica [C] time = 0.05, size = 75, normalized size = 0.28

$$-\frac{2cd(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)),x]
```

```
[Out] (-2*c*d*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 2, -1/2, (g*(a*e + c*d*x))/
(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [F] time = 180.63, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(5/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.48, size = 1907, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] [-1/6*(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^3*e*f*g + (c^3*d^
4 + 2*a*c^2*d^2*e^2)*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*g + (2*a*c^2
*d^3*e + a^2*c*d*e^3)*g^2)*x^2 + (a^2*c*d^2*e^2*g^2 + (2*a*c^2*d^3*e + a^2*
c*d*e^3)*f*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a
*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt
(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*
g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 + 14*
a*c*d*e*f*g + 3*a^2*e^2*g^2 + 10*(c^2*d^2*f*g + 2*a*c*d*e*g^2)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^3*d^4*e^2*f^4 - 3*
a^3*c^2*d^3*e^3*f^3*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (c^5*d^
5*e*f^3*g - 3*a*c^4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c^2*d^2
*e^4*g^4)*x^4 + (c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*(a*c^4
*d^5*e + a^2*c^3*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^2*e^4)
*f*g^3 - (a^3*c^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a*c^4*d
^4*e^2)*f^4 - (a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^4*e^2
- a^3*c^2*d^2*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3 - (2*a
^4*c*d^2*e^4 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e + a^2*c^
3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2
*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x), 1/3*
(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^3*e*f*g + (c^3*d^4 + 2*
a*c^2*d^2*e^2)*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*g + (2*a*c^2*d^3*e
 + a^2*c*d*e^3)*g^2)*x^2 + (a^2*c*d^2*e^2*g^2 + (2*a*c^2*d^3*e + a^2*c*d*e^
3)*f*g)*x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*
x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (15*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2
 + 14*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 10*(c^2*d^2*f*g + 2*a*c*d*e*g^2)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^3*d^4*e^2*f^
4 - 3*a^3*c^2*d^3*e^3*f^3*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (
c^5*d^5*e*f^3*g - 3*a*c^4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c
^2*d^2*e^4*g^4)*x^4 + (c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*
(a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^
2*e^4)*f*g^3 - (a^3*c^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a
*c^4*d^4*e^2)*f^4 - (a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^
4*e^2 - a^3*c^2*d^2*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3
- (2*a^4*c*d^2*e^4 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e +
a^2*c^3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a
^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x)
]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 10.12Unable to transpose Er
ror: Bad Argument Value
```

maple [**A**] time = 0.04, size = 424, normalized size = 1.58

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15\sqrt{cdx+ae}\,c^2d^2g^3x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+15\sqrt{cdx+ae}\,acde\,g^3x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+15\sqrt{cdx+ae}\,c^2d^2f\,g^2x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+15\sqrt{cdx+ae}\,acdef\,g^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-15\sqrt{(aeg-cdf)g}\,c^2d^2g^2x^2-20\sqrt{(aeg-cdf)g}\,acde\,g^2x-10\sqrt{(aeg-cdf)g}\,c^2d^2fgx-3\sqrt{(aeg-cdf)g}\,a^2e^2g^2-14\sqrt{(aeg-cdf)g}\,acdefg+2\sqrt{(aeg-cdf)g}\,c^2d^2f^2\right)}{3\sqrt{ex+d}\,(cdx+ae)^2\left(aeg-cdf\right)^3\left(gx+f\right)\sqrt{(aeg-cdf)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] 1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x^2*c^2*d^2*g^3+15*arctanh((c*d
*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*a*c*d*e*g^3*(c*d*x+a*e)^(1/2)+15
*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*(c*d*x+a*e)^(1/2)*x*c
^2*d^2*f*g^2+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*a*c*d*
e*f*g^2*(c*d*x+a*e)^(1/2)-15*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2-20*((a
*e*g-c*d*f)*g)^(1/2)*a*c*d*e*g^2*x-10*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x
-3*((a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f
*g+2*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*
g-c*d*f)^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{5/2}}{\left(f+g\,x\right)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

3.445 $\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=342

$$\frac{35c^2d^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{9/2}}+\frac{35cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}\,(f+gx)(cdf-aeg)^4}+\frac{35g^2\sqrt{x\left(ae^2+cd^2\right)+ade}}{6\sqrt{d+ex}\,(f+gx)^2(cdf-}$$

Rubi [A] time = 0.54, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$\frac{35c^2d^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{9/2}}+\frac{35cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}\,(f+gx)(cdf-aeg)^4}+\frac{35g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{6\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^3}+\frac{14g\sqrt{d+ex}}{3(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)) + (14*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*
x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*g^2*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(6*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^
2) + (35*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a
*e*g)^4*Sqrt[d + e*x]*(f + g*x)) + (35*c^2*d^2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x]
)])/(4*(c*d*f - a*e*g)^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{(7g}{}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{3(c}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{3(c}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{3(c}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{3(c}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{}{3(c}$$

Mathematica [C] time = 0.05, size = 79, normalized size = 0.23

$$-\frac{2c^2d^2(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)),x]
```

```
[Out] (-2*c^2*d^2*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 3, -1/2, (g*(a*e + c*d*
x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^3*((a*e + c*d*x)*(d + e*x))^(3/
2))
```

IntegrateAlgebraic [F] time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(5/2)),x]
```

[Out] $Aborted

fricas [B] time = 0.49, size = 2935, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] [1/24*(105*(c^4*d^4*e*g^3*x^5 + a^2*c^2*d^3*e^2*f^2*g + (2*c^4*d^4*e*f*g^2
+ (c^4*d^5 + 2*a*c^3*d^3*e^2)*g^3)*x^4 + (c^4*d^4*e*f^2*g + 2*(c^4*d^5 + 2*
a*c^3*d^3*e^2)*f*g^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*g^3)*x^3 + (a^2*c^
2*d^3*e^2*g^3 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g + 2*(2*a*c^3*d^4*e + a^2*
c^2*d^2*e^3)*f*g^2)*x^2 + (2*a^2*c^2*d^3*e^2*f*g^2 + (2*a*c^3*d^4*e + a^2*c
^2*d^2*e^3)*f^2*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f
+ 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)
*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x
)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(105*c^3*d^3*g^3*x^3 - 8*c^3*d^3*f^3
 + 80*a*c^2*d^2*e*f^2*g + 39*a^2*c*d*e^2*f*g^2 - 6*a^3*e^3*g^3 + 35*(5*c^3*
d^3*f*g^2 + 4*a*c^2*d^2*e*g^3)*x^2 + 7*(8*c^3*d^3*f^2*g + 34*a*c^2*d^2*e*f*
g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*
e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*
g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e
^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c
^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*
c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2
*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f
^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c
^4*d^4*e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*
a^3*c^3*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*
g^6)*x^3 - (6*a^2*c^4*d^4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^
6*d^7 + 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*
g^2 - 4*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^
4 + 2*a^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2
*c^4*d^4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*
a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d
*e^6)*f^3*g^3 - (6*a^5*c*d^2*e^5 - a^6*e^7)*f^2*g^4)*x), 1/12*(105*(c^4*d^4
*e*g^3*x^5 + a^2*c^2*d^3*e^2*f^2*g + (2*c^4*d^4*e*f*g^2 + (c^4*d^5 + 2*a*c^
3*d^3*e^2)*g^3)*x^4 + (c^4*d^4*e*f^2*g + 2*(c^4*d^5 + 2*a*c^3*d^3*e^2)*f*g^
2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*g^3)*x^3 + (a^2*c^2*d^3*e^2*g^3 + (c^
4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g + 2*(2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f*g^2)
*x^2 + (2*a^2*c^2*d^3*e^2*f*g^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2*g)*
x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*
d*e*g + (c*d^2 + a*e^2)*g*x)) + (105*c^3*d^3*g^3*x^3 - 8*c^3*d^3*f^3 + 80*a
*c^2*d^2*e*f^2*g + 39*a^2*c*d*e^2*f*g^2 - 6*a^3*e^3*g^3 + 35*(5*c^3*d^3*f*g
^2 + 4*a*c^2*d^2*e*g^3)*x^2 + 7*(8*c^3*d^3*f^2*g + 34*a*c^2*d^2*e*f*g^2 + 3
*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d))/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4
*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4
*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^
5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*
e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5
*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^
5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*
c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c^4*d^4*
e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3
*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^
```

```
3 - (6*a^2*c^4*d^4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 +
 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4
*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a
^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2*c^4*d^
4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3
*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f
^3*g^3 - (6*a^5*c*d^2*e^5 - a^6*e^7)*f^2*g^4)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 13.97Unable to transpose Er
ror: Bad Argument Value
```

maple [B] time = 0.04, size = 670, normalized size = 1.96

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(105\sqrt{cdx+ae}\,c^3d^3g^4x^3\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+105\sqrt{cdx+ae}\,a\,c^2d^2e\,g^4x^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+210\sqrt{cdx+ae}\,c^3d^3f\,g^3x^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+210\sqrt{cdx+ae}\,a\,c^2d^2ef\,g^3x\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+105\sqrt{cdx+ae}\,c^3d^3f^2g^2x\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-105\sqrt{(aeg-cdf)g}\,c^3d^3g^3x^3+105\sqrt{cdx+ae}\,a\,c^2d^2e\,f^2g^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-140\sqrt{(aeg-cdf)g}\,a\,c^2d^2e\,g^3x^2-175\sqrt{(aeg-cdf)g}\,c^3d^3f\,g^2x^2-21\sqrt{(aeg-cdf)g}\,a^2cd\,e^2g^3x-238\sqrt{(aeg-cdf)g}\,a\,c^2d^2ef\,g^2x-56\sqrt{(aeg-cdf)g}\,c^3d^3f^2gx+6\sqrt{(aeg-cdf)g}\,a^3e^3g^3-39\sqrt{(aeg-cdf)g}\,a^2cd\,e^2f\,g^2-80\sqrt{(aeg-cdf)g}\,a\,c^2d^2e\,f^2g+8\sqrt{(aeg-cdf)g}\,c^3d^3f^3\right)}{12\sqrt{ex+d}\,(cdx+ae)^2\left(aeg-cdf\right)^4\left(gx+f\right)^2\sqrt{(aeg-cdf)g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -1/12*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(105*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*g^4*(c*d*x+a*e)^(1/2)+105*arctanh(
(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*a*c^2*d^2*e*g^4*(c*d*x+a*e
)^(1/2)+210*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^
3*f*g^3*(c*d*x+a*e)^(1/2)+210*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2)*g)*x*a*c^2*d^2*e*f*g^3*(c*d*x+a*e)^(1/2)+105*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g^2*(c*d*x+a*e)^(1/2)-105*((a*e*g
-c*d*f)*g)^(1/2)*x^3*c^3*d^3*g^3+105*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2)*g)*a*c^2*d^2*e*f^2*g^2*(c*d*x+a*e)^(1/2)-140*((a*e*g-c*d*f)*g)^
(1/2)*x^2*a*c^2*d^2*e*g^3-175*((a*e*g-c*d*f)*g)^(1/2)*x^2*c^3*d^3*f*g^2-21*
((a*e*g-c*d*f)*g)^(1/2)*x*a^2*c*d*e^2*g^3-238*((a*e*g-c*d*f)*g)^(1/2)*x*a*c
^2*d^2*e*f*g^2-56*((a*e*g-c*d*f)*g)^(1/2)*x*c^3*d^3*f^2*g+6*((a*e*g-c*d*f)*
g)^(1/2)*a^3*e^3*g^3-39*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2-80*((a*e*
g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+8*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/
(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x\right)}^{5/2}}{{\left(f+g\,x\right)}^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

3.446 $\int \frac{(f+gx)^4 \sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=336

$$-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^3\left(2ae^2g-cd(5ef-3dg)\right)}{3465c^5d^5e(d+ex)^{3/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/}}{1155c^4d^4e\sqrt{d+ex}}$$

Rubi [A] time = 0.61, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{99c^2d^2(d+ex)^{3/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}{231c^3d^3(d+ex)^{3/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^3}{1155c^4d^4e\sqrt{d+ex}}-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^3\left(2ae^2g-cd(5ef-3dg)\right)}{3465c^5d^5e(d+ex)^{3/2}}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(3465*c^5*d^5*e*(d + e*x)^(3/2)) + (128*g*(c*
d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(1155*c^4*d^4
*e*Sqrt[d + e*x]) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(231*c^3*d^3*(d + e*x)^(3/2)) + (16*(c*d*f - a*e*
g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(99*c^2*d^2*(
d + e*x)^(3/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2))/(11*c*d*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}} + \frac{(8(cdf-aeg))}{}$$

$$= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{99c^2d^2(d+ex)^{3/2}} + \frac{2(}{}$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{231c^3d^3(d+ex)^{3/2}} + \frac{1}{}$$

$$= \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{1155c^4d^4e\sqrt{d+ex}} + \frac{32(cdf-}{}$$

$$= \frac{128(cdf-aeg)^3\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{3465c^4d^4(d+ex)^{3/2}}$$

Mathematica [A] time = 0.18, size = 195, normalized size = 0.58

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(128a^4e^4g^4-64a^3cde^3g^3(11f+3gx)+48a^2c^2d^2e^2g^2\left(33f^2+22fgx+5g^2x^2\right)-8ac^3d^3eg\left(231f^3+297f^2gx+165fg^2x^2+35g^3x^3\right)+c^4d^4\left(1155f^4+2772f^3gx+2970f^2g^2x^2+1540fg^3x^3+315g^4x^4\right)\right)}{3465c^5d^5(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(1
1*f + 3*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(33*f^2 + 22*f*g*x + 5*g^2*x^2) - 8*a
*c^3*d^3*e*g*(231*f^3 + 297*f^2*g*x + 165*f*g^2*x^2 + 35*g^3*x^3) + c^4*d^4
*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x^2 + 1540*f*g^3*x^3 + 315*g^4*x^4
)))/(3465*c^5*d^5*(d + e*x)^(3/2))
```

IntegrateAlgebraic [B] time = 23.69, size = 7594, normalized size = 22.60

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.40, size = 375, normalized size = 1.12

$$\frac{2\left(315\,c^5d^5g^4x^5+1155\,ac^4d^4ef^4-1848\,a^2c^3d^3e^2f^3g+1584\,a^3c^2d^2e^3f^2g^2-704\,a^4cde^4fg^3+128\,a^5e^5g^4+35\left(44\,c^5d^5fg^3+ac^4d^4eg^4\right)x^4+10\left(297\,c^5d^5f^2g^2+22\,ac^4d^4efg^3-4\,a^2c^3d^3e^2g^4\right)x^3+6\left(462\,c^5d^5f^3g+99\,ac^4d^4ef^2g^2-44\,a^2c^3d^3e^2fg^3+8\,a^3c^2d^2e^3g^4\right)x^2+\left(1155\,c^5d^5f^4+924\,ac^4d^4ef^3g-792\,a^2c^3d^3e^2f^2g^2+352\,a^3c^2d^2e^3fg^3-64\,a^4cde^4g^4\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{3465\left(c^5d^5ex+c^5d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^5*d^5*g^4*x^5 + 1155*a*c^4*d^4*e*f^4 - 1848*a^2*c^3*d^3*e^2*f
^3*g + 1584*a^3*c^2*d^2*e^3*f^2*g^2 - 704*a^4*c*d*e^4*f*g^3 + 128*a^5*e^5*g
^4 + 35*(44*c^5*d^5*f*g^3 + a*c^4*d^4*e*g^4)*x^4 + 10*(297*c^5*d^5*f^2*g^2
+ 22*a*c^4*d^4*e*f*g^3 - 4*a^2*c^3*d^3*e^2*g^4)*x^3 + 6*(462*c^5*d^5*f^3*g
+ 99*a*c^4*d^4*e*f^2*g^2 - 44*a^2*c^3*d^3*e^2*f*g^3 + 8*a^3*c^2*d^2*e^3*g^4
```

```
)*x^2 + (1155*c^5*d^5*f^4 + 924*a*c^4*d^4*e*f^3*g - 792*a^2*c^3*d^3*e^2*f^2
*g^2 + 352*a^3*c^2*d^2*e^3*f*g^3 - 64*a^4*c*d*e^4*g^4)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\left(gx + f\right)^4}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^4/sqrt(e*x
+ d), x)
```

maple [A] time = 0.01, size = 283, normalized size = 0.84

$$\frac{2\left(cdx+ae\right)\left(315g^4x^4c^4d^4-280ac^3d^3eg^4x^3+1540c^4d^4fg^3x^3+240a^2c^2d^2e^2g^4x^2-1320ac^3d^3efg^3x^2+2970c^4d^4f^2g^2x^2-192a^3cde^3g^4x+1056a^2c^2d^2e^2fg^3x-2376ac^3d^3ef^2g^2x+2772c^4d^4f^3gx+128a^4e^4g^4-704a^3cde^3fg^3+1584a^2c^2d^2e^2f^2g^2-1848ac^3d^3ef^3g+1155f^4c^4d^4\right)\sqrt{cdex^2+ae^2x+cd^2x+ade}}{3465\sqrt{ex+d}\,c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/3465*(c*d*x+a*e)*(315*c^4*d^4*g^4*x^4-280*a*c^3*d^3*e*g^4*x^3+1540*c^4*d^
4*f*g^3*x^3+240*a^2*c^2*d^2*e^2*g^4*x^2-1320*a*c^3*d^3*e*f*g^3*x^2+2970*c^4
*d^4*f^2*g^2*x^2-192*a^3*c*d*e^3*g^4*x+1056*a^2*c^2*d^2*e^2*f*g^3*x-2376*a*
c^3*d^3*e*f^2*g^2*x+2772*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-704*a^3*c*d*e^3*f*
g^3+1584*a^2*c^2*d^2*e^2*f^2*g^2-1848*a*c^3*d^3*e*f^3*g+1155*c^4*d^4*f^4)*(
c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c^5/d^5/(e*x+d)^(1/2)
```

maxima [A] time = 0.71, size = 320, normalized size = 0.95

$$\frac{2\left(cdx+ae\right)^{\frac{3}{2}}f^4}{3\,cd}+\frac{8\left(3\,c^2d^2x^2+acdex-2\,a^2e^2\right)\sqrt{cdx+ae}\,f^3g}{15\,c^2d^2}+\frac{4\left(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3\right)\sqrt{cdx+ae}\,f^2g^2}{35\,c^3d^3}+\frac{8\left(35\,c^4d^4x^4+5\,ac^3d^3ex^3-6\,a^2c^2d^2e^2x^2+8\,a^3cde^3x-16\,a^4e^4\right)\sqrt{cdx+ae}\,fg^3}{315\,c^4d^4}+\frac{2\left(315\,c^5d^5x^5+35\,ac^4d^4ex^4-40\,a^2c^3d^3e^2x^3+48\,a^3c^2d^2e^3x^2-64\,a^4cde^4x+128\,a^5e^5\right)\sqrt{cdx+ae}\,g^4}{3465\,c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^4/(c*d) + 8/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2) + 4/35*(15*c^3*d^3*x^3 + 3*a*c^2*d^
2*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f^2*g^2/(c^3*d^3)
+ 8/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8*a^3
*c*d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/3465*(315*c^
5*d^5*x^5 + 35*a*c^4*d^4*e*x^4 - 40*a^2*c^3*d^3*e^2*x^3 + 48*a^3*c^2*d^2*e^
3*x^2 - 64*a^4*c*d*e^4*x + 128*a^5*e^5)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

mupad [B] time = 3.60, size = 347, normalized size = 1.03

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^4\,x^5}{11}+\frac{256\,a^5\,e^5\,g^4-1408\,a^4\,c\,d\,e^4\,f\,g^3+3168\,a^3\,c^2\,d^2\,e^3\,f^2\,g^2-3696\,a^2\,c^3\,d^3\,e^2\,f^3\,g+2310\,a\,c^4\,d^4\,e\,f^4}{3465\,c^5\,d^5}+\frac{x\,\left(-128\,a^4\,c\,d\,e^4\,g^4+704\,a^3\,c^2\,d^2\,e^3\,f\,g^3-1584\,a^2\,c^3\,d^3\,e^2\,f^2\,g^2+1848\,a\,c^4\,d^4\,e\,f^3\,g+2310\,c^5\,d^5\,f^4\right)}{3465\,c^5\,d^5}+\frac{4\,g\,x^2\,\left(8\,a^3\,e^3\,g^3-44\,a^2\,c\,d\,e^2\,f\,g^2+99\,a\,c^2\,d^2\,e\,f^2\,g+462\,c^3\,d^3\,f^3\right)}{1155\,c^3\,d^3}+\frac{4\,g^2\,x^3\,\left(-4\,a^2\,e^2\,g^2+22\,a\,c\,d\,e\,f\,g+297\,c^2\,d^2\,f^2\right)}{693\,c^2\,d^2}+\frac{2\,g^3\,x^4\,\left(a\,e\,g+44\,c\,d\,f\right)}{99\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5)/11 + (256*a^5*e
^5*g^4 + 2310*a*c^4*d^4*e*f^4 - 3696*a^2*c^3*d^3*e^2*f^3*g - 1408*a^4*c*d*e
^4*f*g^3 + 3168*a^3*c^2*d^2*e^3*f^2*g^2)/(3465*c^5*d^5) + (x*(2310*c^5*d^5*
f^4 - 128*a^4*c*d*e^4*g^4 + 704*a^3*c^2*d^2*e^3*f*g^3 + 1848*a*c^4*d^4*e*f^
```

```
3*g - 1584*a^2*c^3*d^3*e^2*f^2*g^2))/(3465*c^5*d^5) + (4*g*x^2*(8*a^3*e^3*g
^3 + 462*c^3*d^3*f^3 + 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/(1155*
c^3*d^3) + (4*g^2*x^3*(297*c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*f*g))/(
693*c^2*d^2) + (2*g^3*x^4*(a*e*g + 44*c*d*f))/(99*c*d)))/(d + e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

3.447 $\int \frac{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=269

$$-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2\left(2ae^2g-cd(5ef-3dg)\right)}{315c^4d^4e(d+ex)^{3/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(}{105c^3d^3e\sqrt{d+ex}}$$

Rubi [A] time = 0.39, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{4(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{21c^2d^2(d+ex)^{3/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}{105c^3d^3e\sqrt{d+ex}}-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2\left(2ae^2g-cd(5ef-3dg)\right)}{315c^4d^4e(d+ex)^{3/2}}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2))/(315*c^4*d^4*e*(d + e*x)^(3/2)) + (16*g*(c*d*f
 - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*S
qrt[d + e*x]) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(21*c^2*d^2*(d + e*x)^(3/2)) + (2*(f + g*x)^3*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}} + \frac{\left(2\left(cde^2f+cd^2\right.\right.}{}$$

$$= \frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21c^2d^2(d+ex)^{3/2}} + \frac{2(f}{}$$

$$= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{105c^3d^3e\sqrt{d+ex}} + \frac{4(cdf-a}{}$$

$$= \frac{16(cdf-aeg)^2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{315c^3d^3(d+ex)^{3/2}}$$

Mathematica [A]   time = 0.12, size = 136, normalized size = 0.51

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(-16a^3e^3g^3+24a^2cde^2g^2(3f+gx)-6ac^2d^2eg\left(21f^2+18fgx+5g^2x^2\right)+c^3d^3\left(105f^3+189f^2gx+135fg^2x^2+35g^3x^3\right)\right)}{315c^4d^4(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(3
*f + g*x) - 6*a*c^2*d^2*e*g*(21*f^2 + 18*f*g*x + 5*g^2*x^2) + c^3*d^3*(105*
f^3 + 189*f^2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3)))/(315*c^4*d^4*(d + e*x)^(3
/2))
```

IntegrateAlgebraic [B]   time = 1.25, size = 676, normalized size = 2.51

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*(-105
*c^4*d^5*e^3*f^3 + 105*a*c^3*d^3*e^5*f^3 + 189*c^4*d^6*e^2*f^2*g - 63*a*c^3
*d^4*e^4*f^2*g - 126*a^2*c^2*d^2*e^6*f^2*g - 135*c^4*d^7*e*f*g^2 + 27*a*c^3
*d^5*e^3*f*g^2 + 36*a^2*c^2*d^3*e^5*f*g^2 + 72*a^3*c*d*e^7*f*g^2 + 35*c^4*d
^8*g^3 - 5*a*c^3*d^6*e^2*g^3 - 6*a^2*c^2*d^4*e^4*g^3 - 8*a^3*c*d^2*e^6*g^3
- 16*a^4*e^8*g^3 + 105*c^4*d^4*e^3*f^3*(d + e*x) - 378*c^4*d^5*e^2*f^2*g*(d
 + e*x) + 63*a*c^3*d^3*e^4*f^2*g*(d + e*x) + 405*c^4*d^6*e*f*g^2*(d + e*x)
- 54*a*c^3*d^4*e^3*f*g^2*(d + e*x) - 36*a^2*c^2*d^2*e^5*f*g^2*(d + e*x) - 1
40*c^4*d^7*g^3*(d + e*x) + 15*a*c^3*d^5*e^2*g^3*(d + e*x) + 12*a^2*c^2*d^3*
e^4*g^3*(d + e*x) + 8*a^3*c*d*e^6*g^3*(d + e*x) + 189*c^4*d^4*e^2*f^2*g*(d
+ e*x)^2 - 405*c^4*d^5*e*f*g^2*(d + e*x)^2 + 27*a*c^3*d^3*e^3*f*g^2*(d + e*
x)^2 + 210*c^4*d^6*g^3*(d + e*x)^2 - 15*a*c^3*d^4*e^2*g^3*(d + e*x)^2 - 6*a
^2*c^2*d^2*e^4*g^3*(d + e*x)^2 + 135*c^4*d^4*e*f*g^2*(d + e*x)^3 - 140*c^4*
d^5*g^3*(d + e*x)^3 + 5*a*c^3*d^3*e^2*g^3*(d + e*x)^3 + 35*c^4*d^4*g^3*(d +
 e*x)^4))/(315*c^4*d^4*e^4*Sqrt[d + e*x])
```

fricas [A]   time = 0.41, size = 264, normalized size = 0.98

$$\frac{2\left(35c^4d^4g^3x^4+105ac^3d^3ef^3-126a^2c^2d^2e^2f^2g+72a^3cde^3fg^2-16a^4e^4g^3+5\left(27c^4d^4fg^2+ac^3d^3eg^3\right)x^3+3\left(63c^4d^4f^2g+9ac^3d^3efg^2-2a^2c^2d^2e^2g^3\right)x^2+\left(105c^4d^4f^3+63ac^3d^3ef^2g-36a^2c^2d^2e^2fg^2+8a^3cde^3g^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{315\left(c^4d^4ex+c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 126*a^2*c^2*d^2*e^2*f^2*g
 + 72*a^3*c*d*e^3*f*g^2 - 16*a^4*e^4*g^3 + 5*(27*c^4*d^4*f*g^2 + a*c^3*d^3*
e*g^3)*x^3 + 3*(63*c^4*d^4*f^2*g + 9*a*c^3*d^3*e*f*g^2 - 2*a^2*c^2*d^2*e^2*
g^3)*x^2 + (105*c^4*d^4*f^3 + 63*a*c^3*d^3*e*f^2*g - 36*a^2*c^2*d^2*e^2*f*g
^2 + 8*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(gx + f\right)^3}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^3/sqrt(e*x
+ d), x)
```

maple [A] time = 0.01, size = 188, normalized size = 0.70

$$-\frac{2\left(cdx + ae\right)\left(-35g^3x^3c^3d^3 + 30a\,c^2d^2e\,g^3x^2 - 135c^3d^3f\,g^2x^2 - 24a^2cd\,e^2g^3x + 108a\,c^2d^2ef\,g^2x - 189c^3d^3f^2gx + 16a^3e^3g^3 - 72a^2cd\,e^2f\,g^2 + 126a\,c^2d^2e\,f^2g - 105f^3c^3d^3\right)\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}}{315\sqrt{ex + d}\;c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/315*(c*d*x+a*e)*(-35*c^3*d^3*g^3*x^3+30*a*c^2*d^2*e*g^3*x^2-135*c^3*d^3*
f*g^2*x^2-24*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-189*c^3*d^3*f^2*g*x+
16*a^3*e^3*g^3-72*a^2*c*d*e^2*f*g^2+126*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)*
(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c^4/d^4/(e*x+d)^(1/2)
```

maxima [A] time = 0.65, size = 218, normalized size = 0.81

$$\frac{2\left(cdx + ae\right)^{\frac{3}{2}}f^3}{3\,cd} + \frac{2\left(3\,c^2d^2x^2 + acdex - 2\,a^2e^2\right)\sqrt{cdx + ae}\,f^2g}{5\,c^2d^2} + \frac{2\left(15\,c^3d^3x^3 + 3\,ac^2d^2ex^2 - 4\,a^2cde^2x + 8\,a^3e^3\right)\sqrt{cdx + ae}\,fg^2}{35\,c^3d^3} + \frac{2\left(35\,c^4d^4x^4 + 5\,ac^3d^3ex^3 - 6\,a^2c^2d^2e^2x^2 + 8\,a^3cde^3x - 16\,a^4e^4\right)\sqrt{cdx + ae}\,g^3}{315\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^3/(c*d) + 2/5*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2*
e^2)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2) + 2/35*(15*c^3*d^3*x^3 + 3*a*c^2*d^2
*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f*g^2/(c^3*d^3) + 2
/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8*a^3*c*
d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

mupad [B] time = 3.37, size = 242, normalized size = 0.90

$$\frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}\,\left(\frac{2\,g^3\,x^4}{9} - \frac{32\,a^4\,e^4\,g^3 - 144\,a^3\,c\,d\,e^3\,f\,g^2 + 252\,a^2\,c^2\,d^2\,e^2\,f^2\,g - 210\,a\,c^3\,d^3\,e\,f^3}{315\,c^4\,d^4} + \frac{x\,\left(16\,a^3\,c\,d\,e^3\,g^3 - 72\,a^2\,c^2\,d^2\,e^2\,f\,g^2 + 126\,a\,c^3\,d^3\,e\,f^2\,g + 210\,c^4\,d^4\,f^3\right)}{315\,c^4\,d^4} + \frac{2\,g\,x^2\,\left(-2\,a^2\,e^2\,g^2 + 9\,a\,c\,d\,e\,f\,g + 63\,c^2\,d^2\,f^2\right)}{105\,c^2\,d^2} + \frac{2\,g^2\,x^3\,\left(a\,e\,g + 27\,c\,d\,f\right)}{63\,c\,d}\right)}{\sqrt{d + e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^3*x^4)/9 - (32*a^4*e^4
*g^3 - 210*a*c^3*d^3*e*f^3 + 252*a^2*c^2*d^2*e^2*f^2*g - 144*a^3*c*d*e^3*f*
g^2)/(315*c^4*d^4) + (x*(210*c^4*d^4*f^3 + 16*a^3*c*d*e^3*g^3 - 72*a^2*c^2*
d^2*e^2*f*g^2 + 126*a*c^3*d^3*e*f^2*g))/(315*c^4*d^4) + (2*g*x^2*(63*c^2*d^
2*f^2 - 2*a^2*e^2*g^2 + 9*a*c*d*e*f*g))/(105*c^2*d^2) + (2*g^2*x^3*(a*e*g +
 27*c*d*f))/(63*c*d)))/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}\left(f+gx\right)^3}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3/sqrt(d + e*x), x)
```

3.448 $\int \frac{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=200

$$-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)}{105c^3d^3e(d+ex)^{3/2}}+\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf}{35c^2d^2e\sqrt{d+ex}}$$

Rubi [A] time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{35c^2d^2e\sqrt{d+ex}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)}{105c^3d^3e(d+ex)^{3/2}}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*(d + e*x)^(3/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(35*c^2*d^2*e*Sqrt[d +
 e*x]) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c
*d*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}} + \frac{\left(4\left(cde^2f+cd^2\right.\right.}{}$$

$$= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{35c^2d^2e\sqrt{d+ex}} + \frac{2(f+gx)^2\left(\right.}{}$$

$$= -\frac{8(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{105c^3d^3e(d+ex)^{3/2}}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 0.45

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(8a^2e^2g^2-4acdeg(7f+3gx)+c^2d^2\left(35f^2+42fgx+15g^2x^2\right)\right)}{105c^3d^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(7*f + 3*g*
x) + c^2*d^2*(35*f^2 + 42*f*g*x + 15*g^2*x^2)))/(105*c^3*d^3*(d + e*x)^(3/2
))
```

IntegrateAlgebraic [A] time = 0.64, size = 365, normalized size = 1.82

$$\frac{2\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}\left(8a^3e^6g^2+4a^2cd^2e^4g^2-28a^2cde^5fg-4a^2cde^4g^2(d+ex)+3ac^2d^4e^2g^2-14ac^2d^3e^3fg-6ac^2d^3e^2g^2(d+ex)+35ac^2d^2e^4f^2+14ac^2d^2e^3fg(d+ex)+3ac^2d^2e^2g^2(d+ex)^2-15c^3d^6g^2+42c^3d^5efg+45c^3d^5g^2(d+ex)-35c^3d^4e^2f^2-84c^3d^4efg(d+ex)-45c^3d^4g^2(d+ex)^2+35c^3d^3e^2f^2(d+ex)+42c^3d^3efg(d+ex)^2+15c^3d^3g^2(d+ex)^3\right)}{105c^3d^3e^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*(-35*
c^3*d^4*e^2*f^2 + 35*a*c^2*d^2*e^4*f^2 + 42*c^3*d^5*e*f*g - 14*a*c^2*d^3*e^
3*f*g - 28*a^2*c*d*e^5*f*g - 15*c^3*d^6*g^2 + 3*a*c^2*d^4*e^2*g^2 + 4*a^2*c
*d^2*e^4*g^2 + 8*a^3*e^6*g^2 + 35*c^3*d^3*e^2*f^2*(d + e*x) - 84*c^3*d^4*e*
f*g*(d + e*x) + 14*a*c^2*d^2*e^3*f*g*(d + e*x) + 45*c^3*d^5*g^2*(d + e*x) -
 6*a*c^2*d^3*e^2*g^2*(d + e*x) - 4*a^2*c*d*e^4*g^2*(d + e*x) + 42*c^3*d^3*e
*f*g*(d + e*x)^2 - 45*c^3*d^4*g^2*(d + e*x)^2 + 3*a*c^2*d^2*e^2*g^2*(d + e*
x)^2 + 15*c^3*d^3*g^2*(d + e*x)^3))/(105*c^3*d^3*e^3*Sqrt[d + e*x])
```

fricas [A] time = 0.41, size = 173, normalized size = 0.86

$$\frac{2\left(15\,c^3d^3g^2x^3+35\,ac^2d^2ef^2-28\,a^2cde^2fg+8\,a^3e^3g^2+3\left(14\,c^3d^3fg+ac^2d^2eg^2\right)x^2+\left(35\,c^3d^3f^2+14\,ac^2d^2efg-4\,a^2cde^2g^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{105\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/105*(15*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 28*a^2*c*d*e^2*f*g + 8*a^3
*e^3*g^2 + 3*(14*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 + 14*
a*c^2*d^2*e*f*g - 4*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,(gx + f)^2}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^2/sqrt(e*x
+ d), x)
```

maple [A] time = 0.01, size = 116, normalized size = 0.58

$$\frac{2\,(cdx + ae)\left(15g^2x^2c^2d^2 - 12acde\,g^2x + 42c^2d^2fgx + 8a^2e^2g^2 - 28acdefg + 35f^2c^2d^2\right)\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}}{105\sqrt{ex + d}\;c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/105*(c*d*x+a*e)*(15*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+42*c^2*d^2*f*g*x+8*a
^2*e^2*g^2-28*a*c*d*e*f*g+35*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)
^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

maxima [A] time = 0.59, size = 133, normalized size = 0.66

$$\frac{2\,(cdx + ae)^{\frac{3}{2}}f^2}{3\,cd} + \frac{4\left(3\,c^2d^2x^2 + acdex - 2\,a^2e^2\right)\sqrt{cdx + ae}\,fg}{15\,c^2d^2} + \frac{2\left(15\,c^3d^3x^3 + 3\,ac^2d^2ex^2 - 4\,a^2cde^2x + 8\,a^3e^3\right)\sqrt{cdx + ae}\,g^2}{105\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^2/(c*d) + 4/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) + 2/105*(15*c^3*d^3*x^3 + 3*a*c^2*d^2
*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*g^2/(c^3*d^3)
```

mupad [B] time = 3.25, size = 157, normalized size = 0.78

$$\frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}\,\left(\frac{2\,g^2\,x^3}{7} + \frac{16\,a^3\,e^3\,g^2 - 56\,a^2\,c\,d\,e^2\,f\,g + 70\,a\,c^2\,d^2\,e\,f^2}{105\,c^3\,d^3} + \frac{x\,\left(-8\,a^2\,c\,d\,e^2\,g^2 + 28\,a\,c^2\,d^2\,e\,f\,g + 70\,c^3\,d^3\,f^2\right)}{105\,c^3\,d^3} + \frac{2\,g\,x^2\,\left(a\,e\,g + 14\,c\,d\,f\right)}{35\,c\,d}\right)}{\sqrt{d + e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3)/7 + (16*a^3*e^3
*g^2 + 70*a*c^2*d^2*e*f^2 - 56*a^2*c*d*e^2*f*g)/(105*c^3*d^3) + (x*(70*c^3*
d^3*f^2 - 8*a^2*c*d*e^2*g^2 + 28*a*c^2*d^2*e*f*g))/(105*c^3*d^3) + (2*g*x^2
*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)\,(ae + cdx)}\,\left(f + gx\right)^2}{\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2/sqrt(d + e*x), x)
```

3.449 $\int \frac{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(2ae^2g-cd(5ef-3dg)\right)}{15c^2d^2e(d+ex)^{3/2}}$$

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(2ae^2g-cd(5ef-3dg)\right)}{15c^2d^2e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x],x
]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2))/(15*c^2*d^2*e*(d + e*x)^(3/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}+\frac{1}{5}\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\int \sqrt{}$$

$$=\frac{2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{15cd(d+ex)^{3/2}}+\frac{2g\left(ade+\left(\right.\right.}{}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.43

$$\frac{2((d+ex)(ae+cdx))^{3/2}(cd(5f+3gx)-2aeg)}{15c^2d^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*a*e*g + c*d*(5*f + 3*g*x)))/(15*c^2*
d^2*(d + e*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.32, size = 169, normalized size = 1.35

$$\frac{2\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}\left(-2a^2e^4g-acd^2e^2g+5acde^3f+acde^2g(d+ex)+3c^2d^4g-5c^2d^3ef-6c^2d^3g(d+ex)+5c^2d^2ef(d+ex)+3c^2d^2g(d+ex)^2\right)}{15c^2d^2e^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*(-5*c
^2*d^3*e*f + 5*a*c*d*e^3*f + 3*c^2*d^4*g - a*c*d^2*e^2*g - 2*a^2*e^4*g + 5*
c^2*d^2*e*f*(d + e*x) - 6*c^2*d^3*g*(d + e*x) + a*c*d*e^2*g*(d + e*x) + 3*c
^2*d^2*g*(d + e*x)^2))/(15*c^2*d^2*e^2*Sqrt[d + e*x])
```

fricas [A] time = 0.41, size = 102, normalized size = 0.82

$$\frac{2\left(3\,c^2d^2gx^2+5\,acdef-2\,a^2e^2g+\left(5\,c^2d^2f+acdeg\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{15\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 2*a^2*e^2*g + (5*c^2*d^2*f + a*c*d*e*
g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*
x + c^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)/sqrt(e*x +
d), x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.54

$$-\frac{2\left(cdx+ae\right)\left(-3cdgx+2aeg-5cdf\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{15\sqrt{ex+d}\;c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(-3*c*d*g*x+2*a*e*g-5*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

maxima [A] time = 0.54, size = 65, normalized size = 0.52

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f}{3\,cd}+\frac{2\left(3\,c^2d^2x^2+acdex-2\,a^2e^2\right)\sqrt{cdx+ae}\,g}{15\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f/(c*d) + 2/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2*e
^2)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

mupad [B] time = 3.13, size = 93, normalized size = 0.74

$$\frac{\left(\frac{2\,g\,x^2}{5}-\frac{4\,a^2\,e^2\,g-10\,a\,c\,d\,e\,f}{15\,c^2\,d^2}+\frac{x\left(10\,f\,c^2\,d^2+2\,a\,e\,g\,c\,d\right)}{15\,c^2\,d^2}\right)\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/
2),x)
```

```
[Out] (((2*g*x^2)/5 - (4*a^2*e^2*g - 10*a*c*d*e*f)/(15*c^2*d^2) + (x*(10*c^2*d^2*
f + 2*a*c*d*e*g))/(15*c^2*d^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2))/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(d+ex)\,(ae+cdx)}\,\left(f+gx\right)}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2
),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)/sqrt(d + e*x), x)
```

3.450 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{3/2}}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.00, size = 82, normalized size = 1.71

$$\frac{2\left(ae^2-cd^2+cd(d+ex)\right)\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}}{3cde\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x
],x]
```

```
[Out] (2*(-(c*d^2) + a*e^2 + c*d*(d + e*x))*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d
+ e*x) + (c*d*(d + e*x)^2)/e])/(3*c*d*e*Sqrt[d + e*x])
```

fricas [A] time = 0.43, size = 57, normalized size = 1.19

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(cdx+ae)\sqrt{ex+d}}{3\left(cdex+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d)
/(c*d*e*x + c*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/sqrt(e*x + d), x)
```

maple [A] time = 0.00, size = 50, normalized size = 1.04

$$\frac{2\,(cdx+ae)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3\sqrt{ex+d}\;cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c/d/(e*x+d)^(1/2)
```

maxima [A] time = 0.49, size = 18, normalized size = 0.38

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

mupad [B] time = 3.05, size = 49, normalized size = 1.02

$$\frac{\left(\frac{2\,x}{3}+\frac{2\,a\,e}{3\,c\,d}\right)\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(1/2),x)
```

```
[Out] (((2*x)/3 + (2*a*e)/(3*c*d))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))
/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2),x)
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/sqrt(d + e*x), x)
```

3.451 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)}\,dx$

Optimal. Leaf size=124

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}$$

Rubi [A] time = 0.19, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {864, 874, 205}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)),x
]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt
[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)}\,dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\int \cdots}{e^2g}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left(2e^2(cdf-aeg)\right)\text{Subst}\left(\int \frac{\cdots}{-e\left(cd^2+ae^2\right)\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{2\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ade+\left(cd^2+ae^2\right)\cdots}}{\sqrt{cdf-aeg}\,\sqrt{d\cdots}}\right.}{g^{3/2}}$$

Mathematica [A] time = 0.13, size = 101, normalized size = 0.81

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}-\frac{\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g] - (Sqrt[c*d*f - a*e*g]*ArcTan[(Sq
rt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/Sqrt[a*e + c*d*x]))/(g^(3/2)
*Sqrt[d + e*x])
```

IntegrateAlgebraic [C] time = 4.70, size = 931, normalized size = 7.51

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*(f + g*x)),x]
```

```
[Out] (2*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e])/(g*S
qrt[d + e*x]) - (2*(c*d*Sqrt[e]*f*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*
I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]] - a*e^(3
/2)*g*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]] - I*Sqrt[c]*Sqrt[d]*Sqrt[-(e*f) + d*
g]*Sqrt[c*d*f - a*e*g]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*
Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]])*ArcTanh[(Sqrt[e]*S
qrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e
*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x])/(-(Sqrt[c*d*e]*Sqrt[g]*(d +
e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e
*x)^2)/e])])/((c*d^2 - a*e^2)*g^(5/2)) - (2*(c*d*Sqrt[e]*f*Sqrt[-2*c*d*e*f
+ c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt
[c*d*f - a*e*g]] - a*e^(3/2)*g*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I)*
Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]] + I*Sqrt[c]
*Sqrt[d]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]*Sqrt[-2*c*d*e*f + c*d^2*g +
 a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*
e*g]])*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x])/(-(
Sqrt[c*d*e]*Sqrt[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*
e*(d + e*x) + (c*d*(d + e*x)^2)/e])])/((c*d^2 - a*e^2)*g^(5/2))
```

fricas [A] time = 0.44, size = 318, normalized size = 2.56

$$\left[\frac{(ex+d)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,g\sqrt{-\frac{cdf-aeg}{g}}-\left(cdef-\left(cd^2+2\,ae^2\right)g\right)x}{egx^2+df+\left(ef+dg\right)x}\right)+2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{egx+dg},\frac{2\left((ex+d)\sqrt{\frac{cdf-aeg}{g}}\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{\frac{cdf-aeg}{g}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\right)+\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\right)}{egx+dg}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

```
[Out] [((e*x + d)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*
g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*
d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f
+ d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
e*g*x + d*g), 2*((e*x + d)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqr
t((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g*x + d*g)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 153, normalized size = 1.23

$$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\left(aeg\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-cdf\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{cdx+ae}\,\sqrt{\left(aeg-cdf\right)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{\left(aeg-cdf\right)g}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*a*e*g-arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2)*g)*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x
+a*e)^(1/2)/g/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(f+g\,x\right)\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/2
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}\,\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)/(e*x+d)**(1/2
),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)), x)
```

3.452 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^2}\,dx$

Optimal. Leaf size=132

$$\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\,(f+gx)}$$

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {862, 874, 205}

$$\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\,(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^2)
,x]
```

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)))
+ (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c
*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^2}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,(f+gx)} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,(f+gx)} + \frac{\left(cde^2\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg}\right.}{g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,(f+gx)} + \frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

Mathematica [A]   time = 0.19, size = 110, normalized size = 0.83

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}\sqrt{cdf-aeg}}-\frac{\sqrt{g}}{f+gx}\right)}{g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^2),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]/(f + g*x)) + (c*d*ArcTan[(Sqrt[g]
*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x])))/(g^(3/2)*Sqrt[d + e*x])
```

IntegrateAlgebraic [C]   time = 12.70, size = 1241, normalized size = 9.40

$$-\frac{ic^{3/2}\sqrt{dg-ef}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{cgd^2-2cefd-2i\sqrt{c}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}\sqrt{d}+ae^2g}\sqrt{d+ex}}{e\sqrt{g}\sqrt{-\frac{c(d+ex)d^2}{e}+\frac{c(d+ex)^2d}{e}+ae(d+ex)}-\sqrt{cde}\sqrt{g}(d+ex)}\right)d^{3/2}}{g^{3/2}\sqrt{cdf-aeg}\sqrt{cgd^2-2cefd-2i\sqrt{c}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}\sqrt{d}+ae^2g}}+\frac{ic^{3/2}\sqrt{dg-ef}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{cgd^2-2cefd+2i\sqrt{c}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}\sqrt{d}+ae^2g}\sqrt{d+ex}}{e\sqrt{g}\sqrt{-\frac{c(d+ex)d^2}{e}+\frac{c(d+ex)^2d}{e}+ae(d+ex)}-\sqrt{cde}\sqrt{g}(d+ex)}\right)d^{3/2}}{g^{3/2}\sqrt{cdf-aeg}\sqrt{cgd^2-2cefd+2i\sqrt{c}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}\sqrt{d}+ae^2g}}-\frac{c\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{cgd^2-2cefd-2i\sqrt{c}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}\sqrt{d}+ae^2g}\sqrt{d+ex}}{e\sqrt{g}\sqrt{-\frac{c(d+ex)d^2}{e}+\frac{c(d+ex)^2d}{e}+ae(d+ex)}-\sqrt{cde}\sqrt{g}(d+ex)}\right)d}{g^{3/2}\sqrt{cgd^2-2cefd-2i\sqrt{c}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}\sqrt{d}+ae^2g}}-\frac{c\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{cgd^2-2cefd+2i\sqrt{c}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}\sqrt{d}+ae^2g}\sqrt{d+ex}}{e\sqrt{g}\sqrt{-\frac{c(d+ex)d^2}{e}+\frac{c(d+ex)^2d}{e}+ae(d+ex)}-\sqrt{cde}\sqrt{g}(d+ex)}\right)d}{g^{3/2}\sqrt{cgd^2-2cefd+2i\sqrt{c}\sqrt{e}\sqrt{dg-ef}\sqrt{cdf-aeg}\sqrt{d}+ae^2g}}+\frac{\sqrt{-\frac{c(d+ex)d^2}{e}+\frac{c(d+ex)^2d}{e}+ae(d+ex)}\left(-ae^3+cd^2e-2cd(d+ex)e\right)+\sqrt{cde}\left(-2c(d+ex)d^2+2c(d+ex)^2d+2ae^2(d+ex)\right)}{g\sqrt{d+ex}\left(-cd^2+2c(d+ex)d+ae^2\right)(ef-dg+g(d+ex))-2\sqrt{cde}\,g\sqrt{d+ex}\,(ef-dg+g(d+ex))\sqrt{-\frac{c(d+ex)d^2}{e}+\frac{c(d+ex)^2d}{e}+ae(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*(f + g*x)^2),x]
```

```
[Out] (Sqrt[c*d*e]*(-2*c*d^2*(d + e*x) + 2*a*e^2*(d + e*x) + 2*c*d*(d + e*x)^2) +
 (c*d^2*e - a*e^3 - 2*c*d*e*(d + e*x))*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d
 + e*x) + (c*d*(d + e*x)^2)/e])/(g*Sqrt[d + e*x]*(-(c*d^2) + a*e^2 + 2*c*d*
(d + e*x))*(e*f - d*g + g*(d + e*x)) - 2*Sqrt[c*d*e]*g*Sqrt[d + e*x]*(e*f -
 d*g + g*(d + e*x))*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d +
 e*x)^2)/e]) - (c*d*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*
e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g
]]*Sqrt[d + e*x])/(-(Sqrt[c*d*e]*Sqrt[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d
^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e])])/(g^(3/2)*Sqrt[-2
*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) +
d*g]*Sqrt[c*d*f - a*e*g]]) - (I*c^(3/2)*d^(3/2)*Sqrt[-(e*f) + d*g]*ArcTanh[
(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e
]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x])/(-(Sqrt[c*d*e]*Sqr
t[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (
c*d*(d + e*x)^2)/e])])/(g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[-2*c*d*e*f + c*d^2
*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f
- a*e*g]]) - (c*d*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^
2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]
*Sqrt[d + e*x])/(-(Sqrt[c*d*e]*Sqrt[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2
```

```
*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e)])])/(g^(3/2)*Sqrt[-2*c
*d*e*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*
g]*Sqrt[c*d*f - a*e*g]]) + (I*c^(3/2)*d^(3/2)*Sqrt[-(e*f) + d*g]*ArcTanh[(S
qrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*
Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x])/(-(Sqrt[c*d*e]*Sqrt[
g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*
d*(d + e*x)^2)/e])])/(g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[-2*c*d*e*f + c*d^2*g
 + a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f -
a*e*g]])
```

fricas [B] time = 0.45, size = 562, normalized size = 4.26

$$\left[-\frac{\left(cdegx^2+cd^2f+\left(cdef+cd^2g\right)x\right)\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-\left(cdef-\left(cd^2+2\,ae^2\right)g\right)x-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{-cdfg+aeg^2}\,\sqrt{ex+d}}{egx^2+df+\left(ef+dg\right)x}\right)+2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdfg-aeg^2\right)\sqrt{ex+d}}{2\left(cd^2f^2g^2-adefg^3+\left(cdefg^3-ae^2g^4\right)x^2+\left(cdef^2g^2-adeg^4+\left(cd^2-ae^2\right)fg^3\right)x\right)},\ -\frac{\left(cdegx^2+cd^2f+\left(cdef+cd^2g\right)x\right)\sqrt{cdfg-aeg^2}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{cdfg-aeg^2}\,\sqrt{ex+d}}{cdegx^2+adeg+\left(cd^2+ae^2\right)gx}\right)+\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdfg-aeg^2\right)\sqrt{ex+d}}{cd^2f^2g^2-adefg^3+\left(cdefg^3-ae^2g^4\right)x^2+\left(cdef^2g^2-adeg^4+\left(cd^2-ae^2\right)fg^3\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e*
g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)
*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g
^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c*d^2*f^2*g^2
- a*d*e*f*g^3 + (c*d*e*f*g^3 - a*e^2*g^4)*x^2 + (c*d*e*f^2*g^2 - a*d*e*g^4
+ (c*d^2 - a*e^2)*f*g^3)*x), -((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)
*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 +
 a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*
g^2)*sqrt(e*x + d))/(c*d^2*f^2*g^2 - a*d*e*f*g^3 + (c*d*e*f*g^3 - a*e^2*g^4
)*x^2 + (c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - a*e^2)*f*g^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 161, normalized size = 1.22

$$\frac{\left(-cdgx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-cdf\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{cdx+ae}\,\sqrt{\left(aeg-cdf\right)g}\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(gx+f\right)\sqrt{\left(aeg-cdf\right)g}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x)
```

```
[Out] (-arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c*d*g-arctanh((c*d
*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*
d*f)*g)^(1/2))*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e*x+d)^(1/2)/(c*d*x
+a*e)^(1/2)/g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^2\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(1
/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**2/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**2), x)
```

3.453 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^3}\,dx$

Optimal. Leaf size=207

$$\frac{c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2}$$

Rubi [A] time = 0.27, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^3)
,x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*g*Sqrt[d + e*x]*(f + g*x)^2
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)*
Sqrt[d + e*x]*(f + g*x)) + (c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(3/2)*(c*d
*f - a*e*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
```

```
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{\sqrt{d + ex}\,(f + gx)^3}\,dx = -\frac{\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{2g\sqrt{d + ex}\,(f + gx)^2} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4g}$$

$$= -\frac{\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{2g\sqrt{d + ex}\,(f + gx)^2} + \frac{cd\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{4g(cdf - aeg)\sqrt{d + ex}\,(f + gx)} + \cdots$$

$$= -\frac{\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{2g\sqrt{d + ex}\,(f + gx)^2} + \frac{cd\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{4g(cdf - aeg)\sqrt{d + ex}\,(f + gx)} + \cdots$$

$$= -\frac{\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{2g\sqrt{d + ex}\,(f + gx)^2} + \frac{cd\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{4g(cdf - aeg)\sqrt{d + ex}\,(f + gx)} + \cdots$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.38

$$\frac{2c^2d^2((d + ex)(ae + cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2}, 3; \frac{5}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{3(d + ex)^{3/2}(cdf - aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^3),x]
```

```
[Out] (2*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^3*(d + e*x)^(3/2))
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*(f + g*x)^3),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.45, size = 1056, normalized size = 5.10

$$\left[\frac{\left(c^2d^2eg^2x^3 + c^2d^3f^2 + \left(2c^2d^2efg + c^2d^3g^2\right)x^2 + \left(c^2d^2ef^2 + 2c^2d^3fg\right)x\right)\sqrt{-cdfg + aeg^2}\log\left(-\frac{cdegx^2 - cd^2f + 2adeg - \left(cdef - \left(cd^2 + 2ae^2\right)g\right)x + 2\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{-cdfg + aeg^2}\sqrt{ex + d}}{egx^2 + df + (ef + dg)x}\right) - 2\left(c^2d^2f^2g - 3acdefg^2 + 2a^2e^2g^3 - \left(c^2d^2fg^2 - acdeg^3\right)x\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex + d}}{8\left(c^2d^3f^4g^2 - 2acd^2ef^3g^3 + a^2de^2f^2g^4 + \left(c^2d^2ef^2g^4 - 2acde^2fg^5 + a^2e^3g^6\right)x^3 + \left(2c^2d^2ef^3g^3 + a^2de^2g^6 + \left(c^2d^3 - 4acde^2\right)f^2g^4 - 2\left(acd^2e - a^2e^3\right)fg^5\right)x^2 + \left(c^2d^2ef^4g^2 + 2a^2de^2fg^5 + 2\left(c^2d^3 - acde^2\right)f^3g^3 - \left(4acd^2e - a^2e^3\right)f^2g^4\right)x\right)}, -\frac{\left(c^2d^2eg^2x^3 + c^2d^3f^2 + \left(2c^2d^2efg + c^2d^3g^2\right)x^2 + \left(c^2d^2ef^2 + 2c^2d^3fg\right)x\right)\sqrt{cdfg - aeg^2}\arctan\left(\frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{cdfg - aeg^2}\sqrt{ex + d}}{cdegx^2 + adeg + \left(cd^2 + ae^2\right)gx}\right) + \left(c^2d^2f^2g - 3acdefg^2 + 2a^2e^2g^3 - \left(c^2d^2fg^2 - acdeg^3\right)x\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex + d}}{4\left(c^2d^3f^4g^2 - 2acd^2ef^3g^3 + a^2de^2f^2g^4 + \left(c^2d^2ef^2g^4 - 2acde^2fg^5 + a^2e^3g^6\right)x^3 + \left(2c^2d^2ef^3g^3 + a^2de^2g^6 + \left(c^2d^3 - 4acde^2\right)f^2g^4 - 2\left(acd^2e - a^2e^3\right)fg^5\right)x^2 + \left(c^2d^2ef^4g^2 + 2a^2de^2fg^5 + 2\left(c^2d^3 - acde^2\right)f^3g^3 - \left(4acd^2e - a^2e^3\right)f^2g^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/8*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^
2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e
*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c
```

```
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d
))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(c^2*d^2*f^2*g - 3*a*c*d*e*f*g^2 +
2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g^2 - 2*a*c*d^2*e*f^3*g^3 + a
^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 + a^2*e^3*g^6)*x^
3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^4
- 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c^2*d^2*e*f^4*g^2 + 2*a^2*d*e^2*f*g
^5 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e - a^2*e^3)*f^2*g^4)*x),
 -1/4*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x
^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x +
d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c^2*d^2*f^2*g - 3*a*c*
d*e*f*g^2 + 2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g^2 - 2*a*c*d^2*e
*f^3*g^3 + a^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 + a^2
*e^3*g^6)*x^3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 4*a*c*d*e
^2)*f^2*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c^2*d^2*e*f^4*g^2 + 2*a
^2*d*e^2*f*g^5 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e - a^2*e^3)*
f^2*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 285, normalized size = 1.38

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(c^2d^2g^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+2c^2d^2fgx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+c^2d^2f^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,cdgx-2\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,aeg+\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,cdf\right)}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(aeg-cdf\right)\left(gx+f\right)^2\sqrt{\left(aeg-cdf\right)g}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x)
```

```
[Out] 1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*x^2*c^2*d^2*g^2+2*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2)*g)*x*c^2*d^2*f*g+arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2)*g)*c^2*d^2*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*g*x-2*
((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*(c*
d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+
f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^3\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(1
/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\left(f+gx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**3/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**3), x)
```

3.454 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^4}\,dx$

Optimal. Leaf size=277

$$\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x}}{}$$

Rubi [A] time = 0.35, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^4)
,x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*g*Sqrt[d + e*x]*(f + g*x)^3
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*g)
*Sqrt[d + e*x]*(f + g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(8*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (c^3*d^3*ArcTan[
(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*
Sqrt[d + e*x])])/(8*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
```

```
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^4}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{6g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.29

$$\frac{2c^3d^3((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3(d+ex)^{3/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^4),x]
```

```
[Out] (2*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^4*(d + e*x)^(3/2))
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*(f + g*x)^4),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.50, size = 1732, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g
^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*
d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*
e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
+ d*g)*x)) + 2*(3*c^3*d^3*f^3*g - 17*a*c^2*d^2*e*f^2*g^2 + 22*a^2*c*d*e^2*f
*g^3 - 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 - 2*(4*c^3*d
^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^6*g^2 - 3*a*c^2*d^3*e*f^5
*g^3 + 3*a^2*c*d^2*e^2*f^4*g^4 - a^3*d*e^3*f^3*g^5 + (c^3*d^3*e*f^3*g^5 - 3
*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^4 + (3*c^3*d^
3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^
2*d^3*e - 3*a^2*c*d*e^3)*f^2*g^6 + 3*(a^2*c*d^2*e^2 - a^3*e^4)*f*g^7)*x^3 +
 3*(c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g
^4 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^
2*g^6)*x^2 + (c^3*d^3*e*f^6*g^2 - 3*a^3*d*e^3*f^2*g^6 + 3*(c^3*d^4 - a*c^2*
d^2*e^2)*f^5*g^3 - 3*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^4 + (9*a^2*c*d^2*e
^2 - a^3*e^4)*f^3*g^5)*x), -1/24*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c
^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2
 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x +
d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^3*d^3*f^3*g - 17*a
*c^2*d^2*e*f^2*g^2 + 22*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^
3 - a*c^2*d^2*e*g^4)*x^2 - 2*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2
*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))
/(c^3*d^4*f^6*g^2 - 3*a*c^2*d^3*e*f^5*g^3 + 3*a^2*c*d^2*e^2*f^4*g^4 - a^3*d
*e^3*f^3*g^5 + (c^3*d^3*e*f^3*g^5 - 3*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3
*f*g^7 - a^3*e^4*g^8)*x^4 + (3*c^3*d^3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4
 - 9*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^2*g^6 + 3*(
a^2*c*d^2*e^2 - a^3*e^4)*f*g^7)*x^3 + 3*(c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^
7 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^4 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3
*g^5 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^6)*x^2 + (c^3*d^3*e*f^6*g^2 - 3*a^
3*d*e^3*f^2*g^6 + 3*(c^3*d^4 - a*c^2*d^2*e^2)*f^5*g^3 - 3*(3*a*c^2*d^3*e -
a^2*c*d*e^3)*f^4*g^4 + (9*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.04, size = 453, normalized size = 1.64

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\left(3c^3d^3g^3x^3\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+9c^3d^3f\,g^2x^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+9c^3d^3f^2gx\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3c^3d^3f^3\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-3\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,c^2d^2g^2x^2+2\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,acdeg^2x-8\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,c^2d^2fgx+8\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,a^2e^2g^2-14\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,acdefg+3\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,c^2d^2f^2\right)}{24\sqrt{ex+d}\,\sqrt{\left(aeg-cdf\right)g}\,\left(gx+f\right)^3\left(aeg-cdf\right)^2\sqrt{cdx+ae}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*g^3+9*arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*f*g^2+9*arctanh((c*d*x+a*e)^(1/2)/((a*e*
g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2)*g)*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^
2*d^2*g^2*x^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x-8*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+8*((a*e*g-c*d*f)*g)^
```

```
(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^
(1/2)*a*c*d*e*f*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/
(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)^2/(c*d*x+a*
e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{\sqrt{ex + d}\left(gx + f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{\left(f + g\,x\right)^4\,\sqrt{d + e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(1
/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\,\left(f + gx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**4/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**4), x)
```

3.455 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^5}\,dx$

Optimal. Leaf size=347

$$\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}+\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{96g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}$$

Rubi [A] time = 0.45, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}+\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{96g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{24g\sqrt{d+ex}\,(f+gx)^3(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^5)
,x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*g*Sqrt[d + e*x]*(f + g*x)^4
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*g*(c*d*f - a*e*g)
*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(96*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^3*d^3*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*g*(c*d*f - a*e*g)^3*Sqrt[d
 + e*x]*(f + g*x)) + (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(3/2)*(c*d*f
- a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
```

```
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^5}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{8g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.23

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3(d+ex)^{3/2}(cdf-aeg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^5),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^5*(d + e*x)^(3/2))
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*(f + g*x)^5),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.48, size = 2610, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c
^4*d^4*f^4*g - 133*a*c^3*d^3*e*f^3*g^2 + 254*a^2*c^2*d^2*e^2*f^2*g^3 - 184*
a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x
^3 - 5*(11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*
x^2 - (73*c^4*d^4*f^3*g^2 - 109*a*c^3*d^3*e*f^2*g^3 + 44*a^2*c^2*d^2*e^2*f*
g^4 - 8*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(c^4*d^5*f^8*g^2 - 4*a*c^3*d^4*e*f^7*g^3 + 6*a^2*c^2*d^3*e^2*f^
6*g^4 - 4*a^3*c*d^2*e^3*f^5*g^5 + a^4*d*e^4*f^4*g^6 + (c^4*d^4*e*f^4*g^6 -
4*a*c^3*d^3*e^2*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*e^4*f*g^9 +
 a^4*e^5*g^10)*x^5 + (4*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^4*d^5 - 16*
a*c^3*d^3*e^2)*f^4*g^6 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^7 + 2*(3
*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^8 - 4*(a^3*c*d^2*e^3 - a^4*e^5)*f*g
^9)*x^4 + 2*(3*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^5 - 6*a*c^3
*d^3*e^2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^6 + 12*(a^2
*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^8
)*x^3 + 2*(2*c^4*d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^5 - 8*a*c^3
*d^3*e^2)*f^6*g^4 - 12*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^5 + 2*(9*a^2*c
^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g^6 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^7
)*x^2 + (c^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 - a*c^3*d^3*e
^2)*f^7*g^3 - 2*(8*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^4 + 4*(6*a^2*c^2*
d^3*e^2 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^6)*x),
-1/192*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(c
*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c
*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*
x)) + (15*c^4*d^4*f^4*g - 133*a*c^3*d^3*e*f^3*g^2 + 254*a^2*c^2*d^2*e^2*f^2
*g^3 - 184*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d
^3*e*g^5)*x^3 - 5*(11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^
2*e^2*g^5)*x^2 - (73*c^4*d^4*f^3*g^2 - 109*a*c^3*d^3*e*f^2*g^3 + 44*a^2*c^2
*d^2*e^2*f*g^4 - 8*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^8*g^2 - 4*a*c^3*d^4*e*f^7*g^3 + 6*a^2*c^2
*d^3*e^2*f^6*g^4 - 4*a^3*c*d^2*e^3*f^5*g^5 + a^4*d*e^4*f^4*g^6 + (c^4*d^4*e
*f^4*g^6 - 4*a*c^3*d^3*e^2*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*
e^4*f*g^9 + a^4*e^5*g^10)*x^5 + (4*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^
4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^6 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3
*g^7 + 2*(3*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^8 - 4*(a^3*c*d^2*e^3 - a
^4*e^5)*f*g^9)*x^4 + 2*(3*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^
5 - 6*a*c^3*d^3*e^2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^
6 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 3*a^4*e
^5)*f^2*g^8)*x^3 + 2*(2*c^4*d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^
5 - 8*a*c^3*d^3*e^2)*f^6*g^4 - 12*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^5 +
 2*(9*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g^6 - 2*(6*a^3*c*d^2*e^3 - a^4*e
^5)*f^3*g^7)*x^2 + (c^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 -
a*c^3*d^3*e^2)*f^7*g^3 - 2*(8*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^4 + 4*
(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - a^4*e^5)*f^
4*g^6)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 696, normalized size = 2.01

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15c^4d^4g^4x^4\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+60c^4d^4fg^3x^3\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+90c^4d^4f^2g^2x^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+60c^4d^4f^3gx\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+15c^4d^4f^4\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-15\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,c^3d^3g^3x^3+10\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,ac^2d^2eg^3x^2-55\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,c^3d^3fg^2x^2-8\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,a^2cde^2g^3x+36\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,ac^2d^2efg^2x-73\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,c^3d^3f^2gx-48\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,a^3e^3g^3+136\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,a^2cde^2fg^2-118\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,ac^2d^2ef^2g+15\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,c^3d^3f^3\right)}{192\sqrt{ex+d}\,\sqrt{(aeg-cdf)g}\,(gx+f)^4(aeg-cdf)\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\sqrt{cdx+ae}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x)
```

```
[Out] 1/192*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^4*d^4*g^4+60*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^4*d^4*f*g^3+90*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^4*d^4*f^2*g^2+60*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x*c^4*d^4*f^3*g-15*x^3*c^3*d^3*g^3*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2)*g)*c^4*d^4*f^4+10*x^2*a*c^2*d^2*e*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)
*g)^(1/2)-55*x^2*c^3*d^3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*
x*a^2*c*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*x*a*c^2*d^2*
e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-73*x*c^3*d^3*f^2*g*(c*d*x
+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-48*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^
(1/2)*a^3*e^3*g^3+136*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^2
*f*g^2-118*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+15*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2)/(g*x+f)^4/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d
^2*f^2)/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{\sqrt{ex + d}\left(gx + f\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{{\left(f + g\,x\right)}^5\,\sqrt{d + e\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**5/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

3.456 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^3\left(2ae^2g-cd(7ef-5dg)\right)}{15015c^5d^5e(d+ex)^{5/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex\right.}{3003c^4d^4e(d+ex)}$$

Rubi [A] time = 0.61, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{143c^2d^2(d+ex)^{5/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2}{429c^3d^3(d+ex)^{5/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^3}{3003c^4d^4e(d+ex)^{3/2}}-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^3\left(2ae^2g-cd(7ef-5dg)\right)}{15015c^5d^5e(d+ex)^{5/2}}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2))/(15015*c^5*d^5*e*(d + e*x)^(5/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3003*c^4*d^
4*e*(d + e*x)^(3/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(429*c^3*d^3*(d + e*x)^(5/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(143*c^2*d
^2*(d + e*x)^(5/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2))/(13*c*d*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}} + \frac{(8(cdf-aeg))\int}{}$$

$$= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2(d+ex)^{5/2}} + \frac{2(f}{}$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{429c^3d^3(d+ex)^{5/2}} + \frac{16}{}$$

$$= \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3003c^4d^4e(d+ex)^{3/2}} + \frac{32(cdf-}{}$$

$$= \frac{128(cdf-aeg)^3\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{15015c^4d^4(d+ex)^{5/2}}$$

Mathematica [A] time = 0.23, size = 195, normalized size = 0.58

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(128a^4e^4g^4-64a^3cde^3g^3(13f+5gx)+16a^2c^2d^2e^2g^2\left(143f^2+130fgx+35g^2x^2\right)-8ac^3d^3eg\left(429f^3+715f^2gx+455fg^2x^2+105g^3x^3\right)+c^4d^4\left(3003f^4+8580f^3gx+10010f^2g^2x^2+5460fg^3x^3+1155g^4x^4\right)\right)}{15015c^5d^5(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(1
3*f + 5*g*x) + 16*a^2*c^2*d^2*e^2*g^2*(143*f^2 + 130*f*g*x + 35*g^2*x^2) -
8*a*c^3*d^3*e*g*(429*f^3 + 715*f^2*g*x + 455*f*g^2*x^2 + 105*g^3*x^3) + c^4
*d^4*(3003*f^4 + 8580*f^3*g*x + 10010*f^2*g^2*x^2 + 5460*f*g^3*x^3 + 1155*g
^4*x^4)))/(15015*c^5*d^5*(d + e*x)^(5/2))
```

IntegrateAlgebraic [F] time = 180.10, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(d + e*x)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.43, size = 472, normalized size = 1.40

$$\frac{2\left(1155c^6d^6g^4x^6+3003a^2c^4d^4e^2f^4-3432a^3c^3d^3e^3f^3g+2288a^4c^2d^2e^4f^2g^2-832a^5cde^5fg^3+128a^6e^6g^4+210\left(26c^6d^6fg^3+7ac^5d^5eg^4\right)x^5+35\left(286c^6d^6f^2g^2+208ac^5d^5efg^3+a^2c^4d^4e^2g^4\right)x^4+20\left(429c^6d^6f^3g+715ac^5d^5ef^2g^2+13a^2c^4d^4e^2fg^3-2a^3c^3d^3e^3g^4\right)x^3+3\left(1001c^6d^6f^4+4576ac^5d^5ef^3g+286a^2c^4d^4e^2f^2g^2-104a^3c^3d^3e^3fg^3+16a^4c^2d^2e^4g^4\right)x^2+2\left(3003ac^5d^5ef^4+858a^2c^4d^4e^2f^3g-572a^3c^3d^3e^3f^2g^2+208a^4c^2d^2e^4fg^3-32a^5cde^5g^4\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{15015\left(c^5d^5ex+c^5d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/15015*(1155*c^6*d^6*g^4*x^6 + 3003*a^2*c^4*d^4*e^2*f^4 - 3432*a^3*c^3*d^3
*e^3*f^3*g + 2288*a^4*c^2*d^2*e^4*f^2*g^2 - 832*a^5*c*d*e^5*f*g^3 + 128*a^6
*e^6*g^4 + 210*(26*c^6*d^6*f*g^3 + 7*a*c^5*d^5*e*g^4)*x^5 + 35*(286*c^6*d^6
*f^2*g^2 + 208*a*c^5*d^5*e*f*g^3 + a^2*c^4*d^4*e^2*g^4)*x^4 + 20*(429*c^6*d
^6*f^3*g + 715*a*c^5*d^5*e*f^2*g^2 + 13*a^2*c^4*d^4*e^2*f*g^3 - 2*a^3*c^3*d
^3*e^3*g^4)*x^3 + 3*(1001*c^6*d^6*f^4 + 4576*a*c^5*d^5*e*f^3*g + 286*a^2*c^
```

```
4*d^4*e^2*f^2*g^2 - 104*a^3*c^3*d^3*e^3*f*g^3 + 16*a^4*c^2*d^2*e^4*g^4)*x^2
 + 2*(3003*a*c^5*d^5*e*f^4 + 858*a^2*c^4*d^4*e^2*f^3*g - 572*a^3*c^3*d^3*e^
3*f^2*g^2 + 208*a^4*c^2*d^2*e^4*f*g^3 - 32*a^5*c*d*e^5*g^4)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}} \left(gx + f\right)^4}{(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^4/(e*x +
d)^(3/2), x)
```

maple [**A**] time = 0.01, size = 283, normalized size = 0.84

$$\frac{2 (cdx + ae) \left(1155 g^4 x^4 c^4 d^4 - 840 a c^3 d^3 e g^4 x^3 + 5460 c^4 d^4 f g^3 x^3 + 560 a^2 c^2 d^2 e^2 g^4 x^2 - 3640 a c^3 d^3 e f g^3 x^2 + 10010 c^4 d^4 f^2 g^2 x^2 - 320 a^3 c d e^3 g^4 x + 2080 a^2 c^2 d^2 e^2 f g^3 x - 5720 a c^3 d^3 e f^2 g^2 x + 8580 c^4 d^4 f^3 g x + 128 a^4 e^4 g^4 - 832 a^3 c d e^3 f g^3 + 2288 a^2 c^2 d^2 e^2 f^2 g^2 - 3432 a c^3 d^3 e f^3 g + 3003 f^4 c^4 d^4\right) \left(cde x^2 + a e^2 x + c d^2 x + ade\right)^{\frac{3}{2}}}{15015 (ex + d)^{\frac{3}{2}} c^5 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/15015*(c*d*x+a*e)*(1155*c^4*d^4*g^4*x^4-840*a*c^3*d^3*e*g^4*x^3+5460*c^4*
d^4*f*g^3*x^3+560*a^2*c^2*d^2*e^2*g^4*x^2-3640*a*c^3*d^3*e*f*g^3*x^2+10010*
c^4*d^4*f^2*g^2*x^2-320*a^3*c*d*e^3*g^4*x+2080*a^2*c^2*d^2*e^2*f*g^3*x-5720
*a*c^3*d^3*e*f^2*g^2*x+8580*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-832*a^3*c*d*e^3
*f*g^3+2288*a^2*c^2*d^2*e^2*f^2*g^2-3432*a*c^3*d^3*e*f^3*g+3003*c^4*d^4*f^4
)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c^5/d^5/(e*x+d)^(3/2)
```

maxima [**A**] time = 0.74, size = 413, normalized size = 1.23

$$\frac{2 \left(c^2 d^2 x^2 + 2 acdex + a^2 e^2\right) \sqrt{cdx + ae}\, f^4}{5 cd} + \frac{8 \left(5 c^3 d^3 x^3 + 8 ac^2 d^2 ex^2 + a^2 cde^2 x - 2 a^3 e^3\right) \sqrt{cdx + ae}\, f^3 g}{35 c^2 d^2} + \frac{4 \left(35 c^4 d^4 x^4 + 50 ac^3 d^3 ex^3 + 3 a^2 c^2 d^2 e^2 x^2 - 4 a^3 cde^3 x + 8 a^4 e^4\right) \sqrt{cdx + ae}\, f^2 g^2}{105 c^3 d^3} + \frac{8 \left(105 c^5 d^5 x^5 + 140 ac^4 d^4 ex^4 + 5 a^2 c^3 d^3 e^2 x^3 - 6 a^3 c^2 d^2 e^3 x^2 + 8 a^4 cde^4 x - 16 a^5 e^5\right) \sqrt{cdx + ae}\, fg^3}{1155 c^4 d^4} + \frac{2 \left(1155 c^6 d^6 x^6 + 1470 ac^5 d^5 ex^5 + 35 a^2 c^4 d^4 e^2 x^4 - 40 a^3 c^3 d^3 e^3 x^3 + 48 a^4 c^2 d^2 e^4 x^2 - 64 a^5 cde^5 x + 128 a^6 e^6\right) \sqrt{cdx + ae}\, g^4}{15015 c^5 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^4/(c*d) + 8/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f^3*g/(c^2*d^2) + 4/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a
^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g^2
/(c^3*d^3) + 8/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3*d^3*
e^2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(c*d*x
+ a*e)*f*g^3/(c^4*d^4) + 2/15015*(1155*c^6*d^6*x^6 + 1470*a*c^5*d^5*e*x^5 +
 35*a^2*c^4*d^4*e^2*x^4 - 40*a^3*c^3*d^3*e^3*x^3 + 48*a^4*c^2*d^2*e^4*x^2 -
 64*a^5*c*d*e^5*x + 128*a^6*e^6)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

mupad [**B**] time = 3.80, size = 445, normalized size = 1.32

$$\frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}\,\left(\frac{4\,g^3\,x^5\,\left(7\,a\,e\,g+26\,c\,d\,f\right)}{143} + \frac{256\,a^6\,e^6\,g^4-1664\,a^5\,c\,d\,e^5\,f\,g^3+4576\,a^4\,c^2\,d^2\,e^4\,f^2\,g^2-6864\,a^3\,c^3\,d^3\,e^3\,f^3\,g+6006\,a^2\,c^4\,d^4\,e^2\,f^4}{15015\,c^5\,d^5} + \frac{x^2\,\left(96\,a^4\,c^2\,d^2\,e^4\,g^4-624\,a^3\,c^3\,d^3\,e^3\,f\,g^3+1716\,a^2\,c^4\,d^4\,e^2\,f^2\,g^2+27456\,a\,c^5\,d^5\,e\,f^3\,g+6006\,c^6\,d^6\,f^4\right)}{15015\,c^5\,d^5} + \frac{x\,\left(-128\,a^5\,c\,d\,e^5\,g^4+832\,a^4\,c^2\,d^2\,e^4\,f\,g^3-2288\,a^3\,c^3\,d^3\,e^3\,f^2\,g^2+3432\,a^2\,c^4\,d^4\,e^2\,f^3\,g+12012\,a\,c^5\,d^5\,e\,f^4\right)}{15015\,c^5\,d^5} + \frac{2\,c\,d\,g^4\,x^6}{13} + \frac{8\,g\,x^3\,\left(-2\,a^3\,e^3\,g^3+13\,a^2\,c\,d\,e^2\,f\,g^2+715\,a\,c^2\,d^2\,e\,f^2\,g+429\,c^3\,d^3\,f^3\right)}{3003\,c^2\,d^2} + \frac{2\,g^2\,x^4\,\left(a^2\,e^2\,g^2+208\,a\,c\,d\,e\,f\,g+286\,c^2\,d^2\,f^2\right)}{429\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^3*x^5*(7*a*e*g + 26*c*
d*f))/143 + (256*a^6*e^6*g^4 + 6006*a^2*c^4*d^4*e^2*f^4 - 6864*a^3*c^3*d^3*
e^3*f^3*g - 1664*a^5*c*d*e^5*f*g^3 + 4576*a^4*c^2*d^2*e^4*f^2*g^2)/(15015*c
^5*d^5) + (x^2*(6006*c^6*d^6*f^4 + 96*a^4*c^2*d^2*e^4*g^4 - 624*a^3*c^3*d^3
*e^3*f*g^3 + 27456*a*c^5*d^5*e*f^3*g + 1716*a^2*c^4*d^4*e^2*f^2*g^2))/(1501
5*c^5*d^5) + (x*(12012*a*c^5*d^5*e*f^4 - 128*a^5*c*d*e^5*g^4 + 3432*a^2*c^4
*d^4*e^2*f^3*g + 832*a^4*c^2*d^2*e^4*f*g^3 - 2288*a^3*c^3*d^3*e^3*f^2*g^2))
/(15015*c^5*d^5) + (2*c*d*g^4*x^6)/13 + (8*g*x^3*(429*c^3*d^3*f^3 - 2*a^3*e
^3*g^3 + 715*a*c^2*d^2*e*f^2*g + 13*a^2*c*d*e^2*f*g^2))/(3003*c^2*d^2) + (2
*g^2*x^4*(a^2*e^2*g^2 + 286*c^2*d^2*f^2 + 208*a*c*d*e*f*g))/(429*c*d)))/(d
+ e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

3.457 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2\left(2ae^2g-cd(7ef-5dg)\right)}{1155c^4d^4e(d+ex)^{5/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5}}{231c^3d^3e(d+ex)^{3/2}}$$

Rubi [A] time = 0.41, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{4(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{33c^2d^2(d+ex)^{5/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2}{231c^3d^3e(d+ex)^{3/2}}-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2\left(2ae^2g-cd(7ef-5dg)\right)}{1155c^4d^4e(d+ex)^{5/2}}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(1155*c^4*d^4*e*(d + e*x)^(5/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*c^3*d^3*e*
(d + e*x)^(3/2)) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(5/2))/(33*c^2*d^2*(d + e*x)^(5/2)) + (2*(f + g*x)^3*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}} + \frac{(6(cdf-aeg))\int}{}$$

$$= \frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33c^2d^2(d+ex)^{5/2}} + \frac{2(f}{}$$

$$= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231c^3d^3e(d+ex)^{3/2}} + \frac{4(cdf-ae}{}$$

$$= \frac{16(cdf-aeg)^2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{1155c^3d^3(d+ex)^{5/2}}$$

Mathematica [A] time = 0.17, size = 137, normalized size = 0.51

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(-16a^3e^3g^3+8a^2cde^2g^2(11f+5gx)-2ac^2d^2eg\left(99f^2+110fgx+35g^2x^2\right)+c^3d^3\left(231f^3+495f^2gx+385fg^2x^2+105g^3x^3\right)\right)}{1155c^4d^4(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + 
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(11
*f + 5*g*x) - 2*a*c^2*d^2*e*g*(99*f^2 + 110*f*g*x + 35*g^2*x^2) + c^3*d^3*(
231*f^3 + 495*f^2*g*x + 385*f*g^2*x^2 + 105*g^3*x^3)))/(1155*c^4*d^4*(d + e
*x)^(5/2))
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(d + e*x)^(3/2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.44, size = 340, normalized size = 1.26

$$\frac{2\left(105c^5d^5g^3x^5+231a^2c^3d^3e^2f^3-198a^3c^2d^2e^3f^2g+88a^4cde^4fg^2-16a^5e^5g^3+35\left(11c^5d^5fg^2+4ac^4d^4eg^3\right)x^4+5\left(99c^5d^5f^2g+110ac^4d^4efg^2+a^2c^3d^3e^2g^3\right)x^3+3\left(77c^5d^5f^3+264ac^4d^4ef^2g+11a^2c^3d^3e^2fg^2-2a^3c^2d^2e^3g^3\right)x^2+\left(462ac^4d^4ef^3+99a^2c^3d^3e^2f^2g-44a^3c^2d^2e^3fg^2+8a^4cde^4g^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{1155\left(c^4d^4ex+c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 198*a^3*c^2*d^2*e^3
*f^2*g + 88*a^4*c*d*e^4*f*g^2 - 16*a^5*e^5*g^3 + 35*(11*c^5*d^5*f*g^2 + 4*a
*c^4*d^4*e*g^3)*x^4 + 5*(99*c^5*d^5*f^2*g + 110*a*c^4*d^4*e*f*g^2 + a^2*c^3
*d^3*e^2*g^3)*x^3 + 3*(77*c^5*d^5*f^3 + 264*a*c^4*d^4*e*f^2*g + 11*a^2*c^3*
d^3*e^2*f*g^2 - 2*a^3*c^2*d^2*e^3*g^3)*x^2 + (462*a*c^4*d^4*e*f^3 + 99*a^2*
c^3*d^3*e^2*f^2*g - 44*a^3*c^2*d^2*e^3*f*g^2 + 8*a^4*c*d*e^4*g^3)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^3}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^3/(e*x +
d)^(3/2), x)
```

maple [A] time = 0.01, size = 188, normalized size = 0.70

$$-\frac{2\left(cdx+ae\right)\left(-105g^{3}x^{3}c^{3}d^{3}+70a\,c^{2}d^{2}e\,g^{3}x^{2}-385c^{3}d^{3}f\,g^{2}x^{2}-40a^{2}cd\,e^{2}g^{3}x+220a\,c^{2}d^{2}ef\,g^{2}x-495c^{3}d^{3}f^{2}gx+16a^{3}e^{3}g^{3}-88a^{2}cd\,e^{2}f\,g^{2}+198a\,c^{2}d^{2}e\,f^{2}g-231f^{3}c^{3}d^{3}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{1155\left(ex+d\right)^{\frac{3}{2}}c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/1155*(c*d*x+a*e)*(-105*c^3*d^3*g^3*x^3+70*a*c^2*d^2*e*g^3*x^2-385*c^3*d^
3*f*g^2*x^2-40*a^2*c*d*e^2*g^3*x+220*a*c^2*d^2*e*f*g^2*x-495*c^3*d^3*f^2*g*
x+16*a^3*e^3*g^3-88*a^2*c*d*e^2*f*g^2+198*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3
)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c^4/d^4/(e*x+d)^(3/2)
```

maxima [A] time = 0.69, size = 294, normalized size = 1.09

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdex+a^{2}e^{2}\right)\sqrt{cdx+ae}\,f^{3}}{5\,cd}+\frac{6\left(5\,c^{3}d^{3}x^{3}+8\,ac^{2}d^{2}ex^{2}+a^{2}cde^{2}x-2\,a^{3}e^{3}\right)\sqrt{cdx+ae}\,f^{2}g}{35\,c^{2}d^{2}}+\frac{2\left(35\,c^{4}d^{4}x^{4}+50\,ac^{3}d^{3}ex^{3}+3\,a^{2}c^{2}d^{2}e^{2}x^{2}-4\,a^{3}cde^{3}x+8\,a^{4}e^{4}\right)\sqrt{cdx+ae}\,fg^{2}}{105\,c^{3}d^{3}}+\frac{2\left(105\,c^{5}d^{5}x^{5}+140\,ac^{4}d^{4}ex^{4}+5\,a^{2}c^{3}d^{3}e^{2}x^{3}-6\,a^{3}c^{2}d^{2}e^{3}x^{2}+8\,a^{4}cde^{4}x-16\,a^{5}e^{5}\right)\sqrt{cdx+ae}\,g^{3}}{1155\,c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^3/(c*d) + 6/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f^2*g/(c^2*d^2) + 2/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a
^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^2/(
c^3*d^3) + 2/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3*d^3*e^
2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(c*d*x +
a*e)*g^3/(c^4*d^4)
```

mupad [B] time = 3.65, size = 310, normalized size = 1.15

$$\frac{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}\,\left(\frac{2\,g^{2}\,x^{4}\,\left(4\,a\,e\,g+11\,c\,d\,f\right)}{33}-\frac{32\,a^{5}\,e^{5}\,g^{3}-176\,a^{4}\,c\,d\,e^{4}\,f\,g^{2}+396\,a^{3}\,c^{2}\,d^{2}\,e^{3}\,f^{2}\,g-462\,a^{2}\,c^{3}\,d^{3}\,e^{2}\,f^{3}}{1155\,c^{4}\,d^{4}}+\frac{x^{2}\,\left(-12\,a^{3}\,c^{2}\,d^{2}\,e^{3}\,g^{3}+66\,a^{2}\,c^{3}\,d^{3}\,e^{2}\,f\,g^{2}+1584\,a\,c^{4}\,d^{4}\,e\,f^{2}\,g+462\,c^{5}\,d^{5}\,f^{3}\right)}{1155\,c^{4}\,d^{4}}+\frac{2\,c\,d\,g^{3}\,x^{5}}{11}+\frac{2\,g\,x^{3}\,\left(a^{2}\,e^{2}\,g^{2}+110\,a\,c\,d\,e\,f\,g+99\,c^{2}\,d^{2}\,f^{2}\right)}{231\,c\,d}+\frac{2\,a\,e\,x\,\left(8\,a^{3}\,e^{3}\,g^{3}-44\,a^{2}\,c\,d\,e^{2}\,f\,g^{2}+99\,a\,c^{2}\,d^{2}\,e\,f^{2}\,g+462\,c^{3}\,d^{3}\,f^{3}\right)}{1155\,c^{3}\,d^{3}}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^4*(4*a*e*g + 11*c*
d*f))/33 - (32*a^5*e^5*g^3 - 462*a^2*c^3*d^3*e^2*f^3 + 396*a^3*c^2*d^2*e^3*
f^2*g - 176*a^4*c*d*e^4*f*g^2)/(1155*c^4*d^4) + (x^2*(462*c^5*d^5*f^3 - 12*
a^3*c^2*d^2*e^3*g^3 + 66*a^2*c^3*d^3*e^2*f*g^2 + 1584*a*c^4*d^4*e*f^2*g))/(
1155*c^4*d^4) + (2*c*d*g^3*x^5)/11 + (2*g*x^3*(a^2*e^2*g^2 + 99*c^2*d^2*f^2
 + 110*a*c*d*e*f*g))/(231*c*d) + (2*a*e*x*(8*a^3*e^3*g^3 + 462*c^3*d^3*f^3
+ 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/(1155*c^3*d^3)))/(d + e*x)^
(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

3.458 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)}{315c^3d^3e(d+ex)^{5/2}}+\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf}{63c^2d^2e(d+ex)^{3/2}}$$

Rubi [A] time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{63c^2d^2e(d+ex)^{3/2}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)}{315c^3d^3e(d+ex)^{5/2}}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(5/2))/(315*c^3*d^3*e*(d + e*x)^(5/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(63*c^2*d^2*e*(d + e*x
)^(3/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9
*c*d*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}} + \frac{\left(4\left(cde^2f+c\right.\right.}{}$$

$$= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63c^2d^2e(d+ex)^{3/2}} + \frac{2(f+gx)}{}$$

$$= -\frac{8(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x\right.}{315c^3d^3e(d+ex)^{5/2}}$$

Mathematica [A] time = 0.12, size = 90, normalized size = 0.45

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(8a^2e^2g^2-4acdeg(9f+5gx)+c^2d^2\left(63f^2+90fgx+35g^2x^2\right)\right)}{315c^3d^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(9*f + 5*g*
x) + c^2*d^2*(63*f^2 + 90*f*g*x + 35*g^2*x^2)))/(315*c^3*d^3*(d + e*x)^(5/2
))
```

IntegrateAlgebraic [A] time = 2.27, size = 120, normalized size = 0.60

$$\frac{2(ae+cdx)((d+ex)(ae+cdx))^{3/2}\left(63a^2e^2g^2+90cdfg(ae+cdx)-126acdefg+35g^2(ae+cdx)^2-90aeg^2(ae+cdx)+63c^2d^2f^2\right)}{315c^3d^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(d + e*x)^(3/2),x]
```

```
[Out] (2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/2)*(63*c^2*d^2*f^2 - 126*a*c*
d*e*f*g + 63*a^2*e^2*g^2 + 90*c*d*f*g*(a*e + c*d*x) - 90*a*e*g^2*(a*e + c*d
*x) + 35*g^2*(a*e + c*d*x)^2))/(315*c^3*d^3*(d + e*x)^(3/2))
```

fricas [A] time = 0.42, size = 230, normalized size = 1.15

$$\frac{2\left(35c^4d^4g^2x^4+63a^2c^2d^2e^2f^2-36a^3cde^3fg+8a^4e^4g^2+10\left(9c^4d^4fg+5ac^3d^3eg^2\right)x^3+3\left(21c^4d^4f^2+48ac^3d^3efg+a^2c^2d^2e^2g^2\right)x^2+2\left(63ac^3d^3ef^2+9a^2c^2d^2e^2fg-2a^3cde^3g^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{315\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 36*a^3*c*d*e^3*f*g + 8
*a^4*e^4*g^2 + 10*(9*c^4*d^4*f*g + 5*a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4*f
^2 + 48*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f^2
+ 9*a^2*c^2*d^2*e^2*f*g - 2*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^2}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^2/(e*x +
d)^(3/2), x)
```

maple [A] time = 0.01, size = 116, normalized size = 0.58

$$\frac{2\left(cdx+ae\right)\left(35g^{2}x^{2}c^{2}d^{2}-20acde\,g^{2}x+90c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-36acdefg+63f^{2}c^{2}d^{2}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{315\left(ex+d\right)^{\frac{3}{2}}c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/315*(c*d*x+a*e)*(35*c^2*d^2*g^2*x^2-20*a*c*d*e*g^2*x+90*c^2*d^2*f*g*x+8*a
^2*e^2*g^2-36*a*c*d*e*f*g+63*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)
^(3/2)/c^3/d^3/(e*x+d)^(3/2)
```

maxima [A] time = 0.62, size = 192, normalized size = 0.96

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdex+a^{2}e^{2}\right)\sqrt{cdx+ae}\,f^{2}}{5\,cd}+\frac{4\left(5\,c^{3}d^{3}x^{3}+8\,ac^{2}d^{2}ex^{2}+a^{2}cde^{2}x-2\,a^{3}e^{3}\right)\sqrt{cdx+ae}\,fg}{35\,c^{2}d^{2}}+\frac{2\left(35\,c^{4}d^{4}x^{4}+50\,ac^{3}d^{3}ex^{3}+3\,a^{2}c^{2}d^{2}e^{2}x^{2}-4\,a^{3}cde^{3}x+8\,a^{4}e^{4}\right)\sqrt{cdx+ae}\,g^{2}}{315\,c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f*g/(c^2*d^2) + 2/315*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a^2
*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*g^2/(c^3*
d^3)
```

mupad [B] time = 3.43, size = 206, normalized size = 1.03

$$\frac{\sqrt{c\,d\,e\,x^{2}+\left(c\,d^{2}+a\,e^{2}\right)\,x+a\,d\,e}\,\left(\frac{4\,g\,x^{3}\,\left(5\,a\,e\,g+9\,c\,d\,f\right)}{63}+\frac{16\,a^{4}\,e^{4}\,g^{2}-72\,a^{3}\,c\,d\,e^{3}\,f\,g+126\,a^{2}\,c^{2}\,d^{2}\,e^{2}\,f^{2}}{315\,c^{3}\,d^{3}}+\frac{x^{2}\,\left(6\,a^{2}\,c^{2}\,d^{2}\,e^{2}\,g^{2}+288\,a\,c^{3}\,d^{3}\,e\,f\,g+126\,c^{4}\,d^{4}\,f^{2}\right)}{315\,c^{3}\,d^{3}}+\frac{2\,c\,d\,g^{2}\,x^{4}}{9}+\frac{4\,a\,e\,x\,\left(-2\,a^{2}\,e^{2}\,g^{2}+9\,a\,c\,d\,e\,f\,g+63\,c^{2}\,d^{2}\,f^{2}\right)}{315\,c^{2}\,d^{2}}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g*x^3*(5*a*e*g + 9*c*d*f
))/63 + (16*a^4*e^4*g^2 + 126*a^2*c^2*d^2*e^2*f^2 - 72*a^3*c*d*e^3*f*g)/(31
5*c^3*d^3) + (x^2*(126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 + 288*a*c^3*d^3*
e*f*g))/(315*c^3*d^3) + (2*c*d*g^2*x^4)/9 + (4*a*e*x*(63*c^2*d^2*f^2 - 2*a^
2*e^2*g^2 + 9*a*c*d*e*f*g))/(315*c^2*d^2)))/(d + e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

3.459 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(2ae^2g-cd(7ef-5dg)\right)}{35c^2d^2e(d+ex)^{5/2}}$$

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(2ae^2g-cd(7ef-5dg)\right)}{35c^2d^2e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3/
2),x]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(35*c^2*d^2*e*(d + e*x)^(5/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}+\frac{1}{7}\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)$$

$$=\frac{2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35cd(d+ex)^{5/2}}+\frac{2g\left(a\right.}{}$$

Mathematica [A] time = 0.07, size = 54, normalized size = 0.43

$$\frac{2((d+ex)(ae+cdx))^{5/2}(cd(7f+5gx)-2aeg)}{35c^2d^2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-2*a*e*g + c*d*(7*f + 5*g*x)))/(35*c^2*
d^2*(d + e*x)^(5/2))
```

IntegrateAlgebraic [A] time = 1.31, size = 67, normalized size = 0.54

$$\frac{2(ae+cdx)((d+ex)(ae+cdx))^{3/2}(5g(ae+cdx)-7aeg+7cdf)}{35c^2d^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/2)*(7*c*d*f - 7*a*e*g + 5*g*(
a*e + c*d*x)))/(35*c^2*d^2*(d + e*x)^(3/2))
```

fricas [A] time = 0.41, size = 137, normalized size = 1.10

$$\frac{2\left(5\,c^{3}d^{3}gx^{3}+7\,a^{2}cde^{2}f-2\,a^{3}e^{3}g+\left(7\,c^{3}d^{3}f+8\,ac^{2}d^{2}eg\right)x^{2}+\left(14\,ac^{2}d^{2}ef+a^{2}cde^{2}g\right)x\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\,\sqrt{ex+d}}{35\left(c^{2}d^{2}ex+c^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 2*a^3*e^3*g + (7*c^3*d^3*f + 8*a*
c^2*d^2*e*g)*x^2 + (14*a*c^2*d^2*e*f + a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.00, size = 67, normalized size = 0.54

$$-\frac{2\,(cdx+ae)\left(-5cdgx+2aeg-7cdf\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{35\,(ex+d)^{\frac{3}{2}}\,c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-5*c*d*g*x+2*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(3/2)/c^2/d^2/(e*x+d)^(3/2)
```

maxima [A] time = 0.57, size = 107, normalized size = 0.86

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdex+a^{2}e^{2}\right)\sqrt{cdx+ae}\,f}{5\,cd}+\frac{2\left(5\,c^{3}d^{3}x^{3}+8\,ac^{2}d^{2}ex^{2}+a^{2}cde^{2}x-2\,a^{3}e^{3}\right)\sqrt{cdx+ae}\,g}{35\,c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f/(c*d) + 2/35*
(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*x
+ a*e)*g/(c^2*d^2)
```

mupad [B] time = 3.25, size = 109, normalized size = 0.87

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(x^2\,\left(\frac{16\,a\,e\,g}{35}+\frac{2\,c\,d\,f}{5}\right)-\frac{4\,a^3\,e^3\,g-14\,a^2\,c\,d\,e^2\,f}{35\,c^2\,d^2}+\frac{2\,c\,d\,g\,x^3}{7}+\frac{2\,a\,e\,x\,\left(a\,e\,g+14\,c\,d\,f\right)}{35\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(3/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x^2*((16*a*e*g)/35 + (2*c*d
*f)/5) - (4*a^3*e^3*g - 14*a^2*c*d*e^2*f)/(35*c^2*d^2) + (2*c*d*g*x^3)/7 +
(2*a*e*x*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d + ex\right)\left(ae + cdx\right)\right)^{\frac{3}{2}}\left(f + gx\right)}{\left(d + ex\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2
),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)/(d + e*x)**(3/2), x)
```

3.460 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.00, size = 45, normalized size = 0.94

$$\frac{2(ae+cdx)((d+ex)(ae+cdx))^{3/2}}{5cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^
(3/2),x]
```

```
[Out] (2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(3/2))/(5*c*d*(d + e*x)^(3/2))
```

fricas [**A**] time = 0.40, size = 74, normalized size = 1.54

$$\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{5\left(cdex+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="fricas")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(3/2), x)
```

maple [**A**] time = 0.00, size = 50, normalized size = 1.04

$$\frac{2\,(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{5\,(ex+d)^{\frac{3}{2}}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/5*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c/d/(e*x+d)^(3/2)
```

maxima [**A**] time = 0.51, size = 43, normalized size = 0.90

$$\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}}{5\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)/(c*d)
```

mupad [**B**] time = 3.08, size = 62, normalized size = 1.29

$$\frac{\left(\frac{4\,a\,e\,x}{5}+\frac{2\,c\,d\,x^2}{5}+\frac{2\,a^2\,e^2}{5\,c\,d}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x)
```

```
[Out] (((4*a*e*x)/5 + (2*c*d*x^2)/5 + (2*a^2*e^2)/(5*c*d))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**(3/2), x)
```

3.461 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal. Leaf size=179

$$\frac{2(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{g^2\sqrt{d+ex}}+\frac{2\left(x\left(ae^2+cd^2\right)\cdots\right.}{3g(d+\cdots}$$

Rubi [A] time = 0.30, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {864, 874, 205}

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{g^2\sqrt{d+ex}}+\frac{2(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[
d + e*x]) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x
)^(3/2)) + (2*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\int}{e^2g}$$

$$=-\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+\right.}{3g(d+ex)^{3/2}}$$

$$=-\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+\right.}{3g(d+ex)^{3/2}}$$

$$=-\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+\right.}{3g(d+ex)^{3/2}}$$

Mathematica [**A**] time = 0.26, size = 132, normalized size = 0.74

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\left(\sqrt{g}\sqrt{ae+cdx}\,(4aeg+cd(gx-3f))+3(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{3g^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(4*a*e*g + c*
d*(-3*f + g*x)) + 3*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]
)/Sqrt[c*d*f - a*e*g]]))/(3*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [**A**] time = 7.98, size = 151, normalized size = 0.84

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(\frac{2(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{g^{5/2}}+\frac{2\left(-3cdf\sqrt{ae+cdx}+g(ae+cdx)^{3/2}+3aeg\sqrt{ae+cdx}\right)}{3g^2}\right)}{(d+ex)^{3/2}(ae+cdx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*((2*(-3*c*d*f*Sqrt[a*e + c*d*x] + 3*a*e*g*
Sqrt[a*e + c*d*x] + g*(a*e + c*d*x)^(3/2)))/(3*g^2) + (2*(c*d*f - a*e*g)^(3
/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/g^(5/2)))/((a*
e + c*d*x)^(3/2)*(d + e*x)^(3/2))
```

fricas [**A**] time = 0.44, size = 408, normalized size = 2.28

$$\left[-\frac{3\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\,g\sqrt{-\frac{cdf-aeg}{g}}-\left(cdef-\left(cd^2+2\,ae^2\right)g\right)x}{egx^2+df+(ef+dg)x}\right)-2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdgx-3\,cdf+4\,aeg\right)\sqrt{ex+d}}{3\left(eg^2x+dg^2\right)},-\frac{2\left(3\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{\frac{cdf-aeg}{g}}\arctan\left(\frac{\sqrt{ex+d}\sqrt{\frac{cdf-aeg}{g}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\right)-\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdgx-3\,cdf+4\,aeg\right)\sqrt{ex+d}\right)}{3\left(eg^2x+dg^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="fricas")
```

```
[Out] [-1/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-(c*d*f - a*e*g)/
g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^
2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/(e*g^
2*x + d*g^2), -2/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt((c*d
*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/(e*g^2*x + d*g^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 4.51Unable to transpose Error: Bad Argum
ent Value
```

maple [A] time = 0.02, size = 263, normalized size = 1.47

$$-\frac{2\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\left(3a^{2}e^{2}g^{2}\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-6acdefg\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3c^{2}d^{2}f^{2}\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,cdgx-4\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,aeg+3\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,cdf\right)}{3\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{\left(aeg-cdf\right)g}\,g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x)
```

```
[Out] -2/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*a^2*e^2*g^2-6*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2)*g)*a*c*d*e*f*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2)*g)*c^2*d^2*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*g*x-4*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c
*d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/((a*e*g-c*d*f)*g
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^{2} + ade + \left(cd^{2} + ae^{2}\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{\left(f+g\,x\right)\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/2
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)(ae+cdx))^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}(f+gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*(f + g*x)), x)
```

3.462 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx$

Optimal. Leaf size=178

$$-\frac{3cd\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade}}{g^2\sqrt{d+ex}}$$

Rubi [A] time = 0.25, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 864, 874, 205}

$$-\frac{3cd\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^2),x]
```

```
[Out] (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]) - (
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f + g*x))
- (3*c*d*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
```

```
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)} + \frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)}\,dx}{2g}$$

$$= \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)} -$$

$$= \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)} -$$

$$= \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)} -$$

Mathematica [C] time = 0.05, size = 75, normalized size = 0.42

$$\frac{2cd((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^2),x]
```

```
[Out] (2*c*d*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (g*(a
*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^2*(d + e*x)^(5/2))
```

IntegrateAlgebraic [A] time = 103.69, size = 159, normalized size = 0.89

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(\frac{cd\sqrt{ae+cdx}\,(2g(ae+cdx)-3aeg+3cdf)}{g^2(g(ae+cdx)-aeg+cdf)} - \frac{3cd\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{g^{5/2}}\right)}{(d+ex)^{3/2}(ae+cdx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^2),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*((c*d*Sqrt[a*e + c*d*x]*(3*c*d*f - 3*a*e*g
 + 2*g*(a*e + c*d*x)))/(g^2*(c*d*f - a*e*g + g*(a*e + c*d*x))) - (3*c*d*Sqr
t[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/g
^(5/2)))/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))
```

fricas [A] time = 0.52, size = 444, normalized size = 2.49

$$\left[\frac{3\left(cdegx^2+cd^2f+\left(cdef+cd^2g\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\,g\sqrt{-\frac{cdf-aeg}{g}}-\left(cdef-\left(cd^2+2\,ae^2\right)g\right)x}{egx^2+df+(ef+dg)x}\right)+2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdgx+3\,cdf-aeg\right)\sqrt{ex+d}}{2\left(eg^3x^2+dfg^2+\left(efg^2+dg^3\right)x\right)}, \frac{3\left(cdegx^2+cd^2f+\left(cdef+cd^2g\right)x\right)\sqrt{\frac{cdf-aeg}{g}}\arctan\left(\frac{\sqrt{ex+d}\sqrt{\frac{cdf-aeg}{g}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\right)+\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdgx+3\,cdf-aeg\right)\sqrt{ex+d}}{eg^3x^2+dfg^2+\left(efg^2+dg^3\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="fricas")
```

```
[Out] [1/2*(3*(c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-(c*d*f - a*e*
g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c
*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d))/(e
*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x), (3*(c*d*e*g*x^2 + c*d^2*f + (c*d
*e*f + c*d^2*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqrt((c*d*f
 - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d))/(
e*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 306, normalized size = 1.72

$$\frac{\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\left(-3acde\,g^{2}x\,\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3c^{2}d^{2}fgx\,\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-3acdefg\,\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3c^{2}d^{2}f^{2}\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+2\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,cdgx-\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,aeg+3\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,cdf\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(gx+f\right)\sqrt{\left(aeg-cdf\right)g}\,g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e*x+d)^(1/2)*(-3*arctanh((c*d*x+a*
e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*a*c*d*e*g^2+3*arctanh((c*d*x+a*e)^(1/
2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^2*d^2*f*g-3*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*a*c*d*e*f*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2)*g)*c^2*d^2*f^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d
*g*x-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^(1
/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)/((a*e*g-c*d*f)*g
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**2,x)
```

```
[Out] Timed out
```

3.463 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx$

Optimal. Leaf size=195

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

Rubi [A] time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {862, 874, 205}

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^3),x]
```

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]*(
f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*g*(d + e*x)^(3
/2)*(f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(5/2)*Sqrt[c*d*f
 - a*e*g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2} + \frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^2}dx}{4g}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2} +$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2} +$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2} +$$

Mathematica [A] time = 0.32, size = 135, normalized size = 0.69

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}\sqrt{cdf-aeg}} - \frac{\sqrt{g}\,(2aeg+cd(3f+5gx))}{(f+gx)^2}\right)}{4g^{5/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^3),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(2*a*e*g + c*d*(3*f + 5*g*x)))/(
f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*
e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x])))/(4*g^(5/2)*Sqrt[d + e*x])
```

IntegrateAlgebraic [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^3),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.45, size = 840, normalized size = 4.31

$$\left[-\frac{3\left(c^2d^2eg^2x^3+c^2d^3f^2+\left(2c^2d^2efg+c^2d^3g^2\right)x^2+\left(c^2d^2ef^2+2c^2d^3fg\right)x\right)\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2adeg-\left(cdef-\left(cd^2+2ae^2\right)g\right)x-2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdfg+aeg^2}\sqrt{ex+d}}{egx^2+df+(ef+dg)x}\right)+2\left(3c^2d^2f^2g-acdefg^2-2a^2e^2g^3+5\left(c^2d^2fg^2-acdeg^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{8\left(cd^2f^3g^3-adef^2g^4+\left(cdefg^5-ae^2g^6\right)x^3+\left(2cdef^2g^4-adeg^6+\left(cd^2-2ae^2\right)fg^5\right)x^2+\left(cdef^3g^3-2adefg^5+\left(2cd^2-ae^2\right)f^2g^4\right)x\right)}, -\frac{3\left(c^2d^2eg^2x^3+c^2d^3f^2+\left(2c^2d^2efg+c^2d^3g^2\right)x^2+\left(c^2d^2ef^2+2c^2d^3fg\right)x\right)\sqrt{cdfg-aeg^2}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{cdfg-aeg^2}\sqrt{ex+d}}{cdegx^2+adeg+\left(cd^2+ae^2\right)gx}\right)+\left(3c^2d^2f^2g-acdefg^2-2a^2e^2g^3+5\left(c^2d^2fg^2-acdeg^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{4\left(cd^2f^3g^3-adef^2g^4+\left(cdefg^5-ae^2g^6\right)x^3+\left(2cdef^2g^4-adeg^6+\left(cd^2-2ae^2\right)fg^5\right)x^2+\left(cdef^3g^3-2adefg^5+\left(2cd^2-ae^2\right)f^2g^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
+ d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*f^2*g - a*c*d*e*f*g^2
```

```
 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^3*g^3 - a*d*e*f^2*g^4 + (c*d
*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 - a*d*e*g^6 + (c*d^2 - 2*a*e^2
)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5 + (2*c*d^2 - a*e^2)*f^2*g^4)*
x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g
^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan
(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e
*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^2*d^2*f^2*g -
 a*c*d*e*f*g^2 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^3*g^3 - a*d*e*
f^2*g^4 + (c*d*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 - a*d*e*g^6 + (c
*d^2 - 2*a*e^2)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5 + (2*c*d^2 - a*
e^2)*f^2*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 276, normalized size = 1.42

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3c^2d^2g^2x^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+6c^2d^2fgx\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3c^2d^2f^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+5\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,cdgx+2\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,aeg+3\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,cdf\right)}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(gx+f\right)^2\sqrt{\left(aeg-cdf\right)g}\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^2*d^2*g^2+6*arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*x*c^2*d^2*f*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2)*g)*c^2*d^2*f^2+5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d
*g*x+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)^
2/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^3\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**3,x)
```

```
[Out] Timed out
```

3.464 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx$

Optimal. Leaf size=265

$$\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}-\frac{(}{}$$

Rubi [A] time = 0.35, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)}+\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^4),x]
```

```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]*(f
+ g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*(c
*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)/(3*g*(d + e*x)^(3/2)*(f + g*x)^3) + (c^3*d^3*ArcTan[(Sqrt[g]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*
x])])/(8*g^(5/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
```

```
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{(cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^3}\,dx}{2g}$$

$$=-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+$$

$$=-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)}$$

$$=-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)}$$

$$=-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.30

$$\frac{2c^3d^3((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^4),x]
```

```
[Out] (2*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^4*(d + e*x)^(5/2))
```

IntegrateAlgebraic [F] time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^4),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.47, size = 1434, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="fricas")
```

```
[Out] [1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^
3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d
^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
 d*g)*x)) - 2*(3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*e^2*f*g^3
 + 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(4*c^3*d^3*f
^2*g^2 - 11*a*c^2*d^2*e*f*g^3 + 7*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^5*g^3 - 2*a*c*d^2*e*f^4*g^
4 + a^2*d*e^2*f^3*g^5 + (c^2*d^2*e*f^2*g^6 - 2*a*c*d*e^2*f*g^7 + a^2*e^3*g^
8)*x^4 + (3*c^2*d^2*e*f^3*g^5 + a^2*d*e^2*g^8 + (c^2*d^3 - 6*a*c*d*e^2)*f^2
*g^6 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g^7)*x^3 + 3*(c^2*d^2*e*f^4*g^4 + a^2*d*
e^2*f*g^7 + (c^2*d^3 - 2*a*c*d*e^2)*f^3*g^5 - (2*a*c*d^2*e - a^2*e^3)*f^2*g
^6)*x^2 + (c^2*d^2*e*f^5*g^3 + 3*a^2*d*e^2*f^2*g^6 + (3*c^2*d^3 - 2*a*c*d*e
^2)*f^4*g^4 - (6*a*c*d^2*e - a^2*e^3)*f^3*g^5)*x), -1/24*(3*(c^3*d^3*e*g^3*
x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^
2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*
g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*
g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) +
 (3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*e^2*f*g^3 + 8*a^3*e^3*
g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(4*c^3*d^3*f^2*g^2 - 11*a
*c^2*d^2*e*f*g^3 + 7*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^5*g^3 - 2*a*c*d^2*e*f^4*g^4 + a^2*d*e^2
*f^3*g^5 + (c^2*d^2*e*f^2*g^6 - 2*a*c*d*e^2*f*g^7 + a^2*e^3*g^8)*x^4 + (3*c
^2*d^2*e*f^3*g^5 + a^2*d*e^2*g^8 + (c^2*d^3 - 6*a*c*d*e^2)*f^2*g^6 - (2*a*c
*d^2*e - 3*a^2*e^3)*f*g^7)*x^3 + 3*(c^2*d^2*e*f^4*g^4 + a^2*d*e^2*f*g^7 + (
c^2*d^3 - 2*a*c*d*e^2)*f^3*g^5 - (2*a*c*d^2*e - a^2*e^3)*f^2*g^6)*x^2 + (c^
2*d^2*e*f^5*g^3 + 3*a^2*d*e^2*f^2*g^6 + (3*c^2*d^3 - 2*a*c*d*e^2)*f^4*g^4 -
 (6*a*c*d^2*e - a^2*e^3)*f^3*g^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 453, normalized size = 1.71

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3c^3d^3g^3x^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+9c^3d^3f\,g^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+9c^3d^3f^2gx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+3c^3d^3f^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-3\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,c^2d^2g^2x^2-14\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,acdeg^2x+8\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,c^2d^2fgx-8\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,a^2e^2g^2+2\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,acdefg+3\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,c^2d^2f^2\right)}{24\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(aeg-cdf\right)\left(gx+f\right)^3\sqrt{\left(aeg-cdf\right)g}\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x)
```

```
[Out] 1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*g^3+9*arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*f*g^2+9*arctanh((c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2)*g)*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2
*d^2*g^2*x^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x+8*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x-8*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(
1/2)*a*c*d*e*f*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(
e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g^2/(g*x+f)^3/((a*e*g-c*d*f)*g
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^4\,\left(d + e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**4,x)
```

```
[Out] Timed out
```

3.465 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx$

Optimal. Leaf size=335

$$\frac{3c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}+\frac{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}$$

Rubi [A] time = 0.45, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}+\frac{3c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^5),x]
```

```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*Sqrt[d + e*x]*(f
+ g*x)^3) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^2*(
c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^3*d^3*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x))
 - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*g*(d + e*x)^(3/2)*(f +
g*x)^4) + (3*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(5/2)*(c*d*f - a*e*g)^(5
/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
```

```
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}+\frac{(3cd)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^4}\,dx}{8g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}+$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.24

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^5),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^5*(d + e*x)^(5/2))
```

IntegrateAlgebraic [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^5),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.48, size = 2238, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="fricas")
```

```
[Out] [-1/128*(3*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^
4*d^4*f^4*g - a*c^3*d^3*e*f^3*g^2 - 26*a^2*c^2*d^2*e^2*f^2*g^3 + 40*a^3*c*d
*e^3*f*g^4 - 16*a^4*e^4*g^5 - 3*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 - (11
*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*x^2 + (11*
c^4*d^4*f^3*g^2 - 55*a*c^3*d^3*e*f^2*g^3 + 68*a^2*c^2*d^2*e^2*f*g^4 - 24*a^
3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)
)/(c^3*d^4*f^7*g^3 - 3*a*c^2*d^3*e*f^6*g^4 + 3*a^2*c*d^2*e^2*f^5*g^5 - a^3*
d*e^3*f^4*g^6 + (c^3*d^3*e*f^3*g^7 - 3*a*c^2*d^2*e^2*f^2*g^8 + 3*a^2*c*d*e^
3*f*g^9 - a^3*e^4*g^10)*x^5 + (4*c^3*d^3*e*f^4*g^6 - a^3*d*e^3*g^10 + (c^3*
d^4 - 12*a*c^2*d^2*e^2)*f^3*g^7 - 3*(a*c^2*d^3*e - 4*a^2*c*d*e^3)*f^2*g^8 +
 (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^9)*x^4 + 2*(3*c^3*d^3*e*f^5*g^5 - 2*a^3*
d*e^3*f*g^9 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g^6 - 3*(2*a*c^2*d^3*e - 3*
a^2*c*d*e^3)*f^3*g^7 + 3*(2*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^8)*x^3 + 2*(2*c^
3*d^3*e*f^6*g^4 - 3*a^3*d*e^3*f^2*g^8 + 3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g
^5 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^6 + (9*a^2*c*d^2*e^2 - 2*a^3*e
^4)*f^3*g^7)*x^2 + (c^3*d^3*e*f^7*g^3 - 4*a^3*d*e^3*f^3*g^7 + (4*c^3*d^4 -
3*a*c^2*d^2*e^2)*f^6*g^4 - 3*(4*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^5 + (12*a^
2*c*d^2*e^2 - a^3*e^4)*f^4*g^6)*x), -1/64*(3*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f
^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4
*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*
e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*
x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^4*d^4*f^4*g - a*c^3*d^3*e*f^3*
g^2 - 26*a^2*c^2*d^2*e^2*f^2*g^3 + 40*a^3*c*d*e^3*f*g^4 - 16*a^4*e^4*g^5 -
3*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 - (11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^
3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*x^2 + (11*c^4*d^4*f^3*g^2 - 55*a*c^3*d^3
*e*f^2*g^3 + 68*a^2*c^2*d^2*e^2*f*g^4 - 24*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^7*g^3 - 3*a*c^2*d
^3*e*f^6*g^4 + 3*a^2*c*d^2*e^2*f^5*g^5 - a^3*d*e^3*f^4*g^6 + (c^3*d^3*e*f^3
*g^7 - 3*a*c^2*d^2*e^2*f^2*g^8 + 3*a^2*c*d*e^3*f*g^9 - a^3*e^4*g^10)*x^5 +
(4*c^3*d^3*e*f^4*g^6 - a^3*d*e^3*g^10 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^
7 - 3*(a*c^2*d^3*e - 4*a^2*c*d*e^3)*f^2*g^8 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)
*f*g^9)*x^4 + 2*(3*c^3*d^3*e*f^5*g^5 - 2*a^3*d*e^3*f*g^9 + (2*c^3*d^4 - 9*a
*c^2*d^2*e^2)*f^4*g^6 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^7 + 3*(2*a^
2*c*d^2*e^2 - a^3*e^4)*f^2*g^8)*x^3 + 2*(2*c^3*d^3*e*f^6*g^4 - 3*a^3*d*e^3*
f^2*g^8 + 3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g^5 - 3*(3*a*c^2*d^3*e - 2*a^2*
c*d*e^3)*f^4*g^6 + (9*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^7)*x^2 + (c^3*d^3*e*
f^7*g^3 - 4*a^3*d*e^3*f^3*g^7 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^4 - 3*(
4*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^5 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^6
)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 665, normalized size = 1.99

$$-\frac{\sqrt{c d e \,x^{2}+a \,e^{2} x +c \,d^{2} x +a d e}\, \left(3 c^{4} d^{4} g^{4} x^{4} \operatorname{arctanh}\left(\frac{\sqrt{c d x +a e}\, g}{\sqrt{\left(a e g -c d f \right) g}}\right)+12 c^{4} d^{4} f \,g^{3} x^{3} \operatorname{arctanh}\left(\frac{\sqrt{c d x +a e}\, g}{\sqrt{\left(a e g -c d f \right) g}}\right)+18 c^{4} d^{4} f^{2} g^{2} x^{2} \operatorname{arctanh}\left(\frac{\sqrt{c d x +a e}\, g}{\sqrt{\left(a e g -c d f \right) g}}\right)+12 c^{4} d^{4} f^{3} g x \operatorname{arctanh}\left(\frac{\sqrt{c d x +a e}\, g}{\sqrt{\left(a e g -c d f \right) g}}\right)+3 c^{4} d^{4} f^{4} \operatorname{arctanh}\left(\frac{\sqrt{c d x +a e}\, g}{\sqrt{\left(a e g -c d f \right) g}}\right)-3 \sqrt{c d x +a e}\, \sqrt{\left(a e g -c d f \right) g}\, c^{3} d^{3} g^{3} x^{3}+2 \sqrt{c d x +a e}\, \sqrt{\left(a e g -c d f \right) g}\, a \,c^{2} d^{2} e \,g^{3} x^{2}-11 \sqrt{c d x +a e}\, \sqrt{\left(a e g -c d f \right) g}\, c^{3} d^{3} f \,g^{2} x^{2}+24 \sqrt{c d x +a e}\, \sqrt{\left(a e g -c d f \right) g}\, a^{2} c d \,e^{2} g^{3} x -44 \sqrt{c d x +a e}\, \sqrt{\left(a e g -c d f \right) g}\, a \,c^{2} d^{2} e f \,g^{2} x +11 \sqrt{c d x +a e}\, \sqrt{\left(a e g -c d f \right) g}\, c^{3} d^{3} f^{2} g x +16 \sqrt{\left(a e g -c d f \right) g}\, \sqrt{c d x +a e}\, a^{3} e^{3} g^{3}-24 \sqrt{\left(a e g -c d f \right) g}\, \sqrt{c d x +a e}\, a^{2} c d \,e^{2} f \,g^{2}+2 \sqrt{\left(a e g -c d f \right) g}\, \sqrt{c d x +a e}\, a \,c^{2} d^{2} e \,f^{2} g +3 \sqrt{\left(a e g -c d f \right) g}\, \sqrt{c d x +a e}\, c^{3} d^{3} f^{3}\right)}{64 \sqrt{e x +d}\, \sqrt{\left(a e g -c d f \right) g}\, \left(g x +f \right)^{4} \left(a e g -c d f \right)^{2} \sqrt{c d x +a e}\, g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x)
```

```
[Out] -1/64*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^4*d^4*g^4+12*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^4*d^4*f*g^3+18*arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*x^2*c^4*d^4*f^2*g^2+12*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x*c^4*d^4*f^3*g-3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*
g)^(1/2)*c^3*d^3*g^3*x^3+3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
)*g)*c^4*d^4*f^4+2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*g^
3*x^2-11*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f*g^2*x^2+24*(c*
d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*g^3*x-44*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f*g^2*x+11*(c*d*x+a*e)^(1/2)*((a*e*g
-c*d*f)*g)^(1/2)*c^3*d^3*f^2*g*x+16*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)*a^3*e^3*g^3-24*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^2*f*g
^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+3*((a*e*g-
c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2)/(g*x+f)^4/g^2/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^{2} + ade + \left(cd^{2} + ae^{2}\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^5\,(d + e\,x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**5,x)
```

```
[Out] Timed out
```

3.466 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$

Optimal. Leaf size=405

$$\frac{3c^5d^5\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}}+\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{64g^2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg}$$

Rubi [A] time = 0.56, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{80g^2\sqrt{d+ex}\,(f+gx)^3(cdf-aeg)}+\frac{3c^5d^5\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^6),x]
```

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(40*g^2*Sqrt[d + e*x]*
(f + g*x)^4) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(80*g^
2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)
^2) + (3*c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^2*(c*d
*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)/(5*g*(d + e*x)^(3/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
e*x])])/(128*g^(5/2)*(c*d*f - a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^5}\,dx}{10g}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.20

$$\frac{2c^5d^5((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^6),x]
```

```
[Out] (2*c^5*d^5*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^6*(d + e*x)^(5/2))
```

IntegrateAlgebraic [F] time = 180.34, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^6),x]
```

[Out] $Aborted

fricas [B] time = 0.53, size = 3204, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="fricas")
```

```
[Out] [1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6
*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3
*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 +
 (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*
g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d)
)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f^
4*g^2 - 258*a^2*c^3*d^3*e^2*f^3*g^3 + 584*a^3*c^2*d^2*e^3*f^2*g^4 - 464*a^4
*c*d*e^4*f*g^5 + 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4
 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 -
 2*(64*c^5*d^5*f^3*g^3 - 87*a*c^4*d^4*e*f^2*g^4 + 27*a^2*c^3*d^3*e^2*f*g^5
- 4*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 268*a*c^4*d^4*e*f^3*
g^3 + 489*a^2*c^3*d^3*e^2*f^2*g^4 - 344*a^3*c^2*d^2*e^3*f*g^5 + 88*a^4*c*d*
e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4
*d^5*f^9*g^3 - 4*a*c^3*d^4*e*f^8*g^4 + 6*a^2*c^2*d^3*e^2*f^7*g^5 - 4*a^3*c*
d^2*e^3*f^6*g^6 + a^4*d*e^4*f^5*g^7 + (c^4*d^4*e*f^4*g^8 - 4*a*c^3*d^3*e^2*
f^3*g^9 + 6*a^2*c^2*d^2*e^3*f^2*g^10 - 4*a^3*c*d*e^4*f*g^11 + a^4*e^5*g^12)
*x^6 + (5*c^4*d^4*e*f^5*g^7 + a^4*d*e^4*g^12 + (c^4*d^5 - 20*a*c^3*d^3*e^2)
*f^4*g^8 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^9 + 2*(3*a^2*c^2*d^
3*e^2 - 10*a^3*c*d*e^4)*f^2*g^10 - (4*a^3*c*d^2*e^3 - 5*a^4*e^5)*f*g^11)*x^
5 + 5*(2*c^4*d^4*e*f^6*g^6 + a^4*d*e^4*f*g^11 + (c^4*d^5 - 8*a*c^3*d^3*e^2)
*f^5*g^7 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^8 + 2*(3*a^2*c^2*d^3*e
^2 - 4*a^3*c*d*e^4)*f^3*g^9 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^10)*x^4 +
 10*(c^4*d^4*e*f^7*g^5 + a^4*d*e^4*f^2*g^10 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f
^6*g^6 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^7 + 2*(3*a^2*c^2*d^3*e
^2 - 2*a^3*c*d*e^4)*f^4*g^8 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^9)*x^3 + 5*
(c^4*d^4*e*f^8*g^4 + 2*a^4*d*e^4*f^3*g^9 + 2*(c^4*d^5 - 2*a*c^3*d^3*e^2)*f^
7*g^5 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^6 + 4*(3*a^2*c^2*d^3*e^
2 - a^3*c*d*e^4)*f^5*g^7 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^8)*x^2 + (c^4*
d^4*e*f^9*g^3 + 5*a^4*d*e^4*f^4*g^8 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^8*g^4
 - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^5 + 2*(15*a^2*c^2*d^3*e^2 -
 2*a^3*c*d*e^4)*f^6*g^6 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^7)*x), -1/640*
(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6*g^5)*x^
5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3*g^2 + c
^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 + (c^5*d^
5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*
g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e
*f^4*g^2 - 258*a^2*c^3*d^3*e^2*f^3*g^3 + 584*a^3*c^2*d^2*e^3*f^2*g^4 - 464*
a^4*c*d*e^4*f*g^5 + 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*
x^4 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^
3 - 2*(64*c^5*d^5*f^3*g^3 - 87*a*c^4*d^4*e*f^2*g^4 + 27*a^2*c^3*d^3*e^2*f*g
^5 - 4*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 268*a*c^4*d^4*e*f
^3*g^3 + 489*a^2*c^3*d^3*e^2*f^2*g^4 - 344*a^3*c^2*d^2*e^3*f*g^5 + 88*a^4*c
*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
c^4*d^5*f^9*g^3 - 4*a*c^3*d^4*e*f^8*g^4 + 6*a^2*c^2*d^3*e^2*f^7*g^5 - 4*a^3
*c*d^2*e^3*f^6*g^6 + a^4*d*e^4*f^5*g^7 + (c^4*d^4*e*f^4*g^8 - 4*a*c^3*d^3*e
^2*f^3*g^9 + 6*a^2*c^2*d^2*e^3*f^2*g^10 - 4*a^3*c*d*e^4*f*g^11 + a^4*e^5*g^
12)*x^6 + (5*c^4*d^4*e*f^5*g^7 + a^4*d*e^4*g^12 + (c^4*d^5 - 20*a*c^3*d^3*e
^2)*f^4*g^8 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^9 + 2*(3*a^2*c^2
```

```
*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^10 - (4*a^3*c*d^2*e^3 - 5*a^4*e^5)*f*g^11)
*x^5 + 5*(2*c^4*d^4*e*f^6*g^6 + a^4*d*e^4*f*g^11 + (c^4*d^5 - 8*a*c^3*d^3*e
^2)*f^5*g^7 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^8 + 2*(3*a^2*c^2*d^
3*e^2 - 4*a^3*c*d*e^4)*f^3*g^9 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^10)*x^
4 + 10*(c^4*d^4*e*f^7*g^5 + a^4*d*e^4*f^2*g^10 + (c^4*d^5 - 4*a*c^3*d^3*e^2
)*f^6*g^6 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^7 + 2*(3*a^2*c^2*d^
3*e^2 - 2*a^3*c*d*e^4)*f^4*g^8 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^9)*x^3 +
 5*(c^4*d^4*e*f^8*g^4 + 2*a^4*d*e^4*f^3*g^9 + 2*(c^4*d^5 - 2*a*c^3*d^3*e^2)
*f^7*g^5 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^6 + 4*(3*a^2*c^2*d^3
*e^2 - a^3*c*d*e^4)*f^5*g^7 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^8)*x^2 + (c
^4*d^4*e*f^9*g^3 + 5*a^4*d*e^4*f^4*g^8 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^8*
g^4 - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^5 + 2*(15*a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^6*g^6 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^7)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 955, normalized size = 2.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x)
```

```
[Out] 1/640*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^5*c^5*d^5*g^5+75*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^5*d^5*f*g^4+150*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^5*d^5*f^2*g^3+150*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^5*d^5*f^3*g^2-15*x^4*c^4*d^4*g^4*((a*e*g-c
*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+75*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2)*g)*x*c^5*d^5*f^4*g+10*x^3*a*c^3*d^3*e*g^4*((a*e*g-c*d*f)*g)^(1/2
)*(c*d*x+a*e)^(1/2)-70*x^3*c^4*d^4*f*g^3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e
)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*c^5*d^5*f^5
-8*x^2*a^2*c^2*d^2*e^2*g^4*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+46*x^2
*a*c^3*d^3*e*f*g^3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)-128*x^2*c^4*d^
4*f^2*g^2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)-176*x*a^3*c*d*e^3*g^4*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+512*x*a^2*c^2*d^2*e^2*f*g^3*((a*e*
g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)-466*x*a*c^3*d^3*e*f^2*g^2*((a*e*g-c*d*f
)*g)^(1/2)*(c*d*x+a*e)^(1/2)+70*x*c^4*d^4*f^3*g*((a*e*g-c*d*f)*g)^(1/2)*(c*
d*x+a*e)^(1/2)-128*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^4*e^4*g^4+33
6*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c*d*e^3*f*g^3-248*((a*e*g-c
*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*((a*e*g-c*d*f)*
g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^3*d^3*e*f^3*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*
d*x+a*e)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^5
/g^2/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(f+g\,x\right)}^6\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(3
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**6,x)
```

```
[Out] Timed out
```

3.467 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^3\left(2ae^2g-cd(9ef-7dg)\right)}{45045c^5d^5e(d+ex)^{7/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/}}{6435c^4d^4e(d+ex)^{5/2}}$$

Rubi [A] time = 0.62, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{195c^2d^2(d+ex)^{7/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{715c^3d^3(d+ex)^{7/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^3}{6435c^4d^4e(d+ex)^{5/2}}-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^3\left(2ae^2g-cd(9ef-7dg)\right)}{45045c^5d^5e(d+ex)^{7/2}}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(7/2))/(45045*c^5*d^5*e*(d + e*x)^(7/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(6435*c^4*d^
4*e*(d + e*x)^(5/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(715*c^3*d^3*(d + e*x)^(7/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(195*c^2*d
^2*(d + e*x)^(7/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(7/2))/(15*c*d*(d + e*x)^(7/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}} + \frac{(8(cdf-aeg}{}$$

$$= \frac{16(cdf-aeg)(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{195c^2d^2(d+ex)^{7/2}} +$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{715c^3d^3(d+ex)^{7/2}} +$$

$$= \frac{128g(cdf-aeg)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6435c^4d^4e(d+ex)^{5/2}} + \frac{32(cd}{}$$

$$= \frac{128(cdf-aeg)^3 \left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+c\right.}{45045c^4d^4(d+ex)^{7/2}}$$

Mathematica [A] time = 0.21, size = 205, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(128a^4e^4g^4-64a^3cde^3g^3(15f+7gx)+48a^2c^2d^2e^2g^2\left(65f^2+70fgx+21g^2x^2\right)-8ac^3d^3eg\left(715f^3+1365f^2gx+945fg^2x^2+231g^3x^3\right)+c^4d^4\left(6435f^4+20020f^3gx+24570f^2g^2x^2+13860fg^3x^3+3003g^4x^4\right)\right)}{45045c^5d^5\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^4*g^4 - 64*a^3*
c*d*e^3*g^3*(15*f + 7*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(65*f^2 + 70*f*g*x + 21
*g^2*x^2) - 8*a*c^3*d^3*e*g*(715*f^3 + 1365*f^2*g*x + 945*f*g^2*x^2 + 231*g
^3*x^3) + c^4*d^4*(6435*f^4 + 20020*f^3*g*x + 24570*f^2*g^2*x^2 + 13860*f*g
^3*x^3 + 3003*g^4*x^4)))/(45045*c^5*d^5*Sqrt[d + e*x])
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2))/(d + e*x)^(5/2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.42, size = 567, normalized size = 1.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^7*d^7*g^4*x^7 + 6435*a^3*c^4*d^4*e^3*f^4 - 5720*a^4*c^3*d^3
*e^4*f^3*g + 3120*a^5*c^2*d^2*e^5*f^2*g^2 - 960*a^6*c*d*e^6*f*g^3 + 128*a^7
*e^7*g^4 + 231*(60*c^7*d^7*f*g^3 + 31*a*c^6*d^6*e*g^4)*x^6 + 63*(390*c^7*d^
7*f^2*g^2 + 540*a*c^6*d^6*e*f*g^3 + 71*a^2*c^5*d^5*e^2*g^4)*x^5 + 35*(572*c
^7*d^7*f^3*g + 1794*a*c^6*d^6*e*f^2*g^2 + 636*a^2*c^5*d^5*e^2*f*g^3 + a^3*c
^4*d^4*e^3*g^4)*x^4 + 5*(1287*c^7*d^7*f^4 + 10868*a*c^6*d^6*e*f^3*g + 8814*
```

```
a^2*c^5*d^5*e^2*f^2*g^2 + 60*a^3*c^4*d^4*e^3*f*g^3 - 8*a^4*c^3*d^3*e^4*g^4)
*x^3 + 3*(6435*a*c^6*d^6*e*f^4 + 14300*a^2*c^5*d^5*e^2*f^3*g + 390*a^3*c^4*
d^4*e^3*f^2*g^2 - 120*a^4*c^3*d^3*e^4*f*g^3 + 16*a^5*c^2*d^2*e^5*g^4)*x^2 +
 (19305*a^2*c^5*d^5*e^2*f^4 + 2860*a^3*c^4*d^4*e^3*f^3*g - 1560*a^4*c^3*d^3
*e^4*f^2*g^2 + 480*a^5*c^2*d^2*e^5*f*g^3 - 64*a^6*c*d*e^6*g^4)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 17.12Done
```

maple [A] time = 0.01, size = 283, normalized size = 0.84

$$\frac{2 \left(c d x+a e\right) \left(3003 g^{4} x^{4} c^{4} d^{4}-1848 a \,c^{3} d^{3} e \,g^{4} x^{3}+13860 c^{4} d^{4} f \,g^{3} x^{3}+1008 a^{2} c^{2} d^{2} e^{2} g^{4} x^{2}-7560 a \,c^{3} d^{3} e f \,g^{3} x^{2}+24570 c^{4} d^{4} f^{2} g^{2} x^{2}-448 a^{3} c d \,e^{3} g^{4} x+3360 a^{2} c^{2} d^{2} e^{2} f \,g^{3} x-10920 a \,c^{3} d^{3} e \,f^{2} g^{2} x+20020 c^{4} d^{4} f^{3} g x+128 a^{4} e^{4} g^{4}-960 a^{3} c d \,e^{3} f \,g^{3}+3120 a^{2} c^{2} d^{2} e^{2} f^{2} g^{2}-5720 a \,c^{3} d^{3} e \,f^{3} g+6435 f^{4} c^{4} d^{4}\right) \left(c d e \,x^{2}+a \,e^{2} x+c \,d^{2} x+a d e\right)^{\frac{5}{2}}}{45045 \left(e x+d\right)^{\frac{5}{2}} c^{5} d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/45045*(c*d*x+a*e)*(3003*c^4*d^4*g^4*x^4-1848*a*c^3*d^3*e*g^4*x^3+13860*c^
4*d^4*f*g^3*x^3+1008*a^2*c^2*d^2*e^2*g^4*x^2-7560*a*c^3*d^3*e*f*g^3*x^2+245
70*c^4*d^4*f^2*g^2*x^2-448*a^3*c*d*e^3*g^4*x+3360*a^2*c^2*d^2*e^2*f*g^3*x-1
0920*a*c^3*d^3*e*f^2*g^2*x+20020*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-960*a^3*c*
d*e^3*f*g^3+3120*a^2*c^2*d^2*e^2*f^2*g^2-5720*a*c^3*d^3*e*f^3*g+6435*c^4*d^
4*f^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c^5/d^5/(e*x+d)^(5/2)
```

maxima [A] time = 0.76, size = 498, normalized size = 1.48

$$\frac{2 \left(c^{3} d^{3} x^{3}+3 a c^{2} d^{2} e x^{2}+3 a^{2} c d e^{2} x+a^{3} e^{3}\right) \sqrt{c d x+a e}\, f^{4}}{7 c d}+\frac{8 \left(7 c^{4} d^{4} x^{4}+19 a c^{3} d^{3} e x^{3}+15 a^{2} c^{2} d^{2} e^{2} x^{2}+a^{3} c d e^{3} x-2 a^{4} e^{4}\right) \sqrt{c d x+a e}\, f^{3} g}{63 c^{2} d^{2}}+\frac{4 \left(63 c^{5} d^{5} x^{5}+161 a c^{4} d^{4} e x^{4}+113 a^{2} c^{3} d^{3} e^{2} x^{3}+3 a^{3} c^{2} d^{2} e^{3} x^{2}-4 a^{4} c d e^{4} x+8 a^{5} e^{5}\right) \sqrt{c d x+a e}\, f^{2} g^{2}}{231 c^{3} d^{3}}+\frac{8 \left(231 c^{6} d^{6} x^{6}+567 a c^{5} d^{5} e x^{5}+371 a^{2} c^{4} d^{4} e^{2} x^{4}+5 a^{3} c^{3} d^{3} e^{3} x^{3}-6 a^{4} c^{2} d^{2} e^{4} x^{2}+8 a^{5} c d e^{5} x-16 a^{6} e^{6}\right) \sqrt{c d x+a e}\, f g^{3}}{3003 c^{4} d^{4}}+\frac{2 \left(3003 c^{7} d^{7} x^{7}+7161 a c^{6} d^{6} e x^{6}+4473 a^{2} c^{5} d^{5} e^{2} x^{5}+35 a^{3} c^{4} d^{4} e^{3} x^{4}-40 a^{4} c^{3} d^{3} e^{4} x^{3}+48 a^{5} c^{2} d^{2} e^{5} x^{2}-64 a^{6} c d e^{6} x+128 a^{7} e^{7}\right) \sqrt{c d x+a e}\, g^{4}}{45045 c^{5} d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^4/(c*d) + 8/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2)
+ 4/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3
*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*f^2*g
^2/(c^3*d^3) + 8/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^2*c^4*
d^4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*c*d*e^5
*x - 16*a^6*e^6)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/45045*(3003*c^7*d^7*
x^7 + 7161*a*c^6*d^6*e*x^6 + 4473*a^2*c^5*d^5*e^2*x^5 + 35*a^3*c^4*d^4*e^3*
x^4 - 40*a^4*c^3*d^3*e^4*x^3 + 48*a^5*c^2*d^2*e^5*x^2 - 64*a^6*c*d*e^6*x +
128*a^7*e^7)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

mupad [B] time = 4.09, size = 523, normalized size = 1.56

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^5\,\left(71\,a^2\,e^2\,g^2+540\,a\,c\,d\,e\,f\,g+390\,c^2\,d^2\,f^2\right)}{715}+\frac{256\,a^7\,e^7\,g^4-1920\,a^6\,c\,d\,e^6\,f\,g^3+6240\,a^5\,c^2\,d^2\,e^5\,f^2\,g^2-11440\,a^4\,c^3\,d^3\,e^4\,f^3\,g+12870\,a^3\,c^4\,d^4\,e^3\,f^4}{45045\,c^5\,d^5}+\frac{x^3\,\left(-80\,a^4\,c^3\,d^3\,e^4\,g^4+600\,a^3\,c^4\,d^4\,e^3\,f\,g^3+88140\,a^2\,c^5\,d^5\,e^2\,f^2\,g^2+108680\,a\,c^6\,d^6\,e\,f^3\,g+12870\,c^7\,d^7\,f^4\right)}{45045\,c^5\,d^5}+\frac{2\,c^2\,d^2\,g^4\,x^7}{15}+\frac{2\,c\,d\,g^3\,x^6\,\left(31\,a\,e\,g+60\,c\,d\,f\right)}{195}+\frac{2\,g\,x^4\,\left(a^3\,e^3\,g^3+636\,a^2\,c\,d\,e^2\,f\,g^2+1794\,a\,c^2\,d^2\,e\,f^2\,g+572\,c^3\,d^3\,f^3\right)}{1287\,c\,d}+\frac{2\,a^2\,e^2\,x\,\left(-64\,a^4\,e^4\,g^4+480\,a^3\,c\,d\,e^3\,f\,g^3-1560\,a^2\,c^2\,d^2\,e^2\,f^2\,g^2+2860\,a\,c^3\,d^3\,e\,f^3\,g+19305\,c^4\,d^4\,f^4\right)}{45045\,c^4\,d^4}+\frac{2\,a\,e\,x^2\,\left(16\,a^4\,e^4\,g^4-120\,a^3\,c\,d\,e^3\,f\,g^3+390\,a^2\,c^2\,d^2\,e^2\,f^2\,g^2+14300\,a\,c^3\,d^3\,e\,f^3\,g+6435\,c^4\,d^4\,f^4\right)}{15015\,c^3\,d^3}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^5*(71*a^2*e^2*g^2
+ 390*c^2*d^2*f^2 + 540*a*c*d*e*f*g))/715 + (256*a^7*e^7*g^4 + 12870*a^3*c^
4*d^4*e^3*f^4 - 11440*a^4*c^3*d^3*e^4*f^3*g - 1920*a^6*c*d*e^6*f*g^3 + 6240
*a^5*c^2*d^2*e^5*f^2*g^2)/(45045*c^5*d^5) + (x^3*(12870*c^7*d^7*f^4 - 80*a^
4*c^3*d^3*e^4*g^4 + 600*a^3*c^4*d^4*e^3*f*g^3 + 108680*a*c^6*d^6*e*f^3*g +
88140*a^2*c^5*d^5*e^2*f^2*g^2))/(45045*c^5*d^5) + (2*c^2*d^2*g^4*x^7)/15 +
(2*c*d*g^3*x^6*(31*a*e*g + 60*c*d*f))/195 + (2*g*x^4*(a^3*e^3*g^3 + 572*c^3
*d^3*f^3 + 1794*a*c^2*d^2*e*f^2*g + 636*a^2*c*d*e^2*f*g^2))/(1287*c*d) + (2
*a^2*e^2*x*(19305*c^4*d^4*f^4 - 64*a^4*e^4*g^4 + 2860*a*c^3*d^3*e*f^3*g + 4
80*a^3*c*d*e^3*f*g^3 - 1560*a^2*c^2*d^2*e^2*f^2*g^2))/(45045*c^4*d^4) + (2*
a*e*x^2*(16*a^4*e^4*g^4 + 6435*c^4*d^4*f^4 + 14300*a*c^3*d^3*e*f^3*g - 120*
a^3*c*d*e^3*f*g^3 + 390*a^2*c^2*d^2*e^2*f^2*g^2))/(15015*c^3*d^3)))/(d + e*
x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

3.468 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2\left(2ae^2g-cd(9ef-7dg)\right)}{3003c^4d^4e(d+ex)^{7/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(}{429c^3d^3e(d+ex)^{5/2}}$$

Rubi [A] time = 0.40, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{12(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{143c^2d^2(d+ex)^{7/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{429c^3d^3e(d+ex)^{5/2}}-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2\left(2ae^2g-cd(9ef-7dg)\right)}{3003c^4d^4e(d+ex)^{7/2}}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(3003*c^4*d^4*e*(d + e*x)^(7/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*c^3*d^3*e*
(d + e*x)^(5/2)) + (12*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(143*c^2*d^2*(d + e*x)^(7/2)) + (2*(f + g*x)^3*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*(d + e*x)^(7/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}} + \frac{(6(cdf-aeg}{}$$
$$= \frac{12(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{7/2}} +$$
$$= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429c^3d^3e(d+ex)^{5/2}} + \frac{12(cdf}{}$$
$$= \frac{16(cdf-aeg)^2\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{3003c^3d^3(d+ex)^{7/2}}$$

Mathematica [A] time = 0.16, size = 147, normalized size = 0.55

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(-16a^3e^3g^3+8a^2cde^2g^2(13f+7gx)-2ac^2d^2eg\left(143f^2+182fgx+63g^2x^2\right)+c^3d^3\left(429f^3+1001f^2gx+819fg^2x^2+231g^3x^3\right)\right)}{3003c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c
*d*e^2*g^2*(13*f + 7*g*x) - 2*a*c^2*d^2*e*g*(143*f^2 + 182*f*g*x + 63*g^2*x
^2) + c^3*d^3*(429*f^3 + 1001*f^2*g*x + 819*f*g^2*x^2 + 231*g^3*x^3)))/(300
3*c^4*d^4*Sqrt[d + e*x])
```

IntegrateAlgebraic [F] time = 180.51, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2))/(d + e*x)^(5/2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.40, size = 416, normalized size = 1.55

$$\frac{2\left(231c^6d^6g^3x^6+429a^3c^3d^3e^3f^3-286a^4c^2d^2e^4f^2g+104a^5cde^5fg^2-16a^6e^6g^3+63\left(13c^6d^6fg^2+9ac^5d^5eg^3\right)x^5+7\left(143c^6d^6f^2g+299ac^5d^5efg^2+53a^2c^4d^4e^2g^3\right)x^4+\left(429c^6d^6f^3+2717ac^5d^5ef^2g+1469a^2c^4d^4e^2fg^2+5a^3c^3d^3e^3g^3\right)x^3+3\left(429ac^5d^5ef^3+715a^2c^4d^4e^2f^2g+13a^3c^3d^3e^3fg^2-2a^4c^2d^2e^4g^3\right)x^2+\left(1287a^2c^4d^4e^2f^3+143a^3c^3d^3e^3f^2g-52a^4c^2d^2e^4fg^2+8a^5cde^5g^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{3003\left(c^4d^4ex+c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3003*(231*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 286*a^4*c^2*d^2*e^4
*f^2*g + 104*a^5*c*d*e^5*f*g^2 - 16*a^6*e^6*g^3 + 63*(13*c^6*d^6*f*g^2 + 9*
a*c^5*d^5*e*g^3)*x^5 + 7*(143*c^6*d^6*f^2*g + 299*a*c^5*d^5*e*f*g^2 + 53*a^
2*c^4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 + 2717*a*c^5*d^5*e*f^2*g + 1469*a
^2*c^4*d^4*e^2*f*g^2 + 5*a^3*c^3*d^3*e^3*g^3)*x^3 + 3*(429*a*c^5*d^5*e*f^3
+ 715*a^2*c^4*d^4*e^2*f^2*g + 13*a^3*c^3*d^3*e^3*f*g^2 - 2*a^4*c^2*d^2*e^4*
g^3)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 + 143*a^3*c^3*d^3*e^3*f^2*g - 52*a^4*c
^2*d^2*e^4*f*g^2 + 8*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 12.66Done
```

maple [A] time = 0.01, size = 188, normalized size = 0.70

$$-\frac{2\left(cdx+ae\right)\left(-231g^{3}x^{3}c^{3}d^{3}+126ac^{2}d^{2}eg^{3}x^{2}-819c^{3}d^{3}fg^{2}x^{2}-56a^{2}cde^{2}g^{3}x+364ac^{2}d^{2}efg^{2}x-1001c^{3}d^{3}f^{2}gx+16a^{3}e^{3}g^{3}-104a^{2}cde^{2}fg^{2}+286ac^{2}d^{2}ef^{2}g-429f^{3}c^{3}d^{3}\right)\left(cdex^{2}+ae^{2}x+cd^{2}x+ade\right)^{\frac{5}{2}}}{3003\left(ex+d\right)^{\frac{5}{2}}c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3003*(c*d*x+a*e)*(-231*c^3*d^3*g^3*x^3+126*a*c^2*d^2*e*g^3*x^2-819*c^3*d
^3*f*g^2*x^2-56*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2*x-1001*c^3*d^3*f^2*
g*x+16*a^3*e^3*g^3-104*a^2*c*d*e^2*f*g^2+286*a*c^2*d^2*e*f^2*g-429*c^3*d^3*
f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c^4/d^4/(e*x+d)^(5/2)
```

maxima [A] time = 0.70, size = 362, normalized size = 1.35

$$\frac{2\left(c^{3}d^{3}x^{3}+3\,ac^{2}d^{2}ex^{2}+3\,a^{2}cde^{2}x+a^{3}e^{3}\right)\sqrt{cdx+ae}\,f^{3}}{7\,cd}+\frac{2\left(7\,c^{4}d^{4}x^{4}+19\,ac^{3}d^{3}ex^{3}+15\,a^{2}c^{2}d^{2}e^{2}x^{2}+a^{3}cde^{3}x-2\,a^{4}e^{4}\right)\sqrt{cdx+ae}\,f^{2}g}{21\,c^{2}d^{2}}+\frac{2\left(63\,c^{5}d^{5}x^{5}+161\,ac^{4}d^{4}ex^{4}+113\,a^{2}c^{3}d^{3}e^{2}x^{3}+3\,a^{3}c^{2}d^{2}e^{3}x^{2}-4\,a^{4}cde^{4}x+8\,a^{5}e^{5}\right)\sqrt{cdx+ae}\,fg^{2}}{231\,c^{3}d^{3}}+\frac{2\left(231\,c^{6}d^{6}x^{6}+567\,ac^{5}d^{5}ex^{5}+371\,a^{2}c^{4}d^{4}e^{2}x^{4}+5\,a^{3}c^{3}d^{3}e^{3}x^{3}-6\,a^{4}c^{2}d^{2}e^{4}x^{2}+8\,a^{5}cde^{5}x-16\,a^{6}e^{6}\right)\sqrt{cdx+ae}\,g^{3}}{3003\,c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^3/(c*d) + 2/21*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2)
+ 2/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3
*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*f*g^2
/(c^3*d^3) + 2/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^2*c^4*d^
4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*c*d*e^5*x
 - 16*a^6*e^6)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

mupad [B] time = 3.81, size = 379, normalized size = 1.41

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g\,x^4\,\left(53\,a^2\,e^2\,g^2+299\,a\,c\,d\,e\,f\,g+143\,c^2\,d^2\,f^2\right)}{429}-\frac{32\,a^6\,e^6\,g^3-208\,a^5\,c\,d\,e^5\,f\,g^2+572\,a^4\,c^2\,d^2\,e^4\,f^2\,g-858\,a^3\,c^3\,d^3\,e^3\,f^3}{3003\,c^4\,d^4}+\frac{x^3\,\left(10\,a^3\,c^3\,d^3\,e^3\,g^3+2938\,a^2\,c^4\,d^4\,e^2\,f\,g^2+5434\,a\,c^5\,d^5\,e\,f^2\,g+858\,c^6\,d^6\,f^3\right)}{3003\,c^4\,d^4}+\frac{2\,c^2\,d^2\,g^3\,x^6}{13}+\frac{6\,c\,d\,g^2\,x^5\,\left(9\,a\,e\,g+13\,c\,d\,f\right)}{143}+\frac{2\,a^2\,e^2\,x\,\left(8\,a^3\,e^3\,g^3-52\,a^2\,c\,d\,e^2\,f\,g^2+143\,a\,c^2\,d^2\,e\,f^2\,g+1287\,c^3\,d^3\,f^3\right)}{3003\,c^3\,d^3}+\frac{2\,a\,e\,x^2\,\left(-2\,a^3\,e^3\,g^3+13\,a^2\,c\,d\,e^2\,f\,g^2+715\,a\,c^2\,d^2\,e\,f^2\,g+429\,c^3\,d^3\,f^3\right)}{1001\,c^2\,d^2}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g*x^4*(53*a^2*e^2*g^2 +
143*c^2*d^2*f^2 + 299*a*c*d*e*f*g))/429 - (32*a^6*e^6*g^3 - 858*a^3*c^3*d^3
*e^3*f^3 + 572*a^4*c^2*d^2*e^4*f^2*g - 208*a^5*c*d*e^5*f*g^2)/(3003*c^4*d^4
) + (x^3*(858*c^6*d^6*f^3 + 10*a^3*c^3*d^3*e^3*g^3 + 2938*a^2*c^4*d^4*e^2*f
*g^2 + 5434*a*c^5*d^5*e*f^2*g))/(3003*c^4*d^4) + (2*c^2*d^2*g^3*x^6)/13 + (
6*c*d*g^2*x^5*(9*a*e*g + 13*c*d*f))/143 + (2*a^2*e^2*x*(8*a^3*e^3*g^3 + 128
7*c^3*d^3*f^3 + 143*a*c^2*d^2*e*f^2*g - 52*a^2*c*d*e^2*f*g^2))/(3003*c^3*d^
```

```
3) + (2*a*e*x^2*(429*c^3*d^3*f^3 - 2*a^3*e^3*g^3 + 715*a*c^2*d^2*e*f^2*g +
13*a^2*c*d*e^2*f*g^2))/(1001*c^2*d^2)))/(d + e*x)^(1/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

3.469 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)\left(2ae^2g-cd(9ef-7dg)\right)}{693c^3d^3e(d+ex)^{7/2}}+\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf}{99c^2d^2e(d+ex)^{5/2}}$$

Rubi [A] time = 0.23, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{99c^2d^2e(d+ex)^{5/2}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)\left(2ae^2g-cd(9ef-7dg)\right)}{693c^3d^3e(d+ex)^{7/2}}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(7/2))/(693*c^3*d^3*e*(d + e*x)^(7/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(99*c^2*d^2*e*(d + e*x
)^(5/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(1
1*c*d*(d + e*x)^(7/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}} + \frac{(4(cdf-aeg}{}$$

$$= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99c^2d^2e(d+ex)^{5/2}} + \frac{2(f+gx)}{}$$

$$= \frac{8(cdf-aeg)\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{693c^2d^2(d+ex)^{7/2}}$$

Mathematica [A] time = 0.11, size = 100, normalized size = 0.50

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^2g^2-4acdeg(11f+7gx)+c^2d^2\left(99f^2+154fgx+63g^2x^2\right)\right)}{693c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e
*g*(11*f + 7*g*x) + c^2*d^2*(99*f^2 + 154*f*g*x + 63*g^2*x^2)))/(693*c^3*d^
3*Sqrt[d + e*x])
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2))/(d + e*x)^(5/2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.42, size = 284, normalized size = 1.42

$$\frac{2\left(63c^5d^5g^2x^5+99a^3c^2d^2e^3f^2-44a^4cde^4fg+8a^5e^5g^2+7\left(22c^5d^5fg+23ac^4d^4eg^2\right)x^4+\left(99c^5d^5f^2+418ac^4d^4efg+113a^2c^3d^3e^2g^2\right)x^3+3\left(99ac^4d^4ef^2+110a^2c^3d^3e^2fg+a^3c^2d^2e^3g^2\right)x^2+\left(297a^2c^3d^3e^2f^2+22a^3c^2d^2e^3fg-4a^4cde^4g^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{693\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/693*(63*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 44*a^4*c*d*e^4*f*g + 8
*a^5*e^5*g^2 + 7*(22*c^5*d^5*f*g + 23*a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*f^
2 + 418*a*c^4*d^4*e*f*g + 113*a^2*c^3*d^3*e^2*g^2)*x^3 + 3*(99*a*c^4*d^4*e*
f^2 + 110*a^2*c^3*d^3*e^2*f*g + a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3*d^3
*e^2*f^2 + 22*a^3*c^2*d^2*e^3*f*g - 4*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 9.37Done
```

maple [A] time = 0.01, size = 116, normalized size = 0.58

$$\frac{2\,(cdx+ae)\left(63g^2x^2c^2d^2-28acde\,g^2x+154c^2d^2fgx+8a^2e^2g^2-44acdefg+99f^2c^2d^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{693\,(ex+d)^{\frac{5}{2}}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/693*(c*d*x+a*e)*(63*c^2*d^2*g^2*x^2-28*a*c*d*e*g^2*x+154*c^2*d^2*f*g*x+8*
a^2*e^2*g^2-44*a*c*d*e*f*g+99*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(5/2)/c^3/d^3/(e*x+d)^(5/2)
```

maxima [A] time = 0.64, size = 243, normalized size = 1.22

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}\,f^2}{7\,cd}+\frac{4\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4\right)\sqrt{cdx+ae}\,fg}{63\,c^2d^2}+\frac{2\left(63\,c^5d^5x^5+161\,ac^4d^4ex^4+113\,a^2c^3d^3e^2x^3+3\,a^3c^2d^2e^3x^2-4\,a^4cde^4x+8\,a^5e^5\right)\sqrt{cdx+ae}\,g^2}{693\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^2/(c*d) + 4/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) +
2/693*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3*a
^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*g^2/(c^
3*d^3)
```

mupad [B] time = 3.56, size = 259, normalized size = 1.30

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,a^5\,e^5\,g^2-88\,a^4\,c\,d\,e^4\,f\,g+198\,a^3\,c^2\,d^2\,e^3\,f^2}{693\,c^3\,d^3}+\frac{x^3\,\left(226\,a^2\,c^3\,d^3\,e^2\,g^2+836\,a\,c^4\,d^4\,e\,f\,g+198\,c^5\,d^5\,f^2\right)}{693\,c^3\,d^3}+\frac{2\,c^2\,d^2\,g^2\,x^5}{11}+\frac{2\,c\,d\,g\,x^4\,\left(23\,a\,e\,g+22\,c\,d\,f\right)}{99}+\frac{2\,a^2\,e^2\,x\,\left(-4\,a^2\,e^2\,g^2+22\,a\,c\,d\,e\,f\,g+297\,c^2\,d^2\,f^2\right)}{693\,c^2\,d^2}+\frac{2\,a\,e\,x^2\,\left(a^2\,e^2\,g^2+110\,a\,c\,d\,e\,f\,g+99\,c^2\,d^2\,f^2\right)}{231\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*a^5*e^5*g^2 + 198*a^3*c
^2*d^2*e^3*f^2 - 88*a^4*c*d*e^4*f*g)/(693*c^3*d^3) + (x^3*(198*c^5*d^5*f^2
+ 226*a^2*c^3*d^3*e^2*g^2 + 836*a*c^4*d^4*e*f*g))/(693*c^3*d^3) + (2*c^2*d^
2*g^2*x^5)/11 + (2*c*d*g*x^4*(23*a*e*g + 22*c*d*f))/99 + (2*a^2*e^2*x*(297*
c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*f*g))/(693*c^2*d^2) + (2*a*e*x^2*(
a^2*e^2*g^2 + 99*c^2*d^2*f^2 + 110*a*c*d*e*f*g))/(231*c*d)))/(d + e*x)^(1/2
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

3.470 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(2ae^2g-cd(9ef-7dg)\right)}{63c^2d^2e(d+ex)^{7/2}}$$

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(2ae^2g-cd(9ef-7dg)\right)}{63c^2d^2e(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5/
2),x]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(63*c^2*d^2*e*(d + e*x)^(7/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx &= \frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}+\frac{1}{9}\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\\ &= \frac{2\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63cd(d+ex)^{7/2}}+\frac{2g\left(a\right.}{}\end{aligned}$$

Mathematica [A] time = 0.08, size = 64, normalized size = 0.51

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\,(cd(9f+7gx)-2aeg)}{63c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(9*f + 7*g
*x)))/(63*c^2*d^2*Sqrt[d + e*x])
```

IntegrateAlgebraic [A] time = 1.73, size = 67, normalized size = 0.54

$$\frac{2(ae+cdx)((d+ex)(ae+cdx))^{5/2}(7g(ae+cdx)-9aeg+9cdf)}{63c^2d^2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)
)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2)*(9*c*d*f - 9*a*e*g + 7*g*(
a*e + c*d*x)))/(63*c^2*d^2*(d + e*x)^(5/2))
```

fricas [A] time = 0.42, size = 173, normalized size = 1.38

$$\frac{2\left(7\,c^4d^4gx^4+9\,a^3cde^3f-2\,a^4e^4g+\left(9\,c^4d^4f+19\,ac^3d^3eg\right)x^3+3\left(9\,ac^3d^3ef+5\,a^2c^2d^2e^2g\right)x^2+\left(27\,a^2c^2d^2e^2f+a^3cde^3g\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{63\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="fricas")
```

```
[Out] 2/63*(7*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 2*a^4*e^4*g + (9*c^4*d^4*f + 19*a
*c^3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f + 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2
*c^2*d^2*e^2*f + a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 5.43Done
```

maple [A] time = 0.00, size = 67, normalized size = 0.54

$$-\frac{2\left(cdx+ae\right)\left(-7cdgx+2aeg-9cdf\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{63\left(ex+d\right)^{\frac{5}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/63*(c*d*x+a*e)*(-7*c*d*g*x+2*a*e*g-9*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(5/2)/c^2/d^2/(e*x+d)^(5/2)
```

maxima [A] time = 0.57, size = 141, normalized size = 1.13

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}\,f}{7\,cd}+\frac{2\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4\right)\sqrt{cdx+ae}\,g}{63\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f/(c*d) + 2/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*d^
2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

mupad [B] time = 3.37, size = 134, normalized size = 1.07

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,c^2\,d^2\,g\,x^4}{9}+\frac{2\,a\,e\,x^2\,\left(5\,a\,e\,g+9\,c\,d\,f\right)}{21}+\frac{2\,c\,d\,x^3\,\left(19\,a\,e\,g+9\,c\,d\,f\right)}{63}-\frac{2\,a^3\,e^3\,\left(2\,a\,e\,g-9\,c\,d\,f\right)}{63\,c^2\,d^2}+\frac{2\,a^2\,e^2\,x\,\left(a\,e\,g+27\,c\,d\,f\right)}{63\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(5/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*c^2*d^2*g*x^4)/9 + (2*a*
e*x^2*(5*a*e*g + 9*c*d*f))/21 + (2*c*d*x^3*(19*a*e*g + 9*c*d*f))/63 - (2*a^
3*e^3*(2*a*e*g - 9*c*d*f))/(63*c^2*d^2) + (2*a^2*e^2*x*(a*e*g + 27*c*d*f))/
(63*c*d)))/(d + e*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2
),x)
```

```
[Out] Timed out
```

3.471 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.00, size = 45, normalized size = 0.94

$$\frac{2(ae+cdx)((d+ex)(ae+cdx))^{5/2}}{7cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^
(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2))/(7*c*d*(d + e*x)^(5/2))
```

fricas [B] time = 0.41, size = 91, normalized size = 1.90

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{7\left(cdex+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="fricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 3.17Done
```

maple [A] time = 0.00, size = 50, normalized size = 1.04

$$\frac{2\left(cdx+ae\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{7\left(ex+d\right)^{\frac{5}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/7*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c/d/(e*x+d)^(5/2)
```

maxima [A] time = 0.51, size = 60, normalized size = 1.25

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}}{7\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)/(c*d)
```

mupad [B] time = 3.16, size = 79, normalized size = 1.65

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{6\,a^2\,e^2\,x}{7}+\frac{2\,c^2\,d^2\,x^3}{7}+\frac{2\,a^3\,e^3}{7\,c\,d}+\frac{6\,a\,c\,d\,e\,x^2}{7}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((6*a^2*e^2*x)/7 + (2*c^2*d^
2*x^3)/7 + (2*a^3*e^3)/(7*c*d) + (6*a*c*d*e*x^2)/7))/(d + e*x)^(1/2)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

3.472 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx$

Optimal. Leaf size=236

$$-\frac{2(cdf-aeg)^{5/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{g^3\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right.\right.}{}$$

Rubi [A] time = 0.47, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {864, 874, 205}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{g^3\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{3g^2(d+ex)^{3/2}}-\frac{2(cdf-aeg)^{5/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)),x]
```

```
[Out] (2*(c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt
[d + e*x]) - (2*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(3*g^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2))/(5*g*(d + e*x)^(5/2)) - (2*(c*d*f - a*e*g)^(5/2)*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/g^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d + ex)^{5/2}(f + gx)}\,dx = \frac{2\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{5g(d + ex)^{5/2}} - \frac{\left(cde^2 f + cd^2 eg - e\left(cd^2 + ae^2\right)g\right)\int}{e^2 g}$$

$$= -\frac{2(cdf - aeg)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{3g^2(d + ex)^{3/2}} + \frac{2\left(ade + \left(cd^2 + ae^2\right)x\right.}{5g(d + ex)^{5/}}$$

$$= \frac{2(cdf - aeg)^2\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{g^3\sqrt{d + ex}} - \frac{2(cdf - aeg)\left(ade + \left(cd^2\right.\right.}{3g^2(d + e}$$

$$= \frac{2(cdf - aeg)^2\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{g^3\sqrt{d + ex}} - \frac{2(cdf - aeg)\left(ade + \left(cd^2\right.\right.}{3g^2(d + e}$$

$$= \frac{2(cdf - aeg)^2\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{g^3\sqrt{d + ex}} - \frac{2(cdf - aeg)\left(ade + \left(cd^2\right.\right.}{3g^2(d + e}$$

Mathematica [A] time = 0.36, size = 145, normalized size = 0.61

$$\frac{((d + ex)(ae + cdx))^{5/2}\left(-\frac{10(cdf-aeg)^{5/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{g^{5/2}(ae+cdx)^{5/2}} + \frac{10(aeg-cdf)(4aeg+cd(gx-3f))}{3g^2(ae+cdx)^2} + 2\right)}{5g(d + ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*(2 + (10*(-(c*d*f) + a*e*g)*(4*a*e*g + c*d
*(-3*f + g*x)))/(3*g^2*(a*e + c*d*x)^2) - (10*(c*d*f - a*e*g)^(5/2)*ArcTan[
(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(g^(5/2)*(a*e + c*d*x)^(5
/2))))/(5*g*(d + e*x)^(5/2))
```

IntegrateAlgebraic [A] time = 11.68, size = 189, normalized size = 0.80

$$\frac{((d + ex)(ae + cdx))^{5/2}\left(\frac{2\sqrt{ae+cdx}\left(15a^2e^2g^2-5cdfg(ae+cdx)-30acdefg+3g^2(ae+cdx)^2+5aeg^2(ae+cdx)+15c^2d^2f^2\right)}{15g^3} - \frac{2(cdf-aeg)^{5/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{g^{7/2}}\right)}{(d + ex)^{5/2}(ae + cdx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*((2*Sqrt[a*e + c*d*x]*(15*c^2*d^2*f^2 - 30
*a*c*d*e*f*g + 15*a^2*e^2*g^2 - 5*c*d*f*g*(a*e + c*d*x) + 5*a*e*g^2*(a*e +
c*d*x) + 3*g^2*(a*e + c*d*x)^2))/(15*g^3) - (2*(c*d*f - a*e*g)^(5/2)*ArcTan
[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/g^(7/2)))/((a*e + c*d*x)
^(5/2)*(d + e*x)^(5/2))
```

fricas [A] time = 0.44, size = 587, normalized size = 2.49

$$\left[\frac{15\left(c^2d^3f^2 - 2acd^2efg + a^2de^2g^2 + \left(c^2d^2ef^2 - 2acde^2fg + a^2e^3g^2\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2 - cd^2f + 2adeg - 2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\,g\sqrt{-\frac{cdf-aeg}{g}} - \left(cdef - \left(cd^2+2ae^2\right)g\right)x}{egx^2+df+(ef+dg)x}\right) + 2\left(3c^2d^2g^2x^2 + 15c^2d^2f^2 - 35acdefg + 23a^2e^2g^2 - \left(5c^2d^2fg - 11acdeg^2\right)x\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex + d}}{15\left(eg^3x + dg^3\right)}, \frac{2\left(15\left(c^2d^3f^2 - 2acd^2efg + a^2de^2g^2 + \left(c^2d^2ef^2 - 2acde^2fg + a^2e^3g^2\right)x\right)\sqrt{\frac{cdf-aeg}{g}}\arctan\left(\frac{\sqrt{ex+d}\sqrt{\frac{cdf-aeg}{g}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\right) + \left(3c^2d^2g^2x^2 + 15c^2d^2f^2 - 35acdefg + 23a^2e^2g^2 - \left(5c^2d^2fg - 11acdeg^2\right)x\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex + d}\right)}{15\left(eg^3x + dg^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="fricas")
```

```
[Out] [1/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 -
 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x
^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)
/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 -
 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 11*a*c*d*e*g^2)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^3*x + d*g^3), 2
/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2
*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x +
d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + (
3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c
^2*d^2*f*g - 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d))/(e*g^3*x + d*g^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 431, normalized size = 1.83

$$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15a^3e^3g^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-45a^2cd\,e^2f\,g^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)+45a\,c^2d^2e\,f^2g\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-15c^3d^3f^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)-3\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,c^2d^2g^2x^2-11\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,acde\,g^2x+5\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,c^2d^2fgx-23\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,a^2e^2g^2+35\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,acdefg-15\sqrt{\left(aeg-cdf\right)g}\,\sqrt{cdx+ae}\,c^2d^2f^2\right)}{15\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{\left(aeg-cdf\right)g}\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x)
```

```
[Out] -2/15*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*a^3*e^3*g^3-45*arctanh((c*d*x+a*e)^(1/2)/((a*e*
g-c*d*f)*g)^(1/2)*g)*a^2*c*d*e^2*f*g^2+45*arctanh((c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2)*g)*a*c^2*d^2*e*f^2*g-15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2)*g)*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*
c^2*d^2*g^2*x^2-11*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x+
5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x-23*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+35*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a
*e)^(1/2)*a*c*d*e*f*g-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*
f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{\left(ex + d\right)^{\frac{5}{2}}\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/2
)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
),x)
```

```
[Out] Timed out
```

3.473 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx$

Optimal. Leaf size=235

$$\frac{5cd(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}}-\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{g^3\sqrt{d+ex}}-\frac{\left(x\left(ae^2+cd^2\right)\right.}{g(d+}$$

Rubi [A] time = 0.38, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 864, 874, 205}

$$-\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{g^3\sqrt{d+ex}}+\frac{5cd(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}+\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^2),x]
```

```
[Out] (-5*c*d*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*S
qrt[d + e*x]) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g^
2*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d +
e*x)^(5/2)*(f + g*x)) + (5*c*d*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/g^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
```

```
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx}{2g}$$

$$= \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)\right.}{3g^2(d+ex)}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)\right.}{3g^2(d+ex)}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} + \frac{5cd\left(ade+\left(cd^2+ae^2\right)\right.}{3g^2(d+ex)}$$

Mathematica [C] time = 0.07, size = 75, normalized size = 0.32

$$\frac{2cd((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^2),x]
```

```
[Out] (2*c*d*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (g*(a
*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^2*(d + e*x)^(7/2))
```

IntegrateAlgebraic [A] time = 104.17, size = 215, normalized size = 0.91

$$\frac{((d+ex)(ae+cdx))^{5/2}\left(\frac{5cd(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{g^{7/2}} - \frac{cd\sqrt{ae+cdx}\left(15a^2e^2g^2+10cdfg(ae+cdx)-30acdefg-2g^2(ae+cdx)^2-10aeg^2(ae+cdx)+15c^2d^2f^2\right)}{3g^3(g(ae+cdx)-aeg+cdf)}\right)}{(d+ex)^{5/2}(ae+cdx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^2),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*(-1/3*(c*d*Sqrt[a*e + c*d*x]*(15*c^2*d^2*f
^2 - 30*a*c*d*e*f*g + 15*a^2*e^2*g^2 + 10*c*d*f*g*(a*e + c*d*x) - 10*a*e*g^
2*(a*e + c*d*x) - 2*g^2*(a*e + c*d*x)^2))/(g^3*(c*d*f - a*e*g + g*(a*e + c*
d*x))) + (5*c*d*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sq
rt[c*d*f - a*e*g]])/g^(7/2)))/((a*e + c*d*x)^(5/2)*(d + e*x)^(5/2))
```

fricas [A] time = 0.49, size = 672, normalized size = 2.86

$$\left[-\frac{15\left(c^2d^3f^2-acd^2efg+\left(c^2d^2efg-acde^2g^2\right)x^2+\left(c^2d^2ef^2-acd^2eg^2+\left(c^2d^3-acde^2\right)fg\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2adeg-2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\,g\sqrt{-\frac{cdf-aeg}{g}}-\left(cdef-\left(cd^2+2ae^2\right)g\right)x}{egx^2+df+(ef+dg)x}\right)-2\left(2c^2d^2g^2x^2-15c^2d^2f^2+20acdefg-3a^2e^2g^2-2\left(5c^2d^2fg-7acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{6\left(eg^4x^2+dfg^3+\left(efg^3+dg^4\right)x\right)}, -\frac{15\left(c^2d^3f^2-acd^2efg+\left(c^2d^2efg-acde^2g^2\right)x^2+\left(c^2d^2ef^2-acd^2eg^2+\left(c^2d^3-acde^2\right)fg\right)x\right)\sqrt{\frac{cdf-aeg}{g}}\arctan\left(\frac{\sqrt{ex+d}\sqrt{\frac{cdf-aeg}{g}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\right)-\left(2c^2d^2g^2x^2-15c^2d^2f^2+20acdefg-3a^2e^2g^2-2\left(5c^2d^2fg-7acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{3\left(eg^4x^2+dfg^3+\left(efg^3+dg^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="fricas")
```

```
[Out] [-1/6*(15*(c^2*d^3*f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x^
2 + (c^2*d^2*e*f^2 - a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt(-(c
*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c
*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(2*c^
2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*e^2*g^2 - 2*(5*c^2*
d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), -1/3*(15*(c^2*d^3*
f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x^2 + (c^2*d^2*e*f^2
- a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt((c*d*f - a*e*g)/g)*arc
tan(sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)) - (2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*
e^2*g^2 - 2*(5*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 523, normalized size = 2.23

$$-\frac{\sqrt{cde\,x^2+a e^2 x+c d^2 x+ade}\,\left(15a^2cde^2g^3x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-30ac^2d^2efg^2x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+15c^3d^3f^2gx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+15a^2cde^2fg^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-30ac^2d^2ef^2g\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+15c^3d^3f^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-2\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,c^2d^2g^2x^2-14\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,acdeg^2x+10\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,c^2d^2fgx+3\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,a^2e^2g^2-20\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,acdefg+15\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,c^2d^2f^2\right)}{3\sqrt{ex+d}\,\sqrt{cdx+ae}\,(gx+f)\sqrt{(aeg-cdf)g}\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x)
```

```
[Out] -1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x*a^2*c*d*e^2*g^3-30*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x*a*c^2*d^2*e*f*g^2+15*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*a^2*c*d*e^2*f*g^2-30*arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*a*c^2*d^2*e*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*c^3*d^3*f^3-2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)*c^2*d^2*g^2*x^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2
*x+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+3*((a*e*g-c*d
*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2-20*((a*e*g-c*d*f)*g)^(1/2)*(c*d*
x+a*e)^(1/2)*a*c*d*e*f*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d
^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**2,x)
```

```
[Out] Timed out
```

3.474 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{15c^2d^2\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{7/2}}+\frac{15c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(x\left(ae^2+cd^2\right)\right.}{4g^2(d+ex}$$

Rubi [A] time = 0.34, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 864, 874, 205}

$$-\frac{15c^2d^2\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{7/2}}+\frac{15c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^3),x]
```

```
[Out] (15*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^3*Sqrt[d + e*
x]) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*g^2*(d + e*x
)^(3/2)*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(2*g*(d
+ e*x)^(5/2)*(f + g*x)^2) - (15*c^2*d^2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
 e*x])])/(4*g^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
```

```
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx}{4g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{4g^2(d+ex)^{3/2}(f+gx)}$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{4g^2(d+ex)^{3/2}(f+gx)}$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{4g^2(d+ex)^{3/2}(f+gx)}$$

Mathematica [C] time = 0.07, size = 79, normalized size = 0.32

$$\frac{2c^2d^2((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^3),x]
```

```
[Out] (2*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^3*(d + e*x)^(7/2))
```

IntegrateAlgebraic [F] time = 180.04, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^3),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.62, size = 683, normalized size = 2.78

$$\left[\frac{15\left(c^2d^2eg^2x^3+c^2d^3f^2+\left(2c^2d^2efg+c^2d^3g^2\right)x^2+\left(c^2d^2ef^2+2c^2d^3fg\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2adeg-2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\,g\sqrt{-\frac{cdf-aeg}{g}}-\left(cdef-\left(cd^2+2ae^2\right)g\right)x}{egx^2+df+(ef+dg)x}\right)+2\left(8c^2d^2g^2x^2+15c^2d^2f^2-5acdefg-2a^2e^2g^2+\left(25c^2d^2fg-9acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{8\left(eg^5x^3+df^2g^3+\left(2efg^4+dg^5\right)x^2+\left(ef^2g^3+2dfg^4\right)x\right)}, \frac{15\left(c^2d^2eg^2x^3+c^2d^3f^2+\left(2c^2d^2efg+c^2d^3g^2\right)x^2+\left(c^2d^2ef^2+2c^2d^3fg\right)x\right)\sqrt{\frac{cdf-aeg}{g}}\arctan\left(\frac{\sqrt{ex+d}\sqrt{\frac{cdf-aeg}{g}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\right)+\left(8c^2d^2g^2x^2+15c^2d^2f^2-5acdefg-2a^2e^2g^2+\left(25c^2d^2fg-9acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{4\left(eg^5x^3+df^2g^3+\left(2efg^4+dg^5\right)x^2+\left(ef^2g^3+2dfg^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="fricas")
```

```
[Out] [1/8*(15*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2
)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^
2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^5*x^3 + d*
f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 + 2*d*f*g^4)*x), 1/4*(15*(c^
2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^2 + (c^2*
d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*
sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + (8*c
^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d
^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e*x + d))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 +
2*d*f*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 526, normalized size = 2.14

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15a\,c^2d^2e\,g^3x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-15c^3d^3f\,g^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+30a\,c^2d^2ef\,g^2x\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-30c^3d^3f^2gx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+15a\,c^2d^2e\,f^2g\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-15c^3d^3f^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-8\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,c^2d^2g^2x^2+9\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,acde\,g^2x-25\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,c^2d^2fgx+2\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,a^2e^2g^2+5\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,acdefg-15\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,c^2d^2f^2\right)}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,\left(gx+f\right)^2\sqrt{(aeg-cdf)g}\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*a*c^2*d^2*e*g^3-15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*f*g^2+30*arctanh((c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*g)*x*a*c^2*d^2*e*f*g^2-30*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*a*c^2*d^2*e*f^2*g-15*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*c^3*d^3*f^3-8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(
1/2)*c^2*d^2*g^2*x^2+9*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^
2*x-25*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+2*((a*e*g-c*
d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*
x+a*e)^(1/2)*a*c*d*e*f*g-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d
^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{\left(ex + d\right)^{\frac{5}{2}}\left(gx + f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^3\,\left(d+e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**3,x)
```

```
[Out] Timed out
```

3.475 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx$

Optimal. Leaf size=253

$$\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{12g^2(d+ex)^{3/2}(f+gx)^2}$$

Rubi [A] time = 0.34, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {862, 874, 205}

$$-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^4),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^3*Sqrt[d + e*
x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g
^2*(d + e*x)^(3/2)*(f + g*x)^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)/(3*g*(d + e*x)^(5/2)*(f + g*x)^3) + (5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])
/(8*g^(7/2)*Sqrt[c*d*f - a*e*g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}dx}{6g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{12g^2(d+ex)^{3/2}(f+gx)^2}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{12g^2(d+ex)^{3/2}(f+gx)^2}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{12g^2(d+ex)^{3/2}(f+gx)^2}$$

Mathematica [A] time = 0.37, size = 171, normalized size = 0.68

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{15c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}\sqrt{cdf-aeg}} - \frac{\sqrt{g}\left(8a^2e^2g^2+2acdeg(5f+13gx)+c^2d^2\left(15f^2+40fgx+33g^2x^2\right)\right)}{(f+gx)^3}\right)}{24g^{7/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^4),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(8*a^2*e^2*g^2 + 2*a*c*d*e*g*(5*
f + 13*g*x) + c^2*d^2*(15*f^2 + 40*f*g*x + 33*g^2*x^2)))/(f + g*x)^3) + (15
*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d
*f - a*e*g]*Sqrt[a*e + c*d*x])))/(24*g^(7/2)*Sqrt[d + e*x])
```

IntegrateAlgebraic [F] time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^4),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.46, size = 1140, normalized size = 4.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="fricas")
```

```
[Out] [-1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3
```

```
*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d
*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
 + d*g)*x)) + 2*(15*c^3*d^3*f^3*g - 5*a*c^2*d^2*e*f^2*g^2 - 2*a^2*c*d*e^2*f
*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(20*c^3
*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^4*g^4 - a*d*e*f^3*g^5
+ (c*d*e*f*g^7 - a*e^2*g^8)*x^4 + (3*c*d*e*f^2*g^6 - a*d*e*g^8 + (c*d^2 - 3
*a*e^2)*f*g^7)*x^3 + 3*(c*d*e*f^3*g^5 - a*d*e*f*g^7 + (c*d^2 - a*e^2)*f^2*g
^6)*x^2 + (c*d*e*f^4*g^4 - 3*a*d*e*f^2*g^6 + (3*c*d^2 - a*e^2)*f^3*g^5)*x),
 -1/24*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3
*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g
 + (c*d^2 + a*e^2)*g*x)) + (15*c^3*d^3*f^3*g - 5*a*c^2*d^2*e*f^2*g^2 - 2*a^
2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2
+ 2*(20*c^3*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*d*e^2*g^4)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^4*g^4 - a*d
*e*f^3*g^5 + (c*d*e*f*g^7 - a*e^2*g^8)*x^4 + (3*c*d*e*f^2*g^6 - a*d*e*g^8 +
 (c*d^2 - 3*a*e^2)*f*g^7)*x^3 + 3*(c*d*e*f^3*g^5 - a*d*e*f*g^7 + (c*d^2 - a
*e^2)*f^2*g^6)*x^2 + (c*d*e*f^4*g^4 - 3*a*d*e*f^2*g^6 + (3*c*d^2 - a*e^2)*f
^3*g^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 441, normalized size = 1.74

$$-\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(15c^3d^3g^3x^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+45c^3d^3fg^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+45c^3d^3f^2gx\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+15c^3d^3f^3\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+33\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,c^2d^2g^2x^2+26\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,acdeg^2x+40\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,c^2d^2fgx+8\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,a^2e^2g^2+10\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,acdefg+15\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\,c^2d^2f^2\right)}{24\sqrt{ex+d}\,\sqrt{cdx+ae}\,(gx+f)^3\sqrt{(aeg-cdf)g}\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*g^3+45*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*f*g^2+45*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*
g-c*d*f)*g)^(1/2)*g)*c^3*d^3*f^3+33*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)*c^2*d^2*g^2*x^2+26*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2
*x+40*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+8*((a*e*g-c*d
*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*
x+a*e)^(1/2)*a*c*d*e*f*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d
^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^4\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**4,x)
```

```
[Out] Timed out
```

3.476 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx$

Optimal. Leaf size=323

$$\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}+\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{32g^3\sqrt{d+ex}\,(f+gx)^2}$$

Rubi [A] time = 0.47, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {862, 872, 874, 205}

$$\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2}+\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^5),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e
*x]*(f + g*x)^2) + (5*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(64*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(24*g^2*(d + e*x)^(3/2)*(f + g*x)^3) - (a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(4*g*(d + e*x)^(5/2)*(f + g*x)^4) +
 (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(S
qrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(7/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
```

```
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx}{8g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{24g^2(d+ex)^{3/2}(f+gx)^3}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

Mathematica [C] time = 0.08, size = 79, normalized size = 0.24

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^5),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^5*(d + e*x)^(7/2))
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^5),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.48, size = 1862, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="fricas")
```

```
[Out] [1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c
^4*d^4*f^4*g - 5*a*c^3*d^3*e*f^3*g^2 - 2*a^2*c^2*d^2*e^2*f^2*g^3 - 56*a^3*c
*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 +
(73*c^4*d^4*f^2*g^3 - 191*a*c^3*d^3*e*f*g^4 + 118*a^2*c^2*d^2*e^2*g^5)*x^2
+ (55*c^4*d^4*f^3*g^2 - 19*a*c^3*d^3*e*f^2*g^3 - 172*a^2*c^2*d^2*e^2*f*g^4
+ 136*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d))/(c^2*d^3*f^6*g^4 - 2*a*c*d^2*e*f^5*g^5 + a^2*d*e^2*f^4*g^6 + (c^2
*d^2*e*f^2*g^8 - 2*a*c*d*e^2*f*g^9 + a^2*e^3*g^10)*x^5 + (4*c^2*d^2*e*f^3*g
^7 + a^2*d*e^2*g^10 + (c^2*d^3 - 8*a*c*d*e^2)*f^2*g^8 - 2*(a*c*d^2*e - 2*a^
2*e^3)*f*g^9)*x^4 + 2*(3*c^2*d^2*e*f^4*g^6 + 2*a^2*d*e^2*f*g^9 + 2*(c^2*d^3
 - 3*a*c*d*e^2)*f^3*g^7 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g^8)*x^3 + 2*(2*c^2
*d^2*e*f^5*g^5 + 3*a^2*d*e^2*f^2*g^8 + (3*c^2*d^3 - 4*a*c*d*e^2)*f^4*g^6 -
2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^7)*x^2 + (c^2*d^2*e*f^6*g^4 + 4*a^2*d*e^2*f
^3*g^7 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g^5 - (8*a*c*d^2*e - a^2*e^3)*f^4*g^
6)*x), -1/192*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c
^4*d^5*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*
d^4*e*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)
*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*
e^2)*g*x)) + (15*c^4*d^4*f^4*g - 5*a*c^3*d^3*e*f^3*g^2 - 2*a^2*c^2*d^2*e^2*
f^2*g^3 - 56*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3
*d^3*e*g^5)*x^3 + (73*c^4*d^4*f^2*g^3 - 191*a*c^3*d^3*e*f*g^4 + 118*a^2*c^2
*d^2*e^2*g^5)*x^2 + (55*c^4*d^4*f^3*g^2 - 19*a*c^3*d^3*e*f^2*g^3 - 172*a^2*
c^2*d^2*e^2*f*g^4 + 136*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^6*g^4 - 2*a*c*d^2*e*f^5*g^5 + a^2*d*
e^2*f^4*g^6 + (c^2*d^2*e*f^2*g^8 - 2*a*c*d*e^2*f*g^9 + a^2*e^3*g^10)*x^5 +
(4*c^2*d^2*e*f^3*g^7 + a^2*d*e^2*g^10 + (c^2*d^3 - 8*a*c*d*e^2)*f^2*g^8 - 2
*(a*c*d^2*e - 2*a^2*e^3)*f*g^9)*x^4 + 2*(3*c^2*d^2*e*f^4*g^6 + 2*a^2*d*e^2*
f*g^9 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^7 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g
^8)*x^3 + 2*(2*c^2*d^2*e*f^5*g^5 + 3*a^2*d*e^2*f^2*g^8 + (3*c^2*d^3 - 4*a*c
*d*e^2)*f^4*g^6 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^7)*x^2 + (c^2*d^2*e*f^6*g
^4 + 4*a^2*d*e^2*f^3*g^7 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g^5 - (8*a*c*d^2*e
 - a^2*e^3)*f^4*g^6)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 665, normalized size = 2.06

$$\frac{\sqrt{c d e x^{2}+a e^{2} x+c d^{2} x+a d e}\,\left(15 c^{4} d^{4} g^{4} x^{4} \operatorname{arctanh}\left(\frac{\sqrt{c d x+a e}\, g}{\sqrt{\left(a e g-c d f\right) g}}\right)+60 c^{4} d^{4} f g^{3} x^{3} \operatorname{arctanh}\left(\frac{\sqrt{c d x+a e}\, g}{\sqrt{\left(a e g-c d f\right) g}}\right)+90 c^{4} d^{4} f^{2} g^{2} x^{2} \operatorname{arctanh}\left(\frac{\sqrt{c d x+a e}\, g}{\sqrt{\left(a e g-c d f\right) g}}\right)+60 c^{4} d^{4} f^{3} g x \operatorname{arctanh}\left(\frac{\sqrt{c d x+a e}\, g}{\sqrt{\left(a e g-c d f\right) g}}\right)+15 c^{4} d^{4} f^{4} \operatorname{arctanh}\left(\frac{\sqrt{c d x+a e}\, g}{\sqrt{\left(a e g-c d f\right) g}}\right)-15 \sqrt{c d x+a e}\, \sqrt{\left(a e g-c d f\right) g}\, c^{3} d^{3} g^{3} x^{3}-118 \sqrt{c d x+a e}\, \sqrt{\left(a e g-c d f\right) g}\, a c^{2} d^{2} e g^{3} x^{2}+73 \sqrt{c d x+a e}\, \sqrt{\left(a e g-c d f\right) g}\, c^{3} d^{3} f g^{2} x^{2}-136 \sqrt{c d x+a e}\, \sqrt{\left(a e g-c d f\right) g}\, a^{2} c d e^{2} g^{3} x+36 \sqrt{c d x+a e}\, \sqrt{\left(a e g-c d f\right) g}\, a c^{2} d^{2} e f g^{2} x+55 \sqrt{c d x+a e}\, \sqrt{\left(a e g-c d f\right) g}\, c^{3} d^{3} f^{2} g x-48 \sqrt{\left(a e g-c d f\right) g}\, \sqrt{c d x+a e}\, a^{3} e^{3} g^{3}+8 \sqrt{\left(a e g-c d f\right) g}\, \sqrt{c d x+a e}\, a^{2} c d e^{2} f g^{2}+10 \sqrt{\left(a e g-c d f\right) g}\, \sqrt{c d x+a e}\, a c^{2} d^{2} e f^{2} g+15 \sqrt{\left(a e g-c d f\right) g}\, \sqrt{c d x+a e}\, c^{3} d^{3} f^{3}\right)}{192 \sqrt{e x+d}\, \sqrt{c d x+a e}\, \left(a e g-c d f\right) \left(g x+f\right)^{4} \sqrt{\left(a e g-c d f\right) g}\, g^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x)
```

```
[Out] 1/192*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^4*d^4*g^4+60*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^4*d^4*f*g^3+90*arctanh((c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^4*d^4*f^2*g^2+60*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x*c^4*d^4*f^3*g-15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f
)*g)^(1/2)*c^3*d^3*g^3*x^3+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2)*g)*c^4*d^4*f^4-118*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2
*e*g^3*x^2+73*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f*g^2*x^2-1
36*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*g^3*x+36*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f*g^2*x+55*(c*d*x+a*e)^(1/2)*(
(a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^2*g*x-48*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*
e)^(1/2)*a^3*e^3*g^3+8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^
2*f*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+15*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/(c*d*x+
a*e)^(1/2)/g^3/(a*e*g-c*d*f)/(g*x+f)^4/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}\left(gx + f\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f + g\,x\right)^5\,(d + e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**5,x)
```

```
[Out] Timed out
```

3.477 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

Optimal. Leaf size=393

$$\frac{3c^5d^5\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}}+\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{64g^3\sqrt{d+ex}\,(f+gx)^2(cdf-aeg}$$

Rubi [A] time = 0.57, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+\frac{3c^5d^5\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}}-\frac{cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^6),x]
```

```
[Out] -(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16*g^3*Sqrt[d + e*x
]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*
g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^4*d^4*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f +
 g*x)) - (c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d + e*
x)^(3/2)*(f + g*x)^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*g*
(d + e*x)^(5/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(128*g^(7
/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}+\frac{(cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx}{2g}$$

$$= -\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{8g^2(d+ex)^{3/2}(f+gx)^4}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

Mathematica [C] time = 0.08, size = 79, normalized size = 0.20

$$\frac{2c^5d^5((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^6),x]
```

```
[Out] (2*c^5*d^5*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^6*(d + e*x)^(7/2))
```

IntegrateAlgebraic [F] time = 180.32, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^6),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.53, size = 2750, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^
6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^
3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2
+ (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e
*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d
))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f
^4*g^2 - 2*a^2*c^3*d^3*e^2*f^3*g^3 - 184*a^3*c^2*d^2*e^3*f^2*g^4 + 304*a^4*
c*d*e^4*f*g^5 - 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4
- 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 +
2*(64*c^5*d^5*f^3*g^3 - 297*a*c^4*d^4*e*f^2*g^4 + 357*a^2*c^3*d^3*e^2*f*g^5
 - 124*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 12*a*c^4*d^4*e*f^
3*g^3 - 279*a^2*c^3*d^3*e^2*f^2*g^4 + 424*a^3*c^2*d^2*e^3*f*g^5 - 168*a^4*c
*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
c^3*d^4*f^8*g^4 - 3*a*c^2*d^3*e*f^7*g^5 + 3*a^2*c*d^2*e^2*f^6*g^6 - a^3*d*e
^3*f^5*g^7 + (c^3*d^3*e*f^3*g^9 - 3*a*c^2*d^2*e^2*f^2*g^10 + 3*a^2*c*d*e^3*
f*g^11 - a^3*e^4*g^12)*x^6 + (5*c^3*d^3*e*f^4*g^8 - a^3*d*e^3*g^12 + (c^3*d
^4 - 15*a*c^2*d^2*e^2)*f^3*g^9 - 3*(a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2*g^10 +
 (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*g^11)*x^5 + 5*(2*c^3*d^3*e*f^5*g^7 - a^3*d
*e^3*f*g^11 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^4*g^8 - 3*(a*c^2*d^3*e - 2*a^2*
c*d*e^3)*f^3*g^9 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^2*g^10)*x^4 + 10*(c^3*d^
3*e*f^6*g^6 - a^3*d*e^3*f^2*g^10 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^5*g^7 - 3*
(a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^8 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^9)*
x^3 + 5*(c^3*d^3*e*f^7*g^5 - 2*a^3*d*e^3*f^3*g^9 + (2*c^3*d^4 - 3*a*c^2*d^2
*e^2)*f^6*g^6 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^7 + (6*a^2*c*d^2*e^2
- a^3*e^4)*f^4*g^8)*x^2 + (c^3*d^3*e*f^8*g^4 - 5*a^3*d*e^3*f^4*g^8 + (5*c^3
*d^4 - 3*a*c^2*d^2*e^2)*f^7*g^5 - 3*(5*a*c^2*d^3*e - a^2*c*d*e^3)*f^6*g^6 +
 (15*a^2*c*d^2*e^2 - a^3*e^4)*f^5*g^7)*x), -1/640*(15*(c^5*d^5*e*g^5*x^6 +
c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^
3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(
c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 + (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g
)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2
+ a*e^2)*g*x)) + (15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f^4*g^2 - 2*a^2*c^3*d^3*
e^2*f^3*g^3 - 184*a^3*c^2*d^2*e^3*f^2*g^4 + 304*a^4*c*d*e^4*f*g^5 - 128*a^5
*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4 - 10*(7*c^5*d^5*f^2*g^4
 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 + 2*(64*c^5*d^5*f^3*g^3 -
 297*a*c^4*d^4*e*f^2*g^4 + 357*a^2*c^3*d^3*e^2*f*g^5 - 124*a^3*c^2*d^2*e^3*
g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 12*a*c^4*d^4*e*f^3*g^3 - 279*a^2*c^3*d^3
*e^2*f^2*g^4 + 424*a^3*c^2*d^2*e^3*f*g^5 - 168*a^4*c*d*e^4*g^6)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^8*g^4 - 3*a*c
^2*d^3*e*f^7*g^5 + 3*a^2*c*d^2*e^2*f^6*g^6 - a^3*d*e^3*f^5*g^7 + (c^3*d^3*e
*f^3*g^9 - 3*a*c^2*d^2*e^2*f^2*g^10 + 3*a^2*c*d*e^3*f*g^11 - a^3*e^4*g^12)*
x^6 + (5*c^3*d^3*e*f^4*g^8 - a^3*d*e^3*g^12 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*
f^3*g^9 - 3*(a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2*g^10 + (3*a^2*c*d^2*e^2 - 5*a
^3*e^4)*f*g^11)*x^5 + 5*(2*c^3*d^3*e*f^5*g^7 - a^3*d*e^3*f*g^11 + (c^3*d^4
- 6*a*c^2*d^2*e^2)*f^4*g^8 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^3*g^9 + (3*a
^2*c*d^2*e^2 - 2*a^3*e^4)*f^2*g^10)*x^4 + 10*(c^3*d^3*e*f^6*g^6 - a^3*d*e^3
*f^2*g^10 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^5*g^7 - 3*(a*c^2*d^3*e - a^2*c*d*
e^3)*f^4*g^8 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^9)*x^3 + 5*(c^3*d^3*e*f^7*
```

```
g^5 - 2*a^3*d*e^3*f^3*g^9 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^6 - 3*(2*a*
c^2*d^3*e - a^2*c*d*e^3)*f^5*g^7 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^8)*x^2
 + (c^3*d^3*e*f^8*g^4 - 5*a^3*d*e^3*f^4*g^8 + (5*c^3*d^4 - 3*a*c^2*d^2*e^2)
*f^7*g^5 - 3*(5*a*c^2*d^3*e - a^2*c*d*e^3)*f^6*g^6 + (15*a^2*c*d^2*e^2 - a^
3*e^4)*f^5*g^7)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 924, normalized size = 2.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x)
```

```
[Out] -1/640*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^5*c^5*d^5*g^5+75*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^5*d^5*f*g^4+150*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^5*d^5*f^2*g^3+150*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^5*d^5*f^3*g^2-15*((a*e*g-c*d*f)*g)^(1/2)*
(c*d*x+a*e)^(1/2)*c^4*d^4*g^4*x^4+75*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2)*g)*x*c^5*d^5*f^4*g+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)
*a*c^3*d^3*e*g^4*x^3-70*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^4*d^4*f
*g^3*x^3+15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*c^5*d^5*f^
5+248*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^2*d^2*e^2*g^4*x^2-466
*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^3*d^3*e*f*g^3*x^2+128*((a*e*
g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^4*d^4*f^2*g^2*x^2+336*((a*e*g-c*d*f)*
g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c*d*e^3*g^4*x-512*((a*e*g-c*d*f)*g)^(1/2)*(c
*d*x+a*e)^(1/2)*a^2*c^2*d^2*e^2*f*g^3*x+46*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a
*e)^(1/2)*a*c^3*d^3*e*f^2*g^2*x+70*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2
)*c^4*d^4*f^3*g*x+128*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^4*e^4*g^4
-176*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c*d*e^3*f*g^3+8*((a*e*g-
c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^3*d^3*e*f^3*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c
*d*x+a*e)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^
5/g^3/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}\left(gx + f\right)^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^6\,\left(d+e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**6,x)
```

```
[Out] Timed out
```

3.478 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

Optimal. Leaf size=463

$$\frac{5c^6d^6\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{512g^{7/2}(cdf-aeg)^{7/2}}+\frac{5c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{768g^3\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}$$

Rubi [A] time = 0.72, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{5c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{768g^3\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{192g^3\sqrt{d+ex}\,(f+gx)^3(cdf-aeg)}-\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4}+\frac{5c^6d^6\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{512g^{7/2}(cdf-aeg)^{7/2}}-\frac{cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^7),x]
```

```
[Out] -(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e*x
]*(f + g*x)^4) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(192
*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^4*d^4*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(768*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f
+ g*x)^2) + (5*c^5*d^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*g^
3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (c*d*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)*(f + g*x)^5) - (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(6*g*(d + e*x)^(5/2)*(f + g*x)^6) + (5*c^
6*d^6*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*
d*f - a*e*g]*Sqrt[d + e*x])])/(512*g^(7/2)*(c*d*f - a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx}{12g}$$

$$= -\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{12g^2(d+ex)^{3/2}(f+gx)^5}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+}$$

Mathematica [C] time = 0.08, size = 79, normalized size = 0.17

$$\frac{2c^6d^6((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},7;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^7),x]
```

```
[Out] (2*c^6*d^6*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 7, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^7*(d + e*x)^(7/2))
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^7),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.55, size = 3872, normalized size = 8.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="fricas")
```

```
[Out] [1/3072*(15*(c^6*d^6*e*g^6*x^7 + c^6*d^7*f^6 + (6*c^6*d^6*e*f*g^5 + c^6*d^7
*g^6)*x^6 + 3*(5*c^6*d^6*e*f^2*g^4 + 2*c^6*d^7*f*g^5)*x^5 + 5*(4*c^6*d^6*e*
f^3*g^3 + 3*c^6*d^7*f^2*g^4)*x^4 + 5*(3*c^6*d^6*e*f^4*g^2 + 4*c^6*d^7*f^3*g
^3)*x^3 + 3*(2*c^6*d^6*e*f^5*g + 5*c^6*d^7*f^4*g^2)*x^2 + (c^6*d^6*e*f^6 +
6*c^6*d^7*f^5*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f +
2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
 (e*f + d*g)*x)) - 2*(15*c^6*d^6*f^6*g - 5*a*c^5*d^5*e*f^5*g^2 - 2*a^2*c^4*
d^4*e^2*f^4*g^3 - 440*a^3*c^3*d^3*e^3*f^3*g^4 + 1072*a^4*c^2*d^2*e^4*f^2*g^
5 - 896*a^5*c*d*e^5*f*g^6 + 256*a^6*e^6*g^7 - 15*(c^6*d^6*f*g^6 - a*c^5*d^5
*e*g^7)*x^5 - 5*(17*c^6*d^6*f^2*g^5 - 19*a*c^5*d^5*e*f*g^6 + 2*a^2*c^4*d^4*
e^2*g^7)*x^4 - 2*(99*c^6*d^6*f^3*g^4 - 127*a*c^5*d^5*e*f^2*g^5 + 32*a^2*c^4
*d^4*e^2*f*g^6 - 4*a^3*c^3*d^3*e^3*g^7)*x^3 + 6*(33*c^6*d^6*f^4*g^3 - 231*a
*c^5*d^5*e*f^3*g^4 + 410*a^2*c^4*d^4*e^2*f^2*g^5 - 284*a^3*c^3*d^3*e^3*f*g^
6 + 72*a^4*c^2*d^2*e^4*g^7)*x^2 + (85*c^6*d^6*f^5*g^2 - 29*a*c^5*d^5*e*f^4*
g^3 - 1328*a^2*c^4*d^4*e^2*f^3*g^4 + 2968*a^3*c^3*d^3*e^3*f^2*g^5 - 2336*a^
4*c^2*d^2*e^4*f*g^6 + 640*a^5*c*d*e^5*g^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^10*g^4 - 4*a*c^3*d^4*e*f^9*g^5 + 6
*a^2*c^2*d^3*e^2*f^8*g^6 - 4*a^3*c*d^2*e^3*f^7*g^7 + a^4*d*e^4*f^6*g^8 + (c
^4*d^4*e*f^4*g^10 - 4*a*c^3*d^3*e^2*f^3*g^11 + 6*a^2*c^2*d^2*e^3*f^2*g^12 -
 4*a^3*c*d*e^4*f*g^13 + a^4*e^5*g^14)*x^7 + (6*c^4*d^4*e*f^5*g^9 + a^4*d*e^
4*g^14 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^10 - 4*(a*c^3*d^4*e - 9*a^2*c^2
*d^2*e^3)*f^3*g^11 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^2*g^12 - 2*(2*a^
3*c*d^2*e^3 - 3*a^4*e^5)*f*g^13)*x^6 + 3*(5*c^4*d^4*e*f^6*g^8 + 2*a^4*d*e^4
*f*g^13 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^9 - 2*(4*a*c^3*d^4*e - 15*a^
2*c^2*d^2*e^3)*f^4*g^10 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^3*c*d*e^4)*f^3*g^11 -
(8*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^12)*x^5 + 5*(4*c^4*d^4*e*f^7*g^7 + 3*a^
4*d*e^4*f^2*g^12 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^8 - 12*(a*c^3*d^4*e
 - 2*a^2*c^2*d^2*e^3)*f^5*g^9 + 2*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^4*g
^10 - 4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^11)*x^4 + 5*(3*c^4*d^4*e*f^8*g^6
+ 4*a^4*d*e^4*f^3*g^11 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*g^7 - 2*(8*a*c^3
*d^4*e - 9*a^2*c^2*d^2*e^3)*f^6*g^8 + 12*(2*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*
f^5*g^9 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^10)*x^3 + 3*(2*c^4*d^4*e*f^9
*g^5 + 5*a^4*d*e^4*f^4*g^10 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8*g^6 - 4*(5*
a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^7 + 2*(15*a^2*c^2*d^3*e^2 - 4*a^3*c*
d*e^4)*f^6*g^8 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^9)*x^2 + (c^4*d^4*e*f
^10*g^4 + 6*a^4*d*e^4*f^5*g^9 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9*g^5 - 6
*(4*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^6 + 4*(9*a^2*c^2*d^3*e^2 - a^3*c*d
*e^4)*f^7*g^7 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^8)*x), -1/1536*(15*(c^6*
d^6*e*g^6*x^7 + c^6*d^7*f^6 + (6*c^6*d^6*e*f*g^5 + c^6*d^7*g^6)*x^6 + 3*(5*
c^6*d^6*e*f^2*g^4 + 2*c^6*d^7*f*g^5)*x^5 + 5*(4*c^6*d^6*e*f^3*g^3 + 3*c^6*d
^7*f^2*g^4)*x^4 + 5*(3*c^6*d^6*e*f^4*g^2 + 4*c^6*d^7*f^3*g^3)*x^3 + 3*(2*c^
6*d^6*e*f^5*g + 5*c^6*d^7*f^4*g^2)*x^2 + (c^6*d^6*e*f^6 + 6*c^6*d^7*f^5*g)*
x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 +
a*e^2)*g*x)) + (15*c^6*d^6*f^6*g - 5*a*c^5*d^5*e*f^5*g^2 - 2*a^2*c^4*d^4*e^
```

```
2*f^4*g^3 - 440*a^3*c^3*d^3*e^3*f^3*g^4 + 1072*a^4*c^2*d^2*e^4*f^2*g^5 - 89
6*a^5*c*d*e^5*f*g^6 + 256*a^6*e^6*g^7 - 15*(c^6*d^6*f*g^6 - a*c^5*d^5*e*g^7
)*x^5 - 5*(17*c^6*d^6*f^2*g^5 - 19*a*c^5*d^5*e*f*g^6 + 2*a^2*c^4*d^4*e^2*g^
7)*x^4 - 2*(99*c^6*d^6*f^3*g^4 - 127*a*c^5*d^5*e*f^2*g^5 + 32*a^2*c^4*d^4*e
^2*f*g^6 - 4*a^3*c^3*d^3*e^3*g^7)*x^3 + 6*(33*c^6*d^6*f^4*g^3 - 231*a*c^5*d
^5*e*f^3*g^4 + 410*a^2*c^4*d^4*e^2*f^2*g^5 - 284*a^3*c^3*d^3*e^3*f*g^6 + 72
*a^4*c^2*d^2*e^4*g^7)*x^2 + (85*c^6*d^6*f^5*g^2 - 29*a*c^5*d^5*e*f^4*g^3 -
1328*a^2*c^4*d^4*e^2*f^3*g^4 + 2968*a^3*c^3*d^3*e^3*f^2*g^5 - 2336*a^4*c^2*
d^2*e^4*f*g^6 + 640*a^5*c*d*e^5*g^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^10*g^4 - 4*a*c^3*d^4*e*f^9*g^5 + 6*a^2*c
^2*d^3*e^2*f^8*g^6 - 4*a^3*c*d^2*e^3*f^7*g^7 + a^4*d*e^4*f^6*g^8 + (c^4*d^4
*e*f^4*g^10 - 4*a*c^3*d^3*e^2*f^3*g^11 + 6*a^2*c^2*d^2*e^3*f^2*g^12 - 4*a^3
*c*d*e^4*f*g^13 + a^4*e^5*g^14)*x^7 + (6*c^4*d^4*e*f^5*g^9 + a^4*d*e^4*g^14
 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^10 - 4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e
^3)*f^3*g^11 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^2*g^12 - 2*(2*a^3*c*d^
2*e^3 - 3*a^4*e^5)*f*g^13)*x^6 + 3*(5*c^4*d^4*e*f^6*g^8 + 2*a^4*d*e^4*f*g^1
3 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^9 - 2*(4*a*c^3*d^4*e - 15*a^2*c^2*
d^2*e^3)*f^4*g^10 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^3*c*d*e^4)*f^3*g^11 - (8*a^3
*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^12)*x^5 + 5*(4*c^4*d^4*e*f^7*g^7 + 3*a^4*d*e^
4*f^2*g^12 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^8 - 12*(a*c^3*d^4*e - 2*a
^2*c^2*d^2*e^3)*f^5*g^9 + 2*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^4*g^10 -
4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^11)*x^4 + 5*(3*c^4*d^4*e*f^8*g^6 + 4*a^
4*d*e^4*f^3*g^11 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*g^7 - 2*(8*a*c^3*d^4*e
 - 9*a^2*c^2*d^2*e^3)*f^6*g^8 + 12*(2*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^
9 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^10)*x^3 + 3*(2*c^4*d^4*e*f^9*g^5 +
 5*a^4*d*e^4*f^4*g^10 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8*g^6 - 4*(5*a*c^3*
d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^7 + 2*(15*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)
*f^6*g^8 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^9)*x^2 + (c^4*d^4*e*f^10*g^
4 + 6*a^4*d*e^4*f^5*g^9 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9*g^5 - 6*(4*a*
c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^6 + 4*(9*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*
f^7*g^7 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^8)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 1261, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x)
```

```
[Out] 1/1536*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*c^6*d^6*f^6-15*x^5*c^5*d^5*g^5*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)-85*x^4*c^5*d^5*f*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)-198*x^3*c^5*d^5*f^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)+198*x^2*c^5*d^5*f^3*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+85*
x*c^5*d^5*f^4*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+10*x^4*a*c^4*d^4*
e*g^5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^6*c^6*d^6*g^6+90*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^5*c^6*d^6*f*g^5+225*arctanh((c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2)*g)*x^4*c^6*d^6*f^2*g^4+300*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^6*d^6*f^3*g^3+225*arctanh((c*d*x+a*e)^(1/
```

```
2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^6*d^6*f^4*g^2+90*arctanh((c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^6*d^6*f^5*g-8*x^3*a^2*c^3*d^3*e^2*g^5*(c
*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-432*x^2*a^3*c^2*d^2*e^3*g^5*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-640*x*a^4*c*d*e^4*g^5*(c*d*x+a*e)^(1/2)*
((a*e*g-c*d*f)*g)^(1/2)+640*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^4*c
*d*e^4*f*g^4-432*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c^2*d^2*e^3*
f^2*g^3+8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^3*d^3*e^2*f^3*g^2
+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^4*d^4*e*f^4*g+56*x^3*a*c^
4*d^4*e*f*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+1272*x^2*a^2*c^3*d^
3*e^2*f*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1188*x^2*a*c^4*d^4*e*
f^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+1696*x*a^3*c^2*d^2*e^3*f*
g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1272*x*a^2*c^3*d^3*e^2*f^2*g^
3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+56*x*a*c^4*d^4*e*f^3*g^2*(c*d*x
+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-256*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)
^(1/2)*a^5*e^5*g^5+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^5*d^5*f^5
)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^6/g^3/(a*e*g-c*d*f)/(a^2*e^
2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f + g\,x\right)^7\,(d + e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(5
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**7,x)
```

```
[Out] Timed out
```

3.479 $\int \frac{\sqrt{d+ex}\,(f+gx)^{5/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=313

$$\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8c^3d^3\sqrt{d+ex}}$$

Rubi [A] time = 0.56, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {870, 891, 63, 217, 206}

$$\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8c^3d^3\sqrt{d+ex}}+\frac{5(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{12c^2d^2\sqrt{d+ex}}+\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*c^3*d^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^2*d^2*Sqrt[d + e*x]) + ((f + g*
x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x])
 + (5*(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(7/2)*d^(7/2)*Sqrt[
g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^{5/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cd\sqrt{d+ex}} + \frac{\left(5\left(cde^2f+cd^2eg-e\left(cd^2+\right.\right.\right.}{6}$$

$$= \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade}}{3c}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+}{}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+}{}$$

Mathematica [A] time = 0.61, size = 269, normalized size = 0.86

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\sqrt{ae+cdx}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\left(15a^2e^2g^2-10acdeg(4f+gx)+c^2d^2\left(33f^2+26fgx+8g^2x^2\right)\right)+15\sqrt{cd}\,(cdf-aeg)^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)\right)}{24c^{7/2}d^{7/2}\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[f + g*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqr
t[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*(15*a^2*e^2*g^2 - 10*a
*c*d*e*g*(4*f + g*x) + c^2*d^2*(33*f^2 + 26*f*g*x + 8*g^2*x^2)) + 15*Sqrt[c
*d]*(c*d*f - a*e*g)^(5/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x
])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(24*c^(7/2)*d^(7/2)*Sqrt[g]*Sqrt[(a*e
 + c*d*x)*(d + e*x)]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)])
```

IntegrateAlgebraic [A] time = 8.64, size = 238, normalized size = 0.76

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{5(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}}+\frac{(cdf-aeg)^3\left(\frac{33c^2d^2\sqrt{ae+cdx}}{\sqrt{f+gx}}+\frac{15g^2(ae+cdx)^{5/2}}{(f+gx)^{5/2}}-\frac{40cdg(ae+cdx)^{3/2}}{(f+gx)^{3/2}}\right)}{24c^3d^3\left(cd-\frac{g(ae+cdx)}{f+gx}\right)^3}\right)}{\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(((c*d*f - a*e*g)^3*((15*g^2*(a*e + c*d*x)
^(5/2))/(f + g*x)^(5/2) - (40*c*d*g*(a*e + c*d*x)^(3/2))/(f + g*x)^(3/2) +
(33*c^2*d^2*Sqrt[a*e + c*d*x])/Sqrt[f + g*x]))/(24*c^3*d^3*(c*d - (g*(a*e +
 c*d*x))/(f + g*x))^3) + (5*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[g]*Sqrt[a*e + c
*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(7/2)*d^(7/2)*Sqrt[g])))/Sqrt
[(a*e + c*d*x)*(d + e*x)]
```

fricas [A] time = 1.56, size = 841, normalized size = 2.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 33*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 15*
a^2*c*d*e^2*g^3 + 2*(13*c^3*d^3*f*g^2 - 5*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^
3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*
e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(
c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^
2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^
3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
+ (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*g*x + c^4*d^5*g),
1/48*(2*(8*c^3*d^3*g^3*x^2 + 33*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 15*a
^2*c*d*e^2*g^3 + 2*(13*c^3*d^3*f*g^2 - 5*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^3
 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e
*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-
c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sq
rt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (
2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*g*x + c^4*d^5*g)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.04, size = 511, normalized size = 1.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/48*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/
2))*a^3*e^3*g^3-45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1
/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*c*d*g*x+a*
e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^2
*d^2*e*f^2*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)
*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^3*d^3*f^3-16*x^2*c^2*d^2*g^2*((g*x+f)*(c*d
*x+a*e))^(1/2)*(d*g*c)^(1/2)+20*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*x
*a*c*d*e*g^2-52*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*x*c^2*d^2*f*g-30*
((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*e^2*g^2+80*((g*x+f)*(c*d*x+a*
e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*f*g-66*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/c^3/d^3/((g*x+f)*(c*d*x+a*e))^(1/2)/(d*g*c
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^{\frac{5}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^(5/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^{5/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

3.480 $\int \frac{\sqrt{d+ex}\,(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=244

$$\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4c^2d^2\sqrt{d+ex}}+$$

Rubi [A] time = 0.37, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {870, 891, 63, 217, 206}

$$\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4c^2d^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(4*c^2*d^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sq
rt[f + g*x])])/(4*c^(5/2)*d^(5/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2cd\sqrt{d+ex}} + \frac{\left(3\left(cde^2f+cd^2eg-e\left(cd^2+a\right.\right.\right.}{4}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+}}{2cd}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+}}{2cd}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+}}{2cd}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+}}{2cd}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+}}{2cd}$$

Mathematica [A] time = 0.47, size = 234, normalized size = 0.96

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\sqrt{ae+cdx}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}(cd(5f+2gx)-3aeg)+3\sqrt{cd}\,(cdf-aeg)^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)\right)}{4c^{5/2}d^{5/2}\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[f + g*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqr
t[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*(-3*a*e*g + c*d*(5*f +
 2*g*x)) + 3*Sqrt[c*d]*(c*d*f - a*e*g)^(3/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[
g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(4*c^(5/2)*d^(5/2)
*Sqrt[g]*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)
])
```

IntegrateAlgebraic [A] time = 7.75, size = 206, normalized size = 0.84

$$\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\left(\frac{3(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}}+\frac{(cdf-aeg)^2\left(\frac{5cd\sqrt{ae+cdx}}{\sqrt{f+gx}}-\frac{3g(ae+cdx)^{3/2}}{(f+gx)^{3/2}}\right)}{4c^2d^2\left(cd-\frac{g(ae+cdx)}{f+gx}\right)^2}\right)}{\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(((c*d*f - a*e*g)^2*((-3*g*(a*e + c*d*x)^(
3/2))/(f + g*x)^(3/2) + (5*c*d*Sqrt[a*e + c*d*x])/Sqrt[f + g*x]))/(4*c^2*d^
2*(c*d - (g*(a*e + c*d*x))/(f + g*x))^2) + (3*(c*d*f - a*e*g)^2*ArcTanh[(Sq
rt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*c^(5/2)*d^(5/
2)*Sqrt[g])))/Sqrt[(a*e + c*d*x)*(d + e*x)]
```

fricas [A] time = 1.24, size = 655, normalized size = 2.68

$$\left[\frac{4\left(2c^2d^2g^2x+5c^2d^2fg-3acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f}+3\left(c^2d^3f^2-2acd^2efg+a^2de^2g^2+\left(c^2d^2ef^2-2acde^2fg+a^2e^3g^2\right)x\right)\sqrt{cdg}\,\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdgx+cdf+aeg\right)\sqrt{cdg}\,\sqrt{ex+d}\,\sqrt{gx+f}+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+\left(c^2d^2ef^2+2\left(4c^2d^3+3acde^2\right)fg+\left(8acd^2e+a^2e^3\right)g^2\right)x}{ex+d}\right)}{16\left(c^3d^3egx+c^3d^4g\right)},\ \frac{2\left(2c^2d^2g^2x+5c^2d^2fg-3acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f}-3\left(c^2d^3f^2-2acd^2efg+a^2de^2g^2+\left(c^2d^2ef^2-2acde^2fg+a^2e^3g^2\right)x\right)\sqrt{-cdg}\,\arctan\left(\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{-cdg}\,\sqrt{ex+d}\,\sqrt{gx+f}}{2cdegx^2+cd^2f+adeg+\left(cdef+\left(2cd^2+ae^2\right)g\right)x}\right)}{8\left(c^3d^3egx+c^3d^4g\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e*g^2)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 -
2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^
3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e
*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*
e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*
a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g*x
 + c^3*d^4*g), 1/8*(2*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e*g^2)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c
^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2
*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d
^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*g*x + c^3*
d^4*g)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 328, normalized size = 1.34

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\left(3a^2e^2g^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-6acdefg\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+3c^2d^2f^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+4\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,cdgx-6\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,aeg+10\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,cdf\right)}{8\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/8*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*c*d*
g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
*a^2*e^2*g^2-6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*
(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+4*(c
*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*x*c*d*g-6*(c*d*g)^(1/2)*((g*x+f)*(c
*d*x+a*e))^(1/2)*a*e*g+10*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/
(e*x+d)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/c^2/d^2/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{3/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\left(f+gx\right)^{\frac{3}{2}}}{\sqrt{\left(d+ex\right)\left(ae+cdx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.481 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Rubi [A] time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {870, 891, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e
*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)\cdots\right.}{2\cdots}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)\cdots\right.\right.}{2cde^2\sqrt{ade\cdots}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)\cdots\right.\right.}{c^2\cdots}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)\cdots\right.\right.}{c^2d^2e^2\cdots}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\cdots}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{ade+\left(cd^2+\cdots\right.}}$$

Mathematica [A] time = 0.19, size = 213, normalized size = 1.26

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\sqrt{ae+cdx}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}+\sqrt{cd}\sqrt{cdf-aeg}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[f + g*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqr
t[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)] + Sqrt[c*d]*Sqrt[c*d*f
 - a*e*g]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sq
rt[c*d*f - a*e*g])]))/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[(a*e + c*d*x)*(d + e*x)
]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)])
```

IntegrateAlgebraic [A] time = 0.71, size = 171, normalized size = 1.01

$$\frac{\sqrt{d+ex}\sqrt{aeg+cdgx}\left(\frac{\sqrt{f+gx}\sqrt{aeg+cd(f+gx)-cdf}}{cd\sqrt{g}}-\frac{\sqrt{cd}\,(cdf-aeg)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{c^2d^2\sqrt{g}}\right)}{\sqrt{g}\sqrt{\frac{(dg+egx)(aeg+cdgx)}{g^2}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2],x]
```

```
[Out] (Sqrt[d + e*x]*Sqrt[a*e*g + c*d*g*x]*((Sqrt[f + g*x]*Sqrt[-(c*d*f) + a*e*g
+ c*d*(f + g*x)])/(c*d*Sqrt[g]) - (Sqrt[c*d]*(c*d*f - a*e*g)*Log[-(Sqrt[c*d
]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]])/(c^2*d^2*Sqrt[g
])))/(Sqrt[g]*Sqrt[((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2])
```

fricas [A] time = 1.15, size = 521, normalized size = 3.08

$$\left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}\,cdg-\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{cdg}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2-4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdgx+cdf+aeg\right)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+\left(c^2d^2ef^2+2\left(4c^2d^3+3acde^2\right)fg+\left(8acd^2e+a^2e^3\right)g^2\right)x}{ex+d}\right)}{4\left(c^2d^2egx+c^2d^3g\right)},\ \frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}\,cdg-\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{-cdg}\arctan\left(\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdg}\sqrt{ex+d}\sqrt{gx+f}}{2cdegx^2+cd^2f+adeg+\left(cdef+\left(2cd^2+ae^2\right)g\right)x}\right)}{2\left(c^2d^2egx+c^2d^3g\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(-(
8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c
*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2
)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*
e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g*x + c^2*d^3*g), 1/2*(2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*g - (
c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/
(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(
c^2*d^2*e*g*x + c^2*d^3*g)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 201, normalized size = 1.19

$$-\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(aeg\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-cdf\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-2\sqrt{(gx+f)\,(cdx+ae)}\,\sqrt{cdg}\right)}{2\sqrt{ex+d}\,\sqrt{(gx+f)\,(cdx+ae)}\,\sqrt{cdg}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(e*x+d)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/2*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(ln(1/2*(2*c*d*g
*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*
a*e*g-ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2))/(c*d*g)^(1/2))*c*d*f-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(e*
x+d)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/c/d/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,\sqrt{gx+f}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*sqrt(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.482 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=105

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {891, 63, 217, 206}

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae+cdx}\right)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.11, size = 160, normalized size = 1.52

$$\frac{2\sqrt{cd}\sqrt{d+ex}\sqrt{ae+cdx}\sqrt{cdf-aeg}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (2*Sqrt[c*d]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[(c*d*
(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[(a*e +
 c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.75, size = 167, normalized size = 1.59

$$-\frac{2\sqrt{d+ex}\sqrt{aeg+cdgx}\left(\sqrt{aeg+cdgx}-\sqrt{cd}\sqrt{f+gx}\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{g\sqrt{\frac{(dg+egx)(aeg+cdgx)}{g^2}}\left(\sqrt{cd}\sqrt{aeg+cdgx}-cd\sqrt{f+gx}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[a*e*g + c*d*g*x]*(-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[
a*e*g + c*d*g*x])*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*g +
c*d*(f + g*x)]])/(g*Sqrt[((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2]*(-(c*d*Sqrt
[f + g*x]) + Sqrt[c*d]*Sqrt[a*e*g + c*d*g*x]))
```

fricas [A] time = 1.04, size = 343, normalized size = 3.27

$$\left[\frac{\sqrt{cdg}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdgx+cdf+aeg\right)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+\left(c^2d^2ef^2+2\left(4c^2d^3+3acde^2\right)fg+\left(8acd^2e+a^2e^3\right)g^2\right)x}{ex+d}\right)}{2cdg}, -\frac{\sqrt{-cdg}\arctan\left(\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdg}\sqrt{ex+d}\sqrt{gx+f}}{2cdegx^2+cd^2f+adeg+\left(cdef+\left(2cd^2+ae^2\right)g\right)x}\right)}{cdg}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
+ a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
+ c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g
 + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d
*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d))/(c*d*g), -sqrt(-c*d*
g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e
*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*
d^2 + a*e^2)*g)*x))/(c*d*g)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 120, normalized size = 1.14

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)}{\sqrt{ex+d}\,\sqrt{cdg}\,\sqrt{cdg\,x^2+aegx+cdfx+aef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/(e*x+d)^(1/2)*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))/(c*d*g)^(1/2)/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)), x)
```

3.483 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)}$$

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {860}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)*Sqrt[d + e
*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,\sqrt{f+gx}}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.82

$$\frac{2\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/((c*d*f - a*e*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])
```

IntegrateAlgebraic [B] time = 0.64, size = 143, normalized size = 2.34

$$\frac{2\sqrt{d+ex}\,(ef+egx)^{3/2}\sqrt{ae^2+cdex}\,\sqrt{ae^2-cd^2+cd(d+ex)}}{e^2\sqrt{\frac{(d+ex)\left(ae^2+cdex\right)}{e}}\,\sqrt{g(d+ex)-dg+ef}\,\left(\frac{g(d+ex)-dg+ef}{e}\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[d + e*x]*Sqrt[a*e^2 + c*d*e*x]*(e*f + e*g*x)^(3/2)*Sqrt[-(c*d^2) +
a*e^2 + c*d*(d + e*x)])/(e^2*(c*d*f - a*e*g)*Sqrt[((d + e*x)*(a*e^2 + c*d*e
*x))/e]*Sqrt[e*f - d*g + g*(d + e*x)]*((e*f - d*g + g*(d + e*x))/e)^(3/2))
```

fricas [B] time = 0.43, size = 114, normalized size = 1.87

$$\frac{2\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\sqrt{ex + d}\,\sqrt{gx + f}}{cd^2f^2 - adefg + \left(cdefg - ae^2g^2\right)x^2 + \left(cdef^2 - adeg^2 + \left(cd^2 - ae^2\right)fg\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(
c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^
2 + (c*d^2 - a*e^2)*f*g)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 63, normalized size = 1.03

$$-\frac{2\,(cdx + ae)\,\sqrt{ex + d}}{\sqrt{gx + f}\,\left(aeg - cdf\right)\sqrt{cde\,x^2 + a\,e^2x + c\,d^2x + ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/(g*x+f)^(1/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(e*x+d)^(1/2)/(c*d*e*x^2+a*e^2*x
+c*d^2*x+a*d*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex + d}}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\left(gx + f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(3/2)), x)
```

mupad [B] time = 4.64, size = 100, normalized size = 1.64

$$-\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(x\,\sqrt{f+g\,x}-\frac{\sqrt{f+g\,x}\,\left(c\,d^2\,f-a\,d\,e\,g\right)}{a\,e^2\,g-c\,d\,e\,f}\right)\,\left(a\,e^2\,g-c\,d\,e\,f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((x*(f +
 g*x)^(1/2) - ((f + g*x)^(1/2)*(c*d^2*f - a*d*e*g))/(a*e^2*g - c*d*e*f))*(a
*e^2*g - c*d*e*f))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)), x)
```

3.484 $\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)}$$

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{3/2}}+\frac{(2cd)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right.}}}{3(cdf-aeg)}$$

$$=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{3/2}}+\frac{4cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{3(cdf-aeg)^2\sqrt{d+ex}\sqrt{f}}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\,(cd(3f+2gx)-aeg)}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a*e*g) + c*d*(3*f + 2*g*x)))/(3*(c*d*f
- a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2))
```

IntegrateAlgebraic [A] time = 3.44, size = 196, normalized size = 1.52

$$\frac{2\sqrt{d+ex}\,(ef+egx)^{5/2}\sqrt{ae^2+cdex}\left(\frac{3cd\sqrt{ae^2-cd^2+cd(d+ex)}}{\sqrt{g(d+ex)-dg+ef}}-\frac{g\left(ae^2-cd^2+cd(d+ex)\right)^{3/2}}{(g(d+ex)-dg+ef)^{3/2}}\right)}{3e^3\sqrt{\frac{(d+ex)\left(ae^2+cdex\right)}{e}}\left(\frac{g(d+ex)-dg+ef}{e}\right)^{5/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[d + e*x]*Sqrt[a*e^2 + c*d*e*x]*(e*f + e*g*x)^(5/2)*(-((g*(-(c*d^2)
+ a*e^2 + c*d*(d + e*x))^(3/2))/(e*f - d*g + g*(d + e*x))^(3/2)) + (3*c*d*S
qrt[-(c*d^2) + a*e^2 + c*d*(d + e*x)])/Sqrt[e*f - d*g + g*(d + e*x)]))/(3*e
^3*(c*d*f - a*e*g)^2*Sqrt[((d + e*x)*(a*e^2 + c*d*e*x))/e]*((e*f - d*g + g*
(d + e*x))/e)^(5/2))
```

fricas [B] time = 0.42, size = 288, normalized size = 2.23

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdgx+3\,cdf-aeg\right)\sqrt{ex+d}\,\sqrt{gx+f}}{3\left(c^2d^3f^4-2\,acd^2ef^3g+a^2de^2f^2g^2+\left(c^2d^2ef^2g^2-2\,acde^2fg^3+a^2e^3g^4\right)x^3+\left(2\,c^2d^2ef^3g+a^2de^2g^4+\left(c^2d^3-4\,acde^2\right)f^2g^2-2\left(acd^2e-a^2e^3\right)fg^3\right)x^2+\left(c^2d^2ef^4+2\,a^2de^2fg^3+2\left(c^2d^3-acde^2\right)f^3g-\left(4\,acd^2e-a^2e^3\right)f^2g^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*
g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^4 - 2*a*c*d^2*e*f^3*g + a^2*d*e^2
*f^2*g^2 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*g^3 + a^2*e^3*g^4)*x^3 + (2*c
^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^2 - 2*(a*c*d
^2*e - a^2*e^3)*f*g^3)*x^2 + (c^2*d^2*e*f^4 + 2*a^2*d*e^2*f*g^3 + 2*(c^2*d^
3 - a*c*d*e^2)*f^3*g - (4*a*c*d^2*e - a^2*e^3)*f^2*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 98, normalized size = 0.76

$$-\frac{2\,(cdx+ae)\left(-2cdgx+aeg-3cdf\right)\sqrt{ex+d}}{3\left(gx+f\right)^{\frac{3}{2}}\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-2*c*d*g*x+a*e*g-3*c*d*f)*(e*x+d)^(1/2)/(g*x+f)^(3/2)/(a^
2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(5/2)), x)
```

mupad [B] time = 4.90, size = 147, normalized size = 1.14

$$-\frac{\left(\frac{\left(2\,a\,e\,g-6\,c\,d\,f\right)\,\sqrt{d+e\,x}}{3\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{4\,c\,d\,x\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{d\,f\,\sqrt{f+g\,x}}{e\,g}+\frac{x\,\sqrt{f+g\,x}\,\left(d\,g+e\,f\right)}{e\,g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((((2*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*e*g*(a*e*g - c*d*f)^2) - (4*c*d
*x*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c
*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (d*f*(f + g*x)^(1/2))/(e*g) + (x*(f
 + g*x)^(1/2)*(d*g + e*f))/(e*g))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

3.485 $\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^3}+\frac{8cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)}$$

Rubi [A] time = 0.22, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^3}+\frac{8cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(5/2)) + (8*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(15*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) + (16*c^2*d^2*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*
Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{5/2}} + \frac{(4cd)\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right.}}}{5(cdf-aeg)}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{5/2}} + \frac{8cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{15(cdf-aeg)^2\sqrt{d+ex}\,(f+}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{5/2}} + \frac{8cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{15(cdf-aeg)^2\sqrt{d+ex}\,(f+}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3a^2e^2g^2-2acdeg(5f+2gx)+c^2d^2\left(15f^2+20fgx+8g^2x^2\right)\right)}{15\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3*a^2*e^2*g^2 - 2*a*c*d*e*g*(5*f + 2*g*x)
 + c^2*d^2*(15*f^2 + 20*f*g*x + 8*g^2*x^2)))/(15*(c*d*f - a*e*g)^3*Sqrt[d +
 e*x]*(f + g*x)^(5/2))
```

IntegrateAlgebraic [A] time = 8.50, size = 137, normalized size = 0.69

$$\frac{2\sqrt{d+ex}\,\sqrt{ae+cdx}\left(\frac{15c^2d^2\sqrt{ae+cdx}}{\sqrt{f+gx}}+\frac{3g^2(ae+cdx)^{5/2}}{(f+gx)^{5/2}}-\frac{10cdg(ae+cdx)^{3/2}}{(f+gx)^{3/2}}\right)}{15\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((3*g^2*(a*e + c*d*x)^(5/2))/(f + g*x)^(
5/2) - (10*c*d*g*(a*e + c*d*x)^(3/2))/(f + g*x)^(3/2) + (15*c^2*d^2*Sqrt[a*
e + c*d*x])/Sqrt[f + g*x]))/(15*(c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d + e
*x)])
```

fricas [B] time = 0.45, size = 572, normalized size = 2.89

$$\frac{2\left(8c^2d^2g^2x^2+15c^2d^2f^2-10acdefg+3a^2e^2g^2+4\left(5c^2d^2fg-acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f}}{15\left(c^3d^4f^6-3ac^2d^3ef^5g+3a^2cd^2e^2f^4g^2-a^3de^3f^3g^3+\left(c^3d^3ef^3g^3-3ac^2d^2e^2f^2g^4+3a^2cde^3fg^5-a^3e^4g^6\right)x^4+\left(3c^3d^3ef^4g^2-a^3de^3g^6+\left(c^3d^4-9ac^2d^2e^2\right)f^3g^3-3\left(ac^2d^3e-3a^2cde^3\right)f^2g^4+3\left(a^2cd^2e^2-a^3e^4\right)fg^5\right)x^3+3\left(c^3d^3ef^5g-a^3de^3fg^5+\left(c^3d^4-3ac^2d^2e^2\right)f^4g^2-3\left(ac^2d^3e-a^2cde^3\right)f^3g^3+\left(3a^2cd^2e^2-a^3e^4\right)f^2g^4\right)x^2+\left(c^3d^3ef^6-3a^3de^3f^2g^4+3\left(c^3d^4-ac^2d^2e^2\right)f^5g-3\left(3ac^2d^3e-a^2cde^3\right)f^4g^2+\left(9a^2cd^2e^2-a^3e^4\right)f^3g^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 3*a^2*e^2*g^2 +
 4*(5*c^2*d^2*f*g - a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^6 - 3*a*c^2*d^3*e*f^5*g + 3*a^2
*c*d^2*e^2*f^4*g^2 - a^3*d*e^3*f^3*g^3 + (c^3*d^3*e*f^3*g^3 - 3*a*c^2*d^2*e
^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^4 + (3*c^3*d^3*e*f^4*g^2
- a^3*d*e^3*g^6 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g^3 - 3*(a*c^2*d^3*e - 3*
a^2*c*d*e^3)*f^2*g^4 + 3*(a^2*c*d^2*e^2 - a^3*e^4)*f*g^5)*x^3 + 3*(c^3*d^3*
```

```
e*f^5*g - a^3*d*e^3*f*g^5 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^2 - 3*(a*c^2*
d^3*e - a^2*c*d*e^3)*f^3*g^3 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^4)*x^2 + (
c^3*d^3*e*f^6 - 3*a^3*d*e^3*f^2*g^4 + 3*(c^3*d^4 - a*c^2*d^2*e^2)*f^5*g - 3
*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^2 + (9*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^
3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 169, normalized size = 0.85

$$-\frac{2\left(cdx+ae\right)\left(8g^{2}x^{2}c^{2}d^{2}-4acde\,g^{2}x+20c^{2}d^{2}fgx+3a^{2}e^{2}g^{2}-10acdefg+15f^{2}c^{2}d^{2}\right)\sqrt{ex+d}}{15\left(gx+f\right)^{\frac{5}{2}}\left(a^{3}e^{3}g^{3}-3a^{2}cd\,e^{2}f\,g^{2}+3a\,c^{2}d^{2}e\,f^{2}g-f^{3}c^{3}d^{3}\right)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(7/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-4*a*c*d*e*g^2*x+20*c^2*d^2*f*g*x+3*a^2
*e^2*g^2-10*a*c*d*e*f*g+15*c^2*d^2*f^2)*(e*x+d)^(1/2)/(g*x+f)^(5/2)/(a^3*e^
3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(c*d*e*x^2+a*e^2
*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(7/2)), x)
```

mupad [B] time = 5.17, size = 242, normalized size = 1.22

$$-\frac{\left(\frac{\sqrt{d+e\,x}\,\left(6\,a^2\,e^2\,g^2-20\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2\right)}{15\,e\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^2\,d^2\,x^2\,\sqrt{d+e\,x}}{15\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c\,d\,x\,\left(a\,e\,g-5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{15\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3\,\sqrt{f+g\,x}+\frac{d\,f^2\,\sqrt{f+g\,x}}{e\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(d\,g+2\,e\,f\right)}{e\,g}+\frac{f\,x\,\sqrt{f+g\,x}\,\left(2\,d\,g+e\,f\right)}{e\,g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((((d + e*x)^(1/2)*(6*a^2*e^2*g^2 + 30*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(15*
e*g^2*(a*e*g - c*d*f)^3) + (16*c^2*d^2*x^2*(d + e*x)^(1/2))/(15*e*(a*e*g -
c*d*f)^3) - (8*c*d*x*(a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*e*g*(a*e*g - c*
d*f)^3))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^(1/2
```

```
) + (d*f^2*(f + g*x)^(1/2))/(e*g^2) + (x^2*(f + g*x)^(1/2)*(d*g + 2*e*f))/(
e*g) + (f*x*(f + g*x)^(1/2)*(2*d*g + e*f))/(e*g^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

3.486 $\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^4}+\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}+\frac{12cd\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{35\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg}$$

Rubi [A] time = 0.31, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{32c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^4}+\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}+\frac{12cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7\sqrt{d+ex}\,(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(7/2)) + (12*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(35*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) + (16*c^2*d^2*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*(c*d*f - a*e*g)^3*Sqrt[d + e*x]
*(f + g*x)^(3/2)) + (32*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(35*(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{(6cd)\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right.}}}{7(cdf-aeg)}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{35(cdf-aeg)^2\sqrt{d+ex}\,(f}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{35(cdf-aeg)^2\sqrt{d+ex}\,(f+}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+}}{35(cdf-aeg)^2\sqrt{d+ex}\,(f+}$$

Mathematica [A] time = 0.12, size = 152, normalized size = 0.57

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(-5a^3e^3g^3+3a^2cde^2g^2(7f+2gx)-ac^2d^2eg\left(35f^2+28fgx+8g^2x^2\right)+c^3d^3\left(35f^3+70f^2gx+56fg^2x^2+16g^3x^3\right)\right)}{35\sqrt{d+ex}\,(f+gx)^{7/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-5*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(7*f +
 2*g*x) - a*c^2*d^2*e*g*(35*f^2 + 28*f*g*x + 8*g^2*x^2) + c^3*d^3*(35*f^3 +
 70*f^2*g*x + 56*f*g^2*x^2 + 16*g^3*x^3)))/(35*(c*d*f - a*e*g)^4*Sqrt[d + e
*x]*(f + g*x)^(7/2))
```

IntegrateAlgebraic [A] time = 8.73, size = 169, normalized size = 0.63

$$\frac{2\sqrt{d+ex}\,\sqrt{ae+cdx}\left(\frac{35c^3d^3\sqrt{ae+cdx}}{\sqrt{f+gx}}-\frac{35c^2d^2g(ae+cdx)^{3/2}}{(f+gx)^{3/2}}-\frac{5g^3(ae+cdx)^{7/2}}{(f+gx)^{7/2}}+\frac{21cdg^2(ae+cdx)^{5/2}}{(f+gx)^{5/2}}\right)}{35\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((-5*g^3*(a*e + c*d*x)^(7/2))/(f + g*x)^
(7/2) + (21*c*d*g^2*(a*e + c*d*x)^(5/2))/(f + g*x)^(5/2) - (35*c^2*d^2*g*(a
*e + c*d*x)^(3/2))/(f + g*x)^(3/2) + (35*c^3*d^3*Sqrt[a*e + c*d*x])/Sqrt[f
+ g*x]))/(35*(c*d*f - a*e*g)^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [B] time = 0.46, size = 953, normalized size = 3.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 35*a*c^2*d^2*e*f^2*g + 21*a^2*c
*d*e^2*f*g^2 - 5*a^3*e^3*g^3 + 8*(7*c^3*d^3*f*g^2 - a*c^2*d^2*e*g^3)*x^2 +
2*(35*c^3*d^3*f^2*g - 14*a*c^2*d^2*e*f*g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d
```

```
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^
8 - 4*a*c^3*d^4*e*f^7*g + 6*a^2*c^2*d^3*e^2*f^6*g^2 - 4*a^3*c*d^2*e^3*f^5*g
^3 + a^4*d*e^4*f^4*g^4 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a
^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^5 + (4*c^4*d^
4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^4 - 4*(a*c
^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^
4)*f^2*g^6 - 4*(a^3*c*d^2*e^3 - a^4*e^5)*f*g^7)*x^4 + 2*(3*c^4*d^4*e*f^6*g^
2 + 2*a^4*d*e^4*f*g^7 + 2*(c^4*d^5 - 6*a*c^3*d^3*e^2)*f^5*g^3 - 2*(4*a*c^3*
d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^4 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3
*g^5 - (8*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^6)*x^3 + 2*(2*c^4*d^4*e*f^7*g +
3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^6*g^2 - 12*(a*c^3*d^4
*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g
^4 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x^2 + (c^4*d^4*e*f^8 + 4*a^4*d*
e^4*f^3*g^5 + 4*(c^4*d^5 - a*c^3*d^3*e^2)*f^7*g - 2*(8*a*c^3*d^4*e - 3*a^2*
c^2*d^2*e^3)*f^6*g^2 + 4*(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^3 - (16*a^
3*c*d^2*e^3 - a^4*e^5)*f^4*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 260, normalized size = 0.97

$$-\frac{2 \left(c d x+a e\right) \left(-16 g^{3} x^{3} c^{3} d^{3}+8 a \,c^{2} d^{2} e \,g^{3} x^{2}-56 c^{3} d^{3} f \,g^{2} x^{2}-6 a^{2} c d \,e^{2} g^{3} x+28 a \,c^{2} d^{2} e f \,g^{2} x-70 c^{3} d^{3} f^{2} g x+5 a^{3} e^{3} g^{3}-21 a^{2} c d \,e^{2} f \,g^{2}+35 a \,c^{2} d^{2} e \,f^{2} g-35 f^{3} c^{3} d^{3}\right) \sqrt{e x +d}}{35 \left(g x +f\right)^{\frac{7}{2}} \left(g^{4} e^{4} a^{4}-4 a^{3} c d \,e^{3} f \,g^{3}+6 a^{2} c^{2} d^{2} e^{2} f^{2} g^{2}-4 a \,c^{3} d^{3} e \,f^{3} g +f^{4} c^{4} d^{4}\right) \sqrt{c d e \,x^{2}+a \,e^{2} x +c \,d^{2} x +a d e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(9/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+8*a*c^2*d^2*e*g^3*x^2-56*c^3*d^3*f*g
^2*x^2-6*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*g^2*x-70*c^3*d^3*f^2*g*x+5*a^3*
e^3*g^3-21*a^2*c*d*e^2*f*g^2+35*a*c^2*d^2*e*f^2*g-35*c^3*d^3*f^3)*(e*x+d)^(
1/2)/(g*x+f)^(7/2)/(a^4*e^4*g^4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g
^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e x + d}}{\sqrt{c d e x^{2} + a d e + \left(c d^{2} + a e^{2}\right)x} \left(g x + f\right)^{\frac{9}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(9/2)), x)
```

mupad [B] time = 5.51, size = 357, normalized size = 1.34

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(10\,a^3\,e^3\,g^3-42\,a^2\,c\,d\,e^2\,f\,g^2+70\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3\right)}{35\,e\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{32\,c^3\,d^3\,x^3\,\sqrt{d+e\,x}}{35\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{4\,c\,d\,x\,\sqrt{d+e\,x}\,\left(3\,a^2\,e^2\,g^2-14\,a\,c\,d\,e\,f\,g+35\,c^2\,d^2\,f^2\right)}{35\,e\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{16\,c^2\,d^2\,x^2\,\left(a\,e\,g-7\,c\,d\,f\right)\,\sqrt{d+e\,x}}{35\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^4\,\sqrt{f+g\,x}+\frac{d\,f^3\,\sqrt{f+g\,x}}{e\,g^3}+\frac{x^3\,\sqrt{f+g\,x}\,\left(d\,g+3\,e\,f\right)}{e\,g}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\left(d\,g+e\,f\right)}{e\,g^2}+\frac{f^2\,x\,\sqrt{f+g\,x}\,\left(3\,d\,g+e\,f\right)}{e\,g^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(9/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(10*a^3*e
^3*g^3 - 70*c^3*d^3*f^3 + 70*a*c^2*d^2*e*f^2*g - 42*a^2*c*d*e^2*f*g^2))/(35
*e*g^3*(a*e*g - c*d*f)^4) - (32*c^3*d^3*x^3*(d + e*x)^(1/2))/(35*e*(a*e*g -
 c*d*f)^4) - (4*c*d*x*(d + e*x)^(1/2)*(3*a^2*e^2*g^2 + 35*c^2*d^2*f^2 - 14*
a*c*d*e*f*g))/(35*e*g^2*(a*e*g - c*d*f)^4) + (16*c^2*d^2*x^2*(a*e*g - 7*c*d
*f)*(d + e*x)^(1/2))/(35*e*g*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (d
*f^3*(f + g*x)^(1/2))/(e*g^3) + (x^3*(f + g*x)^(1/2)*(d*g + 3*e*f))/(e*g) +
 (3*f*x^2*(f + g*x)^(1/2)*(d*g + e*f))/(e*g^2) + (f^2*x*(f + g*x)^(1/2)*(3*
d*g + e*f))/(e*g^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

3.487 $\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{15\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{7/2}d^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{15g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf\cdot}{4c^3d^3\sqrt{d+ex}}$$

Rubi [A] time = 0.47, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {866, 870, 891, 63, 217, 206}

$$\frac{5g(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2c^2d^2\sqrt{d+ex}}+\frac{15g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4c^3d^3\sqrt{d+ex}}+\frac{15\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{7/2}d^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(5/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (15*g*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(4*c^3*d^3*Sqrt[d + e*x]) + (5*g*(f + g*x)^(3/2)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c^2*d^2*Sqrt[d + e*x]) + (15*Sqrt[g
]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a
*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*c^(7/2)*d^(7/2)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(5g)\int \frac{\sqrt{d+ex}\,(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x}}{2c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(c}}{4c^3d^3\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(c}}{4c^3d^3\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(c}}{4c^3d^3\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(c}}{4c^3d^3\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(c}}{4c^3d^3\sqrt{d+ex}}$$

Mathematica [C] time = 0.12, size = 100, normalized size = 0.33

$$-\frac{2\sqrt{d+ex}(f+gx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(5/2)*Hypergeometric2F1[-5/2, -1/2, 1/2, (g*(a*
e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(
f + g*x))/(c*d*f - a*e*g))^(5/2))
```

IntegrateAlgebraic [A] time = 3.86, size = 342, normalized size = 1.14

$$\frac{(d+ex)^{3/2}(aeg+cdgx)^{3/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(15a^2e^2g^{5/2}\sqrt{f+gx}+5acdeg^{3/2}(f+gx)^{3/2}-30acdefg^{3/2}\sqrt{f+gx}+15c^2d^2f^2\sqrt{g}\sqrt{f+gx}-2c^2d^2\sqrt{g}(f+gx)^{5/2}-5c^2d^2f\sqrt{g}(f+gx)^{3/2}\right)}{4c^3d^3(-aeg-cd(f+gx)+cdf)}-\frac{15\sqrt{cd}\left(a^2e^2g^{5/2}-2acdefg^{3/2}+c^2d^2f^2\sqrt{g}\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{4c^4d^4}\right)}{g^{3/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)^(3/2)*(a*e*g + c*d*g*x)^(3/2)*((Sqrt[-(c*d*f) + a*e*g + c*d*(f +
 g*x)]*(15*c^2*d^2*f^2*Sqrt[g]*Sqrt[f + g*x] - 30*a*c*d*e*f*g^(3/2)*Sqrt[f
+ g*x] + 15*a^2*e^2*g^(5/2)*Sqrt[f + g*x] - 5*c^2*d^2*f*Sqrt[g]*(f + g*x)^(
3/2) + 5*a*c*d*e*g^(3/2)*(f + g*x)^(3/2) - 2*c^2*d^2*Sqrt[g]*(f + g*x)^(5/2
)))/(4*c^3*d^3*(c*d*f - a*e*g - c*d*(f + g*x))) - (15*Sqrt[c*d]*(c^2*d^2*f^
2*Sqrt[g] - 2*a*c*d*e*f*g^(3/2) + a^2*e^2*g^(5/2))*Log[-(Sqrt[c*d]*Sqrt[f +
 g*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]])/(4*c^4*d^4)))/(g^(3/2)*((
(a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2))
```

fricas [A] time = 1.23, size = 971, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 15*a^2*e^2*g
^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(a*c^2*d^3*e*f^2 - 2*a^2*c*d^2*e^
2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*c^2*d^2*e^2*f*g + a^2*c*d*e^3*
g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 2*(a*c^2*d^3*e + a^2*c*d*e^3)*f
*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*
x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c
^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(
g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*
c^3*d^3*e^2)*x), 1/8*(2*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25*a*c*d*e*f*g
 - 15*a^2*e^2*g^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a*c^2*d^3*e*f^2 -
 2*a^2*c*d^2*e^2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*c^2*d^2*e^2*f*g
 + a^2*c*d*e^3*g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 2*(a*c^2*d^3*e +
 a^2*c*d*e^3)*f*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(-g/(c*d))*arctan
(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*
```

```
c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2
 + a*e^2)*g)*x)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*
x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.47Unable to transpose Err
or: Bad Argument Value
```

maple [B] time = 0.04, size = 648, normalized size = 2.15

$$\frac{\left(15a^{2}cde^{2}g^{3}x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-30ac^{2}d^{2}efg^{2}x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+15c^{3}d^{3}f^{2}gx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+15a^{3}e^{3}g^{3}\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-30a^{2}cde^{2}fg^{2}\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+15ac^{2}d^{2}ef^{2}g\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+4\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^{2}d^{2}g^{2}x^{2}-10\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,acdeg^{2}x+18\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,c^{2}d^{2}fgx-30\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,a^{2}e^{2}g^{2}+50\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,acdefg-16\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^{2}d^{2}f^{2}\right)\sqrt{cde\,x^{2}+ae^{2}x+cd^{2}x+ade}\,\sqrt{gx+f}}{8\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,(cdx+ae)\sqrt{ex+d}\,c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 1/8*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*x*a^2*c*d*e^2*g^3-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*a*c^2*d^2*e*f*
g^2+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*x*c^3*d^3*f^2*g+15*a^3*e^3*g^3*ln(1/2*(2*c*d*g*x+a*e*
g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-30*a^2*
c*d*e^2*f*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(
c*d*g)^(1/2))/(c*d*g)^(1/2))+15*a*c^2*d^2*e*f^2*g*ln(1/2*(2*c*d*g*x+a*e*g+c
*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+4*((g*x+f)
*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*g^2*x^2-10*(c*d*g)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+18*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1
/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+
50*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-16*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(
1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/(c*d*x+a*e)/c^
3/d^3/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{5}{2}}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^{5/2}\,{\left(d+e\,x\right)}^{3/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

3.488 $\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=227

$$\frac{3\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2\sqrt{d+ex}}-\frac{2\sqrt{d}}{cd\sqrt{x\left(ae^2\right.}}$$

Rubi [A] time = 0.31, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {866, 870, 891, 63, 217, 206}

$$\frac{3g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2\sqrt{d+ex}}+\frac{3\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (3*g*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(c^2*d^2*Sqrt[d + e*x]) + (3*Sqrt[g]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqr
t[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*
x])])/(c^(5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(3g)\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3g\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3g\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3g\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3g\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3g\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)}}{c^2d^2\sqrt{d+ex}}$$

Mathematica [C] time = 0.08, size = 100, normalized size = 0.44

$$-\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(3/2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (g*(a*
e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(
f + g*x))/(c*d*f - a*e*g))^(3/2))
```

IntegrateAlgebraic [A] time = 2.26, size = 238, normalized size = 1.05

$$\frac{(d+ex)^{3/2}(aeg+cdgx)^{3/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(-3aeg^{3/2}\sqrt{f+gx}-cd\sqrt{g}\,(f+gx)^{3/2}+3cdf\sqrt{g}\sqrt{f+gx}\right)}{c^2d^2(-aeg-cd(f+gx)+cdf)}-\frac{3\sqrt{cd}\left(cdf\sqrt{g}-aeg^{3/2}\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{c^3d^3}\right)}{g^{3/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)^(3/2)*(a*e*g + c*d*g*x)^(3/2)*((Sqrt[-(c*d*f) + a*e*g + c*d*(f +
 g*x)]*(3*c*d*f*Sqrt[g]*Sqrt[f + g*x] - 3*a*e*g^(3/2)*Sqrt[f + g*x] - c*d*S
qrt[g]*(f + g*x)^(3/2)))/(c^2*d^2*(c*d*f - a*e*g - c*d*(f + g*x))) - (3*Sqr
t[c*d]*(c*d*f*Sqrt[g] - a*e*g^(3/2))*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[
-(c*d*f) + a*e*g + c*d*(f + g*x)]])/(c^3*d^3)))/(g^(3/2)*(((a*e*g + c*d*g*x
)*(d*g + e*g*x))/g^2)^(3/2))
```

fricas [A] time = 1.14, size = 725, normalized size = 3.19

$$\left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(cdgx-2cdf+3aeg)\sqrt{ex+d}\sqrt{gx+f}-3\left(acd^2ef-a^2de^2g+(c^2d^2ef-acde^2g)x^2+((c^2d^3+acde^2)f-(acd^2e+a^2e^3)g)x\right)\sqrt{\frac{g}{cd}}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+8(c^2d^2efg+(c^2d^3+acde^2)g^2)x^2-4(2c^2d^2gx+c^2d^2f+acdeg)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{\frac{g}{cd}}+(c^2d^2ef^2+2(4c^2d^3+3acde^2)fg+(8acd^2e+a^2e^3)g^2)x}{ex+d}\right)}{4(c^3d^3ex^2+ac^2d^3e+(c^3d^4+ac^2d^2e^2)x)},\ \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(cdgx-2cdf+3aeg)\sqrt{ex+d}\sqrt{gx+f}-3\left(acd^2ef-a^2de^2g+(c^2d^2ef-acde^2g)x^2+((c^2d^3+acde^2)f-(acd^2e+a^2e^3)g)x\right)\sqrt{-\frac{g}{cd}}\arctan\left(\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}\,cd\sqrt{-\frac{g}{cd}}}{2cdegx^2+cd^2f+adeg+(cdef+(2cd^2+ae^2)g)x}\right)}{2(c^3d^3ex^2+ac^2d^3e+(c^3d^4+ac^2d^2e^2)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3*
a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^
2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)
*g)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*
f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 - 4
*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4
*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(
c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x), 1/2*(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3*a*e*g)*sqrt(e*x
+ d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^
2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)*sqrt(-g/(
c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sq
rt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*
f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c
^2*d^2*e^2)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.26Unable to transpose Err
or: Bad Argument Value
```

maple [B] time = 0.03, size = 396, normalized size = 1.74

$$-\frac{\left(3acde\,g^{2}x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-3c^{2}d^{2}fgx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+3a^{2}e^{2}g^{2}\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-3acdefg\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-2\sqrt{cdg}\,\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,cdgx-6\sqrt{cdg}\,\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,aeg+4\sqrt{cdg}\,\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,cdf\right)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\sqrt{gx+f}}{2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\left(cdx+ae\right)\sqrt{cdg}\,\sqrt{ex+d}\,c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -1/2*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*x*a*c*d*e*g^2-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*
x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c^2*d^2*f*g+3*a^2*e
^2*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))-3*a*c*d*e*f*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f
)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-2*(c*d*g)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)*c*d*g*x-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e
*g+4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)*(c*d*e*x^2+a*e^2*x+c*
d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)/(c
*d*g)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{3}{2}}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{3/2}\,(d+e\,x)^{3/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

3.489 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Rubi [A] time = 0.20, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {866, 891, 63, 217, 206}

$$\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[
a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{g\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,d}{cd}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(g\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{}{\sqrt{ae+cd}}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+c}}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(2g\sqrt{ae+cdx}\sqrt{d+ex}\right)\mathrm{Subst}\Big(}{c^2d^2\sqrt{ade+\left(cd^2\right.}}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(2g\sqrt{ae+cdx}\sqrt{d+ex}\right)\mathrm{Subst}\Big(}{c^2d^2\sqrt{ade+\left(cd^2+ae\right.}}$$

$$= -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\sqrt{g}\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}}{c^{3/2}d^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)}}$$

Mathematica [A] time = 0.37, size = 176, normalized size = 1.09

$$\frac{2\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{cdf-aeg}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)-(cd)^{3/2}(f+gx)\right)}{(cd)^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-((c*d)^(3/2)*(f + g*x)) + Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[c
*d*f - a*e*g]*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSi
nh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*
g])]))/((c*d)^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.97, size = 162, normalized size = 1.01

$$\frac{(d+ex)^{3/2}(aeg+cdgx)^{3/2}\left(-\frac{2\sqrt{g}\sqrt{cd}\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{c^2d^2}-\frac{2\sqrt{g}\sqrt{f+gx}}{cd\sqrt{aeg+cd(f+gx)-cdf}}\right)}{g^{3/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] ((d + e*x)^(3/2)*(a*e*g + c*d*g*x)^(3/2)*((-2*Sqrt[g]*Sqrt[f + g*x])/(c*d*S
qrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]) - (2*Sqrt[c*d]*Sqrt[g]*Log[-(Sqrt[c*
d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]])/(c^2*d^2)))/(g
^(3/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2))
```

fricas [A] time = 1.10, size = 569, normalized size = 3.53

$$\left[\frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)\sqrt{\frac{g}{cd}}\log\left(-\frac{8c^{2}d^{2}eg^{2}x^{3}+c^{2}d^{3}f^{2}+6acd^{2}efg+a^{2}de^{2}g^{2}+8\left(c^{2}d^{2}efg+\left(c^{2}d^{3}+acde^{2}\right)g^{2}\right)x^{2}+4\left(2c^{2}d^{2}gx+c^{2}d^{2}f+acdeg\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{\frac{g}{cd}}+\left(c^{2}d^{2}ef^{2}+2\left(4c^{2}d^{3}+3acde^{2}\right)fg+\left(8acd^{2}e+a^{2}e^{3}\right)g^{2}\right)x}{ex+d}\right)-4\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\sqrt{ex+d}\sqrt{gx+f}}{2\left(c^{2}d^{2}ex^{2}+acd^{2}e+\left(c^{2}d^{3}+acde^{2}\right)x\right)}, -\frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)\sqrt{-\frac{g}{cd}}\arctan\left(\frac{2\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\sqrt{ex+d}\sqrt{gx+f}cd\sqrt{-\frac{g}{cd}}}{2cdegx^{2}+cd^{2}f+adeg+\left(cdef+\left(2cd^{2}+ae^{2}\right)g\right)x}\right)+2\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\sqrt{ex+d}\sqrt{gx+f}}{c^{2}d^{2}ex^{2}+acd^{2}e+\left(c^{2}d^{3}+acde^{2}\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2
*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f
*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*
e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f
)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c
*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^
3 + a*c*d*e^2)*x), -((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-g/(c*d))
*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*
x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (
2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2
)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.34Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.03, size = 210, normalized size = 1.30

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\left(cdgx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+aeg\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}\right)}{\sqrt{cdg}\,\left(cdx+ae\right)\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{ex+d}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] (g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(ln(1/2*(2*c*d*g*x+a*
e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c*d
*g+a*e*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g
)^(1/2)/(c*d*x+a*e)/((g*x+f)*(c*d*x+a*e))^(1/2)/d/c/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{\frac{3}{2}}\sqrt{gx+f}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*sqrt(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x
)
```

3.490 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {860}

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.82

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[(a*e + c*d*x)*(d + e
*x)])
```

IntegrateAlgebraic [A] time = 0.85, size = 118, normalized size = 1.93

$$\frac{2(d+ex)^{3/2}\sqrt{f+gx}\,(aeg+cdgx)^{3/2}\sqrt{aeg+cd(f+gx)-cdf}}{g(cdf-aeg)\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}(-aeg-cd(f+gx)+cdf)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*Sqrt[f + g*x]*(a*e*g + c*d*g*x)^(3/2)*Sqrt[-(c*d*f) + a*
e*g + c*d*(f + g*x)])/(g*(c*d*f - a*e*g)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))
/g^2)^(3/2)*(c*d*f - a*e*g - c*d*(f + g*x)))
```

fricas [B] time = 0.44, size = 125, normalized size = 2.05

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f}}{acd^2ef-a^2de^2g+\left(c^2d^2ef-acde^2g\right)x^2+\left(\left(c^2d^3+acde^2\right)f-\left(acd^2e+a^2e^3\right)g\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/
(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 +
a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 63, normalized size = 1.03

$$\frac{2\sqrt{gx+f}\;(cdx+ae)\,(ex+d)^{\frac{3}{2}}}{\left(aeg-cdf\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(e*x+d)^(3/2)/(c*d*e*x^2+a*e^2*x+
c*d^2*x+a*d*e)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sq
rt(g*x + f)), x)
```

mupad [B] time = 4.68, size = 147, normalized size = 2.41

$$\frac{\left(\frac{2 f \sqrt{d+e x}}{c d e \left(a e g-c d f\right)}+\frac{2 g x \sqrt{d+e x}}{c d e \left(a e g-c d f\right)}\right) \sqrt{c d e x^2+\left(c d^2+a e^2\right) x+a d e}}{x^2 \sqrt{f+g x}+\frac{a \sqrt{f+g x}}{c}+\frac{x \sqrt{f+g x} \left(c d^2+a e^2\right)}{c d e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] (((2*f*(d + e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)) + (2*g*x*(d + e*x)^(1/2))/(
c*d*e*(a*e*g - c*d*f)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^2
*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)^(1/2)*(a*e^2 + c*d^
2))/(c*d*e))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^{\frac{3}{2}}}{((d + ex)(ae + cdx))^{\frac{3}{2}} \sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x)),
 x)
```

3.491 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} dx$

Optimal. Leaf size=124

$$-\frac{4g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

Rubi [A] time = 0.15, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {868, 860}

$$-\frac{4g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]) - (4*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c
*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(2}{}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{4g}{(}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.52

$$-\frac{2\sqrt{d+ex}\,(aeg+cd(f+2gx))}{\sqrt{f+gx}\,\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(a*e*g + c*d*(f + 2*g*x)))/((c*d*f - a*e*g)^2*Sqrt[(a*e +
 c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.88, size = 132, normalized size = 1.06

$$-\frac{2(d+ex)^{3/2}(aeg+cdgx)^{3/2}\left(aeg^{3/2}+2cd\sqrt{g}\,(f+gx)-cdf\sqrt{g}\right)}{g^{3/2}\sqrt{f+gx}\,(cdf-aeg)^2\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}\sqrt{aeg+cd(f+gx)-cdf}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(a*e*g + c*d*g*x)^(3/2)*(-(c*d*f*Sqrt[g]) + a*e*g^(3/2)
 + 2*c*d*Sqrt[g]*(f + g*x)))/(g^(3/2)*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*(((a*
e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2)*Sqrt[-(c*d*f) + a*e*g + c*d*(f + g
*x)])
```

fricas [B] time = 0.44, size = 325, normalized size = 2.62

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdgx+cdf+aeg\right)\sqrt{ex+d}\,\sqrt{gx+f}}{ac^2d^3ef^3-2\,a^2cd^2e^2f^2g+a^3de^3fg^2+\left(c^3d^3ef^2g-2\,ac^2d^2e^2fg^2+a^2cde^3g^3\right)x^3+\left(c^3d^3ef^3+\left(c^3d^4-ac^2d^2e^2\right)f^2g-\left(2\,ac^2d^3e+a^2cde^3\right)fg^2+\left(a^2cd^2e^2+a^3e^4\right)g^3\right)x^2+\left(a^3de^3g^3+\left(c^3d^4+ac^2d^2e^2\right)f^3-\left(ac^2d^3e+2\,a^2cde^3\right)f^2g-\left(a^2cd^2e^2-a^3e^4\right)fg^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(e*x + d)*sqrt(g*x + f)/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^3*
d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3)*x
^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e + a^
2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3 + (c
^3*d^4 + a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^2*c*
d^2*e^2 - a^3*e^4)*f*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 97, normalized size = 0.78

$$-\frac{2\,(cdx+ae)\left(2cdgx+aeg+cdf\right)(ex+d)^{\frac{3}{2}}}{\sqrt{gx+f}\;\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2*(c*d*x+a*e)*(2*c*d*g*x+a*e*g+c*d*f)*(e*x+d)^(3/2)/(g*x+f)^(1/2)/(a^2*e^2
*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(3/2)), x)
```

mupad [B] time = 4.98, size = 151, normalized size = 1.22

$$-\frac{\left(\frac{4\,g\,x\,\sqrt{d+e\,x}}{e\,\left(a\,e\,g-c\,d\,f\right)^2}+\frac{\left(2\,a\,e\,g+2\,c\,d\,f\right)\,\sqrt{d+e\,x}}{c\,d\,e\,\left(a\,e\,g-c\,d\,f\right)^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{a\,\sqrt{f+g\,x}}{c}+\frac{x\,\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)}{c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] -(((4*g*x*(d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^2) + ((2*a*e*g + 2*c*d*f)*(d
+ e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)^(1
/2)*(a*e^2 + c*d^2))/(c*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

3.492 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3}-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}$$

Rubi [A] time = 0.25, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {868, 872, 860}

$$-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3}-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (8*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) - (16*c*d*g*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.55

$$-\frac{2\sqrt{d+ex}\left(-a^2e^2g^2+2acdeg(3f+2gx)+c^2d^2\left(3f^2+12fgx+8g^2x^2\right)\right)}{3(f+gx)^{3/2}\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(-(a^2*e^2*g^2) + 2*a*c*d*e*g*(3*f + 2*g*x) + c^2*d^2*(3*
f^2 + 12*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d +
e*x)]*(f + g*x)^(3/2))
```

IntegrateAlgebraic [A] time = 1.09, size = 198, normalized size = 1.03

$$-\frac{2(d+ex)^{3/2}(aeg+cdgx)^{3/2}\left(-a^2e^2g^{5/2}+4acdeg^{3/2}(f+gx)+2acdefg^{3/2}-c^2d^2f^2\sqrt{g}+8c^2d^2\sqrt{g}(f+gx)^2-4c^2d^2f\sqrt{g}(f+gx)\right)}{3g^{3/2}(f+gx)^{3/2}(cdf-aeg)^3\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}\sqrt{aeg+cd(f+gx)-cdf}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(a*e*g + c*d*g*x)^(3/2)*(-(c^2*d^2*f^2*Sqrt[g]) + 2*a*c
*d*e*f*g^(3/2) - a^2*e^2*g^(5/2) - 4*c^2*d^2*f*Sqrt[g]*(f + g*x) + 4*a*c*d*
e*g^(3/2)*(f + g*x) + 8*c^2*d^2*Sqrt[g]*(f + g*x)^2))/(3*g^(3/2)*(c*d*f - a
*e*g)^3*(f + g*x)^(3/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2)*Sqrt[
-(c*d*f) + a*e*g + c*d*(f + g*x)])
```

fricas [B] time = 0.46, size = 649, normalized size = 3.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*c^2*d^2*g^2*x^2 + 3*c^2*d^2*f^2 + 6*a*c*d*e*f*g - a^2*e^2*g^2 + 4*(
3*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^2*f^4*g + 3
*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2 - 3*a*c^3*d
^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4 + (2*c^4*d^
```

```
4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e - a^2*c^2*
d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^3*c*d^2*e^3
 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d^5 - a*c^3*
d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (3*a^2*c^2*d
^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*g^4)*x^2 -
(2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*e + 3*a^2*c
^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 - (5*a^3*c*d^
2*e^3 - a^4*e^5)*f^2*g^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 168, normalized size = 0.88

$$-\frac{2 (cdx+ae) \left(-8g^2x^2c^2d^2-4acde\,g^2x-12c^2d^2fgx+a^2e^2g^2-6acdefg-3f^2c^2d^2\right) (ex+d)^{\frac{3}{2}}}{3 \left(gx+f\right)^{\frac{3}{2}} \left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3\right) \left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-8*c^2*d^2*g^2*x^2-4*a*c*d*e*g^2*x-12*c^2*d^2*f*g*x+a^2*e
^2*g^2-6*a*c*d*e*f*g-3*c^2*d^2*f^2)*(e*x+d)^(3/2)/(g*x+f)^(3/2)/(a^3*e^3*g^
3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(c*d*e*x^2+a*e^2*x+c
*d^2*x+a*d*e)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(5/2)), x)
```

mupad [B] time = 5.33, size = 268, normalized size = 1.40

$$\frac{\left(\frac{8x\left(aeg+3cdf\right)\sqrt{d+ex}}{3e\left(aeg-cdf\right)^3}+\frac{\sqrt{d+ex}\left(-2a^2e^2g^2+12acdefg+6c^2d^2f^2\right)}{3cdeg\left(aeg-cdf\right)^3}+\frac{16cdgx^2\sqrt{d+ex}}{3e\left(aeg-cdf\right)^3}\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3\sqrt{f+gx}+\frac{af\sqrt{f+gx}}{cg}+\frac{x\sqrt{f+gx}\left(cfd^2+agde+afe^2\right)}{cdeg}+\frac{x^2\sqrt{f+gx}\left(cgd^2+cfde+age^2\right)}{cdeg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] (((8*x*(a*e*g + 3*c*d*f)*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3) + ((d + e
*x)^(1/2)*(6*c^2*d^2*f^2 - 2*a^2*e^2*g^2 + 12*a*c*d*e*f*g))/(3*c*d*e*g*(a*e
```

```
*g - c*d*f)^3) + (16*c*d*g*x^2*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3))*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^(1/2) + (a*f*(f
 + g*x)^(1/2))/(c*g) + (x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + a*d*e*g))/(c
*d*e*g) + (x^2*(f + g*x)^(1/2)*(a*e^2*g + c*d^2*g + c*d*e*f))/(c*d*e*g))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

3.493 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{32c^2d^2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4}-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}-\frac{12g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2}$$

Rubi [A] time = 0.33, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {868, 872, 860}

$$-\frac{32c^2d^2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4}-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}-\frac{12g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (12*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(5*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) - (16*c*d*g*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g
*x)^(3/2)) - (32*c^2*d^2*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*
(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \cdots$$

Mathematica [A] time = 0.09, size = 150, normalized size = 0.57

$$-\frac{2\sqrt{d+ex}\left(a^3e^3g^3-a^2cde^2g^2(5f+2gx)+ac^2d^2eg\left(15f^2+20fgx+8g^2x^2\right)+c^3d^3\left(5f^3+30f^2gx+40fg^2x^2+16g^3x^3\right)\right)}{5(f+gx)^{5/2}\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(a^3*e^3*g^3 - a^2*c*d*e^2*g^2*(5*f + 2*g*x) + a*c^2*d^2*
e*g*(15*f^2 + 20*f*g*x + 8*g^2*x^2) + c^3*d^3*(5*f^3 + 30*f^2*g*x + 40*f*g^
2*x^2 + 16*g^3*x^3)))/(5*(c*d*f - a*e*g)^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f
 + g*x)^(5/2))
```

IntegrateAlgebraic [A] time = 1.31, size = 289, normalized size = 1.10

$$-\frac{2(d+ex)^{3/2}(aeg+cdgx)^{3/2}\left(a^3e^3g^{7/2}-2a^2cde^2g^{5/2}(f+gx)-3a^2cde^2fg^{5/2}+3ac^2d^2ef^2g^{3/2}+8ac^2d^2eg^{3/2}(f+gx)^2+4ac^2d^2efg^{3/2}(f+gx)-c^3d^3f^3\sqrt{g}-2c^3d^3f^2\sqrt{g}(f+gx)+16c^3d^3\sqrt{g}(f+gx)^3-8c^3d^3f\sqrt{g}(f+gx)^2\right)}{5g^{3/2}(f+gx)^{5/2}(cdf-aeg)^4\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}\sqrt{aeg+cd(f+gx)-cdf}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(a*e*g + c*d*g*x)^(3/2)*(-(c^3*d^3*f^3*Sqrt[g]) + 3*a*c
^2*d^2*e*f^2*g^(3/2) - 3*a^2*c*d*e^2*f*g^(5/2) + a^3*e^3*g^(7/2) - 2*c^3*d^
3*f^2*Sqrt[g]*(f + g*x) + 4*a*c^2*d^2*e*f*g^(3/2)*(f + g*x) - 2*a^2*c*d*e^2
*g^(5/2)*(f + g*x) - 8*c^3*d^3*f*Sqrt[g]*(f + g*x)^2 + 8*a*c^2*d^2*e*g^(3/2
)*(f + g*x)^2 + 16*c^3*d^3*Sqrt[g]*(f + g*x)^3))/(5*g^(3/2)*(c*d*f - a*e*g)
^4*(f + g*x)^(5/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2)*Sqrt[-(c*d
*f) + a*e*g + c*d*(f + g*x)])
```

fricas [B] time = 0.51, size = 1062, normalized size = 4.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*c^3*d^3*g^3*x^3 + 5*c^3*d^3*f^3 + 15*a*c^2*d^2*e*f^2*g - 5*a^2*c*d
*e^2*f*g^2 + a^3*e^3*g^3 + 8*(5*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 2*(1
5*c^3*d^3*f^2*g + 10*a*c^2*d^2*e*f*g^2 - a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^4*d^5*e*f^7
- 4*a^2*c^3*d^4*e^2*f^6*g + 6*a^3*c^2*d^3*e^3*f^5*g^2 - 4*a^4*c*d^2*e^4*f^4
*g^3 + a^5*d*e^5*f^3*g^4 + (c^5*d^5*e*f^4*g^3 - 4*a*c^4*d^4*e^2*f^3*g^4 + 6
*a^2*c^3*d^3*e^3*f^2*g^5 - 4*a^3*c^2*d^2*e^4*f*g^6 + a^4*c*d*e^5*g^7)*x^5 +
 (3*c^5*d^5*e*f^5*g^2 + (c^5*d^6 - 11*a*c^4*d^4*e^2)*f^4*g^3 - 2*(2*a*c^4*d
^5*e - 7*a^2*c^3*d^3*e^3)*f^3*g^4 + 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f
^2*g^5 - (4*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^6 + (a^4*c*d^2*e^4 + a^5*e^6
)*g^7)*x^4 + (3*c^5*d^5*e*f^6*g + a^5*d*e^5*g^7 + 3*(c^5*d^6 - 3*a*c^4*d^4*
e^2)*f^5*g^2 - (11*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^4*g^3 + 2*(7*a^2*c^3*
d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g^4 - 3*(2*a^3*c^2*d^3*e^3 + 3*a^4*c*d*e^5
)*f^2*g^5 - (a^4*c*d^2*e^4 - 3*a^5*e^6)*f*g^6)*x^3 + (c^5*d^5*e*f^7 + 3*a^5
*d*e^5*f*g^6 + (3*c^5*d^6 - a*c^4*d^4*e^2)*f^6*g - 3*(3*a*c^4*d^5*e + 2*a^2
*c^3*d^3*e^3)*f^5*g^2 + 2*(3*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4)*f^4*g^3 +
 (6*a^3*c^2*d^3*e^3 - 11*a^4*c*d*e^5)*f^3*g^4 - 3*(3*a^4*c*d^2*e^4 - a^5*e^
6)*f^2*g^5)*x^2 + (3*a^5*d*e^5*f^2*g^5 + (c^5*d^6 + a*c^4*d^4*e^2)*f^7 - (a
*c^4*d^5*e + 4*a^2*c^3*d^3*e^3)*f^6*g - 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^
4)*f^5*g^2 + 2*(7*a^3*c^2*d^3*e^3 - 2*a^4*c*d*e^5)*f^4*g^3 - (11*a^4*c*d^2*
e^4 - a^5*e^6)*f^3*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 259, normalized size = 0.99

$$-\frac{2 \left(c d x+a e\right) \left(16 g^{3} x^{3} c^{3} d^{3}+8 a \,c^{2} d^{2} e \,g^{3} x^{2}+40 c^{3} d^{3} f \,g^{2} x^{2}-2 a^{2} c d \,e^{2} g^{3} x +20 a \,c^{2} d^{2} e f \,g^{2} x +30 c^{3} d^{3} f^{2} g x +a^{3} e^{3} g^{3}-5 a^{2} c d \,e^{2} f \,g^{2}+15 a \,c^{2} d^{2} e \,f^{2} g +5 f^{3} c^{3} d^{3}\right) \left(e x +d \right)^{\frac{3}{2}}}{5 \left(g x +f \right)^{\frac{5}{2}} \left(g^{4} e^{4} a^{4}-4 a^{3} c d \,e^{3} f \,g^{3}+6 a^{2} c^{2} d^{2} e^{2} f^{2} g^{2}-4 a \,c^{3} d^{3} e \,f^{3} g +f^{4} c^{4} d^{4}\right) \left(c d e \,x^{2}+a \,e^{2} x +c \,d^{2} x +a d e \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(7/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2),x)
```

```
[Out] -2/5*(c*d*x+a*e)*(16*c^3*d^3*g^3*x^3+8*a*c^2*d^2*e*g^3*x^2+40*c^3*d^3*f*g^2
*x^2-2*a^2*c*d*e^2*g^3*x+20*a*c^2*d^2*e*f*g^2*x+30*c^3*d^3*f^2*g*x+a^3*e^3*
g^3-5*a^2*c*d*e^2*f*g^2+15*a*c^2*d^2*e*f^2*g+5*c^3*d^3*f^3)*(e*x+d)^(3/2)/(
g*x+f)^(5/2)/(a^4*e^4*g^4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a
*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{\frac{3}{2}}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(7/2)), x)
```

mupad [B] time = 5.70, size = 414, normalized size = 1.58

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,x\,\sqrt{d+e\,x}\,\left(-a^2\,e^2\,g^2+10\,a\,c\,d\,e\,f\,g+15\,c^2\,d^2\,f^2\right)}{5\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{\sqrt{d+e\,x}\,\left(\frac{2\,a^3\,e^3\,g^3}{5}-2\,a^2\,c\,d\,e^2\,f\,g^2+6\,a\,c^2\,d^2\,e\,f^2\,g+2\,c^3\,d^3\,f^3\right)}{c\,d\,e\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{32\,c^2\,d^2\,g\,x^3\,\sqrt{d+e\,x}}{5\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{16\,c\,d\,x^2\,\left(a\,e\,g+5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{5\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^4\,\sqrt{f+g\,x}+\frac{a\,f^2\,\sqrt{f+g\,x}}{c\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(2\,c\,d^2\,f\,g+c\,d\,e\,f^2+a\,d\,e\,g^2+2\,a\,e^2\,f\,g\right)}{c\,d\,e\,g^2}+\frac{x^3\,\sqrt{f+g\,x}\,\left(c\,g\,d^2+2\,c\,f\,d\,e+a\,g\,e^2\right)}{c\,d\,e\,g}+\frac{f\,x\,\sqrt{f+g\,x}\,\left(c\,f\,d^2+2\,a\,g\,d\,e+a\,f\,e^2\right)}{c\,d\,e\,g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*x*(d + e*x)^(1/2)*(15*c
^2*d^2*f^2 - a^2*e^2*g^2 + 10*a*c*d*e*f*g))/(5*e*g*(a*e*g - c*d*f)^4) + ((d
 + e*x)^(1/2)*((2*a^3*e^3*g^3)/5 + 2*c^3*d^3*f^3 + 6*a*c^2*d^2*e*f^2*g - 2*
a^2*c*d*e^2*f*g^2))/(c*d*e*g^2*(a*e*g - c*d*f)^4) + (32*c^2*d^2*g*x^3*(d +
e*x)^(1/2))/(5*e*(a*e*g - c*d*f)^4) + (16*c*d*x^2*(a*e*g + 5*c*d*f)*(d + e*
x)^(1/2))/(5*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (a*f^2*(f + g*x)
^(1/2))/(c*g^2) + (x^2*(f + g*x)^(1/2)*(a*d*e*g^2 + c*d*e*f^2 + 2*a*e^2*f*g
 + 2*c*d^2*f*g))/(c*d*e*g^2) + (x^3*(f + g*x)^(1/2)*(a*e^2*g + c*d^2*g + 2*
c*d*e*f))/(c*d*e*g) + (f*x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + 2*a*d*e*g))
/(c*d*e*g^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

3.494 $\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=289

$$\frac{5g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5g^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3\sqrt{d+ex}}-\frac{10}{3c^2d^2\sqrt{x}}$$

Rubi [A] time = 0.43, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {866, 870, 891, 63, 217, 206}

$$\frac{5g^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3\sqrt{d+ex}}+\frac{5g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{10g\sqrt{d+ex}\,(f+gx)^{3/2}}{3c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(5/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (10*g*Sqrt[d + e*x]*(f + g*x)^(3/2))/(3*c^2*d^2*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3*Sqrt[d + e*x]) + (5*g^(3/2)*(c*d*f -
a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/
(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(7/2)*d^(7/2)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(5g)\int\frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}}{3cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{10g\sqrt{d+ex}\,(f+gx)^{3/}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{10g\sqrt{d+ex}\,(f+gx)^{3/}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{10g\sqrt{d+ex}\,(f+gx)^{3/}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{10g\sqrt{d+ex}\,(f+gx)^{3/}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{10g\sqrt{d+ex}\,(f+gx)^{3/}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{10g\sqrt{d+ex}\,(f+gx)^{3/}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

Mathematica [C] time = 0.14, size = 102, normalized size = 0.35

$$-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd((d+ex)(ae+cdx))^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(5/2)*Hypergeometric2F1[-5/2, -3/2, -1/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*
((c*d*(f + g*x))/(c*d*f - a*e*g))^(5/2))
```

IntegrateAlgebraic [**A**] time = 4.55, size = 319, normalized size = 1.10

$$\frac{(d+ex)^{5/2}(aeg+cdgx)^{5/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(15a^2e^2g^{7/2}\sqrt{f+gx}+20acdeg^{5/2}(f+gx)^{3/2}-30acdefg^{5/2}\sqrt{f+gx}+15c^2d^2f^2g^{3/2}\sqrt{f+gx}+3c^2d^2g^{3/2}(f+gx)^{5/2}-20c^2d^2fg^{3/2}(f+gx)^{3/2}\right)}{3c^3d^3(-aeg-cd(f+gx)+cdf)^2}-\frac{5\sqrt{cd}\left(cdfg^{3/2}-aeg^{5/2}\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{c^4d^4}\right)}{g^{5/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)^(5/2)*(a*e*g + c*d*g*x)^(5/2)*((Sqrt[-(c*d*f) + a*e*g + c*d*(f +
 g*x)]*(15*c^2*d^2*f^2*g^(3/2)*Sqrt[f + g*x] - 30*a*c*d*e*f*g^(5/2)*Sqrt[f
+ g*x] + 15*a^2*e^2*g^(7/2)*Sqrt[f + g*x] - 20*c^2*d^2*f*g^(3/2)*(f + g*x)^
(3/2) + 20*a*c*d*e*g^(5/2)*(f + g*x)^(3/2) + 3*c^2*d^2*g^(3/2)*(f + g*x)^(5
/2)))/(3*c^3*d^3*(c*d*f - a*e*g - c*d*(f + g*x))^2) - (5*Sqrt[c*d]*(c*d*f*g
^(3/2) - a*e*g^(5/2))*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*
g + c*d*(f + g*x)]])/(c^4*d^4)))/(g^(5/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x)
)/g^2)^(5/2))
```

fricas [**A**] time = 1.15, size = 1055, normalized size = 3.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a^2*e^2*g
^2 - 2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g - a^3*d*e^3
*g^2 + (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^
2)*f*g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3*e + a^2*c*d
*e^3)*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d
^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e
*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 - 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*
d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a
*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 +
(c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x), 1/6
*(2*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a^2*e^2*g^2 -
2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g - a^3*d*e^3*g^2
+ (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*
g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3*e + a^2*c*d*e^3)
*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(-g/(c*d))*arctan(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*sqrt(-g
/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g
)*x)))/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 +
 (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 7.72Unable to transpose Err
or: Bad Argument Value
```

maple [B] time = 0.04, size = 652, normalized size = 2.26

$$-\frac{\left(15a c^2d^2e g^3x^2 \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-15c^3d^3f g^2x^2 \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+30a^2cd e^2g^3x \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-30a c^2d^2ef g^2x \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+15a^3e^3g^3 \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-15a^2cd e^2f g^2 \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-6\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^2d^2g^2x^2-40\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,acdeg^2x+28\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,c^2d^2fgx-30\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,a^2e^2g^2+20\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,acdefg+4\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^2d^2f^2\right)\sqrt{cde x^2+a e^2x+c d^2x+ade}\,\sqrt{gx+f}}{6\sqrt{(gx+f)(cdx+ae)}\,(cdx+ae)^2\sqrt{cdg}\,\sqrt{ex+d}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -1/6*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*x^2*a*c^2*d^2*e*g^3-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*
f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x^2*c^3*d^3*f
*g^2+30*a^2*c*d*e^2*g^3*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-30*a*c^2*d^2*e*f*g^2*x*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))+15*a^3*e^3*g^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1
/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-15*a^2*c*d*e^2*f*g^2*ln(1/2*(2*c*d*g*x+a*
e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-6*((g
*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*g^2*x^2-40*(c*d*g)^(1/2)*((g
*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+28*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e
))^(1/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2
*g^2+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+4*((g*x+f)*(c
*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)^2/(c*d*g)^(1
/2)/c^3/d^3/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^{5/2}\,{\left(d+e\,x\right)}^{5/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.495 $\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{2g^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2g\sqrt{d+ex}\,\sqrt{f+gx}}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx}{3cd\left(x\left(ae^2+cd^2\right)+ade\right.}$$

Rubi [A] time = 0.29, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {866, 891, 63, 217, 206}

$$\frac{2g^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2g\sqrt{d+ex}\,\sqrt{f+gx}}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (2*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(c^2*d^2*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(
5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{g\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

Mathematica [C] time = 0.11, size = 102, normalized size = 0.47

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd((d+ex)(ae+cdx))^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2)*Hypergeometric2F1[-3/2, -3/2, -1/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*
((c*d*(f + g*x))/(c*d*f - a*e*g))^(3/2))
```

IntegrateAlgebraic [A] time = 2.67, size = 227, normalized size = 1.04

$$\frac{(d+ex)^{5/2}(aeg+cdgx)^{5/2}\left(\frac{2\sqrt{aeg+cd(f+gx)-cdf}\left(-3aeg^{5/2}\sqrt{f+gx}-4cdg^{3/2}(f+gx)^{3/2}+3cdfg^{3/2}\sqrt{f+gx}\right)}{3c^2d^2(-aeg-cd(f+gx)+cdf)^2}-\frac{2g^{3/2}\sqrt{cd}\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{c^3d^3}\right)}{g^{5/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)^(5/2)*(a*e*g + c*d*g*x)^(5/2)*((2*Sqrt[-(c*d*f) + a*e*g + c*d*(f
 + g*x)]*(3*c*d*f*g^(3/2)*Sqrt[f + g*x] - 3*a*e*g^(5/2)*Sqrt[f + g*x] - 4*c
*d*g^(3/2)*(f + g*x)^(3/2)))/(3*c^2*d^2*(c*d*f - a*e*g - c*d*(f + g*x))^2)
- (2*Sqrt[c*d]*g^(3/2)*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e
*g + c*d*(f + g*x)]])/(c^3*d^3)))/(g^(5/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x
))/g^2)^(5/2))
```

fricas [A] time = 1.13, size = 755, normalized size = 3.45

$$\left[-\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(4cdgx+cdf+3aeg\right)\sqrt{ex+d}\sqrt{gx+f}-3\left(c^2d^2egx^3+a^2de^2g+\left(c^2d^3+2acde^2\right)gx^2+\left(2acd^2e+a^2e^3\right)gx\right)\sqrt{\frac{g}{cd}}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+4\left(2c^2d^2gx+c^2d^2f+acdeg\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{\frac{g}{cd}}+\left(c^2d^2ef^2+2\left(4c^2d^3+3acde^2\right)fg+\left(8acd^2e+a^2e^3\right)g^2\right)x}{ex+d}\right)}{6\left(c^4d^4ex^3+a^2c^2d^3e^2+\left(c^4d^5+2ac^3d^3e^2\right)x^2+\left(2ac^3d^4e+a^2c^2d^2e^3\right)x\right)},\ -\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(4cdgx+cdf+3aeg\right)\sqrt{ex+d}\sqrt{gx+f}+3\left(c^2d^2egx^3+a^2de^2g+\left(c^2d^3+2acde^2\right)gx^2+\left(2acd^2e+a^2e^3\right)gx\right)\sqrt{-\frac{g}{cd}}\arctan\left(\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}\,cd\sqrt{-\frac{g}{cd}}}{2cdegx^2+cd^2f+adeg+\left(cdef+\left(2cd^2+ae^2\right)g\right)x}\right)}{3\left(c^4d^4ex^3+a^2c^2d^3e^2+\left(c^4d^5+2ac^3d^3e^2\right)x^2+\left(2ac^3d^4e+a^2c^2d^2e^3\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + c*d*f + 3
*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g + (c
^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(g/(c*d))*lo
g(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8
*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d
^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)
*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)
*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^3 + a^2*c^2
*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e
^3)*x), -1/3*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + c*
d*f + 3*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^2*e*g*x^3 + a^2*d*e^2
*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(-g/(
c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sq
rt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*
f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 +
 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 5.17Unable to transpose Err
or: Bad Argument Value
```

maple [A] time = 0.03, size = 343, normalized size = 1.57

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\left(3c^2d^2g^2x^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+6acde\,g^2x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+3a^2e^2g^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-8\sqrt{cdg}\,\sqrt{\left(gx+f\right)(cdx+ae)}\,cdgx-6\sqrt{cdg}\,\sqrt{\left(gx+f\right)(cdx+ae)}\,aeg-2\sqrt{cdg}\,\sqrt{\left(gx+f\right)(cdx+ae)}\,cdf\right)}{3\sqrt{cdg}\,\left(cdx+ae\right)^2\sqrt{\left(gx+f\right)(cdx+ae)}\,\sqrt{ex+d}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] 1/3*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*c*d*
g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
*x^2*c^2*d^2*g^2+6*a*c*d*e*g^2*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(
```

```
c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+3*a^2*e^2*g^2*ln(1/2*(2*c*d
*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)
)-8*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*g*x-6*(c*d*g)^(1/2)*((g*x
+f)*(c*d*x+a*e))^(1/2)*a*e*g-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*
d*f)/(c*d*g)^(1/2)/(c*d*x+a*e)^2/((g*x+f)*(c*d*x+a*e))^(1/2)/d^2/c^2/(e*x+d
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{3/2}\,\left(d+e\,x\right)^{5/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.496 $\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.83

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*((a*e + c*d*x)*(d +
 e*x))^(3/2))
```

IntegrateAlgebraic [A] time = 1.09, size = 99, normalized size = 1.57

$$\frac{2(d+ex)^{5/2}(f+gx)^{3/2}(aeg+cdgx)^{5/2}}{3g(aeg-cdf)\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}(aeg+cd(f+gx)-cdf)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(5/2)*(f + g*x)^(3/2)*(a*e*g + c*d*g*x)^(5/2))/(3*g*(-(c*d*f)
+ a*e*g)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(5/2)*(-(c*d*f) + a*e*g +
c*d*(f + g*x))^(3/2))
```

fricas [**B**] time = 0.41, size = 193, normalized size = 3.06

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\left(gx+f\right)^{\frac{3}{2}}}{3\left(a^2cd^2e^2f-a^3de^3g+\left(c^3d^3ef-ac^2d^2e^2g\right)x^3+\left(\left(c^3d^4+2\,ac^2d^2e^2\right)f-\left(ac^2d^3e+2\,a^2cde^3\right)g\right)x^2+\left(\left(2\,ac^2d^3e+a^2cde^3\right)f-\left(2\,a^2cd^2e^2+a^3e^4\right)g\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f)^(3
/2)/(a^2*c*d^2*e^2*f - a^3*d*e^3*g + (c^3*d^3*e*f - a*c^2*d^2*e^2*g)*x^3 +
((c^3*d^4 + 2*a*c^2*d^2*e^2)*f - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g)*x^2 + ((2
*a*c^2*d^3*e + a^2*c*d*e^3)*f - (2*a^2*c*d^2*e^2 + a^3*e^4)*g)*x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.22Unable to transpose Err
or: Bad Argument Value
```

maple [**A**] time = 0.01, size = 63, normalized size = 1.00

$$\frac{2\left(gx+f\right)^{\frac{3}{2}}\left(cdx+ae\right)\left(ex+d\right)^{\frac{5}{2}}}{3\left(aeg-cdf\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] 2/3*(g*x+f)^(3/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(e*x+d)^(5/2)/(c*d*e*x^2+a*e^2*
x+c*d^2*x+a*d*e)^(5/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex+d\right)^{\frac{5}{2}}\sqrt{gx+f}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^(5/2), x)
```

mupad [B] time = 4.32, size = 169, normalized size = 2.68

$$\frac{\left(\frac{2 f \sqrt{f+g x}\ \sqrt{d+e x}}{3 c^2 d^2 e \left(a e g-c d f\right)}+\frac{2 g x \sqrt{f+g x}\ \sqrt{d+e x}}{3 c^2 d^2 e \left(a e g-c d f\right)}\right) \sqrt{c d e x^2+\left(c d^2+a e^2\right) x+a d e}}{x^3+\frac{a^2 e}{c^2 d}+\frac{a x \left(2 c d^2+a e^2\right)}{c^2 d^2}+\frac{x^2 \left(c d^2+2 a e^2\right)}{c d e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] (((2*f*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)) + (2*
g*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)))*(x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^
2 + 2*c*d^2))/(c^2*d^2) + (x^2*(2*a*e^2 + c*d^2))/(c*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.497 $\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{4g\sqrt{d+ex}\,\sqrt{f+gx}}{3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Rubi [A] time = 0.14, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {868, 860}

$$\frac{4g\sqrt{d+ex}\,\sqrt{f+gx}}{3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*Sqrt[f + g*x])/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(3*(c*d*f - a
*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{(2g)\int \frac{}{\sqrt{f+gx}}}{3}$$

$$=-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{4}{3(cdf-aeg)}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 0.53

$$\frac{2(d+ex)^{3/2}\sqrt{f+gx}\,(3aeg-cd(f-2gx))}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*Sqrt[f + g*x]*(3*a*e*g - c*d*(f - 2*g*x)))/(3*(c*d*f - a
*e*g)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

IntegrateAlgebraic [A] time = 1.03, size = 119, normalized size = 0.93

$$\frac{2(d+ex)^{5/2}\sqrt{f+gx}\,(aeg+cdgx)^{5/2}(3aeg+2cd(f+gx)-3cdf)}{3g(cdf-aeg)^2\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}(aeg+cd(f+gx)-cdf)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(5/2)*Sqrt[f + g*x]*(a*e*g + c*d*g*x)^(5/2)*(-3*c*d*f + 3*a*e*
g + 2*c*d*(f + g*x)))/(3*g*(c*d*f - a*e*g)^2*(((a*e*g + c*d*g*x)*(d*g + e*g
*x))/g^2)^(5/2)*(-(c*d*f) + a*e*g + c*d*(f + g*x))^(3/2))
```

fricas [B] time = 0.45, size = 318, normalized size = 2.48

$$\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\left(2cdgx-cdf+3aeg\right)\sqrt{ex+d}\sqrt{gx+f}}{3\left(a^2c^2d^3e^2f^2-2a^3cd^2e^3fg+a^4de^4g^2+\left(c^4d^4ef^2-2ac^3d^3e^2fg+a^2c^2d^2e^3g^2\right)x^3+\left(\left(c^4d^5+2ac^3d^3e^2\right)f^2-2\left(ac^3d^4e+2a^2c^2d^2e^3\right)fg+\left(a^2c^2d^3e^2+2a^3cde^4\right)g^2\right)x^2+\left(\left(2ac^3d^4e+a^2c^2d^2e^3\right)f^2-2\left(2a^2c^2d^3e^2+a^3cde^4\right)fg+\left(2a^3cd^2e^3+a^4e^5\right)g^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x - c*d*f + 3*a*e*
g)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*c*d^2*e^3*f*g +
 a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*c^2*d^2*e^3*g^2
)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e + 2*a^2*c^2*d^2*e
^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)*x^2 + ((2*a*c^3*d^4*e + a^
2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g + (2*a^3*c*d^2
*e^3 + a^4*e^5)*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$\frac{2\sqrt{gx+f}\,\left(cdx+ae\right)\left(2cdgx+3aeg-cdf\right)\left(ex+d\right)^{\frac{5}{2}}}{3\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(1/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] 2/3*(g*x+f)^(1/2)*(c*d*x+a*e)*(2*c*d*g*x+3*a*e*g-c*d*f)*(e*x+d)^(5/2)/(a^2*
e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sq
rt(g*x + f)), x)
```

mupad [B] time = 5.06, size = 246, normalized size = 1.92

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{\left(2\,c\,d\,f^2-6\,a\,e\,f\,g\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{x\,\left(6\,a\,e\,g^2+2\,c\,d\,f\,g\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^2*x^2*(d + e*x)^(1/2))
/(3*c*d*e*(a*e*g - c*d*f)^2) - ((2*c*d*f^2 - 6*a*e*f*g)*(d + e*x)^(1/2))/(3
*c^2*d^2*e*(a*e*g - c*d*f)^2) + (x*(6*a*e*g^2 + 2*c*d*f*g)*(d + e*x)^(1/2))
/(3*c^2*d^2*e*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^(1/2) + (a^2*e*(f + g*x)^
(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^2))/(c*d*e) + (a*x*(f
+ g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.498 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^3}+\frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{}{3\sqrt{f+gx}\,\left(x\left(ae^2\right.\right.}$$

Rubi [A] time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {868, 860}

$$\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^3}+\frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}\,\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (8*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*Sqrt[f
 + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \cdots$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \cdots$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \cdots$$

Mathematica [A] time = 0.07, size = 103, normalized size = 0.53

$$\frac{2(d+ex)^{3/2}\left(3a^2e^2g^2+6acdeg(f+2gx)+c^2d^2\left(-f^2+4fgx+8g^2x^2\right)\right)}{3\sqrt{f+gx}\,((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(3*a^2*e^2*g^2 + 6*a*c*d*e*g*(f + 2*g*x) + c^2*d^2*(-f^2
 + 4*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*((a*e + c*d*x)*(d + e*x))^(3
/2)*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 1.20, size = 198, normalized size = 1.02

$$\frac{2(d+ex)^{5/2}(aeg+cdgx)^{5/2}\left(3a^2e^2g^{7/2}+12acdeg^{5/2}(f+gx)-6acdefg^{5/2}+3c^2d^2f^2g^{3/2}+8c^2d^2g^{3/2}(f+gx)^2-12c^2d^2fg^{3/2}(f+gx)\right)}{3g^{5/2}\sqrt{f+gx}\,(cdf-aeg)^3\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}(aeg+cd(f+gx)-cdf)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(5/2)*(a*e*g + c*d*g*x)^(5/2)*(3*c^2*d^2*f^2*g^(3/2) - 6*a*c*d
*e*f*g^(5/2) + 3*a^2*e^2*g^(7/2) - 12*c^2*d^2*f*g^(3/2)*(f + g*x) + 12*a*c*
d*e*g^(5/2)*(f + g*x) + 8*c^2*d^2*g^(3/2)*(f + g*x)^2))/(3*g^(5/2)*(c*d*f -
 a*e*g)^3*Sqrt[f + g*x]*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(5/2)*(-(c*
d*f) + a*e*g + c*d*(f + g*x))^(3/2))
```

fricas [B] time = 0.45, size = 667, normalized size = 3.44

$$\frac{2\left(8c^2d^2g^2x^2-c^2d^2f^2+6acdefg+3a^2e^2g^2+4\left(c^2d^2fg+3acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f}}{3\left(a^2c^3d^4e^2f^4-3a^3c^2d^3e^3f^3g+3a^4cd^2e^4f^2g^2-a^5de^5fg^3+\left(c^5d^5ef^3g-3ac^4d^4e^2f^2g^2+3a^2c^3d^3e^3fg^3-a^3c^2d^2e^4g^4\right)x^4+\left(c^5d^5ef^4+\left(c^5d^6-ac^4d^4e^2\right)f^3g-3\left(ac^4d^5e+a^2c^3d^3e^3\right)f^2g^2+\left(3a^2c^3d^4e^2+5a^3c^2d^2e^4\right)fg^3-\left(a^3c^2d^3e^3+2a^4cde^5\right)g^4\right)x^3+\left(\left(c^5d^6+2ac^4d^4e^2\right)f^4-\left(ac^4d^5e+5a^2c^3d^3e^3\right)f^3g-3\left(a^2c^3d^4e^2-a^3c^2d^2e^4\right)f^2g^2+\left(5a^3c^2d^3e^3+a^4cde^5\right)fg^3-\left(2a^4cd^2e^4+a^5e^6\right)g^4\right)x^2-\left(a^5de^5g^4-\left(2ac^4d^5e+a^2c^3d^3e^3\right)f^4+\left(5a^2c^3d^4e^2+3a^3c^2d^2e^4\right)f^3g-3\left(a^3c^2d^3e^3+a^4cde^5\right)f^2g^2-\left(a^4cd^2e^4-a^5e^6\right)fg^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 + 6*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 4*(c
^2*d^2*f*g + 3*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^3*d^4*e^2*f^4 - 3*a^3*c^2*d^3*e^3*f^3*g
+ 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (c^5*d^5*e*f^3*g - 3*a*c^4*d^
4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c^2*d^2*e^4*g^4)*x^4 + (c^5*d
^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*(a*c^4*d^5*e + a^2*c^3*d^3*e
```

```
^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^2*e^4)*f*g^3 - (a^3*c^2*d^3*
e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a*c^4*d^4*e^2)*f^4 - (a*c^4*d
^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f^2
*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3 - (2*a^4*c*d^2*e^4 + a^5*e^6
)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^4 + (5*a^
2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2*d^3*e^3 + a^4*c*d*e^5
)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 169, normalized size = 0.87

$$-\frac{2\left(cdx+ae\right)\left(8g^{2}x^{2}c^{2}d^{2}+12acde\,g^{2}x+4c^{2}d^{2}fgx+3a^{2}e^{2}g^{2}+6acdefg-f^{2}c^{2}d^{2}\right)\left(ex+d\right)^{\frac{5}{2}}}{3\sqrt{gx+f}\,\left(a^{3}e^{3}g^{3}-3a^{2}cd\,e^{2}f\,g^{2}+3a\,c^{2}d^{2}e\,f^{2}g-f^{3}c^{3}d^{3}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2+12*a*c*d*e*g^2*x+4*c^2*d^2*f*g*x+3*a^2*
e^2*g^2+6*a*c*d*e*f*g-c^2*d^2*f^2)*(e*x+d)^(5/2)/(g*x+f)^(1/2)/(a^3*e^3*g^3
-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(c*d*e*x^2+a*e^2*x+c*
d^2*x+a*d*e)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{\frac{5}{2}}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(3/2)), x)
```

mupad [B] time = 5.28, size = 255, normalized size = 1.31

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{\sqrt{d+e\,x}\,\left(6\,a^2\,e^2\,g^2+12\,a\,c\,d\,e\,f\,g-2\,c^2\,d^2\,f^2\right)}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{8\,g\,x\,\left(3\,a\,e\,g+c\,d\,f\right)\,\sqrt{d+e\,x}}{3\,c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g^2*x^2*(d + e*x)^(1/2
))/(3*e*(a*e*g - c*d*f)^3) + ((d + e*x)^(1/2)*(6*a^2*e^2*g^2 - 2*c^2*d^2*f^
```

```
2 + 12*a*c*d*e*f*g))/(3*c^2*d^2*e*(a*e*g - c*d*f)^3) + (8*g*x*(3*a*e*g + c*
d*f)*(d + e*x)^(1/2))/(3*c*d*e*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2) +
(a^2*e*(f + g*x)^(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^2))/(
c*d*e) + (a*x*(f + g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.499 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=260

$$\frac{32cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^4}+\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}+\frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+\cdots}}$$

Rubi [A] time = 0.31, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {868, 872, 860}

$$\frac{32cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^4}+\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}+\frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*(f + g
*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f +
g*x)^(3/2)) + (32*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(
c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} -$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} +$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} +$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} +$$

Mathematica [A] time = 0.10, size = 152, normalized size = 0.58

$$\frac{2(d+ex)^{3/2}\left(-a^3e^3g^3+3a^2cde^2g^2(3f+2gx)+3ac^2d^2eg\left(3f^2+12fgx+8g^2x^2\right)+c^3d^3\left(-f^3+6f^2gx+24fg^2x^2+16g^3x^3\right)\right)}{3(f+gx)^{3/2}((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(-(a^3*e^3*g^3) + 3*a^2*c*d*e^2*g^2*(3*f + 2*g*x) + 3*a*
c^2*d^2*e*g*(3*f^2 + 12*f*g*x + 8*g^2*x^2) + c^3*d^3*(-f^3 + 6*f^2*g*x + 24
*f*g^2*x^2 + 16*g^3*x^3)))/(3*(c*d*f - a*e*g)^4*((a*e + c*d*x)*(d + e*x))^(
3/2)*(f + g*x)^(3/2))
```

IntegrateAlgebraic [F] time = 180.09, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.51, size = 1065, normalized size = 4.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*c^3*d^3*g^3*x^3 - c^3*d^3*f^3 + 9*a*c^2*d^2*e*f^2*g + 9*a^2*c*d*e^2
*f*g^2 - a^3*e^3*g^3 + 24*(c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 6*(c^3*d^
3*f^2*g + 6*a*c^2*d^2*e*f*g^2 + a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^4*d^5*e^2*f^6 - 4*a
^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3
+ a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*
c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 +
```

```
(2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e -
 a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g
^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2
*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e
^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c^4*d^4*e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e
^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f
^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^3 - (6*a^2*c^4*d^4*e^3*f^5*g -
2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 + 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*
c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d
^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a
^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e
^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f
^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f^3*g^3 - (6*a^5*c*d^2*e^5 - a^6
*e^7)*f^2*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 258, normalized size = 0.99

$$-\frac{2 \left(c d x+a e\right) \left(-16 g^{3} x^{3} c^{3} d^{3}-24 a \,c^{2} d^{2} e \,g^{3} x^{2}-24 c^{3} d^{3} f \,g^{2} x^{2}-6 a^{2} c d \,e^{2} g^{3} x-36 a \,c^{2} d^{2} e f \,g^{2} x-6 c^{3} d^{3} f^{2} g x+a^{3} e^{3} g^{3}-9 a^{2} c d \,e^{2} f \,g^{2}-9 a \,c^{2} d^{2} e \,f^{2} g+f^{3} c^{3} d^{3}\right) \left(e x+d\right)^{\frac{5}{2}}}{3 \left(g x+f\right)^{\frac{3}{2}} \left(g^{4} e^{4} a^{4}-4 a^{3} c d \,e^{3} f \,g^{3}+6 a^{2} c^{2} d^{2} e^{2} f^{2} g^{2}-4 a \,c^{3} d^{3} e \,f^{3} g+f^{4} c^{4} d^{4}\right) \left(c d e \,x^{2}+a \,e^{2} x+c \,d^{2} x+a d e\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(5/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3-24*a*c^2*d^2*e*g^3*x^2-24*c^3*d^3*f*g
^2*x^2-6*a^2*c*d*e^2*g^3*x-36*a*c^2*d^2*e*f*g^2*x-6*c^3*d^3*f^2*g*x+a^3*e^3
*g^3-9*a^2*c*d*e^2*f*g^2-9*a*c^2*d^2*e*f^2*g+c^3*d^3*f^3)*(e*x+d)^(5/2)/(g*
x+f)^(3/2)/(a^4*e^4*g^4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c
^3*d^3*e*f^3*g+c^4*d^4*f^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{\frac{5}{2}}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(5/2)), x)
```

mupad [B] time = 5.86, size = 416, normalized size = 1.60

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,g\,x^2\,\left(a\,e\,g+c\,d\,f\right)\,\sqrt{d+e\,x}}{e\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{\sqrt{d+e\,x}\,\left(2\,a^3\,e^3\,g^3-18\,a^2\,c\,d\,e^2\,f\,g^2-18\,a\,c^2\,d^2\,e\,f^2\,g+2\,c^3\,d^3\,f^3\right)}{3\,c^2\,d^2\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{32\,c\,d\,g^2\,x^3\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{4\,x\,\sqrt{d+e\,x}\,\left(a^2\,e^2\,g^2+6\,a\,c\,d\,e\,f\,g+c^2\,d^2\,f^2\right)}{c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^4\,\sqrt{f+g\,x}+\frac{x^2\,\sqrt{f+g\,x}\,\left(g\,a^2\,e^3+2\,g\,a\,c\,d^2\,e+2\,f\,a\,c\,d\,e^2+f\,c^2\,d^3\right)}{c^2\,d^2\,e\,g}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,f\,d^2+a\,g\,d\,e+a\,f\,e^2\right)}{c^2\,d^2\,g}+\frac{a^2\,e\,f\,\sqrt{f+g\,x}}{c^2\,d\,g}+\frac{x^3\,\sqrt{f+g\,x}\,\left(c\,g\,d^2+c\,f\,d\,e+2\,a\,g\,e^2\right)}{c\,d\,e\,g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g*x^2*(a*e*g + c*d*f)*(
d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^4) - ((d + e*x)^(1/2)*(2*a^3*e^3*g^3 + 2
*c^3*d^3*f^3 - 18*a*c^2*d^2*e*f^2*g - 18*a^2*c*d*e^2*f*g^2))/(3*c^2*d^2*e*g
*(a*e*g - c*d*f)^4) + (32*c*d*g^2*x^3*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)
^4) + (4*x*(d + e*x)^(1/2)*(a^2*e^2*g^2 + c^2*d^2*f^2 + 6*a*c*d*e*f*g))/(c*
d*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (x^2*(f + g*x)^(1/2)*(a^2*e
^3*g + c^2*d^3*f + 2*a*c*d*e^2*f + 2*a*c*d^2*e*g))/(c^2*d^2*e*g) + (a*x*(f
+ g*x)^(1/2)*(a*e^2*f + 2*c*d^2*f + a*d*e*g))/(c^2*d^2*g) + (a^2*e*f*(f + g
*x)^(1/2))/(c^2*d*g) + (x^3*(f + g*x)^(1/2)*(2*a*e^2*g + c*d^2*g + c*d*e*f)
)/(c*d*e*g))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

3.500 $\int \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=385

$$-\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{64c^3d^3g\sqrt{d+ex}}$$

Rubi [A] time = 0.72, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64c^3d^3g\sqrt{d+ex}}-\frac{5(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{96c^2d^2g\sqrt{d+ex}}+\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}+\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{24\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c^3*d^3*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(96*c^2*d^2*g*Sqrt[d + e*x]) +
(((a*e)/(c*d) - f/g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(24*Sqrt[d + e*x]) + ((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(4*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(7/2)*d^(7/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
```

```
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}} - \frac{(cdf-aeg)\int -\cdots}{\sqrt{\cdots}}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+g\cdots}{\cdots}$$

$$= -\frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \cdots$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(\cdots}{\cdots}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(\cdots}{\cdots}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(\cdots}{\cdots}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(\cdots}{\cdots}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(\cdots}{\cdots}$$

Mathematica [A] time = 1.16, size = 300, normalized size = 0.78

$$\frac{\sqrt{cd}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)\left(15a^3e^3g^3-5a^2cde^2g^2(11f+2gx)+ac^2d^2eg\left(73f^2+36fgx+8g^2x^2\right)+c^3d^3\left(15f^3+118f^2gx+136fg^2x^2+48g^3x^3\right)\right)-15\sqrt{ae+cdx}(cdf-aeg)^{9/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)\right)}{192c^{9/2}d^{9/2}g^{3/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqr
t[d + e*x],x]
```

```
[Out] (Sqrt[c*d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*d*x)*(
f + g*x)*(15*a^3*e^3*g^3 - 5*a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g
*(73*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(15*f^3 + 118*f^2*g*x + 136*f*g^
2*x^2 + 48*g^3*x^3)) - 15*(c*d*f - a*e*g)^(9/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d
*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*
d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(192*c^(9/2)*d^(9/2)*g^(3/2)*Sqrt[
(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 8.58, size = 259, normalized size = 0.67

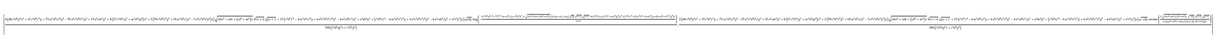
$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{\sqrt{ae+cdx}(cdf-aeg)^4\left(\frac{73c^2d^2g(ae+cdx)}{f+gx}+\frac{15g^3(ae+cdx)^3}{(f+gx)^3}-\frac{55cdg^2(ae+cdx)^2}{(f+gx)^2}+15c^3d^3\right)}{192c^3d^3g\sqrt{f+gx}\left(cd-\frac{g(ae+cdx)}{f+gx}\right)^4}-\frac{5(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}}\right)}{\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/Sqrt[d + e*x],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(((c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]*(15*
c^3*d^3 + (15*g^3*(a*e + c*d*x)^3)/(f + g*x)^3 - (55*c*d*g^2*(a*e + c*d*x)^
2)/(f + g*x)^2 + (73*c^2*d^2*g*(a*e + c*d*x))/(f + g*x)))/(192*c^3*d^3*g*Sq
rt[f + g*x]*(c*d - (g*(a*e + c*d*x))/(f + g*x))^4) - (5*(c*d*f - a*e*g)^4*A
rcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(64*c^
(7/2)*d^(7/2)*g^(3/2))))/Sqrt[(a*e + c*d*x)*(d + e*x)]
```

fricas [A] time = 2.69, size = 1065, normalized size = 2.77



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g + 73*a*c^3*d^3*e*f^2*g^2 -
 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(17*c^4*d^4*f*g^3 + a*c^
3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*a*c^3*d^3*e*f*g^3 - 5*a^2*c^2
*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f
^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d
^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^
4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
 + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
 + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*
g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*
d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*g^2*x +
 c^4*d^5*g^2), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g + 73*a*c^3*d
^3*e*f^2*g^2 - 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(17*c^4*d^
4*f*g^3 + a*c^3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*a*c^3*d^3*e*f*g
^3 - 5*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2
*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f
^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^
3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*
f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*g^2*x + c^4*d
^5*g^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 870, normalized size = 2.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/384*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-96*x^3*c^3*d
^3*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))
/(c*d*g)^(1/2))*a^4*e^4*g^4-60*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a
*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3
+90*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)
*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-60*ln(1/2*(2*c*d*g*x
+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a
*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4-16*x^
2*a*c^2*d^2*e*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)-272
*x^2*c^3*d^3*f*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+20
*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^2*c*d*e^2*g^3-72
*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c^2*d^2*e*f*g^2-
236*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^3*d^3*f^2*g-3
0*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*e^3*g^3+110*(c*
d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*f*g^2-146*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f^2*g-30*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1
/2)/g/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/c^3/d^3/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(gx + f\right)^{\frac{5}{2}}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(5/2)/sqrt(
e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{5/2}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

3.501 $\int \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=313

$$-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8c^2d^2g\sqrt{d+ex}}+\ldots (f$$

Rubi [A] time = 0.52, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8c^2d^2g\sqrt{d+ex}}+\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{12\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] -((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(8*c^2*d^2*g*Sqrt[d + e*x]) + (((a*e)/(c*d) - f/g)*(f + g*x)^(3/2)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*Sqrt[d + e*x]) + ((f + g*x)^(5
/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d + e*x]) - ((c*
d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(5/2)*d^(5/2)*g^(3/2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}} - \frac{(cdf-aeg)\int -}{\sqrt{}}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12\sqrt{d+ex}} + \frac{(f+g}{}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}\right.}{}$$

Mathematica [A] time = 0.84, size = 255, normalized size = 0.81

$$\frac{\sqrt{cd}\sqrt{d+ex}\left(-\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)\left(3a^2e^2g^2-2acdeg(4f+gx)-c^2d^2\left(3f^2+14fgx+8g^2x^2\right)\right)-3\sqrt{ae+cdx}(cdf-aeg)^{7/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)\right)}{24c^{7/2}d^{7/2}g^{3/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqr
t[d + e*x],x]
```

```
[Out] (Sqrt[c*d]*Sqrt[d + e*x]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*d*x)
*(f + g*x)*(3*a^2*e^2*g^2 - 2*a*c*d*e*g*(4*f + g*x) - c^2*d^2*(3*f^2 + 14*f
*g*x + 8*g^2*x^2))) - 3*(c*d*f - a*e*g)^(7/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(
f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*
x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(24*c^(7/2)*d^(7/2)*g^(3/2)*Sqrt[(a*
e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 7.68, size = 231, normalized size = 0.74

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{\sqrt{ae+cdx}\,(cdf-aeg)^3\left(-\frac{3g^2(ae+cdx)^2}{(f+gx)^2}+\frac{8cdg(ae+cdx)}{f+gx}+3c^2d^2\right)}{24c^2d^2g\sqrt{f+gx}\left(cd-\frac{g(ae+cdx)}{f+gx}\right)^3}-\frac{(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}}\right)}{\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/Sqrt[d + e*x],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(((c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*(3*c
^2*d^2 - (3*g^2*(a*e + c*d*x)^2)/(f + g*x)^2 + (8*c*d*g*(a*e + c*d*x))/(f +
 g*x)))/(24*c^2*d^2*g*Sqrt[f + g*x]*(c*d - (g*(a*e + c*d*x))/(f + g*x))^3)
- ((c*d*f - a*e*g)^3*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*S
qrt[f + g*x]])])/(8*c^(5/2)*d^(5/2)*g^(3/2))))/Sqrt[(a*e + c*d*x)*(d + e*x)]
```

fricas [A] time = 1.50, size = 847, normalized size = 2.71

$$\left[\frac{4\left(8c^3d^3g^3x^2+3c^3d^3f^2g+8ac^2d^2efg^2-3a^2cde^2g^3+2\left(7c^3d^3fg^2+ac^2d^2eg^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}-3\left(c^3d^4f^3-3ac^2d^3ef^2g+3a^2cd^2e^2fg^2-a^3de^3g^3+\left(c^3d^3ef^3-3ac^2d^2e^2f^2g+3a^2cde^3fg^2-a^3e^4g^3\right)x\right)\sqrt{cdg}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdgx+cdf+aeg\right)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+\left(c^2d^2ef^2+2\left(4c^2d^3+3acde^2\right)fg+\left(8acd^2e+a^2e^3\right)g^2\right)x}{ex+d}\right)}{96\left(c^3d^3eg^2x+c^3d^4g^2\right)},\ \frac{2\left(8c^3d^3g^3x^2+3c^3d^3f^2g+8ac^2d^2efg^2-3a^2cde^2g^3+2\left(7c^3d^3fg^2+ac^2d^2eg^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}+3\left(c^3d^4f^3-3ac^2d^3ef^2g+3a^2cd^2e^2fg^2-a^3de^3g^3+\left(c^3d^3ef^3-3ac^2d^2e^2f^2g+3a^2cde^3fg^2-a^3e^4g^3\right)x\right)\sqrt{-cdg}\arctan\left(\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdg}\sqrt{ex+d}\sqrt{gx+f}}{2cdegx^2+cd^2f+adeg+\left(cdef+\left(2cd^2+ae^2\right)g\right)x}\right)}{48\left(c^3d^3eg^2x+c^3d^4g^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 - 3*a^2
*c*d*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a
*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 -
 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*d*g)*
log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 +
 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c
*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^2*x + c^3*d^4*g^2), 1/4
8*(2*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 - 3*a^2*c*d
*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^3*d^4*f^3 - 3*a*c^2
*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 - 3*a
*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*d*g)*arc
tan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d
)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 +
a*e^2)*g)*x)))/(c^3*d^3*e*g^2*x + c^3*d^4*g^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 602, normalized size = 1.92

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(3a^3e^3g^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-9a^2cd\,e^2f\,g^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+9a\,c^2d^2e\,f^2g\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-3c^3d^3f^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+16\sqrt{cdg}\,\sqrt{cdg\,x^2+aegx+cdfx+aef}\,c^2d^2g^2x^2+4\sqrt{cdg}\,\sqrt{cdg\,x^2+aegx+cdfx+aef}\,acde\,g^2x+28\sqrt{cdg}\,\sqrt{cdg\,x^2+aegx+cdfx+aef}\,c^2d^2fgx-6\sqrt{cdg}\,\sqrt{cdg\,x^2+aegx+cdfx+aef}\,a^2e^2g^2+16\sqrt{cdg}\,\sqrt{cdg\,x^2+aegx+cdfx+aef}\,acdefg+6\sqrt{cdg}\,\sqrt{cdg\,x^2+aegx+cdfx+aef}\,c^2d^2f^2\right)}{48\sqrt{ex+d}\,\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,c^2d^2g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 1/48*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*c*d
*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*a^3*e^3*g^3-9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*
g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+9*
ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*
d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*
f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c
^3*d^3*f^3+16*x^2*c^2*d^2*g^2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*
f)^(1/2)+4*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c*d*e*
g^2+28*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^2*d^2*f*g-
6*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*e^2*g^2+16*a*c*
d*e*f*g*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)+6*(c*d*g)^(1/
2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/g/(c*
d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/d^2/c^2/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\left(gx + f\right)^{\frac{3}{2}}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt(
e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f + g\,x\right)^{3/2}\,\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{\sqrt{d + e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

3.502 $\int \frac{\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=241

$$-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}+\frac{\sqrt{f+gx}\ \cdots}{}$$

Rubi [A] time = 0.35, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}+\frac{\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*
x],x]
```

```
[Out] (((a*e)/(c*d) - f/g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(4*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(2*g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt
[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])])/(4*c^(3/2)*d^(3/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{\sqrt{\ldots}}{\sqrt{ade+(\ldots}}}{4g}\ldots$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{\ldots}}{\ldots}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{\ldots}}{\ldots}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{\ldots}}{\ldots}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{\ldots}}{\ldots}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{\ldots}}{\ldots}$$

Mathematica [A] time = 0.59, size = 215, normalized size = 0.89

$$\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)(aeg+cd(f+2gx))-\sqrt{ae+cdx}(cdf-aeg)^{5/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)\right)}{4g^{3/2}(cd)^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[
d + e*x],x]
```

[Out] (Sqrt[c]*Sqrt[d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*d*x)*(f + g*x)*(a*e*g + c*d*(f + 2*g*x)) - (c*d*f - a*e*g)^(5/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(4*(c*d)^(5/2)*g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])

IntegrateAlgebraic [A] time = 0.78, size = 227, normalized size = 0.94

$$\frac{\sqrt{g}\sqrt{\frac{(dg+egx)(aeg+cdgx)}{g^2}}\left(\frac{\sqrt{cd}\left(a^2e^2g^2-2acdefg+c^2d^2f^2\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{4c^2d^2g^{3/2}}+\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(aeg\sqrt{f+gx}+2cd(f+gx)^{3/2}-cdf\sqrt{f+gx}\right)}{4cdg^{3/2}}\right)}{\sqrt{d+ex}\sqrt{aeg+cdgx}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x],x]

[Out] (Sqrt[g]*Sqrt[((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2]*((Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]*(-(c*d*f*Sqrt[f + g*x]) + a*e*g*Sqrt[f + g*x] + 2*c*d*(f + g*x)^(3/2)))/(4*c*d*g^(3/2)) + (Sqrt[c*d]*(c^2*d^2*f^2 - 2*a*c*d*e*f*g + a^2*e^2*g^2)*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]])/(4*c^2*d^2*g^(3/2))))/(Sqrt[d + e*x]*Sqrt[a*e*g + c*d*g*x])

fricas [A] time = 1.20, size = 657, normalized size = 2.73

$$\left[\frac{4\left(2c^2d^2g^2x+c^2d^2fg+acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}+\left(c^2d^3f^2-2acd^2efg+a^2de^2g^2+\left(c^2d^2ef^2-2acde^2fg+a^2e^3g^2\right)x\right)\sqrt{cdg}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2-4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdgx+cdf+aeg\right)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+\left(c^2d^2ef^2+2\left(4c^2d^3+3acde^2\right)fg+\left(8acd^2e+a^2e^3\right)g^2\right)x}{ex+d}\right)}{16\left(c^2d^2eg^2x+c^2d^3g^2\right)}, \frac{2\left(2c^2d^2g^2x+c^2d^2fg+acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}+\left(c^2d^3f^2-2acd^2efg+a^2de^2g^2+\left(c^2d^2ef^2-2acde^2fg+a^2e^3g^2\right)x\right)\sqrt{-cdg}\arctan\left(\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdg}\sqrt{ex+d}\sqrt{gx+f}}{2cdegx^2+cd^2f+adeg+\left(cdef+\left(2cd^2+ae^2\right)g\right)x}\right)}{8\left(c^2d^2eg^2x+c^2d^3g^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] [1/16*(4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^2*x + c^2*d^3*g^2), 1/8*(2*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^2*d^2*e*g^2*x + c^2*d^3*g^2)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.02, size = 385, normalized size = 1.60

$$-\frac{\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(a^2e^2g^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdgx^2+aegx+cdfx+aef}\sqrt{cdg}}{2\sqrt{cdg}}\right)-2acdefg\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdgx^2+aegx+cdfx+aef}\sqrt{cdg}}{2\sqrt{cdg}}\right)+c^2d^2f^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdgx^2+aegx+cdfx+aef}\sqrt{cdg}}{2\sqrt{cdg}}\right)-4\sqrt{cdg}\sqrt{cdgx^2+aegx+cdfx+aef}\,cdgx-2\sqrt{cdg}\sqrt{cdgx^2+aegx+cdfx+aef}\,aeg-2\sqrt{cdg}\sqrt{cdgx^2+aegx+cdfx+aef}\,cdf\right)}{8\sqrt{ex+d}\sqrt{cdgx^2+aegx+cdfx+aef}\sqrt{cdg}\,cdg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/8*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(ln(1/2*(2*c*d*g
*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*
d*g)^(1/2))*a^2*e^2*g^2-2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*
x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+ln(1/2*(2*
c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)
)/(c*d*g)^(1/2))*c^2*d^2*f^2-4*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e
*f)^(1/2)*x*c*d*g-2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a
*e*g-2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c*d*f)/(e*x+d)
^(1/2)/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/d/g/c/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\sqrt{gx + f}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(e*
x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\,\sqrt{f+gx}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)/sqrt(d + e*x), x)
```

3.503 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$

Optimal. Leaf size=167

$$\frac{\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{\sqrt{c}\,\sqrt{d}\,g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Rubi [A] time = 0.21, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {864, 891, 63, 217, 206}

$$\frac{\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{\sqrt{c}\,\sqrt{d}\,g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f + g*x
]),x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x
]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] && !IGtQ[n, 0] && !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae\right.}}}{2g}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right.}{2g\sqrt{ade+\left(cd^2+ae^2\right)}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right.}{cdg\sqrt{ade+}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right.}{cdg\sqrt{ade+\left(cd^2\right.}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}}{\sqrt{c}\sqrt{d}\,g^{3/2}\sqrt{ade+\left(cd^2+\right.}}$$

Mathematica [A] time = 0.81, size = 173, normalized size = 1.04

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}(f+gx)-\frac{\sqrt{c}\sqrt{d}(cdf-aeg)^{3/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f
 + g*x]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g]*(f + g*x) - (Sqrt[c]*Sqrt[d]*(c*d*f
 - a*e*g)^(3/2)*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt
[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])/((c*d)^(3/
2)*Sqrt[a*e + c*d*x])))/(g^(3/2)*Sqrt[d + e*x]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.71, size = 164, normalized size = 0.98

$$\frac{\sqrt{g}\sqrt{\frac{(dg+egx)(aeg+cdgx)}{g^2}}\left(\frac{\sqrt{f+gx}\sqrt{aeg+cd(f+gx)-cdf}}{g^{3/2}}+\frac{\sqrt{cd}(cdf-aeg)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{cdg^{3/2}}\right)}{\sqrt{d+ex}\sqrt{aeg+cdgx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[g]*Sqrt[((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2]*((Sqrt[f + g*x]*Sqrt[-
(c*d*f) + a*e*g + c*d*(f + g*x)])/g^(3/2) + (Sqrt[c*d]*(c*d*f - a*e*g)*Log[
-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]])/(c*d*
g^(3/2))))/(Sqrt[d + e*x]*Sqrt[a*e*g + c*d*g*x])
```

fricas [A] time = 1.14, size = 516, normalized size = 3.09

$$\left[\frac{4 \sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x} \sqrt{ex+d} \sqrt{gx+f}\, cdg - \left(cd^2 f - adeg + \left(cdef - ae^2 g\right)x\right)\sqrt{cdg} \log\left(-\frac{8 c^2 d^2 e g^2 x^3 + c^2 d^3 f^2 + 6 acd^2 efg + a^2 de^2 g^2 + 4 \sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2 cdgx + cdf + aeg\right)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f} + 8\left(c^2 d^2 efg + \left(c^2 d^3 + acde^2\right)g^2\right)x^2 + \left(c^2 d^2 ef^2 + 2\left(4 c^2 d^3 + 3 acde^2\right)fg + \left(8 acd^2 e + a^2 e^3\right)g^2\right)x}{ex+d}\right)}{4\left(cdeg^2 x + cd^2 g^2\right)}, \frac{2 \sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x} \sqrt{ex+d} \sqrt{gx+f}\, cdg + \left(cd^2 f - adeg + \left(cdef - ae^2 g\right)x\right)\sqrt{-cdg} \arctan\left(\frac{2 \sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdg}\sqrt{ex+d}\sqrt{gx+f}}{2 cdegx^2 + cd^2 f + adeg + \left(cdef + \left(2 cd^2 + ae^2\right)g\right)x}\right)}{2\left(cdeg^2 x + cd^2 g^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(-(
8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c
*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2
)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*
e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^2*x + c*d^2*g^2), 1/2*(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*g + (c*
d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2
*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c*
d*e*g^2*x + c*d^2*g^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 198, normalized size = 1.19

$$\frac{\sqrt{cde\,x^2 + a\,e^2 x + c\,d^2 x + ade}\,\sqrt{gx+f}\left(aeg \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right) - cdf \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right) + 2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\right)}{2\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 1/2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)/(e*x+d)^(1/2)*(a*
e*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/
2))/(c*d*g)^(1/2))-c*d*f*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*
e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*
g)^(1/2))/((g*x+f)*(c*d*x+a*e))^(1/2)/g/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{\sqrt{ex+d}\,\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*sqrt(g
*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x
)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.504 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{2\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}$$

Rubi [A] time = 0.19, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {862, 891, 63, 217, 206}

$$\frac{2\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]*Sqrt[f +
g*x]) + (2*Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]
*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(3/2)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx = -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{(cd)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{g}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\left(cd\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\,\sqrt{f+\ldots}}}{g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\left(2\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{\ldots}{\sqrt{f-\ldots}}\ldots\right)}{g\sqrt{ade+\left(cd^2+ae^2\right)\ldots}}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\left(2\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\ldots\right)}{g\sqrt{ade+\left(cd^2+ae^2\right)x+c\ldots}}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{2\sqrt{c}\,\sqrt{d}\,\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\ldots\right)}{g^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+c\ldots}}$$

Mathematica [A] time = 0.79, size = 169, normalized size = 1.07

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{cdf-aeg}\,\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\,\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cd}\,\sqrt{cdf-aeg}}\right)}{\sqrt{cd}\,\sqrt{ae+cdx}}-\sqrt{g}\right)}{g^{3/2}\sqrt{d+ex}\,\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-Sqrt[g] + (Sqrt[c]*Sqrt[d]*Sqrt[c*d*f -
a*e*g]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[
g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])/(Sqrt[c*d]*Sqrt[a*e
 + c*d*x])))/(g^(3/2)*Sqrt[d + e*x]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 1.97, size = 213, normalized size = 1.35

$$\frac{\sqrt{d+ex}\,(ef+egx)^{3/2}\sqrt{ae^2+cdex}\left(\frac{2\sqrt{c}\,\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae^2-cd^2+cd(d+ex)}}{\sqrt{c}\,\sqrt{d}\,\sqrt{g(d+ex)-dg+ef}}\right)}{g^{3/2}}-\frac{2\sqrt{ae^2-cd^2+cd(d+ex)}}{g\sqrt{g(d+ex)-dg+ef}}\right)}{e^2\sqrt{\frac{(d+ex)\left(ae^2+cdex\right)}{e}}\left(\frac{g(d+ex)-dg+ef}{e}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*(f + g*x)^(3/2)),x]
```

```
[Out] (Sqrt[d + e*x]*Sqrt[a*e^2 + c*d*e*x]*(e*f + e*g*x)^(3/2)*((-2*Sqrt[-(c*d^2)
 + a*e^2 + c*d*(d + e*x)])/(g*Sqrt[e*f - d*g + g*(d + e*x)]) + (2*Sqrt[c]*S
qrt[d]*ArcTanh[(Sqrt[g]*Sqrt[-(c*d^2) + a*e^2 + c*d*(d + e*x)])/(Sqrt[c]*Sq
rt[d]*Sqrt[e*f - d*g + g*(d + e*x)])])/g^(3/2)))/(e^2*Sqrt[((d + e*x)*(a*e^
2 + c*d*e*x))/e]*((e*f - d*g + g*(d + e*x))/e)^(3/2))
```

fricas [A] time = 1.07, size = 521, normalized size = 3.30

$$\left[\frac{\left(egx^2+df+\left(ef+dg\right)x\right)\sqrt{\frac{cd}{g}}\log\left(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2efg+a^2de^2g^2+4\left(2\,cdg^2x+cdfg+aeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{\frac{cd}{g}}+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+\left(c^2d^2ef^2+2\left(4\,c^2d^3+3\,acde^2\right)fg+\left(8\,acd^2e+a^2e^3\right)g^2\right)x}{ex+d}\right)-4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}}{2\left(eg^2x^2+dfg+\left(efg+dg^2\right)x\right)},-\frac{\left(egx^2+df+\left(ef+dg\right)x\right)\sqrt{-\frac{cd}{g}}\arctan\left(\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{-\frac{cd}{g}}\,g}{2\,cdegx^2+cd^2f+adeg+\left(cdef+\left(2\,cd^2+ae^2\right)g\right)x}\right)+2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}}{eg^2x^2+dfg+\left(efg+dg^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*((e*g*x^2 + d*f + (e*f + d*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3
 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g
 + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 +
(c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*
g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d)*sqrt(g*x + f))/(e*g^2*x^2 + d*f*g + (e*f*g + d*g^2)*x), -((e*g*x^2 +
d*f + (e*f + d*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d
^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e*g^2*x^2 + d*f*g +
 (e*f*g + d*g^2)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 197, normalized size = 1.25

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\left(cdgx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+cdf\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-2\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{cdg}\right)}{\sqrt{cdg}\,\sqrt{\left(gx+f\right)\left(cdx+ae\right)}\,\sqrt{ex+d}\,\sqrt{gx+f}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(c*d*g*x*ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+c*d*f*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)/((g*x+
f)*(c*d*x+a*e))^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\,\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^{3/2}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*x
)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(3/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

3.505 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(5
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{5/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(5/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f +
 g*x)^(3/2))
```

IntegrateAlgebraic [B] time = 2.66, size = 145, normalized size = 2.30

$$\frac{2\sqrt{d+ex}\,(ef+egx)^{5/2}\sqrt{ae^2+cdex}\,\left(ae^2-cd^2+cd(d+ex)\right)^{3/2}}{3e^3\sqrt{\frac{(d+ex)\left(ae^2+cdex\right)}{e}}\,(g(d+ex)-dg+ef)^{3/2}\left(\frac{g(d+ex)-dg+ef}{e}\right)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*(f + g*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[d + e*x]*Sqrt[a*e^2 + c*d*e*x]*(e*f + e*g*x)^(5/2)*(-(c*d^2) + a*e^
2 + c*d*(d + e*x))^(3/2))/(3*e^3*(c*d*f - a*e*g)*Sqrt[((d + e*x)*(a*e^2 + c
*d*e*x))/e]*(e*f - d*g + g*(d + e*x))^(3/2)*((e*f - d*g + g*(d + e*x))/e)^(
5/2))
```

fricas [B] time = 0.43, size = 169, normalized size = 2.68

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(cdx+ae)\sqrt{ex+d}\,\sqrt{gx+f}}{3\left(cd^2f^3-adef^2g+\left(cdefg^2-ae^2g^3\right)x^3+\left(2\,cdef^2g-adeg^3+\left(cd^2-2\,ae^2\right)fg^2\right)x^2+\left(cdef^3-2\,adefg^2+\left(2\,cd^2-ae^2\right)f^2g\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d)
*sqrt(g*x + f)/(c*d^2*f^3 - a*d*e*f^2*g + (c*d*e*f*g^2 - a*e^2*g^3)*x^3 + (
2*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 - 2*a*e^2)*f*g^2)*x^2 + (c*d*e*f^3 - 2*a
*d*e*f*g^2 + (2*c*d^2 - a*e^2)*f^2*g)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 63, normalized size = 1.00

$$-\frac{2\,(cdx+ae)\,\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3\left(gx+f\right)^{\frac{3}{2}}\left(aeg-cdf\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/3/(g*x+f)^(3/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(1/2)/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(5/2)), x)
```

mupad [B] time = 3.92, size = 136, normalized size = 2.16

$$-\frac{\left(\frac{2\,a\,e}{3\,a\,e\,g^2-3\,c\,d\,f\,g}+\frac{2\,c\,d\,x}{3\,a\,e\,g^2-3\,c\,d\,f\,g}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(3\,c\,d\,f^2-3\,a\,e\,f\,g\right)\,\sqrt{d+e\,x}}{3\,a\,e\,g^2-3\,c\,d\,f\,g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(5/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] -(((2*a*e)/(3*a*e*g^2 - 3*c*d*f*g) + (2*c*d*x)/(3*a*e*g^2 - 3*c*d*f*g))*(x*
(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x*(f + g*x)^(1/2)*(d + e*x)^(1
/2) - ((f + g*x)^(1/2)*(3*c*d*f^2 - 3*a*e*f*g)*(d + e*x)^(1/2))/(3*a*e*g^2
- 3*c*d*f*g))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}\,\left(f+gx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(5/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(5/2)), x)
```

3.506 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(7
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(15*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{7/2}}\,dx&=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{(2cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{5/2}}\,dx}{5(cdf-aeg)}\\&=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{3/2}(cd(5f+2gx)-3aeg)}{15(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(7/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-3*a*e*g + c*d*(5*f + 2*g*x)))/(15*(c*d
*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2))
```

IntegrateAlgebraic [A] time = 3.13, size = 196, normalized size = 1.52

$$\frac{2\sqrt{d+ex}\,(ef+egx)^{7/2}\sqrt{ae^2+cdex}\left(\frac{5cd\left(ae^2-cd^2+cd(d+ex)\right)^{3/2}}{(g(d+ex)-dg+ef)^{3/2}}-\frac{3g\left(ae^2-cd^2+cd(d+ex)\right)^{5/2}}{(g(d+ex)-dg+ef)^{5/2}}\right)}{15e^4\sqrt{\frac{(d+ex)\left(ae^2+cdex\right)}{e}}\left(\frac{g(d+ex)-dg+ef}{e}\right)^{7/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*(f + g*x)^(7/2)),x]
```

```
[Out] (2*Sqrt[d + e*x]*Sqrt[a*e^2 + c*d*e*x]*(e*f + e*g*x)^(7/2)*((-3*g*(-(c*d^2)
 + a*e^2 + c*d*(d + e*x))^(5/2))/(e*f - d*g + g*(d + e*x))^(5/2) + (5*c*d*(
-(c*d^2) + a*e^2 + c*d*(d + e*x))^(3/2))/(e*f - d*g + g*(d + e*x))^(3/2)))/
(15*e^4*(c*d*f - a*e*g)^2*Sqrt[((d + e*x)*(a*e^2 + c*d*e*x))/e]*((e*f - d*g
 + g*(d + e*x))/e)^(7/2))
```

fricas [B] time = 0.44, size = 402, normalized size = 3.12

$$\frac{2\left(2c^2d^2gx^2+5acdef-3a^2e^2g+\left(5c^2d^2f-acdeg\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}}{15\left(c^2d^3f^5-2acd^2ef^4g+a^2de^2f^3g^2+\left(c^2d^2ef^2g^3-2acde^2fg^4+a^2e^3g^5\right)x^4+\left(3c^2d^2ef^3g^2+a^2de^2g^5+\left(c^2d^3-6acde^2\right)f^2g^3-\left(2acd^2e-3a^2e^3\right)fg^4\right)x^3+3\left(c^2d^2ef^4g+a^2de^2fg^4+\left(c^2d^3-2acde^2\right)f^3g^2-\left(2acd^2e-a^2e^3\right)f^2g^3\right)x^2+\left(c^2d^2ef^5+3a^2de^2f^2g^3+\left(3c^2d^3-2acde^2\right)f^4g-\left(6acd^2e-a^2e^3\right)f^3g^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 3*a^2*e^2*g + (5*c^2*d^2*f - a*c*d*e*
g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
f)/(c^2*d^3*f^5 - 2*a*c*d^2*e*f^4*g + a^2*d*e^2*f^3*g^2 + (c^2*d^2*e*f^2*g^
3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^4 + (3*c^2*d^2*e*f^3*g^2 + a^2*d*e^2
*g^5 + (c^2*d^3 - 6*a*c*d*e^2)*f^2*g^3 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g^4)*x
^3 + 3*(c^2*d^2*e*f^4*g + a^2*d*e^2*f*g^4 + (c^2*d^3 - 2*a*c*d*e^2)*f^3*g^2
 - (2*a*c*d^2*e - a^2*e^3)*f^2*g^3)*x^2 + (c^2*d^2*e*f^5 + 3*a^2*d*e^2*f^2*
g^3 + (3*c^2*d^3 - 2*a*c*d*e^2)*f^4*g - (6*a*c*d^2*e - a^2*e^3)*f^3*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$-\frac{2\,(cdx+ae)\left(-2cdgx+3aeg-5cdf\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{15\left(gx+f\right)^{\frac{5}{2}}\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(-2*c*d*g*x+3*a*e*g-5*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/(g*x+f)^(5/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{\sqrt{ex + d}\left(gx + f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(7/2)), x)
```

mupad [B] time = 4.08, size = 187, normalized size = 1.45

$$\frac{\left(\frac{x\left(10c^2d^2f-2acdeg\right)}{15g^2\left(aeg-cdf\right)^2}-\frac{6a^2e^2g-10acdef}{15g^2\left(aeg-cdf\right)^2}+\frac{4c^2d^2x^2}{15g\left(aeg-cdf\right)^2}\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^2\sqrt{f+gx}\,\sqrt{d+ex}+\frac{f^2\sqrt{f+gx}\,\sqrt{d+ex}}{g^2}+\frac{2fx\sqrt{f+gx}\,\sqrt{d+ex}}{g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(7/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] (((x*(10*c^2*d^2*f - 2*a*c*d*e*g))/(15*g^2*(a*e*g - c*d*f)^2) - (6*a^2*e^2*
g - 10*a*c*d*e*f)/(15*g^2*(a*e*g - c*d*f)^2) + (4*c^2*d^2*x^2)/(15*g*(a*e*g
 - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g*x)
^(1/2)*(d + e*x)^(1/2) + (f^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (2*f*x
*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(7/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

3.507 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{35(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right.}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-a}$$

Rubi [A] time = 0.22, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {872, 860}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{35(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(9
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(7/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{9/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}}+\frac{(4cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{7/2}}\,dx}{7(cdf-aeg)}$$

$$=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx}$$

$$=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(15a^2e^2g^2-6acdeg(7f+2gx)+c^2d^2\left(35f^2+28fgx+8g^2x^2\right)\right)}{105(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(15*a^2*e^2*g^2 - 6*a*c*d*e*g*(7*f + 2*g
*x) + c^2*d^2*(35*f^2 + 28*f*g*x + 8*g^2*x^2)))/(105*(c*d*f - a*e*g)^3*(d +
 e*x)^(3/2)*(f + g*x)^(7/2))
```

IntegrateAlgebraic [A] time = 8.34, size = 137, normalized size = 0.69

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{35c^2d^2(ae+cdx)^{3/2}}{(f+gx)^{3/2}}+\frac{15g^2(ae+cdx)^{7/2}}{(f+gx)^{7/2}}-\frac{42cdg(ae+cdx)^{5/2}}{(f+gx)^{5/2}}\right)}{105\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*(f + g*x)^(9/2)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((15*g^2*(a*e + c*d*x)^(7/2))/(f + g*x)^
(7/2) - (42*c*d*g*(a*e + c*d*x)^(5/2))/(f + g*x)^(5/2) + (35*c^2*d^2*(a*e +
 c*d*x)^(3/2))/(f + g*x)^(3/2)))/(105*(c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*
(d + e*x)])
```

fricas [B] time = 0.45, size = 748, normalized size = 3.78

$$\frac{2\left(8c^3d^3g^2x^3+35ac^2d^2ef^2-42a^2cde^2fg+15a^3e^3g^2+4\left(7c^3d^3fg-ac^2d^2eg^2\right)x^2+\left(35c^3d^3f^2-14ac^2d^2efg+3a^2cde^2g^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}}{105\left(c^3d^4f^7-3ac^2d^3ef^6g+3a^2cd^2e^2f^5g^2-a^3de^3f^4g^3+\left(c^3d^3ef^3g^4-3ac^2d^2e^2f^2g^5+3a^2cde^3fg^6-a^3e^4g^7\right)x^5+\left(4c^3d^3ef^4g^3-a^3de^3g^7+\left(c^3d^4-12ac^2d^2e^2\right)f^3g^4-3\left(ac^2d^3e-4a^2cde^3\right)f^2g^5+\left(3a^2cd^2e^2-4a^3e^4\right)fg^6\right)x^4+2\left(3c^3d^3ef^5g^2-2a^3de^3fg^6+\left(2c^3d^4-9ac^2d^2e^2\right)f^4g^3-3\left(2ac^2d^3e-3a^2cde^3\right)f^3g^4+3\left(2a^2cd^2e^2-a^3e^4\right)f^2g^5\right)x^3+2\left(2c^3d^3ef^6g-3a^3de^3f^2g^5+3\left(c^3d^4-2ac^2d^2e^2\right)f^5g^2-3\left(3ac^2d^3e-2a^2cde^3\right)f^4g^3+\left(9a^2cd^2e^2-2a^3e^4\right)f^3g^4\right)x^2+\left(c^3d^3ef^7-4a^3de^3f^3g^4+\left(4c^3d^4-3ac^2d^2e^2\right)f^6g-3\left(4ac^2d^3e-a^2cde^3\right)f^5g^2+\left(12a^2cd^2e^2-a^3e^4\right)f^4g^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/105*(8*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 42*a^2*c*d*e^2*f*g + 15*a^3
*e^3*g^2 + 4*(7*c^3*d^3*f*g - a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 - 14*a
*c^2*d^2*e*f*g + 3*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^7 - 3*a*c^2*d^3*e*f^6*g + 3*
a^2*c*d^2*e^2*f^5*g^2 - a^3*d*e^3*f^4*g^3 + (c^3*d^3*e*f^3*g^4 - 3*a*c^2*d^
2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^5 + (4*c^3*d^3*e*f^4*g
^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2*d^3*e
- 4*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^6)*x^4 + 2*(3*
c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g
^3 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^4 + 3*(2*a^2*c*d^2*e^2 - a^3*e
^4)*f^2*g^5)*x^3 + 2*(2*c^3*d^3*e*f^6*g - 3*a^3*d*e^3*f^2*g^5 + 3*(c^3*d^4
- 2*a*c^2*d^2*e^2)*f^5*g^2 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^3 + (9
*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^4)*x^2 + (c^3*d^3*e*f^7 - 4*a^3*d*e^3*f^3
*g^4 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g - 3*(4*a*c^2*d^3*e - a^2*c*d*e^3
)*f^5*g^2 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 169, normalized size = 0.85

$$-\frac{2 \left(c d x+a e\right) \left(8 g^{2} x^{2} c^{2} d^{2}-12 a c d e \,g^{2} x +28 c^{2} d^{2} f g x +15 a^{2} e^{2} g^{2}-42 a c d e f g +35 f^{2} c^{2} d^{2}\right) \sqrt{c d e \,x^{2}+a \,e^{2} x +c \,d^{2} x +a d e}}{105 \left(g x +f \right)^{\frac{7}{2}} \left(a^{3} e^{3} g^{3}-3 a^{2} c d \,e^{2} f \,g^{2}+3 a \,c^{2} d^{2} e \,f^{2} g -f^{3} c^{3} d^{3}\right) \sqrt{e x +d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/105*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+28*c^2*d^2*f*g*x+15*
a^2*e^2*g^2-42*a*c*d*e*f*g+35*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(1/2)/(g*x+f)^(7/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-
c^3*d^3*f^3)/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^{2} + ade + \left(cd^{2} + ae^{2}\right)x}}{\sqrt{ex + d}\left(gx + f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(9/2)), x)
```

mupad [B] time = 4.29, size = 289, normalized size = 1.46

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{30\,a^3\,e^3\,g^2-84\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x\,\left(6\,a^2\,c\,d\,e^2\,g^2-28\,a\,c^2\,d^2\,e\,f\,g+70\,c^3\,d^3\,f^2\right)}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^3\,d^3\,x^3}{105\,g\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c^2\,d^2\,x^2\,\left(a\,e\,g-7\,c\,d\,f\right)}{105\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(9/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((30*a^3*e^3*g^2 + 70*a*c^2
*d^2*e*f^2 - 84*a^2*c*d*e^2*f*g)/(105*g^3*(a*e*g - c*d*f)^3) + (x*(70*c^3*d
^3*f^2 + 6*a^2*c*d*e^2*g^2 - 28*a*c^2*d^2*e*f*g))/(105*g^3*(a*e*g - c*d*f)^
3) + (16*c^3*d^3*x^3)/(105*g*(a*e*g - c*d*f)^3) - (8*c^2*d^2*x^2*(a*e*g - 7
*c*d*f))/(105*g^2*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2)
 + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x^2*(f + g*x)^(1/2)*(d
+ e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(9/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

3.508 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cde\right.}{21(d+ex)^{3/2}(f+gx)^{7/2}(cdf-}$$

Rubi [A] time = 0.31, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{21(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(1
1/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(21*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(5/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(315*(c*d*f - a*e*g)^4*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{11/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{(2cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{9/2}}\,dx}{3(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)}$$

Mathematica [A] time = 0.14, size = 152, normalized size = 0.57

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(-35a^3e^3g^3+15a^2cde^2g^2(9f+2gx)-3ac^2d^2eg\left(63f^2+36fgx+8g^2x^2\right)+c^3d^3\left(105f^3+126f^2gx+72fg^2x^2+16g^3x^3\right)\right)}{315(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(11/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-35*a^3*e^3*g^3 + 15*a^2*c*d*e^2*g^2*(9
*f + 2*g*x) - 3*a*c^2*d^2*e*g*(63*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(10
5*f^3 + 126*f^2*g*x + 72*f*g^2*x^2 + 16*g^3*x^3)))/(315*(c*d*f - a*e*g)^4*(
d + e*x)^(3/2)*(f + g*x)^(9/2))
```

IntegrateAlgebraic [A] time = 8.88, size = 169, normalized size = 0.63

$$\frac{2\sqrt{d+ex}\,\sqrt{ae+cdx}\left(\frac{105c^3d^3(ae+cdx)^{3/2}}{(f+gx)^{3/2}}-\frac{189c^2d^2g(ae+cdx)^{5/2}}{(f+gx)^{5/2}}-\frac{35g^3(ae+cdx)^{9/2}}{(f+gx)^{9/2}}+\frac{135cdg^2(ae+cdx)^{7/2}}{(f+gx)^{7/2}}\right)}{315\sqrt{(d+ex)(ae+cdx)}\,(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*
x]*(f + g*x)^(11/2)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((-35*g^3*(a*e + c*d*x)^(9/2))/(f + g*x)
^(9/2) + (135*c*d*g^2*(a*e + c*d*x)^(7/2))/(f + g*x)^(7/2) - (189*c^2*d^2*g
*(a*e + c*d*x)^(5/2))/(f + g*x)^(5/2) + (105*c^3*d^3*(a*e + c*d*x)^(3/2))/(
f + g*x)^(3/2)))/(315*(c*d*f - a*e*g)^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

fricas [B] time = 0.47, size = 1179, normalized size = 4.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 189*a^2*c^2*d^2*e^2*f^2*g
 + 135*a^3*c*d*e^3*f*g^2 - 35*a^4*e^4*g^3 + 8*(9*c^4*d^4*f*g^2 - a*c^3*d^3*
e*g^3)*x^3 + 6*(21*c^4*d^4*f^2*g - 6*a*c^3*d^3*e*f*g^2 + a^2*c^2*d^2*e^2*g^
3)*x^2 + (105*c^4*d^4*f^3 - 63*a*c^3*d^3*e*f^2*g + 27*a^2*c^2*d^2*e^2*f*g^2
```

```
 - 5*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e
*x + d)*sqrt(g*x + f)/(c^4*d^5*f^9 - 4*a*c^3*d^4*e*f^8*g + 6*a^2*c^2*d^3*e^
2*f^7*g^2 - 4*a^3*c*d^2*e^3*f^6*g^3 + a^4*d*e^4*f^5*g^4 + (c^4*d^4*e*f^4*g^
5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*a^3*c*d*e^4*f*g
^8 + a^4*e^5*g^9)*x^6 + (5*c^4*d^4*e*f^5*g^4 + a^4*d*e^4*g^9 + (c^4*d^5 - 2
0*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^6 +
 2*(3*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e^3 - 5*a^4*
e^5)*f*g^8)*x^5 + 5*(2*c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c^4*d^5 - 8*a
*c^3*d^3*e^2)*f^5*g^4 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^5 + 2*(3*
a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^3*g^6 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^
2*g^7)*x^4 + 10*(c^4*d^4*e*f^7*g^2 + a^4*d*e^4*f^2*g^7 + (c^4*d^5 - 4*a*c^3
*d^3*e^2)*f^6*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^4 + 2*(3*a^
2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^
6)*x^3 + 5*(c^4*d^4*e*f^8*g + 2*a^4*d*e^4*f^3*g^6 + 2*(c^4*d^5 - 2*a*c^3*d^
3*e^2)*f^7*g^2 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^3 + 4*(3*a^2*c
^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^4 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^5)*x^
2 + (c^4*d^4*e*f^9 + 5*a^4*d*e^4*f^4*g^5 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^
8*g - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^2 + 2*(15*a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^6*g^3 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 260, normalized size = 0.97

$$-\frac{2 (cdx+ae) \left(-16g^3x^3c^3d^3+24a c^2d^2e g^3x^2-72c^3d^3f g^2x^2-30a^2cd e^2g^3x+108a c^2d^2ef g^2x-126c^3d^3f^2gx+35a^3e^3g^3-135a^2cd e^2f g^2+189a c^2d^2e f^2g-105f^3c^3d^3\right) \sqrt{cde x^2+a e^2x+c d^2x+ade}}{315 (gx+f)^{\frac{9}{2}} \left(g^4e^4a^4-4a^3cd e^3f g^3+6a^2c^2d^2e^2f^2g^2-4a c^3d^3e f^3g+f^4c^4d^4\right) \sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/315*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+24*a*c^2*d^2*e*g^3*x^2-72*c^3*d^3*f
*g^2*x^2-30*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-126*c^3*d^3*f^2*g*x+3
5*a^3*e^3*g^3-135*a^2*c*d*e^2*f*g^2+189*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)*
(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(g*x+f)^(9/2)/(a^4*e^4*g^4-4*a^3*c*
d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(e*x
+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(11/2)), x)
```

mupad [B] time = 4.50, size = 409, normalized size = 1.53

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{x\,\left(-10\,a^3\,c\,d\,e^3\,g^3+54\,a^2\,c^2\,d^2\,e^2\,f\,g^2-126\,a\,c^3\,d^3\,e\,f^2\,g+210\,c^4\,d^4\,f^3\right)}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{70\,a^4\,e^4\,g^3-270\,a^3\,c\,d\,e^3\,f\,g^2+378\,a^2\,c^2\,d^2\,e^2\,f^2\,g-210\,a\,c^3\,d^3\,e\,f^3}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{32\,c^4\,d^4\,x^4}{315\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{4\,c^2\,d^2\,x^2\,\left(a^2\,e^2\,g^2-6\,a\,c\,d\,e\,f\,g+21\,c^2\,d^2\,f^2\right)}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{16\,c^3\,d^3\,x^3\,\left(a\,e\,g-9\,c\,d\,f\right)}{315\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{4\,f^3\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{6\,f^2\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(11/2)*(d + e*
x)^(1/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((x*(210*c^4*d^4*f^3 - 10*a^
3*c*d*e^3*g^3 + 54*a^2*c^2*d^2*e^2*f*g^2 - 126*a*c^3*d^3*e*f^2*g))/(315*g^4
*(a*e*g - c*d*f)^4) - (70*a^4*e^4*g^3 - 210*a*c^3*d^3*e*f^3 + 378*a^2*c^2*d
^2*e^2*f^2*g - 270*a^3*c*d*e^3*f*g^2)/(315*g^4*(a*e*g - c*d*f)^4) + (32*c^4
*d^4*x^4)/(315*g*(a*e*g - c*d*f)^4) + (4*c^2*d^2*x^2*(a^2*e^2*g^2 + 21*c^2*
d^2*f^2 - 6*a*c*d*e*f*g))/(105*g^3*(a*e*g - c*d*f)^4) - (16*c^3*d^3*x^3*(a*
e*g - 9*c*d*f))/(315*g^2*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2)*(d + e*x
)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*x^3*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (6
*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(11/2)/(e*x+
d)**(1/2),x)
```

```
[Out] Timed out
```

3.509 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=382

$$\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64c^2d^2g^2\sqrt{d+ex}}+\cdots$$

Rubi [A] time = 0.71, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {864, 870, 891, 63, 217, 206}

$$\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64c^2d^2g^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{32cdg^2\sqrt{d+ex}}-\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{8g^2\sqrt{d+ex}}+\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x]
```

```
[Out] (3*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(64*c^2*d^2*g^2*Sqrt[d + e*x]) + ((c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*c*d*g^2*Sqrt[d + e*x]) - ((c
*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(8*g^2*Sqrt[d + e*x]) + ((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2))/(4*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(5/2)*d^(5/2)*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
```

```
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{(3(cdf-ae}{}$$

$$=-\frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}}+$$

$$=\frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32cdg^2\sqrt{d+ex}}-$$

$$=\frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}}+$$

$$=\frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}}+$$

$$=\frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}}+$$

$$=\frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}}+$$

$$=\frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}}+$$

Mathematica [A] time = 1.17, size = 302, normalized size = 0.79

$$\frac{\sqrt{cd}\sqrt{d+ex}\left(3\sqrt{ae+cdx}\,(cdf-aeg)^{9/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)-\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}\,(f+gx)(ae+cdx)\left(3a^3e^3g^3-a^2cde^2g^2(11f+2gx)-ac^2d^2eg\left(11f^2+44fgx+24g^2x^2\right)+c^3d^3\left(3f^3-2f^2gx-24fg^2x^2-16g^3x^3\right)\right)\right)}{64c^{7/2}d^{7/2}g^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
d + e*x)^(3/2),x]
```

```
[Out] (Sqrt[c*d]*Sqrt[d + e*x]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*d*x)
*(f + g*x)*(3*a^3*e^3*g^3 - a^2*c*d*e^2*g^2*(11*f + 2*g*x) - a*c^2*d^2*e*g*
(11*f^2 + 44*f*g*x + 24*g^2*x^2) + c^3*d^3*(3*f^3 - 2*f^2*g*x - 24*f*g^2*x^
2 - 16*g^3*x^3))) + 3*(c*d*f - a*e*g)^(9/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(f
+ g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]
)/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(64*c^(7/2)*d^(7/2)*g^(5/2)*Sqrt[(a*e
+ c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 2.05, size = 431, normalized size = 1.13

$$\frac{g^{3/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(-3a^3e^3g^3\sqrt{f+gx}+2a^2cde^2g^2(f+gx)^{3/2}+9a^2cde^2fg^2\sqrt{f+gx}-9ac^2d^2ef^2g\sqrt{f+gx}+24ac^2d^2eg(f+gx)^{5/2}-4ac^2d^2efg(f+gx)^{3/2}+3c^3d^3f^3\sqrt{f+gx}+2c^3d^3f^2(f+gx)^{3/2}+16c^3d^3(f+gx)^{7/2}-24c^3d^3f(f+gx)^{5/2}\right)}{64c^2d^2g^{5/2}}-\frac{3\sqrt{cd}\left(a^4e^4g^4-4a^3cde^3fg^3+6a^2c^2d^2e^2f^2g^2-4ac^3d^3ef^3g+c^4d^4f^4\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{64c^3d^3g^{5/2}}\right)}{(d+ex)^{3/2}(aeg+cdgx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(d + e*x)^(3/2),x]
```

```
[Out] (g^(3/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2)*((Sqrt[-(c*d*f) + a*
e*g + c*d*(f + g*x)]*(3*c^3*d^3*f^3*Sqrt[f + g*x] - 9*a*c^2*d^2*e*f^2*g*Sqr
t[f + g*x] + 9*a^2*c*d*e^2*f*g^2*Sqrt[f + g*x] - 3*a^3*e^3*g^3*Sqrt[f + g*x
] + 2*c^3*d^3*f^2*(f + g*x)^(3/2) - 4*a*c^2*d^2*e*f*g*(f + g*x)^(3/2) + 2*a
^2*c*d*e^2*g^2*(f + g*x)^(3/2) - 24*c^3*d^3*f*(f + g*x)^(5/2) + 24*a*c^2*d^
2*e*g*(f + g*x)^(5/2) + 16*c^3*d^3*(f + g*x)^(7/2)))/(64*c^2*d^2*g^(5/2)) -
 (3*Sqrt[c*d]*(c^4*d^4*f^4 - 4*a*c^3*d^3*e*f^3*g + 6*a^2*c^2*d^2*e^2*f^2*g^
2 - 4*a^3*c*d*e^3*f*g^3 + a^4*e^4*g^4)*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqr
t[-(c*d*f) + a*e*g + c*d*(f + g*x)]])/(64*c^3*d^3*g^(5/2))))/((d + e*x)^(3/
2)*(a*e*g + c*d*g*x)^(3/2))
```

fricas [A] time = 2.68, size = 1059, normalized size = 2.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/256*(4*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3*d^3*e*f^2*g^2 +
11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^4*f*g^3 + a*c^3*d^
3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3 + a^2*c^2*d^2*e^2*
g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f) + 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4
*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3
*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*sqrt
(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e
^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
 a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d
^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
 + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^3*x + c^3*d^4*g
^3), 1/128*(2*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3*d^3*e*f^2*g^
2 + 11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^4*f*g^3 + a*c^
3*d^3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3 + a^2*c^2*d^2*
```

```
e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f) - 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2
 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2
*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*
sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d
*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e
*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*g^3*x + c^3*d^4*g^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 870, normalized size = 2.28

$$\frac{\sqrt{gx+f}\,\sqrt{cde\,x^2+a e^2x+c d^2x+ade}\,\left(3a^4e^4g^4\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-12a^3cde^3fg^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+18a^2c^2d^2e^2f^2g^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-12ac^3d^3ef^3g\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+3c^4d^4f^4\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+32\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,c^3d^3g^3x^3+48\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,ac^2d^2eg^3x^2+48\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,c^3d^3fg^2x^2+4\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,a^2cde^2g^3x+88\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,ac^2d^2efg^2x+4\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,c^3d^3f^2gx-6\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,a^3e^3g^3+22\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,a^2cde^2fg^2+22\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,ac^2d^2ef^2g-6\sqrt{cdg\,x^2+aegx+cdfx+aef}\,\sqrt{cdg}\,c^3d^3f^3\right)}{128\sqrt{ex+d}\,\sqrt{cdg\,x^2+aegx+cdfx+aef}\,c^2d^2g^2\sqrt{cdg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 1/128*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(32*(c*d*g*x^2+
a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*c^3*d^3*g^3*x^3+3*a^4*e^4*g^4*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))-12*a^3*c*d*e^3*f*g^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+18
*a^2*c^2*d^2*e^2*f^2*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x
+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-12*a*c^3*d^3*e*f^3*g*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))+3*c^4*d^4*f^4*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d
*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+48*(c*d*g
*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*g^3*x^2+48*(c*d
*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*c^3*d^3*f*g^2*x^2+4*(c*d*
g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*g^3*x+88*(c*d*
g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f*g^2*x+4*(c*d
*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^2*g*x-6*(c*d*g)
^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*e^3*g^3+22*(c*d*g)^(1/2)
*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*f*g^2+22*(c*d*g)^(1/2)
*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f^2*g-6*(c*d*g)^(1/2)*
(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/c^2/d^2/
g^2/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}\left(gx + f\right)^{\frac{3}{2}}}{\left(ex + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^(3/2)/(e*
x + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{(d+e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d
)**(3/2),x)
```

```
[Out] Timed out
```

3.510 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=310

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8cdg^2\sqrt{d+ex}}\ \ (\ldots$$

Rubi [A] time = 0.53, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8cdg^2\sqrt{d+ex}}-\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4g^2\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)
^(3/2),x]
```

```
[Out] ((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(8*c*d*g^2*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)) + ((c*
d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(3/2)*d^(3/2)*g^(5/2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] && !IGtQ[n, 0] && !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} - \frac{(cdf-aeg)\int \frac{\sqrt{\ldots}}{\ldots}}{\ldots}$$

$$= -\frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{(f+\ldots}{\ldots}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-\ldots}{\ldots}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-\ldots}{\ldots}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-\ldots}{\ldots}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-\ldots}{\ldots}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-\ldots}{\ldots}$$

Mathematica [A] time = 0.83, size = 254, normalized size = 0.82

$$\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}\,(f+gx)(ae+cdx)\left(3a^2e^2g^2+2acdeg(4f+7gx)+c^2d^2\left(-3f^2+2fgx+8g^2x^2\right)\right)+3\sqrt{ae+cdx}\,(cdf-aeg)^{7/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\,\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)\right)}{24g^{5/2}(cd)^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d
+ e*x)^(3/2),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*
d*x)*(f + g*x)*(3*a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + 7*g*x) + c^2*d^2*(-3*f^2
 + 2*f*g*x + 8*g^2*x^2)) + 3*(c*d*f - a*e*g)^(7/2)*Sqrt[a*e + c*d*x]*Sqrt[(
c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e +
 c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(24*(c*d)^(5/2)*g^(5/2)*Sqrt[(a
*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 1.79, size = 316, normalized size = 1.02

$$\frac{g^{3/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(3a^2e^2g^2\sqrt{f+gx}+14acdeg(f+gx)^{3/2}-6acdefg\sqrt{f+gx}+3c^2d^2f^2\sqrt{f+gx}+8c^2d^2(f+gx)^{5/2}-14c^2d^2f(f+gx)^{3/2}\right)}{24cdg^{5/2}}-\frac{\sqrt{cd}\left(-a^3e^3g^3+3a^2cde^2fg^2-3ac^2d^2ef^2g+c^3d^3f^3\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{8c^2d^2g^{5/2}}\right)}{(d+ex)^{3/2}(aeg+cdgx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/(d + e*x)^(3/2),x]
```

```
[Out] (g^(3/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2)*((Sqrt[-(c*d*f) + a*
e*g + c*d*(f + g*x)]*(3*c^2*d^2*f^2*Sqrt[f + g*x] - 6*a*c*d*e*f*g*Sqrt[f +
g*x] + 3*a^2*e^2*g^2*Sqrt[f + g*x] - 14*c^2*d^2*f*(f + g*x)^(3/2) + 14*a*c*
d*e*g*(f + g*x)^(3/2) + 8*c^2*d^2*(f + g*x)^(5/2)))/(24*c*d*g^(5/2)) - (Sqr
t[c*d]*(c^3*d^3*f^3 - 3*a*c^2*d^2*e*f^2*g + 3*a^2*c*d*e^2*f*g^2 - a^3*e^3*g
^3)*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]
])/(8*c^2*d^2*g^(5/2))))/((d + e*x)^(3/2)*(a*e*g + c*d*g*x)^(3/2))
```

fricas [A] time = 1.51, size = 847, normalized size = 2.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 + 3*a^2
*c*d*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a
*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 -
 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*d*g)*
log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 -
 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c
*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^3*x + c^2*d^3*g^3), 1/4
8*(2*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 + 3*a^2*c*d
*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a*c^2
*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 - 3*a
*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*d*g)*arc
tan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d
)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 +
a*e^2)*g)*x)))/(c^2*d^2*e*g^3*x + c^2*d^3*g^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 602, normalized size = 1.94

$$-\frac{\sqrt{gx+f}\,\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\,\left(3a^{3}e^{3}g^{3}\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-9a^{2}cd\,e^{2}fg^{2}\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+9a\,c^{2}d^{2}e\,f^{2}g\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-3c^{3}d^{3}f^{3}\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-16\sqrt{cdg}\,\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,c^{2}d^{2}g^{2}x^{2}-28\sqrt{cdg}\,\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,acdeg^{2}x-4\sqrt{cdg}\,\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,c^{2}d^{2}fgx-6\sqrt{cdg}\,\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,a^{2}e^{2}g^{2}-16\sqrt{cdg}\,\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,acdefg+6\sqrt{cdg}\,\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,c^{2}d^{2}f^{2}\right)}{48\sqrt{ex+d}\,\sqrt{cdg\,x^{2}+aegx+cdfx+aef}\,\sqrt{cdg}\,cd\,g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2),x)

[Out] -1/48*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*a^3*e^3*g^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-9*a^2*c*d*e^2*f*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+9*a*c^2*d^2*e*f^2*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-3*c^3*d^3*f^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-16*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^2*d^2*g^2*x^2-28*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c*d*e*g^2*x-4*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^2*d^2*f*g*x-6*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*e^2*g^2-16*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c*d*e*f*g+6*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/c/d/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/g^2/(c*d*g)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}\sqrt{gx+f}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")

[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sqrt(g*x + f)/(e*x + d)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(3/2),x)

[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}\sqrt{f+gx}}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d
)**(3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x)/(d + e*x)**(3/2), x
)
```

3.511 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=238

$$\frac{3\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}\,g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4g^2\sqrt{d+ex}}+$$

Rubi [A] time = 0.35, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {864, 891, 63, 217, 206}

$$-\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4g^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}\,g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sqrt[f +
 g*x]),x]
```

```
[Out] (-3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(4*g^2*Sqrt[d + e*x]) + (Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2))/(2*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(4*Sqrt[c]*Sqrt[d]*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}} - \frac{(3(cdf-aeg))\int \frac{\sqrt{ade+(c}}{\sqrt{d-}}}{4g}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\,(a}{}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\,(a}{}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\,(a}{}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\,(a}{}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\,(a}{}$$

Mathematica [A] time = 0.77, size = 193, normalized size = 0.81

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}\,(f+gx)(5aeg+cd(2gx-3f))+\frac{3\sqrt{c}\sqrt{d}\,(cdf-aeg)^{5/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}}\right)}{4g^{5/2}\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sq
rt[f + g*x]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g]*(f + g*x)*(5*a*e*g + c*d*(-3*f + 2*
g*x)) + (3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)^(5/2)*Sqrt[(c*d*(f + g*x))/(c*d*
f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*
Sqrt[c*d*f - a*e*g])])/((c*d)^(3/2)*Sqrt[a*e + c*d*x])))/(4*g^(5/2)*Sqrt[d
+ e*x]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 1.38, size = 222, normalized size = 0.93

$$\frac{g^{3/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(5aeg\sqrt{f+gx}+2cd(f+gx)^{3/2}-5cdf\sqrt{f+gx}\right)}{4g^{5/2}}-\frac{3\sqrt{cd}\left(a^2e^2g^2-2acdefg+c^2d^2f^2\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{4cdg^{5/2}}\right)}{(d+ex)^{3/2}(aeg+cdgx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (g^(3/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2)*((Sqrt[-(c*d*f) + a*
e*g + c*d*(f + g*x)]*(-5*c*d*f*Sqrt[f + g*x] + 5*a*e*g*Sqrt[f + g*x] + 2*c*
d*(f + g*x)^(3/2)))/(4*g^(5/2)) - (3*Sqrt[c*d]*(c^2*d^2*f^2 - 2*a*c*d*e*f*g
 + a^2*e^2*g^2)*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*g + c*
d*(f + g*x)]])/(4*c*d*g^(5/2))))/((d + e*x)^(3/2)*(a*e*g + c*d*g*x)^(3/2))
```

fricas [A] time = 1.21, size = 651, normalized size = 2.74

$$\left[\frac{4\left(2c^2d^2g^2x-3c^2d^2fg+5acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}+3\left(c^2d^3f^2-2acd^2efg+a^2de^2g^2+\left(c^2d^2ef^2-2acde^2fg+a^2e^3g^2\right)x\right)\sqrt{cdg}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdgx+cdf+aeg\right)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+\left(c^2d^2ef^2+2\left(4c^2d^3+3acde^2\right)fg+\left(8acd^2e+a^2e^3\right)g^2\right)x}{ex+d}\right)}{16\left(cdeg^3x+cd^2g^3\right)},\ \frac{2\left(2c^2d^2g^2x-3c^2d^2fg+5acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}-3\left(c^2d^3f^2-2acd^2efg+a^2de^2g^2+\left(c^2d^2ef^2-2acde^2fg+a^2e^3g^2\right)x\right)\sqrt{-cdg}\arctan\left(\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdg}\sqrt{ex+d}\sqrt{gx+f}}{2cdegx^2+cd^2f+adeg+\left(cdef+\left(2cd^2+ae^2\right)g\right)x}\right)}{8\left(cdeg^3x+cd^2g^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g^2)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 -
2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^
3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e
*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*
e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*
a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^3*x +
 c*d^2*g^3), 1/8*(2*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g^2)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^2
*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f
*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2
*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c*d*e*g^3*x + c*d^2*g^
3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 325, normalized size = 1.37

$$\frac{\sqrt{cdex^2+ae^2x+cd^2x+ade}\,\sqrt{gx+f}\left(3a^2e^2g^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-6acdefg\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+3c^2d^2f^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+4\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,cdgx+10\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,aeg-6\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,cdf\right)}{8\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)*(3*a^2*e^2*g^2*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c
```

```
*d*g)^(1/2))-6*a*c*d*e*f*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+
a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+3*c^2*d^2*f^2*ln(1/2*(2*c*d*g*x+a
*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+4*(c
*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*g*x+10*(c*d*g)^(1/2)*((g*x+f)*(
c*d*x+a*e))^(1/2)*a*e*g-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/
(e*x+d)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*sq
rt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\sqrt{f + g\,x}\,\left(d + e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*x
)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(1/2),x)
```

```
[Out] Timed out
```

3.512 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{3\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3cd\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2\,(x}{}$$

Rubi [A] time = 0.30, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {862, 864, 891, 63, 217, 206}

$$\frac{3cd\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}-\frac{3\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(3/2)),x]
```

```
[Out] (3*c*d*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt
[d + e*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(g*(d + e*x)
^(3/2)*Sqrt[f + g*x]) - (3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
 + g*x])])/(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}+\frac{(3cd)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{g}$$

$$=\frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{g(d+ex)^{3/2}\sqrt{f+}}$$

$$=\frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{g(d+ex)^{3/2}\sqrt{f+}}$$

$$=\frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{g(d+ex)^{3/2}\sqrt{f+}}$$

$$=\frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{g(d+ex)^{3/2}\sqrt{f+}}$$

$$=\frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x\right.}{g(d+ex)^{3/2}\sqrt{f+}}$$

Mathematica [C] time = 0.16, size = 102, normalized size = 0.46

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5cd(d+ex)^{5/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(3/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*((c*d*(f + g*x))/(c*d*f - a*e*g))^(3/2)*
Hypergeometric2F1[3/2, 5/2, 7/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*
c*d*(d + e*x)^(5/2)*(f + g*x)^(3/2))
```

IntegrateAlgebraic [A] time = 1.45, size = 178, normalized size = 0.80

$$\frac{g^{3/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\,(-2aeg+cd(f+gx)+2cdf)}{g^{5/2}\sqrt{f+gx}}+\frac{3\sqrt{cd}\,(cdf-aeg)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\,\sqrt{f+gx}\right)}{g^{5/2}}\right)}{(d+ex)^{3/2}(aeg+cdgx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^(3/2)),x]
```

```
[Out] (g^(3/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2)*(((2*c*d*f - 2*a*e*g
 + c*d*(f + g*x))*Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)])/(g^(5/2)*Sqrt[f +
 g*x]) + (3*Sqrt[c*d]*(c*d*f - a*e*g)*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt
[-(c*d*f) + a*e*g + c*d*(f + g*x)]])/g^(5/2)))/((d + e*x)^(3/2)*(a*e*g + c*
d*g*x)^(3/2))
```

fricas [A] time = 1.12, size = 663, normalized size = 2.99

$$\left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(cdgx+3cdf-2aeg)\sqrt{ex+d}\,\sqrt{gx+f}-3\left(cd^2f^2-adefg+(cdefg-ae^2g^2)x^2+(cdef^2-adeg^2+(cd^2-ae^2)fg)x\right)\sqrt{\frac{cd}{g}}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4(2cdg^2x+cdfg+aeg^2)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,\sqrt{ex+d}\,\sqrt{gx+f}\,\sqrt{\frac{cd}{g}}+8(c^2d^2efg+(c^2d^3+acde^2)g^2)x^2+(c^2d^2ef^2+2(4c^2d^3+3acde^2)fg+(8acd^2e+a^2e^3)g^2)x}{ex+d}\right)}{4(eg^3x^2+dfg^2+(efg^2+dg^3)x)},\ \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(cdgx+3cdf-2aeg)\sqrt{ex+d}\,\sqrt{gx+f}+3\left(cd^2f^2-adefg+(cdefg-ae^2g^2)x^2+(cdef^2-adeg^2+(cd^2-ae^2)fg)x\right)\sqrt{-\frac{cd}{g}}\arctan\left(\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,\sqrt{ex+d}\,\sqrt{gx+f}\,\sqrt{-\frac{cd}{g}}\,g}{2cdegx^2+cd^2f+adeg+(cdef+(2cd^2+ae^2)g)x}\right)}{2(eg^3x^2+dfg^2+(efg^2+dg^3)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2*
a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*g
- a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g)*x)*sqrt(c*
d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*
g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^
2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*
f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^3*x^2 + d*f*g^2 + (e
*f*g^2 + d*g^3)*x), 1/2*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d
*g*x + 3*c*d*f - 2*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c*d^2*f^2 - a*d*
e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e
^2)*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*
d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^3*x^2 + d*f*g^2 + (e*f*g^
2 + d*g^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 383, normalized size = 1.73

$$\frac{\left(3acde\,g^2x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-3c^2d^2fgx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+3acdefg\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-3c^2d^2f^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+2\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,cdgx-4\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,aeg+6\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,cdf\right)\sqrt{cdex^2+ae^2x+cd^2x+ade}}{2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,\sqrt{gx+f}\,\sqrt{ex+d}\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/2),x)
```

```
[Out] 1/2*(3*a*c*d*e*g^2*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^
(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-3*c^2*d^2*f*g*x*ln(1/2*(2*c*d*g*x+a*e*g
+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+3*a*c*d*
e*f*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(
1/2))/(c*d*g)^(1/2))-3*c^2*d^2*f^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)
*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+2*(c*d*g)^(1/2)*((g*x+f)*
(c*d*x+a*e))^(1/2)*c*d*g*x-4*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e*
g+6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d
^2*x+a*d*e)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^2/(g*x+f)^(1/
2)/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(f + g\,x\right)^{3/2}\,\left(d + e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*x
)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(3/2),x)
```

```
[Out] Timed out
```

3.513 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=214

$$\frac{2c^{3/2}d^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+c\right.}{3g(d+ex)^{3/2}(f+gx)}$$

Rubi [A] time = 0.28, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {862, 891, 63, 217, 206}

$$\frac{2c^{3/2}d^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(5/2)),x]
```

```
[Out] (-2*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]*Sqr
t[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e
*x)^(3/2)*(f + g*x)^(3/2)) + (2*c^(3/2)*d^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d +
e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/
(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{(cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx}{g}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

Mathematica [A] time = 1.06, size = 188, normalized size = 0.88

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{3\sqrt{c}\,\sqrt{d}\,(cdf-aeg)^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cd}\,\sqrt{cdf-aeg}}\right)}{\sqrt{cd}\,\sqrt{ae+cdx}}-\sqrt{g}\,(aeg+cd(3f+4gx))\right)}{3g^{5/2}\sqrt{d+ex}\,(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]*(a*e*g + c*d*(3*f + 4*g*x))) +
(3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)^(3/2)*((c*d*(f + g*x))/(c*d*f - a*e*g))^
(3/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c
*d*f - a*e*g])])/(Sqrt[c*d]*Sqrt[a*e + c*d*x])))/(3*g^(5/2)*Sqrt[d + e*x]*(
f + g*x)^(3/2))
```

IntegrateAlgebraic [A] time = 1.54, size = 173, normalized size = 0.81

$$\frac{g^{3/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}\left(\frac{2(-aeg-4cd(f+gx)+cdf)\sqrt{aeg+cd(f+gx)-cdf}}{3g^{5/2}(f+gx)^{3/2}}-\frac{2cd\sqrt{cd}\,\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\,\sqrt{f+gx}\right)}{g^{5/2}}\right)}{(d+ex)^{3/2}(aeg+cdgx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^(5/2)),x]
```

```
[Out] (g^(3/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2)*((2*(c*d*f - a*e*g -
 4*c*d*(f + g*x))*Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)])/(3*g^(5/2)*(f + g
*x)^(3/2)) - (2*c*d*Sqrt[c*d]*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f
) + a*e*g + c*d*(f + g*x)]])/g^(5/2)))/((d + e*x)^(3/2)*(a*e*g + c*d*g*x)^(
3/2))
```

fricas [A] time = 1.08, size = 685, normalized size = 3.20

$$\left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(4cdgx+3cdf+aeg)\sqrt{ex+d}\,\sqrt{gx+f}-3\left(cdeg^2x^3+cd^2f^2+(2cdefg+cd^2g^2)x^2+(cdef^2+2cd^2fg)x\right)\sqrt{\frac{cd}{g}}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4(2cdg^2x+cdfg+aeg^2)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,\sqrt{ex+d}\,\sqrt{gx+f}\,\sqrt{\frac{cd}{g}}+8\left(c^2d^2efg+(c^2d^3+acde^2)g^2\right)x^2+\left(c^2d^2ef^2+2(4c^2d^3+3acde^2)fg+(8acd^2e+a^2e^3)g^2\right)x}{ex+d}\right)}{6\left(eg^4x^3+df^2g^2+(2efg^3+dg^4)x^2+(ef^2g^2+2dfg^3)x\right)},\ -\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(4cdgx+3cdf+aeg)\sqrt{ex+d}\,\sqrt{gx+f}+3\left(cdeg^2x^3+cd^2f^2+(2cdefg+cd^2g^2)x^2+(cdef^2+2cd^2fg)x\right)\sqrt{-\frac{cd}{g}}\arctan\left(\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,\sqrt{ex+d}\,\sqrt{gx+f}\,\sqrt{-\frac{cd}{g}}\,g}{2cdegx^2+cd^2f+adeg+\left(cdef+(2cd^2+ae^2)g\right)x}\right)}{3\left(eg^4x^3+df^2g^2+(2efg^3+dg^4)x^2+(ef^2g^2+2dfg^3)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f +
 a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c*d
*e*f*g + c*d^2*g^2)*x^2 + (c*d*e*f^2 + 2*c*d^2*f*g)*x)*sqrt(c*d/g)*log(-(8*
c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*
d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d
*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*
d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^4*x^3 + d*f^2*g^2 + (2*e*f*g^3 +
d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x), -1/3*(2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f + a*e*g)*sqrt(e*x + d)*sqrt(g*x + f)
 + 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c*d*e*f*g + c*d^2*g^2)*x^2 + (c*d*e*f^
2 + 2*c*d^2*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^
2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x^3 + d*f^2*g^2
 + (2*e*f*g^3 + d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 331, normalized size = 1.55

$$\frac{\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3c^2d^2g^2x^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+6c^2d^2fgx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+3c^2d^2f^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-8\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,cdgx-2\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,aeg-6\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,cdf\right)}{3\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,(gx+f)^{\frac{3}{2}}\sqrt{ex+d}\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/2),x)
```

```
[Out] 1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*c^2*d^2*g^2*x^2*ln(1/2*(2*c*
d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2
))+6*c^2*d^2*f*g*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1
/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+3*c^2*d^2*f^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d
*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-8*(c*d*g)^(1
/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*g*x-2*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e
```

```
))^(1/2)*a*e*g-6*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/(c*d*g)^(
1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^{5/2}\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*x
)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(5/2),x)
```

```
[Out] Timed out
```

3.514 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(7/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(7/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f +
 g*x)^(5/2))
```

IntegrateAlgebraic [B] time = 1.40, size = 168, normalized size = 2.67

$$-\frac{2\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}\sqrt{aeg+cd(f+gx)-cdf}\left(a^2e^2g^2+2acdeg(f+gx)-2acdefg+c^2d^2f^2+c^2d^2(f+gx)^2-2c^2d^2f(f+gx)\right)}{5g(d+ex)^{3/2}(f+gx)^{5/2}(aeg-cdf)(aeg+cdgx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^(7/2)),x]
```

```
[Out] (-2*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2)*Sqrt[-(c*d*f) + a*e*g + c
*d*(f + g*x)]*(c^2*d^2*f^2 - 2*a*c*d*e*f*g + a^2*e^2*g^2 - 2*c^2*d^2*f*(f +
 g*x) + 2*a*c*d*e*g*(f + g*x) + c^2*d^2*(f + g*x)^2))/(5*g*(-(c*d*f) + a*e*
g)*(d + e*x)^(3/2)*(f + g*x)^(5/2)*(a*e*g + c*d*g*x)^(3/2))
```

fricas [B] time = 0.44, size = 232, normalized size = 3.68

$$\frac{2\left(c^{2} d^{2} x^{2}+2 \, a c d e x+a^{2} e^{2}\right) \sqrt{c d e x^{2}+a d e+\left(c d^{2}+a e^{2}\right) x} \sqrt{e x+d} \sqrt{g x+f}}{5\left(c d^{2} f^{4}-a d e f^{3} g+\left(c d e f g^{3}-a e^{2} g^{4}\right) x^{4}+\left(3 \, c d e f^{2} g^{2}-a d e g^{4}+\left(c d^{2}-3 \, a e^{2}\right) f g^{3}\right) x^{3}+3\left(c d e f^{3} g-a d e f g^{3}+\left(c d^{2}-a e^{2}\right) f^{2} g^{2}\right) x^{2}+\left(c d e f^{4}-3 \, a d e f^{2} g^{2}+\left(3 \, c d^{2}-a e^{2}\right) f^{3} g\right) x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="fricas")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^4 - a*d*e*f^3*g + (c*d*e*f*
g^3 - a*e^2*g^4)*x^4 + (3*c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - 3*a*e^2)*f*g
^3)*x^3 + 3*(c*d*e*f^3*g - a*d*e*f*g^3 + (c*d^2 - a*e^2)*f^2*g^2)*x^2 + (c*
d*e*f^4 - 3*a*d*e*f^2*g^2 + (3*c*d^2 - a*e^2)*f^3*g)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 63, normalized size = 1.00

$$-\frac{2\,(c d x+a e)\left(c d e\,x^{2}+a\,e^{2} x+c\,d^{2} x+a d e\right)^{\frac{3}{2}}}{5\left(g x+f\right)^{\frac{5}{2}}\left(a e g-c d f\right)(e x+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/2),x)
```

```
[Out] -2/5/(g*x+f)^(5/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(3/2)/(e*x+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c d e x^{2}+a d e+\left(c d^{2}+a e^{2}\right) x\right)^{\frac{3}{2}}}{(e x+d)^{\frac{3}{2}}\left(g x+f\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(7/2)), x)
```

mupad [B] time = 4.07, size = 232, normalized size = 3.68

$$-\frac{\left(\frac{2a^2e^2}{5aeg^3-5cdfg^2}+\frac{2c^2d^2x^2}{5aeg^3-5cdfg^2}+\frac{4acdex}{5aeg^3-5cdfg^2}\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^2\sqrt{f+gx}\sqrt{d+ex}-\frac{\sqrt{f+gx}\left(5cdf^3-5aef^2g\right)\sqrt{d+ex}}{5aeg^3-5cdfg^2}+\frac{x\sqrt{f+gx}\left(10aefg^2-10cdf^2g\right)\sqrt{d+ex}}{5aeg^3-5cdfg^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(7/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] -((((2*a^2*e^2)/(5*a*e*g^3 - 5*c*d*f*g^2) + (2*c^2*d^2*x^2)/(5*a*e*g^3 - 5*c
*d*f*g^2) + (4*a*c*d*e*x)/(5*a*e*g^3 - 5*c*d*f*g^2))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*x)^
(1/2)*(5*c*d*f^3 - 5*a*e*f^2*g)*(d + e*x)^(1/2))/(5*a*e*g^3 - 5*c*d*f*g^2)
+ (x*(f + g*x)^(1/2)*(10*a*e*f*g^2 - 10*c*d*f^2*g)*(d + e*x)^(1/2))/(5*a*e*
g^3 - 5*c*d*f*g^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(7/2),x)
```

```
[Out] Timed out
```

3.515 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(9/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{(2cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx}{7(cdf-aeg)}\\ &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 69, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{5/2}(cd(7f+2gx)-5aeg)}{35(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-5*a*e*g + c*d*(7*f + 2*g*x)))/(35*(c*d
*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2))
```

IntegrateAlgebraic [A] time = 1.71, size = 249, normalized size = 1.93

$$\frac{2\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{3/2}\sqrt{aeg+cd(f+gx)-cdf}\,\left(-5a^3e^3g^3-8a^2cde^2g^2(f+gx)+15a^2cde^2fg^2-15ac^2d^2ef^2g-ac^2d^2eg(f+gx)^2+16ac^2d^2efg(f+gx)+5c^3d^3f^3-8c^3d^3f^2(f+gx)+2c^3d^3(f+gx)^3+c^3d^3f(f+gx)^2\right)}{35g(d+ex)^{3/2}(f+gx)^{7/2}(aeg-cdf)^2(aeg+cdgx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^(9/2)),x]
```

```
[Out] (2*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(3/2)*Sqrt[-(c*d*f) + a*e*g + c*
d*(f + g*x)]*(5*c^3*d^3*f^3 - 15*a*c^2*d^2*e*f^2*g + 15*a^2*c*d*e^2*f*g^2 -
 5*a^3*e^3*g^3 - 8*c^3*d^3*f^2*(f + g*x) + 16*a*c^2*d^2*e*f*g*(f + g*x) - 8
*a^2*c*d*e^2*g^2*(f + g*x) + c^3*d^3*f*(f + g*x)^2 - a*c^2*d^2*e*g*(f + g*x
)^2 + 2*c^3*d^3*(f + g*x)^3))/(35*g*(-(c*d*f) + a*e*g)^2*(d + e*x)^(3/2)*(f
 + g*x)^(7/2)*(a*e*g + c*d*g*x)^(3/2))
```

fricas [B] time = 0.47, size = 526, normalized size = 4.08

$$\frac{2\left(2c^3d^3gx^3+7a^2cde^2f-5a^3e^3g+\left(7c^3d^3f-ac^2d^2eg\right)x^2+2\left(7ac^2d^2ef-4a^2cde^2g\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f}}{35\left(c^2d^3f^6-2acd^2ef^5g+a^2de^2f^4g^2+\left(c^2d^2ef^2g^4-2acde^2fg^5+a^2e^3g^6\right)x^5+\left(4c^2d^2ef^3g^3+a^2de^2g^6+\left(c^2d^3-8acde^2\right)f^2g^4-2\left(acd^2e-2a^2e^3\right)fg^5\right)x^4+2\left(3c^2d^2ef^4g^2+2a^2de^2fg^5+2\left(c^2d^3-3acde^2\right)f^3g^3-\left(4acd^2e-3a^2e^3\right)f^2g^4\right)x^3+2\left(2c^2d^2ef^5g+3a^2de^2f^2g^4+\left(3c^2d^3-4acde^2\right)f^4g^2-2\left(3acd^2e-a^2e^3\right)f^3g^3\right)x^2+\left(c^2d^2ef^6+4a^2de^2f^3g^3+2\left(2c^2d^3-acde^2\right)f^5g-\left(8acd^2e-a^2e^3\right)f^4g^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 5*a^3*e^3*g + (7*c^3*d^3*f - a*c^
2*d^2*e*g)*x^2 + 2*(7*a*c^2*d^2*e*f - 4*a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^6 - 2*a*c
*d^2*e*f^5*g + a^2*d*e^2*f^4*g^2 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 +
 a^2*e^3*g^6)*x^5 + (4*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 8*a*c
*d*e^2)*f^2*g^4 - 2*(a*c*d^2*e - 2*a^2*e^3)*f*g^5)*x^4 + 2*(3*c^2*d^2*e*f^4
*g^2 + 2*a^2*d*e^2*f*g^5 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e
 - 3*a^2*e^3)*f^2*g^4)*x^3 + 2*(2*c^2*d^2*e*f^5*g + 3*a^2*d*e^2*f^2*g^4 + (
3*c^2*d^3 - 4*a*c*d*e^2)*f^4*g^2 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^3)*x^2 +
 (c^2*d^2*e*f^6 + 4*a^2*d*e^2*f^3*g^3 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g - (
8*a*c*d^2*e - a^2*e^3)*f^4*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$-\frac{2\,(cdx+ae)\left(-2cdgx+5aeg-7cdf\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{35\left(gx+f\right)^{\frac{7}{2}}\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)(ex+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-2*c*d*g*x+5*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(3/2)/(g*x+f)^(7/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(9/2)), x)
```

mupad [B] time = 4.31, size = 247, normalized size = 1.91

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^2\,e^2\,\left(5\,a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{4\,c^3\,d^3\,x^3}{35\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,c^2\,d^2\,x^2\,\left(a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{4\,a\,c\,d\,e\,x\,\left(4\,a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(9/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^2*e^2*(5*a*e*g - 7*c*
d*f))/(35*g^3*(a*e*g - c*d*f)^2) - (4*c^3*d^3*x^3)/(35*g^2*(a*e*g - c*d*f)^
2) + (2*c^2*d^2*x^2*(a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2) + (4*a*c*
d*e*x*(4*a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^(1/2
)*(d + e*x)^(1/2) + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x^2*(f
 + g*x)^(1/2)*(d + e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)^(1/2)
)/g^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(9/2),x)
```

```
[Out] Timed out
```

3.516 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}$$

Rubi [A] time = 0.23, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(11/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(9/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(315*(c*d*f - a*e*g)^3*(d
+ e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{(4cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx}{9(cdf-aeg)}\\
&= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7}}\\
&= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7}}
\end{aligned}$$

Mathematica [**A**] time = 0.13, size = 105, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(35a^2e^2g^2-10acdeg(9f+2gx)+c^2d^2\left(63f^2+36fgx+8g^2x^2\right)\right)}{315(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(11/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(35*a^2*e^2*g^2 - 10*a*c*d*e*g*(9*f + 2*
g*x) + c^2*d^2*(63*f^2 + 36*f*g*x + 8*g^2*x^2)))/(315*(c*d*f - a*e*g)^3*(d
+ e*x)^(5/2)*(f + g*x)^(9/2))
```

IntegrateAlgebraic [**F**] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^(11/2)),x]
```

```
[Out] $Aborted
```

fricas [**B**] time = 0.46, size = 918, normalized size = 4.64

$$\frac{2\left(8c^4d^4g^2x^4+63a^2c^2d^2e^2f^2-90a^3cde^3fg+35a^4e^4g^2+4\left(9c^4d^4fg-ac^3d^3eg^2\right)x^3+3\left(21c^4d^4f^2-6ac^3d^3efg+a^2c^2d^2e^2g^2\right)x^2+2\left(63ac^3d^3ef^2-72a^2c^2d^2e^2fg+25a^3cde^3g^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}}{315\left(c^3d^4f^8-3ac^2d^3ef^7g+3a^2cd^2e^2f^6g^2-a^3de^3f^5g^3+\left(c^3d^3ef^3g^5-3ac^2d^2e^2f^2g^6+3a^2cde^3fg^7-a^3e^4g^8\right)x^6+\left(5c^3d^3ef^4g^4-a^3de^3g^8+\left(c^3d^4-15ac^2d^2e^2\right)f^3g^5-3\left(ac^2d^3e-5a^2cde^3\right)f^2g^6+\left(3a^2cd^2e^2-5a^3e^4\right)fg^7\right)x^5+5\left(2c^3d^3ef^5g^3-a^3de^3fg^7+\left(c^3d^4-6ac^2d^2e^2\right)f^4g^4-3\left(ac^2d^3e-2a^2cde^3\right)f^3g^5+\left(3a^2cd^2e^2-2a^3e^4\right)f^2g^6\right)x^4+10\left(c^3d^3ef^6g^2-a^3de^3f^2g^6+\left(c^3d^4-3ac^2d^2e^2\right)f^5g^3-3\left(ac^2d^3e-a^2cde^3\right)f^4g^4+\left(3a^2cd^2e^2-a^3e^4\right)f^3g^5\right)x^3+5\left(c^3d^3ef^7g-2a^3de^3f^3g^5+\left(2c^3d^4-3ac^2d^2e^2\right)f^6g^2-3\left(2ac^2d^3e-a^2cde^3\right)f^5g^3+\left(6a^2cd^2e^2-a^3e^4\right)f^4g^4\right)x^2+\left(c^3d^3ef^8-5a^3de^3f^4g^4+\left(5c^3d^4-3ac^2d^2e^2\right)f^7g-3\left(5ac^2d^3e-a^2cde^3\right)f^6g^2+\left(15a^2cd^2e^2-a^3e^4\right)f^5g^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="fricas")
```

```
[Out] 2/315*(8*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 90*a^3*c*d*e^3*f*g + 35
*a^4*e^4*g^2 + 4*(9*c^4*d^4*f*g - a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4*f^2
- 6*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f^2 - 72
*a^2*c^2*d^2*e^2*f*g + 25*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^8 - 3*a*c^2*d^3*e*f^7
*g + 3*a^2*c*d^2*e^2*f^6*g^2 - a^3*d*e^3*f^5*g^3 + (c^3*d^3*e*f^3*g^5 - 3*a
*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^6 + (5*c^3*d^3*
e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^2
*d^3*e - 5*a^2*c*d*e^3)*f^2*g^6 + (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*g^7)*x^5
+ 5*(2*c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^
4*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e^2 - 2*a^3*
e^4)*f^2*g^6)*x^4 + 10*(c^3*d^3*e*f^6*g^2 - a^3*d*e^3*f^2*g^6 + (c^3*d^4 -
3*a*c^2*d^2*e^2)*f^5*g^3 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^4 + (3*a^2*c
*d^2*e^2 - a^3*e^4)*f^3*g^5)*x^3 + 5*(c^3*d^3*e*f^7*g - 2*a^3*d*e^3*f^3*g^5
 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*
f^5*g^3 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^4)*x^2 + (c^3*d^3*e*f^8 - 5*a^3
*d*e^3*f^4*g^4 + (5*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^7*g - 3*(5*a*c^2*d^3*e - a
^2*c*d*e^3)*f^6*g^2 + (15*a^2*c*d^2*e^2 - a^3*e^4)*f^5*g^3)*x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 169, normalized size = 0.85

$$-\frac{2\left(cdx+ae\right)\left(8g^{2}x^{2}c^{2}d^{2}-20acde\,g^{2}x+36c^{2}d^{2}fgx+35a^{2}e^{2}g^{2}-90acdefg+63f^{2}c^{2}d^{2}\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{315\left(gx+f\right)^{\frac{9}{2}}\left(a^{3}e^{3}g^{3}-3a^{2}cd\,e^{2}f\,g^{2}+3a\,c^{2}d^{2}e\,f^{2}g-f^{3}c^{3}d^{3}\right)\left(ex+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11/2),x)
```

```
[Out] -2/315*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-20*a*c*d*e*g^2*x+36*c^2*d^2*f*g*x+35*
a^2*e^2*g^2-90*a*c*d*e*f*g+63*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(3/2)/(g*x+f)^(9/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-
c^3*d^3*f^3)/(e*x+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(11/2)), x)
```

mupad [B] time = 4.48, size = 377, normalized size = 1.90

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{70\,a^4\,e^4\,g^2-180\,a^3\,c\,d\,e^3\,f\,g+126\,a^2\,c^2\,d^2\,e^2\,f^2}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x^2\,\left(6\,a^2\,c^2\,d^2\,e^2\,g^2-36\,a\,c^3\,d^3\,e\,f\,g+126\,c^4\,d^4\,f^2\right)}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^4\,d^4\,x^4}{315\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c^3\,d^3\,x^3\,\left(a\,e\,g-9\,c\,d\,f\right)}{315\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{4\,a\,c\,d\,e\,x\,\left(25\,a^2\,e^2\,g^2-72\,a\,c\,d\,e\,f\,g+63\,c^2\,d^2\,f^2\right)}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{4\,f^3\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{6\,f^2\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(11/2)*(d + e*
x)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((70*a^4*e^4*g^2 + 126*a^2*
c^2*d^2*e^2*f^2 - 180*a^3*c*d*e^3*f*g)/(315*g^4*(a*e*g - c*d*f)^3) + (x^2*(
126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 - 36*a*c^3*d^3*e*f*g))/(315*g^4*(a*
e*g - c*d*f)^3) + (16*c^4*d^4*x^4)/(315*g^2*(a*e*g - c*d*f)^3) - (8*c^3*d^3
*x^3*(a*e*g - 9*c*d*f))/(315*g^3*(a*e*g - c*d*f)^3) + (4*a*c*d*e*x*(25*a^2*
e^2*g^2 + 63*c^2*d^2*f^2 - 72*a*c*d*e*f*g))/(315*g^4*(a*e*g - c*d*f)^3)))/(
x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))
/g^4 + (4*f*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/
2)*(d + e*x)^(1/2))/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(11/2),x)
```

```
[Out] Timed out
```

3.517 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+c\right.}{33(d+ex)^{5/2}(f+gx)^{9/2}(cd}$$

Rubi [A] time = 0.32, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{33(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(13/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(11/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(33*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*(c*d*f - a*e*g)^3*(
d + e*x)^(5/2)*(f + g*x)^(7/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(5/2))/(1155*(c*d*f - a*e*g)^4*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{(6cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx}{11(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx}$$

Mathematica [A] time = 0.18, size = 152, normalized size = 0.57

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(-105a^3e^3g^3+35a^2cde^2g^2(11f+2gx)-5ac^2d^2eg\left(99f^2+44fgx+8g^2x^2\right)+c^3d^3\left(231f^3+198f^2gx+88fg^2x^2+16g^3x^3\right)\right)}{1155(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(13/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-105*a^3*e^3*g^3 + 35*a^2*c*d*e^2*g^2*(
11*f + 2*g*x) - 5*a*c^2*d^2*e*g*(99*f^2 + 44*f*g*x + 8*g^2*x^2) + c^3*d^3*(
231*f^3 + 198*f^2*g*x + 88*f*g^2*x^2 + 16*g^3*x^3)))/(1155*(c*d*f - a*e*g)^
4*(d + e*x)^(5/2)*(f + g*x)^(11/2))
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)
^(3/2)*(f + g*x)^(13/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.50, size = 1420, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 495*a^3*c^2*d^2*e^3*
f^2*g + 385*a^4*c*d*e^4*f*g^2 - 105*a^5*e^5*g^3 + 8*(11*c^5*d^5*f*g^2 - a*c
^4*d^4*e*g^3)*x^4 + 2*(99*c^5*d^5*f^2*g - 22*a*c^4*d^4*e*f*g^2 + 3*a^2*c^3*
d^3*e^2*g^3)*x^3 + (231*c^5*d^5*f^3 - 99*a*c^4*d^4*e*f^2*g + 33*a^2*c^3*d^3
*e^2*f*g^2 - 5*a^3*c^2*d^2*e^3*g^3)*x^2 + 2*(231*a*c^4*d^4*e*f^3 - 396*a^2*
c^3*d^3*e^2*f^2*g + 275*a^3*c^2*d^2*e^3*f*g^2 - 70*a^4*c*d*e^4*g^3)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^
5*f^10 - 4*a*c^3*d^4*e*f^9*g + 6*a^2*c^2*d^3*e^2*f^8*g^2 - 4*a^3*c*d^2*e^3*
f^7*g^3 + a^4*d*e^4*f^6*g^4 + (c^4*d^4*e*f^4*g^6 - 4*a*c^3*d^3*e^2*f^3*g^7
+ 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*e^4*f*g^9 + a^4*e^5*g^10)*x^7 + (6*
c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^6 -
```

```
4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^7 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c
*d*e^4)*f^2*g^8 - 2*(2*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^9)*x^6 + 3*(5*c^4*d^4
*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^5 - 2
*(4*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^4*g^6 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^
3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^8)*x^5 + 5*(4*c^4*
d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^
4 - 12*(a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3)*f^5*g^5 + 2*(9*a^2*c^2*d^3*e^2 - 8
*a^3*c*d*e^4)*f^4*g^6 - 4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^7)*x^4 + 5*(3*c
^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*
g^3 - 2*(8*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^6*g^4 + 12*(2*a^2*c^2*d^3*e^2
 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^6)*x^3 + 3*(
2*c^4*d^4*e*f^9*g + 5*a^4*d*e^4*f^4*g^6 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8
*g^2 - 4*(5*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^3 + 2*(15*a^2*c^2*d^3*e^
2 - 4*a^3*c*d*e^4)*f^6*g^4 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^5)*x^2 +
(c^4*d^4*e*f^10 + 6*a^4*d*e^4*f^5*g^5 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9
*g - 6*(4*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^2 + 4*(9*a^2*c^2*d^3*e^2 - a
^3*c*d*e^4)*f^7*g^3 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^4)*x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.01, size = 260, normalized size = 0.97

$$-\frac{2 (cdx+ae) \left(-16g^3x^3c^3d^3+40a c^2d^2e g^3x^2-88c^3d^3 f g^2x^2-70a^2cd e^2g^3x+220a c^2d^2ef g^2x-198c^3d^3 f^2gx+105a^3e^3g^3-385a^2cd e^2 f g^2+495a c^2d^2e f^2g-231 f^3c^3d^3\right) \left(cde x^2+a e^2x+c d^2x+ade\right)^{\frac{3}{2}}}{1155 \left(gx+f\right)^{\frac{11}{2}} \left(g^4e^4a^4-4a^3cd e^3 f g^3+6a^2c^2d^2e^2 f^2g^2-4a c^3d^3e f^3g+f^4c^4d^4\right) (ex+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13/2),x)
```

```
[Out] -2/1155*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+40*a*c^2*d^2*e*g^3*x^2-88*c^3*d^3*
f*g^2*x^2-70*a^2*c*d*e^2*g^3*x+220*a*c^2*d^2*e*f*g^2*x-198*c^3*d^3*f^2*g*x+
105*a^3*e^3*g^3-385*a^2*c*d*e^2*f*g^2+495*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3
)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/(g*x+f)^(11/2)/(a^4*e^4*g^4-4*a^3
*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(
e*x+d)^(3/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(13/2)), x)
```

mupad [**B**] time = 4.83, size = 519, normalized size = 1.94

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{210\,a^5\,e^5\,g^3-770\,a^4\,c\,d\,e^4\,f\,g^2+990\,a^3\,c^2\,d^2\,e^3\,f^2\,g-462\,a^2\,c^3\,d^3\,e^2\,f^3}{1155\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{x^2\,\left(-10\,a^3\,c^2\,d^2\,e^3\,g^3+66\,a^2\,c^3\,d^3\,e^2\,f\,g^2-198\,a\,c^4\,d^4\,e\,f^2\,g+462\,c^5\,d^5\,f^3\right)}{1155\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{32\,c^5\,d^5\,x^5}{1155\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{4\,c^3\,d^3\,x^3\,\left(3\,a^2\,e^2\,g^2-22\,a\,c\,d\,e\,f\,g+99\,c^2\,d^2\,f^2\right)}{1155\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{16\,c^4\,d^4\,x^4\,\left(a\,e\,g-11\,c\,d\,f\right)}{1155\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{4\,a\,c\,d\,e\,x\,\left(70\,a^3\,e^3\,g^3-275\,a^2\,c\,d\,e^2\,f\,g^2+396\,a\,c^2\,d^2\,e\,f^2\,g-231\,c^3\,d^3\,f^3\right)}{1155\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^5}+\frac{5\,f\,x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{5\,f^4\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{10\,f^2\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}+\frac{10\,f^3\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(13/2)*(d + e*
x)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((210*a^5*e^5*g^3 - 462*a^2
*c^3*d^3*e^2*f^3 + 990*a^3*c^2*d^2*e^3*f^2*g - 770*a^4*c*d*e^4*f*g^2)/(1155
*g^5*(a*e*g - c*d*f)^4) - (x^2*(462*c^5*d^5*f^3 - 10*a^3*c^2*d^2*e^3*g^3 +
66*a^2*c^3*d^3*e^2*f*g^2 - 198*a*c^4*d^4*e*f^2*g))/(1155*g^5*(a*e*g - c*d*f
)^4) - (32*c^5*d^5*x^5)/(1155*g^2*(a*e*g - c*d*f)^4) - (4*c^3*d^3*x^3*(3*a^
2*e^2*g^2 + 99*c^2*d^2*f^2 - 22*a*c*d*e*f*g))/(1155*g^4*(a*e*g - c*d*f)^4)
+ (16*c^4*d^4*x^4*(a*e*g - 11*c*d*f))/(1155*g^3*(a*e*g - c*d*f)^4) + (4*a*c
*d*e*x*(70*a^3*e^3*g^3 - 231*c^3*d^3*f^3 + 396*a*c^2*d^2*e*f^2*g - 275*a^2*
c*d*e^2*f*g^2))/(1155*g^5*(a*e*g - c*d*f)^4)))/(x^5*(f + g*x)^(1/2)*(d + e*
x)^(1/2) + (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(f + g*x)^(
1/2)*(d + e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (
10*f^2*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(f + g*x)^(1/
2)*(d + e*x)^(1/2))/g^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(13/2),x)
```

```
[Out] Timed out
```

3.518 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=448

$$-\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^5\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{128c^2d^2g^3\sqrt{d+ex}}$$

Rubi [A] time = 0.89, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^4}{128c^2d^2g^3\sqrt{d+ex}}-\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^5\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}}+\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{16g^3\sqrt{d+ex}}-\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{8g^2(d+ex)^{3/2}}+\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x]
```

```
[Out] (-3*(c*d*f - a*e*g)^4*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(128*c^2*d^2*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^3*(f + g*x)^(3/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (
(c*d*f - a*e*g)^2*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(16*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(5/2)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(5/2
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)) - (3
*(c*d*f - a*e*g)^5*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*
e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(128*c^(5/2)*d^(5/2)*g^(7/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] && !IGtQ[n, 0] && !(IntegerQ
```

```
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{(cdf-aeg)}{}$$

$$= -\frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}}$$

$$= \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}} -$$

$$= -\frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} +$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} -$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} -$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} -$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} -$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} -$$

Mathematica [A] time = 6.01, size = 285, normalized size = 0.64

$$\frac{\sqrt{f+gx}\,((d+ex)(ae+cdx))^{7/2}\left(-\frac{15\sqrt{c}\sqrt{d}\sqrt{cd}\,(cdf-aeg)^{9/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{g^{7/2}(ae+cdx)^{7/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}+\frac{15cd(cdf-aeg)^4}{g^3(ae+cdx)^3}-\frac{10cd(cdf-aeg)^3}{g^2(ae+cdx)^2}+\frac{8cd(cdf-aeg)^2}{g(ae+cdx)}+48cd(cdf-aeg)+128c^2d^2(f+gx)\right)}{640c^3d^3(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(
d + e*x)^(5/2),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(7/2)*Sqrt[f + g*x]*(48*c*d*(c*d*f - a*e*g) + (1
5*c*d*(c*d*f - a*e*g)^4)/(g^3*(a*e + c*d*x)^3) - (10*c*d*(c*d*f - a*e*g)^3)
/(g^2*(a*e + c*d*x)^2) + (8*c*d*(c*d*f - a*e*g)^2)/(g*(a*e + c*d*x)) + 128*
c^2*d^2*(f + g*x) - (15*Sqrt[c]*Sqrt[d]*Sqrt[c*d]*(c*d*f - a*e*g)^(9/2)*Arc
Sinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*
e*g])])/(g^(7/2)*(a*e + c*d*x)^(7/2)*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)])
))/(640*c^3*d^3*(d + e*x)^(7/2))
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(d + e*x)^(5/2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 5.79, size = 1331, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/2560*(4*(128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3*g^2
 + 128*a^2*c^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d*e^4*
g^5 + 16*(11*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2*g^3 +
 64*a*c^4*d^4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f^3*g^2
- 23*a*c^4*d^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2*d^2*e^3*g^
5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
f) - 15*(c^5*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*g^2 - 1
0*a^3*c^2*d^3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 + (c^5*d^
5*e*f^5 - 5*a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*a^3*c^2*d
^2*e^4*f^2*g^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*sqrt(c*d*g)*log(-(8*
c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d
*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*
g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^4*x + c^3*d^4*g^4), 1/1280*(2*(
128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3*g^2 + 128*a^2*c
^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d*e^4*g^5 + 16*(11
*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2*g^3 + 64*a*c^4*d^
4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f^3*g^2 - 23*a*c^4*d
^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2*d^2*e^3*g^5)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^5
*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*g^2 - 10*a^3*c^2*d^
3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 + (c^5*d^5*e*f^5 - 5*
a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*a^3*c^2*d^2*e^4*f^2*g
^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/
(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(
c^3*d^3*e*g^4*x + c^3*d^4*g^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 16.59Done
```

maple [B] time = 0.03, size = 1191, normalized size = 2.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 1/1280*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(256*x^4*c^4*d
^4*g^4*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+672*x^3*a*c^3*
d^3*e*g^4*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+352*x^3*c^4
*d^4*f*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*
(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*a^5*e^5*g^5-75*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x
^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*c*d*e^4*f
*g^4+150*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^
(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c^2*d^2*e^3*f^2*g^3-150*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))
/(c*d*g)^(1/2))*a^2*c^3*d^3*e^2*f^3*g^2+75*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*
(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^4
*d^4*e*f^4*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+
a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^5*d^5*f^5+496*x^2*a^2*c^2*d^2*
e^2*g^4*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+1024*x^2*a*c^
3*d^3*e*f*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+16*x^2*
c^4*d^4*f^2*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+20*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^3*c*d*e^3*g^4+932*(
c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^2*c^2*d^2*e^2*f*g^
3+92*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c^3*d^3*e*f^
2*g^2-20*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^4*d^4*f^
3*g-30*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^4*e^4*g^4+14
0*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*c*d*e^3*f*g^3+2
56*a^2*c^2*d^2*e^2*f^2*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^
(1/2)-140*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^3*d^3*e
*f^3*g+30*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^4*d^4*f^4
)/(e*x+d)^(1/2)/g^3/c^2/d^2/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/(c*d*g)
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}\left(gx + f\right)^{\frac{3}{2}}}{(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^(3/2)/(e*
x + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^{3/2}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d
)**(5/2),x)
```

```
[Out] Timed out
```

3.519 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=376

$$-\frac{5\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{64cdg^3\sqrt{d+ex}}$$

Rubi [A] time = 0.68, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{5\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}}+\frac{5(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{32g^3\sqrt{d+ex}}-\frac{5(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{24g^2(d+ex)^{3/2}}+\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)
^(5/2),x]
```

```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e*x]) - (5*(c*d*
f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
24*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2))/(4*g*(d + e*x)^(5/2)) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(64*c^(3/2)*d^(3/2)*g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
```

```
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{(5(cdf-aeg))\int}{}$$

$$=-\frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}}+\frac{(f}{}$$

$$=\frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}}-\frac{5(cd}{}$$

$$=-\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}}+\frac{5(cd}{}$$

$$=-\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}}+\frac{5(cd}{}$$

$$=-\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}}+\frac{5(cd}{}$$

$$=-\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}}+\frac{5(cd}{}$$

$$=-\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}}+\frac{5(cd}{}$$

Mathematica [A] time = 1.13, size = 299, normalized size = 0.80

$$\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)\left(15a^3e^3g^3+a^2cde^2g^2(73f+118gx)+ac^2d^2eg\left(-55f^2+36fgx+136g^2x^2\right)+c^3d^3\left(15f^3-10f^2gx+8fg^2x^2+48g^3x^3\right)\right)-15\sqrt{ae+cdx}(cdf-aeg)^{9/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)\right)}{192g^{7/2}(cd)^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d
+ e*x)^(5/2),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*
d*x)*(f + g*x)*(15*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(73*f + 118*g*x) + a*c^2*d
^2*e*g*(-55*f^2 + 36*f*g*x + 136*g^2*x^2) + c^3*d^3*(15*f^3 - 10*f^2*g*x +
8*f*g^2*x^2 + 48*g^3*x^3)) - 15*(c*d*f - a*e*g)^(9/2)*Sqrt[a*e + c*d*x]*Sqr
t[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*
e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(192*(c*d)^(5/2)*g^(7/2)*Sqr
t[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 2.59, size = 431, normalized size = 1.15

$$\frac{g^{5/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(15a^3e^3g^3\sqrt{f+gx}+118a^2cde^2g^2(f+gx)^{3/2}-45a^2cde^2fg^2\sqrt{f+gx}+45ac^2d^2ef^2g\sqrt{f+gx}+136ac^2d^2eg(f+gx)^{5/2}-236ac^2d^2efg(f+gx)^{3/2}-15c^3d^3f^3\sqrt{f+gx}+118c^3d^3f^2(f+gx)^{3/2}+48c^3d^3(f+gx)^{7/2}-136c^3d^3f(f+gx)^{5/2}\right)}{192cdg^{7/2}}+\frac{5\sqrt{cd}\left(a^4e^4g^4-4a^3cde^3fg^3+6a^2c^2d^2e^2f^2g^2-4ac^3d^3ef^3g+c^4d^4f^4\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{64c^2d^2g^{7/2}}\right)}{(d+ex)^{5/2}(aeg+cdgx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2))/(d + e*x)^(5/2),x]
```

```
[Out] (g^(5/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(5/2)*((Sqrt[-(c*d*f) + a*
e*g + c*d*(f + g*x)]*(-15*c^3*d^3*f^3*Sqrt[f + g*x] + 45*a*c^2*d^2*e*f^2*g*
Sqrt[f + g*x] - 45*a^2*c*d*e^2*f*g^2*Sqrt[f + g*x] + 15*a^3*e^3*g^3*Sqrt[f
+ g*x] + 118*c^3*d^3*f^2*(f + g*x)^(3/2) - 236*a*c^2*d^2*e*f*g*(f + g*x)^(3
/2) + 118*a^2*c*d*e^2*g^2*(f + g*x)^(3/2) - 136*c^3*d^3*f*(f + g*x)^(5/2) +
 136*a*c^2*d^2*e*g*(f + g*x)^(5/2) + 48*c^3*d^3*(f + g*x)^(7/2)))/(192*c*d*
g^(7/2)) + (5*Sqrt[c*d]*(c^4*d^4*f^4 - 4*a*c^3*d^3*e*f^3*g + 6*a^2*c^2*d^2*
e^2*f^2*g^2 - 4*a^3*c*d*e^3*f*g^3 + a^4*e^4*g^4)*Log[-(Sqrt[c*d]*Sqrt[f + g
*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]])/(64*c^2*d^2*g^(7/2))))/((d
+ e*x)^(5/2)*(a*e*g + c*d*g*x)^(5/2))
```

fricas [A] time = 2.71, size = 1065, normalized size = 2.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g - 55*a*c^3*d^3*e*f^2*g^2 +
 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(c^4*d^4*f*g^3 + 17*a*c^
3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a*c^3*d^3*e*f*g^3 - 59*a^2*c^2
*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f
^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d
^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^
4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
 + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
 + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*
g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*
d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^4*x +
 c^2*d^3*g^4), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g - 55*a*c^3*d
^3*e*f^2*g^2 + 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(c^4*d^4*f
*g^3 + 17*a*c^3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a*c^3*d^3*e*f*g^
```

```
3 - 59*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2
*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f
^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^
3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*
f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^2*d^2*e*g^4*x + c^2*d
^3*g^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueEvaluation time: 7.91Done
```

maple [B] time = 0.02, size = 870, normalized size = 2.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -1/384*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-96*(c*d*g*x^
2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*c^3*d^3*g^3*x^3+15*a^4*e^4*g^4
*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c
*d*g)^(1/2))/(c*d*g)^(1/2))-60*a^3*c*d*e^3*f*g^3*ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
+90*a^2*c^2*d^2*e^2*f^2*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*
g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-60*a*c^3*d^3*e*f^3*g
*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c
*d*g)^(1/2))/(c*d*g)^(1/2))+15*c^4*d^4*f^4*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*
(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-272*(
c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*a*c^2*d^2*e*g^3*x^2-16
*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)*c^3*d^3*f*g^2*x^2-23
6*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*g^3*x-7
2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f*g^2*x
+20*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^2*g*x-3
0*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*e^3*g^3-146*(c*
d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*f*g^2+110*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f^2*g-30*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1
/2)/c/d/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/g^3/(c*d*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}\sqrt{gx+f}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)/(e*x
+ d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d
)**(5/2),x)
```

```
[Out] Timed out
```

3.520 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=304

$$-\frac{5\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8\sqrt{c}\sqrt{d}\,g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8g^3\sqrt{d+ex}}-$$

Rubi [A] time = 0.49, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {864, 891, 63, 217, 206}

$$\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8g^3\sqrt{d+ex}}-\frac{5\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{12g^2(d+ex)^{3/2}}-\frac{5\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8\sqrt{c}\sqrt{d}\,g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sqrt[f +
 g*x]),x]
```

```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*g^3*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)) + (Sqrt[f + g*x]
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*g*(d + e*x)^(5/2)) - (5*
(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e
 + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])]])/(8*Sqrt[c]*Sqrt[d]*g^(7/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}} - \frac{(5(cdf-aeg))\int \frac{\left(ade+(cd\cdots\right.}{(d+\cdots}}{6g}$$

$$= -\frac{5(cdf-aeg)\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}} + \frac{\sqrt{f+gx}\,(\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\cdots}{\cdots}$$

Mathematica [A] time = 1.00, size = 229, normalized size = 0.75

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}\,(f+gx)\left(33a^2e^2g^2+2acdeg(13gx-20f)+c^2d^2\left(15f^2-10fgx+8g^2x^2\right)\right)-\frac{15\sqrt{c}\,\sqrt{d}\,(cdf-aeg)^{7/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\,\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cd}\,\sqrt{cdf-aeg}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}}\right)}{24g^{7/2}\sqrt{d+ex}\,\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sq
rt[f + g*x]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g]*(f + g*x)*(33*a^2*e^2*g^2 + 2*a*c*d
*e*g*(-20*f + 13*g*x) + c^2*d^2*(15*f^2 - 10*f*g*x + 8*g^2*x^2)) - (15*Sqrt
[c]*Sqrt[d]*(c*d*f - a*e*g)^(7/2)*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*Arc
Sinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*
e*g])])/((c*d)^(3/2)*Sqrt[a*e + c*d*x])))/(24*g^(7/2)*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

IntegrateAlgebraic [A] time = 1.84, size = 310, normalized size = 1.02

$$\frac{g^{5/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(33a^2e^2g^2\sqrt{f+gx}+26acdeg(f+gx)^{3/2}-66acdefg\sqrt{f+gx}+33c^2d^2f^2\sqrt{f+gx}+8c^2d^2(f+gx)^{5/2}-26c^2d^2f(f+gx)^{3/2}\right)}{24g^{7/2}}+\frac{5\sqrt{cd}\left(-a^3e^3g^3+3a^2cde^2fg^2-3ac^2d^2ef^2g+c^3d^3f^3\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{8cdg^{7/2}}\right)}{(d+ex)^{5/2}(aeg+cdgx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (g^(5/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(5/2)*((Sqrt[-(c*d*f) + a*
e*g + c*d*(f + g*x)]*(33*c^2*d^2*f^2*Sqrt[f + g*x] - 66*a*c*d*e*f*g*Sqrt[f
+ g*x] + 33*a^2*e^2*g^2*Sqrt[f + g*x] - 26*c^2*d^2*f*(f + g*x)^(3/2) + 26*a
*c*d*e*g*(f + g*x)^(3/2) + 8*c^2*d^2*(f + g*x)^(5/2)))/(24*g^(7/2)) + (5*Sq
rt[c*d]*(c^3*d^3*f^3 - 3*a*c^2*d^2*e*f^2*g + 3*a^2*c*d*e^2*f*g^2 - a^3*e^3*
g^3)*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)
]])/(8*c*d*g^(7/2))))/((d + e*x)^(5/2)*(a*e*g + c*d*g*x)^(5/2))
```

fricas [A] time = 1.56, size = 837, normalized size = 2.75

$$\left[\frac{4\left(8c^3d^3g^3x^2+15c^3d^3f^2g-40ac^2d^2efg^2+33a^2cde^2g^3-2\left(5c^3d^3fg^2-13ac^2d^2eg^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}-15\left(c^3d^4f^3-3ac^2d^3ef^2g+3a^2cd^2e^2fg^2-a^3de^3g^3+\left(c^3d^3ef^3-3ac^2d^2e^2f^2g+3a^2cde^3fg^2-a^3e^4g^3\right)x\right)\sqrt{cdg}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdgx+cdf+aeg\right)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+\left(c^2d^2ef^2+2\left(4c^2d^3+3acde^2\right)fg+\left(8acd^2e+a^2e^3\right)g^2\right)x}{ex+d}\right)}{96\left(cdeg^4x+cd^2g^4\right)},\ \frac{2\left(8c^3d^3g^3x^2+15c^3d^3f^2g-40ac^2d^2efg^2+33a^2cde^2g^3-2\left(5c^3d^3fg^2-13ac^2d^2eg^3\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}+15\left(c^3d^4f^3-3ac^2d^3ef^2g+3a^2cd^2e^2fg^2-a^3de^3g^3+\left(c^3d^3ef^3-3ac^2d^2e^2f^2g+3a^2cde^3fg^2-a^3e^4g^3\right)x\right)\sqrt{-cdg}\arctan\left(\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdg}\sqrt{ex+d}\sqrt{gx+f}}{2cdegx^2+cd^2f+adeg+\left(cdef+\left(2cd^2+ae^2\right)g\right)x}\right)}{48\left(cdeg^4x+cd^2g^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 33*
a^2*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^
3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*
e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(
c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^
2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^
3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
+ (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^4*x + c*d^2*g^4), 1/
48*(2*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 33*a^2
*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^3*d^4*f^3 -
 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f
^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*
d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt
(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*
c*d^2 + a*e^2)*g)*x)))/(c*d*e*g^4*x + c*d^2*g^4)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 508, normalized size = 1.67

$$\frac{\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\,\sqrt{gx+f}\left(15a^3e^3g^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)-45a^2cd\,e^2f\,g^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)+45a\,c^2d^2e\,f^2g\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)-15c^3d^3f^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)+16\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^2d^2g^2x^2+52\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,acde\,g^2x-20\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,c^2d^2fgx+66\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,a^2e^2g^2-80\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,acdefg+30\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^2d^2f^2\right)}{48\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x)

[Out] 1/48*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)*(15*a^3*e^3*g^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-45*a^2*c*d*e^2*f*g^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+45*a*c^2*d^2*e*f^2*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-15*c^3*d^3*f^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+16*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*g^2*x^2+52*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-20*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x+66*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-80*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/g^3/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="maxima")

[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*sqrt(g*x + f)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\sqrt{f + g\,x}\,\left(d + e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)),x)

[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f)**(1/2),x)

[Out] Timed out

3.521 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=294

$$\frac{15\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{15cd\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf}{4g^3\sqrt{d+ex}}$$

Rubi [A] time = 0.43, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {862, 864, 891, 63, 217, 206}

$$\frac{5cd\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{15cd\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4g^3\sqrt{d+ex}}+\frac{15\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(3/2)),x]
```

```
[Out] (-15*c*d*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(4*g^3*Sqrt[d + e*x]) + (5*c*d*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(2*g^2*(d + e*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(5/2))/(g*(d + e*x)^(5/2)*Sqrt[f + g*x]) + (15*Sqrt[c]*
Sqrt[d]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*
Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*g^(7/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,d}{g}$$

$$= \frac{5cd\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{2\left(ade+\left(cd^2+ae^2\right)\right.}{g(d+ex)^{5/2}\sqrt{}}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+}}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+}}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+}}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+}}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+}}{}$$

Mathematica [C] time = 0.10, size = 112, normalized size = 0.38

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7cd\sqrt{d+ex}\,(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(3/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f + g*x))/(c*d*f -
a*e*g))^(3/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (g*(a*e + c*d*x))/(-(c*d*f)
+ a*e*g)])/(7*c*d*Sqrt[d + e*x]*(f + g*x)^(3/2))
```

IntegrateAlgebraic [A] time = 2.06, size = 255, normalized size = 0.87

$$\frac{g^{5/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(-8a^2e^2g^2+9acdeg(f+gx)+16acdefg-8c^2d^2f^2+2c^2d^2(f+gx)^2-9c^2d^2f(f+gx)\right)}{4g^{7/2}\sqrt{f+gx}}-\frac{15\sqrt{cd}\left(a^2e^2g^2-2acdefg+c^2d^2f^2\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{4g^{7/2}}\right)}{(d+ex)^{5/2}(aeg+cdgx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^(3/2)),x]
```

```
[Out] (g^(5/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(5/2)*((Sqrt[-(c*d*f) + a*
e*g + c*d*(f + g*x)]*(-8*c^2*d^2*f^2 + 16*a*c*d*e*f*g - 8*a^2*e^2*g^2 - 9*c
^2*d^2*f*(f + g*x) + 9*a*c*d*e*g*(f + g*x) + 2*c^2*d^2*(f + g*x)^2))/(4*g^(
7/2)*Sqrt[f + g*x]) - (15*Sqrt[c*d]*(c^2*d^2*f^2 - 2*a*c*d*e*f*g + a^2*e^2*
g^2)*Log[-(Sqrt[c*d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)
]])/(4*g^(7/2))))/((d + e*x)^(5/2)*(a*e*g + c*d*g*x)^(5/2))
```

fricas [A] time = 1.20, size = 915, normalized size = 3.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*g
^2 - (5*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2*g
+ a^2*d*e^2*f*g^2 + (c^2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)*x^2
 + (c^2*d^2*e*f^3 + a^2*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2*a*c*
d^2*e - a^2*e^3)*f*g^2)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*
f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a*e*g^2)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sq
rt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*
f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*
x + d)))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), 1/8*(2*(2*c^2*d^2*g^2
*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 9
*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) - 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2*g + a^2*d*e^2*f*g^2 + (c^
2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)*x^2 + (c^2*d^2*e*f^3 + a^2
*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2*a*c*d^2*e - a^2*e^3)*f*g^2)
*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g +
(c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^
4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 635, normalized size = 2.16

$$\frac{\left(15a^{2}cd\,e^{2}g^{3}x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-30a\,c^{2}d^{2}ef\,g^{2}x\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+15c^{3}d^{3}f^{2}gx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+15a^{2}cd\,e^{2}f\,g^{2}\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-30a\,c^{2}d^{2}e\,f^{2}g\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+15c^{3}d^{3}f^{3}\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+4\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^{2}d^{2}g^{2}x^{2}+18\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,acde\,g^{2}x-10\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,c^{2}d^{2}fgx-16\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,a^{2}e^{2}g^{2}+50\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,acdefg-30\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^{2}d^{2}f^{2}\right)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}{8\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,\sqrt{gx+f}\,\sqrt{ex+d}\,g^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/2),x)
```

```
[Out] 1/8*(15*a^2*c*d*e^2*g^3*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-30*a*c^2*d^2*e*f*g^2*x*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))+15*c^3*d^3*f^2*g*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e)
)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+15*a^2*c*d*e^2*f*g^2*ln(1/2*(2*c*d*g*
x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-3
0*a*c^2*d^2*e*f^2*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(
1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+15*c^3*d^3*f^3*ln(1/2*(2*c*d*g*x+a*e*g+c
*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+4*((g*x+f)
*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*g^2*x^2+18*(c*d*g)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-10*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1
/2)*c^2*d^2*f*g*x-16*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+
50*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-30*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(
1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(1/2)/(e*x+d)^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^{2} + ade + \left(cd^{2} + ae^{2}\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(f+g\,x\right)}^{3/2}\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*x
)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(3/2),x)
```

```
[Out] Timed out
```

3.522 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{5c^{3/2}d^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5c^2d^2\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}}$$

Rubi [A] time = 0.40, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {862, 864, 891, 63, 217, 206}

$$\frac{5c^2d^2\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}}-\frac{5c^{3/2}d^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{10cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(5/2)),x]
```

```
[Out] (5*c^2*d^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*
Sqrt[d + e*x]) - (10*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*Sqrt[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(3*g*(d + e*x)^(5/2)*(f + g*x)^(3/2)) - (5*c^(3/2)*d^(3/2)*(c*d
*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] && !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx}{3g}$$

$$= -\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\cdots\right.}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)\cdots\right.}{3g^2(d+ex)^{3/2}\sqrt{f\cdots}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)\cdots\right.}{3g^2(d+ex)^{3/2}\sqrt{f\cdots}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)\cdots\right.}{3g^2(d+ex)^{3/2}\sqrt{f\cdots}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)\cdots\right.}{3g^2(d+ex)^{3/2}\sqrt{f\cdots}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)\cdots\right.}{3g^2(d+ex)^{3/2}\sqrt{f\cdots}}$$

Mathematica [C] time = 0.12, size = 112, normalized size = 0.39

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7cd\sqrt{d+ex}\,(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(5/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f + g*x))/(c*d*f -
a*e*g))^(5/2)*Hypergeometric2F1[5/2, 7/2, 9/2, (g*(a*e + c*d*x))/(-(c*d*f)
+ a*e*g)])/(7*c*d*Sqrt[d + e*x]*(f + g*x)^(5/2))
```

IntegrateAlgebraic [A] time = 2.24, size = 240, normalized size = 0.85

$$\frac{g^{5/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}\left(\frac{\sqrt{aeg+cd(f+gx)-cdf}\left(-2a^2e^2g^2-14acdeg(f+gx)+4acdefg-2c^2d^2f^2+3c^2d^2(f+gx)^2+14c^2d^2f(f+gx)\right)}{3g^{7/2}(f+gx)^{3/2}}+\frac{5\sqrt{cd}\left(c^2d^2f-acdeg\right)\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{g^{7/2}}\right)}{(d+ex)^{5/2}(aeg+cdgx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^(5/2)),x]
```

```
[Out] (g^(5/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(5/2)*((Sqrt[-(c*d*f) + a*
e*g + c*d*(f + g*x)]*(-2*c^2*d^2*f^2 + 4*a*c*d*e*f*g - 2*a^2*e^2*g^2 + 14*c
^2*d^2*f*(f + g*x) - 14*a*c*d*e*g*(f + g*x) + 3*c^2*d^2*(f + g*x)^2))/(3*g^
(7/2)*(f + g*x)^(3/2)) + (5*Sqrt[c*d]*(c^2*d^2*f - a*c*d*e*g)*Log[-(Sqrt[c*
d]*Sqrt[f + g*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]])/g^(7/2)))/((d
+ e*x)^(5/2)*(a*e*g + c*d*g*x)^(5/2))
```

fricas [A] time = 1.15, size = 973, normalized size = 3.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g - 2*a^2*e^2*g
^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^3*f^3 - a*c*d^2*e*f^2*g
 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x^3 + (2*c^2*d^2*e*f^2*g - a*c*d^2*e*g
^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x^2 + (c^2*d^2*e*f^3 - 2*a*c*d^2*e*f*g^
2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3
 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g
 + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 +
(c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*
g^2)*x)/(e*x + d)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f
^2*g^3 + 2*d*f*g^4)*x), 1/6*(2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c
*d*e*f*g - 2*a^2*e^2*g^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^3
*f^3 - a*c*d^2*e*f^2*g + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x^3 + (2*c^2*d^2
*e*f^2*g - a*c*d^2*e*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x^2 + (c^2*d^2*e*
f^3 - 2*a*c*d^2*e*f*g^2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x)*sqrt(-c*d/g)*ar
ctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2
 + a*e^2)*g)*x)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2
*g^3 + 2*d*f*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="giac")
```

[Out] Timed out

maple [B] time = 0.03, size = 638, normalized size = 2.25

$$\frac{\left(15a c^2 d^2 e g^3 x^2 \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-15c^3 d^3 f g^2 x^2 \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+30a c^2 d^2 e f g^2 x \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-30c^3 d^3 f^2 g x \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+15a c^2 d^2 e f^2 g \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)-15c^3 d^3 f^3 \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)+6\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^2 d^2 g^2 x^2-28\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,acdeg^2 x+40\sqrt{cdg}\,\sqrt{(gx+f)(cdx+ae)}\,c^2 d^2 f g x-4\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,a^2 e^2 g^2-20\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,acdefg+30\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,c^2 d^2 f^2\right)\sqrt{cdex^2+ae^2x+cd^2x+ade}}{6\sqrt{(gx+f)(cdx+ae)}\,\sqrt{cdg}\,(gx+f)^{\frac{3}{2}}\sqrt{ex+d}\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/2),x)
```

```
[Out] 1/6*(15*a*c^2*d^2*e*g^3*x^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x
+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-15*c^3*d^3*f*g^2*x^2*ln(1/2*(2*c
*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))+30*a*c^2*d^2*e*f*g^2*x*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+
a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-30*c^3*d^3*f^2*g*x*ln(1/2*(2*c*d*
g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
+15*a*c^2*d^2*e*f^2*g*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))
^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))-15*c^3*d^3*f^3*ln(1/2*(2*c*d*g*x+a*e*g
+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+6*((g*x+
f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*g^2*x^2-28*(c*d*g)^(1/2)*((g*x+
f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x+40*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^
(1/2)*c^2*d^2*f*g*x-4*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2
-20*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+30*((g*x+f)*(c*d*
x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^
(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(3/2)/(e*x+d)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^{5/2}\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*x
)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(5/2),x)
```

```
[Out] Timed out
```

3.523 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}} \, dx$

Optimal. Leaf size=274

$$\frac{2c^{5/2}d^{5/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(x\left(ae^2+cd^2\right)+\right.}{3g^2(d+ex)^{3/2}(}$$

Rubi [A] time = 0.37, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {862, 891, 63, 217, 206}

$$-\frac{2c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{2c^{5/2}d^{5/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(7/2)),x]
```

```
[Out] (-2*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt[d + e*x]
*Sqrt[f + g*x]) - (2*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*(f + g*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)*(f + g*x)^(5/2)) + (2*c^(5/2)*d^(5/2)*Sq
rt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*
Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}+\frac{(cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx}{g}$$

$$= -\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

Mathematica [A] time = 1.37, size = 224, normalized size = 0.82

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{15\sqrt{c}\,\sqrt{d}\,(cdf-aeg)^{5/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cd}\,\sqrt{cdf-aeg}}\right)}{\sqrt{cd}\,\sqrt{ae+cdx}}-\sqrt{g}\left(3a^2e^2g^2+acdeg(5f+11gx)+c^2d^2\left(15f^2+35fgx+23g^2x^2\right)\right)\right)}{15g^{7/2}\sqrt{d+ex}\,(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(7/2)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]*(3*a^2*e^2*g^2 + a*c*d*e*g*(5*f
 + 11*g*x) + c^2*d^2*(15*f^2 + 35*f*g*x + 23*g^2*x^2))) + (15*Sqrt[c]*Sqrt[
d]*(c*d*f - a*e*g)^(5/2)*((c*d*(f + g*x))/(c*d*f - a*e*g))^(5/2)*ArcSinh[(S
qrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])
/(Sqrt[c*d]*Sqrt[a*e + c*d*x]))))/(15*g^(7/2)*Sqrt[d + e*x]*(f + g*x)^(5/2))
```

IntegrateAlgebraic [A] time = 2.43, size = 230, normalized size = 0.84

$$\frac{g^{5/2}\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}\left(-\frac{2\sqrt{aeg+cd(f+gx)-cdf}\left(3a^2e^2g^2+11acdeg(f+gx)-6acdefg+3c^2d^2f^2+23c^2d^2(f+gx)^2-11c^2d^2f(f+gx)\right)}{15g^{7/2}(f+gx)^{5/2}}-\frac{2c^2d^2\sqrt{cd}\log\left(\sqrt{aeg+cd(f+gx)-cdf}-\sqrt{cd}\sqrt{f+gx}\right)}{g^{7/2}}\right)}{(d+ex)^{5/2}(aeg+cdgx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^(7/2)),x]
```

```
[Out] (g^(5/2)*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(5/2)*((-2*Sqrt[-(c*d*f) +
 a*e*g + c*d*(f + g*x)]*(3*c^2*d^2*f^2 - 6*a*c*d*e*f*g + 3*a^2*e^2*g^2 - 11
*c^2*d^2*f*(f + g*x) + 11*a*c*d*e*g*(f + g*x) + 23*c^2*d^2*(f + g*x)^2))/(1
5*g^(7/2)*(f + g*x)^(5/2)) - (2*c^2*d^2*Sqrt[c*d]*Log[-(Sqrt[c*d]*Sqrt[f +
g*x]) + Sqrt[-(c*d*f) + a*e*g + c*d*(f + g*x)]])/g^(7/2)))/((d + e*x)^(5/2)
*(a*e*g + c*d*g*x)^(5/2))
```

fricas [A] time = 1.10, size = 933, normalized size = 3.41

$$\left[-\frac{4\left(23c^2d^2g^2x^2+15c^2d^2f^2+5acdefg+3a^2e^2g^2+\left(35c^2d^2fg+11acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}-15\left(c^2d^2eg^3x^4+c^2d^3f^3+\left(3c^2d^2efg^2+c^2d^3g^3\right)x^3+3\left(c^2d^2ef^2g+c^2d^3fg^2\right)x^2+\left(c^2d^2ef^3+3c^2d^3f^2g\right)x\right)\sqrt{\frac{cd}{g}}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4\left(2cdg^2x+cdfg+aeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{\frac{cd}{g}}+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+\left(c^2d^2ef^2+2\left(4c^2d^3+3acde^2\right)fg+\left(8acd^2e+a^2e^3\right)g^2\right)x}{ex+d}\right)}{30\left(eg^6x^4+df^3g^3+\left(3efg^5+dg^6\right)x^3+3\left(ef^2g^4+dfg^5\right)x^2+\left(ef^3g^3+3df^2g^4\right)x\right)}, -\frac{2\left(23c^2d^2g^2x^2+15c^2d^2f^2+5acdefg+3a^2e^2g^2+\left(35c^2d^2fg+11acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}+15\left(c^2d^2eg^3x^4+c^2d^3f^3+\left(3c^2d^2efg^2+c^2d^3g^3\right)x^3+3\left(c^2d^2ef^2g+c^2d^3fg^2\right)x^2+\left(c^2d^2ef^3+3c^2d^3f^2g\right)x\right)\sqrt{-\frac{cd}{g}}\arctan\left(\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{-\frac{cd}{g}}\,g}{2cdegx^2+cd^2f+adeg+\left(cdef+\left(2cd^2+ae^2\right)g\right)x}\right)}{15\left(eg^6x^4+df^3g^3+\left(3efg^5+dg^6\right)x^3+3\left(ef^2g^4+dfg^5\right)x^2+\left(ef^3g^3+3df^2g^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="fricas")
```

```
[Out] [-1/30*(4*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e^2*
g^2 + (35*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^2*e*g^3*x^4 + c^2*d^3*f
^3 + (3*c^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2*d^3*f
*g^2)*x^2 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^
2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*
x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*
g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)
*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x), -1/15*(2
*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e^2*g^2 + (35
*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^2*e*g^3*x^4 + c^2*d^3*f^3 + (3*c
^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2*d^3*f*g^2)*x^2
 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(
2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e
*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^
2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 511, normalized size = 1.86

$$\frac{\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15c^3d^3g^3x^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)+45c^3d^3fg^2x^2\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)+45c^3d^3f^2gx\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)+15c^3d^3f^3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}}{2\sqrt{cdg}}\right)-46\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}\,c^2d^2g^2x^2-22\sqrt{cdg}\sqrt{(gx+f)(cdx+ae)}\,acdeg^2x-70\sqrt{cdg}\sqrt{(gx+f)(cdx+ae)}\,c^2d^2fgx-6\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}\,a^2e^2g^2-10\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}\,acdefg-30\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}\,c^2d^2f^2\right)}{15\sqrt{(gx+f)(cdx+ae)}\sqrt{cdg}\,(gx+f)^{\frac{5}{2}}\sqrt{ex+d}\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/2),x)
```

```
[Out] 1/15*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x^3*c^3*d^3
*g^3+45*c^3*d^3*f*g^2*x^2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a
*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+45*c^3*d^3*f^2*g*x*ln(1/2*(2*c*d*g
*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))+
15*c^3*d^3*f^3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*
(c*d*g)^(1/2))/(c*d*g)^(1/2))-46*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*
c^2*d^2*g^2*x^2-22*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*c*d*e*g^2*x-
70*(c*d*g)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*(c*d*
x+a*e))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-10*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d
*g)^(1/2)*a*c*d*e*f*g-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*
f^2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(5/2)/(e*x+d)^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^{7/2}\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*x
)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(7/2),x)
```

```
[Out] Timed out
```

3.524 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(9/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

Mathematica [A] time = 0.08, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f +
 g*x)^(7/2))
```

IntegrateAlgebraic [B] time = 1.88, size = 248, normalized size = 3.94

$$-\frac{2\left(\frac{(dg+egx)(aeg+cdgx)}{g^2}\right)^{5/2}\sqrt{aeg+cd(f+gx)-cdf}\,\left(a^3e^3g^3+3a^2cde^2g^2(f+gx)-3a^2cde^2fg^2+3ac^2d^2ef^2g+3ac^2d^2eg(f+gx)^2-6ac^2d^2efg(f+gx)-c^3d^3f^3+3c^3d^3f^2(f+gx)+c^3d^3(f+gx)^3-3c^3d^3f(f+gx)^2\right)}{7g(d+ex)^{5/2}(f+gx)^{7/2}(aeg-cdf)(aeg+cdgx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^(9/2)),x]
```

```
[Out] (-2*(((a*e*g + c*d*g*x)*(d*g + e*g*x))/g^2)^(5/2)*Sqrt[-(c*d*f) + a*e*g + c
*d*(f + g*x)]*(-(c^3*d^3*f^3) + 3*a*c^2*d^2*e*f^2*g - 3*a^2*c*d*e^2*f*g^2 +
 a^3*e^3*g^3 + 3*c^3*d^3*f^2*(f + g*x) - 6*a*c^2*d^2*e*f*g*(f + g*x) + 3*a^
2*c*d*e^2*g^2*(f + g*x) - 3*c^3*d^3*f*(f + g*x)^2 + 3*a*c^2*d^2*e*g*(f + g*
x)^2 + c^3*d^3*(f + g*x)^3))/(7*g*(-(c*d*f) + a*e*g)*(d + e*x)^(5/2)*(f + g
*x)^(7/2)*(a*e*g + c*d*g*x)^(5/2))
```

fricas [B] time = 0.44, size = 299, normalized size = 4.75

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f}}{7\left(cd^2f^5-adef^4g+\left(cdefg^4-ae^2g^5\right)x^5+\left(4\,cdef^2g^3-adeg^5+\left(cd^2-4\,ae^2\right)fg^4\right)x^4+2\left(3\,cdef^3g^2-2\,adefg^4+\left(2\,cd^2-3\,ae^2\right)f^2g^3\right)x^3+2\left(2\,cdef^4g-3\,adef^2g^3+\left(3\,cd^2-2\,ae^2\right)f^3g^2\right)x^2+\left(cdef^5-4\,adef^3g^2+\left(4\,cd^2-ae^2\right)f^4g\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="fricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^5 -
 a*d*e*f^4*g + (c*d*e*f*g^4 - a*e^2*g^5)*x^5 + (4*c*d*e*f^2*g^3 - a*d*e*g^5
 + (c*d^2 - 4*a*e^2)*f*g^4)*x^4 + 2*(3*c*d*e*f^3*g^2 - 2*a*d*e*f*g^4 + (2*c
*d^2 - 3*a*e^2)*f^2*g^3)*x^3 + 2*(2*c*d*e*f^4*g - 3*a*d*e*f^2*g^3 + (3*c*d^
2 - 2*a*e^2)*f^3*g^2)*x^2 + (c*d*e*f^5 - 4*a*d*e*f^3*g^2 + (4*c*d^2 - a*e^2
)*f^4*g)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 63, normalized size = 1.00

$$-\frac{2\left(cdx+ae\right)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{7\left(gx+f\right)^{\frac{7}{2}}\left(aeg-cdf\right)\left(ex+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/2),x)
```

```
[Out] -2/7/(g*x+f)^(7/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(5/2)/(e*x+d)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{5}{2}}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(9/2)), x)
```

mupad [B] time = 4.34, size = 325, normalized size = 5.16

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^3\,e^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{2\,c^3\,d^3\,x^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a^2\,c\,d\,e^2\,x}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a\,c^2\,d^2\,e\,x^2}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}\right)}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(7\,c\,d\,f^4-7\,a\,e\,f^3\,g\right)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{x^2\,\sqrt{f+g\,x}\,\left(21\,a\,e\,f\,g^3-21\,c\,d\,f^2\,g^2\right)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}-\frac{x\,\sqrt{f+g\,x}\,\left(21\,c\,d\,f^3\,g-21\,a\,e\,f^2\,g^2\right)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(9/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3)/(7*a*e*g^4 - 7
*c*d*f*g^3) + (2*c^3*d^3*x^3)/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a^2*c*d*e^2*x)
/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a*c^2*d^2*e*x^2)/(7*a*e*g^4 - 7*c*d*f*g^3))
)/(x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*x)^(1/2)*(7*c*d*f^4 - 7*a*
e*f^3*g)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3) + (x^2*(f + g*x)^(1/2)*
(21*a*e*f*g^3 - 21*c*d*f^2*g^2)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3)
- (x*(f + g*x)^(1/2)*(21*c*d*f^3*g - 21*a*e*f^2*g^2)*(d + e*x)^(1/2))/(7*a*
e*g^4 - 7*c*d*f*g^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(9/2),x)
```

```
[Out] Timed out
```

3.525 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(11/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(7/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{(2cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx}{9(cdf-aeg)}\\ &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\,(cd(9f+2gx)-7aeg)}{63\sqrt{d+ex}\,(f+gx)^{9/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(11/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-7*a*e*g + c*d*(9*f + 2*g
*x)))/(63*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(9/2))
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^(11/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.47, size = 639, normalized size = 4.95

$$\frac{2\left(2c^4d^4gx^4+9a^3cde^3f-7a^4e^4g+\left(9c^4d^4f-ac^3d^3eg\right)x^3+3\left(9ac^3d^3ef-5a^2c^2d^2e^2g\right)x^2+\left(27a^2c^2d^2e^2f-19a^3cde^3g\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}}{63\left(c^2d^3f^7-2acd^2ef^6g+a^2de^2f^5g^2+\left(c^2d^2ef^2g^5-2acde^2fg^6+a^2e^3g^7\right)x^6+\left(5c^2d^2ef^3g^4+a^2de^2g^7+\left(c^2d^3-10acde^2\right)f^2g^5-\left(2acd^2e-5a^2e^3\right)fg^6\right)x^5+5\left(2c^2d^2ef^4g^3+a^2de^2fg^6+\left(c^2d^3-4acde^2\right)f^3g^4-2\left(acd^2e-a^2e^3\right)f^2g^5\right)x^4+10\left(c^2d^2ef^5g^2+a^2de^2f^2g^5+\left(c^2d^3-2acde^2\right)f^4g^3-\left(2acd^2e-a^2e^3\right)f^3g^4\right)x^3+5\left(c^2d^2ef^6g+2a^2de^2f^3g^4+2\left(c^2d^3-acde^2\right)f^5g^2-\left(4acd^2e-a^2e^3\right)f^4g^3\right)x^2+\left(c^2d^2ef^7+5a^2de^2f^4g^3+\left(5c^2d^3-2acde^2\right)f^6g-\left(10acd^2e-a^2e^3\right)f^5g^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 7*a^4*e^4*g + (9*c^4*d^4*f - a*c^
3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f - 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2*c^
2*d^2*e^2*f - 19*a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^7 - 2*a*c*d^2*e*f^6*g + a^2*d*e^
2*f^5*g^2 + (c^2*d^2*e*f^2*g^5 - 2*a*c*d*e^2*f*g^6 + a^2*e^3*g^7)*x^6 + (5*
c^2*d^2*e*f^3*g^4 + a^2*d*e^2*g^7 + (c^2*d^3 - 10*a*c*d*e^2)*f^2*g^5 - (2*a
*c*d^2*e - 5*a^2*e^3)*f*g^6)*x^5 + 5*(2*c^2*d^2*e*f^4*g^3 + a^2*d*e^2*f*g^6
 + (c^2*d^3 - 4*a*c*d*e^2)*f^3*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f^2*g^5)*x^4 +
 10*(c^2*d^2*e*f^5*g^2 + a^2*d*e^2*f^2*g^5 + (c^2*d^3 - 2*a*c*d*e^2)*f^4*g^
3 - (2*a*c*d^2*e - a^2*e^3)*f^3*g^4)*x^3 + 5*(c^2*d^2*e*f^6*g + 2*a^2*d*e^2
*f^3*g^4 + 2*(c^2*d^3 - a*c*d*e^2)*f^5*g^2 - (4*a*c*d^2*e - a^2*e^3)*f^4*g^
3)*x^2 + (c^2*d^2*e*f^7 + 5*a^2*d*e^2*f^4*g^3 + (5*c^2*d^3 - 2*a*c*d*e^2)*f
^6*g - (10*a*c*d^2*e - a^2*e^3)*f^5*g^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$-\frac{2\left(cdx+ae\right)\left(-2cdgx+7aeg-9cdf\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{63\left(gx+f\right)^{\frac{9}{2}}\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)\left(ex+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11/2),x)
```

```
[Out] -2/63*(c*d*x+a*e)*(-2*c*d*g*x+7*a*e*g-9*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(5/2)/(g*x+f)^(9/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(11/2)), x)
```

mupad [B] time = 4.54, size = 315, normalized size = 2.44

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^3\,e^3\,\left(7\,a\,e\,g-9\,c\,d\,f\right)}{63\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{4\,c^4\,d^4\,x^4}{63\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,c^3\,d^3\,x^3\,\left(a\,e\,g-9\,c\,d\,f\right)}{63\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,a^2\,c\,d\,e^2\,x\,\left(19\,a\,e\,g-27\,c\,d\,f\right)}{63\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,a\,c^2\,d^2\,e\,x^2\,\left(5\,a\,e\,g-9\,c\,d\,f\right)}{21\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{4\,f^3\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{6\,f^2\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(11/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3*(7*a*e*g - 9*c*
d*f))/(63*g^4*(a*e*g - c*d*f)^2) - (4*c^4*d^4*x^4)/(63*g^3*(a*e*g - c*d*f)^
2) + (2*c^3*d^3*x^3*(a*e*g - 9*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a^2*
c*d*e^2*x*(19*a*e*g - 27*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a*c^2*d^2*
e*x^2*(5*a*e*g - 9*c*d*f))/(21*g^4*(a*e*g - c*d*f)^2)))/(x^4*(f + g*x)^(1/2
)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*x^3*(f
 + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x)^(1/2)
)/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(11/2),x)
```

```
[Out] Timed out
```

3.526 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right.}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-}$$

Rubi [A] time = 0.23, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(13/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(11/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(99*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(693*(c*d*f - a*e*g)^3*(
d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{(4cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}}{11(cdf-aeg)}$$

$$=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{99(cdf-aeg)^2(d+ex)^{7/2}(f+}$$

$$=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{99(cdf-aeg)^2(d+ex)^{7/2}(f+}$$

Mathematica [A] time = 0.11, size = 115, normalized size = 0.58

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(63a^2e^2g^2-14acdeg(11f+2gx)+c^2d^2\left(99f^2+44fgx+8g^2x^2\right)\right)}{693\sqrt{d+ex}\,(f+gx)^{11/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(13/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(63*a^2*e^2*g^2 - 14*a*c*d
*e*g*(11*f + 2*g*x) + c^2*d^2*(99*f^2 + 44*f*g*x + 8*g^2*x^2)))/(693*(c*d*f
 - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^(11/2))
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^(13/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.48, size = 1101, normalized size = 5.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="fricas")
```

```
[Out] 2/693*(8*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 154*a^4*c*d*e^4*f*g + 6
3*a^5*e^5*g^2 + 4*(11*c^5*d^5*f*g - a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*f^2
- 22*a*c^4*d^4*e*f*g + 3*a^2*c^3*d^3*e^2*g^2)*x^3 + (297*a*c^4*d^4*e*f^2 -
330*a^2*c^3*d^3*e^2*f*g + 113*a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3*d^3*e
^2*f^2 - 418*a^3*c^2*d^2*e^3*f*g + 161*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^9 - 3*a*
c^2*d^3*e*f^8*g + 3*a^2*c*d^2*e^2*f^7*g^2 - a^3*d*e^3*f^6*g^3 + (c^3*d^3*e*
f^3*g^6 - 3*a*c^2*d^2*e^2*f^2*g^7 + 3*a^2*c*d*e^3*f*g^8 - a^3*e^4*g^9)*x^7
+ (6*c^3*d^3*e*f^4*g^5 - a^3*d*e^3*g^9 + (c^3*d^4 - 18*a*c^2*d^2*e^2)*f^3*g
^6 - 3*(a*c^2*d^3*e - 6*a^2*c*d*e^3)*f^2*g^7 + 3*(a^2*c*d^2*e^2 - 2*a^3*e^4
)*f*g^8)*x^6 + 3*(5*c^3*d^3*e*f^5*g^4 - 2*a^3*d*e^3*f*g^8 + (2*c^3*d^4 - 15
*a*c^2*d^2*e^2)*f^4*g^5 - 3*(2*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^3*g^6 + (6*a^
2*c*d^2*e^2 - 5*a^3*e^4)*f^2*g^7)*x^5 + 5*(4*c^3*d^3*e*f^6*g^3 - 3*a^3*d*e^
3*f^2*g^7 + 3*(c^3*d^4 - 4*a*c^2*d^2*e^2)*f^5*g^4 - 3*(3*a*c^2*d^3*e - 4*a^
2*c*d*e^3)*f^4*g^5 + (9*a^2*c*d^2*e^2 - 4*a^3*e^4)*f^3*g^6)*x^4 + 5*(3*c^3*
d^3*e*f^7*g^2 - 4*a^3*d*e^3*f^3*g^6 + (4*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^6*g^3
 - 3*(4*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^5*g^4 + 3*(4*a^2*c*d^2*e^2 - a^3*e^4
)*f^4*g^5)*x^3 + 3*(2*c^3*d^3*e*f^8*g - 5*a^3*d*e^3*f^4*g^5 + (5*c^3*d^4 -
6*a*c^2*d^2*e^2)*f^7*g^2 - 3*(5*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^6*g^3 + (15*
a^2*c*d^2*e^2 - 2*a^3*e^4)*f^5*g^4)*x^2 + (c^3*d^3*e*f^9 - 6*a^3*d*e^3*f^5*
g^4 + 3*(2*c^3*d^4 - a*c^2*d^2*e^2)*f^8*g - 3*(6*a*c^2*d^3*e - a^2*c*d*e^3)
*f^7*g^2 + (18*a^2*c*d^2*e^2 - a^3*e^4)*f^6*g^3)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 169, normalized size = 0.85

$$-\frac{2\,(cdx+ae)\left(8g^2x^2c^2d^2-28acde\,g^2x+44c^2d^2fgx+63a^2e^2g^2-154acdefg+99f^2c^2d^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{693\left(gx+f\right)^{\frac{11}{2}}\left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3\right)(ex+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13/2),x)
```

```
[Out] -2/693*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-28*a*c*d*e*g^2*x+44*c^2*d^2*f*g*x+63*
a^2*e^2*g^2-154*a*c*d*e*f*g+99*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(5/2)/(g*x+f)^(11/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*
g-c^3*d^3*f^3)/(e*x+d)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(13/2)), x)
```

mupad [B] time = 4.82, size = 465, normalized size = 2.35

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{126\,a^5\,e^5\,g^2-308\,a^4\,c\,d\,e^4\,f\,g+198\,a^3\,c^2\,d^2\,e^3\,f^2}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x^3\,\left(6\,a^2\,c^3\,d^3\,e^2\,g^2-44\,a\,c^4\,d^4\,e\,f\,g+198\,c^5\,d^5\,f^2\right)}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^5\,d^5\,x^5}{693\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c^4\,d^4\,x^4\,\left(a\,e\,g-11\,c\,d\,f\right)}{693\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{2\,a^2\,c\,d\,e^2\,x\,\left(161\,a^2\,e^2\,g^2-418\,a\,c\,d\,e\,f\,g+297\,c^2\,d^2\,f^2\right)}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{2\,a\,c^2\,d^2\,e\,x^2\,\left(113\,a^2\,e^2\,g^2-330\,a\,c\,d\,e\,f\,g+297\,c^2\,d^2\,f^2\right)}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^5}+\frac{5\,f\,x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{5\,f^4\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{10\,f^2\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}+\frac{10\,f^3\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(13/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((126*a^5*e^5*g^2 + 198*a^3
*c^2*d^2*e^3*f^2 - 308*a^4*c*d*e^4*f*g)/(693*g^5*(a*e*g - c*d*f)^3) + (x^3*
(198*c^5*d^5*f^2 + 6*a^2*c^3*d^3*e^2*g^2 - 44*a*c^4*d^4*e*f*g))/(693*g^5*(a
*e*g - c*d*f)^3) + (16*c^5*d^5*x^5)/(693*g^3*(a*e*g - c*d*f)^3) - (8*c^4*d^
4*x^4*(a*e*g - 11*c*d*f))/(693*g^4*(a*e*g - c*d*f)^3) + (2*a^2*c*d*e^2*x*(1
61*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 418*a*c*d*e*f*g))/(693*g^5*(a*e*g - c*d*
f)^3) + (2*a*c^2*d^2*e*x^2*(113*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 330*a*c*d*e
*f*g))/(693*g^5*(a*e*g - c*d*f)^3)))/(x^5*(f + g*x)^(1/2)*(d + e*x)^(1/2) +
 (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(f + g*x)^(1/2)*(d +
e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (10*f^2*x^3
*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(f + g*x)^(1/2)*(d + e*
x)^(1/2))/g^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(13/2),x)
```

```
[Out] Timed out
```

3.527 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3003(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^3}+\frac{12cd\left(x\left(ae^2+cd^2\right)+ade+\right.}{143(d+ex)^{7/2}(f+gx)^{11/2}(c}$$

Rubi [A] time = 0.32, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3003(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^3}+\frac{12cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{143(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(15/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(13/2)) + (12*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(7/2))/(143*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(11/2)) + (16*c
^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*(c*d*f - a*e*g)^
3*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(7/2))/(3003*(c*d*f - a*e*g)^4*(d + e*x)^(7/2)*(f + g*x)^(7/2
))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{(6cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}dx}{13(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{143(cdf-aeg)^2(d+ex)^{7/2}(f+g}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{143(cdf-aeg)^2(d+ex)^{7/2}(f+g}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{143(cdf-aeg)^2(d+ex)^{7/2}(f+g}$$

Mathematica [A] time = 0.14, size = 162, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(-231a^3e^3g^3+63a^2cde^2g^2(13f+2gx)-7ac^2d^2eg\left(143f^2+52fgx+8g^2x^2\right)+c^3d^3\left(429f^3+286f^2gx+104fg^2x^2+16g^3x^3\right)\right)}{3003\sqrt{d+ex}\,(f+gx)^{13/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(15/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-231*a^3*e^3*g^3 + 63*a^2
*c*d*e^2*g^2*(13*f + 2*g*x) - 7*a*c^2*d^2*e*g*(143*f^2 + 52*f*g*x + 8*g^2*x
^2) + c^3*d^3*(429*f^3 + 286*f^2*g*x + 104*f*g^2*x^2 + 16*g^3*x^3)))/(3003*
(c*d*f - a*e*g)^4*Sqrt[d + e*x]*(f + g*x)^(13/2))
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)
^(5/2)*(f + g*x)^(15/2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.54, size = 1648, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 1001*a^4*c^2*d^2*e^4
*f^2*g + 819*a^5*c*d*e^5*f*g^2 - 231*a^6*e^6*g^3 + 8*(13*c^6*d^6*f*g^2 - a*
c^5*d^5*e*g^3)*x^5 + 2*(143*c^6*d^6*f^2*g - 26*a*c^5*d^5*e*f*g^2 + 3*a^2*c^
4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 - 143*a*c^5*d^5*e*f^2*g + 39*a^2*c^4*
d^4*e^2*f*g^2 - 5*a^3*c^3*d^3*e^3*g^3)*x^3 + (1287*a*c^5*d^5*e*f^3 - 2145*a
^2*c^4*d^4*e^2*f^2*g + 1469*a^3*c^3*d^3*e^3*f*g^2 - 371*a^4*c^2*d^2*e^4*g^3
)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 - 2717*a^3*c^3*d^3*e^3*f^2*g + 2093*a^4*c
^2*d^2*e^4*f*g^2 - 567*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^11 - 4*a*c^3*d^4*e*f^10*
g + 6*a^2*c^2*d^3*e^2*f^9*g^2 - 4*a^3*c*d^2*e^3*f^8*g^3 + a^4*d*e^4*f^7*g^4
 + (c^4*d^4*e*f^4*g^7 - 4*a*c^3*d^3*e^2*f^3*g^8 + 6*a^2*c^2*d^2*e^3*f^2*g^9
```

```
 - 4*a^3*c*d*e^4*f*g^10 + a^4*e^5*g^11)*x^8 + (7*c^4*d^4*e*f^5*g^6 + a^4*d*
e^4*g^11 + (c^4*d^5 - 28*a*c^3*d^3*e^2)*f^4*g^7 - 2*(2*a*c^3*d^4*e - 21*a^2
*c^2*d^2*e^3)*f^3*g^8 + 2*(3*a^2*c^2*d^3*e^2 - 14*a^3*c*d*e^4)*f^2*g^9 - (4
*a^3*c*d^2*e^3 - 7*a^4*e^5)*f*g^10)*x^7 + 7*(3*c^4*d^4*e*f^6*g^5 + a^4*d*e^
4*f*g^10 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^5*g^6 - 2*(2*a*c^3*d^4*e - 9*a^2*
c^2*d^2*e^3)*f^4*g^7 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^8 - (4*a^3
*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^9)*x^6 + 7*(5*c^4*d^4*e*f^7*g^4 + 3*a^4*d*e^4
*f^2*g^9 + (3*c^4*d^5 - 20*a*c^3*d^3*e^2)*f^6*g^5 - 6*(2*a*c^3*d^4*e - 5*a^
2*c^2*d^2*e^3)*f^5*g^6 + 2*(9*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^4*g^7 - (
12*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^3*g^8)*x^5 + 35*(c^4*d^4*e*f^8*g^3 + a^4*d*
e^4*f^3*g^8 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^7*g^4 - 2*(2*a*c^3*d^4*e - 3*a^
2*c^2*d^2*e^3)*f^6*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^5*g^6 - (4
*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^7)*x^4 + 7*(3*c^4*d^4*e*f^9*g^2 + 5*a^4*d*e
^4*f^4*g^7 + (5*c^4*d^5 - 12*a*c^3*d^3*e^2)*f^8*g^3 - 2*(10*a*c^3*d^4*e - 9
*a^2*c^2*d^2*e^3)*f^7*g^4 + 6*(5*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^6*g^5 -
 (20*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^5*g^6)*x^3 + 7*(c^4*d^4*e*f^10*g + 3*a^4*
d*e^4*f^5*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^9*g^2 - 6*(2*a*c^3*d^4*e -
a^2*c^2*d^2*e^3)*f^8*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^7*g^4 -
(12*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^5)*x^2 + (c^4*d^4*e*f^11 + 7*a^4*d*e^4*f
^6*g^5 + (7*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^10*g - 2*(14*a*c^3*d^4*e - 3*a^2*c
^2*d^2*e^3)*f^9*g^2 + 2*(21*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^8*g^3 - (28*
a^3*c*d^2*e^3 - a^4*e^5)*f^7*g^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 260, normalized size = 0.97

$$-\frac{2 (cdx+ae) \left(-16 g^3 x^3 c^3 d^3+56 a c^2 d^2 e g^3 x^2-104 c^3 d^3 f g^2 x^2-126 a^2 c d e^2 g^3 x+364 a c^2 d^2 e f g^2 x-286 c^3 d^3 f^2 g x+231 a^3 e^3 g^3-819 a^2 c d e^2 f g^2+1001 a c^2 d^2 e f^2 g-429 f^3 c^3 d^3\right) \left(cde x^2+a e^2 x+c d^2 x+ade\right)^{\frac{5}{2}}}{3003 \left(gx+f\right)^{\frac{13}{2}} \left(g^4 e^4 a^4-4 a^3 c d e^3 f g^3+6 a^2 c^2 d^2 e^2 f^2 g^2-4 a c^3 d^3 e f^3 g+f^4 c^4 d^4\right) (ex+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15/2),x)
```

```
[Out] -2/3003*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+56*a*c^2*d^2*e*g^3*x^2-104*c^3*d^3
*f*g^2*x^2-126*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2*x-286*c^3*d^3*f^2*g*
x+231*a^3*e^3*g^3-819*a^2*c*d*e^2*f*g^2+1001*a*c^2*d^2*e*f^2*g-429*c^3*d^3*
f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/(g*x+f)^(13/2)/(a^4*e^4*g^4-4*
a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4
)/(e*x+d)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(15/2)), x)
```

mupad [B] time = 5.12, size = 627, normalized size = 2.35

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{462\,a^6\,e^6\,g^3-1638\,a^5\,c\,d\,e^5\,f\,g^2+2002\,a^4\,c^2\,d^2\,e^4\,f^2\,g-858\,a^3\,c^3\,d^3\,e^3\,f^3}{3003\,g^6\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{x^3\,\left(-10\,a^3\,c^3\,d^3\,e^3\,g^3+78\,a^2\,c^4\,d^4\,e^2\,f\,g^2-286\,a\,c^5\,d^5\,e\,f^2\,g+858\,c^6\,d^6\,f^3\right)}{3003\,g^6\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{32\,c^6\,d^6\,x^6}{3003\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{4\,c^4\,d^4\,x^4\,\left(3\,a^2\,e^2\,g^2-26\,a\,c\,d\,e\,f\,g+143\,c^2\,d^2\,f^2\right)}{3003\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{16\,c^5\,d^5\,x^5\,\left(a\,e\,g-13\,c\,d\,f\right)}{3003\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{2\,a^2\,c\,d\,e^2\,x\,\left(567\,a^3\,e^3\,g^3-2093\,a^2\,c\,d\,e^2\,f\,g^2+2717\,a\,c^2\,d^2\,e\,f^2\,g-1287\,c^3\,d^3\,f^3\right)}{3003\,g^6\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{2\,a\,c^2\,d^2\,e\,x^2\,\left(371\,a^3\,e^3\,g^3-1469\,a^2\,c\,d\,e^2\,f\,g^2+2145\,a\,c^2\,d^2\,e\,f^2\,g-1287\,c^3\,d^3\,f^3\right)}{3003\,g^6\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^6\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^6\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^6}+\frac{6\,f\,x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{6\,f^5\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^5}+\frac{15\,f^2\,x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}+\frac{20\,f^3\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{15\,f^4\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(15/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((462*a^6*e^6*g^3 - 858*a^3
*c^3*d^3*e^3*f^3 + 2002*a^4*c^2*d^2*e^4*f^2*g - 1638*a^5*c*d*e^5*f*g^2)/(30
03*g^6*(a*e*g - c*d*f)^4) - (x^3*(858*c^6*d^6*f^3 - 10*a^3*c^3*d^3*e^3*g^3
+ 78*a^2*c^4*d^4*e^2*f*g^2 - 286*a*c^5*d^5*e*f^2*g))/(3003*g^6*(a*e*g - c*d
*f)^4) - (32*c^6*d^6*x^6)/(3003*g^3*(a*e*g - c*d*f)^4) - (4*c^4*d^4*x^4*(3*
a^2*e^2*g^2 + 143*c^2*d^2*f^2 - 26*a*c*d*e*f*g))/(3003*g^5*(a*e*g - c*d*f)^
4) + (16*c^5*d^5*x^5*(a*e*g - 13*c*d*f))/(3003*g^4*(a*e*g - c*d*f)^4) + (2*
a^2*c*d*e^2*x*(567*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2717*a*c^2*d^2*e*f^2*g
- 2093*a^2*c*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4) + (2*a*c^2*d^2*e*x^
2*(371*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2145*a*c^2*d^2*e*f^2*g - 1469*a^2*c
*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4)))/(x^6*(f + g*x)^(1/2)*(d + e*x
)^(1/2) + (f^6*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^6 + (6*f*x^5*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g + (6*f^5*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (1
5*f^2*x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (20*f^3*x^3*(f + g*x)^(1/2
)*(d + e*x)^(1/2))/g^3 + (15*f^4*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(15/2),x)
```

```
[Out] Timed out
```

3.528 $\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \, dx$

Optimal. Leaf size=343

$$-\frac{6(d+ex)^{m-1}(cdf-aeg)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^4d^4e(1-m)(2-m)(3-m)(4-m)}+\frac{6g(d+ex)^m}{}$$

Rubi [A] time = 0.45, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {870, 794, 648}

$$-\frac{6(d+ex)^{m-1}(cdf-aeg)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^4d^4e(1-m)(2-m)(3-m)(4-m)}+\frac{6g(d+ex)^m(cdf-aeg)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{c^3d^3e(2-m)(3-m)(4-m)}+\frac{3(f+gx)^2(d+ex)^{m-1}(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{c^2d^2(3-m)(4-m)}+\frac{(f+gx)^3(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] (-6*(c*d*f - a*e*g)^2*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)
^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^4*d^4*e*(1 -
m)*(2 - m)*(3 - m)*(4 - m)) + (6*g*(c*d*f - a*e*g)^2*(d + e*x)^m*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(2 - m)*(3 - m)*(4 - m))
+ (3*(c*d*f - a*e*g)*(d + e*x)^(-1 + m)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*(3 - m)*(4 - m)) + ((d + e*x)^(-1 + m)*(
f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(4 - m))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(d+ex)^{-1+m}(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1}}{cd(4-m)}$$

$$= \frac{3(cdf-aeg)(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)\right.}{c^2d^2(3-m)(4-m)}$$

$$= \frac{6g(cdf-aeg)^2(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{c^3d^3e(2-m)(3-m)(4-m)}$$

$$= -\frac{6(cdf-aeg)^2\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(}{c^4d^4e(1-m)(2-m)(}$$

Mathematica [A] time = 0.17, size = 134, normalized size = 0.39

$$\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\left(\frac{3g^2(ae+cdx)^2(aeg-cdf)}{m-3}-\frac{3g(ae+cdx)(cdf-aeg)^2}{m-2}-\frac{(cdf-aeg)^3}{m-1}-\frac{g^3(ae+cdx)^3}{m-4}\right)}{c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(-((c*d*f - a*e*g)^3/
(-1 + m)) - (3*g*(c*d*f - a*e*g)^2*(a*e + c*d*x))/(-2 + m) + (3*g^2*(-(c*d*
f) + a*e*g)*(a*e + c*d*x)^2)/(-3 + m) - (g^3*(a*e + c*d*x)^3)/(-4 + m)))/(c
^4*d^4)
```

IntegrateAlgebraic [F] time = 0.69, size = 0, normalized size = 0.00

$$\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^m,x]
```

```
[Out] Defer[IntegrateAlgebraic][((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^m, x]
```

fricas [B] time = 0.44, size = 705, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] -(a*c^3*d^3*e*f^3*m^3 - 24*a*c^3*d^3*e*f^3 + 36*a^2*c^2*d^2*e^2*f^2*g - 24*
a^3*c*d*e^3*f*g^2 + 6*a^4*e^4*g^3 + (c^4*d^4*g^3*m^3 - 6*c^4*d^4*g^3*m^2 +
11*c^4*d^4*g^3*m - 6*c^4*d^4*g^3)*x^4 - (24*c^4*d^4*f*g^2 - (3*c^4*d^4*f*g^
2 + a*c^3*d^3*e*g^3)*m^3 + 3*(7*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m^2 - 2*(2
1*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m)*x^3 - 3*(3*a*c^3*d^3*e*f^3 - a^2*c^2*
d^2*e^2*f^2*g)*m^2 - 3*(12*c^4*d^4*f^2*g - (c^4*d^4*f^2*g + a*c^3*d^3*e*f*g
^2)*m^3 + (8*c^4*d^4*f^2*g + 5*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g^3)*m^2
 - (19*c^4*d^4*f^2*g + 4*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g^3)*m)*x^2 +
(26*a*c^3*d^3*e*f^3 - 21*a^2*c^2*d^2*e^2*f^2*g + 6*a^3*c*d*e^3*f*g^2)*m - (
24*c^4*d^4*f^3 - (c^4*d^4*f^3 + 3*a*c^3*d^3*e*f^2*g)*m^3 + 3*(3*c^4*d^4*f^3
```

```
 + 7*a*c^3*d^3*e*f^2*g - 2*a^2*c^2*d^2*e^2*f*g^2)*m^2 - 2*(13*c^4*d^4*f^3 +
 18*a*c^3*d^3*e*f^2*g - 12*a^2*c^2*d^2*e^2*f*g^2 + 3*a^3*c*d*e^3*g^3)*m)*x)
*(e*x + d)^m/((c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 - 50*c^4*d^4*m
 + 24*c^4*d^4)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

giac [B] time = 0.33, size = 2024, normalized size = 5.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] -((x*e + d)^m*c^4*d^4*g^3*m^3*x^4*e^(-m*log(c*d*x + a*e) - m*log(x*e + d))
+ 3*(x*e + d)^m*c^4*d^4*f*g^2*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d)) - 6*(x*e + d)^m*c^4*d^4*g^3*m^2*x^4*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d)) + (x*e + d)^m*a*c^3*d^3*g^3*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*
e + d) + 1) + 3*(x*e + d)^m*c^4*d^4*f^2*g*m^3*x^2*e^(-m*log(c*d*x + a*e) -
m*log(x*e + d)) - 21*(x*e + d)^m*c^4*d^4*f*g^2*m^2*x^3*e^(-m*log(c*d*x + a*
e) - m*log(x*e + d)) + 11*(x*e + d)^m*c^4*d^4*g^3*m*x^4*e^(-m*log(c*d*x + a
*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a*c^3*d^3*f*g^2*m^3*x^2*e^(-m*log(c*d
*x + a*e) - m*log(x*e + d) + 1) - 3*(x*e + d)^m*a*c^3*d^3*g^3*m^2*x^3*e^(-m
*log(c*d*x + a*e) - m*log(x*e + d) + 1) + (x*e + d)^m*c^4*d^4*f^3*m^3*x*e^(
-m*log(c*d*x + a*e) - m*log(x*e + d)) - 24*(x*e + d)^m*c^4*d^4*f^2*g*m^2*x^
2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 42*(x*e + d)^m*c^4*d^4*f*g^2*m
*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - 6*(x*e + d)^m*c^4*d^4*g^3*x
^4*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a*c^3*d^3*f^2*g
*m^3*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 15*(x*e + d)^m*a*c^3*
d^3*f*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) + 2*(x*e + d
)^m*a*c^3*d^3*g^3*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 9*(x
*e + d)^m*c^4*d^4*f^3*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 57*(
x*e + d)^m*c^4*d^4*f^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - 2
4*(x*e + d)^m*c^4*d^4*f*g^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) +
3*(x*e + d)^m*a^2*c^2*d^2*g^3*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 2) + (x*e + d)^m*a*c^3*d^3*f^3*m^3*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d) + 1) - 21*(x*e + d)^m*a*c^3*d^3*f^2*g*m^2*x*e^(-m*log(c*d*x + a*e) - m
*log(x*e + d) + 1) + 12*(x*e + d)^m*a*c^3*d^3*f*g^2*m*x^2*e^(-m*log(c*d*x +
 a*e) - m*log(x*e + d) + 1) + 26*(x*e + d)^m*c^4*d^4*f^3*m*x*e^(-m*log(c*d*
x + a*e) - m*log(x*e + d)) - 36*(x*e + d)^m*c^4*d^4*f^2*g*x^2*e^(-m*log(c*d
*x + a*e) - m*log(x*e + d)) + 6*(x*e + d)^m*a^2*c^2*d^2*f*g^2*m^2*x*e^(-m*l
og(c*d*x + a*e) - m*log(x*e + d) + 2) - 3*(x*e + d)^m*a^2*c^2*d^2*g^3*m*x^2
*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) - 9*(x*e + d)^m*a*c^3*d^3*f^3
*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) + 36*(x*e + d)^m*a*c^3*d^
3*f^2*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 24*(x*e + d)^m*c
^4*d^4*f^3*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a^2*c
^2*d^2*f^2*g*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) - 24*(x*e + d
)^m*a^2*c^2*d^2*f*g^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) + 26
*(x*e + d)^m*a*c^3*d^3*f^3*m*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) +
 6*(x*e + d)^m*a^3*c*d*g^3*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 3)
 - 21*(x*e + d)^m*a^2*c^2*d^2*f^2*g*m*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 2) - 24*(x*e + d)^m*a*c^3*d^3*f^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
 d) + 1) + 6*(x*e + d)^m*a^3*c*d*f*g^2*m*e^(-m*log(c*d*x + a*e) - m*log(x*e
 + d) + 3) + 36*(x*e + d)^m*a^2*c^2*d^2*f^2*g*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 2) - 24*(x*e + d)^m*a^3*c*d*f*g^2*e^(-m*log(c*d*x + a*e) - m*l
og(x*e + d) + 3) + 6*(x*e + d)^m*a^4*g^3*e^(-m*log(c*d*x + a*e) - m*log(x*e
 + d) + 4))/(c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 - 50*c^4*d^4*m +
 24*c^4*d^4)
```

maple [A] time = 0.01, size = 527, normalized size = 1.54

$$-\frac{\left(c^3d^3g^3m^3x^3+3c^3d^3fg^2m^3x^2-6c^3d^3g^3m^2x^3+3ac^2d^2eg^3m^2x^2+3c^3d^3f^2gm^3x-21c^3d^3fg^2m^2x^2+11c^3d^3g^3mx^3+6ac^2d^2efg^2m^2x-9ac^2d^2eg^3mx^2+c^3d^3f^3m^3-24c^3d^3f^2gm^2x+42c^3d^3fg^2mx^2-6g^3x^3c^3d^3+6a^2cde^2g^3mx+3ac^2d^2ef^2gm^2-30ac^2d^2efg^2mx+6ac^2d^2eg^3x^2-9c^3d^3f^3m^2+57c^3d^3f^2gmx-24c^3d^3fg^2x^2+6a^2cde^2fg^2m-6a^2cde^2g^3x-21ac^2d^2ef^2gm+24ac^2d^2efg^2x+26c^3d^3f^3m-36c^3d^3f^2gx+6a^3e^3g^3-24a^2cde^2fg^2+36ac^2d^2ef^2g-24f^3c^3d^3\right)(cdx+ae)(ex+d)^m\left(cdex^2+ae^2x+cd^2x+ade\right)^{-m}}{\left(m^4-10m^3+35m^2-50m+24\right)c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^3/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] -(e*x+d)^m*(c^3*d^3*g^3*m^3*x^3+3*c^3*d^3*f*g^2*m^3*x^2-6*c^3*d^3*g^3*m^2*x
^3+3*a*c^2*d^2*e*g^3*m^2*x^2+3*c^3*d^3*f^2*g*m^3*x-21*c^3*d^3*f*g^2*m^2*x^2
+11*c^3*d^3*g^3*m*x^3+6*a*c^2*d^2*e*f*g^2*m^2*x-9*a*c^2*d^2*e*g^3*m*x^2+c^3
*d^3*f^3*m^3-24*c^3*d^3*f^2*g*m^2*x+42*c^3*d^3*f*g^2*m*x^2-6*c^3*d^3*g^3*x^
3+6*a^2*c*d*e^2*g^3*m*x+3*a*c^2*d^2*e*f^2*g*m^2-30*a*c^2*d^2*e*f*g^2*m*x+6*
a*c^2*d^2*e*g^3*x^2-9*c^3*d^3*f^3*m^2+57*c^3*d^3*f^2*g*m*x-24*c^3*d^3*f*g^2
*x^2+6*a^2*c*d*e^2*f*g^2*m-6*a^2*c*d*e^2*g^3*x-21*a*c^2*d^2*e*f^2*g*m+24*a*
c^2*d^2*e*f*g^2*x+26*c^3*d^3*f^3*m-36*c^3*d^3*f^2*g*x+6*a^3*e^3*g^3-24*a^2*
c*d*e^2*f*g^2+36*a*c^2*d^2*e*f^2*g-24*c^3*d^3*f^3)*(c*d*x+a*e)/((c*d*e*x^2+
a*e^2*x+c*d^2*x+a*d*e)^m)/c^4/d^4/(m^4-10*m^3+35*m^2-50*m+24)
```

maxima [A] time = 0.61, size = 331, normalized size = 0.97

$$-\frac{(cdx+ae)f^3}{(cdx+ae)^m cd(m-1)} - \frac{3\left(c^2d^2(m-1)x^2 + acdemx + a^2e^2\right)f^2g}{\left(m^2-3m+2\right)(cdx+ae)^m c^2d^2} - \frac{3\left(\left(m^2-3m+2\right)c^3d^3x^3 + \left(m^2-m\right)ac^2d^2ex^2 + 2a^2cde^2mx + 2a^3e^3\right)fg^2}{\left(m^3-6m^2+11m-6\right)(cdx+ae)^m c^3d^3} - \frac{\left(\left(m^3-6m^2+11m-6\right)c^4d^4x^4 + \left(m^3-3m^2+2m\right)ac^3d^3ex^3 + 3\left(m^2-m\right)a^2c^2d^2e^2x^2 + 6a^3cde^3mx + 6a^4e^4\right)g^3}{\left(m^4-10m^3+35m^2-50m+24\right)(cdx+ae)^m c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] -(c*d*x + a*e)*f^3/((c*d*x + a*e)^m*c*d*(m - 1)) - 3*(c^2*d^2*(m - 1)*x^2 +
 a*c*d*e*m*x + a^2*e^2)*f^2*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - 3
*((m^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^2*m
*x + 2*a^3*e^3)*f*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3) -
((m^3 - 6*m^2 + 11*m - 6)*c^4*d^4*x^4 + (m^3 - 3*m^2 + 2*m)*a*c^3*d^3*e*x^3
 + 3*(m^2 - m)*a^2*c^2*d^2*e^2*x^2 + 6*a^3*c*d*e^3*m*x + 6*a^4*e^4)*g^3/((m
^4 - 10*m^3 + 35*m^2 - 50*m + 24)*(c*d*x + a*e)^m*c^4*d^4)
```

mupad [B] time = 3.75, size = 615, normalized size = 1.79

$$-\frac{\frac{g^3x^4(d+ex)^m\left(m^3-6m^2+11m-6\right)}{m^4-10m^3+35m^2-50m+24} + \frac{x(d+ex)^m\left(6a^3cde^3g^3m+6a^2c^2d^2e^2fg^2m^2-24a^2c^2d^2e^2fg^2m+3ac^3d^3ef^2gm^3-21ac^3d^3ef^2gm^2+36ac^3d^3ef^2gm+c^4d^4f^3m^3-9c^4d^4f^3m^2+26c^4d^4f^3m-24c^4d^4f^3\right)}{c^4d^4\left(m^4-10m^3+35m^2-50m+24\right)} + \frac{ae(d+ex)^m\left(6a^3e^3g^3+6a^2cde^2fg^2m-24a^2cde^2fg^2+3ac^2d^2ef^2gm^2-21ac^2d^2ef^2gm+36ac^2d^2ef^2g+c^3d^3f^3m^3-9c^3d^3f^3m^2+26c^3d^3f^3m-24c^3d^3f^3\right)}{c^4d^4\left(m^4-10m^3+35m^2-50m+24\right)} + \frac{3gx^2(m-1)(d+ex)^m\left(a^2e^2g^2m+acdefgm^2-4acdefgm+c^2d^2f^2m^2-7c^2d^2f^2m+12c^2d^2f^2\right)}{c^2d^2\left(m^4-10m^3+35m^2-50m+24\right)} + \frac{g^2x^3(d+ex)^m\left(aegm-12cdf+3cdfm\right)\left(m^2-3m+2\right)}{cd\left(m^4-10m^3+35m^2-50m+24\right)}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((g^3*x^4*(d + e*x)^m*(11*m - 6*m^2 + m^3 - 6))/(35*m^2 - 50*m - 10*m^3 +
m^4 + 24) + (x*(d + e*x)^m*(26*c^4*d^4*f^3*m - 24*c^4*d^4*f^3 - 9*c^4*d^4*f
^3*m^2 + c^4*d^4*f^3*m^3 + 6*a^3*c*d*e^3*g^3*m - 24*a^2*c^2*d^2*e^2*f*g^2*m
 + 36*a*c^3*d^3*e*f^2*g*m + 6*a^2*c^2*d^2*e^2*f*g^2*m^2 - 21*a*c^3*d^3*e*f^
2*g*m^2 + 3*a*c^3*d^3*e*f^2*g*m^3))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 + m^4
+ 24)) + (a*e*(d + e*x)^m*(6*a^3*e^3*g^3 - 24*c^3*d^3*f^3 + 26*c^3*d^3*f^3*
m - 9*c^3*d^3*f^3*m^2 + c^3*d^3*f^3*m^3 + 36*a*c^2*d^2*e*f^2*g - 24*a^2*c*d
*e^2*f*g^2 - 21*a*c^2*d^2*e*f^2*g*m + 6*a^2*c*d*e^2*f*g^2*m + 3*a*c^2*d^2*e
*f^2*g*m^2))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)) + (3*g*x^2*(m -
1)*(d + e*x)^m*(12*c^2*d^2*f^2 + a^2*e^2*g^2*m - 7*c^2*d^2*f^2*m + c^2*d^2*
f^2*m^2 - 4*a*c*d*e*f*g*m + a*c*d*e*f*g*m^2))/(c^2*d^2*(35*m^2 - 50*m - 10*
m^3 + m^4 + 24)) + (g^2*x^3*(d + e*x)^m*(a*e*g*m - 12*c*d*f + 3*c*d*f*m)*(m
^2 - 3*m + 2))/(c*d*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)))/(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)^m
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

3.529 $\int (d+ex)^m (f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=246

$$-\frac{2(d+ex)^{m-1}(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^3d^3e(1-m)(2-m)(3-m)}+\frac{2g(d+ex)^m(}{}$$

Rubi [A] time = 0.20, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {870, 794, 648}

$$-\frac{2(d+ex)^{m-1}(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^3d^3e(1-m)(2-m)(3-m)}+\frac{2g(d+ex)^m(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{c^2d^2e(2-m)(3-m)}+\frac{(f+gx)^2(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(
-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(1 - m)
*(2 - m)*(3 - m)) + (2*g*(c*d*f - a*e*g)*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(2 - m)*(3 - m)) + ((d + e*x)^(-1 + m
)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(3 - m)
)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rubi steps

$$\int (d+ex)^m (f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1}}{cd(3-m)}$$
$$= \frac{2g(cdf-aeg)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{c^2d^2e(2-m)(3-m)}$$
$$= -\frac{2(cdf-aeg)\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d}{c^3d^3e(1-m)(2-}$$

Mathematica [A] time = 0.11, size = 131, normalized size = 0.53

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\left(2a^2e^2g^2+2acdeg(f(m-3)+g(m-1)x)+c^2d^2\left(f^2\left(m^2-5m+6\right)+2fg\left(m^2-4m+3\right)x+g^2\left(m^2-3m+2\right)x^2\right)\right)}{c^3d^3(m-3)(m-2)(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(2*a^2*e^2*g^2 + 2*
a*c*d*e*g*(f*(-3 + m) + g*(-1 + m)*x) + c^2*d^2*(f^2*(6 - 5*m + m^2) + 2*f*
g*(3 - 4*m + m^2)*x + g^2*(2 - 3*m + m^2)*x^2)))/(c^3*d^3*(-3 + m)*(-2 + m)
*(-1 + m)))
```

IntegrateAlgebraic [F] time = 0.41, size = 0, normalized size = 0.00

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^m,x]
```

```
[Out] Defer[IntegrateAlgebraic][((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^m, x]
```

fricas [A] time = 0.46, size = 350, normalized size = 1.42

$$-\frac{\left(ac^2d^2ef^2m^2+6\,ac^2d^2ef^2-6\,a^2cde^2fg+2\,a^3e^3g^2+\left(c^3d^3g^2m^2-3\,c^3d^3g^2m+2\,c^3d^3g^2\right)x^3+\left(6\,c^3d^3fg+\left(2\,c^3d^3fg+ac^2d^2eg^2\right)m^2-\left(8\,c^3d^3fg+ac^2d^2eg^2\right)m\right)x^2-\left(5\,ac^2d^2ef^2-2\,a^2cde^2fg\right)m+\left(6\,c^3d^3f^2+\left(c^3d^3f^2+2\,ac^2d^2efg\right)m^2-\left(5\,c^3d^3f^2+6\,ac^2d^2efg-2\,a^2cde^2g^2\right)m\right)x\right)(ex+d)^m}{\left(c^3d^3m^3-6\,c^3d^3m^2+11\,c^3d^3m-6\,c^3d^3\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] -(a*c^2*d^2*e*f^2*m^2 + 6*a*c^2*d^2*e*f^2 - 6*a^2*c*d*e^2*f*g + 2*a^3*e^3*g
^2 + (c^3*d^3*g^2*m^2 - 3*c^3*d^3*g^2*m + 2*c^3*d^3*g^2)*x^3 + (6*c^3*d^3*f
*g + (2*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*m^2 - (8*c^3*d^3*f*g + a*c^2*d^2*e*g
^2)*m)*x^2 - (5*a*c^2*d^2*e*f^2 - 2*a^2*c*d*e^2*f*g)*m + (6*c^3*d^3*f^2 + (
c^3*d^3*f^2 + 2*a*c^2*d^2*e*f*g)*m^2 - (5*c^3*d^3*f^2 + 6*a*c^2*d^2*e*f*g -
 2*a^2*c*d*e^2*g^2)*m)*x)*(e*x + d)^m/((c^3*d^3*m^3 - 6*c^3*d^3*m^2 + 11*c^
3*d^3*m - 6*c^3*d^3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

giac [B] time = 0.28, size = 981, normalized size = 3.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] -((x*e + d)^m*c^3*d^3*g^2*m^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d))
+ 2*(x*e + d)^m*c^3*d^3*f*g*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) - 3*(x*e + d)^m*c^3*d^3*g^2*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) + (x*e + d)^m*a*c^2*d^2*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 1) + (x*e + d)^m*c^3*d^3*f^2*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d)) - 8*(x*e + d)^m*c^3*d^3*f*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d)) + 2*(x*e + d)^m*c^3*d^3*g^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d)) + 2*(x*e + d)^m*a*c^2*d^2*f*g*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d) + 1) - (x*e + d)^m*a*c^2*d^2*g^2*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 1) - 5*(x*e + d)^m*c^3*d^3*f^2*m*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d)) + 6*(x*e + d)^m*c^3*d^3*f*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d)) + (x*e + d)^m*a*c^2*d^2*f^2*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*
e + d) + 1) - 6*(x*e + d)^m*a*c^2*d^2*f*g*m*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) + 6*(x*e + d)^m*c^3*d^3*f^2*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d)) + 2*(x*e + d)^m*a^2*c*d*g^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 2) - 5*(x*e + d)^m*a*c^2*d^2*f^2*m*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) + 2*(x*e + d)^m*a^2*c*d*f*g*m*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 2) + 6*(x*e + d)^m*a*c^2*d^2*f^2*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) - 6*(x*e + d)^m*a^2*c*d*f*g*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 2) + 2*(x*e + d)^m*a^3*g^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 3))/(c^3*d^3*m^3 - 6*c^3*d^3*m^2 + 11*c^3*d^3*m - 6*c^3*d^3)
```

maple [A] time = 0.01, size = 235, normalized size = 0.96

$$-\frac{(cdx+ae)\left(c^2d^2g^2m^2x^2+2c^2d^2fg\,m^2x-3c^2d^2g^2m\,x^2+2acde\,g^2mx+c^2d^2f^2m^2-8c^2d^2fgmx+2g^2x^2c^2d^2+2acdefgm-2acde\,g^2x-5c^2d^2f^2m+6c^2d^2fgx+2a^2e^2g^2-6acdefg+6f^2c^2d^2\right)(ex+d)^m\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{\left(m^3-6m^2+11m-6\right)c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] -(c*d*x+a*e)*(c^2*d^2*g^2*m^2*x^2+2*c^2*d^2*f*g*m^2*x-3*c^2*d^2*g^2*m*x^2+2
*a*c*d*e*g^2*m*x+c^2*d^2*f^2*m^2-8*c^2*d^2*f*g*m*x+2*c^2*d^2*g^2*x^2+2*a*c*
d*e*f*g*m-2*a*c*d*e*g^2*x-5*c^2*d^2*f^2*m+6*c^2*d^2*f*g*x+2*a^2*e^2*g^2-6*a
*c*d*e*f*g+6*c^2*d^2*f^2)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)/c
^3/d^3/(m^3-6*m^2+11*m-6)
```

maxima [A] time = 0.56, size = 193, normalized size = 0.78

$$-\frac{(cdx+ae)f^2}{(cdx+ae)^m cd(m-1)}-\frac{2\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)fg}{\left(m^2-3\,m+2\right)(cdx+ae)^m c^2d^2}-\frac{\left(\left(m^2-3\,m+2\right)c^3d^3x^3+\left(m^2-m\right)ac^2d^2ex^2+2\,a^2cde^2mx+2\,a^3e^3\right)g^2}{\left(m^3-6\,m^2+11\,m-6\right)(cdx+ae)^m c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] -(c*d*x + a*e)*f^2/((c*d*x + a*e)^m*c*d*(m - 1)) - 2*(c^2*d^2*(m - 1)*x^2 +
 a*c*d*e*m*x + a^2*e^2)*f*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - ((m
^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^2*m*x +
 2*a^3*e^3)*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3)
```

mupad [B] time = 3.52, size = 327, normalized size = 1.33

$$-\frac{\frac{g^2\,x^3\,{\left(d+e\,x\right)}^m\,\left(m^2-3\,m+2\right)}{m^3-6\,m^2+11\,m-6}+\frac{x\,{\left(d+e\,x\right)}^m\,\left(2\,a^2\,c\,d\,e^2\,g^2\,m+2\,a\,c^2\,d^2\,e\,f\,g\,m^2-6\,a\,c^2\,d^2\,e\,f\,g\,m+c^3\,d^3\,f^2\,m^2-5\,c^3\,d^3\,f^2\,m+6\,c^3\,d^3\,f^2\right)}{c^3\,d^3\,\left(m^3-6\,m^2+11\,m-6\right)}+\frac{a\,e\,{\left(d+e\,x\right)}^m\,\left(2\,a^2\,e^2\,g^2+2\,a\,c\,d\,e\,f\,g\,m-6\,a\,c\,d\,e\,f\,g+c^2\,d^2\,f^2\,m^2-5\,c^2\,d^2\,f^2\,m+6\,c^2\,d^2\,f^2\right)}{c^3\,d^3\,\left(m^3-6\,m^2+11\,m-6\right)}+\frac{g\,x^2\,\left(m-1\right)\,{\left(d+e\,x\right)}^m\,\left(a\,e\,g\,m-6\,c\,d\,f+2\,c\,d\,f\,m\right)}{c\,d\,\left(m^3-6\,m^2+11\,m-6\right)}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((g^2*x^3*(d + e*x)^m*(m^2 - 3*m + 2))/(11*m - 6*m^2 + m^3 - 6) + (x*(d +
e*x)^m*(6*c^3*d^3*f^2 - 5*c^3*d^3*f^2*m + c^3*d^3*f^2*m^2 + 2*a^2*c*d*e^2*g
^2*m + 2*a*c^2*d^2*e*f*g*m^2 - 6*a*c^2*d^2*e*f*g*m))/(c^3*d^3*(11*m - 6*m^2
 + m^3 - 6)) + (a*e*(d + e*x)^m*(2*a^2*e^2*g^2 + 6*c^2*d^2*f^2 - 5*c^2*d^2*
f^2*m + c^2*d^2*f^2*m^2 - 6*a*c*d*e*f*g + 2*a*c*d*e*f*g*m))/(c^3*d^3*(11*m
- 6*m^2 + m^3 - 6)) + (g*x^2*(m - 1)*(d + e*x)^m*(a*e*g*m - 6*c*d*f + 2*c*d
*f*m))/(c*d*(11*m - 6*m^2 + m^3 - 6)))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^m
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

3.530 $\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=150

$$\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-$$

Rubi [A] time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {794, 648}

$$\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^2d^2e(1-m)(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(-1 + m)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(1 - m)*(2 - m))) + (g*(
d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*e*(2 - m))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\frac{g(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(ae^2}{}$$

$$=-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}}{c^2d^2e(1-m)(2-m}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 0.45

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}(aeg+cd(f(m-2)+g(m-1)x))}{c^2d^2(m-2)(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(a*e*g + c*d*(f*(-2
 + m) + g*(-1 + m)*x)))/(c^2*d^2*(-2 + m)*(-1 + m)))
```

IntegrateAlgebraic [F] time = 0.26, size = 0, normalized size = 0.00

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^m,x]
```

```
[Out] Defer[IntegrateAlgebraic][((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^m, x]
```

fricas [A] time = 0.42, size = 145, normalized size = 0.97

$$-\frac{\left(acdefm-2\,acdef+a^2e^2g+\left(c^2d^2gm-c^2d^2g\right)x^2-\left(2\,c^2d^2f-\left(c^2d^2f+acdeg\right)m\right)x\right)(ex+d)^m}{\left(c^2d^2m^2-3\,c^2d^2m+2\,c^2d^2\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="fricas")
```

```
[Out] -(a*c*d*e*f*m - 2*a*c*d*e*f + a^2*e^2*g + (c^2*d^2*g*m - c^2*d^2*g)*x^2 - (
2*c^2*d^2*f - (c^2*d^2*f + a*c*d*e*g)*m)*x)*(e*x + d)^m/((c^2*d^2*m^2 - 3*c
^2*d^2*m + 2*c^2*d^2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

giac [B] time = 0.25, size = 369, normalized size = 2.46

$$-\frac{(xe+d)^m c^2d^2gmx^2e^{\left(-m\log(cdx+ae)-m\log(xe+d)\right)}+(xe+d)^m c^2d^2fmxe^{\left(-m\log(cdx+ae)-m\log(xe+d)\right)}-(xe+d)^m c^2d^2gx^2e^{\left(-m\log(cdx+ae)-m\log(xe+d)\right)}+(xe+d)^m acdgmxe^{\left(-m\log(cdx+ae)-m\log(xe+d)+1\right)}-2\,(xe+d)^m c^2d^2fxe^{\left(-m\log(cdx+ae)-m\log(xe+d)\right)}+(xe+d)^m acdfme^{\left(-m\log(cdx+ae)-m\log(xe+d)+1\right)}-2\,(xe+d)^m acdfe^{\left(-m\log(cdx+ae)-m\log(xe+d)+1\right)}+(xe+d)^m a^2ge^{\left(-m\log(cdx+ae)-m\log(xe+d)+2\right)}}{c^2d^2m^2-3\,c^2d^2m+2\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="giac")
```

```
[Out] -((x*e + d)^m*c^2*d^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x
*e + d)^m*c^2*d^2*f*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - (x*e + d
)^m*c^2*d^2*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*a*
c*d*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 2*(x*e + d)^m*c^2*
d^2*f*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*a*c*d*f*m*e^
(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 2*(x*e + d)^m*a*c*d*f*e^(-m*lo
g(c*d*x + a*e) - m*log(x*e + d) + 1) + (x*e + d)^m*a^2*g*e^(-m*log(c*d*x +
a*e) - m*log(x*e + d) + 2))/(c^2*d^2*m^2 - 3*c^2*d^2*m + 2*c^2*d^2)
```

maple [A] time = 0.00, size = 89, normalized size = 0.59

$$-\frac{\left(cdgmx+cdfm-cdgx+aeg-2cdf\right)(cdx+ae)\,(ex+d)^m\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{\left(m^2-3m+2\right)c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] -(e*x+d)^m*(c*d*g*m*x+c*d*f*m-c*d*g*x+a*e*g-2*c*d*f)*(c*d*x+a*e)/((c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^m)/c^2/d^2/(m^2-3*m+2)
```

maxima [A] time = 0.52, size = 94, normalized size = 0.63

$$-\frac{(cdx+ae)f}{(cdx+ae)^m cd(m-1)}-\frac{\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)g}{\left(m^2-3\,m+2\right)(cdx+ae)^m c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="maxima")
```

```
[Out] -(c*d*x + a*e)*f/((c*d*x + a*e)^m*c*d*(m - 1)) - (c^2*d^2*(m - 1)*x^2 + a*c
*d*e*m*x + a^2*e^2)*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2)
```

mupad [B] time = 3.36, size = 139, normalized size = 0.93

$$-\frac{\frac{g\,x^2\,(m-1)\,(d+e\,x)^m}{m^2-3\,m+2}+\frac{x\,(d+e\,x)^m\,\left(a\,e\,g\,m-2\,c\,d\,f+c\,d\,f\,m\right)}{c\,d\,\left(m^2-3\,m+2\right)}+\frac{a\,e\,(d+e\,x)^m\,\left(a\,e\,g-2\,c\,d\,f+c\,d\,f\,m\right)}{c^2\,d^2\,\left(m^2-3\,m+2\right)}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((g*x^2*(m - 1)*(d + e*x)^m)/(m^2 - 3*m + 2) + (x*(d + e*x)^m*(a*e*g*m - 2
*c*d*f + c*d*f*m))/(c*d*(m^2 - 3*m + 2)) + (a*e*(d + e*x)^m*(a*e*g - 2*c*d*
f + c*d*f*m))/(c^2*d^2*(m^2 - 3*m + 2)))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^m
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Exception raised: TypeError
```

3.531 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=54

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {648}

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(
1 - m))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.78

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}}{cd(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m))/(c*d*(-1 + m)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^m, x]
```

fricas [A] time = 0.42, size = 57, normalized size = 1.06

$$-\frac{(cdx+ae)(ex+d)^m}{(cdm-cd)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fri
cas")
```

```
[Out] -(c*d*x + a*e)*(e*x + d)^m/((c*d*m - c*d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^m)
```

giac [A] time = 0.22, size = 87, normalized size = 1.61

$$-\frac{(xe+d)^m cdxe^{\left(-m\log(cdx+ae)-m\log(xe+d)\right)}+(xe+d)^m ae^{\left(-m\log(cdx+ae)-m\log(xe+d)+1\right)}}{cdm-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="gia
c")
```

```
[Out] -((x*e + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*
a*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1))/(c*d*m - c*d)
```

maple [A] time = 0.00, size = 57, normalized size = 1.06

$$-\frac{(cdx+ae)\,(ex+d)^m\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{(m-1)\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] -(c*d*x+a*e)/c/d/(-1+m)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)
```

maxima [A] time = 0.47, size = 33, normalized size = 0.61

$$-\frac{cdx+ae}{(cdx+ae)^m cd(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="max
ima")
```

```
[Out] -(c*d*x + a*e)/((c*d*x + a*e)^m*c*d*(m - 1))
```

mupad [B] time = 3.25, size = 57, normalized size = 1.06

$$-\frac{(a\,e+c\,d\,x)\,\left(d+e\,x\right)^m}{c\,d\,\left(m-1\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((a*e + c*d*x)*(d + e*x)^m)/(c*d*(m - 1)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*
e*x^2)^m)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
[Out] Exception raised: TypeError
```

3.532

$$\int (ae + cdx)^n (d + ex)^m \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^{-m} dx$$

Optimal. Leaf size=65

$$\frac{(d + ex)^{m-1} \left(x \left(ae^2 + cd^2\right) + ade + cdex^2\right)^{1-m} (ae + cdx)^n}{cd(-m + n + 1)}$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {858}

$$\frac{(d + ex)^{m-1} \left(x \left(ae^2 + cd^2\right) + ade + cdex^2\right)^{1-m} (ae + cdx)^n}{cd(-m + n + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] ((a*e + c*d*x)^n*(d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(1 - m))/(c*d*(1 - m + n))
```

Rule 858

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0] && EqQ[c*e*f + c*d*g - b
*e*g, 0] && NeQ[m - n - 1, 0]
```

Rubi steps

$$\int (ae + cdx)^n (d + ex)^m \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^{-m} dx = \frac{(ae + cdx)^n (d + ex)^{-1+m} \left(ade + \left(cd^2 + ae^2\right) x + c\right.}{cd(1 - m + n)}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.82

$$\frac{(d + ex)^m ((d + ex)(ae + cdx))^{-m} (ae + cdx)^{n+1}}{-cdm + cdn + cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m,x]
```

```
[Out] ((a*e + c*d*x)^(1 + n)*(d + e*x)^m)/((c*d - c*d*m + c*d*n)*((a*e + c*d*x)*(
d + e*x))^m)
```

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (ae + cdx)^n (d + ex)^m \left(ade + \left(cd^2 + ae^2\right) x + cdex^2\right)^{-m} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^m,x]
```

```
[Out] Defer[IntegrateAlgebraic][((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^m, x]
```

fricas [A] time = 0.42, size = 66, normalized size = 1.02

$$-\frac{(cdx + ae)(cdx + ae)^n (ex + d)^m e^{\left(-m \log(cdx+ae) - m \log(ex+d)\right)}}{cdm - cdn - cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] -(c*d*x + a*e)*(c*d*x + a*e)^n*(e*x + d)^m*e^(-m*log(c*d*x + a*e) - m*log(e
*x + d))/(c*d*m - c*d*n - c*d)
```

giac [A] time = 0.25, size = 114, normalized size = 1.75

$$-\frac{(cdx + ae)^n (xe + d)^m cdxe^{\left(-m \log(cdx+ae) - m \log(xe+d)\right)} + (cdx + ae)^n (xe + d)^m ae^{\left(-m \log(cdx+ae) - m \log(xe+d) + 1\right)}}{cdm - cdn - cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] -((c*d*x + a*e)^n*(x*e + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) + (c*d*x + a*e)^n*(x*e + d)^m*a*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) +
 1))/(c*d*m - c*d*n - c*d)
```

maple [A] time = 0.00, size = 64, normalized size = 0.98

$$-\frac{(ex + d)^m (cdx + ae)^{n+1} \left(cde\,x^2 + a\,e^2 x + c\,d^2 x + ade\right)^{-m}}{(m - n - 1)\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+a*e)^n*(e*x+d)^m/((c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] -(c*d*x+a*e)^(n+1)/c/d/(-1+m-n)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^m)
```

maxima [A] time = 0.50, size = 49, normalized size = 0.75

$$-\frac{(cdx + ae)e^{\left(-m \log(cdx+ae) + n \log(cdx+ae)\right)}}{cd(m - n - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] -(c*d*x + a*e)*e^(-m*log(c*d*x + a*e) + n*log(c*d*x + a*e))/(c*d*(m - n - 1
))
```

mupad [B] time = 3.54, size = 63, normalized size = 0.97

$$\frac{(a\,e + c\,d\,x)^{n+1}\,(d + e\,x)^m}{c\,d\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^m\,(n - m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*e + c*d*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
,x)
```

```
[Out] ((a*e + c*d*x)^(n + 1)*(d + e*x)^m)/(c*d*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^m*(n - m + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)**n*(e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

3.533 $\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex\right.$

Optimal. Leaf size=78

$$-\frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \log(ae+cdx)\left(-ae^3g-cde^2gx\right)^m}{cde^2g}$$

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}} = 0.041$, Rules used = $\{891, 23, 31\}$

$$-\frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \log(ae+cdx)\left(-ae^3g-cde^2gx\right)^m}{cde^2g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1 + m))/
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((d + e*x)^m*(-(a*e^3*g) - c*d*e^2*g*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \left((ae+cdx)^m(d+ex)^m \left(\right.\right.$$

$$= \left((d+ex)^m \left(cd^2eg-e\left(c\right.\right.\right.$$

$$= -\frac{(d+ex)^m \left(-ae^3g-cd\right.}{}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.82

$$-\frac{(d+ex)^m((d+ex)(ae+cdx))^{-m}\log(ae+cdx)\left(-e^2g(ae+cdx)\right)^m}{cde^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1
+ m))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((-(e^2*g*(a*e + c*d*x)))^m*(d + e*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*((a*
e + c*d*x)*(d + e*x))^m))
```

IntegrateAlgebraic [A] time = 0.39, size = 64, normalized size = 0.82

$$-\frac{(d+ex)^m((d+ex)(ae+cdx))^{-m}\log(ae+cdx)\left(-e^2g(ae+cdx)\right)^m}{cde^2g}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*
g*x)^(-1 + m))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((-(e^2*g*(a*e + c*d*x)))^m*(d + e*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*((a*
e + c*d*x)*(d + e*x))^m))
```

fricas [A] time = 0.43, size = 35, normalized size = 0.45

$$-\frac{\log\left(cdx+ae\right)}{cde^2g\left(-\frac{1}{e^2g}\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fricas")
```

```
[Out] -log(c*d*x + a*e)/(c*d*e^2*g*(-1/(e^2*g))^m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-cde^2gx+cd^2eg-\left(cd^2+ae^2\right)eg\right)^{m-1}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="giac")
```

```
[Out] integrate((-c*d*e^2*g*x + c*d^2*e*g - (c*d^2 + a*e^2)*e*g)^(m - 1)*(e*x + d
)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(cde\,x^2+ade+\left(a\,e^2+c\,d^2\right)x\right)^{-m}\left(-cd\,e^2gx+c\,d^2eg-\left(a\,e^2+c\,d^2\right)eg\right)^{m-1}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(m-1)/((c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^m),x)
```

```
[Out] int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(m-1)/((c*d*e*x^2+a
*d*e+(a*e^2+c*d^2)*x)^m),x)
```

maxima [A] time = 0.50, size = 32, normalized size = 0.41

$$-\frac{e^{2m-2}\left(-g\right)^{m}\log\left(cdx+ae\right)}{cdg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="maxima")
```

```
[Out] -e^(2*m - 2)*(-g)^m*log(c*d*x + a*e)/(c*d*g)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m\,{\left(c\,d^2\,e\,g-e\,g\,\left(c\,d^2+a\,e^2\right)-c\,d\,e^2\,g\,x\right)}^{m-1}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*d**2*e*g-e*(a*e**2+c*d**2)*g-c*d*e**2*g*x)**(-1+m)/
((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Timed out
```

3.534 $\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=501

$$\frac{128\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{3465c^6d^6eg\sqrt{d+ex}}-\frac{128\sqrt{d+ex}\ldots}{\ldots}$$

Rubi [A] time = 0.89, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {880, 870, 794, 648}

$$-\frac{2(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(10ae^2g+cd(ef-11dg)\right)}{99c^2d^2g\sqrt{d+ex}}-\frac{16(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(10ae^2g+cd(ef-11dg)\right)}{693c^3d^3g\sqrt{d+ex}}-\frac{32(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2\left(10ae^2g+cd(ef-11dg)\right)}{1155c^4d^4g\sqrt{d+ex}}-\frac{128\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)}{3465c^5d^5e}+\frac{128\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{3465c^6d^6eg\sqrt{d+ex}}+\frac{2e(f+gx)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(2*a*e^2*g - c*d*(
3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^6*d^6*e*
g*Sqrt[d + e*x]) - (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))
*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^5*d^5*e
) - (32*(c*d*f - a*e*g)^2*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^2*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1155*c^4*d^4*g*Sqrt[d + e*x]) -
(16*(c*d*f - a*e*g)*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^3*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(693*c^3*d^3*g*Sqrt[d + e*x]) - (2*(10*
a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(99*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^5*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(11*c*d*g*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{11cdg\sqrt{d+ex}} - \frac{1}{11}\left(-11d+\frac{10ae^2}{cd}+\frac{ef}{g}\right)\int$$

$$= -\frac{2\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{99c^2d^2g\sqrt{d+ex}} + \frac{2e}{}$$

$$= -\frac{16(cdf-aeg)\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+}}{693c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{32(cdf-aeg)^2\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x}}{1155c^4d^4g\sqrt{d+ex}}$$

$$= -\frac{128(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x}}{3465c^5d^5e}$$

$$= \frac{128(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+}}{3465c^6d^6eg\sqrt{d+ex}}$$

Mathematica [A] time = 0.42, size = 246, normalized size = 0.49

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3465\left(cd^2-ae^2\right)(cdf-aeg)^4-385g^3(ae+cdx)^4\left(5ae^2g-cd(dg+4ef)\right)+990g^2(ae+cdx)^3(cdf-aeg)\left(cd(2dg+3ef)-5ae^2g\right)+1386g(ae+cdx)^2(cdf-aeg)^2\left(cd(3dg+2ef)-5ae^2g\right)+1155(ae+cdx)(cdf-aeg)^3\left(cd(4dg+ef)-5ae^2g\right)+315eg^4(ae+cdx)^5\right)}{3465c^6d^6\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3465*(c*d^2 - a*e^2)*(c*d*f - a*e*g)^4 +
1155*(c*d*f - a*e*g)^3*(-5*a*e^2*g + c*d*(e*f + 4*d*g))*(a*e + c*d*x) + 138
6*g*(c*d*f - a*e*g)^2*(-5*a*e^2*g + c*d*(2*e*f + 3*d*g))*(a*e + c*d*x)^2 +
990*g^2*(c*d*f - a*e*g)*(-5*a*e^2*g + c*d*(3*e*f + 2*d*g))*(a*e + c*d*x)^3
- 385*g^3*(5*a*e^2*g - c*d*(4*e*f + d*g))*(a*e + c*d*x)^4 + 315*e*g^4*(a*e
+ c*d*x)^5))/(3465*c^6*d^6*Sqrt[d + e*x])
```

IntegrateAlgebraic [B] time = 38.39, size = 8325, normalized size = 16.62

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2],x]
```

```
[Out] Result too large to show
```

fricas [A]   time = 0.42, size = 597, normalized size = 1.19

$$\frac{2\left(315\,c^5d^5eg^4x^5 + 1155\left(3\,c^5d^6 - 2\,ac^4d^4e^2\right)f^4 - 1848\left(5\,ac^4d^5e - 4\,a^2c^3d^3e^3\right)f^3g + 1584\left(7\,a^2c^3d^4e^2 - 6\,a^3c^2d^2e^4\right)f^2g^2 - 704\left(9\,a^3c^2d^3e^3 - 8\,a^4cde^5\right)fg^3 + 128\left(11\,a^4cd^2e^4 - 10\,a^5e^6\right)g^4 + 35\left(44\,c^5d^5efg^3 + \left(11\,c^5d^6 - 10\,ac^4d^4e^2\right)g^4\right)x^4 + 10\left(297\,c^5d^5ef^2g^2 + 22\left(9\,c^5d^6 - 8\,ac^4d^4e^2\right)fg^3 - 4\left(11\,ac^4d^5e - 10\,a^2c^3d^3e^3\right)g^4\right)x^3 + 6\left(462\,c^5d^5ef^3g + 99\left(7\,c^5d^6 - 6\,ac^4d^4e^2\right)f^2g^2 - 44\left(9\,ac^4d^5e - 8\,a^2c^3d^3e^3\right)fg^3 + 8\left(11\,a^2c^3d^4e^2 - 10\,a^3c^2d^2e^4\right)g^4\right)x^2 + \left(1155\,c^5d^5ef^4 + 924\left(5\,c^5d^6 - 4\,ac^4d^4e^2\right)f^3g - 792\left(7\,ac^4d^5e - 6\,a^2c^3d^3e^3\right)f^2g^2 + 352\left(9\,a^2c^3d^4e^2 - 8\,a^3c^2d^2e^4\right)fg^3 - 64\left(11\,a^3c^2d^3e^3 - 10\,a^4cde^5\right)g^4\right)x\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\sqrt{ex + d}}{3465\left(c^6d^6ex + c^6d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^5*d^5*e*g^4*x^5 + 1155*(3*c^5*d^6 - 2*a*c^4*d^4*e^2)*f^4 - 18
48*(5*a*c^4*d^5*e - 4*a^2*c^3*d^3*e^3)*f^3*g + 1584*(7*a^2*c^3*d^4*e^2 - 6*
a^3*c^2*d^2*e^4)*f^2*g^2 - 704*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*f*g^3 +
128*(11*a^4*c*d^2*e^4 - 10*a^5*e^6)*g^4 + 35*(44*c^5*d^5*e*f*g^3 + (11*c^5*
d^6 - 10*a*c^4*d^4*e^2)*g^4)*x^4 + 10*(297*c^5*d^5*e*f^2*g^2 + 22*(9*c^5*d^
6 - 8*a*c^4*d^4*e^2)*f*g^3 - 4*(11*a*c^4*d^5*e - 10*a^2*c^3*d^3*e^3)*g^4)*x
^3 + 6*(462*c^5*d^5*e*f^3*g + 99*(7*c^5*d^6 - 6*a*c^4*d^4*e^2)*f^2*g^2 - 44
*(9*a*c^4*d^5*e - 8*a^2*c^3*d^3*e^3)*f*g^3 + 8*(11*a^2*c^3*d^4*e^2 - 10*a^3
*c^2*d^2*e^4)*g^4)*x^2 + (1155*c^5*d^5*e*f^4 + 924*(5*c^5*d^6 - 4*a*c^4*d^4
*e^2)*f^3*g - 792*(7*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^2*g^2 + 352*(9*a^2*
c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*f*g^3 - 64*(11*a^3*c^2*d^3*e^3 - 10*a^4*c*
d*e^5)*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c
^6*d^6*e*x + c^6*d^7)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^4}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^4/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

maple [A]   time = 0.01, size = 641, normalized size = 1.28

$$-\frac{2\left(cdx + ae\right)\left(-315e g^4x^5c^5d^5 + 350a c^4d^4e^2g^4x^4 - 385c^5d^6g^4x^4 - 1540c^5d^5ef g^3x^4 - 400a^2c^3d^3e^3g^4x^3 + 440a c^4d^5e g^4x^3 + 1760a c^4d^4e^2f g^3x^3 - 1980c^5d^6f g^3x^3 - 2970c^5d^5e f^2g^2x^3 + 480a^3c^2d^2e^4g^4x^2 - 528a^2c^3d^4e^2g^4x^2 - 2112a^2c^3d^3e^3f g^3x^2 + 2376a c^4d^5ef g^3x^2 + 3564a c^4d^4e^2f^2g^2x^2 - 4158c^5d^6f^2g^2x^2 - 2772c^5d^5e f^3gx^2 - 640a^4cd e^5g^4x + 704a^3c^2d^3e^3g^4x + 2816a^3c^2d^2e^4f g^3x - 3168a^2c^3d^4e^2f g^3x - 4752a^2c^3d^3e^3f^2g^2x + 5544a c^4d^5e f^2g^2x + 3696a c^4d^4e^2f^3gx - 4620c^5d^6f^3gx - 1155c^5d^5e f^4x + 1280a^5e^6g^4 - 1408a^4c d^2e^4g^4 - 5632a^4cd e^5f g^3 + 6336a^3c^2d^3e^3f g^3 + 9504a^3c^2d^2e^4f^2g^2 - 11088a^2c^3d^4e^2f^2g^2 - 7392a^2c^3d^3e^3f^3g + 9240a c^4d^5e f^3g + 2310a c^4d^4e^2f^4 - 3465d^6f^4c^5\right)\sqrt{ex + d}}{3465\sqrt{cdex^2 + ae^2x + cd^2x + ade}\;c^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^4/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/3465*(c*d*x+a*e)*(-315*c^5*d^5*e*g^4*x^5+350*a*c^4*d^4*e^2*g^4*x^4-385*c
^5*d^6*g^4*x^4-1540*c^5*d^5*e*f*g^3*x^4-400*a^2*c^3*d^3*e^3*g^4*x^3+440*a*c
^4*d^5*e*g^4*x^3+1760*a*c^4*d^4*e^2*f*g^3*x^3-1980*c^5*d^6*f*g^3*x^3-2970*c
^5*d^5*e*f^2*g^2*x^3+480*a^3*c^2*d^2*e^4*g^4*x^2-528*a^2*c^3*d^4*e^2*g^4*x^
2-2112*a^2*c^3*d^3*e^3*f*g^3*x^2+2376*a*c^4*d^5*e*f*g^3*x^2+3564*a*c^4*d^4*
e^2*f^2*g^2*x^2-4158*c^5*d^6*f^2*g^2*x^2-2772*c^5*d^5*e*f^3*g*x^2-640*a^4*c
*d*e^5*g^4*x+704*a^3*c^2*d^3*e^3*g^4*x+2816*a^3*c^2*d^2*e^4*f*g^3*x-3168*a^
2*c^3*d^4*e^2*f*g^3*x-4752*a^2*c^3*d^3*e^3*f^2*g^2*x+5544*a*c^4*d^5*e*f^2*g
^2*x+3696*a*c^4*d^4*e^2*f^3*g*x-4620*c^5*d^6*f^3*g*x-1155*c^5*d^5*e*f^4*x+1
280*a^5*e^6*g^4-1408*a^4*c*d^2*e^4*g^4-5632*a^4*c*d*e^5*f*g^3+6336*a^3*c^2*
d^3*e^3*f*g^3+9504*a^3*c^2*d^2*e^4*f^2*g^2-11088*a^2*c^3*d^4*e^2*f^2*g^2-73
92*a^2*c^3*d^3*e^3*f^3*g+9240*a*c^4*d^5*e*f^3*g+2310*a*c^4*d^4*e^2*f^4-3465
*c^5*d^6*f^4)*(e*x+d)^(1/2)/c^6/d^6/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A]   time = 0.74, size = 693, normalized size = 1.38

$$\frac{2\left(c^2d^2ex^2 + 3\,acd^2e - 2\,a^2e^3 + \left(3\,c^2d^3 - acde^2\right)x\right)f^4}{3\,\sqrt{cdx + ae}\,c^2d^2} + \frac{8\left(3\,c^3d^3ex^3 - 10\,a^2cd^2e^2 + 8\,a^3e^4 + \left(5\,c^3d^4 - ac^2d^2e^2\right)x^2 - \left(5\,ac^2d^3e - 4\,a^2cde^3\right)x\right)f^3g}{15\,\sqrt{cdx + ae}\,c^3d^3} + \frac{4\left(15\,c^4d^4ex^4 + 56\,a^3cd^2e^3 - 48\,a^4e^5 + 3\left(7\,c^4d^5 - ac^3d^3e^2\right)x^3 - \left(7\,ac^3d^4e - 6\,a^2c^2d^2e^3\right)x^2 + 4\left(7\,a^2c^2d^3e^2 - 6\,a^3cde^4\right)x\right)f^2g^2}{35\,\sqrt{cdx + ae}\,c^4d^4} + \frac{8\left(35\,c^5d^5ex^5 - 144\,a^4cd^2e^4 + 128\,a^5e^6 + 5\left(9\,c^5d^6 - ac^4d^4e^2\right)x^4 - \left(9\,ac^4d^5e - 8\,a^2c^3d^3e^3\right)x^3 + 2\left(9\,a^2c^3d^4e^2 - 8\,a^3c^2d^2e^4\right)x^2 - 8\left(9\,a^3c^2d^3e^3 - 8\,a^4cde^5\right)x\right)fg^3}{315\,\sqrt{cdx + ae}\,c^5d^5} + \frac{2\left(315\,c^6d^6ex^6 + 1408\,a^5cd^2e^5 - 1280\,a^6e^7 + 35\left(11\,c^6d^7 - ac^5d^5e^2\right)x^5 - 5\left(11\,ac^5d^6e - 10\,a^2c^4d^4e^3\right)x^4 + 8\left(11\,a^2c^4d^5e^2 - 10\,a^3c^3d^3e^4\right)x^3 - 16\left(11\,a^3c^3d^4e^3 - 10\,a^4c^2d^2e^5\right)x^2 + 64\left(11\,a^4c^2d^3e^4 - 10\,a^5cde^6\right)x\right)g^4}{3465\,\sqrt{cdx + ae}\,c^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^4/(sqrt(c*d*x + a*e)*c^2*d^2) + 8/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e
^3)*x)*f^3*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 4/35*(15*c^4*d^4*e*x^4 + 56*a^3*
c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
 - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f^2*g^
2/(sqrt(c*d*x + a*e)*c^4*d^4) + 8/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2*e^4
 + 128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2
*c^3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^
3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*f*g^3/(sqrt(c*d*x + a*e)*c^5*d^5) + 2/346
5*(315*c^6*d^6*e*x^6 + 1408*a^5*c*d^2*e^5 - 1280*a^6*e^7 + 35*(11*c^6*d^7 -
 a*c^5*d^5*e^2)*x^5 - 5*(11*a*c^5*d^6*e - 10*a^2*c^4*d^4*e^3)*x^4 + 8*(11*a
^2*c^4*d^5*e^2 - 10*a^3*c^3*d^3*e^4)*x^3 - 16*(11*a^3*c^3*d^4*e^3 - 10*a^4*
c^2*d^2*e^5)*x^2 + 64*(11*a^4*c^2*d^3*e^4 - 10*a^5*c*d*e^6)*x)*g^4/(sqrt(c*
d*x + a*e)*c^6*d^6)
```

mupad [B] time = 4.09, size = 653, normalized size = 1.30

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^4\,x^5\,\sqrt{d+e\,x}}{11\,c\,d}-\frac{\sqrt{d+e\,x}\,\left(2560\,a^5\,e^6\,g^4-2816\,a^4\,c\,d^2\,e^4\,g^4-11264\,a^4\,c\,d\,e^5\,f\,g^3+12672\,a^3\,c^2\,d^3\,e^3\,f\,g^3+19008\,a^3\,c^2\,d^2\,e^4\,f^2\,g^2-22176\,a^2\,c^3\,d^4\,e^2\,f^2\,g^2-14784\,a^2\,c^3\,d^3\,e^3\,f^3\,g+18480\,a\,c^4\,d^5\,e\,f^3\,g+4620\,a\,c^4\,d^4\,e^2\,f^4-6930\,c^5\,d^6\,f^4\right)}{3465\,c^6\,d^6\,e}+\frac{x\,\sqrt{d+e\,x}\,\left(1280\,a^4\,c\,d\,e^5\,g^4-1408\,a^3\,c^2\,d^3\,e^3\,g^4-5632\,a^3\,c^2\,d^2\,e^4\,f\,g^3+6336\,a^2\,c^3\,d^4\,e^2\,f\,g^3+9504\,a^2\,c^3\,d^3\,e^3\,f^2\,g^2-11088\,a\,c^4\,d^5\,e\,f^2\,g^2-7392\,a\,c^4\,d^4\,e^2\,f^3\,g+9240\,c^5\,d^6\,f^3\,g+2310\,c^5\,d^5\,e\,f^4\right)}{3465\,c^6\,d^6\,e}+\frac{x^2\,\sqrt{d+e\,x}\,\left(-960\,a^3\,c^2\,d^2\,e^4\,g^4+1056\,a^2\,c^3\,d^4\,e^2\,g^4+4224\,a^2\,c^3\,d^3\,e^3\,f\,g^3-4752\,a\,c^4\,d^5\,e\,f\,g^3-7128\,a\,c^4\,d^4\,e^2\,f^2\,g^2+8316\,c^5\,d^6\,f^2\,g^2+5544\,c^5\,d^5\,e\,f^3\,g\right)}{3465\,c^6\,d^6\,e}+\frac{4\,g^2\,x^3\,\sqrt{d+e\,x}\,\left(40\,a^2\,e^3\,g^2-44\,a\,c\,d^2\,e\,g^2-176\,a\,c\,d\,e^2\,f\,g+198\,c^2\,d^3\,f\,g+297\,c^2\,d^2\,e\,f^2\right)}{693\,c^3\,d^3\,e}+\frac{2\,g^3\,x^4\,\sqrt{d+e\,x}\,\left(11\,c\,g\,d^2+44\,c\,f\,d\,e-10\,a\,g\,e^2\right)}{99\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5*(d + e*x)^(1/2))
/(11*c*d) - ((d + e*x)^(1/2)*(2560*a^5*e^6*g^4 - 6930*c^5*d^6*f^4 + 4620*a*
c^4*d^4*e^2*f^4 - 2816*a^4*c*d^2*e^4*g^4 - 14784*a^2*c^3*d^3*e^3*f^3*g + 12
672*a^3*c^2*d^3*e^3*f*g^3 + 18480*a*c^4*d^5*e*f^3*g - 11264*a^4*c*d*e^5*f*g
^3 - 22176*a^2*c^3*d^4*e^2*f^2*g^2 + 19008*a^3*c^2*d^2*e^4*f^2*g^2))/(3465*
c^6*d^6*e) + (x*(d + e*x)^(1/2)*(2310*c^5*d^5*e*f^4 + 9240*c^5*d^6*f^3*g -
1408*a^3*c^2*d^3*e^3*g^4 + 1280*a^4*c*d*e^5*g^4 - 7392*a*c^4*d^4*e^2*f^3*g
- 11088*a*c^4*d^5*e*f^2*g^2 + 6336*a^2*c^3*d^4*e^2*f*g^3 - 5632*a^3*c^2*d^2
*e^4*f*g^3 + 9504*a^2*c^3*d^3*e^3*f^2*g^2))/(3465*c^6*d^6*e) + (x^2*(d + e*
x)^(1/2)*(8316*c^5*d^6*f^2*g^2 + 1056*a^2*c^3*d^4*e^2*g^4 - 960*a^3*c^2*d^2
*e^4*g^4 + 5544*c^5*d^5*e*f^3*g - 7128*a*c^4*d^4*e^2*f^2*g^2 + 4224*a^2*c^3
*d^3*e^3*f*g^3 - 4752*a*c^4*d^5*e*f*g^3))/(3465*c^6*d^6*e) + (4*g^2*x^3*(d
+ e*x)^(1/2)*(40*a^2*e^3*g^2 + 297*c^2*d^2*e*f^2 + 198*c^2*d^3*f*g - 44*a*c
*d^2*e*g^2 - 176*a*c*d*e^2*f*g))/(693*c^3*d^3*e) + (2*g^3*x^4*(d + e*x)^(1/
2)*(11*c*d^2*g - 10*a*e^2*g + 44*c*d*e*f))/(99*c^2*d^2*e)))/(x + d/e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.535 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=412

$$\frac{16\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{315c^5d^5eg\sqrt{d+ex}}-\frac{16\sqrt{d+ex}\,\sqrt{x\left(\right.}}{}$$

Rubi [A] time = 0.63, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {880, 870, 794, 648}

$$-\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(8ae^2g+cd(ef-9dg)\right)}{63c^2d^2g\sqrt{d+ex}}-\frac{4(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(8ae^2g+cd(ef-9dg)\right)}{105c^3d^3g\sqrt{d+ex}}-\frac{16\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)}{315c^4d^4e}+\frac{16\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{315c^5d^5eg\sqrt{d+ex}}+\frac{2e(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(2*a*e^2*g - c*d*(3*e
*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^5*d^5*e*g*Sq
rt[d + e*x]) - (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*Sqrt[d
 + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^4*d^4*e) - (4*(
c*d*f - a*e*g)*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*g*Sqrt[d + e*x]) - (2*(8*a*e^2*g
+ c*d*(e*f - 9*d*g))*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(63*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(9*c*d*g*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
```

```
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{9cdg\sqrt{d+ex}} - \frac{1}{9}\left(-9d+\frac{8ae^2}{cd}+\frac{ef}{g}\right)\int \frac{}{\sqrt{a}}$$

$$= -\frac{2\left(8ae^2g+cd(ef-9dg)\right)(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{63c^2d^2g\sqrt{d+ex}} + \frac{2e(f}{}$$

$$= -\frac{4(cdf-aeg)\left(8ae^2g+cd(ef-9dg)\right)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+ca}}{105c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{16(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{315c^4d^4e}$$

$$= \frac{16(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+(ca}}{315c^5d^5eg\sqrt{d+ex}}$$

Mathematica [A] time = 0.27, size = 264, normalized size = 0.64

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(128a^4e^5g^3-16a^3cde^3g^2(9dg+27ef+4egx)+24a^2c^2d^2e^2g\left(3dg(7f+gx)+e\left(21f^2+9fgx+2g^2x^2\right)\right)-2ac^3d^3e\left(9dg\left(35f^2+14fgx+3g^2x^2\right)+e\left(105f^3+126f^2gx+81fg^2x^2+20g^3x^3\right)\right)+c^4d^4\left(9d\left(35f^3+35f^2gx+21fg^2x^2+5g^3x^3\right)+ex\left(105f^3+189f^2gx+135fg^2x^2+35g^3x^3\right)\right)\right)}{315c^5d^5\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^5*g^3 - 16*a^3*c*d*e^3*g^2*(27*
e*f + 9*d*g + 4*e*g*x) + 24*a^2*c^2*d^2*e^2*g*(3*d*g*(7*f + g*x) + e*(21*f^
2 + 9*f*g*x + 2*g^2*x^2)) - 2*a*c^3*d^3*e*(9*d*g*(35*f^2 + 14*f*g*x + 3*g^2
*x^2) + e*(105*f^3 + 126*f^2*g*x + 81*f*g^2*x^2 + 20*g^3*x^3)) + c^4*d^4*(9
*d*(35*f^3 + 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3) + e*x*(105*f^3 + 189*f^
2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3))))/(315*c^5*d^5*Sqrt[d + e*x])
```

IntegrateAlgebraic [A] time = 1.40, size = 676, normalized size = 1.64

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*(210*
c^4*d^5*e^3*f^3 - 210*a*c^3*d^3*e^5*f^3 - 126*c^4*d^6*e^2*f^2*g - 378*a*c^3
*d^4*e^4*f^2*g + 504*a^2*c^2*d^2*e^6*f^2*g + 54*c^4*d^7*e*f*g^2 + 90*a*c^3*
d^5*e^3*f*g^2 + 288*a^2*c^2*d^3*e^5*f*g^2 - 432*a^3*c*d*e^7*f*g^2 - 10*c^4*
```

```
d^8*g^3 - 14*a*c^3*d^6*e^2*g^3 - 24*a^2*c^2*d^4*e^4*g^3 - 80*a^3*c*d^2*e^6*
g^3 + 128*a^4*e^8*g^3 + 105*c^4*d^4*e^3*f^3*(d + e*x) - 63*c^4*d^5*e^2*f^2*
g*(d + e*x) - 252*a*c^3*d^3*e^4*f^2*g*(d + e*x) + 27*c^4*d^6*e*f*g^2*(d + e
*x) + 72*a*c^3*d^4*e^3*f*g^2*(d + e*x) + 216*a^2*c^2*d^2*e^5*f*g^2*(d + e*x
) - 5*c^4*d^7*g^3*(d + e*x) - 12*a*c^3*d^5*e^2*g^3*(d + e*x) - 24*a^2*c^2*d
^3*e^4*g^3*(d + e*x) - 64*a^3*c*d*e^6*g^3*(d + e*x) + 189*c^4*d^4*e^2*f^2*g
*(d + e*x)^2 - 216*c^4*d^5*e*f*g^2*(d + e*x)^2 - 162*a*c^3*d^3*e^3*f*g^2*(d
 + e*x)^2 + 75*c^4*d^6*g^3*(d + e*x)^2 + 66*a*c^3*d^4*e^2*g^3*(d + e*x)^2 +
 48*a^2*c^2*d^2*e^4*g^3*(d + e*x)^2 + 135*c^4*d^4*e*f*g^2*(d + e*x)^3 - 95*
c^4*d^5*g^3*(d + e*x)^3 - 40*a*c^3*d^3*e^2*g^3*(d + e*x)^3 + 35*c^4*d^4*g^3
*(d + e*x)^4))/(315*c^5*d^5*e^3*Sqrt[d + e*x])
```

fricas [A] time = 0.42, size = 408, normalized size = 0.99

$$\frac{2\left(35\,c^{4}d^{4}eg^{3}x^{4}+105\left(3\,c^{4}d^{5}-2\,ac^{3}d^{3}e^{2}\right)f^{3}-126\left(5\,ac^{3}d^{4}e-4\,a^{2}c^{2}d^{2}e^{3}\right)f^{2}g+72\left(7\,a^{2}c^{2}d^{3}e^{2}-6\,a^{3}cde^{4}\right)fg^{2}-16\left(9\,a^{3}cd^{2}e^{3}-8\,a^{4}e^{5}\right)g^{3}+5\left(27\,c^{4}d^{4}efg^{2}+\left(9\,c^{4}d^{5}-8\,ac^{3}d^{3}e^{2}\right)g^{3}\right)x^{3}+3\left(63\,c^{4}d^{4}ef^{2}g+9\left(7\,c^{4}d^{5}-6\,ac^{3}d^{3}e^{2}\right)fg^{2}-2\left(9\,ac^{3}d^{4}e-8\,a^{2}c^{2}d^{2}e^{3}\right)g^{3}\right)x^{2}+\left(105\,c^{4}d^{4}ef^{3}+63\left(5\,c^{4}d^{5}-4\,ac^{3}d^{3}e^{2}\right)f^{2}g-36\left(7\,ac^{3}d^{4}e-6\,a^{2}c^{2}d^{2}e^{3}\right)fg^{2}+8\left(9\,a^{2}c^{2}d^{3}e^{2}-8\,a^{3}cde^{4}\right)g^{3}\right)x\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\,\sqrt{ex+d}}{315\left(c^{5}d^{5}ex+c^{5}d^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*e*g^3*x^4 + 105*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^3 - 126*(
5*a*c^3*d^4*e - 4*a^2*c^2*d^2*e^3)*f^2*g + 72*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*
d*e^4)*f*g^2 - 16*(9*a^3*c*d^2*e^3 - 8*a^4*e^5)*g^3 + 5*(27*c^4*d^4*e*f*g^2
 + (9*c^4*d^5 - 8*a*c^3*d^3*e^2)*g^3)*x^3 + 3*(63*c^4*d^4*e*f^2*g + 9*(7*c^
4*d^5 - 6*a*c^3*d^3*e^2)*f*g^2 - 2*(9*a*c^3*d^4*e - 8*a^2*c^2*d^2*e^3)*g^3)
*x^2 + (105*c^4*d^4*e*f^3 + 63*(5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^2*g - 36*(7*
a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f*g^2 + 8*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e
^4)*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*
d^5*e*x + c^5*d^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{3}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^3/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

maple [A] time = 0.01, size = 425, normalized size = 1.03

$$\frac{2\left(cdx+ae\right)\left(35eg^{3}x^{4}c^{4}d^{4}-40a\,c^{3}d^{3}e^{2}g^{3}x^{3}+45c^{4}d^{5}g^{3}x^{3}+135c^{4}d^{4}ef\,g^{2}x^{3}+48a^{2}c^{2}d^{2}e^{3}g^{3}x^{2}-54a\,c^{3}d^{4}e\,g^{3}x^{2}-162a\,c^{3}d^{3}e^{2}f\,g^{2}x^{2}+189c^{4}d^{5}f\,g^{2}x^{2}+189c^{4}d^{4}e\,f^{2}g\,x^{2}-64a^{3}cd\,e^{4}g^{3}x+72a^{2}c^{2}d^{3}e^{2}g^{3}x+216a^{2}c^{2}d^{2}e^{3}f\,g^{2}x-252a\,c^{3}d^{4}ef\,g^{2}x-252a\,c^{3}d^{3}e^{2}f^{2}gx+315c^{4}d^{5}f^{2}gx+105c^{4}d^{4}e\,f^{3}x+128a^{4}e^{5}g^{3}-144a^{3}c\,d^{2}e^{3}g^{3}-432a^{3}cd\,e^{4}f\,g^{2}+504a^{2}c^{2}d^{3}e^{2}f\,g^{2}+504a^{2}c^{2}d^{2}e^{3}f^{2}g-630a\,c^{3}d^{4}e\,f^{2}g-210a\,c^{3}d^{3}e^{2}f^{3}+315d^{5}f^{3}c^{4}\right)\sqrt{ex+d}}{315\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\;c^{5}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 2/315*(c*d*x+a*e)*(35*c^4*d^4*e*g^3*x^4-40*a*c^3*d^3*e^2*g^3*x^3+45*c^4*d^5
*g^3*x^3+135*c^4*d^4*e*f*g^2*x^3+48*a^2*c^2*d^2*e^3*g^3*x^2-54*a*c^3*d^4*e*
g^3*x^2-162*a*c^3*d^3*e^2*f*g^2*x^2+189*c^4*d^5*f*g^2*x^2+189*c^4*d^4*e*f^2
*g*x^2-64*a^3*c*d*e^4*g^3*x+72*a^2*c^2*d^3*e^2*g^3*x+216*a^2*c^2*d^2*e^3*f*
g^2*x-252*a*c^3*d^4*e*f*g^2*x-252*a*c^3*d^3*e^2*f^2*g*x+315*c^4*d^5*f^2*g*x
+105*c^4*d^4*e*f^3*x+128*a^4*e^5*g^3-144*a^3*c*d^2*e^3*g^3-432*a^3*c*d*e^4*
f*g^2+504*a^2*c^2*d^3*e^2*f*g^2+504*a^2*c^2*d^2*e^3*f^2*g-630*a*c^3*d^4*e*f
^2*g-210*a*c^3*d^3*e^2*f^3+315*c^4*d^5*f^3)*(e*x+d)^(1/2)/c^5/d^5/(c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.69, size = 484, normalized size = 1.17

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)f^3}{3\,\sqrt{cdx+ae}\,c^2d^2}+\frac{2\left(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+\left(5\,c^3d^4-ac^2d^2e^2\right)x^2-\left(5\,ac^2d^3e-4\,a^2cde^3\right)x\right)f^2g}{5\,\sqrt{cdx+ae}\,c^3d^3}+\frac{2\left(15\,c^4d^4ex^4+56\,a^3cd^2e^3-48\,a^4e^5+3\left(7\,c^4d^5-ac^3d^3e^2\right)x^3-\left(7\,ac^3d^4e-6\,a^2c^2d^2e^3\right)x^2+4\left(7\,a^2c^2d^3e^2-6\,a^3cde^4\right)x\right)fg^2}{35\,\sqrt{cdx+ae}\,c^4d^4}+\frac{2\left(35\,c^5d^5ex^5-144\,a^4cd^2e^4+128\,a^5e^6+5\left(9\,c^5d^6-ac^4d^4e^2\right)x^4-\left(9\,ac^4d^5e-8\,a^2c^3d^3e^3\right)x^3+2\left(9\,a^2c^3d^4e^2-8\,a^3c^2d^2e^4\right)x^2-8\left(9\,a^3c^2d^3e^3-8\,a^4cde^5\right)x\right)g^3}{315\,\sqrt{cdx+ae}\,c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^3/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^
3)*x)*f^2*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(15*c^4*d^4*e*x^4 + 56*a^3*c
*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
- 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f*g^2/(
sqrt(c*d*x + a*e)*c^4*d^4) + 2/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2*e^4 +
128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2*c^
3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^3*c
^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*g^3/(sqrt(c*d*x + a*e)*c^5*d^5)
```

mupad [B] time = 3.86, size = 438, normalized size = 1.06

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(256\,a^4\,e^5\,g^3-288\,a^3\,c\,d^2\,e^3\,g^3-864\,a^3\,c\,d\,e^4\,f\,g^2+1008\,a^2\,c^2\,d^3\,e^2\,f\,g^2+1008\,a^2\,c^2\,d^2\,e^3\,f^2\,g-1260\,a\,c^3\,d^4\,e\,f^2\,g-420\,a\,c^3\,d^3\,e^2\,f^3+630\,c^4\,d^5\,f^3\right)}{315\,c^5\,d^5\,e}+\frac{2\,g^3\,x^4\,\sqrt{d+e\,x}}{9\,c\,d}+\frac{x\,\sqrt{d+e\,x}\,\left(-128\,a^3\,c\,d\,e^4\,g^3+144\,a^2\,c^2\,d^3\,e^2\,g^3+432\,a^2\,c^2\,d^2\,e^3\,f\,g^2-504\,a\,c^3\,d^4\,e\,f\,g^2-504\,a\,c^3\,d^3\,e^2\,f^2\,g+630\,c^4\,d^5\,f^2\,g+210\,c^4\,d^4\,e\,f^3\right)}{315\,c^5\,d^5\,e}+\frac{2\,g\,x^2\,\sqrt{d+e\,x}\,\left(16\,a^2\,e^3\,g^2-18\,a\,c\,d^2\,e\,g^2-54\,a\,c\,d\,e^2\,f\,g+63\,c^2\,d^3\,f\,g+63\,c^2\,d^2\,e\,f^2\right)}{105\,c^3\,d^3\,e}+\frac{2\,g^2\,x^3\,\sqrt{d+e\,x}\,\left(9\,c\,g\,d^2+27\,c\,f\,d\,e-8\,a\,g\,e^2\right)}{63\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(256*a^4*e
^5*g^3 + 630*c^4*d^5*f^3 - 420*a*c^3*d^3*e^2*f^3 - 288*a^3*c*d^2*e^3*g^3 +
1008*a^2*c^2*d^2*e^3*f^2*g + 1008*a^2*c^2*d^3*e^2*f*g^2 - 1260*a*c^3*d^4*e*
f^2*g - 864*a^3*c*d*e^4*f*g^2))/(315*c^5*d^5*e) + (2*g^3*x^4*(d + e*x)^(1/2
))/(9*c*d) + (x*(d + e*x)^(1/2)*(210*c^4*d^4*e*f^3 + 630*c^4*d^5*f^2*g + 14
4*a^2*c^2*d^3*e^2*g^3 - 128*a^3*c*d*e^4*g^3 - 504*a*c^3*d^3*e^2*f^2*g + 432
*a^2*c^2*d^2*e^3*f*g^2 - 504*a*c^3*d^4*e*f*g^2))/(315*c^5*d^5*e) + (2*g*x^2
*(d + e*x)^(1/2)*(16*a^2*e^3*g^2 + 63*c^2*d^2*e*f^2 + 63*c^2*d^3*f*g - 18*a
*c*d^2*e*g^2 - 54*a*c*d*e^2*f*g))/(105*c^3*d^3*e) + (2*g^2*x^3*(d + e*x)^(1
/2)*(9*c*d^2*g - 8*a*e^2*g + 27*c*d*e*f))/(63*c^2*d^2*e)))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\left(f+gx\right)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.536 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=321

$$\frac{8\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{105c^4d^4eg\sqrt{d+ex}}-\frac{8\sqrt{d+ex}\,\sqrt{x\left(ae^2\right.}}{}$$

Rubi [A] time = 0.42, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {880, 870, 794, 648}

$$-\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(6ae^2g+cd(ef-7dg)\right)}{35c^2d^2g\sqrt{d+ex}}-\frac{8\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)}{105c^3d^3e}+\frac{8\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{105c^4d^4eg\sqrt{d+ex}}+\frac{2e(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cdg\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*(2*a*e^2*g - c*d*(3*e*f
- d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^4*d^4*e*g*Sqrt[
d + e*x]) - (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*Sqrt[d + e*x
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*e) - (2*(6*a*e^
2*g + c*d*(e*f - 7*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(35*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^3*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(7*c*d*g*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
```

```
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p - 1, 0] && !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7cdg\sqrt{d+ex}} - \frac{1}{7}\left(-7d+\frac{6ae^2}{cd}+\frac{ef}{g}\right)\int \frac{}{\sqrt{a}}$$

$$= -\frac{2\left(6ae^2g+cd(ef-7dg)\right)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^2d^2g\sqrt{d+ex}} + \frac{2e(f}{}$$

$$= -\frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\sqrt{d+ex}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{105c^3d^3e}$$

$$= \frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2\right.}}{105c^4d^4eg\sqrt{d+ex}}$$

Mathematica [A] time = 0.18, size = 169, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(-48a^3e^4g^2+8a^2cde^2g(7dg+14ef+3egx)-2ac^2d^2e\left(14dg(5f+gx)+e\left(35f^2+28fgx+9g^2x^2\right)\right)+c^3d^3\left(7d\left(15f^2+10fgx+3g^2x^2\right)+ex\left(35f^2+42fgx+15g^2x^2\right)\right)\right)}{105c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-48*a^3*e^4*g^2 + 8*a^2*c*d*e^2*g*(14*e*f
 + 7*d*g + 3*e*g*x) - 2*a*c^2*d^2*e*(14*d*g*(5*f + g*x) + e*(35*f^2 + 28*f*
g*x + 9*g^2*x^2)) + c^3*d^3*(7*d*(15*f^2 + 10*f*g*x + 3*g^2*x^2) + e*x*(35*
f^2 + 42*f*g*x + 15*g^2*x^2))))/(105*c^4*d^4*Sqrt[d + e*x])
```

IntegrateAlgebraic [A] time = 0.80, size = 365, normalized size = 1.14

$$\frac{2\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}\left(-48a^3e^6g^2+32a^2cd^2e^4g^2+112a^2cde^5fg+24a^2cde^4g^2(d+ex)+10ac^2d^4e^2g^2-84ac^2d^3e^3fg+8ac^2d^3e^2g^2(d+ex)-70ac^2d^2e^4f^2-56ac^2d^2e^3fg(d+ex)-18ac^2d^2e^2g^2(d+ex)^2+6c^3d^6g^2-28c^3d^5efg+3c^3d^5g^2(d+ex)+70c^3d^4e^2f^2-14c^3d^4efg(d+ex)-24c^3d^4g^2(d+ex)^2+35c^3d^3e^2f^2(d+ex)+42c^3d^3efg(d+ex)^2+15c^3d^3g^2(d+ex)^3\right)}{105c^4d^4e^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*(70*c
^3*d^4*e^2*f^2 - 70*a*c^2*d^2*e^4*f^2 - 28*c^3*d^5*e*f*g - 84*a*c^2*d^3*e^3
*f*g + 112*a^2*c*d*e^5*f*g + 6*c^3*d^6*g^2 + 10*a*c^2*d^4*e^2*g^2 + 32*a^2*
c*d^2*e^4*g^2 - 48*a^3*e^6*g^2 + 35*c^3*d^3*e^2*f^2*(d + e*x) - 14*c^3*d^4*
e*f*g*(d + e*x) - 56*a*c^2*d^2*e^3*f*g*(d + e*x) + 3*c^3*d^5*g^2*(d + e*x)
+ 8*a*c^2*d^3*e^2*g^2*(d + e*x) + 24*a^2*c*d*e^4*g^2*(d + e*x) + 42*c^3*d^3
*e*f*g*(d + e*x)^2 - 24*c^3*d^4*g^2*(d + e*x)^2 - 18*a*c^2*d^2*e^2*g^2*(d +
 e*x)^2 + 15*c^3*d^3*g^2*(d + e*x)^3))/(105*c^4*d^4*e^2*Sqrt[d + e*x])
```

fricas [A] time = 0.43, size = 256, normalized size = 0.80

$$\frac{2\left(15\,c^3d^3eg^2x^3+35\left(3\,c^3d^4-2\,ac^2d^2e^2\right)f^2-28\left(5\,ac^2d^3e-4\,a^2cde^3\right)fg+8\left(7\,a^2cd^2e^2-6\,a^3e^4\right)g^2+3\left(14\,c^3d^3efg+\left(7\,c^3d^4-6\,ac^2d^2e^2\right)g^2\right)x^2+\left(35\,c^3d^3ef^2+14\left(5\,c^3d^4-4\,ac^2d^2e^2\right)fg-4\left(7\,ac^2d^3e-6\,a^2cde^3\right)g^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{105\left(c^4d^4ex+c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/105*(15*c^3*d^3*e*g^2*x^3 + 35*(3*c^3*d^4 - 2*a*c^2*d^2*e^2)*f^2 - 28*(5*
a*c^2*d^3*e - 4*a^2*c*d*e^3)*f*g + 8*(7*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^2 + 3*
(14*c^3*d^3*e*f*g + (7*c^3*d^4 - 6*a*c^2*d^2*e^2)*g^2)*x^2 + (35*c^3*d^3*e*
f^2 + 14*(5*c^3*d^4 - 4*a*c^2*d^2*e^2)*f*g - 4*(7*a*c^2*d^3*e - 6*a^2*c*d*e
^3)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*
d^4*e*x + c^4*d^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{2}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^2/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

maple [A] time = 0.01, size = 255, normalized size = 0.79

$$-\frac{2\left(cdx+ae\right)\left(-15e\,g^{2}x^{3}c^{3}d^{3}+18a\,c^{2}d^{2}e^{2}g^{2}x^{2}-21c^{3}d^{4}g^{2}x^{2}-42c^{3}d^{3}efg\,x^{2}-24a^{2}cd\,e^{3}g^{2}x+28a\,c^{2}d^{3}e\,g^{2}x+56a\,c^{2}d^{2}e^{2}fgx-70c^{3}d^{4}fgx-35c^{3}d^{3}e\,f^{2}x+48a^{3}e^{4}g^{2}-56a^{2}c\,d^{2}e^{2}g^{2}-112a^{2}cd\,e^{3}fg+140a\,c^{2}d^{3}efg+70a\,c^{2}d^{2}e^{2}f^{2}-105d^{4}f^{2}c^{3}\right)\sqrt{ex+d}}{105\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\;c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/105*(c*d*x+a*e)*(-15*c^3*d^3*e*g^2*x^3+18*a*c^2*d^2*e^2*g^2*x^2-21*c^3*d
^4*g^2*x^2-42*c^3*d^3*e*f*g*x^2-24*a^2*c*d*e^3*g^2*x+28*a*c^2*d^3*e*g^2*x+5
6*a*c^2*d^2*e^2*f*g*x-70*c^3*d^4*f*g*x-35*c^3*d^3*e*f^2*x+48*a^3*e^4*g^2-56
*a^2*c*d^2*e^2*g^2-112*a^2*c*d*e^3*f*g+140*a*c^2*d^3*e*f*g+70*a*c^2*d^2*e^2
*f^2-105*c^3*d^4*f^2)*(e*x+d)^(1/2)/c^4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(1/2)
```

maxima [A] time = 0.63, size = 309, normalized size = 0.96

$$\frac{2\left(c^{2}d^{2}ex^{2}+3\,acd^{2}e-2\,a^{2}e^{3}+\left(3\,c^{2}d^{3}-acde^{2}\right)x\right)f^{2}}{3\,\sqrt{cdx+ae}\,c^{2}d^{2}}+\frac{4\left(3\,c^{3}d^{3}ex^{3}-10\,a^{2}cd^{2}e^{2}+8\,a^{3}e^{4}+\left(5\,c^{3}d^{4}-ac^{2}d^{2}e^{2}\right)x^{2}-\left(5\,ac^{2}d^{3}e-4\,a^{2}cde^{3}\right)x\right)fg}{15\,\sqrt{cdx+ae}\,c^{3}d^{3}}+\frac{2\left(15\,c^{4}d^{4}ex^{4}+56\,a^{3}cd^{2}e^{3}-48\,a^{4}e^{5}+3\left(7\,c^{4}d^{5}-ac^{3}d^{3}e^{2}\right)x^{3}-\left(7\,ac^{3}d^{4}e-6\,a^{2}c^{2}d^{2}e^{3}\right)x^{2}+4\left(7\,a^{2}c^{2}d^{3}e^{2}-6\,a^{3}cde^{4}\right)x\right)g^{2}}{105\,\sqrt{cdx+ae}\,c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^2/(sqrt(c*d*x + a*e)*c^2*d^2) + 4/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e
^3)*x)*f*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/105*(15*c^4*d^4*e*x^4 + 56*a^3*c
*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
- 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*g^2/(sq
rt(c*d*x + a*e)*c^4*d^4)
```

mupad [B] time = 3.71, size = 279, normalized size = 0.87

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^3\,\sqrt{d+e\,x}}{7\,c\,d}-\frac{\sqrt{d+e\,x}\,\left(96\,a^3\,e^4\,g^2-112\,a^2\,c\,d^2\,e^2\,g^2-224\,a^2\,c\,d\,e^3\,f\,g+280\,a\,c^2\,d^3\,e\,f\,g+140\,a\,c^2\,d^2\,e^2\,f^2-210\,c^3\,d^4\,f^2\right)}{105\,c^4\,d^4\,e}+\frac{x\,\sqrt{d+e\,x}\,\left(48\,a^2\,c\,d\,e^3\,g^2-56\,a\,c^2\,d^3\,e\,g^2-112\,a\,c^2\,d^2\,e^2\,f\,g+140\,c^3\,d^4\,f\,g+70\,c^3\,d^3\,e\,f^2\right)}{105\,c^4\,d^4\,e}+\frac{2\,g\,x^2\,\sqrt{d+e\,x}\,\left(7\,c\,g\,d^2+14\,c\,f\,d\,e-6\,a\,g\,e^2\right)}{35\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3*(d + e*x)^(1/2))
/(7*c*d) - ((d + e*x)^(1/2)*(96*a^3*e^4*g^2 - 210*c^3*d^4*f^2 + 140*a*c^2*d
^2*e^2*f^2 - 112*a^2*c*d^2*e^2*g^2 + 280*a*c^2*d^3*e*f*g - 224*a^2*c*d*e^3*
f*g))/(105*c^4*d^4*e) + (x*(d + e*x)^(1/2)*(70*c^3*d^3*e*f^2 + 140*c^3*d^4*
f*g - 56*a*c^2*d^3*e*g^2 + 48*a^2*c*d*e^3*g^2 - 112*a*c^2*d^2*e^2*f*g))/(10
5*c^4*d^4*e) + (2*g*x^2*(d + e*x)^(1/2)*(7*c*d^2*g - 6*a*e^2*g + 14*c*d*e*f
))/(35*c^2*d^2*e)))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\left(f+gx\right)^{2}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.537 $\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=209

$$-\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(4ae^2g-cd(5ef-dg)\right)}{15c^3d^3e\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15c^2d^2e}$$

Rubi [A] time = 0.20, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {794, 656, 648}

$$-\frac{2\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(4ae^2g-cd(5ef-dg)\right)}{15c^2d^2e}-\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,\left(4ae^2g-cd(5ef-dg)\right)}{15c^3d^3e\sqrt{d+ex}}+\frac{2g(d+ex)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cde}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-4*(c*d^2 - a*e^2)*(4*a*e^2*g - c*d*(5*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) - (2*(4*a*e^2*g - c*d*(5
*e*f - d*g))*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15
*c^2*d^2*e) + (2*g*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(5*c*d*e)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2g(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5cde} + \frac{1}{5}\left(5f-\frac{dg}{e}-\frac{4aeg}{cd}\right)\int \frac{}{\sqrt{a}}$$

$$= -\frac{2\left(4ae^2g-cd(5ef-dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^2d^2e} + \frac{2g(d}{}$$

$$= -\frac{4\left(cd^2-ae^2\right)\left(4ae^2g-cd(5ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^3d^3e\sqrt{d+ex}} - \frac{2(}{}$$

Mathematica [A] time = 0.10, size = 96, normalized size = 0.46

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^3g-2acde(5dg+5ef+2egx)+c^2d^2(5d(3f+gx)+ex(5f+3gx))\right)}{15c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^3*g - 2*a*c*d*e*(5*e*f + 5*d*g +
2*e*g*x) + c^2*d^2*(5*d*(3*f + g*x) + e*x*(5*f + 3*g*x))))/(15*c^3*d^3*Sqrt
[d + e*x])
```

IntegrateAlgebraic [A] time = 0.41, size = 170, normalized size = 0.81

$$\frac{2\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}\left(8a^2e^4g-6acd^2e^2g-10acde^3f-4acde^2g(d+ex)-2c^2d^4g+10c^2d^3ef-c^2d^3g(d+ex)+5c^2d^2ef(d+ex)+3c^2d^2g(d+ex)^2\right)}{15c^3d^3e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*(10*c
^2*d^3*e*f - 10*a*c*d*e^3*f - 2*c^2*d^4*g - 6*a*c*d^2*e^2*g + 8*a^2*e^4*g +
 5*c^2*d^2*e*f*(d + e*x) - c^2*d^3*g*(d + e*x) - 4*a*c*d*e^2*g*(d + e*x) +
3*c^2*d^2*g*(d + e*x)^2))/(15*c^3*d^3*e*Sqrt[d + e*x])
```

fricas [A] time = 0.41, size = 141, normalized size = 0.67

$$\frac{2\left(3\,c^2d^2egx^2+5\left(3\,c^2d^3-2\,acde^2\right)f-2\left(5\,acd^2e-4\,a^2e^3\right)g+\left(5\,c^2d^2ef+\left(5\,c^2d^3-4\,acde^2\right)g\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,\sqrt{ex+d}}{15\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*e*g*x^2 + 5*(3*c^2*d^3 - 2*a*c*d*e^2)*f - 2*(5*a*c*d^2*e -
4*a^2*e^3)*g + (5*c^2*d^2*e*f + (5*c^2*d^3 - 4*a*c*d*e^2)*g)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

maple [A] time = 0.00, size = 131, normalized size = 0.63

$$\frac{2\left(cdx+ae\right)\left(3eg\,x^{2}c^{2}d^{2}-4acd\,e^{2}gx+5c^{2}d^{3}gx+5c^{2}d^{2}efx+8a^{2}e^{3}g-10ac\,d^{2}eg-10acd\,e^{2}f+15d^{3}f\,c^{2}\right)\sqrt{ex+d}}{15\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}\;c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 2/15*(c*d*x+a*e)*(3*c^2*d^2*e*g*x^2-4*a*c*d*e^2*g*x+5*c^2*d^3*g*x+5*c^2*d^2
*e*f*x+8*a^2*e^3*g-10*a*c*d^2*e*g-10*a*c*d*e^2*f+15*c^2*d^3*f)*(e*x+d)^(1/2
)/c^3/d^3/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.57, size = 168, normalized size = 0.80

$$\frac{2\left(c^{2}d^{2}ex^{2}+3\,acd^{2}e-2\,a^{2}e^{3}+\left(3\,c^{2}d^{3}-acde^{2}\right)x\right)f}{3\,\sqrt{cdx+ae}\,c^{2}d^{2}}+\frac{2\left(3\,c^{3}d^{3}ex^{3}-10\,a^{2}cd^{2}e^{2}+8\,a^{3}e^{4}+\left(5\,c^{3}d^{4}-ac^{2}d^{2}e^{2}\right)x^{2}-\left(5\,ac^{2}d^{3}e-4\,a^{2}cde^{3}\right)x\right)g}{15\,\sqrt{cdx+ae}\,c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 + 8
*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^3
)*x)*g/(sqrt(c*d*x + a*e)*c^3*d^3)
```

mupad [B] time = 3.48, size = 152, normalized size = 0.73

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,g\,a^2\,e^3-20\,g\,a\,c\,d^2\,e-20\,f\,a\,c\,d\,e^2+30\,f\,c^2\,d^3\right)}{15\,c^3\,d^3\,e}+\frac{2\,g\,x^2\,\sqrt{d+e\,x}}{5\,c\,d}+\frac{2\,x\,\sqrt{d+e\,x}\,\left(5\,c\,g\,d^2+5\,c\,f\,d\,e-4\,a\,g\,e^2\right)}{15\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e^
3*g + 30*c^2*d^3*f - 20*a*c*d*e^2*f - 20*a*c*d^2*e*g))/(15*c^3*d^3*e) + (2*
g*x^2*(d + e*x)^(1/2))/(5*c*d) + (2*x*(d + e*x)^(1/2)*(5*c*d^2*g - 4*a*e^2*
g + 5*c*d*e*f))/(15*c^2*d^2*e)))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^{\frac{3}{2}}\left(f+gx\right)}{\sqrt{\left(d+ex\right)\left(ae+cdx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.538 $\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=109

$$\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (4*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*
Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(3*c*d)
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx &= \frac{2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cd}+\frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x}}}{3d} \\ &= \frac{4\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae\right.}}{3cd}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.50

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(cd(3d+ex)-2ae^2\right)}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(3*d + e*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

IntegrateAlgebraic [A] time = 0.00, size = 80, normalized size = 0.73

$$\frac{2\left(-2ae^2+2cd^2+cd(d+ex)\right)\sqrt{ae(d+ex)-\frac{cd^2(d+ex)}{e}+\frac{cd(d+ex)^2}{e}}}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

```
[Out] (2*(2*c*d^2 - 2*a*e^2 + c*d*(d + e*x))*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d
 + e*x) + (c*d*(d + e*x)^2)/e])/(3*c^2*d^2*Sqrt[d + e*x])
```

fricas [A] time = 0.41, size = 73, normalized size = 0.67

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdex+3\,cd^2-2\,ae^2\right)\sqrt{ex+d}}{3\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x + 3*c*d^2 - 2*a*e^
2)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

maple [A] time = 0.00, size = 69, normalized size = 0.63

$$-\frac{2\,(cdx+ae)\left(-cdex+2a\,e^2-3c\,d^2\right)\sqrt{ex+d}}{3\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\;c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c*d*e*x+2*a*e^2-3*c*d^2)*(e*x+d)^(1/2)/c^2/d^2/(c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

maxima [A] time = 0.50, size = 65, normalized size = 0.60

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)}{3\sqrt{cdx+ae}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

mupad [B] time = 3.36, size = 85, normalized size = 0.78

$$\frac{\left(\frac{2\,x\,\sqrt{d+e\,x}}{3\,c\,d}-\frac{\left(4\,a\,e^2-6\,c\,d^2\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

```
[Out] (((2*x*(d + e*x)^(1/2))/(3*c*d) - ((4*a*e^2 - 6*c*d^2)*(d + e*x)^(1/2))/(3*
c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

3.539 $\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=139

$$\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

Rubi [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {880, 874, 205}

$$\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] (2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*Sqrt[d + e*x]) - (
2*(e*f - d*g)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}dx = \frac{2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{\left(2\left(\frac{1}{2}cde^2f-\frac{1}{2}cd^2eg\right)\right)\int \frac{}{(f+gx}}{cdeg}$$

$$= \frac{2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{\left(2e^2(ef-dg)\right)\text{Subst}\left(\int \frac{}{-e\left(cd^2\right.}\right.}{}$$

$$= \frac{2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2\right.}}{\sqrt{cdf-aeg}}\right.}{g^{3/2}\sqrt{cdf-aeg}}$$

Mathematica [A] time = 0.11, size = 140, normalized size = 1.01

$$\frac{2\sqrt{d+ex}\left(e\sqrt{g}(ae+cdx)\sqrt{cdf-aeg}+cd(dg-ef)\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{cdg^{3/2}\sqrt{(d+ex)(ae+cdx)}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (2*Sqrt[d + e*x]*(e*Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*(-(e*f)
 + d*g)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a
*e*g]]))/(c*d*g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [C] time = 20.82, size = 2866, normalized size = 20.62

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]),x]
```

```
[Out] (-(e*Sqrt[d + e*x]*(c*d^2 - 2*a*e^2 - 2*c*d*(d + e*x))) - 2*e*Sqrt[c*d*e]*S
qrt[d + e*x]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2
)/e])/(-(c*d*Sqrt[c*d*e]*g*(d + e*x)) + c*d*e*g*Sqrt[-((c*d^2*(d + e*x))/e)
 + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]) - ((2*I)*Sqrt[c]*Sqrt[d]*(-(e*f) +
 d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sq
rt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x]
)/(-(Sqrt[c*d*e]*Sqrt[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e)
 + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e])])/(g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqr
t[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f
) + d*g]*Sqrt[c*d*f - a*e*g]]) + ((2*I)*Sqrt[c]*Sqrt[d]*(-(e*f) + d*g)^(3/2
)*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sqrt
[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x])/(-(Sqrt[
c*d*e]*Sqrt[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d
+ e*x) + (c*d*(d + e*x)^2)/e])])/(g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[-2*c*d*e
*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*S
qrt[c*d*f - a*e*g]]) + (-((d*e*Sqrt[d + e*x])/g) - (2*e^(3/2)*Sqrt[c*d*e]*f
*(d + e*x)*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqr
t[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x])
/(-(Sqrt[c*d*e]*Sqrt[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e)
+ a*e*(d + e*x) + (c*d*(d + e*x)^2)/e])])/(g^(3/2)*Sqrt[-2*c*d*e*f + c*d^2*
```

```
g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f -
 a*e*g]]) + (2*d*Sqrt[e]*Sqrt[c*d*e]*(d + e*x)*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d
*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]
*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x]])/(-(Sqrt[c*d*e]*Sqrt[g]*(d + e*x)) + e*
Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e])
])/(Sqrt[g]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqr
t[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]) + (2*e^(5/2)*f*Sqrt[-((c*d^2*
(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*ArcTanh[(Sqrt[e]*Sqrt[
-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f)
+ d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x]])/(-(Sqrt[c*d*e]*Sqrt[g]*(d + e*x)
) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^
2)/e])])/(g^(3/2)*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]) - (2*d*e^(3/2)*Sqrt[-((
c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*ArcTanh[(Sqrt[e]
*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-
(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x]])/(-(Sqrt[c*d*e]*Sqrt[g]*(d
+ e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d +
 e*x)^2)/e])])/(Sqrt[g]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g - (2*I)*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]) - (2*e^(3/2)*Sqrt
[c*d*e]*f*(d + e*x)*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g +
(2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[
d + e*x]])/(-(Sqrt[c*d*e]*Sqrt[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d +
e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e])])/(g^(3/2)*Sqrt[-2*c*d*e*f
 + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqr
t[c*d*f - a*e*g]]) + (2*d*Sqrt[e]*Sqrt[c*d*e]*(d + e*x)*ArcTanh[(Sqrt[e]*Sq
rt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*
f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x]])/(-(Sqrt[c*d*e]*Sqrt[g]*(d + e
*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*
x)^2)/e])])/(Sqrt[g]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]) + (2*e^(5/2)*f*Sqrt[
-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*ArcTanh[(Sqrt
[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x]])/(-(Sqrt[c*d*e]*Sqrt[g]*
(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(
d + e*x)^2)/e])])/(g^(3/2)*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]) - (2*d*e^(3/2)
*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(d + e*x)^2)/e]*ArcTanh
[(Sqrt[e]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I)*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]*Sqrt[d + e*x]])/(-(Sqrt[c*d*e]*Sq
rt[g]*(d + e*x)) + e*Sqrt[g]*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) +
(c*d*(d + e*x)^2)/e])])/(Sqrt[g]*Sqrt[-2*c*d*e*f + c*d^2*g + a*e^2*g + (2*I
)*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g]]))/(-(Sqrt
[c*d*e]*(d + e*x)) + e*Sqrt[-((c*d^2*(d + e*x))/e) + a*e*(d + e*x) + (c*d*(
d + e*x)^2)/e])
```

fricas [A] time = 0.44, size = 511, normalized size = 3.68

$$\left[\frac{\left(cd^2ef - cd^3g + \left(cde^2f - cd^2eg\right)x\right)\sqrt{-cdfg + aeg^2}\log\left(-\frac{cdegx^2 - cd^2f + 2\,adeg - \left(cdef - \left(cd^2 + 2\,ae^2\right)g\right)x - 2\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\sqrt{-cdfg + aeg^2}\,\sqrt{ex+d}}{egx^2 + df + \left(ef + dg\right)x}\right) + 2\left(cdefg - ae^2g^2\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\sqrt{ex+d}}{c^2d^3fg^2 - acd^2eg^3 + \left(c^2d^2efg^2 - acde^2g^3\right)x}, \frac{2\left(\left(cd^2ef - cd^3g + \left(cde^2f - cd^2eg\right)x\right)\sqrt{cdfg - aeg^2}\arctan\left(\frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\sqrt{cdfg - aeg^2}\,\sqrt{ex+d}}{cdegx^2 + adeg + \left(cd^2 + ae^2\right)gx}\right) + \left(cdefg - ae^2g^2\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\,\sqrt{ex+d}\right)}{c^2d^3fg^2 - acd^2eg^3 + \left(c^2d^2efg^2 - acde^2g^3\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] [((c*d^2*e*f - c*d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(-c*d*f*g + a*e*g^2
)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)
*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)
*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(c*d*e*f*g - a*e^2*g^2
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f*g^2
 - a*c*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x), 2*((c*d^2*e*f - c*
```

```
d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c
*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c*d*e*f*g - a*e^2*g^2)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f*g^2 - a*c
*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)), x)
```

maple [A] time = 0.02, size = 163, normalized size = 1.17

$$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}\left(c\,d^2g\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-cdef\,\mathrm{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,e\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,cdg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)*g)*c*d^2*g-arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2)*g)*c*d*e*f-(c*d*x+a*e)^(1/2)*e*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/
(c*d*x+a*e)^(1/2)/c/d/g/((a*e*g-c*d*f)*g)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)(ae+cdx)}\,(f+gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

3.540 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=170

$$-\frac{\left(2ae^2g-cd(dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\,(f+gx)(cdf-aeg)}$$

Rubi [A] time = 0.23, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {878, 874, 205}

$$-\frac{\left(2ae^2g-cd(dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\,(f+gx)(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -(((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f - a*e
*g)*Sqrt[d + e*x]*(f + g*x))) - ((2*a*e^2*g - c*d*(e*f + d*g))*ArcTan[(Sqrt
[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[
d + e*x])])/(g^(3/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p + 1))/(g*(n + 1)*(c*e*f + c*d*g - b*
e*g)), x] - Dist[(e*(b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3)))/
(g*(n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{\left(e\left(\frac{1}{2}cde^2f+\frac{3}{2}cd\right.\right.}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}\,(f+gx)} - \frac{\left(e^2\left(2ae^2g-cd(e\right.\right.}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}\,(f+gx)} - \frac{\left(2ae^2g-cd(ef+\right.}{}$$

Mathematica [A] time = 0.15, size = 155, normalized size = 0.91

$$\frac{\sqrt{d+ex}\left(-\frac{\sqrt{ae+cdx}\left(cd(dg+ef)-2ae^2g\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}}-\frac{\sqrt{g}(dg-ef)(ae+cdx)}{f+gx}\right)}{g^{3/2}\sqrt{(d+ex)(ae+cdx)}\,(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(-((Sqrt[g]*(-(e*f) + d*g)*(a*e + c*d*x))/(f + g*x)) - ((-2*
a*e^2*g + c*d*(e*f + d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d
*x])/Sqrt[c*d*f - a*e*g]])/Sqrt[c*d*f - a*e*g]))/(g^(3/2)*(-(c*d*f) + a*e*g
)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.44, size = 896, normalized size = 5.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^2*e*f^2 + (c*d^3 - 2*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2
*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2 + 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*
e^2)*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
 d*g)*x)) + 2*(c*d*e*f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2*a*c*d^
2*e*f^2*g^3 + a^2*d*e^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^
```

```
2*e^3*g^5)*x^2 + (c^2*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2*a*c*d*e^
2)*f^2*g^3 - (2*a*c*d^2*e - a^2*e^3)*f*g^4)*x), -((c*d^2*e*f^2 + (c*d^3 - 2
*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2
+ 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*e^2)*g^2)*x)*sqrt(c*d*f*g - a*e*
g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*
g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c*d*e*
f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2*a*c*d^2*e*f^2*g^3 + a^2*d*e
^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^2 + (c^2
*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g^3 - (2*a*c*d
^2*e - a^2*e^3)*f*g^4)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 347, normalized size = 2.04

$$\frac{\left(-2a e^2 g^2 x \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\, g}{\sqrt{(aeg-cdf)g}}\right)+c d^2 g^2 x \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\, g}{\sqrt{(aeg-cdf)g}}\right)+cdefgx \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\, g}{\sqrt{(aeg-cdf)g}}\right)-2a e^2 fg \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\, g}{\sqrt{(aeg-cdf)g}}\right)+c d^2 fg \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\, g}{\sqrt{(aeg-cdf)g}}\right)+cde f^2 \operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\, g}{\sqrt{(aeg-cdf)g}}\right)-\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\, dg+\sqrt{(aeg-cdf)g}\,\sqrt{cdx+ae}\, ef\right)\sqrt{cde x^2+a e^2 x+c d^2 x+ade}}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,(aeg-cdf)(gx+f)\sqrt{(aeg-cdf)g}\, g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^2/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] (-2*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*a*e^2*g^2+arctan
h((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c*d^2*g^2+arctanh((c*d*x+a
*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c*d*e*f*g-2*arctanh((c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)*g)*a*e^2*f*g+arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2)*g)*c*d^2*f*g+arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
)*g)*c*d*e*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*d*g+((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*e*f)/(e*x+d)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.541 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3 \sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=261

$$-\frac{cd\left(4ae^2g-cd(3dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+\right.}}{}$$

Rubi [A] time = 0.36, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {878, 872, 874, 205}

$$-\frac{cd\left(4ae^2g-cd(3dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(4ae^2g-cd(3dg+ef)\right)}{4g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*g*(c*d*f - a*
e*g)*Sqrt[d + e*x]*(f + g*x)^2) - ((4*a*e^2*g - c*d*(e*f + 3*d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(
f + g*x)) - (c*d*(4*a*e^2*g - c*d*(e*f + 3*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4
*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p + 1))/(g*(n + 1)*(c*e*f + c*d*g - b*
e*g)), x] - Dist[(e*(b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3)))/
(g*(n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
```

```
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+\frac{\left(e\left(\frac{1}{2}cde^2f+\frac{7}{2}cd\right.\right.}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(4ae^2g-cd(ef+\right.}{4g(cdf}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(4ae^2g-cd(ef+\right.}{4g(cdf}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(4ae^2g-cd(ef+\right.}{4g(cdf}$$

Mathematica [A] time = 0.42, size = 189, normalized size = 0.72

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd\left(2ae^2g-\frac{1}{2}cd(3dg+ef)\right)\left(\frac{cdf-aeg}{cdf+cdgx}+\frac{\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{ae+cdx}}\right)}{(cdf-aeg)^2}+\frac{ef-dg}{(f+gx)^2}\right)}{2g\sqrt{d+ex}\,(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((e*f - d*g)/(f + g*x)^2 + (c*d*(2*a*e^2*g -
 (c*d*(e*f + 3*d*g))/2)*((c*d*f - a*e*g)/(c*d*f + c*d*g*x) + (Sqrt[c*d*f -
a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[g]*Sq
rt[a*e + c*d*x])))/(c*d*f - a*e*g)^2))/(2*g*(-(c*d*f) + a*e*g)*Sqrt[d + e*x
])
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.47, size = 1704, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/8*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g + (c^2*d^2*e^2*f*g
^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e^2*f^2*g + (7*c^2*d
^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*g^3)*x^2 + (c^2*d^2
*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*d^4 - 4*a*c*d^2*e^2
)*f*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*
g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
d*g)*x)) - 2*(c^2*d^2*e*f^3*g - 2*a^2*d*e^2*g^4 - (5*c^2*d^3 - a*c*d*e^2)*f
^2*g^2 + (7*a*c*d^2*e - 2*a^2*e^3)*f*g^3 - (c^2*d^2*e*f^2*g^2 + (3*c^2*d^3
- 5*a*c*d*e^2)*f*g^3 - (3*a*c*d^2*e - 4*a^2*e^3)*g^4)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g^2 - 3*a*c^2*d^3*e*f
^4*g^3 + 3*a^2*c*d^2*e^2*f^3*g^4 - a^3*d*e^3*f^2*g^5 + (c^3*d^3*e*f^3*g^4 -
 3*a*c^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^3 + (2*c^3*
d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*
c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^6)*x
^2 + (c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)
*f^4*g^3 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^4 + (6*a^2*c*d^2*e^2 - a^3
*e^4)*f^2*g^5)*x), -1/4*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g
 + (c^2*d^2*e^2*f*g^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e
^2*f^2*g + (7*c^2*d^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*
g^3)*x^2 + (c^2*d^2*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*
d^4 - 4*a*c*d^2*e^2)*f*g^2)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e
*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c^2*d^2*e*f^3*g - 2*a^2*d*e^2*g
^4 - (5*c^2*d^3 - a*c*d*e^2)*f^2*g^2 + (7*a*c*d^2*e - 2*a^2*e^3)*f*g^3 - (c
^2*d^2*e*f^2*g^2 + (3*c^2*d^3 - 5*a*c*d*e^2)*f*g^3 - (3*a*c*d^2*e - 4*a^2*e
^3)*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3
*d^4*f^5*g^2 - 3*a*c^2*d^3*e*f^4*g^3 + 3*a^2*c*d^2*e^2*f^3*g^4 - a^3*d*e^3*
f^2*g^5 + (c^3*d^3*e*f^3*g^4 - 3*a*c^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^
6 - a^3*e^4*g^7)*x^3 + (2*c^3*d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 6*
a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c
*d^2*e^2 - 2*a^3*e^4)*f*g^6)*x^2 + (c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 +
 (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^3 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^
3*g^4 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^5)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 673, normalized size = 2.58

$$\frac{\sqrt{cde\,x^2+a e^2 x+c d^2 x+ade}\left(4acde^2g^3x^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-3c^2d^3g^3x^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-c^2d^2efg^2x^2\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+8acde^2fg^2x\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-6c^2d^3fg^2x\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-2c^2d^2ef^2gx\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)+4acde^2f^2g\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-3c^2d^3f^2g\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-c^2d^2ef^3\operatorname{arctanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{(aeg-cdf)g}}\right)-4\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,ae^2g^2x+3\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,cd^2g^2x+\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,cdefgx-2\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,adeg^2-2\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,ae^2fg+5\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,cd^2fg-\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)g}\,cdef^2\right)}{4\sqrt{ex+d}\,\sqrt{(aeg-cdf)g}\,(gx+f)^2(aeg-cdf)^2\sqrt{cdx+ae}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^3/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] 1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(4*arctanh((c*d*x+a*e)^(1/2)/((
a*e*g-c*d*f)*g)^(1/2)*g)*x^2*a*c*d*e^2*g^3-3*arctanh((c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2)*g)*x^2*c^2*d^3*g^3-arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*
```

```
d*f)*g)^(1/2)*g)*x^2*c^2*d^2*e*f*g^2+8*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2)*g)*x*a*c*d*e^2*f*g^2-6*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2)*g)*x*c^2*d^3*f*g^2-2*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2)*g)*x*c^2*d^2*e*f^2*g+4*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2)*g)*a*c*d*e^2*f^2*g-3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
)*g)*c^2*d^3*f^2*g-arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*c^2
*d^2*e*f^3-4*x*a*e^2*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*x*c*d^
2*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+x*c*d*e*f*g*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)-2*a*d*e*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)-2*a*e^2*f*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+5*c*d^2*f*g*(c*
d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-c*d*e*f^2*(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^2/g/(a*e*g-c
*d*f)^2/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\left(f+g\,x\right)^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.542 $\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=351

$$-\frac{c^2d^2\left(6ae^2g-cd(5dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(6ae^2g-cd(5dg\right.}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^3}$$

Rubi [A] time = 0.56, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {878, 872, 874, 205}

$$-\frac{c^2d^2\left(6ae^2g-cd(5dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(6ae^2g-cd(5dg+ef)\right)}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^3}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(6ae^2g-cd(5dg+ef)\right)}{12g\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*(c*d*f - a*
e*g)*Sqrt[d + e*x]*(f + g*x)^3) - ((6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*
(f + g*x)^2) - (c*d*(6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(8*g*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (c^
2*d^2*(6*a*e^2*g - c*d*(e*f + 5*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*g^(3/2)*(c
*d*f - a*e*g)^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p + 1))/(g*(n + 1)*(c*e*f + c*d*g - b*
e*g)), x] - Dist[(e*(b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3)))/
```

```
(g*(n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{\left(e\left(\frac{1}{2}cde^2f+\frac{11}{2}c\right.\right.}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} - \frac{\left(6ae^2g-cd(ef+\right.}{12g(cd}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} - \frac{\left(6ae^2g-cd(ef+\right.}{12g(cd}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} - \frac{\left(6ae^2g-cd(ef+\right.}{12g(cd}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} - \frac{\left(6ae^2g-cd(ef+\right.}{12g(cd}$$

Mathematica [C] time = 0.10, size = 132, normalized size = 0.38

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{ef-dg}{(f+gx)^3}-\frac{c^2d^2\left(cd(5dg+ef)-6ae^2g\right){}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{(cdf-aeg)^3}\right)}{3g\sqrt{d+ex}\,(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((e*f - d*g)/(f + g*x)^3 - (c^2*d^2*(-6*a*e^
2*g + c*d*(e*f + 5*d*g))*Hypergeometric2F1[1/2, 3, 3/2, (g*(a*e + c*d*x))/(
-(c*d*f) + a*e*g)])/(c*d*f - a*e*g)^3))/(3*g*(-(c*d*f) + a*e*g)*Sqrt[d + e*
x])
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.49, size = 2736, normalized size = 7.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/48*(3*(c^3*d^4*e*f^4 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^3*g + (c^3*d^3*e
^2*f*g^3 + (5*c^3*d^4*e - 6*a*c^2*d^2*e^3)*g^4)*x^4 + (3*c^3*d^3*e^2*f^2*g^
2 + 2*(8*c^3*d^4*e - 9*a*c^2*d^2*e^3)*f*g^3 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)
*g^4)*x^3 + 3*(c^3*d^3*e^2*f^3*g + 6*(c^3*d^4*e - a*c^2*d^2*e^3)*f^2*g^2 +
(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f*g^3)*x^2 + (c^3*d^3*e^2*f^4 + 2*(4*c^3*d^4*
e - 3*a*c^2*d^2*e^3)*f^3*g + 3*(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^2*g^2)*x)*sq
rt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(
3*c^3*d^3*e*f^4*g + 8*a^3*d*e^3*g^5 - (33*c^3*d^4 - 13*a*c^2*d^2*e^2)*f^3*g
^2 + (59*a*c^2*d^3*e - 20*a^2*c*d*e^3)*f^2*g^3 - 2*(17*a^2*c*d^2*e^2 - 2*a^
3*e^4)*f*g^4 - 3*(c^3*d^3*e*f^2*g^3 + (5*c^3*d^4 - 7*a*c^2*d^2*e^2)*f*g^4 -
 (5*a*c^2*d^3*e - 6*a^2*c*d*e^3)*g^5)*x^2 - 2*(4*c^3*d^3*e*f^3*g^2 + (20*c^
3*d^4 - 29*a*c^2*d^2*e^2)*f^2*g^3 - (25*a*c^2*d^3*e - 31*a^2*c*d*e^3)*f*g^4
 + (5*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g^2 - 4*a*c^3*d^4*e*f^6*g^3 + 6*a^2*c
^2*d^3*e^2*f^5*g^4 - 4*a^3*c*d^2*e^3*f^4*g^5 + a^4*d*e^4*f^3*g^6 + (c^4*d^4
*e*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*a^3*c*
d*e^4*f*g^8 + a^4*e^5*g^9)*x^4 + (3*c^4*d^4*e*f^5*g^4 + a^4*d*e^4*g^9 + (c^
4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f
^3*g^6 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e^3 - 3
*a^4*e^5)*f*g^8)*x^3 + 3*(c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c^4*d^5 -
4*a*c^3*d^3*e^2)*f^5*g^4 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^5 +
2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^6 - (4*a^3*c*d^2*e^3 - a^4*e^5)
*f^2*g^7)*x^2 + (c^4*d^4*e*f^7*g^2 + 3*a^4*d*e^4*f^2*g^7 + (3*c^4*d^5 - 4*a
*c^3*d^3*e^2)*f^6*g^3 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^4 + 2*(9*
a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (12*a^3*c*d^2*e^3 - a^4*e^5)*f^3
*g^6)*x), -1/24*(3*(c^3*d^4*e*f^4 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^3*g + (
c^3*d^3*e^2*f*g^3 + (5*c^3*d^4*e - 6*a*c^2*d^2*e^3)*g^4)*x^4 + (3*c^3*d^3*e
^2*f^2*g^2 + 2*(8*c^3*d^4*e - 9*a*c^2*d^2*e^3)*f*g^3 + (5*c^3*d^5 - 6*a*c^2
*d^3*e^2)*g^4)*x^3 + 3*(c^3*d^3*e^2*f^3*g + 6*(c^3*d^4*e - a*c^2*d^2*e^3)*f
^2*g^2 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f*g^3)*x^2 + (c^3*d^3*e^2*f^4 + 2*(4
*c^3*d^4*e - 3*a*c^2*d^2*e^3)*f^3*g + 3*(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^2*g
^2)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^
2 + a*e^2)*g*x)) + (3*c^3*d^3*e*f^4*g + 8*a^3*d*e^3*g^5 - (33*c^3*d^4 - 13*
a*c^2*d^2*e^2)*f^3*g^2 + (59*a*c^2*d^3*e - 20*a^2*c*d*e^3)*f^2*g^3 - 2*(17*
a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^4 - 3*(c^3*d^3*e*f^2*g^3 + (5*c^3*d^4 - 7*a*
c^2*d^2*e^2)*f*g^4 - (5*a*c^2*d^3*e - 6*a^2*c*d*e^3)*g^5)*x^2 - 2*(4*c^3*d^
3*e*f^3*g^2 + (20*c^3*d^4 - 29*a*c^2*d^2*e^2)*f^2*g^3 - (25*a*c^2*d^3*e - 3
1*a^2*c*d*e^3)*f*g^4 + (5*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^5)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g^2 - 4*a*c^3*d^4
*e*f^6*g^3 + 6*a^2*c^2*d^3*e^2*f^5*g^4 - 4*a^3*c*d^2*e^3*f^4*g^5 + a^4*d*e^
4*f^3*g^6 + (c^4*d^4*e*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^
3*f^2*g^7 - 4*a^3*c*d*e^4*f*g^8 + a^4*e^5*g^9)*x^4 + (3*c^4*d^4*e*f^5*g^4 +
 a^4*d*e^4*g^9 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e -
9*a^2*c^2*d^2*e^3)*f^3*g^6 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^7 -
(4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^8)*x^3 + 3*(c^4*d^4*e*f^6*g^3 + a^4*d*e^4
*f*g^8 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^4 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2
*d^2*e^3)*f^4*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^6 - (4*a^3*
c*d^2*e^3 - a^4*e^5)*f^2*g^7)*x^2 + (c^4*d^4*e*f^7*g^2 + 3*a^4*d*e^4*f^2*g^
7 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^3 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*
e^3)*f^5*g^4 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (12*a^3*c*d^
2*e^3 - a^4*e^5)*f^3*g^6)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.05, size = 1142, normalized size = 3.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^4/(c*d*e*x^2+a*d*e+(a*e^2+c*d^2)*x)^(1/2),x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-3*c^2*d^2*e*f^3*(c*d*x+a*e)
^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2)*g)*x^3*c^3*d^4*g^4-15*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2)*g)*c^3*d^4*f^3*g-3*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*
g)*c^3*d^3*e*f^4-50*x*a*c*d*e^2*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(
1/2)-45*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^4*f*
g^3-45*arctanh((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^4*f^2*g
^2+15*x^2*c^2*d^3*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+12*x*a^2*e^
3*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+8*a^2*d*e^2*g^3*(c*d*x+a*e)
^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+4*a^2*e^3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*
d*f)*g)^(1/2)+33*c^2*d^3*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-18
*x^2*a*c*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+40*x*c^2*d^3*f
*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+18*arctanh((c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*a*c^2*d^2*e^2*g^4-3*arctanh((c*d*x+a*e)^(1/
2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^3*c^3*d^3*e*f*g^3-9*arctanh((c*d*x+a*e)^(1/
2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*c^3*d^3*e*f^2*g^2-9*arctanh((c*d*x+a*e)^(
1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*c^3*d^3*e*f^3*g+18*arctanh((c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*a*c^2*d^2*e^2*f^3*g+3*x^2*c^2*d^2*e*f*g^2*(c
*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-10*x*a*c*d^2*e*g^3*(c*d*x+a*e)^(1/2
)*((a*e*g-c*d*f)*g)^(1/2)+8*x*c^2*d^2*e*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d
*f)*g)^(1/2)-26*a*c*d^2*e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1
6*a*c*d*e^2*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+54*arctanh((c*d
*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x^2*a*c^2*d^2*e^2*f*g^3+54*arctanh
((c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)*g)*x*a*c^2*d^2*e^2*f^2*g^2)/(e*x
+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*
a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\left(f+g\,x\right)^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

3.543 $\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=324

$$-\frac{b\sqrt{1-d^2x^2}\left(45a^2d^4+60acd^2+10b^2d^2+24c^2\right)}{15d^6}-\frac{x\sqrt{1-d^2x^2}\left(24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c\right.}{16d^6}$$

Rubi [A] time = 0.93, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {899, 1815, 641, 216}

$$-\frac{x\sqrt{1-d^2x^2}\left(24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3\right)}{16d^6}-\frac{b\sqrt{1-d^2x^2}\left(45a^2d^4+60acd^2+10b^2d^2+24c^2\right)}{15d^6}+\frac{\sin^{-1}(dx)\left(24a^2cd^4+16a^3d^6+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3\right)}{16d^7}-\frac{cx^3\sqrt{1-d^2x^2}\left(18acd^2+18b^2d^2+5c^2\right)}{24d^4}-\frac{bx^2\sqrt{1-d^2x^2}\left(30acd^2+5b^2d^2+12c^2\right)}{15d^4}-\frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2}-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(b*(24*c^2 + 10*b^2*d^2 + 60*a*c*d^2 + 45*a^2*d^4)*Sqrt[1 - d^2*x^2])/(15*
d^6) - ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4)
*x*Sqrt[1 - d^2*x^2])/(16*d^6) - (b*(12*c^2 + 5*b^2*d^2 + 30*a*c*d^2)*x^2*S
qrt[1 - d^2*x^2])/(15*d^4) - (c*(5*c^2 + 18*b^2*d^2 + 18*a*c*d^2)*x^3*Sqrt[
1 - d^2*x^2])/(24*d^4) - (3*b*c^2*x^4*Sqrt[1 - d^2*x^2])/(5*d^2) - (c^3*x^5
*Sqrt[1 - d^2*x^2])/(6*d^2) + ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*
b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x])/(16*d^7)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{(a+bx+cx^2)^3}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} - \frac{\int \frac{-6a^3d^2-18a^2bd^2x-18a(b^2+ac)d^2x^2-6b(b^2+6ac)d^2x^3-c(5c^2+18b^2d^2+18acd^2)x^4-18b}{\sqrt{1-d^2x^2}}}{6d^2}$$

$$= -\frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} + \frac{\int \frac{30a^3d^4+90a^2bd^4x+90a(b^2+ac)d^4x^2+6bd^2(12c^2+5b^2d^2+30}{\sqrt{1-d^2x^2}}}{30d^4}$$

$$= -\frac{c(5c^2+18b^2d^2+18acd^2)x^3\sqrt{1-d^2x^2}}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} - \int -$$

$$= -\frac{b(12c^2+5b^2d^2+30acd^2)x^2\sqrt{1-d^2x^2}}{15d^4} - \frac{c(5c^2+18b^2d^2+18acd^2)x^3\sqrt{1-d^2x^2}}{24d^4} -$$

$$= -\frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)x\sqrt{1-d^2x^2}}{16d^6} - \frac{b(12c^2+5b^2d^2+30}{15d}$$

$$= -\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\sqrt{1-d^2x^2}}{15d^6} - \frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab}{16d^6}$$

$$= -\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\sqrt{1-d^2x^2}}{15d^6} - \frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab}{16d^6}$$

Mathematica [A] time = 0.26, size = 229, normalized size = 0.71

$$\frac{15\sin^{-1}(dx)\left(16a^3d^6+24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3\right)-d\sqrt{1-d^2x^2}\left(48b\left(15a^2d^4+10acd^2\left(d^2x^2+2\right)+c^2\left(3d^4x^4+4d^2x^2+8\right)\right)+5cx\left(72a^2d^4+18acd^2\left(2d^2x^2+3\right)+c^2\left(8d^4x^4+10d^2x^2+15\right)\right)+90b^2d^2x\left(4ad^2+c\left(2d^2x^2+3\right)\right)+80b^3d^2\left(d^2x^2+2\right)\right)}{240d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*Sqrt[1 - d^2*x^2]*(80*b^3*d^2*(2 + d^2*x^2) + 90*b^2*d^2*x*(4*a*d^2 +
c*(3 + 2*d^2*x^2)) + 48*b*(15*a^2*d^4 + 10*a*c*d^2*(2 + d^2*x^2) + c^2*(8 +
 4*d^2*x^2 + 3*d^4*x^4)) + 5*c*x*(72*a^2*d^4 + 18*a*c*d^2*(3 + 2*d^2*x^2) +
 c^2*(15 + 10*d^2*x^2 + 8*d^4*x^4)))) + 15*(5*c^3 + 18*b^2*c*d^2 + 18*a*c^2
*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x])/(240*d^7)
```

IntegrateAlgebraic [B] time = 0.63, size = 1219, normalized size = 3.76

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[1 - d*x]*(-165*c^3 - 720*b*c^2*d - 450*b^2*c*d^2 - 450*a*c^2*d^2 - 24
0*b^3*d^3 - 1440*a*b*c*d^3 - 360*a*b^2*d^4 - 360*a^2*c*d^4 - 720*a^2*b*d^5
+ (165*c^3*(1 - d*x)^5)/(1 + d*x)^5 - (720*b*c^2*d*(1 - d*x)^5)/(1 + d*x)^5
 + (450*b^2*c*d^2*(1 - d*x)^5)/(1 + d*x)^5 + (450*a*c^2*d^2*(1 - d*x)^5)/(1
 + d*x)^5 - (240*b^3*d^3*(1 - d*x)^5)/(1 + d*x)^5 - (1440*a*b*c*d^3*(1 - d*
x)^5)/(1 + d*x)^5 + (360*a*b^2*d^4*(1 - d*x)^5)/(1 + d*x)^5 + (360*a^2*c*d^
4*(1 - d*x)^5)/(1 + d*x)^5 - (720*a^2*b*d^5*(1 - d*x)^5)/(1 + d*x)^5 - (25*
c^3*(1 - d*x)^4)/(1 + d*x)^4 - (1680*b*c^2*d*(1 - d*x)^4)/(1 + d*x)^4 + (63
0*b^2*c*d^2*(1 - d*x)^4)/(1 + d*x)^4 + (630*a*c^2*d^2*(1 - d*x)^4)/(1 + d*x
)^4 - (880*b^3*d^3*(1 - d*x)^4)/(1 + d*x)^4 - (5280*a*b*c*d^3*(1 - d*x)^4)/
(1 + d*x)^4 + (1080*a*b^2*d^4*(1 - d*x)^4)/(1 + d*x)^4 + (1080*a^2*c*d^4*(1
```

```
 - d*x)^4)/(1 + d*x)^4 - (3600*a^2*b*d^5*(1 - d*x)^4)/(1 + d*x)^4 + (450*c^
3*(1 - d*x)^3)/(1 + d*x)^3 - (3744*b*c^2*d*(1 - d*x)^3)/(1 + d*x)^3 + (180*
b^2*c*d^2*(1 - d*x)^3)/(1 + d*x)^3 + (180*a*c^2*d^2*(1 - d*x)^3)/(1 + d*x)^
3 - (1440*b^3*d^3*(1 - d*x)^3)/(1 + d*x)^3 - (8640*a*b*c*d^3*(1 - d*x)^3)/(
1 + d*x)^3 + (720*a*b^2*d^4*(1 - d*x)^3)/(1 + d*x)^3 + (720*a^2*c*d^4*(1 -
d*x)^3)/(1 + d*x)^3 - (7200*a^2*b*d^5*(1 - d*x)^3)/(1 + d*x)^3 - (450*c^3*(
1 - d*x)^2)/(1 + d*x)^2 - (3744*b*c^2*d*(1 - d*x)^2)/(1 + d*x)^2 - (180*b^2
*c*d^2*(1 - d*x)^2)/(1 + d*x)^2 - (180*a*c^2*d^2*(1 - d*x)^2)/(1 + d*x)^2 -
 (1440*b^3*d^3*(1 - d*x)^2)/(1 + d*x)^2 - (8640*a*b*c*d^3*(1 - d*x)^2)/(1 +
 d*x)^2 - (720*a*b^2*d^4*(1 - d*x)^2)/(1 + d*x)^2 - (720*a^2*c*d^4*(1 - d*x
)^2)/(1 + d*x)^2 - (7200*a^2*b*d^5*(1 - d*x)^2)/(1 + d*x)^2 + (25*c^3*(1 -
d*x))/(1 + d*x) - (1680*b*c^2*d*(1 - d*x))/(1 + d*x) - (630*b^2*c*d^2*(1 -
d*x))/(1 + d*x) - (630*a*c^2*d^2*(1 - d*x))/(1 + d*x) - (880*b^3*d^3*(1 - d
*x))/(1 + d*x) - (5280*a*b*c*d^3*(1 - d*x))/(1 + d*x) - (1080*a*b^2*d^4*(1
- d*x))/(1 + d*x) - (1080*a^2*c*d^4*(1 - d*x))/(1 + d*x) - (3600*a^2*b*d^5*
(1 - d*x))/(1 + d*x)))/(120*d^7*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))^6)
+ ((-5*c^3 - 18*b^2*c*d^2 - 18*a*c^2*d^2 - 24*a*b^2*d^4 - 24*a^2*c*d^4 - 16
*a^3*d^6)*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/(8*d^7)
```

fricas [A] time = 0.42, size = 251, normalized size = 0.77

$$-\frac{\left(40\,c^{3}d^{5}x^{5}+144\,bc^{2}d^{5}x^{4}+720\,a^{2}bd^{5}+384\,bc^{2}d+160\left(b^{3}+6\,abc\right)d^{3}+10\left(5\,c^{3}d^{3}+18\left(b^{2}c+ac^{2}\right)d^{5}\right)x^{3}+16\left(12\,bc^{2}d^{3}+5\left(b^{3}+6\,abc\right)d^{5}\right)x^{2}+15\left(24\left(ab^{2}+a^{2}c\right)d^{5}+5\,c^{3}d+18\left(b^{2}c+ac^{2}\right)d^{3}\right)x\right)\sqrt{dx+1}\sqrt{-dx+1}+30\left(16\,a^{3}d^{6}+24\left(ab^{2}+a^{2}c\right)d^{4}+5\,c^{3}+18\left(b^{2}c+ac^{2}\right)d^{2}\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{240\,d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/240*((40*c^3*d^5*x^5 + 144*b*c^2*d^5*x^4 + 720*a^2*b*d^5 + 384*b*c^2*d +
 160*(b^3 + 6*a*b*c)*d^3 + 10*(5*c^3*d^3 + 18*(b^2*c + a*c^2)*d^5)*x^3 + 16
*(12*b*c^2*d^3 + 5*(b^3 + 6*a*b*c)*d^5)*x^2 + 15*(24*(a*b^2 + a^2*c)*d^5 +
5*c^3*d + 18*(b^2*c + a*c^2)*d^3)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(16*
a^3*d^6 + 24*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 18*(b^2*c + a*c^2)*d^2)*arctan((
sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^7
```

giac [A] time = 0.62, size = 412, normalized size = 1.27

$$-\frac{\left(\left(2\left((dx+1)\left(4\,(dx+1)\left(\frac{5\,(dx+1)c^{3}}{d^{6}}+\frac{18\,bc^{2}d^{37}-25\,c^{3}d^{36}}{d^{42}}\right)+\frac{9\left(10\,b^{2}cd^{38}+10\,ac^{2}d^{38}-32\,bc^{2}d^{37}+25\,c^{3}d^{36}\right)}{d^{42}}\right)+\frac{40\,b^{3}d^{39}+240\,abcd^{39}-270\,b^{2}cd^{38}-270\,ac^{2}d^{38}+528\,bc^{2}d^{37}-275\,c^{3}d^{36}}{d^{42}}\right)(dx+1)+\frac{5\left(72\,ab^{2}d^{40}+72\,a^{2}cd^{40}-32\,b^{3}d^{39}-192\,abcd^{39}+162\,b^{2}cd^{38}+162\,ac^{2}d^{38}-192\,bc^{2}d^{37}+85\,c^{3}d^{36}\right)}{d^{42}}\right)(dx+1)+\frac{15\left(48\,a^{2}bd^{41}-24\,ab^{2}d^{40}-24\,a^{2}cd^{40}+16\,b^{3}d^{39}+96\,abcd^{39}-30\,b^{2}cd^{38}-30\,ac^{2}d^{38}+48\,bc^{2}d^{37}-11\,c^{3}d^{36}\right)}{d^{42}}\right)\sqrt{dx+1}\sqrt{-dx+1}-\frac{30\left(16\,a^{3}d^{6}+24\,ab^{2}d^{4}+24\,a^{2}cd^{4}+18\,b^{2}cd^{2}+18\,ac^{2}d^{2}+5\,c^{3}\right)\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{d^{6}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/240*(((2*((d*x + 1)*(4*(d*x + 1)*(5*(d*x + 1)*c^3/d^6 + (18*b*c^2*d^37 -
 25*c^3*d^36)/d^42) + 9*(10*b^2*c*d^38 + 10*a*c^2*d^38 - 32*b*c^2*d^37 + 25
*c^3*d^36)/d^42) + (40*b^3*d^39 + 240*a*b*c*d^39 - 270*b^2*c*d^38 - 270*a*c
^2*d^38 + 528*b*c^2*d^37 - 275*c^3*d^36)/d^42)*(d*x + 1) + 5*(72*a*b^2*d^40
 + 72*a^2*c*d^40 - 32*b^3*d^39 - 192*a*b*c*d^39 + 162*b^2*c*d^38 + 162*a*c^
2*d^38 - 192*b*c^2*d^37 + 85*c^3*d^36)/d^42)*(d*x + 1) + 15*(48*a^2*b*d^41
- 24*a*b^2*d^40 - 24*a^2*c*d^40 + 16*b^3*d^39 + 96*a*b*c*d^39 - 30*b^2*c*d^
38 - 30*a*c^2*d^38 + 48*b*c^2*d^37 - 11*c^3*d^36)/d^42)*sqrt(d*x + 1)*sqrt(
-d*x + 1) - 30*(16*a^3*d^6 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 18*b^2*c*d^2 + 1
8*a*c^2*d^2 + 5*c^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^6)/d
```

maple [C] time = 0.06, size = 602, normalized size = 1.86

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(40\sqrt{-d^{2}x^{2}+1}\,c^{3}d^{5}x^{5}\,\mathrm{csgn}(d)+144\sqrt{-d^{2}x^{2}+1}\,b\,c^{2}d^{5}x^{4}\,\mathrm{csgn}(d)+180\sqrt{-d^{2}x^{2}+1}\,a\,c^{2}d^{5}x^{3}\,\mathrm{csgn}(d)+180\sqrt{-d^{2}x^{2}+1}\,b^{2}c\,d^{5}x^{3}\,\mathrm{csgn}(d)+480\sqrt{-d^{2}x^{2}+1}\,abc\,d^{5}x^{2}\,\mathrm{csgn}(d)+80\sqrt{-d^{2}x^{2}+1}\,b^{3}d^{5}x^{2}\,\mathrm{csgn}(d)-240a^{3}d^{6}\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)+360\sqrt{-d^{2}x^{2}+1}\,a^{2}c\,d^{5}x\,\mathrm{csgn}(d)+360\sqrt{-d^{2}x^{2}+1}\,a\,b^{2}d^{5}x\,\mathrm{csgn}(d)+50\sqrt{-d^{2}x^{2}+1}\,c^{3}d^{3}x^{3}\,\mathrm{csgn}(d)+720\sqrt{-d^{2}x^{2}+1}\,a^{2}b\,d^{5}\,\mathrm{csgn}(d)+192\sqrt{-d^{2}x^{2}+1}\,b\,c^{2}d^{3}x^{2}\,\mathrm{csgn}(d)-360a^{2}c\,d^{4}\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)-360a\,b^{2}d^{4}\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)+270\sqrt{-d^{2}x^{2}+1}\,a\,c^{2}d^{3}x\,\mathrm{csgn}(d)+270\sqrt{-d^{2}x^{2}+1}\,b^{2}c\,d^{3}x\,\mathrm{csgn}(d)+960\sqrt{-d^{2}x^{2}+1}\,abc\,d^{3}\,\mathrm{csgn}(d)+160\sqrt{-d^{2}x^{2}+1}\,b^{3}d^{3}\,\mathrm{csgn}(d)-270a\,c^{2}d^{2}\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)-270b^{2}c\,d^{2}\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)+75\sqrt{-d^{2}x^{2}+1}\,c^{3}dx\,\mathrm{csgn}(d)+384\sqrt{-d^{2}x^{2}+1}\,b\,c^{2}d\,\mathrm{csgn}(d)-75c^{3}\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)\right)\mathrm{csgn}(d)}{240\sqrt{-d^{2}x^{2}+1}\,d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/240*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(40*csgn(d)*x^5*c^3*d^5*(-d^2*x^2+1)^(1
/2)+144*csgn(d)*x^4*b*c^2*d^5*(-d^2*x^2+1)^(1/2)+180*csgn(d)*x^3*a*c^2*d^5*
(-d^2*x^2+1)^(1/2)+180*csgn(d)*x^3*b^2*c*d^5*(-d^2*x^2+1)^(1/2)+480*csgn(d)
*x^2*a*b*c*d^5*(-d^2*x^2+1)^(1/2)+80*csgn(d)*x^2*b^3*d^5*(-d^2*x^2+1)^(1/2)
+50*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x^3*c^3+360*csgn(d)*d^5*(-d^2*x^2+1)^(1/
2)*x*a^2*c+360*csgn(d)*d^5*(-d^2*x^2+1)^(1/2)*x*a*b^2+192*csgn(d)*d^3*(-d^2
*x^2+1)^(1/2)*x^2*b*c^2+720*(-d^2*x^2+1)^(1/2)*csgn(d)*d^5*a^2*b-240*arctan
(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a^3*d^6+270*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)
*x*a*c^2+270*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*b^2*c+960*(-d^2*x^2+1)^(1/2)*
csgn(d)*d^3*a*b*c+160*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*b^3-360*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*a^2*c*d^4-360*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2
))*a*b^2*d^4+75*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c^3+384*(-d^2*x^2+1)^(1/2)*c
sgn(d)*d*b*c^2-270*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*c^2*d^2-270*arc
tan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b^2*c*d^2-75*arctan(csgn(d)*d*x/(-d^2*x
^2+1)^(1/2))*c^3)*csgn(d)/d^7/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 0.98, size = 365, normalized size = 1.13

$$-\frac{\sqrt{-d^2x^2+1}\,c^3x^5}{6\,d^2}-\frac{3\sqrt{-d^2x^2+1}\,bc^2x^4}{5\,d^2}+\frac{a^3\arcsin(dx)}{d}-\frac{5\sqrt{-d^2x^2+1}\,c^3x^3}{24\,d^4}-\frac{3\sqrt{-d^2x^2+1}\,(b^2c+ac^2)x^3}{4\,d^2}-\frac{3\sqrt{-d^2x^2+1}\,a^2b}{d^2}-\frac{4\sqrt{-d^2x^2+1}\,bc^2x^2}{5\,d^4}-\frac{\sqrt{-d^2x^2+1}\,(b^3+6\,abc)x^2}{3\,d^2}-\frac{3\sqrt{-d^2x^2+1}\,(ab^2+a^2c)x}{2\,d^2}+\frac{3\,(ab^2+a^2c)\arcsin(dx)}{2\,d^3}-\frac{5\sqrt{-d^2x^2+1}\,c^3x}{16\,d^6}-\frac{9\sqrt{-d^2x^2+1}\,(b^2c+ac^2)x}{8\,d^4}-\frac{8\sqrt{-d^2x^2+1}\,bc^2}{5\,d^6}-\frac{2\sqrt{-d^2x^2+1}\,(b^3+6\,abc)}{3\,d^4}+\frac{5\,c^3\arcsin(dx)}{16\,d^7}+\frac{9\,(b^2c+ac^2)\arcsin(dx)}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*sqrt(-d^2*x^2 + 1)*c^3*x^5/d^2 - 3/5*sqrt(-d^2*x^2 + 1)*b*c^2*x^4/d^2
+ a^3*arcsin(d*x)/d - 5/24*sqrt(-d^2*x^2 + 1)*c^3*x^3/d^4 - 3/4*sqrt(-d^2*x
^2 + 1)*(b^2*c + a*c^2)*x^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*a^2*b/d^2 - 4/5*sqrt
(-d^2*x^2 + 1)*b*c^2*x^2/d^4 - 1/3*sqrt(-d^2*x^2 + 1)*(b^3 + 6*a*b*c)*x^2/d
^2 - 3/2*sqrt(-d^2*x^2 + 1)*(a*b^2 + a^2*c)*x/d^2 + 3/2*(a*b^2 + a^2*c)*arc
sin(d*x)/d^3 - 5/16*sqrt(-d^2*x^2 + 1)*c^3*x/d^6 - 9/8*sqrt(-d^2*x^2 + 1)*(
b^2*c + a*c^2)*x/d^4 - 8/5*sqrt(-d^2*x^2 + 1)*b*c^2/d^6 - 2/3*sqrt(-d^2*x^2
 + 1)*(b^3 + 6*a*b*c)/d^4 + 5/16*c^3*arcsin(d*x)/d^7 + 9/8*(b^2*c + a*c^2)*
arcsin(d*x)/d^5
```

mupad [B] time = 31.33, size = 1768, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((((1 - d*x)^(1/2) - 1)^23*((5*c^3)/4 + 6*a*b^2*d^4 + (9*a*c^2*d^2)/2 + 6
*a^2*c*d^4 + (9*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^23 - (((1 - d*x)^(1/2)
 - 1)*((5*c^3)/4 + 6*a*b^2*d^4 + (9*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (9*b^2*c*d
^2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^3*((175*c^3)/12 + 6*
a*b^2*d^4 + (105*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (105*b^2*c*d^2)/2))/((d*x + 1
)^(1/2) - 1)^3 + (((1 - d*x)^(1/2) - 1)^21*((175*c^3)/12 + 6*a*b^2*d^4 + (1
05*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (105*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^2
1 + (((1 - d*x)^(1/2) - 1)^5*(126*a*b^2*d^4 - (311*c^3)/4 + (669*a*c^2*d^2)
/2 + 126*a^2*c*d^4 + (669*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d
*x)^(1/2) - 1)^19*(126*a*b^2*d^4 - (311*c^3)/4 + (669*a*c^2*d^2)/2 + 126*a^
2*c*d^4 + (669*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^19 + (((1 - d*x)^(1/2)
- 1)^7*((8361*c^3)/4 + 510*a*b^2*d^4 + (1533*a*c^2*d^2)/2 + 510*a^2*c*d^4 +
 (1533*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^17*(
(8361*c^3)/4 + 510*a*b^2*d^4 + (1533*a*c^2*d^2)/2 + 510*a^2*c*d^4 + (1533*b
^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^17 + (((1 - d*x)^(1/2) - 1)^11*((25295*
c^3)/2 + 420*a*b^2*d^4 - 549*a*c^2*d^2 + 420*a^2*c*d^4 - 549*b^2*c*d^2))/((
d*x + 1)^(1/2) - 1)^11 - (((1 - d*x)^(1/2) - 1)^13*((25295*c^3)/2 + 420*a*b
^2*d^4 - 549*a*c^2*d^2 + 420*a^2*c*d^4 - 549*b^2*c*d^2))/((d*x + 1)^(1/2) -
 1)^13 - (((1 - d*x)^(1/2) - 1)^9*((42259*c^3)/6 - 804*a*b^2*d^4 + 165*a*c^
2*d^2 - 804*a^2*c*d^4 + 165*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*
```

```
x)^(1/2) - 1)^15*((42259*c^3)/6 - 804*a*b^2*d^4 + 165*a*c^2*d^2 - 804*a^2*c
*d^4 + 165*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^6*
((1024*b^3*d^3)/3 + 1080*a^2*b*d^5 + 2048*b*c^2*d + 2048*a*b*c*d^3))/((d*x
+ 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^18*((1024*b^3*d^3)/3 + 1080*a^2*
b*d^5 + 2048*b*c^2*d + 2048*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^18 + (((1 - d
*x)^(1/2) - 1)^10*(1024*b^3*d^3 + 5040*a^2*b*d^5 + (6144*b*c^2*d)/5 + 6144*
a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^10 + (((1 - d*x)^(1/2) - 1)^14*(1024*b^3*
d^3 + 5040*a^2*b*d^5 + (6144*b*c^2*d)/5 + 6144*a*b*c*d^3))/((d*x + 1)^(1/2)
 - 1)^14 + (((1 - d*x)^(1/2) - 1)^12*((3200*b^3*d^3)/3 + 6048*a^2*b*d^5 + (
32768*b*c^2*d)/5 + 6400*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^
(1/2) - 1)^4*(64*b^3*d^3 + 240*a^2*b*d^5 + 384*a*b*c*d^3))/((d*x + 1)^(1/2)
 - 1)^4 + (((1 - d*x)^(1/2) - 1)^20*(64*b^3*d^3 + 240*a^2*b*d^5 + 384*a*b*c
*d^3))/((d*x + 1)^(1/2) - 1)^20 + (((1 - d*x)^(1/2) - 1)^8*(768*b^3*d^3 + 2
880*a^2*b*d^5 + 4608*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2
) - 1)^16*(768*b^3*d^3 + 2880*a^2*b*d^5 + 4608*a*b*c*d^3))/((d*x + 1)^(1/2)
 - 1)^16 + (24*a^2*b*d^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 +
 (24*a^2*b*d^5*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22)/(d^7 + (
12*d^7*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (66*d^7*((1 - d*x
)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (220*d^7*((1 - d*x)^(1/2) - 1)^6)
/((d*x + 1)^(1/2) - 1)^6 + (495*d^7*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/
2) - 1)^8 + (792*d^7*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (
924*d^7*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (792*d^7*((1 -
 d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (495*d^7*((1 - d*x)^(1/2) -
 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (220*d^7*((1 - d*x)^(1/2) - 1)^18)/((d*x
 + 1)^(1/2) - 1)^18 + (66*d^7*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) -
1)^20 + (12*d^7*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22 + (d^7*(
(1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24) - (atan(((1 - d*x)^(1/2)
 - 1)/((d*x + 1)^(1/2) - 1))*(5*c^3 + 16*a^3*d^6 + 24*a*b^2*d^4 + 18*a*c^2*
d^2 + 24*a^2*c*d^4 + 18*b^2*c*d^2))/(4*d^7)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.544 $\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=166

$$\frac{\sin^{-1}(dx)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{8d^5}-\frac{x\sqrt{1-d^2x^2}\left(c\left(8a+\frac{3c}{d^2}\right)+4b^2\right)}{8d^2}-\frac{2b\sqrt{1-d^2x^2}\left(3ad^2+2c\right)}{3d^4}-\frac{2bcx^2\sqrt{1}}{3d^2}$$

Rubi [A] time = 0.32, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {899, 1815, 641, 216}

$$\frac{\sin^{-1}(dx)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{8d^5}-\frac{x\sqrt{1-d^2x^2}\left(c\left(8a+\frac{3c}{d^2}\right)+4b^2\right)}{8d^2}-\frac{2b\sqrt{1-d^2x^2}\left(3ad^2+2c\right)}{3d^4}-\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-2*b*(2*c + 3*a*d^2)*Sqrt[1 - d^2*x^2])/(3*d^4) - ((4*b^2 + c*(8*a + (3*c)
/d^2))*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (2*b*c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2)
 - (c^2*x^3*Sqrt[1 - d^2*x^2])/(4*d^2) + ((3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 +
8*a^2*d^4)*ArcSin[d*x])/(8*d^5)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx=\int \frac{\left(a+bx+cx^2\right)^2}{\sqrt{1-d^2x^2}}\,dx$$

$$=-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}-\frac{\int \frac{-4a^2d^2-8abd^2x-\left(3c^2+4b^2d^2+8acd^2\right)x^2-8bcd^2x^3}{\sqrt{1-d^2x^2}}dx}{4d^2}$$

$$=-\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}+\frac{\int \frac{12a^2d^4+8bd^2\left(2c+3ad^2\right)x+3d^2\left(3c^2+4b^2d^2+8acd^2\right)x^2}{\sqrt{1-d^2x^2}}dx}{12d^4}$$

$$=-\frac{\left(3c^2+4b^2d^2+8acd^2\right)x\sqrt{1-d^2x^2}}{8d^4}-\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}-\frac{\int \frac{-3d^2(\cdots}{\cdots}}{\cdots}$$

$$=-\frac{2b\left(2c+3ad^2\right)\sqrt{1-d^2x^2}}{3d^4}-\frac{\left(3c^2+4b^2d^2+8acd^2\right)x\sqrt{1-d^2x^2}}{8d^4}-\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}\cdots$$

$$=-\frac{2b\left(2c+3ad^2\right)\sqrt{1-d^2x^2}}{3d^4}-\frac{\left(3c^2+4b^2d^2+8acd^2\right)x\sqrt{1-d^2x^2}}{8d^4}-\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}\cdots$$

Mathematica [A] time = 0.12, size = 114, normalized size = 0.69

$$\frac{3\sin^{-1}(dx)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)-d\sqrt{1-d^2x^2}\left(16b\left(3ad^2+cd^2x^2+2c\right)+3cx\left(8ad^2+2cd^2x^2+3c\right)+12b^2d^2x\right)}{24d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*Sqrt[1 - d^2*x^2]*(12*b^2*d^2*x + 16*b*(2*c + 3*a*d^2 + c*d^2*x^2) + 3
*c*x*(3*c + 8*a*d^2 + 2*c*d^2*x^2))) + 3*(3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 + 8
*a^2*d^4)*ArcSin[d*x])/(24*d^5)
```

IntegrateAlgebraic [B] time = 0.29, size = 446, normalized size = 2.69

$$\frac{\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)\left(-8a^2d^4-8acd^2-4b^2d^2-3c^2\right)}{4d^5}+\frac{\sqrt{1-dx}\left(-\frac{144abd^3(1-dx)}{dx+1}-\frac{144abd^3(1-dx)^2}{(dx+1)^2}-\frac{48abd^3(1-dx)^3}{(dx+1)^3}-48abd^3-\frac{24acd^2(1-dx)}{dx+1}+\frac{24acd^2(1-dx)^2}{(dx+1)^2}+\frac{24acd^2(1-dx)^3}{(dx+1)^3}-24acd^2-\frac{12b^2d^2(1-dx)}{dx+1}+\frac{12b^2d^2(1-dx)^2}{(dx+1)^2}+\frac{12b^2d^2(1-dx)^3}{(dx+1)^3}-12b^2d^2-\frac{80bcd(1-dx)}{dx+1}-\frac{80bcd(1-dx)^2}{(dx+1)^2}-\frac{48bcd(1-dx)^3}{(dx+1)^3}-48bcd+\frac{9c^2(1-dx)}{dx+1}-\frac{9c^2(1-dx)^2}{(dx+1)^2}+\frac{15c^2(1-dx)^3}{(dx+1)^3}-15c^2\right)}{12d^5\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[1 - d*x]*(-15*c^2 - 48*b*c*d - 12*b^2*d^2 - 24*a*c*d^2 - 48*a*b*d^3 +
 (15*c^2*(1 - d*x)^3)/(1 + d*x)^3 - (48*b*c*d*(1 - d*x)^3)/(1 + d*x)^3 + (1
2*b^2*d^2*(1 - d*x)^3)/(1 + d*x)^3 + (24*a*c*d^2*(1 - d*x)^3)/(1 + d*x)^3 -
 (48*a*b*d^3*(1 - d*x)^3)/(1 + d*x)^3 - (9*c^2*(1 - d*x)^2)/(1 + d*x)^2 - (
80*b*c*d*(1 - d*x)^2)/(1 + d*x)^2 + (12*b^2*d^2*(1 - d*x)^2)/(1 + d*x)^2 +
(24*a*c*d^2*(1 - d*x)^2)/(1 + d*x)^2 - (144*a*b*d^3*(1 - d*x)^2)/(1 + d*x)^
2 + (9*c^2*(1 - d*x))/(1 + d*x) - (80*b*c*d*(1 - d*x))/(1 + d*x) - (12*b^2*
d^2*(1 - d*x))/(1 + d*x) - (24*a*c*d^2*(1 - d*x))/(1 + d*x) - (144*a*b*d^3*
(1 - d*x))/(1 + d*x)))/(12*d^5*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))^4) +
 ((-3*c^2 - 4*b^2*d^2 - 8*a*c*d^2 - 8*a^2*d^4)*ArcTan[Sqrt[1 - d*x]/Sqrt[1
+ d*x]])/(4*d^5)
```

fricas [A] time = 0.42, size = 134, normalized size = 0.81

$$-\frac{\left(6\,c^2d^3x^3+16\,bcd^3x^2+48\,abd^3+32\,bcd+3\left(4\left(b^2+2\,ac\right)d^3+3\,c^2d\right)x\right)\sqrt{dx+1}\,\sqrt{-dx+1}+6\left(8\,a^2d^4+4\left(b^2+2\,ac\right)d^2+3\,c^2\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{24\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/24*((6*c^2*d^3*x^3 + 16*b*c*d^3*x^2 + 48*a*b*d^3 + 32*b*c*d + 3*(4*(b^2
+ 2*a*c)*d^3 + 3*c^2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*a^2*d^4 + 4*
(b^2 + 2*a*c)*d^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x))
)/d^5
```

giac [A] time = 0.42, size = 196, normalized size = 1.18

$$-\frac{\left((dx+1)\left(2(dx+1)\left(\frac{3(dx+1)c^2}{d^4}+\frac{8bcd^{17}-9c^2d^{16}}{d^{20}}\right)+\frac{12b^2d^{18}+24acd^{18}-32bcd^{17}+27c^2d^{16}}{d^{20}}\right)+\frac{3\left(16abd^{19}-4b^2d^{18}-8acd^{18}+16bcd^{17}-5c^2d^{16}\right)}{d^{20}}\right)\sqrt{dx+1}\sqrt{-dx+1}-\frac{6\left(8a^2d^4+4b^2d^2+8acd^2+3c^2\right)\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{d^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x + 1)*c^2/d^4 + (8*b*c*d^17 - 9*c^2*d
^16)/d^20) + (12*b^2*d^18 + 24*a*c*d^18 - 32*b*c*d^17 + 27*c^2*d^16)/d^20)
+ 3*(16*a*b*d^19 - 4*b^2*d^18 - 8*a*c*d^18 + 16*b*c*d^17 - 5*c^2*d^16)/d^20
)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6*(8*a^2*d^4 + 4*b^2*d^2 + 8*a*c*d^2 + 3*c
^2)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4)/d
```

maple [C] time = 0.03, size = 291, normalized size = 1.75

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(6\sqrt{-d^2x^2+1}\,c^2d^3x^3\mathrm{csgn}(d)+16\sqrt{-d^2x^2+1}\,bc\,d^3x^2\mathrm{csgn}(d)-24a^2d^4\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+24\sqrt{-d^2x^2+1}\,ac\,d^3x\,\mathrm{csgn}(d)+12\sqrt{-d^2x^2+1}\,b^2d^3x\,\mathrm{csgn}(d)+48\sqrt{-d^2x^2+1}\,ab\,d^3\mathrm{csgn}(d)-24ac\,d^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-12b^2d^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+9\sqrt{-d^2x^2+1}\,c^2dx\,\mathrm{csgn}(d)+32\sqrt{-d^2x^2+1}\,bcd\,\mathrm{csgn}(d)-9c^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right)\mathrm{csgn}(d)}{24\sqrt{-d^2x^2+1}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/24*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(6*csgn(d)*x^3*c^2*d^3*(-d^2*x^2+1)^(1/2
)+16*csgn(d)*x^2*b*c*d^3*(-d^2*x^2+1)^(1/2)+24*csgn(d)*d^3*(-d^2*x^2+1)^(1/
2)*x*a*c+12*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*b^2+48*(-d^2*x^2+1)^(1/2)*csgn
(d)*d^3*a*b-24*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*a^2*d^4+9*csgn(d)*d
*(-d^2*x^2+1)^(1/2)*x*c^2+32*(-d^2*x^2+1)^(1/2)*csgn(d)*d*b*c-24*arctan(1/(
-d^2*x^2+1)^(1/2)*d*x*csgn(d))*a*c*d^2-12*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*c
sgn(d))*b^2*d^2-9*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*c^2)*csgn(d)/d^5
/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 171, normalized size = 1.03

$$-\frac{\sqrt{-d^2x^2+1}\,c^2x^3}{4d^2}-\frac{2\sqrt{-d^2x^2+1}\,bcx^2}{3d^2}+\frac{a^2\arcsin(dx)}{d}-\frac{2\sqrt{-d^2x^2+1}\,ab}{d^2}-\frac{\sqrt{-d^2x^2+1}\,(b^2+2ac)x}{2d^2}-\frac{3\sqrt{-d^2x^2+1}\,c^2x}{8d^4}+\frac{(b^2+2ac)\arcsin(dx)}{2d^3}-\frac{4\sqrt{-d^2x^2+1}\,bc}{3d^4}+\frac{3c^2\arcsin(dx)}{8d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-d^2*x^2 + 1)*c^2*x^3/d^2 - 2/3*sqrt(-d^2*x^2 + 1)*b*c*x^2/d^2 +
a^2*arcsin(d*x)/d - 2*sqrt(-d^2*x^2 + 1)*a*b/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(
b^2 + 2*a*c)*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*c^2*x/d^4 + 1/2*(b^2 + 2*a*c)*a
rcsin(d*x)/d^3 - 4/3*sqrt(-d^2*x^2 + 1)*b*c/d^4 + 3/8*c^2*arcsin(d*x)/d^5
```

mupad [B] time = 13.85, size = 897, normalized size = 5.40

$$-\frac{\frac{\left(\sqrt{1-dx}-1\right)^{15}\left(2b^2d^2+\frac{3c^2}{2}+4acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{15}}+\frac{\left(\sqrt{1-dx}-1\right)^{3}\left(6b^2d^2-\frac{23c^2}{2}+12acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{3}}-\frac{\left(\sqrt{1-dx}-1\right)^{13}\left(6b^2d^2-\frac{23c^2}{2}+12acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{13}}+\frac{\left(\sqrt{1-dx}-1\right)^{5}\left(30b^2d^2+\frac{333c^2}{2}+60acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{5}}-\frac{\left(\sqrt{1-dx}-1\right)^{11}\left(30b^2d^2+\frac{333c^2}{2}+60acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{11}}+\frac{\left(\sqrt{1-dx}-1\right)^{7}\left(22b^2d^2-\frac{671c^2}{2}+44acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{7}}-\frac{\left(\sqrt{1-dx}-1\right)^{9}\left(22b^2d^2-\frac{671c^2}{2}+44acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{9}}+\frac{\left(\sqrt{1-dx}-1\right)^{4}\left(96abd^3+128bcd\right)}{\left(\sqrt{dx+1}-1\right)^{4}}+\frac{\left(\sqrt{1-dx}-1\right)^{12}\left(96abd^3+128bcd\right)}{\left(\sqrt{dx+1}-1\right)^{12}}+\frac{\left(\sqrt{1-dx}-1\right)^{8}\left(320abd^3+\frac{256bcd}{3}\right)}{\left(\sqrt{dx+1}-1\right)^{8}}+\frac{\left(\sqrt{1-dx}-1\right)^{6}\left(240abd^3+\frac{512bcd}{3}\right)}{\left(\sqrt{dx+1}-1\right)^{6}}+\frac{\left(\sqrt{1-dx}-1\right)^{10}\left(240abd^3+\frac{512bcd}{3}\right)}{\left(\sqrt{dx+1}-1\right)^{10}}-\frac{\left(\sqrt{1-dx}-1\right)\left(2b^2d^2+\frac{3c^2}{2}+4acd^2\right)}{\sqrt{dx+1}-1}+\frac{16abd^3\left(\sqrt{1-dx}-1\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+\frac{16abd^3\left(\sqrt{1-dx}-1\right)^{14}}{\left(\sqrt{dx+1}-1\right)^{14}}}{d^5+\frac{8d^5\left(\sqrt{1-dx}-1\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+\frac{28d^5\left(\sqrt{1-dx}-1\right)^4}{\left(\sqrt{dx+1}-1\right)^4}+\frac{56d^5\left(\sqrt{1-dx}-1\right)^6}{\left(\sqrt{dx+1}-1\right)^6}+\frac{70d^5\left(\sqrt{1-dx}-1\right)^8}{\left(\sqrt{dx+1}-1\right)^8}+\frac{56d^5\left(\sqrt{1-dx}-1\right)^{10}}{\left(\sqrt{dx+1}-1\right)^{10}}+\frac{28d^5\left(\sqrt{1-dx}-1\right)^{12}}{\left(\sqrt{dx+1}-1\right)^{12}}+\frac{8d^5\left(\sqrt{1-dx}-1\right)^{14}}{\left(\sqrt{dx+1}-1\right)^{14}}+\frac{d^5\left(\sqrt{1-dx}-1\right)^{16}}{\left(\sqrt{dx+1}-1\right)^{16}}}-\frac{\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^2/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((((1 - d*x)^(1/2) - 1)^15*((3*c^2)/2 + 2*b^2*d^2 + 4*a*c*d^2))/((d*x + 1
)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^3*(6*b^2*d^2 - (23*c^2)/2 + 12*a*c
```

```
*d^2))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/2) - 1)^13*(6*b^2*d^2 - (23
*c^2)/2 + 12*a*c*d^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*x)^(1/2) - 1)^5*
((333*c^2)/2 + 30*b^2*d^2 + 60*a*c*d^2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d
*x)^(1/2) - 1)^11*((333*c^2)/2 + 30*b^2*d^2 + 60*a*c*d^2))/((d*x + 1)^(1/2)
 - 1)^11 + (((1 - d*x)^(1/2) - 1)^7*(22*b^2*d^2 - (671*c^2)/2 + 44*a*c*d^2)
)/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^9*(22*b^2*d^2 - (671*c^2
)/2 + 44*a*c*d^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*x)^(1/2) - 1)^4*(128*
b*c*d + 96*a*b*d^3))/((d*x + 1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^12*(1
28*b*c*d + 96*a*b*d^3))/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^(1/2) - 1)^8
*((256*b*c*d)/3 + 320*a*b*d^3))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2)
 - 1)^6*((512*b*c*d)/3 + 240*a*b*d^3))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x
)^(1/2) - 1)^10*((512*b*c*d)/3 + 240*a*b*d^3))/((d*x + 1)^(1/2) - 1)^10 - (
((1 - d*x)^(1/2) - 1)*((3*c^2)/2 + 2*b^2*d^2 + 4*a*c*d^2))/((d*x + 1)^(1/2)
 - 1) + (16*a*b*d^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (16*
a*b*d^3*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14)/(d^5 + (8*d^5*(
(1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((1 - d*x)^(1/2)
- 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x +
1)^(1/2) - 1)^6 + (70*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8
+ (56*d^5*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^5*((1
- d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (8*d^5*((1 - d*x)^(1/2) -
1)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)
^(1/2) - 1)^16) - (atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))*(3*c^2
 + 8*a^2*d^4 + 4*b^2*d^2 + 8*a*c*d^2))/(2*d^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.545 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1815, 641, 216}

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\left(-c-2ad^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{\left(c+2ad^2\right)\sin^{-1}(dx)}{2d^3}$$

Mathematica [**A**] time = 0.03, size = 45, normalized size = 0.71

$$\frac{\left(2ad^2+c\right)\sin^{-1}(dx) - d\sqrt{1-d^2x^2}\,(2b+cx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*(2*b + c*x)*Sqrt[1 - d^2*x^2]) + (c + 2*a*d^2)*ArcSin[d*x])/(2*d^3)
```

IntegrateAlgebraic [**A**] time = 0.00, size = 117, normalized size = 1.86

$$\frac{\left(-2ad^2-c\right)\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d^3} - \frac{\sqrt{1-dx}\left(\frac{2bd(1-dx)}{dx+1}+2bd-\frac{c(1-dx)}{dx+1}+c\right)}{d^3\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((Sqrt[1 - d*x]*(c + 2*b*d - (c*(1 - d*x))/(1 + d*x) + (2*b*d*(1 - d*x))/(
1 + d*x)))/(d^3*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))^2)) + ((-c - 2*a*d^
2)*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d^3
```

fricas [**A**] time = 0.42, size = 67, normalized size = 1.06

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{-dx+1}\,+2\left(2\,ad^2+c\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

giac [**A**] time = 0.27, size = 76, normalized size = 1.21

$$-\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\left(2\,ad^2+c\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6
) - 2*(2*a*d^2 + c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)/d
```

maple [C] time = 0.02, size = 117, normalized size = 1.86

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-2a\,d^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+\sqrt{-d^2x^2+1}\,cdx\,\mathrm{csgn}(d)+2\sqrt{-d^2x^2+1}\,bd\,\mathrm{csgn}(d)-c\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right)\mathrm{csgn}(d)}{2\sqrt{-d^2x^2+1}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c-2*a
rctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2
)*b-arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

maxima [A] time = 0.97, size = 57, normalized size = 0.90

$$\frac{a\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,cx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,b}{d^2}+\frac{c\arcsin(dx)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

mupad [B] time = 7.76, size = 232, normalized size = 3.68

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,c\,\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}-\frac{14\,c\,\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,c\,\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}-\frac{2\,c\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

sympy [C] time = 49.71, size = 282, normalized size = 4.48

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{bG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{icG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{cG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) + a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
 ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d) - I*b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/
4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - b*meijerg(((-1, -3/4,
-1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((-3/4, -1/4), (-1/2, -1
/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)
*d**3) + c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-
3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.546 $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=282

$$\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}$$

Rubi [A] time = 0.52, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {899, 985, 725, 206}

$$\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] -((Sqrt[2]*c*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*
c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt
[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2])) + (
Sqrt[2]*c*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2
 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[b^
2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c^2-\left(b-\sqrt{b^2-4ac}\right)^2d^2-x^2}\,dx, x, \frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}} + \frac{(2c)\,\text{Sub}\cdots}{\cdots}$$

$$= -\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}} + \frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{\cdots}{\sqrt{2}\cdots}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2\cdots}}$$

Mathematica [A] time = 0.56, size = 260, normalized size = 0.92

$$\frac{2\sqrt{2}\,c\left(\frac{\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{1-d^2x^2}\sqrt{-2bd^2\left(\sqrt{b^2-4ac}+b\right)+4acd^2+4c^2}}\right)}{2\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}} - \frac{\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{1-d^2x^2}\sqrt{2bd^2\left(\sqrt{b^2-4ac}-b\right)+4acd^2+4c^2}}\right)}{2\sqrt{bd^2\left(\sqrt{b^2-4ac}-b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*Sqrt[2]*c*(-1/2*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^
2 + 4*a*c*d^2 + 2*b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]/Sqrt[
2*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2] + ArcTanh[(2*c + (b + S
qrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^2 + 4*a*c*d^2 - 2*b*(b + Sqrt[b^2 - 4*a*
c])*d^2]*Sqrt[1 - d^2*x^2])]/(2*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2])))/Sqrt[b^2 - 4*a*c]
```

IntegrateAlgebraic [A] time = 2.00, size = 372, normalized size = 1.32

$$\frac{\left(d\sqrt{b^2-4ac}\sqrt{ad^2-bd+c}+2c\sqrt{ad^2-bd+c}-bd\sqrt{ad^2-bd+c}\right)\tan^{-1}\left(\frac{\sqrt{1-dx}\sqrt{ad^2-bd+c}}{\sqrt{dx+1}\sqrt{-d\sqrt{b^2-4ac}+ad^2-c}}\right)}{\sqrt{b^2-4ac}\left(-ad^2+bd-c\right)\sqrt{-d\sqrt{b^2-4ac}+ad^2-c}} + \frac{\left(d\sqrt{b^2-4ac}\sqrt{ad^2-bd+c}-2c\sqrt{ad^2-bd+c}+bd\sqrt{ad^2-bd+c}\right)\tan^{-1}\left(\frac{\sqrt{1-dx}\sqrt{ad^2-bd+c}}{\sqrt{dx+1}\sqrt{d\sqrt{b^2-4ac}+ad^2-c}}\right)}{\sqrt{b^2-4ac}\left(-ad^2+bd-c\right)\sqrt{d\sqrt{b^2-4ac}+ad^2-c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] ((2*c*Sqrt[c - b*d + a*d^2] - b*d*Sqrt[c - b*d + a*d^2] + Sqrt[b^2 - 4*a*c]
*d*Sqrt[c - b*d + a*d^2])*ArcTan[(Sqrt[c - b*d + a*d^2]*Sqrt[1 - d*x])/(Sqr
t[-c - Sqrt[b^2 - 4*a*c]*d + a*d^2]*Sqrt[1 + d*x])])/(Sqrt[b^2 - 4*a*c]*(-c
 + b*d - a*d^2)*Sqrt[-c - Sqrt[b^2 - 4*a*c]*d + a*d^2]) + ((-2*c*Sqrt[c - b
*d + a*d^2] + b*d*Sqrt[c - b*d + a*d^2] + Sqrt[b^2 - 4*a*c]*d*Sqrt[c - b*d
+ a*d^2])*ArcTan[(Sqrt[c - b*d + a*d^2]*Sqrt[1 - d*x])/(Sqrt[-c + Sqrt[b^2
- 4*a*c]*d + a*d^2]*Sqrt[1 + d*x])])/(Sqrt[b^2 - 4*a*c]*(-c + b*d - a*d^2)*
Sqrt[-c + Sqrt[b^2 - 4*a*c]*d + a*d^2])
```

fricas [B] time = 0.61, size = 4313, normalized size = 15.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b
^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^
2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*
a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 -
6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^
3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)
*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*
c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((
a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^
4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*
c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^
6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c
- 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2
*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b
^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*
c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(
-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3
- (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8
- 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*
a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*d^2)))/x) - 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 -
 ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^
2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c +
8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24
*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*
a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4
*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(
b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*
a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^
2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^
2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - sqrt(
2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c
 + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4
/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2
*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b
^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3
*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^
3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*
d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 +
b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2
*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*
c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x) - 1/2*sqrt(2)*sqrt(
-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3
- (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8
- 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*
a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^
2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^
4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c
)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^
6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3
 + 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a
*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4
```

```
*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3
*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2
*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - ((
a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d
^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*
c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*
a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*
b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((
a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*
d^2)))/x) + 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^
3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*
d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 +
b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2
*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*
c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sq
rt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4
 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqr
t(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*
d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d
^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - sqrt(2)*(((a^3*b^3 - 4
*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d
^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^
5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 +
(b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*
c^3 + 8*a^2*c^4)*d^2))*x - ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d
^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^
2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4
*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5
 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b
^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (
b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x)
```

giac [B] time = 1.50, size = 684, normalized size = 2.43

$$-\frac{\left(ad^2-bd+c\right)\left(\frac{\left(ad^2-c+\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}\right)d}{ad^2-bd+c}-d\right)\sqrt{\frac{ad^2-c+\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}}{ad^2-bd+c}}\arctan\left(-\frac{\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}}{2\sqrt{\frac{ad^2-c+\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}}{ad^2-bd+c}}}\right)}{\left(ad^2-c+\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}\right)\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}}+\frac{\left(ad^2-bd+c\right)\left(\frac{\left(ad^2-c-\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}\right)d}{ad^2-bd+c}-d\right)\sqrt{\frac{ad^2-c-\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}}{ad^2-bd+c}}\arctan\left(-\frac{\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}}{2\sqrt{\frac{ad^2-c-\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}}{ad^2-bd+c}}}\right)}{\left(ad^2-c-\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}\right)\sqrt{-\left(ad^2+bd+c\right)\left(ad^2-bd+c\right)+\left(ad^2-c\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*d^2 - b*d + c)*((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c)
+ (a*d^2 - c)^2))*d/(a*d^2 - b*d + c) - d)*sqrt((a*d^2 - c + sqrt(-(a*d^2 +
 b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c))*arctan(-1/
2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt
(-d*x + 1)))/sqrt((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) +
(a*d^2 - c)^2))/(a*d^2 - b*d + c)))/((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(
a*d^2 - b*d + c) + (a*d^2 - c)^2))*sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c
) + (a*d^2 - c)^2)) + (a*d^2 - b*d + c)*((a*d^2 - c - sqrt(-(a*d^2 + b*d +
c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))*d/(a*d^2 - b*d + c) - d)*sqrt((a*d^2
 - c - sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 -
 b*d + c))*arctan(-1/2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x
 + 1)/(sqrt(2) - sqrt(-d*x + 1)))/sqrt((a*d^2 - c - sqrt(-(a*d^2 + b*d + c)
*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c)))/((a*d^2 - c - sqrt
(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))*sqrt(-(a*d^2 + b*d
+ c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))
```

maple [C] time = 0.14, size = 1759, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -32*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*c^2*(ln(2*(x*b*d^2-(-4*a*c+b^2)^
(1/2)*x*d^2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d
^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(2*c*x-(-4*a*c+b^2
)^(1/2)+b))*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c
+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)-ln(2*((-4*a*c+b^2)^(1/2)*x*d^2+x*b*d^
2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*
c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))
*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2
)-2*a*c+b^2)/a^2/c^2)^(1/2)+2*ln(2*(x*b*d^2-(-4*a*c+b^2)^(1/2)*x*d^2+(-d^2*
x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^
(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(2*c*x-(-4*a*c+b^2)^(1/2)+b))*a*c*d^
2*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-
b^2)/a^2/c^2)^(1/2)-ln(2*(x*b*d^2-(-4*a*c+b^2)^(1/2)*x*d^2+(-d^2*x^2+1)^(1/
2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*
c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(2*c*x-(-4*a*c+b^2)^(1/2)+b))*b^2*d^2*(-(b*(-4
*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c
^2)^(1/2)-2*ln(2*((-4*a*c+b^2)^(1/2)*x*d^2+x*b*d^2+(-d^2*x^2+1)^(1/2)*(-(b*
(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^
2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*a*c*d^2*(-(b*(-4*a*c+b^2)
^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2
)+ln(2*((-4*a*c+b^2)^(1/2)*x*d^2+x*b*d^2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^
2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/
2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*b^2*d^2*(-(b*(-4*a*c+b^2)^(1/2)+2*a
*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)+ln(2*(x*
b*d^2-(-4*a*c+b^2)^(1/2)*x*d^2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2
*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)
/(2*c*x-(-4*a*c+b^2)^(1/2)+b))*c^2*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^
2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)-ln(2*((-4*a*c+b^2)^(1/
2)*x*d^2+x*b*d^2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a
^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a
*c+b^2)^(1/2)))*c^2*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*
c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2))/(-d^2*x^2+1)^(1/2)/(b*d-d*(-4*a*c+b
^2)^(1/2)+2*c)/(d*(-4*a*c+b^2)^(1/2)+b*d+2*c)/(b*d-d*(-4*a*c+b^2)^(1/2)-2*c
)/(-4*a*c+b^2)^(1/2)/(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a
*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)/(d*(-4*a*c+b^2)^(1/2)+b*d-2*c)/(-(b
*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a
^2/c^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)\sqrt{dx+1}\,\sqrt{-dx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

mupad [B] time = 82.37, size = 33018, normalized size = 117.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] - atan(((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^
2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 3
```

2*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d
^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16
*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 +
16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^
(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) +
8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2
*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32
*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^
2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*
a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 +
16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(
1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8
*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*
c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*
a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2
+ b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a
^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 1
6*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1
/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*
d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 3628388371660
8*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12
+ 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392
*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12
- 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a
^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*
d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 -
263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 3999473
5460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792
*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c
^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1
/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340
029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5
*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 +
42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*
a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c
^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 -
57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 1985992
8776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b
^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^
5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 158054
7964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4
*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 4831838208
0*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14
- 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a
^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^1
0 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 10812
58016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184
*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c
^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2*(1778116
460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c
^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 2961
8094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b
*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^1
0 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 1556
4961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1
/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 574494825
51296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^1
3 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 12884901888
0*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*

c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 12197707 12064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^ 8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 123695058124 8*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072* a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 3994319 58528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5* b^3*c^2*d^14) - 2147483648*a*b^8*d^12 + (((1 - d*x)^(1/2) - 1)*(26800595927 04*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^ 9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 644245094 40*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4 *d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 4 29496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^ 8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266 704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6 *d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 6871947 6736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c ^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 257698 03776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244096*a^3*c^6*d^8 + 6768868458496*a^4*c ^5*d^10 + 8074538516480*a^5*c^4*d^12 - 137438953472*a*b^2*c^6*d^6 + 3049426 78016*a*b^4*c^4*d^8 - 164282499072*a*b^6*c^2*d^10 - 17179869184*a^2*b^6*c*d ^12 - 15032385536*a^4*b^4*c*d^14 - 1030792151040*a^2*b^2*c^5*d^8 + 11338713 66144*a^2*b^4*c^3*d^10 - 3599182594048*a^3*b^2*c^4*d^10 + 1028644667392*a^3 *b^4*c^2*d^12 - 5720896438272*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^ 14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 348751344435 2*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d ^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 1047972020 2240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728* a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 16 32087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^ 2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b* c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 3006 4771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3 *c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 1073741824*a*b^6*d^12 + 687194 76736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233 382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^ 3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 1 0960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^ 2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 31525 05995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450 944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2 *d^12) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960* a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 14 6028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^1 0 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8)*1i + (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6* a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 +

10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*(1073741824*a*b^6*d^12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889

785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) + 2147483648*a*b^8*d^12 - (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - 2147483648*a^3*b^6*d^14 - 549755813888*a^2*c^7*d^6 + 755914244096*a^3*c^6*d^8 - 6768868458496*a^4*c^5*d^10 - 8074538516480*a^5*c^4*d^12 + 137438953472*a*b^2*c^6*d^6 - 304942678016*a*b^4*c^4*d^8 + 164282499072*a*b^6*c^2*d^10 + 17179869184*a^2*b^6*c*d^12 + 15032385536*a^4*b^4*c*d^14 + 1030792151040*a^2*b^2*c^5*d^8 - 1133871366144*a^2*b^4*c^3*d^10 + 3599182594048*a^3*b^2*c^4*d^10 - 1028644667392*a^3*b^4*c^2*d^12 + 5720896438272*a^4*b^2*c^3*d^12 - 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8)*1i)/((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^

(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2 *c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32 *a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^ 2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16* a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8 *a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2* c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32* a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a ^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 1 6*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1 /2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8* a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c ^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a ^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741 824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283 883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^ 6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 22677 42732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c ^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934 592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4 *c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 417 3634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 14023927214 8992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6 *b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 10 73741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 115448720916 48*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d ^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 68719476 7360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b ^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^1 1 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 374 5211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 34359738368 0*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 21 47483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224* a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 11 5964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8 *c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 54975581 3888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2* c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5 738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208 320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b ^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704* a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^1 0 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 4724464025 6*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5 *d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 37795712 2048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(3 0236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5 *c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778 116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2* b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d

^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672
924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272
*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 773
09411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^
2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 254262
0639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^
3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 305
8016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768
*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) - 2147483648*a*b^8*d^12
+ (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*
b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 1724
8588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b
*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11
- 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 1889785
61024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147
483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 +
44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a
^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2
045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*
b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^
10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 158
57019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1
/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244
096*a^3*c^6*d^8 + 6768868458496*a^4*c^5*d^10 + 8074538516480*a^5*c^4*d^12 -
137438953472*a*b^2*c^6*d^6 + 304942678016*a*b^4*c^4*d^8 - 164282499072*a*b
^6*c^2*d^10 - 17179869184*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 103
0792151040*a^2*b^2*c^5*d^8 + 1133871366144*a^2*b^4*c^3*d^10 - 3599182594048
*a^3*b^2*c^4*d^10 + 1028644667392*a^3*b^4*c^2*d^12 - 5720896438272*a^4*b^2*
c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(139500
53777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*
c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360
777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a
^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(1511828
4881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^
11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388
352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2
*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 22333829939
2*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10
+ 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*
a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d
^12) + 1073741824*a*b^6*d^12 + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) -
1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616
*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7
- 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*
d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b
^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312
512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^1
0 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416
*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 +
149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*
b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)^2*(214
7483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4
*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^
5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^
(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 14602
8888064*a*b*c^4*d^8) - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b
^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^
3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/

2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396
983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4
*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(
16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2
 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2))
)^(1/2)*(1073741824*a*b^6*d^12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b
^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*
d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*
a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^
2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b
^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2
*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^
4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d
^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 -
 d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*
d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 472446
40256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*
c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 177
21035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026
624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b
^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 +
b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*
c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a
^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)
*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^1
4 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a
^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 +
29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*
b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 6
6571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*
b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8
 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263
779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 3999473546
0352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^
5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*
d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2)
 - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029
796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*
c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42
949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2
*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*
d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57
999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 1985992877
6704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*
c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b
^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 158054796
4928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^
14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a
*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8
589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*
b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 -
 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 10812580
16768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^
5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*
d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)*(302365697638
40*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 -
 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a

*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7
 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 149464
8619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a
^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^
3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*
b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 32
98534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5
*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 61
8475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*
a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^
3*d^12 - 240518168576*a^5*b^3*c^2*d^14) + 2147483648*a*b^8*d^12 - (((1 - d*
x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 -
 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*
a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 +
 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 141733920
76800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7
*c*d^11))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*
b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401
664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12
 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 204547817472
0*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14
- 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 742084
44940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832
*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2
- 2147483648*a^3*b^6*d^14 - 549755813888*a^2*c^7*d^6 + 755914244096*a^3*c^6
*d^8 - 6768868458496*a^4*c^5*d^10 - 8074538516480*a^5*c^4*d^12 + 1374389534
72*a*b^2*c^6*d^6 - 304942678016*a*b^4*c^4*d^8 + 164282499072*a*b^6*c^2*d^10
 + 17179869184*a^2*b^6*c*d^12 + 15032385536*a^4*b^4*c*d^14 + 1030792151040*
a^2*b^2*c^5*d^8 - 1133871366144*a^2*b^4*c^3*d^10 + 3599182594048*a^3*b^2*c^
4*d^10 - 1028644667392*a^3*b^4*c^2*d^12 + 5720896438272*a^4*b^2*c^3*d^12 -
25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^
2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 +
14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a
^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*
d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2
*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 377957
1220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2
*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 +
 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4
*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 123695058
1248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d
^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 6871
9476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 22
33382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*
b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x
 + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 +
(((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 +
 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*
b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 315
2505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2
) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 64424
50944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c
^2*d^12) + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a
^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 214748364
8*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8) +
 283467841536*a*c^4*d^8 + (2*((1 - d*x)^(1/2) - 1)^2*(519691042816*a*c^4*d^
8 + 1073741824*a*b^2*c^2*d^10))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a*b^2*
c^2*d^10 + (34359738368*a*b*c^3*d^9*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2)
 - 1)))*(-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) +

8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*2i - atan(((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 2149201634918 4*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c

^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) - 2147483648*a*b^8*d^12 + (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244096*a^3*c^6*d^8 + 6768868458496*a^4*c^5*d^10 + 8074538516480*a^5*c^4*d^12 - 137438953472*a*b^2*c^6*d^6 + 304942678016*a*b^4*c^4*d^8 - 164282499072*a*b^6*c^2*d^10 - 17179869184*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 1030792151040*a^2*b^2*c^5*d^8 + 1133871366144*a^2*b^4*c^3*d^10 - 3599182594048*a^3*b^2*c^4*d^10 + 1028644667392*a^3*b^4*c^2*d^12 - 5720896438272*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 1073741824*a*b^6*d^12 + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(18897856

10240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/(
(d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^
10 + 146028888064*a*b*c^4*d^8)*1i + (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4
+ b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c
^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*(((
(1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d
^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*
b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^
2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 3
2*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*
a*b^4*c*d^2)))^(1/2)*(1073741824*a*b^6*d^12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*
d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(1
6*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2
+ 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))
^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) +
8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^
2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 3
2*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d
^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16
*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 +
16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^
(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 284498633687
04*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*
d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 4724464
0256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*
c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10
+ 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 37795
7122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2
+ b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2
)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c
^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c
d^2)))^(1/2)((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 214748364
8*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 362
83883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^
6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 76
0209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b
^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 181419
41858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^
3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c
^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12
+ 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 247
8196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184
*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1
- d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*
d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758
715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*
c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720
238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488
*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*
c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13
- 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503
724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 107
3741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^
8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 116135915683
84*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 +
48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b
^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 6184
75290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b

^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^
10 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 2149
2016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712
*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)*
(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a
^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 17
78116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^
2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5
*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 46
72924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 35734127902
72*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 7
7309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*
c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542
620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*
c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3
058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 17523466567
68*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) + 2147483648*a*b^8*d^1
2 - (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^
2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17
248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5
*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^1
1 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 18897
8561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^2*(21
47483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6
+ 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928
*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 -
2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^
4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*
d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 1
5857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^
(1/2) - 1)^2 - 2147483648*a^3*b^6*d^14 - 549755813888*a^2*c^7*d^6 + 7559142
44096*a^3*c^6*d^8 - 6768868458496*a^4*c^5*d^10 - 8074538516480*a^5*c^4*d^12
+ 137438953472*a*b^2*c^6*d^6 - 304942678016*a*b^4*c^4*d^8 + 164282499072*a
*b^6*c^2*d^10 + 17179869184*a^2*b^6*c*d^12 + 15032385536*a^4*b^4*c*d^14 + 1
030792151040*a^2*b^2*c^5*d^8 - 1133871366144*a^2*b^4*c^3*d^10 + 35991825940
48*a^3*b^2*c^4*d^10 - 1028644667392*a^3*b^4*c^2*d^12 + 5720896438272*a^4*b^
2*c^3*d^12 - 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(1395
0053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^
5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 3
60777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240
*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118
284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*
d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 99299643
88352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b
^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299
392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^1
0 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 25769803776
0*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2
*d^12) + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3
*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 1245
54051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c
*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^
3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 226774273228
8*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 254
6915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4
*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((
d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2
*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676
416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 1

88978561024*a^2*c^4*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1
) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a
*b*c^4*d^8)*1i)/((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3
)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2
 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*
c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^
2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^
2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*
c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*
c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2
- 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c
*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2
 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2
)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c
^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c
d^2)))^(1/2)((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 -
 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*
d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2
+ b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)
/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^
3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*
d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648
*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 3628
3883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6
*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760
209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^
8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 1814194
1858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3
*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^
4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12
+ 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478
196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*
a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1
- d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d
^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 421937587
15904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c
^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 107202
38370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*
a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c
^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13
- 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 1675037
24544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073
741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8
 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 1161359156838
4*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 4
8318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^
6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 61847
5290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^
6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^1
0 - 1081258016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492
016349184*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*
a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2
*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 176737904230
4*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^
12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244
096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^
5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^

12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 - 30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 + 910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^3*c^2*d^14) - 2147483648*a*b^8*d^12 + (((1 - d*x)^(1/2) - 1)*(2680059592704*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9 + 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 85796266704896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 68719476736*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 + 2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a^3*b^6*d^14 + 549755813888*a^2*c^7*d^6 - 755914244096*a^3*c^6*d^8 + 6768868458496*a^4*c^5*d^10 + 8074538516480*a^5*c^4*d^12 - 137438953472*a*b^2*c^6*d^6 + 304942678016*a*b^4*c^4*d^8 - 164282499072*a*b^6*c^2*d^10 - 17179869184*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 1030792151040*a^2*b^2*c^5*d^8 + 1133871366144*a^2*b^4*c^3*d^10 - 3599182594048*a^3*b^2*c^4*d^10 + 1028644667392*a^3*b^4*c^2*d^12 - 5720896438272*a^4*b^2*c^3*d^12 + 25769803776*a^5*b^2*c^2*d^14) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^10 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 10479720202240*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 30064771072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 1073741824*a*b^6*d^12 + 68719476736*a*c^6*d^6 - (((1 - d*x)^(1/2) - 1)*(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 - 197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8 + 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 919123001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4

*d^9 + 146028888064*a*b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 34359738368*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8) - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(2147483648*a*b^3*c^2*d^10 + 42949672960*a^2*b*c^3*d^10 + 1709396983808*a*b*c^4*d^8))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*(1073741824*a*b^6*d^12 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1778116460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 29618094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^10 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 15564961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - (-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 36283883716608*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12 - 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 - 263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 39994735460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 + 42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 - 57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 19859928776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 1580547964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 48318382080*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^10 - 1799591297024*a^3*b^4*c^4*d

^10 + 5738076307456*a^4*b^2*c^5*d^10 - 1081258016768*a^3*b^6*c^2*d^12 + 824
6337208320*a^4*b^4*c^3*d^12 - 21492016349184*a^5*b^2*c^4*d^12 + 94918777241
6*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c
^2*d^16) + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 57449482551
296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^13
+ 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 128849018880*
a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*c^
4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 - 4
260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 1219770712
064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^8*
c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 1236950581248*
a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + 3298534883328*a^4*b*c^5*d^10 -
30064771072*a^2*b^7*c*d^12 + 2542620639232*a^5*b*c^4*d^12 + 30064771072*a^
4*b^5*c*d^14 + 481036337152*a^6*b*c^3*d^14 - 618475290624*a^2*b^3*c^5*d^8 +
910533066752*a^2*b^5*c^3*d^10 - 3058016714752*a^3*b^3*c^4*d^10 + 399431958
528*a^3*b^5*c^2*d^12 - 1752346656768*a^4*b^3*c^3*d^12 - 240518168576*a^5*b^
3*c^2*d^14) + 2147483648*a*b^8*d^12 - (((1 - d*x)^(1/2) - 1)*(2680059592704
*a*b^3*c^5*d^7 - 10720238370816*a^2*b*c^6*d^7 - 962072674304*a*b^5*c^3*d^9
+ 5772436045824*a^3*b*c^5*d^9 + 17248588660736*a^4*b*c^4*d^11 + 64424509440
*a^3*b^5*c*d^13 + 687194767360*a^5*b*c^3*d^13 + 2405181685760*a^2*b^3*c^4*d
^9 + 3221225472000*a^2*b^5*c^2*d^11 - 14173392076800*a^3*b^3*c^3*d^11 - 429
496729600*a^4*b^3*c^2*d^13 - 188978561024*a*b^7*c*d^11))/((d*x + 1)^(1/2) -
1) - (((1 - d*x)^(1/2) - 1)^2*(2147483648*a^3*b^6*d^14 - 2147483648*a*b^8*
d^12 - 18141941858304*a^2*c^7*d^6 + 44598940401664*a^3*c^6*d^8 + 8579626670
4896*a^4*c^5*d^10 + 23055384444928*a^5*c^4*d^12 + 4535485464576*a*b^2*c^6*d
^6 + 1267015352320*a*b^4*c^4*d^8 - 2045478174720*a*b^6*c^2*d^10 - 687194767
36*a^2*b^6*c*d^12 - 15032385536*a^4*b^4*c*d^14 - 16217796509696*a^2*b^2*c^5
*d^8 + 21371757264896*a^2*b^4*c^3*d^10 - 74208444940288*a^3*b^2*c^4*d^10 +
2832530931712*a^3*b^4*c^2*d^12 - 15857019256832*a^4*b^2*c^3*d^12 + 25769803
776*a^5*b^2*c^2*d^14))/((d*x + 1)^(1/2) - 1)^2 - 2147483648*a^3*b^6*d^14 -
549755813888*a^2*c^7*d^6 + 755914244096*a^3*c^6*d^8 - 6768868458496*a^4*c^5
*d^10 - 8074538516480*a^5*c^4*d^12 + 137438953472*a*b^2*c^6*d^6 - 304942678
016*a*b^4*c^4*d^8 + 164282499072*a*b^6*c^2*d^10 + 17179869184*a^2*b^6*c*d^1
2 + 15032385536*a^4*b^4*c*d^14 + 1030792151040*a^2*b^2*c^5*d^8 - 1133871366
144*a^2*b^4*c^3*d^10 + 3599182594048*a^3*b^2*c^4*d^10 - 1028644667392*a^3*b
^4*c^2*d^12 + 5720896438272*a^4*b^2*c^3*d^12 - 25769803776*a^5*b^2*c^2*d^14
) + (((1 - d*x)^(1/2) - 1)^2*(13950053777408*a^2*b*c^5*d^8 - 3487513444352*
a*b^3*c^4*d^8 + 1730871820288*a*b^5*c^2*d^10 + 14224931684352*a^3*b*c^4*d^1
0 + 47244640256*a^2*b^5*c*d^12 + 360777252864*a^4*b*c^3*d^12 - 104797202022
40*a^2*b^3*c^3*d^10 - 279172874240*a^3*b^3*c^2*d^12))/((d*x + 1)^(1/2) - 1)
^2 + (((1 - d*x)^(1/2) - 1)*(15118284881920*a^2*c^6*d^7 + 13606456393728*a^
3*c^5*d^9 - 1511828488192*a^4*c^4*d^11 - 3779571220480*a*b^2*c^5*d^7 + 1632
087572480*a*b^4*c^3*d^9 - 9929964388352*a^2*b^2*c^4*d^9 - 944892805120*a^2*
b^4*c^2*d^11 + 2095944040448*a^3*b^2*c^3*d^11 + 128849018880*a*b^6*c*d^11))
/((d*x + 1)^(1/2) - 1) - 223338299392*a*b^3*c^4*d^8 + 893353197568*a^2*b*c^
5*d^8 + 124554051584*a*b^5*c^2*d^10 + 1236950581248*a^3*b*c^4*d^10 + 300647
71072*a^2*b^5*c*d^12 + 257698037760*a^4*b*c^3*d^12 - 807453851648*a^2*b^3*c
^3*d^10 - 184683593728*a^3*b^3*c^2*d^12) + 68719476736*a*c^6*d^6 - (((1 - d
x)^(1/2) - 1)(231928233984*a*b^3*c^3*d^9 - 2233382993920*a^2*b*c^4*d^9 -
197568495616*a^3*b*c^3*d^11 + 124554051584*a^2*b^3*c^2*d^11 + 1340029796352
*a*b*c^5*d^7 - 21474836480*a*b^5*c*d^11))/((d*x + 1)^(1/2) - 1) + 687194767
360*a^2*c^5*d^8 + 1859720839168*a^3*c^4*d^10 + (((1 - d*x)^(1/2) - 1)^2*(10
73741824*a*b^6*d^12 - 2267742732288*a*c^6*d^6 + 10960756539392*a^2*c^5*d^8
+ 6000069312512*a^3*c^4*d^10 - 2546915606528*a*b^2*c^4*d^8 + 505732399104*a
*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 3152505995264*a^2*b^2*c^3*d^10
+ 9663676416*a^3*b^2*c^2*d^12))/((d*x + 1)^(1/2) - 1)^2 - 330712481792*a*b^
2*c^4*d^8 + 149250113536*a*b^4*c^2*d^10 - 6442450944*a^2*b^4*c*d^12 - 91912
3001344*a^2*b^2*c^3*d^10 + 9663676416*a^3*b^2*c^2*d^12) + (((1 - d*x)^(1/2)
- 1)*(1889785610240*a*c^5*d^7 - 188978561024*a^2*c^4*d^9 + 146028888064*a*

```
b^2*c^3*d^9))/((d*x + 1)^(1/2) - 1) - 2147483648*a*b^3*c^2*d^10 + 343597383
68*a^2*b*c^3*d^10 + 146028888064*a*b*c^4*d^8) + 283467841536*a*c^4*d^8 + (2
*((1 - d*x)^(1/2) - 1)^2*(519691042816*a*c^4*d^8 + 1073741824*a*b^2*c^2*d^1
0))/((d*x + 1)^(1/2) - 1)^2 + 2147483648*a*b^2*c^2*d^10 + (34359738368*a*b*
c^3*d^9*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))*(-(8*a*c^3 - 2*b^2*c
^2 + b^4*d^2 - b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d
^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3
*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4
*c*d^2)))^(1/2)*2i
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-dx+1}\sqrt{dx+1}\,(a+bx+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*x + 1)*sqrt(d*x + 1)*(a + b*x + c*x**2)), x)
```

3.547 $\int \frac{1}{\sqrt{1-dx}\,\sqrt{1+dx}\,\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=571

$$-\frac{c\left(-cd^2\left(-8a^2d^2-b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-\cdots}\right.}{\sqrt{2}\,\sqrt{1-d^2x^2}\,\sqrt{-bd^2\left(b-\cdots\right.}}\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\,\left(b^2d^2-\left(ad^2+c\right)^2\right)}$$

Rubi [A] time = 5.23, antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 975, 1034, 725, 206}

$$-\frac{c\left(-cd^2\left(-8a^2d^2-b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\,\sqrt{1-d^2x^2}\,\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\,\left(b^2d^2-\left(ad^2+c\right)^2\right)}+\frac{c\left(-4cd^2\left(b^2-2a^2d^2\right)-bd^2\left(\sqrt{b^2-4ac}+b\right)\left(c-ad^2\right)-2ab^2d^4+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\,\sqrt{1-d^2x^2}\,\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}\,\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{\sqrt{1-d^2x^2}\left(b\left(b^2d^2-c\left(3ad^2+c\right)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)\right)}{\left(b^2-4ac\right)\left(b^2d^2-\left(ad^2+c\right)^2\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] -(((b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)*Sqrt
[1 - d^2*x^2])/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^2)))
 - (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 - c*d^2*(5*b^
2 - b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2))*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c]
)*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*S
qrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b
*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) + (c*(4*c^3 + 12*a
*c^2*d^2 - 2*a*b^2*d^4 - b*(b + Sqrt[b^2 - 4*a*c])*d^2*(c - a*d^2) - 4*c*d^
2*(b^2 - 2*a^2*d^2))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]
*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
```

```
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-dx}\,\sqrt{1+dx}\,\left(a+bx+cx^2\right)^2}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^2\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)} - \frac{\int -\ldots}{\ldots}$$

$$= -\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)} + \frac{\left(c\left(\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)} - \frac{\left(c\left(\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)} - \frac{c\left(4\ldots\right.}{\ldots}$$

Mathematica [A] time = 1.28, size = 508, normalized size = 0.89

$$\frac{\frac{c\left(cd^2\left(8a^2d^2+b\sqrt{b^2-4ac}-5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{1-d^2x^2}\sqrt{2bd^2\left(\sqrt{b^2-4ac}-b\right)+4acd^2+4c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bd^2\left(\sqrt{b^2-4ac}-b\right)+2acd^2+2c^2}}+\frac{c\left(cd^2\left(-8a^2d^2+b\sqrt{b^2-4ac}+5b^2\right)+abd^4\left(b-\sqrt{b^2-4ac}\right)-12ac^2d^2-4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{1-d^2x^2}\sqrt{-2bd^2\left(\sqrt{b^2-4ac}+b\right)+4acd^2+4c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}+\frac{\sqrt{1-d^2x^2}\left(-bc\left(3ad^2+c\right)-2c^2x\left(ad^2+c\right)+b^3d^2+b^2cd^2x\right)}{a+x(b+cx)}}{\left(b^2-4ac\right)\left(\left(ad^2+c\right)^2-b^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] (((b^3*d^2 - b*c*(c + 3*a*d^2) + b^2*c*d^2*x - 2*c^2*(c + a*d^2)*x)*Sqrt[1
- d^2*x^2])/(a + x*(b + c*x)) + (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + Sqrt[b^
2 - 4*a*c])*d^4 + c*d^2*(-5*b^2 + b*Sqrt[b^2 - 4*a*c] + 8*a^2*d^2))*ArcTanh
[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^2 + 4*a*c*d^2 + 2*b*(-b +
Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqr
t[2*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2]) + (c*(-4*c^3 - 12*a*
```

```
c^2*d^2 + a*b*(b - Sqrt[b^2 - 4*a*c])*d^4 + c*d^2*(5*b^2 + b*Sqrt[b^2 - 4*a
*c] - 8*a^2*d^2))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^2
 + 4*a*c*d^2 - 2*b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[
2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2
]))/((b^2 - 4*a*c)*(-(b^2*d^2) + (c + a*d^2)^2))
```

IntegrateAlgebraic [A] time = 50.06, size = 1064, normalized size = 1.86

$$\frac{2d\sqrt{1-dx}\left(\frac{2(1-dx)c^3}{dx+1}-2c^3-2ad^2c^2-bdc^2+\frac{2ad^2(1-dx)c^2}{dx+1}-\frac{bd(1-dx)c^2}{dx+1}-3abd^3c+b^2d^2c-\frac{3abd^3(1-dx)c}{dx+1}-\frac{b^2d^2(1-dx)c}{dx+1}+b^3d^3+\frac{b^3d^3(1-dx)}{dx+1}\right)}{\left(b^2-4ac\right)\left(-ad^2+bd-c\right)\left(ad^2+bd+c\right)\sqrt{dx+1}\left(-ad^2-\frac{2a(1-dx)d^2}{dx+1}-\frac{a(1-dx)^2d^2}{(dx+1)^2}-bd+\frac{b(1-dx)^2d}{(dx+1)^2}-c+\frac{2c(1-dx)}{dx+1}-\frac{c(1-dx)^2}{(dx+1)^2}\right)}+\frac{\left(ab^3d^5-6a^2bcd^5-ab^2\sqrt{b^2-4ac}\,d^5+4a^2c\sqrt{b^2-4ac}\,d^5+8a^2c^2d^4-ab^2cd^4+abc\sqrt{b^2-4ac}\,d^4-4abc^2d^3+2b^3cd^3+6ac^2\sqrt{b^2-4ac}\,d^3-2b^2c\sqrt{b^2-4ac}\,d^3+12ac^3d^2-5b^2c^2d^2-bc^2\sqrt{b^2-4ac}\,d^2-2bc^3d+2c^3\sqrt{b^2-4ac}\,d+4c^4\right)\tan^{-1}\left(\frac{\sqrt{ad^2-bd+c}\,\sqrt{1-dx}}{\sqrt{ad^2-\sqrt{b^2-4ac}\,d-c}\,\sqrt{dx+1}}\right)}{\left(b^2-4ac\right)^{3/2}\left(ad^2-bd+c\right)^{3/2}\left(ad^2+bd+c\right)\sqrt{ad^2-\sqrt{b^2-4ac}\,d-c}}+\frac{\left(-ab^3d^5+6a^2bcd^5-ab^2\sqrt{b^2-4ac}\,d^5+4a^2c\sqrt{b^2-4ac}\,d^5-8a^2c^2d^4+ab^2cd^4+abc\sqrt{b^2-4ac}\,d^4+4abc^2d^3-2b^3cd^3+6ac^2\sqrt{b^2-4ac}\,d^3-2b^2c\sqrt{b^2-4ac}\,d^3-12ac^3d^2+5b^2c^2d^2-bc^2\sqrt{b^2-4ac}\,d^2+2bc^3d+2c^3\sqrt{b^2-4ac}\,d-4c^4\right)\tan^{-1}\left(\frac{\sqrt{ad^2-bd+c}\,\sqrt{1-dx}}{\sqrt{ad^2+\sqrt{b^2-4ac}\,d-c}\,\sqrt{dx+1}}\right)}{\left(b^2-4ac\right)^{3/2}\left(ad^2-bd+c\right)^{3/2}\left(ad^2+bd+c\right)\sqrt{ad^2+\sqrt{b^2-4ac}\,d-c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] (2*d*Sqrt[1 - d*x]*(-2*c^3 - b*c^2*d + b^2*c*d^2 - 2*a*c^2*d^2 + b^3*d^3 -
3*a*b*c*d^3 + (2*c^3*(1 - d*x))/(1 + d*x) - (b*c^2*d*(1 - d*x))/(1 + d*x) -
 (b^2*c*d^2*(1 - d*x))/(1 + d*x) + (2*a*c^2*d^2*(1 - d*x))/(1 + d*x) + (b^3
*d^3*(1 - d*x))/(1 + d*x) - (3*a*b*c*d^3*(1 - d*x))/(1 + d*x)))/((b^2 - 4*a
*c)*(-c + b*d - a*d^2)*(c + b*d + a*d^2)*Sqrt[1 + d*x]*(-c - b*d - a*d^2 -
(c*(1 - d*x)^2)/(1 + d*x)^2 + (b*d*(1 - d*x)^2)/(1 + d*x)^2 - (a*d^2*(1 - d
*x)^2)/(1 + d*x)^2 + (2*c*(1 - d*x))/(1 + d*x) - (2*a*d^2*(1 - d*x))/(1 + d
*x))) + ((4*c^4 - 2*b*c^3*d + 2*c^3*Sqrt[b^2 - 4*a*c]*d - 5*b^2*c^2*d^2 + 1
2*a*c^3*d^2 - b*c^2*Sqrt[b^2 - 4*a*c]*d^2 + 2*b^3*c*d^3 - 4*a*b*c^2*d^3 - 2
*b^2*c*Sqrt[b^2 - 4*a*c]*d^3 + 6*a*c^2*Sqrt[b^2 - 4*a*c]*d^3 - a*b^2*c*d^4
+ 8*a^2*c^2*d^4 + a*b*c*Sqrt[b^2 - 4*a*c]*d^4 + a*b^3*d^5 - 6*a^2*b*c*d^5 -
 a*b^2*Sqrt[b^2 - 4*a*c]*d^5 + 4*a^2*c*Sqrt[b^2 - 4*a*c]*d^5)*ArcTan[(Sqrt[
c - b*d + a*d^2]*Sqrt[1 - d*x])/(Sqrt[-c - Sqrt[b^2 - 4*a*c]*d + a*d^2]*Sqr
t[1 + d*x])])/((b^2 - 4*a*c)^(3/2)*(c - b*d + a*d^2)^(3/2)*(c + b*d + a*d^2
)*Sqrt[-c - Sqrt[b^2 - 4*a*c]*d + a*d^2]) + ((-4*c^4 + 2*b*c^3*d + 2*c^3*Sq
rt[b^2 - 4*a*c]*d + 5*b^2*c^2*d^2 - 12*a*c^3*d^2 - b*c^2*Sqrt[b^2 - 4*a*c]*
d^2 - 2*b^3*c*d^3 + 4*a*b*c^2*d^3 - 2*b^2*c*Sqrt[b^2 - 4*a*c]*d^3 + 6*a*c^2
*Sqrt[b^2 - 4*a*c]*d^3 + a*b^2*c*d^4 - 8*a^2*c^2*d^4 + a*b*c*Sqrt[b^2 - 4*a
*c]*d^4 - a*b^3*d^5 + 6*a^2*b*c*d^5 - a*b^2*Sqrt[b^2 - 4*a*c]*d^5 + 4*a^2*c
*Sqrt[b^2 - 4*a*c]*d^5)*ArcTan[(Sqrt[c - b*d + a*d^2]*Sqrt[1 - d*x])/(Sqrt[
-c + Sqrt[b^2 - 4*a*c]*d + a*d^2]*Sqrt[1 + d*x])])/((b^2 - 4*a*c)^(3/2)*(c
- b*d + a*d^2)^(3/2)*(c + b*d + a*d^2)*Sqrt[-c + Sqrt[b^2 - 4*a*c]*d + a*d^
2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [C] time = 0.81, size = 41837, normalized size = 73.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)^2\sqrt{dx+1}\,\sqrt{-dx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.548 $\int \frac{\left(a+bx+cx^2\right)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal. Leaf size=276

$$\frac{x\left(ad^2+c\right)\left(a^2d^4+2acd^2+3b^2d^2+c^2\right)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}-\frac{3\sin^{-1}(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2\right.}{8d^7}$$

Rubi [A] time = 0.60, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 1814, 1815, 641, 216}

$$\frac{x\left(ad^2+c\right)\left(a^2d^4+2acd^2+3b^2d^2+c^2\right)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}-\frac{3\sin^{-1}(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2+12b^2cd^2+5c^3\right)}{8d^7}+\frac{cx\sqrt{1-d^2x^2}\left(12acd^2+12b^2d^2+7c^2\right)}{8d^6}+\frac{b\sqrt{1-d^2x^2}\left(6acd^2+b^2d^2+5c^2\right)}{d^6}+\frac{bc^2x^2\sqrt{1-d^2x^2}}{d^4}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (b*(3*a^2 + (3*c^2)/d^4 + b^2/d^2 + (6*a*c)/d^2)*d^4 + (c + a*d^2)*(c^2 + 3
*b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^6*Sqrt[1 - d^2*x^2]) + (b*(5*c^2 + b^
2*d^2 + 6*a*c*d^2)*Sqrt[1 - d^2*x^2])/d^6 + (c*(7*c^2 + 12*b^2*d^2 + 12*a*c
*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^6) + (b*c^2*x^2*Sqrt[1 - d^2*x^2])/d^4 + (c
^3*x^3*Sqrt[1 - d^2*x^2])/(4*d^4) - (3*(5*c^3 + 12*b^2*c*d^2 + 12*a*c^2*d^2
 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*ArcSin[d*x])/(8*d^7)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x]},
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{\left(a+bx+cx^2\right)^3}{\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} - \int \frac{c^3+3ac^2d^2 \cdots}{\cdots}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{c^3x^3\sqrt{1-\cdots}}{4d^4}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{bc^2x^2\sqrt{1-\cdots}}{d^4}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{c\left(7c^2+12\cdots\right.}{\cdots}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{b\left(5c^2+b^2\cdots\right.}{\cdots}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}} + \frac{b\left(5c^2+b^2\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.24, size = 239, normalized size = 0.87

$$\frac{-3\sqrt{1-d^2x^2}\sin^{-1}(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2+12b^2cd^2+5c^3\right)-8b\left(-3a^2d^5+6acd^3\left(d^2x^2-2\right)+c^2d\left(d^4x^4+4d^2x^2-8\right)\right)+dx\left(8a^3d^6+24a^2cd^4-12ac^2d^2\left(d^2x^2-3\right)+c^3\left(-2d^4x^4-5d^2x^2+15\right)\right)-12b^2d^3x\left(c\left(d^2x^2-3\right)-2ad^2\right)-8b^3d^3\left(d^2x^2-2\right)}{8d^7\sqrt{1-d^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (-8*b^3*d^3*(-2 + d^2*x^2) - 12*b^2*d^3*x*(-2*a*d^2 + c*(-3 + d^2*x^2)) + d
*x*(24*a^2*c*d^4 + 8*a^3*d^6 - 12*a*c^2*d^2*(-3 + d^2*x^2) + c^3*(15 - 5*d^
2*x^2 - 2*d^4*x^4)) - 8*b*(-3*a^2*d^5 + 6*a*c*d^3*(-2 + d^2*x^2) + c^2*d*(-
8 + 4*d^2*x^2 + d^4*x^4)) - 3*(5*c^3 + 12*b^2*c*d^2 + 12*a*c^2*d^2 + 8*a*b^
2*d^4 + 8*a^2*c*d^4)*Sqrt[1 - d^2*x^2]*ArcSin[d*x])/(8*d^7*Sqrt[1 - d^2*x^2
])
```

IntegrateAlgebraic [B] time = 0.60, size = 1332, normalized size = 4.83

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (Sqrt[1 + d*x]*(2*c^3 + 6*b*c^2*d + 6*b^2*c*d^2 + 6*a*c^2*d^2 + 2*b^3*d^3 +
 12*a*b*c*d^3 + 6*a*b^2*d^4 + 6*a^2*c*d^4 + 6*a^2*b*d^5 + 2*a^3*d^6 - (2*c^
3*(1 - d*x)^5)/(1 + d*x)^5 + (6*b*c^2*d*(1 - d*x)^5)/(1 + d*x)^5 - (6*b^2*c
*d^2*(1 - d*x)^5)/(1 + d*x)^5 - (6*a*c^2*d^2*(1 - d*x)^5)/(1 + d*x)^5 + (2*
b^3*d^3*(1 - d*x)^5)/(1 + d*x)^5 + (12*a*b*c*d^3*(1 - d*x)^5)/(1 + d*x)^5 -
 (6*a*b^2*d^4*(1 - d*x)^5)/(1 + d*x)^5 - (6*a^2*c*d^4*(1 - d*x)^5)/(1 + d*x
)^5 + (6*a^2*b*d^5*(1 - d*x)^5)/(1 + d*x)^5 - (2*a^3*d^6*(1 - d*x)^5)/(1 +
d*x)^5 - (15*c^3*(1 - d*x)^4)/(1 + d*x)^4 + (78*b*c^2*d*(1 - d*x)^4)/(1 + d
```

```
*x)^4 - (30*b^2*c*d^2*(1 - d*x)^4)/(1 + d*x)^4 - (30*a*c^2*d^2*(1 - d*x)^4)
/(1 + d*x)^4 + (18*b^3*d^3*(1 - d*x)^4)/(1 + d*x)^4 + (108*a*b*c*d^3*(1 - d
*x)^4)/(1 + d*x)^4 - (18*a*b^2*d^4*(1 - d*x)^4)/(1 + d*x)^4 - (18*a^2*c*d^4
*(1 - d*x)^4)/(1 + d*x)^4 + (30*a^2*b*d^5*(1 - d*x)^4)/(1 + d*x)^4 - (6*a^3
*d^6*(1 - d*x)^4)/(1 + d*x)^4 - (5*c^3*(1 - d*x)^3)/(1 + d*x)^3 + (172*b*c^
2*d*(1 - d*x)^3)/(1 + d*x)^3 - (24*b^2*c*d^2*(1 - d*x)^3)/(1 + d*x)^3 - (24
*a*c^2*d^2*(1 - d*x)^3)/(1 + d*x)^3 + (44*b^3*d^3*(1 - d*x)^3)/(1 + d*x)^3
+ (264*a*b*c*d^3*(1 - d*x)^3)/(1 + d*x)^3 - (12*a*b^2*d^4*(1 - d*x)^3)/(1 +
 d*x)^3 - (12*a^2*c*d^4*(1 - d*x)^3)/(1 + d*x)^3 + (60*a^2*b*d^5*(1 - d*x)^
3)/(1 + d*x)^3 - (4*a^3*d^6*(1 - d*x)^3)/(1 + d*x)^3 + (5*c^3*(1 - d*x)^2)/
(1 + d*x)^2 + (172*b*c^2*d*(1 - d*x)^2)/(1 + d*x)^2 + (24*b^2*c*d^2*(1 - d*
x)^2)/(1 + d*x)^2 + (24*a*c^2*d^2*(1 - d*x)^2)/(1 + d*x)^2 + (44*b^3*d^3*(1
 - d*x)^2)/(1 + d*x)^2 + (264*a*b*c*d^3*(1 - d*x)^2)/(1 + d*x)^2 + (12*a*b^
2*d^4*(1 - d*x)^2)/(1 + d*x)^2 + (12*a^2*c*d^4*(1 - d*x)^2)/(1 + d*x)^2 + (
60*a^2*b*d^5*(1 - d*x)^2)/(1 + d*x)^2 + (4*a^3*d^6*(1 - d*x)^2)/(1 + d*x)^2
 + (15*c^3*(1 - d*x))/(1 + d*x) + (78*b*c^2*d*(1 - d*x))/(1 + d*x) + (30*b^
2*c*d^2*(1 - d*x))/(1 + d*x) + (30*a*c^2*d^2*(1 - d*x))/(1 + d*x) + (18*b^3
*d^3*(1 - d*x))/(1 + d*x) + (108*a*b*c*d^3*(1 - d*x))/(1 + d*x) + (18*a*b^2
*d^4*(1 - d*x))/(1 + d*x) + (18*a^2*c*d^4*(1 - d*x))/(1 + d*x) + (30*a^2*b*
d^5*(1 - d*x))/(1 + d*x) + (6*a^3*d^6*(1 - d*x))/(1 + d*x)))/(4*d^7*Sqrt[1
- d*x]*(1 + (1 - d*x)/(1 + d*x))^4) + (3*(5*c^3 + 12*b^2*c*d^2 + 12*a*c^2*d
^2 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/(4*d^7
)
```

fricas [A] time = 0.42, size = 376, normalized size = 1.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/8*(24*a^2*b*d^5 + 64*b*c^2*d + 16*(b^3 + 6*a*b*c)*d^3 - 8*(3*a^2*b*d^7 +
 8*b*c^2*d^3 + 2*(b^3 + 6*a*b*c)*d^5)*x^2 - (2*c^3*d^5*x^5 + 8*b*c^2*d^5*x^
4 - 24*a^2*b*d^5 - 64*b*c^2*d - 16*(b^3 + 6*a*b*c)*d^3 + (5*c^3*d^3 + 12*(b
^2*c + a*c^2)*d^5)*x^3 + 8*(4*b*c^2*d^3 + (b^3 + 6*a*b*c)*d^5)*x^2 - (8*a^3
*d^7 + 24*(a*b^2 + a^2*c)*d^5 + 15*c^3*d + 36*(b^2*c + a*c^2)*d^3)*x)*sqrt(
d*x + 1)*sqrt(-d*x + 1) + 6*(8*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 12*(b^2*c + a*
c^2)*d^2 - (8*(a*b^2 + a^2*c)*d^6 + 5*c^3*d^2 + 12*(b^2*c + a*c^2)*d^4)*x^2
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^9*x^2 - d^7)
```

giac [B] time = 0.61, size = 732, normalized size = 2.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(((d*x + 1)*(2*(d*x + 1)*((d*x + 1)*c^3/d^7 + (4*b*c^2*d^36 - 5*c^3*d^3
5)/d^42) + (12*b^2*c*d^37 + 12*a*c^2*d^37 - 32*b*c^2*d^36 + 25*c^3*d^35)/d^
42) + (8*b^3*d^38 + 48*a*b*c*d^38 - 36*b^2*c*d^37 - 36*a*c^2*d^37 + 80*b*c^
2*d^36 - 35*c^3*d^35)/d^42)*(d*x + 1) - 2*(2*a^3*d^41 + 6*a^2*b*d^40 + 6*a*
b^2*d^39 + 6*a^2*c*d^39 + 10*b^3*d^38 + 60*a*b*c*d^38 - 6*b^2*c*d^37 - 6*a*
c^2*d^37 + 54*b*c^2*d^36 - 7*c^3*d^35)/d^42)*sqrt(d*x + 1)*sqrt(-d*x + 1)/(
d*x - 1) - 3/4*(8*a*b^2*d^4 + 8*a^2*c*d^4 + 12*b^2*c*d^2 + 12*a*c^2*d^2 + 5
*c^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^7 + 1/4*(a^3*d^6*(sqrt(2) - sqrt(
-d*x + 1))/sqrt(d*x + 1) - 3*a^2*b*d^5*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x
+ 1) + 3*a*b^2*d^4*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a^2*c*d^4*(
sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - b^3*d^3*(sqrt(2) - sqrt(-d*x + 1)
```

```
)/sqrt(d*x + 1) - 6*a*b*c*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*
b^2*c*d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a*c^2*d^2*(sqrt(2) -
 sqrt(-d*x + 1))/sqrt(d*x + 1) - 3*b*c^2*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(
d*x + 1) + c^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^7 - 1/4*(a^3*d^6
 - 3*a^2*b*d^5 + 3*a*b^2*d^4 + 3*a^2*c*d^4 - b^3*d^3 - 6*a*b*c*d^3 + 3*b^2*
c*d^2 + 3*a*c^2*d^2 - 3*b*c^2*d + c^3)*sqrt(d*x + 1)/(d^7*(sqrt(2) - sqrt(-
d*x + 1)))
```

maple [C]   time = 0.04, size = 755, normalized size = 2.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x)
```

```
[Out] 1/8*(-d*x+1)^(1/2)*(-16*(-d^2*x^2+1)^(1/2)*b^3*d^3*csgn(d)+24*a^2*c*d^4*arc
tan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+24*a*b^2*d^4*arctan(1/(-d^2*x^2+1)^(1
/2)*d*x*csgn(d))+36*a*c^2*d^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+36*b
^2*c*d^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+15*c^3*arctan(1/(-d^2*x^2
+1)^(1/2)*d*x*csgn(d))-15*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2*c^3*
d^2+2*(-d^2*x^2+1)^(1/2)*c^3*d^5*x^5*csgn(d)-36*arctan(1/(-d^2*x^2+1)^(1/2)
*d*x*csgn(d))*x^2*a*c^2*d^4-36*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2
*b^2*c*d^4-24*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2*a^2*c*d^6-24*arc
tan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2*a*b^2*d^6-8*csgn(d)*d^7*(-d^2*x^2
+1)^(1/2)*x*a^3-15*(-d^2*x^2+1)^(1/2)*c^3*d*x*csgn(d)-64*(-d^2*x^2+1)^(1/2)
*b*c^2*d*csgn(d)+8*(-d^2*x^2+1)^(1/2)*b^3*d^5*x^2*csgn(d)+5*(-d^2*x^2+1)^(1
/2)*c^3*d^3*x^3*csgn(d)-24*(-d^2*x^2+1)^(1/2)*a^2*b*d^5*csgn(d)+8*(-d^2*x^2
+1)^(1/2)*b*c^2*d^5*x^4*csgn(d)+12*(-d^2*x^2+1)^(1/2)*a*c^2*d^5*x^3*csgn(d)
+12*(-d^2*x^2+1)^(1/2)*b^2*c*d^5*x^3*csgn(d)-24*(-d^2*x^2+1)^(1/2)*a^2*c*d^
5*x*csgn(d)-24*(-d^2*x^2+1)^(1/2)*a*b^2*d^5*x*csgn(d)+32*(-d^2*x^2+1)^(1/2)
*b*c^2*d^3*x^2*csgn(d)-36*(-d^2*x^2+1)^(1/2)*a*c^2*d^3*x*csgn(d)-36*(-d^2*x
^2+1)^(1/2)*b^2*c*d^3*x*csgn(d)-96*(-d^2*x^2+1)^(1/2)*a*b*c*d^3*csgn(d)+48*
(-d^2*x^2+1)^(1/2)*a*b*c*d^5*x^2*csgn(d))*csgn(d)/(d*x-1)/(-d^2*x^2+1)^(1/2
)/d^7/(d*x+1)^(1/2)
```

maxima [A]   time = 0.98, size = 371, normalized size = 1.34

$$-\frac{c^3x^5}{4\sqrt{-d^2x^2+1}\,d^2}-\frac{bc^2x^4}{\sqrt{-d^2x^2+1}\,d^2}+\frac{a^3x}{\sqrt{-d^2x^2+1}}-\frac{5c^3x^3}{8\sqrt{-d^2x^2+1}\,d^4}-\frac{3\left(b^2c+ac^2\right)x^3}{2\sqrt{-d^2x^2+1}\,d^2}+\frac{3a^2b}{\sqrt{-d^2x^2+1}\,d^2}-\frac{4bc^2x^2}{\sqrt{-d^2x^2+1}\,d^4}-\frac{\left(b^3+6abc\right)x^2}{\sqrt{-d^2x^2+1}\,d^2}+\frac{3\left(ab^2+a^2c\right)x}{\sqrt{-d^2x^2+1}\,d^2}-\frac{3\left(ab^2+a^2c\right)\arcsin(dx)}{d^3}+\frac{15c^3x}{8\sqrt{-d^2x^2+1}\,d^6}+\frac{9\left(b^2c+ac^2\right)x}{2\sqrt{-d^2x^2+1}\,d^4}-\frac{15c^3\arcsin(dx)}{8d^7}-\frac{9\left(b^2c+ac^2\right)\arcsin(dx)}{2d^5}+\frac{8bc^2}{\sqrt{-d^2x^2+1}\,d^6}+\frac{2\left(b^3+6abc\right)}{\sqrt{-d^2x^2+1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/4*c^3*x^5/(sqrt(-d^2*x^2 + 1)*d^2) - b*c^2*x^4/(sqrt(-d^2*x^2 + 1)*d^2)
+ a^3*x/sqrt(-d^2*x^2 + 1) - 5/8*c^3*x^3/(sqrt(-d^2*x^2 + 1)*d^4) - 3/2*(b^
2*c + a*c^2)*x^3/(sqrt(-d^2*x^2 + 1)*d^2) + 3*a^2*b/(sqrt(-d^2*x^2 + 1)*d^2
) - 4*b*c^2*x^2/(sqrt(-d^2*x^2 + 1)*d^4) - (b^3 + 6*a*b*c)*x^2/(sqrt(-d^2*x
^2 + 1)*d^2) + 3*(a*b^2 + a^2*c)*x/(sqrt(-d^2*x^2 + 1)*d^2) - 3*(a*b^2 + a^
2*c)*arcsin(d*x)/d^3 + 15/8*c^3*x/(sqrt(-d^2*x^2 + 1)*d^6) + 9/2*(b^2*c + a
*c^2)*x/(sqrt(-d^2*x^2 + 1)*d^4) - 15/8*c^3*arcsin(d*x)/d^7 - 9/2*(b^2*c +
a*c^2)*arcsin(d*x)/d^5 + 8*b*c^2/(sqrt(-d^2*x^2 + 1)*d^6) + 2*(b^3 + 6*a*b*
c)/(sqrt(-d^2*x^2 + 1)*d^4)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^3}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
[Out] int((a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
[Out] Timed out
```

3.549 $\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}} \, dx$

Optimal. Leaf size=135

$$\frac{x\left(a^2d^4+2acd^2+b^2d^2+c^2\right)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}-\frac{\sin^{-1}(dx)\left(c\left(4a+\frac{3c}{d^2}\right)+2b^2\right)}{2d^3}+\frac{2bc\sqrt{1-d^2x^2}}{d^4}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

Rubi [A] time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 1814, 1815, 641, 216}

$$\frac{x\left(a^2d^4+2acd^2+b^2d^2+c^2\right)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}-\frac{\sin^{-1}(dx)\left(c\left(4a+\frac{3c}{d^2}\right)+2b^2\right)}{2d^3}+\frac{2bc\sqrt{1-d^2x^2}}{d^4}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (2*b*(a + c/d^2)*d^2 + (c^2 + b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^4*Sqrt[1
 - d^2*x^2]) + (2*b*c*Sqrt[1 - d^2*x^2])/d^4 + (c^2*x*Sqrt[1 - d^2*x^2])/(2
*d^4) - ((2*b^2 + c*(4*a + (3*c)/d^2))*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx+cx^2\right)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx &= \int \frac{\left(a+bx+cx^2\right)^2}{\left(1-d^2x^2\right)^{3/2}}\,dx \\
&= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} - \int \frac{\frac{c^2+b^2d^2+2acd^2}{d^4}+\frac{2bcx}{d^2}+\frac{c^2x^2}{d^2}}{\sqrt{1-d^2x^2}}\,dx \\
&= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} + \frac{\int \frac{-2b^2-c\left(4a+\frac{3c}{d^2}\right.\cdots}{\sqrt{1-d^2x^2}}\cdots}{2d^2} \\
&= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} + \cdots \\
&= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} + \cdots
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 127, normalized size = 0.94

$$
\frac{dx\left(2a^2d^4+4acd^2+c^2\left(3-d^2x^2\right)\right)-\sqrt{1-d^2x^2}\,\sin^{-1}(dx)\left(4acd^2+2b^2d^2+3c^2\right)+4bd\left(ad^2+c\left(2-d^2x^2\right)\right)+2b^2d^3x}{2d^5\sqrt{1-d^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (2*b^2*d^3*x + 4*b*d*(a*d^2 + c*(2 - d^2*x^2)) + d*x*(4*a*c*d^2 + 2*a^2*d^4
 + c^2*(3 - d^2*x^2)) - (3*c^2 + 2*b^2*d^2 + 4*a*c*d^2)*Sqrt[1 - d^2*x^2]*A
rcSin[d*x])/(2*d^5*Sqrt[1 - d^2*x^2])
```

IntegrateAlgebraic [B] time = 0.29, size = 504, normalized size = 3.73

$$
\frac{\sqrt{dx+1}\left(\frac{a^2d^4(1-dx)}{dx+1}-\frac{a^2d^4(1-dx)^2}{(dx+1)^2}-\frac{a^2d^4(1-dx)^3}{(dx+1)^3}+a^2d^4+\frac{6abd^3(1-dx)}{dx+1}+\frac{6abd^3(1-dx)^2}{(dx+1)^2}+\frac{2abd^3(1-dx)^3}{(dx+1)^3}+2abd^3+\frac{2acd^2(1-dx)}{dx+1}-\frac{2acd^2(1-dx)^2}{(dx+1)^2}-\frac{2acd^2(1-dx)^3}{(dx+1)^3}+2acd^2+\frac{b^2d^2(1-dx)}{dx+1}-\frac{b^2d^2(1-dx)^2}{(dx+1)^2}-\frac{b^2d^2(1-dx)^3}{(dx+1)^3}+b^2d^2+\frac{14bcd(1-dx)}{dx+1}+\frac{14bcd(1-dx)^2}{(dx+1)^2}+\frac{2bcd(1-dx)^3}{(dx+1)^3}+2bcd+\frac{3c^2(1-dx)}{dx+1}-\frac{3c^2(1-dx)^2}{(dx+1)^2}-\frac{c^2(1-dx)^3}{(dx+1)^3}+c^2\right)}{2d^5\sqrt{1-dx}\left(\frac{1-dx}{dx+1}+1\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)\left(4acd^2+2b^2d^2+3c^2\right)}{d^5}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (Sqrt[1 + d*x]*(c^2 + 2*b*c*d + b^2*d^2 + 2*a*c*d^2 + 2*a*b*d^3 + a^2*d^4 -
 (c^2*(1 - d*x)^3)/(1 + d*x)^3 + (2*b*c*d*(1 - d*x)^3)/(1 + d*x)^3 - (b^2*d
^2*(1 - d*x)^3)/(1 + d*x)^3 - (2*a*c*d^2*(1 - d*x)^3)/(1 + d*x)^3 + (2*a*b*
d^3*(1 - d*x)^3)/(1 + d*x)^3 - (a^2*d^4*(1 - d*x)^3)/(1 + d*x)^3 - (3*c^2*(
1 - d*x)^2)/(1 + d*x)^2 + (14*b*c*d*(1 - d*x)^2)/(1 + d*x)^2 - (b^2*d^2*(1
- d*x)^2)/(1 + d*x)^2 - (2*a*c*d^2*(1 - d*x)^2)/(1 + d*x)^2 + (6*a*b*d^3*(1
 - d*x)^2)/(1 + d*x)^2 - (a^2*d^4*(1 - d*x)^2)/(1 + d*x)^2 + (3*c^2*(1 - d*
x))/(1 + d*x) + (14*b*c*d*(1 - d*x))/(1 + d*x) + (b^2*d^2*(1 - d*x))/(1 + d
*x) + (2*a*c*d^2*(1 - d*x))/(1 + d*x) + (6*a*b*d^3*(1 - d*x))/(1 + d*x) + (
a^2*d^4*(1 - d*x))/(1 + d*x)))/(2*d^5*Sqrt[1 - d*x]*(1 + (1 - d*x)/(1 + d*x
))^2) + ((3*c^2 + 2*b^2*d^2 + 4*a*c*d^2)*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]
])/d^5
```

fricas [A] time = 0.41, size = 204, normalized size = 1.51

$$
-\frac{4\,abd^3+8\,bcd-4\left(abd^5+2\,bcd^3\right)x^2-\left(c^2d^3x^3+4\,bcd^3x^2-4\,abd^3-8\,bcd-\left(2\,a^2d^5+2\left(b^2+2\,ac\right)d^3+3\,c^2d\right)x\right)\sqrt{dx+1}\,\sqrt{-dx+1}+2\left(2\left(b^2+2\,ac\right)d^2-\left(2\left(b^2+2\,ac\right)d^4+3\,c^2d^2\right)x^2+3\,c^2\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\left(d^7x^2-d^5\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/2*(4*a*b*d^3 + 8*b*c*d - 4*(a*b*d^5 + 2*b*c*d^3)*x^2 - (c^2*d^3*x^3 + 4*
b*c*d^3*x^2 - 4*a*b*d^3 - 8*b*c*d - (2*a^2*d^5 + 2*(b^2 + 2*a*c)*d^3 + 3*c^
2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*(b^2 + 2*a*c)*d^2 - (2*(b^2 + 2
*a*c)*d^4 + 3*c^2*d^2)*x^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) -
1)/(d*x)))/(d^7*x^2 - d^5)
```

giac [B] time = 0.39, size = 387, normalized size = 2.87

$$\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left((dx+1)\left(\frac{(dx+1)c^2}{d^5}+\frac{4\,bcd^{16}-3\,c^2d^{15}}{d^{20}}\right)-\frac{a^2d^{19}+2\,abd^{18}+b^2d^{17}+2\,acd^{17}+10\,bcd^{16}-c^2d^{15}}{d^{20}}\right)}{2\,(dx-1)}-\frac{\left(2\,b^2d^2+4\,acd^2+3\,c^2\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^5}+\frac{\frac{a^2d^4\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}-\frac{2\,abd^3\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{b^2d^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{2\,acd^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}-\frac{2\,bcd\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{c^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}}{4\,d^5}-\frac{\left(a^2d^4-2\,abd^3+b^2d^2+2\,acd^2-2\,bcd+c^2\right)\sqrt{dx+1}}{4\,d^5\left(\sqrt{2}-\sqrt{-dx+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*((d*x + 1)*c^2/d^5 + (4*b*c*d^1
6 - 3*c^2*d^15)/d^20) - (a^2*d^19 + 2*a*b*d^18 + b^2*d^17 + 2*a*c*d^17 + 10
*b*c*d^16 - c^2*d^15)/d^20)/(d*x - 1) - (2*b^2*d^2 + 4*a*c*d^2 + 3*c^2)*arc
sin(1/2*sqrt(2)*sqrt(d*x + 1))/d^5 + 1/4*(a^2*d^4*(sqrt(2) - sqrt(-d*x + 1)
)/sqrt(d*x + 1) - 2*a*b*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + b^2*
d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 2*a*c*d^2*(sqrt(2) - sqrt(-d
*x + 1))/sqrt(d*x + 1) - 2*b*c*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) +
 c^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^5 - 1/4*(a^2*d^4 - 2*a*b*d
^3 + b^2*d^2 + 2*a*c*d^2 - 2*b*c*d + c^2)*sqrt(d*x + 1)/(d^5*(sqrt(2) - sqr
t(-d*x + 1)))
```

maple [C] time = 0.03, size = 380, normalized size = 2.81

$$\frac{\sqrt{-dx+1}\,\left(-2\sqrt{-d^2x^2+1}\,a^2d^5x\,\mathrm{csgn}(d)-4ac\,d^4x^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-2b^2d^4x^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+\sqrt{-d^2x^2+1}\,c^2d^3x^3\mathrm{csgn}(d)+4\sqrt{-d^2x^2+1}\,bc\,d^3x^2\mathrm{csgn}(d)-4\sqrt{-d^2x^2+1}\,ac\,d^3x\,\mathrm{csgn}(d)-2\sqrt{-d^2x^2+1}\,b^2d^3x\,\mathrm{csgn}(d)-3c^2d^2x^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-4\sqrt{-d^2x^2+1}\,ab\,d^3\mathrm{csgn}(d)+4ac\,d^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+2b^2d^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-3\sqrt{-d^2x^2+1}\,c^2dx\,\mathrm{csgn}(d)-8\sqrt{-d^2x^2+1}\,bcd\,\mathrm{csgn}(d)+3c^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right)\mathrm{csgn}(d)}{2\,(dx-1)\,\sqrt{-d^2x^2+1}\,\sqrt{dx+1}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x)
```

```
[Out] 1/2*(-d*x+1)^(1/2)*((-d^2*x^2+1)^(1/2)*c^2*d^3*x^3*csgn(d)-2*csgn(d)*d^5*(-
d^2*x^2+1)^(1/2)*x*a^2+4*(-d^2*x^2+1)^(1/2)*b*c*d^3*x^2*csgn(d)-4*arctan(1/
(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2*a*c*d^4-2*arctan(1/(-d^2*x^2+1)^(1/2)*d
*x*csgn(d))*x^2*b^2*d^4-4*(-d^2*x^2+1)^(1/2)*a*c*d^3*x*csgn(d)-2*(-d^2*x^2+
1)^(1/2)*b^2*d^3*x*csgn(d)-4*(-d^2*x^2+1)^(1/2)*a*b*d^3*csgn(d)-3*arctan(1/
(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x^2*c^2*d^2-3*(-d^2*x^2+1)^(1/2)*c^2*d*x*cs
gn(d)-8*(-d^2*x^2+1)^(1/2)*b*c*d*csgn(d)+4*a*c*d^2*arctan(1/(-d^2*x^2+1)^(1
/2)*d*x*csgn(d))+2*b^2*d^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+3*c^2*a
rctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d)))*csgn(d)/(d*x-1)/(-d^2*x^2+1)^(1/2)
/d^5/(d*x+1)^(1/2)
```

maxima [A] time = 0.97, size = 176, normalized size = 1.30

$$\frac{a^2x}{\sqrt{-d^2x^2+1}}-\frac{c^2x^3}{2\,\sqrt{-d^2x^2+1}\,d^2}-\frac{2\,bcx^2}{\sqrt{-d^2x^2+1}\,d^2}+\frac{2\,ab}{\sqrt{-d^2x^2+1}\,d^2}+\frac{\left(b^2+2\,ac\right)x}{\sqrt{-d^2x^2+1}\,d^2}-\frac{\left(b^2+2\,ac\right)\arcsin\left(dx\right)}{d^3}+\frac{3\,c^2x}{2\,\sqrt{-d^2x^2+1}\,d^4}-\frac{3\,c^2\arcsin\left(dx\right)}{2\,d^5}+\frac{4\,bc}{\sqrt{-d^2x^2+1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] a^2*x/sqrt(-d^2*x^2 + 1) - 1/2*c^2*x^3/(sqrt(-d^2*x^2 + 1)*d^2) - 2*b*c*x^2
/(sqrt(-d^2*x^2 + 1)*d^2) + 2*a*b/(sqrt(-d^2*x^2 + 1)*d^2) + (b^2 + 2*a*c)*
x/(sqrt(-d^2*x^2 + 1)*d^2) - (b^2 + 2*a*c)*arcsin(d*x)/d^3 + 3/2*c^2*x/(sqr
t(-d^2*x^2 + 1)*d^4) - 3/2*c^2*arcsin(d*x)/d^5 + 4*b*c/(sqrt(-d^2*x^2 + 1)*
d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2+b\,x+a\right)^2}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
[Out] int((a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
[Out] Timed out
```

3.550 $\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{x\left(ad^2+c\right)+b}{d^2\sqrt{1-d^2x^2}}-\frac{c\sin^{-1}(dx)}{d^3}$$

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1814, 12, 216}

$$\frac{x\left(ad^2+c\right)+b}{d^2\sqrt{1-d^2x^2}}-\frac{c\sin^{-1}(dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (b + (c + a*d^2)*x)/(d^2*Sqrt[1 - d^2*x^2]) - (c*ArcSin[d*x])/d^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{a+bx+cx^2}{\left(1-d^2x^2\right)^{3/2}}\,dx$$
$$= \frac{b+\left(c+ad^2\right)x}{d^2\sqrt{1-d^2x^2}} - \int \frac{c}{d^2\sqrt{1-d^2x^2}}\,dx$$
$$= \frac{b+\left(c+ad^2\right)x}{d^2\sqrt{1-d^2x^2}} - \frac{c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{d^2}$$
$$= \frac{b+\left(c+ad^2\right)x}{d^2\sqrt{1-d^2x^2}} - \frac{c\sin^{-1}(dx)}{d^3}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.98

$$\frac{\frac{d\left(x\left(ad^2+c\right)+b\right)}{\sqrt{1-d^2x^2}} - c\sin^{-1}(dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] ((d*(b + (c + a*d^2)*x))/Sqrt[1 - d^2*x^2] - c*ArcSin[d*x])/d^3
```

IntegrateAlgebraic [B] time = 0.13, size = 115, normalized size = 2.88

$$\frac{\sqrt{dx+1}\left(-\frac{ad^2(1-dx)}{dx+1}+ad^2+\frac{bd(1-dx)}{dx+1}+bd-\frac{c(1-dx)}{dx+1}+c\right)}{2d^3\sqrt{1-dx}} + \frac{2c\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (Sqrt[1 + d*x]*(c + b*d + a*d^2 - (c*(1 - d*x))/(1 + d*x) + (b*d*(1 - d*x))
/(1 + d*x) - (a*d^2*(1 - d*x))/(1 + d*x)))/(2*d^3*Sqrt[1 - d*x]) + (2*c*Arc
Tan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d^3
```

fricas [B] time = 0.42, size = 101, normalized size = 2.52

$$\frac{bd^3x^2-\left(bd+\left(ad^3+cd\right)x\right)\sqrt{dx+1}\,\sqrt{-dx+1}-bd+2\left(cd^2x^2-c\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{d^5x^2-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d^3*x^2 - (b*d + (a*d^3 + c*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - b*d + 2
*(c*d^2*x^2 - c)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*x^2
 - d^3)
```

giac [B] time = 0.30, size = 182, normalized size = 4.55

$$-\frac{2\,c\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{d^3} + \frac{\frac{ad^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}} - \frac{bd\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}} + \frac{c\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}}{4\,d^3} - \frac{\left(ad^2-bd+c\right)\sqrt{dx+1}}{4\,d^3\left(\sqrt{2}-\sqrt{-dx+1}\right)} - \frac{\left(ad^5+bd^4+cd^3\right)\sqrt{dx+1}\,\sqrt{-dx+1}}{2\,(dx-1)d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -2*c*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3 + 1/4*(a*d^2*(sqrt(2) - sqrt(-d*
x + 1))/sqrt(d*x + 1) - b*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + c*(s
qrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^3 - 1/4*(a*d^2 - b*d + c)*sqrt(d*
x + 1)/(d^3*(sqrt(2) - sqrt(-d*x + 1))) - 1/2*(a*d^5 + b*d^4 + c*d^3)*sqrt(
d*x + 1)*sqrt(-d*x + 1)/((d*x - 1)*d^6)
```

maple [C] time = 0.03, size = 151, normalized size = 3.78

$$\frac{\left(-\sqrt{-d^2x^2+1}\, a\, d^3 x \,\mathrm{csgn}(d) - c\, d^2 x^2 \arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-(dx-1)(dx+1)}}\right) - \sqrt{-d^2x^2+1}\, c d x \,\mathrm{csgn}(d) - \sqrt{-d^2x^2+1}\, b d \,\mathrm{csgn}(d) + c \arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-(dx-1)(dx+1)}}\right)\right)\sqrt{-dx+1}\,\mathrm{csgn}(d)}{(dx-1)\sqrt{-d^2x^2+1}\,\sqrt{dx+1}\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x)
```

```
[Out] (-(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*x*a-arctan(csgn(d)*d*x/(-(d*x-1)*(d*x+1))^
(1/2))*x^2*c*d^2-(-d^2*x^2+1)^(1/2)*c*d*x*csgn(d)-(-d^2*x^2+1)^(1/2)*b*d*cs
gn(d)+arctan(csgn(d)*d*x/(-(d*x-1)*(d*x+1))^(1/2))*c)*(-d*x+1)^(1/2)*csgn(d
)/(d*x-1)/(-d^2*x^2+1)^(1/2)/d^3/(d*x+1)^(1/2)
```

maxima [A] time = 0.97, size = 61, normalized size = 1.52

$$\frac{ax}{\sqrt{-d^2x^2+1}} + \frac{cx}{\sqrt{-d^2x^2+1}\,d^2} - \frac{c \arcsin(dx)}{d^3} + \frac{b}{\sqrt{-d^2x^2+1}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] a*x/sqrt(-d^2*x^2 + 1) + c*x/(sqrt(-d^2*x^2 + 1)*d^2) - c*arcsin(d*x)/d^3 +
 b/(sqrt(-d^2*x^2 + 1)*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c\,x^2+b\,x+a}{(1-d\,x)^{3/2}\,(d\,x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

```
[Out] Timed out
```

3.551 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)}\,dx$

Optimal. Leaf size=443

$$\frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{c\left(-bd^2\left(b-\sqrt{b^2-4a\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}\ldots}$$

Rubi [A] time = 1.44, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 976, 1034, 725, 206}

$$\frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{c\left(-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2\right)\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}+\frac{d^2\left(b-x\left(ad^2+c\right)\right)}{\sqrt{1-d^2x^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (d^2*(b - (c + a*d^2)*x))/((b^2*d^2 - (c + a*d^2)^2)*Sqrt[1 - d^2*x^2]) + (
c*(2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b - S
qrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2
- 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) - (c
*(2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b + Sq
rt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
```

```
 + q + 4))))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} - \frac{\int \frac{2d^2\left(c^2-b^2d^2+acd^2\right)-2bcd^4x}{\left(a+bx+cx^2\right)\sqrt{1-d^2x^2}}\,dx}{2d^2\left(b^2d^2-\left(c+ad^2\right)^2\right)}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} + \frac{\left(c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right.\right.\right.}{\sqrt{b^2-4ac}\,\left(b^2c\right.}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} - \frac{\left(c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right.\right.\right.}{\sqrt{}}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} + \frac{c\left(2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right.\right.}{\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{2c^2+2acd^2}}$$

Mathematica [A] time = 2.39, size = 335, normalized size = 0.76

$$\frac{d^2\left(x\left(ad^2+c\right)-b\right)}{\sqrt{1-d^2x^2}\left(a^2d^4+2acd^2-b^2d^2+c^2\right)} - \frac{2\sqrt{2}\,c^3\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{1-d^2x^2}\sqrt{2bd^2\left(\sqrt{b^2-4ac}-b\right)+4acd^2+4c^2}}\right)}{\sqrt{b^2-4ac}\left(bd^2\left(\sqrt{b^2-4ac}-b\right)+2acd^2+2c^2\right)^{3/2}} + \frac{2\sqrt{2}\,c^3\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{1-d^2x^2}\sqrt{-2bd^2\left(\sqrt{b^2-4ac}+b\right)+4acd^2+4c^2}}\right)}{\sqrt{b^2-4ac}\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (d^2*(-b + (c + a*d^2)*x))/((c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*Sqrt[1 -
d^2*x^2]) - (2*Sqrt[2]*c^3*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(S
qrt[4*c^2 + 4*a*c*d^2 + 2*b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2]
)])/(Sqrt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2)
^(3/2)) + (2*Sqrt[2]*c^3*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqr
t[4*c^2 + 4*a*c*d^2 - 2*b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])
/(Sqrt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2)^(3/
2))
```

IntegrateAlgebraic [A] time = 9.49, size = 615, normalized size = 1.39

$$\frac{\left(-c^2d\sqrt{b^2-4ac}-acd^3\sqrt{b^2-4ac}+b^2d^3\sqrt{b^2-4ac}-bcd^2\sqrt{b^2-4ac}+3abcd^3-2ac^2d^2-b^3d^3+b^2cd^2+bc^2d-2c^3\right)\tan^{-1}\left(\frac{\sqrt{1-dx}\sqrt{ad^2-bd+c}}{\sqrt{dx+1}\sqrt{-d\sqrt{b^2-4ac}+ad^2-c}}\right)}{\sqrt{b^2-4ac}\left(ad^2-bd+c\right)^{3/2}\left(ad^2+bd+c\right)\sqrt{-d\sqrt{b^2-4ac}+ad^2-c}}+\frac{\left(-c^2d\sqrt{b^2-4ac}-acd^3\sqrt{b^2-4ac}+b^2d^3\sqrt{b^2-4ac}-bcd^2\sqrt{b^2-4ac}-3abcd^3+2ac^2d^2+b^3d^3-b^2cd^2-bc^2d+2c^3\right)\tan^{-1}\left(\frac{\sqrt{1-dx}\sqrt{ad^2-bd+c}}{\sqrt{dx+1}\sqrt{d\sqrt{b^2-4ac}+ad^2-c}}\right)}{\sqrt{b^2-4ac}\left(ad^2-bd+c\right)^{3/2}\left(ad^2+bd+c\right)\sqrt{d\sqrt{b^2-4ac}+ad^2-c}}-\frac{d\sqrt{dx+1}\left(\frac{ad^2(1-dx)}{dx+1}-ad^2+\frac{bd(1-dx)}{dx+1}+bd+\frac{c(1-dx)}{dx+1}-c\right)}{2\sqrt{1-dx}\left(ad^2-bd+c\right)\left(ad^2+bd+c\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*(d*Sqrt[1 + d*x]*(-c + b*d - a*d^2 + (c*(1 - d*x))/(1 + d*x) + (b*d*(1
 - d*x))/(1 + d*x) + (a*d^2*(1 - d*x))/(1 + d*x)))/((c - b*d + a*d^2)*(c +
b*d + a*d^2)*Sqrt[1 - d*x]) + ((-2*c^3 + b*c^2*d - c^2*Sqrt[b^2 - 4*a*c]*d
+ b^2*c*d^2 - 2*a*c^2*d^2 - b*c*Sqrt[b^2 - 4*a*c]*d^2 - b^3*d^3 + 3*a*b*c*d
^3 + b^2*Sqrt[b^2 - 4*a*c]*d^3 - a*c*Sqrt[b^2 - 4*a*c]*d^3)*ArcTan[(Sqrt[c
- b*d + a*d^2]*Sqrt[1 - d*x])/(Sqrt[-c - Sqrt[b^2 - 4*a*c]*d + a*d^2]*Sqrt[
1 + d*x])])/(Sqrt[b^2 - 4*a*c]*(c - b*d + a*d^2)^(3/2)*(c + b*d + a*d^2)*Sq
rt[-c - Sqrt[b^2 - 4*a*c]*d + a*d^2]) + ((2*c^3 - b*c^2*d - c^2*Sqrt[b^2 -
4*a*c]*d - b^2*c*d^2 + 2*a*c^2*d^2 - b*c*Sqrt[b^2 - 4*a*c]*d^2 + b^3*d^3 -
3*a*b*c*d^3 + b^2*Sqrt[b^2 - 4*a*c]*d^3 - a*c*Sqrt[b^2 - 4*a*c]*d^3)*ArcTan
[(Sqrt[c - b*d + a*d^2]*Sqrt[1 - d*x])/(Sqrt[-c + Sqrt[b^2 - 4*a*c]*d + a*d
^2]*Sqrt[1 + d*x])])/(Sqrt[b^2 - 4*a*c]*(c - b*d + a*d^2)^(3/2)*(c + b*d +
a*d^2)*Sqrt[-c + Sqrt[b^2 - 4*a*c]*d + a*d^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.12, size = 11141, normalized size = 25.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)(dx+1)^{\frac{3}{2}}(-dx+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(1-d\,x)^{3/2}\,(d\,x+1)^{3/2}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)),x)
[Out] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a),x)
[Out] Timed out
```

3.552 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2} dx$

Optimal. Leaf size=939

$$-\frac{\left(b\left(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3\right)-\left(2c^4-d^2\left(b^2+6a^2d^2\right)c^2-\left(4a^3d^6+6ab^2d^4\right)c+b^2d^4\left(2b^2\right.\right.}{\left(b^2-4ac\right)\left(ad^2-bd+c\right)^2\left(ad^2+bd+c\right)^2\sqrt{1-d^2x^2}}$$

Rubi [A] time = 11.84, antiderivative size = 938, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {899, 975, 1062, 1034, 725, 206}

Antiderivative was successfully verified.

```
[In] Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((d^2*(b*(c^3 + 2*b^2*c*d^2 - 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) -
 (2*c^4 + b^2*d^4*(2*b^2 + a^2*d^2) - c^2*d^2*(b^2 + 6*a^2*d^2) - c*(6*a*b^
2*d^4 + 4*a^3*d^6))*x))/((b^2 - 4*a*c)*(c - b*d + a*d^2)^2*(c + b*d + a*d^2
)^2*Sqrt[1 - d^2*x^2])) - (b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d
^2 + 2*a*c*d^2)*x)/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^
2)*Sqrt[1 - d^2*x^2]) + (c*(4*c^5 + 24*a*c^4*d^2 + 3*a*b^3*(b + Sqrt[b^2 -
4*a*c])*d^6 - c^3*d^2*(9*b^2 - b*Sqrt[b^2 - 4*a*c] - 36*a^2*d^2) - 2*a*c^2*
d^4*(7*b^2 + 5*b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2) + b*c*d^4*(2*b^3 + 2*b^2*Sq
rt[b^2 - 4*a*c] - 17*a^2*b*d^2 - 11*a^2*Sqrt[b^2 - 4*a*c]*d^2))*ArcTanh[(2*
c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b -
 Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*
Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*
a*c*d^2 + a^2*d^4)^2) - (c*(4*c^5*d^2 + 24*a*c^4*d^4 + 6*a*b^4*d^8 + 4*b^2*
c*d^6*(b^2 - 7*a^2*d^2) - b*(b + Sqrt[b^2 - 4*a*c])*d^4*(c^3 + 2*b^2*c*d^2
- 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) - 4*c^3*(2*b^2*d^4 - 9*a^2*d^6
) - 8*c^2*(3*a*b^2*d^6 - 2*a^3*d^8))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])
*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sq
rt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*d^2*Sqrt[2*c^2 + 2*a*c*d^2
- b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1062

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d
+ e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e) + (-(a*B))*(2*c^2*d - c*(2*a*
f)) + c*(A*(2*c^2*d - c*(2*a*f)) - B*(-2*a*c*e) + C*(-2*a*(c*d - a*f)))*x))
/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((-4*a*c)*(a*c*e
^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*S
imp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p + 1) + (2*(A*c*(c*
d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*
c - a*C)*(2*a*c*e) + (-(a*B))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2) -
(2*f*((A*c - a*C)*(2*a*c*e) + (-(a*B))*(2*c^2*d - c*((Plus[2])*a*f)))*(p +
q + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(-(c*e*(2*p + q
+ 4))))*x - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*
q + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2
- 4*d*f, 0] && LtQ[p, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !Intege
rQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^2\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)\sqrt{1-d^2x^2}} - \frac{\int \ldots}{\ldots}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d \ldots\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ \ldots\right.\right.}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d \ldots\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ \ldots\right.\right.}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d \ldots\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ \ldots\right.\right.}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d \ldots\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ \ldots\right.\right.}$$

Mathematica [A] time = 9.22, size = 890, normalized size = 0.95

$$\frac{\frac{\sqrt{2}\left(-3ab\left(b+\sqrt{b^2-4ac}\right)d^4+20ac^2d^2-c\left(7b^2-3\sqrt{b^2-4ac}\,b-16a^2d^2\right)d^2+4c^3\right)\tanh^{-1}\left(\frac{\left(b-\sqrt{b^2-4ac}\right)xd^2+2c}{\sqrt{2}\sqrt{2c^2+2ad^2c-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)c^3}{\sqrt{b^2-4ac}\left(2c^2+2ad^2c-b\left(b-\sqrt{b^2-4ac}\right)d^2\right)^{3/2}}-\frac{\sqrt{2}\left(-3ab\left(b-\sqrt{b^2-4ac}\right)d^4+20ac^2d^2-c\left(7b^2+3\sqrt{b^2-4ac}\,b-16a^2d^2\right)d^2+4c^3\right)\tanh^{-1}\left(\frac{\left(b+\sqrt{b^2-4ac}\right)xd^2+2c}{\sqrt{2}\sqrt{2c^2+2ad^2c-b\left(b+\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)c^3}{\sqrt{b^2-4ac}\left(2c^2+2ad^2c-b\left(b+\sqrt{b^2-4ac}\right)d^2\right)^{3/2}}-\frac{\left(3b\left(c-ad^2\right)d^2+\frac{-3ab^2d^4+16a^2cd^4+20ac^2d^2-7b^2cd^2+4c^3}{\sqrt{b^2-4ac}}\right)\left(2c-\left(b-\sqrt{b^2-4ac}\right)d^2x\right)c}{\left(4c^2-\left(b-\sqrt{b^2-4ac}\right)^2d^2\right)\sqrt{1-d^2x^2}}-\frac{\left(3bd^2\left(c-ad^2\right)-\frac{-3ab^2d^4+16a^2cd^4+20ac^2d^2-7b^2cd^2+4c^3}{\sqrt{b^2-4ac}}\right)\left(2c-\left(b+\sqrt{b^2-4ac}\right)d^2x\right)c}{\left(4c^2-\left(b+\sqrt{b^2-4ac}\right)^2d^2\right)\sqrt{1-d^2x^2}}+\frac{2d^2\left(2c^2+2ad^2c-b^2d^2\right)x}{\sqrt{1-d^2x^2}}}{\left(b^2-4ac\right)\left(\left(ad^2+c\right)^2-b^2d^2\right)}-\frac{-d^2b^3+c\left(3ad^2+c\right)b+c\left(2c^2+2ad^2c-b^2d^2\right)x}{\left(b^2-4ac\right)\left(\left(ad^2+c\right)^2-b^2d^2\right)\left(cx^2+bx+a\right)\sqrt{1-d^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((-(b^3*d^2) + b*c*(c + 3*a*d^2) + c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)/((b^
2 - 4*a*c)*(-(b^2*d^2) + (c + a*d^2)^2)*(a + b*x + c*x^2)*Sqrt[1 - d^2*x^2]
)) + ((2*d^2*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)/Sqrt[1 - d^2*x^2] - (c*(3*b*d
^2*(c - a*d^2) + (4*c^3 - 7*b^2*c*d^2 + 20*a*c^2*d^2 - 3*a*b^2*d^4 + 16*a^2
*c*d^4)/Sqrt[b^2 - 4*a*c])*(2*c - (b - Sqrt[b^2 - 4*a*c])*d^2*x))/((4*c^2 -
 (b - Sqrt[b^2 - 4*a*c])^2*d^2)*Sqrt[1 - d^2*x^2]) - (c*(3*b*d^2*(c - a*d^2
) - (4*c^3 - 7*b^2*c*d^2 + 20*a*c^2*d^2 - 3*a*b^2*d^4 + 16*a^2*c*d^4)/Sqrt[
b^2 - 4*a*c])*(2*c - (b + Sqrt[b^2 - 4*a*c])*d^2*x))/((4*c^2 - (b + Sqrt[b^
2 - 4*a*c])^2*d^2)*Sqrt[1 - d^2*x^2]) + (Sqrt[2]*c^3*(4*c^3 + 20*a*c^2*d^2
- 3*a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 - c*d^2*(7*b^2 - 3*b*Sqrt[b^2 - 4*a*c]
- 16*a^2*d^2))*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[
2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sq
rt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2)^(3/2))
- (Sqrt[2]*c^3*(4*c^3 + 20*a*c^2*d^2 - 3*a*b*(b - Sqrt[b^2 - 4*a*c])*d^4 -
c*d^2*(7*b^2 + 3*b*Sqrt[b^2 - 4*a*c] - 16*a^2*d^2))*ArcTanh[(2*c + (b + Sqr
t[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 - b
*(b + Sqrt[b^2 - 4*a*c])*d^2)^(3/2)))/((b^2 - 4*a*c)*(-(b^2*d^2) + (c + a*d
^2)^2))
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),
x]
```

```
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [C] time = 4.14, size = 108974, normalized size = 116.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2 + bx + a\right)^2 (dx + 1)^{\frac{3}{2}} (-dx + 1)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(1 - d\,x)^{3/2}\,(d\,x + 1)^{3/2}\,\left(c\,x^2 + b\,x + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a)**2,x)
[Out] Timed out
```

3.553 $\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=275

$$\frac{(ef-dg)^2(f+gx)^{n+2}\left(3aeg^2+c\left(2d^2g^2-10defg+5e^2f^2\right)\right)}{g^6(n+2)}-\frac{e(ef-dg)(f+gx)^{n+3}\left(3aeg^2+c\left(7d^2g^2-20defg\right.\right.}{g^6(n+3)}$$

Rubi [A] time = 0.26, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {947}

$$\frac{(ef-dg)^2(f+gx)^{n+2}\left(3aeg^2+c\left(2d^2g^2-10defg+5e^2f^2\right)\right)}{g^6(n+2)}-\frac{e(ef-dg)(f+gx)^{n+3}\left(3aeg^2+c\left(7d^2g^2-20defg+10e^2f^2\right)\right)}{g^6(n+3)}+\frac{e^2(f+gx)^{n+4}\left(aeg^2+c\left(9d^2g^2-20defg+10e^2f^2\right)\right)}{g^6(n+4)}-\frac{(ef-dg)^3(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^6(n+1)}-\frac{5ce^3(ef-dg)(f+gx)^{n+5}}{g^6(n+5)}+\frac{ce^4(f+gx)^{n+6}}{g^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^6*(1 + n
))) + ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(
f + g*x)^(2 + n))/(g^6*(2 + n)) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2
 - 20*d*e*f*g + 7*d^2*g^2))*(f + g*x)^(3 + n))/(g^6*(3 + n)) + (e^2*(a*e*g^
2 + c*(10*e^2*f^2 - 20*d*e*f*g + 9*d^2*g^2))*(f + g*x)^(4 + n))/(g^6*(4 + n
)) - (5*c*e^3*(e*f - d*g)*(f + g*x)^(5 + n))/(g^6*(5 + n)) + (c*e^4*(f + g*
x)^(6 + n))/(g^6*(6 + n))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)dx &= \int\left(\frac{(ef-dg)^3\left(-ag^2-cf(ef-2dg)\right)(f+gx)^n}{g^5}+\frac{(ef-dg)^2\left(3aeg^2\right.}{}\right.\\ &= -\frac{(ef-dg)^3\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^6(1+n)}+\frac{(ef-dg)^2\left(3aeg^2+\right.}{}\end{aligned}$$

Mathematica [A] time = 0.31, size = 249, normalized size = 0.91

$$\frac{(f+gx)^{n+1}\left(\frac{e^2(f+gx)^3\left(aeg^2+c\left(9d^2g^2-20defg+10e^2f^2\right)\right)}{n+4}-\frac{e(f+gx)^2(ef-dg)\left(3aeg^2+c\left(7d^2g^2-20defg+10e^2f^2\right)\right)}{n+3}+\frac{(f+gx)(ef-dg)^2\left(3aeg^2+c\left(2d^2g^2-10defg+5e^2f^2\right)\right)}{n+2}-\frac{(ef-dg)^3\left(ag^2+cf(ef-2dg)\right)}{n+1}-\frac{5ce^3(f+gx)^4(ef-dg)}{n+5}+\frac{ce^4(f+gx)^5}{n+6}\right)}{g^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(-(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n))
+ ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(f +
g*x))/(2 + n) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2 - 20*d*e*f*g + 7*
d^2*g^2))*(f + g*x)^2)/(3 + n) + (e^2*(a*e*g^2 + c*(10*e^2*f^2 - 20*d*e*f*g
 + 9*d^2*g^2))*(f + g*x)^3)/(4 + n) - (5*c*e^3*(e*f - d*g)*(f + g*x)^4)/(5
+ n) + (c*e^4*(f + g*x)^5)/(6 + n)))/g^6
```

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),
x]
```

fricas [B] time = 0.48, size = 2032, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d^3*f*g^5*n^5 - 120*c*e^4*f^6 + 720*c*d*e^3*f^5*g + 720*a*d^3*f*g^5 - 18
0*(9*c*d^2*e^2 + a*e^3)*f^4*g^2 + 240*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 360
*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4 + (c*e^4*g^6*n^5 + 15*c*e^4*g^6*n^4 + 85*c*e
^4*g^6*n^3 + 225*c*e^4*g^6*n^2 + 274*c*e^4*g^6*n + 120*c*e^4*g^6)*x^6 + (72
0*c*d*e^3*g^6 + (c*e^4*f*g^5 + 5*c*d*e^3*g^6)*n^5 + 10*(c*e^4*f*g^5 + 8*c*d
*e^3*g^6)*n^4 + 5*(7*c*e^4*f*g^5 + 95*c*d*e^3*g^6)*n^3 + 50*(c*e^4*f*g^5 +
26*c*d*e^3*g^6)*n^2 + 12*(2*c*e^4*f*g^5 + 135*c*d*e^3*g^6)*n)*x^5 + (20*a*d
^3*f*g^5 - (2*c*d^4 + 3*a*d^2*e)*f^2*g^4)*n^4 + (180*(9*c*d^2*e^2 + a*e^3)*
g^6 + (5*c*d*e^3*f*g^5 + (9*c*d^2*e^2 + a*e^3)*g^6)*n^5 - (5*c*e^4*f^2*g^4
- 60*c*d*e^3*f*g^5 - 17*(9*c*d^2*e^2 + a*e^3)*g^6)*n^4 - (30*c*e^4*f^2*g^4
- 235*c*d*e^3*f*g^5 - 107*(9*c*d^2*e^2 + a*e^3)*g^6)*n^3 - (55*c*e^4*f^2*g^
4 - 360*c*d*e^3*f*g^5 - 307*(9*c*d^2*e^2 + a*e^3)*g^6)*n^2 - 6*(5*c*e^4*f^2
*g^4 - 30*c*d*e^3*f*g^5 - 66*(9*c*d^2*e^2 + a*e^3)*g^6)*n)*x^4 + (155*a*d^3
*f*g^5 + 2*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 18*(2*c*d^4 + 3*a*d^2*e)*f^2*g
^4)*n^3 + (240*(7*c*d^3*e + 3*a*d*e^2)*g^6 + ((9*c*d^2*e^2 + a*e^3)*f*g^5 +
 (7*c*d^3*e + 3*a*d*e^2)*g^6)*n^5 - 2*(10*c*d*e^3*f^2*g^4 - 7*(9*c*d^2*e^2
+ a*e^3)*f*g^5 - 9*(7*c*d^3*e + 3*a*d*e^2)*g^6)*n^4 + (20*c*e^4*f^3*g^3 - 1
80*c*d*e^3*f^2*g^4 + 65*(9*c*d^2*e^2 + a*e^3)*f*g^5 + 121*(7*c*d^3*e + 3*a*
d*e^2)*g^6)*n^3 + 4*(15*c*e^4*f^3*g^3 - 100*c*d*e^3*f^2*g^4 + 28*(9*c*d^2*e
^2 + a*e^3)*f*g^5 + 93*(7*c*d^3*e + 3*a*d*e^2)*g^6)*n^2 + 4*(10*c*e^4*f^3*g
^3 - 60*c*d*e^3*f^2*g^4 + 15*(9*c*d^2*e^2 + a*e^3)*f*g^5 + 127*(7*c*d^3*e +
 3*a*d*e^2)*g^6)*n)*x^3 + (580*a*d^3*f*g^5 - 6*(9*c*d^2*e^2 + a*e^3)*f^4*g^
2 + 30*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 119*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4)
*n^2 + (360*(2*c*d^4 + 3*a*d^2*e)*g^6 + ((7*c*d^3*e + 3*a*d*e^2)*f*g^5 + (2
*c*d^4 + 3*a*d^2*e)*g^6)*n^5 - (3*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 16*(7*c*d
^3*e + 3*a*d*e^2)*f*g^5 - 19*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^4 + (60*c*d*e^3*f
^3*g^3 - 36*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 + 89*(7*c*d^3*e + 3*a*d*e^2)*f*g^
5 + 137*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^3 - (60*c*e^4*f^4*g^2 - 420*c*d*e^3*f^
3*g^3 + 123*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 194*(7*c*d^3*e + 3*a*d*e^2)*f*g
^5 - 461*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^2 - 6*(10*c*e^4*f^4*g^2 - 60*c*d*e^3*
f^3*g^3 + 15*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 20*(7*c*d^3*e + 3*a*d*e^2)*f*g
^5 - 117*(2*c*d^4 + 3*a*d^2*e)*g^6)*n)*x^2 + 2*(60*c*d*e^3*f^5*g + 522*a*d^
3*f*g^5 - 33*(9*c*d^2*e^2 + a*e^3)*f^4*g^2 + 74*(7*c*d^3*e + 3*a*d*e^2)*f^3
*g^3 - 171*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4)*n + (720*a*d^3*g^6 + (a*d^3*g^6 +
 (2*c*d^4 + 3*a*d^2*e)*f*g^5)*n^5 + 2*(10*a*d^3*g^6 - (7*c*d^3*e + 3*a*d*e^
2)*f^2*g^4 + 9*(2*c*d^4 + 3*a*d^2*e)*f*g^5)*n^4 + (155*a*d^3*g^6 + 6*(9*c*d
^2*e^2 + a*e^3)*f^3*g^3 - 30*(7*c*d^3*e + 3*a*d*e^2)*f^2*g^4 + 119*(2*c*d^4
 + 3*a*d^2*e)*f*g^5)*n^3 - 2*(60*c*d*e^3*f^4*g^2 - 290*a*d^3*g^6 - 33*(9*c*
d^2*e^2 + a*e^3)*f^3*g^3 + 74*(7*c*d^3*e + 3*a*d*e^2)*f^2*g^4 - 171*(2*c*d^
4 + 3*a*d^2*e)*f*g^5)*n^2 + 12*(10*c*e^4*f^5*g - 60*c*d*e^3*f^4*g^2 + 87*a*
d^3*g^6 + 15*(9*c*d^2*e^2 + a*e^3)*f^3*g^3 - 20*(7*c*d^3*e + 3*a*d*e^2)*f^2
*g^4 + 30*(2*c*d^4 + 3*a*d^2*e)*f*g^5)*n)*x)*(g*x + f)^n/(g^6*n^6 + 21*g^6*
n^5 + 175*g^6*n^4 + 735*g^6*n^3 + 1624*g^6*n^2 + 1764*g^6*n + 720*g^6)
```

giac [B] time = 0.59, size = 3760, normalized size = 13.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^6*n^5*x^6*e^4 + 5*(g*x + f)^n*c*d*g^6*n^5*x^5*e^3 + 9*(g*x
 + f)^n*c*d^2*g^6*n^5*x^4*e^2 + 7*(g*x + f)^n*c*d^3*g^6*n^5*x^3*e + 2*(g*x
+ f)^n*c*d^4*g^6*n^5*x^2 + (g*x + f)^n*c*f*g^5*n^5*x^5*e^4 + 15*(g*x + f)^n
*c*g^6*n^4*x^6*e^4 + 5*(g*x + f)^n*c*d*f*g^5*n^5*x^4*e^3 + 80*(g*x + f)^n*c
*d*g^6*n^4*x^5*e^3 + 9*(g*x + f)^n*c*d^2*f*g^5*n^5*x^3*e^2 + 153*(g*x + f)^
n*c*d^2*g^6*n^4*x^4*e^2 + 7*(g*x + f)^n*c*d^3*f*g^5*n^5*x^2*e + 126*(g*x +
f)^n*c*d^3*g^6*n^4*x^3*e + 2*(g*x + f)^n*c*d^4*f*g^5*n^5*x + 38*(g*x + f)^n
*c*d^4*g^6*n^4*x^2 + 10*(g*x + f)^n*c*f*g^5*n^4*x^5*e^4 + 85*(g*x + f)^n*c*
g^6*n^3*x^6*e^4 + 60*(g*x + f)^n*c*d*f*g^5*n^4*x^4*e^3 + (g*x + f)^n*a*g^6*
n^5*x^4*e^3 + 475*(g*x + f)^n*c*d*g^6*n^3*x^5*e^3 + 126*(g*x + f)^n*c*d^2*f
*g^5*n^4*x^3*e^2 + 3*(g*x + f)^n*a*d*g^6*n^5*x^3*e^2 + 963*(g*x + f)^n*c*d^
2*g^6*n^3*x^4*e^2 + 112*(g*x + f)^n*c*d^3*f*g^5*n^4*x^2*e + 3*(g*x + f)^n*a
*d^2*g^6*n^5*x^2*e + 847*(g*x + f)^n*c*d^3*g^6*n^3*x^3*e + 36*(g*x + f)^n*c
*d^4*f*g^5*n^4*x + (g*x + f)^n*a*d^3*g^6*n^5*x + 274*(g*x + f)^n*c*d^4*g^6*
n^3*x^2 - 5*(g*x + f)^n*c*f^2*g^4*n^4*x^4*e^4 + 35*(g*x + f)^n*c*f*g^5*n^3*
x^5*e^4 + 225*(g*x + f)^n*c*g^6*n^2*x^6*e^4 - 20*(g*x + f)^n*c*d*f^2*g^4*n^
4*x^3*e^3 + (g*x + f)^n*a*f*g^5*n^5*x^3*e^3 + 235*(g*x + f)^n*c*d*f*g^5*n^3
*x^4*e^3 + 17*(g*x + f)^n*a*g^6*n^4*x^4*e^3 + 1300*(g*x + f)^n*c*d*g^6*n^2*
x^5*e^3 - 27*(g*x + f)^n*c*d^2*f^2*g^4*n^4*x^2*e^2 + 3*(g*x + f)^n*a*d*f*g^
5*n^5*x^2*e^2 + 585*(g*x + f)^n*c*d^2*f*g^5*n^3*x^3*e^2 + 54*(g*x + f)^n*a*
d*g^6*n^4*x^3*e^2 + 2763*(g*x + f)^n*c*d^2*g^6*n^2*x^4*e^2 - 14*(g*x + f)^n
*c*d^3*f^2*g^4*n^4*x*e + 3*(g*x + f)^n*a*d^2*f*g^5*n^5*x*e + 623*(g*x + f)^
n*c*d^3*f*g^5*n^3*x^2*e + 57*(g*x + f)^n*a*d^2*g^6*n^4*x^2*e + 2604*(g*x +
f)^n*c*d^3*g^6*n^2*x^3*e - 2*(g*x + f)^n*c*d^4*f^2*g^4*n^4 + (g*x + f)^n*a*
d^3*f*g^5*n^5 + 238*(g*x + f)^n*c*d^4*f*g^5*n^3*x + 20*(g*x + f)^n*a*d^3*g^
6*n^4*x + 922*(g*x + f)^n*c*d^4*g^6*n^2*x^2 - 30*(g*x + f)^n*c*f^2*g^4*n^3*
x^4*e^4 + 50*(g*x + f)^n*c*f*g^5*n^2*x^5*e^4 + 274*(g*x + f)^n*c*g^6*n*x^6*
e^4 - 180*(g*x + f)^n*c*d*f^2*g^4*n^3*x^3*e^3 + 14*(g*x + f)^n*a*f*g^5*n^4*
x^3*e^3 + 360*(g*x + f)^n*c*d*f*g^5*n^2*x^4*e^3 + 107*(g*x + f)^n*a*g^6*n^3
*x^4*e^3 + 1620*(g*x + f)^n*c*d*g^6*n*x^5*e^3 - 324*(g*x + f)^n*c*d^2*f^2*g
^4*n^3*x^2*e^2 + 48*(g*x + f)^n*a*d*f*g^5*n^4*x^2*e^2 + 1008*(g*x + f)^n*c*
d^2*f*g^5*n^2*x^3*e^2 + 363*(g*x + f)^n*a*d*g^6*n^3*x^3*e^2 + 3564*(g*x + f
)^n*c*d^2*g^6*n*x^4*e^2 - 210*(g*x + f)^n*c*d^3*f^2*g^4*n^3*x*e + 54*(g*x +
 f)^n*a*d^2*f*g^5*n^4*x*e + 1358*(g*x + f)^n*c*d^3*f*g^5*n^2*x^2*e + 411*(g
*x + f)^n*a*d^2*g^6*n^3*x^2*e + 3556*(g*x + f)^n*c*d^3*g^6*n*x^3*e - 36*(g*
x + f)^n*c*d^4*f^2*g^4*n^3 + 20*(g*x + f)^n*a*d^3*f*g^5*n^4 + 684*(g*x + f)
^n*c*d^4*f*g^5*n^2*x + 155*(g*x + f)^n*a*d^3*g^6*n^3*x + 1404*(g*x + f)^n*c
*d^4*g^6*n*x^2 + 20*(g*x + f)^n*c*f^3*g^3*n^3*x^3*e^4 - 55*(g*x + f)^n*c*f^
2*g^4*n^2*x^4*e^4 + 24*(g*x + f)^n*c*f*g^5*n*x^5*e^4 + 120*(g*x + f)^n*c*g^
6*x^6*e^4 + 60*(g*x + f)^n*c*d*f^3*g^3*n^3*x^2*e^3 - 3*(g*x + f)^n*a*f^2*g^
4*n^4*x^2*e^3 - 400*(g*x + f)^n*c*d*f^2*g^4*n^2*x^3*e^3 + 65*(g*x + f)^n*a*
f*g^5*n^3*x^3*e^3 + 180*(g*x + f)^n*c*d*f*g^5*n*x^4*e^3 + 307*(g*x + f)^n*a
*g^6*n^2*x^4*e^3 + 720*(g*x + f)^n*c*d*g^6*x^5*e^3 + 54*(g*x + f)^n*c*d^2*f
^3*g^3*n^3*x*e^2 - 6*(g*x + f)^n*a*d*f^2*g^4*n^4*x*e^2 - 1107*(g*x + f)^n*c
*d^2*f^2*g^4*n^2*x^2*e^2 + 267*(g*x + f)^n*a*d*f*g^5*n^3*x^2*e^2 + 540*(g*x
 + f)^n*c*d^2*f*g^5*n*x^3*e^2 + 1116*(g*x + f)^n*a*d*g^6*n^2*x^3*e^2 + 1620
*(g*x + f)^n*c*d^2*g^6*x^4*e^2 + 14*(g*x + f)^n*c*d^3*f^3*g^3*n^3*e - 3*(g*
x + f)^n*a*d^2*f^2*g^4*n^4*e - 1036*(g*x + f)^n*c*d^3*f^2*g^4*n^2*x*e + 357
*(g*x + f)^n*a*d^2*f*g^5*n^3*x*e + 840*(g*x + f)^n*c*d^3*f*g^5*n*x^2*e + 13
83*(g*x + f)^n*a*d^2*g^6*n^2*x^2*e + 1680*(g*x + f)^n*c*d^3*g^6*x^3*e - 238
*(g*x + f)^n*c*d^4*f^2*g^4*n^2 + 155*(g*x + f)^n*a*d^3*f*g^5*n^3 + 720*(g*x
 + f)^n*c*d^4*f*g^5*n*x + 580*(g*x + f)^n*a*d^3*g^6*n^2*x + 720*(g*x + f)^n
*c*d^4*g^6*x^2 + 60*(g*x + f)^n*c*f^3*g^3*n^2*x^3*e^4 - 30*(g*x + f)^n*c*f^
```

```
2*g^4*n*x^4*e^4 + 420*(g*x + f)^n*c*d*f^3*g^3*n^2*x^2*e^3 - 36*(g*x + f)^n*
a*f^2*g^4*n^3*x^2*e^3 - 240*(g*x + f)^n*c*d*f^2*g^4*n*x^3*e^3 + 112*(g*x +
f)^n*a*f*g^5*n^2*x^3*e^3 + 396*(g*x + f)^n*a*g^6*n*x^4*e^3 + 594*(g*x + f)^
n*c*d^2*f^3*g^3*n^2*x*e^2 - 90*(g*x + f)^n*a*d*f^2*g^4*n^3*x*e^2 - 810*(g*x
 + f)^n*c*d^2*f^2*g^4*n*x^2*e^2 + 582*(g*x + f)^n*a*d*f*g^5*n^2*x^2*e^2 + 1
524*(g*x + f)^n*a*d*g^6*n*x^3*e^2 + 210*(g*x + f)^n*c*d^3*f^3*g^3*n^2*e - 5
4*(g*x + f)^n*a*d^2*f^2*g^4*n^3*e - 1680*(g*x + f)^n*c*d^3*f^2*g^4*n*x*e +
1026*(g*x + f)^n*a*d^2*f*g^5*n^2*x*e + 2106*(g*x + f)^n*a*d^2*g^6*n*x^2*e -
 684*(g*x + f)^n*c*d^4*f^2*g^4*n + 580*(g*x + f)^n*a*d^3*f*g^5*n^2 + 1044*(
g*x + f)^n*a*d^3*g^6*n*x - 60*(g*x + f)^n*c*f^4*g^2*n^2*x^2*e^4 + 40*(g*x +
 f)^n*c*f^3*g^3*n*x^3*e^4 - 120*(g*x + f)^n*c*d*f^4*g^2*n^2*x*e^3 + 6*(g*x
+ f)^n*a*f^3*g^3*n^3*x*e^3 + 360*(g*x + f)^n*c*d*f^3*g^3*n*x^2*e^3 - 123*(g
*x + f)^n*a*f^2*g^4*n^2*x^2*e^3 + 60*(g*x + f)^n*a*f*g^5*n*x^3*e^3 + 180*(g
*x + f)^n*a*g^6*x^4*e^3 - 54*(g*x + f)^n*c*d^2*f^4*g^2*n^2*e^2 + 6*(g*x + f
)^n*a*d*f^3*g^3*n^3*e^2 + 1620*(g*x + f)^n*c*d^2*f^3*g^3*n*x*e^2 - 444*(g*x
 + f)^n*a*d*f^2*g^4*n^2*x*e^2 + 360*(g*x + f)^n*a*d*f*g^5*n*x^2*e^2 + 720*(
g*x + f)^n*a*d*g^6*x^3*e^2 + 1036*(g*x + f)^n*c*d^3*f^3*g^3*n*e - 357*(g*x
+ f)^n*a*d^2*f^2*g^4*n^2*e + 1080*(g*x + f)^n*a*d^2*f*g^5*n*x*e + 1080*(g*x
 + f)^n*a*d^2*g^6*x^2*e - 720*(g*x + f)^n*c*d^4*f^2*g^4 + 1044*(g*x + f)^n*
a*d^3*f*g^5*n + 720*(g*x + f)^n*a*d^3*g^6*x - 60*(g*x + f)^n*c*f^4*g^2*n*x^
2*e^4 - 720*(g*x + f)^n*c*d*f^4*g^2*n*x*e^3 + 66*(g*x + f)^n*a*f^3*g^3*n^2*
x*e^3 - 90*(g*x + f)^n*a*f^2*g^4*n*x^2*e^3 - 594*(g*x + f)^n*c*d^2*f^4*g^2*
n*e^2 + 90*(g*x + f)^n*a*d*f^3*g^3*n^2*e^2 - 720*(g*x + f)^n*a*d*f^2*g^4*n*
x*e^2 + 1680*(g*x + f)^n*c*d^3*f^3*g^3*e - 1026*(g*x + f)^n*a*d^2*f^2*g^4*n
*e + 720*(g*x + f)^n*a*d^3*f*g^5 + 120*(g*x + f)^n*c*f^5*g*n*x*e^4 + 120*(g
*x + f)^n*c*d*f^5*g*n*e^3 - 6*(g*x + f)^n*a*f^4*g^2*n^2*e^3 + 180*(g*x + f)
^n*a*f^3*g^3*n*x*e^3 - 1620*(g*x + f)^n*c*d^2*f^4*g^2*e^2 + 444*(g*x + f)^n
*a*d*f^3*g^3*n*e^2 - 1080*(g*x + f)^n*a*d^2*f^2*g^4*e + 720*(g*x + f)^n*c*d
*f^5*g*e^3 - 66*(g*x + f)^n*a*f^4*g^2*n*e^3 + 720*(g*x + f)^n*a*d*f^3*g^3*e
^2 - 120*(g*x + f)^n*c*f^6*e^4 - 180*(g*x + f)^n*a*f^4*g^2*e^3)/(g^6*n^6 +
21*g^6*n^5 + 175*g^6*n^4 + 735*g^6*n^3 + 1624*g^6*n^2 + 1764*g^6*n + 720*g^
6)
```

maple [B] time = 0.02, size = 2017, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(n+1)*(c*e^4*g^5*n^5*x^5+5*c*d*e^3*g^5*n^5*x^4+15*c*e^4*g^5*n^4*x^5
+9*c*d^2*e^2*g^5*n^5*x^3+80*c*d*e^3*g^5*n^4*x^4-5*c*e^4*f*g^4*n^4*x^4+85*c*
e^4*g^5*n^3*x^5+a*e^3*g^5*n^5*x^3+7*c*d^3*e*g^5*n^5*x^2+153*c*d^2*e^2*g^5*n
^4*x^3-20*c*d*e^3*f*g^4*n^4*x^3+475*c*d*e^3*g^5*n^3*x^4-50*c*e^4*f*g^4*n^3*
x^4+225*c*e^4*g^5*n^2*x^5+3*a*d*e^2*g^5*n^5*x^2+17*a*e^3*g^5*n^4*x^3+2*c*d^
4*g^5*n^5*x+126*c*d^3*e*g^5*n^4*x^2-27*c*d^2*e^2*f*g^4*n^4*x^2+963*c*d^2*e^
2*g^5*n^3*x^3-240*c*d*e^3*f*g^4*n^3*x^3+1300*c*d*e^3*g^5*n^2*x^4+20*c*e^4*f
^2*g^3*n^3*x^3-175*c*e^4*f*g^4*n^2*x^4+274*c*e^4*g^5*n*x^5+3*a*d^2*e*g^5*n^
5*x+54*a*d*e^2*g^5*n^4*x^2-3*a*e^3*f*g^4*n^4*x^2+107*a*e^3*g^5*n^3*x^3+38*c
*d^4*g^5*n^4*x-14*c*d^3*e*f*g^4*n^4*x+847*c*d^3*e*g^5*n^3*x^2-378*c*d^2*e^2
*f*g^4*n^3*x^2+2763*c*d^2*e^2*g^5*n^2*x^3+60*c*d*e^3*f^2*g^3*n^3*x^2-940*c*
d*e^3*f*g^4*n^2*x^3+1620*c*d*e^3*g^5*n*x^4+120*c*e^4*f^2*g^3*n^2*x^3-250*c*
e^4*f*g^4*n*x^4+120*c*e^4*g^5*x^5+a*d^3*g^5*n^5+57*a*d^2*e*g^5*n^4*x-6*a*d*
e^2*f*g^4*n^4*x+363*a*d*e^2*g^5*n^3*x^2-42*a*e^3*f*g^4*n^3*x^2+307*a*e^3*g^
5*n^2*x^3-2*c*d^4*f*g^4*n^4+274*c*d^4*g^5*n^3*x-224*c*d^3*e*f*g^4*n^3*x+260
4*c*d^3*e*g^5*n^2*x^2+54*c*d^2*e^2*f^2*g^3*n^3*x-1755*c*d^2*e^2*f*g^4*n^2*x
^2+3564*c*d^2*e^2*g^5*n*x^3+540*c*d*e^3*f^2*g^3*n^2*x^2-1440*c*d*e^3*f*g^4*
n*x^3+720*c*d*e^3*g^5*x^4-60*c*e^4*f^3*g^2*n^2*x^2+220*c*e^4*f^2*g^3*n*x^3-
120*c*e^4*f*g^4*x^4+20*a*d^3*g^5*n^4-3*a*d^2*e*f*g^4*n^4+411*a*d^2*e*g^5*n^
3*x-96*a*d*e^2*f*g^4*n^3*x+1116*a*d*e^2*g^5*n^2*x^2+6*a*e^3*f^2*g^3*n^3*x-1
```

```
95*a*e^3*f*g^4*n^2*x^2+396*a*e^3*g^5*n*x^3-36*c*d^4*f*g^4*n^3+922*c*d^4*g^5
*n^2*x+14*c*d^3*e*f^2*g^3*n^3-1246*c*d^3*e*f*g^4*n^2*x+3556*c*d^3*e*g^5*n*x
^2+648*c*d^2*e^2*f^2*g^3*n^2*x-3024*c*d^2*e^2*f*g^4*n*x^2+1620*c*d^2*e^2*g^
5*x^3-120*c*d*e^3*f^3*g^2*n^2*x+1200*c*d*e^3*f^2*g^3*n*x^2-720*c*d*e^3*f*g^
4*x^3-180*c*e^4*f^3*g^2*n*x^2+120*c*e^4*f^2*g^3*x^3+155*a*d^3*g^5*n^3-54*a*
d^2*e*f*g^4*n^3+1383*a*d^2*e*g^5*n^2*x+6*a*d*e^2*f^2*g^3*n^3-534*a*d*e^2*f*
g^4*n^2*x+1524*a*d*e^2*g^5*n*x^2+72*a*e^3*f^2*g^3*n^2*x-336*a*e^3*f*g^4*n*x
^2+180*a*e^3*g^5*x^3-238*c*d^4*f*g^4*n^2+1404*c*d^4*g^5*n*x+210*c*d^3*e*f^2
*g^3*n^2-2716*c*d^3*e*f*g^4*n*x+1680*c*d^3*e*g^5*x^2-54*c*d^2*e^2*f^3*g^2*n
^2+2214*c*d^2*e^2*f^2*g^3*n*x-1620*c*d^2*e^2*f*g^4*x^2-840*c*d*e^3*f^3*g^2*
n*x+720*c*d*e^3*f^2*g^3*x^2+120*c*e^4*f^4*g*n*x-120*c*e^4*f^3*g^2*x^2+580*a
*d^3*g^5*n^2-357*a*d^2*e*f*g^4*n^2+2106*a*d^2*e*g^5*n*x+90*a*d*e^2*f^2*g^3*
n^2-1164*a*d*e^2*f*g^4*n*x+720*a*d*e^2*g^5*x^2-6*a*e^3*f^3*g^2*n^2+246*a*e^
3*f^2*g^3*n*x-180*a*e^3*f*g^4*x^2-684*c*d^4*f*g^4*n+720*c*d^4*g^5*x+1036*c*
d^3*e*f^2*g^3*n-1680*c*d^3*e*f*g^4*x-594*c*d^2*e^2*f^3*g^2*n+1620*c*d^2*e^2
*f^2*g^3*x+120*c*d*e^3*f^4*g*n-720*c*d*e^3*f^3*g^2*x+120*c*e^4*f^4*g*x+1044
*a*d^3*g^5*n-1026*a*d^2*e*f*g^4*n+1080*a*d^2*e*g^5*x+444*a*d*e^2*f^2*g^3*n-
720*a*d*e^2*f*g^4*x-66*a*e^3*f^3*g^2*n+180*a*e^3*f^2*g^3*x-720*c*d^4*f*g^4+
1680*c*d^3*e*f^2*g^3-1620*c*d^2*e^2*f^3*g^2+720*c*d*e^3*f^4*g-120*c*e^4*f^5
+720*a*d^3*g^5-1080*a*d^2*e*f*g^4+720*a*d*e^2*f^2*g^3-180*a*e^3*f^3*g^2)/g^
6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)
```

maxima [B] time = 0.60, size = 811, normalized size = 2.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^4/((n^2 + 3*n + 2)*g^2)
 + 7*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d^3*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 3*(g^2*(n + 1)*x^2 + f
*g*n*x - f^2)*(g*x + f)^n*a*d^2*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)
*a*d^3/(g*(n + 1)) + 9*((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2
*n)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*
c*d^2*e^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4) + 3*((n^2 + 3*n + 2)*g^
3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x + f)^n*a*d*e^2/((n^
3 + 6*n^2 + 11*n + 6)*g^3) + 5*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^5*x^5
 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*f*g^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*f^2*g^3*x
^3 + 12*(n^2 + n)*f^3*g^2*x^2 - 24*f^4*g*n*x + 24*f^5)*(g*x + f)^n*c*d*e^3/
((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^5) + ((n^3 + 6*n^2 + 11*
n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 +
6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*a*e^3/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)
*g^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^6*x^6 + (n^5 + 1
0*n^4 + 35*n^3 + 50*n^2 + 24*n)*f*g^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*
f^2*g^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*f^3*g^3*x^3 - 60*(n^2 + n)*f^4*g^2*x^2
 + 120*f^5*g*n*x - 120*f^6)*(g*x + f)^n*c*e^4/((n^6 + 21*n^5 + 175*n^4 + 73
5*n^3 + 1624*n^2 + 1764*n + 720)*g^6)
```

mupad [B] time = 3.90, size = 1943, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)^3*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (x*(f + g*x)^n*(720*a*d^3*g^6 + 580*a*d^3*g^6*n^2 + 155*a*d^3*g^6*n^3 + 20*
a*d^3*g^6*n^4 + a*d^3*g^6*n^5 + 1044*a*d^3*g^6*n + 720*c*d^4*f*g^5*n + 120*
c*e^4*f^5*g*n + 180*a*e^3*f^3*g^3*n + 684*c*d^4*f*g^5*n^2 + 238*c*d^4*f*g^5
*n^3 + 36*c*d^4*f*g^5*n^4 + 2*c*d^4*f*g^5*n^5 + 66*a*e^3*f^3*g^3*n^2 + 6*a*
e^3*f^3*g^3*n^3 - 444*a*d*e^2*f^2*g^4*n^2 - 90*a*d*e^2*f^2*g^4*n^3 - 6*a*d*
```

```
e^2*f^2*g^4*n^4 + 1620*c*d^2*e^2*f^3*g^3*n - 120*c*d*e^3*f^4*g^2*n^2 - 1036
*c*d^3*e*f^2*g^4*n^2 - 210*c*d^3*e*f^2*g^4*n^3 - 14*c*d^3*e*f^2*g^4*n^4 + 1
080*a*d^2*e*f*g^5*n + 594*c*d^2*e^2*f^3*g^3*n^2 + 54*c*d^2*e^2*f^3*g^3*n^3
- 720*a*d*e^2*f^2*g^4*n + 1026*a*d^2*e*f*g^5*n^2 + 357*a*d^2*e*f*g^5*n^3 +
54*a*d^2*e*f*g^5*n^4 + 3*a*d^2*e*f*g^5*n^5 - 720*c*d*e^3*f^4*g^2*n - 1680*c
*d^3*e*f^2*g^4*n))/(g^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n
^6 + 720)) - ((f + g*x)^n*(120*c*e^4*f^6 + 180*a*e^3*f^4*g^2 + 720*c*d^4*f^
2*g^4 - 720*a*d^3*f*g^5 - 720*c*d*e^3*f^5*g - 1044*a*d^3*f*g^5*n - 720*a*d*
e^2*f^3*g^3 + 1080*a*d^2*e*f^2*g^4 - 1680*c*d^3*e*f^3*g^3 - 580*a*d^3*f*g^5
*n^2 - 155*a*d^3*f*g^5*n^3 - 20*a*d^3*f*g^5*n^4 - a*d^3*f*g^5*n^5 + 66*a*e^
3*f^4*g^2*n + 684*c*d^4*f^2*g^4*n + 1620*c*d^2*e^2*f^4*g^2 + 6*a*e^3*f^4*g^
2*n^2 + 238*c*d^4*f^2*g^4*n^2 + 36*c*d^4*f^2*g^4*n^3 + 2*c*d^4*f^2*g^4*n^4
- 90*a*d*e^2*f^3*g^3*n^2 + 357*a*d^2*e*f^2*g^4*n^2 - 6*a*d*e^2*f^3*g^3*n^3
+ 54*a*d^2*e*f^2*g^4*n^3 + 3*a*d^2*e*f^2*g^4*n^4 + 594*c*d^2*e^2*f^4*g^2*n
- 210*c*d^3*e*f^3*g^3*n^2 - 14*c*d^3*e*f^3*g^3*n^3 - 120*c*d*e^3*f^5*g*n +
54*c*d^2*e^2*f^4*g^2*n^2 - 444*a*d*e^2*f^3*g^3*n + 1026*a*d^2*e*f^2*g^4*n -
 1036*c*d^3*e*f^3*g^3*n))/(g^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*
n^5 + n^6 + 720)) + (c*e^4*x^6*(f + g*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n
^4 + n^5 + 120))/(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 72
0) + (x^2*(f + g*x)^n*(n + 1)*(720*c*d^4*g^4 + 238*c*d^4*g^4*n^2 + 36*c*d^4
*g^4*n^3 + 2*c*d^4*g^4*n^4 + 1080*a*d^2*e*g^4 + 684*c*d^4*g^4*n - 60*c*e^4*
f^4*n + 1026*a*d^2*e*g^4*n + 357*a*d^2*e*g^4*n^2 + 54*a*d^2*e*g^4*n^3 + 3*a
*d^2*e*g^4*n^4 - 90*a*e^3*f^2*g^2*n - 33*a*e^3*f^2*g^2*n^2 - 3*a*e^3*f^2*g^
2*n^3 - 810*c*d^2*e^2*f^2*g^2*n + 360*a*d*e^2*f*g^3*n + 360*c*d*e^3*f^3*g*n
 + 840*c*d^3*e*f*g^3*n - 297*c*d^2*e^2*f^2*g^2*n^2 - 27*c*d^2*e^2*f^2*g^2*n
^3 + 222*a*d*e^2*f*g^3*n^2 + 45*a*d*e^2*f*g^3*n^3 + 3*a*d*e^2*f*g^3*n^4 + 6
0*c*d*e^3*f^3*g*n^2 + 518*c*d^3*e*f*g^3*n^2 + 105*c*d^3*e*f*g^3*n^3 + 7*c*d
^3*e*f*g^3*n^4))/(g^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6
 + 720)) + (e*x^3*(f + g*x)^n*(3*n + n^2 + 2)*(840*c*d^3*g^3 + 105*c*d^3*g^
3*n^2 + 7*c*d^3*g^3*n^3 + 360*a*d*e*g^3 + 518*c*d^3*g^3*n + 20*c*e^3*f^3*n
+ 45*a*d*e*g^3*n^2 + 3*a*d*e*g^3*n^3 + 30*a*e^2*f*g^2*n + 11*a*e^2*f*g^2*n^
2 + a*e^2*f*g^2*n^3 + 222*a*d*e*g^3*n - 120*c*d*e^2*f^2*g*n + 270*c*d^2*e*f
*g^2*n - 20*c*d*e^2*f^2*g*n^2 + 99*c*d^2*e*f*g^2*n^2 + 9*c*d^2*e*f*g^2*n^3)
)/(g^3*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (e^2
*x^4*(f + g*x)^n*(11*n + 6*n^2 + n^3 + 6)*(270*c*d^2*g^2 + 30*a*e*g^2 + 9*c
*d^2*g^2*n^2 + 11*a*e*g^2*n + a*e*g^2*n^2 + 99*c*d^2*g^2*n - 5*c*e^2*f^2*n
+ 5*c*d*e*f*g*n^2 + 30*c*d*e*f*g*n))/(g^2*(1764*n + 1624*n^2 + 735*n^3 + 17
5*n^4 + 21*n^5 + n^6 + 720)) + (c*e^3*x^5*(f + g*x)^n*(30*d*g + 5*d*g*n + e
*f*n)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(g*(1764*n + 1624*n^2 + 735*n^3
+ 175*n^4 + 21*n^5 + n^6 + 720))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Timed out
```

3.554 $\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$

Optimal. Leaf size=208

$$-\frac{2(ef-dg)(f+gx)^{n+2}\left(aeg^2+c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{g^5(n+2)}+\frac{e(f+gx)^{n+3}\left(aeg^2+c\left(5d^2g^2-12defg+6e^2f^2\right)\right)}{g^5(n+3)}+$$

Rubi [A] time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {947}

$$-\frac{2(ef-dg)(f+gx)^{n+2}\left(aeg^2+c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{g^5(n+2)}+\frac{e(f+gx)^{n+3}\left(aeg^2+c\left(5d^2g^2-12defg+6e^2f^2\right)\right)}{g^5(n+3)}+\frac{(ef-dg)^2(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^5(n+1)}-\frac{4ce^2(ef-dg)(f+gx)^{n+4}}{g^5(n+4)}+\frac{ce^3(f+gx)^{n+5}}{g^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^5*(1 + n))
 - (2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x)
^(2 + n))/(g^5*(2 + n)) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g
^2))*(f + g*x)^(3 + n))/(g^5*(3 + n)) - (4*c*e^2*(e*f - d*g)*(f + g*x)^(4 +
 n))/(g^5*(4 + n)) + (c*e^3*(f + g*x)^(5 + n))/(g^5*(5 + n))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right)\,dx=\int\left(\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)(f+gx)^n}{g^4}+\frac{2(ef-dg)\left(-aeg^2\right.}{}\right.$$

$$=\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^5(1+n)}-\frac{2(ef-dg)\left(aeg^2+c\right.}{}$$

Mathematica [A] time = 0.20, size = 187, normalized size = 0.90

$$\frac{(f+gx)^{n+1}\left(\frac{e(f+gx)^2\left(aeg^2+c\left(5d^2g^2-12defg+6e^2f^2\right)\right)}{n+3}-\frac{2(f+gx)(ef-dg)\left(aeg^2+c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{n+2}+\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)}{n+1}-\frac{4ce^2(f+gx)^3(ef-dg)}{n+4}+\frac{ce^3(f+gx)^4}{n+5}\right)}{g^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n) - (
2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x))/(2
 + n) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g^2))*(f + g*x)^2)/
(3 + n) - (4*c*e^2*(e*f - d*g)*(f + g*x)^3)/(4 + n) + (c*e^3*(f + g*x)^4)/(
5 + n)))/g^5
```

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),
x]
```

fricas [B] time = 0.44, size = 1122, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d^2*f*g^4*n^4 + 24*c*e^3*f^5 - 120*c*d*e^2*f^4*g + 120*a*d^2*f*g^4 + 40*
(5*c*d^2*e + a*e^2)*f^3*g^2 - 120*(c*d^3 + a*d*e)*f^2*g^3 + (c*e^3*g^5*n^4
+ 10*c*e^3*g^5*n^3 + 35*c*e^3*g^5*n^2 + 50*c*e^3*g^5*n + 24*c*e^3*g^5)*x^5
+ (120*c*d*e^2*g^5 + (c*e^3*f*g^4 + 4*c*d*e^2*g^5)*n^4 + 2*(3*c*e^3*f*g^4 +
 22*c*d*e^2*g^5)*n^3 + (11*c*e^3*f*g^4 + 164*c*d*e^2*g^5)*n^2 + 2*(3*c*e^3*
f*g^4 + 122*c*d*e^2*g^5)*n)*x^4 + 2*(7*a*d^2*f*g^4 - (c*d^3 + a*d*e)*f^2*g^
3)*n^3 + (40*(5*c*d^2*e + a*e^2)*g^5 + (4*c*d*e^2*f*g^4 + (5*c*d^2*e + a*e^
2)*g^5)*n^4 - 4*(c*e^3*f^2*g^3 - 8*c*d*e^2*f*g^4 - 3*(5*c*d^2*e + a*e^2)*g^
5)*n^3 - (12*c*e^3*f^2*g^3 - 68*c*d*e^2*f*g^4 - 49*(5*c*d^2*e + a*e^2)*g^5)
*n^2 - 2*(4*c*e^3*f^2*g^3 - 20*c*d*e^2*f*g^4 - 39*(5*c*d^2*e + a*e^2)*g^5)*
n)*x^3 + (71*a*d^2*f*g^4 + 2*(5*c*d^2*e + a*e^2)*f^3*g^2 - 24*(c*d^3 + a*d*
e)*f^2*g^3)*n^2 + (120*(c*d^3 + a*d*e)*g^5 + ((5*c*d^2*e + a*e^2)*f*g^4 + 2
*(c*d^3 + a*d*e)*g^5)*n^4 - 2*(6*c*d*e^2*f^2*g^3 - 5*(5*c*d^2*e + a*e^2)*f*
g^4 - 13*(c*d^3 + a*d*e)*g^5)*n^3 + (12*c*e^3*f^3*g^2 - 72*c*d*e^2*f^2*g^3
+ 29*(5*c*d^2*e + a*e^2)*f*g^4 + 118*(c*d^3 + a*d*e)*g^5)*n^2 + 2*(6*c*e^3*
f^3*g^2 - 30*c*d*e^2*f^2*g^3 + 10*(5*c*d^2*e + a*e^2)*f*g^4 + 107*(c*d^3 +
a*d*e)*g^5)*n)*x^2 - 2*(12*c*d*e^2*f^4*g - 77*a*d^2*f*g^4 - 9*(5*c*d^2*e +
a*e^2)*f^3*g^2 + 47*(c*d^3 + a*d*e)*f^2*g^3)*n + (120*a*d^2*g^5 + (a*d^2*g^
5 + 2*(c*d^3 + a*d*e)*f*g^4)*n^4 + 2*(7*a*d^2*g^5 - (5*c*d^2*e + a*e^2)*f^2
*g^3 + 12*(c*d^3 + a*d*e)*f*g^4)*n^3 + (24*c*d*e^2*f^3*g^2 + 71*a*d^2*g^5 -
 18*(5*c*d^2*e + a*e^2)*f^2*g^3 + 94*(c*d^3 + a*d*e)*f*g^4)*n^2 - 2*(12*c*e
^3*f^4*g - 60*c*d*e^2*f^3*g^2 - 77*a*d^2*g^5 + 20*(5*c*d^2*e + a*e^2)*f^2*g
^3 - 60*(c*d^3 + a*d*e)*f*g^4)*n)*x)*(g*x + f)^n/(g^5*n^5 + 15*g^5*n^4 + 85
*g^5*n^3 + 225*g^5*n^2 + 274*g^5*n + 120*g^5)
```

giac [B] time = 0.23, size = 2114, normalized size = 10.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^5*n^4*x^5*e^3 + 4*(g*x + f)^n*c*d*g^5*n^4*x^4*e^2 + 5*(g*x
 + f)^n*c*d^2*g^5*n^4*x^3*e + 2*(g*x + f)^n*c*d^3*g^5*n^4*x^2 + (g*x + f)^n
*c*f*g^4*n^4*x^4*e^3 + 10*(g*x + f)^n*c*g^5*n^3*x^5*e^3 + 4*(g*x + f)^n*c*d
*f*g^4*n^4*x^3*e^2 + 44*(g*x + f)^n*c*d*g^5*n^3*x^4*e^2 + 5*(g*x + f)^n*c*d
^2*f*g^4*n^4*x^2*e + 60*(g*x + f)^n*c*d^2*g^5*n^3*x^3*e + 2*(g*x + f)^n*c*d
^3*f*g^4*n^4*x + 26*(g*x + f)^n*c*d^3*g^5*n^3*x^2 + 6*(g*x + f)^n*c*f*g^4*n
^3*x^4*e^3 + 35*(g*x + f)^n*c*g^5*n^2*x^5*e^3 + 32*(g*x + f)^n*c*d*f*g^4*n^
3*x^3*e^2 + (g*x + f)^n*a*g^5*n^4*x^3*e^2 + 164*(g*x + f)^n*c*d*g^5*n^2*x^4
*e^2 + 50*(g*x + f)^n*c*d^2*f*g^4*n^3*x^2*e + 2*(g*x + f)^n*a*d*g^5*n^4*x^2
*e + 245*(g*x + f)^n*c*d^2*g^5*n^2*x^3*e + 24*(g*x + f)^n*c*d^3*f*g^4*n^3*x
 + (g*x + f)^n*a*d^2*g^5*n^4*x + 118*(g*x + f)^n*c*d^3*g^5*n^2*x^2 - 4*(g*x
 + f)^n*c*f^2*g^3*n^3*x^3*e^3 + 11*(g*x + f)^n*c*f*g^4*n^2*x^4*e^3 + 50*(g*
x + f)^n*c*g^5*n*x^5*e^3 - 12*(g*x + f)^n*c*d*f^2*g^3*n^3*x^2*e^2 + (g*x +
f)^n*a*f*g^4*n^4*x^2*e^2 + 68*(g*x + f)^n*c*d*f*g^4*n^2*x^3*e^2 + 12*(g*x +
 f)^n*a*g^5*n^3*x^3*e^2 + 244*(g*x + f)^n*c*d*g^5*n*x^4*e^2 - 10*(g*x + f)^
n*c*d^2*f^2*g^3*n^3*x*e + 2*(g*x + f)^n*a*d*f*g^4*n^4*x*e + 145*(g*x + f)^n
```

```
*c*d^2*f*g^4*n^2*x^2*e + 26*(g*x + f)^n*a*d*g^5*n^3*x^2*e + 390*(g*x + f)^n
*c*d^2*g^5*n*x^3*e - 2*(g*x + f)^n*c*d^3*f^2*g^3*n^3 + (g*x + f)^n*a*d^2*f*
g^4*n^4 + 94*(g*x + f)^n*c*d^3*f*g^4*n^2*x + 14*(g*x + f)^n*a*d^2*g^5*n^3*x
 + 214*(g*x + f)^n*c*d^3*g^5*n*x^2 - 12*(g*x + f)^n*c*f^2*g^3*n^2*x^3*e^3 +
 6*(g*x + f)^n*c*f*g^4*n*x^4*e^3 + 24*(g*x + f)^n*c*g^5*x^5*e^3 - 72*(g*x +
 f)^n*c*d*f^2*g^3*n^2*x^2*e^2 + 10*(g*x + f)^n*a*f*g^4*n^3*x^2*e^2 + 40*(g*
x + f)^n*c*d*f*g^4*n*x^3*e^2 + 49*(g*x + f)^n*a*g^5*n^2*x^3*e^2 + 120*(g*x
+ f)^n*c*d*g^5*x^4*e^2 - 90*(g*x + f)^n*c*d^2*f^2*g^3*n^2*x*e + 24*(g*x + f
)^n*a*d*f*g^4*n^3*x*e + 100*(g*x + f)^n*c*d^2*f*g^4*n*x^2*e + 118*(g*x + f)
^n*a*d*g^5*n^2*x^2*e + 200*(g*x + f)^n*c*d^2*g^5*x^3*e - 24*(g*x + f)^n*c*d
^3*f^2*g^3*n^2 + 14*(g*x + f)^n*a*d^2*f*g^4*n^3 + 120*(g*x + f)^n*c*d^3*f*g
^4*n*x + 71*(g*x + f)^n*a*d^2*g^5*n^2*x + 120*(g*x + f)^n*c*d^3*g^5*x^2 + 1
2*(g*x + f)^n*c*f^3*g^2*n^2*x^2*e^3 - 8*(g*x + f)^n*c*f^2*g^3*n*x^3*e^3 + 2
4*(g*x + f)^n*c*d*f^3*g^2*n^2*x*e^2 - 2*(g*x + f)^n*a*f^2*g^3*n^3*x*e^2 - 6
0*(g*x + f)^n*c*d*f^2*g^3*n*x^2*e^2 + 29*(g*x + f)^n*a*f*g^4*n^2*x^2*e^2 +
78*(g*x + f)^n*a*g^5*n*x^3*e^2 + 10*(g*x + f)^n*c*d^2*f^3*g^2*n^2*e - 2*(g*
x + f)^n*a*d*f^2*g^3*n^3*e - 200*(g*x + f)^n*c*d^2*f^2*g^3*n*x*e + 94*(g*x
+ f)^n*a*d*f*g^4*n^2*x*e + 214*(g*x + f)^n*a*d*g^5*n*x^2*e - 94*(g*x + f)^n
*c*d^3*f^2*g^3*n + 71*(g*x + f)^n*a*d^2*f*g^4*n^2 + 154*(g*x + f)^n*a*d^2*g
^5*n*x + 12*(g*x + f)^n*c*f^3*g^2*n*x^2*e^3 + 120*(g*x + f)^n*c*d*f^3*g^2*n
*x*e^2 - 18*(g*x + f)^n*a*f^2*g^3*n^2*x*e^2 + 20*(g*x + f)^n*a*f*g^4*n*x^2*
e^2 + 40*(g*x + f)^n*a*g^5*x^3*e^2 + 90*(g*x + f)^n*c*d^2*f^3*g^2*n*e - 24*
(g*x + f)^n*a*d*f^2*g^3*n^2*e + 120*(g*x + f)^n*a*d*f*g^4*n*x*e + 120*(g*x
+ f)^n*a*d*g^5*x^2*e - 120*(g*x + f)^n*c*d^3*f^2*g^3 + 154*(g*x + f)^n*a*d^
2*f*g^4*n + 120*(g*x + f)^n*a*d^2*g^5*x - 24*(g*x + f)^n*c*f^4*g*n*x*e^3 -
24*(g*x + f)^n*c*d*f^4*g*n*e^2 + 2*(g*x + f)^n*a*f^3*g^2*n^2*e^2 - 40*(g*x
+ f)^n*a*f^2*g^3*n*x*e^2 + 200*(g*x + f)^n*c*d^2*f^3*g^2*e - 94*(g*x + f)^n
*a*d*f^2*g^3*n*e + 120*(g*x + f)^n*a*d^2*f*g^4 - 120*(g*x + f)^n*c*d*f^4*g*
e^2 + 18*(g*x + f)^n*a*f^3*g^2*n*e^2 - 120*(g*x + f)^n*a*d*f^2*g^3*e + 24*(
g*x + f)^n*c*f^5*e^3 + 40*(g*x + f)^n*a*f^3*g^2*e^2)/(g^5*n^5 + 15*g^5*n^4
+ 85*g^5*n^3 + 225*g^5*n^2 + 274*g^5*n + 120*g^5)
```

maple [B] time = 0.01, size = 1048, normalized size = 5.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(n+1)*(c*e^3*g^4*n^4*x^4+4*c*d*e^2*g^4*n^4*x^3+10*c*e^3*g^4*n^3*x^4
+5*c*d^2*e*g^4*n^4*x^2+44*c*d*e^2*g^4*n^3*x^3-4*c*e^3*f*g^3*n^3*x^3+35*c*e^
3*g^4*n^2*x^4+a*e^2*g^4*n^4*x^2+2*c*d^3*g^4*n^4*x+60*c*d^2*e*g^4*n^3*x^2-12
*c*d*e^2*f*g^3*n^3*x^2+164*c*d*e^2*g^4*n^2*x^3-24*c*e^3*f*g^3*n^2*x^3+50*c*
e^3*g^4*n*x^4+2*a*d*e*g^4*n^4*x+12*a*e^2*g^4*n^3*x^2+26*c*d^3*g^4*n^3*x-10*
c*d^2*e*f*g^3*n^3*x+245*c*d^2*e*g^4*n^2*x^2-96*c*d*e^2*f*g^3*n^2*x^2+244*c*
d*e^2*g^4*n*x^3+12*c*e^3*f^2*g^2*n^2*x^2-44*c*e^3*f*g^3*n*x^3+24*c*e^3*g^4*
x^4+a*d^2*g^4*n^4+26*a*d*e*g^4*n^3*x-2*a*e^2*f*g^3*n^3*x+49*a*e^2*g^4*n^2*x
^2-2*c*d^3*f*g^3*n^3+118*c*d^3*g^4*n^2*x-100*c*d^2*e*f*g^3*n^2*x+390*c*d^2*
e*g^4*n*x^2+24*c*d*e^2*f^2*g^2*n^2*x-204*c*d*e^2*f*g^3*n*x^2+120*c*d*e^2*g^
4*x^3+36*c*e^3*f^2*g^2*n*x^2-24*c*e^3*f*g^3*x^3+14*a*d^2*g^4*n^3-2*a*d*e*f*
g^3*n^3+118*a*d*e*g^4*n^2*x-20*a*e^2*f*g^3*n^2*x+78*a*e^2*g^4*n*x^2-24*c*d^
3*f*g^3*n^2+214*c*d^3*g^4*n*x+10*c*d^2*e*f^2*g^2*n^2-290*c*d^2*e*f*g^3*n*x+
200*c*d^2*e*g^4*x^2+144*c*d*e^2*f^2*g^2*n*x-120*c*d*e^2*f*g^3*x^2-24*c*e^3*
f^3*g*n*x+24*c*e^3*f^2*g^2*x^2+71*a*d^2*g^4*n^2-24*a*d*e*f*g^3*n^2+214*a*d*
e*g^4*n*x+2*a*e^2*f^2*g^2*n^2-58*a*e^2*f*g^3*n*x+40*a*e^2*g^4*x^2-94*c*d^3*
f*g^3*n+120*c*d^3*g^4*x+90*c*d^2*e*f^2*g^2*n-200*c*d^2*e*f*g^3*x-24*c*d*e^2
*f^3*g*n+120*c*d*e^2*f^2*g^2*x-24*c*e^3*f^3*g*x+154*a*d^2*g^4*n-94*a*d*e*f*
g^3*n+120*a*d*e*g^4*x+18*a*e^2*f^2*g^2*n-40*a*e^2*f*g^3*x-120*c*d^3*f*g^3+2
00*c*d^2*e*f^2*g^2-120*c*d*e^2*f^3*g+24*c*e^3*f^4+120*a*d^2*g^4-120*a*d*e*f
*g^3+40*a*e^2*f^2*g^2)/g^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

maxima [B] time = 0.54, size = 512, normalized size = 2.46

$$\frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n cd^3}{\left(n^2+3n+2\right)g^2}+\frac{5\left(\left(n^2+3n+2\right)g^3x^3+\left(n^2+n\right)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n cd^2e}{\left(n^3+6n^2+11n+6\right)g^3}+\frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n ade}{\left(n^2+3n+2\right)g^2}+\frac{(gx+f)^{n+1}ad^2}{g(n+1)}+\frac{4\left(\left(n^3+6n^2+11n+6\right)g^4x^4+\left(n^3+3n^2+2n\right)fg^3x^3-3\left(n^2+n\right)f^2g^2x^2+6f^3gnx-6f^4\right)(gx+f)^n cde^2}{\left(n^4+10n^3+35n^2+50n+24\right)g^4}+\frac{\left(\left(n^2+3n+2\right)g^3x^3+\left(n^2+n\right)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n ae^2}{\left(n^3+6n^2+11n+6\right)g^3}+\frac{\left(\left(n^4+10n^3+35n^2+50n+24\right)g^5x^5+\left(n^4+6n^3+11n^2+6n\right)fg^4x^4-4\left(n^3+3n^2+2n\right)f^2g^3x^3+12\left(n^2+n\right)f^3g^2x^2-24f^4gnx+24f^5\right)(gx+f)^n ce^3}{\left(n^5+15n^4+85n^3+225n^2+274n+120\right)g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^3/((n^2 + 3*n + 2)*g^2)
 + 5*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d^2*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 2*(g^2*(n + 1)*x^2 + f
*g*n*x - f^2)*(g*x + f)^n*a*d*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)*a
*d^2/(g*(n + 1)) + 4*((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n
)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*c*
d*e^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4) + ((n^2 + 3*n + 2)*g^3*x^3
+ (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x + f)^n*a*e^2/((n^3 + 6*n^
2 + 11*n + 6)*g^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^5*x^5 + (n^4 +
6*n^3 + 11*n^2 + 6*n)*f*g^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*f^2*g^3*x^3 + 12*(n
^2 + n)*f^3*g^2*x^2 - 24*f^4*g*n*x + 24*f^5)*(g*x + f)^n*c*e^3/((n^5 + 15*n
^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^5)
```

mupad [B] time = 3.52, size = 1133, normalized size = 5.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)^2*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] ((f + g*x)^n*(24*c*e^3*f^5 + 40*a*e^2*f^3*g^2 - 120*c*d^3*f^2*g^3 + 120*a*d
^2*f*g^4 - 120*a*d*e*f^2*g^3 - 120*c*d*e^2*f^4*g + 154*a*d^2*f*g^4*n + 200*
c*d^2*e*f^3*g^2 + 71*a*d^2*f*g^4*n^2 + 14*a*d^2*f*g^4*n^3 + a*d^2*f*g^4*n^4
 + 18*a*e^2*f^3*g^2*n - 94*c*d^3*f^2*g^3*n + 2*a*e^2*f^3*g^2*n^2 - 24*c*d^3
*f^2*g^3*n^2 - 2*c*d^3*f^2*g^3*n^3 + 10*c*d^2*e*f^3*g^2*n^2 - 94*a*d*e*f^2*
g^3*n - 24*c*d*e^2*f^4*g*n - 24*a*d*e*f^2*g^3*n^2 - 2*a*d*e*f^2*g^3*n^3 + 9
0*c*d^2*e*f^3*g^2*n))/(g^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
 + (x*(f + g*x)^n*(120*a*d^2*g^5 + 71*a*d^2*g^5*n^2 + 14*a*d^2*g^5*n^3 + a*
d^2*g^5*n^4 + 154*a*d^2*g^5*n + 120*c*d^3*f*g^4*n - 24*c*e^3*f^4*g*n - 40*a
*e^2*f^2*g^3*n + 94*c*d^3*f*g^4*n^2 + 24*c*d^3*f*g^4*n^3 + 2*c*d^3*f*g^4*n^
4 - 18*a*e^2*f^2*g^3*n^2 - 2*a*e^2*f^2*g^3*n^3 + 120*a*d*e*f*g^4*n + 24*c*d
*e^2*f^3*g^2*n^2 - 90*c*d^2*e*f^2*g^3*n^2 - 10*c*d^2*e*f^2*g^3*n^3 + 94*a*d
*e*f*g^4*n^2 + 24*a*d*e*f*g^4*n^3 + 2*a*d*e*f*g^4*n^4 + 120*c*d*e^2*f^3*g^2
*n - 200*c*d^2*e*f^2*g^3*n))/(g^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5
+ 120)) + (c*e^3*x^5*(f + g*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*
n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (x^2*(f + g*x)^n*(n + 1)*(120*
c*d^3*g^3 + 24*c*d^3*g^3*n^2 + 2*c*d^3*g^3*n^3 + 120*a*d*e*g^3 + 94*c*d^3*g
^3*n + 12*c*e^3*f^3*n + 24*a*d*e*g^3*n^2 + 2*a*d*e*g^3*n^3 + 20*a*e^2*f*g^2
*n + 9*a*e^2*f*g^2*n^2 + a*e^2*f*g^2*n^3 + 94*a*d*e*g^3*n - 60*c*d*e^2*f^2*
g*n + 100*c*d^2*e*f*g^2*n - 12*c*d*e^2*f^2*g*n^2 + 45*c*d^2*e*f*g^2*n^2 + 5
*c*d^2*e*f*g^2*n^3))/(g^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
+ (e*x^3*(f + g*x)^n*(3*n + n^2 + 2)*(100*c*d^2*g^2 + 20*a*e*g^2 + 5*c*d^2*
g^2*n^2 + 9*a*e*g^2*n + a*e*g^2*n^2 + 45*c*d^2*g^2*n - 4*c*e^2*f^2*n + 4*c*
d*e*f*g*n^2 + 20*c*d*e*f*g*n))/(g^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^
5 + 120)) + (c*e^2*x^4*(f + g*x)^n*(20*d*g + 4*d*g*n + e*f*n)*(11*n + 6*n^2
 + n^3 + 6))/(g*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
```

sympy [A] time = 11.82, size = 11946, normalized size = 57.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + c*d**3*x**2 + 5*c*
d**2*e*x**3/3 + c*d*e**2*x**4 + c*e**3*x**5/5), Eq(g, 0)), (-3*a*d**2*g**4/
(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g*
*9*x**4) - 2*a*d*e*f*g**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**
2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 8*a*d*e*g**4*x/(12*f**4*g**5 + 48*f**3
*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - a*e**2*f**2*
g**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 +
12*g**9*x**4) - 4*a*e**2*f*g**3*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*
g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 6*a*e**2*g**4*x**2/(12*f**4*g*
*5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) -
2*c*d**3*f*g**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g
**8*x**3 + 12*g**9*x**4) - 8*c*d**3*g**4*x/(12*f**4*g**5 + 48*f**3*g**6*x +
 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 5*c*d**2*e*f**2*g**2/
(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g*
*9*x**4) - 20*c*d**2*e*f*g**3*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g*
*7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 30*c*d**2*e*g**4*x**2/(12*f**4*g
**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) -
 12*c*d*e**2*f**3*g/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48
*f*g**8*x**3 + 12*g**9*x**4) - 48*c*d*e**2*f**2*g**2*x/(12*f**4*g**5 + 48*f
**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 72*c*d*e*
*2*f*g**3*x**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g*
*8*x**3 + 12*g**9*x**4) - 48*c*d*e**2*g**4*x**3/(12*f**4*g**5 + 48*f**3*g**
6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 12*c*e**3*f**4*l
og(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*
x**3 + 12*g**9*x**4) + 25*c*e**3*f**4/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f
**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f**3*g*x*log(f/g
 + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 +
 12*g**9*x**4) + 88*c*e**3*f**3*g*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**
2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 72*c*e**3*f**2*g**2*x**2*log
(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x*
*3 + 12*g**9*x**4) + 108*c*e**3*f**2*g**2*x**2/(12*f**4*g**5 + 48*f**3*g**6
*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f*g**3*
x**3*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f
*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f*g**3*x**3/(12*f**4*g**5 + 48*f**3*
g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 12*c*e**3*g**
4*x**4*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48
*f*g**8*x**3 + 12*g**9*x**4), Eq(n, -5)), (-a*d**2*g**4/(3*f**3*g**5 + 9*f*
*2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - a*d*e*f*g**3/(3*f**3*g**5 + 9*f*
*2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*d*e*g**4*x/(3*f**3*g**5 + 9*
f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - a*e**2*f**2*g**2/(3*f**3*g**5
+ 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*e**2*f*g**3*x/(3*f**3*
g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*e**2*g**4*x**2/(3
*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - c*d**3*f*g**3/(
3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*c*d**3*g**4*
x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 5*c*d**2*e*
f**2*g**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 15*
c*d**2*e*f*g**3*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**
3) - 15*c*d**2*e*g**4*x**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3
*g**8*x**3) + 12*c*d*e**2*f**3*g*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x
+ 9*f*g**7*x**2 + 3*g**8*x**3) + 22*c*d*e**2*f**3*g/(3*f**3*g**5 + 9*f**2*g
**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 36*c*d*e**2*f**2*g**2*x*log(f/g + x)
/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 54*c*d*e**2*
f**2*g**2*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 3
6*c*d*e**2*f*g**3*x**2*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7
*x**2 + 3*g**8*x**3) + 36*c*d*e**2*f*g**3*x**2/(3*f**3*g**5 + 9*f**2*g**6*x
 + 9*f*g**7*x**2 + 3*g**8*x**3) + 12*c*d*e**2*g**4*x**3*log(f/g + x)/(3*f**
3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 12*c*e**3*f**4*log(
f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 22*c
*e**3*f**4/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 36
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*c*e**3*f**3*g*x*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2
+ 3*g**8*x**3) - 54*c*e**3*f**3*g*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7
*x**2 + 3*g**8*x**3) - 36*c*e**3*f**2*g**2*x**2*log(f/g + x)/(3*f**3*g**5 +
 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 36*c*e**3*f**2*g**2*x**2/(3
*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 12*c*e**3*f*g**
3*x**3*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x
**3) + 3*c*e**3*g**4*x**4/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*
g**8*x**3), Eq(n, -4)), (-a*d**2*g**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x*
*2) - 2*a*d*e*f*g**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 4*a*d*e*g**
4*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 2*a*e**2*f**2*g**2*log(f/g +
 x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 3*a*e**2*f**2*g**2/(2*f**2*g
**5 + 4*f*g**6*x + 2*g**7*x**2) + 4*a*e**2*f*g**3*x*log(f/g + x)/(2*f**2*g*
*5 + 4*f*g**6*x + 2*g**7*x**2) + 4*a*e**2*f*g**3*x/(2*f**2*g**5 + 4*f*g**6*
x + 2*g**7*x**2) + 2*a*e**2*g**4*x**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*
x + 2*g**7*x**2) - 2*c*d**3*f*g**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2)
 - 4*c*d**3*g**4*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 10*c*d**2*e*f
**2*g**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 15*c*d**2*
e*f**2*g**2/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 20*c*d**2*e*f*g**3*x
*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 20*c*d**2*e*f*g**3
*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 10*c*d**2*e*g**4*x**2*log(f/g
 + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 24*c*d*e**2*f**3*g*log(f/g
 + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 36*c*d*e**2*f**3*g/(2*f**2
*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 48*c*d*e**2*f**2*g**2*x*log(f/g + x)/(2
*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 48*c*d*e**2*f**2*g**2*x/(2*f**2*g*
*5 + 4*f*g**6*x + 2*g**7*x**2) - 24*c*d*e**2*f*g**3*x**2*log(f/g + x)/(2*f*
*2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 8*c*d*e**2*g**4*x**3/(2*f**2*g**5 + 4
*f*g**6*x + 2*g**7*x**2) + 12*c*e**3*f**4*log(f/g + x)/(2*f**2*g**5 + 4*f*g
**6*x + 2*g**7*x**2) + 18*c*e**3*f**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x*
*2) + 24*c*e**3*f**3*g*x*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x*
*2) + 24*c*e**3*f**3*g*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 12*c*e*
*3*f**2*g**2*x**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 4
*c*e**3*f*g**3*x**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + c*e**3*g**4*
x**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2), Eq(n, -3)), (-3*a*d**2*g**4/
(3*f*g**5 + 3*g**6*x) + 6*a*d*e*f*g**3*log(f/g + x)/(3*f*g**5 + 3*g**6*x) +
 6*a*d*e*f*g**3/(3*f*g**5 + 3*g**6*x) + 6*a*d*e*g**4*x*log(f/g + x)/(3*f*g*
*5 + 3*g**6*x) - 6*a*e**2*f**2*g**2*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 6*
a*e**2*f**2*g**2/(3*f*g**5 + 3*g**6*x) - 6*a*e**2*f*g**3*x*log(f/g + x)/(3*
f*g**5 + 3*g**6*x) + 3*a*e**2*g**4*x**2/(3*f*g**5 + 3*g**6*x) + 6*c*d**3*f*
g**3*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 6*c*d**3*f*g**3/(3*f*g**5 + 3*g**
6*x) + 6*c*d**3*g**4*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 30*c*d**2*e*f**
2*g**2*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 30*c*d**2*e*f**2*g**2/(3*f*g**5
 + 3*g**6*x) - 30*c*d**2*e*f*g**3*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 15
*c*d**2*e*g**4*x**2/(3*f*g**5 + 3*g**6*x) + 36*c*d*e**2*f**3*g*log(f/g + x)
/(3*f*g**5 + 3*g**6*x) + 36*c*d*e**2*f**3*g/(3*f*g**5 + 3*g**6*x) + 36*c*d*
e**2*f**2*g**2*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 18*c*d*e**2*f*g**3*x*
*2/(3*f*g**5 + 3*g**6*x) + 6*c*d*e**2*g**4*x**3/(3*f*g**5 + 3*g**6*x) - 12*
c*e**3*f**4*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 12*c*e**3*f**4/(3*f*g**5 +
 3*g**6*x) - 12*c*e**3*f**3*g*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 6*c*e*
*3*f**2*g**2*x**2/(3*f*g**5 + 3*g**6*x) - 2*c*e**3*f*g**3*x**3/(3*f*g**5 +
3*g**6*x) + c*e**3*g**4*x**4/(3*f*g**5 + 3*g**6*x), Eq(n, -2)), (a*d**2*log
(f/g + x)/g - 2*a*d*e*f*log(f/g + x)/g**2 + 2*a*d*e*x/g + a*e**2*f**2*log(f
/g + x)/g**3 - a*e**2*f*x/g**2 + a*e**2*x**2/(2*g) - 2*c*d**3*f*log(f/g + x
)/g**2 + 2*c*d**3*x/g + 5*c*d**2*e*f**2*log(f/g + x)/g**3 - 5*c*d**2*e*f*x/
g**2 + 5*c*d**2*e*x**2/(2*g) - 4*c*d*e**2*f**3*log(f/g + x)/g**4 + 4*c*d*e*
*2*f**2*x/g**3 - 2*c*d*e**2*f*x**2/g**2 + 4*c*d*e**2*x**3/(3*g) + c*e**3*f*
*4*log(f/g + x)/g**5 - c*e**3*f**3*x/g**4 + c*e**3*f**2*x**2/(2*g**3) - c*e
**3*f*x**3/(3*g**2) + c*e**3*x**4/(4*g), Eq(n, -1)), (a*d**2*f*g**4*n**4*(f
 + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g
**5*n + 120*g**5) + 14*a*d**2*f*g**4*n**3*(f + g*x)**n/(g**5*n**5 + 15*g**5
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*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 71*a*d**2*f
*g**4*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) + 154*a*d**2*f*g**4*n*(f + g*x)**n/(g**5*n**
5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) +
120*a*d**2*f*g**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 2
25*g**5*n**2 + 274*g**5*n + 120*g**5) + a*d**2*g**5*n**4*x*(f + g*x)**n/(g*
*5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g*
*5) + 14*a*d**2*g**5*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 71*a*d**2*g**5*n**2*x*(f
 + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g*
*5*n + 120*g**5) + 154*a*d**2*g**5*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n*
*4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*a*d**2*g**
5*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 +
 274*g**5*n + 120*g**5) - 2*a*d*e*f**2*g**3*n**3*(f + g*x)**n/(g**5*n**5 +
15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*a
*d*e*f**2*g**3*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 +
 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 94*a*d*e*f**2*g**3*n*(f + g*x)**n
/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 12
0*g**5) - 120*a*d*e*f**2*g**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g
**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2*a*d*e*f*g**4*n**4*x*(
f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*
g**5*n + 120*g**5) + 24*a*d*e*f*g**4*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g*
*5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 94*a*d*e*
f*g**4*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g
**5*n**2 + 274*g**5*n + 120*g**5) + 120*a*d*e*f*g**4*n*x*(f + g*x)**n/(g**5
*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5
) + 2*a*d*e*g**5*n**4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 26*a*d*e*g**5*n**3*x**2*(f
 + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g
**5*n + 120*g**5) + 118*a*d*e*g**5*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g
**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 214*a*d*
e*g**5*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g
**5*n**2 + 274*g**5*n + 120*g**5) + 120*a*d*e*g**5*x**2*(f + g*x)**n/(g**5*
n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
 + 2*a*e**2*f**3*g**2*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 18*a*e**2*f**3*g**2*n*(f +
 g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**
5*n + 120*g**5) + 40*a*e**2*f**3*g**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**
4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 2*a*e**2*f**2*g
**3*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) - 18*a*e**2*f**2*g**3*n**2*x*(f + g*x)**n/(g
**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g
**5) - 40*a*e**2*f**2*g**3*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*
g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + a*e**2*f*g**4*n**4*x**
2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) + 10*a*e**2*f*g**4*n**3*x**2*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 29
*a*e**2*f*g**4*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 20*a*e**2*f*g**4*n*x**2*(f +
 g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**
5*n + 120*g**5) + a*e**2*g**5*n**4*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n
**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 12*a*e**2*g**
5*n**3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**
5*n**2 + 274*g**5*n + 120*g**5) + 49*a*e**2*g**5*n**2*x**3*(f + g*x)**n/(g*
*5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g*
*5) + 78*a*e**2*g**5*n*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 40*a*e**2*g**5*x**3*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) - 2*c*d**3*f**2*g**3*n**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*
```

```
n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*d**3*f*
*2*g**3*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g*
*5*n**2 + 274*g**5*n + 120*g**5) - 94*c*d**3*f**2*g**3*n*(f + g*x)**n/(g**5
*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5
) - 120*c*d**3*f**2*g**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2*c*d**3*f*g**4*n**4*x*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) + 24*c*d**3*f*g**4*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*
n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 94*c*d**3*f*
g**4*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**
5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d**3*f*g**4*n*x*(f + g*x)**n/(g**5*
n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
 + 2*c*d**3*g**5*n**4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 26*c*d**3*g**5*n**3*x**2*(
f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*
g**5*n + 120*g**5) + 118*c*d**3*g**5*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15
*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 214*c*
d**3*g**5*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 22
5*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d**3*g**5*x**2*(f + g*x)**n/(g
**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g
**5) + 10*c*d**2*e*f**3*g**2*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 +
85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 90*c*d**2*e*f**3*g*
*2*n*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
+ 274*g**5*n + 120*g**5) + 200*c*d**2*e*f**3*g**2*(f + g*x)**n/(g**5*n**5 +
 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 10*
c*d**2*e*f**2*g**3*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*
n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 90*c*d**2*e*f**2*g**3*n**2*
x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) - 200*c*d**2*e*f**2*g**3*n*x*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 5*
c*d**2*e*f*g**4*n**4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*
n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 50*c*d**2*e*f*g**4*n**3*x**
2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) + 145*c*d**2*e*f*g**4*n**2*x**2*(f + g*x)**n/(g**5*n*
*5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) +
 100*c*d**2*e*f*g**4*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 5*c*d**2*e*g**5*n**4*x**3
*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 27
4*g**5*n + 120*g**5) + 60*c*d**2*e*g**5*n**3*x**3*(f + g*x)**n/(g**5*n**5 +
 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 245
*c*d**2*e*g**5*n**2*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 390*c*d**2*e*g**5*n*x**3*(f
+ g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g*
*5*n + 120*g**5) + 200*c*d**2*e*g**5*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5
*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*d*e**2
*f**4*g*n*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*
n**2 + 274*g**5*n + 120*g**5) - 120*c*d*e**2*f**4*g*(f + g*x)**n/(g**5*n**5
 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2
4*c*d*e**2*f**3*g**2*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d*e**2*f**3*g**2*n*
x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 2
74*g**5*n + 120*g**5) - 12*c*d*e**2*f**2*g**3*n**3*x**2*(f + g*x)**n/(g**5*
n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
 - 72*c*d*e**2*f**2*g**3*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 +
 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 60*c*d*e**2*f**2*g
**3*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) + 4*c*d*e**2*f*g**4*n**4*x**3*(f + g*x)**n/(
g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*
g**5) + 32*c*d*e**2*f*g**4*n**3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4
```

```
 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 68*c*d*e**2*f*g*
*4*n**2*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g*
*5*n**2 + 274*g**5*n + 120*g**5) + 40*c*d*e**2*f*g**4*n*x**3*(f + g*x)**n/(
g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*
g**5) + 4*c*d*e**2*g**5*n**4*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 +
85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 44*c*d*e**2*g**5*n*
*3*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n*
*2 + 274*g**5*n + 120*g**5) + 164*c*d*e**2*g**5*n**2*x**4*(f + g*x)**n/(g**
5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**
5) + 244*c*d*e**2*g**5*n*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g
**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*c*d*e**2*g**5*x**4*
(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274
*g**5*n + 120*g**5) + 24*c*e**3*f**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4
 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*e**3*f**4*g
*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
+ 274*g**5*n + 120*g**5) + 12*c*e**3*f**3*g**2*n**2*x**2*(f + g*x)**n/(g**5
*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5
) + 12*c*e**3*f**3*g**2*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*
g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 4*c*e**3*f**2*g**3*n**
3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**
2 + 274*g**5*n + 120*g**5) - 12*c*e**3*f**2*g**3*n**2*x**3*(f + g*x)**n/(g*
*5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g*
*5) - 8*c*e**3*f**2*g**3*n*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85
*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + c*e**3*f*g**4*n**4*x*
*4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 +
274*g**5*n + 120*g**5) + 6*c*e**3*f*g**4*n**3*x**4*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 11
*c*e**3*f*g**4*n**2*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n
**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 6*c*e**3*f*g**4*n*x**4*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) + c*e**3*g**5*n**4*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n*
*4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 10*c*e**3*g**5
*n**3*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) + 35*c*e**3*g**5*n**2*x**5*(f + g*x)**n/(g**
5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**
5) + 50*c*e**3*g**5*n*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5
*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 24*c*e**3*g**5*x**5*(f + g
*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*
n + 120*g**5), True))
```

3.555 $\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=146

$$\frac{(f+gx)^{n+2}\left(aeg^2+c\left(2d^2g^2-6defg+3e^2f^2\right)\right)}{g^4(n+2)}-\frac{(ef-dg)(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^4(n+1)}-\frac{3ce(ef-dg)(f\cdots}{g^4(n+3\cdots}$$

Rubi [A] time = 0.11, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {771}

$$\frac{(f+gx)^{n+2}\left(aeg^2+c\left(2d^2g^2-6defg+3e^2f^2\right)\right)}{g^4(n+2)}-\frac{(ef-dg)(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^4(n+1)}-\frac{3ce(ef-dg)(f+gx)^{n+3}}{g^4(n+3)}+\frac{ce^2(f+gx)^{n+4}}{g^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -(((e*f - d*g)*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^4*(1 + n))
) + ((a*e*g^2 + c*(3*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(2 + n))/(
g^4*(2 + n)) - (3*c*e*(e*f - d*g)*(f + g*x)^(3 + n))/(g^4*(3 + n)) + (c*e^2
*(f + g*x)^(4 + n))/(g^4*(4 + n))
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)(f+gx)^n\left(a+2cdx+cex^2\right)dx &= \int\left(\frac{(ef-dg)\left(-ag^2-cf(ef-2dg)\right)(f+gx)^n}{g^3}+\frac{\left(aeg^2+c\left(3e^2f^2\cdots\right.\right.}{\cdots}\right.\\ &= -\frac{(ef-dg)\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^4(1+n)}+\frac{\left(aeg^2+c\left(3e^2f^2-\cdots\right.\right.}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.28, size = 141, normalized size = 0.97

$$\frac{(f+gx)^{n+1}\left(\frac{2(f+gx)\left(aeg^2(n+3)+c\left(-d^2g^2n-6defg+3e^2f^2\right)\right)}{g^2(n+2)}+\frac{6(dg-ef)\left(ag^2+cf(ef-2dg)\right)}{g^2(n+1)}+(a+cx(2d+ex))(dg(n+6)-3ef+eg(n+3)x)\right)}{g^2(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*((6*(-(e*f) + d*g)*(a*g^2 + c*f*(e*f - 2*d*g)))/(g^2*(1
+ n)) + (2*(a*e*g^2*(3 + n) + c*(3*e^2*f^2 - 6*d*e*f*g - d^2*g^2*n))*(f + g
*x))/(g^2*(2 + n)) + (-3*e*f + d*g*(6 + n) + e*g*(3 + n)*x)*(a + c*x*(2*d +
 e*x))))/(g^2*(3 + n)*(4 + n))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (d+ex)(f+gx)^n\left(a+2cdx+cex^2\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2), x]
```

fricas [B] time = 0.43, size = 549, normalized size = 3.76

$$\frac{\left(adfg^3n^3 - 6ce^2f^4 + 24cdef^3g + 24adfg^3 - 12\left(2cd^2 + ae\right)f^2g^2 + \left(ce^2g^4n^3 + 6ce^2g^4n^2 + 11ce^2g^4n + 6ce^2g^4\right)x^4 + \left(24cdeg^4 + \left(ce^2fg^3 + 3cdeg^4\right)n^3 + 3\left(ce^2fg^3 + 7cdeg^4\right)n^2 + 2\left(ce^2fg^3 + 21cdeg^4\right)n\right)x^3 + \left(9adfg^3 - \left(2cd^2 + ae\right)f^2g^2\right)n^2 + \left(12\left(2cd^2 + ae\right)g^4 + \left(3cdefg^3 + \left(2cd^2 + ae\right)g^4\right)n^3 - \left(3ce^2f^2g^2 - 15cdefg^3 - 8\left(2cd^2 + ae\right)g^4\right)n^2 - \left(3ce^2f^2g^2 - 12cdefg^3 - 19\left(2cd^2 + ae\right)g^4\right)n\right)x^2 + \left(6cdef^3g + 26adfg^3 - 7\left(2cd^2 + ae\right)f^2g^2\right)n + \left(24adg^4 + \left(adg^4 + \left(2cd^2 + ae\right)fg^3\right)n^3 - \left(6cdef^2g^2 - 9adg^4 - 7\left(2cd^2 + ae\right)fg^3\right)n^2 + 2\left(3ce^2f^3g - 12cdef^2g^2 + 13adg^4 + 6\left(2cd^2 + ae\right)fg^3\right)n\right)x\right)(gx + f)^n}{g^4n^4 + 10g^4n^3 + 35g^4n^2 + 50g^4n + 24g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*f*g^3*n^3 - 6*c*e^2*f^4 + 24*c*d*e*f^3*g + 24*a*d*f*g^3 - 12*(2*c*d^2
+ a*e)*f^2*g^2 + (c*e^2*g^4*n^3 + 6*c*e^2*g^4*n^2 + 11*c*e^2*g^4*n + 6*c*e^
2*g^4)*x^4 + (24*c*d*e*g^4 + (c*e^2*f*g^3 + 3*c*d*e*g^4)*n^3 + 3*(c*e^2*f*g
^3 + 7*c*d*e*g^4)*n^2 + 2*(c*e^2*f*g^3 + 21*c*d*e*g^4)*n)*x^3 + (9*a*d*f*g^
3 - (2*c*d^2 + a*e)*f^2*g^2)*n^2 + (12*(2*c*d^2 + a*e)*g^4 + (3*c*d*e*f*g^3
 + (2*c*d^2 + a*e)*g^4)*n^3 - (3*c*e^2*f^2*g^2 - 15*c*d*e*f*g^3 - 8*(2*c*d^
2 + a*e)*g^4)*n^2 - (3*c*e^2*f^2*g^2 - 12*c*d*e*f*g^3 - 19*(2*c*d^2 + a*e)*
g^4)*n)*x^2 + (6*c*d*e*f^3*g + 26*a*d*f*g^3 - 7*(2*c*d^2 + a*e)*f^2*g^2)*n
+ (24*a*d*g^4 + (a*d*g^4 + (2*c*d^2 + a*e)*f*g^3)*n^3 - (6*c*d*e*f^2*g^2 -
9*a*d*g^4 - 7*(2*c*d^2 + a*e)*f*g^3)*n^2 + 2*(3*c*e^2*f^3*g - 12*c*d*e*f^2*
g^2 + 13*a*d*g^4 + 6*(2*c*d^2 + a*e)*f*g^3)*n)*x)*(g*x + f)^n/(g^4*n^4 + 10
*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 24*g^4)
```

giac [B] time = 0.39, size = 1018, normalized size = 6.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^4*n^3*x^4*e^2 + 3*(g*x + f)^n*c*d*g^4*n^3*x^3*e + 2*(g*x +
 f)^n*c*d^2*g^4*n^3*x^2 + (g*x + f)^n*c*f*g^3*n^3*x^3*e^2 + 6*(g*x + f)^n*c
*g^4*n^2*x^4*e^2 + 3*(g*x + f)^n*c*d*f*g^3*n^3*x^2*e + 21*(g*x + f)^n*c*d*g
^4*n^2*x^3*e + 2*(g*x + f)^n*c*d^2*f*g^3*n^3*x + 16*(g*x + f)^n*c*d^2*g^4*n
^2*x^2 + 3*(g*x + f)^n*c*f*g^3*n^2*x^3*e^2 + 11*(g*x + f)^n*c*g^4*n*x^4*e^2
 + 15*(g*x + f)^n*c*d*f*g^3*n^2*x^2*e + (g*x + f)^n*a*g^4*n^3*x^2*e + 42*(g
*x + f)^n*c*d*g^4*n*x^3*e + 14*(g*x + f)^n*c*d^2*f*g^3*n^2*x + (g*x + f)^n*
a*d*g^4*n^3*x + 38*(g*x + f)^n*c*d^2*g^4*n*x^2 - 3*(g*x + f)^n*c*f^2*g^2*n^
2*x^2*e^2 + 2*(g*x + f)^n*c*f*g^3*n*x^3*e^2 + 6*(g*x + f)^n*c*g^4*x^4*e^2 -
 6*(g*x + f)^n*c*d*f^2*g^2*n^2*x*e + (g*x + f)^n*a*f*g^3*n^3*x*e + 12*(g*x
+ f)^n*c*d*f*g^3*n*x^2*e + 8*(g*x + f)^n*a*g^4*n^2*x^2*e + 24*(g*x + f)^n*c
*d*g^4*x^3*e - 2*(g*x + f)^n*c*d^2*f^2*g^2*n^2 + (g*x + f)^n*a*d*f*g^3*n^3
+ 24*(g*x + f)^n*c*d^2*f*g^3*n*x + 9*(g*x + f)^n*a*d*g^4*n^2*x + 24*(g*x +
f)^n*c*d^2*g^4*x^2 - 3*(g*x + f)^n*c*f^2*g^2*n*x^2*e^2 - 24*(g*x + f)^n*c*d
*f^2*g^2*n*x*e + 7*(g*x + f)^n*a*f*g^3*n^2*x*e + 19*(g*x + f)^n*a*g^4*n*x^2
*e - 14*(g*x + f)^n*c*d^2*f^2*g^2*n + 9*(g*x + f)^n*a*d*f*g^3*n^2 + 26*(g*x
 + f)^n*a*d*g^4*n*x + 6*(g*x + f)^n*c*f^3*g*n*x*e^2 + 6*(g*x + f)^n*c*d*f^3
*g*n*e - (g*x + f)^n*a*f^2*g^2*n^2*e + 12*(g*x + f)^n*a*f*g^3*n*x*e + 12*(g
*x + f)^n*a*g^4*x^2*e - 24*(g*x + f)^n*c*d^2*f^2*g^2 + 26*(g*x + f)^n*a*d*f
*g^3*n + 24*(g*x + f)^n*a*d*g^4*x + 24*(g*x + f)^n*c*d*f^3*g*e - 7*(g*x + f
)^n*a*f^2*g^2*n*e + 24*(g*x + f)^n*a*d*f*g^3 - 6*(g*x + f)^n*c*f^4*e^2 - 12
*(g*x + f)^n*a*f^2*g^2*e)/(g^4*n^4 + 10*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 2
4*g^4)
```

maple [B] time = 0.01, size = 449, normalized size = 3.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(n+1)*(c*e^2*g^3*n^3*x^3+3*c*d*e*g^3*n^3*x^2+6*c*e^2*g^3*n^2*x^3+2*
c*d^2*g^3*n^3*x+21*c*d*e*g^3*n^2*x^2-3*c*e^2*f*g^2*n^2*x^2+11*c*e^2*g^3*n*x
^3+a*e*g^3*n^3*x+16*c*d^2*g^3*n^2*x-6*c*d*e*f*g^2*n^2*x+42*c*d*e*g^3*n*x^2-
9*c*e^2*f*g^2*n*x^2+6*c*e^2*g^3*x^3+a*d*g^3*n^3+8*a*e*g^3*n^2*x-2*c*d^2*f*g
^2*n^2+38*c*d^2*g^3*n*x-30*c*d*e*f*g^2*n*x+24*c*d*e*g^3*x^2+6*c*e^2*f^2*g*n
*x-6*c*e^2*f*g^2*x^2+9*a*d*g^3*n^2-a*e*f*g^2*n^2+19*a*e*g^3*n*x-14*c*d^2*f*
g^2*n+24*c*d^2*g^3*x+6*c*d*e*f^2*g*n-24*c*d*e*f*g^2*x+6*c*e^2*f^2*g*x+26*a*
d*g^3*n-7*a*e*f*g^2*n+12*a*e*g^3*x-24*c*d^2*f*g^2+24*c*d*e*f^2*g-6*c*e^2*f^
3+24*a*d*g^3-12*a*e*f*g^2)/g^4/(n^4+10*n^3+35*n^2+50*n+24)
```

maxima [A] time = 0.51, size = 289, normalized size = 1.98

$$\frac{2\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n cd^2}{\left(n^2+3n+2\right)g^2}+\frac{3\left(\left(n^2+3n+2\right)g^3x^3+\left(n^2+n\right)fg^2x^2-2f^2gnx+2f^3\right)(gx+f)^n cde}{\left(n^3+6n^2+11n+6\right)g^3}+\frac{\left(g^2(n+1)x^2+fgnx-f^2\right)(gx+f)^n ae}{\left(n^2+3n+2\right)g^2}+\frac{(gx+f)^{n+1}ad}{g(n+1)}+\frac{\left(\left(n^3+6n^2+11n+6\right)g^4x^4+\left(n^3+3n^2+2n\right)fg^3x^3-3\left(n^2+n\right)f^2g^2x^2+6f^3gnx-6f^4\right)(gx+f)^n ce^2}{\left(n^4+10n^3+35n^2+50n+24\right)g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^2/((n^2 + 3*n + 2)*g^2)
 + 3*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + (g^2*(n + 1)*x^2 + f*g*n
*x - f^2)*(g*x + f)^n*a*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)*a*d/(g*
(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n)*f*g^3*x^
3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*c*e^2/((n^4
+ 10*n^3 + 35*n^2 + 50*n + 24)*g^4)
```

mupad [B] time = 3.29, size = 572, normalized size = 3.92

$$\frac{x\,(f+g\,x)^n\,\left(2\,c\,d^2\,f\,g^3\,n^3+14\,c\,d^2\,f\,g^3\,n^2+24\,c\,d^2\,f\,g^3\,n-6\,c\,d\,e\,f^2\,g^2\,n^2-24\,c\,d\,e\,f^2\,g^2\,n+a\,d\,g^4\,n^3+9\,a\,d\,g^4\,n^2+26\,a\,d\,g^4\,n+24\,a\,d\,g^4+6\,c\,e^2\,f^3\,g\,n+a\,e\,f\,g^3\,n^3+7\,a\,e\,f\,g^3\,n^2+12\,a\,e\,f\,g^3\,n\right)}{g^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{(f+g\,x)^n\,\left(2\,c\,d^2\,f^2\,g^2\,n^2+14\,c\,d^2\,f^2\,g^2\,n+24\,c\,d^2\,f^2\,g^2-6\,c\,d\,e\,f^3\,g\,n-24\,c\,d\,e\,f^3\,g-a\,d\,f\,g^3\,n^3-9\,a\,d\,f\,g^3\,n^2-26\,a\,d\,f\,g^3\,n-24\,a\,d\,f\,g^3+6\,c\,e^2\,f^4+a\,e\,f^2\,g^2\,n^2+7\,a\,e\,f^2\,g^2\,n+12\,a\,e\,f^2\,g^2\right)}{g^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{c\,e^2\,x^4\,(f+g\,x)^n\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}+\frac{x^2\,(f+g\,x)^n\,(n+1)\,\left(2\,c\,d^2\,g^2\,n^2+14\,c\,d^2\,g^2\,n+24\,c\,d^2\,g^2+3\,c\,d\,e\,f\,g\,n^2+12\,c\,d\,e\,f\,g\,n-3\,c\,e^2\,f^2\,n+a\,e\,g^2\,n^2+7\,a\,e\,g^2\,n+12\,a\,e\,g^2\right)}{g^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{c\,e\,x^3\,(f+g\,x)^n\,\left(12\,d\,g+3\,d\,g\,n+e\,f\,n\right)\,\left(n^2+3\,n+2\right)}{g\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (x*(f + g*x)^n*(24*a*d*g^4 + 26*a*d*g^4*n + 9*a*d*g^4*n^2 + a*d*g^4*n^3 + 7
*a*e*f*g^3*n^2 + a*e*f*g^3*n^3 + 24*c*d^2*f*g^3*n + 6*c*e^2*f^3*g*n + 14*c*
d^2*f*g^3*n^2 + 2*c*d^2*f*g^3*n^3 + 12*a*e*f*g^3*n - 24*c*d*e*f^2*g^2*n - 6
*c*d*e*f^2*g^2*n^2))/(g^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - ((f + g*x)
^n*(6*c*e^2*f^4 + 24*c*d^2*f^2*g^2 - 24*a*d*f*g^3 + 12*a*e*f^2*g^2 - 9*a*d*
f*g^3*n^2 - a*d*f*g^3*n^3 + 7*a*e*f^2*g^2*n + a*e*f^2*g^2*n^2 + 14*c*d^2*f^
2*g^2*n - 24*c*d*e*f^3*g - 26*a*d*f*g^3*n + 2*c*d^2*f^2*g^2*n^2 - 6*c*d*e*f
^3*g*n))/(g^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (c*e^2*x^4*(f + g*x)^n
*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 + 24) + (x^2*(f +
g*x)^n*(n + 1)*(24*c*d^2*g^2 + 12*a*e*g^2 + 2*c*d^2*g^2*n^2 + 7*a*e*g^2*n +
 a*e*g^2*n^2 + 14*c*d^2*g^2*n - 3*c*e^2*f^2*n + 3*c*d*e*f*g*n^2 + 12*c*d*e*
f*g*n))/(g^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (c*e*x^3*(f + g*x)^n*(1
2*d*g + 3*d*g*n + e*f*n)*(3*n + n^2 + 2))/(g*(50*n + 35*n^2 + 10*n^3 + n^4
+ 24))
```

sympy [A] time = 5.27, size = 4952, normalized size = 33.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d*x + a*e*x**2/2 + c*d**2*x**2 + c*d*e*x**3 + c*e**2*x**
4/4), Eq(g, 0)), (-2*a*d*g**3/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**
2 + 6*g**7*x**3) - a*e*f*g**2/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**
2 + 6*g**7*x**3) - 3*a*e*g**3*x/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x
**2 + 6*g**7*x**3) - 2*c*d**2*f*g**2/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g
```

```
**6*x**2 + 6*g**7*x**3) - 6*c*d**2*g**3*x/(6*f**3*g**4 + 18*f**2*g**5*x + 1
8*f*g**6*x**2 + 6*g**7*x**3) - 6*c*d*e*f**2*g/(6*f**3*g**4 + 18*f**2*g**5*x
 + 18*f*g**6*x**2 + 6*g**7*x**3) - 18*c*d*e*f*g**2*x/(6*f**3*g**4 + 18*f**2
*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) - 18*c*d*e*g**3*x**2/(6*f**3*g**4 +
 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 6*c*e**2*f**3*log(f/g + x
)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 11*c*e**2
*f**3/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 18*c*
e**2*f**2*g*x*log(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 +
 6*g**7*x**3) + 27*c*e**2*f**2*g*x/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**
6*x**2 + 6*g**7*x**3) + 18*c*e**2*f*g**2*x**2*log(f/g + x)/(6*f**3*g**4 + 1
8*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 18*c*e**2*f*g**2*x**2/(6*f*
*3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 6*c*e**2*g**3*x*
*3*log(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**
3), Eq(n, -4)), (-a*d*g**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - a*e*f
*g**2/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 2*a*e*g**3*x/(2*f**2*g**4
+ 4*f*g**5*x + 2*g**6*x**2) - 2*c*d**2*f*g**2/(2*f**2*g**4 + 4*f*g**5*x + 2
*g**6*x**2) - 4*c*d**2*g**3*x/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 6*
c*d*e*f**2*g*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 9*c*d*
e*f**2*g/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 12*c*d*e*f*g**2*x*log(f
/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 12*c*d*e*f*g**2*x/(2*f**
2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 6*c*d*e*g**3*x**2*log(f/g + x)/(2*f**2
*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 6*c*e**2*f**3*log(f/g + x)/(2*f**2*g**4
 + 4*f*g**5*x + 2*g**6*x**2) - 9*c*e**2*f**3/(2*f**2*g**4 + 4*f*g**5*x + 2*
g**6*x**2) - 12*c*e**2*f**2*g*x*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*
g**6*x**2) - 12*c*e**2*f**2*g*x/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) -
6*c*e**2*f*g**2*x**2*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2)
+ 2*c*e**2*g**3*x**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2), Eq(n, -3)),
(-2*a*d*g**3/(2*f*g**4 + 2*g**5*x) + 2*a*e*f*g**2*log(f/g + x)/(2*f*g**4 +
2*g**5*x) + 2*a*e*f*g**2/(2*f*g**4 + 2*g**5*x) + 2*a*e*g**3*x*log(f/g + x)/
(2*f*g**4 + 2*g**5*x) + 4*c*d**2*f*g**2*log(f/g + x)/(2*f*g**4 + 2*g**5*x)
+ 4*c*d**2*f*g**2/(2*f*g**4 + 2*g**5*x) + 4*c*d**2*g**3*x*log(f/g + x)/(2*f
*g**4 + 2*g**5*x) - 12*c*d*e*f**2*g*log(f/g + x)/(2*f*g**4 + 2*g**5*x) - 12
*c*d*e*f**2*g/(2*f*g**4 + 2*g**5*x) - 12*c*d*e*f*g**2*x*log(f/g + x)/(2*f*g
**4 + 2*g**5*x) + 6*c*d*e*g**3*x**2/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*f**3*l
og(f/g + x)/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*f**3/(2*f*g**4 + 2*g**5*x) + 6
*c*e**2*f**2*g*x*log(f/g + x)/(2*f*g**4 + 2*g**5*x) - 3*c*e**2*f*g**2*x**2/
(2*f*g**4 + 2*g**5*x) + c*e**2*g**3*x**3/(2*f*g**4 + 2*g**5*x), Eq(n, -2)),
 (a*d*log(f/g + x)/g - a*e*f*log(f/g + x)/g**2 + a*e*x/g - 2*c*d**2*f*log(f
/g + x)/g**2 + 2*c*d**2*x/g + 3*c*d*e*f**2*log(f/g + x)/g**3 - 3*c*d*e*f*x/
g**2 + 3*c*d*e*x**2/(2*g) - c*e**2*f**3*log(f/g + x)/g**4 + c*e**2*f**2*x/g
**3 - c*e**2*f*x**2/(2*g**2) + c*e**2*x**3/(3*g), Eq(n, -1)), (a*d*f*g**3*n
**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*
g**4) + 9*a*d*f*g**3*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*
n**2 + 50*g**4*n + 24*g**4) + 26*a*d*f*g**3*n*(f + g*x)**n/(g**4*n**4 + 10*
g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*a*d*f*g**3*(f + g*x)**
n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*d*g**
4*n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n
+ 24*g**4) + 9*a*d*g**4*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 26*a*d*g**4*n*x*(f + g*x)**n/(g**4*n**4
+ 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*a*d*g**4*x*(f + g
*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - a*
e*f**2*g**2*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50
*g**4*n + 24*g**4) - 7*a*e*f**2*g**2*n*(f + g*x)**n/(g**4*n**4 + 10*g**4*n*
*3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 12*a*e*f**2*g**2*(f + g*x)**n/(g
**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*e*f*g**3*
n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n +
24*g**4) + 7*a*e*f*g**3*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 12*a*e*f*g**3*n*x*(f + g*x)**n/(g**4*n**
4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*e*g**4*n**3*x**2
```

```
*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**
4) + 8*a*e*g**4*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*
n**2 + 50*g**4*n + 24*g**4) + 19*a*e*g**4*n*x**2*(f + g*x)**n/(g**4*n**4 +
10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 12*a*e*g**4*x**2*(f +
g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 2
*c*d**2*f**2*g**2*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**
2 + 50*g**4*n + 24*g**4) - 14*c*d**2*f**2*g**2*n*(f + g*x)**n/(g**4*n**4 +
10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 24*c*d**2*f**2*g**2*(f
 + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4)
+ 2*c*d**2*f*g**3*n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n
**2 + 50*g**4*n + 24*g**4) + 14*c*d**2*f*g**3*n**2*x*(f + g*x)**n/(g**4*n**
4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*c*d**2*f*g**3*n
*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g
**4) + 2*c*d**2*g**4*n**3*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 16*c*d**2*g**4*n**2*x**2*(f + g*x)**n/(g
**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 38*c*d**2*g
**4*n*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*
n + 24*g**4) + 24*c*d**2*g**4*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 +
 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*d*e*f**3*g*n*(f + g*x)**n/(g**4*
n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*c*d*e*f**3*g
*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**
4) - 6*c*d*e*f**2*g**2*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g
**4*n**2 + 50*g**4*n + 24*g**4) - 24*c*d*e*f**2*g**2*n*x*(f + g*x)**n/(g**4
*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 3*c*d*e*f*g**3
*n**3*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*
n + 24*g**4) + 15*c*d*e*f*g**3*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*
n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 12*c*d*e*f*g**3*n*x**2*(f + g*
x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 3*c
*d*e*g**4*n**3*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 +
 50*g**4*n + 24*g**4) + 21*c*d*e*g**4*n**2*x**3*(f + g*x)**n/(g**4*n**4 + 1
0*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 42*c*d*e*g**4*n*x**3*(f
 + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4)
+ 24*c*d*e*g**4*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2
+ 50*g**4*n + 24*g**4) - 6*c*e**2*f**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n*
*3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*e**2*f**3*g*n*x*(f + g*x)**n
/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 3*c*e**2
*f**2*g**2*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2
+ 50*g**4*n + 24*g**4) - 3*c*e**2*f**2*g**2*n*x**2*(f + g*x)**n/(g**4*n**4
+ 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + c*e**2*f*g**3*n**3*x
**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*
g**4) + 3*c*e**2*f*g**3*n**2*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 +
35*g**4*n**2 + 50*g**4*n + 24*g**4) + 2*c*e**2*f*g**3*n*x**3*(f + g*x)**n/(
g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + c*e**2*g**
4*n**3*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4
*n + 24*g**4) + 6*c*e**2*g**4*n**2*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n
**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 11*c*e**2*g**4*n*x**4*(f + g*x)
**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*e
**2*g**4*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g*
*4*n + 24*g**4), True))
```

3.556 $\int (f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=84

$$\frac{(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^3(n+1)}-\frac{2c(ef-dg)(f+gx)^{n+2}}{g^3(n+2)}+\frac{ce(f+gx)^{n+3}}{g^3(n+3)}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^3(n+1)}-\frac{2c(ef-dg)(f+gx)^{n+2}}{g^3(n+2)}+\frac{ce(f+gx)^{n+3}}{g^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^3*(1 + n)) - (2*c*(e*f -
 d*g)*(f + g*x)^(2 + n))/(g^3*(2 + n)) + (c*e*(f + g*x)^(3 + n))/(g^3*(3 +
n))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (f+gx)^n \left(a+2cdx+cex^2\right) dx &= \int \left(\frac{\left(ag^2+cf(ef-2dg)\right)(f+gx)^n}{g^2}+\frac{2c(-ef+dg)(f+gx)^{1+n}}{g^2}+\frac{ce(f+gx)^{2+n}}{g^2}\right) dx\\ &= \frac{\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^3(1+n)}-\frac{2c(ef-dg)(f+gx)^{2+n}}{g^3(2+n)}+\frac{ce(f+gx)^{3+n}}{g^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.10, size = 73, normalized size = 0.87

$$\frac{(f+gx)^{n+1}\left(\frac{ag^2+cf(ef-2dg)}{n+1}-\frac{2c(f+gx)(ef-dg)}{n+2}+\frac{ce(f+gx)^2}{n+3}\right)}{g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*((a*g^2 + c*f*(e*f - 2*d*g))/(1 + n) - (2*c*(e*f - d*g)*
(f + g*x))/(2 + n) + (c*e*(f + g*x)^2)/(3 + n)))/g^3
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (f+gx)^n \left(a+2cdx+cex^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(f + g*x)^n*(a + 2*c*d*x + c*e*x^2), x]
```

fricas [B] time = 0.41, size = 218, normalized size = 2.60

$$\frac{\left(afg^2n^2 + 2cef^3 - 6cdf^2g + 6afg^2 + \left(ceg^3n^2 + 3ceg^3n + 2ceg^3\right)x^3 + \left(6cdg^3 + \left(cefg^2 + 2cdg^3\right)n^2 + \left(cefg^2 + 8cdg^3\right)n\right)x^2 - \left(2cdf^2g - 5afg^2\right)n + \left(6ag^3 + \left(2cdfg^2 + ag^3\right)n^2 - \left(2cef^2g - 6cdfg^2 - 5ag^3\right)n\right)x\right)\left(gx + f\right)^n}{g^3n^3 + 6g^3n^2 + 11g^3n + 6g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*f*g^2*n^2 + 2*c*e*f^3 - 6*c*d*f^2*g + 6*a*f*g^2 + (c*e*g^3*n^2 + 3*c*e*g
^3*n + 2*c*e*g^3)*x^3 + (6*c*d*g^3 + (c*e*f*g^2 + 2*c*d*g^3)*n^2 + (c*e*f*g
^2 + 8*c*d*g^3)*n)*x^2 - (2*c*d*f^2*g - 5*a*f*g^2)*n + (6*a*g^3 + (2*c*d*f*
g^2 + a*g^3)*n^2 - (2*c*e*f^2*g - 6*c*d*f*g^2 - 5*a*g^3)*n)*x)*(g*x + f)^n/
(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^3)
```

giac [B] time = 0.19, size = 373, normalized size = 4.44

$$\frac{\left(gx + f\right)^n cg^3n^2x^3e + 2\left(gx + f\right)^n cdg^3n^2x^2 + \left(gx + f\right)^n cfg^2n^2x^2e + 3\left(gx + f\right)^n cg^3nx^3e + 2\left(gx + f\right)^n cdfg^2n^2x + 8\left(gx + f\right)^n cdg^3nx^2 + \left(gx + f\right)^n cfg^2nx^2e + 2\left(gx + f\right)^n cg^3x^3e + 6\left(gx + f\right)^n cdfg^2nx + \left(gx + f\right)^n ag^3n^2x + 6\left(gx + f\right)^n cdg^3x^2 - 2\left(gx + f\right)^n cf^2gnxe - 2\left(gx + f\right)^n cdf^2gn + \left(gx + f\right)^n afg^2n^2 + 5\left(gx + f\right)^n ag^3nx - 6\left(gx + f\right)^n cdf^2g + 5\left(gx + f\right)^n afg^2n + 6\left(gx + f\right)^n ag^3x + 2\left(gx + f\right)^n cf^3e + 6\left(gx + f\right)^n afg^2}{g^3n^3 + 6g^3n^2 + 11g^3n + 6g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^3*n^2*x^3*e + 2*(g*x + f)^n*c*d*g^3*n^2*x^2 + (g*x + f)^n*
c*f*g^2*n^2*x^2*e + 3*(g*x + f)^n*c*g^3*n*x^3*e + 2*(g*x + f)^n*c*d*f*g^2*n
^2*x + 8*(g*x + f)^n*c*d*g^3*n*x^2 + (g*x + f)^n*c*f*g^2*n*x^2*e + 2*(g*x +
 f)^n*c*g^3*x^3*e + 6*(g*x + f)^n*c*d*f*g^2*n*x + (g*x + f)^n*a*g^3*n^2*x +
 6*(g*x + f)^n*c*d*g^3*x^2 - 2*(g*x + f)^n*c*f^2*g*n*x*e - 2*(g*x + f)^n*c*
d*f^2*g*n + (g*x + f)^n*a*f*g^2*n^2 + 5*(g*x + f)^n*a*g^3*n*x - 6*(g*x + f)
^n*c*d*f^2*g + 5*(g*x + f)^n*a*f*g^2*n + 6*(g*x + f)^n*a*g^3*x + 2*(g*x + f
)^n*c*f^3*e + 6*(g*x + f)^n*a*f*g^2)/(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^
3)
```

maple [A] time = 0.00, size = 147, normalized size = 1.75

$$\frac{\left(ce g^2n^2x^2 + 2cd g^2n^2x + 3ce g^2n x^2 + 8cd g^2nx - 2cefgnx + 2ce x^2g^2 + a g^2n^2 - 2cdfgn + 6cd g^2x - 2cefgx + 5a g^2n - 6cdfg + 2ce f^2 + 6a g^2\right)\left(gx + f\right)^{n+1}}{\left(n^3 + 6n^2 + 11n + 6\right)g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(n+1)*(c*e*g^2*n^2*x^2+2*c*d*g^2*n^2*x+3*c*e*g^2*n*x^2+8*c*d*g^2*n*
x-2*c*e*f*g*n*x+2*c*e*g^2*x^2+a*g^2*n^2-2*c*d*f*g*n+6*c*d*g^2*x-2*c*e*f*g*x
+5*a*g^2*n-6*c*d*f*g+2*c*e*f^2+6*a*g^2)/g^3/(n^3+6*n^2+11*n+6)
```

maxima [A] time = 0.47, size = 135, normalized size = 1.61

$$\frac{2\left(g^2(n+1)x^2 + fgnx - f^2\right)\left(gx + f\right)^n cd}{\left(n^2 + 3n + 2\right)g^2} + \frac{\left(\left(n^2 + 3n + 2\right)g^3x^3 + \left(n^2 + n\right)fg^2x^2 - 2f^2gnx + 2f^3\right)\left(gx + f\right)^n ce}{\left(n^3 + 6n^2 + 11n + 6\right)g^3} + \frac{\left(gx + f\right)^{n+1}a}{g(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d/((n^2 + 3*n + 2)*g^2) +
 ((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x
 + f)^n*c*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + (g*x + f)^(n + 1)*a/(g*(n + 1)
)
```

mupad [B] time = 3.07, size = 211, normalized size = 2.51

$$(f+gx)^n\left(\frac{f\,\left(2cef^2-2cdfgn-6cdfg+ag^2n^2+5ag^2n+6ag^2\right)}{g^3\,\left(n^3+6n^2+11n+6\right)}+\frac{x\,\left(-2cef^2gn+2cdfg^2n^2+6cdfg^2n+ag^3n^2+5ag^3n+6ag^3\right)}{g^3\,\left(n^3+6n^2+11n+6\right)}+\frac{cex^3\,\left(n^2+3n+2\right)}{n^3+6n^2+11n+6}+\frac{cx^2\,(n+1)\,\left(6dg+2dgn+efn\right)}{g\,\left(n^3+6n^2+11n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (f + g*x)^n*((f*(6*a*g^2 + a*g^2*n^2 + 2*c*e*f^2 + 5*a*g^2*n - 6*c*d*f*g -
2*c*d*f*g*n))/(g^3*(11*n + 6*n^2 + n^3 + 6)) + (x*(6*a*g^3 + a*g^3*n^2 + 5*
a*g^3*n + 2*c*d*f*g^2*n^2 + 6*c*d*f*g^2*n - 2*c*e*f^2*g*n))/(g^3*(11*n + 6*
n^2 + n^3 + 6)) + (c*e*x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (c*x
^2*(n + 1)*(6*d*g + 2*d*g*n + e*f*n))/(g*(11*n + 6*n^2 + n^3 + 6)))
```

sympy [A] time = 2.21, size = 1489, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*x + c*d*x**2 + c*e*x**3/3), Eq(g, 0)), (-a*g**2/(2*f**2*
g**3 + 4*f*g**4*x + 2*g**5*x**2) - 2*c*d*f*g/(2*f**2*g**3 + 4*f*g**4*x + 2*
g**5*x**2) - 4*c*d*g**2*x/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 2*c*e*
f**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 3*c*e*f**2/(2*
f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x*log(f/g + x)/(2*f**2*g*
*3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x/(2*f**2*g**3 + 4*f*g**4*x + 2*
g**5*x**2) + 2*c*e*g**2*x**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**
5*x**2), Eq(n, -3)), (-a*g**2/(f*g**3 + g**4*x) + 2*c*d*f*g*log(f/g + x)/(f
*g**3 + g**4*x) + 2*c*d*f*g/(f*g**3 + g**4*x) + 2*c*d*g**2*x*log(f/g + x)/(
f*g**3 + g**4*x) - 2*c*e*f**2*log(f/g + x)/(f*g**3 + g**4*x) - 2*c*e*f**2/(
f*g**3 + g**4*x) - 2*c*e*f*g*x*log(f/g + x)/(f*g**3 + g**4*x) + c*e*g**2*x*
*2/(f*g**3 + g**4*x), Eq(n, -2)), (a*log(f/g + x)/g - 2*c*d*f*log(f/g + x)/
g**2 + 2*c*d*x/g + c*e*f**2*log(f/g + x)/g**3 - c*e*f*x/g**2 + c*e*x**2/(2*
g), Eq(n, -1)), (a*f*g**2*n**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g
**3*n + 6*g**3) + 5*a*f*g**2*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g
**3*n + 6*g**3) + 6*a*f*g**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**
3*n + 6*g**3) + a*g**3*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 5*a*g**3*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 6*a*g**3*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3
*n + 6*g**3) - 2*c*d*f**2*g*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) - 6*c*d*f**2*g*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 2*c*d*f*g**2*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2
+ 11*g**3*n + 6*g**3) + 6*c*d*f*g**2*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n
**2 + 11*g**3*n + 6*g**3) + 2*c*d*g**3*n**2*x**2*(f + g*x)**n/(g**3*n**3 +
6*g**3*n**2 + 11*g**3*n + 6*g**3) + 8*c*d*g**3*n*x**2*(f + g*x)**n/(g**3*n*
*3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 6*c*d*g**3*x**2*(f + g*x)**n/(g**3
*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e*f**3*(f + g*x)**n/(g**3*n
**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) - 2*c*e*f**2*g*n*x*(f + g*x)**n/(g*
*3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g**2*n**2*x**2*(f + g*x
)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g**2*n*x**2*(f
+ g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*g**3*n**2*x*
*3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 3*c*e*g**3
*n*x**3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e
*g**3*x**3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3), Tru
e))
```

3.557 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=83

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {893}

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (c*x)/(e*g) + ((c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) - ((
c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx&=\int \left(\frac{c}{eg}+\frac{cd^2-bde+ae^2}{e(ef-dg)(d+ex)}+\frac{cf^2-bfg+ag^2}{g(-ef+dg)(f+gx)}\right)dx\\&=\frac{cx}{eg}+\frac{\left(cd^2-bde+ae^2\right)\log(d+ex)}{e^2(ef-dg)}-\frac{\left(cf^2-bfg+ag^2\right)\log(f+gx)}{g^2(ef-dg)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 1.02

$$-\frac{\log(d+ex)\left(-ae^2+bde-cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (c*x)/(e*g) - ((-(c*d^2) + b*d*e - a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) -
 ((c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)), x]
```

fricas [A] time = 0.40, size = 99, normalized size = 1.19

$$\frac{\left(cd^2 - bde + ae^2\right)g^2 \log\left(ex + d\right) + \left(ce^2fg - cdeg^2\right)x - \left(ce^2f^2 - be^2fg + ae^2g^2\right)\log\left(gx + f\right)}{e^3fg^2 - de^2g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*g^2*log(e*x + d) + (c*e^2*f*g - c*d*e*g^2)*x - (c*
e^2*f^2 - b*e^2*f*g + a*e^2*g^2)*log(g*x + f))/(e^3*f*g^2 - d*e^2*g^3)
```

giac [A] time = 0.18, size = 88, normalized size = 1.06

$$\frac{cxe^{(-1)}}{g} + \frac{\left(cf^2 - bfg + ag^2\right)\log\left(\left|gx + f\right|\right)}{dg^3 - fg^2e} - \frac{\left(cd^2 - bde + ae^2\right)\log\left(\left|xe + d\right|\right)}{dge^2 - fe^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] c*x*e^(-1)/g + (c*f^2 - b*f*g + a*g^2)*log(abs(g*x + f))/(d*g^3 - f*g^2*e)
- (c*d^2 - b*d*e + a*e^2)*log(abs(x*e + d))/(d*g*e^2 - f*e^3)
```

maple [A] time = 0.01, size = 142, normalized size = 1.71

$$-\frac{a \ln\left(ex + d\right)}{dg - ef} + \frac{a \ln\left(gx + f\right)}{dg - ef} + \frac{bd \ln\left(ex + d\right)}{\left(dg - ef\right)e} - \frac{bf \ln\left(gx + f\right)}{\left(dg - ef\right)g} - \frac{c\,d^2 \ln\left(ex + d\right)}{\left(dg - ef\right)e^2} + \frac{c\,f^2 \ln\left(gx + f\right)}{\left(dg - ef\right)g^2} + \frac{cx}{eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x)
```

```
[Out] c*x/e/g+1/(d*g-e*f)*ln(g*x+f)*a-1/g/(d*g-e*f)*ln(g*x+f)*b*f+1/g^2/(d*g-e*f)
*ln(g*x+f)*c*f^2-1/(d*g-e*f)*ln(e*x+d)*a+1/(d*g-e*f)/e*ln(e*x+d)*b*d-1/(d*g
-e*f)/e^2*ln(e*x+d)*c*d^2
```

maxima [A] time = 0.44, size = 87, normalized size = 1.05

$$\frac{\left(cd^2 - bde + ae^2\right)\log\left(ex + d\right)}{e^3f - de^2g} - \frac{\left(cf^2 - bfg + ag^2\right)\log\left(gx + f\right)}{efg^2 - dg^3} + \frac{cx}{eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*log(e*x + d)/(e^3*f - d*e^2*g) - (c*f^2 - b*f*g + a
*g^2)*log(g*x + f)/(e*f*g^2 - d*g^3) + c*x/(e*g)
```

mupad [B] time = 3.42, size = 84, normalized size = 1.01

$$\frac{\ln\left(d + e\,x\right)\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{e^3\,f - d\,e^2\,g} + \frac{\ln\left(f + g\,x\right)\,\left(c\,f^2 - b\,f\,g + a\,g^2\right)}{g^2\,\left(d\,g - e\,f\right)} + \frac{c\,x}{e\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(a*e^2 + c*d^2 - b*d*e))/(e^3*f - d*e^2*g) + (log(f + g*x)*(a
*g^2 + c*f^2 - b*f*g))/(g^2*(d*g - e*f)) + (c*x)/(e*g)
```

sympy [B] time = 9.52, size = 420, normalized size = 5.06

$$\frac{cx}{eg}+\frac{\left(ag^{2}-bfg+cf^{2}\right)\log\left(x+\frac{adeg^{2}+ae^{2}fg-2bdefg+cd^{2}fg+cdef^{2}-\frac{d^{2}eg\left(ag^{2}-bfg+cf^{2}\right)}{dg-ef}+\frac{2de^{2}f\left(ag^{2}-bfg+cf^{2}\right)}{dg-ef}-\frac{e^{3}f^{2}\left(ag^{2}-bfg+cf^{2}\right)}{g\left(dg-ef\right)}}{2ae^{2}g^{2}-bdeg^{2}-be^{2}fg+cd^{2}g^{2}+ce^{2}f^{2}}\right)}{g^{2}\left(dg-ef\right)}-\frac{\left(ae^{2}-bde+cd^{2}\right)\log\left(x+\frac{adeg^{2}+ae^{2}fg-2bdefg+cd^{2}fg+cdef^{2}+\frac{d^{2}g^{3}\left(ae^{2}-bde+cd^{2}\right)}{e\left(dg-ef\right)}-\frac{2dfg^{2}\left(ae^{2}-bde+cd^{2}\right)}{dg-ef}+\frac{ef^{2}g\left(ae^{2}-bde+cd^{2}\right)}{dg-ef}}{2ae^{2}g^{2}-bdeg^{2}-be^{2}fg+cd^{2}g^{2}+ce^{2}f^{2}}\right)}{e^{2}\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f),x)
```

```
[Out] c*x/(e*g) + (a*g**2 - b*f*g + c*f**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*
b*d*e*f*g + c*d**2*f*g + c*d*e*f**2 - d**2*e*g*(a*g**2 - b*f*g + c*f**2)/(d
*g - e*f) + 2*d*e**2*f*(a*g**2 - b*f*g + c*f**2)/(d*g - e*f) - e**3*f**2*(a
*g**2 - b*f*g + c*f**2)/(g*(d*g - e*f)))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e*
*2*f*g + c*d**2*g**2 + c*e**2*f**2))/(g**2*(d*g - e*f)) - (a*e**2 - b*d*e +
 c*d**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*b*d*e*f*g + c*d**2*f*g + c*d*
e*f**2 + d**2*g**3*(a*e**2 - b*d*e + c*d**2)/(e*(d*g - e*f)) - 2*d*f*g**2*(
a*e**2 - b*d*e + c*d**2)/(d*g - e*f) + e*f**2*g*(a*e**2 - b*d*e + c*d**2)/(
d*g - e*f))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e**2*f*g + c*d**2*g**2 + c*e**2
*f**2))/(e**2*(d*g - e*f))
```

3.558 $\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=184

$$\frac{x\left(-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2\left(d^2g^2+defg+e^2f^2\right)\right)}{e^3g^3}+\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)^2}{e^4(ef-dg)}-\frac{\log(f+gx)\left(\right.}{g^4(\ldots}$$

Rubi [A] time = 0.31, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {893}

$$\frac{x\left(-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2\left(d^2g^2+defg+e^2f^2\right)\right)}{e^3g^3}+\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)^2}{e^4(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)^2}{g^4(ef-dg)}-\frac{cx^2(-2beg+cdg+cef)}{2e^2g^2}+\frac{c^2x^3}{3eg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((b^2*e^2*g^2 - 2*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g +
d^2*g^2))*x)/(e^3*g^3) - (c*(c*e*f + c*d*g - 2*b*e*g)*x^2)/(2*e^2*g^2) + (c
^2*x^3)/(3*e*g) + ((c*d^2 - b*d*e + a*e^2)^2*Log[d + e*x])/(e^4*(e*f - d*g)
) - ((c*f^2 - b*f*g + a*g^2)^2*Log[f + g*x])/(g^4*(e*f - d*g))
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)(f+gx)}\,dx=\int \left(\frac{b^2e^2g^2-2ceg(bef+bdg-aeg)+c^2\left(e^2f^2+defg+d^2g^2\right)}{e^3g^3}-\frac{c(cef+cdg-2beg)x}{e^2g^2}\ldots\right.$$

$$=\frac{\left(b^2e^2g^2-2ceg(bef+bdg-aeg)+c^2\left(e^2f^2+defg+d^2g^2\right)\right)x}{e^3g^3}-\frac{c(cef+cdg-2beg)x^2}{2e^2g^2}\ldots$$

Mathematica [A] time = 0.15, size = 177, normalized size = 0.96

$$-\frac{egx(dg-ef)\left(6ceg(2aeg+b(-2dg-2ef+egx))+6b^2e^2g^2+c^2\left(6d^2g^2-3deg(gx-2f)+e^2\left(6f^2-3fgx+2g^2x^2\right)\right)\right)-6g^4\log(d+ex)\left(e(ae-bd)+cd^2\right)^2+6e^4\log(f+gx)\left(g(ag-bf)+cf^2\right)^2}{6e^4g^4(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] -1/6*(e*g*(-(e*f) + d*g)*x*(6*b^2*e^2*g^2 + 6*c*e*g*(2*a*e*g + b*(-2*e*f -
2*d*g + e*g*x)) + c^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*
g*x + 2*g^2*x^2))) - 6*(c*d^2 + e*(-(b*d) + a*e))^2*g^4*Log[d + e*x] + 6*e^
4*(c*f^2 + g*(-(b*f) + a*g))^2*Log[f + g*x])/(e^4*g^4*(e*f - d*g))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^2}{(d+ex)(f+gx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)), x]
```

fricas [**A**] time = 0.71, size = 313, normalized size = 1.70

$$\frac{6\left(c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+\left(b^2+2\,ac\right)d^2e^2\right)g^4\log\left(ex+d\right)+2\left(c^2e^4fg^3-c^2de^3g^4\right)x^3-3\left(c^2e^4f^2g^2-2\,bce^4fg^3-\left(c^2d^2e^2-2\,bcde^3\right)g^4\right)x^2+6\left(c^2e^4f^3g-2\,bce^4f^2g^2+\left(b^2+2\,ac\right)e^4fg^3-\left(c^2d^3e-2\,bcd^2e^2+\left(b^2+2\,ac\right)de^3\right)g^4\right)x-6\left(c^2e^4f^4-2\,bce^4f^3g-2\,abe^4fg^3+a^2e^4g^4+\left(b^2+2\,ac\right)e^4f^2g^2\right)\log\left(gx+f\right)}{6\left(e^5fg^4-de^4g^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e
^2)*g^4*log(e*x + d) + 2*(c^2*e^4*f*g^3 - c^2*d*e^3*g^4)*x^3 - 3*(c^2*e^4*f
^2*g^2 - 2*b*c*e^4*f*g^3 - (c^2*d^2*e^2 - 2*b*c*d*e^3)*g^4)*x^2 + 6*(c^2*e^
4*f^3*g - 2*b*c*e^4*f^2*g^2 + (b^2 + 2*a*c)*e^4*f*g^3 - (c^2*d^3*e - 2*b*c*
d^2*e^2 + (b^2 + 2*a*c)*d*e^3)*g^4)*x - 6*(c^2*e^4*f^4 - 2*b*c*e^4*f^3*g -
2*a*b*e^4*f*g^3 + a^2*e^4*g^4 + (b^2 + 2*a*c)*e^4*f^2*g^2)*log(g*x + f))/(e
^5*f*g^4 - d*e^4*g^5)
```

giac [**A**] time = 0.17, size = 281, normalized size = 1.53

$$\frac{\left(c^2f^4-2\,bcf^3g+b^2f^2g^2+2\,acf^2g^2-2\,abfg^3+a^2g^4\right)\log\left(\left|gx+f\right|\right)}{dg^5-fg^4e}-\frac{\left(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4\right)\log\left(\left|xe+d\right|\right)}{dge^4-fe^5}+\frac{\left(2\,c^2g^2x^3e^2-3\,c^2dg^2x^2e+6\,c^2d^2g^2x-3\,c^2fgx^2e^2+6\,bcg^2x^2e^2+6\,c^2dfgxe-12\,bcdg^2xe+6\,c^2f^2xe^2-12\,bcfgxe^2+6\,b^2g^2xe^2+12\,acg^2xe^2\right)e^{(-3)}}{6\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] (c^2*f^4 - 2*b*c*f^3*g + b^2*f^2*g^2 + 2*a*c*f^2*g^2 - 2*a*b*f*g^3 + a^2*g^
4)*log(abs(g*x + f))/(d*g^5 - f*g^4*e) - (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e
^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*log(abs(x*e + d))/(d*g*e^4 - f*
e^5) + 1/6*(2*c^2*g^2*x^3*e^2 - 3*c^2*d*g^2*x^2*e + 6*c^2*d^2*g^2*x - 3*c^2
*f*g*x^2*e^2 + 6*b*c*g^2*x^2*e^2 + 6*c^2*d*f*g*x*e - 12*b*c*d*g^2*x*e + 6*c
^2*f^2*x*e^2 - 12*b*c*f*g*x*e^2 + 6*b^2*g^2*x*e^2 + 12*a*c*g^2*x*e^2)*e^(-3
)/g^3
```

maple [**B**] time = 0.01, size = 444, normalized size = 2.41

$$-\frac{a^2\ln\left(ex+d\right)}{dg-ef}+\frac{a^2\ln\left(gx+f\right)}{dg-ef}+\frac{2abd\ln\left(ex+d\right)}{\left(dg-ef\right)e}-\frac{2abf\ln\left(gx+f\right)}{\left(dg-ef\right)g}-\frac{2acd^2\ln\left(ex+d\right)}{\left(dg-ef\right)e^2}+\frac{2acf^2\ln\left(gx+f\right)}{\left(dg-ef\right)g^2}-\frac{b^2d^2\ln\left(ex+d\right)}{\left(dg-ef\right)e^2}+\frac{b^2f^2\ln\left(gx+f\right)}{\left(dg-ef\right)g^2}+\frac{2bcd^3\ln\left(ex+d\right)}{\left(dg-ef\right)e^3}-\frac{2bcf^3\ln\left(gx+f\right)}{\left(dg-ef\right)g^3}-\frac{c^2d^4\ln\left(ex+d\right)}{\left(dg-ef\right)e^4}+\frac{c^2x^3}{3eg}+\frac{c^2f^4\ln\left(gx+f\right)}{\left(dg-ef\right)g^4}+\frac{bcx^2}{eg}-\frac{c^2dx^2}{2e^2g}-\frac{c^2fx^2}{2eg^2}+\frac{2acx}{eg}+\frac{b^2x}{eg}-\frac{2bcdx}{e^2g}-\frac{2bcfx}{eg^2}+\frac{c^2d^2x}{e^3g}+\frac{c^2dfx}{e^2g^2}+\frac{c^2f^2x}{eg^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x)
```

```
[Out] 1/3*c^2*x^3/e/g+1/e/g*x^2*b*c-1/2/e^2/g*x^2*c^2*d-1/2/e/g^2*x^2*c^2*f+2/e/g
*a*c*x+1/e/g*b^2*x-2/e^2/g*b*c*d*x-2/e/g^2*b*c*f*x+1/e^3/g*c^2*d^2*x+1/e^2/
g^2*c^2*d*f*x+1/e/g^3*c^2*f^2*x+1/(d*g-e*f)*ln(g*x+f)*a^2-2/g/(d*g-e*f)*ln(
g*x+f)*a*b*f+2/g^2/(d*g-e*f)*ln(g*x+f)*a*c*f^2+1/g^2/(d*g-e*f)*ln(g*x+f)*b^
2*f^2-2/g^3/(d*g-e*f)*ln(g*x+f)*b*c*f^3+1/g^4/(d*g-e*f)*ln(g*x+f)*c^2*f^4-1
/(d*g-e*f)*ln(e*x+d)*a^2+2/e/(d*g-e*f)*ln(e*x+d)*a*b*d-2/e^2/(d*g-e*f)*ln(e
*x+d)*a*c*d^2-1/e^2/(d*g-e*f)*ln(e*x+d)*b^2*d^2+2/e^3/(d*g-e*f)*ln(e*x+d)*b
*c*d^3-1/e^4/(d*g-e*f)*ln(e*x+d)*c^2*d^4
```

maxima [**A**] time = 0.45, size = 255, normalized size = 1.39

$$\frac{\left(c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+\left(b^2+2\,ac\right)d^2e^2\right)\log\left(ex+d\right)}{e^5f-de^4g}-\frac{\left(c^2f^4-2\,bcf^3g-2\,abfg^3+a^2g^4+\left(b^2+2\,ac\right)f^2g^2\right)\log\left(gx+f\right)}{efg^4-dg^5}+\frac{2\,c^2e^2g^2x^3-3\left(c^2e^2fg+\left(c^2de-2\,bce^2\right)g^2\right)x^2+6\left(c^2e^2f^2+\left(c^2de-2\,bce^2\right)fg+\left(c^2d^2-2\,bcde+\left(b^2+2\,ac\right)e^2\right)g^2\right)x}{6\,e^3g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*log
(e*x + d)/(e^5*f - d*e^4*g) - (c^2*f^4 - 2*b*c*f^3*g - 2*a*b*f*g^3 + a^2*g^
4 + (b^2 + 2*a*c)*f^2*g^2)*log(g*x + f)/(e*f*g^4 - d*g^5) + 1/6*(2*c^2*e^2*
g^2*x^3 - 3*(c^2*e^2*f*g + (c^2*d*e - 2*b*c*e^2)*g^2)*x^2 + 6*(c^2*e^2*f^2
+ (c^2*d*e - 2*b*c*e^2)*f*g + (c^2*d^2 - 2*b*c*d*e + (b^2 + 2*a*c)*e^2)*g^2
)*x)/(e^3*g^3)
```

mupad [**B**] time = 3.51, size = 266, normalized size = 1.45

$$x\left(\frac{b^2+2\,a\,c}{e\,g}+\frac{\left(\frac{c^2\,\left(d\,g+e\,f\right)}{e^2\,g^2}-\frac{2\,b\,c}{e\,g}\right)\,\left(d\,g+e\,f\right)}{e\,g}-\frac{c^2\,d\,f}{e^2\,g^2}\right)-x^2\,\left(\frac{c^2\,\left(d\,g+e\,f\right)}{2\,e^2\,g^2}-\frac{b\,c}{e\,g}\right)+\frac{\ln\left(d+e\,x\right)\,\left(e^2\,\left(b^2\,d^2+2\,a\,c\,d^2\right)+a^2\,e^4+c^2\,d^4-2\,a\,b\,d\,e^3-2\,b\,c\,d^3\,e\right)}{e^5\,f-d\,e^4\,g}+\frac{\ln\left(f+g\,x\right)\,\left(g^2\,\left(b^2\,f^2+2\,a\,c\,f^2\right)+a^2\,g^4+c^2\,f^4-2\,a\,b\,f\,g^3-2\,b\,c\,f^3\,g\right)}{d\,g^5-e\,f\,g^4}+\frac{c^2\,x^3}{3\,e\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^2/((f + g*x)*(d + e*x)),x)
```

```
[Out] x*((2*a*c + b^2)/(e*g) + (((c^2*(d*g + e*f))/(e^2*g^2) - (2*b*c)/(e*g))*(d*
g + e*f))/(e*g) - (c^2*d*f)/(e^2*g^2)) - x^2*((c^2*(d*g + e*f))/(2*e^2*g^2)
 - (b*c)/(e*g)) + (log(d + e*x)*(e^2*(b^2*d^2 + 2*a*c*d^2) + a^2*e^4 + c^2*
d^4 - 2*a*b*d*e^3 - 2*b*c*d^3*e))/(e^5*f - d*e^4*g) + (log(f + g*x)*(g^2*(b
^2*f^2 + 2*a*c*f^2) + a^2*g^4 + c^2*f^4 - 2*a*b*f*g^3 - 2*b*c*f^3*g))/(d*g^
5 - e*f*g^4) + (c^2*x^3)/(3*e*g)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

3.559 $\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=531

$$-\frac{x\left(-3ce^2g^2\left(a^2e^2g^2-2abeg(dg+ef)+b^2\left(d^2g^2+defg+e^2f^2\right)\right)+b^2e^3g^3(-3aeg+bdg+bef)-3c^2eg\left(aeg\left(\right.\right.}{}$$

Rubi [A] time = 0.99, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {893}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

```
[Out] -(((b^2*e^3*g^3*(b*e*f + b*d*g - 3*a*e*g) - c^3*(e^4*f^4 + d*e^3*f^3*g + d^
2*e^2*f^2*g^2 + d^3*e*f*g^3 + d^4*g^4) - 3*c*e^2*g^2*(a^2*e^2*g^2 - 2*a*b*e
*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) - 3*c^2*e*g*(a*e*g*(e^2
*f^2 + d*e*f*g + d^2*g^2) - b*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^
3)))*x)/(e^5*g^5)) + ((b^3*e^3*g^3 - 3*b*c*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g
) - c^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3) - 3*c^2*e*g*(a*e*g*
(e*f + d*g) - b*(e^2*f^2 + d*e*f*g + d^2*g^2)))*x^2)/(2*e^4*g^4) + (c*(3*b^
2*e^2*g^2 - 3*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g + d^2*
g^2))*x^3)/(3*e^3*g^3) - (c^2*(c*e*f + c*d*g - 3*b*e*g)*x^4)/(4*e^2*g^2) +
(c^3*x^5)/(5*e*g) + ((c*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/(e^6*(e*f - d*
g)) - ((c*f^2 - b*f*g + a*g^2)^3*Log[f + g*x])/(g^6*(e*f - d*g))
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx = \int \left(\frac{-b^2e^3g^3(bef+bdg-3aeg)+c^3\left(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4\right)+}{}\right.$$

$$= -\frac{\left(b^2e^3g^3(bef+bdg-3aeg)-c^3\left(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4\right)-3ce^2\right.}{}$$

Mathematica [A] time = 0.42, size = 476, normalized size = 0.90

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

```
[Out] -1/60*(e*g*x*(-30*b^2*e^3*g^3*(e*f - d*g)*(6*a*e*g + b*(-2*e*f - 2*d*g + e*
g*x)) + c^3*(60*d^5*g^5 - 30*d^4*e*g^5*x + 20*d^3*e^2*g^5*x^2 - 15*d^2*e^3*
```

```
g^5*x^3 + 12*d*e^4*g^5*x^4 + e^5*f*(-60*f^4 + 30*f^3*g*x - 20*f^2*g^2*x^2 +
 15*f*g^3*x^3 - 12*g^4*x^4)) - 30*c*e^2*g^2*(e*f - d*g)*(6*a^2*e^2*g^2 + 6*
a*b*e*g*(-2*e*f - 2*d*g + e*g*x) + b^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) +
e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2))) + 15*c^2*e*g*(-2*a*e*g*(e*f - d*g)*(6*d
^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)) + b*(-12
*d^4*g^4 + 6*d^3*e*g^4*x - 4*d^2*e^2*g^4*x^2 + 3*d*e^3*g^4*x^3 + e^4*f*(12*
f^3 - 6*f^2*g*x + 4*f*g^2*x^2 - 3*g^3*x^3)))) - 60*(c*d^2 + e*(-(b*d) + a*e
))^3*g^6*Log[d + e*x] + 60*e^6*(c*f^2 + g*(-(b*f) + a*g))^3*Log[f + g*x])/(
e^6*g^6*(e*f - d*g))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)(f+gx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)), x]
```

fricas [A] time = 3.90, size = 736, normalized size = 1.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] 1/60*(60*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*
c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*g^6*log
(e*x + d) + 12*(c^3*e^6*f*g^5 - c^3*d*e^5*g^6)*x^5 - 15*(c^3*e^6*f^2*g^4 -
3*b*c^2*e^6*f*g^5 - (c^3*d^2*e^4 - 3*b*c^2*d*e^5)*g^6)*x^4 + 20*(c^3*e^6*f^
3*g^3 - 3*b*c^2*e^6*f^2*g^4 + 3*(b^2*c + a*c^2)*e^6*f*g^5 - (c^3*d^3*e^3 -
3*b*c^2*d^2*e^4 + 3*(b^2*c + a*c^2)*d*e^5)*g^6)*x^3 - 30*(c^3*e^6*f^4*g^2 -
 3*b*c^2*e^6*f^3*g^3 + 3*(b^2*c + a*c^2)*e^6*f^2*g^4 - (b^3 + 6*a*b*c)*e^6*
f*g^5 - (c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c + a*c^2)*d^2*e^4 - (b^3 +
 6*a*b*c)*d*e^5)*g^6)*x^2 + 60*(c^3*e^6*f^5*g - 3*b*c^2*e^6*f^4*g^2 + 3*(b^
2*c + a*c^2)*e^6*f^3*g^3 - (b^3 + 6*a*b*c)*e^6*f^2*g^4 + 3*(a*b^2 + a^2*c)*
e^6*f*g^5 - (c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c + a*c^2)*d^3*e^3 - (b^3
 + 6*a*b*c)*d^2*e^4 + 3*(a*b^2 + a^2*c)*d*e^5)*g^6)*x - 60*(c^3*e^6*f^6 - 3
*b*c^2*e^6*f^5*g - 3*a^2*b*e^6*f*g^5 + a^3*e^6*g^6 + 3*(b^2*c + a*c^2)*e^6*
f^4*g^2 - (b^3 + 6*a*b*c)*e^6*f^3*g^3 + 3*(a*b^2 + a^2*c)*e^6*f^2*g^4)*log(
g*x + f))/(e^7*f*g^6 - d*e^6*g^7)
```

giac [A] time = 0.17, size = 907, normalized size = 1.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] (c^3*f^6 - 3*b*c^2*f^5*g + 3*b^2*c*f^4*g^2 + 3*a*c^2*f^4*g^2 - b^3*f^3*g^3
- 6*a*b*c*f^3*g^3 + 3*a*b^2*f^2*g^4 + 3*a^2*c*f^2*g^4 - 3*a^2*b*f*g^5 + a^3
*g^6)*log(abs(g*x + f))/(d*g^7 - f*g^6*e) - (c^3*d^6 - 3*b*c^2*d^5*e + 3*b^
2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2
*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*log(abs(x*e + d))/(d*g*e^
6 - f*e^7) + 1/60*(12*c^3*g^4*x^5*e^4 - 15*c^3*d*g^4*x^4*e^3 + 20*c^3*d^2*g
^4*x^3*e^2 - 30*c^3*d^3*g^4*x^2*e + 60*c^3*d^4*g^4*x - 15*c^3*f*g^3*x^4*e^4
 + 45*b*c^2*g^4*x^4*e^4 + 20*c^3*d*f*g^3*x^3*e^3 - 60*b*c^2*d*g^4*x^3*e^3 -
```

```
 30*c^3*d^2*f*g^3*x^2*e^2 + 90*b*c^2*d^2*g^4*x^2*e^2 + 60*c^3*d^3*f*g^3*x*e
 - 180*b*c^2*d^3*g^4*x*e + 20*c^3*f^2*g^2*x^3*e^4 - 60*b*c^2*f*g^3*x^3*e^4
+ 60*b^2*c*g^4*x^3*e^4 + 60*a*c^2*g^4*x^3*e^4 - 30*c^3*d*f^2*g^2*x^2*e^3 +
90*b*c^2*d*f*g^3*x^2*e^3 - 90*b^2*c*d*g^4*x^2*e^3 - 90*a*c^2*d*g^4*x^2*e^3
+ 60*c^3*d^2*f^2*g^2*x*e^2 - 180*b*c^2*d^2*f*g^3*x*e^2 + 180*b^2*c*d^2*g^4*
x*e^2 + 180*a*c^2*d^2*g^4*x*e^2 - 30*c^3*f^3*g*x^2*e^4 + 90*b*c^2*f^2*g^2*x
^2*e^4 - 90*b^2*c*f*g^3*x^2*e^4 - 90*a*c^2*f*g^3*x^2*e^4 + 30*b^3*g^4*x^2*e
^4 + 180*a*b*c*g^4*x^2*e^4 + 60*c^3*d*f^3*g*x*e^3 - 180*b*c^2*d*f^2*g^2*x*e
^3 + 180*b^2*c*d*f*g^3*x*e^3 + 180*a*c^2*d*f*g^3*x*e^3 - 60*b^3*d*g^4*x*e^3
 - 360*a*b*c*d*g^4*x*e^3 + 60*c^3*f^4*x*e^4 - 180*b*c^2*f^3*g*x*e^4 + 180*b
^2*c*f^2*g^2*x*e^4 + 180*a*c^2*f^2*g^2*x*e^4 - 60*b^3*f*g^3*x*e^4 - 360*a*b
*c*f*g^3*x*e^4 + 180*a*b^2*g^4*x*e^4 + 180*a^2*c*g^4*x*e^4)*e^(-5)/g^5
```

maple [B] time = 0.02, size = 1232, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x)
```

```
[Out] 1/5*c^3*x^5/e/g+1/2/g/e*x^2*b^3-6/g/e^2*a*b*c*d*x-6/g^2/e*a*b*c*f*x+3/g^2/e
^2*a*c^2*d*f*x+3/g^2/e^2*b^2*c*d*f*x-3/g^2/e^3*b*c^2*d^2*f*x-6/g^3/(d*g-e*f
)*ln(g*x+f)*a*b*c*f^3+6/e^3/(d*g-e*f)*ln(e*x+d)*a*b*c*d^3+1/(d*g-e*f)*ln(g*
x+f)*a^3-1/(d*g-e*f)*ln(e*x+d)*a^3-3/g/(d*g-e*f)*ln(g*x+f)*a^2*b*f+3/e/(d*g
-e*f)*ln(e*x+d)*a^2*b*d-3/e^2/(d*g-e*f)*ln(e*x+d)*a^2*c*d^2-3/e^2/(d*g-e*f)
*ln(e*x+d)*a*b^2*d^2-3/e^4/(d*g-e*f)*ln(e*x+d)*a*c^2*d^4-3/e^4/(d*g-e*f)*ln
(e*x+d)*b^2*c*d^4+3/e^5/(d*g-e*f)*ln(e*x+d)*b*c^2*d^5+3/2/g^3/e*x^2*b*c^2*f
^2-1/2/g^2/e^3*x^2*c^3*d^2*f-1/2/g^3/e^2*x^2*c^3*d*f^2+3/g/e^3*a*c^2*d^2*x+
3/g^3/e*a*c^2*f^2*x+3/g/e^3*b^2*c*d^2*x+3/g^3/e*b^2*c*f^2*x+3/g^2/(d*g-e*f)
*ln(g*x+f)*a^2*c*f^2+3/g^2/(d*g-e*f)*ln(g*x+f)*a*b^2*f^2+3/g^4/(d*g-e*f)*ln
(g*x+f)*a*c^2*f^4+3/g^4/(d*g-e*f)*ln(g*x+f)*b^2*c*f^4-3/g^5/(d*g-e*f)*ln(g*
x+f)*b*c^2*f^5-1/g/e^2*x^3*b*c^2*d-1/g^2/e*x^3*b*c^2*f+1/3/g^2/e^2*x^3*c^3*
d*f+1/g/e*x^3*a*c^2+1/g^5/e*c^3*f^4*x-3/g/e^4*b*c^2*d^3*x-3/g^4/e*b*c^2*f^3
*x+1/g^2/e^4*c^3*d^3*f*x+1/g^3/e^3*c^3*d^2*f^2*x+1/g^4/e^2*c^3*d*f^3*x+1/g/
e^5*c^3*d^4*x+3/4/g/e*x^4*b*c^2-1/4/g/e^2*x^4*c^3*d-1/4/g^2/e*x^4*c^3*f+1/3
/g/e^3*x^3*c^3*d^2+1/3/g^3/e*x^3*c^3*f^2-1/2/g/e^4*x^2*c^3*d^3-1/2/g^4/e*x^
2*c^3*f^3+3/g/e*a^2*c*x+3/g/e*a*b^2*x-1/g/e^2*b^3*d*x-1/g^2/e*b^3*f*x+1/g/e
*x^3*b^2*c-1/g^3/(d*g-e*f)*ln(g*x+f)*b^3*f^3+1/g^6/(d*g-e*f)*ln(g*x+f)*c^3*
f^6+1/e^3/(d*g-e*f)*ln(e*x+d)*b^3*d^3-1/e^6/(d*g-e*f)*ln(e*x+d)*c^3*d^6+3/g
/e*x^2*a*b*c-3/2/g/e^2*x^2*a*c^2*d-3/2/g^2/e*x^2*a*c^2*f-3/2/g/e^2*x^2*b^2*
c*d-3/2/g^2/e*x^2*b^2*c*f+3/2/g/e^3*x^2*b*c^2*d^2-3/g^3/e^2*b*c^2*d*f^2*x+3
/2/g^2/e^2*x^2*b*c^2*d*f
```

maxima [A] time = 0.49, size = 721, normalized size = 1.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4*
e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(e*x + d)/(e^
7*f - d*e^6*g) - (c^3*f^6 - 3*b*c^2*f^5*g - 3*a^2*b*f*g^5 + a^3*g^6 + 3*(b^
2*c + a*c^2)*f^4*g^2 - (b^3 + 6*a*b*c)*f^3*g^3 + 3*(a*b^2 + a^2*c)*f^2*g^4)
*log(g*x + f)/(e*f*g^6 - d*g^7) + 1/60*(12*c^3*e^4*g^4*x^5 - 15*(c^3*e^4*f*
g^3 + (c^3*d*e^3 - 3*b*c^2*e^4)*g^4)*x^4 + 20*(c^3*e^4*f^2*g^2 + (c^3*d*e^3
 - 3*b*c^2*e^4)*f*g^3 + (c^3*d^2*e^2 - 3*b*c^2*d*e^3 + 3*(b^2*c + a*c^2)*e^
4)*g^4)*x^3 - 30*(c^3*e^4*f^3*g + (c^3*d*e^3 - 3*b*c^2*e^4)*f^2*g^2 + (c^3*
d^2*e^2 - 3*b*c^2*d*e^3 + 3*(b^2*c + a*c^2)*e^4)*f*g^3 + (c^3*d^3*e - 3*b*c
^2*d^2*e^2 + 3*(b^2*c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*g^4)*x^2 + 60*(
c^3*e^4*f^4 + (c^3*d*e^3 - 3*b*c^2*e^4)*f^3*g + (c^3*d^2*e^2 - 3*b*c^2*d*e^
```

3 + 3*(b^2*c + a*c^2)*e^4)*f^2*g^2 + (c^3*d^3*e - 3*b*c^2*d^2*e^2 + 3*(b^2*
c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*f*g^3 + (c^3*d^4 - 3*b*c^2*d^3*e +
3*(b^2*c + a*c^2)*d^2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + 3*(a*b^2 + a^2*c)*e^4)*
g^4)*x)/(e^5*g^5)

mupad [B] time = 4.20, size = 794, normalized size = 1.50

$$x^4\left(\frac{3bc^2}{4eg}-\frac{c^3\left(dg+ef\right)}{4e^2g^2}\right)-x^3\left(\frac{\left(dg+ef\right)\left(\frac{3bc^2}{eg}-\frac{c^3\left(dg+ef\right)}{e^2g^2}\right)}{3eg}-\frac{c\left(b^2+ac\right)}{eg}+\frac{c^3df}{3e^2g^2}\right)+x^2\left(\frac{b^3+6acb}{2eg}+\frac{\left(dg+ef\right)\left(\frac{\left(dg+ef\right)\left(\frac{3bc^2}{eg}-\frac{c^3\left(dg+ef\right)}{e^2g^2}\right)}{eg}-\frac{3c\left(b^2+ac\right)}{eg}+\frac{c^3df}{e^2g^2}\right)}{2eg}-\frac{df\left(\frac{3bc^2}{eg}-\frac{c^3\left(dg+ef\right)}{e^2g^2}\right)}{2eg}\right)+x\left(\frac{3a\left(b^2+ac\right)}{eg}-\frac{\left(dg+ef\right)\left(\frac{b^3+6acb}{eg}+\frac{\left(dg+ef\right)\left(\frac{\left(dg+ef\right)\left(\frac{3bc^2}{eg}-\frac{c^3\left(dg+ef\right)}{e^2g^2}\right)}{eg}-\frac{3c\left(b^2+ac\right)}{eg}+\frac{c^3df}{e^2g^2}\right)}{eg}-\frac{df\left(\frac{3bc^2}{eg}-\frac{c^3\left(dg+ef\right)}{e^2g^2}\right)}{eg}\right)}{eg}+\frac{df\left(\frac{\left(dg+ef\right)\left(\frac{3bc^2}{eg}-\frac{c^3\left(dg+ef\right)}{e^2g^2}\right)}{eg}-\frac{3c\left(b^2+ac\right)}{eg}+\frac{c^3df}{e^2g^2}\right)}{eg}\right)+\frac{\ln\left(d+ex\right)\left(e^4\left(3ca^2d^2+3ab^2d^2\right)+e^2\left(3b^2cd^4+3ac^2d^4\right)-e^3\left(b^3d^3+6acbd^3\right)+a^3e^6+c^3d^6-3a^2bde^5-3bc^2d^5e\right)}{e^7f-de^6g}+\frac{\ln\left(f+gx\right)\left(g^4\left(3ca^2f^2+3ab^2f^2\right)+g^2\left(3b^2cf^4+3ac^2f^4\right)-g^3\left(b^3f^3+6acbf^3\right)+a^3g^6+c^3f^6-3a^2bfg^5-3bc^2f^5g\right)}{dg^7-efg^6}+\frac{c^3x^5}{5eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/((f + g*x)*(d + e*x)),x)
```

```
[Out] x^4*((3*b*c^2)/(4*e*g) - (c^3*(d*g + e*f))/(4*e^2*g^2)) - x^3*(((d*g + e*f)
*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(3*e*g) - (c*(a*c + b^2))
/(e*g) + (c^3*d*f)/(3*e^2*g^2)) + x^2*((b^3 + 6*a*b*c)/(2*e*g) + ((d*g + e*
f)*(((d*g + e*f)*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(e*g) - (
3*c*(a*c + b^2))/(e*g) + (c^3*d*f)/(e^2*g^2)))/(2*e*g) - (d*f*((3*b*c^2)/(e
*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(2*e*g)) + x*((3*a*(a*c + b^2))/(e*g) -
 ((d*g + e*f)*((b^3 + 6*a*b*c)/(e*g) + ((d*g + e*f)*(((d*g + e*f)*((3*b*c^2
)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(e*g) - (3*c*(a*c + b^2))/(e*g) + (
c^3*d*f)/(e^2*g^2)))/(e*g) - (d*f*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2
*g^2)))/(e*g)))/(e*g) + (d*f*(((d*g + e*f)*((3*b*c^2)/(e*g) - (c^3*(d*g + e
*f))/(e^2*g^2)))/(e*g) - (3*c*(a*c + b^2))/(e*g) + (c^3*d*f)/(e^2*g^2)))/(e
*g)) + (log(d + e*x)*(e^4*(3*a*b^2*d^2 + 3*a^2*c*d^2) + e^2*(3*a*c^2*d^4 +
3*b^2*c*d^4) - e^3*(b^3*d^3 + 6*a*b*c*d^3) + a^3*e^6 + c^3*d^6 - 3*a^2*b*d*
e^5 - 3*b*c^2*d^5*e))/(e^7*f - d*e^6*g) + (log(f + g*x)*(g^4*(3*a*b^2*f^2 +
 3*a^2*c*f^2) + g^2*(3*a*c^2*f^4 + 3*b^2*c*f^4) - g^3*(b^3*f^3 + 6*a*b*c*f^
3) + a^3*g^6 + c^3*f^6 - 3*a^2*b*f*g^5 - 3*b*c^2*f^5*g))/(d*g^7 - e*f*g^6)
+ (c^3*x^5)/(5*e*g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

3.560 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=246

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-\frac{\log\left(a+bx+cx^2\right)(-beg+cdg+cef)}{2\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}+\frac{e^2}{(ef-dg)}$$

Rubi [A] time = 0.47, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {893, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-\frac{\log\left(a+bx+cx^2\right)(-beg+cdg+cef)}{2\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}+\frac{e^2\log(d+ex)}{(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{g^2\log(f+gx)}{(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] -(((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g)))) + (e^2*Log[d + e*x])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)) -
(g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)) - ((c*e*f + c*d*g
- b*e*g)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f -
 a*g)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx = \int \left(-\frac{e^3}{\left(cd^2-bde+ae^2\right)(-ef+dg)(d+ex)} - \frac{g^3}{(ef-dg)\left(cf^2-bfg+ag^2\right.}\right.$$

$$= \frac{e^2\log(d+ex)}{\left(cd^2-bde+ae^2\right)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)} + \frac{\int \frac{c^2df+b^2eg-}{}}{\left(cd^2-bd\right.}$$

$$= \frac{e^2\log(d+ex)}{\left(cd^2-bde+ae^2\right)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)} + \frac{(-cef-}{2\left(cd^2-bd\right.}$$

$$= \frac{e^2\log(d+ex)}{\left(cd^2-bde+ae^2\right)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)} - \frac{(cef+cdg}{2\left(cd^2-bde\right.}$$

$$= -\frac{\left(2c^2df+b^2eg-c(bef+bdg+2aeg)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)} + \frac{e^2\log}{\left(cd^2-bde+\right.}$$

Mathematica [A] time = 0.32, size = 246, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{4ac-b^2}\left(e(ae-bd)+cd^2\right)\left(g(ag-bf)+cf^2\right)}+\frac{e^2\log(d+ex)}{(ef-dg)\left(e(ae-bd)+cd^2\right)}-\frac{\log(a+x(b+cx))(-beg+cdg+cef)}{2\left(e(ae-bd)+cd^2\right)\left(g(ag-bf)+cf^2\right)}-\frac{g^2\log(f+gx)}{(ef-dg)\left(g(ag-bf)+cf^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] ((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*(c*f^2 + g
*(-(b*f) + a*g))) + (e^2*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))*(e*f - d
*g)) - (g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))) - ((c*e*
f + c*d*g - b*e*g)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))*(c*f
^2 + g*(-(b*f) + a*g)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.18, size = 392, normalized size = 1.59

$$\frac{g^3\log\left(|gx+f|\right)}{cdf^2g^2-bdfg^3+adg^4-cf^3ge+bf^2g^2e-afg^3e}-\frac{\left(cdg+cfe-bge\right)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2f^2-bcd^2fg+acd^2g^2-bcdf^2e+b^2dfge-abdg^2e+acf^2e^2-abfge^2+a^2g^2e^2\right)}-\frac{e^3\log\left(|xe+d|\right)}{cd^3ge-cd^2fe^2-bd^2ge^2+bdfe^3+adge^3-afe^4}+\frac{\left(2c^2df-bcdg-bcfe+b^2ge-2acge\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\left(c^2d^2f^2-bcd^2fg+acd^2g^2-bcdf^2e+b^2dfge-abdg^2e+acf^2e^2-abfge^2+a^2g^2e^2\right)\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] g^3*log(abs(g*x + f))/(c*d*f^2*g^2 - b*d*f*g^3 + a*d*g^4 - c*f^3*g*e + b*f^
2*g^2*e - a*f*g^3*e) - 1/2*(c*d*g + c*f*e - b*g*e)*log(c*x^2 + b*x + a)/(c^
2*d^2*f^2 - b*c*d^2*f*g + a*c*d^2*g^2 - b*c*d*f^2*e + b^2*d*f*g*e - a*b*d*g
^2*e + a*c*f^2*e^2 - a*b*f*g*e^2 + a^2*g^2*e^2) - e^3*log(abs(x*e + d))/(c*
d^3*g*e - c*d^2*f*e^2 - b*d^2*g*e^2 + b*d*f*e^3 + a*d*g*e^3 - a*f*e^4) + (2
*c^2*d*f - b*c*d*g - b*c*f*e + b^2*g*e - 2*a*c*g*e)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((c^2*d^2*f^2 - b*c*d^2*f*g + a*c*d^2*g^2 - b*c*d*f^2*e + b
^2*d*f*g*e - a*b*d*g^2*e + a*c*f^2*e^2 - a*b*f*g*e^2 + a^2*g^2*e^2)*sqrt(-b
^2 + 4*a*c))
```

maple [B] time = 0.01, size = 606, normalized size = 2.46

$$-\frac{2aceg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a e^2-bde+c d^2\right)\left(a g^2-bfg+c f^2\right)\sqrt{4ac-b^2}}+\frac{b^2eg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a e^2-bde+c d^2\right)\left(a g^2-bfg+c f^2\right)\sqrt{4ac-b^2}}-\frac{bcdg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a e^2-bde+c d^2\right)\left(a g^2-bfg+c f^2\right)\sqrt{4ac-b^2}}-\frac{bcef\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a e^2-bde+c d^2\right)\left(a g^2-bfg+c f^2\right)\sqrt{4ac-b^2}}+\frac{2c^2df\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a e^2-bde+c d^2\right)\left(a g^2-bfg+c f^2\right)\sqrt{4ac-b^2}}+\frac{beg\ln\left(c x^2+bx+a\right)}{2\left(a e^2-bde+c d^2\right)\left(a g^2-bfg+c f^2\right)}-\frac{cdg\ln\left(c x^2+bx+a\right)}{2\left(a e^2-bde+c d^2\right)\left(a g^2-bfg+c f^2\right)}-\frac{cef\ln\left(c x^2+bx+a\right)}{2\left(a e^2-bde+c d^2\right)\left(a g^2-bfg+c f^2\right)}-\frac{e^2\ln\left(ex+d\right)}{\left(a e^2-bde+c d^2\right)\left(dg-ef\right)}+\frac{g^2\ln\left(gx+f\right)}{\left(dg-ef\right)\left(a g^2-bfg+c f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)*ln(c*x^2+b*x+a)*g*e*b-1/2/(a*e^
2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)*c*ln(c*x^2+b*x+a)*g*d-1/2/(a*e^2-b*d*e+c
*d^2)/(a*g^2-b*f*g+c*f^2)*c*ln(c*x^2+b*x+a)*f*e-2/(a*e^2-b*d*e+c*d^2)/(a*g^
2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c*e*
g+1/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*b^2*e*g-1/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c*d*g-1/(a*e^2-b*d*e+
c*d^2)/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b*c*e*f+2/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*d*f+g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2
)*ln(g*x+f)-e^2/(a*e^2-b*d*e+c*d^2)/(d*g-e*f)*ln(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 19.25, size = 12173, normalized size = 49.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)),x)
```

```
[Out] (log(6*a^2*c^4*d^5*g^5 + 6*a^2*c^4*e^5*f^5 - a^3*b^3*e^5*g^5 - a^3*b^2*e^5*
g^5*(b^2 - 4*a*c)^(1/2) - c^5*d^3*e^2*f^5*(b^2 - 4*a*c)^(1/2) - c^5*d^5*f^3
*g^2*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3*d^3*e^2*g^5 + b^2*c^4*d^2*e^3*f^5 - 1
8*a^3*c^3*e^5*f^3*g^2 + b^2*c^4*d^5*f^2*g^3 + 4*a^4*b*c*e^5*g^5 + 4*a^4*c*e
^5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*c^3*d^5*g^5 - 2*a*b^2*c^3*e^5*f^5 + 2*
a*b^5*d^2*e^3*g^5 - 10*a*c^5*d^2*e^3*f^5 + a^2*b^4*d*e^4*g^5 + b*c^5*d^3*e^
2*f^5 - 8*a^4*c^2*d*e^4*g^5 + 2*a*b^5*e^5*f^2*g^3 - 10*a*c^5*d^5*f^2*g^3 +
a^2*b^4*e^5*f*g^4 + b*c^5*d^5*f^3*g^2 - 8*a^4*c^2*e^5*f*g^4 - a^2*b^4*e^5*g
^5*x - 8*a^4*c^2*e^5*g^5*x - 2*b^3*c^3*d^5*g^5*x - 2*b^3*c^3*e^5*f^5*x + 2*
```

b^6*d^2*e^3*g^5*x + 2*c^6*d^3*e^2*f^5*x + 2*b^6*e^5*f^2*g^3*x + 2*c^6*d^5*f
^3*g^2*x - 2*a*b*c^3*d^5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b*c^3*e^5*f^5*(b^2 -
 4*a*c)^(1/2) + 7*a*c^4*d*e^4*f^5*(b^2 - 4*a*c)^(1/2) + 7*a*c^4*d^5*f*g^4*(
b^2 - 4*a*c)^(1/2) + 2*c^5*d^4*e*f^4*g*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*d^5*g^
5*x*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*e^5*f^5*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c
^2*d^4*e*g^5 - 6*a*b^4*c*d^3*e^2*g^5 - 21*a^2*b*c^3*d^4*e*g^5 - 2*a^3*b^2*c
*d*e^4*g^5 + 6*a*b^3*c^2*e^5*f^4*g - 6*a*b^4*c*e^5*f^3*g^2 - 21*a^2*b*c^3*e
^5*f^4*g - 2*a^3*b^2*c*e^5*f*g^4 + 10*a*c^5*d^3*e^2*f^4*g + 10*a*c^5*d^4*e*
f^3*g^2 + 26*a^2*c^4*d*e^4*f^4*g + 26*a^2*c^4*d^4*e*f*g^4 + 6*a^3*b^2*c*e^5
*g^5*x - 3*b*c^5*d^2*e^3*f^5*x + 14*a^2*c^4*d^4*e*g^5*x + 5*b^2*c^4*d*e^4*f
^5*x + 6*b^4*c^2*d^4*e*g^5*x - 6*b^5*c*d^3*e^2*g^5*x - 3*b*c^5*d^5*f^2*g^3*
x + 14*a^2*c^4*e^5*f^4*g*x + 5*b^2*c^4*d^5*f*g^4*x + 6*b^4*c^2*e^5*f^4*g*x
- 6*b^5*c*e^5*f^3*g^2*x + 2*a*b^4*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + a^2*b^3
*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) - b*c^4*d^2*e^3*f^5*(b^2 - 4*a*c)^(1/2) - 7*
a^2*c^3*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*e^5*f^2*g^3*(b^2 - 4*a*c)^(
1/2) + a^2*b^3*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) - b*c^4*d^5*f^2*g^3*(b^2 - 4*a
*c)^(1/2) - 7*a^2*c^3*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) - a^2*b^3*e^5*g^5*x*(b^
2 - 4*a*c)^(1/2) - 2*b^2*c^3*d^5*g^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c^3*e^5*
f^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^5*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) - 5*c^5
*d^2*e^3*f^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^5*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2
) - 5*c^5*d^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c*d^2*e^3*g^5 + 21
*a^3*b*c^2*d^2*e^3*g^5 - 13*a^2*b^3*c*e^5*f^2*g^3 + 21*a^3*b*c^2*e^5*f^2*g^
3 + 2*a^3*c^3*d*e^4*f^2*g^3 + 2*a^3*c^3*d^2*e^3*f*g^4 - b^2*c^4*d^3*e^2*f^4
*g - b^2*c^4*d^4*e*f^3*g^2 - b^3*c^3*d^2*e^3*f^4*g - b^3*c^3*d^4*e*f^2*g^3
- b^5*c*d^2*e^3*f^2*g^3 - 10*a^3*c^3*d^2*e^3*g^5*x - 10*a^3*c^3*e^5*f^2*g^3
*x + 3*a*b*c^4*d*e^4*f^5 + 5*a^3*c^2*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 3*a*
b*c^4*d^5*f*g^4 + 5*a^3*c^2*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 5*a*b^5*d*e^4
*f*g^4 - 2*b*c^5*d^4*e*f^4*g + 7*a*b*c^4*d^5*g^5*x + 7*a*b*c^4*e^5*f^5*x +
a*b^5*d*e^4*g^5*x - 14*a*c^5*d*e^4*f^5*x + a*b^5*e^5*f*g^4*x - 14*a*c^5*d^5
*f*g^4*x - 5*b^6*d*e^4*f*g^4*x - 4*c^6*d^4*e*f^4*g*x + 27*a^2*b^2*c^2*d^3*e
^2*g^5 + 27*a^2*b^2*c^2*e^5*f^3*g^2 - 40*a^2*c^4*d^2*e^3*f^3*g^2 - 40*a^2*c
^4*d^3*e^2*f^2*g^3 + b^3*c^3*d^3*e^2*f^3*g^2 + b^4*c^2*d^2*e^3*f^3*g^2 + b^
4*c^2*d^3*e^2*f^2*g^3 + 32*a*b^3*c^2*d^3*e^2*g^5*x - 35*a^2*b*c^3*d^3*e^2*g
^5*x + 32*a*b^3*c^2*e^5*f^3*g^2*x - 35*a^2*b*c^3*e^5*f^3*g^2*x + 48*a*c^5*d
^3*e^2*f^3*g^2*x + 14*a^2*c^4*d*e^4*f^3*g^2*x + 14*a^2*c^4*d^3*e^2*f*g^4*x
+ 3*b^2*c^4*d^2*e^3*f^4*g*x + 3*b^2*c^4*d^4*e*f^2*g^3*x + 4*b^4*c^2*d*e^4*f
^3*g^2*x + 4*b^4*c^2*d^3*e^2*f*g^4*x + 13*a^2*b*c^2*d^3*e^2*g^5*(b^2 - 4*a*
c)^(1/2) - 7*a^2*b^2*c*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*e^5*f
^3*g^2*(b^2 - 4*a*c)^(1/2) - 7*a^2*b^2*c*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) -
24*a*c^4*d^3*e^2*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 7*a^2*c^3*d*e^4*f^3*g^2*(b^2
 - 4*a*c)^(1/2) - 7*a^2*c^3*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^2
*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2)
+ b^4*c*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 9*a^2*c^3*d^3*e^2*g^5*x*(b^2
- 4*a*c)^(1/2) - 9*a^2*c^3*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^3
*d^2*e^3*f^3*g^2 + 10*a*b^2*c^3*d^3*e^2*f^2*g^3 - 23*a*b^3*c^2*d^2*e^3*f^2*
g^3 + 96*a^2*b*c^3*d^2*e^3*f^2*g^3 - 39*a^2*b^2*c^2*d*e^4*f^2*g^3 - 39*a^2*
b^2*c^2*d^2*e^3*f*g^4 + 27*a^2*b^2*c^2*d^2*e^3*g^5*x + 27*a^2*b^2*c^2*e^5*f
^2*g^3*x - 48*a^2*c^4*d^2*e^3*f^2*g^3*x - 18*b^2*c^4*d^3*e^2*f^3*g^2*x + 17
*b^3*c^3*d^2*e^3*f^3*g^2*x + 17*b^3*c^3*d^3*e^2*f^2*g^3*x - 27*b^4*c^2*d^2*
e^3*f^2*g^3*x - 4*a^3*b*c*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*b*c*e^5*f*g
^4*(b^2 - 4*a*c)^(1/2) - 5*a*b^4*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) + 4*a^3*b*
c*e^5*g^5*x*(b^2 - 4*a*c)^(1/2) + a*b^4*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) + 5
*b*c^4*d*e^4*f^5*x*(b^2 - 4*a*c)^(1/2) + a*b^4*e^5*f*g^4*x*(b^2 - 4*a*c)^(1
/2) + 5*b*c^4*d^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 5*b^5*d*e^4*f*g^4*x*(b^2 -
4*a*c)^(1/2) + 7*a*b*c^4*d^2*e^3*f^4*g + 7*a*b*c^4*d^4*e*f^2*g^3 - 10*a*b^2
*c^3*d*e^4*f^4*g - 10*a*b^2*c^3*d^4*e*f*g^4 + 10*a*b^4*c*d*e^4*f^2*g^3 + 10
*a*b^4*c*d^2*e^3*f*g^4 + 19*a^2*b^3*c*d*e^4*f*g^4 + 2*a^3*b*c^2*d*e^4*f*g^4
 + 24*a^2*c^3*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) - b^2*c^3*d^3*e^2*f^3*g^2
*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) - b^3*c^

2*d^3*e^2*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 26*a*b^2*c^3*d^4*e*g^5*x - 14*a*b^4*c*d^2*e^3*g^5*x - 5*a^2*b^3*c*d*e^4*g^5*x + 4*a^3*b*c^2*d*e^4*g^5*x - 26*a*b^2*c^3*e^5*f^4*g*x - 14*a*b^4*c*e^5*f^2*g^3*x - 5*a^2*b^3*c*e^5*f*g^4*x + 4*a^3*b*c^2*e^5*f*g^4*x - 6*a*c^5*d^2*e^3*f^4*g*x - 6*a*c^5*d^4*e*f^2*g^3*x + 12*a^3*c^3*d*e^4*f*g^4*x + 3*b*c^5*d^3*e^2*f^4*g*x + 3*b*c^5*d^4*e*f^3*g^2*x - 12*b^3*c^3*d*e^4*f^4*g*x - 12*b^3*c^3*d^4*e*f*g^4*x + 8*b^5*c*d*e^4*f^2*g^3*x + 8*b^5*c*d^2*e^3*f*g^4*x + 6*a*b^2*c^2*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c*d^3*e^2*g^5*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c*e^5*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*d^2*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 6*a^3*c^2*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) + b*c^4*d^3*e^2*f^4*g*(b^2 - 4*a*c)^(1/2) + b*c^4*d^4*e*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) + 6*b^3*c^2*d^4*e*g^5*x*(b^2 - 4*a*c)^(1/2) - 6*b^4*c*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 6*b^3*c^2*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 6*b^4*c*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 5*c^5*d^3*e^2*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 5*c^5*d^4*e*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 16*a*b*c^4*d^3*e^2*f^3*g^2 + 2*a*b^3*c^2*d*e^4*f^3*g^2 + 2*a*b^3*c^2*d^3*e^2*f*g^4 - 5*a^2*b*c^3*d*e^4*f^3*g^2 - 5*a^2*b*c^3*d^3*e^2*f*g^4 + 15*b^2*c^3*d^2*e^3*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 15*b^2*c^3*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 25*b^3*c^2*d^2*e^3*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c*d*e^4*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) - 10*a*b^3*c*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*b^2*c*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) - 10*a*b^3*c*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*b^2*c*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 5*b*c^4*d^2*e^3*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 5*b*c^4*d^4*e*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 12*b^2*c^3*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 12*b^2*c^3*d^4*e*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 8*b^4*c*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 8*b^4*c*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 60*a*b*c^4*d^2*e^3*f^3*g^2*x - 60*a*b*c^4*d^3*e^2*f^2*g^3*x - 18*a*b^2*c^3*d*e^4*f^3*g^2*x - 18*a*b^2*c^3*d^3*e^2*f*g^4*x - 38*a*b^3*c^2*d*e^4*f^2*g^3*x - 38*a*b^3*c^2*d^2*e^3*f*g^4*x + 27*a^2*b*c^3*d*e^4*f^2*g^3*x + 27*a^2*b*c^3*d^2*e^3*f*g^4*x - 36*a^2*b^2*c^2*d*e^4*f*g^4*x + 20*a*b*c^3*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 20*a*b*c^3*d^3*e^2*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d*e^4*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*d*e^4*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) + 20*a*b^2*c^2*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) + 20*a*b^2*c^2*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 41*a*b*c^4*d*e^4*f^4*g*x + 41*a*b*c^4*d^4*e*f*g^4*x + 28*a*b^4*c*d*e^4*f*g^4*x - 20*a*c^4*d^2*e^3*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 20*a*c^4*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + a^2*c^3*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + a^2*c^3*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 20*b*c^4*d^3*e^2*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 4*b^3*c^2*d*e^4*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 4*b^3*c^2*d^3*e^2*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 114*a*b^2*c^3*d^2*e^3*f^2*g^3*x - 14*a*b*c^3*d*e^4*f^4*g*(b^2 - 4*a*c)^(1/2) - 14*a*b*c^3*d^4*e*f*g^4*(b^2 - 4*a*c)^(1/2) - 14*a*b*c^3*d^4*e*g^5*x*(b^2 - 4*a*c)^(1/2) - 14*a*b*c^3*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 13*a*c^4*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 13*a*c^4*d^4*e*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 27*a*b^2*c^2*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 60*a*b*c^3*d^2*e^3*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 26*a*b^2*c^2*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 26*a*b^2*c^2*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 18*a*b^3*c*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 6*a*b*c^3*d*e^4*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b*c^3*d^3*e^2*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*c^2*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2))*(b^2*c*d*g - 4*a*c^2*d*g - 4*a*c^2*e*f - b^3*e*g + b^2*c*e*f - 2*c^2*d*f*(b^2 - 4*a*c)^(1/2) - b^2*e*g*(b^2 - 4*a*c)^(1/2) + 4*a*b*c*e*g + 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*d*g*(b^2 - 4*a*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2 - a^2*b^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^2 + a*b^3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b

^2*c*d^2*g^2 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*
c*d*e*g^2 - 4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g)) - (
log(6*a^2*c^4*d^5*g^5 + 6*a^2*c^4*e^5*f^5 - a^3*b^3*e^5*g^5 + a^3*b^2*e^5*g
^5*(b^2 - 4*a*c)^(1/2) + c^5*d^3*e^2*f^5*(b^2 - 4*a*c)^(1/2) + c^5*d^5*f^3*
g^2*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3*d^3*e^2*g^5 + b^2*c^4*d^2*e^3*f^5 - 18
*a^3*c^3*e^5*f^3*g^2 + b^2*c^4*d^5*f^2*g^3 + 4*a^4*b*c*e^5*g^5 - 4*a^4*c*e^
5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*c^3*d^5*g^5 - 2*a*b^2*c^3*e^5*f^5 + 2*a
*b^5*d^2*e^3*g^5 - 10*a*c^5*d^2*e^3*f^5 + a^2*b^4*d*e^4*g^5 + b*c^5*d^3*e^2
*f^5 - 8*a^4*c^2*d*e^4*g^5 + 2*a*b^5*e^5*f^2*g^3 - 10*a*c^5*d^5*f^2*g^3 + a
^2*b^4*e^5*f*g^4 + b*c^5*d^5*f^3*g^2 - 8*a^4*c^2*e^5*f*g^4 - a^2*b^4*e^5*g^
5*x - 8*a^4*c^2*e^5*g^5*x - 2*b^3*c^3*d^5*g^5*x - 2*b^3*c^3*e^5*f^5*x + 2*b
^6*d^2*e^3*g^5*x + 2*c^6*d^3*e^2*f^5*x + 2*b^6*e^5*f^2*g^3*x + 2*c^6*d^5*f^
3*g^2*x + 2*a*b*c^3*d^5*g^5*(b^2 - 4*a*c)^(1/2) + 2*a*b*c^3*e^5*f^5*(b^2 -
4*a*c)^(1/2) - 7*a*c^4*d*e^4*f^5*(b^2 - 4*a*c)^(1/2) - 7*a*c^4*d^5*f*g^4*(b
^2 - 4*a*c)^(1/2) - 2*c^5*d^4*e*f^4*g*(b^2 - 4*a*c)^(1/2) - 3*a*c^4*d^5*g^5
x(b^2 - 4*a*c)^(1/2) - 3*a*c^4*e^5*f^5*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c^
2*d^4*e*g^5 - 6*a*b^4*c*d^3*e^2*g^5 - 21*a^2*b*c^3*d^4*e*g^5 - 2*a^3*b^2*c*
d*e^4*g^5 + 6*a*b^3*c^2*e^5*f^4*g - 6*a*b^4*c*e^5*f^3*g^2 - 21*a^2*b*c^3*e^
5*f^4*g - 2*a^3*b^2*c*e^5*f*g^4 + 10*a*c^5*d^3*e^2*f^4*g + 10*a*c^5*d^4*e*f
^3*g^2 + 26*a^2*c^4*d*e^4*f^4*g + 26*a^2*c^4*d^4*e*f*g^4 + 6*a^3*b^2*c*e^5*
g^5*x - 3*b*c^5*d^2*e^3*f^5*x + 14*a^2*c^4*d^4*e*g^5*x + 5*b^2*c^4*d*e^4*f^
5*x + 6*b^4*c^2*d^4*e*g^5*x - 6*b^5*c*d^3*e^2*g^5*x - 3*b*c^5*d^5*f^2*g^3*x
 + 14*a^2*c^4*e^5*f^4*g*x + 5*b^2*c^4*d^5*f*g^4*x + 6*b^4*c^2*e^5*f^4*g*x -
 6*b^5*c*e^5*f^3*g^2*x - 2*a*b^4*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) - a^2*b^3*
d*e^4*g^5*(b^2 - 4*a*c)^(1/2) + b*c^4*d^2*e^3*f^5*(b^2 - 4*a*c)^(1/2) + 7*a
^2*c^3*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*e^5*f^2*g^3*(b^2 - 4*a*c)^(1
/2) - a^2*b^3*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) + b*c^4*d^5*f^2*g^3*(b^2 - 4*a*
c)^(1/2) + 7*a^2*c^3*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) + a^2*b^3*e^5*g^5*x*(b^2
 - 4*a*c)^(1/2) + 2*b^2*c^3*d^5*g^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^2*c^3*e^5*f
^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) + 5*c^5*
d^2*e^3*f^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2)
 + 5*c^5*d^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c*d^2*e^3*g^5 + 21*
a^3*b*c^2*d^2*e^3*g^5 - 13*a^2*b^3*c*e^5*f^2*g^3 + 21*a^3*b*c^2*e^5*f^2*g^3
 + 2*a^3*c^3*d*e^4*f^2*g^3 + 2*a^3*c^3*d^2*e^3*f*g^4 - b^2*c^4*d^3*e^2*f^4*
g - b^2*c^4*d^4*e*f^3*g^2 - b^3*c^3*d^2*e^3*f^4*g - b^3*c^3*d^4*e*f^2*g^3 -
 b^5*c*d^2*e^3*f^2*g^3 - 10*a^3*c^3*d^2*e^3*g^5*x - 10*a^3*c^3*e^5*f^2*g^3*
x + 3*a*b*c^4*d*e^4*f^5 - 5*a^3*c^2*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 3*a*b
*c^4*d^5*f*g^4 - 5*a^3*c^2*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 5*a*b^5*d*e^4*
f*g^4 - 2*b*c^5*d^4*e*f^4*g + 7*a*b*c^4*d^5*g^5*x + 7*a*b*c^4*e^5*f^5*x + a
*b^5*d*e^4*g^5*x - 14*a*c^5*d*e^4*f^5*x + a*b^5*e^5*f*g^4*x - 14*a*c^5*d^5*
f*g^4*x - 5*b^6*d*e^4*f*g^4*x - 4*c^6*d^4*e*f^4*g*x + 27*a^2*b^2*c^2*d^3*e^
2*g^5 + 27*a^2*b^2*c^2*e^5*f^3*g^2 - 40*a^2*c^4*d^2*e^3*f^3*g^2 - 40*a^2*c^
4*d^3*e^2*f^2*g^3 + b^3*c^3*d^3*e^2*f^3*g^2 + b^4*c^2*d^2*e^3*f^3*g^2 + b^4
*c^2*d^3*e^2*f^2*g^3 + 32*a*b^3*c^2*d^3*e^2*g^5*x - 35*a^2*b*c^3*d^3*e^2*g^
5*x + 32*a*b^3*c^2*e^5*f^3*g^2*x - 35*a^2*b*c^3*e^5*f^3*g^2*x + 48*a*c^5*d^
3*e^2*f^3*g^2*x + 14*a^2*c^4*d*e^4*f^3*g^2*x + 14*a^2*c^4*d^3*e^2*f*g^4*x +
 3*b^2*c^4*d^2*e^3*f^4*g*x + 3*b^2*c^4*d^4*e*f^2*g^3*x + 4*b^4*c^2*d*e^4*f^
3*g^2*x + 4*b^4*c^2*d^3*e^2*f*g^4*x - 13*a^2*b*c^2*d^3*e^2*g^5*(b^2 - 4*a*c
)^(1/2) + 7*a^2*b^2*c*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*e^5*f^
3*g^2*(b^2 - 4*a*c)^(1/2) + 7*a^2*b^2*c*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 2
4*a*c^4*d^3*e^2*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 7*a^2*c^3*d*e^4*f^3*g^2*(b^2
 - 4*a*c)^(1/2) + 7*a^2*c^3*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) - b^2*c^3*d^2*
e^3*f^4*g*(b^2 - 4*a*c)^(1/2) - b^2*c^3*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2) -
 b^4*c*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 9*a^2*c^3*d^3*e^2*g^5*x*(b^2 -
 4*a*c)^(1/2) + 9*a^2*c^3*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^3*
d^2*e^3*f^3*g^2 + 10*a*b^2*c^3*d^3*e^2*f^2*g^3 - 23*a*b^3*c^2*d^2*e^3*f^2*g
^3 + 96*a^2*b*c^3*d^2*e^3*f^2*g^3 - 39*a^2*b^2*c^2*d*e^4*f^2*g^3 - 39*a^2*b
^2*c^2*d^2*e^3*f*g^4 + 27*a^2*b^2*c^2*d^2*e^3*g^5*x + 27*a^2*b^2*c^2*e^5*f^
2*g^3*x - 48*a^2*c^4*d^2*e^3*f^2*g^3*x - 18*b^2*c^4*d^3*e^2*f^3*g^2*x + 17*

b^3*c^3*d^2*e^3*f^3*g^2*x + 17*b^3*c^3*d^3*e^2*f^2*g^3*x - 27*b^4*c^2*d^2*e^3*f^2*g^3*x + 4*a^3*b*c*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) + 4*a^3*b*c*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) + 5*a*b^4*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) - 4*a^3*b*c*e^5*g^5*x*(b^2 - 4*a*c)^(1/2) - a*b^4*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) - 5*b*c^4*d*e^4*f^5*x*(b^2 - 4*a*c)^(1/2) - a*b^4*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 5*b*c^4*d^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 5*b^5*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 7*a*b*c^4*d^2*e^3*f^4*g + 7*a*b*c^4*d^4*e*f^2*g^3 - 10*a*b^2*c^3*d*e^4*f^4*g - 10*a*b^2*c^3*d^4*e*f*g^4 + 10*a*b^4*c*d*e^4*f^2*g^3 + 10*a*b^4*c*d^2*e^3*f*g^4 + 19*a^2*b^3*c*d*e^4*f*g^4 + 2*a^3*b*c^2*d*e^4*f*g^4 - 24*a^2*c^3*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^3*e^2*f^3*g^2*(b^2 - 4*a*c)^(1/2) + b^3*c^2*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) + b^3*c^2*d^3*e^2*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 26*a*b^2*c^3*d^4*e*g^5*x - 14*a*b^4*c*d^2*e^3*g^5*x - 5*a^2*b^3*c*d*e^4*g^5*x + 4*a^3*b*c^2*d*e^4*g^5*x - 26*a*b^2*c^3*e^5*f^4*g*x - 14*a*b^4*c*e^5*f^2*g^3*x - 5*a^2*b^3*c*e^5*f*g^4*x + 4*a^3*b*c^2*e^5*f*g^4*x - 6*a*c^5*d^2*e^3*f^4*g*x - 6*a*c^5*d^4*e*f^2*g^3*x + 12*a^3*c^3*d*e^4*f*g^4*x + 3*b*c^5*d^3*e^2*f^4*g*x + 3*b*c^5*d^4*e*f^3*g^2*x - 12*b^3*c^3*d*e^4*f^4*g*x - 12*b^3*c^3*d^4*e*f*g^4*x + 8*b^5*c*d*e^4*f^2*g^3*x + 8*b^5*c*d^2*e^3*f*g^4*x - 6*a*b^2*c^2*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c*d^3*e^2*g^5*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c*e^5*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 3*a*c^4*d^2*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) - 3*a*c^4*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 6*a^3*c^2*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) - b*c^4*d^3*e^2*f^4*g*(b^2 - 4*a*c)^(1/2) - b*c^4*d^4*e*f^3*g^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) - 6*b^3*c^2*d^4*e*g^5*x*(b^2 - 4*a*c)^(1/2) + 6*b^4*c*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 6*b^3*c^2*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 6*b^4*c*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 5*c^5*d^3*e^2*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 5*c^5*d^4*e*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 16*a*b*c^4*d^3*e^2*f^3*g^2 + 2*a*b^3*c^2*d*e^4*f^3*g^2 + 2*a*b^3*c^2*d^3*e^2*f*g^4 - 5*a^2*b*c^3*d*e^4*f^3*g^2 - 5*a^2*b*c^3*d^3*e^2*f*g^4 - 15*b^2*c^3*d^2*e^3*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 15*b^2*c^3*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 25*b^3*c^2*d^2*e^3*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c*d*e^4*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) - 17*a^2*b^2*c*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*b^2*c*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^3*c*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*b^2*c*e^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 5*b*c^4*d^2*e^3*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 5*b*c^4*d^4*e*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 12*b^2*c^3*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) + 12*b^2*c^3*d^4*e*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 8*b^4*c*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 8*b^4*c*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 60*a*b*c^4*d^2*e^3*f^3*g^2*x - 60*a*b*c^4*d^3*e^2*f^2*g^3*x - 18*a*b^2*c^3*d*e^4*f^3*g^2*x - 18*a*b^2*c^3*d^3*e^2*f*g^4*x - 38*a*b^3*c^2*d*e^4*f^2*g^3*x - 38*a*b^3*c^2*d^2*e^3*f*g^4*x + 27*a^2*b*c^3*d*e^4*f^2*g^3*x + 27*a^2*b*c^3*d^2*e^3*f*g^4*x - 36*a^2*b^2*c^2*d*e^4*f*g^4*x - 20*a*b*c^3*d^2*e^3*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 20*a*b*c^3*d^3*e^2*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*d*e^4*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*d*e^4*f^2*g^3*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*d^2*e^3*f*g^4*(b^2 - 4*a*c)^(1/2) - 20*a*b^2*c^2*d^3*e^2*g^5*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) - 20*a*b^2*c^2*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 41*a*b*c^4*d*e^4*f^4*g*x + 41*a*b*c^4*d^4*e*f*g^4*x + 28*a*b^4*c*d*e^4*f*g^4*x + 20*a*c^4*d^2*e^3*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 20*a*c^4*d^3*e^2*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - a^2*c^3*d*e^4*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - a^2*c^3*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 20*b*c^4*d^3*e^2*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 4*b^3*c^2*d*e^4*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) - 4*b^3*c^2*d^3*e^2*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 114*a*b^2*c^3*d^2*e^3*f^2*g^3*x + 14*a*b*c^3*d*e^4*f^4*g*(b^2 - 4*a*c)^(1/2) + 14*a*b*c^3*d^4*e*f*g^4*(b^2 - 4*a*c)^(1/2) + 14*a*b*c^3*d^4*e*g^5*x*(b^2 - 4*a*c)^(1/2) + 14*a*b*c^3*e^5*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 13*a*c^4*d*e^4*f^4*g*x*(b^2 - 4*a*c)^(1/2) - 13*a*c^4*d^4*e*f*g^4*x*(b^2 -

```
 4*a*c)^(1/2) + 27*a*b^2*c^2*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 60*a*b*c
^3*d^2*e^3*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) + 26*a*b^2*c^2*d*e^4*f^2*g^3*x*(b^
2 - 4*a*c)^(1/2) + 26*a*b^2*c^2*d^2*e^3*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 18*a*
b^3*c*d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 6*a*b*c^3*d*e^4*f^3*g^2*x*(b^2 -
4*a*c)^(1/2) + 6*a*b*c^3*d^3*e^2*f*g^4*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*b*c^2*
d*e^4*f*g^4*x*(b^2 - 4*a*c)^(1/2))*(b^3*e*g + 4*a*c^2*d*g + 4*a*c^2*e*f - b
^2*c*d*g - b^2*c*e*f - 2*c^2*d*f*(b^2 - 4*a*c)^(1/2) - b^2*e*g*(b^2 - 4*a*c
)^(1/2) - 4*a*b*c*e*g + 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*d*g*(b^2 - 4*a*
c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*
g^2 - a^2*b^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2
*f^2 + a*b^3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^
2*c*d^2*g^2 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c
*d*e*g^2 - 4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g)) + (e
^2*log(d + e*x))/(a*e^3*f - c*d^3*g - a*d*e^2*g - b*d*e^2*f + b*d^2*e*g + c
*d^2*e*f) + (g^2*log(f + g*x))/(a*d*g^3 - c*e*f^3 - a*e*f*g^2 - b*d*f*g^2 +
 b*e*f^2*g + c*d*f^2*g)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.561 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=644

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ceg\left(a^2e^2g^2+abeg(dg+ef)-b^2(dg+ef)^2\right)+b^2e^2g^2(-2aeg+bdg+bef)-c^2\left(4ade^2fg^2\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-\right.}$$

Rubi [A] time = 2.05, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {893, 638, 618, 206, 634, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ceg\left(a^2e^2g^2+abeg(dg+ef)-b^2(dg+ef)^2\right)+b^2e^2g^2(-2aeg+bdg+bef)-c^2\left(4ade^2fg^2-b\left(5d^2efg^2+d^3g^3+5de^2f^2g+e^3f^3\right)\right)-2c^3df\left(d^2g^2+defg+e^2f^2\right)\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-ag)\right)^2}-\frac{cx\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)+bc(cdf-3aeg)+2ac^2(dg+ef)-b^2c(dg+ef)+b^3eg}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-\frac{\log\left(a+bx+cx^2\right)(-beg+cdg+cef)\left(eg(2aeg-b(dg+ef))+c\left(d^2g^2+e^2f^2\right)\right)}{2\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-ag)\right)^2}+\frac{e^4\log(d+ex)}{(ef-dg)\left(ae^2-bde+cd^2\right)^2}-\frac{g^4\log(f+gx)}{(ef-dg)\left(ag^2-bfg+cf^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((b^3*e*g - b^2*c*(e*f + d*g) + 2*a*c^2*(e*f + d*g) + b*c*(c*d*f - 3*a*e*g
) + c*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x)/((b^2 - 4*a*c)
*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a*g))*(a + b*x + c*x^2))) + (2*c
*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g))) + ((b^2*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*c^3*d*f*(e^2*f^2
 + d*e*f*g + d^2*g^2) + 2*c*e*g*(a^2*e^2*g^2 + a*b*e*g*(e*f + d*g) - b^2*(e
*f + d*g)^2) - c^2*(4*a*d*e^2*f*g^2 - b*(e^3*f^3 + 5*d*e^2*f^2*g + 5*d^2*e*
f*g^2 + d^3*g^3)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2) + (e^4*Log[d + e*x]
)/((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)
*(c*f^2 - b*f*g + a*g^2)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^
2) + e*g*(2*a*e*g - b*(e*f + d*g)))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e
 + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \int\Bigg(-\frac{e^5}{\left(cd^2-bde+ae^2\right)^2(-ef+dg)(d+ex)} - \frac{g^5}{(ef-dg)\left(cf^2-bfg+ag\right.}$$

$$= \frac{e^4\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2(ef-dg)} - \frac{g^4\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)^2} + \frac{\int \frac{-b^2e^2g^2}{}}{}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c\left(2c^2df+b^2\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+\right.}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c\left(2c^2df+b^2\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+\right.}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c\left(2c^2df+b^2\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+\right.}$$

Mathematica [A] time = 2.63, size = 710, normalized size = 1.10

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-(b^3*e*g) + b^2*c*(d*g + e*(f - g*x)) - 2*c^2*(a*d*g + c*d*f*x + a*e*(f -
 g*x)) + b*c*(3*a*e*g + c*(-(d*f) + e*f*x + d*g*x)))/((b^2 - 4*a*c)*(-(c*d^
2) + e*(b*d - a*e))*(-(c*f^2) + g*(b*f - a*g))*(a + x*(b + c*x))) + ((4*c^5
*d^3*f^3 + b^4*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*b^2*c*e*g*(-6*a^2*e^2*
g^2 + 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) + 2*c^4*d*
f*(-3*b*d*f*(e*f + d*g) + 2*a*(3*e^2*f^2 + d*e*f*g + 3*d^2*g^2)) + c^2*(-12
*a^3*e^3*g^3 - 6*a^2*b*e^2*g^2*(e*f + d*g) + 12*a*b^2*e*g*(e^2*f^2 + d*e*f*
g + d^2*g^2) + b^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3)) - 2*c^3
*(-4*b^2*d^2*e*f^2*g + 2*a^2*e*g*(e^2*f^2 - 5*d*e*f*g + d^2*g^2) + a*b*(3*e
^3*f^3 + 11*d*e^2*f^2*g + 11*d^2*e*f*g^2 + 3*d^3*g^3)))*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2*(c*
f^2 + g*(-(b*f) + a*g))^2) + (e^4*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))
^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g)
```

```
)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(
e*f + d*g)))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^2*(c*f^2 +
 g*(-(b*f) + a*g))^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
[Out] IntegrateAlgebraic[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [B] time = 0.28, size = 3315, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
[Out] g^5*log(abs(g*x + f))/(c^2*d*f^4*g^2 - 2*b*c*d*f^3*g^3 + b^2*d*f^2*g^4 + 2*
a*c*d*f^2*g^4 - 2*a*b*d*f*g^5 + a^2*d*g^6 - c^2*f^5*g*e + 2*b*c*f^4*g^2*e -
 b^2*f^3*g^3*e - 2*a*c*f^3*g^3*e + 2*a*b*f^2*g^4*e - a^2*f*g^5*e) - 1/2*(c^
2*d^3*g^3 + c^2*d^2*f*g^2*e - 2*b*c*d^2*g^3*e + c^2*d*f^2*g*e^2 - 2*b*c*d*f
*g^2*e^2 + b^2*d*g^3*e^2 + 2*a*c*d*g^3*e^2 + c^2*f^3*e^3 - 2*b*c*f^2*g*e^3
+ b^2*f*g^2*e^3 + 2*a*c*f*g^2*e^3 - 2*a*b*g^3*e^3)*log(c*x^2 + b*x + a)/(c^
4*d^4*f^4 - 2*b*c^3*d^4*f^3*g + b^2*c^2*d^4*f^2*g^2 + 2*a*c^3*d^4*f^2*g^2 -
 2*a*b*c^2*d^4*f*g^3 + a^2*c^2*d^4*g^4 - 2*b*c^3*d^3*f^4*e + 4*b^2*c^2*d^3*
f^3*g*e - 2*b^3*c*d^3*f^2*g^2*e - 4*a*b*c^2*d^3*f^2*g^2*e + 4*a*b^2*c*d^3*f
*g^3*e - 2*a^2*b*c*d^3*g^4*e + b^2*c^2*d^2*f^4*e^2 + 2*a*c^3*d^2*f^4*e^2 -
2*b^3*c*d^2*f^3*g*e^2 - 4*a*b*c^2*d^2*f^3*g*e^2 + b^4*d^2*f^2*g^2*e^2 + 4*a
*b^2*c*d^2*f^2*g^2*e^2 + 4*a^2*c^2*d^2*f^2*g^2*e^2 - 2*a*b^3*d^2*f*g^3*e^2
- 4*a^2*b*c*d^2*f*g^3*e^2 + a^2*b^2*d^2*g^4*e^2 + 2*a^3*c*d^2*g^4*e^2 - 2*a
*b*c^2*d*f^4*e^3 + 4*a*b^2*c*d*f^3*g*e^3 - 2*a*b^3*d*f^2*g^2*e^3 - 4*a^2*b*
c*d*f^2*g^2*e^3 + 4*a^2*b^2*d*f*g^3*e^3 - 2*a^3*b*d*g^4*e^3 + a^2*c^2*f^4*e
^4 - 2*a^2*b*c*f^3*g*e^4 + a^2*b^2*f^2*g^2*e^4 + 2*a^3*c*f^2*g^2*e^4 - 2*a^
3*b*f*g^3*e^4 + a^4*g^4*e^4) - e^5*log(abs(x*e + d))/(c^2*d^5*g*e - c^2*d^4
*f*e^2 - 2*b*c*d^4*g*e^2 + 2*b*c*d^3*f*e^3 + b^2*d^3*g*e^3 + 2*a*c*d^3*g*e^
3 - b^2*d^2*f*e^4 - 2*a*c*d^2*f*e^4 - 2*a*b*d^2*g*e^4 + 2*a*b*d*f*e^5 + a^2
*d*g*e^5 - a^2*f*e^6) - (4*c^5*d^3*f^3 - 6*b*c^4*d^3*f^2*g + 12*a*c^4*d^3*f
*g^2 + b^3*c^2*d^3*g^3 - 6*a*b*c^3*d^3*g^3 - 6*b*c^4*d^2*f^3*e + 8*b^2*c^3*
d^2*f^2*g*e + 4*a*c^4*d^2*f^2*g*e + b^3*c^2*d^2*f*g^2*e - 22*a*b*c^3*d^2*f*
g^2*e - 2*b^4*c*d^2*g^3*e + 12*a*b^2*c^2*d^2*g^3*e - 4*a^2*c^3*d^2*g^3*e +
12*a*c^4*d*f^3*e^2 + b^3*c^2*d*f^2*g*e^2 - 22*a*b*c^3*d*f^2*g*e^2 - 2*b^4*c
*d*f*g^2*e^2 + 12*a*b^2*c^2*d*f*g^2*e^2 + 20*a^2*c^3*d*f*g^2*e^2 + b^5*d*g^
3*e^2 - 4*a*b^3*c*d*g^3*e^2 - 6*a^2*b*c^2*d*g^3*e^2 + b^3*c^2*f^3*e^3 - 6*a
*b*c^3*f^3*e^3 - 2*b^4*c*f^2*g*e^3 + 12*a*b^2*c^2*f^2*g*e^3 - 4*a^2*c^3*f^2
*g*e^3 + b^5*f*g^2*e^3 - 4*a*b^3*c*f*g^2*e^3 - 6*a^2*b*c^2*f*g^2*e^3 - 2*a*
b^4*g^3*e^3 + 12*a^2*b^2*c*g^3*e^3 - 12*a^3*c^2*g^3*e^3)*arctan((2*c*x + b)
/sqrt(-b^2 + 4*a*c))/((b^2*c^4*d^4*f^4 - 4*a*c^5*d^4*f^4 - 2*b^3*c^3*d^4*f^
```

```
3*g + 8*a*b*c^4*d^4*f^3*g + b^4*c^2*d^4*f^2*g^2 - 2*a*b^2*c^3*d^4*f^2*g^2 -
 8*a^2*c^4*d^4*f^2*g^2 - 2*a*b^3*c^2*d^4*f*g^3 + 8*a^2*b*c^3*d^4*f*g^3 + a^
2*b^2*c^2*d^4*g^4 - 4*a^3*c^3*d^4*g^4 - 2*b^3*c^3*d^3*f^4*e + 8*a*b*c^4*d^3
*f^4*e + 4*b^4*c^2*d^3*f^3*g*e - 16*a*b^2*c^3*d^3*f^3*g*e - 2*b^5*c*d^3*f^2
*g^2*e + 4*a*b^3*c^2*d^3*f^2*g^2*e + 16*a^2*b*c^3*d^3*f^2*g^2*e + 4*a*b^4*c
*d^3*f*g^3*e - 16*a^2*b^2*c^2*d^3*f*g^3*e - 2*a^2*b^3*c*d^3*g^4*e + 8*a^3*b
*c^2*d^3*g^4*e + b^4*c^2*d^2*f^4*e^2 - 2*a*b^2*c^3*d^2*f^4*e^2 - 8*a^2*c^4*
d^2*f^4*e^2 - 2*b^5*c*d^2*f^3*g*e^2 + 4*a*b^3*c^2*d^2*f^3*g*e^2 + 16*a^2*b*
c^3*d^2*f^3*g*e^2 + b^6*d^2*f^2*g^2*e^2 - 12*a^2*b^2*c^2*d^2*f^2*g^2*e^2 -
16*a^3*c^3*d^2*f^2*g^2*e^2 - 2*a*b^5*d^2*f*g^3*e^2 + 4*a^2*b^3*c*d^2*f*g^3*
e^2 + 16*a^3*b*c^2*d^2*f*g^3*e^2 + a^2*b^4*d^2*g^4*e^2 - 2*a^3*b^2*c*d^2*g^
4*e^2 - 8*a^4*c^2*d^2*g^4*e^2 - 2*a*b^3*c^2*d*f^4*e^3 + 8*a^2*b*c^3*d*f^4*e
^3 + 4*a*b^4*c*d*f^3*g*e^3 - 16*a^2*b^2*c^2*d*f^3*g*e^3 - 2*a*b^5*d*f^2*g^2
*e^3 + 4*a^2*b^3*c*d*f^2*g^2*e^3 + 16*a^3*b*c^2*d*f^2*g^2*e^3 + 4*a^2*b^4*d
*f*g^3*e^3 - 16*a^3*b^2*c*d*f*g^3*e^3 - 2*a^3*b^3*d*g^4*e^3 + 8*a^4*b*c*d*g
^4*e^3 + a^2*b^2*c^2*f^4*e^4 - 4*a^3*c^3*f^4*e^4 - 2*a^2*b^3*c*f^3*g*e^4 +
8*a^3*b*c^2*f^3*g*e^4 + a^2*b^4*f^2*g^2*e^4 - 2*a^3*b^2*c*f^2*g^2*e^4 - 8*a
^4*c^2*f^2*g^2*e^4 - 2*a^3*b^3*f*g^3*e^4 + 8*a^4*b*c*f*g^3*e^4 + a^4*b^2*g^
4*e^4 - 4*a^5*c*g^4*e^4)*sqrt(-b^2 + 4*a*c)) - (b*c^4*d^3*f^3 - 2*b^2*c^3*d
^3*f^2*g + 2*a*c^4*d^3*f^2*g + b^3*c^2*d^3*f*g^2 - a*b*c^3*d^3*f*g^2 - a*b^
2*c^2*d^3*g^3 + 2*a^2*c^3*d^3*g^3 - 2*b^2*c^3*d^2*f^3*e + 2*a*c^4*d^2*f^3*e
 + 4*b^3*c^2*d^2*f^2*g*e - 7*a*b*c^3*d^2*f^2*g*e - 2*b^4*c*d^2*f*g^2*e + 3*
a*b^2*c^2*d^2*f*g^2*e + 2*a^2*c^3*d^2*f*g^2*e + 2*a*b^3*c*d^2*g^3*e - 5*a^2
*b*c^2*d^2*g^3*e + b^3*c^2*d*f^3*e^2 - a*b*c^3*d*f^3*e^2 - 2*b^4*c*d*f^2*g*
e^2 + 3*a*b^2*c^2*d*f^2*g*e^2 + 2*a^2*c^3*d*f^2*g*e^2 + b^5*d*f*g^2*e^2 - a
*b^3*c*d*f*g^2*e^2 - 3*a^2*b*c^2*d*f*g^2*e^2 - a*b^4*d*g^3*e^2 + 2*a^2*b^2*
c*d*g^3*e^2 + 2*a^3*c^2*d*g^3*e^2 - a*b^2*c^2*f^3*e^3 + 2*a^2*c^3*f^3*e^3 +
 2*a*b^3*c*f^2*g*e^3 - 5*a^2*b*c^2*f^2*g*e^3 - a*b^4*f*g^2*e^3 + 2*a^2*b^2*
c*f*g^2*e^3 + 2*a^3*c^2*f*g^2*e^3 + a^2*b^3*g^3*e^3 - 3*a^3*b*c*g^3*e^3 + (
2*c^5*d^3*f^3 - 3*b*c^4*d^3*f^2*g + b^2*c^3*d^3*f*g^2 + 2*a*c^4*d^3*f*g^2 -
 a*b*c^3*d^3*g^3 - 3*b*c^4*d^2*f^3*e + 5*b^2*c^3*d^2*f^2*g*e - 2*a*c^4*d^2*
f^2*g*e - 2*b^3*c^2*d^2*f*g^2*e - a*b*c^3*d^2*f*g^2*e + 2*a*b^2*c^2*d^2*g^3
*e - 2*a^2*c^3*d^2*g^3*e + b^2*c^3*d*f^3*e^2 + 2*a*c^4*d*f^3*e^2 - 2*b^3*c^
2*d*f^2*g*e^2 - a*b*c^3*d*f^2*g*e^2 + b^4*c*d*f*g^2*e^2 + 2*a^2*c^3*d*f*g^2
*e^2 - a*b^3*c*d*g^3*e^2 + a^2*b*c^2*d*g^3*e^2 - a*b*c^3*f^3*e^3 + 2*a*b^2*
c^2*f^2*g*e^3 - 2*a^2*c^3*f^2*g*e^3 - a*b^3*c*f*g^2*e^3 + a^2*b*c^2*f*g^2*e
^3 + a^2*b^2*c*g^3*e^3 - 2*a^3*c^2*g^3*e^3)*x)/((c*d^2 - b*d*e + a*e^2)^2*(
c*f^2 - b*f*g + a*g^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

maple [B] time = 0.05, size = 9103, normalized size = 14.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 32.63, size = 130035, normalized size = 201.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^2),x)
```

```
[Out] ((b^3*e*g + 2*a*c^2*d*g + 2*a*c^2*e*f + b*c^2*d*f - b^2*c*d*g - b^2*c*e*f -
 3*a*b*c*e*g)/(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2 - a^2*b^2*e^2*g^2 + 4*a^2*
c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^2 + a*b^3*d*e*g^2 + b^3*c*d
*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^2*c*d^2*g^2 - a*b^2*c*e^2*f^2
- b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c*d*e*g^2 - 4*a*b*c^2*d^2*f*g -
 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g) - (x*(2*a*c^2*e*g - 2*c^3*d*f + b*c
^2*d*g + b*c^2*e*f - b^2*c*e*g))/(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2 - a^2*b
^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^2 + a*b^
3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^2*c*d^2*g^2
 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c*d*e*g^2 -
4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g))/(a + b*x + c*x^
2) + symsum(log((12*a^2*c^5*e^6*g^6 - 3*b^2*c^5*d^2*e^4*g^6 - 3*b^2*c^5*e^6
*f^2*g^4 + 4*c^7*d^2*e^4*f^2*g^4 - 2*a*b^2*c^4*e^6*g^6 + 16*a*c^6*d^2*e^4*g
^6 + 3*b^3*c^4*d*e^5*g^6 + 16*a*c^6*e^6*f^2*g^4 + 3*b^3*c^4*e^6*f*g^5 - 4*b
*c^6*d*e^5*f^2*g^4 - 4*b*c^6*d^2*e^4*f*g^5 - 16*a*b*c^5*d*e^5*g^6 - 16*a*b*
c^5*e^6*f*g^5 + 16*a*c^6*d*e^5*f*g^5)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4
 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e
^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4
- 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32
*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c
^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2
*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4
- 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^
3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b
*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^
3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c
^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*
c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^
2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e
^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^
2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2
*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3
*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^
4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*
a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f
^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a
*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f
^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5
*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g
- 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*
d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*
a^4*b^2*c^2*d*e^3*f*g^3) - root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6
*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^
5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f
^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*
z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 5
12*a^4*b*c^9*d^9*e*f^6*g^4*z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10
*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^1
0*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*
e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g
^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4
+ 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a
^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^
```

5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9
*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*
z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*
a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8
*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f
^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*
z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16
*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*
c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3
*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*
z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*
b^9*c^4*d^4*e^6*f^9*g*z^4 - 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5
*b^2*c^7*d^6*e^4*f^6*g^4*z^4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*
a^6*b^2*c^6*d^4*e^6*f^6*g^4*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 56
32*a^6*b^2*c^6*d^7*e^3*f^3*g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 +
5232*a^5*b^4*c^5*d^6*e^4*f^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^
4 + 4808*a^4*b^6*c^4*d^5*e^5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7
*z^4 - 4288*a^6*b^4*c^4*d^3*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*
g^5*z^4 - 4288*a^4*b^4*c^6*d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f
^4*g^6*z^4 + 3968*a^6*b^3*c^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^
4*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b^2*c^5*d^5
*e^5*f^3*g^7*z^4 + 3840*a^7*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*
d^7*e^3*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776*a^6*b^4*c
^4*d^4*e^6*f^4*g^6*z^4 + 3776*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3456*a^6*b^
2*c^6*d^5*e^5*f^5*g^5*z^4 + 3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4 + 3440*a^6
*b^4*c^4*d^2*e^8*f^6*g^4*z^4 + 3440*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z^4 + 3440*
a^4*b^4*c^6*d^4*e^6*f^8*g^2*z^4 - 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^8*z^4 - 33
60*a^8*b^2*c^4*d^2*e^8*f^4*g^6*z^4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6*g^4*z^4 -
3360*a^4*b^2*c^8*d^6*e^4*f^8*g^2*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*f^3*g^7*z^
4 - 2944*a^3*b^4*c^7*d^7*e^3*f^7*g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e^5*f^3*g^7
*z^4 + 2512*a^5*b^6*c^3*d^3*e^7*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^7*e^3*f^5*
g^5*z^4 + 2512*a^3*b^6*c^5*d^5*e^5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3*d^4*e^6*f
^2*g^8*z^4 + 2312*a^7*b^4*c^3*d^2*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*c^7*d^8*e^
2*f^6*g^4*z^4 + 2312*a^3*b^4*c^7*d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b^6*c^2*d^3
*e^7*f^3*g^7*z^4 + 1952*a^2*b^6*c^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^5*b^4*c^5*
d^7*e^3*f^3*g^7*z^4 - 1920*a^5*b^4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828*a^5*b^6*c
^3*d^6*e^4*f^2*g^8*z^4 - 1828*a^5*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1828*a^3*b^
6*c^5*d^8*e^2*f^4*g^6*z^4 - 1828*a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4 + 1740*a^5
*b^4*c^5*d^8*e^2*f^2*g^8*z^4 + 1740*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z^4 - 1728*
a^7*b^2*c^5*d^6*e^4*f^2*g^8*z^4 - 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^4*z^4 - 17
28*a^5*b^2*c^7*d^8*e^2*f^4*g^6*z^4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8*g^2*z^4 -
1716*a^4*b^6*c^4*d^6*e^4*f^4*g^6*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*f^6*g^4*z^
4 - 1664*a^9*b^2*c^3*d^2*e^8*f^2*g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e^2*f^8*g^2
*z^4 - 1600*a^6*b^3*c^5*d^7*e^3*f^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^2*e^8*f^7*
g^3*z^4 - 1600*a^5*b^3*c^6*d^8*e^2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6*d^3*e^7*f
^8*g^2*z^4 - 1553*a^4*b^6*c^4*d^8*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*c^4*d^2*e^
8*f^8*g^2*z^4 + 1536*a^8*b^2*c^4*d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b^2*c^8*d^7
*e^3*f^7*g^3*z^4 + 1408*a^7*b^3*c^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^7*b^3*c^4*
d^3*e^7*f^4*g^6*z^4 - 1408*a^6*b^3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408*a^6*b^3*c
^5*d^3*e^7*f^6*g^4*z^4 - 1408*a^5*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1408*a^5*b^
3*c^6*d^4*e^6*f^7*g^3*z^4 + 1408*a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4 + 1408*a^4
*b^3*c^7*d^6*e^4*f^7*g^3*z^4 - 1360*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z^4 - 1360*
a^6*b^5*c^3*d^2*e^8*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^8*e^2*f^5*g^5*z^4 - 13
60*a^3*b^5*c^6*d^5*e^5*f^8*g^2*z^4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4*g^6*z^4 -
1248*a^5*b^5*c^4*d^4*e^6*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*f^5*g^5*z^
4 - 1248*a^4*b^5*c^5*d^5*e^5*f^6*g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e^7*f^2*g^8
*z^4 + 1088*a^8*b^3*c^3*d^2*e^8*f^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^8*e^2*f^7*
g^3*z^4 + 1088*a^3*b^3*c^8*d^7*e^3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2*d^2*e^8*f
^2*g^8*z^4 + 1056*a^2*b^4*c^8*d^8*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c^2*d^3*e^7

*f^2*g^8*z^4 - 912*a^7*b^5*c^2*d^2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*c^7*d^8*e^
2*f^7*g^3*z^4 - 912*a^2*b^5*c^7*d^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6*c^3*d^4*e
^6*f^4*g^6*z^4 - 848*a^3*b^6*c^5*d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^3*c^4*d^5*
e^5*f^2*g^8*z^4 + 832*a^7*b^3*c^4*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^8
*e^2*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*b^7*c^2*d^
5*e^5*f^2*g^8*z^4 + 828*a^5*b^7*c^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d
^8*e^2*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^3*b^8*c^3*
d^5*e^5*f^5*g^5*z^4 - 696*a^4*b^8*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a^4*b^8*c^2
*d^3*e^7*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*a^2*b^8*c^
4*d^5*e^5*f^7*g^3*z^4 - 694*a^6*b^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694*a^6*b^6*c
^2*d^2*e^8*f^4*g^6*z^4 - 694*a^2*b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 694*a^2*b^6*
c^6*d^6*e^4*f^8*g^2*z^4 + 692*a^4*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 692*a^4*b^7
*c^3*d^2*e^8*f^7*g^3*z^4 + 692*a^3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 + 692*a^3*b^
7*c^4*d^3*e^7*f^8*g^2*z^4 + 672*a^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 + 672*a^4*b
^6*c^4*d^3*e^7*f^7*g^3*z^4 + 600*a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4 + 600*a^2*
b^8*c^4*d^6*e^4*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4 + 544*a^3
*b^8*c^3*d^6*e^4*f^4*g^6*z^4 + 544*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^4 - 544*a^
3*b^8*c^3*d^3*e^7*f^7*g^3*z^4 - 536*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z^4 - 536*a
^4*b^7*c^3*d^4*e^6*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*z^4 - 536*
a^3*b^7*c^4*d^5*e^5*f^6*g^4*z^4 - 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7*z^4 - 504
*a^5*b^7*c^2*d^3*e^7*f^4*g^6*z^4 - 504*a^2*b^7*c^5*d^7*e^3*f^6*g^4*z^4 - 50
4*a^2*b^7*c^5*d^6*e^4*f^7*g^3*z^4 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g^8*z^4 + 4
16*a^3*b^8*c^3*d^2*e^8*f^8*g^2*z^4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*g^7*z^4 -
352*a^6*b^5*c^3*d^3*e^7*f^4*g^6*z^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6*g^4*z^4 -
352*a^3*b^5*c^6*d^6*e^4*f^7*g^3*z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^2*g^8*z^4
- 248*a^3*b^9*c^2*d^2*e^8*f^7*g^3*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f^3*g^7*z^4
- 248*a^2*b^9*c^3*d^3*e^7*f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*f^2*g^8*z^
4 + 246*a^4*b^8*c^2*d^2*e^8*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2*f^4*g^6*z
^4 + 246*a^2*b^8*c^4*d^4*e^6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^2*f^2*g^8*
z^4 + 208*a^6*b^2*c^6*d^2*e^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*e^3*f^3*g^
7*z^4 + 168*a^2*b^10*c^2*d^3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^5*e^5*f^4*
g^6*z^4 + 160*a^3*b^9*c^2*d^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^6*e^4*f^5
*g^5*z^4 + 160*a^2*b^9*c^3*d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*d^7*e^3*f^
2*g^8*z^4 + 144*a^5*b^5*c^4*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5*d^8*e^2*f
^3*g^7*z^4 + 144*a^4*b^5*c^5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c^2*d^6*e^4
*f^4*g^6*z^4 - 144*a^2*b^10*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7*c^3*d^6*e
^4*f^3*g^7*z^4 + 120*a^4*b^7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^7*c^4*d^7*
e^3*f^4*g^6*z^4 + 120*a^3*b^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^5*c^4*d^6*
e^4*f^3*g^7*z^4 + 96*a^5*b^5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5*c^5*d^7*e
^3*f^4*g^6*z^4 + 96*a^4*b^5*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*c^2*d^6*e^
4*f^3*g^7*z^4 + 64*a^3*b^9*c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c^3*d^7*e^3
*f^4*g^6*z^4 + 64*a^2*b^9*c^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c^2*d^8*e^2
*f^2*g^8*z^4 - 36*a^2*b^10*c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*c^2*d^5*e^
5*f^5*g^5*z^4 - 24*a^9*b^4*c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e*f^9*g*z^4
+ 2688*a^7*b^2*c^5*d^7*e^3*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^7*g^3*z^4
+ 2688*a^5*b^2*c^7*d^9*e*f^3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f^9*g*z^4 -
2560*a^7*b^3*c^4*d^6*e^4*f*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*g^4*z^4 -
2560*a^4*b^3*c^7*d^9*e*f^4*g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9*g*z^4 + 2
112*a^8*b^2*c^4*d^5*e^5*f*g^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^5*z^4 + 21
12*a^4*b^2*c^8*d^9*e*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g*z^4 + 166
4*a^6*b^5*c^3*d^6*e^4*f*g^9*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*z^4 + 1664
*a^3*b^5*c^6*d^9*e*f^4*g^6*z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z^4 + 1536*
a^8*b*c^5*d^4*e^6*f^3*g^7*z^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^4 + 1536*a
^7*b*c^6*d^5*e^5*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4 + 1536*a^
6*b*c^7*d^6*e^4*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4 + 1536*a^5
*b*c^8*d^7*e^3*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 - 1408*a^8*
b^3*c^3*d^4*e^6*f*g^9*z^4 - 1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 - 1408*a^3*b
^3*c^8*d^9*e*f^6*g^4*z^4 - 1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1280*a^7*b*
c^6*d^7*e^3*f^2*g^8*z^4 - 1280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 1280*a^6*b*c

^7*d^8*e^2*f^3*g^7*z^4 - 1280*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 1152*a^6*b^3*
c^5*d^8*e^2*f*g^9*z^4 - 1152*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152*a^5*b^3*c
^6*d^9*e*f^2*g^8*z^4 - 1152*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*a^5*b^5*c^
4*d^8*e^2*f*g^9*z^4 + 1056*a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a^4*b^5*c^5
*d^9*e*f^2*g^8*z^4 + 1056*a^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7*b^5*c^2*d
^4*e^6*f*g^9*z^4 + 864*a^7*b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^5*c^7*d^9*
e*f^6*g^4*z^4 + 864*a^2*b^5*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c^4*d^7*e^3
*f*g^9*z^4 - 800*a^6*b^4*c^4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*d^9*e*f^3*
g^7*z^4 - 800*a^4*b^4*c^6*d^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e^5*f^2*g^8
*z^4 - 768*a^8*b*c^5*d^2*e^8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*f^5*g^5*z^
4 - 768*a^5*b*c^8*d^5*e^5*f^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f*g^9*z^4 +
 640*a^9*b^2*c^3*d*e^9*f^3*g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^3*z^4 + 64
0*a^3*b^2*c^9*d^7*e^3*f^9*g*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z^4 + 512*a
^7*b*c^6*d^3*e^7*f^6*g^4*z^4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4 + 512*a^6*
b*c^7*d^4*e^6*f^7*g^3*z^4 - 480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 480*a*b^8*c
^5*d^7*e^3*f^7*g^3*z^4 - 400*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*a^7*b^4*c^
3*d*e^9*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d
^5*e^5*f^9*g*z^4 - 372*a^6*b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^6*c^2*d*e^
9*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^5*e^5
*f^9*g*z^4 - 328*a^5*b^6*c^3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*d*e^9*f^7*
g^3*z^4 - 328*a^3*b^6*c^5*d^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3*e^7*f^9*g
*z^4 - 288*a^8*b^4*c^2*d^3*e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*f^3*g^7*z^
4 - 288*a^5*b^7*c^2*d^6*e^4*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6*g^4*z^4 -
 288*a^2*b^7*c^5*d^9*e*f^4*g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*g*z^4 - 28
8*a^2*b^4*c^8*d^9*e*f^7*g^3*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z^4 - 280*a
^4*b^7*c^3*d^8*e^2*f*g^9*z^4 - 280*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4 - 280*a^3*
b^7*c^4*d^9*e*f^2*g^8*z^4 - 280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 256*a^9*b*c
^4*d^3*e^7*f^2*g^8*z^4 + 256*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*a^4*b*c^9*
d^8*e^2*f^7*g^3*z^4 + 256*a^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7*b^6*c*d^2
*e^8*f^2*g^8*z^4 - 248*a*b^6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^7*c*d^3*e^
7*f^2*g^8*z^4 + 236*a^6*b^7*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6*d^8*e^2*f
^7*g^3*z^4 + 236*a*b^7*c^6*d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^4*e^6*f^3*
g^7*z^4 + 200*a^4*b^9*c*d^3*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*e^6*f^4*g^
6*z^4 - 200*a*b^10*c^3*d^6*e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^3*f^6*g^4*
z^4 + 200*a*b^9*c^4*d^6*e^4*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f^2*g^8*z^4
 - 196*a^4*b^9*c*d^2*e^8*f^5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*g^5*z^4 -
196*a*b^9*c^4*d^5*e^5*f^8*g^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9*z^4 - 192
*a^9*b^3*c^2*d*e^9*f^2*g^8*z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^4 - 192*a^
2*b^3*c^9*d^8*e^2*f^9*g*z^4 + 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 + 156*a^4*b
^8*c^2*d*e^9*f^7*g^3*z^4 + 156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 156*a^2*b^8*
c^4*d^3*e^7*f^9*g*z^4 + 96*a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5*b^8*c*d^2
*e^8*f^4*g^6*z^4 + 96*a*b^8*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^5*d^6*e^4*
f^8*g^2*z^4 + 88*a^3*b^10*c*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d^3*e^7*f^5
*g^5*z^4 + 88*a*b^10*c^3*d^7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*e^5*f^7*g^
3*z^4 - 36*a^2*b^11*c*d^6*e^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7*f^6*g^4*z
^4 - 36*a*b^11*c^2*d^7*e^3*f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^7*g^3*z^4
+ 28*a^3*b^10*c*d^6*e^4*f^2*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g^4*z^4 + 2
8*a*b^10*c^3*d^8*e^2*f^4*g^6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*z^4 + 24*a
^3*b^9*c^2*d^8*e^2*f*g^9*z^4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 + 24*a^2*b^
11*c*d^7*e^3*f^2*g^8*z^4 + 24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24*a^2*b^9*c
^3*d^9*e*f^2*g^8*z^4 + 24*a^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^11*c^2*d^8
*e^2*f^3*g^7*z^4 + 24*a*b^11*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^11*c*d^5*e^
5*f^4*g^6*z^4 + 12*a^2*b^11*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2*d^6*e^4*f
^5*g^5*z^4 + 12*a*b^11*c^2*d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*e^3*f^7*g^
3*z^4 + 20*b^12*c^2*d^6*e^4*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6*g^4*z^4 -
 20*b^11*c^3*d^6*e^4*f^7*g^3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4 - 20*b^9*
c^5*d^7*e^3*f^8*g^2*z^4 + 20*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^11*c^3*d^8*
e^2*f^5*g^5*z^4 + 16*b^11*c^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^8*e^2*f^4*
g^6*z^4 - 6*b^12*c^2*d^4*e^6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6*g^4*z^4 -

5*b^10*c^4*d^6*e^4*f^8*g^2*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4 - 4*b^12*c^2*d^5*e^5*f^7*g^3*z^4 - 4608*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*a^7*c^7*d^6*e^4*f^4*g^6*z^4 + 3328*a^7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c^6*d^5*e^5*f^3*g^7*z^4 + 3072*a^8*c^6*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^3*e^7*f^5*g^5*z^4 - 3072*a^6*c^8*d^7*e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4*f^6*g^4*z^4 - 3072*a^6*c^8*d^5*e^5*f^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*g^7*z^4 - 2048*a^7*c^7*d^7*e^3*f^3*g^7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z^4 - 2048*a^5*c^9*d^7*e^3*f^7*g^3*z^4 + 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 + 1792*a^8*c^6*d^2*e^8*f^6*g^4*z^4 + 1792*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*a^6*c^8*d^4*e^6*f^8*g^2*z^4 + 1408*a^9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c^5*d^2*e^8*f^4*g^6*z^4 + 1408*a^5*c^9*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^6*e^4*f^8*g^2*z^4 + 1088*a^7*c^7*d^8*e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8*f^8*g^2*z^4 + 512*a^10*c^4*d^2*e^8*f^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*g^2*z^4 + 40*a^4*b^10*d^3*e^7*f^3*g^7*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4 - 20*a^5*b^9*d^3*e^7*f^2*g^8*z^4 - 20*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*b^11*d^4*e^6*f^3*g^7*z^4 - 20*a^3*b^11*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^4*e^6*f^4*g^6*z^4 + 16*a^3*b^11*d^5*e^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f^5*g^5*z^4 - 6*a^2*b^12*d^6*e^4*f^2*g^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^4 - 5*a^4*b^10*d^4*e^6*f^2*g^8*z^4 - 5*a^4*b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2*b^12*d^5*e^5*f^3*g^7*z^4 - 4*a^2*b^12*d^3*e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^4*e^10*f^6*g^4*z^4 - 440*a^7*b^4*c^3*e^10*f^6*g^4*z^4 + 320*a^8*b^3*c^3*e^10*f^5*g^5*z^4 + 320*a^7*b^3*c^4*e^10*f^7*g^3*z^4 - 240*a^8*b^4*c^2*e^10*f^4*g^6*z^4 - 240*a^6*b^4*c^4*e^10*f^8*g^2*z^4 + 192*a^9*b^3*c^2*e^10*f^3*g^7*z^4 + 192*a^9*b^2*c^3*e^10*f^4*g^6*z^4 + 192*a^7*b^2*c^5*e^10*f^8*g^2*z^4 + 90*a^6*b^6*c^2*e^10*f^6*g^4*z^4 + 68*a^5*b^6*c^3*e^10*f^8*g^2*z^4 - 48*a^10*b^2*c^2*e^10*f^2*g^8*z^4 + 48*a^7*b^5*c^2*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c^3*e^10*f^7*g^3*z^4 - 36*a^5*b^7*c^2*e^10*f^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*f^8*g^2*z^4 + 480*a^4*b^2*c^8*d^10*f^4*g^6*z^4 - 440*a^3*b^4*c^7*d^10*f^4*g^6*z^4 + 320*a^4*b^3*c^7*d^10*f^3*g^7*z^4 + 320*a^3*b^3*c^8*d^10*f^5*g^5*z^4 - 240*a^4*b^4*c^6*d^10*f^2*g^8*z^4 - 240*a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 192*a^5*b^2*c^7*d^10*f^2*g^8*z^4 + 192*a^3*b^2*c^9*d^10*f^6*g^4*z^4 + 192*a^2*b^3*c^9*d^10*f^7*g^3*z^4 + 90*a^2*b^6*c^6*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c^5*d^10*f^2*g^8*z^4 + 48*a^3*b^5*c^6*d^10*f^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10*f^5*g^5*z^4 - 48*a^2*b^2*c^10*d^10*f^8*g^2*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g^7*z^4 - 6*a^2*b^8*c^4*d^10*f^2*g^8*z^4 + 480*a^8*b^2*c^4*d^6*e^4*g^10*z^4 - 440*a^7*b^4*c^3*d^6*e^4*g^10*z^4 + 320*a^8*b^3*c^3*d^5*e^5*g^10*z^4 + 320*a^7*b^3*c^4*d^7*e^3*g^10*z^4 - 240*a^8*b^4*c^2*d^4*e^6*g^10*z^4 - 240*a^6*b^4*c^4*d^8*e^2*g^10*z^4 + 192*a^9*b^3*c^2*d^3*e^7*g^10*z^4 + 192*a^9*b^2*c^3*d^4*e^6*g^10*z^4 + 192*a^7*b^2*c^5*d^8*e^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6*e^4*g^10*z^4 + 68*a^5*b^6*c^3*d^8*e^2*g^10*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g^10*z^4 + 48*a^7*b^5*c^2*d^5*e^5*g^10*z^4 + 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4 - 36*a^5*b^7*c^2*d^7*e^3*g^10*z^4 - 6*a^4*b^8*c^2*d^8*e^2*g^10*z^4 + 480*a^4*b^2*c^8*d^4*e^6*f^10*z^4 - 440*a^3*b^4*c^7*d^4*e^6*f^10*z^4 + 320*a^4*b^3*c^7*d^3*e^7*f^10*z^4 + 320*a^3*b^3*c^8*d^5*e^5*f^10*z^4 - 240*a^4*b^4*c^6*d^2*e^8*f^10*z^4 - 240*a^2*b^4*c^8*d^6*e^4*f^10*z^4 + 192*a^5*b^2*c^7*d^2*e^8*f^10*z^4 + 192*a^3*b^2*c^9*d^6*e^4*f^10*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f^10*z^4 + 90*a^2*b^6*c^6*d^4*e^6*f^10*z^4 + 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4 + 48*a^3*b^5*c^6*d^3*e^7*f^10*z^4 + 48*a^2*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a^2*b^2*c^10*d^8*e^2*f^10*z^4 - 36*a^2*b^7*c^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*c^4*d^2*e^8*f^10*z^4 + 16*b^9*c^5*d^9*e*f^6*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^9*g*z^4 - 14*b^10*c^4*d^9*e*f^5*g^5*z^4 - 14*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4*b^13*c*d^7*e^3*f^4*g^6*z^4 - 4*b^13*c*d^6*e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e^5*f^6*g^4*z^4 + 4*b^13*c*d^4*e^6*f^7*g^3*z^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^4 + 4*b^11*c^3*d^4*e^6*f^9*g*z^4 - 4*b^8*c^6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*d^7*e^3*f^9*g*z^4 - 4*b^7*c^7*d^9*e*f^8*g^2*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z^4 - 768*a^9*c^5*d^5*e^5*f*g^9*z^4 - 768*a^9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^5*c^9*d^9*e*f^5*g^5*z^4 - 768*a^5*c^9*d^5*e^5*f^9*g*z^4 - 512*a^10*c^4*d^3*e^7*f*g^9*z^4 - 512*a^10*c^4*d*e^9*f^3*g^7*z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*z^4 - 512*a^8*c^6*d*e^9*f^7*g^3*z^4 - 512*a^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a^6*c^8*d^3 *e^7*f^9*g*z^4 - 512*a^4*c^10*d^9*e*f^7*g^3*z^4 - 512*a^4*c^10*d^7*e^3*f^9*g*z^4 + 16*a^5*b^9*d^4*e^6*f*g^9*z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z^4 - 14*a^4*b^10*d^5*e^5*f*g^9*z^4 - 14*a^4*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*b^7*d^2*e^8*f*g^9*z^4 - 4*a^7*b^7*d*e^9*f^2*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g^9*z^4 - 4*a^6*b^8*d*e^9*f^3*g^7*z^4 + 4*a^3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3*b^11*d*e^9*f^6*g^4*z^4 + 4*a*b^13*d^6*e^4*f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f^4*g^6*z^4 - 4*a*b^13*d^4*e^6*f^5*g^5*z^4 + 4*a*b^13*d^3*e^7*f^6*g^4*z^4 - 768*a^9*b*c^4*e^10*f^5*g^5*z^4 - 768*a^8*b*c^5*e^10*f^7*g^3*z^4 - 256*a^10*b*c^3*e^10*f^3*g^7*z^4 + 192*a^6*b^3*c^5*e^10*f^9*g*z^4 + 68*a^7*b^6*c*e^10*f^4*g^6*z^4 - 48*a^8*b^5*c*e^10*f^3*g^7*z^4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^4 - 36*a^6*b^7*c*e^10*f^5*g^5*z^4 + 12*a^9*b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b^9*c*e^10*f^7*g^3*z^4 + 4*a^4*b^7*c^3*e^10*f^9*g*z^4 - 768*a^5*b*c^8*d^10*f^3*g^7*z^4 - 768*a^4*b*c^9*d^10*f^5*g^5*z^4 - 256*a^3*b*c^10*d^10*f^7*g^3*z^4 + 192*a^5*b^3*c^6*d^10*f*g^9*z^4 + 68*a*b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^4*b^5*c^5*d^10*f*g^9*z^4 - 48*a*b^5*c^8*d^10*f^7*g^3*z^4 - 36*a*b^7*c^6*d^10*f^5*g^5*z^4 + 12*a*b^4*c^9*d^10*f^8*g^2*z^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^4 + 4*a*b^9*c^4*d^10*f^3*g^7*z^4 - 768*a^9*b*c^4*d^5*e^5*g^10*z^4 - 768*a^8*b*c^5*d^7*e^3*g^10*z^4 - 256*a^10*b*c^3*d^3*e^7*g^10*z^4 + 192*a^6*b^3*c^5*d^9*e*g^10*z^4 + 68*a^7*b^6*c*d^4*e^6*g^10*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10*z^4 - 48*a^5*b^5*c^4*d^9*e*g^10*z^4 - 36*a^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a^9*b^4*c*d^2*e^8*g^10*z^4 + 4*a^4*b^9*c*d^7*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^9*e*g^10*z^4 - 768*a^5*b*c^8*d^3*e^7*f^10*z^4 - 768*a^4*b*c^9*d^5*e^5*f^10*z^4 - 256*a^3*b*c^10*d^7*e^3*f^10*z^4 + 192*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68*a*b^6*c^7*d^6*e^4*f^10*z^4 - 48*a^4*b^5*c^5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*d^7*e^3*f^10*z^4 - 36*a*b^7*c^6*d^5*e^5*f^10*z^4 + 12*a*b^4*c^9*d^8*e^2*f^10*z^4 + 4*a^3*b^7*c^4*d*e^9*f^10*z^4 + 4*a*b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6*c^8*d^9*e*f^9*g*z^4 - 128*a^11*c^3*d*e^9*f*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g^9*z^4 - 128*a^7*c^7*d*e^9*f^9*g*z^4 - 128*a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8*b^6*d*e^9*f*g^9*z^4 - 256*a^7*b*c^6*e^10*f^9*g*z^4 - 256*a^6*b*c^7*d^10*f*g^9*z^4 - 256*a^7*b*c^6*d^9*e*g^10*z^4 - 256*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b^14*d^5*e^5*f^5*g^5*z^4 + 384*a^9*c^5*e^10*f^6*g^4*z^4 + 256*a^10*c^4*e^10*f^4*g^6*z^4 + 256*a^8*c^6*e^10*f^8*g^2*z^4 + 64*a^11*c^3*e^10*f^2*g^8*z^4 - 6*b^8*c^6*d^10*f^6*g^4*z^4 + 4*b^9*c^5*d^10*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f^7*g^3*z^4 + 384*a^5*c^9*d^10*f^4*g^6*z^4 + 256*a^6*c^8*d^10*f^2*g^8*z^4 + 256*a^4*c^10*d^10*f^6*g^4*z^4 + 64*a^3*c^11*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^10*f^4*g^6*z^4 + 4*a^7*b^7*e^10*f^3*g^7*z^4 + 4*a^5*b^9*e^10*f^5*g^5*z^4 + 384*a^9*c^5*d^6*e^4*g^10*z^4 + 256*a^10*c^4*d^4*e^6*g^10*z^4 + 256*a^8*c^6*d^8*e^2*g^10*z^4 + 64*a^11*c^3*d^2*e^8*g^10*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^4 + 4*b^9*c^5*d^5*e^5*f^10*z^4 + 4*b^7*c^7*d^7*e^3*f^10*z^4 + 384*a^5*c^9*d^4*e^6*f^10*z^4 + 256*a^6*c^8*d^2*e^8*f^10*z^4 + 256*a^4*c^10*d^6*e^4*f^10*z^4 + 64*a^3*c^11*d^8*e^2*f^10*z^4 - 6*a^6*b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7*d^3*e^7*g^10*z^4 + 4*a^5*b^9*d^5*e^5*g^10*z^4 - 48*a^6*b^2*c^6*e^10*f^10*z^4 - 48*a^6*b^2*c^6*d^10*g^10*z^4 + 12*a^5*b^4*c^5*e^10*f^10*z^4 + 12*a^5*b^4*c^5*d^10*g^10*z^4 + 64*a^7*c^7*e^10*f^10*z^4 + 64*a^7*c^7*d^10*g^10*z^4 - b^14*d^6*e^4*f^4*g^6*z^4 - b^14*d^4*e^6*f^6*g^4*z^4 - b^10*c^4*d^10*f^4*g^6*z^4 - b^6*c^8*d^10*f^8*g^2*z^4 - a^8*b^6*e^10*f^2*g^8*z^4 - a^4*b^10*e^10*f^6*g^4*z^4 - b^10*c^4*d^4*e^6*f^10*z^4 - b^6*c^8*d^8*e^2*f^10*z^4 - a^8*b^6*d^2*e^8*g^10*z^4 - a^4*b^10*d^6*e^4*g^10*z^4 - a^4*b^6*c^4*e^10*f^10*z^4 - a^4*b^6*c^4*d^10*g^10*z^4 + 272*a^5*b^2*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b^4*c^2*d*e^7*f*g^7*z^2 - 164*a^5*b*c^4*d^2*e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*e^7*f^2*g^6*z^2 + 120*a^2*b^2*c^6*d^7*e*f*g^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f^7*g*z^2 + 120*a*b^2*c^7*d^7*e*f^3*g^5*z^2 + 120*a*b^2*c^7*d^3*e^5*f^7*g*z^2 - 76*a^4*b*c^5*d^4*e^4*f*g^7*z^2 - 76*a^4*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^3*b*c^6*d^6*e^2*f*g^7*z^2 - 76*a^3*b*c^6*d*e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d^7*e*f^2*g^6*z^2 - 64*a*b^3*c^6*d^2*e^6*f^7*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*g^6*z^2 - 60*a^2*b*c^7*d^2*e^6*f^7*g*z^2 + 44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 + 44*a*b*c^8*d^5*e^3*f^6*g^2*z^2 + 22*a*b^5*c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5*c^4*d*e^7*f^6*g^2*z^2 - 20*a^2*b^7*c*d^2*e^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^7*f^2*g^6*z^2 + 8*a*b^8*c*d^2*e^6*f^2*g^6*z^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z^2 - 8*a*b

^6*c^3*d*e^7*f^5*g^3*z^2 + 2*a*b^7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^7*c^2*d*e^
7*f^4*g^4*z^2 - 590*a^2*b^2*c^6*d^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4*c^4*d^3*e
^5*f^3*g^5*z^2 - 346*a^3*b^2*c^5*d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^2*c^5*d^2*
e^6*f^4*g^4*z^2 - 274*a^4*b^2*c^4*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b^2*c^5*d^3
*e^5*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^
3*e^5*f^4*g^4*z^2 + 204*a^3*b^3*c^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3*b^3*c^4*d
^2*e^6*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^2*b^2*c^6*
d^3*e^5*f^5*g^3*z^2 + 71*a^2*b^4*c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2*b^4*c^4*d
^2*e^6*f^4*g^4*z^2 - 56*a^2*b^3*c^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*b^3*c^5*d^
2*e^6*f^5*g^3*z^2 + 18*a^2*b^2*c^6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b^2*c^6*d^2
*e^6*f^6*g^2*z^2 - 16*a^3*b^4*c^3*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^3*
e^5*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*c^2*d^2*e^
6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e*f*g^7*z^2
 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 - 4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4*a*b^8*c*d
*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^4*f^7*g*z^
2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3*g^5*z^2 +
 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^3*z^2 - 20
8*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 - 208*a^3*b^4*c^3*d*e^7*f^3*g^5*z^2 - 164*a
^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 164*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2 + 140*a^2*
b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 122*a*b^2*c
^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*a^2*b^3*c^
5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b^3*c^6*d^5
*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c^3*d^3*e^5
*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^2*e^6*f^6*
g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4*f^3*g^5*z
^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*g^3*z^2 -
54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2 - 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z^2 - 52*a*
b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 + 48*a^3*b^5
*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^2*b^6*c^2*
d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3*c^3*d^2*e
^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^6*e^2*f^3*
g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2*f^3*g^5*z
^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*g^5*z^2 -
32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2 - 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z^2 - 20*a*
b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 + 20*a*b^4*c
^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*b^5*c^4*d^
2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c^3*d*e^7*f
^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^7*f^5*g^3*
z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g^4*z^2 - 5
0*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 32*b^4*c^6*d^5*e^3*f^5*g^3*z^2 + 24*b^3*c^7
*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^6*d^6*e^2*
f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2*f^2*g^6*z
^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z^2 + 8*b^6
*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*c^5*d^4*e^
4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5*f^3*g^5*z
^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^2 - 2*b^7*
c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c^5*d^6*e^2
*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^6*f^2*g^6*
z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g^6*z^2 + 3
76*a^4*c^6*d^2*e^6*f^4*g^4*z^2 + 320*a^3*c^7*d^4*e^4*f^4*g^4*z^2 - 280*a^3*
c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a^2*c^8*d^5
*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*d^2*e^6*f^
6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4*f^6*g^2*z
^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^4*z^2 - 31
0*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 232*a^5*b^2*c^3*e^8*f^2*g^6*z^2 + 229*a^2*b
^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^4*b^3*c^3*
e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^2*e^8*f^4*
g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5*g^3*z^2 -
 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2 + 436*a^3*b^4*c^3*d^4*e^4*g^8*z^2 - 310*a^

3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 - 4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2 - 168*a^5*c^5*d*e^7*f^3*g^5*z^2 - 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^2 + 548*a^5*b*c^4*e^8*f^3*g^5*z^2 - 182*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*z^2 - 68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 58*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2 + 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*z^2 - 24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d*e^7*f*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 204*a^3*b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*f^7*g*z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2 - 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^2*b*c^7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8*z^2 + 128*a^6*b*c^3*d*e^7*g^8*z^2 + 48*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^5*c*d*e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2 + 6*a*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^6*f^4*g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^9*d^8*f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*b^7*c^3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 - 12*b*c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*z^2 - 2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8*f^8*z^2 - 12*a*b^4*c^5*d^8*g^8*z^2 - 8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^8*f^6*g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 - 92*a^6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2*g^6*z^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 + 36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c^2*d^6*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^2 - 188*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^4*e^4*f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*a^2*c^8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^8*z^2 + 45*a^2*b^2*c^6*e^8*f^8*z^2 + 45*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^8*f^6*g^2*z^2 + b^10*e^8*f^4*g^4*z^2 + 4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^4*g^8*z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*z^2 - 48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*c^7*d^8*g^8*z^2 + a^4*b^6*e^8*g^8*z^2 - b^10*d^2*e^6*f^2*g^6*z^2 + 108*a^2*b^2*c^4*d^2*e^5*f*g^6*z + 108*a^2*b^2*c^4*d*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^3*e^4*f^2*g^5*z + 60*a*b^2*c^5*d^2*e^5*f^3*g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2*g^5*z - 44*a*b^3*c^4*d^2*e^5*f^2*g^5*z - 120*a^2*b*c^5*d^3*e^4*f*g^6*z - 120*a^2*b*c^5*d*e^6*f^3*g^4*z - 96*a*b*c^6*d^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^3*d*e^6*f*g^6*z + 32*a*b^3*c^4*d^3*e^4*f*g^6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4*z - 28*a*b^4*c^3*d^2*e^5*f*g^6*z - 28*a*b^4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2*c^5*d^4*e^3*f*g^6*z - 18*a*b^2*c^5*d*e^6*f^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2*g^5*z + 4*a*b*c^6*d^2*e^5*f^4*g^3*z + 24*a*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*b*c^4*d*e^6*f*g^6*z - 8*a*b*c^6*d^5*e^2*f*g^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z - 13*b^2*c^6*d^6*e*f*g^6*z - 13*b^2*c^6*d*e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*g^5*z + 8*b*c^7*d^2*e^5*f^6*g*z + 9*b^2*c^6*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d^3*e^4*f^4*g^3*z + 8*b^5*c^3*d^2*e^5*f^2*g^5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*z - 6*b^4*c^4*d^2*e^5*f^3*g^4*z - 6*b^3*c^5*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d^2*e^5*f^4

*g^3*z + 4*b^3*c^5*d^3*e^4*f^3*g^4*z + b^2*c^6*d^5*e^2*f^2*g^5*z + b^2*c^6*
d^2*e^5*f^5*g^2*z + 16*a^2*c^6*d^3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2*e^5*f^3*g
^4*z - 112*a^2*b^3*c^3*e^7*f^2*g^5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4*z - 112*a
^2*b^3*c^3*d^2*e^5*g^7*z - 12*a^2*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c*d*e^6*f*g
^6*z + 8*a*c^7*d^6*e*f*g^6*z + 8*a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6*e^7*f^6*g
*z - 10*a*b^6*c*e^7*f*g^6*z + 52*a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c*d*e^6*g^7
*z + 14*b^3*c^5*d^5*e^2*f*g^6*z + 14*b^3*c^5*d*e^6*f^5*g^2*z - 12*b*c^7*d^5
*e^2*f^3*g^4*z - 12*b*c^7*d^3*e^4*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f*g^6*z - 5
*b^4*c^4*d*e^6*f^4*g^3*z + b^6*c^2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^6*f^2*g^5*
z + 52*a^2*c^6*d^4*e^3*f*g^6*z + 52*a^2*c^6*d*e^6*f^4*g^3*z + 24*a*c^7*d^4*
e^3*f^3*g^4*z + 24*a*c^7*d^3*e^4*f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2*g^5*z - 1
6*a*c^7*d^2*e^5*f^5*g^2*z + 8*a^3*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5*d*e^6*f^2
*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^5*z + 144*a^2*b*c^5*e^7*f^4*g^3*z - 42*a*b
^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^2*e^7*f*g^
6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z - 10*a*b^3*c^4*e^7*f^4*g^3*z + 4*a*b^4*c^3
*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^2*e^5*g^7*z + 144*a^2*b*c^5*d^4*e^3*g^7*z
- 42*a*b^2*c^5*d^5*e^2*g^7*z + 32*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^2*b^4*c^2*
d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g^7*z + 4*a
*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*z + 11*b^4
*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^4*z - 136*
a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5*e^2*g^7*z
 - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*c^2*d^3*e^4*g^7*z - 136*a^3*c^5*d^3*e^4*g^
7*z - 68*a^2*c^6*d^5*e^2*g^7*z - 96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8*d^6*e*f^3
*g^4*z + 4*c^8*d^3*e^4*f^6*g*z - 10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c*e^7*f^2*g
^5*z - 128*a^4*c^4*e^7*f*g^6*z - 10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c*d^2*e^5*g
^7*z - 128*a^4*c^4*d*e^6*g^7*z + 128*a^4*b*c^3*e^7*g^7*z + 24*a^2*b^5*c*e^7
*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^7*f^7*z +
3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g^6*z + b^8*d*e^6*g^7*z - 16*a*c^7*e^7*f^7*
z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5 + 20*a*b*c
^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b*c^5*d*e^5
*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5 - 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5*e^6*f^2*g
^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6*g^6 - 4*c
^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^6 - 36*a^2
*c^4*e^6*g^6, z, k)*((13*a^2*b^5*c^2*e^7*g^7 - 56*a^3*b^3*c^3*e^7*g^7 + 24*
a^2*c^7*d^5*e^2*g^7 - 2*b^4*c^5*d^5*e^2*g^7 + b^5*c^4*d^4*e^3*g^7 + b^6*c^3
*d^3*e^4*g^7 - 2*b^7*c^2*d^2*e^5*g^7 + 24*a^2*c^7*e^7*f^5*g^2 - 2*b^4*c^5*e
^7*f^5*g^2 + b^5*c^4*e^7*f^4*g^3 + b^6*c^3*e^7*f^3*g^4 - 2*b^7*c^2*e^7*f^2*
g^5 - a*b^7*c*e^7*g^7 + b^8*c*d*e^6*g^7 + b^8*c*e^7*f*g^6 + 80*a^4*b*c^4*e^
7*g^7 - 28*a^4*c^5*d*e^6*g^7 + b^3*c^6*d^6*e*g^7 - 28*a^4*c^5*e^7*f*g^6 + b
^3*c^6*e^7*f^6*g + 4*c^9*d^3*e^4*f^6*g + 4*c^9*d^6*e*f^3*g^4 - 12*a*b^6*c^2
*d*e^6*g^7 - 12*a*b^6*c^2*e^7*f*g^6 - 4*b*c^8*d^2*e^5*f^6*g - 4*b*c^8*d^6*e
*f^2*g^5 - b^2*c^7*d*e^6*f^6*g - b^2*c^7*d^6*e*f*g^6 - 2*b^7*c^2*d*e^6*f*g^
6 + 2*a*b^2*c^6*d^5*e^2*g^7 + 10*a*b^3*c^5*d^4*e^3*g^7 - 20*a*b^4*c^4*d^3*e
^4*g^7 + 25*a*b^5*c^3*d^2*e^5*g^7 - 56*a^2*b*c^6*d^4*e^3*g^7 + 44*a^2*b^4*c
^3*d*e^6*g^7 + 76*a^3*b*c^5*d^2*e^5*g^7 - 40*a^3*b^2*c^4*d*e^6*g^7 + 2*a*b^
2*c^6*e^7*f^5*g^2 + 10*a*b^3*c^5*e^7*f^4*g^3 - 20*a*b^4*c^4*e^7*f^3*g^4 + 2
5*a*b^5*c^3*e^7*f^2*g^5 - 56*a^2*b*c^6*e^7*f^4*g^3 + 44*a^2*b^4*c^3*e^7*f*g
^6 + 76*a^3*b*c^5*e^7*f^2*g^5 - 40*a^3*b^2*c^4*e^7*f*g^6 + 16*a*c^8*d^2*e^5
*f^5*g^2 + 24*a*c^8*d^3*e^4*f^4*g^3 + 24*a*c^8*d^4*e^3*f^3*g^4 + 16*a*c^8*d
^5*e^2*f^2*g^5 + 28*a^2*c^7*d*e^6*f^4*g^3 + 28*a^2*c^7*d^4*e^3*f*g^6 - 80*a
^3*c^6*d*e^6*f^2*g^5 - 80*a^3*c^6*d^2*e^5*f*g^6 - 12*b*c^8*d^3*e^4*f^5*g^2
- 12*b*c^8*d^5*e^2*f^3*g^4 + 6*b^3*c^6*d*e^6*f^5*g^2 + 6*b^3*c^6*d^5*e^2*f*
g^6 - 9*b^4*c^5*d*e^6*f^4*g^3 - 9*b^4*c^5*d^4*e^3*f*g^6 + 4*b^5*c^4*d*e^6*f
^3*g^4 + 4*b^5*c^4*d^3*e^4*f*g^6 + b^6*c^3*d*e^6*f^2*g^5 + b^6*c^3*d^2*e^5*
f*g^6 - 4*a*b*c^7*d^6*e*g^7 - 4*a*b*c^7*e^7*f^6*g + 8*a*c^8*d*e^6*f^6*g + 8
*a*c^8*d^6*e*f*g^6 + 65*a^2*b^2*c^5*d^3*e^4*g^7 - 88*a^2*b^3*c^4*d^2*e^5*g^
7 + 65*a^2*b^2*c^5*e^7*f^3*g^4 - 88*a^2*b^3*c^4*e^7*f^2*g^5 + 68*a^2*c^7*d^
2*e^5*f^3*g^4 + 68*a^2*c^7*d^3*e^4*f^2*g^5 + 8*b^2*c^7*d^2*e^5*f^5*g^2 + 9*
b^2*c^7*d^3*e^4*f^4*g^3 + 9*b^2*c^7*d^4*e^3*f^3*g^4 + 8*b^2*c^7*d^5*e^2*f^2

*g^5 - b^3*c^6*d^2*e^5*f^4*g^3 + 4*b^3*c^6*d^3*e^4*f^3*g^4 - b^3*c^6*d^4*e^3*f^2*g^5 - 9*b^4*c^5*d^2*e^5*f^3*g^4 - 9*b^4*c^5*d^3*e^4*f^2*g^5 + 7*b^5*c^4*d^2*e^5*f^2*g^5 + 74*a*b^2*c^6*d^2*e^5*f^3*g^4 + 74*a*b^2*c^6*d^3*e^4*f^2*g^5 - 28*a*b^3*c^5*d^2*e^5*f^2*g^5 - 120*a^2*b*c^6*d^2*e^5*f^2*g^5 + 159*a^2*b^2*c^5*d*e^6*f^2*g^5 + 159*a^2*b^2*c^5*d^2*e^5*f*g^6 - 36*a*b*c^7*d*e^6*f^5*g^2 - 36*a*b*c^7*d^5*e^2*f*g^6 + 28*a*b^5*c^3*d*e^6*f*g^6 + 104*a^3*b*c^5*d*e^6*f*g^6 - 56*a*b*c^7*d^2*e^5*f^4*g^3 - 96*a*b*c^7*d^3*e^4*f^3*g^4 - 56*a*b*c^7*d^4*e^3*f^2*g^5 + 44*a*b^2*c^6*d*e^6*f^4*g^3 + 44*a*b^2*c^6*d^4*e^3*f*g^6 - 32*a*b^4*c^4*d*e^6*f^2*g^5 - 32*a*b^4*c^4*d^2*e^5*f*g^6 - 116*a^2*b*c^6*d*e^6*f^3*g^4 - 116*a^2*b*c^6*d^3*e^4*f*g^6 - 112*a^2*b^3*c^4*d*e^6*f*g^6)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3) - root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 512*a^4*b*c^9*d^9*e*f^6*g^4*z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^10*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4 + 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*b^9*c^4*d^4*e^6*f^9*g*z^4 -

9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5*b^2*c^7*d^6*e^4*f^6*g^4*z^
4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*a^6*b^2*c^6*d^4*e^6*f^6*g^4
*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 5632*a^6*b^2*c^6*d^7*e^3*f^3*
g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 + 5232*a^5*b^4*c^5*d^6*e^4*f
^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^4 + 4808*a^4*b^6*c^4*d^5*e^
5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7*z^4 - 4288*a^6*b^4*c^4*d^3
*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*
d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f^4*g^6*z^4 + 3968*a^6*b^3*c
^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^4*f^5*g^5*z^4 + 3968*a^5*b^
3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b^2*c^5*d^5*e^5*f^3*g^7*z^4 + 3840*a^7
*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^7*e^3*f^5*g^5*z^4 + 3840*
a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776*a^6*b^4*c^4*d^4*e^6*f^4*g^6*z^4 + 37
76*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3456*a^6*b^2*c^6*d^5*e^5*f^5*g^5*z^4 +
3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4 + 3440*a^6*b^4*c^4*d^2*e^8*f^6*g^4*z^
4 + 3440*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z^4 + 3440*a^4*b^4*c^6*d^4*e^6*f^8*g^2
*z^4 - 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^8*z^4 - 3360*a^8*b^2*c^4*d^2*e^8*f^4*
g^6*z^4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6*g^4*z^4 - 3360*a^4*b^2*c^8*d^6*e^4*f
^8*g^2*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*f^3*g^7*z^4 - 2944*a^3*b^4*c^7*d^7*e^
3*f^7*g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e^5*f^3*g^7*z^4 + 2512*a^5*b^6*c^3*d^3
*e^7*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^7*e^3*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*
d^5*e^5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3*d^4*e^6*f^2*g^8*z^4 + 2312*a^7*b^4*c
^3*d^2*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*c^7*d^8*e^2*f^6*g^4*z^4 + 2312*a^3*b^
4*c^7*d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b^6*c^2*d^3*e^7*f^3*g^7*z^4 + 1952*a^2
*b^6*c^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^5*b^4*c^5*d^7*e^3*f^3*g^7*z^4 - 1920*
a^5*b^4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828*a^5*b^6*c^3*d^6*e^4*f^2*g^8*z^4 - 18
28*a^5*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1828*a^3*b^6*c^5*d^8*e^2*f^4*g^6*z^4 -
1828*a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4 + 1740*a^5*b^4*c^5*d^8*e^2*f^2*g^8*z^
4 + 1740*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z^4 - 1728*a^7*b^2*c^5*d^6*e^4*f^2*g^8
*z^4 - 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^4*z^4 - 1728*a^5*b^2*c^7*d^8*e^2*f^4*
g^6*z^4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8*g^2*z^4 - 1716*a^4*b^6*c^4*d^6*e^4*f
^4*g^6*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*f^6*g^4*z^4 - 1664*a^9*b^2*c^3*d^2*e^
8*f^2*g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e^2*f^8*g^2*z^4 - 1600*a^6*b^3*c^5*d^7
*e^3*f^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^2*e^8*f^7*g^3*z^4 - 1600*a^5*b^3*c^6*
d^8*e^2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6*d^3*e^7*f^8*g^2*z^4 - 1553*a^4*b^6*c
^4*d^8*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*c^4*d^2*e^8*f^8*g^2*z^4 + 1536*a^8*b^
2*c^4*d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b^2*c^8*d^7*e^3*f^7*g^3*z^4 + 1408*a^7
*b^3*c^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^7*b^3*c^4*d^3*e^7*f^4*g^6*z^4 - 1408*
a^6*b^3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408*a^6*b^3*c^5*d^3*e^7*f^6*g^4*z^4 - 14
08*a^5*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1408*a^5*b^3*c^6*d^4*e^6*f^7*g^3*z^4 +
1408*a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4 + 1408*a^4*b^3*c^7*d^6*e^4*f^7*g^3*z^
4 - 1360*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z^4 - 1360*a^6*b^5*c^3*d^2*e^8*f^5*g^5
*z^4 - 1360*a^3*b^5*c^6*d^8*e^2*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^5*e^5*f^8*
g^2*z^4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4*g^6*z^4 - 1248*a^5*b^5*c^4*d^4*e^6*f
^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^5*e^
5*f^6*g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e^7*f^2*g^8*z^4 + 1088*a^8*b^3*c^3*d^2
*e^8*f^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^8*e^2*f^7*g^3*z^4 + 1088*a^3*b^3*c^8*
d^7*e^3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2*d^2*e^8*f^2*g^8*z^4 + 1056*a^2*b^4*c
^8*d^8*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c^2*d^3*e^7*f^2*g^8*z^4 - 912*a^7*b^5*
c^2*d^2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*c^7*d^8*e^2*f^7*g^3*z^4 - 912*a^2*b^5
*c^7*d^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6*c^3*d^4*e^6*f^4*g^6*z^4 - 848*a^3*b^
6*c^5*d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^3*c^4*d^5*e^5*f^2*g^8*z^4 + 832*a^7*b
^3*c^4*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^8*e^2*f^5*g^5*z^4 + 832*a^4*
b^3*c^7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*b^7*c^2*d^5*e^5*f^2*g^8*z^4 + 828*a^5
*b^7*c^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^8*e^2*f^5*g^5*z^4 + 828*a^
2*b^7*c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^3*b^8*c^3*d^5*e^5*f^5*g^5*z^4 - 696*a
^4*b^8*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a^4*b^8*c^2*d^3*e^7*f^5*g^5*z^4 - 696*
a^2*b^8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^5*e^5*f^7*g^3*z^4 - 694
*a^6*b^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694*a^6*b^6*c^2*d^2*e^8*f^4*g^6*z^4 - 69
4*a^2*b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 694*a^2*b^6*c^6*d^6*e^4*f^8*g^2*z^4 + 6

92*a^4*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 692*a^4*b^7*c^3*d^2*e^8*f^7*g^3*z^4 +
692*a^3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 + 692*a^3*b^7*c^4*d^3*e^7*f^8*g^2*z^4 +
672*a^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 + 672*a^4*b^6*c^4*d^3*e^7*f^7*g^3*z^4
+ 600*a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4 + 600*a^2*b^8*c^4*d^6*e^4*f^6*g^4*z^4
- 544*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4 + 544*a^3*b^8*c^3*d^6*e^4*f^4*g^6*z^
4 + 544*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^3*e^7*f^7*g^3*z
^4 - 536*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z^4 - 536*a^4*b^7*c^3*d^4*e^6*f^5*g^5*
z^4 - 536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^5*e^5*f^6*g^4
*z^4 - 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7*z^4 - 504*a^5*b^7*c^2*d^3*e^7*f^4*g^
6*z^4 - 504*a^2*b^7*c^5*d^7*e^3*f^6*g^4*z^4 - 504*a^2*b^7*c^5*d^6*e^4*f^7*g
^3*z^4 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g^8*z^4 + 416*a^3*b^8*c^3*d^2*e^8*f^8*
g^2*z^4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*g^7*z^4 - 352*a^6*b^5*c^3*d^3*e^7*f^4
*g^6*z^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6*g^4*z^4 - 352*a^3*b^5*c^6*d^6*e^4*f^
7*g^3*z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^2*g^8*z^4 - 248*a^3*b^9*c^2*d^2*e^8*f
^7*g^3*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f^3*g^7*z^4 - 248*a^2*b^9*c^3*d^3*e^7*
f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*f^2*g^8*z^4 + 246*a^4*b^8*c^2*d^2*e^8
*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2*f^4*g^6*z^4 + 246*a^2*b^8*c^4*d^4*e^
6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^2*f^2*g^8*z^4 + 208*a^6*b^2*c^6*d^2*e
^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*e^3*f^3*g^7*z^4 + 168*a^2*b^10*c^2*d^
3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^5*e^5*f^4*g^6*z^4 + 160*a^3*b^9*c^2*d
^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^6*e^4*f^5*g^5*z^4 + 160*a^2*b^9*c^3*
d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*d^7*e^3*f^2*g^8*z^4 + 144*a^5*b^5*c^4
*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5*d^8*e^2*f^3*g^7*z^4 + 144*a^4*b^5*c^
5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c^2*d^6*e^4*f^4*g^6*z^4 - 144*a^2*b^10
*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7*c^3*d^6*e^4*f^3*g^7*z^4 + 120*a^4*b^
7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^7*c^4*d^7*e^3*f^4*g^6*z^4 + 120*a^3*b
^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^5*c^4*d^6*e^4*f^3*g^7*z^4 + 96*a^5*b^
5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5*c^5*d^7*e^3*f^4*g^6*z^4 + 96*a^4*b^5
*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*c^2*d^6*e^4*f^3*g^7*z^4 + 64*a^3*b^9*
c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c^3*d^7*e^3*f^4*g^6*z^4 + 64*a^2*b^9*c
^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c^2*d^8*e^2*f^2*g^8*z^4 - 36*a^2*b^10*
c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*c^2*d^5*e^5*f^5*g^5*z^4 - 24*a^9*b^4*
c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e*f^9*g*z^4 + 2688*a^7*b^2*c^5*d^7*e^3
*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^7*g^3*z^4 + 2688*a^5*b^2*c^7*d^9*e*f^
3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f^9*g*z^4 - 2560*a^7*b^3*c^4*d^6*e^4*f
*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*g^4*z^4 - 2560*a^4*b^3*c^7*d^9*e*f^4*
g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9*g*z^4 + 2112*a^8*b^2*c^4*d^5*e^5*f*g
^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^9*e*f^5*g^
5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g*z^4 + 1664*a^6*b^5*c^3*d^6*e^4*f*g^9
*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*z^4 + 1664*a^3*b^5*c^6*d^9*e*f^4*g^6*
z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z^4 + 1536*a^8*b*c^5*d^4*e^6*f^3*g^7*z
^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^5*e^5*f^4*g^6*z^
4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^6*e^4*f^5*g^5*z^4
+ 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^7*e^3*f^6*g^4*z^4
+ 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 - 1408*a^8*b^3*c^3*d^4*e^6*f*g^9*z^4 -
1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 - 1408*a^3*b^3*c^8*d^9*e*f^6*g^4*z^4 -
1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1280*a^7*b*c^6*d^7*e^3*f^2*g^8*z^4 - 1
280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 1280*a^6*b*c^7*d^8*e^2*f^3*g^7*z^4 - 12
80*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 1152*a^6*b^3*c^5*d^8*e^2*f*g^9*z^4 - 115
2*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152*a^5*b^3*c^6*d^9*e*f^2*g^8*z^4 - 1152
*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*a^5*b^5*c^4*d^8*e^2*f*g^9*z^4 + 1056*
a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a^4*b^5*c^5*d^9*e*f^2*g^8*z^4 + 1056*a
^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7*b^5*c^2*d^4*e^6*f*g^9*z^4 + 864*a^7*
b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^5*c^7*d^9*e*f^6*g^4*z^4 + 864*a^2*b^5
*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c^4*d^7*e^3*f*g^9*z^4 - 800*a^6*b^4*c^
4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*d^9*e*f^3*g^7*z^4 - 800*a^4*b^4*c^6*d
^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e^5*f^2*g^8*z^4 - 768*a^8*b*c^5*d^2*e^
8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*f^5*g^5*z^4 - 768*a^5*b*c^8*d^5*e^5*f
^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f*g^9*z^4 + 640*a^9*b^2*c^3*d*e^9*f^3*

g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^3*z^4 + 640*a^3*b^2*c^9*d^7*e^3*f^9*g
*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z^4 + 512*a^7*b*c^6*d^3*e^7*f^6*g^4*z^
4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4 + 512*a^6*b*c^7*d^4*e^6*f^7*g^3*z^4 -
480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 480*a*b^8*c^5*d^7*e^3*f^7*g^3*z^4 - 40
0*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*a^7*b^4*c^3*d*e^9*f^5*g^5*z^4 - 400*a
^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^5*e^5*f^9*g*z^4 - 372*a^6*
b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^6*c^2*d*e^9*f^5*g^5*z^4 - 372*a^2*b^6
*c^6*d^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^5*e^5*f^9*g*z^4 - 328*a^5*b^6*c^
3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*d*e^9*f^7*g^3*z^4 - 328*a^3*b^6*c^5*d
^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3*e^7*f^9*g*z^4 - 288*a^8*b^4*c^2*d^3*
e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*f^3*g^7*z^4 - 288*a^5*b^7*c^2*d^6*e^4
*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6*g^4*z^4 - 288*a^2*b^7*c^5*d^9*e*f^4*
g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*g*z^4 - 288*a^2*b^4*c^8*d^9*e*f^7*g^3
*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z^4 - 280*a^4*b^7*c^3*d^8*e^2*f*g^9*z^
4 - 280*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4 - 280*a^3*b^7*c^4*d^9*e*f^2*g^8*z^4 -
280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 256*a^9*b*c^4*d^3*e^7*f^2*g^8*z^4 + 25
6*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*a^4*b*c^9*d^8*e^2*f^7*g^3*z^4 + 256*a
^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7*b^6*c*d^2*e^8*f^2*g^8*z^4 - 248*a*b^
6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^7*c*d^3*e^7*f^2*g^8*z^4 + 236*a^6*b^7
*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6*d^8*e^2*f^7*g^3*z^4 + 236*a*b^7*c^6*
d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^4*e^6*f^3*g^7*z^4 + 200*a^4*b^9*c*d^3
*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*e^6*f^4*g^6*z^4 - 200*a*b^10*c^3*d^6*
e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^3*f^6*g^4*z^4 + 200*a*b^9*c^4*d^6*e^4
*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f^2*g^8*z^4 - 196*a^4*b^9*c*d^2*e^8*f^
5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*g^5*z^4 - 196*a*b^9*c^4*d^5*e^5*f^8*g
^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9*z^4 - 192*a^9*b^3*c^2*d*e^9*f^2*g^8*
z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^4 - 192*a^2*b^3*c^9*d^8*e^2*f^9*g*z^4
+ 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 + 156*a^4*b^8*c^2*d*e^9*f^7*g^3*z^4 +
156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 156*a^2*b^8*c^4*d^3*e^7*f^9*g*z^4 + 96*
a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5*b^8*c*d^2*e^8*f^4*g^6*z^4 + 96*a*b^8
*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^5*d^6*e^4*f^8*g^2*z^4 + 88*a^3*b^10*c
*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d^3*e^7*f^5*g^5*z^4 + 88*a*b^10*c^3*d^
7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*e^5*f^7*g^3*z^4 - 36*a^2*b^11*c*d^6*e
^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7*f^6*g^4*z^4 - 36*a*b^11*c^2*d^7*e^3*
f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^7*g^3*z^4 + 28*a^3*b^10*c*d^6*e^4*f^2
*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g^4*z^4 + 28*a*b^10*c^3*d^8*e^2*f^4*g^
6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*z^4 + 24*a^3*b^9*c^2*d^8*e^2*f*g^9*z^
4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 + 24*a^2*b^11*c*d^7*e^3*f^2*g^8*z^4 +
24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24*a^2*b^9*c^3*d^9*e*f^2*g^8*z^4 + 24*a
^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^11*c^2*d^8*e^2*f^3*g^7*z^4 + 24*a*b^1
1*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^11*c*d^5*e^5*f^4*g^6*z^4 + 12*a^2*b^11
*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2*d^6*e^4*f^5*g^5*z^4 + 12*a*b^11*c^2*
d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*e^3*f^7*g^3*z^4 + 20*b^12*c^2*d^6*e^4
*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6*g^4*z^4 - 20*b^11*c^3*d^6*e^4*f^7*g^
3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4 - 20*b^9*c^5*d^7*e^3*f^8*g^2*z^4 + 2
0*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^11*c^3*d^8*e^2*f^5*g^5*z^4 + 16*b^11*c
^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^8*e^2*f^4*g^6*z^4 - 6*b^12*c^2*d^4*e^
6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6*g^4*z^4 - 5*b^10*c^4*d^6*e^4*f^8*g^2
*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4 - 4*b^12*c^2*d^5*e^5*f^7*g^3*z^4 - 46
08*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*a^7*c^7*d^6*e^4*f^4*g^6*z^4 + 3328*a^
7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c^6*d^5*e^5*f^3*g^7*z^4 + 3072*a^8*c^6
*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^3*e^7*f^5*g^5*z^4 - 3072*a^6*c^8*d^7*
e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4*f^6*g^4*z^4 - 3072*a^6*c^8*d^5*e^5*f
^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*g^7*z^4 - 2048*a^7*c^7*d^7*e^3*f^3*g^
7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z^4 - 2048*a^5*c^9*d^7*e^3*f^7*g^3*z^4
+ 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 + 1792*a^8*c^6*d^2*e^8*f^6*g^4*z^4 + 17
92*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*a^6*c^8*d^4*e^6*f^8*g^2*z^4 + 1408*a^
9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c^5*d^2*e^8*f^4*g^6*z^4 + 1408*a^5*c^9
*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^6*e^4*f^8*g^2*z^4 + 1088*a^7*c^7*d^8*

e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8*f^8*g^2*z^4 + 512*a^10*c^4*d^2*e^8*f
^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*g^2*z^4 + 40*a^4*b^10*d^3*e^7*f^3*g^7
*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4 - 20*a^5*b^9*d^3*e^7*f^2*g^8*z^4 - 20
*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*b^11*d^4*e^6*f^3*g^7*z^4 - 20*a^3*b^1
1*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^4*e^6*f^4*g^6*z^4 + 16*a^3*b^11*d^5*e
^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f^5*g^5*z^4 - 6*a^2*b^12*d^6*e^4*f^2*g
^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^4 - 5*a^4*b^10*d^4*e^6*f^2*g^8*z^4 -
5*a^4*b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2*b^12*d^5*e^5*f^3*g^7*z^4 - 4*a^2*b^1
2*d^3*e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^4*e^10*f^6*g^4*z^4 - 440*a^7*b^4*c^3*
e^10*f^6*g^4*z^4 + 320*a^8*b^3*c^3*e^10*f^5*g^5*z^4 + 320*a^7*b^3*c^4*e^10*
f^7*g^3*z^4 - 240*a^8*b^4*c^2*e^10*f^4*g^6*z^4 - 240*a^6*b^4*c^4*e^10*f^8*g
^2*z^4 + 192*a^9*b^3*c^2*e^10*f^3*g^7*z^4 + 192*a^9*b^2*c^3*e^10*f^4*g^6*z^
4 + 192*a^7*b^2*c^5*e^10*f^8*g^2*z^4 + 90*a^6*b^6*c^2*e^10*f^6*g^4*z^4 + 68
*a^5*b^6*c^3*e^10*f^8*g^2*z^4 - 48*a^10*b^2*c^2*e^10*f^2*g^8*z^4 + 48*a^7*b
^5*c^2*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c^3*e^10*f^7*g^3*z^4 - 36*a^5*b^7*c^2*
e^10*f^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*f^8*g^2*z^4 + 480*a^4*b^2*c^8*d^10*f^
4*g^6*z^4 - 440*a^3*b^4*c^7*d^10*f^4*g^6*z^4 + 320*a^4*b^3*c^7*d^10*f^3*g^7
*z^4 + 320*a^3*b^3*c^8*d^10*f^5*g^5*z^4 - 240*a^4*b^4*c^6*d^10*f^2*g^8*z^4
- 240*a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 192*a^5*b^2*c^7*d^10*f^2*g^8*z^4 + 192
*a^3*b^2*c^9*d^10*f^6*g^4*z^4 + 192*a^2*b^3*c^9*d^10*f^7*g^3*z^4 + 90*a^2*b
^6*c^6*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c^5*d^10*f^2*g^8*z^4 + 48*a^3*b^5*c^6*
d^10*f^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10*f^5*g^5*z^4 - 48*a^2*b^2*c^10*d^10*f
^8*g^2*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g^7*z^4 - 6*a^2*b^8*c^4*d^10*f^2*g^8*z
^4 + 480*a^8*b^2*c^4*d^6*e^4*g^10*z^4 - 440*a^7*b^4*c^3*d^6*e^4*g^10*z^4 +
320*a^8*b^3*c^3*d^5*e^5*g^10*z^4 + 320*a^7*b^3*c^4*d^7*e^3*g^10*z^4 - 240*a
^8*b^4*c^2*d^4*e^6*g^10*z^4 - 240*a^6*b^4*c^4*d^8*e^2*g^10*z^4 + 192*a^9*b^
3*c^2*d^3*e^7*g^10*z^4 + 192*a^9*b^2*c^3*d^4*e^6*g^10*z^4 + 192*a^7*b^2*c^5
*d^8*e^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6*e^4*g^10*z^4 + 68*a^5*b^6*c^3*d^8*e^
2*g^10*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g^10*z^4 + 48*a^7*b^5*c^2*d^5*e^5*g^10
*z^4 + 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4 - 36*a^5*b^7*c^2*d^7*e^3*g^10*z^4 -
6*a^4*b^8*c^2*d^8*e^2*g^10*z^4 + 480*a^4*b^2*c^8*d^4*e^6*f^10*z^4 - 440*a^3
*b^4*c^7*d^4*e^6*f^10*z^4 + 320*a^4*b^3*c^7*d^3*e^7*f^10*z^4 + 320*a^3*b^3*
c^8*d^5*e^5*f^10*z^4 - 240*a^4*b^4*c^6*d^2*e^8*f^10*z^4 - 240*a^2*b^4*c^8*d
^6*e^4*f^10*z^4 + 192*a^5*b^2*c^7*d^2*e^8*f^10*z^4 + 192*a^3*b^2*c^9*d^6*e^
4*f^10*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f^10*z^4 + 90*a^2*b^6*c^6*d^4*e^6*f^10
*z^4 + 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4 + 48*a^3*b^5*c^6*d^3*e^7*f^10*z^4 +
48*a^2*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a^2*b^2*c^10*d^8*e^2*f^10*z^4 - 36*a^2
*b^7*c^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*c^4*d^2*e^8*f^10*z^4 + 16*b^9*c^5*d^9
*e*f^6*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^9*g*z^4 - 14*b^10*c^4*d^9*e*f^5*g^5*z
^4 - 14*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4*b^13*c*d^7*e^3*f^4*g^6*z^4 - 4*b^13*
c*d^6*e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e^5*f^6*g^4*z^4 + 4*b^13*c*d^4*e^6*f^7
*g^3*z^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^4 + 4*b^11*c^3*d^4*e^6*f^9*g*z^4 - 4*
b^8*c^6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*d^7*e^3*f^9*g*z^4 - 4*b^7*c^7*d^9*e*f
^8*g^2*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z^4 - 768*a^9*c^5*d^5*e^5*f*g^9*z^4 -
768*a^9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^5*c^9*d^9*e*f^5*g^5*z^4 - 768*a^5*c^9
*d^5*e^5*f^9*g*z^4 - 512*a^10*c^4*d^3*e^7*f*g^9*z^4 - 512*a^10*c^4*d*e^9*f^
3*g^7*z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*z^4 - 512*a^8*c^6*d*e^9*f^7*g^3*z^4 -
512*a^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a^6*c^8*d^3*e^7*f^9*g*z^4 - 512*a^4*c^
10*d^9*e*f^7*g^3*z^4 - 512*a^4*c^10*d^7*e^3*f^9*g*z^4 + 16*a^5*b^9*d^4*e^6*
f*g^9*z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z^4 - 14*a^4*b^10*d^5*e^5*f*g^9*z^4 -
14*a^4*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*b^7*d^2*e^8*f*g^9*z^4 - 4*a^7*b^7*d*e
^9*f^2*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g^9*z^4 - 4*a^6*b^8*d*e^9*f^3*g^7*z^4
+ 4*a^3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3*b^11*d*e^9*f^6*g^4*z^4 + 4*a*b^13*d^
6*e^4*f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f^4*g^6*z^4 - 4*a*b^13*d^4*e^6*f^5*g^5
*z^4 + 4*a*b^13*d^3*e^7*f^6*g^4*z^4 - 768*a^9*b*c^4*e^10*f^5*g^5*z^4 - 768*
a^8*b*c^5*e^10*f^7*g^3*z^4 - 256*a^10*b*c^3*e^10*f^3*g^7*z^4 + 192*a^6*b^3*
c^5*e^10*f^9*g*z^4 + 68*a^7*b^6*c*e^10*f^4*g^6*z^4 - 48*a^8*b^5*c*e^10*f^3*
g^7*z^4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^4 - 36*a^6*b^7*c*e^10*f^5*g^5*z^4 + 1
2*a^9*b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b^9*c*e^10*f^7*g^3*z^4 + 4*a^4*b^7*c^3

*e^10*f^9*g*z^4 - 768*a^5*b*c^8*d^10*f^3*g^7*z^4 - 768*a^4*b*c^9*d^10*f^5*g
^5*z^4 - 256*a^3*b*c^10*d^10*f^7*g^3*z^4 + 192*a^5*b^3*c^6*d^10*f*g^9*z^4 +
68*a*b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^4*b^5*c^5*d^10*f*g^9*z^4 - 48*a*b^5*c
^8*d^10*f^7*g^3*z^4 - 36*a*b^7*c^6*d^10*f^5*g^5*z^4 + 12*a*b^4*c^9*d^10*f^8
*g^2*z^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^4 + 4*a*b^9*c^4*d^10*f^3*g^7*z^4 - 76
8*a^9*b*c^4*d^5*e^5*g^10*z^4 - 768*a^8*b*c^5*d^7*e^3*g^10*z^4 - 256*a^10*b*
c^3*d^3*e^7*g^10*z^4 + 192*a^6*b^3*c^5*d^9*e*g^10*z^4 + 68*a^7*b^6*c*d^4*e^
6*g^10*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10*z^4 - 48*a^5*b^5*c^4*d^9*e*g^10*z^4
- 36*a^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a^9*b^4*c*d^2*e^8*g^10*z^4 + 4*a^4*b^9
*c*d^7*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^9*e*g^10*z^4 - 768*a^5*b*c^8*d^3*e^7*
f^10*z^4 - 768*a^4*b*c^9*d^5*e^5*f^10*z^4 - 256*a^3*b*c^10*d^7*e^3*f^10*z^4
+ 192*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68*a*b^6*c^7*d^6*e^4*f^10*z^4 - 48*a^4*
b^5*c^5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*d^7*e^3*f^10*z^4 - 36*a*b^7*c^6*d^5*e
^5*f^10*z^4 + 12*a*b^4*c^9*d^8*e^2*f^10*z^4 + 4*a^3*b^7*c^4*d*e^9*f^10*z^4
+ 4*a*b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6*c^8*d^9*e*f^9*g*z^4 - 128*a^11*c^3*d
*e^9*f*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g^9*z^4 - 128*a^7*c^7*d*e^9*f^9*g*z^4
- 128*a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8*b^6*d*e^9*f*g^9*z^4 - 256*a^7*b*c^6*
e^10*f^9*g*z^4 - 256*a^6*b*c^7*d^10*f*g^9*z^4 - 256*a^7*b*c^6*d^9*e*g^10*z^
4 - 256*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b^14*d^5*e^5*f^5*g^5*z^4 + 384*a^9*c^5
*e^10*f^6*g^4*z^4 + 256*a^10*c^4*e^10*f^4*g^6*z^4 + 256*a^8*c^6*e^10*f^8*g^
2*z^4 + 64*a^11*c^3*e^10*f^2*g^8*z^4 - 6*b^8*c^6*d^10*f^6*g^4*z^4 + 4*b^9*c
^5*d^10*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f^7*g^3*z^4 + 384*a^5*c^9*d^10*f^4*g^6
*z^4 + 256*a^6*c^8*d^10*f^2*g^8*z^4 + 256*a^4*c^10*d^10*f^6*g^4*z^4 + 64*a^
3*c^11*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^10*f^4*g^6*z^4 + 4*a^7*b^7*e^10*f^3*g
^7*z^4 + 4*a^5*b^9*e^10*f^5*g^5*z^4 + 384*a^9*c^5*d^6*e^4*g^10*z^4 + 256*a^
10*c^4*d^4*e^6*g^10*z^4 + 256*a^8*c^6*d^8*e^2*g^10*z^4 + 64*a^11*c^3*d^2*e^
8*g^10*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^4 + 4*b^9*c^5*d^5*e^5*f^10*z^4 + 4*b^
7*c^7*d^7*e^3*f^10*z^4 + 384*a^5*c^9*d^4*e^6*f^10*z^4 + 256*a^6*c^8*d^2*e^8
*f^10*z^4 + 256*a^4*c^10*d^6*e^4*f^10*z^4 + 64*a^3*c^11*d^8*e^2*f^10*z^4 -
6*a^6*b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7*d^3*e^7*g^10*z^4 + 4*a^5*b^9*d^5*e^5
*g^10*z^4 - 48*a^6*b^2*c^6*e^10*f^10*z^4 - 48*a^6*b^2*c^6*d^10*g^10*z^4 + 1
2*a^5*b^4*c^5*e^10*f^10*z^4 + 12*a^5*b^4*c^5*d^10*g^10*z^4 + 64*a^7*c^7*e^1
0*f^10*z^4 + 64*a^7*c^7*d^10*g^10*z^4 - b^14*d^6*e^4*f^4*g^6*z^4 - b^14*d^4
*e^6*f^6*g^4*z^4 - b^10*c^4*d^10*f^4*g^6*z^4 - b^6*c^8*d^10*f^8*g^2*z^4 - a
^8*b^6*e^10*f^2*g^8*z^4 - a^4*b^10*e^10*f^6*g^4*z^4 - b^10*c^4*d^4*e^6*f^10
*z^4 - b^6*c^8*d^8*e^2*f^10*z^4 - a^8*b^6*d^2*e^8*g^10*z^4 - a^4*b^10*d^6*e
^4*g^10*z^4 - a^4*b^6*c^4*e^10*f^10*z^4 - a^4*b^6*c^4*d^10*g^10*z^4 + 272*a
^5*b^2*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b^4*c^2*d*e^7*f*g^7*z^2 - 164*a^5*b*c^
4*d^2*e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*e^7*f^2*g^6*z^2 + 120*a^2*b^2*c^6*d^7
*e*f*g^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f^7*g*z^2 + 120*a*b^2*c^7*d^7*e*f^3*g^
5*z^2 + 120*a*b^2*c^7*d^3*e^5*f^7*g*z^2 - 76*a^4*b*c^5*d^4*e^4*f*g^7*z^2 -
76*a^4*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^3*b*c^6*d^6*e^2*f*g^7*z^2 - 76*a^3*b*
c^6*d*e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d^7*e*f^2*g^6*z^2 - 64*a*b^3*c^6*d^2*e
^6*f^7*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*g^6*z^2 - 60*a^2*b*c^7*d^2*e^6*f^7*g*
z^2 + 44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 + 44*a*b*c^8*d^5*e^3*f^6*g^2*z^2 + 22*
a*b^5*c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5*c^4*d*e^7*f^6*g^2*z^2 - 20*a^2*b^7*c
*d^2*e^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^7*f^2*g^6*z^2 + 8*a*b^8*c*d^2*e^6*f^2
*g^6*z^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z^2 - 8*a*b^6*c^3*d*e^7*f^5*g^3*z^2 +
2*a*b^7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^7*c^2*d*e^7*f^4*g^4*z^2 - 590*a^2*b^2
*c^6*d^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4*c^4*d^3*e^5*f^3*g^5*z^2 - 346*a^3*b^
2*c^5*d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^2*c^5*d^2*e^6*f^4*g^4*z^2 - 274*a^4*b
^2*c^4*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b^2*c^5*d^3*e^5*f^3*g^5*z^2 + 250*a^2*
b^3*c^5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^3*e^5*f^4*g^4*z^2 + 204*a^3
*b^3*c^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3*b^3*c^4*d^2*e^6*f^3*g^5*z^2 + 136*a^
2*b^2*c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^3*e^5*f^5*g^3*z^2 + 71*a^
2*b^4*c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2*b^4*c^4*d^2*e^6*f^4*g^4*z^2 - 56*a^2
*b^3*c^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*b^3*c^5*d^2*e^6*f^5*g^3*z^2 + 18*a^2*
b^2*c^6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b^2*c^6*d^2*e^6*f^6*g^2*z^2 - 16*a^3*b
^4*c^3*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^3*e^5*f^2*g^6*z^2 + 16*a^2*b^

5*c^3*d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*c^2*d^2*e^6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e*f*g^7*z^2 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 - 4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4*a*b^8*c*d*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^4*f^7*g*z^2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3*g^5*z^2 + 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^3*z^2 - 208*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 - 208*a^3*b^4*c^3*d*e^7*f^3*g^5*z^2 - 164*a^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 164*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2 + 140*a^2*b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 122*a*b^2*c^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*a^2*b^3*c^5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b^3*c^6*d^5*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c^3*d^3*e^5*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^2*e^6*f^6*g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4*f^3*g^5*z^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*g^3*z^2 - 54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2 - 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z^2 - 52*a*b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 + 48*a^3*b^5*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^2*b^6*c^2*d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3*c^3*d^2*e^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^6*e^2*f^3*g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2*f^3*g^5*z^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*g^5*z^2 - 32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2 - 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z^2 - 20*a*b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 + 20*a*b^4*c^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*b^5*c^4*d^2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c^3*d*e^7*f^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^7*f^5*g^3*z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g^4*z^2 - 50*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 32*b^4*c^6*d^5*e^3*f^5*g^3*z^2 + 24*b^3*c^7*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^6*d^6*e^2*f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2*f^2*g^6*z^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z^2 + 8*b^6*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*c^5*d^4*e^4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5*f^3*g^5*z^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^2 - 2*b^7*c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c^5*d^6*e^2*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^6*f^2*g^6*z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g^6*z^2 + 376*a^4*c^6*d^2*e^6*f^4*g^4*z^2 + 320*a^3*c^7*d^4*e^4*f^4*g^4*z^2 - 280*a^3*c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a^2*c^8*d^5*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*d^2*e^6*f^6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4*f^6*g^2*z^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^4*z^2 - 310*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 232*a^5*b^2*c^3*e^8*f^2*g^6*z^2 + 229*a^2*b^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^4*b^3*c^3*e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^2*e^8*f^4*g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5*g^3*z^2 - 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2 + 436*a^3*b^4*c^3*d^4*e^4*g^8*z^2 - 310*a^3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 - 4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2 - 168*a^5*c^5*d*e^7*f^3*g^5*z^2 - 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^

2 + 548*a^5*b*c^4*e^8*f^3*g^5*z^2 - 182*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*
b^3*c^2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2
*g^6*z^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18
*a*b^2*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*
d^3*e^5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*
z^2 - 68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 58*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*
b^7*c*d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6
*f^8*z^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2 + 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6
*c^4*d*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*
z^2 - 24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d
*e^7*f*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 20
4*a^3*b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*
f^7*g*z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2 - 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^
2*b*c^7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8
*z^2 + 128*a^6*b*c^3*d*e^7*g^8*z^2 + 48*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^
5*c*d*e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2
 + 6*a*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^
6*f^4*g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^
9*d^8*f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*
b^7*c^3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 -
 12*b*c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*
z^2 - 2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8
*f^8*z^2 - 12*a*b^4*c^5*d^8*g^8*z^2 - 8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^
8*f^6*g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 -
 92*a^6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2
*g^6*z^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 + 36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c
^2*d^6*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^
2 - 188*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^
4*e^4*f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*
a^2*c^8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^
8*z^2 + 45*a^2*b^2*c^6*e^8*f^8*z^2 + 45*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^
8*f^6*g^2*z^2 + b^10*e^8*f^4*g^4*z^2 + 4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^
4*g^8*z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*
z^2 - 48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*c^7*d^8*g^8*z^2 + a^4*b^6*e^8*g^8*z^2
 - b^10*d^2*e^6*f^2*g^6*z^2 + 108*a^2*b^2*c^4*d^2*e^5*f*g^6*z + 108*a^2*b^2
*c^4*d*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^3*e^4*f^2*g^5*z + 60*a*b^2*c^5*d^2*e^
5*f^3*g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2*g^5*z - 44*a*b^3*c^4*d^2*e^5*f^2*g^5
*z - 120*a^2*b*c^5*d^3*e^4*f*g^6*z - 120*a^2*b*c^5*d*e^6*f^3*g^4*z - 96*a*b
*c^6*d^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^3*d*e^6*f*g^6*z + 32*a*b^3*c^4*d^3*e^
4*f*g^6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4*z - 28*a*b^4*c^3*d^2*e^5*f*g^6*z - 2
8*a*b^4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2*c^5*d^4*e^3*f*g^6*z - 18*a*b^2*c^5*d
*e^6*f^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2*g^5*z + 4*a*b*c^6*d^2*e^5*f^4*g^3*z
+ 24*a*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*b*c^4*d*e^6*f*g^6*z - 8*a*b*c^6*d^5*e
^2*f*g^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z - 13*b^2*c^6*d^6*e*f*g^6*z - 13*b^2*
c^6*d*e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*g^5*z + 8*b*c^7*d^2*e^5*f^6*g*z + 9*b
^2*c^6*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d^3*e^4*f^4*g^3*z + 8*b^5*c^3*d^2*e^5*
f^2*g^5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*z - 6*b^4*c^4*d^2*e^5*f^3*g^4*z - 6*b
^3*c^5*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d^2*e^5*f^4*g^3*z + 4*b^3*c^5*d^3*e^4*
f^3*g^4*z + b^2*c^6*d^5*e^2*f^2*g^5*z + b^2*c^6*d^2*e^5*f^5*g^2*z + 16*a^2*
c^6*d^3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2*e^5*f^3*g^4*z - 112*a^2*b^3*c^3*e^7*
f^2*g^5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4*z - 112*a^2*b^3*c^3*d^2*e^5*g^7*z -
12*a^2*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c*d*e^6*f*g^6*z + 8*a*c^7*d^6*e*f*g^6*
z + 8*a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6*e^7*f^6*g*z - 10*a*b^6*c*e^7*f*g^6*z
 + 52*a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c*d*e^6*g^7*z + 14*b^3*c^5*d^5*e^2*f*g
^6*z + 14*b^3*c^5*d*e^6*f^5*g^2*z - 12*b*c^7*d^5*e^2*f^3*g^4*z - 12*b*c^7*d
^3*e^4*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f*g^6*z - 5*b^4*c^4*d*e^6*f^4*g^3*z +
b^6*c^2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^6*f^2*g^5*z + 52*a^2*c^6*d^4*e^3*f*g^
6*z + 52*a^2*c^6*d*e^6*f^4*g^3*z + 24*a*c^7*d^4*e^3*f^3*g^4*z + 24*a*c^7*d^
3*e^4*f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2*g^5*z - 16*a*c^7*d^2*e^5*f^5*g^2*z +

8*a^3*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5*d*e^6*f^2*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^5*z + 144*a^2*b*c^5*e^7*f^4*g^3*z - 42*a*b^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^2*e^7*f*g^6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z - 10*a*b^3*c^4*e^7*f^4*g^3*z + 4*a*b^4*c^3*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^2*e^5*g^7*z + 144*a^2*b*c^5*d^4*e^3*g^7*z - 42*a*b^2*c^5*d^5*e^2*g^7*z + 32*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^2*b^4*c^2*d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g^7*z + 4*a*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*z + 11*b^4*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^4*z - 136*a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5*e^2*g^7*z - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*c^2*d^3*e^4*g^7*z - 136*a^3*c^5*d^3*e^4*g^7*z - 68*a^2*c^6*d^5*e^2*g^7*z - 96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8*d^6*e*f^3*g^4*z + 4*c^8*d^3*e^4*f^6*g*z - 10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c*e^7*f^2*g^5*z - 128*a^4*c^4*e^7*f*g^6*z - 10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c*d^2*e^5*g^7*z - 128*a^4*c^4*d*e^6*g^7*z + 128*a^4*b*c^3*e^7*g^7*z + 24*a^2*b^5*c*e^7*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^7*f^7*z + 3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g^6*z + b^8*d*e^6*g^7*z - 16*a*c^7*e^7*f^7*z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5 + 20*a*b*c^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b*c^5*d*e^5*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5 - 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5*e^6*f^2*g^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6*g^6 - 4*c^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^6 - 36*a^2*c^4*e^6*g^6, z, k)*(root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 512*a^4*b*c^9*d^9*e*f^6*g^4*z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^10*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4 + 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*b^9*c^4*d^4*e^6*f^9*g*z^4 - 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5*b^2*c^7*d^6*e^4*f^6*g^4*z^4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*a^6*b^2*c^6*d^4*e^6*f^6*g^4*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^5*z^4 + 5632*a^6*b^2*c^6*d^7*e^3*f^3*g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7*g^3*z^4 + 5232*a^5*b^4*c^5*d^6*e^4*f^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*f^6*g^4*z^4 + 4808*a^4*b^6*c^4*d^5*e^5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e^5*f^3*g^7*z^4 - 4288*a^6*b^4*c^4*d^3*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^7*e^3*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5*d^5*e^5*f^4*g^6*z^4 + 3968*a^6*b^3*c^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^6*e^4*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b^2*c^5*d^5*e^5*f^3*g^7*z^4 + 3840*a^7*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^7*e^3*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776*a^6*b^4*c^4*d^4*e^6*f^4*g^6*z^4 + 3776*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3456*a^6*b^2*c^6*d^5*e^5*f^5*g^5*z^4 + 3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4 + 3440*a^6*b^4*c^4*d^2*e^8*f^6*g^4*z^4 + 34

```
40*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z^4 + 3440*a^4*b^4*c^6*d^4*e^6*f^8*g^2*z^4 -
 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^8*z^4 - 3360*a^8*b^2*c^4*d^2*e^8*f^4*g^6*z^
4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6*g^4*z^4 - 3360*a^4*b^2*c^8*d^6*e^4*f^8*g^2
*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*f^3*g^7*z^4 - 2944*a^3*b^4*c^7*d^7*e^3*f^7*
g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e^5*f^3*g^7*z^4 + 2512*a^5*b^6*c^3*d^3*e^7*f
^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^7*e^3*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^5*e^
5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3*d^4*e^6*f^2*g^8*z^4 + 2312*a^7*b^4*c^3*d^2
*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*c^7*d^8*e^2*f^6*g^4*z^4 + 2312*a^3*b^4*c^7*
d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b^6*c^2*d^3*e^7*f^3*g^7*z^4 + 1952*a^2*b^6*c
^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^5*b^4*c^5*d^7*e^3*f^3*g^7*z^4 - 1920*a^5*b^
4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828*a^5*b^6*c^3*d^6*e^4*f^2*g^8*z^4 - 1828*a^5
*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1828*a^3*b^6*c^5*d^8*e^2*f^4*g^6*z^4 - 1828*
a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4 + 1740*a^5*b^4*c^5*d^8*e^2*f^2*g^8*z^4 + 17
40*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z^4 - 1728*a^7*b^2*c^5*d^6*e^4*f^2*g^8*z^4 -
 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^4*z^4 - 1728*a^5*b^2*c^7*d^8*e^2*f^4*g^6*z^
4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8*g^2*z^4 - 1716*a^4*b^6*c^4*d^6*e^4*f^4*g^6
*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*f^6*g^4*z^4 - 1664*a^9*b^2*c^3*d^2*e^8*f^2*
g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e^2*f^8*g^2*z^4 - 1600*a^6*b^3*c^5*d^7*e^3*f
^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^2*e^8*f^7*g^3*z^4 - 1600*a^5*b^3*c^6*d^8*e^
2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6*d^3*e^7*f^8*g^2*z^4 - 1553*a^4*b^6*c^4*d^8
*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*c^4*d^2*e^8*f^8*g^2*z^4 + 1536*a^8*b^2*c^4*
d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b^2*c^8*d^7*e^3*f^7*g^3*z^4 + 1408*a^7*b^3*c
^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^7*b^3*c^4*d^3*e^7*f^4*g^6*z^4 - 1408*a^6*b^
3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408*a^6*b^3*c^5*d^3*e^7*f^6*g^4*z^4 - 1408*a^5
*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1408*a^5*b^3*c^6*d^4*e^6*f^7*g^3*z^4 + 1408*
a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4 + 1408*a^4*b^3*c^7*d^6*e^4*f^7*g^3*z^4 - 13
60*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z^4 - 1360*a^6*b^5*c^3*d^2*e^8*f^5*g^5*z^4 -
 1360*a^3*b^5*c^6*d^8*e^2*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^5*e^5*f^8*g^2*z^
4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4*g^6*z^4 - 1248*a^5*b^5*c^4*d^4*e^6*f^5*g^5
*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^5*e^5*f^6*
g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e^7*f^2*g^8*z^4 + 1088*a^8*b^3*c^3*d^2*e^8*f
^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^8*e^2*f^7*g^3*z^4 + 1088*a^3*b^3*c^8*d^7*e^
3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2*d^2*e^8*f^2*g^8*z^4 + 1056*a^2*b^4*c^8*d^8
*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c^2*d^3*e^7*f^2*g^8*z^4 - 912*a^7*b^5*c^2*d^
2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*c^7*d^8*e^2*f^7*g^3*z^4 - 912*a^2*b^5*c^7*d
^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6*c^3*d^4*e^6*f^4*g^6*z^4 - 848*a^3*b^6*c^5*
d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^3*c^4*d^5*e^5*f^2*g^8*z^4 + 832*a^7*b^3*c^4
*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^8*e^2*f^5*g^5*z^4 + 832*a^4*b^3*c^
7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*b^7*c^2*d^5*e^5*f^2*g^8*z^4 + 828*a^5*b^7*c
^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^8*e^2*f^5*g^5*z^4 + 828*a^2*b^7*
c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^3*b^8*c^3*d^5*e^5*f^5*g^5*z^4 - 696*a^4*b^8
*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a^4*b^8*c^2*d^3*e^7*f^5*g^5*z^4 - 696*a^2*b^
8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^5*e^5*f^7*g^3*z^4 - 694*a^6*b
^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694*a^6*b^6*c^2*d^2*e^8*f^4*g^6*z^4 - 694*a^2*
b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 694*a^2*b^6*c^6*d^6*e^4*f^8*g^2*z^4 + 692*a^4
*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 692*a^4*b^7*c^3*d^2*e^8*f^7*g^3*z^4 + 692*a^
3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 + 692*a^3*b^7*c^4*d^3*e^7*f^8*g^2*z^4 + 672*a
^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 + 672*a^4*b^6*c^4*d^3*e^7*f^7*g^3*z^4 + 600*
a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4 + 600*a^2*b^8*c^4*d^6*e^4*f^6*g^4*z^4 - 544
*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4 + 544*a^3*b^8*c^3*d^6*e^4*f^4*g^6*z^4 + 54
4*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^3*e^7*f^7*g^3*z^4 - 5
36*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z^4 - 536*a^4*b^7*c^3*d^4*e^6*f^5*g^5*z^4 -
536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^5*e^5*f^6*g^4*z^4 -
 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7*z^4 - 504*a^5*b^7*c^2*d^3*e^7*f^4*g^6*z^4
- 504*a^2*b^7*c^5*d^7*e^3*f^6*g^4*z^4 - 504*a^2*b^7*c^5*d^6*e^4*f^7*g^3*z^4
 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g^8*z^4 + 416*a^3*b^8*c^3*d^2*e^8*f^8*g^2*z^
4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*g^7*z^4 - 352*a^6*b^5*c^3*d^3*e^7*f^4*g^6*z
^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6*g^4*z^4 - 352*a^3*b^5*c^6*d^6*e^4*f^7*g^3*
z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^2*g^8*z^4 - 248*a^3*b^9*c^2*d^2*e^8*f^7*g^3
```

*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f^3*g^7*z^4 - 248*a^2*b^9*c^3*d^3*e^7*f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*f^2*g^8*z^4 + 246*a^4*b^8*c^2*d^2*e^8*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2*f^4*g^6*z^4 + 246*a^2*b^8*c^4*d^4*e^6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^2*f^2*g^8*z^4 + 208*a^6*b^2*c^6*d^2*e^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*e^3*f^3*g^7*z^4 + 168*a^2*b^10*c^2*d^3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^5*e^5*f^4*g^6*z^4 + 160*a^3*b^9*c^2*d^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^6*e^4*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*d^7*e^3*f^2*g^8*z^4 + 144*a^5*b^5*c^4*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5*d^8*e^2*f^3*g^7*z^4 + 144*a^4*b^5*c^5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c^2*d^6*e^4*f^4*g^6*z^4 - 144*a^2*b^10*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7*c^3*d^6*e^4*f^3*g^7*z^4 + 120*a^4*b^7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^7*c^4*d^7*e^3*f^4*g^6*z^4 + 120*a^3*b^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^5*c^4*d^6*e^4*f^3*g^7*z^4 + 96*a^5*b^5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5*c^5*d^7*e^3*f^4*g^6*z^4 + 96*a^4*b^5*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*c^2*d^6*e^4*f^3*g^7*z^4 + 64*a^3*b^9*c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c^3*d^7*e^3*f^4*g^6*z^4 + 64*a^2*b^9*c^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c^2*d^8*e^2*f^2*g^8*z^4 - 36*a^2*b^10*c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*c^2*d^5*e^5*f^5*g^5*z^4 - 24*a^9*b^4*c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e*f^9*g*z^4 + 2688*a^7*b^2*c^5*d^7*e^3*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^7*g^3*z^4 + 2688*a^5*b^2*c^7*d^9*e*f^3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f^9*g*z^4 - 2560*a^7*b^3*c^4*d^6*e^4*f*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*g^4*z^4 - 2560*a^4*b^3*c^7*d^9*e*f^4*g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9*g*z^4 + 2112*a^8*b^2*c^4*d^5*e^5*f*g^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^9*e*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g*z^4 + 1664*a^6*b^5*c^3*d^6*e^4*f*g^9*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*z^4 + 1664*a^3*b^5*c^6*d^9*e*f^4*g^6*z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z^4 + 1536*a^8*b*c^5*d^4*e^6*f^3*g^7*z^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^5*e^5*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^6*e^4*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^7*e^3*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 - 1408*a^8*b^3*c^3*d^4*e^6*f*g^9*z^4 - 1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 - 1408*a^3*b^3*c^8*d^9*e*f^6*g^4*z^4 - 1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1280*a^7*b*c^6*d^7*e^3*f^2*g^8*z^4 - 1280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 1280*a^6*b*c^7*d^8*e^2*f^3*g^7*z^4 - 1280*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 1152*a^6*b^3*c^5*d^8*e^2*f*g^9*z^4 - 1152*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152*a^5*b^3*c^6*d^9*e*f^2*g^8*z^4 - 1152*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*a^5*b^5*c^4*d^8*e^2*f*g^9*z^4 + 1056*a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a^4*b^5*c^5*d^9*e*f^2*g^8*z^4 + 1056*a^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7*b^5*c^2*d^4*e^6*f*g^9*z^4 + 864*a^7*b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^5*c^7*d^9*e*f^6*g^4*z^4 + 864*a^2*b^5*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c^4*d^7*e^3*f*g^9*z^4 - 800*a^6*b^4*c^4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*d^9*e*f^3*g^7*z^4 - 800*a^4*b^4*c^6*d^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e^5*f^2*g^8*z^4 - 768*a^8*b*c^5*d^2*e^8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*f^5*g^5*z^4 - 768*a^5*b*c^8*d^5*e^5*f^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f*g^9*z^4 + 640*a^9*b^2*c^3*d*e^9*f^3*g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^3*z^4 + 640*a^3*b^2*c^9*d^7*e^3*f^9*g*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z^4 + 512*a^7*b*c^6*d^3*e^7*f^6*g^4*z^4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4 + 512*a^6*b*c^7*d^4*e^6*f^7*g^3*z^4 - 480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 480*a*b^8*c^5*d^7*e^3*f^7*g^3*z^4 - 400*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*a^7*b^4*c^3*d*e^9*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^5*e^5*f^9*g*z^4 - 372*a^6*b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^6*c^2*d*e^9*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^5*e^5*f^9*g*z^4 - 328*a^5*b^6*c^3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*d*e^9*f^7*g^3*z^4 - 328*a^3*b^6*c^5*d^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3*e^7*f^9*g*z^4 - 288*a^8*b^4*c^2*d^3*e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*f^3*g^7*z^4 - 288*a^5*b^7*c^2*d^6*e^4*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6*g^4*z^4 - 288*a^2*b^7*c^5*d^9*e*f^4*g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*g*z^4 - 288*a^2*b^4*c^8*d^9*e*f^7*g^3*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z^4 - 280*a^4*b^7*c^3*d^8*e^2*f*g^9*z^4 - 28

0*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4 - 280*a^3*b^7*c^4*d^9*e*f^2*g^8*z^4 - 280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 256*a^9*b*c^4*d^3*e^7*f^2*g^8*z^4 + 256*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*a^4*b*c^9*d^8*e^2*f^7*g^3*z^4 + 256*a^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7*b^6*c*d^2*e^8*f^2*g^8*z^4 - 248*a*b^6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^7*c*d^3*e^7*f^2*g^8*z^4 + 236*a^6*b^7*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6*d^8*e^2*f^7*g^3*z^4 + 236*a*b^7*c^6*d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^4*e^6*f^3*g^7*z^4 + 200*a^4*b^9*c*d^3*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*e^6*f^4*g^6*z^4 - 200*a*b^10*c^3*d^6*e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^3*f^6*g^4*z^4 + 200*a*b^9*c^4*d^6*e^4*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f^2*g^8*z^4 - 196*a^4*b^9*c*d^2*e^8*f^5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*g^5*z^4 - 196*a*b^9*c^4*d^5*e^5*f^8*g^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9*z^4 - 192*a^9*b^3*c^2*d*e^9*f^2*g^8*z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^4 - 192*a^2*b^3*c^9*d^8*e^2*f^9*g*z^4 + 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 + 156*a^4*b^8*c^2*d*e^9*f^7*g^3*z^4 + 156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 156*a^2*b^8*c^4*d^3*e^7*f^9*g*z^4 + 96*a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5*b^8*c*d^2*e^8*f^4*g^6*z^4 + 96*a*b^8*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^5*d^6*e^4*f^8*g^2*z^4 + 88*a^3*b^10*c*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d^3*e^7*f^5*g^5*z^4 + 88*a*b^10*c^3*d^7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*e^5*f^7*g^3*z^4 - 36*a^2*b^11*c*d^6*e^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7*f^6*g^4*z^4 - 36*a*b^11*c^2*d^7*e^3*f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^7*g^3*z^4 + 28*a^3*b^10*c*d^6*e^4*f^2*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g^4*z^4 + 28*a*b^10*c^3*d^8*e^2*f^4*g^6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*z^4 + 24*a^3*b^9*c^2*d^8*e^2*f*g^9*z^4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 + 24*a^2*b^11*c*d^7*e^3*f^2*g^8*z^4 + 24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24*a^2*b^9*c^3*d^9*e*f^2*g^8*z^4 + 24*a^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^11*c^2*d^8*e^2*f^3*g^7*z^4 + 24*a*b^11*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^11*c*d^5*e^5*f^4*g^6*z^4 + 12*a^2*b^11*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2*d^6*e^4*f^5*g^5*z^4 + 12*a*b^11*c^2*d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*e^3*f^7*g^3*z^4 + 20*b^12*c^2*d^6*e^4*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6*g^4*z^4 - 20*b^11*c^3*d^6*e^4*f^7*g^3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4 - 20*b^9*c^5*d^7*e^3*f^8*g^2*z^4 + 20*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^11*c^3*d^8*e^2*f^5*g^5*z^4 + 16*b^11*c^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^8*e^2*f^4*g^6*z^4 - 6*b^12*c^2*d^4*e^6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6*g^4*z^4 - 5*b^10*c^4*d^6*e^4*f^8*g^2*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4 - 4*b^12*c^2*d^5*e^5*f^7*g^3*z^4 - 4608*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*a^7*c^7*d^6*e^4*f^4*g^6*z^4 + 3328*a^7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c^6*d^5*e^5*f^3*g^7*z^4 + 3072*a^8*c^6*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^3*e^7*f^5*g^5*z^4 - 3072*a^6*c^8*d^7*e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4*f^6*g^4*z^4 - 3072*a^6*c^8*d^5*e^5*f^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*g^7*z^4 - 2048*a^7*c^7*d^7*e^3*f^3*g^7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z^4 - 2048*a^5*c^9*d^7*e^3*f^7*g^3*z^4 + 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 + 1792*a^8*c^6*d^2*e^8*f^6*g^4*z^4 + 1792*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*a^6*c^8*d^4*e^6*f^8*g^2*z^4 + 1408*a^9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c^5*d^2*e^8*f^4*g^6*z^4 + 1408*a^5*c^9*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^6*e^4*f^8*g^2*z^4 + 1088*a^7*c^7*d^8*e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8*f^8*g^2*z^4 + 512*a^10*c^4*d^2*e^8*f^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*g^2*z^4 + 40*a^4*b^10*d^3*e^7*f^3*g^7*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4 - 20*a^5*b^9*d^3*e^7*f^2*g^8*z^4 - 20*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*b^11*d^4*e^6*f^3*g^7*z^4 - 20*a^3*b^11*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^4*e^6*f^4*g^6*z^4 + 16*a^3*b^11*d^5*e^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f^5*g^5*z^4 - 6*a^2*b^12*d^6*e^4*f^2*g^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^4 - 5*a^4*b^10*d^4*e^6*f^2*g^8*z^4 - 5*a^4*b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2*b^12*d^5*e^5*f^3*g^7*z^4 - 4*a^2*b^12*d^3*e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^4*e^10*f^6*g^4*z^4 - 440*a^7*b^4*c^3*e^10*f^6*g^4*z^4 + 320*a^8*b^3*c^3*e^10*f^5*g^5*z^4 + 320*a^7*b^3*c^4*e^10*f^7*g^3*z^4 - 240*a^8*b^4*c^2*e^10*f^4*g^6*z^4 - 240*a^6*b^4*c^4*e^10*f^8*g^2*z^4 + 192*a^9*b^3*c^2*e^10*f^3*g^7*z^4 + 192*a^9*b^2*c^3*e^10*f^4*g^6*z^4 + 192*a^7*b^2*c^5*e^10*f^8*g^2*z^4 + 90*a^6*b^6*c^2*e^10*f^6*g^4*z^4 + 68*a^5*b^6*c^3*e^10*f^8*g^2*z^4 - 48*a^10*b^2*c^2*e^10*f^2*g^8*z^4 + 48*a^7*b^5*c^2

*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c^3*e^10*f^7*g^3*z^4 - 36*a^5*b^7*c^2*e^10*f
^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*f^8*g^2*z^4 + 480*a^4*b^2*c^8*d^10*f^4*g^6*
z^4 - 440*a^3*b^4*c^7*d^10*f^4*g^6*z^4 + 320*a^4*b^3*c^7*d^10*f^3*g^7*z^4 +
320*a^3*b^3*c^8*d^10*f^5*g^5*z^4 - 240*a^4*b^4*c^6*d^10*f^2*g^8*z^4 - 240*
a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 192*a^5*b^2*c^7*d^10*f^2*g^8*z^4 + 192*a^3*b
^2*c^9*d^10*f^6*g^4*z^4 + 192*a^2*b^3*c^9*d^10*f^7*g^3*z^4 + 90*a^2*b^6*c^6
*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c^5*d^10*f^2*g^8*z^4 + 48*a^3*b^5*c^6*d^10*f
^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10*f^5*g^5*z^4 - 48*a^2*b^2*c^10*d^10*f^8*g^2
*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g^7*z^4 - 6*a^2*b^8*c^4*d^10*f^2*g^8*z^4 + 4
80*a^8*b^2*c^4*d^6*e^4*g^10*z^4 - 440*a^7*b^4*c^3*d^6*e^4*g^10*z^4 + 320*a^
8*b^3*c^3*d^5*e^5*g^10*z^4 + 320*a^7*b^3*c^4*d^7*e^3*g^10*z^4 - 240*a^8*b^4
*c^2*d^4*e^6*g^10*z^4 - 240*a^6*b^4*c^4*d^8*e^2*g^10*z^4 + 192*a^9*b^3*c^2*
d^3*e^7*g^10*z^4 + 192*a^9*b^2*c^3*d^4*e^6*g^10*z^4 + 192*a^7*b^2*c^5*d^8*e
^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6*e^4*g^10*z^4 + 68*a^5*b^6*c^3*d^8*e^2*g^10
*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g^10*z^4 + 48*a^7*b^5*c^2*d^5*e^5*g^10*z^4 +
48*a^6*b^5*c^3*d^7*e^3*g^10*z^4 - 36*a^5*b^7*c^2*d^7*e^3*g^10*z^4 - 6*a^4*
b^8*c^2*d^8*e^2*g^10*z^4 + 480*a^4*b^2*c^8*d^4*e^6*f^10*z^4 - 440*a^3*b^4*c
^7*d^4*e^6*f^10*z^4 + 320*a^4*b^3*c^7*d^3*e^7*f^10*z^4 + 320*a^3*b^3*c^8*d^
5*e^5*f^10*z^4 - 240*a^4*b^4*c^6*d^2*e^8*f^10*z^4 - 240*a^2*b^4*c^8*d^6*e^4
*f^10*z^4 + 192*a^5*b^2*c^7*d^2*e^8*f^10*z^4 + 192*a^3*b^2*c^9*d^6*e^4*f^10
*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f^10*z^4 + 90*a^2*b^6*c^6*d^4*e^6*f^10*z^4 +
68*a^3*b^6*c^5*d^2*e^8*f^10*z^4 + 48*a^3*b^5*c^6*d^3*e^7*f^10*z^4 + 48*a^2
*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a^2*b^2*c^10*d^8*e^2*f^10*z^4 - 36*a^2*b^7*c
^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*c^4*d^2*e^8*f^10*z^4 + 16*b^9*c^5*d^9*e*f^6
*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^9*g*z^4 - 14*b^10*c^4*d^9*e*f^5*g^5*z^4 - 1
4*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4*b^13*c*d^7*e^3*f^4*g^6*z^4 - 4*b^13*c*d^6*
e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e^5*f^6*g^4*z^4 + 4*b^13*c*d^4*e^6*f^7*g^3*z
^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^4 + 4*b^11*c^3*d^4*e^6*f^9*g*z^4 - 4*b^8*c^
6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*d^7*e^3*f^9*g*z^4 - 4*b^7*c^7*d^9*e*f^8*g^2
*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z^4 - 768*a^9*c^5*d^5*e^5*f*g^9*z^4 - 768*a^
9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^5*c^9*d^9*e*f^5*g^5*z^4 - 768*a^5*c^9*d^5*e
^5*f^9*g*z^4 - 512*a^10*c^4*d^3*e^7*f*g^9*z^4 - 512*a^10*c^4*d*e^9*f^3*g^7*
z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*z^4 - 512*a^8*c^6*d*e^9*f^7*g^3*z^4 - 512*a
^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a^6*c^8*d^3*e^7*f^9*g*z^4 - 512*a^4*c^10*d^9
*e*f^7*g^3*z^4 - 512*a^4*c^10*d^7*e^3*f^9*g*z^4 + 16*a^5*b^9*d^4*e^6*f*g^9*
z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z^4 - 14*a^4*b^10*d^5*e^5*f*g^9*z^4 - 14*a^4
*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*b^7*d^2*e^8*f*g^9*z^4 - 4*a^7*b^7*d*e^9*f^2
*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g^9*z^4 - 4*a^6*b^8*d*e^9*f^3*g^7*z^4 + 4*a^
3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3*b^11*d*e^9*f^6*g^4*z^4 + 4*a*b^13*d^6*e^4*
f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f^4*g^6*z^4 - 4*a*b^13*d^4*e^6*f^5*g^5*z^4 +
4*a*b^13*d^3*e^7*f^6*g^4*z^4 - 768*a^9*b*c^4*e^10*f^5*g^5*z^4 - 768*a^8*b*
c^5*e^10*f^7*g^3*z^4 - 256*a^10*b*c^3*e^10*f^3*g^7*z^4 + 192*a^6*b^3*c^5*e^
10*f^9*g*z^4 + 68*a^7*b^6*c*e^10*f^4*g^6*z^4 - 48*a^8*b^5*c*e^10*f^3*g^7*z^
4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^4 - 36*a^6*b^7*c*e^10*f^5*g^5*z^4 + 12*a^9*
b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b^9*c*e^10*f^7*g^3*z^4 + 4*a^4*b^7*c^3*e^10*
f^9*g*z^4 - 768*a^5*b*c^8*d^10*f^3*g^7*z^4 - 768*a^4*b*c^9*d^10*f^5*g^5*z^4
- 256*a^3*b*c^10*d^10*f^7*g^3*z^4 + 192*a^5*b^3*c^6*d^10*f*g^9*z^4 + 68*a*
b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^4*b^5*c^5*d^10*f*g^9*z^4 - 48*a*b^5*c^8*d^1
0*f^7*g^3*z^4 - 36*a*b^7*c^6*d^10*f^5*g^5*z^4 + 12*a*b^4*c^9*d^10*f^8*g^2*z
^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^4 + 4*a*b^9*c^4*d^10*f^3*g^7*z^4 - 768*a^9*
b*c^4*d^5*e^5*g^10*z^4 - 768*a^8*b*c^5*d^7*e^3*g^10*z^4 - 256*a^10*b*c^3*d^
3*e^7*g^10*z^4 + 192*a^6*b^3*c^5*d^9*e*g^10*z^4 + 68*a^7*b^6*c*d^4*e^6*g^10
*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10*z^4 - 48*a^5*b^5*c^4*d^9*e*g^10*z^4 - 36*a
^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a^9*b^4*c*d^2*e^8*g^10*z^4 + 4*a^4*b^9*c*d^7
*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^9*e*g^10*z^4 - 768*a^5*b*c^8*d^3*e^7*f^10*z
^4 - 768*a^4*b*c^9*d^5*e^5*f^10*z^4 - 256*a^3*b*c^10*d^7*e^3*f^10*z^4 + 192
*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68*a*b^6*c^7*d^6*e^4*f^10*z^4 - 48*a^4*b^5*c^
5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*d^7*e^3*f^10*z^4 - 36*a*b^7*c^6*d^5*e^5*f^1
0*z^4 + 12*a*b^4*c^9*d^8*e^2*f^10*z^4 + 4*a^3*b^7*c^4*d*e^9*f^10*z^4 + 4*a*

b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6*c^8*d^9*e*f^9*g*z^4 - 128*a^11*c^3*d*e^9*f
*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g^9*z^4 - 128*a^7*c^7*d*e^9*f^9*g*z^4 - 128*
a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8*b^6*d*e^9*f*g^9*z^4 - 256*a^7*b*c^6*e^10*f
^9*g*z^4 - 256*a^6*b*c^7*d^10*f*g^9*z^4 - 256*a^7*b*c^6*d^9*e*g^10*z^4 - 25
6*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b^14*d^5*e^5*f^5*g^5*z^4 + 384*a^9*c^5*e^10*
f^6*g^4*z^4 + 256*a^10*c^4*e^10*f^4*g^6*z^4 + 256*a^8*c^6*e^10*f^8*g^2*z^4
+ 64*a^11*c^3*e^10*f^2*g^8*z^4 - 6*b^8*c^6*d^10*f^6*g^4*z^4 + 4*b^9*c^5*d^1
0*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f^7*g^3*z^4 + 384*a^5*c^9*d^10*f^4*g^6*z^4 +
256*a^6*c^8*d^10*f^2*g^8*z^4 + 256*a^4*c^10*d^10*f^6*g^4*z^4 + 64*a^3*c^11
*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^10*f^4*g^6*z^4 + 4*a^7*b^7*e^10*f^3*g^7*z^4
+ 4*a^5*b^9*e^10*f^5*g^5*z^4 + 384*a^9*c^5*d^6*e^4*g^10*z^4 + 256*a^10*c^4
*d^4*e^6*g^10*z^4 + 256*a^8*c^6*d^8*e^2*g^10*z^4 + 64*a^11*c^3*d^2*e^8*g^10
*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^4 + 4*b^9*c^5*d^5*e^5*f^10*z^4 + 4*b^7*c^7*
d^7*e^3*f^10*z^4 + 384*a^5*c^9*d^4*e^6*f^10*z^4 + 256*a^6*c^8*d^2*e^8*f^10*
z^4 + 256*a^4*c^10*d^6*e^4*f^10*z^4 + 64*a^3*c^11*d^8*e^2*f^10*z^4 - 6*a^6*
b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7*d^3*e^7*g^10*z^4 + 4*a^5*b^9*d^5*e^5*g^10*
z^4 - 48*a^6*b^2*c^6*e^10*f^10*z^4 - 48*a^6*b^2*c^6*d^10*g^10*z^4 + 12*a^5*
b^4*c^5*e^10*f^10*z^4 + 12*a^5*b^4*c^5*d^10*g^10*z^4 + 64*a^7*c^7*e^10*f^10
*z^4 + 64*a^7*c^7*d^10*g^10*z^4 - b^14*d^6*e^4*f^4*g^6*z^4 - b^14*d^4*e^6*f
^6*g^4*z^4 - b^10*c^4*d^10*f^4*g^6*z^4 - b^6*c^8*d^10*f^8*g^2*z^4 - a^8*b^6
*e^10*f^2*g^8*z^4 - a^4*b^10*e^10*f^6*g^4*z^4 - b^10*c^4*d^4*e^6*f^10*z^4 -
b^6*c^8*d^8*e^2*f^10*z^4 - a^8*b^6*d^2*e^8*g^10*z^4 - a^4*b^10*d^6*e^4*g^1
0*z^4 - a^4*b^6*c^4*e^10*f^10*z^4 - a^4*b^6*c^4*d^10*g^10*z^4 + 272*a^5*b^2
*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b^4*c^2*d*e^7*f*g^7*z^2 - 164*a^5*b*c^4*d^2*
e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*e^7*f^2*g^6*z^2 + 120*a^2*b^2*c^6*d^7*e*f*g
^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f^7*g*z^2 + 120*a*b^2*c^7*d^7*e*f^3*g^5*z^2
+ 120*a*b^2*c^7*d^3*e^5*f^7*g*z^2 - 76*a^4*b*c^5*d^4*e^4*f*g^7*z^2 - 76*a^4
*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^3*b*c^6*d^6*e^2*f*g^7*z^2 - 76*a^3*b*c^6*d*
e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d^7*e*f^2*g^6*z^2 - 64*a*b^3*c^6*d^2*e^6*f^7
*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*g^6*z^2 - 60*a^2*b*c^7*d^2*e^6*f^7*g*z^2 +
44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 + 44*a*b*c^8*d^5*e^3*f^6*g^2*z^2 + 22*a*b^5*
c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5*c^4*d*e^7*f^6*g^2*z^2 - 20*a^2*b^7*c*d^2*e
^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^7*f^2*g^6*z^2 + 8*a*b^8*c*d^2*e^6*f^2*g^6*z
^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z^2 - 8*a*b^6*c^3*d*e^7*f^5*g^3*z^2 + 2*a*b^
7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^7*c^2*d*e^7*f^4*g^4*z^2 - 590*a^2*b^2*c^6*d
^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4*c^4*d^3*e^5*f^3*g^5*z^2 - 346*a^3*b^2*c^5*
d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^2*c^5*d^2*e^6*f^4*g^4*z^2 - 274*a^4*b^2*c^4
*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b^2*c^5*d^3*e^5*f^3*g^5*z^2 + 250*a^2*b^3*c^
5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^3*e^5*f^4*g^4*z^2 + 204*a^3*b^3*c
^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3*b^3*c^4*d^2*e^6*f^3*g^5*z^2 + 136*a^2*b^2*
c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^3*e^5*f^5*g^3*z^2 + 71*a^2*b^4*
c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2*b^4*c^4*d^2*e^6*f^4*g^4*z^2 - 56*a^2*b^3*c
^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*b^3*c^5*d^2*e^6*f^5*g^3*z^2 + 18*a^2*b^2*c^
6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b^2*c^6*d^2*e^6*f^6*g^2*z^2 - 16*a^3*b^4*c^3
*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^3*e^5*f^2*g^6*z^2 + 16*a^2*b^5*c^3*
d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*c^2*d^2*e^6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^
7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e*f*g^7*z^2 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 -
4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4*a*b^8*c*d*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*
e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^4*f^7*g*z^2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7
*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3*g^5*z^2 + 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^
2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^3*z^2 - 208*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 -
208*a^3*b^4*c^3*d*e^7*f^3*g^5*z^2 - 164*a^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 16
4*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2 + 140*a^2*b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a
^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 122*a*b^2*c^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^
2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*a^2*b^3*c^5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3
*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b^3*c^6*d^5*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*
d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c^3*d^3*e^5*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e
^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^2*e^6*f^6*g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^
2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4*f^3*g^5*z^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4

*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*g^3*z^2 - 54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2
- 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z^2 - 52*a*b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*
a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 + 48*a^3*b^5*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b
^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^2*b^6*c^2*d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^
2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3*c^3*d^2*e^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e
^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^6*e^2*f^3*g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^
6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2*f^3*g^5*z^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2
*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*g^5*z^2 - 32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2
- 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z^2 - 20*a*b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*
a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 + 20*a*b^4*c^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5
*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*b^5*c^4*d^2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3
*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c^3*d*e^7*f^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^
3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^7*f^5*g^3*z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6
*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g^4*z^2 - 50*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 3
2*b^4*c^6*d^5*e^3*f^5*g^3*z^2 + 24*b^3*c^7*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7
*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^6*d^6*e^2*f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*
f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2*f^2*g^6*z^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z
^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z^2 + 8*b^6*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*
c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*c^5*d^4*e^4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4
*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5*f^3*g^5*z^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^
2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^2 - 2*b^7*c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c
^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c^5*d^6*e^2*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*
f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^6*f^2*g^6*z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5
*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g^6*z^2 + 376*a^4*c^6*d^2*e^6*f^4*g^4*z^2 +
320*a^3*c^7*d^4*e^4*f^4*g^4*z^2 - 280*a^3*c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3
*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a^2*c^8*d^5*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^
6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*d^2*e^6*f^6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f
^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4*f^6*g^2*z^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*
z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^4*z^2 - 310*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 2
32*a^5*b^2*c^3*e^8*f^2*g^6*z^2 + 229*a^2*b^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*
b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^4*b^3*c^3*e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5
*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^2*e^8*f^4*g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*
g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5*g^3*z^2 - 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2
+ 436*a^3*b^4*c^3*d^4*e^4*g^8*z^2 - 310*a^3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a
^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*
c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6
*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*
z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^
3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3
*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 -
4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5
*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2
- 168*a^5*c^5*d*e^7*f^3*g^5*z^2 - 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c
^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^
7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^2 + 54
8*a^5*b*c^4*e^8*f^3*g^5*z^2 - 182*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^
2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z
^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2
*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^
5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*z^2 -
68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 58*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*
d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z
^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2 + 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d
*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*z^2 -
24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d*e^7*f
*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 204*a^3*
b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*f^7*g*
z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2 - 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^2*b*c^
7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8*z^2 +

128*a^6*b*c^3*d*e^7*g^8*z^2 + 48*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^5*c*d*
e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2 + 6*a
*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^6*f^4*
g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^9*d^8*
f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*b^7*c^
3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 - 12*b*
c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*z^2 -
2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8*f^8*z
^2 - 12*a*b^4*c^5*d^8*g^8*z^2 - 8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^8*f^6*
g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 - 92*a^
6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2*g^6*z
^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 + 36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c^2*d^6
*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^2 - 18
8*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^4*e^4*
f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*a^2*c^
8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^8*z^2
+ 45*a^2*b^2*c^6*e^8*f^8*z^2 + 45*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^8*f^6*
g^2*z^2 + b^10*e^8*f^4*g^4*z^2 + 4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^4*g^8*
z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*z^2 -
48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*c^7*d^8*g^8*z^2 + a^4*b^6*e^8*g^8*z^2 - b^1
0*d^2*e^6*f^2*g^6*z^2 + 108*a^2*b^2*c^4*d^2*e^5*f*g^6*z + 108*a^2*b^2*c^4*d
*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^3*e^4*f^2*g^5*z + 60*a*b^2*c^5*d^2*e^5*f^3*
g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2*g^5*z - 44*a*b^3*c^4*d^2*e^5*f^2*g^5*z - 1
20*a^2*b*c^5*d^3*e^4*f*g^6*z - 120*a^2*b*c^5*d*e^6*f^3*g^4*z - 96*a*b*c^6*d
^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^3*d*e^6*f*g^6*z + 32*a*b^3*c^4*d^3*e^4*f*g^
6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4*z - 28*a*b^4*c^3*d^2*e^5*f*g^6*z - 28*a*b^
4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2*c^5*d^4*e^3*f*g^6*z - 18*a*b^2*c^5*d*e^6*f
^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2*g^5*z + 4*a*b*c^6*d^2*e^5*f^4*g^3*z + 24*a
*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*b*c^4*d*e^6*f*g^6*z - 8*a*b*c^6*d^5*e^2*f*g
^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z - 13*b^2*c^6*d^6*e*f*g^6*z - 13*b^2*c^6*d*
e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*g^5*z + 8*b*c^7*d^2*e^5*f^6*g*z + 9*b^2*c^6
*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d^3*e^4*f^4*g^3*z + 8*b^5*c^3*d^2*e^5*f^2*g^
5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*z - 6*b^4*c^4*d^2*e^5*f^3*g^4*z - 6*b^3*c^5
*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d^2*e^5*f^4*g^3*z + 4*b^3*c^5*d^3*e^4*f^3*g^
4*z + b^2*c^6*d^5*e^2*f^2*g^5*z + b^2*c^6*d^2*e^5*f^5*g^2*z + 16*a^2*c^6*d^
3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2*e^5*f^3*g^4*z - 112*a^2*b^3*c^3*e^7*f^2*g^
5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4*z - 112*a^2*b^3*c^3*d^2*e^5*g^7*z - 12*a^2
*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c*d*e^6*f*g^6*z + 8*a*c^7*d^6*e*f*g^6*z + 8*
a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6*e^7*f^6*g*z - 10*a*b^6*c*e^7*f*g^6*z + 52*
a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c*d*e^6*g^7*z + 14*b^3*c^5*d^5*e^2*f*g^6*z +
14*b^3*c^5*d*e^6*f^5*g^2*z - 12*b*c^7*d^5*e^2*f^3*g^4*z - 12*b*c^7*d^3*e^4
*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f*g^6*z - 5*b^4*c^4*d*e^6*f^4*g^3*z + b^6*c^
2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^6*f^2*g^5*z + 52*a^2*c^6*d^4*e^3*f*g^6*z +
52*a^2*c^6*d*e^6*f^4*g^3*z + 24*a*c^7*d^4*e^3*f^3*g^4*z + 24*a*c^7*d^3*e^4*
f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2*g^5*z - 16*a*c^7*d^2*e^5*f^5*g^2*z + 8*a^3
*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5*d*e^6*f^2*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^
5*z + 144*a^2*b*c^5*e^7*f^4*g^3*z - 42*a*b^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2
*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^2*e^7*f*g^6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z
- 10*a*b^3*c^4*e^7*f^4*g^3*z + 4*a*b^4*c^3*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^
2*e^5*g^7*z + 144*a^2*b*c^5*d^4*e^3*g^7*z - 42*a*b^2*c^5*d^5*e^2*g^7*z + 32
*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^2*b^4*c^2*d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^
5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g^7*z + 4*a*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*
d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*z + 11*b^4*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^
7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^4*z - 136*a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^
6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5*e^2*g^7*z - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*
c^2*d^3*e^4*g^7*z - 136*a^3*c^5*d^3*e^4*g^7*z - 68*a^2*c^6*d^5*e^2*g^7*z -
96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8*d^6*e*f^3*g^4*z + 4*c^8*d^3*e^4*f^6*g*z -
10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c*e^7*f^2*g^5*z - 128*a^4*c^4*e^7*f*g^6*z -
10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c*d^2*e^5*g^7*z - 128*a^4*c^4*d*e^6*g^7*z +

128*a^4*b*c^3*e^7*g^7*z + 24*a^2*b^5*c*e^7*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*
c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^7*f^7*z + 3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g
^6*z + b^8*d*e^6*g^7*z - 16*a*c^7*e^7*f^7*z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*
e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5 + 20*a*b*c^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g
^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b*c^5*d*e^5*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5
- 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5*e^6*f^2*g^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c
^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6*g^6 - 4*c^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^
6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^6 - 36*a^2*c^4*e^6*g^6, z, k)*((64*a^6*c^7*
d^7*e^2*g^9 + 64*a^7*c^6*d^5*e^4*g^9 - 64*a^8*c^5*d^3*e^6*g^9 + 64*a^6*c^7*
e^9*f^7*g^2 + 64*a^7*c^6*e^9*f^5*g^4 - 64*a^8*c^5*e^9*f^3*g^6 - 64*a^9*c^4*
d*e^8*g^9 - 64*a^9*c^4*e^9*f*g^8 + 16*a^5*b*c^7*d^8*e*g^9 + a^6*b^6*c*d*e^8
*g^9 + 16*a^5*b*c^7*e^9*f^8*g + a^6*b^6*c*e^9*f*g^8 - 128*a^5*c^8*d*e^8*f^8
*g - 128*a^5*c^8*d^8*e*f*g^8 + a^3*b^5*c^5*d^8*e*g^9 + a^3*b^9*c*d^4*e^5*g^
9 - 8*a^4*b^3*c^6*d^8*e*g^9 - a^4*b^8*c*d^3*e^6*g^9 - a^5*b^7*c*d^2*e^7*g^9
- 144*a^6*b*c^6*d^6*e^3*g^9 - 80*a^7*b*c^5*d^4*e^5*g^9 - 12*a^7*b^4*c^2*d*
e^8*g^9 + 80*a^8*b*c^4*d^2*e^7*g^9 + 48*a^8*b^2*c^3*d*e^8*g^9 + a^3*b^5*c^5
*e^9*f^8*g + a^3*b^9*c*e^9*f^4*g^5 - 8*a^4*b^3*c^6*e^9*f^8*g - a^4*b^8*c*e^
9*f^3*g^6 - a^5*b^7*c*e^9*f^2*g^7 - 144*a^6*b*c^6*e^9*f^6*g^3 - 80*a^7*b*c^
5*e^9*f^4*g^5 - 12*a^7*b^4*c^2*e^9*f*g^8 + 80*a^8*b*c^4*e^9*f^2*g^7 + 48*a^
8*b^2*c^3*e^9*f*g^8 - 128*a^3*c^10*d^5*e^4*f^8*g - 128*a^3*c^10*d^8*e*f^5*g
^4 - 256*a^4*c^9*d^3*e^6*f^8*g - 256*a^4*c^9*d^8*e*f^3*g^6 - 448*a^6*c^7*d*
e^8*f^6*g^3 - 448*a^6*c^7*d^6*e^3*f*g^8 - 576*a^7*c^6*d*e^8*f^4*g^5 - 576*a
^7*c^6*d^4*e^5*f*g^8 - 320*a^8*c^5*d*e^8*f^2*g^7 - 320*a^8*c^5*d^2*e^7*f*g^
8 + b^5*c^8*d^6*e^3*f^8*g + b^5*c^8*d^8*e*f^6*g^3 - b^6*c^7*d^5*e^4*f^8*g -
b^6*c^7*d^8*e*f^5*g^4 - b^7*c^6*d^4*e^5*f^8*g - b^7*c^6*d^8*e*f^4*g^5 + b^
8*c^5*d^3*e^6*f^8*g + b^8*c^5*d^8*e*f^3*g^6 + b^12*c*d^3*e^6*f^4*g^5 + b^12
*c*d^4*e^5*f^3*g^6 - 4*a^3*b^6*c^4*d^7*e^2*g^9 + 6*a^3*b^7*c^3*d^6*e^3*g^9
- 4*a^3*b^8*c^2*d^5*e^4*g^9 + 36*a^4*b^4*c^5*d^7*e^2*g^9 - 57*a^4*b^5*c^4*d
^6*e^3*g^9 + 37*a^4*b^6*c^3*d^5*e^4*g^9 - 7*a^4*b^7*c^2*d^4*e^5*g^9 - 96*a^
5*b^2*c^6*d^7*e^2*g^9 + 168*a^5*b^3*c^5*d^6*e^3*g^9 - 100*a^5*b^4*c^4*d^5*e
^4*g^9 + 3*a^5*b^5*c^3*d^4*e^5*g^9 + 10*a^5*b^6*c^2*d^3*e^6*g^9 + 48*a^6*b^
2*c^5*d^5*e^4*g^9 + 56*a^6*b^3*c^4*d^4*e^5*g^9 - 36*a^6*b^4*c^3*d^3*e^6*g^9
+ 13*a^6*b^5*c^2*d^2*e^7*g^9 + 64*a^7*b^2*c^4*d^3*e^6*g^9 - 56*a^7*b^3*c^3
*d^2*e^7*g^9 - 4*a^3*b^6*c^4*e^9*f^7*g^2 + 6*a^3*b^7*c^3*e^9*f^6*g^3 - 4*a^
3*b^8*c^2*e^9*f^5*g^4 + 36*a^4*b^4*c^5*e^9*f^7*g^2 - 57*a^4*b^5*c^4*e^9*f^6
*g^3 + 37*a^4*b^6*c^3*e^9*f^5*g^4 - 7*a^4*b^7*c^2*e^9*f^4*g^5 - 96*a^5*b^2*
c^6*e^9*f^7*g^2 + 168*a^5*b^3*c^5*e^9*f^6*g^3 - 100*a^5*b^4*c^4*e^9*f^5*g^4
+ 3*a^5*b^5*c^3*e^9*f^4*g^5 + 10*a^5*b^6*c^2*e^9*f^3*g^6 + 48*a^6*b^2*c^5*
e^9*f^5*g^4 + 56*a^6*b^3*c^4*e^9*f^4*g^5 - 36*a^6*b^4*c^3*e^9*f^3*g^6 + 13*
a^6*b^5*c^2*e^9*f^2*g^7 + 64*a^7*b^2*c^4*e^9*f^3*g^6 - 56*a^7*b^3*c^3*e^9*f
^2*g^7 + 64*a^3*c^10*d^6*e^3*f^7*g^2 + 64*a^3*c^10*d^7*e^2*f^6*g^3 + 192*a^
4*c^9*d^4*e^5*f^7*g^2 - 320*a^4*c^9*d^5*e^4*f^6*g^3 - 320*a^4*c^9*d^6*e^3*f
^5*g^4 + 192*a^4*c^9*d^7*e^2*f^4*g^5 + 192*a^5*c^8*d^2*e^7*f^7*g^2 - 832*a^
5*c^8*d^3*e^6*f^6*g^3 - 192*a^5*c^8*d^4*e^5*f^5*g^4 - 192*a^5*c^8*d^5*e^4*f
^4*g^5 - 832*a^5*c^8*d^6*e^3*f^3*g^6 + 192*a^5*c^8*d^7*e^2*f^2*g^7 + 64*a^6
*c^7*d^2*e^7*f^5*g^4 - 960*a^6*c^7*d^3*e^6*f^4*g^5 - 960*a^6*c^7*d^4*e^5*f^
3*g^6 + 64*a^6*c^7*d^5*e^4*f^2*g^7 - 448*a^7*c^6*d^2*e^7*f^3*g^6 - 448*a^7*
c^6*d^3*e^6*f^2*g^7 - 2*b^5*c^8*d^7*e^2*f^7*g^2 + 2*b^6*c^7*d^6*e^3*f^7*g^2
+ 2*b^6*c^7*d^7*e^2*f^6*g^3 - 2*b^7*c^6*d^5*e^4*f^7*g^2 - 6*b^7*c^6*d^6*e^
3*f^6*g^3 - 2*b^7*c^6*d^7*e^2*f^5*g^4 + 6*b^8*c^5*d^4*e^5*f^7*g^2 + 8*b^8*c
^5*d^5*e^4*f^6*g^3 + 8*b^8*c^5*d^6*e^3*f^5*g^4 + 6*b^8*c^5*d^7*e^2*f^4*g^5
- 4*b^9*c^4*d^3*e^6*f^7*g^2 - 11*b^9*c^4*d^4*e^5*f^6*g^3 - 10*b^9*c^4*d^5*e
^4*f^5*g^4 - 11*b^9*c^4*d^6*e^3*f^4*g^5 - 4*b^9*c^4*d^7*e^2*f^3*g^6 + 6*b^1
0*c^3*d^3*e^6*f^6*g^3 + 9*b^10*c^3*d^4*e^5*f^5*g^4 + 9*b^10*c^3*d^5*e^4*f^4
*g^5 + 6*b^10*c^3*d^6*e^3*f^3*g^6 - 4*b^11*c^2*d^3*e^6*f^5*g^4 - 4*b^11*c^2
*d^4*e^5*f^4*g^5 - 4*b^11*c^2*d^5*e^4*f^3*g^6 + 16*a*b^3*c^9*d^7*e^2*f^7*g^
2 - 12*a*b^4*c^8*d^6*e^3*f^7*g^2 - 12*a*b^4*c^8*d^7*e^2*f^6*g^3 + 30*a*b^5*
c^7*d^5*e^4*f^7*g^2 + 30*a*b^5*c^7*d^6*e^3*f^6*g^3 + 30*a*b^5*c^7*d^7*e^2*f
^5*g^4 - 100*a*b^6*c^6*d^4*e^5*f^7*g^2 - 56*a*b^6*c^6*d^5*e^4*f^6*g^3 - 56*

a*b^6*c^6*d^6*e^3*f^5*g^4 - 100*a*b^6*c^6*d^7*e^2*f^4*g^5 + 62*a*b^7*c^5*d^3*e^6*f^7*g^2 + 128*a*b^7*c^5*d^4*e^5*f^6*g^3 + 42*a*b^7*c^5*d^5*e^4*f^5*g^4 + 128*a*b^7*c^5*d^6*e^3*f^4*g^5 + 62*a*b^7*c^5*d^7*e^2*f^3*g^6 + 4*a*b^8*c^4*d^2*e^7*f^7*g^2 - 76*a*b^8*c^4*d^3*e^6*f^6*g^3 - 48*a*b^8*c^4*d^4*e^5*f^5*g^4 - 48*a*b^8*c^4*d^5*e^4*f^4*g^5 - 76*a*b^8*c^4*d^6*e^3*f^3*g^6 + 4*a*b^8*c^4*d^7*e^2*f^2*g^7 - 6*a*b^9*c^3*d^2*e^7*f^6*g^3 + 28*a*b^9*c^3*d^3*e^6*f^5*g^4 - 20*a*b^9*c^3*d^4*e^5*f^4*g^5 + 28*a*b^9*c^3*d^5*e^4*f^3*g^6 - 6*a*b^9*c^3*d^6*e^3*f^2*g^7 + 4*a*b^10*c^2*d^2*e^7*f^5*g^4 + 14*a*b^10*c^2*d^3*e^6*f^4*g^5 + 14*a*b^10*c^2*d^4*e^5*f^3*g^6 + 4*a*b^10*c^2*d^5*e^4*f^2*g^7 - 32*a^2*b*c^10*d^7*e^2*f^7*g^2 + 48*a^2*b^2*c^9*d^5*e^4*f^8*g + 48*a^2*b^2*c^9*d^8*e*f^5*g^4 - 168*a^2*b^3*c^8*d^4*e^5*f^8*g - 168*a^2*b^3*c^8*d^8*e*f^4*g^5 + 80*a^2*b^4*c^7*d^3*e^6*f^8*g + 80*a^2*b^4*c^7*d^8*e*f^3*g^6 + 27*a^2*b^5*c^6*d^2*e^7*f^8*g + 27*a^2*b^5*c^6*d^8*e*f^2*g^7 + 4*a^2*b^7*c^4*d*e^8*f^7*g^2 + 4*a^2*b^7*c^4*d^7*e^2*f*g^8 - 6*a^2*b^8*c^3*d*e^8*f^6*g^3 - 6*a^2*b^8*c^3*d^6*e^3*f*g^8 + 4*a^2*b^9*c^2*d*e^8*f^5*g^4 + 4*a^2*b^9*c^2*d^5*e^4*f*g^8 + 16*a^2*b^10*c*d^2*e^7*f^3*g^6 + 16*a^2*b^10*c*d^3*e^6*f^2*g^7 + 224*a^3*b*c^9*d^5*e^4*f^7*g^2 - 288*a^3*b*c^9*d^6*e^3*f^6*g^3 + 224*a^3*b*c^9*d^7*e^2*f^5*g^4 - 32*a^3*b^2*c^8*d^3*e^6*f^8*g - 32*a^3*b^2*c^8*d^8*e*f^3*g^6 - 168*a^3*b^3*c^7*d^2*e^7*f^8*g - 168*a^3*b^3*c^7*d^8*e*f^2*g^7 - 14*a^3*b^5*c^5*d*e^8*f^7*g^2 - 14*a^3*b^5*c^5*d^7*e^2*f*g^8 + 40*a^3*b^6*c^4*d*e^8*f^6*g^3 + 40*a^3*b^6*c^4*d^6*e^3*f*g^8 - 44*a^3*b^7*c^3*d*e^8*f^5*g^4 - 44*a^3*b^7*c^3*d^5*e^4*f*g^8 + 24*a^3*b^8*c^2*d*e^8*f^4*g^5 + 24*a^3*b^8*c^2*d^4*e^5*f*g^8 - 30*a^3*b^9*c*d^2*e^7*f^2*g^7 + 544*a^4*b*c^8*d^3*e^6*f^7*g^2 + 256*a^4*b*c^8*d^4*e^5*f^6*g^3 + 1632*a^4*b*c^8*d^5*e^4*f^5*g^4 + 256*a^4*b*c^8*d^6*e^3*f^4*g^5 + 544*a^4*b*c^8*d^7*e^2*f^3*g^6 - 80*a^4*b^3*c^6*d*e^8*f^7*g^2 - 80*a^4*b^3*c^6*d^7*e^2*f*g^8 - 60*a^4*b^4*c^5*d*e^8*f^6*g^3 - 60*a^4*b^4*c^5*d^6*e^3*f*g^8 + 234*a^4*b^5*c^4*d*e^8*f^5*g^4 + 234*a^4*b^5*c^4*d^5*e^4*f*g^8 - 208*a^4*b^6*c^3*d*e^8*f^4*g^5 - 208*a^4*b^6*c^3*d^4*e^5*f*g^8 + 50*a^4*b^7*c^2*d*e^8*f^3*g^6 + 50*a^4*b^7*c^2*d^3*e^6*f*g^8 + 416*a^5*b*c^7*d^2*e^7*f^6*g^3 + 2592*a^5*b*c^7*d^3*e^6*f^5*g^4 + 1056*a^5*b*c^7*d^4*e^5*f^4*g^5 + 2592*a^5*b*c^7*d^5*e^4*f^3*g^6 + 416*a^5*b*c^7*d^6*e^3*f^2*g^7 + 96*a^5*b^2*c^6*d*e^8*f^6*g^3 + 96*a^5*b^2*c^6*d^6*e^3*f*g^8 - 784*a^5*b^3*c^5*d*e^8*f^5*g^4 - 784*a^5*b^3*c^5*d^5*e^4*f*g^8 + 732*a^5*b^4*c^4*d*e^8*f^4*g^5 + 732*a^5*b^4*c^4*d^4*e^5*f*g^8 - 18*a^5*b^5*c^3*d*e^8*f^3*g^6 - 18*a^5*b^5*c^3*d^3*e^6*f*g^8 - 184*a^5*b^6*c^2*d*e^8*f^2*g^7 - 184*a^5*b^6*c^2*d^2*e^7*f*g^8 + 1024*a^6*b*c^6*d^2*e^7*f^4*g^5 + 3552*a^6*b*c^6*d^3*e^6*f^3*g^6 + 1024*a^6*b*c^6*d^4*e^5*f^2*g^7 - 736*a^6*b^2*c^5*d*e^8*f^4*g^5 - 736*a^6*b^2*c^5*d^4*e^5*f*g^8 - 720*a^6*b^3*c^4*d*e^8*f^3*g^6 - 720*a^6*b^3*c^4*d^3*e^6*f*g^8 + 684*a^6*b^4*c^3*d*e^8*f^2*g^7 + 684*a^6*b^4*c^3*d^2*e^7*f*g^8 + 992*a^7*b*c^5*d^2*e^7*f^2*g^7 - 736*a^7*b^2*c^4*d*e^8*f^2*g^7 - 736*a^7*b^2*c^4*d^2*e^7*f*g^8 - 10*a^5*b^7*c*d*e^8*f*g^8 + 608*a^8*b*c^4*d*e^8*f*g^8 - 144*a^2*b^3*c^8*d^5*e^4*f^7*g^2 + 48*a^2*b^3*c^8*d^6*e^3*f^6*g^3 - 144*a^2*b^3*c^8*d^7*e^2*f^5*g^4 + 524*a^2*b^4*c^7*d^4*e^5*f^7*g^2 + 44*a^2*b^4*c^7*d^5*e^4*f^6*g^3 + 44*a^2*b^4*c^7*d^6*e^3*f^5*g^4 + 524*a^2*b^4*c^7*d^7*e^2*f^4*g^5 - 270*a^2*b^5*c^6*d^3*e^6*f^7*g^2 - 480*a^2*b^5*c^6*d^4*e^5*f^6*g^3 + 246*a^2*b^5*c^6*d^5*e^4*f^5*g^4 - 480*a^2*b^5*c^6*d^6*e^3*f^4*g^5 - 270*a^2*b^5*c^6*d^7*e^2*f^3*g^6 - 90*a^2*b^6*c^5*d^2*e^7*f^7*g^2 + 286*a^2*b^6*c^5*d^3*e^6*f^6*g^3 - 180*a^2*b^6*c^5*d^4*e^5*f^5*g^4 - 180*a^2*b^6*c^5*d^5*e^4*f^4*g^5 + 286*a^2*b^6*c^5*d^6*e^3*f^3*g^6 - 90*a^2*b^6*c^5*d^7*e^2*f^2*g^7 + 104*a^2*b^7*c^4*d^2*e^7*f^6*g^3 + 4*a^2*b^7*c^4*d^3*e^6*f^5*g^4 + 520*a^2*b^7*c^4*d^4*e^5*f^4*g^5 + 4*a^2*b^7*c^4*d^5*e^4*f^3*g^6 + 104*a^2*b^7*c^4*d^6*e^3*f^2*g^7 - 30*a^2*b^8*c^3*d^2*e^7*f^5*g^4 - 186*a^2*b^8*c^3*d^3*e^6*f^4*g^5 - 186*a^2*b^8*c^3*d^4*e^5*f^3*g^6 - 30*a^2*b^8*c^3*d^5*e^4*f^2*g^7 - 27*a^2*b^9*c^2*d^2*e^7*f^4*g^5 + 70*a^2*b^9*c^2*d^3*e^6*f^3*g^6 - 27*a^2*b^9*c^2*d^4*e^5*f^2*g^7 - 928*a^3*b^2*c^8*d^4*e^5*f^7*g^2 + 288*a^3*b^2*c^8*d^5*e^4*f^6*g^3 + 288*a^3*b^2*c^8*d^6*e^3*f^5*g^4 - 928*a^3*b^2*c^8*d^7*e^2*f^4*g^5 + 208*a^3*b^3*c^7*d^3*e^6*f^7*g^2 + 512*a^3*b^3*c^7*d^4*e^5*f^6*g^3 - 1424*a^3*b^3*c^7*d^5*e^4*f^5*g^4 + 512*a^3*b^3*c^7*d^6*e^3*f^4*g^5 + 208*a^3*b^3*c^7*d^7*e^2*f^3*g^6 + 540*a^3*b

^4*c^6*d^2*e^7*f^7*g^2 - 228*a^3*b^4*c^6*d^3*e^6*f^6*g^3 + 1428*a^3*b^4*c^6
*d^4*e^5*f^5*g^4 + 1428*a^3*b^4*c^6*d^5*e^4*f^4*g^5 - 228*a^3*b^4*c^6*d^6*e
^3*f^3*g^6 + 540*a^3*b^4*c^6*d^7*e^2*f^2*g^7 - 518*a^3*b^5*c^5*d^2*e^7*f^6*
g^3 - 190*a^3*b^5*c^5*d^3*e^6*f^5*g^4 - 2110*a^3*b^5*c^5*d^4*e^5*f^4*g^5 -
190*a^3*b^5*c^5*d^5*e^4*f^3*g^6 - 518*a^3*b^5*c^5*d^6*e^3*f^2*g^7 - 88*a^3*
b^6*c^4*d^2*e^7*f^5*g^4 + 368*a^3*b^6*c^4*d^3*e^6*f^4*g^5 + 368*a^3*b^6*c^4
*d^4*e^5*f^3*g^6 - 88*a^3*b^6*c^4*d^5*e^4*f^2*g^7 + 404*a^3*b^7*c^3*d^2*e^7
*f^4*g^5 + 12*a^3*b^7*c^3*d^3*e^6*f^3*g^6 + 404*a^3*b^7*c^3*d^4*e^5*f^2*g^7
 - 140*a^3*b^8*c^2*d^2*e^7*f^3*g^6 - 140*a^3*b^8*c^2*d^3*e^6*f^2*g^7 - 1024
*a^4*b^2*c^7*d^2*e^7*f^7*g^2 - 128*a^4*b^2*c^7*d^3*e^6*f^6*g^3 - 2016*a^4*b
^2*c^7*d^4*e^5*f^5*g^4 - 2016*a^4*b^2*c^7*d^5*e^4*f^4*g^5 - 128*a^4*b^2*c^7
*d^6*e^3*f^3*g^6 - 1024*a^4*b^2*c^7*d^7*e^2*f^2*g^7 + 688*a^4*b^3*c^6*d^2*e
^7*f^6*g^3 - 720*a^4*b^3*c^6*d^3*e^6*f^5*g^4 + 2160*a^4*b^3*c^6*d^4*e^5*f^4
*g^5 - 720*a^4*b^3*c^6*d^5*e^4*f^3*g^6 + 688*a^4*b^3*c^6*d^6*e^3*f^2*g^7 +
1124*a^4*b^4*c^5*d^2*e^7*f^5*g^4 + 1060*a^4*b^4*c^5*d^3*e^6*f^4*g^5 + 1060*
a^4*b^4*c^5*d^4*e^5*f^3*g^6 + 1124*a^4*b^4*c^5*d^5*e^4*f^2*g^7 - 1616*a^4*b
^5*c^4*d^2*e^7*f^4*g^5 - 674*a^4*b^5*c^4*d^3*e^6*f^3*g^6 - 1616*a^4*b^5*c^4
*d^4*e^5*f^2*g^7 + 186*a^4*b^6*c^3*d^2*e^7*f^3*g^6 + 186*a^4*b^6*c^3*d^3*e^
6*f^2*g^7 + 334*a^4*b^7*c^2*d^2*e^7*f^2*g^7 - 2208*a^5*b^2*c^6*d^2*e^7*f^5*
g^4 - 2592*a^5*b^2*c^6*d^3*e^6*f^4*g^5 - 2592*a^5*b^2*c^6*d^4*e^5*f^3*g^6 -
 2208*a^5*b^2*c^6*d^5*e^4*f^2*g^7 + 1728*a^5*b^3*c^5*d^2*e^7*f^4*g^5 - 304*
a^5*b^3*c^5*d^3*e^6*f^3*g^6 + 1728*a^5*b^3*c^5*d^4*e^5*f^2*g^7 + 1108*a^5*b
^4*c^4*d^2*e^7*f^3*g^6 + 1108*a^5*b^4*c^4*d^3*e^6*f^2*g^7 - 1170*a^5*b^5*c^
3*d^2*e^7*f^2*g^7 - 2432*a^6*b^2*c^5*d^2*e^7*f^3*g^6 - 2432*a^6*b^2*c^5*d^3
*e^6*f^2*g^7 + 1008*a^6*b^3*c^4*d^2*e^7*f^2*g^7 - 8*a*b^3*c^9*d^6*e^3*f^8*g
 - 8*a*b^3*c^9*d^8*e*f^6*g^3 + 27*a*b^5*c^7*d^4*e^5*f^8*g + 27*a*b^5*c^7*d^
8*e*f^4*g^5 - 18*a*b^6*c^6*d^3*e^6*f^8*g - 18*a*b^6*c^6*d^8*e*f^3*g^6 - a*b
^7*c^5*d^2*e^7*f^8*g - a*b^7*c^5*d^8*e*f^2*g^7 - a*b^11*c*d^2*e^7*f^4*g^5 -
 10*a*b^11*c*d^3*e^6*f^3*g^6 - a*b^11*c*d^4*e^5*f^2*g^7 + 16*a^2*b*c^10*d^6
*e^3*f^8*g + 16*a^2*b*c^10*d^8*e*f^6*g^3 - a^2*b^6*c^5*d*e^8*f^8*g - a^2*b^
6*c^5*d^8*e*f*g^8 - a^2*b^10*c*d*e^8*f^4*g^5 - a^2*b^10*c*d^4*e^5*f*g^8 + 3
04*a^3*b*c^9*d^4*e^5*f^8*g + 304*a^3*b*c^9*d^8*e*f^4*g^5 - 6*a^3*b^9*c*d*e^
8*f^3*g^6 - 6*a^3*b^9*c*d^3*e^6*f*g^8 + 304*a^4*b*c^8*d^2*e^7*f^8*g + 304*a
^4*b*c^8*d^8*e*f^2*g^7 + 48*a^4*b^2*c^7*d*e^8*f^8*g + 48*a^4*b^2*c^7*d^8*e*
f*g^8 + 16*a^4*b^8*c*d*e^8*f^2*g^7 + 16*a^4*b^8*c*d^2*e^7*f*g^8 + 288*a^5*b
*c^7*d*e^8*f^7*g^2 + 288*a^5*b*c^7*d^7*e^2*f*g^8 + 1184*a^6*b*c^6*d*e^8*f^5
*g^4 + 1184*a^6*b*c^6*d^5*e^4*f*g^8 + 118*a^6*b^5*c^2*d*e^8*f*g^8 + 1504*a^
7*b*c^5*d*e^8*f^3*g^6 + 1504*a^7*b*c^5*d^3*e^6*f*g^8 - 464*a^7*b^3*c^3*d*e^
8*f*g^8)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^
4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4
 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2
*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^
2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e
^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^
4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3
*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2
*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*
d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*
d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d
^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^
4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f
^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^
3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f
^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*
e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c
^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b
^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^
3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e
^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*

b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f
^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5
*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2
+ 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*
e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3) - (x
*(128*a^9*c^4*e^9*g^9 + 24*a^7*b^4*c^2*e^9*g^9 - 96*a^8*b^2*c^3*e^9*g^9 + 2
88*a^6*c^7*d^6*e^3*g^9 + 416*a^7*c^6*d^4*e^5*g^9 + 352*a^8*c^5*d^2*e^7*g^9
+ 288*a^6*c^7*e^9*f^6*g^3 + 416*a^7*c^6*e^9*f^4*g^5 + 352*a^8*c^5*e^9*f^2*g
^7 - 2*a^6*b^6*c*e^9*g^9 + 96*a^5*c^8*d^8*e*g^9 + 96*a^5*c^8*e^9*f^8*g + 6*
a^5*b^7*c*d*e^8*g^9 - 384*a^8*b*c^4*d*e^8*g^9 + 6*a^5*b^7*c*e^9*f*g^8 - 384
*a^8*b*c^4*e^9*f*g^8 + 64*a^8*c^5*d*e^8*f*g^8 - 2*a^2*b^6*c^5*d^8*e*g^9 - 2
*a^2*b^10*c*d^4*e^5*g^9 + 22*a^3*b^4*c^6*d^8*e*g^9 + 6*a^3*b^9*c*d^3*e^6*g^
9 - 80*a^4*b^2*c^7*d^8*e*g^9 - 8*a^4*b^8*c*d^2*e^7*g^9 - 416*a^5*b*c^7*d^7*
e^2*g^9 - 960*a^6*b*c^6*d^5*e^4*g^9 - 72*a^6*b^5*c^2*d*e^8*g^9 - 928*a^7*b*
c^5*d^3*e^6*g^9 + 288*a^7*b^3*c^3*d*e^8*g^9 - 2*a^2*b^6*c^5*e^9*f^8*g - 2*a
^2*b^10*c*e^9*f^4*g^5 + 22*a^3*b^4*c^6*e^9*f^8*g + 6*a^3*b^9*c*e^9*f^3*g^6
- 80*a^4*b^2*c^7*e^9*f^8*g - 8*a^4*b^8*c*e^9*f^2*g^7 - 416*a^5*b*c^7*e^9*f^
7*g^2 - 960*a^6*b*c^6*e^9*f^5*g^4 - 72*a^6*b^5*c^2*e^9*f*g^8 - 928*a^7*b*c^
5*e^9*f^3*g^6 + 288*a^7*b^3*c^3*e^9*f*g^8 - 32*a^2*c^11*d^6*e^3*f^8*g - 32*
a^2*c^11*d^8*e*f^6*g^3 + 32*a^3*c^10*d^4*e^5*f^8*g + 32*a^3*c^10*d^8*e*f^4*
g^5 + 160*a^4*c^9*d^2*e^7*f^8*g + 160*a^4*c^9*d^8*e*f^2*g^7 + 64*a^5*c^8*d*
e^8*f^7*g^2 + 64*a^5*c^8*d^7*e^2*f*g^8 + 192*a^6*c^7*d*e^8*f^5*g^4 + 192*a^
6*c^7*d^5*e^4*f*g^8 + 192*a^7*c^6*d*e^8*f^3*g^6 + 192*a^7*c^6*d^3*e^6*f*g^8
- 2*b^4*c^9*d^6*e^3*f^8*g - 2*b^4*c^9*d^8*e*f^6*g^3 + 6*b^5*c^8*d^5*e^4*f^
8*g + 6*b^5*c^8*d^8*e*f^5*g^4 - 8*b^6*c^7*d^4*e^5*f^8*g - 8*b^6*c^7*d^8*e*f
^4*g^5 + 6*b^7*c^6*d^3*e^6*f^8*g + 6*b^7*c^6*d^8*e*f^3*g^6 - 2*b^8*c^5*d^2*
e^7*f^8*g - 2*b^8*c^5*d^8*e*f^2*g^7 - 2*b^12*c*d^2*e^7*f^4*g^5 + 2*b^12*c*d
^3*e^6*f^3*g^6 - 2*b^12*c*d^4*e^5*f^2*g^7 + 8*a^2*b^7*c^4*d^7*e^2*g^9 - 12*
a^2*b^8*c^3*d^6*e^3*g^9 + 8*a^2*b^9*c^2*d^5*e^4*g^9 - 90*a^3*b^5*c^5*d^7*e^
2*g^9 + 132*a^3*b^6*c^4*d^6*e^3*g^9 - 76*a^3*b^7*c^3*d^5*e^4*g^9 + 6*a^3*b^
8*c^2*d^4*e^5*g^9 + 336*a^4*b^3*c^6*d^7*e^2*g^9 - 462*a^4*b^4*c^5*d^6*e^3*g
^9 + 164*a^4*b^5*c^4*d^5*e^4*g^9 + 106*a^4*b^6*c^3*d^4*e^5*g^9 - 56*a^4*b^7
*c^2*d^3*e^6*g^9 + 432*a^5*b^2*c^6*d^6*e^3*g^9 + 288*a^5*b^3*c^5*d^5*e^4*g^
9 - 598*a^5*b^4*c^4*d^4*e^5*g^9 + 102*a^5*b^5*c^3*d^3*e^6*g^9 + 90*a^5*b^6*
c^2*d^2*e^7*g^9 + 720*a^6*b^2*c^5*d^4*e^5*g^9 + 336*a^6*b^3*c^4*d^3*e^6*g^9
- 314*a^6*b^4*c^3*d^2*e^7*g^9 + 240*a^7*b^2*c^4*d^2*e^7*g^9 + 8*a^2*b^7*c^
4*e^9*f^7*g^2 - 12*a^2*b^8*c^3*e^9*f^6*g^3 + 8*a^2*b^9*c^2*e^9*f^5*g^4 - 90
*a^3*b^5*c^5*e^9*f^7*g^2 + 132*a^3*b^6*c^4*e^9*f^6*g^3 - 76*a^3*b^7*c^3*e^9
*f^5*g^4 + 6*a^3*b^8*c^2*e^9*f^4*g^5 + 336*a^4*b^3*c^6*e^9*f^7*g^2 - 462*a^
4*b^4*c^5*e^9*f^6*g^3 + 164*a^4*b^5*c^4*e^9*f^5*g^4 + 106*a^4*b^6*c^3*e^9*f
^4*g^5 - 56*a^4*b^7*c^2*e^9*f^3*g^6 + 432*a^5*b^2*c^6*e^9*f^6*g^3 + 288*a^5
*b^3*c^5*e^9*f^5*g^4 - 598*a^5*b^4*c^4*e^9*f^4*g^5 + 102*a^5*b^5*c^3*e^9*f^
3*g^6 + 90*a^5*b^6*c^2*e^9*f^2*g^7 + 720*a^6*b^2*c^5*e^9*f^4*g^5 + 336*a^6*
b^3*c^4*e^9*f^3*g^6 - 314*a^6*b^4*c^3*e^9*f^2*g^7 + 240*a^7*b^2*c^4*e^9*f^2
*g^7 + 64*a^2*c^11*d^7*e^2*f^7*g^2 + 192*a^3*c^10*d^5*e^4*f^7*g^2 - 320*a^3
*c^10*d^6*e^3*f^6*g^3 + 192*a^3*c^10*d^7*e^2*f^5*g^4 + 192*a^4*c^9*d^3*e^6*
f^7*g^2 - 256*a^4*c^9*d^4*e^5*f^6*g^3 + 576*a^4*c^9*d^5*e^4*f^5*g^4 - 256*a
^4*c^9*d^6*e^3*f^4*g^5 + 192*a^4*c^9*d^7*e^2*f^3*g^6 + 320*a^5*c^8*d^2*e^7*
f^6*g^3 + 576*a^5*c^8*d^3*e^6*f^5*g^4 - 192*a^5*c^8*d^4*e^5*f^4*g^5 + 576*a
^5*c^8*d^5*e^4*f^3*g^6 + 320*a^5*c^8*d^6*e^3*f^2*g^7 + 512*a^6*c^7*d^2*e^7*
f^4*g^5 + 576*a^6*c^7*d^3*e^6*f^3*g^6 + 512*a^6*c^7*d^4*e^5*f^2*g^7 + 704*a
^7*c^6*d^2*e^7*f^2*g^7 + 4*b^4*c^9*d^7*e^2*f^7*g^2 - 6*b^5*c^8*d^6*e^3*f^7*
g^2 - 6*b^5*c^8*d^7*e^2*f^6*g^3 - 6*b^6*c^7*d^5*e^4*f^7*g^2 + 26*b^6*c^7*d^
6*e^3*f^6*g^3 - 6*b^6*c^7*d^7*e^2*f^5*g^4 + 22*b^7*c^6*d^4*e^5*f^7*g^2 - 22
*b^7*c^6*d^5*e^4*f^6*g^3 - 22*b^7*c^6*d^6*e^3*f^5*g^4 + 22*b^7*c^6*d^7*e^2*
f^4*g^5 - 22*b^8*c^5*d^3*e^6*f^7*g^2 - 12*b^8*c^5*d^4*e^5*f^6*g^3 + 42*b^8*
c^5*d^5*e^4*f^5*g^4 - 12*b^8*c^5*d^6*e^3*f^4*g^5 - 22*b^8*c^5*d^7*e^2*f^3*g
^6 + 8*b^9*c^4*d^2*e^7*f^7*g^2 + 28*b^9*c^4*d^3*e^6*f^6*g^3 - 16*b^9*c^4*d^
4*e^5*f^5*g^4 - 16*b^9*c^4*d^5*e^4*f^4*g^5 + 28*b^9*c^4*d^6*e^3*f^3*g^6 + 8

*b^9*c^4*d^7*e^2*f^2*g^7 - 12*b^10*c^3*d^2*e^7*f^6*g^3 - 12*b^10*c^3*d^3*e^
6*f^5*g^4 + 18*b^10*c^3*d^4*e^5*f^4*g^5 - 12*b^10*c^3*d^5*e^4*f^3*g^6 - 12*
b^10*c^3*d^6*e^3*f^2*g^7 + 8*b^11*c^2*d^2*e^7*f^5*g^4 - 2*b^11*c^2*d^3*e^6*
f^4*g^5 - 2*b^11*c^2*d^4*e^5*f^3*g^6 + 8*b^11*c^2*d^5*e^4*f^2*g^7 - 32*a*b^
2*c^10*d^7*e^2*f^7*g^2 + 48*a*b^3*c^9*d^6*e^3*f^7*g^2 + 48*a*b^3*c^9*d^7*e^
2*f^6*g^3 + 60*a*b^4*c^8*d^5*e^4*f^7*g^2 - 228*a*b^4*c^8*d^6*e^3*f^6*g^3 +
60*a*b^4*c^8*d^7*e^2*f^5*g^4 - 214*a*b^5*c^7*d^4*e^5*f^7*g^2 + 194*a*b^5*c^
7*d^5*e^4*f^6*g^3 + 194*a*b^5*c^7*d^6*e^3*f^5*g^4 - 214*a*b^5*c^7*d^7*e^2*f
^4*g^5 + 216*a*b^6*c^6*d^3*e^6*f^7*g^2 + 148*a*b^6*c^6*d^4*e^5*f^6*g^3 - 40
8*a*b^6*c^6*d^5*e^4*f^5*g^4 + 148*a*b^6*c^6*d^6*e^3*f^4*g^5 + 216*a*b^6*c^6
*d^7*e^2*f^3*g^6 - 62*a*b^7*c^5*d^2*e^7*f^7*g^2 - 302*a*b^7*c^5*d^3*e^6*f^6
*g^3 + 150*a*b^7*c^5*d^4*e^5*f^5*g^4 + 150*a*b^7*c^5*d^5*e^4*f^4*g^5 - 302*
a*b^7*c^5*d^6*e^3*f^3*g^6 - 62*a*b^7*c^5*d^7*e^2*f^2*g^7 + 100*a*b^8*c^4*d^
2*e^7*f^6*g^3 + 136*a*b^8*c^4*d^3*e^6*f^5*g^4 - 200*a*b^8*c^4*d^4*e^5*f^4*g
^5 + 136*a*b^8*c^4*d^5*e^4*f^3*g^6 + 100*a*b^8*c^4*d^6*e^3*f^2*g^7 - 68*a*b
^9*c^3*d^2*e^7*f^5*g^4 + 32*a*b^9*c^3*d^3*e^6*f^4*g^5 + 32*a*b^9*c^3*d^4*e^
5*f^3*g^6 - 68*a*b^9*c^3*d^5*e^4*f^2*g^7 + 14*a*b^10*c^2*d^2*e^7*f^4*g^5 -
32*a*b^10*c^2*d^3*e^6*f^3*g^6 + 14*a*b^10*c^2*d^4*e^5*f^2*g^7 - 96*a^2*b*c^
10*d^6*e^3*f^7*g^2 - 96*a^2*b*c^10*d^7*e^2*f^6*g^3 - 144*a^2*b^2*c^9*d^4*e^
5*f^8*g - 144*a^2*b^2*c^9*d^8*e*f^4*g^5 + 128*a^2*b^3*c^8*d^3*e^6*f^8*g + 1
28*a^2*b^3*c^8*d^8*e*f^3*g^6 - 6*a^2*b^4*c^7*d^2*e^7*f^8*g - 6*a^2*b^4*c^7*
d^8*e*f^2*g^7 + 174*a^2*b^6*c^5*d*e^8*f^7*g^2 + 174*a^2*b^6*c^5*d^7*e^2*f*g
^8 - 260*a^2*b^7*c^4*d*e^8*f^6*g^3 - 260*a^2*b^7*c^4*d^6*e^3*f*g^8 + 156*a^
2*b^8*c^3*d*e^8*f^5*g^4 + 156*a^2*b^8*c^3*d^5*e^4*f*g^8 - 18*a^2*b^9*c^2*d*
e^8*f^4*g^5 - 18*a^2*b^9*c^2*d^4*e^5*f*g^8 - 6*a^2*b^10*c*d^2*e^7*f^2*g^7 -
608*a^3*b*c^9*d^4*e^5*f^7*g^2 + 288*a^3*b*c^9*d^5*e^4*f^6*g^3 + 288*a^3*b*
c^9*d^6*e^3*f^5*g^4 - 608*a^3*b*c^9*d^7*e^2*f^4*g^5 - 112*a^3*b^2*c^8*d^2*e
^7*f^8*g - 112*a^3*b^2*c^8*d^8*e*f^2*g^7 - 620*a^3*b^4*c^6*d*e^8*f^7*g^2 -
620*a^3*b^4*c^6*d^7*e^2*f*g^8 + 894*a^3*b^5*c^5*d*e^8*f^6*g^3 + 894*a^3*b^5
*c^5*d^6*e^3*f*g^8 - 384*a^3*b^6*c^4*d*e^8*f^5*g^4 - 384*a^3*b^6*c^4*d^5*e^
4*f*g^8 - 140*a^3*b^7*c^3*d*e^8*f^4*g^5 - 140*a^3*b^7*c^3*d^4*e^5*f*g^8 + 9
2*a^3*b^8*c^2*d*e^8*f^3*g^6 + 92*a^3*b^8*c^2*d^3*e^6*f*g^8 - 928*a^4*b*c^8*
d^2*e^7*f^7*g^2 - 160*a^4*b*c^8*d^3*e^6*f^6*g^3 - 672*a^4*b*c^8*d^4*e^5*f^5
*g^4 - 672*a^4*b*c^8*d^5*e^4*f^4*g^5 - 160*a^4*b*c^8*d^6*e^3*f^3*g^6 - 928*
a^4*b*c^8*d^7*e^2*f^2*g^7 + 704*a^4*b^2*c^7*d*e^8*f^7*g^2 + 704*a^4*b^2*c^7
*d^7*e^2*f*g^8 - 816*a^4*b^3*c^6*d*e^8*f^6*g^3 - 816*a^4*b^3*c^6*d^6*e^3*f*
g^8 - 308*a^4*b^4*c^5*d*e^8*f^5*g^4 - 308*a^4*b^4*c^5*d^5*e^4*f*g^8 + 898*a
^4*b^5*c^4*d*e^8*f^4*g^5 + 898*a^4*b^5*c^4*d^4*e^5*f*g^8 - 150*a^4*b^6*c^3*
d*e^8*f^3*g^6 - 150*a^4*b^6*c^3*d^3*e^6*f*g^8 - 154*a^4*b^7*c^2*d*e^8*f^2*g
^7 - 154*a^4*b^7*c^2*d^2*e^7*f*g^8 - 1824*a^5*b*c^7*d^2*e^7*f^5*g^4 - 1056*
a^5*b*c^7*d^3*e^6*f^4*g^5 - 1056*a^5*b*c^7*d^4*e^5*f^3*g^6 - 1824*a^5*b*c^7
*d^5*e^4*f^2*g^7 + 1440*a^5*b^2*c^6*d*e^8*f^5*g^4 + 1440*a^5*b^2*c^6*d^5*e^
4*f*g^8 - 976*a^5*b^3*c^5*d*e^8*f^4*g^5 - 976*a^5*b^3*c^5*d^4*e^5*f*g^8 - 6
44*a^5*b^4*c^4*d*e^8*f^3*g^6 - 644*a^5*b^4*c^4*d^3*e^6*f*g^8 + 498*a^5*b^5*
c^3*d*e^8*f^2*g^7 + 498*a^5*b^5*c^3*d^2*e^7*f*g^8 - 1888*a^6*b*c^6*d^2*e^7*
f^3*g^6 - 1888*a^6*b*c^6*d^3*e^6*f^2*g^7 + 1600*a^6*b^2*c^5*d*e^8*f^3*g^6 +
1600*a^6*b^2*c^5*d^3*e^6*f*g^8 - 176*a^6*b^3*c^4*d*e^8*f^2*g^7 - 176*a^6*b
^3*c^4*d^2*e^7*f*g^8 + 4*a*b^7*c^5*d*e^8*f^8*g + 4*a*b^7*c^5*d^8*e*f*g^8 +
4*a*b^11*c*d*e^8*f^4*g^5 + 4*a*b^11*c*d^4*e^5*f*g^8 - 160*a^4*b*c^8*d*e^8*f
^8*g - 160*a^4*b*c^8*d^8*e*f*g^8 - 14*a^4*b^8*c*d*e^8*f*g^8 - 192*a^2*b^2*c
^9*d^5*e^4*f^7*g^2 + 576*a^2*b^2*c^9*d^6*e^3*f^6*g^3 - 192*a^2*b^2*c^9*d^7*
e^2*f^5*g^4 + 656*a^2*b^3*c^8*d^4*e^5*f^7*g^2 - 496*a^2*b^3*c^8*d^5*e^4*f^6
*g^3 - 496*a^2*b^3*c^8*d^6*e^3*f^5*g^4 + 656*a^2*b^3*c^8*d^7*e^2*f^4*g^5 -
660*a^2*b^4*c^7*d^3*e^6*f^7*g^2 - 624*a^2*b^4*c^7*d^4*e^5*f^6*g^3 + 1284*a^
2*b^4*c^7*d^5*e^4*f^5*g^4 - 624*a^2*b^4*c^7*d^6*e^3*f^4*g^5 - 660*a^2*b^4*c
^7*d^7*e^2*f^3*g^6 + 54*a^2*b^5*c^6*d^2*e^7*f^7*g^2 + 1062*a^2*b^5*c^6*d^3*
e^6*f^6*g^3 - 474*a^2*b^5*c^6*d^4*e^5*f^5*g^4 - 474*a^2*b^5*c^6*d^5*e^4*f^4
*g^5 + 1062*a^2*b^5*c^6*d^6*e^3*f^3*g^6 + 54*a^2*b^5*c^6*d^7*e^2*f^2*g^7 -
130*a^2*b^6*c^5*d^2*e^7*f^6*g^3 - 482*a^2*b^6*c^5*d^3*e^6*f^5*g^4 + 850*a^2

*b^6*c^5*d^4*e^5*f^4*g^5 - 482*a^2*b^6*c^5*d^5*e^4*f^3*g^6 - 130*a^2*b^6*c^5*d^6*e^3*f^2*g^7 + 108*a^2*b^7*c^4*d^2*e^7*f^5*g^4 - 228*a^2*b^7*c^4*d^3*e^6*f^4*g^5 - 228*a^2*b^7*c^4*d^4*e^5*f^3*g^6 + 108*a^2*b^7*c^4*d^5*e^4*f^2*g^7 - 18*a^2*b^8*c^3*d^2*e^7*f^4*g^5 + 192*a^2*b^8*c^3*d^3*e^6*f^3*g^6 - 18*a^2*b^8*c^3*d^4*e^5*f^2*g^7 - 2*a^2*b^9*c^2*d^2*e^7*f^3*g^6 - 2*a^2*b^9*c^2*d^3*e^6*f^2*g^7 + 544*a^3*b^2*c^8*d^3*e^6*f^7*g^2 + 960*a^3*b^2*c^8*d^4*e^5*f^6*g^3 - 1440*a^3*b^2*c^8*d^5*e^4*f^5*g^4 + 960*a^3*b^2*c^8*d^6*e^3*f^4*g^5 + 544*a^3*b^2*c^8*d^7*e^2*f^3*g^6 + 496*a^3*b^3*c^7*d^2*e^7*f^7*g^2 - 1168*a^3*b^3*c^7*d^3*e^6*f^6*g^3 + 688*a^3*b^3*c^7*d^4*e^5*f^5*g^4 + 688*a^3*b^3*c^7*d^5*e^4*f^4*g^5 - 1168*a^3*b^3*c^7*d^6*e^3*f^3*g^6 + 496*a^3*b^3*c^7*d^7*e^2*f^2*g^7 - 668*a^3*b^4*c^6*d^2*e^7*f^6*g^3 + 436*a^3*b^4*c^6*d^3*e^6*f^5*g^4 - 1820*a^3*b^4*c^6*d^4*e^5*f^4*g^5 + 436*a^3*b^4*c^6*d^5*e^4*f^3*g^6 - 668*a^3*b^4*c^6*d^6*e^3*f^2*g^7 + 238*a^3*b^5*c^5*d^2*e^7*f^5*g^4 + 734*a^3*b^5*c^5*d^3*e^6*f^4*g^5 + 734*a^3*b^5*c^5*d^4*e^5*f^3*g^6 + 238*a^3*b^5*c^5*d^5*e^4*f^2*g^7 + 144*a^3*b^6*c^4*d^2*e^7*f^4*g^5 - 416*a^3*b^6*c^4*d^3*e^6*f^3*g^6 + 144*a^3*b^6*c^4*d^4*e^5*f^2*g^7 - 156*a^3*b^7*c^3*d^2*e^7*f^3*g^6 - 156*a^3*b^7*c^3*d^3*e^6*f^2*g^7 + 44*a^3*b^8*c^2*d^2*e^7*f^2*g^7 + 1344*a^4*b^2*c^7*d^2*e^7*f^6*g^3 + 192*a^4*b^2*c^7*d^3*e^6*f^5*g^4 + 1920*a^4*b^2*c^7*d^4*e^5*f^4*g^5 + 192*a^4*b^2*c^7*d^5*e^4*f^3*g^6 + 1344*a^4*b^2*c^7*d^6*e^3*f^2*g^7 + 80*a^4*b^3*c^6*d^2*e^7*f^5*g^4 - 560*a^4*b^3*c^6*d^3*e^6*f^4*g^5 - 560*a^4*b^3*c^6*d^4*e^5*f^3*g^6 + 80*a^4*b^3*c^6*d^5*e^4*f^2*g^7 - 1280*a^4*b^4*c^5*d^2*e^7*f^4*g^5 - 220*a^4*b^4*c^5*d^3*e^6*f^3*g^6 - 1280*a^4*b^4*c^5*d^4*e^5*f^2*g^7 + 714*a^4*b^5*c^4*d^2*e^7*f^3*g^6 + 714*a^4*b^5*c^4*d^3*e^6*f^2*g^7 + 58*a^4*b^6*c^3*d^2*e^7*f^2*g^7 + 2304*a^5*b^2*c^6*d^2*e^7*f^4*g^5 + 1248*a^5*b^2*c^6*d^3*e^6*f^3*g^6 + 2304*a^5*b^2*c^6*d^4*e^5*f^2*g^7 - 272*a^5*b^3*c^5*d^2*e^7*f^3*g^6 - 272*a^5*b^3*c^5*d^3*e^6*f^2*g^7 - 996*a^5*b^4*c^4*d^2*e^7*f^2*g^7 + 1600*a^6*b^2*c^5*d^2*e^7*f^2*g^7 + 16*a*b^2*c^10*d^6*e^3*f^8*g + 16*a*b^2*c^10*d^8*e*f^6*g^3 - 48*a*b^3*c^9*d^5*e^4*f^8*g - 48*a*b^3*c^9*d^8*e*f^5*g^4 + 66*a*b^4*c^8*d^4*e^5*f^8*g + 66*a*b^4*c^8*d^8*e*f^4*g^5 - 52*a*b^5*c^7*d^3*e^6*f^8*g - 52*a*b^5*c^7*d^8*e*f^3*g^6 + 14*a*b^6*c^6*d^2*e^7*f^8*g + 14*a*b^6*c^6*d^8*e*f^2*g^7 - 16*a*b^8*c^4*d*e^8*f^7*g^2 - 16*a*b^8*c^4*d^7*e^2*f*g^8 + 24*a*b^9*c^3*d*e^8*f^6*g^3 + 24*a*b^9*c^3*d^6*e^3*f*g^8 - 16*a*b^10*c^2*d*e^8*f^5*g^4 - 16*a*b^10*c^2*d^5*e^4*f*g^8 + 2*a*b^11*c*d^2*e^7*f^3*g^6 + 2*a*b^11*c*d^3*e^6*f^2*g^7 + 96*a^2*b*c^10*d^5*e^4*f^8*g + 96*a^2*b*c^10*d^8*e*f^5*g^4 - 42*a^2*b^5*c^6*d*e^8*f^8*g - 42*a^2*b^5*c^6*d^8*e*f*g^8 - 10*a^2*b^10*c*d*e^8*f^3*g^6 - 10*a^2*b^10*c*d^3*e^6*f*g^8 - 64*a^3*b*c^9*d^3*e^6*f^8*g - 64*a^3*b*c^9*d^8*e*f^3*g^6 + 144*a^3*b^3*c^7*d*e^8*f^8*g + 144*a^3*b^3*c^7*d^8*e*f*g^8 + 14*a^3*b^9*c*d*e^8*f^2*g^7 + 14*a^3*b^9*c*d^2*e^7*f*g^8 - 544*a^5*b*c^7*d*e^8*f^6*g^3 - 544*a^5*b*c^7*d^6*e^3*f*g^8 + 168*a^5*b^6*c^2*d*e^8*f*g^8 - 992*a^6*b*c^6*d*e^8*f^4*g^5 - 992*a^6*b*c^6*d^4*e^5*f*g^8 - 668*a^6*b^4*c^3*d*e^8*f*g^8 - 992*a^7*b*c^5*d*e^8*f^2*g^7 - 992*a^7*b*c^5*d^2*e^7*f*g^8 + 864*a^7*b^2*c^4*d*e^8*f*g^8))/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c

^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3)) - (48*a^6*b^2*c^3*e^8*g^8 - 12*a^5*b^4*c^2*e^8*g^8 - 64*a^7*c^4*e^8*g^8 + 40*a^4*c^7*d^6*e^2*g^8 + 80*a^5*c^6*d^4*e^4*g^8 - 24*a^6*c^5*d^2*e^6*g^8 + 40*a^4*c^7*e^8*f^6*g^2 + 80*a^5*c^6*e^8*f^4*g^4 - 24*a^6*c^5*e^8*f^2*g^6 + a^4*b^6*c*e^8*g^8 + a*b^5*c^5*d^7*e*g^8 + a*b^9*c*d^3*e^5*g^8 + 20*a^3*b*c^7*d^7*e*g^8 + a*b^5*c^5*e^8*f^7*g + a*b^9*c*e^8*f^3*g^5 + 20*a^3*b*c^7*e^8*f^7*g + 8*a*c^10*d^5*e^3*f^7*g + 8*a*c^10*d^7*e*f^5*g^3 + 8*a^3*c^8*d*e^7*f^7*g + 8*a^3*c^8*d^7*e*f*g^7 + 304*a^6*c^5*d*e^7*f*g^7 - b^6*c^5*d*e^7*f^7*g - b^6*c^5*d^7*e*f*g^7 - b^10*c*d*e^7*f^3*g^5 - b^10*c*d^3*e^5*f*g^7 - 4*a*b^6*c^4*d^6*e^2*g^8 + 6*a*b^7*c^3*d^5*e^3*g^8 - 4*a*b^8*c^2*d^4*e^4*g^8 - 9*a^2*b^3*c^6*d^7*e*g^8 - 2*a^2*b^8*c*d^2*e^6*g^8 - 88*a^4*b*c^6*d^5*e^3*g^8 - 172*a^5*b*c^5*d^3*e^5*g^8 - 4*a*b^6*c^4*e^8*f^6*g^2 + 6*a*b^7*c^3*e^8*f^5*g^3 - 4*a*b^8*c^2*e^8*f^4*g^4 - 9*a^2*b^3*c^6*e^8*f^7*g - 2*a^2*b^8*c*e^8*f^2*g^6 - 88*a^4*b*c^6*e^8*f^5*g^3 - 172*a^5*b*c^5*e^8*f^3*g^5 - 16*a*c^10*d^6*e^2*f^6*g^2 + 16*a^2*c^9*d^3*e^5*f^7*g + 16*a^2*c^9*d^7*e*f^3*g^5 + 192*a^4*c^7*d*e^7*f^5*g^3 + 192*a^4*c^7*d^5*e^3*f*g^7 + 488*a^5*c^6*d*e^7*f^3*g^5 + 488*a^5*c^6*d^3*e^5*f*g^7 - 2*b^2*c^9*d^5*e^3*f^7*g - 2*b^2*c^9*d^7*e*f^5*g^3 + 3*b^3*c^8*d^4*e^4*f^7*g + 3*b^3*c^8*d^7*e*f^4*g^4 + 4*b^7*c^4*d*e^7*f^6*g^2 + 4*b^7*c^4*d^6*e^2*f*g^7 - 6*b^8*c^3*d*e^7*f^5*g^3 - 6*b^8*c^3*d^5*e^3*f*g^7 + 4*b^9*c^2*d*e^7*f^4*g^4 + 4*b^9*c^2*d^4*e^4*f*g^7 - b^10*c*d^2*e^6*f^2*g^6 + 38*a^2*b^4*c^5*d^6*e^2*g^8 - 58*a^2*b^5*c^4*d^5*e^3*g^8 + 36*a^2*b^6*c^3*d^4*e^4*g^8 - 5*a^2*b^7*c^2*d^3*e^5*g^8 - 98*a^3*b^2*c^6*d^6*e^2*g^8 + 158*a^3*b^3*c^5*d^5*e^3*g^8 - 80*a^3*b^4*c^4*d^4*e^4*g^8 - 22*a^3*b^5*c^3*d^3*e^5*g^8 + 22*a^3*b^6*c^2*d^2*e^6*g^8 - 20*a^4*b^2*c^5*d^4*e^4*g^8 + 147*a^4*b^3*c^4*d^3*e^5*g^8 - 80*a^4*b^4*c^3*d^2*e^6*g^8 + 102*a^5*b^2*c^4*d^2*e^6*g^8 + 38*a^2*b^4*c^5*e^8*f^6*g^2 - 58*a^2*b^5*c^4*e^8*f^5*g^3 + 36*a^2*b^6*c^3*e^8*f^4*g^4 - 5*a^2*b^7*c^2*e^8*f^3*g^5 - 98*a^3*b^2*c^6*e^8*f^6*g^2 + 158*a^3*b^3*c^5*e^8*f^5*g^3 - 80*a^3*b^4*c^4*e^8*f^4*g^4 - 22*a^3*b^5*c^3*e^8*f^3*g^5 + 22*a^3*b^6*c^2*e^8*f^2*g^6 - 20*a^4*b^2*c^5*e^8*f^4*g^4 + 147*a^4*b^3*c^4*e^8*f^3*g^5 - 80*a^4*b^4*c^3*e^8*f^2*g^6 + 102*a^5*b^2*c^4*e^8*f^2*g^6 - 56*a^2*c^9*d^4*e^4*f^6*g^2 + 80*a^2*c^9*d^5*e^3*f^5*g^3 - 56*a^2*c^9*d^6*e^2*f^4*g^4 + 264*a^3*c^8*d^3*e^5*f^5*g^3 - 96*a^3*c^8*d^4*e^4*f^4*g^4 + 264*a^3*c^8*d^5*e^3*f^3*g^5 + 40*a^4*c^7*d^2*e^6*f^4*g^4 + 736*a^4*c^7*d^3*e^5*f^3*g^5 + 40*a^4*c^7*d^4*e^4*f^2*g^6 + 16*a^5*c^6*d^2*e^6*f^2*g^6 + 4*b^2*c^9*d^6*e^2*f^6*g^2 - 3*b^3*c^8*d^5*e^3*f^6*g^2 - 3*b^3*c^8*d^6*e^2*f^5*g^3 - 4*b^4*c^7*d^4*e^4*f^6*g^2 + 8*b^4*c^7*d^5*e^3*f^5*g^3 - 4*b^4*c^7*d^6*e^2*f^4*g^4 - b^6*c^5*d^2*e^6*f^6*g^2 - b^6*c^5*d^3*e^5*f^5*g^3 - b^6*c^5*d^4*e^4*f^4*g^4 - b^6*c^5*d^5*e^3*f^3*g^5 - b^6*c^5*d^6*e^2*f^2*g^6 + 4*b^7*c^4*d^2*e^6*f^5*g^3 + 4*b^7*c^4*d^3*e^5*f^4*g^4 + 4*b^7*c^4*d^4*e^4*f^3*g^5 + 4*b^7*c^4*d^5*e^3*f^2*g^6 - 6*b^8*c^3*d^2*e^6*f^4*g^4 - 6*b^8*c^3*d^3*e^5*f^3*g^5 - 6*b^8*c^3*d^4*e^4*f^2*g^6 + 4*b^9*c^2*d^2*e^6*f^3*g^5 + 4*b^9*c^2*d^3*e^5*f^2*g^6 + 30*a*b^2*c^8*d^4*e^4*f^6*g^2 - 52*a*b^2*c^8*d^5*e^3*f^5*g^3 + 30*a*b^2*c^8*d^6*e^2*f^4*g^4 - 6*a*b^3*c^7*d^3*e^5*f^6*g^2 + 8*a*b^3*c^7*d^4*e^4*f^5*g^3 + 8*a*b^3*c^7*d^5*e^3*f^4*g^4 - 6*a*b^3*c^7*d^6*e^2*f^3*g^5 + 20*a*b^4*c^6*d^2*e^6*f^6*g^2 + 26*a*b^4*c^6*d^3*e^5*f^5*g^3 - 28*a*b^4*c^6*d^4*e^4*f^4*g^4 + 26*a*b^4*c^6*d^5*e^3*f^3*g^5 + 20*a*b^4*c^6*d^6*e^2*f^2*g^6 - 61*a*b^5*c^5*d^2*e^6*f^5*g^3 - 43*a*b^5*c^5*d^3*e^5*f^4*g^4 - 43*a*b^5*c^5*d^4*e^4*f^3*g^5 - 61*a*b^5*c^5*d^5*e^3*f^2*g^6 + 80*a*b^6*c^4*d^2*e^6*f^4*g^4 + 68*a*b^6*c^4*d^3*e^5*f^3*g^5 + 80*a*b^6*c^4*d^4*e^4*f^2*g^6 - 44*a*b^7*c^3*d^2*e^6*f^3*g^5 - 44*a*b^7*c^3*d^3*e^5*f^2*g^6 + 4*a*b^8*c^2*d^2*e^6*f^2*g^6 + 24*

a^2*b*c^8*d^3*e^5*f^6*g^2 - 32*a^2*b*c^8*d^4*e^4*f^5*g^3 - 32*a^2*b*c^8*d^5
*e^3*f^4*g^4 + 24*a^2*b*c^8*d^6*e^2*f^3*g^5 + 113*a^2*b^3*c^6*d*e^7*f^6*g^2
 + 113*a^2*b^3*c^6*d^6*e^2*f*g^7 - 152*a^2*b^4*c^5*d*e^7*f^5*g^3 - 152*a^2*
b^4*c^5*d^5*e^3*f*g^7 + 34*a^2*b^5*c^4*d*e^7*f^4*g^4 + 34*a^2*b^5*c^4*d^4*e
^4*f*g^7 + 64*a^2*b^6*c^3*d*e^7*f^3*g^5 + 64*a^2*b^6*c^3*d^3*e^5*f*g^7 - 31
*a^2*b^7*c^2*d*e^7*f^2*g^6 - 31*a^2*b^7*c^2*d^2*e^6*f*g^7 - 260*a^3*b*c^7*d
^2*e^6*f^5*g^3 - 476*a^3*b*c^7*d^3*e^5*f^4*g^4 - 476*a^3*b*c^7*d^4*e^4*f^3*
g^5 - 260*a^3*b*c^7*d^5*e^3*f^2*g^6 - 16*a^3*b^2*c^6*d*e^7*f^5*g^3 - 16*a^3
*b^2*c^6*d^5*e^3*f*g^7 + 282*a^3*b^3*c^5*d*e^7*f^4*g^4 + 282*a^3*b^3*c^5*d^
4*e^4*f*g^7 - 316*a^3*b^4*c^4*d*e^7*f^3*g^5 - 316*a^3*b^4*c^4*d^3*e^5*f*g^7
 + 70*a^3*b^5*c^3*d*e^7*f^2*g^6 + 70*a^3*b^5*c^3*d^2*e^6*f*g^7 - 928*a^4*b*
c^6*d^2*e^6*f^3*g^5 - 928*a^4*b*c^6*d^3*e^5*f^2*g^6 + 246*a^4*b^2*c^5*d*e^7
*f^3*g^5 + 246*a^4*b^2*c^5*d^3*e^5*f*g^7 + 173*a^4*b^3*c^4*d*e^7*f^2*g^6 +
173*a^4*b^3*c^4*d^2*e^6*f*g^7 - 12*a*b*c^9*d^4*e^4*f^7*g - 12*a*b*c^9*d^7*e
*f^4*g^4 + 10*a*b^4*c^6*d*e^7*f^7*g + 10*a*b^4*c^6*d^7*e*f*g^7 + 3*a*b^9*c*
d*e^7*f^2*g^6 + 3*a*b^9*c*d^2*e^6*f*g^7 - 2*a^2*b^8*c*d*e^7*f*g^7 - 64*a^2*
b^2*c^7*d^2*e^6*f^6*g^2 - 154*a^2*b^2*c^7*d^3*e^5*f^5*g^3 + 152*a^2*b^2*c^7
*d^4*e^4*f^4*g^4 - 154*a^2*b^2*c^7*d^5*e^3*f^3*g^5 - 64*a^2*b^2*c^7*d^6*e^2
*f^2*g^6 + 245*a^2*b^3*c^6*d^2*e^6*f^5*g^3 + 227*a^2*b^3*c^6*d^3*e^5*f^4*g^
4 + 227*a^2*b^3*c^6*d^4*e^4*f^3*g^5 + 245*a^2*b^3*c^6*d^5*e^3*f^2*g^6 - 346
*a^2*b^4*c^5*d^2*e^6*f^4*g^4 - 280*a^2*b^4*c^5*d^3*e^5*f^3*g^5 - 346*a^2*b^
4*c^5*d^4*e^4*f^2*g^6 + 120*a^2*b^5*c^4*d^2*e^6*f^3*g^5 + 120*a^2*b^5*c^4*d
^3*e^5*f^2*g^6 + 70*a^2*b^6*c^3*d^2*e^6*f^2*g^6 + 478*a^3*b^2*c^6*d^2*e^6*f
^4*g^4 + 232*a^3*b^2*c^6*d^3*e^5*f^3*g^5 + 478*a^3*b^2*c^6*d^4*e^4*f^2*g^6
+ 200*a^3*b^3*c^5*d^2*e^6*f^3*g^5 + 200*a^3*b^3*c^5*d^3*e^5*f^2*g^6 - 528*a
^3*b^4*c^4*d^2*e^6*f^2*g^6 + 988*a^4*b^2*c^5*d^2*e^6*f^2*g^6 + 12*a*b*c^9*d
^5*e^3*f^6*g^2 + 12*a*b*c^9*d^6*e^2*f^5*g^3 - 4*a*b^2*c^8*d^3*e^5*f^7*g - 4
*a*b^2*c^8*d^7*e*f^3*g^5 - 2*a*b^3*c^7*d^2*e^6*f^7*g - 2*a*b^3*c^7*d^7*e*f^
2*g^6 - 41*a*b^5*c^5*d*e^7*f^6*g^2 - 41*a*b^5*c^5*d^6*e^2*f*g^7 + 60*a*b^6*
c^4*d*e^7*f^5*g^3 + 60*a*b^6*c^4*d^5*e^3*f*g^7 - 34*a*b^7*c^3*d*e^7*f^4*g^4
 - 34*a*b^7*c^3*d^4*e^4*f*g^7 + 2*a*b^8*c^2*d*e^7*f^3*g^5 + 2*a*b^8*c^2*d^3
*e^5*f*g^7 + 8*a^2*b*c^8*d^2*e^6*f^7*g + 8*a^2*b*c^8*d^7*e*f^2*g^6 - 26*a^2
*b^2*c^7*d*e^7*f^7*g - 26*a^2*b^2*c^7*d^7*e*f*g^7 - 52*a^3*b*c^7*d*e^7*f^6*
g^2 - 52*a^3*b*c^7*d^6*e^2*f*g^7 + 24*a^3*b^6*c^2*d*e^7*f*g^7 - 520*a^4*b*c
^6*d*e^7*f^4*g^4 - 520*a^4*b*c^6*d^4*e^4*f*g^7 - 80*a^4*b^4*c^3*d*e^7*f*g^7
 - 596*a^5*b*c^5*d*e^7*f^2*g^6 - 596*a^5*b*c^5*d^2*e^6*f*g^7 - 12*a^5*b^2*c
^4*d*e^7*f*g^7)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4
+ 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d
^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^
4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 +
 b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^
5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5
*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4
- 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*
b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2
*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5
*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*
b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b
*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*
d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d
^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^
2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4
*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^
3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2
+ 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g
- 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^
3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2
 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*
d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*

a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3) + (x*(48*a^4*b^5*c^2*e^8*g^8 - 192*a^5*b^3*c^3*e^8*g^8 - 256*a^4*c^7*d^5*e^3*g^8 - 464*a^5*c^6*d^3*e^5*g^8 - 256*a^4*c^7*e^8*f^5*g^3 - 464*a^5*c^6*e^8*f^3*g^5 - 4*a^3*b^7*c*e^8*g^8 + 256*a^6*b*c^4*e^8*g^8 - 48*a^3*c^8*d^7*e*g^8 - 256*a^6*c^5*d*e^7*g^8 - 48*a^3*c^8*e^8*f^7*g - 256*a^6*c^5*e^8*f*g^7 - 2*a*b^4*c^6*d^7*e*g^8 - 2*a*b^9*c*d^2*e^6*g^8 + 6*a^2*b^8*c*d*e^7*g^8 - 2*a*b^4*c^6*e^8*f^7*g - 2*a*b^9*c*e^8*f^2*g^6 + 6*a^2*b^8*c*e^8*f*g^7 - 16*a*c^10*d^4*e^4*f^7*g - 16*a*c^10*d^7*e*f^4*g^4 + 2*b^5*c^6*d*e^7*f^7*g + 2*b^5*c^6*d^7*e*f*g^7 + 2*b^10*c*d*e^7*f^2*g^6 + 2*b^10*c*d^2*e^6*f*g^7 + 6*a*b^5*c^5*d^6*e^2*g^8 - 4*a*b^6*c^4*d^5*e^3*g^8 - 4*a*b^7*c^3*d^4*e^4*g^8 + 6*a*b^8*c^2*d^3*e^5*g^8 + 20*a^2*b^2*c^7*d^7*e*g^8 + 144*a^3*b*c^7*d^6*e^2*g^8 - 68*a^3*b^6*c^2*d*e^7*g^8 + 640*a^4*b*c^6*d^4*e^4*g^8 + 240*a^4*b^4*c^3*d*e^7*g^8 + 848*a^5*b*c^5*d^2*e^6*g^8 - 192*a^5*b^2*c^4*d*e^7*g^8 + 6*a*b^5*c^5*e^8*f^6*g^2 - 4*a*b^6*c^4*e^8*f^5*g^3 - 4*a*b^7*c^3*e^8*f^4*g^4 + 6*a*b^8*c^2*e^8*f^3*g^5 + 20*a^2*b^2*c^7*e^8*f^7*g + 144*a^3*b*c^7*e^8*f^6*g^2 - 68*a^3*b^6*c^2*e^8*f*g^7 + 640*a^4*b*c^6*e^8*f^4*g^4 + 240*a^4*b^4*c^3*e^8*f*g^7 + 848*a^5*b*c^5*e^8*f^2*g^6 - 192*a^5*b^2*c^4*e^8*f*g^7 + 16*a*c^10*d^5*e^3*f^6*g^2 + 16*a*c^10*d^6*e^2*f^5*g^3 - 64*a^2*c^9*d^2*e^6*f^7*g - 64*a^2*c^9*d^7*e*f^2*g^6 + 48*a^3*c^8*d*e^7*f^6*g^2 + 48*a^3*c^8*d^6*e^2*f*g^7 - 304*a^5*c^6*d*e^7*f^2*g^6 - 304*a^5*c^6*d^2*e^6*f*g^7 + 4*b^2*c^9*d^4*e^4*f^7*g + 4*b^2*c^9*d^7*e*f^4*g^4 - 8*b^3*c^8*d^3*e^5*f^7*g - 8*b^3*c^8*d^7*e*f^3*g^5 + 2*b^4*c^7*d^2*e^6*f^7*g + 2*b^4*c^7*d^7*e*f^2*g^6 - 6*b^6*c^5*d*e^7*f^6*g^2 - 6*b^6*c^5*d^6*e^2*f*g^7 + 4*b^7*c^4*d*e^7*f^5*g^3 + 4*b^7*c^4*d^5*e^3*f*g^7 + 4*b^8*c^3*d*e^7*f^4*g^4 + 4*b^8*c^3*d^4*e^4*f*g^7 - 6*b^9*c^2*d*e^7*f^3*g^5 - 6*b^9*c^2*d^3*e^5*f*g^7 - 60*a^2*b^3*c^6*d^6*e^2*g^8 + 30*a^2*b^4*c^5*d^5*e^3*g^8 + 64*a^2*b^5*c^4*d^4*e^4*g^8 - 72*a^2*b^6*c^3*d^3*e^5*g^8 + 12*a^2*b^7*c^2*d^2*e^6*g^8 + 8*a^3*b^2*c^6*d^5*e^3*g^8 - 352*a^3*b^3*c^5*d^4*e^4*g^8 + 268*a^3*b^4*c^4*d^3*e^5*g^8 + 52*a^3*b^5*c^3*d^2*e^6*g^8 - 188*a^4*b^2*c^5*d^3*e^5*g^8 - 484*a^4*b^3*c^4*d^2*e^6*g^8 - 60*a^2*b^3*c^6*e^8*f^6*g^2 + 30*a^2*b^4*c^5*e^8*f^5*g^3 + 64*a^2*b^5*c^4*e^8*f^4*g^4 - 72*a^2*b^6*c^3*e^8*f^3*g^5 + 12*a^2*b^7*c^2*e^8*f^2*g^6 + 8*a^3*b^2*c^6*e^8*f^5*g^3 - 352*a^3*b^3*c^5*e^8*f^4*g^4 + 268*a^3*b^4*c^4*e^8*f^3*g^5 + 52*a^3*b^5*c^3*e^8*f^2*g^6 - 188*a^4*b^2*c^5*e^8*f^3*g^5 - 484*a^4*b^3*c^4*e^8*f^2*g^6 + 64*a^2*c^9*d^3*e^5*f^6*g^2 + 64*a^2*c^9*d^6*e^2*f^3*g^5 - 272*a^3*c^8*d^2*e^6*f^5*g^3 + 16*a^3*c^8*d^3*e^5*f^4*g^4 + 16*a^3*c^8*d^4*e^4*f^3*g^5 - 272*a^3*c^8*d^5*e^3*f^2*g^6 - 512*a^4*c^7*d^2*e^6*f^3*g^5 - 512*a^4*c^7*d^3*e^5*f^2*g^6 - 4*b^2*c^9*d^5*e^3*f^6*g^2 - 4*b^2*c^9*d^6*e^2*f^5*g^3 - 4*b^3*c^8*d^4*e^4*f^6*g^2 + 24*b^3*c^8*d^5*e^3*f^5*g^3 - 4*b^3*c^8*d^6*e^2*f^4*g^4 + 22*b^4*c^7*d^3*e^5*f^6*g^2 - 24*b^4*c^7*d^4*e^4*f^5*g^3 - 24*b^4*c^7*d^5*e^3*f^4*g^4 + 22*b^4*c^7*d^6*e^2*f^3*g^5 - 8*b^5*c^6*d^2*e^6*f^6*g^2 - 14*b^5*c^6*d^3*e^5*f^5*g^3 + 40*b^5*c^6*d^4*e^4*f^4*g^4 - 14*b^5*c^6*d^5*e^3*f^3*g^5 - 8*b^5*c^6*d^6*e^2*f^2*g^6 + 14*b^6*c^5*d^2*e^6*f^5*g^3 - 4*b^6*c^5*d^3*e^5*f^4*g^4 - 4*b^6*c^5*d^4*e^4*f^3*g^5 + 14*b^6*c^5*d^5*e^3*f^2*g^6 - 16*b^7*c^4*d^2*e^6*f^4*g^4 - 4*b^7*c^4*d^3*e^5*f^3*g^5 - 16*b^7*c^4*d^4*e^4*f^2*g^6 + 14*b^8*c^3*d^2*e^6*f^3*g^5 + 14*b^8*c^3*d^3*e^5*f^2*g^6 - 8*b^9*c^2*d^2*e^6*f^2*g^6 - 8*a*b^9*c*d*e^7*f*g^7 - 104*a*b^2*c^8*d^3*e^5*f^6*g^2 + 96*a*b^2*c^8*d^4*e^4*f^5*g^3 + 96*a*b^2*c^8*d^5*e^3*f^4*g^4 - 104*a*b^2*c^8*d^6*e^2*f^3*g^5 + 104*a*b^3*c^7*d^3*e^5*f^5*g^3 - 160*a*b^3*c^7*d^4*e^4*f^4*g^4 + 104*a*b^3*c^7*d^5*e^3*f^3*g^5 - 78*a*b^4*c^6*d^2*e^6*f^5*g^3 - 42*a*b^4*c^6*d^3*e^5*f^4*g^4 - 42*a*b^4*c^6*d^4*e^4*f^3*g^5 - 78*a*b^4*c^6*d^5*e^3*f^2*g^6 + 166*a*b^5*c^5*d^2*e^6*f^4*g^4 + 88*a*b^5*c^5*d^3*e^5*f^3*g^5 + 166*a*b^5*c^5*d^4*e^4*f^2*g^6 - 148*a*b^6*c^4*d^2*e^6*f^3*g^5 - 148*a*b^6*c^4*d^3*e^5*f^2*g^6 + 60*a*b^7*c^3*d^2*e^6*f^2*g^6 + 128*a^2*b*c^8*d^2*e^6*f^6*g^2 - 192*a^2*b*c^8*d^3*e^5*f^5*g^3 - 192*a^2*b*c^8*d^5*e^3*f^3*g^5 + 128*a^2*b*c^8*d^6*e^2*f^2*g^6 - 212*a^2*b^2*c^7*d*e^7*f^6*g^2 - 212*a^2*b^2*c^7*d^6*e^2*f*g^7 + 96*a^2*b^3*c^6*d*e^7*f^5*g^3 + 96*a^2*b^3*c^6*d^5*e^3*f*g^7 + 266*a^2*b^4*c^5*d*e^7*f^4*g^4 + 266*a^2*b^

4*c^5*d^4*e^4*f*g^7 - 196*a^2*b^5*c^4*d*e^7*f^3*g^5 - 196*a^2*b^5*c^4*d^3*e
^5*f*g^7 - 108*a^2*b^6*c^3*d*e^7*f^2*g^6 - 108*a^2*b^6*c^3*d^2*e^6*f*g^7 +
656*a^3*b*c^7*d^2*e^6*f^4*g^4 - 64*a^3*b*c^7*d^3*e^5*f^3*g^5 + 656*a^3*b*c^
7*d^4*e^4*f^2*g^6 - 488*a^3*b^2*c^6*d*e^7*f^4*g^4 - 488*a^3*b^2*c^6*d^4*e^4
*f*g^7 + 16*a^3*b^3*c^5*d*e^7*f^3*g^5 + 16*a^3*b^3*c^5*d^3*e^5*f*g^7 + 612*
a^3*b^4*c^4*d*e^7*f^2*g^6 + 612*a^3*b^4*c^4*d^2*e^6*f*g^7 + 1536*a^4*b*c^6*
d^2*e^6*f^2*g^6 - 772*a^4*b^2*c^5*d*e^7*f^2*g^6 - 772*a^4*b^2*c^5*d^2*e^6*f
*g^7 + 32*a*b*c^9*d^3*e^5*f^7*g + 32*a*b*c^9*d^7*e*f^3*g^5 - 24*a*b^3*c^7*d
*e^7*f^7*g - 24*a*b^3*c^7*d^7*e*f*g^7 + 64*a^2*b*c^8*d*e^7*f^7*g + 64*a^2*b
*c^8*d^7*e*f*g^7 + 608*a^5*b*c^5*d*e^7*f*g^7 + 156*a^2*b^2*c^7*d^2*e^6*f^5*
g^3 + 228*a^2*b^2*c^7*d^3*e^5*f^4*g^4 + 228*a^2*b^2*c^7*d^4*e^4*f^3*g^5 + 1
56*a^2*b^2*c^7*d^5*e^3*f^2*g^6 - 572*a^2*b^3*c^6*d^2*e^6*f^4*g^4 - 272*a^2*
b^3*c^6*d^3*e^5*f^3*g^5 - 572*a^2*b^3*c^6*d^4*e^4*f^2*g^6 + 424*a^2*b^4*c^5
*d^2*e^6*f^3*g^5 + 424*a^2*b^4*c^5*d^3*e^5*f^2*g^6 + 24*a^2*b^5*c^4*d^2*e^6
*f^2*g^6 - 96*a^3*b^2*c^6*d^2*e^6*f^3*g^5 - 96*a^3*b^2*c^6*d^3*e^5*f^2*g^6
- 928*a^3*b^3*c^5*d^2*e^6*f^2*g^6 + 16*a*b*c^9*d^4*e^4*f^6*g^2 - 96*a*b*c^9
*d^5*e^3*f^5*g^3 + 16*a*b*c^9*d^6*e^2*f^4*g^4 + 8*a*b^2*c^8*d^2*e^6*f^7*g +
8*a*b^2*c^8*d^7*e*f^2*g^6 + 74*a*b^4*c^6*d*e^7*f^6*g^2 + 74*a*b^4*c^6*d^6*
e^2*f*g^7 - 48*a*b^5*c^5*d*e^7*f^5*g^3 - 48*a*b^5*c^5*d^5*e^3*f*g^7 - 52*a*
b^6*c^4*d*e^7*f^4*g^4 - 52*a*b^6*c^4*d^4*e^4*f*g^7 + 64*a*b^7*c^3*d*e^7*f^3
*g^5 + 64*a*b^7*c^3*d^3*e^5*f*g^7 - 6*a*b^8*c^2*d*e^7*f^2*g^6 - 6*a*b^8*c^2
*d^2*e^6*f*g^7 + 84*a^2*b^7*c^2*d*e^7*f*g^7 + 128*a^3*b*c^7*d*e^7*f^5*g^3 +
128*a^3*b*c^7*d^5*e^3*f*g^7 - 248*a^3*b^5*c^3*d*e^7*f*g^7 + 512*a^4*b*c^6*
d*e^7*f^3*g^5 + 512*a^4*b*c^6*d^3*e^5*f*g^7 + 8*a^4*b^3*c^4*d*e^7*f*g^7))/(
16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*
f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*
c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e
^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*
f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2
+ b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*
b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f
*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4
- 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4
- 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4
+ 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g -
32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g +
16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2
*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^
7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^
2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 -
6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3
*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3
*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*
e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2
+ 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*
e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*
a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2
*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b
^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^
2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3)) + (x*(b^8*c*e
^7*g^7 + 104*a^4*c^5*e^7*g^7 + 50*a^2*b^4*c^3*e^7*g^7 - 96*a^3*b^2*c^4*e^7*
g^7 + 36*a^2*c^7*d^4*e^3*g^7 + 72*a^3*c^6*d^2*e^5*g^7 - 2*b^3*c^6*d^5*e^2*g
^7 + b^4*c^5*d^4*e^3*g^7 + b^6*c^3*d^2*e^5*g^7 + 36*a^2*c^7*e^7*f^4*g^3 + 7
2*a^3*c^6*e^7*f^2*g^5 - 2*b^3*c^6*e^7*f^5*g^2 + b^4*c^5*e^7*f^4*g^3 + b^6*c
^3*e^7*f^2*g^5 - 12*a*b^6*c^2*e^7*g^7 + b^2*c^7*d^6*e*g^7 - 2*b^7*c^2*d*e^6
*g^7 + b^2*c^7*e^7*f^6*g - 2*b^7*c^2*e^7*f*g^6 + 4*c^9*d^2*e^5*f^6*g + 4*c^
9*d^6*e*f^2*g^5 - 4*a*b*c^7*d^5*e^2*g^7 + 22*a*b^5*c^3*d*e^6*g^7 - 16*a^3*b
*c^5*d*e^6*g^7 - 4*a*b*c^7*e^7*f^5*g^2 + 22*a*b^5*c^3*e^7*f*g^6 - 16*a^3*b*
c^5*e^7*f*g^6 + 8*a*c^8*d*e^6*f^5*g^2 + 8*a*c^8*d^5*e^2*f*g^6 - 112*a^3*c^6

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*d*e^6*f*g^6 + 4*b^6*c^3*d*e^6*f*g^6 + 2*a*b^2*c^6*d^4*e^3*g^7 + 10*a*b^3*c
^5*d^3*e^4*g^7 - 18*a*b^4*c^4*d^2*e^5*g^7 - 80*a^2*b*c^6*d^3*e^4*g^7 - 56*a
^2*b^3*c^4*d*e^6*g^7 + 2*a*b^2*c^6*e^7*f^4*g^3 + 10*a*b^3*c^5*e^7*f^3*g^4 -
 18*a*b^4*c^4*e^7*f^2*g^5 - 80*a^2*b*c^6*e^7*f^3*g^4 - 56*a^2*b^3*c^4*e^7*f
*g^6 + 40*a*c^8*d^2*e^5*f^4*g^3 + 40*a*c^8*d^4*e^3*f^2*g^5 + 16*a^2*c^7*d*e
^6*f^3*g^4 + 16*a^2*c^7*d^3*e^4*f*g^6 - 12*b*c^8*d^2*e^5*f^5*g^2 - 12*b*c^8
*d^5*e^2*f^2*g^5 + 10*b^2*c^7*d*e^6*f^5*g^2 + 10*b^2*c^7*d^5*e^2*f*g^6 - 14
*b^4*c^5*d*e^6*f^3*g^4 - 14*b^4*c^5*d^3*e^4*f*g^6 + 6*b^5*c^4*d*e^6*f^2*g^5
 + 6*b^5*c^4*d^2*e^5*f*g^6 - 4*b*c^8*d*e^6*f^6*g - 4*b*c^8*d^6*e*f*g^6 + 54
*a^2*b^2*c^5*d^2*e^5*g^7 + 54*a^2*b^2*c^5*e^7*f^2*g^5 + 168*a^2*c^7*d^2*e^5
*f^2*g^5 + 5*b^2*c^7*d^2*e^5*f^4*g^3 + 5*b^2*c^7*d^4*e^3*f^2*g^5 + 10*b^3*c
^6*d^2*e^5*f^3*g^4 + 10*b^3*c^6*d^3*e^4*f^2*g^5 - 12*b^4*c^5*d^2*e^5*f^2*g^
5 + 36*a*b^2*c^6*d^2*e^5*f^2*g^5 - 60*a*b*c^7*d*e^6*f^4*g^3 - 60*a*b*c^7*d^
4*e^3*f*g^6 - 72*a*b^4*c^4*d*e^6*f*g^6 - 80*a*b*c^7*d^2*e^5*f^3*g^4 - 80*a*
b*c^7*d^3*e^4*f^2*g^5 + 92*a*b^2*c^6*d*e^6*f^3*g^4 + 92*a*b^2*c^6*d^3*e^4*f
*g^6 + 6*a*b^3*c^5*d*e^6*f^2*g^5 + 6*a*b^3*c^5*d^2*e^5*f*g^6 - 192*a^2*b*c^
6*d*e^6*f^2*g^5 - 192*a^2*b*c^6*d^2*e^5*f*g^6 + 276*a^2*b^2*c^5*d*e^6*f*g^6
))/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*
e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*
b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d
^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*
e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2
*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*
a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e
^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3
*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*
g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*
g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3
*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*
g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2
 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g -
2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 1
6*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^
4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4
*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d
*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*
c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2
*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*
d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g -
 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2
*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a
^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^
2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3)) + (x*(4*b
^3*c^4*e^6*g^6 - 16*a*b*c^5*e^6*g^6 + 16*a*c^6*d*e^5*g^6 + 16*a*c^6*e^6*f*g
^5 - 4*b^2*c^5*d*e^5*g^6 - 4*b^2*c^5*e^6*f*g^5))/(16*a^2*c^6*d^4*f^4 + a^4*
b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 1
6*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c
^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e
^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2
 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 +
b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5
*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*
g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4
 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4
+ 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g
- 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 -
 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 -
32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*
a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7
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*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*
a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 +
 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*
g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*
f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^
2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f
^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*
c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 +
 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2
*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a
^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*
f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3))*root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4
+ 1120*a^6*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*
a^5*b^4*c^5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c
^4*d*e^9*f^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^
9*f^8*g^2*z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9
*g*z^4 + 512*a^4*b*c^9*d^9*e*f^6*g^4*z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4
+ 256*a^10*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256
*a^3*b*c^10*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*
b^7*c*d^4*e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d
^9*e*f^6*g^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*
f*g^9*z^4 + 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z
^4 + 144*a^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 14
4*a*b^8*c^5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^
2*c^10*d^9*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^
9*f^3*g^7*z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g
*z^4 + 48*a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48
*a*b^5*c^8*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c
*d^6*e^4*f^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*
e^3*f*g^9*z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^
9*z^4 - 16*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 -
16*a*b^12*c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^
10*c^3*d^3*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^
9*f^6*g^4*z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*
z^4 - 8*a*b^9*c^4*d^4*e^6*f^9*g*z^4 - 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4
- 9984*a^5*b^2*c^7*d^6*e^4*f^6*g^4*z^4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z
^4 - 8640*a^6*b^2*c^6*d^4*e^6*f^6*g^4*z^4 - 8544*a^5*b^4*c^5*d^5*e^5*f^5*g^
5*z^4 + 5632*a^6*b^2*c^6*d^7*e^3*f^3*g^7*z^4 + 5632*a^6*b^2*c^6*d^3*e^7*f^7
*g^3*z^4 + 5232*a^5*b^4*c^5*d^6*e^4*f^4*g^6*z^4 + 5232*a^5*b^4*c^5*d^4*e^6*
f^6*g^4*z^4 + 4808*a^4*b^6*c^4*d^5*e^5*f^5*g^5*z^4 - 4288*a^6*b^4*c^4*d^5*e
^5*f^3*g^7*z^4 - 4288*a^6*b^4*c^4*d^3*e^7*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^
7*e^3*f^5*g^5*z^4 - 4288*a^4*b^4*c^6*d^5*e^5*f^7*g^3*z^4 + 3968*a^6*b^3*c^5
*d^5*e^5*f^4*g^6*z^4 + 3968*a^6*b^3*c^5*d^4*e^6*f^5*g^5*z^4 + 3968*a^5*b^3*
c^6*d^6*e^4*f^5*g^5*z^4 + 3968*a^5*b^3*c^6*d^5*e^5*f^6*g^4*z^4 + 3840*a^7*b
^2*c^5*d^5*e^5*f^3*g^7*z^4 + 3840*a^7*b^2*c^5*d^3*e^7*f^5*g^5*z^4 + 3840*a^
5*b^2*c^7*d^7*e^3*f^5*g^5*z^4 + 3840*a^5*b^2*c^7*d^5*e^5*f^7*g^3*z^4 + 3776
*a^6*b^4*c^4*d^4*e^6*f^4*g^6*z^4 + 3776*a^4*b^4*c^6*d^6*e^4*f^6*g^4*z^4 + 3
456*a^6*b^2*c^6*d^5*e^5*f^5*g^5*z^4 + 3440*a^6*b^4*c^4*d^6*e^4*f^2*g^8*z^4
+ 3440*a^6*b^4*c^4*d^2*e^8*f^6*g^4*z^4 + 3440*a^4*b^4*c^6*d^8*e^2*f^4*g^6*z
^4 + 3440*a^4*b^4*c^6*d^4*e^6*f^8*g^2*z^4 - 3360*a^8*b^2*c^4*d^4*e^6*f^2*g^
8*z^4 - 3360*a^8*b^2*c^4*d^2*e^8*f^4*g^6*z^4 - 3360*a^4*b^2*c^8*d^8*e^2*f^6
*g^4*z^4 - 3360*a^4*b^2*c^8*d^6*e^4*f^8*g^2*z^4 - 2944*a^7*b^4*c^3*d^3*e^7*
f^3*g^7*z^4 - 2944*a^3*b^4*c^7*d^7*e^3*f^7*g^3*z^4 + 2512*a^5*b^6*c^3*d^5*e
^5*f^3*g^7*z^4 + 2512*a^5*b^6*c^3*d^3*e^7*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^
7*e^3*f^5*g^5*z^4 + 2512*a^3*b^6*c^5*d^5*e^5*f^7*g^3*z^4 + 2312*a^7*b^4*c^3
*d^4*e^6*f^2*g^8*z^4 + 2312*a^7*b^4*c^3*d^2*e^8*f^4*g^6*z^4 + 2312*a^3*b^4*
c^7*d^8*e^2*f^6*g^4*z^4 + 2312*a^3*b^4*c^7*d^6*e^4*f^8*g^2*z^4 + 1952*a^6*b
^6*c^2*d^3*e^7*f^3*g^7*z^4 + 1952*a^2*b^6*c^6*d^7*e^3*f^7*g^3*z^4 - 1920*a^
5*b^4*c^5*d^7*e^3*f^3*g^7*z^4 - 1920*a^5*b^4*c^5*d^3*e^7*f^7*g^3*z^4 - 1828
```

*a^5*b^6*c^3*d^6*e^4*f^2*g^8*z^4 - 1828*a^5*b^6*c^3*d^2*e^8*f^6*g^4*z^4 - 1
828*a^3*b^6*c^5*d^8*e^2*f^4*g^6*z^4 - 1828*a^3*b^6*c^5*d^4*e^6*f^8*g^2*z^4
+ 1740*a^5*b^4*c^5*d^8*e^2*f^2*g^8*z^4 + 1740*a^5*b^4*c^5*d^2*e^8*f^8*g^2*z
^4 - 1728*a^7*b^2*c^5*d^6*e^4*f^2*g^8*z^4 - 1728*a^7*b^2*c^5*d^2*e^8*f^6*g^
4*z^4 - 1728*a^5*b^2*c^7*d^8*e^2*f^4*g^6*z^4 - 1728*a^5*b^2*c^7*d^4*e^6*f^8
*g^2*z^4 - 1716*a^4*b^6*c^4*d^6*e^4*f^4*g^6*z^4 - 1716*a^4*b^6*c^4*d^4*e^6*
f^6*g^4*z^4 - 1664*a^9*b^2*c^3*d^2*e^8*f^2*g^8*z^4 - 1664*a^3*b^2*c^9*d^8*e
^2*f^8*g^2*z^4 - 1600*a^6*b^3*c^5*d^7*e^3*f^2*g^8*z^4 - 1600*a^6*b^3*c^5*d^
2*e^8*f^7*g^3*z^4 - 1600*a^5*b^3*c^6*d^8*e^2*f^3*g^7*z^4 - 1600*a^5*b^3*c^6
*d^3*e^7*f^8*g^2*z^4 - 1553*a^4*b^6*c^4*d^8*e^2*f^2*g^8*z^4 - 1553*a^4*b^6*
c^4*d^2*e^8*f^8*g^2*z^4 + 1536*a^8*b^2*c^4*d^3*e^7*f^3*g^7*z^4 + 1536*a^4*b
^2*c^8*d^7*e^3*f^7*g^3*z^4 + 1408*a^7*b^3*c^4*d^4*e^6*f^3*g^7*z^4 + 1408*a^
7*b^3*c^4*d^3*e^7*f^4*g^6*z^4 - 1408*a^6*b^3*c^5*d^6*e^4*f^3*g^7*z^4 - 1408
*a^6*b^3*c^5*d^3*e^7*f^6*g^4*z^4 - 1408*a^5*b^3*c^6*d^7*e^3*f^4*g^6*z^4 - 1
408*a^5*b^3*c^6*d^4*e^6*f^7*g^3*z^4 + 1408*a^4*b^3*c^7*d^7*e^3*f^6*g^4*z^4
+ 1408*a^4*b^3*c^7*d^6*e^4*f^7*g^3*z^4 - 1360*a^6*b^5*c^3*d^5*e^5*f^2*g^8*z
^4 - 1360*a^6*b^5*c^3*d^2*e^8*f^5*g^5*z^4 - 1360*a^3*b^5*c^6*d^8*e^2*f^5*g^
5*z^4 - 1360*a^3*b^5*c^6*d^5*e^5*f^8*g^2*z^4 - 1248*a^5*b^5*c^4*d^5*e^5*f^4
*g^6*z^4 - 1248*a^5*b^5*c^4*d^4*e^6*f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^6*e^4*
f^5*g^5*z^4 - 1248*a^4*b^5*c^5*d^5*e^5*f^6*g^4*z^4 + 1088*a^8*b^3*c^3*d^3*e
^7*f^2*g^8*z^4 + 1088*a^8*b^3*c^3*d^2*e^8*f^3*g^7*z^4 + 1088*a^3*b^3*c^8*d^
8*e^2*f^7*g^3*z^4 + 1088*a^3*b^3*c^8*d^7*e^3*f^8*g^2*z^4 + 1056*a^8*b^4*c^2
*d^2*e^8*f^2*g^8*z^4 + 1056*a^2*b^4*c^8*d^8*e^2*f^8*g^2*z^4 - 912*a^7*b^5*c
^2*d^3*e^7*f^2*g^8*z^4 - 912*a^7*b^5*c^2*d^2*e^8*f^3*g^7*z^4 - 912*a^2*b^5*
c^7*d^8*e^2*f^7*g^3*z^4 - 912*a^2*b^5*c^7*d^7*e^3*f^8*g^2*z^4 - 848*a^5*b^6
*c^3*d^4*e^6*f^4*g^6*z^4 - 848*a^3*b^6*c^5*d^6*e^4*f^6*g^4*z^4 + 832*a^7*b^
3*c^4*d^5*e^5*f^2*g^8*z^4 + 832*a^7*b^3*c^4*d^2*e^8*f^5*g^5*z^4 + 832*a^4*b
^3*c^7*d^8*e^2*f^5*g^5*z^4 + 832*a^4*b^3*c^7*d^5*e^5*f^8*g^2*z^4 + 828*a^5*
b^7*c^2*d^5*e^5*f^2*g^8*z^4 + 828*a^5*b^7*c^2*d^2*e^8*f^5*g^5*z^4 + 828*a^2
*b^7*c^5*d^8*e^2*f^5*g^5*z^4 + 828*a^2*b^7*c^5*d^5*e^5*f^8*g^2*z^4 - 800*a^
3*b^8*c^3*d^5*e^5*f^5*g^5*z^4 - 696*a^4*b^8*c^2*d^5*e^5*f^3*g^7*z^4 - 696*a
^4*b^8*c^2*d^3*e^7*f^5*g^5*z^4 - 696*a^2*b^8*c^4*d^7*e^3*f^5*g^5*z^4 - 696*
a^2*b^8*c^4*d^5*e^5*f^7*g^3*z^4 - 694*a^6*b^6*c^2*d^4*e^6*f^2*g^8*z^4 - 694
*a^6*b^6*c^2*d^2*e^8*f^4*g^6*z^4 - 694*a^2*b^6*c^6*d^8*e^2*f^6*g^4*z^4 - 69
4*a^2*b^6*c^6*d^6*e^4*f^8*g^2*z^4 + 692*a^4*b^7*c^3*d^7*e^3*f^2*g^8*z^4 + 6
92*a^4*b^7*c^3*d^2*e^8*f^7*g^3*z^4 + 692*a^3*b^7*c^4*d^8*e^2*f^3*g^7*z^4 +
692*a^3*b^7*c^4*d^3*e^7*f^8*g^2*z^4 + 672*a^4*b^6*c^4*d^7*e^3*f^3*g^7*z^4 +
672*a^4*b^6*c^4*d^3*e^7*f^7*g^3*z^4 + 600*a^4*b^8*c^2*d^4*e^6*f^4*g^6*z^4
+ 600*a^2*b^8*c^4*d^6*e^4*f^6*g^4*z^4 - 544*a^3*b^8*c^3*d^7*e^3*f^3*g^7*z^4
+ 544*a^3*b^8*c^3*d^6*e^4*f^4*g^6*z^4 + 544*a^3*b^8*c^3*d^4*e^6*f^6*g^4*z^
4 - 544*a^3*b^8*c^3*d^3*e^7*f^7*g^3*z^4 - 536*a^4*b^7*c^3*d^5*e^5*f^4*g^6*z
^4 - 536*a^4*b^7*c^3*d^4*e^6*f^5*g^5*z^4 - 536*a^3*b^7*c^4*d^6*e^4*f^5*g^5*
z^4 - 536*a^3*b^7*c^4*d^5*e^5*f^6*g^4*z^4 - 504*a^5*b^7*c^2*d^4*e^6*f^3*g^7
*z^4 - 504*a^5*b^7*c^2*d^3*e^7*f^4*g^6*z^4 - 504*a^2*b^7*c^5*d^7*e^3*f^6*g^
4*z^4 - 504*a^2*b^7*c^5*d^6*e^4*f^7*g^3*z^4 + 416*a^3*b^8*c^3*d^8*e^2*f^2*g
^8*z^4 + 416*a^3*b^8*c^3*d^2*e^8*f^8*g^2*z^4 - 352*a^6*b^5*c^3*d^4*e^6*f^3*
g^7*z^4 - 352*a^6*b^5*c^3*d^3*e^7*f^4*g^6*z^4 - 352*a^3*b^5*c^6*d^7*e^3*f^6
*g^4*z^4 - 352*a^3*b^5*c^6*d^6*e^4*f^7*g^3*z^4 - 248*a^3*b^9*c^2*d^7*e^3*f^
2*g^8*z^4 - 248*a^3*b^9*c^2*d^2*e^8*f^7*g^3*z^4 - 248*a^2*b^9*c^3*d^8*e^2*f
^3*g^7*z^4 - 248*a^2*b^9*c^3*d^3*e^7*f^8*g^2*z^4 + 246*a^4*b^8*c^2*d^6*e^4*
f^2*g^8*z^4 + 246*a^4*b^8*c^2*d^2*e^8*f^6*g^4*z^4 + 246*a^2*b^8*c^4*d^8*e^2
*f^4*g^6*z^4 + 246*a^2*b^8*c^4*d^4*e^6*f^8*g^2*z^4 + 208*a^6*b^2*c^6*d^8*e^
2*f^2*g^8*z^4 + 208*a^6*b^2*c^6*d^2*e^8*f^8*g^2*z^4 + 168*a^2*b^10*c^2*d^7*
e^3*f^3*g^7*z^4 + 168*a^2*b^10*c^2*d^3*e^7*f^7*g^3*z^4 + 160*a^3*b^9*c^2*d^
5*e^5*f^4*g^6*z^4 + 160*a^3*b^9*c^2*d^4*e^6*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d
^6*e^4*f^5*g^5*z^4 + 160*a^2*b^9*c^3*d^5*e^5*f^6*g^4*z^4 + 144*a^5*b^5*c^4*
d^7*e^3*f^2*g^8*z^4 + 144*a^5*b^5*c^4*d^2*e^8*f^7*g^3*z^4 + 144*a^4*b^5*c^5
*d^8*e^2*f^3*g^7*z^4 + 144*a^4*b^5*c^5*d^3*e^7*f^8*g^2*z^4 - 144*a^2*b^10*c
^2*d^6*e^4*f^4*g^6*z^4 - 144*a^2*b^10*c^2*d^4*e^6*f^6*g^4*z^4 + 120*a^4*b^7

*c^3*d^6*e^4*f^3*g^7*z^4 + 120*a^4*b^7*c^3*d^3*e^7*f^6*g^4*z^4 + 120*a^3*b^
7*c^4*d^7*e^3*f^4*g^6*z^4 + 120*a^3*b^7*c^4*d^4*e^6*f^7*g^3*z^4 + 96*a^5*b^
5*c^4*d^6*e^4*f^3*g^7*z^4 + 96*a^5*b^5*c^4*d^3*e^7*f^6*g^4*z^4 + 96*a^4*b^5
*c^5*d^7*e^3*f^4*g^6*z^4 + 96*a^4*b^5*c^5*d^4*e^6*f^7*g^3*z^4 + 64*a^3*b^9*
c^2*d^6*e^4*f^3*g^7*z^4 + 64*a^3*b^9*c^2*d^3*e^7*f^6*g^4*z^4 + 64*a^2*b^9*c
^3*d^7*e^3*f^4*g^6*z^4 + 64*a^2*b^9*c^3*d^4*e^6*f^7*g^3*z^4 - 36*a^2*b^10*c
^2*d^8*e^2*f^2*g^8*z^4 - 36*a^2*b^10*c^2*d^2*e^8*f^8*g^2*z^4 + 24*a^2*b^10*
c^2*d^5*e^5*f^5*g^5*z^4 - 24*a^9*b^4*c*d*e^9*f*g^9*z^4 - 24*a*b^4*c^9*d^9*e
*f^9*g*z^4 + 2688*a^7*b^2*c^5*d^7*e^3*f*g^9*z^4 + 2688*a^7*b^2*c^5*d*e^9*f^
7*g^3*z^4 + 2688*a^5*b^2*c^7*d^9*e*f^3*g^7*z^4 + 2688*a^5*b^2*c^7*d^3*e^7*f
^9*g*z^4 - 2560*a^7*b^3*c^4*d^6*e^4*f*g^9*z^4 - 2560*a^7*b^3*c^4*d*e^9*f^6*
g^4*z^4 - 2560*a^4*b^3*c^7*d^9*e*f^4*g^6*z^4 - 2560*a^4*b^3*c^7*d^4*e^6*f^9
*g*z^4 + 2112*a^8*b^2*c^4*d^5*e^5*f*g^9*z^4 + 2112*a^8*b^2*c^4*d*e^9*f^5*g^
5*z^4 + 2112*a^4*b^2*c^8*d^9*e*f^5*g^5*z^4 + 2112*a^4*b^2*c^8*d^5*e^5*f^9*g
*z^4 + 1664*a^6*b^5*c^3*d^6*e^4*f*g^9*z^4 + 1664*a^6*b^5*c^3*d*e^9*f^6*g^4*
z^4 + 1664*a^3*b^5*c^6*d^9*e*f^4*g^6*z^4 + 1664*a^3*b^5*c^6*d^4*e^6*f^9*g*z
^4 + 1536*a^8*b*c^5*d^4*e^6*f^3*g^7*z^4 + 1536*a^8*b*c^5*d^3*e^7*f^4*g^6*z^
4 + 1536*a^7*b*c^6*d^5*e^5*f^4*g^6*z^4 + 1536*a^7*b*c^6*d^4*e^6*f^5*g^5*z^4
 + 1536*a^6*b*c^7*d^6*e^4*f^5*g^5*z^4 + 1536*a^6*b*c^7*d^5*e^5*f^6*g^4*z^4
+ 1536*a^5*b*c^8*d^7*e^3*f^6*g^4*z^4 + 1536*a^5*b*c^8*d^6*e^4*f^7*g^3*z^4 -
 1408*a^8*b^3*c^3*d^4*e^6*f*g^9*z^4 - 1408*a^8*b^3*c^3*d*e^9*f^4*g^6*z^4 -
1408*a^3*b^3*c^8*d^9*e*f^6*g^4*z^4 - 1408*a^3*b^3*c^8*d^6*e^4*f^9*g*z^4 - 1
280*a^7*b*c^6*d^7*e^3*f^2*g^8*z^4 - 1280*a^7*b*c^6*d^2*e^8*f^7*g^3*z^4 - 12
80*a^6*b*c^7*d^8*e^2*f^3*g^7*z^4 - 1280*a^6*b*c^7*d^3*e^7*f^8*g^2*z^4 - 115
2*a^6*b^3*c^5*d^8*e^2*f*g^9*z^4 - 1152*a^6*b^3*c^5*d*e^9*f^8*g^2*z^4 - 1152
*a^5*b^3*c^6*d^9*e*f^2*g^8*z^4 - 1152*a^5*b^3*c^6*d^2*e^8*f^9*g*z^4 + 1056*
a^5*b^5*c^4*d^8*e^2*f*g^9*z^4 + 1056*a^5*b^5*c^4*d*e^9*f^8*g^2*z^4 + 1056*a
^4*b^5*c^5*d^9*e*f^2*g^8*z^4 + 1056*a^4*b^5*c^5*d^2*e^8*f^9*g*z^4 + 864*a^7
*b^5*c^2*d^4*e^6*f*g^9*z^4 + 864*a^7*b^5*c^2*d*e^9*f^4*g^6*z^4 + 864*a^2*b^
5*c^7*d^9*e*f^6*g^4*z^4 + 864*a^2*b^5*c^7*d^6*e^4*f^9*g*z^4 - 800*a^6*b^4*c
^4*d^7*e^3*f*g^9*z^4 - 800*a^6*b^4*c^4*d*e^9*f^7*g^3*z^4 - 800*a^4*b^4*c^6*
d^9*e*f^3*g^7*z^4 - 800*a^4*b^4*c^6*d^3*e^7*f^9*g*z^4 - 768*a^8*b*c^5*d^5*e
^5*f^2*g^8*z^4 - 768*a^8*b*c^5*d^2*e^8*f^5*g^5*z^4 - 768*a^5*b*c^8*d^8*e^2*
f^5*g^5*z^4 - 768*a^5*b*c^8*d^5*e^5*f^8*g^2*z^4 + 640*a^9*b^2*c^3*d^3*e^7*f
*g^9*z^4 + 640*a^9*b^2*c^3*d*e^9*f^3*g^7*z^4 + 640*a^3*b^2*c^9*d^9*e*f^7*g^
3*z^4 + 640*a^3*b^2*c^9*d^7*e^3*f^9*g*z^4 + 512*a^7*b*c^6*d^6*e^4*f^3*g^7*z
^4 + 512*a^7*b*c^6*d^3*e^7*f^6*g^4*z^4 + 512*a^6*b*c^7*d^7*e^3*f^4*g^6*z^4
+ 512*a^6*b*c^7*d^4*e^6*f^7*g^3*z^4 - 480*a^5*b^8*c*d^3*e^7*f^3*g^7*z^4 - 4
80*a*b^8*c^5*d^7*e^3*f^7*g^3*z^4 - 400*a^7*b^4*c^3*d^5*e^5*f*g^9*z^4 - 400*
a^7*b^4*c^3*d*e^9*f^5*g^5*z^4 - 400*a^3*b^4*c^7*d^9*e*f^5*g^5*z^4 - 400*a^3
*b^4*c^7*d^5*e^5*f^9*g*z^4 - 372*a^6*b^6*c^2*d^5*e^5*f*g^9*z^4 - 372*a^6*b^
6*c^2*d*e^9*f^5*g^5*z^4 - 372*a^2*b^6*c^6*d^9*e*f^5*g^5*z^4 - 372*a^2*b^6*c
^6*d^5*e^5*f^9*g*z^4 - 328*a^5*b^6*c^3*d^7*e^3*f*g^9*z^4 - 328*a^5*b^6*c^3*
d*e^9*f^7*g^3*z^4 - 328*a^3*b^6*c^5*d^9*e*f^3*g^7*z^4 - 328*a^3*b^6*c^5*d^3
*e^7*f^9*g*z^4 - 288*a^8*b^4*c^2*d^3*e^7*f*g^9*z^4 - 288*a^8*b^4*c^2*d*e^9*
f^3*g^7*z^4 - 288*a^5*b^7*c^2*d^6*e^4*f*g^9*z^4 - 288*a^5*b^7*c^2*d*e^9*f^6
*g^4*z^4 - 288*a^2*b^7*c^5*d^9*e*f^4*g^6*z^4 - 288*a^2*b^7*c^5*d^4*e^6*f^9*
g*z^4 - 288*a^2*b^4*c^8*d^9*e*f^7*g^3*z^4 - 288*a^2*b^4*c^8*d^7*e^3*f^9*g*z
^4 - 280*a^4*b^7*c^3*d^8*e^2*f*g^9*z^4 - 280*a^4*b^7*c^3*d*e^9*f^8*g^2*z^4
- 280*a^3*b^7*c^4*d^9*e*f^2*g^8*z^4 - 280*a^3*b^7*c^4*d^2*e^8*f^9*g*z^4 + 2
56*a^9*b*c^4*d^3*e^7*f^2*g^8*z^4 + 256*a^9*b*c^4*d^2*e^8*f^3*g^7*z^4 + 256*
a^4*b*c^9*d^8*e^2*f^7*g^3*z^4 + 256*a^4*b*c^9*d^7*e^3*f^8*g^2*z^4 - 248*a^7
*b^6*c*d^2*e^8*f^2*g^8*z^4 - 248*a*b^6*c^7*d^8*e^2*f^8*g^2*z^4 + 236*a^6*b^
7*c*d^3*e^7*f^2*g^8*z^4 + 236*a^6*b^7*c*d^2*e^8*f^3*g^7*z^4 + 236*a*b^7*c^6
*d^8*e^2*f^7*g^3*z^4 + 236*a*b^7*c^6*d^7*e^3*f^8*g^2*z^4 + 200*a^4*b^9*c*d^
4*e^6*f^3*g^7*z^4 + 200*a^4*b^9*c*d^3*e^7*f^4*g^6*z^4 - 200*a^3*b^10*c*d^4*
e^6*f^4*g^6*z^4 - 200*a*b^10*c^3*d^6*e^4*f^6*g^4*z^4 + 200*a*b^9*c^4*d^7*e^
3*f^6*g^4*z^4 + 200*a*b^9*c^4*d^6*e^4*f^7*g^3*z^4 - 196*a^4*b^9*c*d^5*e^5*f
^2*g^8*z^4 - 196*a^4*b^9*c*d^2*e^8*f^5*g^5*z^4 - 196*a*b^9*c^4*d^8*e^2*f^5*

g^5*z^4 - 196*a*b^9*c^4*d^5*e^5*f^8*g^2*z^4 - 192*a^9*b^3*c^2*d^2*e^8*f*g^9
*z^4 - 192*a^9*b^3*c^2*d*e^9*f^2*g^8*z^4 - 192*a^2*b^3*c^9*d^9*e*f^8*g^2*z^
4 - 192*a^2*b^3*c^9*d^8*e^2*f^9*g*z^4 + 156*a^4*b^8*c^2*d^7*e^3*f*g^9*z^4 +
 156*a^4*b^8*c^2*d*e^9*f^7*g^3*z^4 + 156*a^2*b^8*c^4*d^9*e*f^3*g^7*z^4 + 15
6*a^2*b^8*c^4*d^3*e^7*f^9*g*z^4 + 96*a^5*b^8*c*d^4*e^6*f^2*g^8*z^4 + 96*a^5
*b^8*c*d^2*e^8*f^4*g^6*z^4 + 96*a*b^8*c^5*d^8*e^2*f^6*g^4*z^4 + 96*a*b^8*c^
5*d^6*e^4*f^8*g^2*z^4 + 88*a^3*b^10*c*d^5*e^5*f^3*g^7*z^4 + 88*a^3*b^10*c*d
^3*e^7*f^5*g^5*z^4 + 88*a*b^10*c^3*d^7*e^3*f^5*g^5*z^4 + 88*a*b^10*c^3*d^5*
e^5*f^7*g^3*z^4 - 36*a^2*b^11*c*d^6*e^4*f^3*g^7*z^4 - 36*a^2*b^11*c*d^3*e^7
*f^6*g^4*z^4 - 36*a*b^11*c^2*d^7*e^3*f^4*g^6*z^4 - 36*a*b^11*c^2*d^4*e^6*f^
7*g^3*z^4 + 28*a^3*b^10*c*d^6*e^4*f^2*g^8*z^4 + 28*a^3*b^10*c*d^2*e^8*f^6*g
^4*z^4 + 28*a*b^10*c^3*d^8*e^2*f^4*g^6*z^4 + 28*a*b^10*c^3*d^4*e^6*f^8*g^2*
z^4 + 24*a^3*b^9*c^2*d^8*e^2*f*g^9*z^4 + 24*a^3*b^9*c^2*d*e^9*f^8*g^2*z^4 +
 24*a^2*b^11*c*d^7*e^3*f^2*g^8*z^4 + 24*a^2*b^11*c*d^2*e^8*f^7*g^3*z^4 + 24
*a^2*b^9*c^3*d^9*e*f^2*g^8*z^4 + 24*a^2*b^9*c^3*d^2*e^8*f^9*g*z^4 + 24*a*b^
11*c^2*d^8*e^2*f^3*g^7*z^4 + 24*a*b^11*c^2*d^3*e^7*f^8*g^2*z^4 + 12*a^2*b^1
1*c*d^5*e^5*f^4*g^6*z^4 + 12*a^2*b^11*c*d^4*e^6*f^5*g^5*z^4 + 12*a*b^11*c^2
*d^6*e^4*f^5*g^5*z^4 + 12*a*b^11*c^2*d^5*e^5*f^6*g^4*z^4 + 40*b^10*c^4*d^7*
e^3*f^7*g^3*z^4 + 20*b^12*c^2*d^6*e^4*f^6*g^4*z^4 - 20*b^11*c^3*d^7*e^3*f^6
*g^4*z^4 - 20*b^11*c^3*d^6*e^4*f^7*g^3*z^4 - 20*b^9*c^5*d^8*e^2*f^7*g^3*z^4
 - 20*b^9*c^5*d^7*e^3*f^8*g^2*z^4 + 20*b^8*c^6*d^8*e^2*f^8*g^2*z^4 + 16*b^1
1*c^3*d^8*e^2*f^5*g^5*z^4 + 16*b^11*c^3*d^5*e^5*f^8*g^2*z^4 - 6*b^12*c^2*d^
8*e^2*f^4*g^6*z^4 - 6*b^12*c^2*d^4*e^6*f^8*g^2*z^4 - 5*b^10*c^4*d^8*e^2*f^6
*g^4*z^4 - 5*b^10*c^4*d^6*e^4*f^8*g^2*z^4 - 4*b^12*c^2*d^7*e^3*f^5*g^5*z^4
 - 4*b^12*c^2*d^5*e^5*f^7*g^3*z^4 - 4608*a^7*c^7*d^5*e^5*f^5*g^5*z^4 + 3328*
a^7*c^7*d^6*e^4*f^4*g^6*z^4 + 3328*a^7*c^7*d^4*e^6*f^6*g^4*z^4 - 3072*a^8*c
^6*d^5*e^5*f^3*g^7*z^4 + 3072*a^8*c^6*d^4*e^6*f^4*g^6*z^4 - 3072*a^8*c^6*d^
3*e^7*f^5*g^5*z^4 - 3072*a^6*c^8*d^7*e^3*f^5*g^5*z^4 + 3072*a^6*c^8*d^6*e^4
*f^6*g^4*z^4 - 3072*a^6*c^8*d^5*e^5*f^7*g^3*z^4 - 2048*a^9*c^5*d^3*e^7*f^3*
g^7*z^4 - 2048*a^7*c^7*d^7*e^3*f^3*g^7*z^4 - 2048*a^7*c^7*d^3*e^7*f^7*g^3*z
^4 - 2048*a^5*c^9*d^7*e^3*f^7*g^3*z^4 + 1792*a^8*c^6*d^6*e^4*f^2*g^8*z^4 +
1792*a^8*c^6*d^2*e^8*f^6*g^4*z^4 + 1792*a^6*c^8*d^8*e^2*f^4*g^6*z^4 + 1792*
a^6*c^8*d^4*e^6*f^8*g^2*z^4 + 1408*a^9*c^5*d^4*e^6*f^2*g^8*z^4 + 1408*a^9*c
^5*d^2*e^8*f^4*g^6*z^4 + 1408*a^5*c^9*d^8*e^2*f^6*g^4*z^4 + 1408*a^5*c^9*d^
6*e^4*f^8*g^2*z^4 + 1088*a^7*c^7*d^8*e^2*f^2*g^8*z^4 + 1088*a^7*c^7*d^2*e^8
*f^8*g^2*z^4 + 512*a^10*c^4*d^2*e^8*f^2*g^8*z^4 + 512*a^4*c^10*d^8*e^2*f^8*
g^2*z^4 + 40*a^4*b^10*d^3*e^7*f^3*g^7*z^4 + 20*a^6*b^8*d^2*e^8*f^2*g^8*z^4
 - 20*a^5*b^9*d^3*e^7*f^2*g^8*z^4 - 20*a^5*b^9*d^2*e^8*f^3*g^7*z^4 - 20*a^3*
b^11*d^4*e^6*f^3*g^7*z^4 - 20*a^3*b^11*d^3*e^7*f^4*g^6*z^4 + 20*a^2*b^12*d^
4*e^6*f^4*g^6*z^4 + 16*a^3*b^11*d^5*e^5*f^2*g^8*z^4 + 16*a^3*b^11*d^2*e^8*f
^5*g^5*z^4 - 6*a^2*b^12*d^6*e^4*f^2*g^8*z^4 - 6*a^2*b^12*d^2*e^8*f^6*g^4*z^
4 - 5*a^4*b^10*d^4*e^6*f^2*g^8*z^4 - 5*a^4*b^10*d^2*e^8*f^4*g^6*z^4 - 4*a^2
*b^12*d^5*e^5*f^3*g^7*z^4 - 4*a^2*b^12*d^3*e^7*f^5*g^5*z^4 + 480*a^8*b^2*c^
4*e^10*f^6*g^4*z^4 - 440*a^7*b^4*c^3*e^10*f^6*g^4*z^4 + 320*a^8*b^3*c^3*e^1
0*f^5*g^5*z^4 + 320*a^7*b^3*c^4*e^10*f^7*g^3*z^4 - 240*a^8*b^4*c^2*e^10*f^4
*g^6*z^4 - 240*a^6*b^4*c^4*e^10*f^8*g^2*z^4 + 192*a^9*b^3*c^2*e^10*f^3*g^7*
z^4 + 192*a^9*b^2*c^3*e^10*f^4*g^6*z^4 + 192*a^7*b^2*c^5*e^10*f^8*g^2*z^4 +
 90*a^6*b^6*c^2*e^10*f^6*g^4*z^4 + 68*a^5*b^6*c^3*e^10*f^8*g^2*z^4 - 48*a^1
0*b^2*c^2*e^10*f^2*g^8*z^4 + 48*a^7*b^5*c^2*e^10*f^5*g^5*z^4 + 48*a^6*b^5*c
^3*e^10*f^7*g^3*z^4 - 36*a^5*b^7*c^2*e^10*f^7*g^3*z^4 - 6*a^4*b^8*c^2*e^10*
f^8*g^2*z^4 + 480*a^4*b^2*c^8*d^10*f^4*g^6*z^4 - 440*a^3*b^4*c^7*d^10*f^4*g
^6*z^4 + 320*a^4*b^3*c^7*d^10*f^3*g^7*z^4 + 320*a^3*b^3*c^8*d^10*f^5*g^5*z^
4 - 240*a^4*b^4*c^6*d^10*f^2*g^8*z^4 - 240*a^2*b^4*c^8*d^10*f^6*g^4*z^4 + 1
92*a^5*b^2*c^7*d^10*f^2*g^8*z^4 + 192*a^3*b^2*c^9*d^10*f^6*g^4*z^4 + 192*a^
2*b^3*c^9*d^10*f^7*g^3*z^4 + 90*a^2*b^6*c^6*d^10*f^4*g^6*z^4 + 68*a^3*b^6*c
^5*d^10*f^2*g^8*z^4 + 48*a^3*b^5*c^6*d^10*f^3*g^7*z^4 + 48*a^2*b^5*c^7*d^10
*f^5*g^5*z^4 - 48*a^2*b^2*c^10*d^10*f^8*g^2*z^4 - 36*a^2*b^7*c^5*d^10*f^3*g
^7*z^4 - 6*a^2*b^8*c^4*d^10*f^2*g^8*z^4 + 480*a^8*b^2*c^4*d^6*e^4*g^10*z^4
 - 440*a^7*b^4*c^3*d^6*e^4*g^10*z^4 + 320*a^8*b^3*c^3*d^5*e^5*g^10*z^4 + 320

*a^7*b^3*c^4*d^7*e^3*g^10*z^4 - 240*a^8*b^4*c^2*d^4*e^6*g^10*z^4 - 240*a^6*
b^4*c^4*d^8*e^2*g^10*z^4 + 192*a^9*b^3*c^2*d^3*e^7*g^10*z^4 + 192*a^9*b^2*c
^3*d^4*e^6*g^10*z^4 + 192*a^7*b^2*c^5*d^8*e^2*g^10*z^4 + 90*a^6*b^6*c^2*d^6
*e^4*g^10*z^4 + 68*a^5*b^6*c^3*d^8*e^2*g^10*z^4 - 48*a^10*b^2*c^2*d^2*e^8*g
^10*z^4 + 48*a^7*b^5*c^2*d^5*e^5*g^10*z^4 + 48*a^6*b^5*c^3*d^7*e^3*g^10*z^4
 - 36*a^5*b^7*c^2*d^7*e^3*g^10*z^4 - 6*a^4*b^8*c^2*d^8*e^2*g^10*z^4 + 480*a
^4*b^2*c^8*d^4*e^6*f^10*z^4 - 440*a^3*b^4*c^7*d^4*e^6*f^10*z^4 + 320*a^4*b^
3*c^7*d^3*e^7*f^10*z^4 + 320*a^3*b^3*c^8*d^5*e^5*f^10*z^4 - 240*a^4*b^4*c^6
*d^2*e^8*f^10*z^4 - 240*a^2*b^4*c^8*d^6*e^4*f^10*z^4 + 192*a^5*b^2*c^7*d^2*
e^8*f^10*z^4 + 192*a^3*b^2*c^9*d^6*e^4*f^10*z^4 + 192*a^2*b^3*c^9*d^7*e^3*f
^10*z^4 + 90*a^2*b^6*c^6*d^4*e^6*f^10*z^4 + 68*a^3*b^6*c^5*d^2*e^8*f^10*z^4
 + 48*a^3*b^5*c^6*d^3*e^7*f^10*z^4 + 48*a^2*b^5*c^7*d^5*e^5*f^10*z^4 - 48*a
^2*b^2*c^10*d^8*e^2*f^10*z^4 - 36*a^2*b^7*c^5*d^3*e^7*f^10*z^4 - 6*a^2*b^8*
c^4*d^2*e^8*f^10*z^4 + 16*b^9*c^5*d^9*e*f^6*g^4*z^4 + 16*b^9*c^5*d^6*e^4*f^
9*g*z^4 - 14*b^10*c^4*d^9*e*f^5*g^5*z^4 - 14*b^10*c^4*d^5*e^5*f^9*g*z^4 + 4
*b^13*c*d^7*e^3*f^4*g^6*z^4 - 4*b^13*c*d^6*e^4*f^5*g^5*z^4 - 4*b^13*c*d^5*e
^5*f^6*g^4*z^4 + 4*b^13*c*d^4*e^6*f^7*g^3*z^4 + 4*b^11*c^3*d^9*e*f^4*g^6*z^
4 + 4*b^11*c^3*d^4*e^6*f^9*g*z^4 - 4*b^8*c^6*d^9*e*f^7*g^3*z^4 - 4*b^8*c^6*
d^7*e^3*f^9*g*z^4 - 4*b^7*c^7*d^9*e*f^8*g^2*z^4 - 4*b^7*c^7*d^8*e^2*f^9*g*z
^4 - 768*a^9*c^5*d^5*e^5*f*g^9*z^4 - 768*a^9*c^5*d*e^9*f^5*g^5*z^4 - 768*a^
5*c^9*d^9*e*f^5*g^5*z^4 - 768*a^5*c^9*d^5*e^5*f^9*g*z^4 - 512*a^10*c^4*d^3*
e^7*f*g^9*z^4 - 512*a^10*c^4*d*e^9*f^3*g^7*z^4 - 512*a^8*c^6*d^7*e^3*f*g^9*
z^4 - 512*a^8*c^6*d*e^9*f^7*g^3*z^4 - 512*a^6*c^8*d^9*e*f^3*g^7*z^4 - 512*a
^6*c^8*d^3*e^7*f^9*g*z^4 - 512*a^4*c^10*d^9*e*f^7*g^3*z^4 - 512*a^4*c^10*d^
7*e^3*f^9*g*z^4 + 16*a^5*b^9*d^4*e^6*f*g^9*z^4 + 16*a^5*b^9*d*e^9*f^4*g^6*z
^4 - 14*a^4*b^10*d^5*e^5*f*g^9*z^4 - 14*a^4*b^10*d*e^9*f^5*g^5*z^4 - 4*a^7*
b^7*d^2*e^8*f*g^9*z^4 - 4*a^7*b^7*d*e^9*f^2*g^8*z^4 - 4*a^6*b^8*d^3*e^7*f*g
^9*z^4 - 4*a^6*b^8*d*e^9*f^3*g^7*z^4 + 4*a^3*b^11*d^6*e^4*f*g^9*z^4 + 4*a^3
*b^11*d*e^9*f^6*g^4*z^4 + 4*a*b^13*d^6*e^4*f^3*g^7*z^4 - 4*a*b^13*d^5*e^5*f
^4*g^6*z^4 - 4*a*b^13*d^4*e^6*f^5*g^5*z^4 + 4*a*b^13*d^3*e^7*f^6*g^4*z^4 -
768*a^9*b*c^4*e^10*f^5*g^5*z^4 - 768*a^8*b*c^5*e^10*f^7*g^3*z^4 - 256*a^10*
b*c^3*e^10*f^3*g^7*z^4 + 192*a^6*b^3*c^5*e^10*f^9*g*z^4 + 68*a^7*b^6*c*e^10
*f^4*g^6*z^4 - 48*a^8*b^5*c*e^10*f^3*g^7*z^4 - 48*a^5*b^5*c^4*e^10*f^9*g*z^
4 - 36*a^6*b^7*c*e^10*f^5*g^5*z^4 + 12*a^9*b^4*c*e^10*f^2*g^8*z^4 + 4*a^4*b
^9*c*e^10*f^7*g^3*z^4 + 4*a^4*b^7*c^3*e^10*f^9*g*z^4 - 768*a^5*b*c^8*d^10*f
^3*g^7*z^4 - 768*a^4*b*c^9*d^10*f^5*g^5*z^4 - 256*a^3*b*c^10*d^10*f^7*g^3*z
^4 + 192*a^5*b^3*c^6*d^10*f*g^9*z^4 + 68*a*b^6*c^7*d^10*f^6*g^4*z^4 - 48*a^
4*b^5*c^5*d^10*f*g^9*z^4 - 48*a*b^5*c^8*d^10*f^7*g^3*z^4 - 36*a*b^7*c^6*d^1
0*f^5*g^5*z^4 + 12*a*b^4*c^9*d^10*f^8*g^2*z^4 + 4*a^3*b^7*c^4*d^10*f*g^9*z^
4 + 4*a*b^9*c^4*d^10*f^3*g^7*z^4 - 768*a^9*b*c^4*d^5*e^5*g^10*z^4 - 768*a^8
*b*c^5*d^7*e^3*g^10*z^4 - 256*a^10*b*c^3*d^3*e^7*g^10*z^4 + 192*a^6*b^3*c^5
*d^9*e*g^10*z^4 + 68*a^7*b^6*c*d^4*e^6*g^10*z^4 - 48*a^8*b^5*c*d^3*e^7*g^10
*z^4 - 48*a^5*b^5*c^4*d^9*e*g^10*z^4 - 36*a^6*b^7*c*d^5*e^5*g^10*z^4 + 12*a
^9*b^4*c*d^2*e^8*g^10*z^4 + 4*a^4*b^9*c*d^7*e^3*g^10*z^4 + 4*a^4*b^7*c^3*d^
9*e*g^10*z^4 - 768*a^5*b*c^8*d^3*e^7*f^10*z^4 - 768*a^4*b*c^9*d^5*e^5*f^10*
z^4 - 256*a^3*b*c^10*d^7*e^3*f^10*z^4 + 192*a^5*b^3*c^6*d*e^9*f^10*z^4 + 68
*a*b^6*c^7*d^6*e^4*f^10*z^4 - 48*a^4*b^5*c^5*d*e^9*f^10*z^4 - 48*a*b^5*c^8*
d^7*e^3*f^10*z^4 - 36*a*b^7*c^6*d^5*e^5*f^10*z^4 + 12*a*b^4*c^9*d^8*e^2*f^1
0*z^4 + 4*a^3*b^7*c^4*d*e^9*f^10*z^4 + 4*a*b^9*c^4*d^3*e^7*f^10*z^4 + 2*b^6
*c^8*d^9*e*f^9*g*z^4 - 128*a^11*c^3*d*e^9*f*g^9*z^4 - 128*a^7*c^7*d^9*e*f*g
^9*z^4 - 128*a^7*c^7*d*e^9*f^9*g*z^4 - 128*a^3*c^11*d^9*e*f^9*g*z^4 + 2*a^8
*b^6*d*e^9*f*g^9*z^4 - 256*a^7*b*c^6*e^10*f^9*g*z^4 - 256*a^6*b*c^7*d^10*f*
g^9*z^4 - 256*a^7*b*c^6*d^9*e*g^10*z^4 - 256*a^6*b*c^7*d*e^9*f^10*z^4 + 2*b
^14*d^5*e^5*f^5*g^5*z^4 + 384*a^9*c^5*e^10*f^6*g^4*z^4 + 256*a^10*c^4*e^10*
f^4*g^6*z^4 + 256*a^8*c^6*e^10*f^8*g^2*z^4 + 64*a^11*c^3*e^10*f^2*g^8*z^4 -
 6*b^8*c^6*d^10*f^6*g^4*z^4 + 4*b^9*c^5*d^10*f^5*g^5*z^4 + 4*b^7*c^7*d^10*f
^7*g^3*z^4 + 384*a^5*c^9*d^10*f^4*g^6*z^4 + 256*a^6*c^8*d^10*f^2*g^8*z^4 +
256*a^4*c^10*d^10*f^6*g^4*z^4 + 64*a^3*c^11*d^10*f^8*g^2*z^4 - 6*a^6*b^8*e^
10*f^4*g^6*z^4 + 4*a^7*b^7*e^10*f^3*g^7*z^4 + 4*a^5*b^9*e^10*f^5*g^5*z^4 +

384*a^9*c^5*d^6*e^4*g^10*z^4 + 256*a^10*c^4*d^4*e^6*g^10*z^4 + 256*a^8*c^6*
d^8*e^2*g^10*z^4 + 64*a^11*c^3*d^2*e^8*g^10*z^4 - 6*b^8*c^6*d^6*e^4*f^10*z^
4 + 4*b^9*c^5*d^5*e^5*f^10*z^4 + 4*b^7*c^7*d^7*e^3*f^10*z^4 + 384*a^5*c^9*d
^4*e^6*f^10*z^4 + 256*a^6*c^8*d^2*e^8*f^10*z^4 + 256*a^4*c^10*d^6*e^4*f^10*
z^4 + 64*a^3*c^11*d^8*e^2*f^10*z^4 - 6*a^6*b^8*d^4*e^6*g^10*z^4 + 4*a^7*b^7
*d^3*e^7*g^10*z^4 + 4*a^5*b^9*d^5*e^5*g^10*z^4 - 48*a^6*b^2*c^6*e^10*f^10*z
^4 - 48*a^6*b^2*c^6*d^10*g^10*z^4 + 12*a^5*b^4*c^5*e^10*f^10*z^4 + 12*a^5*b
^4*c^5*d^10*g^10*z^4 + 64*a^7*c^7*e^10*f^10*z^4 + 64*a^7*c^7*d^10*g^10*z^4
- b^14*d^6*e^4*f^4*g^6*z^4 - b^14*d^4*e^6*f^6*g^4*z^4 - b^10*c^4*d^10*f^4*g
^6*z^4 - b^6*c^8*d^10*f^8*g^2*z^4 - a^8*b^6*e^10*f^2*g^8*z^4 - a^4*b^10*e^1
0*f^6*g^4*z^4 - b^10*c^4*d^4*e^6*f^10*z^4 - b^6*c^8*d^8*e^2*f^10*z^4 - a^8*
b^6*d^2*e^8*g^10*z^4 - a^4*b^10*d^6*e^4*g^10*z^4 - a^4*b^6*c^4*e^10*f^10*z^
4 - a^4*b^6*c^4*d^10*g^10*z^4 + 272*a^5*b^2*c^3*d*e^7*f*g^7*z^2 - 192*a^4*b
^4*c^2*d*e^7*f*g^7*z^2 - 164*a^5*b*c^4*d^2*e^6*f*g^7*z^2 - 164*a^5*b*c^4*d*
e^7*f^2*g^6*z^2 + 120*a^2*b^2*c^6*d^7*e*f*g^7*z^2 + 120*a^2*b^2*c^6*d*e^7*f
^7*g*z^2 + 120*a*b^2*c^7*d^7*e*f^3*g^5*z^2 + 120*a*b^2*c^7*d^3*e^5*f^7*g*z^
2 - 76*a^4*b*c^5*d^4*e^4*f*g^7*z^2 - 76*a^4*b*c^5*d*e^7*f^4*g^4*z^2 - 76*a^
3*b*c^6*d^6*e^2*f*g^7*z^2 - 76*a^3*b*c^6*d*e^7*f^6*g^2*z^2 - 64*a*b^3*c^6*d
^7*e*f^2*g^6*z^2 - 64*a*b^3*c^6*d^2*e^6*f^7*g*z^2 - 60*a^2*b*c^7*d^7*e*f^2*
g^6*z^2 - 60*a^2*b*c^7*d^2*e^6*f^7*g*z^2 + 44*a*b*c^8*d^6*e^2*f^5*g^3*z^2 +
44*a*b*c^8*d^5*e^3*f^6*g^2*z^2 + 22*a*b^5*c^4*d^6*e^2*f*g^7*z^2 + 22*a*b^5
*c^4*d*e^7*f^6*g^2*z^2 - 20*a^2*b^7*c*d^2*e^6*f*g^7*z^2 - 20*a^2*b^7*c*d*e^
7*f^2*g^6*z^2 + 8*a*b^8*c*d^2*e^6*f^2*g^6*z^2 - 8*a*b^6*c^3*d^5*e^3*f*g^7*z
^2 - 8*a*b^6*c^3*d*e^7*f^5*g^3*z^2 + 2*a*b^7*c^2*d^4*e^4*f*g^7*z^2 + 2*a*b^
7*c^2*d*e^7*f^4*g^4*z^2 - 590*a^2*b^2*c^6*d^4*e^4*f^4*g^4*z^2 - 352*a^2*b^4
*c^4*d^3*e^5*f^3*g^5*z^2 - 346*a^3*b^2*c^5*d^4*e^4*f^2*g^6*z^2 - 346*a^3*b^
2*c^5*d^2*e^6*f^4*g^4*z^2 - 274*a^4*b^2*c^4*d^2*e^6*f^2*g^6*z^2 + 272*a^3*b
^2*c^5*d^3*e^5*f^3*g^5*z^2 + 250*a^2*b^3*c^5*d^4*e^4*f^3*g^5*z^2 + 250*a^2*
b^3*c^5*d^3*e^5*f^4*g^4*z^2 + 204*a^3*b^3*c^4*d^3*e^5*f^2*g^6*z^2 + 204*a^3
*b^3*c^4*d^2*e^6*f^3*g^5*z^2 + 136*a^2*b^2*c^6*d^5*e^3*f^3*g^5*z^2 + 136*a^
2*b^2*c^6*d^3*e^5*f^5*g^3*z^2 + 71*a^2*b^4*c^4*d^4*e^4*f^2*g^6*z^2 + 71*a^2
*b^4*c^4*d^2*e^6*f^4*g^4*z^2 - 56*a^2*b^3*c^5*d^5*e^3*f^2*g^6*z^2 - 56*a^2*
b^3*c^5*d^2*e^6*f^5*g^3*z^2 + 18*a^2*b^2*c^6*d^6*e^2*f^2*g^6*z^2 + 18*a^2*b
^2*c^6*d^2*e^6*f^6*g^2*z^2 - 16*a^3*b^4*c^3*d^2*e^6*f^2*g^6*z^2 + 16*a^2*b^
5*c^3*d^3*e^5*f^2*g^6*z^2 + 16*a^2*b^5*c^3*d^2*e^6*f^3*g^5*z^2 - 4*a^2*b^6*
c^2*d^2*e^6*f^2*g^6*z^2 + 48*a^3*b^6*c*d*e^7*f*g^7*z^2 - 20*a*b^4*c^5*d^7*e
*f*g^7*z^2 - 20*a*b^4*c^5*d*e^7*f^7*g*z^2 - 4*a*b^8*c*d^3*e^5*f*g^7*z^2 - 4
*a*b^8*c*d*e^7*f^3*g^5*z^2 + 4*a*b*c^8*d^7*e*f^4*g^4*z^2 + 4*a*b*c^8*d^4*e^
4*f^7*g*z^2 + 368*a^4*b^2*c^4*d^3*e^5*f*g^7*z^2 + 368*a^4*b^2*c^4*d*e^7*f^3
*g^5*z^2 + 264*a^3*b^2*c^5*d^5*e^3*f*g^7*z^2 + 264*a^3*b^2*c^5*d*e^7*f^5*g^
3*z^2 - 208*a^3*b^4*c^3*d^3*e^5*f*g^7*z^2 - 208*a^3*b^4*c^3*d*e^7*f^3*g^5*z
^2 - 164*a^4*b*c^5*d^3*e^5*f^2*g^6*z^2 - 164*a^4*b*c^5*d^2*e^6*f^3*g^5*z^2
+ 140*a^2*b*c^7*d^5*e^3*f^4*g^4*z^2 + 140*a^2*b*c^7*d^4*e^4*f^5*g^3*z^2 - 1
22*a*b^2*c^7*d^6*e^2*f^4*g^4*z^2 - 122*a*b^2*c^7*d^4*e^4*f^6*g^2*z^2 - 108*
a^2*b^3*c^5*d^6*e^2*f*g^7*z^2 - 108*a^2*b^3*c^5*d*e^7*f^6*g^2*z^2 + 102*a*b
^3*c^6*d^5*e^3*f^4*g^4*z^2 + 102*a*b^3*c^6*d^4*e^4*f^5*g^3*z^2 + 80*a*b^6*c
^3*d^3*e^5*f^3*g^5*z^2 + 68*a*b^4*c^5*d^6*e^2*f^2*g^6*z^2 + 68*a*b^4*c^5*d^
2*e^6*f^6*g^2*z^2 - 60*a^3*b*c^6*d^5*e^3*f^2*g^6*z^2 + 60*a^3*b*c^6*d^4*e^4
*f^3*g^5*z^2 + 60*a^3*b*c^6*d^3*e^5*f^4*g^4*z^2 - 60*a^3*b*c^6*d^2*e^6*f^5*
g^3*z^2 - 54*a^3*b^3*c^4*d^4*e^4*f*g^7*z^2 - 54*a^3*b^3*c^4*d*e^7*f^4*g^4*z
^2 - 52*a*b^4*c^5*d^5*e^3*f^3*g^5*z^2 - 52*a*b^4*c^5*d^3*e^5*f^5*g^3*z^2 +
48*a^3*b^5*c^2*d^2*e^6*f*g^7*z^2 + 48*a^3*b^5*c^2*d*e^7*f^2*g^6*z^2 + 48*a^
2*b^6*c^2*d^3*e^5*f*g^7*z^2 + 48*a^2*b^6*c^2*d*e^7*f^3*g^5*z^2 + 44*a^4*b^3
*c^3*d^2*e^6*f*g^7*z^2 + 44*a^4*b^3*c^3*d*e^7*f^2*g^6*z^2 - 44*a^2*b*c^7*d^
6*e^2*f^3*g^5*z^2 - 44*a^2*b*c^7*d^3*e^5*f^6*g^2*z^2 - 44*a*b^3*c^6*d^6*e^2
*f^3*g^5*z^2 - 44*a*b^3*c^6*d^3*e^5*f^6*g^2*z^2 - 32*a*b^5*c^4*d^4*e^4*f^3*
g^5*z^2 - 32*a*b^5*c^4*d^3*e^5*f^4*g^4*z^2 - 32*a*b^2*c^7*d^5*e^3*f^5*g^3*z
^2 - 20*a*b^7*c^2*d^3*e^5*f^2*g^6*z^2 - 20*a*b^7*c^2*d^2*e^6*f^3*g^5*z^2 +
20*a*b^4*c^5*d^4*e^4*f^4*g^4*z^2 - 14*a*b^5*c^4*d^5*e^3*f^2*g^6*z^2 - 14*a*

b^5*c^4*d^2*e^6*f^5*g^3*z^2 + 4*a^2*b^5*c^3*d^4*e^4*f*g^7*z^2 + 4*a^2*b^5*c
^3*d*e^7*f^4*g^4*z^2 - 4*a^2*b^4*c^4*d^5*e^3*f*g^7*z^2 - 4*a^2*b^4*c^4*d*e^
7*f^5*g^3*z^2 + 2*a*b^6*c^3*d^4*e^4*f^2*g^6*z^2 + 2*a*b^6*c^3*d^2*e^6*f^4*g
^4*z^2 - 50*b^2*c^8*d^6*e^2*f^6*g^2*z^2 - 32*b^4*c^6*d^5*e^3*f^5*g^3*z^2 +
24*b^3*c^7*d^6*e^2*f^5*g^3*z^2 + 24*b^3*c^7*d^5*e^3*f^6*g^2*z^2 + 23*b^4*c^
6*d^6*e^2*f^4*g^4*z^2 + 23*b^4*c^6*d^4*e^4*f^6*g^2*z^2 - 11*b^6*c^4*d^6*e^2
*f^2*g^6*z^2 - 11*b^6*c^4*d^2*e^6*f^6*g^2*z^2 + 8*b^6*c^4*d^5*e^3*f^3*g^5*z
^2 + 8*b^6*c^4*d^3*e^5*f^5*g^3*z^2 - 8*b^5*c^5*d^5*e^3*f^4*g^4*z^2 - 8*b^5*
c^5*d^4*e^4*f^5*g^3*z^2 + 5*b^6*c^4*d^4*e^4*f^4*g^4*z^2 - 4*b^8*c^2*d^3*e^5
*f^3*g^5*z^2 + 4*b^7*c^3*d^5*e^3*f^2*g^6*z^2 + 4*b^7*c^3*d^2*e^6*f^5*g^3*z^
2 - 2*b^7*c^3*d^4*e^4*f^3*g^5*z^2 - 2*b^7*c^3*d^3*e^5*f^4*g^4*z^2 - 2*b^5*c
^5*d^6*e^2*f^3*g^5*z^2 - 2*b^5*c^5*d^3*e^5*f^6*g^2*z^2 + 416*a^5*c^5*d^2*e^
6*f^2*g^6*z^2 - 392*a^4*c^6*d^3*e^5*f^3*g^5*z^2 + 376*a^4*c^6*d^4*e^4*f^2*g
^6*z^2 + 376*a^4*c^6*d^2*e^6*f^4*g^4*z^2 + 320*a^3*c^7*d^4*e^4*f^4*g^4*z^2
- 280*a^3*c^7*d^5*e^3*f^3*g^5*z^2 - 280*a^3*c^7*d^3*e^5*f^5*g^3*z^2 - 200*a
^2*c^8*d^5*e^3*f^5*g^3*z^2 + 160*a^3*c^7*d^6*e^2*f^2*g^6*z^2 + 160*a^3*c^7*
d^2*e^6*f^6*g^2*z^2 + 120*a^2*c^8*d^6*e^2*f^4*g^4*z^2 + 120*a^2*c^8*d^4*e^4
*f^6*g^2*z^2 - 471*a^4*b^2*c^4*e^8*f^4*g^4*z^2 + 436*a^3*b^4*c^3*e^8*f^4*g^
4*z^2 - 310*a^3*b^3*c^4*e^8*f^5*g^3*z^2 - 232*a^5*b^2*c^3*e^8*f^2*g^6*z^2 +
229*a^2*b^4*c^4*e^8*f^6*g^2*z^2 + 216*a^4*b^4*c^2*e^8*f^2*g^6*z^2 - 204*a^
4*b^3*c^3*e^8*f^3*g^5*z^2 - 150*a^3*b^2*c^5*e^8*f^6*g^2*z^2 - 91*a^2*b^6*c^
2*e^8*f^4*g^4*z^2 - 72*a^3*b^5*c^2*e^8*f^3*g^5*z^2 - 44*a^2*b^5*c^3*e^8*f^5
*g^3*z^2 - 471*a^4*b^2*c^4*d^4*e^4*g^8*z^2 + 436*a^3*b^4*c^3*d^4*e^4*g^8*z^
2 - 310*a^3*b^3*c^4*d^5*e^3*g^8*z^2 - 232*a^5*b^2*c^3*d^2*e^6*g^8*z^2 + 229
*a^2*b^4*c^4*d^6*e^2*g^8*z^2 + 216*a^4*b^4*c^2*d^2*e^6*g^8*z^2 - 204*a^4*b^
3*c^3*d^3*e^5*g^8*z^2 - 150*a^3*b^2*c^5*d^6*e^2*g^8*z^2 - 91*a^2*b^6*c^2*d^
4*e^4*g^8*z^2 - 72*a^3*b^5*c^2*d^3*e^5*g^8*z^2 - 44*a^2*b^5*c^3*d^5*e^3*g^8
*z^2 - 26*b^3*c^7*d^7*e*f^4*g^4*z^2 - 26*b^3*c^7*d^4*e^4*f^7*g*z^2 + 16*b^2
*c^8*d^7*e*f^5*g^3*z^2 + 16*b^2*c^8*d^5*e^3*f^7*g*z^2 + 10*b^5*c^5*d^7*e*f^
2*g^6*z^2 + 10*b^5*c^5*d^2*e^6*f^7*g*z^2 - 4*b^4*c^6*d^7*e*f^3*g^5*z^2 - 4*
b^4*c^6*d^3*e^5*f^7*g*z^2 + 2*b^9*c*d^3*e^5*f^2*g^6*z^2 + 2*b^9*c*d^2*e^6*f
^3*g^5*z^2 - 168*a^5*c^5*d^3*e^5*f*g^7*z^2 - 168*a^5*c^5*d*e^7*f^3*g^5*z^2
- 120*a^4*c^6*d^5*e^3*f*g^7*z^2 - 120*a^4*c^6*d*e^7*f^5*g^3*z^2 - 56*a^2*c^
8*d^7*e*f^3*g^5*z^2 - 56*a^2*c^8*d^3*e^5*f^7*g*z^2 + 32*a*c^9*d^6*e^2*f^6*g
^2*z^2 + 624*a^4*b*c^5*e^8*f^5*g^3*z^2 + 548*a^5*b*c^4*e^8*f^3*g^5*z^2 - 18
2*a^2*b^3*c^5*e^8*f^7*g*z^2 - 96*a^5*b^3*c^2*e^8*f*g^7*z^2 - 68*a*b^6*c^3*e
^8*f^6*g^2*z^2 - 58*a^3*b^6*c*e^8*f^2*g^6*z^2 + 38*a^2*b^7*c*e^8*f^3*g^5*z^
2 + 36*a*b^7*c^2*e^8*f^5*g^3*z^2 + 18*a*b^2*c^7*d^8*f^2*g^6*z^2 + 624*a^4*b
*c^5*d^5*e^3*g^8*z^2 + 548*a^5*b*c^4*d^3*e^5*g^8*z^2 - 182*a^2*b^3*c^5*d^7*
e*g^8*z^2 - 96*a^5*b^3*c^2*d*e^7*g^8*z^2 - 68*a*b^6*c^3*d^6*e^2*g^8*z^2 - 5
8*a^3*b^6*c*d^2*e^6*g^8*z^2 + 38*a^2*b^7*c*d^3*e^5*g^8*z^2 + 36*a*b^7*c^2*d
^5*e^3*g^8*z^2 + 18*a*b^2*c^7*d^2*e^6*f^8*z^2 + 12*b*c^9*d^7*e*f^6*g^2*z^2
+ 12*b*c^9*d^6*e^2*f^7*g*z^2 - 72*a^6*c^4*d*e^7*f*g^7*z^2 - 40*a*c^9*d^7*e*
f^5*g^3*z^2 - 40*a*c^9*d^5*e^3*f^7*g*z^2 - 24*a^3*c^7*d^7*e*f*g^7*z^2 - 24*
a^3*c^7*d*e^7*f^7*g*z^2 - 4*a^2*b^8*d*e^7*f*g^7*z^2 + 2*a*b^9*d^2*e^6*f*g^7
*z^2 + 2*a*b^9*d*e^7*f^2*g^6*z^2 + 204*a^3*b*c^6*e^8*f^7*g*z^2 + 128*a^6*b*
c^3*e^8*f*g^7*z^2 + 48*a*b^5*c^4*e^8*f^7*g*z^2 + 24*a^4*b^5*c*e^8*f*g^7*z^2
- 48*a*b*c^8*d^8*f^3*g^5*z^2 - 36*a^2*b*c^7*d^8*f*g^7*z^2 + 6*a*b^3*c^6*d^
8*f*g^7*z^2 + 204*a^3*b*c^6*d^7*e*g^8*z^2 + 128*a^6*b*c^3*d*e^7*g^8*z^2 + 4
8*a*b^5*c^4*d^7*e*g^8*z^2 + 24*a^4*b^5*c*d*e^7*g^8*z^2 - 48*a*b*c^8*d^3*e^5
*f^8*z^2 - 36*a^2*b*c^7*d*e^7*f^8*z^2 + 6*a*b^3*c^6*d*e^7*f^8*z^2 - b^8*c^2
*d^4*e^4*f^2*g^6*z^2 - b^8*c^2*d^2*e^6*f^4*g^4*z^2 - 4*b^9*c*e^8*f^5*g^3*z^
2 - 4*b^7*c^3*e^8*f^7*g*z^2 - 12*b*c^9*d^8*f^5*g^3*z^2 + 24*a*c^9*d^8*f^4*g
^4*z^2 - 4*b^9*c*d^5*e^3*g^8*z^2 - 4*b^7*c^3*d^7*e*g^8*z^2 - 4*a*b^9*e^8*f^
3*g^5*z^2 - 2*a^3*b^7*e^8*f*g^7*z^2 - 12*b*c^9*d^5*e^3*f^8*z^2 + 24*a*c^9*d
^4*e^4*f^8*z^2 - 4*a*b^9*d^3*e^5*g^8*z^2 - 2*a^3*b^7*d*e^7*g^8*z^2 - 12*a^5
*b^4*c*e^8*g^8*z^2 - 12*a*b^4*c^5*e^8*f^8*z^2 - 12*a*b^4*c^5*d^8*g^8*z^2 -
8*c^10*d^7*e*f^7*g*z^2 + 6*b^8*c^2*e^8*f^6*g^2*z^2 - 232*a^5*c^5*e^8*f^4*g^
4*z^2 - 188*a^4*c^6*e^8*f^6*g^2*z^2 - 92*a^6*c^4*e^8*f^2*g^6*z^2 + 9*b^2*c^

8*d^8*f^4*g^4*z^2 - 3*b^4*c^6*d^8*f^2*g^6*z^2 + 2*b^3*c^7*d^8*f^3*g^5*z^2 +
 36*a^2*c^8*d^8*f^2*g^6*z^2 + 6*b^8*c^2*d^6*e^2*g^8*z^2 + 5*a^2*b^8*e^8*f^2
*g^6*z^2 - 232*a^5*c^5*d^4*e^4*g^8*z^2 - 188*a^4*c^6*d^6*e^2*g^8*z^2 - 92*a
^6*c^4*d^2*e^6*g^8*z^2 + 9*b^2*c^8*d^4*e^4*f^8*z^2 - 3*b^4*c^6*d^2*e^6*f^8*
z^2 + 2*b^3*c^7*d^3*e^5*f^8*z^2 + 36*a^2*c^8*d^2*e^6*f^8*z^2 + 5*a^2*b^8*d^
2*e^6*g^8*z^2 + 48*a^6*b^2*c^2*e^8*g^8*z^2 + 45*a^2*b^2*c^6*e^8*f^8*z^2 + 4
5*a^2*b^2*c^6*d^8*g^8*z^2 + 4*c^10*d^8*f^6*g^2*z^2 + b^10*e^8*f^4*g^4*z^2 +
 4*c^10*d^6*e^2*f^8*z^2 + b^10*d^4*e^4*g^8*z^2 - 64*a^7*c^3*e^8*g^8*z^2 + b
^6*c^4*e^8*f^8*z^2 + b^6*c^4*d^8*g^8*z^2 - 48*a^3*c^7*e^8*f^8*z^2 - 48*a^3*
c^7*d^8*g^8*z^2 + a^4*b^6*e^8*g^8*z^2 - b^10*d^2*e^6*f^2*g^6*z^2 + 108*a^2*
b^2*c^4*d^2*e^5*f*g^6*z + 108*a^2*b^2*c^4*d*e^6*f^2*g^5*z + 60*a*b^2*c^5*d^
3*e^4*f^2*g^5*z + 60*a*b^2*c^5*d^2*e^5*f^3*g^4*z - 48*a^2*b*c^5*d^2*e^5*f^2
*g^5*z - 44*a*b^3*c^4*d^2*e^5*f^2*g^5*z - 120*a^2*b*c^5*d^3*e^4*f*g^6*z - 1
20*a^2*b*c^5*d*e^6*f^3*g^4*z - 96*a*b*c^6*d^3*e^4*f^3*g^4*z - 64*a^2*b^3*c^
3*d*e^6*f*g^6*z + 32*a*b^3*c^4*d^3*e^4*f*g^6*z + 32*a*b^3*c^4*d*e^6*f^3*g^4
*z - 28*a*b^4*c^3*d^2*e^5*f*g^6*z - 28*a*b^4*c^3*d*e^6*f^2*g^5*z - 18*a*b^2
*c^5*d^4*e^3*f*g^6*z - 18*a*b^2*c^5*d*e^6*f^4*g^3*z + 4*a*b*c^6*d^4*e^3*f^2
*g^5*z + 4*a*b*c^6*d^2*e^5*f^4*g^3*z + 24*a*b^5*c^2*d*e^6*f*g^6*z - 16*a^3*
b*c^4*d*e^6*f*g^6*z - 8*a*b*c^6*d^5*e^2*f*g^6*z - 8*a*b*c^6*d*e^6*f^5*g^2*z
 - 13*b^2*c^6*d^6*e*f*g^6*z - 13*b^2*c^6*d*e^6*f^6*g*z + 8*b*c^7*d^6*e*f^2*
g^5*z + 8*b*c^7*d^2*e^5*f^6*g*z + 9*b^2*c^6*d^4*e^3*f^3*g^4*z + 9*b^2*c^6*d
^3*e^4*f^4*g^3*z + 8*b^5*c^3*d^2*e^5*f^2*g^5*z - 6*b^4*c^4*d^3*e^4*f^2*g^5*
z - 6*b^4*c^4*d^2*e^5*f^3*g^4*z - 6*b^3*c^5*d^4*e^3*f^2*g^5*z - 6*b^3*c^5*d
^2*e^5*f^4*g^3*z + 4*b^3*c^5*d^3*e^4*f^3*g^4*z + b^2*c^6*d^5*e^2*f^2*g^5*z
 + b^2*c^6*d^2*e^5*f^5*g^2*z + 16*a^2*c^6*d^3*e^4*f^2*g^5*z + 16*a^2*c^6*d^2
*e^5*f^3*g^4*z - 112*a^2*b^3*c^3*e^7*f^2*g^5*z - 12*a^2*b^2*c^4*e^7*f^3*g^4
*z - 112*a^2*b^3*c^3*d^2*e^5*g^7*z - 12*a^2*b^2*c^4*d^3*e^4*g^7*z - 2*b^7*c
*d*e^6*f*g^6*z + 8*a*c^7*d^6*e*f*g^6*z + 8*a*c^7*d*e^6*f^6*g*z + 52*a*b*c^6
*e^7*f^6*g*z - 10*a*b^6*c*e^7*f*g^6*z + 52*a*b*c^6*d^6*e*g^7*z - 10*a*b^6*c
*d*e^6*g^7*z + 14*b^3*c^5*d^5*e^2*f*g^6*z + 14*b^3*c^5*d*e^6*f^5*g^2*z - 12
*b*c^7*d^5*e^2*f^3*g^4*z - 12*b*c^7*d^3*e^4*f^5*g^2*z - 5*b^4*c^4*d^4*e^3*f
*g^6*z - 5*b^4*c^4*d*e^6*f^4*g^3*z + b^6*c^2*d^2*e^5*f*g^6*z + b^6*c^2*d*e^
6*f^2*g^5*z + 52*a^2*c^6*d^4*e^3*f*g^6*z + 52*a^2*c^6*d*e^6*f^4*g^3*z + 24*
a*c^7*d^4*e^3*f^3*g^4*z + 24*a*c^7*d^3*e^4*f^4*g^3*z - 16*a*c^7*d^5*e^2*f^2
*g^5*z - 16*a*c^7*d^2*e^5*f^5*g^2*z + 8*a^3*c^5*d^2*e^5*f*g^6*z + 8*a^3*c^5
*d*e^6*f^2*g^5*z + 200*a^3*b*c^4*e^7*f^2*g^5*z + 144*a^2*b*c^5*e^7*f^4*g^3*
z - 42*a*b^2*c^5*e^7*f^5*g^2*z + 32*a^3*b^2*c^3*e^7*f*g^6*z + 24*a^2*b^4*c^
2*e^7*f*g^6*z + 24*a*b^5*c^2*e^7*f^2*g^5*z - 10*a*b^3*c^4*e^7*f^4*g^3*z + 4
*a*b^4*c^3*e^7*f^3*g^4*z + 200*a^3*b*c^4*d^2*e^5*g^7*z + 144*a^2*b*c^5*d^4*
e^3*g^7*z - 42*a*b^2*c^5*d^5*e^2*g^7*z + 32*a^3*b^2*c^3*d*e^6*g^7*z + 24*a^
2*b^4*c^2*d*e^6*g^7*z + 24*a*b^5*c^2*d^2*e^5*g^7*z - 10*a*b^3*c^4*d^4*e^3*g
^7*z + 4*a*b^4*c^3*d^3*e^4*g^7*z + 4*b*c^7*d^7*f*g^6*z + 4*b*c^7*d*e^6*f^7*
z + 11*b^4*c^4*e^7*f^5*g^2*z - 4*b^5*c^3*e^7*f^4*g^3*z + b^6*c^2*e^7*f^3*g^
4*z - 136*a^3*c^5*e^7*f^3*g^4*z - 68*a^2*c^6*e^7*f^5*g^2*z + 11*b^4*c^4*d^5
*e^2*g^7*z - 4*b^5*c^3*d^4*e^3*g^7*z + b^6*c^2*d^3*e^4*g^7*z - 136*a^3*c^5*
d^3*e^4*g^7*z - 68*a^2*c^6*d^5*e^2*g^7*z - 96*a^3*b^3*c^2*e^7*g^7*z + 4*c^8
*d^6*e*f^3*g^4*z + 4*c^8*d^3*e^4*f^6*g*z - 10*b^3*c^5*e^7*f^6*g*z - 2*b^7*c
*e^7*f^2*g^5*z - 128*a^4*c^4*e^7*f*g^6*z - 10*b^3*c^5*d^6*e*g^7*z - 2*b^7*c
*d^2*e^5*g^7*z - 128*a^4*c^4*d*e^6*g^7*z + 128*a^4*b*c^3*e^7*g^7*z + 24*a^2
*b^5*c*e^7*g^7*z - 4*c^8*d^7*f^2*g^5*z - 4*c^8*d^2*e^5*f^7*z + 3*b^2*c^6*e^
7*f^7*z + 3*b^2*c^6*d^7*g^7*z + b^8*e^7*f*g^6*z + b^8*d*e^6*g^7*z - 16*a*c^
7*e^7*f^7*z - 16*a*c^7*d^7*g^7*z - 2*a*b^7*e^7*g^7*z - 8*a*c^5*d*e^5*f*g^5
+ 20*a*b*c^4*e^6*f*g^5 + 20*a*b*c^4*d*e^5*g^6 + 4*b*c^5*d^2*e^4*f*g^5 + 4*b
*c^5*d*e^5*f^2*g^4 - 2*b^2*c^4*d*e^5*f*g^5 - 4*b^3*c^3*e^6*f*g^5 - 16*a*c^5
*e^6*f^2*g^4 - 4*b^3*c^3*d*e^5*g^6 - 16*a*c^5*d^2*e^4*g^6 + 8*a*b^2*c^3*e^6
*g^6 - 4*c^6*d^2*e^4*f^2*g^4 + 3*b^2*c^4*e^6*f^2*g^4 + 3*b^2*c^4*d^2*e^4*g^
6 - 36*a^2*c^4*e^6*g^6, z, k), k, 1, 4)

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

3.562 $\int \frac{(d+ex)^3(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=287

$$-\frac{2e(f+gx)^{7/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}+\frac{2(f+gx)^{5/2}(ef-dg)\left(3eg(-aeg-bdg\right.}{5g}$$

Rubi [A] time = 0.50, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {897, 1153}

$$-\frac{2e(f+gx)^{7/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}+\frac{2(f+gx)^{5/2}(ef-dg)\left(3eg(-aeg-bdg+2bef)-c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}-\frac{2\sqrt{f+gx}(ef-dg)^3\left(ag^2-bfg+cf^2\right)}{g^6}+\frac{2(f+gx)^{3/2}(ef-dg)^2(cf(5ef-2dg)-g(-3aeg-bdg+4bef))}{3g^6}-\frac{2e^2(f+gx)^{9/2}(-beg-3cdg+5cef)}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*
g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*(f + g*x)^(3/2))
/(3*g^6) + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2
- 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6) - (2*e*(e*g*(4*b*e*f - 3*b
*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/2))/(
7*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)dx,x,\sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(-ef+dg)^3\left(cf^2-bfg+ag^2\right)}{g^5}+\frac{(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))x^2}{g^5}+\frac{(ef-dg)(\ldots}{}\right.\right.}{}$$

$$= -\frac{2(ef-dg)^3\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef\ldots}{3g^6}$$

Mathematica [A] time = 0.41, size = 249, normalized size = 0.87

$$\frac{2\sqrt{f+gx}\left(-495e(f+gx)^3\left(c\left(-3d^2g^2+12defg-10e^2f^2\right)-eg(aeg+3bdg-4bef)\right)+693(f+gx)^2(ef-dg)\left(-3eg(aeg+bdg-2bef)-c\left(d^2g^2-8defg+10e^2f^2\right)\right)-3465(ef-dg)^3\left(g(ag-bf)+cf^2\right)+1155(f+gx)(ef-dg)^2(g(3aeg+bdg-4bef)+cf(5ef-2dg))-385e^2(f+gx)^4(-beg-3cdg+5cef)+315ce^3(f+gx)^5\right)}{3465g^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-3465*(e*f - d*g)^3*(c*f^2 + g*(-(b*f) + a*g)) + 1155*(e*
f - d*g)^2*(c*f*(5*e*f - 2*d*g) + g*(-4*b*e*f + b*d*g + 3*a*e*g))*(f + g*x)
 + 693*(e*f - d*g)*(-3*e*g*(-2*b*e*f + b*d*g + a*e*g) - c*(10*e^2*f^2 - 8*d
*e*f*g + d^2*g^2))*(f + g*x)^2 - 495*e*(-(e*g*(-4*b*e*f + 3*b*d*g + a*e*g))
 + c*(-10*e^2*f^2 + 12*d*e*f*g - 3*d^2*g^2))*(f + g*x)^3 - 385*e^2*(5*c*e*f
 - 3*c*d*g - b*e*g)*(f + g*x)^4 + 315*c*e^3*(f + g*x)^5))/(3465*g^6)
```

IntegrateAlgebraic [**B**] time = 0.35, size = 634, normalized size = 2.21

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-3465*c*e^3*f^5 + 10395*c*d*e^2*f^4*g + 3465*b*e^3*f^4*g
- 10395*c*d^2*e*f^3*g^2 - 10395*b*d*e^2*f^3*g^2 - 3465*a*e^3*f^3*g^2 + 3465
*c*d^3*f^2*g^3 + 10395*b*d^2*e*f^2*g^3 + 10395*a*d*e^2*f^2*g^3 - 3465*b*d^3
*f*g^4 - 10395*a*d^2*e*f*g^4 + 3465*a*d^3*g^5 + 5775*c*e^3*f^4*(f + g*x) -
13860*c*d*e^2*f^3*g*(f + g*x) - 4620*b*e^3*f^3*g*(f + g*x) + 10395*c*d^2*e*
f^2*g^2*(f + g*x) + 10395*b*d*e^2*f^2*g^2*(f + g*x) + 3465*a*e^3*f^2*g^2*(f
 + g*x) - 2310*c*d^3*f*g^3*(f + g*x) - 6930*b*d^2*e*f*g^3*(f + g*x) - 6930*
a*d*e^2*f*g^3*(f + g*x) + 1155*b*d^3*g^4*(f + g*x) + 3465*a*d^2*e*g^4*(f +
g*x) - 6930*c*e^3*f^3*(f + g*x)^2 + 12474*c*d*e^2*f^2*g*(f + g*x)^2 + 4158*
b*e^3*f^2*g*(f + g*x)^2 - 6237*c*d^2*e*f*g^2*(f + g*x)^2 - 6237*b*d*e^2*f*g
^2*(f + g*x)^2 - 2079*a*e^3*f*g^2*(f + g*x)^2 + 693*c*d^3*g^3*(f + g*x)^2 +
 2079*b*d^2*e*g^3*(f + g*x)^2 + 2079*a*d*e^2*g^3*(f + g*x)^2 + 4950*c*e^3*f
^2*(f + g*x)^3 - 5940*c*d*e^2*f*g*(f + g*x)^3 - 1980*b*e^3*f*g*(f + g*x)^3
+ 1485*c*d^2*e*g^2*(f + g*x)^3 + 1485*b*d*e^2*g^2*(f + g*x)^3 + 495*a*e^3*g
^2*(f + g*x)^3 - 1925*c*e^3*f*(f + g*x)^4 + 1155*c*d*e^2*g*(f + g*x)^4 + 38
5*b*e^3*g*(f + g*x)^4 + 315*c*e^3*(f + g*x)^5))/(3465*g^6)
```

fricas [**A**] time = 0.56, size = 429, normalized size = 1.49

$$\frac{2\left(315\,ce^{3}g^{5}x^{5} - 1280\,ce^{3}f^{5} + 3465\,ad^{3}g^{5} + 1408\left(3\,cde^{2} + be^{3}\right)f^{4}g - 1584\left(3\,cd^{2}e + 3\,bde^{2} + ae^{3}\right)f^{3}g^{2} + 1848\left(cd^{3} + 3\,bd^{2}e + 3\,ade^{2}\right)f^{2}g^{3} - 2310\left(bd^{3} + 3\,ad^{2}e\right)fg^{4} - 35\left(10\,ce^{3}fg^{4} - 11\left(3\,cde^{2} + be^{3}\right)g^{5}\right)x^{4} + 5\left(80\,ce^{3}f^{2}g^{3} - 88\left(3\,cde^{2} + be^{3}\right)fg^{4} + 99\left(3\,cd^{2}e + 3\,bde^{2} + ae^{3}\right)g^{5}\right)x^{3} - 3\left(160\,ce^{3}f^{3}g^{2} - 176\left(3\,cde^{2} + be^{3}\right)f^{2}g^{3} + 198\left(3\,cd^{2}e + 3\,bde^{2} + ae^{3}\right)fg^{4} - 231\left(cd^{3} + 3\,bd^{2}e + 3\,ade^{2}\right)g^{5}\right)x^{2} + \left(640\,ce^{3}f^{4}g - 704\left(3\,cde^{2} + be^{3}\right)f^{3}g^{2} + 792\left(3\,cd^{2}e + 3\,bde^{2} + ae^{3}\right)f^{2}g^{3} - 924\left(cd^{3} + 3\,bd^{2}e + 3\,ade^{2}\right)fg^{4} + 1155\left(bd^{3} + 3\,ad^{2}e\right)g^{5}\right)x\right)\sqrt{gx + f}}{3465\,g^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 3465*a*d^3*g^5 + 1408*(3*c*d*e
^2 + b*e^3)*f^4*g - 1584*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 + 1848*(c*
d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 - 2310*(b*d^3 + 3*a*d^2*e)*f*g^4 - 35*
(10*c*e^3*f*g^4 - 11*(3*c*d*e^2 + b*e^3)*g^5)*x^4 + 5*(80*c*e^3*f^2*g^3 - 8
8*(3*c*d*e^2 + b*e^3)*f*g^4 + 99*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*x^3 -
 3*(160*c*e^3*f^3*g^2 - 176*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 198*(3*c*d^2*e +
3*b*d*e^2 + a*e^3)*f*g^4 - 231*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*x^2 + (
640*c*e^3*f^4*g - 704*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 792*(3*c*d^2*e + 3*b*d*
e^2 + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^4 + 1155*(b*
d^3 + 3*a*d^2*e)*g^5)*x)*sqrt(g*x + f)/g^6
```

giac [**B**] time = 0.20, size = 565, normalized size = 1.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(g*x + f)*a*d^3 + 1155*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*
f)*b*d^3/g + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d^2*e/g + 231*(3*
(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^3/g^2 +
693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d^2
*e/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x +
 f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 297*(5*(g*x + f)^(7/2)
- 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*d
*e^2/g^3 + 99*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2
)*f^2 - 35*sqrt(g*x + f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x
 + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqr
t(g*x + f)*f^4)*c*d*e^2/g^4 + 11*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*
f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f
^4)*b*e^3/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x +
 f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*s
qrt(g*x + f)*f^5)*c*e^3/g^5)/g
```

maple [B]   time = 0.01, size = 540, normalized size = 1.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3465*(g*x+f)^(1/2)*(315*c*e^3*g^5*x^5+385*b*e^3*g^5*x^4+1155*c*d*e^2*g^5*
x^4-350*c*e^3*f*g^4*x^4+495*a*e^3*g^5*x^3+1485*b*d*e^2*g^5*x^3-440*b*e^3*f*
g^4*x^3+1485*c*d^2*e*g^5*x^3-1320*c*d*e^2*f*g^4*x^3+400*c*e^3*f^2*g^3*x^3+2
079*a*d*e^2*g^5*x^2-594*a*e^3*f*g^4*x^2+2079*b*d^2*e*g^5*x^2-1782*b*d*e^2*f
*g^4*x^2+528*b*e^3*f^2*g^3*x^2+693*c*d^3*g^5*x^2-1782*c*d^2*e*f*g^4*x^2+158
4*c*d*e^2*f^2*g^3*x^2-480*c*e^3*f^3*g^2*x^2+3465*a*d^2*e*g^5*x-2772*a*d*e^2
*f*g^4*x+792*a*e^3*f^2*g^3*x+1155*b*d^3*g^5*x-2772*b*d^2*e*f*g^4*x+2376*b*d
*e^2*f^2*g^3*x-704*b*e^3*f^3*g^2*x-924*c*d^3*f*g^4*x+2376*c*d^2*e*f^2*g^3*x
-2112*c*d*e^2*f^3*g^2*x+640*c*e^3*f^4*g*x+3465*a*d^3*g^5-6930*a*d^2*e*f*g^4
+5544*a*d*e^2*f^2*g^3-1584*a*e^3*f^3*g^2-2310*b*d^3*f*g^4+5544*b*d^2*e*f^2*
g^3-4752*b*d*e^2*f^3*g^2+1408*b*e^3*f^4*g+1848*c*d^3*f^2*g^3-4752*c*d^2*e*f
^3*g^2+4224*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6
```

maxima [A]   time = 0.46, size = 429, normalized size = 1.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - (3*c*d*e^2 + b*e^3)*g
)*(g*x + f)^(9/2) + 495*(10*c*e^3*f^2 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^
2*e + 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(7/2) - 693*(10*c*e^3*f^3 - 6*(3*c*
d*e^2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3
*b*d^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 4*(3*c*d*e
^2 + b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*b*d^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*(g*x + f)^(3/2) - 3
465*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*c*d^2*e + 3*b*d
*e^2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + (b*d^3 +
3*a*d^2*e)*f*g^4)*sqrt(g*x + f))/g^6
```

mupad [B]   time = 0.15, size = 283, normalized size = 0.99

$$\frac{(f+g\,x)^{9/2}\,\left(2\,b\,e^3\,g-10\,c\,e^3\,f+6\,c\,d\,e^2\,g\right)}{9\,g^6}+\frac{(f+g\,x)^{7/2}\,\left(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+6\,b\,d\,e^2\,g^2+20\,c\,e^3\,f^2-8\,b\,e^3\,f\,g+2\,a\,e^3\,g^2\right)}{7\,g^6}+\frac{2\,(f+g\,x)^{5/2}\,(d\,g-e\,f)\,\left(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+3\,b\,d\,e\,g^2+10\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{5\,g^6}+\frac{2\,\sqrt{f+g\,x}\,{(d\,g-e\,f)}^3\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{g^6}+\frac{2\,(f+g\,x)^{3/2}\,{(d\,g-e\,f)}^2\,\left(3\,a\,e\,g^2+b\,d\,g^2+5\,c\,e\,f^2-4\,b\,e\,f\,g-2\,c\,d\,f\,g\right)}{3\,g^6}+\frac{2\,c\,e^3\,(f+g\,x)^{11/2}}{11\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(9/2)*(2*b*e^3*g - 10*c*e^3*f + 6*c*d*e^2*g))/(9*g^6) + ((f + g*
x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 - 8*b*e^3*f*g + 6*b*d*e^2*g^2 + 6*c*d^
2*e*g^2 - 24*c*d*e^2*f*g))/(7*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(3*a*e^
2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 + 3*b*d*e*g^2 - 6*b*e^2*f*g - 8*c*d*e*f*g)
)/(5*g^6) + (2*(f + g*x)^(1/2)*(d*g - e*f)^3*(a*g^2 + c*f^2 - b*f*g))/g^6 +
 (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*e*g^2 + b*d*g^2 + 5*c*e*f^2 - 4*b*e*
f*g - 2*c*d*f*g))/(3*g^6) + (2*c*e^3*(f + g*x)^(11/2))/(11*g^6)
```

sympy [A] time = 164.70, size = 1544, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d**3*f/sqrt(f + g*x) - 2*a*d**3*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) - 6*a*d**2*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 6*a*d**2*e*(
f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 6*a*d*e**2
*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 6*a
*d*e**2*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) -
(f + g*x)**(5/2)/5)/g**2 - 2*a*e**3*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f
+ g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 - 2*a*e**3*(f**4/sqr
t(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)
**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*b*d**3*f*(-f/sqrt(f + g*x) - sqrt(
f + g*x))/g - 2*b*d**3*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)*
*(3/2)/3)/g - 6*b*d**2*e*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g
*x)**(3/2)/3)/g**2 - 6*b*d**2*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x)
 + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 6*b*d*e**2*f*(-f**3/sqrt
(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)
/g**3 - 6*b*d*e**2*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f +
 g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*b*e**3
*f*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4
*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 - 2*b*e**3*(-f**5/sqrt(f +
 g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x
)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4 - 2*c*d**3*f*(
f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d**
3*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g
*x)**(5/2)/5)/g**2 - 6*c*d**2*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*
x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 - 6*c*d**2*e*(f**4/sqrt(
f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**
(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 6*c*d*e**2*f*(f**4/sqrt(f + g*x) + 4*f
**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f +
 g*x)**(7/2)/7)/g**4 - 6*c*d*e**2*(-f**5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*
x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5*f*(f + g*x)**
(7/2)/7 - (f + g*x)**(9/2)/9)/g**4 - 2*c*e**3*f*(-f**5/sqrt(f + g*x) - 5*f*
*4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5
*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**5 - 2*c*e**3*(f**6/sqrt(f +
g*x) + 6*f**5*sqrt(f + g*x) - 5*f**4*(f + g*x)**(3/2) + 4*f**3*(f + g*x)**(
5/2) - 15*f**2*(f + g*x)**(7/2)/7 + 2*f*(f + g*x)**(9/2)/3 - (f + g*x)**(11
/2)/11)/g**5)/g, Ne(g, 0)), ((a*d**3*x + c*e**3*x**6/6 + x**5*(b*e**3 + 3*c
*d*e**2)/5 + x**4*(a*e**3 + 3*b*d*e**2 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 +
 3*b*d**2*e + c*d**3)/3 + x**2*(3*a*d**2*e + b*d**3)/2)/sqrt(f), True))
```

3.563 $\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=212

$$-\frac{2(f+gx)^{5/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\,(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5}-\frac{2(\cdots}{}$$

Rubi [A] time = 0.34, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {897, 1153}

$$-\frac{2(f+gx)^{5/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\,(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5}-\frac{2(f+gx)^{3/2}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{3g^5}-\frac{2e(f+gx)^{7/2}(-beg-2cdg+4cef)}{7g^5}+\frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^5 - (2*(e*f - d*g
)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*(f + g*x)^(3/2))/(3
*g^5) - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^
2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*
x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx &= \frac{2\,\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)dx,x,\sqrt{f+gx}\right)}{g}\\ &= \frac{2\,\text{Subst}\left(\int \left(\frac{(-ef+dg)^2\left(cf^2-bfg+ag^2\right)}{g^4}+\frac{(ef-dg)(-2cf(2ef-dg)+g(3bef-bdg-2aeg))x^2}{g^4}+\frac{(-eg(3b\cdots}{}\right.\right.}{g}\\ &= \frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^5}-\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-\cdots}{3g^5}\end{aligned}$$

Mathematica [A] time = 0.34, size = 184, normalized size = 0.87

$$\frac{2\sqrt{f+gx}\left(-63(f+gx)^2\left(-eg(aeg+2bdg-3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)+315(ef-dg)^2\left(g(ag-bf)+cf^2\right)-105(f+gx)(ef-dg)(g(2aeg+bdg-3bef)+2cf(2ef-dg))-45e(f+gx)^3(-beg-2cdg+4cef)+35ce^2(f+gx)^4\right)}{315g^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(315*(e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g)) - 105*(e*f -
 d*g)*(2*c*f*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*(f + g*x) - 63
*(-(e*g*(-3*b*e*f + 2*b*d*g + a*e*g)) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2)
)*(f + g*x)^2 - 45*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^3 + 35*c*e^2*(f
+ g*x)^4))/(315*g^5)
```

IntegrateAlgebraic [A] time = 0.21, size = 368, normalized size = 1.74

$$\frac{2\sqrt{f+gx}\left(315ad^2g^4+210adeg^3(f+gx)-630adefg^3+315ae^2f^2g^2-210ae^2fg^2(f+gx)+63ae^2g^2(f+gx)^2+105bd^2g^3(f+gx)-315bd^2fg^3+630bdef^2g^2-420bdefg^2(f+gx)+126bdeg^2(f+gx)^2-315be^2f^3g+315be^2f^2g(f+gx)-189be^2fg(f+gx)^2+45be^2g(f+gx)^3+315cd^2f^2g^2-210cd^2fg^2(f+gx)+63cd^2g^2(f+gx)^2-630cdef^3g+630cdef^2g(f+gx)-378cdefg(f+gx)^2+90cdeg(f+gx)^3+315ce^2f^4-420ce^2f^3(f+gx)+378ce^2f^2(f+gx)^2-180ce^2f(f+gx)^3+35ce^2(f+gx)^4\right)}{315g^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(315*c*e^2*f^4 - 630*c*d*e*f^3*g - 315*b*e^2*f^3*g + 315*c
*d^2*f^2*g^2 + 630*b*d*e*f^2*g^2 + 315*a*e^2*f^2*g^2 - 315*b*d^2*f*g^3 - 63
0*a*d*e*f*g^3 + 315*a*d^2*g^4 - 420*c*e^2*f^3*(f + g*x) + 630*c*d*e*f^2*g*(
f + g*x) + 315*b*e^2*f^2*g*(f + g*x) - 210*c*d^2*f*g^2*(f + g*x) - 420*b*d*
e*f*g^2*(f + g*x) - 210*a*e^2*f*g^2*(f + g*x) + 105*b*d^2*g^3*(f + g*x) + 2
10*a*d*e*g^3*(f + g*x) + 378*c*e^2*f^2*(f + g*x)^2 - 378*c*d*e*f*g*(f + g*x
)^2 - 189*b*e^2*f*g*(f + g*x)^2 + 63*c*d^2*g^2*(f + g*x)^2 + 126*b*d*e*g^2*
(f + g*x)^2 + 63*a*e^2*g^2*(f + g*x)^2 - 180*c*e^2*f*(f + g*x)^3 + 90*c*d*e
*g*(f + g*x)^3 + 45*b*e^2*g*(f + g*x)^3 + 35*c*e^2*(f + g*x)^4))/(315*g^5)
```

fricas [A] time = 0.54, size = 260, normalized size = 1.23

$$\frac{2\left(35ce^2g^4x^4+128ce^2f^4+315ad^2g^4-144\left(2cde+be^2\right)f^3g+168\left(cd^2+2bde+ae^2\right)f^2g^2-210\left(bd^2+2ade\right)fg^3-5\left(8ce^2fg^3-9\left(2cde+be^2\right)g^4\right)x^3+3\left(16ce^2f^2g^2-18\left(2cde+be^2\right)fg^3+21\left(cd^2+2bde+ae^2\right)g^4\right)x^2-\left(64ce^2f^3g-72\left(2cde+be^2\right)f^2g^2+84\left(cd^2+2bde+ae^2\right)fg^3-105\left(bd^2+2ade\right)g^4\right)x\right)\sqrt{gx+f}}{315g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 + 315*a*d^2*g^4 - 144*(2*c*d*e + b*
e^2)*f^3*g + 168*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g^2 - 210*(b*d^2 + 2*a*d*e)*
f*g^3 - 5*(8*c*e^2*f*g^3 - 9*(2*c*d*e + b*e^2)*g^4)*x^3 + 3*(16*c*e^2*f^2*g
^2 - 18*(2*c*d*e + b*e^2)*f*g^3 + 21*(c*d^2 + 2*b*d*e + a*e^2)*g^4)*x^2 - (
64*c*e^2*f^3*g - 72*(2*c*d*e + b*e^2)*f^2*g^2 + 84*(c*d^2 + 2*b*d*e + a*e^2
)*f*g^3 - 105*(b*d^2 + 2*a*d*e)*g^4)*x)*sqrt(g*x + f)/g^5
```

giac [A] time = 0.23, size = 363, normalized size = 1.71

$$\frac{2\left(315\sqrt{gx+f}\,ad^2+\frac{105\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)bd^2}{g}+\frac{210\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)ade}{g}+\frac{21\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)cd^2}{g^2}+\frac{42\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)bde}{g^2}+\frac{21\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)ae^2}{g^2}+\frac{18\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^2-35\sqrt{gx+f}\,f^3\right)cde}{g^3}+\frac{9\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^2-35\sqrt{gx+f}\,f^3\right)be^2}{g^3}+\frac{\left(35(gx+f)^{\frac{9}{2}}-180(gx+f)^{\frac{7}{2}}f+378(gx+f)^{\frac{5}{2}}f^2-420(gx+f)^{\frac{3}{2}}f^3+315\sqrt{gx+f}\,f^4\right)ce^2}{g^4}\right)}{315g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(g*x + f)*a*d^2 + 105*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*
b*d^2/g + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d*e/g + 21*(3*(g*x +
f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^2/g^2 + 42*(3*(
g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d*e/g^2 + 2
1*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*e^2/g
^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2
- 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + 9*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/
2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*e^2/g^3 + (35*(g*x
+ f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f
)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

maple [A] time = 0.01, size = 315, normalized size = 1.49

$$\frac{2\sqrt{gx+f}\,\left(35e^{2}c\,x^{4}g^{4}+45b\,e^{2}g^{4}x^{3}+90cde\,g^{4}x^{3}-40c\,e^{2}f\,g^{3}x^{3}+63a\,e^{2}g^{4}x^{2}+126bde\,g^{4}x^{2}-54b\,e^{2}f\,g^{3}x^{2}+63c\,d^{2}g^{4}x^{2}-108cdef\,g^{3}x^{2}+48c\,e^{2}f^{2}g^{2}x^{2}+210ade\,g^{4}x-84a\,e^{2}f\,g^{3}x+105b\,d^{2}g^{4}x-168bdef\,g^{3}x+72b\,e^{2}f^{2}g^{2}x-84c\,d^{2}f\,g^{3}x+144cde\,f^{2}g^{2}x-64c\,e^{2}f^{3}gx+315a\,d^{2}g^{4}-420adef\,g^{3}+168a\,e^{2}f^{2}g^{2}-210b\,d^{2}f\,g^{3}+336bde\,f^{2}g^{2}-144b\,e^{2}f^{3}g+168c\,d^{2}f^{2}g^{2}-288cde\,f^{3}g+128c\,e^{2}f^{4}\right)}{315g^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/315*(g*x+f)^(1/2)*(35*c*e^2*g^4*x^4+45*b*e^2*g^4*x^3+90*c*d*e*g^4*x^3-40*
c*e^2*f*g^3*x^3+63*a*e^2*g^4*x^2+126*b*d*e*g^4*x^2-54*b*e^2*f*g^3*x^2+63*c*
d^2*g^4*x^2-108*c*d*e*f*g^3*x^2+48*c*e^2*f^2*g^2*x^2+210*a*d*e*g^4*x-84*a*e
^2*f*g^3*x+105*b*d^2*g^4*x-168*b*d*e*f*g^3*x+72*b*e^2*f^2*g^2*x-84*c*d^2*f*
g^3*x+144*c*d*e*f^2*g^2*x-64*c*e^2*f^3*g*x+315*a*d^2*g^4-420*a*d*e*f*g^3+16
8*a*e^2*f^2*g^2-210*b*d^2*f*g^3+336*b*d*e*f^2*g^2-144*b*e^2*f^3*g+168*c*d^2
*f^2*g^2-288*c*d*e*f^3*g+128*c*e^2*f^4)/g^5
```

maxima [A] time = 0.45, size = 261, normalized size = 1.23

$$\frac{2\left(35\left(gx+f\right)^{\frac{9}{2}}ce^{2}-45\left(4\,ce^{2}f-\left(2\,cde+be^{2}\right)g\right)\left(gx+f\right)^{\frac{7}{2}}+63\left(6\,ce^{2}f^{2}-3\left(2\,cde+be^{2}\right)fg+\left(cd^{2}+2\,bde+ae^{2}\right)g^{2}\right)\left(gx+f\right)^{\frac{5}{2}}-105\left(4\,ce^{2}f^{3}-3\left(2\,cde+be^{2}\right)f^{2}g+2\left(cd^{2}+2\,bde+ae^{2}\right)fg^{2}-\left(bd^{2}+2\,ade\right)g^{3}\right)\left(gx+f\right)^{\frac{3}{2}}+315\left(ce^{2}f^{4}+ad^{2}g^{4}-\left(2\,cde+be^{2}\right)f^{3}g+\left(cd^{2}+2\,bde+ae^{2}\right)f^{2}g^{2}-\left(bd^{2}+2\,ade\right)fg^{3}\right)\sqrt{gx+f}\right)}{315\,g^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(g*x + f)^(9/2)*c*e^2 - 45*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g*x
 + f)^(7/2) + 63*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e
+ a*e^2)*g^2)*(g*x + f)^(5/2) - 105*(4*c*e^2*f^3 - 3*(2*c*d*e + b*e^2)*f^2*
g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*(g*x + f)^(3
/2) + 315*(c*e^2*f^4 + a*d^2*g^4 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 + 2*b*d
*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)*sqrt(g*x + f))/g^5
```

mupad [B] time = 3.17, size = 204, normalized size = 0.96

$$\frac{\left(f+g\,x\right)^{7/2}\,\left(2\,b\,e^2\,g-8\,c\,e^2\,f+4\,c\,d\,e\,g\right)}{7\,g^5}+\frac{\left(f+g\,x\right)^{5/2}\,\left(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+12\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+2\,a\,e^2\,g^2\right)}{5\,g^5}+\frac{2\,\left(f+g\,x\right)^{3/2}\,\left(d\,g-e\,f\right)\,\left(2\,a\,e\,g^2+b\,d\,g^2+4\,c\,e\,f^2-3\,b\,e\,f\,g-2\,c\,d\,f\,g\right)}{3\,g^5}+\frac{2\,\sqrt{f+g\,x}\,{\left(d\,g-e\,f\right)}^2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{g^5}+\frac{2\,c\,e^2\,{\left(f+g\,x\right)}^{9/2}}{9\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*b*e^2*g - 8*c*e^2*f + 4*c*d*e*g))/(7*g^5) + ((f + g*x)^
(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 + 4*b*d*e*g^2 - 6*b*e^2*f*g
 - 12*c*d*e*f*g))/(5*g^5) + (2*(f + g*x)^(3/2)*(d*g - e*f)*(2*a*e*g^2 + b*d
*g^2 + 4*c*e*f^2 - 3*b*e*f*g - 2*c*d*f*g))/(3*g^5) + (2*(f + g*x)^(1/2)*(d*
g - e*f)^2*(a*g^2 + c*f^2 - b*f*g))/g^5 + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

sympy [A] time = 105.54, size = 1001, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d**2*f/sqrt(f + g*x) - 2*a*d**2*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) - 4*a*d*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 4*a*d*e*(f**2/s
qrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*a*e**2*f*(f**2
/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*a*e**2*(-
f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)*
*(5/2)/5)/g**2 - 2*b*d**2*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*b*d**2
*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 4*b*d*e*
f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 4*b*
d*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f +
 g*x)**(5/2)/5)/g**2 - 2*b*e**2*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*
```

```
x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 - 2*b*e**2*(f**4/sqrt(f
+ g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5
/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*c*d**2*f*(f**2/sqrt(f + g*x) + 2*f*sqr
t(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d**2*(-f**3/sqrt(f + g*x) - 3*f
**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 4*c*d*e
*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f +
g*x)**(5/2)/5)/g**3 - 4*c*d*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) -
2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3
 - 2*c*e**2*f*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)
**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 - 2*c*e**2*(-f*
*5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f*
*2*(f + g*x)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4)/g,
 Ne(g, 0)), ((a*d**2*x + c*e**2*x**5/5 + x**4*(b*e**2 + 2*c*d*e)/4 + x**3*(
a*e**2 + 2*b*d*e + c*d**2)/3 + x**2*(2*a*d*e + b*d**2)/2)/sqrt(f), True))
```

3.564 $\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=137

$$-\frac{2\sqrt{f+gx}\,(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4}+\frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4}-\frac{2(f+gx)^{5/2}(-be\cdots}{5g\cdots}$$

Rubi [A] time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {771}

$$-\frac{2\sqrt{f+gx}\,(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4}+\frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4}-\frac{2(f+gx)^{5/2}(-beg-cdg+3cef)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(c*f*(3*e*f
 - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*(f + g*x)^(3/2))/(3*g^4) - (2*(3*c
*e*f - c*d*g - b*e*g)*(f + g*x)^(5/2))/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7
*g^4)
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx &= \int \left(\frac{(-ef+dg)\left(cf^2-bfg+ag^2\right)}{g^3\sqrt{f+gx}}+\frac{(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+\cdots}}{g^3}\cdots\right.\\ &= -\frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^4}+\frac{2(cf(3ef-2dg)-g(2bef-bdg-ae\cdots}{3g^4}\cdots\end{aligned}$$

Mathematica [A] time = 0.19, size = 131, normalized size = 0.96

$$\frac{2\sqrt{f+gx}\left(7g\left(5ag(3dg-2ef+egx)+5bdg(gx-2f)+be\left(8f^2-4fgx+3g^2x^2\right)\right)+c\left(7dg\left(8f^2-4fgx+3g^2x^2\right)-3e\left(16f^3-8f^2gx+6fg^2x^2-5g^3x^3\right)\right)\right)}{105g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(7*g*(5*b*d*g*(-2*f + g*x) + 5*a*g*(-2*e*f + 3*d*g + e*g*x
) + b*e*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + c*(7*d*g*(8*f^2 - 4*f*g*x + 3*g^2*
x^2) - 3*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3))))/(105*g^4)
```

IntegrateAlgebraic [A] time = 0.13, size = 168, normalized size = 1.23

$$\frac{2\sqrt{f+gx}\left(105adg^3+35aeg^2(f+gx)-105aefg^2+35bdg^2(f+gx)-105bdfg^2+105bef^2g-70befg(f+gx)+21beg(f+gx)^2+105cdf^2g-70cdfg(f+gx)+21cdg(f+gx)^2-105cef^3+105cef^2(f+gx)-63cef(f+gx)^2+15ce(f+gx)^3\right)}{105g^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-105*c*e*f^3 + 105*c*d*f^2*g + 105*b*e*f^2*g - 105*b*d*f*
g^2 - 105*a*e*f*g^2 + 105*a*d*g^3 + 105*c*e*f^2*(f + g*x) - 70*c*d*f*g*(f +
 g*x) - 70*b*e*f*g*(f + g*x) + 35*b*d*g^2*(f + g*x) + 35*a*e*g^2*(f + g*x)
- 63*c*e*f*(f + g*x)^2 + 21*c*d*g*(f + g*x)^2 + 21*b*e*g*(f + g*x)^2 + 15*c
*e*(f + g*x)^3))/(105*g^4)
```

fricas [A] time = 0.56, size = 125, normalized size = 0.91

$$\frac{2\left(15\,ceg^3x^3 - 48\,cef^3 + 105\,adg^3 + 56\,(cd+be)f^2g - 70\,(bd+ae)fg^2 - 3\left(6\,cefg^2 - 7\,(cd+be)g^3\right)x^2 + \left(24\,cef^2g - 28\,(cd+be)fg^2 + 35\,(bd+ae)g^3\right)x\right)\sqrt{gx+f}}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 105*a*d*g^3 + 56*(c*d + b*e)*f^2*g - 7
0*(b*d + a*e)*f*g^2 - 3*(6*c*e*f*g^2 - 7*(c*d + b*e)*g^3)*x^2 + (24*c*e*f^2
*g - 28*(c*d + b*e)*f*g^2 + 35*(b*d + a*e)*g^3)*x)*sqrt(g*x + f)/g^4
```

giac [A] time = 0.19, size = 199, normalized size = 1.45

$$\frac{2\left(105\,\sqrt{gx+f}\,ad + \frac{35\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}\,f\right)bd}{g} + \frac{35\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}\,f\right)ae}{g} + \frac{7\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(gx+f)^{\frac{3}{2}}f + 15\,\sqrt{gx+f}\,f^2\right)cd}{g^2} + \frac{7\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(gx+f)^{\frac{3}{2}}f + 15\,\sqrt{gx+f}\,f^2\right)be}{g^2} + \frac{3\left(5\,(gx+f)^{\frac{7}{2}} - 21\,(gx+f)^{\frac{5}{2}}f + 35\,(gx+f)^{\frac{3}{2}}f^2 - 35\,\sqrt{gx+f}\,f^3\right)ce}{g^3}\right)}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b*d
/g + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e/g + 7*(3*(g*x + f)^(5/2)
- 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d/g^2 + 7*(3*(g*x + f)^(5/
2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*e/g^2 + 3*(5*(g*x + f)^
(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^
3)*c*e/g^3)/g
```

maple [A] time = 0.01, size = 144, normalized size = 1.05

$$\frac{2\sqrt{gx+f}\,\left(15ce\,x^3g^3 + 21be\,g^3x^2 + 21cd\,g^3x^2 - 18cef\,g^2x^2 + 35ae\,g^3x + 35bd\,g^3x - 28bef\,g^2x - 28cdf\,g^2x + 24ce\,f^2gx + 105ad\,g^3 - 70aef\,g^2 - 70bdf\,g^2 + 56be\,f^2g + 56cd\,f^2g - 48ce\,f^3\right)}{105g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(15*c*e*g^3*x^3+21*b*e*g^3*x^2+21*c*d*g^3*x^2-18*c*e*f*
g^2*x^2+35*a*e*g^3*x+35*b*d*g^3*x-28*b*e*f*g^2*x-28*c*d*f*g^2*x+24*c*e*f^2*
g*x+105*a*d*g^3-70*a*e*f*g^2-70*b*d*f*g^2+56*b*e*f^2*g+56*c*d*f^2*g-48*c*e*
f^3)/g^4
```

maxima [A] time = 0.46, size = 129, normalized size = 0.94

$$\frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce - 21\,\left(3\,cef - (cd+be)g\right)(gx+f)^{\frac{5}{2}} + 35\,\left(3\,cef^2 - 2\,(cd+be)fg + (bd+ae)g^2\right)(gx+f)^{\frac{3}{2}} - 105\,\left(cef^3 - adg^3 - (cd+be)f^2g + (bd+ae)fg^2\right)\sqrt{gx+f}\right)}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^(5/2
) + 35*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*(g*x + f)^(3/2) -
105*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)*sqrt(g*x +
f))/g^4
```

mupad [B] time = 0.08, size = 125, normalized size = 0.91

$$\frac{\left(f+g\,x\right)^{5/2}\,\left(2\,b\,e\,g+2\,c\,d\,g-6\,c\,e\,f\right)}{5\,g^4}+\frac{\left(f+g\,x\right)^{3/2}\,\left(2\,a\,e\,g^2+2\,b\,d\,g^2+6\,c\,e\,f^2-4\,b\,e\,f\,g-4\,c\,d\,f\,g\right)}{3\,g^4}+\frac{2\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{g^4}+\frac{2\,c\,e\,\left(f+g\,x\right)^{7/2}}{7\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*b*e*g + 2*c*d*g - 6*c*e*f))/(5*g^4) + ((f + g*x)^(3/2)*
(2*a*e*g^2 + 2*b*d*g^2 + 6*c*e*f^2 - 4*b*e*f*g - 4*c*d*f*g))/(3*g^4) + (2*(
f + g*x)^(1/2)*(d*g - e*f)*(a*g^2 + c*f^2 - b*f*g))/g^4 + (2*c*e*(f + g*x)^
(7/2))/(7*g^4)
```

sympy [A] time = 55.83, size = 549, normalized size = 4.01

$$\begin{cases} \frac{-\frac{2adf}{\sqrt{f+gx}} - 2ad\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) - \frac{2aef\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} - \frac{2ae\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} - \frac{2bdf\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} - \frac{2bd\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} - \frac{2bef\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2be\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2} - \frac{2cdf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2cd\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2} - \frac{2cef\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^3} - \frac{2ce\left(\frac{f^4}{\sqrt{f+gx}} + 4f^3\sqrt{f+gx} - 2f^2(f+gx)^{\frac{3}{2}} + \frac{4f(f+gx)^{\frac{5}{2}}}{5} - \frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^3}}{g} & \text{for}\: g \neq 0 \\ \frac{adx + \frac{cex^4}{4} + \frac{x^3(be+cd)}{3} + \frac{x^2(ae+bd)}{2}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d*f/sqrt(f + g*x) - 2*a*d*(-f/sqrt(f + g*x) - sqrt(f + g*x
)) - 2*a*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*a*e*(f**2/sqrt(f + g*
x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*b*d*f*(-f/sqrt(f + g*x)
- sqrt(f + g*x))/g - 2*b*d*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g
*x)**(3/2)/3)/g - 2*b*e*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*
x)**(3/2)/3)/g**2 - 2*b*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(
f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 2*c*d*f*(f**2/sqrt(f + g*x) +
2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d*(-f**3/sqrt(f + g*x) -
 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 2*c
*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f
+ g*x)**(5/2)/5)/g**3 - 2*c*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) -
2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3
)/g, Ne(g, 0)), ((a*d*x + c*e*x**4/4 + x**3*(b*e + c*d)/3 + x**2*(a*e + b*d
)/2)/sqrt(f), True))
```

3.565 $\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{f+gx}\,\left(ag^2-bfg+cf^2\right)}{g^3}-\frac{2(f+gx)^{3/2}(2cf-bg)}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {698}

$$\frac{2\sqrt{f+gx}\,\left(ag^2-bfg+cf^2\right)}{g^3}-\frac{2(f+gx)^{3/2}(2cf-bg)}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^3 - (2*(2*c*f - b*g)*(f + g*x)^
(3/2))/(3*g^3) + (2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx &= \int \left(\frac{cf^2-bfg+ag^2}{g^2\sqrt{f+gx}}+\frac{(-2cf+bg)\sqrt{f+gx}}{g^2}+\frac{c(f+gx)^{3/2}}{g^2}\right)dx\\ &= \frac{2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^3}-\frac{2(2cf-bg)(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.74

$$\frac{2\sqrt{f+gx}\,\left(5g(3ag-2bf+bgx)+c\left(8f^2-4fgx+3g^2x^2\right)\right)}{15g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(5*g*(-2*b*f + 3*a*g + b*g*x) + c*(8*f^2 - 4*f*g*x + 3*g^2
*x^2)))/(15*g^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 62, normalized size = 0.85

$$\frac{2\sqrt{f+gx}\,\left(15ag^2+5bg(f+gx)-15bfg+15cf^2-10cf(f+gx)+3c(f+gx)^2\right)}{15g^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(15*c*f^2 - 15*b*f*g + 15*a*g^2 - 10*c*f*(f + g*x) + 5*b*g
*(f + g*x) + 3*c*(f + g*x)^2))/(15*g^3)
```

fricas [A] time = 0.72, size = 54, normalized size = 0.74

$$\frac{2\left(3\,cg^2x^2+8\,cf^2-10\,bfg+15\,ag^2-\left(4\,cfg-5\,bg^2\right)x\right)\sqrt{gx+f}}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*g^2*x^2 + 8*c*f^2 - 10*b*f*g + 15*a*g^2 - (4*c*f*g - 5*b*g^2)*x)*
sqrt(g*x + f)/g^3
```

giac [A] time = 0.17, size = 77, normalized size = 1.05

$$\frac{2\left(15\,\sqrt{gx+f}\,a+\frac{5\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}\,f\right)b}{g}+\frac{\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (3
*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

maple [A] time = 0.00, size = 53, normalized size = 0.73

$$\frac{2\sqrt{gx+f}\,\left(3c\,x^2g^2+5b\,g^2x-4cfgx+15a\,g^2-10bfg+8c\,f^2\right)}{15g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(3*c*g^2*x^2+5*b*g^2*x-4*c*f*g*x+15*a*g^2-10*b*f*g+8*c*f
^2)/g^3
```

maxima [A] time = 0.44, size = 77, normalized size = 1.05

$$\frac{2\left(15\,\sqrt{gx+f}\,a+\frac{5\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}\,f\right)b}{g}+\frac{\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (3
*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

mupad [B] time = 3.12, size = 58, normalized size = 0.79

$$\frac{2\,\sqrt{f+g\,x}\,\left(3\,c\,{\left(f+g\,x\right)}^2+15\,a\,g^2+15\,c\,f^2+5\,b\,g\,\left(f+g\,x\right)-10\,c\,f\,\left(f+g\,x\right)-15\,b\,f\,g\right)}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(f + g*x)^(1/2),x)
```

```
[Out] (2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 + 5*b*g*(f + g*x)
 - 10*c*f*(f + g*x) - 15*b*f*g))/(15*g^3)
```

sympy [A] time = 10.87, size = 223, normalized size = 3.05

$$\begin{cases} \frac{-\frac{2af}{\sqrt{f+gx}} - 2a\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) - \frac{2bf\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} - \frac{2b\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} - \frac{2cf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2c\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for}\: g \neq 0 \\ \frac{ax + \frac{bx^2}{2} + \frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*f/sqrt(f + g*x) - 2*a*(-f/sqrt(f + g*x) - sqrt(f + g*x)) -
 2*b*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*b*(f**2/sqrt(f + g*x) + 2*f
*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*c*f*(f**2/sqrt(f + g*x) + 2*f*sq
rt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*(-f**3/sqrt(f + g*x) - 3*f**2*
sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2)/g, Ne(g, 0))
, ((a*x + b*x**2/2 + c*x**3/3)/sqrt(f), True))
```

3.566 $\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=116

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{2\sqrt{f+gx}\,(beg-c(dg+ef))}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

Rubi **[A]** time = 0.17, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {897, 1153, 208}

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{2\sqrt{f+gx}\,(beg-c(dg+ef))}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*(b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))
/(3*e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqr
t[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*g])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{beg-c(ef+dg)}{e^2g}+\frac{cx^2}{eg}+\frac{cd^2-bde+ae^2}{e^2\left(d-\frac{ef}{g}+\frac{ex^2}{g}\right)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}+\frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}\cdots}\right.}{e^2g}$$

$$= \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2\left(cd^2-bde+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+g\cdots}}{\sqrt{ef-dg}}\right.}{e^{5/2}\sqrt{ef-dg}}$$

Mathematica [A] time = 0.19, size = 118, normalized size = 1.02

$$\frac{2\left(-\frac{g^2\left(cd^2-e(bd-ae)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{\sqrt{f+gx}\,(beg-c(dg+ef))}{e^2}+\frac{c(f+gx)^{3/2}}{3e}\right)}{g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*(((b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/e^2 + (c*(f + g*x)^(3/2))/(3*e)
 - ((c*d^2 - e*(b*d - a*e))*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
d*g]])/(e^(5/2)*Sqrt[e*f - d*g])))/g^2
```

IntegrateAlgebraic [A] time = 0.17, size = 117, normalized size = 1.01

$$\frac{2\sqrt{f+gx}\,(3beg-3cdg+ce(f+gx)-3cef)}{3e^2g^2}-\frac{2\left(ae^2-bde+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)}{e^{5/2}\sqrt{dg-ef}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(-3*c*e*f - 3*c*d*g + 3*b*e*g + c*e*(f + g*x)))/(3*e^2*g^2
) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTan[(Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[f +
g*x])/(e*f - d*g)])/(e^(5/2)*Sqrt[-(e*f) + d*g])
```

fricas [A] time = 0.57, size = 341, normalized size = 2.94

$$\left[\frac{3\left(cd^2-bde+ae^2\right)\sqrt{e^2f-deg}\,g^2\log\left(\frac{egx+2ef-dg-2\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2ce^3f^2+\left(cde^2-3be^3\right)fg-3\left(cd^2e-bde^2\right)g^2-\left(ce^3fg-cde^2g^2\right)x\right)\sqrt{gx+f}}{3\left(e^4fg^2-de^3g^3\right)}, \frac{2\left(3\left(cd^2-bde+ae^2\right)\sqrt{-e^2f+deg}\,g^2\arctan\left(\frac{\sqrt{-e^2f+deg}\sqrt{gx+f}}{egx+ef}\right)-\left(2ce^3f^2+\left(cde^2-3be^3\right)fg-3\left(cd^2e-bde^2\right)g^2-\left(ce^3fg-cde^2g^2\right)x\right)\sqrt{gx+f}\right)}{3\left(e^4fg^2-de^3g^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c*d^2 - b*d*e + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f
- d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2 +
(c*d*e^2 - 3*b*e^3)*f*g - 3*(c*d^2*e - b*d*e^2)*g^2 - (c*e^3*f*g - c*d*e^2*
```

```
g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 - d*e^3*g^3), 2/3*(3*(c*d^2 - b*d*e + a*e
^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g
*x + e*f)) - (2*c*e^3*f^2 + (c*d*e^2 - 3*b*e^3)*f*g - 3*(c*d^2*e - b*d*e^2)
*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 - d*e^3*g^3)]
```

giac [A] time = 0.17, size = 128, normalized size = 1.10

$$\frac{2\left(cd^{2}-bde+ae^{2}\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^{2}}}\right)e^{(-2)}}{\sqrt{dge-fe^{2}}}-\frac{2\left(3\sqrt{gx+f}\,cdg^{5}e-\left(gx+f\right)^{\frac{3}{2}}cg^{4}e^{2}+3\sqrt{gx+f}\,cfg^{4}e^{2}-3\sqrt{gx+f}\,bg^{5}e^{2}\right)e^{(-3)}}{3\,g^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))*e^(-2
)/sqrt(d*g*e - f*e^2) - 2/3*(3*sqrt(g*x + f)*c*d*g^5*e - (g*x + f)^(3/2)*c*
g^4*e^2 + 3*sqrt(g*x + f)*c*f*g^4*e^2 - 3*sqrt(g*x + f)*b*g^5*e^2)*e^(-3)/g
^6
```

maple [A] time = 0.01, size = 189, normalized size = 1.63

$$\frac{2a\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\sqrt{\left(dg-ef\right)e}}-\frac{2bd\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\sqrt{\left(dg-ef\right)e}\,e}+\frac{2c\,d^{2}\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\sqrt{\left(dg-ef\right)e}\,e^{2}}+\frac{2\sqrt{gx+f}\,b}{eg}-\frac{2\sqrt{gx+f}\,cd}{e^{2}g}-\frac{2\sqrt{gx+f}\,cf}{e\,g^{2}}+\frac{2\left(gx+f\right)^{\frac{3}{2}}c}{3e\,g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3*(g*x+f)^(3/2)*c/e/g^2+2/g/e*b*(g*x+f)^(1/2)-2*(g*x+f)^(1/2)*c*d/e^2/g-2
*(g*x+f)^(1/2)*c/e*f/g^2+2/((d*g-e*f)*e)^(1/2)*a*arctan((g*x+f)^(1/2)/((d*g
-e*f)*e)^(1/2)*e)-2/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e
)^(1/2)*e)*b*d+2/((d*g-e*f)*e)^(1/2)*c*d^2/e^2*arctan((g*x+f)^(1/2)/((d*g-e
*f)*e)^(1/2)*e)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.14, size = 117, normalized size = 1.01

$$\sqrt{f+g\,x}\,\left(\frac{2\,b\,g-4\,c\,f}{e\,g^{2}}-\frac{2\,c\,\left(d\,g^{3}-e\,f\,g^{2}\right)}{e^{2}\,g^{4}}\right)+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(c\,d^{2}-b\,d\,e+a\,e^{2}\right)}{e^{5/2}\,\sqrt{d\,g-e\,f}}+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e\,g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

```
[Out] (f + g*x)^(1/2)*((2*b*g - 4*c*f)/(e*g^2) - (2*c*(d*g^3 - e*f*g^2))/(e^2*g^4
)) + (2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2 -
b*d*e))/(e^(5/2)*(d*g - e*f)^(1/2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

sympy [A] time = 37.63, size = 112, normalized size = 0.97

$$\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3eg^{2}}-\frac{2\left(ae^{2}-bde+cd^{2}\right)\operatorname{atan}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{e^{2}\sqrt{\frac{e}{dg-ef}}\left(dg-ef\right)}+\frac{2\sqrt{f+gx}\left(beg-cdg-cef\right)}{e^{2}g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*e*g**2) - 2*(a*e**2 - b*d*e + c*d**2)*atan(1/(sqrt(
e/(d*g - e*f))*sqrt(f + g*x)))/(e**2*sqrt(e/(d*g - e*f))*(d*g - e*f)) + 2*s
qrt(f + g*x)*(b*e*g - c*d*g - c*e*f)/(e**2*g**2)
```

3.567 $\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=140

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}(ef-dg)^{3/2}}-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{2c\sqrt{f+gx}}{e^2g}$$

Rubi [A] time = 0.29, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {897, 1157, 388, 208}

$$-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e
*f - d*g)*(d + e*x)) + ((c*d*(4*e*f - 3*d*g) - e*(2*b*e*f - b*d*g - a*e*g))
*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(3/
2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} + \frac{\text{Subst}\left(\int \frac{-a+\frac{cd^2}{e^2}-\frac{bd}{e}-\frac{2cf^2}{g^2}+\frac{2bf}{g}+\frac{2c(ef-dg)x^2}{eg^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}dx, x, \sqrt{f+g\right.}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} - \frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))}{e^2g(ef}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)} + \frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))}{e^{5/2}(ef-dg)^{3/2}}$$

Mathematica [A] time = 0.56, size = 150, normalized size = 1.07

$$\frac{\sqrt{f+gx}\left(eg(bd-ae)+c\left(-3d^2g+2de(f-gx)+2e^2fx\right)\right)}{e^2g(d+ex)(ef-dg)} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-aeg-bdg+2bef)+cd(3dg-4ef))}{e^{5/2}(ef-dg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*(e*(b*d - a*e)*g + c*(-3*d^2*g + 2*e^2*f*x + 2*d*e*(f - g*x)
)))/(e^2*g*(e*f - d*g)*(d + e*x)) - ((c*d*(-4*e*f + 3*d*g) + e*(2*b*e*f - b
*d*g - a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(
e*f - d*g)^(3/2))
```

IntegrateAlgebraic [A] time = 0.55, size = 200, normalized size = 1.43

$$\frac{\sqrt{f+gx}\left(ae^2g^2-bdeg^2+3cd^2g^2+2cdeg(f+gx)-4cdefg+2ce^2f^2-2ce^2f(f+gx)\right)}{e^2g(ef-dg)(-dg-e(f+gx)+ef)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)\left(-ae^2g-bdeg+2be^2f+3cd^2g-4cdef\right)}{e^{5/2}(dg-ef)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*(2*c*e^2*f^2 - 4*c*d*e*f*g + 3*c*d^2*g^2 - b*d*e*g^2 + a*e^2
*g^2 - 2*c*e^2*f*(f + g*x) + 2*c*d*e*g*(f + g*x)))/(e^2*g*(e*f - d*g)*(e*f
- d*g - e*(f + g*x))) + ((-4*c*d*e*f + 2*b*e^2*f + 3*c*d^2*g - b*d*e*g - a*
e^2*g)*ArcTan[(Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[f + g*x])/(e*f - d*g)])/(e^(
5/2)*(-(e*f) + d*g)^(3/2))
```

fricas [B] time = 0.65, size = 637, normalized size = 4.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(e^2*f - d*e*g)*(2*(2*c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - b*d^2*
e - a*d*e^2)*g^2 + (2*(2*c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - b*d*e^2 - a*e^
3)*g^2)*x)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/
(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^2 - b*d*e^3 + a*e^4)*f*g + (3*c*
```

```
d^3*e - b*d^2*e^2 + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2
*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3 + (e^6
*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -(sqrt(-e^2*f + d*e*g)*(2*(2*c*d^
2*e - b*d*e^2)*f*g - (3*c*d^3 - b*d^2*e - a*d*e^2)*g^2 + (2*(2*c*d*e^2 - b*
e^3)*f*g - (3*c*d^2*e - b*d*e^2 - a*e^3)*g^2)*x)*arctan(sqrt(-e^2*f + d*e*g
)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*d*e^3*f^2 - (5*c*d^2*e^2 - b*d*e^3 +
a*e^4)*f*g + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e
^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 +
d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x)]
```

giac [A] time = 0.18, size = 175, normalized size = 1.25

$$\frac{2\,\sqrt{gx+f}\,ce^{(-2)}}{g}-\frac{\left(3\,cd^2g-4\,cdfe-bdge+2\,bfe^2-age^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(dge^2-fe^3\right)\sqrt{dge-fe^2}}+\frac{\sqrt{gx+f}\,cd^2g-\sqrt{gx+f}\,bdge+\sqrt{gx+f}\,age^2}{\left(dge^2-fe^3\right)\left(dg+\left(gx+f\right)e-fe\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(g*x + f)*c*e^(-2)/g - (3*c*d^2*g - 4*c*d*f*e - b*d*g*e + 2*b*f*e^2 -
 a*g*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d*g*e^2 - f*e^3)*sq
rt(d*g*e - f*e^2)) + (sqrt(g*x + f)*c*d^2*g - sqrt(g*x + f)*b*d*g*e + sqrt(
g*x + f)*a*g*e^2)/((d*g*e^2 - f*e^3)*(d*g + (g*x + f)*e - f*e))
```

maple [B] time = 0.02, size = 371, normalized size = 2.65

$$\frac{ag\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}}+\frac{bdg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e}-\frac{2bf\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}}-\frac{3c\,d^2g\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e^2}+\frac{4cdf\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)\sqrt{\left(dg-ef\right)e}\,e}+\frac{\sqrt{gx+f}\,ag}{\left(dg-ef\right)\left(egx+dg\right)}-\frac{\sqrt{gx+f}\,bdg}{\left(dg-ef\right)\left(egx+dg\right)e}+\frac{\sqrt{gx+f}\,c\,d^2g}{\left(dg-ef\right)\left(egx+dg\right)e^2}+\frac{2\sqrt{gx+f}\,c}{e^2g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*c/e^2/g+g/(d*g-e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*a-g/e/(d*g-e*
f)*(g*x+f)^(1/2)/(e*g*x+d*g)*b*d+g/e^2/(d*g-e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*
c*d^2+g/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1
/2)*e)*a+g/e/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*
e)^(1/2)*e)*b*d-2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-
e*f)*e)^(1/2)*e)*b*f-3*g/e^2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(
1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d^2+4/e/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(
(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*d*c*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.23, size = 146, normalized size = 1.04

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(a\,e^2\,g-2\,b\,e^2\,f-3\,c\,d^2\,g+b\,d\,e\,g+4\,c\,d\,e\,f\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}}+\frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2-b\,g\,d\,e+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)}+\frac{2\,c\,\sqrt{f+g\,x}}{e^2\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 2*b*e^2*f - 3
*c*d^2*g + b*d*e*g + 4*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)^(
1/2)*(a*e^2*g + c*d^2*g - b*d*e*g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f + d
*e^2*g)) + (2*c*(f + g*x)^(1/2))/(e^2*g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.568 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=206

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(eg(-3aeg-bdg+4bef)-c\left(3d^2g^2-8defg+8e^2f^2\right)\right)}{4e^{5/2}(ef-dg)^{5/2}}-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}\,(cd}{}$$

Rubi [A] time = 0.39, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {897, 1157, 385, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(eg(-3aeg-bdg+4bef)-c\left(3d^2g^2-8defg+8e^2f^2\right)\right)}{4e^{5/2}(ef-dg)^{5/2}}-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}\,(cd(8ef-5dg)-e(-3aeg-bdg+4bef))}{4e^2(d+ex)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -((a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(2*(e*f - d*g)*(d + e*x)^2) + ((
c*d*(8*e*f - 5*d*g) - e*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/(4*e^2*
(e*f - d*g)^2*(d + e*x)) + ((e*g*(4*b*e*f - b*d*g - 3*a*e*g) - c*(8*e^2*f^2
 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(4*e^(5/2)*(e*f - d*g)^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-3a+\frac{cd^2}{e^2}-\frac{bd}{e}-\frac{4cf^2}{g^2}+\frac{4bf}{g}+\frac{4c(ef-dg)x^2}{eg^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}dx, x, \sqrt{f+\cdots}\right)}{2(ef-dg)}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} - \cdots$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2} + \frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} + \cdots$$

Mathematica [A] time = 0.65, size = 297, normalized size = 1.44

$$\frac{-\frac{2\sqrt{e}\sqrt{f+gx}\left(e(ae-bd)+cd^2\right)}{(d+ex)^2(ef-dg)}-\frac{3g\left(e(ae-bd)+cd^2\right)\left(g(d+ex)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)-\sqrt{e}\sqrt{f+gx}\sqrt{ef-dg}\right)}{(d+ex)(ef-dg)^{5/2}}-\frac{4\sqrt{e}\sqrt{f+gx}(be-2cd)}{(d+ex)(ef-dg)}-\frac{4g(2cd-be)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}-\frac{8c\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}}{4e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] ((-2*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[f + g*x])/((e*f - d*g)*(d + e*
x)^2) - (4*Sqrt[e]*(-2*c*d + b*e)*Sqrt[f + g*x])/((e*f - d*g)*(d + e*x)) -
(4*(2*c*d - b*e)*g*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e*f -
 d*g)^(3/2) - (8*c*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e
*f - d*g] - (3*(c*d^2 + e*(-(b*d) + a*e))*g*(-(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt
[f + g*x]) + g*(d + e*x)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]))
/((e*f - d*g)^(5/2)*(d + e*x)))/(4*e^(5/2))
```

IntegrateAlgebraic [A] time = 0.90, size = 293, normalized size = 1.42

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)\left(-3ae^2g^2-bdeg^2+4be^2fg-3cd^2g^2+8cdefg-8ce^2f^2\right)}{4e^{5/2}(dg-ef)^{5/2}}-\frac{g\sqrt{f+gx}\left(-5ade^2g^2-3ae^3g(f+gx)+5ae^3fg+bd^2eg^2-bde^2g(f+gx)+3bde^2fg-4be^3f^2+4be^3f(f+gx)+3cd^3g^2+5cd^2eg(f+gx)-11cd^2efg+8cde^2f^2-8cde^2f(f+gx)\right)}{4e^2(ef-dg)^2(-dg-e(f+gx)+ef)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -1/4*(g*Sqrt[f + g*x]*(8*c*d*e^2*f^2 - 4*b*e^3*f^2 - 11*c*d^2*e*f*g + 3*b*d
*e^2*f*g + 5*a*e^3*f*g + 3*c*d^3*g^2 + b*d^2*e*g^2 - 5*a*d*e^2*g^2 - 8*c*d*
e^2*f*(f + g*x) + 4*b*e^3*f*(f + g*x) + 5*c*d^2*e*g*(f + g*x) - b*d*e^2*g*(
f + g*x) - 3*a*e^3*g*(f + g*x)))/(e^2*(e*f - d*g)^2*(e*f - d*g - e*(f + g*x
))^2) + ((-8*c*e^2*f^2 + 8*c*d*e*f*g + 4*b*e^2*f*g - 3*c*d^2*g^2 - b*d*e*g^
2 - 3*a*e^2*g^2)*ArcTan[(Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[f + g*x])/(e*f - d
*g)])/(4*e^(5/2)*(-(e*f) + d*g)^(5/2))
```

fricas [B] time = 0.66, size = 1096, normalized size = 5.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((8*c*d^2*e^2*f^2 - 4*(2*c*d^3*e + b*d^2*e^2)*f*g + (3*c*d^4 + b*d^3*e
 + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 4*(2*c*d*e^3 + b*e^4)*f*g + (3*c*d^2*e
^2 + b*d*e^3 + 3*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 - 4*(2*c*d^2*e^2 + b*d*
e^3)*f*g + (3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*
log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d))
+ 2*(2*(3*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^2 - (9*c*d^3*e^2 - b*d^2*e^3 - 7*a
*d*e^4)*f*g + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*g^2 + (4*(2*c*d*e^4 - b
*e^5)*f^2 - (13*c*d^2*e^3 - 5*b*d*e^4 - 3*a*e^5)*f*g + (5*c*d^3*e^2 - b*d^2
*e^3 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3
*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 -
 d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*
e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 4*(2*c*d^3*e + b*d^2*e^2)*f*g + (3*c*d
^4 + b*d^3*e + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 4*(2*c*d*e^3 + b*e^4)*f*g
+ (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 - 4*(2*c*d^
2*e^2 + b*d*e^3)*f*g + (3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*g^2)*x)*sqrt(-e^
2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*
(3*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^2 - (9*c*d^3*e^2 - b*d^2*e^3 - 7*a*d*e^4)
*f*g + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*g^2 + (4*(2*c*d*e^4 - b*e^5)*f
^2 - (13*c*d^2*e^3 - 5*b*d*e^4 - 3*a*e^5)*f*g + (5*c*d^3*e^2 - b*d^2*e^3 -
3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3*d^4*e^
4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 - d^3*e^
5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*e^4*g^3
)*x)]
```

giac [**B**] time = 0.20, size = 373, normalized size = 1.81

$$\frac{\left(3\,cd^2g^2 - 8\,cdfge + bdg^2e + 8\,cf^2e^2 - 4\,bfge^2 + 3\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{4\left(d^2g^2e^2 - 2\,dfge^3 + f^2e^4\right)\sqrt{dge-fe^2}} - \frac{3\,\sqrt{gx+f}\,cd^3g^3 + 5\left(gx+f\right)^{\frac{3}{2}}cd^2g^2e - 11\,\sqrt{gx+f}\,cd^2fg^2e + \sqrt{gx+f}\,bd^2g^3e - 8\left(gx+f\right)^{\frac{3}{2}}cdfge^2 + 8\,\sqrt{gx+f}\,cdf^2ge^2 - \left(gx+f\right)^{\frac{3}{2}}bdg^2e^2 + 3\,\sqrt{gx+f}\,bdfg^2e^2 - 5\,\sqrt{gx+f}\,adg^3e^2 + 4\left(gx+f\right)^{\frac{3}{2}}bfge^3 - 4\,\sqrt{gx+f}\,bf^2ge^3 - 3\left(gx+f\right)^{\frac{3}{2}}ag^2e^3 + 5\,\sqrt{gx+f}\,afg^2e^3}{4\left(d^2g^2e^2 - 2\,dfge^3 + f^2e^4\right)\left(dg + \left(gx+f\right)e - fe\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*g^2 - 8*c*d*f*g*e + b*d*g^2*e + 8*c*f^2*e^2 - 4*b*f*g*e^2 + 3*
a*g^2*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e^2 - 2*d*
f*g*e^3 + f^2*e^4)*sqrt(d*g*e - f*e^2)) - 1/4*(3*sqrt(g*x + f)*c*d^3*g^3 +
5*(g*x + f)^(3/2)*c*d^2*g^2*e - 11*sqrt(g*x + f)*c*d^2*f*g^2*e + sqrt(g*x +
 f)*b*d^2*g^3*e - 8*(g*x + f)^(3/2)*c*d*f*g*e^2 + 8*sqrt(g*x + f)*c*d*f^2*g
*e^2 - (g*x + f)^(3/2)*b*d*g^2*e^2 + 3*sqrt(g*x + f)*b*d*f*g^2*e^2 - 5*sqrt
(g*x + f)*a*d*g^3*e^2 + 4*(g*x + f)^(3/2)*b*f*g*e^3 - 4*sqrt(g*x + f)*b*f^2
*g*e^3 - 3*(g*x + f)^(3/2)*a*g^2*e^3 + 5*sqrt(g*x + f)*a*f*g^2*e^3)/((d^2*g
^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

maple [**B**] time = 0.02, size = 538, normalized size = 2.61

$$\frac{3a\,g^2\arctan\left(\frac{\sqrt{gx+f}\;e}{\sqrt{\left(dg-ef\right)e}}\right)}{4\left(d^2g^2-2defg+e^2f^2\right)\sqrt{\left(dg-ef\right)e}} + \frac{bd\,g^2\arctan\left(\frac{\sqrt{gx+f}\;e}{\sqrt{\left(dg-ef\right)e}}\right)}{4\left(d^2g^2-2defg+e^2f^2\right)\sqrt{\left(dg-ef\right)e}\;e} - \frac{bfg\arctan\left(\frac{\sqrt{gx+f}\;e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(d^2g^2-2defg+e^2f^2\right)\sqrt{\left(dg-ef\right)e}} + \frac{3c\,d^2g^2\arctan\left(\frac{\sqrt{gx+f}\;e}{\sqrt{\left(dg-ef\right)e}}\right)}{4\left(d^2g^2-2defg+e^2f^2\right)\sqrt{\left(dg-ef\right)e}\;e^2} - \frac{2cdfg\arctan\left(\frac{\sqrt{gx+f}\;e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(d^2g^2-2defg+e^2f^2\right)\sqrt{\left(dg-ef\right)e}\;e} + \frac{2c\,f^2\arctan\left(\frac{\sqrt{gx+f}\;e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(d^2g^2-2defg+e^2f^2\right)\sqrt{\left(dg-ef\right)e}} + \frac{\frac{\left(3a\,e^2g+bdeg-4b\,e^2f-5c\,d^2g+8decf\right)\left(gx+f\right)^{\frac{3}{2}}g}{4\left(d^2g^2-2defg+e^2f^2\right)e} + \frac{\left(5a\,e^2g-bdeg-4b\,e^2f-3c\,d^2g+8decf\right)\sqrt{gx+f}\;g}{4\left(dg-ef\right)e^2}}{\left(dg-ef+\left(gx+f\right)e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x)
```

```
[Out] 2*(1/8*g*(3*a*e^2*g+b*d*e*g-4*b*e^2*f-5*c*d^2*g+8*c*d*e*f)/e/(d^2*g^2-2*d*e
*f*g+e^2*f^2)*(g*x+f)^(3/2)+1/8*(5*a*e^2*g-b*d*e*g-4*b*e^2*f-3*c*d^2*g+8*c*
d*e*f)/e^2*g/(d*g-e*f)*(g*x+f)^(1/2))/(d*g-e*f+(g*x+f)*e)^2+3/4/(d^2*g^2-2*
d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)*a*g^2*arctan((g*x+f)^(1/2)/((d*g-e*f)*
e)^(1/2)*e)+1/4/(d^2*g^2-2*d*e*f*g+e^2*f^2)/e/((d*g-e*f)*e)^(1/2)*arctan((g
*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*b*d*g^2-1/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((
d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*b*f*g+3/4/(d^
2*g^2-2*d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)*c*d^2/e^2*g^2*arctan((g*x+f)^(
1/2)/((d*g-e*f)*e)^(1/2)*e)-2/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/
2)*c*d/e*f*g*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)+2/(d^2*g^2-2*d*e*f
```

```
*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)*c*f^2*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1
/2)*e)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.28, size = 270, normalized size = 1.31

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+b\,d\,e\,g^2+8\,c\,e^2\,f^2-4\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{4\,e^{5/2}\,{\left(d\,g-e\,f\right)}^{5/2}}-\frac{\frac{\sqrt{f+g\,x}\,\left(3\,c\,d^2\,g^2+b\,d\,e\,g^2-8\,c\,f\,d\,e\,g-5\,a\,e^2\,g^2+4\,b\,f\,e^2\,g\right)}{4\,e^2\,\left(d\,g-e\,f\right)}-\frac{{\left(f+g\,x\right)}^{3/2}\,\left(-5\,c\,d^2\,g^2+b\,d\,e\,g^2+8\,c\,f\,d\,e\,g+3\,a\,e^2\,g^2-4\,b\,f\,e^2\,g\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^2\,f-2\,d\,e\,g\right)+d^2\,g^2+e^2\,f^2-2\,d\,e\,f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(3*a*e^2*g^2 + 3*c*d^2*g
^2 + 8*c*e^2*f^2 + b*d*e*g^2 - 4*b*e^2*f*g - 8*c*d*e*f*g))/(4*e^(5/2)*(d*g
- e*f)^(5/2)) - (((f + g*x)^(1/2)*(3*c*d^2*g^2 - 5*a*e^2*g^2 + b*d*e*g^2 +
4*b*e^2*f*g - 8*c*d*e*f*g))/(4*e^2*(d*g - e*f)) - ((f + g*x)^(3/2)*(3*a*e^2
*g^2 - 5*c*d^2*g^2 + b*d*e*g^2 - 4*b*e^2*f*g + 8*c*d*e*f*g))/(4*e*(d*g - e*
f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2 + e^2*f^2
 - 2*d*e*f*g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.569 $\int \frac{(d+ex)^3\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=285

$$-\frac{2e(f+gx)^{5/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}+\frac{2(f+gx)^{3/2}(ef-dg)\left(3eg(-aeg-bdg\right.}{3g}$$

Rubi [A] time = 0.41, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {897, 1261}

$$-\frac{2e(f+gx)^{5/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}+\frac{2(f+gx)^{3/2}(ef-dg)\left(3eg(-aeg-bdg+2bef)-c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{3g^6}+\frac{2(ef-dg)^3\left(ag^2-bfg+cf^2\right)}{g^6\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(ef-dg)^2(cf(5ef-2dg)-g(-3aeg-bdg+4bef))}{g^6}-\frac{2e^2(f+gx)^{7/2}(-beg-3cdg+5cef)}{7g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d
*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/
g^6 + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 - 8*d
*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*d*g
- a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(5*g^6
) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*
(f + g*x)^(9/2))/(9*g^6)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx=\frac{2\,\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx,x,\sqrt{f+gx}\right)}{g}$$

$$=\frac{2\,\text{Subst}\left(\int \left(\frac{(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))}{g^5}+\frac{(-ef+dg)^3\left(cf^2-bfg+ag^2\right)}{g^5x^2}+\frac{(ef-dg)(3e\right.\right.}{}$$

$$=\frac{2(ef-dg)^3\left(cf^2-bfg+ag^2\right)}{g^6\sqrt{f+gx}}+\frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3\right.}{g^6}$$

Mathematica [**A**] time = 0.73, size = 249, normalized size = 0.87

$$\frac{2\left(-63e(f+gx)^3\left(c\left(-3d^2g^2+12defg-10e^2f^2\right)-eg(aeg+3bdg-4bef)\right)+105(f+gx)^2(ef-dg)\left(-3eg(aeg+bdg-2bef)-c\left(d^2g^2-8defg+10e^2f^2\right)\right)+315(ef-dg)^3\left(g(ag-bf)+cf^2\right)+315(f+gx)(ef-dg)^2(g(3aeg+bdg-4bef)+cf(5ef-2dg))-45e^2(f+gx)^4(-beg-3cdg+5cef)+35ce^3(f+gx)^5\right)}{315g^6\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(315*(e*f - d*g)^3*(c*f^2 + g*(-(b*f) + a*g)) + 315*(e*f - d*g)^2*(c*f*(
5*e*f - 2*d*g) + g*(-4*b*e*f + b*d*g + 3*a*e*g))*(f + g*x) + 105*(e*f - d*g
)*(-3*e*g*(-2*b*e*f + b*d*g + a*e*g) - c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2)
)*(f + g*x)^2 - 63*e*(-(e*g*(-4*b*e*f + 3*b*d*g + a*e*g)) + c*(-10*e^2*f^2
+ 12*d*e*f*g - 3*d^2*g^2))*(f + g*x)^3 - 45*e^2*(5*c*e*f - 3*c*d*g - b*e*g)
*(f + g*x)^4 + 35*c*e^3*(f + g*x)^5))/(315*g^6*Sqrt[f + g*x])
```

IntegrateAlgebraic [**B**] time = 0.30, size = 634, normalized size = 2.22

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(315*c*e^3*f^5 - 945*c*d*e^2*f^4*g - 315*b*e^3*f^4*g + 945*c*d^2*e*f^3*g
^2 + 945*b*d*e^2*f^3*g^2 + 315*a*e^3*f^3*g^2 - 315*c*d^3*f^2*g^3 - 945*b*d^
2*e*f^2*g^3 - 945*a*d*e^2*f^2*g^3 + 315*b*d^3*f*g^4 + 945*a*d^2*e*f*g^4 - 3
15*a*d^3*g^5 + 1575*c*e^3*f^4*(f + g*x) - 3780*c*d*e^2*f^3*g*(f + g*x) - 12
60*b*e^3*f^3*g*(f + g*x) + 2835*c*d^2*e*f^2*g^2*(f + g*x) + 2835*b*d*e^2*f^
2*g^2*(f + g*x) + 945*a*e^3*f^2*g^2*(f + g*x) - 630*c*d^3*f*g^3*(f + g*x) -
 1890*b*d^2*e*f*g^3*(f + g*x) - 1890*a*d*e^2*f*g^3*(f + g*x) + 315*b*d^3*g^
4*(f + g*x) + 945*a*d^2*e*g^4*(f + g*x) - 1050*c*e^3*f^3*(f + g*x)^2 + 1890
*c*d*e^2*f^2*g*(f + g*x)^2 + 630*b*e^3*f^2*g*(f + g*x)^2 - 945*c*d^2*e*f*g^
2*(f + g*x)^2 - 945*b*d*e^2*f*g^2*(f + g*x)^2 - 315*a*e^3*f*g^2*(f + g*x)^2
 + 105*c*d^3*g^3*(f + g*x)^2 + 315*b*d^2*e*g^3*(f + g*x)^2 + 315*a*d*e^2*g^
3*(f + g*x)^2 + 630*c*e^3*f^2*(f + g*x)^3 - 756*c*d*e^2*f*g*(f + g*x)^3 - 2
52*b*e^3*f*g*(f + g*x)^3 + 189*c*d^2*e*g^2*(f + g*x)^3 + 189*b*d*e^2*g^2*(f
 + g*x)^3 + 63*a*e^3*g^2*(f + g*x)^3 - 225*c*e^3*f*(f + g*x)^4 + 135*c*d*e^
2*g*(f + g*x)^4 + 45*b*e^3*g*(f + g*x)^4 + 35*c*e^3*(f + g*x)^5))/(315*g^6*
Sqrt[f + g*x])
```

fricas [**A**] time = 0.62, size = 438, normalized size = 1.54

$$\frac{2\left(35\,ce^3g^5x^5+1280\,ce^3f^5-315\,ad^3g^5-1152\left(3\,cde^2+be^3\right)f^4g+1008\left(3\,cd^2e+3\,bde^2+ae^3\right)f^3g^2-840\left(cd^3+3\,bd^2e+3\,ade^2\right)f^2g^3+630\left(bd^3+3\,ad^2e\right)fg^4-5\left(10\,ce^3fg^4-9\left(3\,cde^2+be^3\right)g^5\right)x^4+\left(80\,ce^3f^2g^3-72\left(3\,cde^2+be^3\right)fg^4+63\left(3\,cd^2e+3\,bde^2+ae^3\right)g^5\right)x^3-\left(160\,ce^3f^3g^2-144\left(3\,cde^2+be^3\right)f^2g^3+126\left(3\,cd^2e+3\,bde^2+ae^3\right)fg^4-105\left(cd^3+3\,bd^2e+3\,ade^2\right)g^5\right)x^2+\left(640\,ce^3f^4g-576\left(3\,cde^2+be^3\right)f^3g^2+504\left(3\,cd^2e+3\,bde^2+ae^3\right)f^2g^3-420\left(cd^3+3\,bd^2e+3\,ade^2\right)fg^4+315\left(bd^3+3\,ad^2e\right)g^5\right)x\right)\sqrt{gx+f}}{315\left(g^7x+fg^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 315*a*d^3*g^5 - 1152*(3*c*d*e^2
+ b*e^3)*f^4*g + 1008*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 - 840*(c*d^3
+ 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + 630*(b*d^3 + 3*a*d^2*e)*f*g^4 - 5*(10*c*
e^3*f*g^4 - 9*(3*c*d*e^2 + b*e^3)*g^5)*x^4 + (80*c*e^3*f^2*g^3 - 72*(3*c*d*
e^2 + b*e^3)*f*g^4 + 63*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*x^3 - (160*c*e
^3*f^3*g^2 - 144*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 126*(3*c*d^2*e + 3*b*d*e^2 +
 a*e^3)*f*g^4 - 105*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f
^4*g - 576*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 504*(3*c*d^2*e + 3*b*d*e^2 + a*e^3
)*f^2*g^3 - 420*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^4 + 315*(b*d^3 + 3*a*d^
2*e)*g^5)*x)*sqrt(g*x + f)/(g^7*x + f*g^6)
```

giac [**B**] time = 0.24, size = 669, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^3*f^2*g^3 - b*d^3*f*g^4 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e + 3*b*d^2*f
^2*g^3*e - 3*a*d^2*f*g^4*e + 3*c*d*f^4*g*e^2 - 3*b*d*f^3*g^2*e^2 + 3*a*d*f^
2*g^3*e^2 - c*f^5*e^3 + b*f^4*g*e^3 - a*f^3*g^2*e^3)/(sqrt(g*x + f)*g^6) +
2/315*(105*(g*x + f)^(3/2)*c*d^3*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^51 + 31
5*sqrt(g*x + f)*b*d^3*g^52 + 189*(g*x + f)^(5/2)*c*d^2*g^50*e - 945*(g*x +
f)^(3/2)*c*d^2*f*g^50*e + 2835*sqrt(g*x + f)*c*d^2*f^2*g^50*e + 315*(g*x +
f)^(3/2)*b*d^2*g^51*e - 1890*sqrt(g*x + f)*b*d^2*f*g^51*e + 945*sqrt(g*x +
f)*a*d^2*g^52*e + 135*(g*x + f)^(7/2)*c*d*g^49*e^2 - 756*(g*x + f)^(5/2)*c*
d*f*g^49*e^2 + 1890*(g*x + f)^(3/2)*c*d*f^2*g^49*e^2 - 3780*sqrt(g*x + f)*c
*d*f^3*g^49*e^2 + 189*(g*x + f)^(5/2)*b*d*g^50*e^2 - 945*(g*x + f)^(3/2)*b*
d*f*g^50*e^2 + 2835*sqrt(g*x + f)*b*d*f^2*g^50*e^2 + 315*(g*x + f)^(3/2)*a*
d*g^51*e^2 - 1890*sqrt(g*x + f)*a*d*f*g^51*e^2 + 35*(g*x + f)^(9/2)*c*g^48*
e^3 - 225*(g*x + f)^(7/2)*c*f*g^48*e^3 + 630*(g*x + f)^(5/2)*c*f^2*g^48*e^3
 - 1050*(g*x + f)^(3/2)*c*f^3*g^48*e^3 + 1575*sqrt(g*x + f)*c*f^4*g^48*e^3
+ 45*(g*x + f)^(7/2)*b*g^49*e^3 - 252*(g*x + f)^(5/2)*b*f*g^49*e^3 + 630*(g
*x + f)^(3/2)*b*f^2*g^49*e^3 - 1260*sqrt(g*x + f)*b*f^3*g^49*e^3 + 63*(g*x
+ f)^(5/2)*a*g^50*e^3 - 315*(g*x + f)^(3/2)*a*f*g^50*e^3 + 945*sqrt(g*x + f
)*a*f^2*g^50*e^3)/g^54
```

maple [B] time = 0.01, size = 540, normalized size = 1.89

$$-\frac{2\left(-35e^{3}cx^{5}g^{5}-45be^{3}g^{5}x^{4}-135cde^{2}g^{5}x^{4}+50ce^{3}fg^{4}x^{4}-63ae^{3}g^{5}x^{3}-189bde^{2}g^{5}x^{3}+72be^{3}fg^{4}x^{3}-189cd^{2}eg^{5}x^{3}+216cde^{2}fg^{4}x^{3}-80ce^{3}f^{2}g^{3}x^{3}-315ade^{2}g^{5}x^{2}+126ae^{3}fg^{4}x^{2}-315bd^{2}eg^{5}x^{2}+378bde^{2}fg^{4}x^{2}-144be^{3}f^{2}g^{3}x^{2}-105cd^{3}g^{5}x^{2}+378cd^{2}efg^{4}x^{2}-432cde^{2}f^{2}g^{3}x^{2}+160ce^{3}f^{3}g^{2}x^{2}-945ad^{2}eg^{5}x+1260ade^{2}fg^{4}x-504ae^{3}f^{2}g^{3}x-315bd^{3}g^{5}x+1260bd^{2}efg^{4}x-1512bde^{2}f^{2}g^{3}x+576be^{3}f^{3}g^{2}x+420cd^{3}fg^{4}x-1512cd^{2}ef^{2}g^{3}x+1728cde^{2}f^{3}g^{2}x-640ce^{3}f^{4}gx+315d^{3}ag^{5}-1890ad^{2}efg^{4}+2520ade^{2}f^{2}g^{3}-1008ae^{3}f^{3}g^{2}-630bd^{3}fg^{4}+2520bd^{2}ef^{2}g^{3}-3024bde^{2}f^{3}g^{2}+1152be^{3}f^{4}g+840cd^{3}f^{2}g^{3}-3024cd^{2}ef^{3}g^{2}+3456cde^{2}f^{4}g-1280ce^{3}f^{5}\right)}{315\sqrt{gx+f}\,g^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/315/(g*x+f)^(1/2)*(-35*c*e^3*g^5*x^5-45*b*e^3*g^5*x^4-135*c*d*e^2*g^5*x^
4+50*c*e^3*f*g^4*x^4-63*a*e^3*g^5*x^3-189*b*d*e^2*g^5*x^3+72*b*e^3*f*g^4*x^
3-189*c*d^2*e*g^5*x^3+216*c*d*e^2*f*g^4*x^3-80*c*e^3*f^2*g^3*x^3-315*a*d*e^
2*g^5*x^2+126*a*e^3*f*g^4*x^2-315*b*d^2*e*g^5*x^2+378*b*d*e^2*f*g^4*x^2-144
*b*e^3*f^2*g^3*x^2-105*c*d^3*g^5*x^2+378*c*d^2*e*f*g^4*x^2-432*c*d*e^2*f^2*
g^3*x^2+160*c*e^3*f^3*g^2*x^2-945*a*d^2*e*g^5*x+1260*a*d*e^2*f*g^4*x-504*a*
e^3*f^2*g^3*x-315*b*d^3*g^5*x+1260*b*d^2*e*f*g^4*x-1512*b*d*e^2*f^2*g^3*x+5
76*b*e^3*f^3*g^2*x+420*c*d^3*f*g^4*x-1512*c*d^2*e*f^2*g^3*x+1728*c*d*e^2*f^
3*g^2*x-640*c*e^3*f^4*g*x+315*a*d^3*g^5-1890*a*d^2*e*f*g^4+2520*a*d*e^2*f^2
*g^3-1008*a*e^3*f^3*g^2-630*b*d^3*f*g^4+2520*b*d^2*e*f^2*g^3-3024*b*d*e^2*f
^3*g^2+1152*b*e^3*f^4*g+840*c*d^3*f^2*g^3-3024*c*d^2*e*f^3*g^2+3456*c*d*e^2
*f^4*g-1280*c*e^3*f^5)/g^6
```

maxima [A] time = 0.46, size = 437, normalized size = 1.53

$$\frac{2\left(\frac{35\,(gx+f)^{\frac{9}{2}}ce^{3}-45\left(5\,ce^{3}f-\left(3\,cde^{2}+be^{3}\right)g\right)(gx+f)^{\frac{7}{2}}+63\left(10\,ce^{3}f^{2}-4\left(3\,cde^{2}+be^{3}\right)fg+\left(3\,cd^{2}e+3\,bde^{2}+ae^{3}\right)g^{2}\right)(gx+f)^{\frac{5}{2}}-105\left(10\,ce^{3}f^{3}-6\left(3\,cde^{2}+be^{3}\right)f^{2}g+3\left(3\,cd^{2}e+3\,bde^{2}+ae^{3}\right)fg^{2}-\left(cd^{3}+3\,bd^{2}e+3\,ade^{2}\right)g^{3}\right)(gx+f)^{\frac{3}{2}}+315\left(5\,ce^{3}f^{4}-4\left(3\,cde^{2}+be^{3}\right)f^{3}g+3\left(3\,cd^{2}e+3\,bde^{2}+ae^{3}\right)f^{2}g^{2}-2\left(cd^{3}+3\,bd^{2}e+3\,ade^{2}\right)fg^{3}+\left(bd^{3}+3\,ad^{2}e\right)g^{4}\right)\sqrt{gx+f}}{g^{5}}+\frac{315\left(ce^{3}f^{5}-ad^{3}g^{5}-\left(3\,cde^{2}+be^{3}\right)f^{4}g+\left(3\,cd^{2}e+3\,bde^{2}+ae^{3}\right)f^{3}g^{2}-\left(cd^{3}+3\,bd^{2}e+3\,ade^{2}\right)f^{2}g^{3}+\left(bd^{3}+3\,ad^{2}e\right)fg^{4}\right)}{\sqrt{gx+f}\,g^{5}}\right)}{315\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - (3*c*d*e^2 + b*e^3)*g)*(
g*x + f)^(7/2) + 63*(10*c*e^3*f^2 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^2*e
+ 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(5/2) - 105*(10*c*e^3*f^3 - 6*(3*c*d*e^
2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 4*(3*c*d*e^2 +
b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*sqrt(g*x + f))/g^5 + 315
*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*c*d^2*e + 3*b*d*e^
2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + (b*d^3 + 3*a
*d^2*e)*f*g^4)/(sqrt(g*x + f)*g^5))/g
```

mupad [B] time = 0.12, size = 394, normalized size = 1.38

$$\frac{\left(f+g\,x\right)^{7/2}\,\left(2\,b\,e^3\,g-10\,c\,e^3\,f+6\,c\,d\,e^2\,g\right)}{7\,g^6}-\frac{2\,c\,d^3\,f^2\,g^3-2\,b\,d^3\,f\,g^4+2\,a\,d^3\,g^5-6\,c\,d^2\,e\,f^3\,g^2+6\,b\,d^2\,e\,f^2\,g^3-6\,a\,d^2\,e\,f\,g^4+6\,c\,d\,e^2\,f^4\,g-6\,b\,d\,e^2\,f^3\,g^2+6\,a\,d\,e^2\,f^2\,g^3-2\,c\,e^3\,f^5+2\,b\,e^3\,f^4\,g-2\,a\,e^3\,f^3\,g^2}{g^6\,\sqrt{f+g\,x}}+\frac{\left(f+g\,x\right)^{5/2}\,\left(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+6\,b\,d\,e^2\,g^2+20\,c\,e^3\,f^2-8\,b\,e^3\,f\,g+2\,a\,e^3\,g^2\right)}{5\,g^6}+\frac{2\,\left(f+g\,x\right)^{3/2}\,\left(d\,g-e\,f\right)\,\left(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+3\,b\,d\,e\,g^2+10\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{3\,g^6}+\frac{2\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)^2\,\left(3\,a\,e\,g^2+b\,d\,g^2+5\,c\,e\,f^2-4\,b\,e\,f\,g-2\,c\,d\,f\,g\right)}{g^6}+\frac{2\,c\,e^3\,\left(f+g\,x\right)^{9/2}}{9\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*b*e^3*g - 10*c*e^3*f + 6*c*d*e^2*g))/(7*g^6) - (2*a*d^3
*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2*g^3 - 2*b*d^3*f*g^4 + 2*
b*e^3*f^4*g - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*b*d
*e^2*f^3*g^2 + 6*b*d^2*e*f^2*g^3 - 6*c*d^2*e*f^3*g^2)/(g^6*(f + g*x)^(1/2))
 + ((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 - 8*b*e^3*f*g + 6*b*d*e^2*g
^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f*g))/(5*g^6) + (2*(f + g*x)^(3/2)*(d*g - e
*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 + 3*b*d*e*g^2 - 6*b*e^2*f*g - 8
*c*d*e*f*g))/(3*g^6) + (2*(f + g*x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2 + b*d*g^
2 + 5*c*e*f^2 - 4*b*e*f*g - 2*c*d*f*g))/g^6 + (2*c*e^3*(f + g*x)^(9/2))/(9*
g^6)
```

sympy [A] time = 158.56, size = 452, normalized size = 1.59

$$\frac{2ce^3\left(f+gx\right)^{\frac{9}{2}}}{9g^6}+\frac{\left(f+gx\right)^{\frac{7}{2}}\left(2be^3g+6cde^2g-10ce^3f\right)}{7g^6}+\frac{\left(f+gx\right)^{\frac{5}{2}}\left(2ae^3g^2+6bde^2g^2-8be^3fg+6cd^2eg^2-24cde^2fg+20ce^3f^2\right)}{5g^6}+\frac{\left(f+gx\right)^{\frac{3}{2}}\left(6ade^2g^3-6ae^3fg^2+6bd^2eg^3-18bde^2fg^2+12be^3f^2g+2cd^3g^3-18cd^2efg^2+36cde^2f^2g-20ce^3f^3\right)}{3g^6}+\frac{\sqrt{f+gx}\left(6ad^2eg^4-12ade^2fg^3+6ae^3f^2g^2+2bd^3g^4-12bd^2efg^3+18bde^2f^2g^2-8be^3f^3g-4cd^3fg^3+18cd^2ef^2g^2-24cde^2f^3g+10ce^3f^4\right)}{g^6}-\frac{2\left(dg-ef\right)^3\left(ag^2-bfg+cf^2\right)}{g^6\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**3*(f + g*x)**(9/2)/(9*g**6) + (f + g*x)**(7/2)*(2*b*e**3*g + 6*c*d*e
**2*g - 10*c*e**3*f)/(7*g**6) + (f + g*x)**(5/2)*(2*a*e**3*g**2 + 6*b*d*e**
2*g**2 - 8*b*e**3*f*g + 6*c*d**2*e*g**2 - 24*c*d*e**2*f*g + 20*c*e**3*f**2)
/(5*g**6) + (f + g*x)**(3/2)*(6*a*d*e**2*g**3 - 6*a*e**3*f*g**2 + 6*b*d**2*
e*g**3 - 18*b*d*e**2*f*g**2 + 12*b*e**3*f**2*g + 2*c*d**3*g**3 - 18*c*d**2*
e*f*g**2 + 36*c*d*e**2*f**2*g - 20*c*e**3*f**3)/(3*g**6) + sqrt(f + g*x)*(6
*a*d**2*e*g**4 - 12*a*d*e**2*f*g**3 + 6*a*e**3*f**2*g**2 + 2*b*d**3*g**4 -
12*b*d**2*e*f*g**3 + 18*b*d*e**2*f**2*g**2 - 8*b*e**3*f**3*g - 4*c*d**3*f*g
**3 + 18*c*d**2*e*f**2*g**2 - 24*c*d*e**2*f**3*g + 10*c*e**3*f**4)/g**6 - 2
*(d*g - e*f)**3*(a*g**2 - b*f*g + c*f**2)/(g**6*sqrt(f + g*x))
```

3.570 $\int \frac{(d+ex)^2(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=210

$$-\frac{2(f+gx)^{3/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5\sqrt{f+gx}}-\frac{2\sqrt{f+gx}\,(\cdots}{}$$

Rubi [A] time = 0.29, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {897, 1261}

$$-\frac{2(f+gx)^{3/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5\sqrt{f+gx}}-\frac{2\sqrt{f+gx}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{g^5}-\frac{2e(f+gx)^{5/2}(-beg-2cdg+4cef)}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2))/(g^5*Sqrt[f + g*x]) - (2*(e*f -
d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x])/g
^5 - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g
^2))*(f + g*x)^(3/2))/(3*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^
(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx,x,\sqrt{f+gx}\right)}{g}\\
&= \frac{2\,\text{Subst}\left(\int \left(\frac{(ef-dg)(-2cf(2ef-dg)+g(3bef-bdg-2aeg))}{g^4}+\frac{(-ef+dg)^2\left(cf^2-bfg+ag^2\right)}{g^4x^2}+\frac{(-eg(3bef\cdots}{}\right.\right.}{g}\\
&= -\frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)}{g^5\sqrt{f+gx}}-\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg-2\cdots}{g^5}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 184, normalized size = 0.88

$$\frac{2\left(-35(f+gx)^2\left(-eg(aeg+2bdg-3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)-105(ef-dg)^2\left(g(ag-bf)+cf^2\right)-105(f+gx)(ef-dg)(g(2aeg+bdg-3bef)+2cf(2ef-dg))-21e(f+gx)^3(-beg-2cdg+4cef)+15ce^2(f+gx)^4\right)}{105g^5\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-105*(e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g)) - 105*(e*f - d*g)*(2*c*f*
(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*(f + g*x) - 35*(-(e*g*(-3*b
*e*f + 2*b*d*g + a*e*g)) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^2
 - 21*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^3 + 15*c*e^2*(f + g*x)^4))/(1
05*g^5*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.20, size = 368, normalized size = 1.75

$$\frac{2\left(-105ad^2g^4+210adeg^3(f+gx)+210adefg^3-105ae^2f^2g^2-210ae^2fg^2(f+gx)+35ae^2g^2(f+gx)^2+105bd^2g^3(f+gx)+105bd^2fg^3-210bdef^2g^2-420bdefg^2(f+gx)+70bdeg^2(f+gx)^2+105be^2f^3g+315be^2f^2g(f+gx)-105be^2fg(f+gx)^2+21be^2g(f+gx)^3-105cd^2f^2g^2-210cd^2fg^2(f+gx)+35cd^2g^2(f+gx)^2+210cdef^3g+630cdef^2g(f+gx)-210cdefg(f+gx)^2+42cdeg(f+gx)^3-105ce^2f^4-420ce^2f^3(f+gx)+210ce^2f^2(f+gx)^2-84ce^2f(f+gx)^3+15ce^2(f+gx)^4\right)}{105g^5\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-105*c*e^2*f^4 + 210*c*d*e*f^3*g + 105*b*e^2*f^3*g - 105*c*d^2*f^2*g^2
- 210*b*d*e*f^2*g^2 - 105*a*e^2*f^2*g^2 + 105*b*d^2*f*g^3 + 210*a*d*e*f*g^3
 - 105*a*d^2*g^4 - 420*c*e^2*f^3*(f + g*x) + 630*c*d*e*f^2*g*(f + g*x) + 31
5*b*e^2*f^2*g*(f + g*x) - 210*c*d^2*f*g^2*(f + g*x) - 420*b*d*e*f*g^2*(f +
g*x) - 210*a*e^2*f*g^2*(f + g*x) + 105*b*d^2*g^3*(f + g*x) + 210*a*d*e*g^3*
(f + g*x) + 210*c*e^2*f^2*(f + g*x)^2 - 210*c*d*e*f*g*(f + g*x)^2 - 105*b*e
^2*f*g*(f + g*x)^2 + 35*c*d^2*g^2*(f + g*x)^2 + 70*b*d*e*g^2*(f + g*x)^2 +
35*a*e^2*g^2*(f + g*x)^2 - 84*c*e^2*f*(f + g*x)^3 + 42*c*d*e*g*(f + g*x)^3
+ 21*b*e^2*g*(f + g*x)^3 + 15*c*e^2*(f + g*x)^4))/(105*g^5*Sqrt[f + g*x])
```

fricas [A] time = 0.61, size = 269, normalized size = 1.28

$$\frac{2\left(15ce^2g^4x^4-384ce^2f^4-105ad^2g^4+336\left(2cde+be^2\right)f^3g-280\left(cd^2+2bde+ae^2\right)f^2g^2+210\left(bd^2+2ade\right)fg^3-3\left(8ce^2fg^3-7\left(2cde+be^2\right)g^4\right)x^3+\left(48ce^2f^2g^2-42\left(2cde+be^2\right)fg^3+35\left(cd^2+2bde+ae^2\right)g^4\right)x^2-\left(192ce^2f^3g-168\left(2cde+be^2\right)f^2g^2+140\left(cd^2+2bde+ae^2\right)fg^3-105\left(bd^2+2ade\right)g^4\right)x\right)\sqrt{gx+f}}{105\left(g^6x+fg^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 - 105*a*d^2*g^4 + 336*(2*c*d*e + b*
e^2)*f^3*g - 280*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g^2 + 210*(b*d^2 + 2*a*d*e)*
f*g^3 - 3*(8*c*e^2*f*g^3 - 7*(2*c*d*e + b*e^2)*g^4)*x^3 + (48*c*e^2*f^2*g^2
 - 42*(2*c*d*e + b*e^2)*f*g^3 + 35*(c*d^2 + 2*b*d*e + a*e^2)*g^4)*x^2 - (19
2*c*e^2*f^3*g - 168*(2*c*d*e + b*e^2)*f^2*g^2 + 140*(c*d^2 + 2*b*d*e + a*e^
2)*f*g^3 - 105*(b*d^2 + 2*a*d*e)*g^4)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

giac [B] time = 0.23, size = 404, normalized size = 1.92

$$-\frac{2\left(cd^2f^2g^2-bd^2fg^3+ad^2g^4-2cdf^3ge+2bdf^2g^2e-2adfg^3e+cf^4e^2-bf^3ge^2+af^2g^2e^2\right)}{\sqrt{gx+f}\,g^5}+\frac{2\left(35\left(gx+f\right)^{\frac{3}{2}}cd^2g^{32}-210\sqrt{gx+f}\,cd^2fg^{32}+105\sqrt{gx+f}\,bd^2g^{33}+42\left(gx+f\right)^{\frac{5}{2}}cdg^{31}e-210\left(gx+f\right)^{\frac{3}{2}}cdfg^{31}e+630\sqrt{gx+f}\,cdf^2g^{31}e+70\left(gx+f\right)^{\frac{3}{2}}bdg^{32}e-420\sqrt{gx+f}\,bdfg^{32}e+210\sqrt{gx+f}\,adg^{33}e+15\left(gx+f\right)^{\frac{7}{2}}cg^{30}e^2-84\left(gx+f\right)^{\frac{5}{2}}cfg^{30}e^2+210\left(gx+f\right)^{\frac{3}{2}}cf^2g^{30}e^2-420\sqrt{gx+f}\,cf^3g^{30}e^2+21\left(gx+f\right)^{\frac{5}{2}}bg^{31}e^2-105\left(gx+f\right)^{\frac{3}{2}}bfg^{31}e^2+315\sqrt{gx+f}\,bf^2g^{31}e^2+35\left(gx+f\right)^{\frac{3}{2}}ag^{32}e^2-210\sqrt{gx+f}\,afg^{32}e^2\right)}{105\,g^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*f^2*g^2 - b*d^2*f*g^3 + a*d^2*g^4 - 2*c*d*f^3*g*e + 2*b*d*f^2*g^2
*e - 2*a*d*f*g^3*e + c*f^4*e^2 - b*f^3*g*e^2 + a*f^2*g^2*e^2)/(sqrt(g*x + f
)*g^5) + 2/105*(35*(g*x + f)^(3/2)*c*d^2*g^32 - 210*sqrt(g*x + f)*c*d^2*f*g
^32 + 105*sqrt(g*x + f)*b*d^2*g^33 + 42*(g*x + f)^(5/2)*c*d*g^31*e - 210*(g
*x + f)^(3/2)*c*d*f*g^31*e + 630*sqrt(g*x + f)*c*d*f^2*g^31*e + 70*(g*x + f
)^(3/2)*b*d*g^32*e - 420*sqrt(g*x + f)*b*d*f*g^32*e + 210*sqrt(g*x + f)*a*d
*g^33*e + 15*(g*x + f)^(7/2)*c*g^30*e^2 - 84*(g*x + f)^(5/2)*c*f*g^30*e^2 +
 210*(g*x + f)^(3/2)*c*f^2*g^30*e^2 - 420*sqrt(g*x + f)*c*f^3*g^30*e^2 + 21
*(g*x + f)^(5/2)*b*g^31*e^2 - 105*(g*x + f)^(3/2)*b*f*g^31*e^2 + 315*sqrt(g
*x + f)*b*f^2*g^31*e^2 + 35*(g*x + f)^(3/2)*a*g^32*e^2 - 210*sqrt(g*x + f)*
a*f*g^32*e^2)/g^35
```

maple [A] time = 0.01, size = 315, normalized size = 1.50

$$-\frac{2\left(-15 e^{2} c \,x^{4} g^{4}-21 b \,e^{2} g^{4} x^{3}-42 c d e \,g^{4} x^{3}+24 c \,e^{2} f \,g^{3} x^{3}-35 a \,e^{2} g^{4} x^{2}-70 b d e \,g^{4} x^{2}+42 b \,e^{2} f \,g^{3} x^{2}-35 c \,d^{2} g^{4} x^{2}+84 c d e f \,g^{3} x^{2}-48 c \,e^{2} f^{2} g^{2} x^{2}-210 a d e \,g^{4} x+140 a \,e^{2} f \,g^{3} x-105 b \,d^{2} g^{4} x+280 b d e f \,g^{3} x-168 b \,e^{2} f^{2} g^{2} x+140 c \,d^{2} f \,g^{3} x-336 c d e \,f^{2} g^{2} x+192 c \,e^{2} f^{3} g x+105 a \,d^{2} g^{4}-420 a d e f \,g^{3}+280 a \,e^{2} f^{2} g^{2}-210 b \,d^{2} f \,g^{3}+560 b d e \,f^{2} g^{2}-336 b \,e^{2} f^{3} g+280 c \,d^{2} f^{2} g^{2}-672 c d e \,f^{3} g+384 c \,e^{2} f^{4}\right)}{105 \sqrt{g x+f}\, g^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/105/(g*x+f)^(1/2)*(-15*c*e^2*g^4*x^4-21*b*e^2*g^4*x^3-42*c*d*e*g^4*x^3+2
4*c*e^2*f*g^3*x^3-35*a*e^2*g^4*x^2-70*b*d*e*g^4*x^2+42*b*e^2*f*g^3*x^2-35*c
*d^2*g^4*x^2+84*c*d*e*f*g^3*x^2-48*c*e^2*f^2*g^2*x^2-210*a*d*e*g^4*x+140*a*
e^2*f*g^3*x-105*b*d^2*g^4*x+280*b*d*e*f*g^3*x-168*b*e^2*f^2*g^2*x+140*c*d^2
*f*g^3*x-336*c*d*e*f^2*g^2*x+192*c*e^2*f^3*g*x+105*a*d^2*g^4-420*a*d*e*f*g^
3+280*a*e^2*f^2*g^2-210*b*d^2*f*g^3+560*b*d*e*f^2*g^2-336*b*e^2*f^3*g+280*c
*d^2*f^2*g^2-672*c*d*e*f^3*g+384*c*e^2*f^4)/g^5
```

maxima [A] time = 0.45, size = 269, normalized size = 1.28

$$\frac{2\left(\frac{15\,(gx+f)^{\frac{7}{2}} ce^2-21\left(4\,ce^2f-\left(2\,cde+be^2\right)g\right)(gx+f)^{\frac{5}{2}}+35\left(6\,ce^2f^2-3\left(2\,cde+be^2\right)fg+\left(cd^2+2\,bde+ae^2\right)g^2\right)(gx+f)^{\frac{3}{2}}-105\left(4\,ce^2f^3-3\left(2\,cde+be^2\right)f^2g+2\left(cd^2+2\,bde+ae^2\right)fg^2-\left(bd^2+2\,ade\right)g^3\right)\sqrt{gx+f}}{g^4}-\frac{105\left(ce^2f^4+ad^2g^4-\left(2\,cde+be^2\right)f^3g+\left(cd^2+2\,bde+ae^2\right)f^2g^2-\left(bd^2+2\,ade\right)fg^3\right)}{\sqrt{gx+f}\,g^4}\right)}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 - 21*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g*
x + f)^(5/2) + 35*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e
 + a*e^2)*g^2)*(g*x + f)^(3/2) - 105*(4*c*e^2*f^3 - 3*(2*c*d*e + b*e^2)*f^2
*g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*sqrt(g*x +
f))/g^4 - 105*(c*e^2*f^4 + a*d^2*g^4 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 + 2
*b*d*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)/(sqrt(g*x + f)*g^4))/g
```

mupad [B] time = 3.13, size = 270, normalized size = 1.29

$$\frac{{\left(f+g\,x\right)}^{5/2}\,\left(2\,b\,e^2\,g-8\,c\,e^2\,f+4\,c\,d\,e\,g\right)}{5\,g^5}-\frac{2\,a\,d^2\,g^4+2\,c\,e^2\,f^4+2\,a\,e^2\,f^2\,g^2+2\,c\,d^2\,f^2\,g^2-2\,b\,d^2\,f\,g^3-2\,b\,e^2\,f^3\,g+4\,b\,d\,e\,f^2\,g^2-4\,a\,d\,e\,f\,g^3-4\,c\,d\,e\,f^3\,g}{g^5\,\sqrt{f+g\,x}}+\frac{{\left(f+g\,x\right)}^{3/2}\,\left(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+12\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+2\,a\,e^2\,g^2\right)}{3\,g^5}+\frac{2\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)\,\left(2\,a\,e\,g^2+b\,d\,g^2+4\,c\,e\,f^2-3\,b\,e\,f\,g-2\,c\,d\,f\,g\right)}{g^5}+\frac{2\,c\,e^2\,{\left(f+g\,x\right)}^{7/2}}{7\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*b*e^2*g - 8*c*e^2*f + 4*c*d*e*g))/(5*g^5) - (2*a*d^2*g^
4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2 - 2*b*d^2*f*g^3 - 2*b*e
^2*f^3*g + 4*b*d*e*f^2*g^2 - 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*x)^
(1/2)) + ((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 + 4*b*d
*e*g^2 - 6*b*e^2*f*g - 12*c*d*e*f*g))/(3*g^5) + (2*(f + g*x)^(1/2)*(d*g - e
*f)*(2*a*e*g^2 + b*d*g^2 + 4*c*e*f^2 - 3*b*e*f*g - 2*c*d*f*g))/g^5 + (2*c*e
^2*(f + g*x)^(7/2))/(7*g^5)
```

sympy [A] time = 79.49, size = 272, normalized size = 1.30

$$\frac{2 c e^{2} \left(f + g x\right)^{\frac{7}{2}}}{7 g^{5}} + \frac{\left(f + g x\right)^{\frac{5}{2}} \left(2 b e^{2} g + 4 c d e g - 8 c e^{2} f\right)}{5 g^{5}} + \frac{\left(f + g x\right)^{\frac{3}{2}} \left(2 a e^{2} g^{2} + 4 b d e g^{2} - 6 b e^{2} f g + 2 c d^{2} g^{2} - 12 c d e f g + 12 c e^{2} f^{2}\right)}{3 g^{5}} + \frac{\sqrt{f + g x} \left(4 a d e g^{3} - 4 a e^{2} f g^{2} + 2 b d^{2} g^{3} - 8 b d e f g^{2} + 6 b e^{2} f^{2} g - 4 c d^{2} f g^{2} + 12 c d e f^{2} g - 8 c e^{2} f^{3}\right)}{g^{5}} - \frac{2 \left(d g - e f\right)^{2} \left(a g^{2} - b f g + c f^{2}\right)}{g^{5} \sqrt{f + g x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**2*(f + g*x)**(7/2)/(7*g**5) + (f + g*x)**(5/2)*(2*b*e**2*g + 4*c*d*e
*g - 8*c*e**2*f)/(5*g**5) + (f + g*x)**(3/2)*(2*a*e**2*g**2 + 4*b*d*e*g**2
- 6*b*e**2*f*g + 2*c*d**2*g**2 - 12*c*d*e*f*g + 12*c*e**2*f**2)/(3*g**5) +
sqrt(f + g*x)*(4*a*d*e*g**3 - 4*a*e**2*f*g**2 + 2*b*d**2*g**3 - 8*b*d*e*f*g
**2 + 6*b*e**2*f**2*g - 4*c*d**2*f*g**2 + 12*c*d*e*f**2*g - 8*c*e**2*f**3)/
g**5 - 2*(d*g - e*f)**2*(a*g**2 - b*f*g + c*f**2)/(g**5*sqrt(f + g*x))
```

3.571 $\int \frac{(d+ex)\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{2(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\,(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4}-\frac{2(f+gx)^{3/2}(-beg-cdg+}{3g^4}$$

Rubi [A] time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {771}

$$\frac{2(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\,(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4}-\frac{2(f+gx)^{3/2}(-beg-cdg+3cef)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(c*f*(3*e*
f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*Sqrt[f + g*x])/g^4 - (2*(3*c*e*f
- c*d*g - b*e*g)*(f + g*x)^(3/2))/(3*g^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{(-ef+dg)\left(cf^2-bfg+ag^2\right)}{g^3(f+gx)^{3/2}}+\frac{cf(3ef-2dg)-g(2bef-bdg-aeg)}{g^3\sqrt{f+gx}}+\frac{(\right.\\ &= \frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+}}{g^4}\end{aligned}$$

Mathematica [A] time = 0.18, size = 128, normalized size = 0.95

$$\frac{2\left(5g\left(3ag(-dg+2ef+egx)+3bdg(2f+gx)+be\left(-8f^2-4fgx+g^2x^2\right)\right)+c\left(5dg\left(-8f^2-4fgx+g^2x^2\right)+3e\left(16f^3+8f^2gx-2fg^2x^2+g^3x^3\right)\right)\right)}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(5*g*(3*b*d*g*(2*f + g*x) + 3*a*g*(2*e*f - d*g + e*g*x) + b*e*(-8*f^2 -
4*f*g*x + g^2*x^2)) + c*(5*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 3*e*(16*f^3 +
 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))))/(15*g^4*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.11, size = 168, normalized size = 1.24

$$\frac{2\left(-15adg^3+15aeg^2(f+gx)+15aefg^2+15bdg^2(f+gx)+15bdfg^2-15bef^2g-30befg(f+gx)+5beg(f+gx)^2-15cdf^2g-30cdfg(f+gx)+5cdg(f+gx)^2+15cef^3+45cef^2(f+gx)-15cef(f+gx)^2+3ce(f+gx)^3\right)}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(15*c*e*f^3 - 15*c*d*f^2*g - 15*b*e*f^2*g + 15*b*d*f*g^2 + 15*a*e*f*g^2
- 15*a*d*g^3 + 45*c*e*f^2*(f + g*x) - 30*c*d*f*g*(f + g*x) - 30*b*e*f*g*(f
+ g*x) + 15*b*d*g^2*(f + g*x) + 15*a*e*g^2*(f + g*x) - 15*c*e*f*(f + g*x)^2
 + 5*c*d*g*(f + g*x)^2 + 5*b*e*g*(f + g*x)^2 + 3*c*e*(f + g*x)^3))/(15*g^4*
Sqrt[f + g*x])
```

fricas [A] time = 0.39, size = 135, normalized size = 1.00

$$\frac{2\left(3\,ceg^{3}x^{3}+48\,cef^{3}-15\,adg^{3}-40\,(cd+be)f^{2}g+30\,(bd+ae)fg^{2}-\left(6\,cefg^{2}-5\,(cd+be)g^{3}\right)x^{2}+\left(24\,cef^{2}g-20\,(cd+be)fg^{2}+15\,(bd+ae)g^{3}\right)x\right)\sqrt{gx+f}}{15\left(g^{5}x+fg^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 15*a*d*g^3 - 40*(c*d + b*e)*f^2*g + 30*(
b*d + a*e)*f*g^2 - (6*c*e*f*g^2 - 5*(c*d + b*e)*g^3)*x^2 + (24*c*e*f^2*g -
20*(c*d + b*e)*f*g^2 + 15*(b*d + a*e)*g^3)*x)*sqrt(g*x + f)/(g^5*x + f*g^4)
```

giac [A] time = 0.18, size = 204, normalized size = 1.51

$$-\frac{2\left(cdf^{2}g-bdfg^{2}+adg^{3}-cf^{3}e+bf^{2}ge-afg^{2}e\right)}{\sqrt{gx+f}\,g^{4}}+\frac{2\left(5\left(gx+f\right)^{\frac{3}{2}}cdg^{17}-30\sqrt{gx+f}\,cdfg^{17}+15\sqrt{gx+f}\,bdg^{18}+3\left(gx+f\right)^{\frac{5}{2}}cg^{16}e-15\left(gx+f\right)^{\frac{3}{2}}cfg^{16}e+45\sqrt{gx+f}\,cf^{2}g^{16}e+5\left(gx+f\right)^{\frac{3}{2}}bg^{17}e-30\sqrt{gx+f}\,bfg^{17}e+15\sqrt{gx+f}\,ag^{18}e\right)}{15\,g^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d*f^2*g - b*d*f*g^2 + a*d*g^3 - c*f^3*e + b*f^2*g*e - a*f*g^2*e)/(sqr
t(g*x + f)*g^4) + 2/15*(5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f
*g^17 + 15*sqrt(g*x + f)*b*d*g^18 + 3*(g*x + f)^(5/2)*c*g^16*e - 15*(g*x +
f)^(3/2)*c*f*g^16*e + 45*sqrt(g*x + f)*c*f^2*g^16*e + 5*(g*x + f)^(3/2)*b*g
^17*e - 30*sqrt(g*x + f)*b*f*g^17*e + 15*sqrt(g*x + f)*a*g^18*e)/g^20
```

maple [A] time = 0.00, size = 144, normalized size = 1.07

$$-\frac{2\left(-3ce\,x^{3}g^{3}-5be\,g^{3}x^{2}-5cd\,g^{3}x^{2}+6cef\,g^{2}x^{2}-15ae\,g^{3}x-15bd\,g^{3}x+20bef\,g^{2}x+20cdf\,g^{2}x-24ce\,f^{2}gx+15ad\,g^{3}-30aef\,g^{2}-30bdf\,g^{2}+40be\,f^{2}g+40cd\,f^{2}g-48ce\,f^{3}\right)}{15\sqrt{gx+f}\,g^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/15/(g*x+f)^(1/2)*(-3*c*e*g^3*x^3-5*b*e*g^3*x^2-5*c*d*g^3*x^2+6*c*e*f*g^2
*x^2-15*a*e*g^3*x-15*b*d*g^3*x+20*b*e*f*g^2*x+20*c*d*f*g^2*x-24*c*e*f^2*g*x
+15*a*d*g^3-30*a*e*f*g^2-30*b*d*f*g^2+40*b*e*f^2*g+40*c*d*f^2*g-48*c*e*f^3)
/g^4
```

maxima [A] time = 0.44, size = 137, normalized size = 1.01

$$\frac{2\left(\frac{3\left(gx+f\right)^{\frac{5}{2}}ce-5\left(3\,cef-(cd+be)g\right)\left(gx+f\right)^{\frac{3}{2}}+15\left(3\,cef^{2}-2\,(cd+be)fg+(bd+ae)g^{2}\right)\sqrt{gx+f}}{g^{3}}+\frac{15\left(cef^{3}-adg^{3}-(cd+be)f^{2}g+(bd+ae)fg^{2}\right)}{\sqrt{gx+f}\,g^{3}}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^(3/2)
+ 15*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*sqrt(g*x + f))/g^3 +
 15*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)/(sqrt(g*x +
 f)*g^3))/g
```

mupad [B] time = 3.13, size = 147, normalized size = 1.09

$$\frac{(f+gx)^{3/2}\,(2beg+2cdg-6cef)}{3g^4}-\frac{2adg^3-2cef^3-2aefg^2-2bdfg^2+2bef^2g+2cdf^2g}{g^4\sqrt{f+gx}}+\frac{\sqrt{f+gx}\,(2aeg^2+2bdg^2+6cef^2-4befg-4cdfg)}{g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*b*e*g + 2*c*d*g - 6*c*e*f))/(3*g^4) - (2*a*d*g^3 - 2*c*
e*f^3 - 2*a*e*f*g^2 - 2*b*d*f*g^2 + 2*b*e*f^2*g + 2*c*d*f^2*g)/(g^4*(f + g*
x)^(1/2)) + ((f + g*x)^(1/2)*(2*a*e*g^2 + 2*b*d*g^2 + 6*c*e*f^2 - 4*b*e*f*g
 - 4*c*d*f*g))/g^4 + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

sympy [A] time = 34.53, size = 141, normalized size = 1.04

$$\frac{2ce\left(f+gx\right)^{\frac{5}{2}}}{5g^4}+\frac{\left(f+gx\right)^{\frac{3}{2}}\left(2beg+2cdg-6cef\right)}{3g^4}+\frac{\sqrt{f+gx}\left(2aeg^2+2bdg^2-4befg-4cdfg+6cef^2\right)}{g^4}-\frac{2\left(dg-ef\right)\left(ag^2-bfg+cf^2\right)}{g^4\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e*(f + g*x)**(5/2)/(5*g**4) + (f + g*x)**(3/2)*(2*b*e*g + 2*c*d*g - 6*c
*e*f)/(3*g**4) + sqrt(f + g*x)*(2*a*e*g**2 + 2*b*d*g**2 - 4*b*e*f*g - 4*c*d
*f*g + 6*c*e*f**2)/g**4 - 2*(d*g - e*f)*(a*g**2 - b*f*g + c*f**2)/(g**4*sqr
t(f + g*x))
```

3.572 $\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2\left(ag^2-bfg+cf^2\right)}{g^3\sqrt{f+gx}}-\frac{2\sqrt{f+gx}\,(2cf-bg)}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {698}

$$-\frac{2\left(ag^2-bfg+cf^2\right)}{g^3\sqrt{f+gx}}-\frac{2\sqrt{f+gx}\,(2cf-bg)}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g^3*Sqrt[f + g*x]) - (2*(2*c*f - b*g)*Sqrt[f
+ g*x])/g^3 + (2*c*(f + g*x)^(3/2))/(3*g^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{cf^2-bfg+ag^2}{g^2(f+gx)^{3/2}}+\frac{-2cf+bg}{g^2\sqrt{f+gx}}+\frac{c\sqrt{f+gx}}{g^2}\right)dx\\ &= -\frac{2\left(cf^2-bfg+ag^2\right)}{g^3\sqrt{f+gx}}-\frac{2(2cf-bg)\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 0.76

$$\frac{6g(-ag+2bf+bgx)+2c\left(-8f^2-4fgx+g^2x^2\right)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (6*g*(2*b*f - a*g + b*g*x) + 2*c*(-8*f^2 - 4*f*g*x + g^2*x^2))/(3*g^3*Sqrt[
f + g*x])
```

IntegrateAlgebraic [A] time = 0.04, size = 61, normalized size = 0.86

$$\frac{2\left(-3ag^2+3bg(f+gx)+3bfg-3cf^2-6cf(f+gx)+c(f+gx)^2\right)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-3*c*f^2 + 3*b*f*g - 3*a*g^2 - 6*c*f*(f + g*x) + 3*b*g*(f + g*x) + c*(f
 + g*x)^2))/(3*g^3*Sqrt[f + g*x])
```

fricas [A] time = 0.40, size = 63, normalized size = 0.89

$$\frac{2\left(cg^{2}x^{2}-8\,cf^{2}+6\,bfg-3\,ag^{2}-\left(4\,cfg-3\,bg^{2}\right)x\right)\sqrt{gx+f}}{3\left(g^{4}x+fg^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*g^2*x^2 - 8*c*f^2 + 6*b*f*g - 3*a*g^2 - (4*c*f*g - 3*b*g^2)*x)*sqrt(
g*x + f)/(g^4*x + f*g^3)
```

giac [A] time = 0.15, size = 74, normalized size = 1.04

$$-\frac{2\left(cf^{2}-bfg+ag^{2}\right)}{\sqrt{gx+f}\,g^{3}}+\frac{2\left(\left(gx+f\right)^{\frac{3}{2}}cg^{6}-6\,\sqrt{gx+f}\,cfg^{6}+3\,\sqrt{gx+f}\,bg^{7}\right)}{3\,g^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 - b*f*g + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6
 - 6*sqrt(g*x + f)*c*f*g^6 + 3*sqrt(g*x + f)*b*g^7)/g^9
```

maple [A] time = 0.00, size = 53, normalized size = 0.75

$$-\frac{2\left(-c\,x^{2}g^{2}-3b\,g^{2}x+4cfgx+3a\,g^{2}-6bfg+8c\,f^{2}\right)}{3\sqrt{gx+f}\;g^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/3/(g*x+f)^(1/2)*(-c*g^2*x^2-3*b*g^2*x+4*c*f*g*x+3*a*g^2-6*b*f*g+8*c*f^2)
/g^3
```

maxima [A] time = 0.44, size = 66, normalized size = 0.93

$$\frac{2\left(\frac{\left(gx+f\right)^{\frac{3}{2}}c-3\left(2\,cf-bg\right)\sqrt{gx+f}}{g^{2}}-\frac{3\left(cf^{2}-bfg+ag^{2}\right)}{\sqrt{gx+f}\,g^{2}}\right)}{3\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((g*x + f)^(3/2)*c - 3*(2*c*f - b*g)*sqrt(g*x + f))/g^2 - 3*(c*f^2 - b
*f*g + a*g^2)/(sqrt(g*x + f)*g^2))/g
```

mupad [B] time = 0.06, size = 58, normalized size = 0.82

$$\frac{2\,c\left(f+g\,x\right)^{2}-6\,a\,g^{2}-6\,c\,f^{2}+6\,b\,g\,\left(f+g\,x\right)-12\,c\,f\,\left(f+g\,x\right)+6\,b\,f\,g}{3\,g^{3}\,\sqrt{f+g\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(f + g*x)^(3/2),x)
```

```
[Out] (2*c*(f + g*x)^2 - 6*a*g^2 - 6*c*f^2 + 6*b*g*(f + g*x) - 12*c*f*(f + g*x) +
 6*b*f*g)/(3*g^3*(f + g*x)^(1/2))
```

sympy [**A**] time = 13.12, size = 70, normalized size = 0.99

$$\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3g^3}+\frac{\sqrt{f+gx}\left(2bg-4cf\right)}{g^3}-\frac{2\left(ag^2-bfg+cf^2\right)}{g^3\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*g**3) + sqrt(f + g*x)*(2*b*g - 4*c*f)/g**3 - 2*(a*g
**2 - b*f*g + c*f**2)/(g**3*sqrt(f + g*x))
```

3.573 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2-bfg+cf^2\right)}{g^2\sqrt{f+gx}\,(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

Rubi [A] time = 0.22, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {897, 1261, 208}

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2-bfg+cf^2\right)}{g^2\sqrt{f+gx}\,(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f +
 g*x])/(e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])
/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int\left(\frac{c}{eg}+\frac{cf^2-bfg+ag^2}{g(-ef+dg)x^2}-\frac{(cd^2-bde+ae^2)g}{e(ef-dg)(ef-dg-ex^2)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\left(cf^2-bfg+ag^2\right)}{g^2(ef-dg)\sqrt{f+gx}} + \frac{2c\sqrt{f+gx}}{eg^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int\frac{1}{ef-dg-ex^2}dx, x, \right.}{e(ef-dg)}$$

$$= \frac{2\left(cf^2-bfg+ag^2\right)}{g^2(ef-dg)\sqrt{f+gx}} + \frac{2c\sqrt{f+gx}}{eg^2} - \frac{2\left(cd^2-bde+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

Mathematica [**A**] time = 0.29, size = 124, normalized size = 1.02

$$\frac{2\left(-\frac{g^2\left(cd^2-e(bd-ae)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{cf^2-g(bf-ag)}{\sqrt{f+gx}\,(ef-dg)}+\frac{c\sqrt{f+gx}}{e}\right)}{g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*((c*f^2 - g*(b*f - a*g))/((e*f - d*g)*Sqrt[f + g*x]) + (c*Sqrt[f + g*x])
/e - ((c*d^2 - e*(b*d - a*e))*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f
- d*g]])/(e^(3/2)*(e*f - d*g)^(3/2))))/g^2
```

IntegrateAlgebraic [**A**] time = 0.19, size = 139, normalized size = 1.14

$$\frac{2\left(ae^2-bde+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)}{e^{3/2}(dg-ef)^{3/2}}+\frac{2\left(aeg^2-befg-cdg(f+gx)+cef^2+cef(f+gx)\right)}{eg^2\sqrt{f+gx}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*e*f^2 - b*e*f*g + a*e*g^2 + c*e*f*(f + g*x) - c*d*g*(f + g*x)))/(e*g^
2*(e*f - d*g)*Sqrt[f + g*x]) + (2*(c*d^2 - b*d*e + a*e^2)*ArcTan[(Sqrt[e]*S
qrt[-(e*f) + d*g]*Sqrt[f + g*x])/(e*f - d*g)])/(e^(3/2)*(-(e*f) + d*g)^(3/2
))
```

fricas [**B**] time = 0.44, size = 540, normalized size = 4.43

$$\left[-\frac{\left(\left(cd^2-bde+ae^2\right)g^3x+\left(cd^2-bde+ae^2\right)fg^2\right)\sqrt{e^2f-deg}\log\left(\frac{egx+2ef-dg+2\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2ce^3f^3-ade^2g^3-\left(3cde^2+be^3\right)f^2g+\left(cd^2e+bde^2+ae^3\right)fg^2+\left(ce^3f^2g-2cde^2fg^2+cd^2eg^3\right)x\right)\sqrt{gx+f}}{e^4f^3g^2-2de^3f^2g^3+d^2e^2fg^4+\left(e^4f^2g^3-2de^3fg^4+d^2e^2g^5\right)x}, \frac{2\left(\left(\left(cd^2-bde+ae^2\right)g^3x+\left(cd^2-bde+ae^2\right)fg^2\right)\sqrt{-e^2f+deg}\arctan\left(\frac{\sqrt{-e^2f+deg}\sqrt{gx+f}}{egx+ef}\right)+\left(2ce^3f^3-ade^2g^3-\left(3cde^2+be^3\right)f^2g+\left(cd^2e+bde^2+ae^3\right)fg^2+\left(ce^3f^2g-2cde^2fg^2+cd^2eg^3\right)x\right)\sqrt{gx+f}\right)}{e^4f^3g^2-2de^3f^2g^3+d^2e^2fg^4+\left(e^4f^2g^3-2de^3fg^4+d^2e^2g^5\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(((c*d^2 - b*d*e + a*e^2)*g^3*x + (c*d^2 - b*d*e + a*e^2)*f*g^2)*sqrt(e^2
*f - d*e*g)*log((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))
/(e*x + d)) - 2*(2*c*e^3*f^3 - a*d*e^2*g^3 - (3*c*d*e^2 + b*e^3)*f^2*g + (c
*d^2*e + b*d*e^2 + a*e^3)*f*g^2 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*
```

```
g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^
4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x), 2*(((c*d^2 - b*d*e + a*e^2)*g^
3*x + (c*d^2 - b*d*e + a*e^2)*f*g^2)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*
f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*e^3*f^3 - a*d*e^2*g^3 - (3*c
*d*e^2 + b*e^3)*f^2*g + (c*d^2*e + b*d*e^2 + a*e^3)*f*g^2 + (c*e^3*f^2*g -
2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*f^2
*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x)]
```

giac [A] time = 0.25, size = 112, normalized size = 0.92

$$-\frac{2\left(cd^2-bde+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(dge-fe^2\right)^{\frac{3}{2}}}+\frac{2\sqrt{gx+f}\,ce^{(-1)}}{g^2}-\frac{2\left(cf^2-bfg+ag^2\right)}{\left(dg^3-fg^2e\right)\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/(d*g
*e - f*e^2)^(3/2) + 2*sqrt(g*x + f)*c*e^(-1)/g^2 - 2*(c*f^2 - b*f*g + a*g^2
)/((d*g^3 - f*g^2*e)*sqrt(g*x + f))
```

maple [B] time = 0.01, size = 237, normalized size = 1.94

$$-\frac{2ae\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}+\frac{2bd\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}-\frac{2cd^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}\,e}-\frac{2a}{(dg-ef)\sqrt{gx+f}}+\frac{2bf}{(dg-ef)\sqrt{gx+f}\,g}-\frac{2cf^2}{(dg-ef)\sqrt{gx+f}\,g^2}+\frac{2\sqrt{gx+f}\,c}{eg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*c/e/g^2-2/(d*g-e*f)*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1
/2)/((d*g-e*f)*e)^(1/2)*e)*a+2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)
^(1/2)/((d*g-e*f)*e)^(1/2)*e)*b*d-2/(d*g-e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(
(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*c*d^2-2/(d*g-e*f)/(g*x+f)^(1/2)*a+2/g/
(d*g-e*f)/(g*x+f)^(1/2)*b*f-2/g^2/(d*g-e*f)/(g*x+f)^(1/2)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 3.21, size = 162, normalized size = 1.33

$$\frac{2c\sqrt{f+gx}}{eg^2}+\frac{2\,\mathrm{atan}\left(\frac{2\sqrt{f+gx}\left(e^2f-deg\right)\left(cd^2-bde+ae^2\right)}{\sqrt{e}\left(dg-ef\right)^{3/2}\left(2cd^2-2bde+2ae^2\right)}\right)\left(cd^2-bde+ae^2\right)}{e^{3/2}\left(dg-ef\right)^{3/2}}-\frac{2\left(cef^2-befg+aeg^2\right)}{eg^2\sqrt{f+gx}\,\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

```
[Out] (2*c*(f + g*x)^(1/2))/(e*g^2) + (2*atan((2*(f + g*x)^(1/2)*(e^2*f - d*e*g)*
(a*e^2 + c*d^2 - b*d*e))/(e^(1/2)*(d*g - e*f)^(3/2)*(2*a*e^2 + 2*c*d^2 - 2*
b*d*e)))*(a*e^2 + c*d^2 - b*d*e))/(e^(3/2)*(d*g - e*f)^(3/2)) - (2*(a*e*g^2
 + c*e*f^2 - b*e*f*g))/(e*g^2*(f + g*x)^(1/2)*(d*g - e*f))
```

sympy [A] time = 52.23, size = 116, normalized size = 0.95

$$\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2\left(ag^2-bfg+cf^2\right)}{g^2\sqrt{f+gx}\left(dg-ef\right)}-\frac{2\left(ae^2-bde+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*sqrt(f + g*x)/(e*g**2) - 2*(a*g**2 - b*f*g + c*f**2)/(g**2*sqrt(f + g*x
)*(d*g - e*f)) - 2*(a*e**2 - b*d*e + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g -
 e*f)/e))/(e**2*sqrt((d*g - e*f)/e)*(d*g - e*f))
```

3.574 $\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg)-e(-3aeg+bdg+2bef))}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2-bfg+cf^2\right)}{g\sqrt{f+gx}\,(ef-dg)^2}$$

Rubi [A] time = 0.37, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {897, 1259, 453, 208}

$$-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg)-e(-3aeg+bdg+2bef))}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2-bfg+cf^2\right)}{g\sqrt{f+gx}\,(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 - b*
d*e + a*e^2)*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((c*d*(4*e*f - d*
g) - e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{g^3\,\text{Subst}\left(\int \frac{\frac{2e^2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^5}-\frac{e\left(e(bd-ae)g^2+c\left(2e^2f^2-4defg+d^2 \cdots\right.\right.}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\cdots\right)}{e^2(ef-dg)^2 \cdots}$$

$$= -\frac{2\left(cf^2-bfg+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{(cd(4ef-dg)-e(2bef+bdg \cdots}{\cdots}$$

$$= -\frac{2\left(cf^2-bfg+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(cd(4ef-dg)-e(2bef+bdg \cdots}{e^{3/2}(ef \cdots}$$

Mathematica [A] time = 0.41, size = 176, normalized size = 1.07

$$-\frac{eg(2adg+ae(f+3gx)-b(3df+dgx+2efx))+c\left(d^2g(f+gx)+2def^2+2e^2f^2x\right)}{eg(d+ex)\sqrt{f+gx}\,(ef-dg)^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-3aeg+bdg+2bef)+cd(dg-4ef))}{e^{3/2}(ef-dg)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] -((c*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x)) + e*g*(2*a*d*g + a*e*(f +
3*g*x) - b*(3*d*f + 2*e*f*x + d*g*x)))/(e*g*(e*f - d*g)^2*(d + e*x)*Sqrt[f
+ g*x])) - ((c*d*(-4*e*f + d*g) + e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTanh[(S
qrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2))
```

IntegrateAlgebraic [A] time = 0.69, size = 268, normalized size = 1.62

$$\frac{2adeg^3+3ae^2g^2(f+gx)-2ae^2fg^2-bdeg^2(f+gx)-2bdefg^2+2be^2f^2g-2be^2fg(f+gx)+cd^2g^2(f+gx)+2cdef^2g-2ce^2f^3+2ce^2f^2(f+gx)}{eg\sqrt{f+gx}\,(ef-dg)^2(-dg-e(f+gx)+ef)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)\left(3ae^2g-bdeg-2be^2f-cd^2g+4cdef\right)}{e^{3/2}(ef-dg)^2\sqrt{dg-ef}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*c*e^2*f^3 + 2*c*d*e*f^2*g + 2*b*e^2*f^2*g - 2*b*d*e*f*g^2 - 2*a*e^2*f*g
^2 + 2*a*d*e*g^3 + 2*c*e^2*f^2*(f + g*x) - 2*b*e^2*f*g*(f + g*x) + c*d^2*g^
2*(f + g*x) - b*d*e*g^2*(f + g*x) + 3*a*e^2*g^2*(f + g*x))/(e*g*(e*f - d*g)
^2*Sqrt[f + g*x]*(e*f - d*g - e*(f + g*x))) + ((4*c*d*e*f - 2*b*e^2*f - c*d
^2*g - b*d*e*g + 3*a*e^2*g)*ArcTan[(Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[f + g*x
])/(e*f - d*g)])/(e^(3/2)*(e*f - d*g)^2*Sqrt[-(e*f) + d*g])
```

fricas [B] time = 0.46, size = 1088, normalized size = 6.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((2*(2*c*d^2*e - b*d*e^2)*f^2*g - (c*d^3 + b*d^2*e - 3*a*d*e^2)*f*g^2
+ (2*(2*c*d*e^2 - b*e^3)*f*g^2 - (c*d^2*e + b*d*e^2 - 3*a*e^3)*g^3)*x^2 + (
```

```
2*(2*c*d*e^2 - b*e^3)*f^2*g + 3*(c*d^2*e - b*d*e^2 + a*e^3)*f*g^2 - (c*d^3
+ b*d^2*e - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g
 + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3 - 2*a
*d^2*e^2*g^3 - (c*d^2*e^2 + 3*b*d*e^3 - a*e^4)*f^2*g - (c*d^3*e - 3*b*d^2*e
^2 - a*d*e^3)*f*g^2 + (2*c*e^4*f^3 - 2*(c*d*e^3 + b*e^4)*f^2*g + (c*d^2*e^2
 + b*d*e^3 + 3*a*e^4)*f*g^2 - (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*g^3)*x)*sqr
t(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*
f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2
 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x), -((2*(
2*c*d^2*e - b*d*e^2)*f^2*g - (c*d^3 + b*d^2*e - 3*a*d*e^2)*f*g^2 + (2*(2*c*
d*e^2 - b*e^3)*f*g^2 - (c*d^2*e + b*d*e^2 - 3*a*e^3)*g^3)*x^2 + (2*(2*c*d*e
^2 - b*e^3)*f^2*g + 3*(c*d^2*e - b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + b*d^2*e
- 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(
g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*e^2*g^3 - (c*d^2*e^2 + 3
*b*d*e^3 - a*e^4)*f^2*g - (c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*f*g^2 + (2*c*e^
4*f^3 - 2*(c*d*e^3 + b*e^4)*f^2*g + (c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*f*g^2 -
 (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*
d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^
5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g
^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x)]
```

giac [A] time = 0.23, size = 282, normalized size = 1.71

$$\frac{\left(cd^2g - 4\,cdfe + bdge + 2\,bfe^2 - 3\,age^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(d^2g^2e - 2\,dfge^2 + f^2e^3\right)\sqrt{dge - fe^2}} - \frac{(gx+f)cd^2g^2 + 2\,cdf^2ge - (gx+f)bdg^2e - 2\,bdfg^2e + 2\,adg^3e + 2\,(gx+f)cf^2e^2 - 2\,cf^3e^2 - 2\,(gx+f)bfge^2 + 2\,bf^2ge^2 + 3\,(gx+f)ag^2e^2 - 2\,afg^2e^2}{\left(d^2g^3e - 2\,dfg^2e^2 + f^2ge^3\right)\left(\sqrt{gx+f}\,dg + (gx+f)^{\frac{3}{2}}e - \sqrt{gx+f}\,fe\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] (c*d^2*g - 4*c*d*f*e + b*d*g*e + 2*b*f*e^2 - 3*a*g*e^2)*arctan(sqrt(g*x + f
)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e - 2*d*f*g*e^2 + f^2*e^3)*sqrt(d*g*e -
f*e^2)) - ((g*x + f)*c*d^2*g^2 + 2*c*d*f^2*g*e - (g*x + f)*b*d*g^2*e - 2*b*
d*f*g^2*e + 2*a*d*g^3*e + 2*(g*x + f)*c*f^2*e^2 - 2*c*f^3*e^2 - 2*(g*x + f)
*b*f*g*e^2 + 2*b*f^2*g*e^2 + 3*(g*x + f)*a*g^2*e^2 - 2*a*f*g^2*e^2)/((d^2*g
^3*e - 2*d*f*g^2*e^2 + f^2*g*e^3)*(sqrt(g*x + f)*d*g + (g*x + f)^(3/2)*e -
sqrt(g*x + f)*f*e))
```

maple [B] time = 0.02, size = 418, normalized size = 2.53

$$-\frac{3aeg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2\sqrt{(dg-ef)e}} + \frac{bdg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2\sqrt{(dg-ef)e}} + \frac{2bef\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2\sqrt{(dg-ef)e}} + \frac{c\,d^2g\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2\sqrt{(dg-ef)e}\,e} - \frac{4cdf\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2\sqrt{(dg-ef)e}} - \frac{\sqrt{gx+f}\,aeg}{(dg-ef)^2(egx+dg)} + \frac{\sqrt{gx+f}\,bdg}{(dg-ef)^2(egx+dg)} - \frac{\sqrt{gx+f}\,c\,d^2g}{(dg-ef)^2(egx+dg)e} - \frac{2ag}{(dg-ef)^2\sqrt{gx+f}} + \frac{2bf}{(dg-ef)^2\sqrt{gx+f}} - \frac{2c\,f^2}{(dg-ef)^2\sqrt{gx+f}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x)
```

```
[Out] -1/(d*g-e*f)^2*(g*x+f)^(1/2)/(e*g*x+d*g)*a*e*g+g/(d*g-e*f)^2*(g*x+f)^(1/2)/
(e*g*x+d*g)*b*d-1/(d*g-e*f)^2*(g*x+f)^(1/2)/(e*g*x+d*g)*c*d^2/e*g-3/(d*g-e*
f)^2/((d*g-e*f)*e)^(1/2)*a*e*g*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)+
g/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*
e)*b*d+2/(d*g-e*f)^2*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*
e)^(1/2)*e)*b*f+1/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)*c*d^2/e*g*arctan((g*x+f)^
(1/2)/((d*g-e*f)*e)^(1/2)*e)-4/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)*c*d*f*arctan
((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)-2/(d*g-e*f)^2/(g*x+f)^(1/2)*a*g+2/(d*
g-e*f)^2/(g*x+f)^(1/2)*b*f-2/(d*g-e*f)^2/(g*x+f)^(1/2)*c*f^2/g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 0.30, size = 218, normalized size = 1.32

$$\frac{\operatorname{atan}\left(\frac{\sqrt{f+g\,x}\,\left(d^2\,e\,g^2-2\,d\,e^2\,f\,g+e^3\,f^2\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{5/2}}\right)\,\left(2\,b\,e^2\,f-3\,a\,e^2\,g+c\,d^2\,g+b\,d\,e\,g-4\,c\,d\,e\,f\right)}{e^{3/2}\,{\left(d\,g-e\,f\right)}^{5/2}}-\frac{\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{d\,g-e\,f}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2-b\,d\,e\,g^2+2\,c\,e^2\,f^2-2\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{e\,{\left(d\,g-e\,f\right)}^2}}{\sqrt{f+g\,x}\,\left(d\,g^2-e\,f\,g\right)+e\,g\,{\left(f+g\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*g))/(e^(1/2)*(d*g -
 e*f)^(5/2)))*(2*b*e^2*f - 3*a*e^2*g + c*d^2*g + b*d*e*g - 4*c*d*e*f))/(e^(
3/2)*(d*g - e*f)^(5/2)) - ((2*(a*g^2 + c*f^2 - b*f*g))/(d*g - e*f) + ((f +
g*x)*(3*a*e^2*g^2 + c*d^2*g^2 + 2*c*e^2*f^2 - b*d*e*g^2 - 2*b*e^2*f*g))/(e*
(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(f + g*x)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

3.575 $\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e(d+ex)^2(ef-dg)^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(3eg(5aeg-b(dg+4ef))+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)}{4e^{3/2}(ef-dg)^{7/2}}+\frac{2(}{\sqrt{}}$$

Rubi [A] time = 0.63, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {897, 1259, 456, 453, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(3eg(5aeg-b(dg+4ef))+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)}{4e^{3/2}(ef-dg)^{7/2}}-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e(d+ex)^2(ef-dg)^2}+\frac{2\left(ag^2-bfg+cf^2\right)}{\sqrt{f+gx}(ef-dg)^3}+\frac{\sqrt{f+gx}(cd(8ef-dg)-e(-7aeg+3bdg+4bef))}{4e(d+ex)(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 - b*d*e
 + a*e^2)*Sqrt[f + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((c*d*(8*e*f - d
*g) - e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d
 + e*x)) - ((c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2) + 3*e*g*(5*a*e*g - b*(4*e*
f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(3/2)*(e*
f - d*g)^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} - \frac{g^3\,\text{Subst}\left(\int \frac{\frac{4e^2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^5}-\frac{e\left(3e(bd-ae)g^2+c\left(4e^2f^2-8defg+\cdots\right.\right.}{g^5}\cdots}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\cdots\right)}{2e^2(ef-dg)^2}\cdots$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg-7aeg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} + \cdots$$

$$= \frac{2\left(cf^2-bfg+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg\cdots}{4e(ef-dg)^3(d+\cdots}$$

$$= \frac{2\left(cf^2-bfg+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg\cdots}{4e(ef-dg)^3(d+\cdots}$$

Mathematica [A] time = 1.10, size = 290, normalized size = 1.17

$$\frac{1}{4}\left(-\frac{2\sqrt{f+gx}\left(e(ae-bd)+cd^2\right)}{e(d+ex)^2(ef-dg)^2}+\frac{g\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-7aeg+3bdg+4bef)+cd(dg-8ef))}{e^{3/2}(ef-dg)^{7/2}}+\frac{8\left(g(ag-bf)+cf^2\right)}{\sqrt{f+gx}(ef-dg)^3}-\frac{8\sqrt{e}\left(g(ag-bf)+cf^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{7/2}}-\frac{\sqrt{f+gx}(e(-7aeg+3bdg+4bef)+cd(dg-8ef))}{e(d+ex)(ef-dg)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] ((8*(c*f^2 + g*(-(b*f) + a*g)))/((e*f - d*g)^3*Sqrt[f + g*x]) - (2*(c*d^2 +
 e*(-(b*d) + a*e))*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)^2) - ((c*d*(-8
*e*f + d*g) + e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*Sqrt[f + g*x])/(e*(e*f - d*g
)^3*(d + e*x)) - (8*Sqrt[e]*(c*f^2 + g*(-(b*f) + a*g))*ArcTanh[(Sqrt[e]*Sqr
t[f + g*x])/Sqrt[e*f - d*g]])/(e*f - d*g)^(7/2) + (g*(c*d*(-8*e*f + d*g) +
e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
 d*g]])/(e^(3/2)*(e*f - d*g)^(7/2)))/4
```

IntegrateAlgebraic [B] time = 1.20, size = 497, normalized size = 2.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}\sqrt{dg-ef}}{ef-dg}\right)\left(-15ae^2g^2+3bdeg^2+12be^2fg+cd^2g^2-8cdefg-8ce^2f^2\right)}{4e^{3/2}(ef-dg)^3\sqrt{dg-ef}}+\frac{8ad^2eg^4+25ade^2g^3(f+gx)-16ade^2fg^3+8ae^3f^2g^2-25ae^3fg^2(f+gx)+15ae^3g^2(f+gx)^2-5bd^2eg^3(f+gx)-8bd^2efg^3+16bde^2f^2g^2-15bde^2fg^2(f+gx)-3bde^2g^2(f+gx)^2-8be^3f^3g+20be^3f^2g(f+gx)-12be^3fg(f+gx)^2+cd^3g^3(f+gx)+8cd^2ef^2g^2+7cd^2efg^2(f+gx)-cd^2eg^2(f+gx)^2-16cde^2f^3g+8cde^2f^2g(f+gx)+8cde^2fg(f+gx)^2+8ce^3f^4-16ce^3f^3(f+gx)+8ce^3f^2(f+gx)^2}{4e\sqrt{f+gx}(ef-dg)^3(-dg-e(f+gx)+ef)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (8*c*e^3*f^4 - 16*c*d*e^2*f^3*g - 8*b*e^3*f^3*g + 8*c*d^2*e*f^2*g^2 + 16*b*
d*e^2*f^2*g^2 + 8*a*e^3*f^2*g^2 - 8*b*d^2*e*f*g^3 - 16*a*d*e^2*f*g^3 + 8*a*
d^2*e*g^4 - 16*c*e^3*f^3*(f + g*x) + 8*c*d*e^2*f^2*g*(f + g*x) + 20*b*e^3*f
^2*g*(f + g*x) + 7*c*d^2*e*f*g^2*(f + g*x) - 15*b*d*e^2*f*g^2*(f + g*x) - 2
5*a*e^3*f*g^2*(f + g*x) + c*d^3*g^3*(f + g*x) - 5*b*d^2*e*g^3*(f + g*x) + 2
5*a*d*e^2*g^3*(f + g*x) + 8*c*e^3*f^2*(f + g*x)^2 + 8*c*d*e^2*f*g*(f + g*x)
^2 - 12*b*e^3*f*g*(f + g*x)^2 - c*d^2*e*g^2*(f + g*x)^2 - 3*b*d*e^2*g^2*(f
+ g*x)^2 + 15*a*e^3*g^2*(f + g*x)^2)/(4*e*(e*f - d*g)^3*Sqrt[f + g*x]*(e*f
- d*g - e*(f + g*x))^2) + ((-8*c*e^2*f^2 - 8*c*d*e*f*g + 12*b*e^2*f*g + c*d
^2*g^2 + 3*b*d*e*g^2 - 15*a*e^2*g^2)*ArcTan[(Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqr
t[f + g*x])/(e*f - d*g)])/(4*e^(3/2)*(e*f - d*g)^3*Sqrt[-(e*f) + d*g])
```

fricas [B] time = 0.50, size = 1883, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((8*c*d^2*e^2*f^3 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f^2*g - (c*d^4 + 3*b*
d^3*e - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 4*(2*c*d*e^3 - 3*b*e^4)*f*g^
2 - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 12*(2*c*d*
e^3 - b*e^4)*f^2*g + 3*(5*c*d^2*e^2 - 9*b*d*e^3 + 5*a*e^4)*f*g^2 - 2*(c*d^3
*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*(c*d^2*e^2 -
 b*d*e^3)*f^2*g + 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*f*g^2 - (c*d^4 + 3*
b*d^3*e - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*
g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(8*a*d^3*e^2*g^3 -
2*(7*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^3 + (13*c*d^3*e^2 + 11*b*d^2*e^3 - 11*a
*d*e^4)*f^2*g + (c*d^4*e - 13*b*d^3*e^2 + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 -
 12*b*e^5*f^2*g - 3*(3*c*d^2*e^3 - 3*b*d*e^4 - 5*a*e^5)*f*g^2 + (c*d^3*e^2
+ 3*b*d^2*e^3 - 15*a*d*e^4)*g^3)*x^2 - (4*(6*c*d*e^4 - b*e^5)*f^3 - (19*c*d
^2*e^3 + 17*b*d*e^4 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 4*b*d^2*e^3 - 5*a*d*e
^4)*f*g^2 - (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(
d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6
*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f
*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*
d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d
^2*e^6*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^
6*e^2*g^5)*x), 1/4*((8*c*d^2*e^2*f^3 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f^2*g -
(c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 4*(2*c*d*e^3 -
3*b*e^4)*f*g^2 - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3
 + 12*(2*c*d*e^3 - b*e^4)*f^2*g + 3*(5*c*d^2*e^2 - 9*b*d*e^3 + 5*a*e^4)*f*g
^2 - 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24
*(c*d^2*e^2 - b*d*e^3)*f^2*g + 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*f*g^2
- (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sq
rt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (8*a*d^3*e^2*g^3 - 2*(7*c
*d^2*e^3 - b*d*e^4 - a*e^5)*f^3 + (13*c*d^3*e^2 + 11*b*d^2*e^3 - 11*a*d*e^4
)*f^2*g + (c*d^4*e - 13*b*d^3*e^2 + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 - 12*b*
e^5*f^2*g - 3*(3*c*d^2*e^3 - 3*b*d*e^4 - 5*a*e^5)*f*g^2 + (c*d^3*e^2 + 3*b*
d^2*e^3 - 15*a*d*e^4)*g^3)*x^2 - (4*(6*c*d*e^4 - b*e^5)*f^3 - (19*c*d^2*e^3
 + 17*b*d*e^4 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 4*b*d^2*e^3 - 5*a*d*e^4)*f*
g^2 - (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^
6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f
*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 +
 d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^
5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6
*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*
g^5)*x)]
```

giac [B] time = 0.26, size = 462, normalized size = 1.86

$$\frac{\left(cd^2g^2-8\,cdfge+3\,bdg^2e-8\,cf^2e^2+12\,bfge^2-15\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{4\left(d^3g^3e-3\,d^2fg^2e^2+3\,df^2ge^3-f^3e^4\right)\sqrt{dge-fe^2}}-\frac{2\left(cf^2-bfg+ag^2\right)}{\left(d^3g^3-3\,d^2fg^2e+3\,df^2ge^2-f^3e^3\right)\sqrt{gx+f}}-\frac{\sqrt{gx+f}\,cd^3g^3-\left(gx+f\right)^{\frac{3}{2}}cd^2g^2e+7\,\sqrt{gx+f}\,cd^2fg^2e-5\,\sqrt{gx+f}\,bd^2g^3e+8\left(gx+f\right)^{\frac{3}{2}}cdfge^2-8\,\sqrt{gx+f}\,cdf^2ge^2-3\left(gx+f\right)^{\frac{3}{2}}bdg^2e^2+\sqrt{gx+f}\,bdfg^2e^2+9\,\sqrt{gx+f}\,adg^3e^2-4\left(gx+f\right)^{\frac{3}{2}}bfge^3+4\,\sqrt{gx+f}\,bf^2ge^3+7\left(gx+f\right)^{\frac{3}{2}}ag^2e^3-9\,\sqrt{gx+f}\,afg^2e^3}{4\left(d^3g^3e-3\,d^2fg^2e^2+3\,df^2ge^3-f^3e^4\right)\left(dg+\left(gx+f\right)e-fe\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(c*d^2*g^2 - 8*c*d*f*g*e + 3*b*d*g^2*e - 8*c*f^2*e^2 + 12*b*f*g*e^2 - 1
5*a*g^2*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d^3*g^3*e - 3*d^
2*f*g^2*e^2 + 3*d*f^2*g*e^3 - f^3*e^4)*sqrt(d*g*e - f*e^2)) - 2*(c*f^2 - b*
f*g + a*g^2)/((d^3*g^3 - 3*d^2*f*g^2*e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(g*x
+ f)) - 1/4*(sqrt(g*x + f)*c*d^3*g^3 - (g*x + f)^(3/2)*c*d^2*g^2*e + 7*sqrt
(g*x + f)*c*d^2*f*g^2*e - 5*sqrt(g*x + f)*b*d^2*g^3*e + 8*(g*x + f)^(3/2)*c
*d*f*g*e^2 - 8*sqrt(g*x + f)*c*d*f^2*g*e^2 - 3*(g*x + f)^(3/2)*b*d*g^2*e^2
+ sqrt(g*x + f)*b*d*f*g^2*e^2 + 9*sqrt(g*x + f)*a*d*g^3*e^2 - 4*(g*x + f)^(
3/2)*b*f*g*e^3 + 4*sqrt(g*x + f)*b*f^2*g*e^3 + 7*(g*x + f)^(3/2)*a*g^2*e^3
- 9*sqrt(g*x + f)*a*f*g^2*e^3)/((d^3*g^3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^
3 - f^3*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

maple [B] time = 0.03, size = 847, normalized size = 3.42

$$-\frac{9\sqrt{gx+f}\,adeg^3}{4\left(dg-ef\right)^3\left(egx+dg\right)^2}+\frac{9\sqrt{gx+f}\,ae^2fg^2}{4\left(dg-ef\right)^3\left(egx+dg\right)^2}+\frac{5\sqrt{gx+f}\,bd^2g^3}{4\left(dg-ef\right)^3\left(egx+dg\right)^2}-\frac{\sqrt{gx+f}\,bdefg^2}{4\left(dg-ef\right)^3\left(egx+dg\right)^2}-\frac{\sqrt{gx+f}\,be^2f^2g}{\left(dg-ef\right)^3\left(egx+dg\right)^2}-\frac{\sqrt{gx+f}\,cd^3g^3}{4\left(dg-ef\right)^3\left(egx+dg\right)^2e}-\frac{7\sqrt{gx+f}\,cd^2fg^2}{4\left(dg-ef\right)^3\left(egx+dg\right)^2}+\frac{2\sqrt{gx+f}\,cdef^2g}{\left(dg-ef\right)^3\left(egx+dg\right)^2}-\frac{7\left(gx+f\right)^{\frac{3}{2}}ae^2g^2}{4\left(dg-ef\right)^3\left(egx+dg\right)^2}-\frac{15aeg^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{4\left(dg-ef\right)^3\sqrt{\left(dg-ef\right)e}}+\frac{3\left(gx+f\right)^{\frac{3}{2}}bdeg^2}{4\left(dg-ef\right)^3\left(egx+dg\right)^2}+\frac{3bdg^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{4\left(dg-ef\right)^3\sqrt{\left(dg-ef\right)e}}+\frac{\left(gx+f\right)^{\frac{3}{2}}be^2fg}{\left(dg-ef\right)^3\left(egx+dg\right)^2}+\frac{3befg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^3\sqrt{\left(dg-ef\right)e}}+\frac{cd^2g^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{4\left(dg-ef\right)^3\sqrt{\left(dg-ef\right)e}\,e}+\frac{\left(gx+f\right)^{\frac{3}{2}}cd^2g^2}{4\left(dg-ef\right)^3\left(egx+dg\right)^2}-\frac{2\left(gx+f\right)^{\frac{3}{2}}cdefg}{\left(dg-ef\right)^3\left(egx+dg\right)^2}-\frac{2cdfg\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^3\sqrt{\left(dg-ef\right)e}}-\frac{2cef^2\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^3\sqrt{\left(dg-ef\right)e}}-\frac{2ag^2}{\left(dg-ef\right)^3\sqrt{gx+f}}+\frac{2bfg}{\left(dg-ef\right)^3\sqrt{gx+f}}-\frac{2cf^2}{\left(dg-ef\right)^3\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x)
```

```
[Out] -7/4/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*a*e^2*g^2+3/4/(d*g-e*f)^3/(e*g
*x+d*g)^2*(g*x+f)^(3/2)*b*d*e*g^2+1/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)
*b*e^2*f*g+1/4/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*c*d^2*g^2-2/(d*g-e*f
)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*c*d*e*f*g-9/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3
*e*(g*x+f)^(1/2)*a*d+9/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^2*e^2*(g*x+f)^(1/2)*a*
f+5/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3*(g*x+f)^(1/2)*b*d^2-1/4/(d*g-e*f)^3/(e*
g*x+d*g)^2*g^2*e*(g*x+f)^(1/2)*f*d*b-1/(d*g-e*f)^3/(e*g*x+d*g)^2*g*e^2*(g*x
+f)^(1/2)*b*f^2-1/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3/e*(g*x+f)^(1/2)*c*d^3-7/4
/(d*g-e*f)^3/(e*g*x+d*g)^2*g^2*(g*x+f)^(1/2)*c*d^2*f+2/(d*g-e*f)^3/(e*g*x+d
*g)^2*g*e*(g*x+f)^(1/2)*c*d*f^2-15/4/(d*g-e*f)^3*e/((d*g-e*f)*e)^(1/2)*arct
an((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*a*g^2+3/4/(d*g-e*f)^3/((d*g-e*f)*e)
^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*b*d*g^2+3/(d*g-e*f)^3*e/
((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e)*b*f*g+1/4/(
d*g-e*f)^3/e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)*e
)*c*d^2*g^2-2/(d*g-e*f)^3/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(1/2)/((d*g-e*
f)*e)^(1/2)*e)*c*d*f*g-2/(d*g-e*f)^3*e/((d*g-e*f)*e)^(1/2)*arctan((g*x+f)^(
1/2)/((d*g-e*f)*e)^(1/2)*e)*c*f^2-2/(d*g-e*f)^3/(g*x+f)^(1/2)*a*g^2+2/(d*g-
e*f)^3/(g*x+f)^(1/2)*b*f*g-2/(d*g-e*f)^3/(g*x+f)^(1/2)*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f positive or negative?
```

mupad [B] time = 3.41, size = 363, normalized size = 1.46

$$\frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g-3\,b\,d\,e\,g^2+8\,c\,e^2\,f^2-12\,b\,e^2\,f\,g+15\,a\,e^2\,g^2\right)}{4\,e^{3/2}\,{\left(d\,g-e\,f\right)}^{7/2}}-\frac{\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{d\,g-e\,f}+\frac{{\left(f+g\,x\right)}^2\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g-3\,b\,d\,e\,g^2+8\,c\,e^2\,f^2-12\,b\,e^2\,f\,g+15\,a\,e^2\,g^2\right)}{4\,{\left(d\,g-e\,f\right)}^3}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2+8\,c\,d\,e\,f\,g-5\,b\,d\,e\,g^2+16\,c\,e^2\,f^2-20\,b\,e^2\,f\,g+25\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^{5/2}-{\left(f+g\,x\right)}^{3/2}\,\left(2\,e^2\,f-2\,d\,e\,g\right)+\sqrt{f+g\,x}\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^2
*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 -
 3*b*d*e*g^2 - 12*b*e^2*f*g + 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)) -
 ((2*(a*g^2 + c*f^2 - b*f*g))/(d*g - e*f) + ((f + g*x)^2*(15*a*e^2*g^2 - c*
d^2*g^2 + 8*c*e^2*f^2 - 3*b*d*e*g^2 - 12*b*e^2*f*g + 8*c*d*e*f*g))/(4*(d*g
- e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f^2 - 5*b*d*e*g
^2 - 20*b*e^2*f*g + 8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x)^(5/2)
 - (f + g*x)^(3/2)*(2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 + e^2*f^2
 - 2*d*e*f*g))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

3.576 $\int \frac{\sqrt{-1+x}\,\sqrt{1+x}}{1+x-x^2}\,dx$

Optimal. Leaf size=91

$$\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{\sqrt{5}-2}\,\sqrt{x-1}}\right)-\cosh^{-1}(x)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2+\sqrt{5}}\,\sqrt{x-1}}\right)$$

Rubi [B] time = 0.14, antiderivative size = 191, normalized size of antiderivative = 2.10, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {901, 991, 217, 206, 1034, 725, 204}

$$\frac{\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\sqrt{x-1}\,\sqrt{x+1}\,\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\,\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}-\frac{\sqrt{x-1}\,\sqrt{x+1}\,\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}-\frac{\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\sqrt{x-1}\,\sqrt{x+1}\,\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[5])/10]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTan[(2 - (1 - Sqrt[5])
*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2] - (Sqrt[-1 + x
]*Sqrt[1 + x]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^2] - (Sqrt[(1 + Sqrt[5
])/10]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 +
Sqrt[5])]*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 991

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*
```

```
e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f},
x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx &= \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{\sqrt{-1+x^2}}{1+x-x^2}\,dx}{\sqrt{-1+x^2}}\\
&= -\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^2}}+\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{x}{\left(1+x-x^2\right)\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^2}}\\
&= -\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}}+\frac{\left(\left(5-\sqrt{5}\right)\sqrt{-1+x}\sqrt{1+x}\right)\int \cdots}{5\sqrt{-1+x^2}}\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}}-\frac{\left(\left(5-\sqrt{5}\right)\sqrt{-1+x}\sqrt{1+x}\right)\mathrm{Subst}\left(\int \frac{1}{-4+(1-\cdots}\cdots\right)}{5\sqrt{-1+x^2}}\\
&= \frac{\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\sqrt{-1+x}\sqrt{1+x}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}}-\frac{\sqrt{-1+x}\sqrt{1+x}\tanh\cdots}{\sqrt{-1+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 113, normalized size = 1.24

$$-\frac{1}{5}\sqrt{\sqrt{5}-2}\left(5+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\sqrt{5}-2}\sqrt{\frac{x-1}{x+1}}\right)-2\tanh^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{1}{5}\left(\sqrt{5}-5\right)\sqrt{2+\sqrt{5}}\tanh^{-1}\left(\sqrt{2+\sqrt{5}}\sqrt{\frac{x-1}{x+1}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] -1/5*(Sqrt[-2 + Sqrt[5]]*(5 + Sqrt[5])*ArcTan[Sqrt[-2 + Sqrt[5]]*Sqrt[(-1 +
 x)/(1 + x)]]) - 2*ArcTanh[Sqrt[(-1 + x)/(1 + x)]] - ((-5 + Sqrt[5])*Sqrt[2
 + Sqrt[5]]*ArcTanh[Sqrt[2 + Sqrt[5]]*Sqrt[(-1 + x)/(1 + x)]])/5
```

IntegrateAlgebraic [A] time = 0.31, size = 110, normalized size = 1.21

$$-\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\tan^{-1}\left(\frac{\sqrt{\sqrt{5}-2}\sqrt{x-1}}{\sqrt{x+1}}\right)-2\tanh^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{x+1}}\right)+\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{x-1}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] -(Sqrt[(-2 + 2*Sqrt[5])/5]*ArcTan[(Sqrt[-2 + Sqrt[5]]*Sqrt[-1 + x])/Sqrt[1
+ x]]) - 2*ArcTanh[Sqrt[-1 + x]/Sqrt[1 + x]] + Sqrt[(2 + 2*Sqrt[5])/5]*ArcT
anh[(Sqrt[2 + Sqrt[5]]*Sqrt[-1 + x])/Sqrt[1 + x]]
```

fricas [B] time = 0.43, size = 214, normalized size = 2.35

$$\frac{2}{5}\sqrt{5}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{1}{8}\sqrt{-4\left(2x+\sqrt{5}-1\right)\sqrt{x+1}\sqrt{x-1}+8x^2+4\sqrt{5}x-4x}\sqrt{2\sqrt{5}-2}\left(\sqrt{5}+1\right)-\frac{1}{4}\left(\sqrt{x+1}\sqrt{x-1}\left(\sqrt{5}+1\right)-\sqrt{5}x-x-2\right)\sqrt{2\sqrt{5}-2}\right)+\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(2\sqrt{x+1}\sqrt{x-1}-2x+\sqrt{5}+\sqrt{2\sqrt{5}+2}+1\right)-\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(2\sqrt{x+1}\sqrt{x-1}-2x+\sqrt{5}-\sqrt{2\sqrt{5}+2}+1\right)+\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(2*sqrt(5) - 2)*arctan(1/8*sqrt(-4*(2*x + sqrt(5) - 1)*sqrt
(x + 1)*sqrt(x - 1) + 8*x^2 + 4*sqrt(5)*x - 4*x)*sqrt(2*sqrt(5) - 2)*(sqrt(
5) + 1) - 1/4*(sqrt(x + 1)*sqrt(x - 1)*(sqrt(5) + 1) - sqrt(5)*x - x - 2)*s
qrt(2*sqrt(5) - 2)) + 1/10*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(2*sqrt(x + 1)*sq
rt(x - 1) - 2*x + sqrt(5) + sqrt(2*sqrt(5) + 2) + 1) - 1/10*sqrt(5)*sqrt(2*
sqrt(5) + 2)*log(2*sqrt(x + 1)*sqrt(x - 1) - 2*x + sqrt(5) - sqrt(2*sqrt(5)
 + 2) + 1) + log(sqrt(x + 1)*sqrt(x - 1) - x)
```

giac [A] time = 0.20, size = 16, normalized size = 0.18

$$\log\left(\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="giac")
```

```
[Out] log((sqrt(x + 1) - sqrt(x - 1))^2)
```

maple [B] time = 0.09, size = 231, normalized size = 2.54

$$-\frac{\sqrt{x-1}\,\sqrt{x+1}\,\sqrt{5}\left(-\sqrt{5}\,\sqrt{-2+2\sqrt{5}}\,\operatorname{arctanh}\left(\frac{\sqrt{5}\,x+x-2}{\sqrt{2\sqrt{5}+2}\,\sqrt{x^2-1}}\right)-\sqrt{-2+2\sqrt{5}}\,\operatorname{arctanh}\left(\frac{\sqrt{5}\,x+x-2}{\sqrt{2\sqrt{5}+2}\,\sqrt{x^2-1}}\right)-\sqrt{5}\,\sqrt{2\sqrt{5}+2}\,\arctan\left(\frac{\sqrt{5}\,x-x+2}{\sqrt{-2+2\sqrt{5}}\,\sqrt{x^2-1}}\right)+\sqrt{2\sqrt{5}+2}\,\arctan\left(\frac{\sqrt{5}\,x-x+2}{\sqrt{-2+2\sqrt{5}}\,\sqrt{x^2-1}}\right)+\sqrt{5}\,\sqrt{2\sqrt{5}+2}\,\sqrt{-2+2\sqrt{5}}\,\ln\left(x+\sqrt{x^2-1}\right)\right)}{5\sqrt{x^2-1}\,\sqrt{2\sqrt{5}+2}\,\sqrt{-2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)*(x+1)^(1/2)/(-x^2+x+1),x)
```

```
[Out] -1/5*(x-1)^(1/2)*(x+1)^(1/2)*5^(1/2)*(5^(1/2)*ln(x+(x^2-1)^(1/2))*(2*5^(1/2
)+2)^(1/2)*(-2+2*5^(1/2))^(1/2)-5^(1/2)*arctan((x*5^(1/2)-x+2)/(-2+2*5^(1/2
))^(1/2)/(x^2-1)^(1/2))*(2*5^(1/2)+2)^(1/2)-5^(1/2)*arctanh((x*5^(1/2)+x-2)
/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))*(-2+2*5^(1/2))^(1/2)+arctan((x*5^(1/2)-
x+2)/(-2+2*5^(1/2))^(1/2)/(x^2-1)^(1/2))*(2*5^(1/2)+2)^(1/2)-arctanh((x*5^(
1/2)+x-2)/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))*(-2+2*5^(1/2))^(1/2))/(x^2-1)^
(1/2)/(2*5^(1/2)+2)^(1/2)/(-2+2*5^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{x+1}\,\sqrt{x-1}}{x^2-x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x + 1)*sqrt(x - 1)/(x^2 - x - 1), x)
```

mupad [B] time = 5.02, size = 916, normalized size = 10.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^(1/2)*(x + 1)^(1/2))/(x - x^2 + 1),x)
```

```
[Out] - 4*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) - (10^(1/2)*atan((34083
70*10^(1/2)*(5^(1/2) + 1)^(1/2) - 10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2
)*300730i - 3408370*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) - 1771398*5^
(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2) + 7836865*10^(1/2)*x*(5^(1/2) + 1)^(1/2)
 + 3066340*10^(1/2)*x^2*(5^(1/2) + 1)^(1/2) - 1294942*5^(1/2)*10^(1/2)*x^2*
(5^(1/2) + 1)^(1/2) + 10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1
/2)*300730i - 5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*134482i +
1771398*5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) - 10^(1/2)*x*(5^
(1/2) + 1)^(1/2)*(x - 1)^(1/2)*300730i - 6132680*10^(1/2)*x*(5^(1/2) + 1)^(
1/2)*(x + 1)^(1/2) - 3475583*5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2) + 5^(1/
2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*134482i + 10^(1
/2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*150365i - 5^(1/2)*10^
(1/2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*134482i + 2589884*5^(1/2)*10^(1/2
)*x*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) + 5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1
/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*67241i)/(29119280*x - 24066900*x*(x + 1)^(1
/2) - 11518800*5^(1/2)*x - 10104760*(x + 1)^(1/2) - 7067880*5^(1/2) - 39924
30*5^(1/2)*x^2 + 12033450*x^2 + 7067880*5^(1/2)*(x + 1)^(1/2) + 7984860*5^(
1/2)*x*(x + 1)^(1/2) + 10104760))*(5^(1/2) + 1)^(1/2)*1i)/5 - (10^(1/2)*ata
n((3408370*10^(1/2)*(1 - 5^(1/2))^(1/2) + 3066340*10^(1/2)*x^2*(1 - 5^(1/2)
)^(1/2) - 10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*300730i - 3408370*10^(
1/2)*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) + 1771398*5^(1/2)*10^(1/2)*(1 - 5^(1
/2))^(1/2) + 7836865*10^(1/2)*x*(1 - 5^(1/2))^(1/2) + 3475583*5^(1/2)*10^(1
/2)*x*(1 - 5^(1/2))^(1/2) + 1294942*5^(1/2)*10^(1/2)*x^2*(1 - 5^(1/2))^(1/2
) + 10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*300730i + 5^(1
/2)*10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*134482i - 1771398*5^(1/2)*10
^(1/2)*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) - 10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(
x - 1)^(1/2)*300730i - 6132680*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2)
 - 5^(1/2)*10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*134482i
 + 10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*150365i + 5^(
1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*134482i - 2589884*5^(1/2)
*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) - 5^(1/2)*10^(1/2)*x*(1 - 5^(
1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*67241i)/(29119280*x - 24066900*x*(x
 + 1)^(1/2) + 11518800*5^(1/2)*x - 10104760*(x + 1)^(1/2) + 7067880*5^(1/2)
 + 3992430*5^(1/2)*x^2 + 12033450*x^2 - 7067880*5^(1/2)*(x + 1)^(1/2) - 798
4860*5^(1/2)*x*(x + 1)^(1/2) + 10104760))*(1 - 5^(1/2))^(1/2)*1i)/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x-1}\,\sqrt{x+1}}{x^2-x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)*(1+x)**(1/2)/(-x**2+x+1),x)
```

```
[Out] -Integral(sqrt(x - 1)*sqrt(x + 1)/(x**2 - x - 1), x)
```

3.577 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$

Optimal. Leaf size=164

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,(-4beg+5cdg+3c}{4e^2g^2}$$

Rubi [A] time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {951, 80, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,(-4beg+5cdg+3cef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) +
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g +
3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
```

```
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}e(3cef+5cdg-4beg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2de\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2de\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2de\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2de\right.\right.}{}$$

Mathematica [A] time = 0.77, size = 173, normalized size = 1.05

$$\frac{\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)+e\sqrt{g}\sqrt{d+ex}(f+gx)(4beg+c(-3dg-3ef+2egx))}{4e^3g^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x))
 + Sqrt[e*f - d*g]*(c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g
- b*(e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d +
 e*x])/Sqrt[e*f - d*g]])/(4*e^3*g^(5/2)*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.39, size = 229, normalized size = 1.40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)\left(8ae^2g^2-4bdeg^2-4be^2fg+3cd^2g^2+2cdefg+3ce^2f^2\right)}{4e^{5/2}g^{5/2}}+\frac{\sqrt{f+gx}(ef-dg)\left(\frac{4be^2g(f+gx)}{d+ex}-4beg^2-\frac{3ce^2f(f+gx)}{d+ex}-\frac{5cdeg(f+gx)}{d+ex}+3cdg^2+5cefg\right)}{4e^2g^2\sqrt{d+ex}\left(\frac{e(f+gx)}{d+ex}-g\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] ((e*f - d*g)*Sqrt[f + g*x]*(5*c*e*f*g + 3*c*d*g^2 - 4*b*e*g^2 - (3*c*e^2*f*
(f + g*x))/(d + e*x) - (5*c*d*e*g*(f + g*x))/(d + e*x) + (4*b*e^2*g*(f + g*
x))/(d + e*x)))/(4*e^2*g^2*Sqrt[d + e*x]*(-g + (e*(f + g*x))/(d + e*x))^2)
+ ((3*c*e^2*f^2 + 2*c*d*e*f*g - 4*b*e^2*f*g + 3*c*d^2*g^2 - 4*b*d*e*g^2 + 8
*a*e^2*g^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^
(5/2)*g^(5/2))
```

fricas [A] time = 0.50, size = 380, normalized size = 2.32

$$\left[\frac{\left(3ce^2f^2+2\left(cde-2be^2\right)fg+\left(3cd^2-4bde+8ae^2\right)g^2\right)\sqrt{eg}\log\left(8e^2g^2x^2+e^2f^2+6defg+d^2g^2+4\left(2egx+ef+dg\right)\sqrt{eg}\sqrt{ex+d}\sqrt{gx+f}+8\left(e^2fg+deg^2\right)x\right)+4\left(2ce^2g^2x-3ce^2fg-\left(3cde-4be^2\right)g^2\right)\sqrt{ex+d}\sqrt{gx+f}}{16e^3g^3}, -\frac{\left(3ce^2f^2+2\left(cde-2be^2\right)fg+\left(3cd^2-4bde+8ae^2\right)g^2\right)\sqrt{-eg}\arctan\left(\frac{\left(2egx+ef+dg\right)\sqrt{-eg}\sqrt{ex+d}\sqrt{gx+f}}{2\left(e^2g^2x^2+defg+\left(e^2fg+deg^2\right)x\right)}\right)-2\left(2ce^2g^2x-3ce^2fg-\left(3cde-4be^2\right)g^2\right)\sqrt{ex+d}\sqrt{gx+f}}{8e^3g^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^
2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*
e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e
*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^2)*g^2)*sqrt(e
*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*
f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e
*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (
e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^
2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

giac [A] time = 0.26, size = 179, normalized size = 1.09

$$\frac{1}{4}\sqrt{(xe+d)ge-dge+fe^2}\,\sqrt{xe+d}\left(\frac{2\,(xe+d)ce^{(-3)}}{g}-\frac{\left(5\,cdg^2e^5+3\,cfge^6-4\,bg^2e^6\right)e^{(-8)}}{g^3}\right)-\frac{\left(3\,cd^2g^2+2\,cdfge-4\,bdg^2e+3\,cf^2e^2-4\,bfge^2+8\,ag^2e^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-\sqrt{xe+d}\,\sqrt{g}\,e^{\frac{1}{2}}+\sqrt{(xe+d)ge-dge+fe^2}\right|\right)}{4\,g^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*(2*(x*e + d)*c*e^(-3)
/g - (5*c*d*g^2*e^5 + 3*c*f*g*e^6 - 4*b*g^2*e^6)*e^(-8)/g^3) - 1/4*(3*c*d^2
*g^2 + 2*c*d*f*g*e - 4*b*d*g^2*e + 3*c*f^2*e^2 - 4*b*f*g*e^2 + 8*a*g^2*e^2)
*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e^(1/2) + sqrt((x*e + d)*g*e - d*g
*e + f*e^2)))/g^(5/2)
```

maple [B] time = 0.03, size = 425, normalized size = 2.59

$$\frac{\left(8ae^2g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-4bdeg^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-4be^2fg\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+3cd^2g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+2cdefg\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+3ce^2f^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+4\sqrt{eg}\,\sqrt{(ex+d)(gx+f)}\,cegx+8\sqrt{eg}\,\sqrt{(ex+d)(gx+f)}\,beg-6\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,cdg-6\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,cef\right)\sqrt{ex+d}\,\sqrt{gx+f}}{8\sqrt{eg}\,\sqrt{(ex+d)(gx+f)}\,e^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(8*a*e^2*g^2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1
/2))/(e*g)^(1/2))-4*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)
^(1/2))/(e*g)^(1/2))*b*d*e*g^2-4*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f)
)^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*e^2*f*g+3*c*d^2*g^2*ln(1/2*(2*e*g*x+d*g
+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))+2*c*d*e*f*g*ln(1/2
*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))+3*c*e
^2*f^2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)
^(1/2))+4*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e*g*x+8*(e*g)^(1/2)*((e*x+d
)*(g*x+f))^(1/2)*b*e*g-6*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*d*g-6*((e*x+
d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2
)/g^2/e^2/((e*x+d)*(g*x+f))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [B] time = 22.38, size = 833, normalized size = 5.08

$$\frac{\frac{(2bdg+2bef)\left(\sqrt{d+ex}-\sqrt{d}\right)}{g^3\left(\sqrt{f+gx}-\sqrt{f}\right)}+\frac{(2bdg+2bef)\left(\sqrt{d+ex}-\sqrt{d}\right)^3}{e\,g^2\left(\sqrt{f+gx}-\sqrt{f}\right)^3}-\frac{8b\sqrt{d}\sqrt{f}\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g^2\left(\sqrt{f+gx}-\sqrt{f}\right)^2}}{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{\left(\sqrt{f+gx}-\sqrt{f}\right)^4}+\frac{e^2}{g^2}-\frac{2e\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g\left(\sqrt{f+gx}-\sqrt{f}\right)^2}}-\frac{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)\left(\frac{3ce^3f^2}{2}+\frac{3cd^2eg^2}{2}+cde^2fg\right)}{g^6\left(\sqrt{f+gx}-\sqrt{f}\right)}-\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^3\left(\frac{11cd^2g^2}{2}+\frac{11ce^2f^2}{2}+25cdefg\right)}{g^5\left(\sqrt{f+gx}-\sqrt{f}\right)^3}+\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^7\left(\frac{3cd^2g^2}{2}+\frac{3ce^2f^2}{2}+cdefg\right)}{e^2g^3\left(\sqrt{f+gx}-\sqrt{f}\right)^7}-\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^5\left(\frac{11cd^2g^2}{2}+\frac{11ce^2f^2}{2}+25cdefg\right)}{e\,g^4\left(\sqrt{f+gx}-\sqrt{f}\right)^5}+\frac{\sqrt{d}\sqrt{f}\,(32cdg+32cef)\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{g^4\left(\sqrt{f+gx}-\sqrt{f}\right)^4}}{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^8}{\left(\sqrt{f+gx}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4e\left(\sqrt{d+ex}-\sqrt{d}\right)^6}{g\left(\sqrt{f+gx}-\sqrt{f}\right)^6}-\frac{4e^3\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g^3\left(\sqrt{f+gx}-\sqrt{f}\right)^2}+\frac{6e^2\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{g^2\left(\sqrt{f+gx}-\sqrt{f}\right)^4}}-\frac{4a\,\mathrm{atan}\left(\frac{e\left(\sqrt{f+gx}-\sqrt{f}\right)}{\sqrt{-eg}\left(\sqrt{d+ex}-\sqrt{d}\right)}\right)}{\sqrt{-eg}}-\frac{2b\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+gx}-\sqrt{f}\right)}\right)(dg+ef)}{e^{3/2}g^{3/2}}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+gx}-\sqrt{f}\right)}\right)\left(3d^2g^2+2defg+3e^2f^2\right)}{2e^{5/2}g^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*((f + g*x)^(1/2) -
f^(1/2))) + ((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(e*g^2*((f
+ g*x)^(1/2) - f^(1/2))^3) - (8*b*d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2
))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))^4/(
(f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2))^2
)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*
e^3*f^2)/2 + (3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1
/2))) - (((d + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2
 + 25*c*d*e*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) -
 d^(1/2))^7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f +
 g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/
2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5)
 + (d^(1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g
^4*((f + g*x)^(1/2) - f^(1/2))^4))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x
)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f
 + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((
f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*(
(f + g*x)^(1/2) - f^(1/2))^4)) - (4*a*atan((e*((f + g*x)^(1/2) - f^(1/2)))/
((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(-e*g)^(1/2) - (2*b*atanh((g^(
1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d
*g + e*f))/(e^(3/2)*g^(3/2)) + (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)
))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*
g))/(2*e^(5/2)*g^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{\sqrt{d + ex}\,\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.578 $\int \frac{(d+ex)^{3/2}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=333

$$-\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,(ef-dg)\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)}{64e^2g^4}+\frac{(d+ex)^{3/2}\sqrt{f+gx}\,\left(8eg\right.}{}$$

Rubi [A] time = 0.35, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{(d+ex)^{3/2}\sqrt{f+gx}\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)}{96e^2g^3}-\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,(ef-dg)\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)}{64e^2g^4}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)}{64e^{5/2}g^{9/2}}-\frac{(d+ex)^{5/2}\sqrt{f+gx}\,(-8beg+9cdg+7cef)}{24e^2g^2}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] -((e*f - d*g)*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*g - b
*(5*e*f + d*g)))*Sqrt[d + e*x]*Sqrt[f + g*x])/(64*e^2*g^4) + ((c*(35*e^2*f^
2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*g - b*(5*e*f + d*g)))*(d + e*x)^
(3/2)*Sqrt[f + g*x])/(96*e^2*g^3) - ((7*c*e*f + 9*c*d*g - 8*b*e*g)*(d + e*x
)^(5/2)*Sqrt[f + g*x])/(24*e^2*g^2) + (c*(d + e*x)^(7/2)*Sqrt[f + g*x])/(4*
e^2*g) + ((e*f - d*g)^2*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6
*a*e*g - b*(5*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f
+ g*x])])/(64*e^(5/2)*g^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} + \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(8ae^2g-cd(7ef+dg)\right)-\frac{1}{2}e(7cef+9cdg-8beg)x\right)}{\sqrt{f+gx}}\,dx}{4e^2g}$$

$$= -\frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2} + \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} + \frac{\left(c\left(35\right.\right.}{}$$

$$= \frac{\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f+g}}{96e^2g^3}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+}}{64e^2g^4}$$

Mathematica [A] time = 1.54, size = 313, normalized size = 0.94

$$\frac{3(ef-dg)^{5/2}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)-e\sqrt{g}\sqrt{d+ex}(f+gx)\left(c\left(9d^3g^3+3d^2eg^2(5f-2gx)+de^2g\left(-145f^2+92fgx-72g^2x^2\right)+e^3\left(105f^3-70f^2gx+56fg^2x^2-48g^3x^3\right)\right)-8eg\left(6aeg(5dg-3ef+2egx)+b\left(3d^2g^2+2deg(7gx-11f)+e^2\left(15f^2-10fgx+8g^2x^2\right)\right)\right)\right)}{192e^3g^{9/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-(e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(c*(9*d^3*g^3 + 3*d^2*e*g^2*(5*f - 2*g
*x) + d*e^2*g*(-145*f^2 + 92*f*g*x - 72*g^2*x^2) + e^3*(105*f^3 - 70*f^2*g*
x + 56*f*g^2*x^2 - 48*g^3*x^3)) - 8*e*g*(6*a*e*g*(-3*e*f + 5*d*g + 2*e*g*x)
 + b*(3*d^2*g^2 + 2*d*e*g*(-11*f + 7*g*x) + e^2*(15*f^2 - 10*f*g*x + 8*g^2*
x^2))))) + 3*(e*f - d*g)^(5/2)*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8
*e*g*(6*a*e*g - b*(5*e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(
Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(192*e^3*g^(9/2)*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.73, size = 643, normalized size = 1.93

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] -1/192*((e*f - d*g)^2*Sqrt[f + g*x]*(-279*c*e^2*f^2*g^3 + 30*c*d*e*f*g^4 +
264*b*e^2*f*g^4 + 9*c*d^2*g^5 - 24*b*d*e*g^5 - 240*a*e^2*g^5 + (511*c*e^3*f
^2*g^2*(f + g*x))/(d + e*x) + (146*c*d*e^2*f*g^3*(f + g*x))/(d + e*x) - (58
4*b*e^3*f*g^3*(f + g*x))/(d + e*x) - (33*c*d^2*e*g^4*(f + g*x))/(d + e*x) -
 (40*b*d*e^2*g^4*(f + g*x))/(d + e*x) + (624*a*e^3*g^4*(f + g*x))/(d + e*x)
 - (385*c*e^4*f^2*g*(f + g*x)^2)/(d + e*x)^2 - (110*c*d*e^3*f*g^2*(f + g*x)
^2)/(d + e*x)^2 + (440*b*e^4*f*g^2*(f + g*x)^2)/(d + e*x)^2 - (33*c*d^2*e^2
*g^3*(f + g*x)^2)/(d + e*x)^2 + (88*b*d*e^3*g^3*(f + g*x)^2)/(d + e*x)^2 -
(528*a*e^4*g^3*(f + g*x)^2)/(d + e*x)^2 + (105*c*e^5*f^2*(f + g*x)^3)/(d +
e*x)^3 + (30*c*d*e^4*f*g*(f + g*x)^3)/(d + e*x)^3 - (120*b*e^5*f*g*(f + g*x
)^3)/(d + e*x)^3 + (9*c*d^2*e^3*g^2*(f + g*x)^3)/(d + e*x)^3 - (24*b*d*e^4*
g^2*(f + g*x)^3)/(d + e*x)^3 + (144*a*e^5*g^2*(f + g*x)^3)/(d + e*x)^3))/(e
^2*g^4*Sqrt[d + e*x]*(-g + (e*(f + g*x))/(d + e*x))^4) + ((e*f - d*g)^2*(35
*c*e^2*f^2 + 10*c*d*e*f*g - 40*b*e^2*f*g + 3*c*d^2*g^2 - 8*b*d*e*g^2 + 48*a
*e^2*g^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(64*e^(
5/2)*g^(9/2))
```

fricas [A] time = 0.66, size = 852, normalized size = 2.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*c*e^4*f^4 - 20*(3*c*d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2 +
 12*b*d*e^3 + 8*a*e^4)*f^2*g^2 + 4*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g
^3 + (3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*g^4)*sqrt(e*g)*log(8*e^2*g^2*x^2
+ e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*
x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(48*c*e^4*g^4*x^3 - 105
*c*e^4*f^3*g + 5*(29*c*d*e^3 + 24*b*e^4)*f^2*g^2 - (15*c*d^2*e^2 + 176*b*d*
e^3 + 144*a*e^4)*f*g^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 - 80*a*d*e^3)*g^4 - 8*(
7*c*e^4*f*g^3 - (9*c*d*e^3 + 8*b*e^4)*g^4)*x^2 + 2*(35*c*e^4*f^2*g^2 - 2*(2
3*c*d*e^3 + 20*b*e^4)*f*g^3 + (3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*g^4)*x)
*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^5), -1/384*(3*(35*c*e^4*f^4 - 20*(3*c*
d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2 + 12*b*d*e^3 + 8*a*e^4)*f^2*g^2 + 4
*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g^3 + (3*c*d^4 - 8*b*d^3*e + 48*a*d
^2*e^2)*g^4)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*
x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(
48*c*e^4*g^4*x^3 - 105*c*e^4*f^3*g + 5*(29*c*d*e^3 + 24*b*e^4)*f^2*g^2 - (1
5*c*d^2*e^2 + 176*b*d*e^3 + 144*a*e^4)*f*g^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 -
 80*a*d*e^3)*g^4 - 8*(7*c*e^4*f*g^3 - (9*c*d*e^3 + 8*b*e^4)*g^4)*x^2 + 2*(3
5*c*e^4*f^2*g^2 - 2*(23*c*d*e^3 + 20*b*e^4)*f*g^3 + (3*c*d^2*e^2 + 56*b*d*e
^3 + 48*a*e^4)*g^4)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^5)]
```

giac [A] time = 0.42, size = 448, normalized size = 1.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*(2*(4*(x*e + d)*(6*(x*e + d)*c*e^
(-3)/g - (9*c*d*g^6*e^6 + 7*c*f*g^5*e^7 - 8*b*g^6*e^7)*e^(-9)/g^7) + (3*c*d
^2*g^6*e^6 + 10*c*d*f*g^5*e^7 - 8*b*d*g^6*e^7 + 35*c*f^2*g^4*e^8 - 40*b*f*g
^5*e^8 + 48*a*g^6*e^8)*e^(-9)/g^7)*(x*e + d) + 3*(3*c*d^3*g^6*e^6 + 7*c*d^2
*f*g^5*e^7 - 8*b*d^2*g^6*e^7 + 25*c*d*f^2*g^4*e^8 - 32*b*d*f*g^5*e^8 + 48*a
*d*g^6*e^8 - 35*c*f^3*g^3*e^9 + 40*b*f^2*g^4*e^9 - 48*a*f*g^5*e^9)*e^(-9)/g
^7)*sqrt(x*e + d) - 1/64*(3*c*d^4*g^4 + 4*c*d^3*f*g^3*e - 8*b*d^3*g^4*e + 1
8*c*d^2*f^2*g^2*e^2 - 24*b*d^2*f*g^3*e^2 + 48*a*d^2*g^4*e^2 - 60*c*d*f^3*g*
e^3 + 72*b*d*f^2*g^2*e^3 - 96*a*d*f*g^3*e^3 + 35*c*f^4*e^4 - 40*b*f^3*g*e^4
 + 48*a*f^2*g^2*e^4)*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e^(1/2) + sqrt
((x*e + d)*g*e - d*g*e + f*e^2)))/g^(9/2)
```

maple [B] time = 0.03, size = 1207, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/384*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(9*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*
x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^4*g^4+105*ln(1/2*(2*e*g*x+d*g+e*f
+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*e^4*f^4-72*ln(1/2*(2
*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d^2*e^
2*f*g^3-30*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d^2*e*f*g^2+224*(e*g)^(1/2
)*((e*x+d)*(g*x+f))^(1/2)*x*b*d*e^2*g^3-160*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(
1/2)*x*b*e^3*f*g^2+12*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*x*c*d^2*e*g^3+140
*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*x*c*e^3*f^2*g-112*x^2*c*e^3*f*g^2*((e*
x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-352*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*b*d
*e^2*f*g^2+290*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d*e^2*f^2*g+144*x^2*c*
d*e^2*g^3*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+144*ln(1/2*(2*e*g*x+d*g+e*f+2
*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*a*d^2*e^2*g^4-184*(e*g)^
(1/2)*((e*x+d)*(g*x+f))^(1/2)*x*c*d*e^2*f*g^2+144*ln(1/2*(2*e*g*x+d*g+e*f+2
*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*a*e^4*f^2*g^2-120*ln(1/2
*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*e^4
*f^3*g-210*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e^3*f^3-24*ln(1/2*(2*e*g*x
+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d^3*e*g^4-18
*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d^3*g^3+216*ln(1/2*(2*e*g*x+d*g+e*f+
2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d*e^3*f^2*g^2+54*ln(1
/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d
^2*e^2*f^2*g^2-180*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^
(1/2))/(e*g)^(1/2))*c*d*e^3*f^3*g+480*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*a
*d*e^2*g^3-288*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*a*e^3*f*g^2+240*(e*g)^(1
/2)*((e*x+d)*(g*x+f))^(1/2)*b*e^3*f^2*g-288*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x
+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*a*d*e^3*f*g^3+48*(e*g)^(1/2)*(
(e*x+d)*(g*x+f))^(1/2)*b*d^2*e*g^3+192*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*
x*a*e^3*g^3+96*x^3*c*e^3*g^3*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+128*x^2*b*
e^3*g^3*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+12*ln(1/2*(2*e*g*x+d*g+e*f+2*((
e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^3*e*f*g^3)/e^2/g^4/((e*
x+d)*(g*x+f))^(1/2)/(e*g)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}\,\left(c\,x^2+b\,x+a\right)}{\sqrt{f+g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(3/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
[Out] int(((d + e*x)^(3/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
[Out] Timed out
```

3.579 $\int \frac{\sqrt{d+ex}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)}{8e^2g^3}-\frac{(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(2eg(4$$

Rubi [A] time = 0.26, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)}{8e^2g^3}-\frac{(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)}{8e^{5/2}g^{7/2}}-\frac{(d+ex)^{3/2}\sqrt{f+gx}(-6beg+7cdg+5cef)}{12e^2g^2}+\frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] ((c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*
Sqrt[d + e*x]*Sqrt[f + g*x])/(8*e^2*g^3) - ((5*c*e*f + 7*c*d*g - 6*b*e*g)*(
d + e*x)^(3/2)*Sqrt[f + g*x])/(12*e^2*g^2) + (c*(d + e*x)^(5/2)*Sqrt[f + g*
x])/(3*e^2*g) - ((e*f - d*g)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(
4*a*e*g - b*(3*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f
 + g*x])])/(8*e^(5/2)*g^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} + \frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(6ae^2g-cd(5ef+dg)\right)-\frac{1}{2}e(5cef+7cdg-6beg)x\right)}{\sqrt{f+gx}}\,dx}{3e^2g}$$

$$= -\frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} + \frac{\left(c\left(5e^2f^2+\right.\right.}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5ce}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5ce}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5ce}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5ce}{}$$

Mathematica [A] time = 1.01, size = 225, normalized size = 0.91

$$\frac{-e\sqrt{g}\sqrt{d+ex}(f+gx)\left(c\left(3d^2g^2-2deg(gx-2f)+e^2\left(-15f^2+10fgx-8g^2x^2\right)\right)-6eg(4aeg+b(dg-3ef+2egx))\right)-3(ef-dg)^{3/2}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)}{24e^3g^{7/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-(e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(-6*e*g*(4*a*e*g + b*(-3*e*f + d*g + 2
*e*g*x)) + c*(3*d^2*g^2 - 2*d*e*g*(-2*f + g*x) + e^2*(-15*f^2 + 10*f*g*x -
8*g^2*x^2)))) - 3*(e*f - d*g)^(3/2)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) +
2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[
(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(24*e^3*g^(7/2)*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 1.09, size = 357, normalized size = 1.45

$$\frac{\sqrt{d+\frac{e(f+gx)}{g}-\frac{ef}{g}}\left(24ae^2g^2\sqrt{f+gx}+6bdeg^2\sqrt{f+gx}+12be^2g(f+gx)^{3/2}-30be^2fg\sqrt{f+gx}-3cd^2g^2\sqrt{f+gx}+2cdeg(f+gx)^{3/2}-6cdefg\sqrt{f+gx}+33ce^2f^2\sqrt{f+gx}+8ce^2(f+gx)^{5/2}-26ce^2f(f+gx)^{3/2}\right)}{24e^2g^3}+\frac{\sqrt{\frac{e}{g}}\log\left(\sqrt{d+\frac{e(f+gx)}{g}-\frac{ef}{g}}-\sqrt{\frac{e}{g}}\sqrt{f+gx}\right)\left(-8ade^2g^3+8ae^3fg^2+2bd^2eg^3+4bde^2fg^2-6be^3f^2g-cd^3g^3-cd^2efg^2-3cde^2f^2g+5ce^3f^3\right)}{8e^3g^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[d - (e*f)/g + (e*(f + g*x))/g]*(33*c*e^2*f^2*Sqrt[f + g*x] - 6*c*d*e*
f*g*Sqrt[f + g*x] - 30*b*e^2*f*g*Sqrt[f + g*x] - 3*c*d^2*g^2*Sqrt[f + g*x]
+ 6*b*d*e*g^2*Sqrt[f + g*x] + 24*a*e^2*g^2*Sqrt[f + g*x] - 26*c*e^2*f*(f +
g*x)^(3/2) + 2*c*d*e*g*(f + g*x)^(3/2) + 12*b*e^2*g*(f + g*x)^(3/2) + 8*c*e
^2*(f + g*x)^(5/2)))/(24*e^2*g^3) + (Sqrt[e/g]*(5*c*e^3*f^3 - 3*c*d*e^2*f^2
*g - 6*b*e^3*f^2*g - c*d^2*e*f*g^2 + 4*b*d*e^2*f*g^2 + 8*a*e^3*f*g^2 - c*d^
3*g^3 + 2*b*d^2*e*g^3 - 8*a*d*e^2*g^3)*Log[-(Sqrt[e/g]*Sqrt[f + g*x]) + Sqr
t[d - (e*f)/g + (e*(f + g*x))/g]])/(8*e^3*g^3)
```

fricas [A] time = 0.52, size = 576, normalized size = 2.34

$$\left[-\frac{3\left(5ce^3f^3-3\left(cde^2+2be^3\right)f^2g-\left(cd^2e-4bde^2-8ae^3\right)fg^2-\left(cd^3-2bd^2e+8ade^2\right)g^3\right)\sqrt{eg}\log\left(8e^2g^2x^2+e^2f^2+6defg+d^2g^2+4\left(2egx+ef+dg\right)\sqrt{eg}\sqrt{ex+d}\sqrt{gx+f}+8\left(e^2fg+deg^2\right)x\right)-4\left(8ce^3g^3x^2+15ce^3f^2g-2\left(2cde^2+9be^3\right)fg^2-3\left(cd^2e-2bde^2-8ae^3\right)g^3-2\left(5ce^3fg^2-\left(cde^2+6be^3\right)g^3\right)x\right)\sqrt{ex+d}\sqrt{gx+f}}{96e^3g^4}, \ldots\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*c*e^3*f^3 - 3*(c*d*e^2 + 2*b*e^3)*f^2*g - (c*d^2*e - 4*b*d*e^2
 - 8*a*e^3)*f*g^2 - (c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*g^3)*sqrt(e*g)*log(8*e^
2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*
g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) - 4*(8*c*e^3*g^3*
x^2 + 15*c*e^3*f^2*g - 2*(2*c*d*e^2 + 9*b*e^3)*f*g^2 - 3*(c*d^2*e - 2*b*d*e
^2 - 8*a*e^3)*g^3 - 2*(5*c*e^3*f*g^2 - (c*d*e^2 + 6*b*e^3)*g^3)*x)*sqrt(e*x
 + d)*sqrt(g*x + f))/(e^3*g^4), 1/48*(3*(5*c*e^3*f^3 - 3*(c*d*e^2 + 2*b*e^3
)*f^2*g - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)*f*g^2 - (c*d^3 - 2*b*d^2*e + 8*a*
d*e^2)*g^3)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x
 + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) + 2*(8
*c*e^3*g^3*x^2 + 15*c*e^3*f^2*g - 2*(2*c*d*e^2 + 9*b*e^3)*f*g^2 - 3*(c*d^2*
e - 2*b*d*e^2 - 8*a*e^3)*g^3 - 2*(5*c*e^3*f*g^2 - (c*d*e^2 + 6*b*e^3)*g^3)*
x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^4)]
```

giac [A] time = 0.35, size = 291, normalized size = 1.18

$$\frac{1}{24}\sqrt{(xe+d)ge-dge+fe^2}\left(2\left(xe+d\right)\left(\frac{4\left(xe+d\right)ce^{(-3)}}{g}-\frac{\left(7cdg^4e^6+5cfg^3e^7-6bg^4e^7\right)e^{(-9)}}{g^5}\right)+\frac{3\left(cd^2g^4e^6+2cdfg^3e^7-2bdg^4e^7+5cf^2g^2e^8-6bfg^3e^8+8ag^4e^8\right)e^{(-9)}}{g^5}\right)\sqrt{xe+d}-\frac{\left(cd^3g^3+cd^2fg^2e-2bd^2g^3e+3cdf^2ge^2-4bdfg^2e^2+8adg^3e^2-5cf^3e^3+6bf^2ge^3-8afg^2e^3\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-\sqrt{xe+d}\sqrt{g}e^{\frac{1}{2}}+\sqrt{(xe+d)ge-dge+fe^2}\right|\right)}{8g^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*(2*(x*e + d)*(4*(x*e + d)*c*e^(-3)
/g - (7*c*d*g^4*e^6 + 5*c*f*g^3*e^7 - 6*b*g^4*e^7)*e^(-9)/g^5) + 3*(c*d^2*g
^4*e^6 + 2*c*d*f*g^3*e^7 - 2*b*d*g^4*e^7 + 5*c*f^2*g^2*e^8 - 6*b*f*g^3*e^8
+ 8*a*g^4*e^8)*e^(-9)/g^5)*sqrt(x*e + d) - 1/8*(c*d^3*g^3 + c*d^2*f*g^2*e -
 2*b*d^2*g^3*e + 3*c*d*f^2*g*e^2 - 4*b*d*f*g^2*e^2 + 8*a*d*g^3*e^2 - 5*c*f^
3*e^3 + 6*b*f^2*g*e^3 - 8*a*f*g^2*e^3)*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt
(g)*e^(1/2) + sqrt((x*e + d)*g*e - d*g*e + f*e^2)))/g^(7/2)
```

maple [B] time = 0.02, size = 763, normalized size = 3.10

$$\frac{\sqrt{ex+d}\,\sqrt{gx+f}\left(24ade^2g^3\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\sqrt{eg}}{2\sqrt{eg}}\right)-24ae^3fg^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\sqrt{eg}}{2\sqrt{eg}}\right)-\ldots\text{[illegible]}\right)}{48\sqrt{(ex+d)(gx+f)}\sqrt{eg}\,\text{[illegible]}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/48*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(16*x^2*c*e^2*g^2*((e*x+d)*(g*x+f))^(1/2)*
(e*g)^(1/2)+24*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2
))/(e*g)^(1/2))*a*d*e^2*g^3-24*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^
(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*a*e^3*f*g^2-6*ln(1/2*(2*e*g*x+d*g+e*f+2*((e
```

```
*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d^2*e*g^3-12*ln(1/2*(2*e*g
*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d*e^2*f*g^
2+18*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(
1/2))*b*e^3*f^2*g+3*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)
^(1/2))/(e*g)^(1/2))*c*d^3*g^3+3*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f)
)^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^2*e*f*g^2+9*ln(1/2*(2*e*g*x+d*g+e*f+2
*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d*e^2*f^2*g-15*ln(1/2*
(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*e^3*
f^3+24*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*x*b*e^2*g^2+4*(e*g)^(1/2)*((e*x+
d)*(g*x+f))^(1/2)*x*c*d*e*g^2-20*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*x*c*e^
2*f*g+48*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*a*e^2*g^2+12*(e*g)^(1/2)*((e*x
+d)*(g*x+f))^(1/2)*b*d*e*g^2-36*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*b*e^2*f
*g-6*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d^2*g^2-8*(e*g)^(1/2)*((e*x+d)*(
g*x+f))^(1/2)*c*d*e*f*g+30*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e^2*f^2)/g
^3/((e*x+d)*(g*x+f))^(1/2)/e^2/(e*g)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g>0)', see `assume?` for more detai
ls)Is g positive, negative or zero?
```

mupad [B] time = 74.34, size = 1832, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] (((2*a*d*g + 2*a*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(g^2*((f + g*x)^(1/2)
- f^(1/2))^3) + ((2*a*e^2*f + 2*a*d*e*g)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*
((f + g*x)^(1/2) - f^(1/2))) - (8*a*d^(1/2)*e*f^(1/2)*((d + e*x)^(1/2) - d^
(1/2))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))
^4/((f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2
))^2)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((
c*d^3*e^3*g^3)/4 - (5*c*e^6*f^3)/4 + (3*c*d*e^5*f^2*g)/4 + (c*d^2*e^4*f*g^2
)/4))/(g^9*((f + g*x)^(1/2) - f^(1/2))) - (((d + e*x)^(1/2) - d^(1/2))^5*((
33*c*e^4*f^3)/2 + (19*c*d^3*e*g^3)/2 + (313*c*d*e^3*f^2*g)/2 + (275*c*d^2*e
^2*f*g^2)/2))/(g^7*((f + g*x)^(1/2) - f^(1/2))^5) - (((d + e*x)^(1/2) - d^(
1/2))^7*((19*c*d^3*g^3)/2 + (33*c*e^3*f^3)/2 + (313*c*d*e^2*f^2*g)/2 + (275
*c*d^2*e*f*g^2)/2))/(g^6*((f + g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2)
 - d^(1/2))^3*((17*c*d^3*e^2*g^3)/12 - (85*c*e^5*f^3)/12 + (17*c*d*e^4*f^2*
g)/4 + (91*c*d^2*e^3*f*g^2)/4))/(g^8*((f + g*x)^(1/2) - f^(1/2))^3) + (((d
+ e*x)^(1/2) - d^(1/2))^11*((c*d^3*g^3)/4 - (5*c*e^3*f^3)/4 + (3*c*d*e^2*f^
2*g)/4 + (c*d^2*e*f*g^2)/4))/(e^2*g^4*((f + g*x)^(1/2) - f^(1/2))^11) - (((
d + e*x)^(1/2) - d^(1/2))^9*((17*c*d^3*g^3)/12 - (85*c*e^3*f^3)/12 + (17*c*
d*e^2*f^2*g)/4 + (91*c*d^2*e*f*g^2)/4))/(e*g^5*((f + g*x)^(1/2) - f^(1/2))^
9) + (d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^6*(128*c*e^3*f^2 + 64*c*d
^2*e*g^2 + (704*c*d*e^2*f*g)/3))/(g^6*((f + g*x)^(1/2) - f^(1/2))^6) + (d^(
1/2)*f^(1/2)*(32*c*d^2*g + 96*c*d*e*f)*((d + e*x)^(1/2) - d^(1/2))^8)/(g^4*
((f + g*x)^(1/2) - f^(1/2))^8) + (d^(1/2)*f^(1/2)*(96*c*d*e^3*f + 32*c*d^2*
e^2*g)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^6*((f + g*x)^(1/2) - f^(1/2))^4))/
(((d + e*x)^(1/2) - d^(1/2))^12/((f + g*x)^(1/2) - f^(1/2))^12 + e^6/g^6 -
(6*e*((d + e*x)^(1/2) - d^(1/2))^10)/(g*((f + g*x)^(1/2) - f^(1/2))^10) - (
6*e^5*((d + e*x)^(1/2) - d^(1/2))^2)/(g^5*((f + g*x)^(1/2) - f^(1/2))^2) +
```

```
(15*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/2) - f^(1/2))^4)
- (20*e^3*((d + e*x)^(1/2) - d^(1/2))^6)/(g^3*((f + g*x)^(1/2) - f^(1/2))^6
) + (15*e^2*((d + e*x)^(1/2) - d^(1/2))^8)/(g^2*((f + g*x)^(1/2) - f^(1/2))
^8)) + ((((d + e*x)^(1/2) - d^(1/2))*((b*d^2*e^2*g^2)/2 - (3*b*e^4*f^2)/2 +
 b*d*e^3*f*g))/(g^6*((f + g*x)^(1/2) - f^(1/2))) + (((d + e*x)^(1/2) - d^(1
/2))^3*((11*b*e^3*f^2)/2 + (7*b*d^2*e*g^2)/2 + 23*b*d*e^2*f*g))/(g^5*((f +
g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) - d^(1/2))^5*((7*b*d^2*g^2)/2
+ (11*b*e^2*f^2)/2 + 23*b*d*e*f*g))/(g^4*((f + g*x)^(1/2) - f^(1/2))^5) + (
((d + e*x)^(1/2) - d^(1/2))^7*((b*d^2*g^2)/2 - (3*b*e^2*f^2)/2 + b*d*e*f*g)
)/(e*g^3*((f + g*x)^(1/2) - f^(1/2))^7) - (d^(1/2)*f^(1/2)*(32*b*e^2*f + 16
*b*d*e*g)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/2) - f^(1/2))^4
) - (8*b*d^(3/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^6)/(g^2*((f + g*x)^(1/
2) - f^(1/2))^6) - (8*b*d^(3/2)*e^2*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^2)/
(g^4*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g
*x)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*(
(f + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*
((f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2
*((f + g*x)^(1/2) - f^(1/2))^4)) + (2*a*atanh((g^(1/2)*((d + e*x)^(1/2) - d
^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d*g - e*f))/(e^(1/2)*g^(3/
2)) - (b*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1
/2) - f^(1/2))))*(d*g - e*f)*(d*g + 3*e*f))/(2*e^(3/2)*g^(5/2)) + (c*atanh(
(g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2)))
)*(d*g - e*f)*(d^2*g^2 + 5*e^2*f^2 + 2*d*e*f*g))/(4*e^(5/2)*g^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.580 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=164

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}\,(-4beg+5cdg+3c}{4e^2g^2}$$

Rubi [A] time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {951, 80, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}\,(-4beg+5cdg+3cef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) +
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g +
3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
```

```
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}e(3cef+5cdg-4beg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2de\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2de\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2de\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2de\right.\right.}{}$$

Mathematica [A] time = 0.62, size = 173, normalized size = 1.05

$$\frac{\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)+e\sqrt{g}\sqrt{d+ex}(f+gx)(4beg+c(-3dg-3ef+2egx))}{4e^3g^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x))
 + Sqrt[e*f - d*g]*(c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g
- b*(e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d +
 e*x])/Sqrt[e*f - d*g]])/(4*e^3*g^(5/2)*Sqrt[f + g*x])
```

IntegrateAlgebraic [A] time = 0.00, size = 229, normalized size = 1.40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)\left(8ae^2g^2-4bdeg^2-4be^2fg+3cd^2g^2+2cdefg+3ce^2f^2\right)}{4e^{5/2}g^{5/2}} + \frac{\sqrt{f+gx}(ef-dg)\left(\frac{4be^2g(f+gx)}{d+ex}-4beg^2-\frac{3ce^2f(f+gx)}{d+ex}-\frac{5cdeg(f+gx)}{d+ex}+3cdg^2+5cefg\right)}{4e^2g^2\sqrt{d+ex}\left(\frac{e(f+gx)}{d+ex}-g\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] ((e*f - d*g)*Sqrt[f + g*x]*(5*c*e*f*g + 3*c*d*g^2 - 4*b*e*g^2 - (3*c*e^2*f*
(f + g*x))/(d + e*x) - (5*c*d*e*g*(f + g*x))/(d + e*x) + (4*b*e^2*g*(f + g*
x))/(d + e*x)))/(4*e^2*g^2*Sqrt[d + e*x]*(-g + (e*(f + g*x))/(d + e*x))^2)
+ ((3*c*e^2*f^2 + 2*c*d*e*f*g - 4*b*e^2*f*g + 3*c*d^2*g^2 - 4*b*d*e*g^2 + 8
*a*e^2*g^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^
(5/2)*g^(5/2))
```

fricas [A] time = 0.51, size = 380, normalized size = 2.32

$$\left[\frac{\left(3ce^2f^2+2\left(cde-2be^2\right)fg+\left(3cd^2-4bde+8ae^2\right)g^2\right)\sqrt{eg}\log\left(8e^2g^2x^2+e^2f^2+6defg+d^2g^2+4\left(2egx+ef+dg\right)\sqrt{eg}\sqrt{ex+d}\sqrt{gx+f}+8\left(e^2fg+deg^2\right)x\right)+4\left(2ce^2g^2x-3ce^2fg-\left(3cde-4be^2\right)g^2\right)\sqrt{ex+d}\sqrt{gx+f}}{16e^3g^3}, -\frac{\left(3ce^2f^2+2\left(cde-2be^2\right)fg+\left(3cd^2-4bde+8ae^2\right)g^2\right)\sqrt{-eg}\arctan\left(\frac{\left(2egx+ef+dg\right)\sqrt{-eg}\sqrt{ex+d}\sqrt{gx+f}}{2\left(e^2g^2x^2+defg+\left(e^2fg+deg^2\right)x\right)}\right)-2\left(2ce^2g^2x-3ce^2fg-\left(3cde-4be^2\right)g^2\right)\sqrt{ex+d}\sqrt{gx+f}}{8e^3g^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^
2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*
e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e
*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^2)*g^2)*sqrt(e
*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*
f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e
*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (
e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^
2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

giac [A] time = 0.25, size = 179, normalized size = 1.09

$$\frac{1}{4}\sqrt{(xe+d)ge-dge+fe^2}\,\sqrt{xe+d}\left(\frac{2(xe+d)ce^{(-3)}}{g}-\frac{\left(5cdg^2e^5+3cfge^6-4bg^2e^6\right)e^{(-8)}}{g^3}\right)-\frac{\left(3cd^2g^2+2cdfge-4bdg^2e+3cf^2e^2-4bfge^2+8ag^2e^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-\sqrt{xe+d}\,\sqrt{g}\,e^{\frac{1}{2}}+\sqrt{(xe+d)ge-dge+fe^2}\right|\right)}{4g^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*(2*(x*e + d)*c*e^(-3)
/g - (5*c*d*g^2*e^5 + 3*c*f*g*e^6 - 4*b*g^2*e^6)*e^(-8)/g^3) - 1/4*(3*c*d^2
*g^2 + 2*c*d*f*g*e - 4*b*d*g^2*e + 3*c*f^2*e^2 - 4*b*f*g*e^2 + 8*a*g^2*e^2)
*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e^(1/2) + sqrt((x*e + d)*g*e - d*g
*e + f*e^2)))/g^(5/2)
```

maple [B] time = 0.00, size = 425, normalized size = 2.59

$$\frac{\left(8ae^2g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-4bdeg^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)-4be^2fg\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+3cd^2g^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+2cdefg\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+3ce^2f^2\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right)+4\sqrt{eg}\,\sqrt{(ex+d)(gx+f)}\,cegx+8\sqrt{eg}\,\sqrt{(ex+d)(gx+f)}\,beg-6\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,cdg-6\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,cef\right)\sqrt{ex+d}\,\sqrt{gx+f}}{8\sqrt{eg}\,\sqrt{(ex+d)(gx+f)}\,e^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(8*a*e^2*g^2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1
/2))/(e*g)^(1/2))-4*b*d*e*g^2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(
1/2)*(e*g)^(1/2))/(e*g)^(1/2))-4*b*e^2*f*g*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+
d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))+3*c*d^2*g^2*ln(1/2*(2*e*g*x+d*g
+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))+2*c*d*e*f*g*ln(1/2
*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))+3*c*e
^2*f^2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)
^(1/2))+4*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e*g*x+8*(e*g)^(1/2)*((e*x+d
)*(g*x+f))^(1/2)*b*e*g-6*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*d*g-6*((e*x+
d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2
)/((e*x+d)*(g*x+f))^(1/2)/e^2/g^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [B] time = 0.00, size = 833, normalized size = 5.08

$$\frac{\frac{(2bdg+2bef)\left(\sqrt{d+ex}-\sqrt{d}\right)}{g^3\left(\sqrt{f+gx}-\sqrt{f}\right)}+\frac{(2bdg+2bef)\left(\sqrt{d+ex}-\sqrt{d}\right)^3}{e\,g^2\left(\sqrt{f+gx}-\sqrt{f}\right)^3}-\frac{8b\sqrt{d}\sqrt{f}\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g^2\left(\sqrt{f+gx}-\sqrt{f}\right)^2}}{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{\left(\sqrt{f+gx}-\sqrt{f}\right)^4}+\frac{e^2}{g^2}-\frac{2e\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g\left(\sqrt{f+gx}-\sqrt{f}\right)^2}}-\frac{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)\left(\frac{3ce^3f^2}{2}+\frac{3cd^2eg^2}{2}+cde^2fg\right)}{g^6\left(\sqrt{f+gx}-\sqrt{f}\right)}-\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^3\left(\frac{11cd^2g^2}{2}+\frac{11ce^2f^2}{2}+25cdefg\right)}{g^5\left(\sqrt{f+gx}-\sqrt{f}\right)^3}+\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^7\left(\frac{3cd^2g^2}{2}+\frac{3ce^2f^2}{2}+cdefg\right)}{e^2g^3\left(\sqrt{f+gx}-\sqrt{f}\right)^7}-\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^5\left(\frac{11cd^2g^2}{2}+\frac{11ce^2f^2}{2}+25cdefg\right)}{e\,g^4\left(\sqrt{f+gx}-\sqrt{f}\right)^5}+\frac{\sqrt{d}\sqrt{f}\,(32cdg+32cef)\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{g^4\left(\sqrt{f+gx}-\sqrt{f}\right)^4}}{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^8}{\left(\sqrt{f+gx}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4e\left(\sqrt{d+ex}-\sqrt{d}\right)^6}{g\left(\sqrt{f+gx}-\sqrt{f}\right)^6}-\frac{4e^3\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g^3\left(\sqrt{f+gx}-\sqrt{f}\right)^2}+\frac{6e^2\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{g^2\left(\sqrt{f+gx}-\sqrt{f}\right)^4}}-\frac{4a\,\mathrm{atan}\left(\frac{e\left(\sqrt{f+gx}-\sqrt{f}\right)}{\sqrt{-eg}\left(\sqrt{d+ex}-\sqrt{d}\right)}\right)}{\sqrt{-eg}}-\frac{2b\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+gx}-\sqrt{f}\right)}\right)(dg+ef)}{e^{3/2}g^{3/2}}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+gx}-\sqrt{f}\right)}\right)\left(3d^2g^2+2defg+3e^2f^2\right)}{2e^{5/2}g^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*((f + g*x)^(1/2) -
f^(1/2))) + ((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(e*g^2*((f
+ g*x)^(1/2) - f^(1/2))^3) - (8*b*d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2
))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))^4/(
(f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2))^2
)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*
e^3*f^2)/2 + (3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1
/2))) - (((d + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2
 + 25*c*d*e*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) -
 d^(1/2))^7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f +
 g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/
2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5)
 + (d^(1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g
^4*((f + g*x)^(1/2) - f^(1/2))^4))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x
)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f
 + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((
f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*(
(f + g*x)^(1/2) - f^(1/2))^4)) - (4*a*atan((e*((f + g*x)^(1/2) - f^(1/2)))/
((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(-e*g)^(1/2) - (2*b*atanh((g^(
1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d
*g + e*f))/(e^(3/2)*g^(3/2)) + (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)
))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*
g))/(2*e^(5/2)*g^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{\sqrt{d + ex}\,\sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

3.581 $\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=129

$$-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}\,(ef-dg)}-\frac{(-2beg+3cdg+cef)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}$$

Rubi [A] time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {949, 80, 63, 217, 206}

$$-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}\,(ef-dg)}-\frac{(-2beg+3cdg+cef)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e*f - d*g)*Sqrt[d + e*x]) +
(c*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g) - ((c*e*f + 3*c*d*g - 2*b*e*g)*ArcT
anh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*g^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
```

```
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} - \frac{2\int \frac{\frac{(cd-be)(ef-dg)}{2e^2}-\frac{c(ef-dg)x}{2e}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{ef-dg}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\int \frac{1}{\sqrt{d+ex}\sqrt{\ldots}}}{2e^2g}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\,\mathrm{Subst}\left(\int \ldots\right.}{e^3g}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\,\mathrm{Subst}\left(\int \ldots\right.}{e^3g}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)\tanh^{-1}\left(\ldots\right.}{e^{5/2}g^{3/2}}$$

Mathematica [C] time = 0.59, size = 222, normalized size = 1.72

$$-\frac{2\sqrt{f+gx}\left(e\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\left(g^2(ae-bd)+cf(2dg-ef)\right)+e\sqrt{g}\sqrt{d+ex}(2cf-bg)(ef-dg)\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)+c(ef-dg)^{5/2}{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{g(d+ex)}{dg-ef}\right)\right)}{e^2g^2\sqrt{d+ex}(ef-dg)^{3/2}\sqrt{\frac{e(f+gx)}{ef-dg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(e*Sqrt[e*f - d*g]*((-(b*d) + a*e)*g^2 + c*f*(-(e*f) + 2*
d*g))*Sqrt[(e*(f + g*x))/(e*f - d*g)] + e*Sqrt[g]*(2*c*f - b*g)*(e*f - d*g)
*Sqrt[d + e*x]*ArcSinh[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]] + c*(e*f -
d*g)^(5/2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (g*(d + e*x))/(-(e*f) + d*g)]
))/(e^2*g^2*(e*f - d*g)^(3/2)*Sqrt[d + e*x]*Sqrt[(e*(f + g*x))/(e*f - d*g)]
)
```

IntegrateAlgebraic [A] time = 0.34, size = 216, normalized size = 1.67

$$\frac{(2beg-3cdg-cef)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{e^{5/2}g^{3/2}}-\frac{\sqrt{f+gx}\left(\frac{2ae^3g(f+gx)}{d+ex}-2ae^2g^2-\frac{2bde^2g(f+gx)}{d+ex}+2bdeg^2+\frac{2cd^2eg(f+gx)}{d+ex}-3cd^2g^2+2cdefg-ce^2f^2\right)}{e^2g\sqrt{d+ex}(ef-dg)\left(\frac{e(f+gx)}{d+ex}-g\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] -((Sqrt[f + g*x]*(-(c*e^2*f^2) + 2*c*d*e*f*g - 3*c*d^2*g^2 + 2*b*d*e*g^2 -
2*a*e^2*g^2 + (2*c*d^2*e*g*(f + g*x))/(d + e*x) - (2*b*d*e^2*g*(f + g*x))/(
d + e*x) + (2*a*e^3*g*(f + g*x))/(d + e*x)))/(e^2*g*(e*f - d*g)*Sqrt[d + e*
x]*(-g + (e*(f + g*x))/(d + e*x)))) + ((-(c*e*f) - 3*c*d*g + 2*b*e*g)*ArcTa
nh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(e^(5/2)*g^(3/2))
```

fricas [B] time = 1.44, size = 588, normalized size = 4.56

$$\left[-\frac{\left(c d e^{2} f^{2} + 2 \left(c d^{2} e - b d e^{2}\right) f g - \left(3 c d^{3} - 2 b d^{2} e\right) g^{2} + \left(c e^{3} f^{2} + 2 \left(c d e^{2} - b e^{3}\right) f g - \left(3 c d^{2} e - 2 b d e^{2}\right) g^{2}\right) x\right) \sqrt{e g} \log\left(8 e^{2} g^{2} x^{2} + e^{2} f^{2} + 6 d e f g + d^{2} g^{2} + 4 \left(2 e g x + e f + d g\right) \sqrt{e g} \sqrt{e x + d} \sqrt{g x + f} + 8 \left(e^{2} f g + d e g^{2}\right) x\right) - 4 \left(c d e^{2} f g - \left(3 c d^{2} e - 2 b d e^{2} + 2 a e^{3}\right) g^{2} + \left(c e^{3} f g - c d e^{2} g^{2}\right) x\right) \sqrt{e x + d} \sqrt{g x + f}}{4 \left(d e^{4} f g^{2} - d^{2} e^{3} g^{3} + \left(e^{5} f g^{2} - d e^{4} g^{3}\right) x\right)}, \frac{\left(c d e^{2} f^{2} + 2 \left(c d^{2} e - b d e^{2}\right) f g - \left(3 c d^{3} - 2 b d^{2} e\right) g^{2} + \left(c e^{3} f^{2} + 2 \left(c d e^{2} - b e^{3}\right) f g - \left(3 c d^{2} e - 2 b d e^{2}\right) g^{2}\right) x\right) \sqrt{-e g} \arctan\left(\frac{\left(2 e g x + e f + d g\right) \sqrt{-e g} \sqrt{e x + d} \sqrt{g x + f}}{2 \left(e^{2} g^{2} x^{2} + d e f g + \left(e^{2} f g + d e g^{2}\right) x\right)}\right) + 2 \left(c d e^{2} f g - \left(3 c d^{2} e - 2 b d e^{2} + 2 a e^{3}\right) g^{2} + \left(c e^{3} f g - c d e^{2} g^{2}\right) x\right) \sqrt{e x + d} \sqrt{g x + f}}{2 \left(d e^{4} f g^{2} - d^{2} e^{3} g^{3} + \left(e^{5} f g^{2} - d e^{4} g^{3}\right) x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - 2*b*d^2*e)*g^2
 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - 2*b*d*e^2)*g^2)*x)*s
qrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e
*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x)
- 4*(c*d*e^2*f*g - (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*g^2 + (c*e^3*f*g - c*d
*e^2*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d*e^4*f*g^2 - d^2*e^3*g^3 + (e^5
*f*g^2 - d*e^4*g^3)*x), 1/2*((c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*
c*d^3 - 2*b*d^2*e)*g^2 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e
- 2*b*d*e^2)*g^2)*x)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)
*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x
)) + 2*(c*d*e^2*f*g - (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*g^2 + (c*e^3*f*g -
c*d*e^2*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d*e^4*f*g^2 - d^2*e^3*g^3 + (
e^5*f*g^2 - d*e^4*g^3)*x)]
```

giac [A] time = 0.39, size = 201, normalized size = 1.56

$$\frac{\sqrt{(xe+d)ge - dge + fe^{2}}\,\sqrt{xe+d}\,c e^{(-3)}}{g} + \frac{4\left(cd^{2}\sqrt{g}e^{\frac{1}{2}} - bd\sqrt{g}e^{\frac{3}{2}} + a\sqrt{g}e^{\frac{5}{2}}\right)e^{(-2)}}{dge + \left(\sqrt{xe+d}\,\sqrt{g}e^{\frac{1}{2}} - \sqrt{(xe+d)ge - dge + fe^{2}}\right)^{2} - fe^{2}} + \frac{\left(3cdg^{\frac{3}{2}}e^{\frac{1}{2}} + cf\sqrt{g}e^{\frac{3}{2}} - 2bg^{\frac{3}{2}}e^{\frac{3}{2}}\right)e^{(-3)}\log\left(\left(\sqrt{xe+d}\,\sqrt{g}e^{\frac{1}{2}} - \sqrt{(xe+d)ge - dge + fe^{2}}\right)^{2}\right)}{2g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*c*e^(-3)/g + 4*(c*d^2*sqr
t(g)*e^(1/2) - b*d*sqrt(g)*e^(3/2) + a*sqrt(g)*e^(5/2))*e^(-2)/(d*g*e + (sq
rt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2 - f*e^
2) + 1/2*(3*c*d*g^(3/2)*e^(1/2) + c*f*sqrt(g)*e^(3/2) - 2*b*g^(3/2)*e^(3/2)
)*e^(-3)*log((sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e +
f*e^2))^2)/g^2
```

maple [B] time = 0.03, size = 697, normalized size = 5.40

$$\frac{\sqrt{gx+f}\left(2bde^{2}g^{2}x\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right) - 2be^{3}fgx\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right) - 3cd^{2}eg^{2}x\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right) + 2cde^{2}fgx\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right) + ce^{3}f^{2}x\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right) + 2bd^{2}eg^{2}\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right) - 2bde^{2}fg\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right) - 3cd^{3}g^{2}\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right) + 2cd^{2}efg\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right) + cde^{2}f^{2}\ln\left(\frac{2egx+dg+ef+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right) + 2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,cdegx - 2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,ce^{2}fx + 4\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,ae^{2}g - 4\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,bdeg + 6\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,cd^{2}g - 2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,cdef\right)}{2\sqrt{eg}\,(dg-ef)\sqrt{(ex+d)(gx+f)}\,\sqrt{ex+d}\,e^{2}g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g
)^(1/2))/(e*g)^(1/2))*x*b*d*e^2*g^2-2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g
*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x*b*e^3*f*g-3*ln(1/2*(2*e*g*x+d*g+e*
f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x*c*d^2*e*g^2+2*ln(1/
2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x*c*
d*e^2*f*g+ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e
*g)^(1/2))*x*c*e^3*f^2+2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*
(e*g)^(1/2))/(e*g)^(1/2))*b*d^2*e*g^2-2*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*
(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*b*d*e^2*f*g-3*ln(1/2*(2*e*g*x+d*g+
e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^3*g^2+2*ln(1/2*
(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^2*
e*f*g+ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^
(1/2))*c*d*e^2*f^2+2*x*c*d*e*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-2*x*c*e^
2*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+4*a*e^2*g*((e*x+d)*(g*x+f))^(1/2)*(
e*g)^(1/2)-4*b*d*e*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+6*c*d^2*g*((e*x+d)
```

```
*(g*x+f))^(1/2)*(e*g)^(1/2)-2*c*d*e*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/
(e*g)^(1/2)/g/(d*g-e*f)/((e*x+d)*(g*x+f))^(1/2)/e^2/(e*x+d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g>0)', see `assume?` for more detai
ls)Is g positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^2+b\,x+a}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^{2}}{\left(d + ex\right)^{\frac{3}{2}} \sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(3/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

3.582 $\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=160

$$\frac{2\sqrt{f+gx}\left(c\left(6def-4d^2g\right)-e(-2aeg-bdg+3bef)\right)}{3e^2\sqrt{d+ex}\,(ef-dg)^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

Rubi [A] time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {949, 78, 63, 217, 206}

$$\frac{2\sqrt{f+gx}\left(c\left(6def-4d^2g\right)-e(-2aeg-bdg+3bef)\right)}{3e^2\sqrt{d+ex}\,(ef-dg)^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(3*(e*f - d*g)*(d + e*x)^(3/2)
) + (2*(c*(6*d*e*f - 4*d^2*g) - e*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x
])/(3*e^2*(e*f - d*g)^2*Sqrt[d + e*x]) + (2*c*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*Sqrt[g])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
```

```
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} - \frac{2\int \frac{\frac{cd(3ef-dg)-e(3bef-bdg-2aeg)}{2e^2}-\frac{3}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{3(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}}$$

Mathematica [C] time = 0.23, size = 173, normalized size = 1.08

$$\frac{2\sqrt{f+gx}\left(2g(d+ex)\left(g(ag-bf)+cf^2\right)-(ef-dg)\left(g(ag-bf)+cf^2\right)+(f+gx)(2cf-bg)(ef-dg)-\frac{c(ef-dg)^3\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{g(d+ex)}{dg-ef}\right)}{e^2\sqrt{\frac{e(f+gx)}{ef-dg}}}\right)}{3g^2(d+ex)^{3/2}(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(-((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))) + 2*g*(c*f^2 +
g*(-(b*f) + a*g))*(d + e*x) + (2*c*f - b*g)*(e*f - d*g)*(f + g*x) - (c*(e*f
 - d*g)^3*Hypergeometric2F1[-3/2, -3/2, -1/2, (g*(d + e*x))/(-(e*f) + d*g)]
)/(e^2*Sqrt[(e*(f + g*x))/(e*f - d*g)])))/(3*g^2*(e*f - d*g)^2*(d + e*x)^(3
/2))
```

IntegrateAlgebraic [A] time = 0.23, size = 161, normalized size = 1.01

$$\frac{2c\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{e^{5/2}\sqrt{g}} - \frac{2\sqrt{f+gx}\left(\frac{ae^3(f+gx)}{d+ex}-3ae^2g-\frac{bde^2(f+gx)}{d+ex}+3be^2f+\frac{cd^2e(f+gx)}{d+ex}+3cd^2g-6cdef\right)}{3e^2\sqrt{d+ex}\,(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(-6*c*d*e*f + 3*b*e^2*f + 3*c*d^2*g - 3*a*e^2*g + (c*d^2*
e*(f + g*x))/(d + e*x) - (b*d*e^2*(f + g*x))/(d + e*x) + (a*e^3*(f + g*x))/
(d + e*x)))/(3*e^2*(e*f - d*g)^2*Sqrt[d + e*x]) + (2*c*ArcTanh[(Sqrt[e]*Sqr
t[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(e^(5/2)*Sqrt[g])
```

fricas [B] time = 3.55, size = 792, normalized size = 4.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(c*d^2*e^2*f^2 - 2*c*d^3*e*f*g + c*d^4*g^2 + (c*e^4*f^2 - 2*c*d*e^3
*f*g + c*d^2*e^2*g^2)*x^2 + 2*(c*d*e^3*f^2 - 2*c*d^2*e^2*f*g + c*d^3*e*g^2)
*x)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*
x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2
)*x) + 4*((5*c*d^2*e^2 - 2*b*d*e^3 - a*e^4)*f*g - 3*(c*d^3*e - a*d*e^3)*g^2
 + (3*(2*c*d*e^3 - b*e^4)*f*g - (4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*g^2)*x)*s
qrt(e*x + d)*sqrt(g*x + f))/(d^2*e^5*f^2*g - 2*d^3*e^4*f*g^2 + d^4*e^3*g^3
+ (e^7*f^2*g - 2*d*e^6*f*g^2 + d^2*e^5*g^3)*x^2 + 2*(d*e^6*f^2*g - 2*d^2*e^
5*f*g^2 + d^3*e^4*g^3)*x), -1/3*(3*(c*d^2*e^2*f^2 - 2*c*d^3*e*f*g + c*d^4*g
^2 + (c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x^2 + 2*(c*d*e^3*f^2 - 2*c
*d^2*e^2*f*g + c*d^3*e*g^2)*x)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*
sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g +
d*e*g^2)*x)) - 2*((5*c*d^2*e^2 - 2*b*d*e^3 - a*e^4)*f*g - 3*(c*d^3*e - a*d*
e^3)*g^2 + (3*(2*c*d*e^3 - b*e^4)*f*g - (4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*g
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d^2*e^5*f^2*g - 2*d^3*e^4*f*g^2 + d^4*
e^3*g^3 + (e^7*f^2*g - 2*d*e^6*f*g^2 + d^2*e^5*g^3)*x^2 + 2*(d*e^6*f^2*g -
2*d^2*e^5*f*g^2 + d^3*e^4*g^3)*x)]
```

giac [B] time = 0.68, size = 504, normalized size = 3.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] -c*e^(-5/2)*log((sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
 + f*e^2))^2)/sqrt(g) - 4/3*(4*c*d^3*g^(5/2)*e^(5/2) + 6*(sqrt(x*e + d)*sqr
t(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^2*g^(3/2)*e^(3/2)
 + 6*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^
4*c*d*sqrt(g)*e^(1/2) - 10*c*d^2*f*g^(3/2)*e^(7/2) - b*d^2*g^(5/2)*e^(7/2)
- 12*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^
2*c*d*f*sqrt(g)*e^(5/2) - 3*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)
*g*e - d*g*e + f*e^2))^4*b*sqrt(g)*e^(3/2) + 6*c*d*f^2*sqrt(g)*e^(9/2) + 4*
b*d*f*g^(3/2)*e^(9/2) - 2*a*d*g^(5/2)*e^(9/2) + 6*(sqrt(x*e + d)*sqrt(g)*e^
(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*b*f*sqrt(g)*e^(7/2) - 6*(sqr
t(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*g^(3/
2)*e^(7/2) - 3*b*f^2*sqrt(g)*e^(11/2) + 2*a*f*g^(3/2)*e^(11/2))*e^(-2)/(d*g
*e + (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^
2 - f*e^2)^3
```

maple [B] time = 0.03, size = 773, normalized size = 4.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/3*(g*x+f)^(1/2)*(3*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g
)^(1/2))/(e*g)^(1/2))*x^2*c*d^2*e^2*g^2-6*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d
)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x^2*c*d*e^3*f*g+3*ln(1/2*(2*e*g*
x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x^2*c*e^4*f^2
```

```
+6*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/
2))*x*c*d^3*e*g^2-12*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g
)^(1/2))/(e*g)^(1/2))*x*c*d^2*e^2*f*g+6*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*
(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*x*c*d*e^3*f^2+3*ln(1/2*(2*e*g*x+d*
g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^4*g^2-6*ln(1/
2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2))*c*d^
3*e*f*g+3*ln(1/2*(2*e*g*x+d*g+e*f+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e
*g)^(1/2))*c*d^2*e^2*f^2+4*x*a*e^3*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+2*
x*b*d*e^2*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-6*x*b*e^3*f*((e*x+d)*(g*x+f
))^(1/2)*(e*g)^(1/2)-8*x*c*d^2*e*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+12*x
*c*d*e^2*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+6*a*d*e^2*g*((e*x+d)*(g*x+f)
)^(1/2)*(e*g)^(1/2)-2*a*e^3*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-4*b*d*e^2
*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-6*c*d^3*g*((e*x+d)*(g*x+f))^(1/2)*(e
*g)^(1/2)+10*c*d^2*e*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2)/(d*
g-e*f)^2/((e*x+d)*(g*x+f))^(1/2)/e^2/(e*x+d)^(3/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g>0)', see `assume?` for more detai
ls)Is g positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^2+b\,x+a}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)),x)
```

```
[Out] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(5/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.583 $\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=198

$$\frac{2\sqrt{f+gx}\left(2eg(-4aeg-bdg+5bef)-c\left(3d^2g^2-10defg+15e^2f^2\right)\right)}{15e^2\sqrt{d+ex}\,(ef-dg)^3}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}+\frac{2\sqrt{f+gx}\,(2cd(\ldots}{15\ldots}$$

Rubi [A] time = 0.21, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {949, 78, 37}

$$\frac{2\sqrt{f+gx}\left(2eg(-4aeg-bdg+5bef)-c\left(3d^2g^2-10defg+15e^2f^2\right)\right)}{15e^2\sqrt{d+ex}\,(ef-dg)^3}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}+\frac{2\sqrt{f+gx}\,(2cd(5ef-3dg)-e(-4aeg-bdg+5bef))}{15e^2(d+ex)^{3/2}(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(5*(e*f - d*g)*(d + e*x)^(5/2)
) + (2*(2*c*d*(5*e*f - 3*d*g) - e*(5*b*e*f - b*d*g - 4*a*e*g))*Sqrt[f + g*x
])/(15*e^2*(e*f - d*g)^2*(d + e*x)^(3/2)) + (2*(2*e*g*(5*b*e*f - b*d*g - 4*
a*e*g) - c*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(15*e^2*(e
*f - d*g)^3*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} - \frac{2\int \frac{\frac{cd(5ef-dg)-e(5bef-bdg-4aeg)}{2e^2}-\frac{5}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx}{5(ef-dg)}$$
$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} + \frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}}$$
$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} + \frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}}$$

Mathematica [A] time = 0.23, size = 178, normalized size = 0.90

$$-\frac{2\sqrt{f+gx}\left(a\left(15d^2g^2-10deg(f-2gx)+e^2\left(3f^2-4fgx+8g^2x^2\right)\right)+b\left(5d^2g(gx-2f)+2de\left(f^2-13fgx+g^2x^2\right)+5e^2fx(f-2gx)\right)+c\left(d^2\left(8f^2-4fgx+3g^2x^2\right)+10defx(2f-gx)+15e^2f^2x^2\right)\right)}{15(d+ex)^{5/2}(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(b*(5*e^2*f*x*(f - 2*g*x) + 5*d^2*g*(-2*f + g*x) + 2*d*e*
(f^2 - 13*f*g*x + g^2*x^2)) + c*(15*e^2*f^2*x^2 + 10*d*e*f*x*(2*f - g*x) +
d^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + a*(15*d^2*g^2 - 10*d*e*g*(f - 2*g*x) +
 e^2*(3*f^2 - 4*f*g*x + 8*g^2*x^2))))/(15*(e*f - d*g)^3*(d + e*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 177, normalized size = 0.89

$$-\frac{2\sqrt{f+gx}\left(\frac{3ae^2(f+gx)^2}{(d+ex)^2}-\frac{10aeg(f+gx)}{d+ex}+15ag^2+\frac{5bef(f+gx)}{d+ex}-\frac{3bde(f+gx)^2}{(d+ex)^2}+\frac{5bdg(f+gx)}{d+ex}-15bfg+\frac{3cd^2(f+gx)^2}{(d+ex)^2}-\frac{10cdf(f+gx)}{d+ex}+15cf^2\right)}{15\sqrt{d+ex}\,(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(15*c*f^2 - 15*b*f*g + 15*a*g^2 - (10*c*d*f*(f + g*x))/(d
 + e*x) + (5*b*e*f*(f + g*x))/(d + e*x) + (5*b*d*g*(f + g*x))/(d + e*x) - (
10*a*e*g*(f + g*x))/(d + e*x) + (3*c*d^2*(f + g*x)^2)/(d + e*x)^2 - (3*b*d*
e*(f + g*x)^2)/(d + e*x)^2 + (3*a*e^2*(f + g*x)^2)/(d + e*x)^2))/(15*(e*f -
 d*g)^3*Sqrt[d + e*x])
```

fricas [A] time = 10.81, size = 353, normalized size = 1.78

$$-\frac{2\left(15\,ad^2g^2+\left(8\,cd^2+2\,bde+3\,ae^2\right)f^2-10\left(bd^2+ade\right)fg+\left(15\,ce^2f^2-10\left(cde+be^2\right)fg+\left(3\,cd^2+2\,bde+8\,ae^2\right)g^2\right)x^2+\left(5\left(4\,cde+be^2\right)f^2-2\left(2\,cd^2+13\,bde+2\,ae^2\right)fg+5\left(bd^2+4\,ade\right)g^2\right)x\right)\sqrt{ex+d}\,\sqrt{gx+f}}{15\left(d^3e^3f^3-3\,d^4e^2f^2g+3\,d^5efg^2-d^6g^3+\left(e^6f^3-3\,de^5f^2g+3\,d^2e^4fg^2-d^3e^3g^3\right)x^3+3\left(de^5f^3-3\,d^2e^4f^2g+3\,d^3e^3fg^2-d^4e^2g^3\right)x^2+3\left(d^2e^4f^3-3\,d^3e^3f^2g+3\,d^4e^2fg^2-d^5eg^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*a*d^2*g^2 + (8*c*d^2 + 2*b*d*e + 3*a*e^2)*f^2 - 10*(b*d^2 + a*d*e
)*f*g + (15*c*e^2*f^2 - 10*(c*d*e + b*e^2)*f*g + (3*c*d^2 + 2*b*d*e + 8*a*e
^2)*g^2)*x^2 + (5*(4*c*d*e + b*e^2)*f^2 - 2*(2*c*d^2 + 13*b*d*e + 2*a*e^2)*
f*g + 5*(b*d^2 + 4*a*d*e)*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(d^3*e^3*f^3
- 3*d^4*e^2*f^2*g + 3*d^5*e*f*g^2 - d^6*g^3 + (e^6*f^3 - 3*d*e^5*f^2*g + 3*
d^2*e^4*f*g^2 - d^3*e^3*g^3)*x^3 + 3*(d*e^5*f^3 - 3*d^2*e^4*f^2*g + 3*d^3*e
^3*f*g^2 - d^4*e^2*g^3)*x^2 + 3*(d^2*e^4*f^3 - 3*d^3*e^3*f^2*g + 3*d^4*e^2*
f*g^2 - d^5*e*g^3)*x)
```

giac [B] time = 0.85, size = 1080, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(3*c*d^4*g^(9/2)*e^(9/2) + 30*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x
*e + d)*g*e - d*g*e + f*e^2))^4*c*d^2*g^(5/2)*e^(5/2) + 15*(sqrt(x*e + d)*s
qrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c*sqrt(g)*e^(1/2) -
 16*c*d^3*f*g^(7/2)*e^(11/2) + 2*b*d^3*g^(9/2)*e^(11/2) - 20*(sqrt(x*e + d)
*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^2*f*g^(5/2)*e
^(9/2) + 10*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f
*e^2))^2*b*d^2*g^(7/2)*e^(9/2) - 40*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((
x*e + d)*g*e - d*g*e + f*e^2))^4*c*d*f*g^(3/2)*e^(7/2) - 10*(sqrt(x*e + d)*
sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*b*d*g^(5/2)*e^(7/2
) - 60*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2)
)^6*c*f*sqrt(g)*e^(5/2) + 30*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d
)*g*e - d*g*e + f*e^2))^6*b*g^(3/2)*e^(5/2) + 38*c*d^2*f^2*g^(5/2)*e^(13/2)
 - 14*b*d^2*f*g^(7/2)*e^(13/2) + 8*a*d^2*g^(9/2)*e^(13/2) + 80*(sqrt(x*e +
d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d*f^2*g^(3/2)
*e^(11/2) - 60*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
+ f*e^2))^2*b*d*f*g^(5/2)*e^(11/2) + 40*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sq
rt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*d*g^(7/2)*e^(11/2) + 90*(sqrt(x*e +
d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*f^2*sqrt(g)*e
^(9/2) - 70*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f
*e^2))^4*b*f*g^(3/2)*e^(9/2) + 80*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*
e + d)*g*e - d*g*e + f*e^2))^4*a*g^(5/2)*e^(9/2) - 40*c*d*f^3*g^(3/2)*e^(15
/2) + 22*b*d*f^2*g^(5/2)*e^(15/2) - 16*a*d*f*g^(7/2)*e^(15/2) - 60*(sqrt(x*
e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*f^3*sqrt(
g)*e^(13/2) + 50*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*
e + f*e^2))^2*b*f^2*g^(3/2)*e^(13/2) - 40*(sqrt(x*e + d)*sqrt(g)*e^(1/2) -
sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*f*g^(5/2)*e^(13/2) + 15*c*f^4*sqrt
(g)*e^(17/2) - 10*b*f^3*g^(3/2)*e^(17/2) + 8*a*f^2*g^(5/2)*e^(17/2))*e^(-2)
/(d*g*e + (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e
^2))^2 - f*e^2)^5
```

maple [A] time = 0.01, size = 238, normalized size = 1.20

$$\frac{2\sqrt{gx+f}\,\left(8a\,e^{2}g^{2}x^{2}+2bde\,g^{2}x^{2}-10b\,e^{2}fg\,x^{2}+3c\,d^{2}g^{2}x^{2}-10cdefg\,x^{2}+15c\,e^{2}f^{2}x^{2}+20ade\,g^{2}x-4a\,e^{2}fgx+5b\,d^{2}g^{2}x-26bdefgx+5b\,e^{2}f^{2}x-4c\,d^{2}fgx+20cde\,f^{2}x+15a\,d^{2}g^{2}-10adefg+3a\,e^{2}f^{2}-10b\,d^{2}fg+2bde\,f^{2}+8c\,d^{2}f^{2}\right)}{15\left(ex+d\right)^{\frac{5}{2}}\left(g^{3}d^{3}-3d^{2}efg^{2}+3d\,e^{2}f^{2}g-e^{3}f^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(8*a*e^2*g^2*x^2+2*b*d*e*g^2*x^2-10*b*e^2*f*g*x^2+3*c*d^
2*g^2*x^2-10*c*d*e*f*g*x^2+15*c*e^2*f^2*x^2+20*a*d*e*g^2*x-4*a*e^2*f*g*x+5*
b*d^2*g^2*x-26*b*d*e*f*g*x+5*b*e^2*f^2*x-4*c*d^2*f*g*x+20*c*d*e*f^2*x+15*a*
d^2*g^2-10*a*d*e*f*g+3*a*e^2*f^2-10*b*d^2*f*g+2*b*d*e*f^2+8*c*d^2*f^2)/(e*x
+d)^(5/2)/(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [B] time = 4.30, size = 260, normalized size = 1.31

$$\frac{\sqrt{f+g\,x}\,\left(\frac{16\,c\,d^2\,f^2-20\,b\,d^2\,f\,g+30\,a\,d^2\,g^2+4\,b\,d\,e\,f^2-20\,a\,d\,e\,f\,g+6\,a\,e^2\,f^2}{15\,e^2\,{\left(d\,g-e\,f\right)}^3}+\frac{x\,\left(-8\,c\,d^2\,f\,g+10\,b\,d^2\,g^2+40\,c\,d\,e\,f^2-52\,b\,d\,e\,f\,g+40\,a\,d\,e\,g^2+10\,b\,e^2\,f^2-8\,a\,e^2\,f\,g\right)}{15\,e^2\,{\left(d\,g-e\,f\right)}^3}+\frac{x^2\,\left(6\,c\,d^2\,g^2-20\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+30\,c\,e^2\,f^2-20\,b\,e^2\,f\,g+16\,a\,e^2\,g^2\right)}{15\,e^2\,{\left(d\,g-e\,f\right)}^3}\right)}{x^2\,\sqrt{d+e\,x}+\frac{d^2\,\sqrt{d+e\,x}}{e^2}+\frac{2\,d\,x\,\sqrt{d+e\,x}}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(7/2)),x)
```

```
[Out] ((f + g*x)^(1/2)*((30*a*d^2*g^2 + 6*a*e^2*f^2 + 16*c*d^2*f^2 + 4*b*d*e*f^2
- 20*b*d^2*f*g - 20*a*d*e*f*g)/(15*e^2*(d*g - e*f)^3) + (x*(10*b*d^2*g^2 +
10*b*e^2*f^2 + 40*a*d*e*g^2 + 40*c*d*e*f^2 - 8*a*e^2*f*g - 8*c*d^2*f*g - 52
*b*d*e*f*g))/(15*e^2*(d*g - e*f)^3) + (x^2*(16*a*e^2*g^2 + 6*c*d^2*g^2 + 30
*c*e^2*f^2 + 4*b*d*e*g^2 - 20*b*e^2*f*g - 20*c*d*e*f*g))/(15*e^2*(d*g - e*f
)^3)))/(x^2*(d + e*x)^(1/2) + (d^2*(d + e*x)^(1/2))/e^2 + (2*d*x*(d + e*x)^
(1/2))/e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(7/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.584 $\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=281

$$-\frac{4g\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g^2-14defg+35e^2f^2\right)\right)}{105e^2\sqrt{d+ex}\,(ef-dg)^4}+\frac{2\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef)-\right.}{105e^2(d+ex)^{3/2}(\ldots}$$

Rubi [A] time = 0.29, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {949, 78, 45, 37}

$$-\frac{4g\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g^2-14defg+35e^2f^2\right)\right)}{105e^2\sqrt{d+ex}\,(ef-dg)^4}+\frac{2\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g^2-14defg+35e^2f^2\right)\right)}{105e^2(d+ex)^{3/2}(ef-dg)^3}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{7(d+ex)^{7/2}(ef-dg)}+\frac{2\sqrt{f+gx}\,(2cd(7ef-4dg)-e(-6aeg-bdg+7bef))}{35e^2(d+ex)^{5/2}(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(7*(e*f - d*g)*(d + e*x)^(7/2)
) + (2*(2*c*d*(7*e*f - 4*d*g) - e*(7*b*e*f - b*d*g - 6*a*e*g))*Sqrt[f + g*x
])/(35*e^2*(e*f - d*g)^2*(d + e*x)^(5/2)) + (2*(4*e*g*(7*b*e*f - b*d*g - 6*
a*e*g) - c*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(
e*f - d*g)^3*(d + e*x)^(3/2)) - (4*g*(4*e*g*(7*b*e*f - b*d*g - 6*a*e*g) - c
*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(e*f - d*g)
^4*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
```

```
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx &= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} - \frac{2\int \frac{\frac{cd(7ef-dg)-e(7bef-bdg-6aeg)}{2e^2}-\frac{7}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx}{7(ef-dg)} \\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} \\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} \\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.35, size = 332, normalized size = 1.18

$$
\frac{2\sqrt{f+gx}\left(3a\left(35d^3g^3-35d^2eg^2(f-2gx)+7de^2g\left(3f^2-4fgx+8g^2x^2\right)+e^3\left(-5f^3+6f^2gx-8fg^2x^2+16g^3x^3\right)\right)+b\left(35d^3g^2(gx-2f)+7d^2eg\left(4f^2-37fgx+4g^2x^2\right)+de^2\left(-6f^3+101f^2gx-200fg^2x^2+8g^3x^3\right)-7e^3fx\left(3f^2-4fgx+8g^2x^2\right)\right)+c\left(7d^3g\left(8f^2-4fgx+3g^2x^2\right)+d^2e\left(-8f^3+200f^2gx-101fg^2x^2+6g^3x^3\right)-7de^2fx\left(4f^2-37fgx+4g^2x^2\right)-35e^3f^2x^2(f-2gx)\right)\right)}{105(d+ex)^{7/2}(ef-dg)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*(-35*e^3*f^2*x^2*(f - 2*g*x) + 7*d^3*g*(8*f^2 - 4*f*g*x
 + 3*g^2*x^2) - 7*d*e^2*f*x*(4*f^2 - 37*f*g*x + 4*g^2*x^2) + d^2*e*(-8*f^3
+ 200*f^2*g*x - 101*f*g^2*x^2 + 6*g^3*x^3)) + b*(35*d^3*g^2*(-2*f + g*x) +
7*d^2*e*g*(4*f^2 - 37*f*g*x + 4*g^2*x^2) - 7*e^3*f*x*(3*f^2 - 4*f*g*x + 8*g
^2*x^2) + d*e^2*(-6*f^3 + 101*f^2*g*x - 200*f*g^2*x^2 + 8*g^3*x^3)) + 3*a*(
35*d^3*g^3 - 35*d^2*e*g^2*(f - 2*g*x) + 7*d*e^2*g*(3*f^2 - 4*f*g*x + 8*g^2*
x^2) + e^3*(-5*f^3 + 6*f^2*g*x - 8*f*g^2*x^2 + 16*g^3*x^3))))/(105*(e*f - d
*g)^4*(d + e*x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.23, size = 311, normalized size = 1.11

$$
-\frac{2\sqrt{f+gx}\left(\frac{15ae^3(f+gx)^3}{(d+ex)^3}-\frac{63ae^2g(f+gx)^2}{(d+ex)^2}+\frac{105aeg^2(f+gx)}{d+ex}-105ag^3-\frac{15bde^2(f+gx)^3}{(d+ex)^3}+\frac{21be^2f(f+gx)^2}{(d+ex)^2}-\frac{35bdg^2(f+gx)}{d+ex}+\frac{42bdeg(f+gx)^2}{(d+ex)^2}-\frac{70befg(f+gx)}{d+ex}+105bfg^2-\frac{21cd^2g(f+gx)^2}{(d+ex)^2}+\frac{15cd^2e(f+gx)^3}{(d+ex)^3}+\frac{35cef^2(f+gx)}{d+ex}+\frac{70cdfg(f+gx)}{d+ex}-\frac{42cdef(f+gx)^2}{(d+ex)^2}-105cf^2g\right)}{105\sqrt{d+ex}\,(ef-dg)^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(-105*c*f^2*g + 105*b*f*g^2 - 105*a*g^3 + (35*c*e*f^2*(f
+ g*x))/(d + e*x) + (70*c*d*f*g*(f + g*x))/(d + e*x) - (70*b*e*f*g*(f + g*x
))/(d + e*x) - (35*b*d*g^2*(f + g*x))/(d + e*x) + (105*a*e*g^2*(f + g*x))/(
d + e*x) - (42*c*d*e*f*(f + g*x)^2)/(d + e*x)^2 + (21*b*e^2*f*(f + g*x)^2)/
(d + e*x)^2 - (21*c*d^2*g*(f + g*x)^2)/(d + e*x)^2 + (42*b*d*e*g*(f + g*x)^
2)/(d + e*x)^2 - (63*a*e^2*g*(f + g*x)^2)/(d + e*x)^2 + (15*c*d^2*e*(f + g*
x)^3)/(d + e*x)^3 - (15*b*d*e^2*(f + g*x)^3)/(d + e*x)^3 + (15*a*e^3*(f + g
*x)^3)/(d + e*x)^3))/(105*(e*f - d*g)^4*Sqrt[d + e*x])
```

fricas [B] time = 36.00, size = 641, normalized size = 2.28

$$
\frac{2\left(105\,ad^3g^3-\left(8\,cd^2e+6\,bde^2+15\,ae^3\right)f^3+7\left(8\,cd^3+4\,bd^2e+9\,ade^2\right)f^2g-35\left(2\,bd^3+3\,ad^2e\right)fg^2+2\left(35\,ce^3f^2g-14\left(cde^2+2\,be^3\right)fg^2+\left(3\,cd^2e+4\,bde^2+24\,ae^3\right)g^3\right)x^3-\left(35\,ce^3f^3-7\left(37\,cde^2+4\,be^3\right)f^2g+\left(101\,cd^2e+200\,bde^2+24\,ae^3\right)fg^2-7\left(3\,cd^3+4\,bd^2e+24\,ade^2\right)g^3\right)x^2-\left(7\left(4\,cde^2+3\,be^3\right)f^3-\left(200\,cd^2e+101\,bde^2+18\,ae^3\right)f^2g+7\left(4\,cd^3+37\,bd^2e+12\,ade^2\right)fg^2-35\left(bd^3+6\,ad^2e\right)g^3\right)x\right)\sqrt{ex+d}\,\sqrt{gx+f}}{105\left(d^4e^4f^4-4\,d^5e^3f^3g+6\,d^6e^2f^2g^2-4\,d^7efg^3+d^8g^4+\left(e^8f^4-4\,de^7f^3g+6\,d^2e^6f^2g^2-4\,d^3e^5fg^3+d^4e^4g^4\right)x^4+4\left(de^7f^4-4\,d^2e^6f^3g+6\,d^3e^5f^2g^2-4\,d^4e^4fg^3+d^5e^3g^4\right)x^3+6\left(d^2e^6f^4-4\,d^3e^5f^3g+6\,d^4e^4f^2g^2-4\,d^5e^3fg^3+d^6e^2g^4\right)x^2+4\left(d^3e^5f^4-4\,d^4e^4f^3g+6\,d^5e^3f^2g^2-4\,d^6e^2fg^3+d^7eg^4\right)x\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(105*a*d^3*g^3 - (8*c*d^2*e + 6*b*d*e^2 + 15*a*e^3)*f^3 + 7*(8*c*d^3
+ 4*b*d^2*e + 9*a*d*e^2)*f^2*g - 35*(2*b*d^3 + 3*a*d^2*e)*f*g^2 + 2*(35*c*e
^3*f^2*g - 14*(c*d*e^2 + 2*b*e^3)*f*g^2 + (3*c*d^2*e + 4*b*d*e^2 + 24*a*e^3
)*g^3)*x^3 - (35*c*e^3*f^3 - 7*(37*c*d*e^2 + 4*b*e^3)*f^2*g + (101*c*d^2*e
+ 200*b*d*e^2 + 24*a*e^3)*f*g^2 - 7*(3*c*d^3 + 4*b*d^2*e + 24*a*d*e^2)*g^3)
*x^2 - (7*(4*c*d*e^2 + 3*b*e^3)*f^3 - (200*c*d^2*e + 101*b*d*e^2 + 18*a*e^3
)*f^2*g + 7*(4*c*d^3 + 37*b*d^2*e + 12*a*d*e^2)*f*g^2 - 35*(b*d^3 + 6*a*d^2
*e)*g^3)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(d^4*e^4*f^4 - 4*d^5*e^3*f^3*g + 6*
d^6*e^2*f^2*g^2 - 4*d^7*e*f*g^3 + d^8*g^4 + (e^8*f^4 - 4*d*e^7*f^3*g + 6*d^
2*e^6*f^2*g^2 - 4*d^3*e^5*f*g^3 + d^4*e^4*g^4)*x^4 + 4*(d*e^7*f^4 - 4*d^2*e
^6*f^3*g + 6*d^3*e^5*f^2*g^2 - 4*d^4*e^4*f*g^3 + d^5*e^3*g^4)*x^3 + 6*(d^2*
e^6*f^4 - 4*d^3*e^5*f^3*g + 6*d^4*e^4*f^2*g^2 - 4*d^5*e^3*f*g^3 + d^6*e^2*g
^4)*x^2 + 4*(d^3*e^5*f^4 - 4*d^4*e^4*f^3*g + 6*d^5*e^3*f^2*g^2 - 4*d^6*e^2*
f*g^3 + d^7*e*g^4)*x)
```

giac [B] time = 1.27, size = 1868, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(3*c*d^5*g^(13/2)*e^(11/2) + 21*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt
((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^4*g^(11/2)*e^(9/2) - 42*(sqrt(x*e +
d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*d^3*g^(9/2)*e
^(7/2) + 210*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e +
f*e^2))^6*c*d^2*g^(7/2)*e^(5/2) - 105*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt
((x*e + d)*g*e - d*g*e + f*e^2))^8*c*d*g^(5/2)*e^(3/2) + 105*(sqrt(x*e + d)
*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^10*c*g^(3/2)*e^(1/2
) - 23*c*d^4*f*g^(11/2)*e^(13/2) + 4*b*d^4*g^(13/2)*e^(13/2) - 140*(sqrt(x*
e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^3*f*g^(
9/2)*e^(11/2) + 28*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*
g*e + f*e^2))^2*b*d^3*g^(11/2)*e^(11/2) - 42*(sqrt(x*e + d)*sqrt(g)*e^(1/2)
 - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*d^2*f*g^(7/2)*e^(9/2) + 84*(sqr
t(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*b*d^2*g
^(9/2)*e^(9/2) - 140*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e -
d*g*e + f*e^2))^6*c*d*f*g^(5/2)*e^(7/2) - 140*(sqrt(x*e + d)*sqrt(g)*e^(1/2
) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*b*d*g^(7/2)*e^(7/2) - 455*(sqrt(
x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c*f*g^(3/
2)*e^(5/2) + 280*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*
e + f*e^2))^8*b*g^(5/2)*e^(5/2) + 86*c*d^3*f^2*g^(9/2)*e^(15/2) - 40*b*d^3*
f*g^(11/2)*e^(15/2) + 24*a*d^3*g^(13/2)*e^(15/2) + 462*(sqrt(x*e + d)*sqrt(
g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^2*f^2*g^(7/2)*e^(13
/2) - 252*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e
^2))^2*b*d^2*f*g^(9/2)*e^(13/2) + 168*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt
((x*e + d)*g*e - d*g*e + f*e^2))^2*a*d^2*g^(11/2)*e^(13/2) + 714*(sqrt(x*e
+ d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*d*f^2*g^(5/
2)*e^(11/2) - 672*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g
*e + f*e^2))^4*b*d*f*g^(7/2)*e^(11/2) + 504*(sqrt(x*e + d)*sqrt(g)*e^(1/2)
- sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*d*g^(9/2)*e^(11/2) + 770*(sqrt(x
*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c*f^2*g^(3
/2)*e^(9/2) - 700*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g
*e + f*e^2))^6*b*f*g^(5/2)*e^(9/2) + 840*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - s
qrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*g^(7/2)*e^(9/2) - 150*c*d^2*f^3*g^(
7/2)*e^(17/2) + 96*b*d^2*f^2*g^(9/2)*e^(17/2) - 72*a*d^2*f*g^(11/2)*e^(17/2
) - 588*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2
))^2*c*d*f^3*g^(5/2)*e^(15/2) + 420*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((
```

```
x*e + d)*g*e - d*g*e + f*e^2))^2*b*d*f^2*g^(7/2)*e^(15/2) - 336*(sqrt(x*e +
 d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*d*f*g^(9/2)*
e^(15/2) - 630*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
+ f*e^2))^4*c*f^3*g^(3/2)*e^(13/2) + 588*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - s
qrt((x*e + d)*g*e - d*g*e + f*e^2))^4*b*f^2*g^(5/2)*e^(13/2) - 504*(sqrt(x*
e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*f*g^(7/2)
*e^(13/2) + 119*c*d*f^4*g^(5/2)*e^(19/2) - 88*b*d*f^3*g^(7/2)*e^(19/2) + 72
*a*d*f^2*g^(9/2)*e^(19/2) + 245*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e
+ d)*g*e - d*g*e + f*e^2))^2*c*f^4*g^(3/2)*e^(17/2) - 196*(sqrt(x*e + d)*sq
rt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*b*f^3*g^(5/2)*e^(17/
2) + 168*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^
2))^2*a*f^2*g^(7/2)*e^(17/2) - 35*c*f^5*g^(3/2)*e^(21/2) + 28*b*f^4*g^(5/2)
*e^(21/2) - 24*a*f^3*g^(7/2)*e^(21/2))*e^(-1)/(d*g*e + (sqrt(x*e + d)*sqrt(
g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2 - f*e^2)^7
```

maple [A] time = 0.01, size = 468, normalized size = 1.67

$$\frac{2\sqrt{gx+f}\,\left(48a\,e^{3}g^{3}x^{3}+8bd\,e^{2}g^{3}x^{3}-56b\,e^{3}f\,g^{2}x^{3}+6c\,d^{2}e\,g^{3}x^{3}-28cd\,e^{2}f\,g^{2}x^{3}+70c\,e^{3}f^{2}g\,x^{3}+168ad\,e^{2}g^{3}x^{2}-24a\,e^{3}f\,g^{2}x^{2}+28b\,d^{2}e\,g^{3}x^{2}-200bd\,e^{2}f\,g^{2}x^{2}+28b\,e^{3}f^{2}g\,x^{2}+21c\,d^{3}g^{3}x^{2}-101c\,d^{2}ef\,g^{2}x^{2}+259cd\,e^{2}f^{2}g\,x^{2}-35c\,e^{3}f^{3}x^{2}+210a\,d^{2}e\,g^{3}x-84ad\,e^{2}f\,g^{2}x+18a\,e^{3}f^{2}gx+35b\,d^{3}g^{3}x-259b\,d^{2}ef\,g^{2}x+101bd\,e^{2}f^{2}gx-21b\,e^{3}f^{3}x-28c\,d^{3}f\,g^{2}x+200c\,d^{2}e\,f^{2}gx-28cd\,e^{2}f^{3}x+105a\,d^{3}g^{3}-105a\,d^{2}ef\,g^{2}+63ad\,e^{2}f^{2}g-15a\,e^{3}f^{3}-70b\,d^{3}f\,g^{2}+28b\,d^{2}e\,f^{2}g-6bd\,e^{2}f^{3}+56c\,d^{3}f^{2}g-8c\,d^{2}e\,f^{3}\right)}{105\left(ex+d\right)^{\frac{7}{2}}\left(g^{4}d^{4}-4e\,g^{3}f\,d^{3}+6d^{2}e^{2}f^{2}g^{2}-4d\,e^{3}f^{3}g+e^{4}f^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(48*a*e^3*g^3*x^3+8*b*d*e^2*g^3*x^3-56*b*e^3*f*g^2*x^3+
6*c*d^2*e*g^3*x^3-28*c*d*e^2*f*g^2*x^3+70*c*e^3*f^2*g*x^3+168*a*d*e^2*g^3*x
^2-24*a*e^3*f*g^2*x^2+28*b*d^2*e*g^3*x^2-200*b*d*e^2*f*g^2*x^2+28*b*e^3*f^2
*g*x^2+21*c*d^3*g^3*x^2-101*c*d^2*e*f*g^2*x^2+259*c*d*e^2*f^2*g*x^2-35*c*e^
3*f^3*x^2+210*a*d^2*e*g^3*x-84*a*d*e^2*f*g^2*x+18*a*e^3*f^2*g*x+35*b*d^3*g^
3*x-259*b*d^2*e*f*g^2*x+101*b*d*e^2*f^2*g*x-21*b*e^3*f^3*x-28*c*d^3*f*g^2*x
+200*c*d^2*e*f^2*g*x-28*c*d*e^2*f^3*x+105*a*d^3*g^3-105*a*d^2*e*f*g^2+63*a*
d*e^2*f^2*g-15*a*e^3*f^3-70*b*d^3*f*g^2+28*b*d^2*e*f^2*g-6*b*d*e^2*f^3+56*c
*d^3*f^2*g-8*c*d^2*e*f^3)/(e*x+d)^(7/2)/(d^4*g^4-4*d^3*e*f*g^3+6*d^2*e^2*f^
2*g^2-4*d*e^3*f^3*g+e^4*f^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [B] time = 4.65, size = 452, normalized size = 1.61

$$\frac{\sqrt{f+g\,x}\,\left(\frac{x^{3}\left(12\,c\,d^{2}\,e\,g^{3}-56\,c\,d\,e^{2}\,f\,g^{2}+16\,b\,d\,e^{2}\,g^{3}+140\,c\,e^{3}\,f^{2}\,g-112\,b\,e^{3}\,f\,g^{2}+96\,a\,e^{3}\,g^{3}\right)}{105\,e^{3}\,{\left(d\,g-e\,f\right)}^{4}}-\frac{-112\,c\,d^{3}\,f^{2}\,g+140\,b\,d^{3}\,f\,g^{2}-210\,a\,d^{3}\,g^{3}+16\,c\,d^{2}\,e\,f^{3}-56\,b\,d^{2}\,e\,f^{2}\,g+210\,a\,d^{2}\,e\,f\,g^{2}+12\,b\,d\,e^{2}\,f^{3}-126\,a\,d\,e^{2}\,f^{2}\,g+30\,a\,e^{3}\,f^{3}}{105\,e^{3}\,{\left(d\,g-e\,f\right)}^{4}}+\frac{x\,\left(-56\,c\,d^{3}\,f\,g^{2}+70\,b\,d^{3}\,g^{3}+400\,c\,d^{2}\,e\,f^{2}\,g-518\,b\,d^{2}\,e\,f\,g^{2}+420\,a\,d^{2}\,e\,g^{3}-56\,c\,d\,e^{2}\,f^{3}+202\,b\,d\,e^{2}\,f^{2}\,g-168\,a\,d\,e^{2}\,f\,g^{2}-42\,b\,e^{3}\,f^{3}+36\,a\,e^{3}\,f^{2}\,g\right)}{105\,e^{3}\,{\left(d\,g-e\,f\right)}^{4}}+\frac{2\,x^{2}\,\left(7\,d\,g-e\,f\right)\,\left(3\,c\,d^{2}\,g^{2}-14\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^{2}+35\,c\,e^{2}\,f^{2}-28\,b\,e^{2}\,f\,g+24\,a\,e^{2}\,g^{2}\right)}{105\,e^{3}\,{\left(d\,g-e\,f\right)}^{4}}\right)}{x^{3}\,\sqrt{d+e\,x}+\frac{d^{3}\,\sqrt{d+e\,x}}{e^{3}}+\frac{3\,d\,x^{2}\,\sqrt{d+e\,x}}{e}+\frac{3\,d^{2}\,x\,\sqrt{d+e\,x}}{e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(9/2)),x)
```

```
[Out] ((f + g*x)^(1/2)*((x^3*(96*a*e^3*g^3 + 16*b*d*e^2*g^3 + 12*c*d^2*e*g^3 - 11
2*b*e^3*f*g^2 + 140*c*e^3*f^2*g - 56*c*d*e^2*f*g^2))/(105*e^3*(d*g - e*f)^4
) - (30*a*e^3*f^3 - 210*a*d^3*g^3 + 12*b*d*e^2*f^3 + 16*c*d^2*e*f^3 + 140*b
*d^3*f*g^2 - 112*c*d^3*f^2*g - 126*a*d*e^2*f^2*g + 210*a*d^2*e*f*g^2 - 56*b
*d^2*e*f^2*g)/(105*e^3*(d*g - e*f)^4) + (x*(70*b*d^3*g^3 - 42*b*e^3*f^3 + 4
20*a*d^2*e*g^3 - 56*c*d*e^2*f^3 + 36*a*e^3*f^2*g - 56*c*d^3*f*g^2 - 168*a*d
*e^2*f*g^2 + 202*b*d*e^2*f^2*g - 518*b*d^2*e*f*g^2 + 400*c*d^2*e*f^2*g))/(1
```

```
05*e^3*(d*g - e*f)^4) + (2*x^2*(7*d*g - e*f)*(24*a*e^2*g^2 + 3*c*d^2*g^2 +
35*c*e^2*f^2 + 4*b*d*e*g^2 - 28*b*e^2*f*g - 14*c*d*e*f*g))/(105*e^3*(d*g -
e*f)^4)))/(x^3*(d + e*x)^(1/2) + (d^3*(d + e*x)^(1/2))/e^3 + (3*d*x^2*(d +
e*x)^(1/2))/e + (3*d^2*x*(d + e*x)^(1/2))/e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(9/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.585 $\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{(e+fx)^{3/2}}\,dx$

Optimal. Leaf size=249

$$\frac{\sqrt{d+ex}\,\sqrt{e+fx}\,\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{4ef^3\left(e^2-df\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{e+fx}}\right)\left(4ef\left(-2aef-\right.\right.}{4}$$

Rubi [A] time = 0.28, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {949, 80, 50, 63, 217, 206}

$$\frac{\sqrt{d+ex}\,\sqrt{e+fx}\,\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{4ef^3\left(e^2-df\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{e+fx}}\right)\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{4e^{3/2}f^{7/2}}+\frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{\left(e^2-df\right)\sqrt{e+fx}}+\frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(3/2))/((e^2 - d*f)*Sqrt[e + f*x]) +
 ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*Sq
rt[d + e*x]*Sqrt[e + f*x])/(4*e*f^3*(e^2 - d*f)) + (c*(d + e*x)^(3/2)*Sqrt[
e + f*x])/(2*e*f^2) - ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d
*e^2*f - d^2*f^2))*ArcTanh[(Sqrt[f]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[e + f*x])]
)/(4*e^(3/2)*f^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{(e+fx)^{3/2}}\,dx = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{2\int \frac{\sqrt{d+ex}\left(\frac{f\left(3be^2-bdf-2aef\right)-c\left(3e^3-def\right)}{2f^2}-\frac{1}{2}c\left(d-\frac{e^2}{f}\right)x\right)}{\sqrt{e+fx}}\,dx}{e^2-df}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-\right.}{4e}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2f^2\right)\right.}{4ef^3\left(e^2-df\right)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2f^2\right)\right.}{4ef^3\left(e^2-df\right)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2f^2\right)\right.}{4ef^3\left(e^2-df\right)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2f^2\right)\right.}{4ef^3\left(e^2-df\right)}$$

Mathematica [A] time = 1.09, size = 196, normalized size = 0.79

$$\frac{\frac{\sqrt{e^2-df}\sqrt{\frac{e(e+fx)}{e^2-df}}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e^2-df}}\right)\left(4ef\left(2aef+bdf-3be^2\right)+c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{e}+\sqrt{f}\sqrt{d+ex}\left(4ef(-2af+3be+bfx)+c\left(ef\left(d+2fx^2\right)+df^2x-15e^3-5e^2fx\right)\right)}{4ef^{7/2}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (Sqrt[f]*Sqrt[d + e*x]*(4*e*f*(3*b*e - 2*a*f + b*f*x) + c*(-15*e^3 - 5*e^2*
f*x + d*f^2*x + e*f*(d + 2*f*x^2))) + (Sqrt[e^2 - d*f]*(4*e*f*(-3*b*e^2 + b
*d*f + 2*a*e*f) + c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*Sqrt[(e*(e + f*x))/(e^2
 - d*f)]*ArcSinh[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[e^2 - d*f]])/e)/(4*e*f^(7/2)*
Sqrt[e + f*x])
```

IntegrateAlgebraic [A] time = 0.50, size = 382, normalized size = 1.53

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)\left(8ae^2f^2+4bdef^2-12be^3f-cd^2f^2-6cde^2f+15ce^4\right)}{4e^{3/2}f^{7/2}}-\frac{\sqrt{d+ex}\left(-\frac{16ae^2f^3(d+ex)}{e+fx}+\frac{8aef^4(d+ex)^2}{(e+fx)^2}+8ae^3f^2+\frac{20be^3f^2(d+ex)}{e+fx}-\frac{8be^2f^3(d+ex)^2}{(e+fx)^2}+4bde^2f^2-\frac{4bdef^3(d+ex)}{e+fx}-12be^4f-\frac{cd^2f^3(d+ex)}{e+fx}-cd^2ef^2-\frac{25ce^4f(d+ex)}{e+fx}+\frac{8ce^3f^2(d+ex)^2}{(e+fx)^2}-6cde^3f+\frac{10cde^2f^2(d+ex)}{e+fx}+15ce^5\right)}{4ef^3\sqrt{e+fx}\left(e-\frac{f(d+ex)}{e+fx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] -1/4*(Sqrt[d + e*x]*(15*c*e^5 - 6*c*d*e^3*f - 12*b*e^4*f - c*d^2*e*f^2 + 4*
b*d*e^2*f^2 + 8*a*e^3*f^2 + (8*c*e^3*f^2*(d + e*x)^2)/(e + f*x)^2 - (8*b*e^
2*f^3*(d + e*x)^2)/(e + f*x)^2 + (8*a*e*f^4*(d + e*x)^2)/(e + f*x)^2 - (25*
c*e^4*f*(d + e*x))/(e + f*x) + (10*c*d*e^2*f^2*(d + e*x))/(e + f*x) + (20*b
*e^3*f^2*(d + e*x))/(e + f*x) - (c*d^2*f^3*(d + e*x))/(e + f*x) - (4*b*d*e*
f^3*(d + e*x))/(e + f*x) - (16*a*e^2*f^3*(d + e*x))/(e + f*x)))/(e*f^3*Sqrt
[e + f*x]*(e - (f*(d + e*x))/(e + f*x))^2) + ((15*c*e^4 - 6*c*d*e^2*f - 12*
b*e^3*f - c*d^2*f^2 + 4*b*d*e*f^2 + 8*a*e^2*f^2)*ArcTanh[(Sqrt[f]*Sqrt[d +
e*x])/(Sqrt[e]*Sqrt[e + f*x])])/(4*e^(3/2)*f^(7/2))
```

fricas [A] time = 0.98, size = 580, normalized size = 2.33

$$\left[\frac{\left(15ce^5-\left(cd^2e-4bde^2-8ae^3\right)f^2-6\left(cde^3+2be^4\right)f+\left(15ce^4f-\left(cd^2-4bde-8ae^2\right)f^3-6\left(cde^2+2be^3\right)f^2\right)x\right)\sqrt{ef}\log\left(8e^2f^2x^2+e^4+6de^2f+d^2f^2+4\left(2efx+e^2+df\right)\sqrt{ef}\sqrt{ex+d}\sqrt{fx+e}+8\left(e^3f+def^2\right)x\right)+4\left(2ce^2f^3x^2-15ce^4f-8ae^2f^3+\left(cde^2+12be^3\right)f^2-\left(5ce^3f^2-\left(cde+4be^2\right)f^3\right)x\right)\sqrt{ex+d}\sqrt{fx+e}}{16\left(e^2f^5x+e^3f^4\right)},\ -\frac{\left(15ce^5-\left(cd^2e-4bde^2-8ae^3\right)f^2-6\left(cde^3+2be^4\right)f+\left(15ce^4f-\left(cd^2-4bde-8ae^2\right)f^3-6\left(cde^2+2be^3\right)f^2\right)x\right)\sqrt{-ef}\arctan\left(\frac{\left(2efx+e^2+df\right)\sqrt{-ef}\sqrt{ex+d}\sqrt{fx+e}}{2\left(e^2f^2x^2+de^2f+\left(e^3f+def^2\right)x\right)}\right)-2\left(2ce^2f^3x^2-15ce^4f-8ae^2f^3+\left(cde^2+12be^3\right)f^2-\left(5ce^3f^2-\left(cde+4be^2\right)f^3\right)x\right)\sqrt{ex+d}\sqrt{fx+e}}{8\left(e^2f^5x+e^3f^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((15*c*e^5 - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)*f^2 - 6*(c*d*e^3 + 2*b*e
^4)*f + (15*c*e^4*f - (c*d^2 - 4*b*d*e - 8*a*e^2)*f^3 - 6*(c*d*e^2 + 2*b*e^
3)*f^2)*x)*sqrt(e*f)*log(8*e^2*f^2*x^2 + e^4 + 6*d*e^2*f + d^2*f^2 + 4*(2*e
*f*x + e^2 + d*f)*sqrt(e*f)*sqrt(e*x + d)*sqrt(f*x + e) + 8*(e^3*f + d*e*f^
2)*x) + 4*(2*c*e^2*f^3*x^2 - 15*c*e^4*f - 8*a*e^2*f^3 + (c*d*e^2 + 12*b*e^3
)*f^2 - (5*c*e^3*f^2 - (c*d*e + 4*b*e^2)*f^3)*x)*sqrt(e*x + d)*sqrt(f*x + e
))/(e^2*f^5*x + e^3*f^4), -1/8*((15*c*e^5 - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)
*f^2 - 6*(c*d*e^3 + 2*b*e^4)*f + (15*c*e^4*f - (c*d^2 - 4*b*d*e - 8*a*e^2)*
f^3 - 6*(c*d*e^2 + 2*b*e^3)*f^2)*x)*sqrt(-e*f)*arctan(1/2*(2*e*f*x + e^2 +
d*f)*sqrt(-e*f)*sqrt(e*x + d)*sqrt(f*x + e)/(e^2*f^2*x^2 + d*e^2*f + (e^3*f
 + d*e*f^2)*x)) - 2*(2*c*e^2*f^3*x^2 - 15*c*e^4*f - 8*a*e^2*f^3 + (c*d*e^2
+ 12*b*e^3)*f^2 - (5*c*e^3*f^2 - (c*d*e + 4*b*e^2)*f^3)*x)*sqrt(e*x + d)*sq
rt(f*x + e))/(e^2*f^5*x + e^3*f^4)]
```

giac [A] time = 0.44, size = 237, normalized size = 0.95

$$\frac{\left((xe+d)\left(\frac{2(xe+d)ce^{(-1)}}{f}-\frac{\left(3cdf^4e^2-4bf^4e^3+5cf^3e^4\right)e^{(-3)}}{f^5}\right)+\frac{\left(cd^2f^4e^2-4bdf^4e^3+6cdf^3e^4-8af^4e^4+12bf^3e^5-15cf^2e^6\right)e^{(-3)}}{f^5}\right)\sqrt{xe+d}}{4\sqrt{(xe+d)fe-dfe+e^3}}+\frac{\left(cd^2f^2-4bdf^2e+6cdfe^2-8af^2e^2+12bfe^3-15ce^4\right)e^{\left(-\frac{3}{2}\right)}\log\left(\left|-\sqrt{xe+d}\sqrt{f}e^{\frac{1}{2}}+\sqrt{(xe+d)fe-dfe+e^3}\right|\right)}{4f^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((x*e + d)*(2*(x*e + d)*c*e^(-1)/f - (3*c*d*f^4*e^2 - 4*b*f^4*e^3 + 5*c
*f^3*e^4)*e^(-3)/f^5) + (c*d^2*f^4*e^2 - 4*b*d*f^4*e^3 + 6*c*d*f^3*e^4 - 8*
a*f^4*e^4 + 12*b*f^3*e^5 - 15*c*f^2*e^6)*e^(-3)/f^5)*sqrt(x*e + d)/sqrt((x*
e + d)*f*e - d*f*e + e^3) + 1/4*(c*d^2*f^2 - 4*b*d*f^2*e + 6*c*d*f*e^2 - 8*
a*f^2*e^2 + 12*b*f*e^3 - 15*c*e^4)*e^(-3/2)*log(abs(-sqrt(x*e + d)*sqrt(f)*
e^(1/2) + sqrt((x*e + d)*f*e - d*f*e + e^3)))/f^(7/2)
```

maple [B] time = 0.04, size = 834, normalized size = 3.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

```
[Out] 1/8*(e*x+d)^(1/2)*(8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+
d*f+e^2)/(e*f)^(1/2))*x*a*e^2*f^3+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/
2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*x*b*d*e*f^3-12*ln(1/2*(2*e*f*x+2*((e*x
+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*x*b*e^3*f^2-ln(1/2*(2*
e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*x*c*d^2*f
^3-6*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(
1/2))*x*c*d*e^2*f^2+15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2
)+d*f+e^2)/(e*f)^(1/2))*x*c*e^4*f+4*x^2*c*e*f^2*((e*x+d)*(f*x+e))^(1/2)*(e*
f)^(1/2)+8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(
e*f)^(1/2))*a*e^3*f^2+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/
2)+d*f+e^2)/(e*f)^(1/2))*b*d*e^2*f^2-12*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))
^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*e^4*f-ln(1/2*(2*e*f*x+2*((e*x+d)
*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d^2*e*f^2-6*ln(1/2*(2*e
*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d*e^3*f+
15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/
2))*c*e^5+8*x*b*e*f^2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+2*x*c*d*f^2*((e*x
+d)*(f*x+e))^(1/2)*(e*f)^(1/2)-10*x*c*e^2*f*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(
1/2)-16*a*e*f^2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+24*b*e^2*f*((e*x+d)*(f*
x+e))^(1/2)*(e*f)^(1/2)+2*c*d*e*f*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)-30*c*
e^3*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2))/(e*f)^(1/2)/e/((e*x+d)*(f*x+e))^(1
/2)/f^3/(f*x+e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*f-e^2>0)', see `assume?` for more
 details)Is d*f-e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}\,\left(c\,x^2+b\,x+a\right)}{{\left(e+f\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(e + f*x)^(3/2),x)
```

```
[Out] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(e + f*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex} \left(a + bx + cx^{2}\right)}{\left(e + fx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(a + b*x + c*x**2)/(e + f*x)**(3/2), x)
```

3.586 $\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=240

$$\frac{(bd-ae)^2\left(35a^2e^2-90abde+73b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}+\frac{\sqrt{a+bx}\sqrt{d+ex}\,(bd-ae)\left(35a^2e^2-90abde+73b\right.}{8b^4}$$

Rubi [A] time = 0.23, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{a+bx}\,(d+ex)^{3/2}\left(35a^2e^2-90abde+73b^2d^2\right)}{12b^3}+\frac{\sqrt{a+bx}\sqrt{d+ex}\,(bd-ae)\left(35a^2e^2-90abde+73b^2d^2\right)}{8b^4}+\frac{(bd-ae)^2\left(35a^2e^2-90abde+73b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}+\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}+\frac{\sqrt{a+bx}\,(d+ex)^{5/2}(17bd-13ae)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] ((b*d - a*e)*(73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d +
e*x])/(8*b^4) + ((73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*(d +
e*x)^(3/2))/(12*b^3) + ((17*b*d - 13*a*e)*Sqrt[a + b*x]*(d + e*x)^(5/2))/(3
*b^2) + (2*e*(a + b*x)^(3/2)*(d + e*x)^(5/2))/b^2 + ((b*d - a*e)^2*(73*b^2*
d^2 - 90*a*b*d*e + 35*a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqr
t[d + e*x])])/(8*b^(9/2)*Sqrt[e])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\int \frac{(d+ex)^{3/2}\left(4e\left(15b^2d^2-3abde-5a^2e^2\right)+4be^2(17bd-13ae)\cdots}{\sqrt{a+bx}}}{4b^2e}$$

$$= \frac{(17bd-13ae)\sqrt{a+bx}\,(d+ex)^{5/2}}{3b^2} + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\left(73b^2d\cdots\right.}{\cdots}$$

$$= \frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,(d+ex)^{3/2}}{12b^3} + \frac{(17bd-13ae)\sqrt{a+\cdots}}{3b^2}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-\cdots\right.}{\cdots}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-\cdots\right.}{\cdots}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-\cdots\right.}{\cdots}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.85, size = 204, normalized size = 0.85

$$\frac{\sqrt{d+ex}\left(\frac{3\left(35a^2e^2-90abde+73b^2d^2\right)(bd-ae)^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{\sqrt{e}\sqrt{\frac{b(d+ex)}{bd-ae}}}+\sqrt{a+bx}\left(-105a^3e^3+5a^2be^2(89d+14ex)-ab^2e\left(725d^2+292dex+56e^2x^2\right)+b^3\left(501d^3+466d^2ex+232de^2x^2+48e^3x^3\right)\right)\right)}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x
]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[a + b*x]*(-105*a^3*e^3 + 5*a^2*b*e^2*(89*d + 14*e*x) -
 a*b^2*e*(725*d^2 + 292*d*e*x + 56*e^2*x^2) + b^3*(501*d^3 + 466*d^2*e*x +
232*d*e^2*x^2 + 48*e^3*x^3)) + (3*(b*d - a*e)^(3/2)*(73*b^2*d^2 - 90*a*b*d*
e + 35*a^2*e^2)*ArcSinh[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(Sqrt[e]*
Sqrt[(b*(d + e*x))/(b*d - a*e)])))/(24*b^4)
```

IntegrateAlgebraic [A] time = 0.46, size = 367, normalized size = 1.53

$$\frac{\left(35a^2e^2-90abde+73b^2d^2\right)(bd-ae)^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}+\frac{\sqrt{a+bx}\,(bd-ae)^2\left(279a^2b^3e^2-\frac{511a^2b^2e^3(a+bx)}{d+ex}-\frac{105a^2e^5(a+bx)^3}{(d+ex)^3}+\frac{385a^2be^4(a+bx)^2}{(d+ex)^2}-\frac{1037b^4d^2e(a+bx)}{d+ex}-690ab^4de+\frac{803b^3d^2e^2(a+bx)^2}{(d+ex)^2}+\frac{1314ab^3de^2(a+bx)}{d+ex}-\frac{219b^2d^2e^3(a+bx)^3}{(d+ex)^3}-\frac{990ab^2de^3(a+bx)^2}{(d+ex)^2}+\frac{270abde^4(a+bx)^3}{(d+ex)^3}+501b^5d^2\right)}{24b^4\sqrt{d+ex}\left(b-\frac{e(a+bx)}{d+ex}\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a
 + b*x],x]
```

```
[Out] ((b*d - a*e)^2*Sqrt[a + b*x]*(501*b^5*d^2 - 690*a*b^4*d*e + 279*a^2*b^3*e^2
 - (219*b^2*d^2*e^3*(a + b*x)^3)/(d + e*x)^3 + (270*a*b*d*e^4*(a + b*x)^3)/
(d + e*x)^3 - (105*a^2*e^5*(a + b*x)^3)/(d + e*x)^3 + (803*b^3*d^2*e^2*(a +
 b*x)^2)/(d + e*x)^2 - (990*a*b^2*d*e^3*(a + b*x)^2)/(d + e*x)^2 + (385*a^2
*b*e^4*(a + b*x)^2)/(d + e*x)^2 - (1037*b^4*d^2*e*(a + b*x))/(d + e*x) + (1
314*a*b^3*d*e^2*(a + b*x))/(d + e*x) - (511*a^2*b^2*e^3*(a + b*x))/(d + e*x
)))/(24*b^4*Sqrt[d + e*x]*(b - (e*(a + b*x))/(d + e*x))^4) + ((b*d - a*e)^2
*(73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sq
rt[b]*Sqrt[d + e*x])])/(8*b^(9/2)*Sqrt[e])
```

fricas [A] time = 0.46, size = 546, normalized size = 2.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/96*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d^2*e^2 - 160*a^3*b*d*
e^3 + 35*a^4*e^4)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e
^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2
*d*e + a*b*e^2)*x) + 4*(48*b^4*e^4*x^3 + 501*b^4*d^3*e - 725*a*b^3*d^2*e^2
+ 445*a^2*b^2*d*e^3 - 105*a^3*b*e^4 + 8*(29*b^4*d*e^3 - 7*a*b^3*e^4)*x^2 +
2*(233*b^4*d^2*e^2 - 146*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x)*sqrt(b*x + a)*sqr
t(e*x + d))/(b^5*e), -1/48*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d
^2*e^2 - 160*a^3*b*d*e^3 + 35*a^4*e^4)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d
 + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^
2*d*e + a*b*e^2)*x)) - 2*(48*b^4*e^4*x^3 + 501*b^4*d^3*e - 725*a*b^3*d^2*e^
2 + 445*a^2*b^2*d*e^3 - 105*a^3*b*e^4 + 8*(29*b^4*d*e^3 - 7*a*b^3*e^4)*x^2
+ 2*(233*b^4*d^2*e^2 - 146*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x)*sqrt(b*x + a)*s
qrt(e*x + d))/(b^5*e)]
```

giac [B] time = 0.52, size = 717, normalized size = 2.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -1/24*(360*((b^2*d - a*b*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2)
 + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/sqrt(b) - sqrt(b^2*d + (b*x + a)*b
*e - a*b*e)*sqrt(b*x + a))*d^3*abs(b)/b^2 - 28*(sqrt(b^2*d + (b*x + a)*b*e
- a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^
5*e^4)*e^(-4)/b^7) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)*e^(-4
)/b^7) - 3*(b^3*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*e^(-5/2)*log
(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))
/b^(3/2))*d*abs(b)*e^2/b^2 - 210*((b^3*d^2 + 2*a*b^2*d*e - 3*a^2*b*e^2)*e^(
-3/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e -
 a*b*e)))/sqrt(b) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*(2*b*x + (b*d*e - 5
```

```
*a*e^2)*e^(-2) + 2*a)*sqrt(b*x + a))*d^2*abs(b)*e/b^3 - (sqrt(b^2*d + (b*x
+ a)*b*e - a*b*e)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*d*e^5
- 25*a*b^11*e^6)*e^(-6)/b^14) - (5*b^13*d^2*e^4 + 14*a*b^12*d*e^5 - 163*a^2
*b^11*e^6)*e^(-6)/b^14) + 3*(5*b^14*d^3*e^3 + 9*a*b^13*d^2*e^4 + 15*a^2*b^1
2*d*e^5 - 93*a^3*b^11*e^6)*e^(-6)/b^14)*sqrt(b*x + a) + 3*(5*b^4*d^4 + 4*a*
b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 - 35*a^4*e^4)*e^(-7/2)*log(a
bs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/b
^(5/2))*abs(b)*e^3/b^2)/b
```

maple [B] time = 0.04, size = 571, normalized size = 2.38

$$\frac{\sqrt{ex+d}\,\sqrt{bx+a}\,\left(105a^4e^4\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-480a^3bde^3\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+864a^2b^2d^2e^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-708ab^3d^3e\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+219b^4d^4\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+96\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,b^3e^3x^3-112\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,ab^2e^3x^2+464\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,b^3de^2x^2+140\sqrt{be}\,\sqrt{(bx+a)(ex+d)}\,a^2be^3x-584\sqrt{be}\,\sqrt{(bx+a)(ex+d)}\,ab^2de^2x+932\sqrt{be}\,\sqrt{(bx+a)(ex+d)}\,b^3d^2ex-210\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,a^3e^3+890\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,a^2bde^2-1450\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,ab^2d^2e+1002\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,b^3d^3\right)}{48\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/48*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(96*x^3*b^3*e^3*((b*x+a)*(e*x+d))^(1/2)*(b
*e)^(1/2)-112*x^2*a*b^2*e^3*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+464*x^2*b^3
*d*e^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+105*ln(1/2*(2*b*e*x+2*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^4*e^4-480*ln(1/2*(2*b*e*x
+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^3*b*d*e^3+86
4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2
))*a^2*b^2*d^2*e^2-708*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2
)+a*e+b*d)/(b*e)^(1/2))*a*b^3*d^3*e+219*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))
^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^4*d^4+140*(b*e)^(1/2)*((b*x+a)*(
e*x+d))^(1/2)*x*a^2*b*e^3-584*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*x*a*b^2*d
*e^2+932*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*x*b^3*d^2*e-210*((b*x+a)*(e*x+
d))^(1/2)*(b*e)^(1/2)*a^3*e^3+890*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*a^2*b
*d*e^2-1450*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*a*b^2*d^2*e+1002*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)*b^3*d^3)/b^4/((b*x+a)*(e*x+d))^(1/2)/(b*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x\right)}^{3/2}\,\left(15\,d^2+20\,d\,e\,x+8\,e^2\,x^2\right)}{\sqrt{a+b\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(3/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2),x)
```

```
[Out] int(((d + e*x)^(3/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.587 $\int \frac{\sqrt{d+ex}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=176

$$\frac{(bd-ae)\left(5a^2e^2-13abde+11b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}+\frac{\sqrt{a+bx}\sqrt{d+ex}\left(5a^2e^2-13abde+11b^2d^2\right)}{b^3}+\frac{8e(a+$$

Rubi [A] time = 0.17, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{a+bx}\sqrt{d+ex}\left(5a^2e^2-13abde+11b^2d^2\right)}{b^3}+\frac{(bd-ae)\left(5a^2e^2-13abde+11b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}+\frac{2\sqrt{a+bx}(d+ex)^{3/2}(4bd-3ae)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] ((11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^3 + (
2*(4*b*d - 3*a*e)*Sqrt[a + b*x]*(d + e*x)^(3/2))/b^2 + (8*e*(a + b*x)^(3/2)
*(d + e*x)^(3/2))/(3*b^2) + ((b*d - a*e)*(11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e
^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(7/2)*Sqrt
[e])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{\int \frac{\sqrt{d+ex}\left(3e(3bd-2ae)(5bd+2ae)+12be^2(4bd-3ae)x\right)}{\sqrt{a+bx}}\,dx}{3b^2e}$$

$$= \frac{2(4bd-3ae)\sqrt{a+bx}\,(d+ex)^{3/2}}{b^2} + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{\left(11b^2d^2-\right.}{}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}\,(d}{b^2}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}\,(d}{b^2}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}\,(d}{b^2}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}\,(d}{b^2}$$

Mathematica [A] time = 0.44, size = 163, normalized size = 0.93

$$\frac{\sqrt{d+ex}\left(\sqrt{a+bx}\left(15a^2e^2-abe(49d+10ex)+b^2\left(57d^2+32dex+8e^2x^2\right)\right)+\frac{3\sqrt{bd-ae}\left(5a^2e^2-13abde+11b^2d^2\right)\sinh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{\sqrt{e}\sqrt{\frac{b(d+ex)}{bd-ae}}}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[a + b*x]*(15*a^2*e^2 - a*b*e*(49*d + 10*e*x) + b^2*(57
*d^2 + 32*d*e*x + 8*e^2*x^2)) + (3*Sqrt[b*d - a*e]*(11*b^2*d^2 - 13*a*b*d*e
 + 5*a^2*e^2)*ArcSinh[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(Sqrt[e]*Sq
rt[(b*(d + e*x))/(b*d - a*e)])))/(3*b^3)
```

IntegrateAlgebraic [A] time = 0.67, size = 220, normalized size = 1.25

$$\frac{\sqrt{a+\frac{b(d+ex)}{e}-\frac{bd}{e}}\left(15a^2e^2\sqrt{d+ex}-10abe(d+ex)^{3/2}-39abde\sqrt{d+ex}+33b^2d^2\sqrt{d+ex}+8b^2(d+ex)^{5/2}+16b^2d(d+ex)^{3/2}\right)}{3b^3}-\frac{\sqrt{\frac{b}{e}}\left(-5a^3e^3+18a^2bde^2-24ab^2d^2e+11b^3d^3\right)\log\left(\sqrt{a+\frac{b(d+ex)}{e}-\frac{bd}{e}}-\sqrt{\frac{b}{e}}\sqrt{d+ex}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a +
 b*x],x]
```

[Out] (Sqrt[a - (b*d)/e + (b*(d + e*x))/e]*(33*b^2*d^2*Sqrt[d + e*x] - 39*a*b*d*e*Sqrt[d + e*x] + 15*a^2*e^2*Sqrt[d + e*x] + 16*b^2*d*(d + e*x)^(3/2) - 10*a*b*e*(d + e*x)^(3/2) + 8*b^2*(d + e*x)^(5/2)))/(3*b^3) - (Sqrt[b/e]*(11*b^3*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*Log[-(Sqrt[b/e]*Sqrt[d + e*x]) + Sqrt[a - (b*d)/e + (b*(d + e*x))/e]])/b^4

fricas [A] time = 0.46, size = 414, normalized size = 2.35

$$\left[-\frac{3\left(11\,b^{3}d^{3}-24\,ab^{2}d^{2}e+18\,a^{2}bde^{2}-5\,a^{3}e^{3}\right)\sqrt{be}\,\log\left(8\,b^{2}e^{2}x^{2}+b^{2}d^{2}+6\,abde+a^{2}e^{2}-4\,(2\,bex+bd+ae)\sqrt{be}\,\sqrt{bx+a}\,\sqrt{ex+d}+8\left(b^{2}de+abe^{2}\right)x\right)-4\left(8\,b^{3}e^{3}x^{2}+57\,b^{3}d^{2}e-49\,ab^{2}de^{2}+15\,a^{2}be^{3}+2\left(16\,b^{3}de^{2}-5\,ab^{2}e^{3}\right)x\right)\sqrt{bx+a}\,\sqrt{ex+d}}{12\,b^{4}e}, -\frac{3\left(11\,b^{3}d^{3}-24\,ab^{2}d^{2}e+18\,a^{2}bde^{2}-5\,a^{3}e^{3}\right)\sqrt{-be}\,\arctan\left(\frac{(2\,bex+bd+ae)\sqrt{-be}\,\sqrt{bx+a}\,\sqrt{ex+d}}{2\left(b^{2}e^{2}x^{2}+abde+\left(b^{2}de+abe^{2}\right)x\right)}\right)-2\left(8\,b^{3}e^{3}x^{2}+57\,b^{3}d^{2}e-49\,ab^{2}de^{2}+15\,a^{2}be^{3}+2\left(16\,b^{3}de^{2}-5\,ab^{2}e^{3}\right)x\right)\sqrt{bx+a}\,\sqrt{ex+d}}{6\,b^{4}e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/12*(3*(11*b^3*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 - 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) - 4*(8*b^3*e^3*x^2 + 57*b^3*d^2*e - 49*a*b^2*d*e^2 + 15*a^2*b*e^3 + 2*(16*b^3*d*e^2 - 5*a*b^2*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d))/(b^4*e), -1/6*(3*(11*b^3*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - 2*(8*b^3*e^3*x^2 + 57*b^3*d^2*e - 49*a*b^2*d*e^2 + 15*a^2*b*e^3 + 2*(16*b^3*d*e^2 - 5*a*b^2*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d))/(b^4*e)]

giac [B] time = 0.38, size = 441, normalized size = 2.51

$$-\frac{\frac{45\left(\frac{\left(b^{2}d-abe\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}e^{\frac{1}{2}}+\sqrt{b^{2}d+(bx+a)be-abe}\right|\right)}{\sqrt{b}}-\sqrt{b^{2}d+(bx+a)be-abe}\,\sqrt{bx+a}\right)d^{2}|b|}{b^{2}}-\frac{\left(\sqrt{b^{2}d+(bx+a)be-abe}\,\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^{2}}+\frac{\left(b^{6}de^{3}-13\,ab^{5}e^{4}\right)e^{(-4)}}{b^{7}}\right)-\frac{3\left(b^{7}d^{2}e^{2}+2\,ab^{6}de^{3}-11\,a^{2}b^{5}e^{4}\right)e^{(-4)}}{b^{7}}\right)-\frac{3\left(b^{3}d^{3}+ab^{2}d^{2}e+3\,a^{2}bde^{2}-5\,a^{3}e^{3}\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}e^{\frac{1}{2}}+\sqrt{b^{2}d+(bx+a)be-abe}\right|\right)}{b^{\frac{3}{2}}}\right)|b|e^{2}}{b^{2}}-\frac{15\left(\frac{\left(b^{3}d^{2}+2\,ab^{2}de-3\,a^{2}be^{2}\right)e^{\left(-\frac{3}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}e^{\frac{1}{2}}+\sqrt{b^{2}d+(bx+a)be-abe}\right|\right)}{\sqrt{b}}+\sqrt{b^{2}d+(bx+a)be-abe}\left(2\,bx+\left(bde-5\,ae^{2}\right)e^{(-2)}+2\,a\right)\sqrt{bx+a}\right)d|b|e}{b^{3}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algorithm="giac")

[Out] -1/3*(45*((b^2*d - a*b*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/sqrt(b) - sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b*x + a))*d^2*abs(b)/b^2 - (sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^5*e^4)*e^(-4)/b^7) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)*e^(-4)/b^7) - 3*(b^3*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*e^(-5/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/b^(3/2))*abs(b)*e^2/b^2 - 15*((b^3*d^2 + 2*a*b^2*d*e - 3*a^2*b*e^2)*e^(-3/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/sqrt(b) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*(2*b*x + (b*d*e - 5*a*e^2)*e^(-2) + 2*a)*sqrt(b*x + a))*d*abs(b)*e/b^3)/b

maple [B] time = 0.02, size = 392, normalized size = 2.23

$$-\frac{\sqrt{ex+d}\,\sqrt{bx+a}\,\left(15a^{3}e^{3}\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-54a^{2}bd\,e^{2}\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+72a\,b^{2}d^{2}e\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-33b^{3}d^{3}\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-16\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,b^{2}e^{2}x^{2}+20\sqrt{be}\,\sqrt{(bx+a)(ex+d)}\,ab\,e^{2}x-64\sqrt{be}\,\sqrt{(bx+a)(ex+d)}\,b^{2}dex-30\sqrt{be}\,\sqrt{(bx+a)(ex+d)}\,a^{2}e^{2}+98\sqrt{be}\,\sqrt{(bx+a)(ex+d)}\,abde-114\sqrt{be}\,\sqrt{(bx+a)(ex+d)}\,b^{2}d^{2}\right)}{6\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x)

[Out] -1/6*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(-16*x^2*b^2*e^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+15*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*a^3*e^3-54*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*a^2*b*d*e^2+72*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*a*b^2*d^2*e-33*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*b^3*d^3+20*(b*e)

```
^(1/2)*((b*x+a)*(e*x+d))^(1/2)*x*a*b*e^2-64*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(
1/2)*x*b^2*d*e-30*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a^2*e^2+98*(b*e)^(1/2
)*((b*x+a)*(e*x+d))^(1/2)*a*b*d*e-114*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*b
^2*d^2)/b^3/((b*x+a)*(e*x+d))^(1/2)/(b*e)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [B] time = 73.15, size = 1797, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2),x)
```

```
[Out] ((((a + b*x)^(1/2) - a^(1/2))^3*(70*b^2*d^3 + 110*a^2*d*e^2 + 460*a*b*d^2*e
))/(e^3*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))*(10*b
^3*d^3 + 20*a*b^2*d^2*e - 30*a^2*b*d*e^2))/(e^4*((d + e*x)^(1/2) - d^(1/2))
) - (160*a^(1/2)*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2)
 - d^(1/2))^6) + (((a + b*x)^(1/2) - a^(1/2))^7*(10*b^2*d^3 - 30*a^2*d*e^2
+ 20*a*b*d^2*e))/(b^2*e*((d + e*x)^(1/2) - d^(1/2))^7) + (((a + b*x)^(1/2)
- a^(1/2))^5*(70*b^2*d^3 + 110*a^2*d*e^2 + 460*a*b*d^2*e))/(b*e^2*((d + e*x
)^(1/2) - d^(1/2))^5) - (a^(1/2)*d^(1/2)*(320*b*d^2 + 640*a*d*e)*((a + b*x)
^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4) - (160*a^(1/2)*b^2
*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2)
)/(((a + b*x)^(1/2) - a^(1/2))^8/((d + e*x)^(1/2) - d^(1/2))^8 + b^4/e^4 -
(4*b^3*((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2) +
 (6*b^2*((a + b*x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4)
- (4*b*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2) - d^(1/2))^6)) -
((((a + b*x)^(1/2) - a^(1/2))*(2*b^5*d^3 - 10*a^3*b^2*e^3 + 6*a^2*b^3*d*e^2
 + 2*a*b^4*d^2*e))/(e^6*((d + e*x)^(1/2) - d^(1/2))) - (((a + b*x)^(1/2) -
a^(1/2))^5*(132*a^3*e^3 + 76*b^3*d^3 + 1100*a*b^2*d^2*e + 1252*a^2*b*d*e^2)
)/(e^4*((d + e*x)^(1/2) - d^(1/2))^5) - (((a + b*x)^(1/2) - a^(1/2))^3*((34
*b^4*d^3)/3 - (170*a^3*b*e^3)/3 + 34*a^2*b^2*d*e^2 + 182*a*b^3*d^2*e))/(e^5
*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))^11*(2*b^3*d^
3 - 10*a^3*e^3 + 2*a*b^2*d^2*e + 6*a^2*b*d*e^2))/(b^3*e*((d + e*x)^(1/2) -
d^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^9*((34*b^3*d^3)/3 - (170*a^3*e^
3)/3 + 182*a*b^2*d^2*e + 34*a^2*b*d*e^2))/(b^2*e^2*((d + e*x)^(1/2) - d^(1/
2))^9) - (((a + b*x)^(1/2) - a^(1/2))^7*(132*a^3*e^3 + 76*b^3*d^3 + 1100*a*
b^2*d^2*e + 1252*a^2*b*d*e^2))/(b*e^3*((d + e*x)^(1/2) - d^(1/2))^7) + (a^(
1/2)*d^(1/2)*((a + b*x)^(1/2) - a^(1/2))^6*(1024*a^2*e^2 + 512*b^2*d^2 + (5
632*a*b*d*e)/3))/(e^3*((d + e*x)^(1/2) - d^(1/2))^6) + (a^(1/2)*d^(1/2)*(25
6*b*d^2 + 768*a*d*e)*((a + b*x)^(1/2) - a^(1/2))^8)/(e^2*((d + e*x)^(1/2) -
 d^(1/2))^8) + (a^(1/2)*d^(1/2)*(256*b^3*d^2 + 768*a*b^2*d*e)*((a + b*x)^(1
/2) - a^(1/2))^4)/(e^4*((d + e*x)^(1/2) - d^(1/2))^4))/(((a + b*x)^(1/2) -
a^(1/2))^12/((d + e*x)^(1/2) - d^(1/2))^12 + b^6/e^6 - (6*b^5*((a + b*x)^(1
/2) - a^(1/2))^2)/(e^5*((d + e*x)^(1/2) - d^(1/2))^2) + (15*b^4*((a + b*x)^
(1/2) - a^(1/2))^4)/(e^4*((d + e*x)^(1/2) - d^(1/2))^4) - (20*b^3*((a + b*x
)^(1/2) - a^(1/2))^6)/(e^3*((d + e*x)^(1/2) - d^(1/2))^6) + (15*b^2*((a + b
*x)^(1/2) - a^(1/2))^8)/(e^2*((d + e*x)^(1/2) - d^(1/2))^8) - (6*b*((a + b*
x)^(1/2) - a^(1/2))^10)/(e*((d + e*x)^(1/2) - d^(1/2))^10)) + (((30*b*d^3 +
```

```
30*a*d^2*e)*((a + b*x)^(1/2) - a^(1/2)))/(e^2*((d + e*x)^(1/2) - d^(1/2)))
 - (120*a^(1/2)*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d + e*x)^(1/2)
- d^(1/2))^2) + ((30*b*d^3 + 30*a*d^2*e)*((a + b*x)^(1/2) - a^(1/2))^3)/(b*
e*((d + e*x)^(1/2) - d^(1/2))^3))/(((a + b*x)^(1/2) - a^(1/2))^4/((d + e*x)
^(1/2) - d^(1/2))^4 + b^2/e^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d
+ e*x)^(1/2) - d^(1/2))^2)) - (2*atanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2))
)/(b^(1/2)*((d + e*x)^(1/2) - d^(1/2))))*(a*e - b*d)*(5*a^2*e^2 + b^2*d^2 +
 2*a*b*d*e))/(b^(7/2)*e^(1/2)) - (30*d^2*atanh((e^(1/2)*((a + b*x)^(1/2) -
a^(1/2)))/(b^(1/2)*((d + e*x)^(1/2) - d^(1/2))))*(a*e - b*d))/(b^(3/2)*e^(1
/2)) + (10*d*atanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((d + e*x
)^(1/2) - d^(1/2))))*(a*e - b*d)*(3*a*e + b*d))/(b^(5/2)*e^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.588 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=122

$$\frac{2\left(3a^2e^2-8abde+8b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2\sqrt{a+bx}\sqrt{d+ex}\,(7bd-5ae)}{b^2}$$

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {951, 80, 63, 217, 206}

$$\frac{2\left(3a^2e^2-8abde+8b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2\sqrt{a+bx}\sqrt{d+ex}\,(7bd-5ae)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*(7*b*d - 5*a*e)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^2 + (4*e*(a + b*x)^(3/2)*
Sqrt[d + e*x])/b^2 + (2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[e
]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(5/2)*Sqrt[e])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
```

```
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx = \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\int \frac{2e\left(15b^2d^2-6abde-2a^2e^2\right)+4be^2(7bd-5ae)x}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{2b^2e}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\left(8b^2d^2-8abde+3a^2 \ldots\right.}{b \ldots}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\left(2\left(8b^2d^2-8abde+\ldots\right.\right.}{\ldots}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\left(2\left(8b^2d^2-8abde+\ldots\right.\right.}{\ldots}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{2\left(8b^2d^2-8abde+3\ldots\right.}{b^5 \ldots}$$

Mathematica [A] time = 0.41, size = 135, normalized size = 1.11

$$\frac{2\left(\frac{\sqrt{bd-ae}\left(3a^2e^2-8abde+8b^2d^2\right)\sqrt{\frac{b(d+ex)}{bd-ae}}\sinh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{\sqrt{e}}+b\sqrt{a+bx}(d+ex)(-3ae+7bd+2bex)\right)}{b^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*(b*Sqrt[a + b*x]*(d + e*x)*(7*b*d - 3*a*e + 2*b*e*x) + (Sqrt[b*d - a*e]*
(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*Sqrt[(b*(d + e*x))/(b*d - a*e)]*ArcSinh
[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/Sqrt[e]))/(b^3*Sqrt[d + e*x])
```

IntegrateAlgebraic [A] time = 0.29, size = 196, normalized size = 1.61

$$\frac{2\left(3a^2e^2-8abde+8b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{e}\sqrt{a+bx}}\right)}{b^{5/2}\sqrt{e}}+\frac{2\sqrt{d+ex}\left(\frac{5a^2be^2(d+ex)}{a+bx}-3a^2e^3+\frac{7b^3d^2(d+ex)}{a+bx}-\frac{12ab^2de(d+ex)}{a+bx}+8abde^2-5b^2d^2e\right)}{b^2\sqrt{a+bx}\left(\frac{b(d+ex)}{a+bx}-e\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d +
e*x]),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-5*b^2*d^2*e + 8*a*b*d*e^2 - 3*a^2*e^3 + (7*b^3*d^2*(d +
e*x))/(a + b*x) - (12*a*b^2*d*e*(d + e*x))/(a + b*x) + (5*a^2*b*e^2*(d + e*
x))/(a + b*x)))/(b^2*Sqrt[a + b*x]*(-e + (b*(d + e*x))/(a + b*x))^2) + (2*(
8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/(Sqrt[e]
*Sqrt[a + b*x])])/(b^(5/2)*Sqrt[e])
```

fricas [A] time = 0.55, size = 308, normalized size = 2.52

$$\left[\frac{\left(8b^2d^2-8abde+3a^2e^2\right)\sqrt{be}\log\left(8b^2e^2x^2+b^2d^2+6abde+a^2e^2+4(2bex+bd+ae)\sqrt{be}\sqrt{bx+a}\sqrt{ex+d}+8\left(b^2de+abe^2\right)x\right)+4\left(2b^2e^2x+7b^2de-3abe^2\right)\sqrt{bx+a}\sqrt{ex+d}}{2b^3e}, -\frac{\left(8b^2d^2-8abde+3a^2e^2\right)\sqrt{-be}\arctan\left(\frac{(2bex+bd+ae)\sqrt{-be}\sqrt{bx+a}\sqrt{ex+d}}{2\left(b^2e^2x^2+abde+\left(b^2de+abe^2\right)x\right)}\right)-2\left(2b^2e^2x+7b^2de-3abe^2\right)\sqrt{bx+a}\sqrt{ex+d}}{b^3e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/2*((8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2
*d^2 + 6*a*b*d*e + a^2*e^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a
)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) + 4*(2*b^2*e^2*x + 7*b^2*d*e - 3
*a*b*e^2)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*e), -((8*b^2*d^2 - 8*a*b*d*e +
3*a^2*e^2)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x
+ a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - 2*(2*
b^2*e^2*x + 7*b^2*d*e - 3*a*b*e^2)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*e)]
```

giac [A] time = 0.24, size = 145, normalized size = 1.19

$$\frac{2\left(\sqrt{b^2d+(bx+a)be-abe}\,\sqrt{bx+a}\left(\frac{2\,(bx+a)e}{b^3}+\frac{\left(7\,b^6de^2-5\,ab^5e^3\right)e^{(-2)}}{b^8}\right)-\frac{\left(8\,b^2d^2-8\,abde+3\,a^2e^2\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}\,e^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{b^{\frac{5}{2}}}\right)b}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] 2*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*e/b^3 + (
7*b^6*d*e^2 - 5*a*b^5*e^3)*e^(-2)/b^8) - (8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2
)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*
b*e - a*b*e)))/b^(5/2))*b/abs(b)
```

maple [B] time = 0.03, size = 247, normalized size = 2.02

$$\frac{\left(3a^2e^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-8abde\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+8b^2d^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+4\sqrt{be}\,\sqrt{(bx+a)\,(ex+d)}\,bex-6\sqrt{be}\,\sqrt{(bx+a)\,(ex+d)}\,ae+14\sqrt{be}\,\sqrt{(bx+a)\,(ex+d)}\,bd\right)\sqrt{ex+d}\,\sqrt{bx+a}}{\sqrt{be}\,\sqrt{(bx+a)\,(ex+d)}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] (3*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/
2))*a^2*e^2-8*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)
)/(b*e)^(1/2))*a*b*d*e+8*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*
(b*e)^(1/2))/(b*e)^(1/2))*b^2*d^2+4*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*x*b
*e-6*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*e+14*(b*e)^(1/2)*((b*x+a)*(e*x+d
))^(1/2)*b*d)*(e*x+d)^(1/2)*(b*x+a)^(1/2)/(b*e)^(1/2)/b^2/((b*x+a)*(e*x+d))
^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [B] time = 20.64, size = 893, normalized size = 7.32

$$\frac{\frac{(40bd^2+40aed)\left(\sqrt{a+bx}-\sqrt{a}\right)}{e^2\left(\sqrt{d+ex}-\sqrt{d}\right)}-\frac{160\sqrt{a}\,d^{3/2}\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{e\left(\sqrt{d+ex}-\sqrt{d}\right)^2}+\frac{(40bd^2+40aed)\left(\sqrt{a+bx}-\sqrt{a}\right)^3}{be\left(\sqrt{d+ex}-\sqrt{d}\right)^3}}{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{\left(\sqrt{d+ex}-\sqrt{d}\right)^4}+\frac{b^2}{e^2}-\frac{2b\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{e\left(\sqrt{d+ex}-\sqrt{d}\right)^2}}-\frac{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)(12a^2be^2+8ab^2de+12b^3d^2)}{e^4\left(\sqrt{d+ex}-\sqrt{d}\right)}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^3(44a^2e^2+200abde+44b^2d^2)}{e^3\left(\sqrt{d+ex}-\sqrt{d}\right)^3}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^7(12a^2e^2+8abde+12b^2d^2)}{b^2e\left(\sqrt{d+ex}-\sqrt{d}\right)^7}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^5(44a^2e^2+200abde+44b^2d^2)}{be^2\left(\sqrt{d+ex}-\sqrt{d}\right)^5}+\frac{\sqrt{a}\,\sqrt{d}\,(256ae+256bd)\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{e^2\left(\sqrt{d+ex}-\sqrt{d}\right)^4}}{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^8}{\left(\sqrt{d+ex}-\sqrt{d}\right)^8}+\frac{b^4}{e^4}-\frac{4b^3\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{e^3\left(\sqrt{d+ex}-\sqrt{d}\right)^2}+\frac{6b^2\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{e^2\left(\sqrt{d+ex}-\sqrt{d}\right)^4}-\frac{4b\left(\sqrt{a+bx}-\sqrt{a}\right)^6}{e\left(\sqrt{d+ex}-\sqrt{d}\right)^6}}-\frac{60d^2\,\mathrm{atan}\left(\frac{b\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{-be}\left(\sqrt{a+bx}-\sqrt{a}\right)}\right)}{\sqrt{-be}}-\frac{2\ln\left(\frac{\sqrt{e}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{d+ex}-\sqrt{d}}-\sqrt{b}\right)\left(3a^2e^2+2abde+3b^2d^2\right)}{b^{5/2}\sqrt{e}}+\frac{\ln\left(\sqrt{b}+\frac{\sqrt{e}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{d+ex}-\sqrt{d}}\right)\left(6a^2e^2+4abde+6b^2d^2\right)}{b^{5/2}\sqrt{e}}-\frac{40d\,\mathrm{atanh}\left(\frac{\sqrt{e}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{d+ex}-\sqrt{d}\right)}\right)(ae+bd)}{b^{3/2}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((40*b*d^2 + 40*a*d*e)*((a + b*x)^(1/2) - a^(1/2)))/(e^2*((d + e*x)^(1/2)
- d^(1/2))) - (160*a^(1/2)*d^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d +
e*x)^(1/2) - d^(1/2))^2) + ((40*b*d^2 + 40*a*d*e)*((a + b*x)^(1/2) - a^(1/2
))^3)/(b*e*((d + e*x)^(1/2) - d^(1/2))^3))/(((a + b*x)^(1/2) - a^(1/2))^4/(
(d + e*x)^(1/2) - d^(1/2))^4 + b^2/e^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2
)/(e*((d + e*x)^(1/2) - d^(1/2))^2)) - ((((a + b*x)^(1/2) - a^(1/2))*(12*b^
3*d^2 + 12*a^2*b*e^2 + 8*a*b^2*d*e))/(e^4*((d + e*x)^(1/2) - d^(1/2))) - ((
(a + b*x)^(1/2) - a^(1/2))^3*(44*a^2*e^2 + 44*b^2*d^2 + 200*a*b*d*e))/(e^3*
((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))^7*(12*a^2*e^2
 + 12*b^2*d^2 + 8*a*b*d*e))/(b^2*e*((d + e*x)^(1/2) - d^(1/2))^7) - (((a +
b*x)^(1/2) - a^(1/2))^5*(44*a^2*e^2 + 44*b^2*d^2 + 200*a*b*d*e))/(b*e^2*((d
 + e*x)^(1/2) - d^(1/2))^5) + (a^(1/2)*d^(1/2)*(256*a*e + 256*b*d)*((a + b*
x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4))/(((a + b*x)^(1/
2) - a^(1/2))^8/((d + e*x)^(1/2) - d^(1/2))^8 + b^4/e^4 - (4*b^3*((a + b*x)
^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2) + (6*b^2*((a + b*x
)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4) - (4*b*((a + b*x)
^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2) - d^(1/2))^6)) - (60*d^2*atan((b*(
(d + e*x)^(1/2) - d^(1/2)))/((-b*e)^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(-
b*e)^(1/2) - (2*log((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/((d + e*x)^(1/2)
- d^(1/2)) - b^(1/2))*(3*a^2*e^2 + 3*b^2*d^2 + 2*a*b*d*e))/(b^(5/2)*e^(1/2)
) + (log(b^(1/2) + (e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/((d + e*x)^(1/2) -
 d^(1/2)))*(6*a^2*e^2 + 6*b^2*d^2 + 4*a*b*d*e))/(b^(5/2)*e^(1/2)) - (40*d*a
tanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((d + e*x)^(1/2) - d^(1
/2))))*(a*e + b*d))/(b^(3/2)*e^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\,\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*sqrt(d + e*x)),
x)
```

3.589 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}\,(bd-ae)}+\frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}$$

Rubi [A] time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {949, 80, 63, 217, 206}

$$\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}\,(bd-ae)}+\frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/((b*d - a*e)*Sqrt[d + e*x]) + (8*Sqrt[a + b*x]*Sqrt[d
 + e*x])/b + (8*(2*b*d - a*e)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt
[d + e*x])])/(b^(3/2)*Sqrt[e])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
```

```
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{3/2}}\,dx &= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{2\int \frac{6d(bd-ae)+4e(bd-ae)x}{\sqrt{a+bx}\,\sqrt{d+ex}}\,dx}{bd-ae}\\
&= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b}+\frac{(4(2bd-ae))\int \frac{1}{\sqrt{a+bx}\,\sqrt{d+ex}}\,dx}{b}\\
&= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b}+\frac{(8(2bd-ae))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx\right.}{b^2}\\
&= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b}+\frac{(8(2bd-ae))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx,x,\right.}{b^2}\\
&= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b}+\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}
\end{aligned}
$$

Mathematica [A] time = 0.34, size = 134, normalized size = 1.24

$$
\frac{2\left(\frac{b\sqrt{a+bx}\,(bd(7d+4ex)-4ae(d+ex))}{bd-ae}+\frac{4\sqrt{bd-ae}\,(2bd-ae)\sqrt{\frac{b(d+ex)}{bd-ae}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{\sqrt{e}}\right)}{b^2\sqrt{d+ex}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x
]
```

```
[Out] (2*((b*Sqrt[a + b*x]*(-4*a*e*(d + e*x) + b*d*(7*d + 4*e*x)))/(b*d - a*e) +
(4*Sqrt[b*d - a*e]*(2*b*d - a*e)*Sqrt[(b*(d + e*x))/(b*d - a*e)]*ArcSinh[(S
qrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/Sqrt[e]))/(b^2*Sqrt[d + e*x])
```

IntegrateAlgebraic [A] time = 0.24, size = 146, normalized size = 1.35

$$
\frac{2\sqrt{a+bx}\left(4a^2e^2-\frac{3bd^2e(a+bx)}{d+ex}-8abde+7b^2d^2\right)}{b\sqrt{d+ex}\,(bd-ae)\left(b-\frac{e(a+bx)}{d+ex}\right)}+\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)
^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x]*(7*b^2*d^2 - 8*a*b*d*e + 4*a^2*e^2 - (3*b*d^2*e*(a + b*x))
/(d + e*x)))/(b*(b*d - a*e)*Sqrt[d + e*x]*(b - (e*(a + b*x))/(d + e*x))) +
(8*(2*b*d - a*e)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/
(b^(3/2)*Sqrt[e])
```

fricas [B] time = 0.54, size = 463, normalized size = 4.29

$$
\left[-\frac{2\left(\left(2b^2d^3-3abd^2e+a^2de^2+\left(2b^2d^2e-3abde^2+a^2e^3\right)x\right)\sqrt{be}\,\log\left(8b^2e^2x^2+b^2d^2+6abde+a^2e^2-4(2bex+bd+ae)\sqrt{be}\,\sqrt{bx+a}\,\sqrt{ex+d}+8\left(b^2de+abe^2\right)x\right)-\left(7b^2d^2e-4abde^2+4\left(b^2de^2-abe^3\right)x\right)\sqrt{bx+a}\,\sqrt{ex+d}\right)}{b^3d^2e-ab^2de^2+\left(b^3de^2-ab^2e^3\right)x}, -\frac{2\left(2\left(2b^2d^3-3abd^2e+a^2de^2+\left(2b^2d^2e-3abde^2+a^2e^3\right)x\right)\sqrt{-be}\,\arctan\left(\frac{(2bex+bd+ae)\sqrt{-be}\,\sqrt{bx+a}\,\sqrt{ex+d}}{2\left(b^2e^2x^2+abde+\left(b^2de+abe^2\right)x\right)}\right)-\left(7b^2d^2e-4abde^2+4\left(b^2de^2-abe^3\right)x\right)\sqrt{bx+a}\,\sqrt{ex+d}\right)}{b^3d^2e-ab^2de^2+\left(b^3de^2-ab^2e^3\right)x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-2*((2*b^2*d^3 - 3*a*b*d^2*e + a^2*d*e^2 + (2*b^2*d^2*e - 3*a*b*d*e^2 + a^
2*e^3)*x)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 - 4*(
2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a
*b*e^2)*x) - (7*b^2*d^2*e - 4*a*b*d*e^2 + 4*(b^2*d*e^2 - a*b*e^3)*x)*sqrt(b
*x + a)*sqrt(e*x + d))/(b^3*d^2*e - a*b^2*d*e^2 + (b^3*d*e^2 - a*b^2*e^3)*x
), -2*(2*(2*b^2*d^3 - 3*a*b*d^2*e + a^2*d*e^2 + (2*b^2*d^2*e - 3*a*b*d*e^2
+ a^2*e^3)*x)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b
*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - (7
*b^2*d^2*e - 4*a*b*d*e^2 + 4*(b^2*d*e^2 - a*b*e^3)*x)*sqrt(b*x + a)*sqrt(e*
x + d))/(b^3*d^2*e - a*b^2*d*e^2 + (b^3*d*e^2 - a*b^2*e^3)*x)]
```

giac [B] time = 0.36, size = 193, normalized size = 1.79

$$-\frac{8\,(2\,bd-ae)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}\,e^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{b}\,|b|}+\frac{2\,\sqrt{bx+a}\left(\frac{4\left(b^3de^3-ab^2e^4\right)(bx+a)}{b^3d|b|e^2-ab^2|b|e^3}+\frac{7\,b^4d^2e^2-8\,ab^3de^3+4\,a^2b^2e^4}{b^3d|b|e^2-ab^2|b|e^3}\right)}{\sqrt{b^2d+(bx+a)be-abe}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -8*(2*b*d - a*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2
*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b)*abs(b)) + 2*sqrt(b*x + a)*(4*(b^3*d*
e^3 - a*b^2*e^4)*(b*x + a)/(b^3*d*abs(b)*e^2 - a*b^2*abs(b)*e^3) + (7*b^4*d
^2*e^2 - 8*a*b^3*d*e^3 + 4*a^2*b^2*e^4)/(b^3*d*abs(b)*e^2 - a*b^2*abs(b)*e^
3))/sqrt(b^2*d + (b*x + a)*b*e - a*b*e)
```

maple [B] time = 0.03, size = 438, normalized size = 4.06

$$-\frac{2\sqrt{bx+a}\,\left(2a^2e^3x\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-6abd\,e^2x\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+4b^2d^2ex\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+2a^2d\,e^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-6ab\,d^2e\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+4b^2d^3\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-4\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,a\,e^2x+4\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,bdex-4\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,ade+7\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,b\,d^2\right)}{\sqrt{be}\,(ae-bd)\sqrt{(bx+a)(ex+d)}\,\sqrt{ex+d}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)*(2*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)
^(1/2))/(b*e)^(1/2))*x*a^2*e^3-6*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d)
)^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*x*a*b*d*e^2+4*ln(1/2*(2*b*e*x+a*e+b*d+2*(
(b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*x*b^2*d^2*e+2*ln(1/2*(2*b*
e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*a^2*d*e^2-6
*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2)
)*a*b*d^2*e+4*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)
)/(b*e)^(1/2))*b^2*d^3-4*x*a*e^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+4*x*b*
d*e*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)-4*a*d*e*((b*x+a)*(e*x+d))^(1/2)*(b*
e)^(1/2)+7*b*d^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/b/(b*e)^(1/2)/(a*e-b*
d)/((b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{15\,d^2+20\,d\,e\,x+8\,e^2\,x^2}{\sqrt{a+b\,x}\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(3/2)),x)
```

```
[Out] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15d^{2}+20dex+8e^{2}x^{2}}{\sqrt{a+bx}\,(d+ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(3/2)
), x)
```

3.590 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}+\frac{4d\sqrt{a+bx}\,(3bd-2ae)}{\sqrt{d+ex}\,(bd-ae)^2}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

Rubi [A] time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {949, 78, 63, 217, 206}

$$\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}+\frac{4d\sqrt{a+bx}\,(3bd-2ae)}{\sqrt{d+ex}\,(bd-ae)^2}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x]
```

```
[Out] (2*d^2*Sqrt[a + b*x])/((b*d - a*e)*(d + e*x)^(3/2)) + (4*d*(3*b*d - 2*a*e)*
Sqrt[a + b*x])/((b*d - a*e)^2*Sqrt[d + e*x]) + (16*ArcTanh[(Sqrt[e]*Sqrt[a
+ b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(Sqrt[b]*Sqrt[e])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
```

```
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{5/2}}\,dx &= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{2\int \frac{3d(7bd-6ae)+12e(bd-ae)x}{\sqrt{a+bx}\,(d+ex)^{3/2}}\,dx}{3(bd-ae)}\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+8\int \frac{1}{\sqrt{a+bx}\,\sqrt{d+ex}}\,dx\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx,x,\sqrt{\right.}{b}\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{d+ex}}\right)}{b}\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{\sqrt{b}\,\sqrt{e}}
\end{aligned}
$$

Mathematica [A] time = 0.30, size = 128, normalized size = 1.10

$$
\frac{2\left(\frac{8(bd-ae)^{3/2}\left(\frac{b(d+ex)}{bd-ae}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{b^2\sqrt{e}}+\frac{d\sqrt{a+bx}\,(bd(7d+6ex)-ae(5d+4ex))}{(bd-ae)^2}\right)}{(d+ex)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x
]
```

```
[Out] (2*((d*Sqrt[a + b*x]*(-(a*e*(5*d + 4*e*x)) + b*d*(7*d + 6*e*x)))/(b*d - a*e
)^2 + (8*(b*d - a*e)^(3/2)*((b*(d + e*x))/(b*d - a*e))^(3/2)*ArcSinh[(Sqrt[
e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(b^2*Sqrt[e])))/(d + e*x)^(3/2)
```

IntegrateAlgebraic [A] time = 0.17, size = 99, normalized size = 0.85

$$
\frac{2d\sqrt{a+bx}\left(-\frac{de(a+bx)}{d+ex}-4ae+7bd\right)}{\sqrt{d+ex}\,(bd-ae)^2}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{\sqrt{b}\,\sqrt{e}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)
^(5/2)),x]
```

```
[Out] (2*d*Sqrt[a + b*x]*(7*b*d - 4*a*e - (d*e*(a + b*x))/(d + e*x)))/((b*d - a*e
)^2*Sqrt[d + e*x]) + (16*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d +
e*x])])/(Sqrt[b]*Sqrt[e])
```

fricas [B] time = 0.60, size = 665, normalized size = 5.73

$$
\left[\frac{2\left(2\left(b^2d^4-2abd^3e+a^2d^2e^2+\left(b^2d^2e^2-2abde^3+a^2e^4\right)x^2+2\left(b^2d^3e-2abd^2e^2+a^2de^3\right)x\right)\sqrt{be}\,\log\left(8b^2e^2x^2+b^2d^2+6abde+a^2e^2+4(2bex+bd+ae)\sqrt{be}\,\sqrt{bx+a}\,\sqrt{ex+d}+8\left(b^2de+abe^2\right)x\right)+\left(7b^2d^3e-5abd^2e^2+2\left(3b^2d^2e^2-2abde^3\right)x\right)\sqrt{bx+a}\,\sqrt{ex+d}\right)}{b^3d^4e-2ab^2d^3e^2+a^2bd^2e^3+\left(b^3d^2e^3-2ab^2de^4+a^2be^5\right)x^2+2\left(b^3d^3e^2-2ab^2d^2e^3+a^2bde^4\right)x},-\frac{2\left(4\left(b^2d^4-2abd^3e+a^2d^2e^2+\left(b^2d^2e^2-2abde^3+a^2e^4\right)x^2+2\left(b^2d^3e-2abd^2e^2+a^2de^3\right)x\right)\sqrt{-be}\,\arctan\left(\frac{(2bex+bd+ae)\sqrt{-be}\,\sqrt{bx+a}\,\sqrt{ex+d}}{2\left(b^2e^2x^2+abde+\left(b^2de+abe^2\right)x\right)}\right)-\left(7b^2d^3e-5abd^2e^2+2\left(3b^2d^2e^2-2abde^3\right)x\right)\sqrt{bx+a}\,\sqrt{ex+d}\right)}{b^3d^4e-2ab^2d^3e^2+a^2bd^2e^3+\left(b^3d^2e^3-2ab^2de^4+a^2be^5\right)x^2+2\left(b^3d^3e^2-2ab^2d^2e^3+a^2bde^4\right)x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [2*(2*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3 + a
^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*sqrt(b*e)*log(8*
b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(
b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) + (7*b^2*d^3*e
- 5*a*b*d^2*e^2 + 2*(3*b^2*d^2*e^2 - 2*a*b*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x
 + d))/(b^3*d^4*e - 2*a*b^2*d^3*e^2 + a^2*b*d^2*e^3 + (b^3*d^2*e^3 - 2*a*b^
2*d*e^4 + a^2*b*e^5)*x^2 + 2*(b^3*d^3*e^2 - 2*a*b^2*d^2*e^3 + a^2*b*d*e^4)*
x), -2*(4*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3
 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*sqrt(-b*e)*a
rctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2
*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - (7*b^2*d^3*e - 5*a*b*d^2*e^2
 + 2*(3*b^2*d^2*e^2 - 2*a*b*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*d^4
*e - 2*a*b^2*d^3*e^2 + a^2*b*d^2*e^3 + (b^3*d^2*e^3 - 2*a*b^2*d*e^4 + a^2*b
*e^5)*x^2 + 2*(b^3*d^3*e^2 - 2*a*b^2*d^2*e^3 + a^2*b*d*e^4)*x)]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

maple [**B**] time = 0.03, size = 601, normalized size = 5.18

$$\frac{2\sqrt{bx+a}\,\left(4a^2e^4x^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-8abde^3x^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+4b^2d^2e^2x^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+8a^2de^3x\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-16abd^2e^2x\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+8b^2d^3ex\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+4a^2d^2e^2\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-8abd^3e\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)+4b^2d^4\ln\left(\frac{2bex+ae+bd+2\sqrt{(bx+a)(ex+d)}\,\sqrt{be}}{2\sqrt{be}}\right)-4\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,ade^2x+6\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,bd^2ex-5\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,ad^2e+7\sqrt{(bx+a)(ex+d)}\,\sqrt{be}\,bd^3\right)}{\sqrt{be}\,(ae-bd)^2\sqrt{(bx+a)(ex+d)}\,(ex+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 2*(b*x+a)^(1/2)*(4*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^
(1/2))/(b*e)^(1/2))*x^2*a^2*e^4-8*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d
))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*x^2*a*b*d*e^3+4*ln(1/2*(2*b*e*x+a*e+b*d+
2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*x^2*b^2*d^2*e^2+8*ln(1/
2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*x*a^
2*d*e^3-16*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(
b*e)^(1/2))*x*a*b*d^2*e^2+8*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/
2)*(b*e)^(1/2))/(b*e)^(1/2))*x*b^2*d^3*e+4*ln(1/2*(2*b*e*x+a*e+b*d+2*((b*x+
a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*a^2*d^2*e^2-8*ln(1/2*(2*b*e*x+a
*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*a*b*d^3*e+4*ln(1
/2*(2*b*e*x+a*e+b*d+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2))*b^2
*d^4-4*x*a*d*e^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+6*x*b*d^2*e*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)-5*a*d^2*e*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+7*b
*d^3*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2)/(a*e-b*d)^2/((b*x+a)*
(e*x+d))^(1/2)/(e*x+d)^(3/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{15\,d^2+20\,d\,e\,x+8\,e^2\,x^2}{\sqrt{a+b\,x}\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(5/2)),x)
[Out] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(5/2)/(b*x+a)**(1/2),x)
[Out] Timed out
```

3.591 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=133

$$\frac{16\sqrt{a+bx}\,\left(15a^2e^2-35abde+23b^2d^2\right)}{15\sqrt{d+ex}\,(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}+\frac{8d\sqrt{a+bx}\,(8bd-5ae)}{15(d+ex)^{3/2}(bd-ae)^2}$$

Rubi [A] time = 0.13, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {949, 78, 37}

$$\frac{16\sqrt{a+bx}\,\left(15a^2e^2-35abde+23b^2d^2\right)}{15\sqrt{d+ex}\,(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}+\frac{8d\sqrt{a+bx}\,(8bd-5ae)}{15(d+ex)^{3/2}(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (8*d*(8*b*d - 5*a*e
)*Sqrt[a + b*x])/(15*(b*d - a*e)^2*(d + e*x)^(3/2)) + (16*(23*b^2*d^2 - 35*
a*b*d*e + 15*a^2*e^2)*Sqrt[a + b*x])/(15*(b*d - a*e)^3*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{7/2}}\,dx &= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{2\int \frac{6d(6bd-5ae)+20e(bd-ae)x}{\sqrt{a+bx}\,(d+ex)^{5/2}}dx}{5(bd-ae)} \\
&= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}} + \frac{\left(8\left(23b^2d^2-35abde+15a^2e^2\right.\right.}{15(bd-ae} \\
&= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}} + \frac{16\left(23b^2d^2-35abde+15a^2e^2\right.}{15(bd-ae)^3\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 110, normalized size = 0.83

$$
\frac{2\sqrt{a+bx}\left(a^2e^2\left(149d^2+260dex+120e^2x^2\right)-2abde\left(175d^2+306dex+140e^2x^2\right)+b^2d^2\left(225d^2+400dex+184e^2x^2\right)\right)}{15(d+ex)^{5/2}(bd-ae)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x
]
```

```
[Out] (2*Sqrt[a + b*x]*(a^2*e^2*(149*d^2 + 260*d*e*x + 120*e^2*x^2) - 2*a*b*d*e*(
175*d^2 + 306*d*e*x + 140*e^2*x^2) + b^2*d^2*(225*d^2 + 400*d*e*x + 184*e^2
*x^2)))/(15*(b*d - a*e)^3*(d + e*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 115, normalized size = 0.86

$$
\frac{2\sqrt{a+bx}\left(120a^2e^2+\frac{9d^2e^2(a+bx)^2}{(d+ex)^2}-\frac{50bd^2e(a+bx)}{d+ex}+\frac{20ade^2(a+bx)}{d+ex}-300abde+225b^2d^2\right)}{15\sqrt{d+ex}\,(bd-ae)^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)
^(7/2)),x]
```

```
[Out] (2*Sqrt[a + b*x]*(225*b^2*d^2 - 300*a*b*d*e + 120*a^2*e^2 + (9*d^2*e^2*(a +
 b*x)^2)/(d + e*x)^2 - (50*b*d^2*e*(a + b*x))/(d + e*x) + (20*a*d*e^2*(a +
b*x))/(d + e*x)))/(15*(b*d - a*e)^3*Sqrt[d + e*x])
```

fricas [B] time = 1.05, size = 293, normalized size = 2.20

$$
\frac{2\left(225\,b^2d^4-350\,abd^3e+149\,a^2d^2e^2+8\left(23\,b^2d^2e^2-35\,abde^3+15\,a^2e^4\right)x^2+4\left(100\,b^2d^3e-153\,abd^2e^2+65\,a^2de^3\right)x\right)\sqrt{bx+a}\,\sqrt{ex+d}}{15\left(b^3d^6-3\,ab^2d^5e+3\,a^2bd^4e^2-a^3d^3e^3+\left(b^3d^3e^3-3\,ab^2d^2e^4+3\,a^2bde^5-a^3e^6\right)x^3+3\left(b^3d^4e^2-3\,ab^2d^3e^3+3\,a^2bd^2e^4-a^3de^5\right)x^2+3\left(b^3d^5e-3\,ab^2d^4e^2+3\,a^2bd^3e^3-a^3d^2e^4\right)x\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/15*(225*b^2*d^4 - 350*a*b*d^3*e + 149*a^2*d^2*e^2 + 8*(23*b^2*d^2*e^2 - 3
5*a*b*d*e^3 + 15*a^2*e^4)*x^2 + 4*(100*b^2*d^3*e - 153*a*b*d^2*e^2 + 65*a^2
*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d)/(b^3*d^6 - 3*a*b^2*d^5*e + 3*a^2*b*d
^4*e^2 - a^3*d^3*e^3 + (b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*e^5 - a^3
*e^6)*x^3 + 3*(b^3*d^4*e^2 - 3*a*b^2*d^3*e^3 + 3*a^2*b*d^2*e^4 - a^3*d*e^5)
*x^2 + 3*(b^3*d^5*e - 3*a*b^2*d^4*e^2 + 3*a^2*b*d^3*e^3 - a^3*d^2*e^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

maple [A]   time = 0.01, size = 150, normalized size = 1.13

$$-\frac{2\sqrt{bx+a}\,\left(120a^{2}e^{4}x^{2}-280abd\,e^{3}x^{2}+184b^{2}d^{2}e^{2}x^{2}+260a^{2}d\,e^{3}x-612ab\,d^{2}e^{2}x+400b^{2}d^{3}ex+149a^{2}d^{2}e^{2}-350ab\,d^{3}e+225b^{2}d^{4}\right)}{15\left(ex+d\right)^{\frac{5}{2}}\left(a^{3}e^{3}-3a^{2}bd\,e^{2}+3a\,b^{2}d^{2}e-b^{3}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)^(1/2)*(120*a^2*e^4*x^2-280*a*b*d*e^3*x^2+184*b^2*d^2*e^2*x^2+
260*a^2*d*e^3*x-612*a*b*d^2*e^2*x+400*b^2*d^3*e*x+149*a^2*d^2*e^2-350*a*b*d
^3*e+225*b^2*d^4)/(e*x+d)^(5/2)/(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^
3)
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [B]   time = 4.30, size = 268, normalized size = 2.02

$$-\frac{\sqrt{d+e\,x}\,\left(\frac{x^2\,\left(240\,a^3\,e^4-40\,a^2\,b\,d\,e^3-856\,a\,b^2\,d^2\,e^2+800\,b^3\,d^3\,e\right)}{15\,e^3\,{\left(a\,e-b\,d\right)}^3}+\frac{x\,\left(520\,a^3\,d\,e^3-926\,a^2\,b\,d^2\,e^2+100\,a\,b^2\,d^3\,e+450\,b^3\,d^4\right)}{15\,e^3\,{\left(a\,e-b\,d\right)}^3}+\frac{2\,a\,d^2\,\left(149\,a^2\,e^2-350\,a\,b\,d\,e+225\,b^2\,d^2\right)}{15\,e^3\,{\left(a\,e-b\,d\right)}^3}+\frac{16\,b\,x^3\,\left(15\,a^2\,e^2-35\,a\,b\,d\,e+23\,b^2\,d^2\right)}{15\,e\,{\left(a\,e-b\,d\right)}^3}\right)}{x^3\,\sqrt{a+b\,x}+\frac{d^3\,\sqrt{a+b\,x}}{e^3}+\frac{3\,d\,x^2\,\sqrt{a+b\,x}}{e}+\frac{3\,d^2\,x\,\sqrt{a+b\,x}}{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(7/2)),x)
```

```
[Out] -((d + e*x)^(1/2)*((x^2*(240*a^3*e^4 + 800*b^3*d^3*e - 856*a*b^2*d^2*e^2 -
40*a^2*b*d*e^3))/(15*e^3*(a*e - b*d)^3) + (x*(450*b^3*d^4 + 520*a^3*d*e^3 -
 926*a^2*b*d^2*e^2 + 100*a*b^2*d^3*e))/(15*e^3*(a*e - b*d)^3) + (2*a*d^2*(1
49*a^2*e^2 + 225*b^2*d^2 - 350*a*b*d*e))/(15*e^3*(a*e - b*d)^3) + (16*b*x^3
*(15*a^2*e^2 + 23*b^2*d^2 - 35*a*b*d*e))/(15*e*(a*e - b*d)^3)))/(x^3*(a + b
*x)^(1/2) + (d^3*(a + b*x)^(1/2))/e^3 + (3*d*x^2*(a + b*x)^(1/2))/e + (3*d^
2*x*(a + b*x)^(1/2))/e^2)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(7/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.592 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=189

$$\frac{32b\sqrt{a+bx}\left(35a^2e^2-84abde+58b^2d^2\right)}{105\sqrt{d+ex}\,(bd-ae)^4}+\frac{16\sqrt{a+bx}\left(35a^2e^2-84abde+58b^2d^2\right)}{105(d+ex)^{3/2}(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}+\frac{4d}{3}$$

Rubi [A] time = 0.19, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {949, 78, 45, 37}

$$\frac{32b\sqrt{a+bx}\left(35a^2e^2-84abde+58b^2d^2\right)}{105\sqrt{d+ex}\,(bd-ae)^4}+\frac{16\sqrt{a+bx}\left(35a^2e^2-84abde+58b^2d^2\right)}{105(d+ex)^{3/2}(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}+\frac{4d\sqrt{a+bx}\,(23bd-14ae)}{35(d+ex)^{5/2}(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/(7*(b*d - a*e)*(d + e*x)^(7/2)) + (4*d*(23*b*d - 14*a
*e)*Sqrt[a + b*x])/(35*(b*d - a*e)^2*(d + e*x)^(5/2)) + (16*(58*b^2*d^2 - 8
4*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a*e)^3*(d + e*x)^(3/2))
+ (32*b*(58*b^2*d^2 - 84*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a
*e)^4*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
```

```
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{9/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{2\int \frac{3d(17bd-14ae)+28e(bd-ae)x}{\sqrt{a+bx}\,(d+ex)^{7/2}}\,dx}{7(bd-ae)}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{\left(8\left(58b^2d^2-84abde+35a^2e^2 \right.\right.}{35(bd-ae}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{16\left(58b^2d^2-84abde+35a^2e^2 \right.}{105(bd-ae)^3(d+ex)^{}}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{16\left(58b^2d^2-84abde+35a^2e^2 \right.}{105(bd-ae)^3(d+ex)^{}}$$

Mathematica [A] time = 0.12, size = 173, normalized size = 0.92

$$\frac{2\sqrt{a+bx}\left(-a^3e^3\left(409d^2+644dex+280e^2x^2\right)+a^2be^2\left(1953d^3+3890d^2ex+2632de^2x^2+560e^3x^3\right)-ab^2de\left(2975d^3+6664d^2ex+5168de^2x^2+1344e^3x^3\right)+b^3d^2\left(1575d^3+3850d^2ex+3248de^2x^2+928e^3x^3\right)\right)}{105(d+ex)^{7/2}(bd-ae)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x
]
```

```
[Out] (2*Sqrt[a + b*x]*(-(a^3*e^3*(409*d^2 + 644*d*e*x + 280*e^2*x^2)) + a^2*b*e^
2*(1953*d^3 + 3890*d^2*e*x + 2632*d*e^2*x^2 + 560*e^3*x^3) + b^3*d^2*(1575*
d^3 + 3850*d^2*e*x + 3248*d*e^2*x^2 + 928*e^3*x^3) - a*b^2*d*e*(2975*d^3 +
6664*d^2*e*x + 5168*d*e^2*x^2 + 1344*e^3*x^3)))/(105*(b*d - a*e)^4*(d + e*x
)^(7/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 185, normalized size = 0.98

$$\frac{2\sqrt{a+bx}\left(-\frac{280a^2e^3(a+bx)}{d+ex}+840a^2be^2-\frac{875b^2d^2e(a+bx)}{d+ex}-2100ab^2de-\frac{45d^2e^3(a+bx)^3}{(d+ex)^3}+\frac{273bd^2e^2(a+bx)^2}{(d+ex)^2}-\frac{84ade^3(a+bx)^2}{(d+ex)^2}+\frac{840abde^2(a+bx)}{d+ex}+1575b^3d^2\right)}{105\sqrt{d+ex}\,(bd-ae)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)
^(9/2)),x]
```

```
[Out] (2*Sqrt[a + b*x]*(1575*b^3*d^2 - 2100*a*b^2*d*e + 840*a^2*b*e^2 - (45*d^2*e
^3*(a + b*x)^3)/(d + e*x)^3 + (273*b*d^2*e^2*(a + b*x)^2)/(d + e*x)^2 - (84
*a*d*e^3*(a + b*x)^2)/(d + e*x)^2 - (875*b^2*d^2*e*(a + b*x))/(d + e*x) + (
840*a*b*d*e^2*(a + b*x))/(d + e*x) - (280*a^2*e^3*(a + b*x))/(d + e*x)))/(1
05*(b*d - a*e)^4*Sqrt[d + e*x])
```

fricas [B] time = 2.23, size = 487, normalized size = 2.58

$$\frac{2\left(1575\,b^3d^5-2975\,ab^2d^4e+1953\,a^2bd^3e^2-409\,a^3d^2e^3+16\left(58\,b^3d^2e^3-84\,ab^2de^4+35\,a^2be^5\right)x^3+8\left(406\,b^3d^3e^2-646\,ab^2d^2e^3+329\,a^2bde^4-35\,a^3e^5\right)x^2+2\left(1925\,b^3d^4e-3332\,ab^2d^3e^2+1945\,a^2bd^2e^3-322\,a^3de^4\right)x\right)\sqrt{bx+a}\,\sqrt{ex+d}}{105\left(b^4d^8-4\,ab^3d^7e+6\,a^2b^2d^6e^2-4\,a^3bd^5e^3+a^4d^4e^4+\left(b^4d^4e^4-4\,ab^3d^3e^5+6\,a^2b^2d^2e^6-4\,a^3bde^7+a^4e^8\right)x^4+4\left(b^4d^5e^3-4\,ab^3d^4e^4+6\,a^2b^2d^3e^5-4\,a^3bd^2e^6+a^4de^7\right)x^3+6\left(b^4d^6e^2-4\,ab^3d^5e^3+6\,a^2b^2d^4e^4-4\,a^3bd^3e^5+a^4d^2e^6\right)x^2+4\left(b^4d^7e-4\,ab^3d^6e^2+6\,a^2b^2d^5e^3-4\,a^3bd^4e^4+a^4d^3e^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/105*(1575*b^3*d^5 - 2975*a*b^2*d^4*e + 1953*a^2*b*d^3*e^2 - 409*a^3*d^2*e
^3 + 16*(58*b^3*d^2*e^3 - 84*a*b^2*d*e^4 + 35*a^2*b*e^5)*x^3 + 8*(406*b^3*d
```

```
^3*e^2 - 646*a*b^2*d^2*e^3 + 329*a^2*b*d*e^4 - 35*a^3*e^5)*x^2 + 2*(1925*b^
3*d^4*e - 3332*a*b^2*d^3*e^2 + 1945*a^2*b*d^2*e^3 - 322*a^3*d*e^4)*x)*sqrt(
b*x + a)*sqrt(e*x + d)/(b^4*d^8 - 4*a*b^3*d^7*e + 6*a^2*b^2*d^6*e^2 - 4*a^3
*b*d^5*e^3 + a^4*d^4*e^4 + (b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 6*a^2*b^2*d^2*e
^6 - 4*a^3*b*d*e^7 + a^4*e^8)*x^4 + 4*(b^4*d^5*e^3 - 4*a*b^3*d^4*e^4 + 6*a^
2*b^2*d^3*e^5 - 4*a^3*b*d^2*e^6 + a^4*d*e^7)*x^3 + 6*(b^4*d^6*e^2 - 4*a*b^3
*d^5*e^3 + 6*a^2*b^2*d^4*e^4 - 4*a^3*b*d^3*e^5 + a^4*d^2*e^6)*x^2 + 4*(b^4*
d^7*e - 4*a*b^3*d^6*e^2 + 6*a^2*b^2*d^5*e^3 - 4*a^3*b*d^4*e^4 + a^4*d^3*e^5
)*x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.01, size = 248, normalized size = 1.31

$$-\frac{2\sqrt{bx+a}\,\left(-560a^{2}b\,e^{5}x^{3}+1344a\,b^{2}d\,e^{4}x^{3}-928b^{3}d^{2}e^{3}x^{3}+280a^{3}e^{5}x^{2}-2632a^{2}bd\,e^{4}x^{2}+5168a\,b^{2}d^{2}e^{3}x^{2}-3248b^{3}d^{3}e^{2}x^{2}+644a^{3}d\,e^{4}x-3890a^{2}b\,d^{2}e^{3}x+6664a\,b^{2}d^{3}e^{2}x-3850b^{3}d^{4}ex+409a^{3}d^{2}e^{3}-1953a^{2}b\,d^{3}e^{2}+2975a\,b^{2}d^{4}e-1575b^{3}d^{5}\right)}{105\left(ex+d\right)^{\frac{7}{2}}\left(e^{4}a^{4}-4b\,e^{3}d\,a^{3}+6a^{2}b^{2}d^{2}e^{2}-4a\,b^{3}d^{3}e+b^{4}d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/105*(b*x+a)^(1/2)*(-560*a^2*b*e^5*x^3+1344*a*b^2*d*e^4*x^3-928*b^3*d^2*e
^3*x^3+280*a^3*e^5*x^2-2632*a^2*b*d*e^4*x^2+5168*a*b^2*d^2*e^3*x^2-3248*b^3
*d^3*e^2*x^2+644*a^3*d*e^4*x-3890*a^2*b*d^2*e^3*x+6664*a*b^2*d^3*e^2*x-3850
*b^3*d^4*e*x+409*a^3*d^2*e^3-1953*a^2*b*d^3*e^2+2975*a*b^2*d^4*e-1575*b^3*d
^5)/(e*x+d)^(7/2)/(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^
4*d^4)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e-b*d>0)', see `assume?` for more
 details)Is a*e-b*d zero or nonzero?
```

mupad [**B**] time = 4.51, size = 389, normalized size = 2.06

$$\frac{\sqrt{d+e\,x}\,\left(\frac{-818\,a^4\,d^2\,e^3+3906\,a^3\,b\,d^3\,e^2-5950\,a^2\,b^2\,d^4\,e+3150\,a\,b^3\,d^5}{105\,e^4\,{\left(a\,e-b\,d\right)}^4}+\frac{x\,\left(-1288\,a^4\,d\,e^4+6962\,a^3\,b\,d^2\,e^3-9422\,a^2\,b^2\,d^3\,e^2+1750\,a\,b^3\,d^4\,e+3150\,b^4\,d^5\right)}{105\,e^4\,{\left(a\,e-b\,d\right)}^4}-\frac{x^2\,\left(560\,a^4\,e^5-3976\,a^3\,b\,d\,e^4+2556\,a^2\,b^2\,d^2\,e^3+6832\,a\,b^3\,d^3\,e^2-7700\,b^4\,d^4\,e\right)}{105\,e^4\,{\left(a\,e-b\,d\right)}^4}+\frac{32\,b^2\,x^4\,\left(35\,a^2\,e^2-84\,a\,b\,d\,e+58\,b^2\,d^2\right)}{105\,e\,{\left(a\,e-b\,d\right)}^4}+\frac{16\,b\,x^3\,\left(35\,a^3\,e^3+161\,a^2\,b\,d\,e^2-530\,a\,b^2\,d^2\,e+406\,b^3\,d^3\right)}{105\,e^2\,{\left(a\,e-b\,d\right)}^4}\right)}{x^4\,\sqrt{a+b\,x}+\frac{d^4\,\sqrt{a+b\,x}}{e^4}+\frac{6\,d^2\,x^2\,\sqrt{a+b\,x}}{e^2}+\frac{4\,d\,x^3\,\sqrt{a+b\,x}}{e}+\frac{4\,d^3\,x\,\sqrt{a+b\,x}}{e^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(9/2)),x)
```

```
[Out] ((d + e*x)^(1/2)*((3150*a*b^3*d^5 - 818*a^4*d^2*e^3 - 5950*a^2*b^2*d^4*e +
3906*a^3*b*d^3*e^2)/(105*e^4*(a*e - b*d)^4) + (x*(3150*b^4*d^5 - 1288*a^4*d
*e^4 + 6962*a^3*b*d^2*e^3 - 9422*a^2*b^2*d^3*e^2 + 1750*a*b^3*d^4*e))/(105*
e^4*(a*e - b*d)^4) - (x^2*(560*a^4*e^5 - 7700*b^4*d^4*e + 6832*a*b^3*d^3*e^
```

```
2 + 2556*a^2*b^2*d^2*e^3 - 3976*a^3*b*d*e^4))/(105*e^4*(a*e - b*d)^4) + (32
*b^2*x^4*(35*a^2*e^2 + 58*b^2*d^2 - 84*a*b*d*e))/(105*e*(a*e - b*d)^4) + (1
6*b*x^3*(35*a^3*e^3 + 406*b^3*d^3 - 530*a*b^2*d^2*e + 161*a^2*b*d*e^2))/(10
5*e^2*(a*e - b*d)^4)))/(x^4*(a + b*x)^(1/2) + (d^4*(a + b*x)^(1/2))/e^4 + (
6*d^2*x^2*(a + b*x)^(1/2))/e^2 + (4*d*x^3*(a + b*x)^(1/2))/e + (4*d^3*x*(a
+ b*x)^(1/2))/e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(9/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.593 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=417

$$-\frac{2\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right.}{c\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Rubi [A] time = 3.14, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {909, 63, 217, 206, 6728, 93, 208}

$$-\frac{2\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) - (2*(e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqr
t[b^2 - 4*a*c])*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e
*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(c*Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) - (
2*(e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(c*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 909

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[m + 1/2, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx=\int\left(\frac{e^2}{c\sqrt{d+ex}\sqrt{f+gx}}+\frac{cd^2-ae^2+e(2cd-be)x}{c\sqrt{d+ex}\sqrt{f+gx}\left(a+bx+cx^2\right)}\right)dx$$

$$=\frac{\int\frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx}{c}+\frac{e^2\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$=\frac{\int\left(\frac{e(2cd-be)+\frac{2c^2d^2-2bcde+b^2e^2-2ace^2}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{e(2cd-be)-\frac{2c^2d^2-2bcde+b^2e^2-2ace^2}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c}+\frac{(2e)\,\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$=\frac{(2e)\,\mathrm{Subst}\left(\int\frac{1}{1-\frac{gx^2}{e}}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c}+\frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int\frac{\cdots}{\left(b+\sqrt{b^2\cdots}\right.}}{c}$$

$$=\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}+\frac{\left(2\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int\frac{\cdots}{-2cd+\cdots}\right.}{c}$$

$$=\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}-\frac{2\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b\cdots\right.}}{\sqrt{2cd-\left(b\cdots\right.}}\right.}{c\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2\cdots\right.}}}$$

Mathematica [A] time = 1.79, size = 401, normalized size = 0.96

$$\frac{\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)^{3/2}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}}\right)-\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)^{3/2}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\right)}{c\sqrt{b^2-4ac}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}+\frac{2(ef-dg)^{3/2}\left(\frac{e(f+gx)}{ef-dg}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{c\sqrt{g}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*(e*f - d*g)^(3/2)*((e*(f + g*x))/(e*f - d*g))^(3/2)*ArcSinh[(Sqrt[g]*Sqr
t[d + e*x]])/Sqrt[e*f - d*g]])/(c*Sqrt[g]*(f + g*x)^(3/2)) + ((-2*c*d + (b -
```

```
Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]*ArcTa
nh[(Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d +
(b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])] - (-2*c*d + (b + Sqrt[b^2 - 4*a*
c])*e)^(3/2)*Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]*ArcTanh[(Sqrt[-2*c*f
+ (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 -
4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*f + (b - Sqrt[b^
2 - 4*a*c])*g]*Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g])
```

IntegrateAlgebraic [A] time = 6.48, size = 479, normalized size = 1.15

$$\frac{\sqrt{2}\left(e\sqrt{b^2-4ac}-be+2cd\right)\sqrt{ae^2-bde+cd^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f+gx}\sqrt{ae^2-bde+cd^2}}{\sqrt{d+ex}\sqrt{-dg\sqrt{b^2-4ac}+ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}}\right)}{c\sqrt{b^2-4ac}\sqrt{-dg\sqrt{b^2-4ac}+ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}}+\frac{\sqrt{2}\left(e\sqrt{b^2-4ac}+be-2cd\right)\sqrt{ae^2-bde+cd^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f+gx}\sqrt{ae^2-bde+cd^2}}{\sqrt{d+ex}\sqrt{dg\sqrt{b^2-4ac}-ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}}\right)}{c\sqrt{b^2-4ac}\sqrt{dg\sqrt{b^2-4ac}-ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}}+\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{c\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (Sqrt[2]*(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*Sqrt[c*d^2 - b*d*e + a*e^2]*Ar
cTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b
*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqr
t[d + e*x])])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c
]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + (Sqrt[2]*(-2*c*d + b*e
+ Sqrt[b^2 - 4*a*c]*e)*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan[(Sqrt[2]*Sqrt[c*d
^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a*
c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d + e*x])])/(c*Sqrt[
b^2 - 4*a*c]*Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b
^2 - 4*a*c]*d*g - 2*a*e*g]) + (2*e^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(S
qrt[g]*Sqrt[d + e*x])])/(c*Sqrt[g])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.18, size = 11688, normalized size = 28.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cx^2+bx+a\right)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\sqrt{f+g\,x}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

3.594 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$

Optimal. Leaf size=285

$$\frac{2\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2 \ldots}}{\sqrt{f+gx}\sqrt{\ldots}}\right.}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(b-\sqrt{b^2-4 \ldots}\right.}}$$

Rubi [A] time = 0.53, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {909, 93, 208}

$$\frac{2\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b - Sqrt
[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*S
qrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g])
 + (2*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]
*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 909

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeqQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{e+\frac{2cd-be}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e-\frac{2cd-be}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\cdots}\right.$$

$$= \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx + \left(e+\frac{2cd-\cdots}{\sqrt{b^2-\cdots}}\right.$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b+\sqrt{b^2-\cdots}\right.\right.}\right.$$

$$= -\frac{2\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{2\sqrt{2cd-\left(b\cdots\right.}}{\cdots}$$

Mathematica [A] time = 0.87, size = 266, normalized size = 0.93

$$\frac{2\left(\frac{\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\right)}{\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}} - \frac{\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}}\right)}{\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*(-((Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[-2*c*f + (b -
 Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c]
)*e]*Sqrt[f + g*x])])/Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]) + (Sqrt[-2*
c*d + (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a
*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g
*x])])/Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]))/Sqrt[b^2 - 4*a*c]
```

IntegrateAlgebraic [F] time = 180.36, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 37.79, size = 4471, normalized size = 15.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f^
2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g
+ d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 -
```

```
2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4
*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^
2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2*g^2 - 2*(b*d*e - a*e^2)*f^2 + sqrt(2
)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)*d*f*g + ((b^3*c - 4*a*b*c^2)*f^3 - (
b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b
^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3
)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2
 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))*sqrt(e*x + d)*s
qrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f
^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g
 + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 -
 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 -
4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a
^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g - (b*d^2 - 4*a*d*e)*g^2)*x - (2*(b^2
*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g
^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2 - 4*a^2*c)*d*g^3 + ((b^2*c - 4*a*c^2
)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*
f*g^2)*x)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2
*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b
^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x) - 1/4*sqrt(2)*sqrt(((
2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)
*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^
2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c
^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^
2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2))*log(-(2*
b*d^2*f*g - 2*a*d^2*g^2 - 2*(b*d*e - a*e^2)*f^2 - sqrt(2)*((b^2 - 4*a*c)*e*
f^2 - (b^2 - 4*a*c)*d*f*g + ((b^3*c - 4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8
*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*
sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c -
4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2
*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt((
(2*c*d - b*e)*f - (b*d - 2*a*e)*g + ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c
)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c
^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*
c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b
^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^
2*f^2 - 4*a*e^2*f*g - (b*d^2 - 4*a*d*e)*g^2)*x - (2*(b^2*c - 4*a*c^2)*d*f^3
 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*
c^2)*e*f^3 + (a*b^2 - 4*a^2*c)*d*g^3 + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*
c)*e)*f^2*g - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*f*g^2)*x)*sqrt((e^2
*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2
)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g
^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x) + 1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b
*d - 2*a*e)*g - ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a
^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 -
2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*
b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2
 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2
*g^2 - 2*(b*d*e - a*e^2)*f^2 + sqrt(2)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)
*d*f*g - ((b^3*c - 4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3
*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d
*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g +
(b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*
b^2 - 4*a^3*c)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (
b*d - 2*a*e)*g - ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*
a^2*c)*g^2)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 -
 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a
*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2
 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g
```

```
 - (b*d^2 - 4*a*d*e)*g^2)*x + (2*(b^2*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c
)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2
 - 4*a^2*c)*d*g^3 + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3
 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*f*g^2)*x)*sqrt((e^2*f^2 - 2*d*e*f*g +
d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*
a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a
^3*c)*g^4)))/x) - 1/4*sqrt(2)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g - ((b
^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e^
2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^
2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*
g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f
*g + (a*b^2 - 4*a^2*c)*g^2))*log(-(2*b*d^2*f*g - 2*a*d^2*g^2 - 2*(b*d*e - a
*e^2)*f^2 - sqrt(2)*((b^2 - 4*a*c)*e*f^2 - (b^2 - 4*a*c)*d*f*g - ((b^3*c -
4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c
)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(
(b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c -
8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)
))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g - ((
b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((e
^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c
^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f
*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*
f*g + (a*b^2 - 4*a^2*c)*g^2)) - (b*e^2*f^2 - 4*a*e^2*f*g - (b*d^2 - 4*a*d*e
)*g^2)*x + (2*(b^2*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^
2 - 4*a^2*c)*d*f*g^2 + ((b^2*c - 4*a*c^2)*e*f^3 + (a*b^2 - 4*a^2*c)*d*g^3 +
 ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*f^2*g - ((b^3 - 4*a*b*c)*d - (a*
b^2 - 4*a^2*c)*e)*f*g^2)*x)*sqrt((e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((b^2*c^2
- 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2
)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.05, size = 5482, normalized size = 19.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}}{(cx^2+bx+a)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)),x)
[Out] \text{Hanged}
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex}}{\sqrt{f + gx} \left(a + bx + cx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(d + e*x)/(sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

3.595 $\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$

Optimal. Leaf size=287

$$\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4\cdots}\right.}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4\cdots}\right.}}\right.}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-\cdots}}$$

Rubi [A] time = 0.41, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {911, 93, 208}

$$\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-4*c*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
]) + (4*c*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]
*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\left(a+bx+cx^2\right)}dx = \int \left(\frac{2c}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}} - \frac{}{\sqrt{b^2-4a}}\right.$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f}}}{\sqrt{b^2-4ac}}$$

$$= \frac{(4c)\,\text{Subst}\left(\int \frac{1}{-2cd+\left(b-\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b-\sqrt{b^2-4ac}\right)g\right)x^2}dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}} -$$

$$= -\frac{4c\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} -$$

Mathematica [A] time = 0.75, size = 267, normalized size = 0.93

$$\frac{4c\left(\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}}\right)}{\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (4*c*(ArcTanh[(Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqr
t[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]/(Sqrt[-2*c*d + (b - S
qrt[b^2 - 4*a*c])*e]*Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]) - ArcTanh[(S
qrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b +
Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c]
)*e]*Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g])))/Sqrt[b^2 - 4*a*c]
```

IntegrateAlgebraic [A] time = 5.45, size = 566, normalized size = 1.97

$$\frac{\left(\sqrt{2}e\sqrt{b^2-4ac}\sqrt{ae^2-bde+cd^2}-2\sqrt{2}cd\sqrt{ae^2-bde+cd^2}+\sqrt{2}be\sqrt{ae^2-bde+cd^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f+gx}\sqrt{ae^2-bde+cd^2}}{\sqrt{d+ex}\sqrt{-dg\sqrt{b^2-4ac}+ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}}\right)}{\sqrt{b^2-4ac}\left(-ae^2+bde-cd^2\right)\sqrt{-dg\sqrt{b^2-4ac}+ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}}+\frac{\left(\sqrt{2}e\sqrt{b^2-4ac}\sqrt{ae^2-bde+cd^2}+2\sqrt{2}cd\sqrt{ae^2-bde+cd^2}-\sqrt{2}be\sqrt{ae^2-bde+cd^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f+gx}\sqrt{ae^2-bde+cd^2}}{\sqrt{d+ex}\sqrt{dg\sqrt{b^2-4ac}-ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}}\right)}{\sqrt{b^2-4ac}\left(-ae^2+bde-cd^2\right)\sqrt{dg\sqrt{b^2-4ac}-ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] ((-2*Sqrt[2]*c*d*Sqrt[c*d^2 - b*d*e + a*e^2] + Sqrt[2]*b*e*Sqrt[c*d^2 - b*d
*e + a*e^2] + Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[c*d^2 - b*d*e + a*e^2])*ArcT
an[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b*e
*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[
d + e*x])])/(Sqrt[b^2 - 4*a*c]*(-(c*d^2) + b*d*e - a*e^2)*Sqrt[-2*c*d*f + b
*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]) +
((2*Sqrt[2]*c*d*Sqrt[c*d^2 - b*d*e + a*e^2] - Sqrt[2]*b*e*Sqrt[c*d^2 - b*d*
e + a*e^2] + Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[c*d^2 - b*d*e + a*e^2])*ArcTa
n[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b*e*
f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d
```

```
 + e*x])]])/(Sqrt[b^2 - 4*a*c]*(-(c*d^2) + b*d*e - a*e^2)*Sqrt[-2*c*d*f + b*
e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g])
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**B**] time = 0.06, size = 5507, normalized size = 19.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2 + bx + a\right)\sqrt{ex + d}\,\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d + ex}\,\sqrt{f + gx}\,\left(a + bx + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

3.596 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=429

$$-\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}+\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-}}{\sqrt{f+gx}\sqrt{2cd-}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/}}$$

Rubi [A] time = 1.36, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {911, 96, 93, 208}

$$-\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}+\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}+\frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)*(e*f - d*g)*Sqrt[d + e*x]) - (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(e*f - d*g)*Sqrt[d + e*x]) - (8*c^2*ArcTan
h[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) + (8*c^
2*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
```

```
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{2c}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}} - \frac{\ }{\sqrt{b^2-4a}}\right.$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}}}{\sqrt{b^2-4ac}}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{\ }{\sqrt{b^2-4ac}}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{\ }{\sqrt{b^2-4ac}}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{\ }{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 2.06, size = 334, normalized size = 0.78

$$-\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}}\right)}{\sqrt{b^2-4ac}\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)^{3/2}\sqrt{g\left(b-\sqrt{b^2-4ac}\right)-2cf}} + \frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}{\sqrt{f+gx}\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\right)}{\sqrt{b^2-4ac}\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)^{3/2}\sqrt{g\left(\sqrt{b^2-4ac}+b\right)-2cf}} + \frac{2e^2\sqrt{f+gx}}{\sqrt{d+ex}(dg-ef)\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x])/((c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)*Sqrt[d + e
*x]) - (8*c^2*ArcTanh[(Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*
x])/(Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(Sqrt[b^2 -
4*a*c]*(-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[-2*c*f + (b - Sqrt[b
^2 - 4*a*c])*g]) + (8*c^2*ArcTanh[(Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]
*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/
(Sqrt[b^2 - 4*a*c]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[-2*c*f +
 (b + Sqrt[b^2 - 4*a*c])*g])
```

IntegrateAlgebraic [A] time = 4.10, size = 651, normalized size = 1.52

$$\frac{\left(2\sqrt{2}cde\sqrt{b^2-4ac}-\sqrt{2}be^2\sqrt{b^2-4ac}+2\sqrt{2}ace^2-\sqrt{2}b^2e^2+2\sqrt{2}bcde-2\sqrt{2}c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f+gx}\sqrt{ae^2-bde+cd^2}}{\sqrt{d+ex}\sqrt{-dg\sqrt{b^2-4ac}+ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}}\right)}{\sqrt{b^2-4ac}\left(-ae^2+bde-cd^2\right)\sqrt{ae^2-bde+cd^2}\sqrt{-dg\sqrt{b^2-4ac}+ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}} + \frac{\left(2\sqrt{2}cde\sqrt{b^2-4ac}-\sqrt{2}be^2\sqrt{b^2-4ac}-2\sqrt{2}ace^2+\sqrt{2}b^2e^2-2\sqrt{2}bcde+2\sqrt{2}c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f+gx}\sqrt{ae^2-bde+cd^2}}{\sqrt{d+ex}\sqrt{dg\sqrt{b^2-4ac}-ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}}\right)}{\sqrt{b^2-4ac}\left(-ae^2+bde-cd^2\right)\sqrt{ae^2-bde+cd^2}\sqrt{dg\sqrt{b^2-4ac}-ef\sqrt{b^2-4ac}-2aeg+bdg+bef-2cdf}} + \frac{2e^2\sqrt{f+gx}}{\sqrt{d+ex}(dg-ef)\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x])/((c*d^2 - b*d*e + a*e^2)*(-(e*f) + d*g)*Sqrt[d + e*x]
) + ((-2*Sqrt[2]*c^2*d^2 + 2*Sqrt[2]*b*c*d*e + 2*Sqrt[2]*c*Sqrt[b^2 - 4*a*c
```

```
]*d*e - Sqrt[2]*b^2*e^2 + 2*Sqrt[2]*a*c*e^2 - Sqrt[2]*b*Sqrt[b^2 - 4*a*c]*e
^2)*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d
*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*
g]*Sqrt[d + e*x])])/(Sqrt[b^2 - 4*a*c]*(-(c*d^2) + b*d*e - a*e^2)*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g -
Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + ((2*Sqrt[2]*c^2*d^2 - 2*Sqrt[2]*b*c*d*e
 + 2*Sqrt[2]*c*Sqrt[b^2 - 4*a*c]*d*e + Sqrt[2]*b^2*e^2 - 2*Sqrt[2]*a*c*e^2
- Sqrt[2]*b*Sqrt[b^2 - 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g +
 Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d + e*x])])/(Sqrt[b^2 - 4*a*c]*(-(c*
d^2) + b*d*e - a*e^2)*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[-2*c*d*f + b*e*f - S
qrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.34, size = 47351, normalized size = 110.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2 + bx + a\right)(ex + d)^{\frac{3}{2}} \sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}\,\left(c\,x^2+b\,x+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**(3/2)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

3.597 $\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=532

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ce(2cd-be)\left(-4cdg^2(aeg-2bdg+6bef)+5b^2deg^3+16c^2e^2f^3\right)-2g\left(-2ce(bd-a\right.\right.}{128c^{7/2}e^5}$$

Rubi [A] time = 1.70, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bdg+6bef)+5b^3e^3g^3+16bc^2eg\left(d^2g^2-3defg+3e^2f^2\right)+64c^3(ef-dg)^3\right)}{64c^3e^4}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ce(2cd-be)\left(-4cdg^2(aeg-2bdg+6bef)+5b^2deg^3+16c^2e^2f^3\right)-2g\left(-2ce(bd-ae)-\frac{b^2e^2}{2}+4c^2d^2\right)\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)\right)}{128c^{7/2}e^5}+\frac{g^2\left(a+bx+cx^2\right)^{3/2}(-5beg-14cdg+24cef)}{24c^2e^2}+\frac{(ef-dg)^3\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5}+\frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((5*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(6*b*e*f - 2*b*d*g +
 a*e*g) + 16*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(5*b^2*e
^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*
g + d^2*g^2))*x)*Sqrt[a + b*x + c*x^2])/(64*c^3*e^4) + (g^2*(24*c*e*f - 14*
c*d*g - 5*b*e*g)*(a + b*x + c*x^2)^(3/2))/(24*c^2*e^2) + (g^3*(d + e*x)*(a
+ b*x + c*x^2)^(3/2))/(4*c*e^2) - ((4*c*e*(2*c*d - b*e)*(16*c^2*e^2*f^3 + 5
*b^2*d*e*g^3 - 4*c*d*g^2*(6*b*e*f - 2*b*d*g + a*e*g)) - 2*(4*c^2*d^2 - (b^2
*e^2)/2 - 2*c*e*(b*d - a*e))*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g
+ a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*e^5) + (Sqrt[c*d^2 - b*d*e
+ a*e^2]*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{1}{2}e\left(8ce^2f^3-d(3bd+2ae)g^3\right)-eg\left(e(4bd+ae)g^2-3c(\ldots\right.\right.}{d+ex}}{4ce^3}$$

$$= \frac{g^2(24cef-14cdg-5beg)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \int \ldots$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3d\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3d\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3d\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3d\ldots\right.\right.}{\ldots}$$

Mathematica [A] time = 1.01, size = 559, normalized size = 1.05

$$\frac{-\frac{24e^2g(bg-2cf)(ef-dg)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{5/2}}-\frac{48eg\left(b^2-4ac\right)(ef-dg)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}+\frac{e^3g\left(3\left(-4cg(ag+4bf)+5b^2g^2+16c^2f^2\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)+80c^{3/2}g(a+x(b+cx))^{3/2}(2cf-bg)\right)}{c^{7/2}}+\frac{192(ef-dg)^3\left(2\sqrt{c}\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{\sqrt{c}e}+\frac{128e^2g^2(a+x(b+cx))^{3/2}(ef-dg)}{c}+\frac{96eg(b+2cx)\sqrt{a+x(b+cx)}(ef-dg)^2}{c}+384\sqrt{a+x(b+cx)}(ef-dg)^3+\frac{96e^3g^2(f+gx)(a+x(b+cx))^{3/2}}{c}}{384e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (384*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)] + (96*e*g*(e*f - d*g)^2*(b + 2*c*x
)*Sqrt[a + x*(b + c*x)])/c + (128*e^2*g^2*(e*f - d*g)*(a + x*(b + c*x))^(3/
```

```
2))/c + (96*e^3*g^2*(f + g*x)*(a + x*(b + c*x))^(3/2))/c - (48*(b^2 - 4*a*c
)*e*g*(e*f - d*g)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
/c^(3/2) - (24*e^2*g*(-2*c*f + b*g)*(e*f - d*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt
[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])]))/c^(5/2) + (e^3*g*(80*c^(3/2)*g*(2*c*f - b*g)*(a + x*(b + c*
x))^(3/2) + 3*(16*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(4*b*f + a*g))*(2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])])))/c^(7/2) + (192*(e*f - d*g)^3*((-2*c*d + b*e)*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*e))/(384*e^4)
```

IntegrateAlgebraic [A] time = 4.59, size = 788, normalized size = 1.48

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(192*c^3*e^3*f^3 - 576*c^3*d*e^2*f^2*g + 144*b*c^2*e
^3*f^2*g + 576*c^3*d^2*e*f*g^2 - 144*b*c^2*d*e^2*f*g^2 - 72*b^2*c*e^3*f*g^2
 + 192*a*c^2*e^3*f*g^2 - 192*c^3*d^3*g^3 + 48*b*c^2*d^2*e*g^3 + 24*b^2*c*d*
e^2*g^3 - 64*a*c^2*d*e^2*g^3 + 15*b^3*e^3*g^3 - 52*a*b*c*e^3*g^3 + 288*c^3*
e^3*f^2*g*x - 288*c^3*d*e^2*f*g^2*x + 48*b*c^2*e^3*f*g^2*x + 96*c^3*d^2*e*g
^3*x - 16*b*c^2*d*e^2*g^3*x - 10*b^2*c*e^3*g^3*x + 24*a*c^2*e^3*g^3*x + 192
*c^3*e^3*f*g^2*x^2 - 64*c^3*d*e^2*g^3*x^2 + 8*b*c^2*e^3*g^3*x^2 + 48*c^3*e^
3*g^3*x^3))/(192*c^3*e^4) - (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e^3*f^3) +
 3*d*e^2*f^2*g - 3*d^2*e*f*g^2 + d^3*g^3)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x -
 e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^5 + ((128*c^4*
d*e^3*f^3 - 64*b*c^3*e^4*f^3 - 384*c^4*d^2*e^2*f^2*g + 192*b*c^3*d*e^3*f^2*
g + 48*b^2*c^2*e^4*f^2*g - 192*a*c^3*e^4*f^2*g + 384*c^4*d^3*e*f*g^2 - 192*
b*c^3*d^2*e^2*f*g^2 - 48*b^2*c^2*d*e^3*f*g^2 + 192*a*c^3*d*e^3*f*g^2 - 24*b
^3*c*e^4*f*g^2 + 96*a*b*c^2*e^4*f*g^2 - 128*c^4*d^4*g^3 + 64*b*c^3*d^3*e*g^
3 + 16*b^2*c^2*d^2*e^2*g^3 - 64*a*c^3*d^2*e^2*g^3 + 8*b^3*c*d*e^3*g^3 - 32*
a*b*c^2*d*e^3*g^3 + 5*b^4*e^4*g^3 - 24*a*b^2*c*e^4*g^3 + 16*a^2*c^2*e^4*g^3
)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(128*c^(7/2)*e^5)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.04, size = 3941, normalized size = 7.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] -1/e^4*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*g^
3*d^3+1/e*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*f^3+3/e^3*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*d^2*f*g^2-1/16*g^3/e/c^2*a*(c*x^2+b*x+a)^(1/2)*b+1/8*g^3/e^2*b^2/c^2*(c*x
^2+b*x+a)^(1/2)*d-3/8*g^2/e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*f-3/8*g/e*f^2/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-3/2/e^2*ln((1/2*(b*e-2*c
*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))/c^(1/2)*b*d*f^2*g-3/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(x+d/e))*a*d^2*f*g^2+3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*
d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+
d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*d
*f^2*g+3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+
(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2
*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^3*f*g^2-3/e^3/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e
*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/
e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^2*f^2*g-3/e^5/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^4*f*g^2+3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x+d/e))*c*d^3*f^2*g-3/4*g^2/e^2*d*f/c*(c*x^2+b*x+a)^(1/2)*b-3/2*g
^2/e^2*d*f/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/8*g^2/e^
2*d*f/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-3/4*g^2/e*b/c
*x*(c*x^2+b*x+a)^(1/2)*f+1/4*g^3/e^2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*a*d-1/2/e^4*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/
e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*g^3*
d^3-3/e^4*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)
/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^3*f*g^2+3/e^3*ln((1/2*
(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*f^2*g+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x+d/e))*a*g^3*d^3-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e
))*b*d^4*g^3+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2
)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d*f^3+1/e^
6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)
/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+
d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^5*g^3-1/e^3/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^2*f^3+3/16*g^2/e*b^3/c^(5/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f-3/2*g^2/e^2*d*f*x*(c*x^2+b*x+a)^(1/2)-1/
8*g^3/e/c*a*x*(c*x^2+b*x+a)^(1/2)-3/e^2*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f^2*g+1/2/e*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*
c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/c^(1/2)*b*f^3+1/e^5*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*
c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^4*g^3-1/e
^2*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d
/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*f^3-1/e/((a*e^2-b*d*e+c*d^2)/
```

```
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(x+d/e))*a*f^3+5/32*g^3/e*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)+3/16*
g^3/e*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/4*g^3/e^3
*d^2/c*(c*x^2+b*x+a)^(1/2)*b+1/2*g^3/e^3*d^2/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a-1/8*g^3/e^3*d^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*b^2+3/4*g/e*f^2/c*(c*x^2+b*x+a)^(1/2)*b+3/2*g/e*f^2/c^(1/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/16*g^3/e^2*b^3/c^(5/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d-3/4*g^2/e*b/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f+1/4*g^3/e^2*b/c*x*(c*x^2+b*x+a)^(1/2)*
d+3/2/e^3*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)
/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*d^2*f*g^2+3/2*g/e*f^2*
x*(c*x^2+b*x+a)^(1/2)+1/2*g^3/e^3*d^2*x*(c*x^2+b*x+a)^(1/2)+1/4*g^3/e*x*(c*
x^2+b*x+a)^(3/2)/c-5/24*g^3/e*b/c^2*(c*x^2+b*x+a)^(3/2)+5/64*g^3/e*b^3/c^3*
(c*x^2+b*x+a)^(1/2)-5/128*g^3/e*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))-1/8*g^3/e/c^(3/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))-1/3*g^3/e^2*(c*x^2+b*x+a)^(3/2)/c*d+g^2/e*(c*x^2+b*x+a)^(3/2)/c*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^3\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^3*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^{3}\sqrt{a+bx+cx^{2}}}{d+ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**3*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.598 $\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=325

$$-\frac{\sqrt{a+bx+cx^2}\left(b^2e^2g^2-2cegx(-beg-2cdg+4cef)-2bceg(2ef-dg)-8c^2(ef-dg)^2\right)}{8c^2e^3}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right.}{}$$

Rubi [A] time = 0.71, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(g\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)(-beg-2cdg+4cef)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)}{16c^{5/2}e^4}-\frac{\sqrt{a+bx+cx^2}\left(b^2e^2g^2-2cegx(-beg-2cdg+4cef)-2bceg(2ef-dg)-8c^2(ef-dg)^2\right)}{8c^2e^3}+\frac{(ef-dg)^2\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4}+\frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] -((b^2*e^2*g^2 - 8*c^2*(e*f - d*g)^2 - 2*b*c*e*g*(2*e*f - d*g) - 2*c*e*g*(4
*c*e*f - 2*c*d*g - b*e*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c^2*e^3) + (g^2*(a +
 b*x + c*x^2)^(3/2))/(3*c*e) + (((8*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a*e))*
g*(4*c*e*f - 2*c*d*g - b*e*g) - 4*c*e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*g^2))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*e^4) +
(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d -
b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce} + \frac{\int \frac{\left(\frac{3}{2}e\left(2cef^2-bdg^2\right)+\frac{3}{2}eg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{3ce^2}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a}}{8c^2e^3}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a}}{8c^2e^3}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a}}{8c^2e^3}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a}}{8c^2e^3}$$

Mathematica [A] time = 0.41, size = 372, normalized size = 1.14

$$\frac{-\frac{6eg\left(b^2-4ac\right)(ef-dg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}-\frac{3e^2g(bg-2cf)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{5/2}}+\frac{24(ef-dg)^2\left(2\sqrt{c}\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{\sqrt{c}e}+\frac{12eg(b+2cx)\sqrt{a+x(b+cx)}(ef-dg)}{c}+48\sqrt{a+x(b+cx)}(ef-dg)^2+\frac{16e^2g^2(a+x(b+cx))^{3/2}}{c}}{48e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (48*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)] + (12*e*g*(e*f - d*g)*(b + 2*c*x)*S
qrt[a + x*(b + c*x)])/c + (16*e^2*g^2*(a + x*(b + c*x))^(3/2))/c - (6*(b^2
- 4*a*c)*e*g*(e*f - d*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])])/c^(3/2) - (3*e^2*g*(-2*c*f + b*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(
b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*
x)])]))/c^(5/2) + (24*(e*f - d*g)^2*((-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*
ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*
Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*e))/(48*e^3)
```

IntegrateAlgebraic [A] time = 2.03, size = 428, normalized size = 1.32

$$\frac{\sqrt{a+bx+cx^2}\left(8ace^2g^2-3b^2e^2g^2-6bcdeg^2+12bce^2fg+2bce^2g^2x+24c^2d^2g^2-48c^2defg-12c^2deg^2x+24c^2e^2f^2+24c^2e^2fgx+8c^2e^2g^2x^2\right)}{24c^2e^3}+\frac{\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)\left(4abce^3g^2+8ac^2de^2g^2-16ac^2e^3fg-b^3e^3g^2-2b^2cde^2g^2+4b^2ce^3fg-8bc^2d^2eg^2+16bc^2de^2fg-8bc^2e^3f^2+16c^3d^3g^2-32c^3d^2efg+16c^3de^2f^2\right)}{16c^{5/2}e^4}+\frac{2\left(d^2g^2-2defg+e^2f^2\right)\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{-e\sqrt{a+bx+cx^2}+\sqrt{c}d+\sqrt{c}ex}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(24*c^2*e^2*f^2 - 48*c^2*d*e*f*g + 12*b*c*e^2*f*g +
24*c^2*d^2*g^2 - 6*b*c*d*e*g^2 - 3*b^2*e^2*g^2 + 8*a*c*e^2*g^2 + 24*c^2*e^2
*f*g*x - 12*c^2*d*e*g^2*x + 2*b*c*e^2*g^2*x + 8*c^2*e^2*g^2*x^2))/(24*c^2*e
^3) + (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*Arc
Tan[(Sqrt[c]*d + Sqrt[c]*e*x - e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d
*e - a*e^2]])/e^4 + ((16*c^3*d*e^2*f^2 - 8*b*c^2*e^3*f^2 - 32*c^3*d^2*e*f*g
 + 16*b*c^2*d*e^2*f*g + 4*b^2*c*e^3*f*g - 16*a*c^2*e^3*f*g + 16*c^3*d^3*g^2
 - 8*b*c^2*d^2*e*g^2 - 2*b^2*c*d*e^2*g^2 + 8*a*c^2*d*e^2*g^2 - b^3*e^3*g^2
+ 4*a*b*c*e^3*g^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(16*c^
(5/2)*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 2602, normalized size = 8.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/3*g^2*(c*x^2+b*x+a)^(3/2)/c/e+1/e^3*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^2*g^2+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(x+d/e))*c*d^3*f*g+1/e*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*f^2-1/e^2*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e
)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*d*f*g
+2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2
*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/
e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*d*f*g-2/e^3/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^2*f*g-2/e^2*((x+d/e)^2*c+(b*e-2*c*d)/
e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f*g+1/2/e*ln((1/2*(b*e-2*c*d)/e+
(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e
```

```
^2)^(1/2))/c^(1/2)*b*f^2-1/e^4*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x
+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^3
*g^2-1/e^2*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d
)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*f^2-1/e/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*f^2-1/2*g^2/e^2*d*(c*x^2+b*x+a)^(1/2)*x-1/
4*g^2/e*b/c*(c*x^2+b*x+a)^(1/2)*x-1/4*g^2/e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))*a-1/4*g^2/e^2*d/c*(c*x^2+b*x+a)^(1/2)*b-1/2*g^2/e^2*
d/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8*g^2/e^2*d/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/2/e^3*ln((1/2*(b*e-2*c
*d)/e+(x+d/e)*c)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))/c^(1/2)*b*d^2*g^2+2/e^3*ln((1/2*(b*e-2*c*d)/e+(x+d/e)*c)/c
^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c
^(1/2)*d^2*f*g-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*
c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*d^2*g^2+
1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e
*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^3*g^2+1/e^2/((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+
2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d*f^2-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x+d/e))*c*d^4*g^2-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e
))*c*d^2*f^2+1/2*g/e*f/c*(c*x^2+b*x+a)^(1/2)*b+g/e*f/c^(1/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/4*g/e*f/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*b^2+g/e*f*(c*x^2+b*x+a)^(1/2)*x-1/8*g^2/e*b^2/c^2*(c*x^2
+b*x+a)^(1/2)+1/16*g^2/e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^2\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^2*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + g x\right)^{2} \sqrt{a + b x + c x^{2}}}{d + e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**2*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.599 $\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=219

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg)\right)}{8c^{3/2}e^3}+\frac{(ef-dg)\sqrt{ae^2-bde+cd^2}\tanh^{-1}}{e^3}$$

Rubi [A] time = 0.32, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {814, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg)\right)}{8c^{3/2}e^3}+\frac{(ef-dg)\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3}+\frac{\sqrt{a+bx+cx^2}\,(beg-4cdg+4cef+2cegx)}{4ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*Sqrt[a + b*x + c*x^2])/(4*c*e^2) -
 ((b^2*e^2*g + 8*c^2*d*(e*f - d*g) - 4*c*e*(b*e*f - b*d*g + a*e*g))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*e^3) + (Sqrt[c*
d^2 - b*d*e + a*e^2]*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{\int \frac{\frac{1}{2}(4ce(bd-2ae)f+4acdeg-bd(4cd-be)g)+\frac{1}{2}}{(d+ex)\sqrt{a}}}{4c}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} + \frac{\left(\left(cd^2-bde+ae^2\right)(ef-dg)\right)\int \frac{}{(d+}}{e^3}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)(ef-dg)\right)\text{Sul}}{}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{\left(b^2e^2g+8c^2d(ef-dg)-4ce(bef}{8c}$$

Mathematica [A] time = 0.35, size = 216, normalized size = 0.99

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(4ce(aeg-bdg+bef)-b^2e^2g+8c^2d(dg-ef)\right)+2\sqrt{c}\left(4c(dg-ef)\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+e\sqrt{a+x(b+cx)}(beg+2c(-2dg+2ef+egx))\right)}{8c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((-(b^2*e^2*g) + 8*c^2*d*(-(e*f) + d*g) + 4*c*e*(b*e*f - b*d*g + a*e*g))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(e*Sqrt[a
+ x*(b + c*x)]*(b*e*g + 2*c*(2*e*f - 2*d*g + e*g*x)) + 4*c*Sqrt[c*d^2 + e*(
-(b*d) + a*e)]*(-(e*f) + d*g)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2
*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(8*c^(3/2)*e^3)
```

IntegrateAlgebraic [A] time = 1.00, size = 229, normalized size = 1.05

$$\frac{\log\left(-2c^{3/2}\sqrt{a+bx+cx^2}+bc+2c^2x\right)\left(-4ace^2g+b^2e^2g+4bcdeg-4bce^2f-8c^2d^2g+8c^2def\right)}{8c^{3/2}e^3}-\frac{2(dg-ef)\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{-e\sqrt{a+bx+cx^2}+\sqrt{c}d+\sqrt{c}ex}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^3}+\frac{\sqrt{a+bx+cx^2}(beg-4cdg+4cef+2cegx)}{4ce^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*Sqrt[a + b*x + c*x^2])/(4*c*e^2) -
 (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(Sqrt[c]*d + Sqrt[
c]*e*x - e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^3 + ((
8*c^2*d*e*f - 4*b*c*e^2*f - 8*c^2*d^2*g + 4*b*c*d*e*g + b^2*e^2*g - 4*a*c*e
^2*g)*Log[b*c + 2*c^2*x - 2*c^(3/2)*Sqrt[a + b*x + c*x^2]])/(8*c^(3/2)*e^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1559, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*g*(c*x^2+b*x+a)^(1/2)*x+1/4/e*g/c*(c*x^2+b*x+a)^(1/2)*b+1/2/e*g/c^(1/
2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8/e*g/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-1/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e
)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*g+1/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*f-1/2/e^2*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e
)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/c^(1/2)*b*d*g+1/2/e*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c
+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*f+1/e^3*ln
(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*g-1/e^2*ln(((x+d/e)*c+1/2*(b*e-2
*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))*c^(1/2)*d*f+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*
(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d
/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*d*
g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*
d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(
x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*f-1/e^3/((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^2*g+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(x+d/e))*b*d*f+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+
d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)
^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^3*g
-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b
*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*
(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^2*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f + gx)\sqrt{a + bx + cx^2}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.600 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^2}+\frac{\sqrt{a+bx+cx^2}}{e}$$

Rubi [A] time = 0.15, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {734, 843, 621, 206, 724}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^2}+\frac{\sqrt{a+bx+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/e - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x + c*x^2]])])/e^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{\sqrt{a+bx+cx^2}}{e} - \frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e}$$

$$= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{(e(bd-2ae)-d(2cd-be))\int \frac{1}{(d+ex)\sqrt{a+bx+cx}}}{2e^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Sub}}{}$$

$$= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^2} + \frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\left(\frac{bd-2}{2\sqrt{cd^2-bc}}\right.}{e^2}$$

Mathematica [A] time = 0.15, size = 145, normalized size = 0.95

$$\frac{-2\sqrt{e(ae-bd)+cd^2}\,\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+\frac{(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}+2e\sqrt{a+x(b+cx)}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] (2*e*Sqrt[a + x*(b + c*x)] + ((-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])])/Sqrt[c] - 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt
[a + x*(b + c*x)])])/(2*e^2)
```

IntegrateAlgebraic [A] time = 0.00, size = 191, normalized size = 1.26

$$\frac{2\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+bx+cx^2}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^2}+\frac{(2cd-be)\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{2\sqrt{c}\,e^2}+\frac{\sqrt{a+bx+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/e + (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]
*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) + b*d*e -
a*e^2] - (e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^2 + (
(2*c*d - b*e)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[c]*
e^2)
```

fricas [A] time = 1.98, size = 992, normalized size = 6.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*sq
rt(c*d^2 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2
```

```
- (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e +
a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e^2),
 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + sqrt(c*d^2
 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2
*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*s
qrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e^2), 1/4*(4*
sqrt(c*x^2 + b*x + a)*c*e + 4*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sq
rt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*
e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b
*c*d^2 - b^2*d*e + a*b*e^2)*x)) - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))/(c*e^2)
, 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arcta
n(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2
*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)
*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)))/(c*e
^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 715, normalized size = 4.70

$$-\frac{a\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2ae^2-2bde+2cd^2}{e^2}+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\,e}+\frac{bd\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2ae^2-2bde+2cd^2}{e^2}+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\,e^2}-\frac{cd^2\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2ae^2-2bde+2cd^2}{e^2}+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\,e^3}+\frac{b\ln\left(\frac{\left(x+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}\right)}{2\sqrt{c}\,e}-\frac{\sqrt{c}\,d\ln\left(\frac{\left(x+\frac{d}{e}\right)c+\frac{be-2cd}{2e}}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}\right)}{e^2}+\frac{\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2/e
*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/e^2*ln(((x+d/e)*c+1/2*(b*e-2*
c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))*c^(1/2)*d-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/
e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2
*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a+1/e^2/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d
^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d-1/e^3/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(x+d/e))*c*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                            +c*d^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

3.601 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=228

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)}+\frac{\sqrt{c}\ \mathrm{t}}{}$$

Rubi [A] time = 0.33, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {895, 724, 206, 843, 621}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(e*g) + (S
qrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)) - (Sqrt[c*f
^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
 b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
```

```
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx &= -\frac{\int \frac{cdf-bef+aeg-c(ef-dg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{e(ef-dg)} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e(ef-dg)} \\
&= \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{eg} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)} \\
&= \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)} + \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{eg} \\
&= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg} + \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)} - \frac{\sqrt{cf}\ldots}{\ldots}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 218, normalized size = 0.96

$$\frac{g\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+bd-bex+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{ae^2-bde+cd^2}}\right)+\sqrt{c}(ef-dg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-e\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+bf-bgx+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{ag^2-bfg+cf^2}}\right)}{eg(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

```
[Out] (Sqrt[c]*(e*f - d*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
 + Sqrt[c*d^2 - b*d*e + a*e^2]*g*ArcTanh[(b*d - 2*a*e + 2*c*d*x - b*e*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + x*(b + c*x)])] - e*Sqrt[c*f^2 - b*f*g
 + a*g^2]*ArcTanh[(b*f - 2*a*g + 2*c*f*x - b*g*x)/(2*Sqrt[c*f^2 - b*f*g + a
*g^2]*Sqrt[a + x*(b + c*x)])])/(e*g*(e*f - d*g))
```

IntegrateAlgebraic [A] time = 0.70, size = 315, normalized size = 1.38

$$\frac{2\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+bx+cx^2}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{e(ef-dg)}+\frac{2\sqrt{-ag^2+bfg-cf^2}\tan^{-1}\left(-\frac{g\sqrt{a+bx+cx^2}}{\sqrt{-ag^2+bfg-cf^2}}+\frac{\sqrt{c}gx}{\sqrt{-ag^2+bfg-cf^2}}+\frac{\sqrt{c}f}{\sqrt{-ag^2+bfg-cf^2}}\right)}{g(dg-ef)}-\frac{\sqrt{c}\log\left(-2\sqrt{c}eg\sqrt{a+bx+cx^2}+beg+2cegx\right)}{eg}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

```
[Out] (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e
- a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x +
 c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(e*(e*f - d*g)) + (2*Sqrt[-(c*f^2
) + b*f*g - a*g^2]*ArcTan[(Sqrt[c]*f)/Sqrt[-(c*f^2) + b*f*g - a*g^2] + (Sqr
t[c]*g*x)/Sqrt[-(c*f^2) + b*f*g - a*g^2] - (g*Sqrt[a + b*x + c*x^2])/Sqrt[-
(c*f^2) + b*f*g - a*g^2]])/(g*(-(e*f) + d*g)) - (Sqrt[c]*Log[b*e*g + 2*c*e*
g*x - 2*Sqrt[c]*e*g*Sqrt[a + b*x + c*x^2]])/(e*g)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.03, size = 1529, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] 1/(d*g-e*f)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/
2)+1/2/(d*g-e*f)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g
-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)*b-1/(d*g-e*f)/g*l
n((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(1/2)*f-1/(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*
f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2))/(x+f/g))*a+1/(d*g-e*f)/g/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((
2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)
^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(
x+f/g))*b*f-1/(d*g-e*f)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*
f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+
f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f
^2-1/(d*g-e*f)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)-1/2/(d*g-e*f)*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b+1/(d*g-e*f)/
e*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d+1/(d*g-e*f)/((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))/(x+d/e))*a-1/(d*g-e*f)/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*l
n(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/(x+d/e))*b*d+1/(d*g-e*f)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c
*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(
(x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*
c*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{\left(f+g\,x\right)\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)), x)
```

3.602 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx$

Optimal. Leaf size=490

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2}-\frac{e\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^2}+\frac{(2c}{}$$

Rubi [A] time = 0.70, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {960, 734, 843, 621, 206, 724, 732}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2}-\frac{e\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^2}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)\sqrt{ag^2-bfg+cf^2}}+\frac{\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^2}+\frac{e(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/((e*f - d*g)*(f + g*x)) - ((2*c*d - b*e)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^2) + (e*
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[c]*g*(e*f - d*g)^2) - (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(g*(e*f - d*g)) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d
- 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*
x^2])])/(e*f - d*g)^2 + ((2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g
)*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g*(e*f - d*g)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx &= \int \left(\frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^2(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^2} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)}\right)dx \\
&= \frac{e^2\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^2} - \frac{g\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{ef-dg} \\
&= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{e\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} + \frac{e\int \frac{bf-2ag+(2cf-bg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} - \frac{\int \frac{b+2cx}{(f+gx)\sqrt{a+bx+cx^2}}}{2(ef-dg)} \\
&= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} \\
&= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right.}{} \\
&= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,(ef-dg)^2} + \frac{e(2cf-bg)\tanh^{-1}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.}{2\sqrt{c}\,g(ef-dg)^2}
\end{aligned}
$$

Mathematica [A] time = 0.52, size = 222, normalized size = 0.45

$$
\frac{2\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right) - \frac{(2aeg-b(dg+ef)+2cdf)\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\sqrt{g(ag-bf)+cf^2}} + \frac{2\sqrt{a+x(b+cx)}\,(ef-dg)}{f+gx}}{2(ef-dg)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] ((2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + 2*Sqrt[c*d^2 + e*(-(b*d)
 + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d)
 + a*e)]*Sqrt[a + x*(b + c*x)])] - ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*Arc
Tanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqr
t[a + x*(b + c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)])/(2*(e*f - d*g)^2)
```

IntegrateAlgebraic [A] time = 1.26, size = 351, normalized size = 0.72

$$\frac{2\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+bx+cx^2}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2+bde-cd^2}}\right)}{(ef-dg)^2}+\frac{\left(-2cdf\sqrt{-ag^2+bfg-cf^2}+bdg\sqrt{-ag^2+bfg-cf^2}+bef\sqrt{-ag^2+bfg-cf^2}-2aeg\sqrt{-ag^2+bfg-cf^2}\right)\tan^{-1}\left(\frac{-g\sqrt{a+bx+cx^2}+\sqrt{c}\,f+\sqrt{c}\,gx}{\sqrt{-ag^2+bfg-cf^2}}\right)}{(ef-dg)^2\left(ag^2-bfg+cf^2\right)}+\frac{\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/((e*f - d*g)*(f + g*x)) + (2*Sqrt[-(c*d^2) + b*d*e -
a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x)/Sq
rt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*
d*e - a*e^2]])/(e*f - d*g)^2 + ((-2*c*d*f*Sqrt[-(c*f^2) + b*f*g - a*g^2] +
b*e*f*Sqrt[-(c*f^2) + b*f*g - a*g^2] + b*d*g*Sqrt[-(c*f^2) + b*f*g - a*g^2]
 - 2*a*e*g*Sqrt[-(c*f^2) + b*f*g - a*g^2])*ArcTan[(Sqrt[c]*f + Sqrt[c]*g*x
- g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 3162, normalized size = 6.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x)
```

```
[Out] -g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g
+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*((x+f/g)^2*
c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b-1/(d*g-e*f)/(a*g^2
-b*f*g+c*f^2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(
1/2)*c*f-1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)/c
^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c
^(1/2)*f*b+1/g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/
g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2
))*c^(3/2)*f^2-1/2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2
)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g
+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g
```

```
^2)^(1/2))/(x+f/g))*a*b+1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2
)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-
b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2))/(x+f/g))*a*c*f+1/2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*
f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2
*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-
b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b^2*f-3/2/g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)
/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c
*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g
)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b*f^2*c+1/g^2/(d*g-e*f)/(a*g^2
-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(
a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-
2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c^2*f^3+g/(d*g-e*
f)*c/(a*g^2-b*f*g+c*f^2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*
f^2)/g^2)^(1/2)*x+g/(d*g-e*f)*c^(1/2)/(a*g^2-b*f*g+c*f^2)*ln(((x+f/g)*c+1/2
*(b*g-2*c*f)/g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2))*a-1/(d*g-e*f)^2*e*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2
-b*f*g+c*f^2)/g^2)^(1/2)-1/2/(d*g-e*f)^2*e*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)
/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))
/c^(1/2)*b+1/(d*g-e*f)^2*e/g*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)/c^(1/2)+((x+f
/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(1/2)*f+1/(
d*g-e*f)^2*e/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a
*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2
*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a-1/(d*g-e*f)^2*e/
g/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+
c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/
g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b*f+1/(d*g-e*f)^2*e/g^2/((a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/
g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f^2+1/(d*g-e*f)^2*e*((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2/(d*g-e*f)^2*e*ln(((
x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/(d*g-e*f)^2*ln(((x+d/e)*c+1/2*(b*e-
2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))*c^(1/2)*d-1/(d*g-e*f)^2*e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*
e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d
/e))*a+1/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e
)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*
c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d-1/(d*g
-e*f)^2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^
2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*
d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)\left(gx + f\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{\left(f + g\,x\right)^2\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^2*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**2), x)
```

3.603 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3} \, dx$

Optimal. Leaf size=673

$$\frac{g\left(b^2-4ac\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{e\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3}-\frac{e^2\sqrt{ag^2-bf}}{}$$

Rubi [A] time = 0.86, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {960, 734, 843, 621, 206, 724, 720, 732}

$$\frac{g\left(b^2-4ac\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{e\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3}-\frac{e^2\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^3}+\frac{e^2(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^3}-\frac{g\sqrt{a+bx+cx^2}(-2ag+x(2cf-bg)+bf)}{4(f+gx)^2(ef-dg)\left(ag^2-bfg+cf^2\right)}+\frac{e(2cf-bg)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}+\frac{e\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2}-\frac{\sqrt{c}e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2}-\frac{e(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] (e*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*(f + g*x)) - (g*(b*f - 2*a*g + (2*
c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)
*(f + g*x)^2) - (e*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^3) + (e^2*(2*c*f - b*g)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^3) - (Sq
rt[c]*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d
*g)^2) + (e*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e
)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^3
+ ((b^2 - 4*a*c)*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g + a*g
^2)^(3/2)) + (e*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sq
rt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g)^2*Sqrt[
c*f^2 - b*f*g + a*g^2]) - (e^2*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2
*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2
])])/(g*(e*f - d*g)^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \int \left(\frac{e^3\sqrt{a+bx+cx^2}}{(ef-dg)^3(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^3} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)^2} - \frac{e^2g\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+g}\right.$$

$$= \frac{e^3\int \frac{\sqrt{a+bx+cx^2}}{d+ex}dx}{(ef-dg)^3} - \frac{(e^2g)\int \frac{\sqrt{a+bx+cx^2}}{f+gx}dx}{(ef-dg)^3} - \frac{(eg)\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}dx}{(ef-dg)^2} - \frac{g\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^3}dx}{ef-dg}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{e^2\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}}{2(ef-dg)^3}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{(e(2cd-be))\int \frac{1}{\sqrt{a+b}}}{2(ef-dg)^3}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{\left(b^2-4ac\right)g\tanh^{-1}}{8(ef-dg)\left(c\right.}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{e(2cd-be)\tanh^{-1}\left(\right.}{2\sqrt{c}(ef-}$$

Mathematica [A] time = 1.37, size = 609, normalized size = 0.90

$$\frac{\frac{g\left(b^2-4ac\right)(ef-dg)^2\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{3/2}} + 8e\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right) + \frac{2g\sqrt{a+x(b+cx)}(ef-dg)^2(2ag-bf+bgx-2cfx)}{(f+gx)^2\left(g(ag-bf)+cf^2\right)} - \frac{4e(ef-dg)\left(2\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-\frac{(2cf-bg)\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\sqrt{g(ag-bf)+cf^2}}\right)}{g} + \frac{8e\sqrt{a+x(b+cx)}(ef-dg)}{f+gx} + \frac{4e(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} + \frac{4e^2\left((2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\sqrt{g(ag-bf)+cf^2}\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)\right)}{\sqrt{c}g}}{8(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((8*e*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (2*g*(e*f - d*g)^2*(-(
b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f)
+ a*g))*(f + g*x)^2) + (4*e*(-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + x*(b + c*x)])])/Sqrt[c] + 8*e*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan
h[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
 + x*(b + c*x)])] + ((b^2 - 4*a*c)*g*(e*f - d*g)^2*ArcTanh[(-2*a*g + 2*c*f*
x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])
/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (4*e*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*
g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b
+ c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)]))/g + (4*e^2*((2*c*f - b*g)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 +
g*(-(b*f) + a*g)]*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 +
g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*g))/(8*(e*f - d*g)^3)
```

IntegrateAlgebraic [F] time = 180.49, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 4.64, size = 1844, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d^2*g^3 - 4*a*c*d^2*g^3 - 8*c^2*d*f^3*e + 12*b*c*d*f^2*g*e - 6*b^
2*d*f*g^2*e + 4*a*b*d*g^3*e + 4*b*c*f^3*e^2 - 3*b^2*f^2*g*e^2 - 12*a*c*f^2*
g*e^2 + 12*a*b*f*g^2*e^2 - 8*a^2*g^3*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c*d^3*f^2*g^3 -
b*d^3*f*g^4 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e + 3*b*d^2*f^2*g^3*e - 3*a*d^2*f
*g^4*e + 3*c*d*f^4*g*e^2 - 3*b*d*f^3*g^2*e^2 + 3*a*d*f^2*g^3*e^2 - c*f^5*e^
3 + b*f^4*g*e^3 - a*f^3*g^2*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) - 2*(c*d^2*e
 - b*d*e^2 + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c
)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((d^3*g^3 - 3*d^2*f*g^2*e + 3*d*f^2*g*e^
2 - f^3*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/4*(8*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*c^2*d*f^2*g^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*
d*f*g^3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*g^4 + 4*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*a*c*d*g^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b*c*f^2*g^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^3*e + 4*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^3*e - 4*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a*b*g^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)
*d*f^3*g - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^3 - 4*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^3 + 8*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*d*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*c^(5/2)*f^4*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*
f^3*g*e + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f^2*g^2*e + 1
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f^2*g^2*e - 4*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^3*e - 8*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*a^2*sqrt(c)*g^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c
^2*d*f^3*g - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g^2 - 8*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^2 - (sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b^3*d*f*g^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*f*g^
3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*g^4 + 4*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^2*c*d*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*
f^4*e - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f^3*g*e - 16*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3*g*e + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b^3*f^2*g^2*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*g^2*
e - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^3*e - 28*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^2*c*f*g^3*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^2*b*g^4*e + 2*b^2*c^(3/2)*d*f^3*g - b^3*sqrt(c)*d*f^2*g^2 - 4*a*b*c^(3/2
)*d*f^2*g^2 + a*b^2*sqrt(c)*d*f*g^3 + 4*a^2*c^(3/2)*d*f*g^3 + 2*b^2*c^(3/2)
*f^4*e - 3*b^3*sqrt(c)*f^3*g*e - 8*a*b*c^(3/2)*f^3*g*e + 15*a*b^2*sqrt(c)*f
^2*g^2*e + 4*a^2*c^(3/2)*f^2*g^2*e - 20*a^2*b*sqrt(c)*f*g^3*e + 8*a^3*sqrt(
c)*g^4*e)/((c*d^2*f^2*g^3 - b*d^2*f*g^4 + a*d^2*g^5 - 2*c*d*f^3*g^2*e + 2*b
*d*f^2*g^3*e - 2*a*d*f*g^4*e + c*f^4*g*e^2 - b*f^3*g^2*e^2 + a*f^2*g^3*e^2)
*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*sqrt(c)*f + b*f - a*g)^2)
```

maple [B] time = 0.02, size = 6714, normalized size = 9.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)(gx + f)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{\left(f + g\,x\right)^3\,(d + e\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^3*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)\left(f + gx\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**3,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**3), x)
```

3.604 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx$

Optimal. Leaf size=933

$$\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^3}{2\sqrt{c}\,g(ef-dg)^4}-\frac{\sqrt{cf^2-bgf+ag^2}\,\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^3}{g(ef-dg)^4}-\frac{\sqrt{c}\,\tanh^{-1}\Big(\frac{\ \ }{2\sqrt{c}\ldots}}{g(ef-\ldots}$$

Rubi [A] time = 1.22, antiderivative size = 933, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {960, 734, 843, 621, 206, 724, 730, 720, 732}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

```
[Out] (e^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^3*(f + g*x)) - (g*(2*c*f - b*g)*(b
*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*(e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^2*(f + g*x)^2) - (e*g*(b*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt
[a + b*x + c*x^2])/(4*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)^2) +
(g^2*(a + b*x + c*x^2)^(3/2))/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^3) - (e^2*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*(e*f - d*g)^4) + (e^3*(2*c*f - b*g)*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^4) - (Sqrt[c]
*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)
^3) + (e^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)
*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^4 +
 ((b^2 - 4*a*c)*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*
Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(16*(e*f - d*g)*(c*f^2
 - b*f*g + a*g^2)^(5/2)) + ((b^2 - 4*a*c)*e*g*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) + (e^2*(2*c*f - b*g)*ArcTanh[(b*f -
 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x
^2])])/(2*g*(e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e^3*Sqrt[c*f^2 -
b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*
g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
```

```
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \int \left(\frac{e^4\sqrt{a+bx+cx^2}}{(ef-dg)^4(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^4} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)^3} - \frac{e^2g\sqrt{a+bx\cdots}}{(ef-dg)^3(f\cdots}\right.$$

$$= \frac{e^4\int\frac{\sqrt{a+bx+cx^2}}{d+ex}dx}{(ef-dg)^4} - \frac{(e^3g)\int\frac{\sqrt{a+bx+cx^2}}{f+gx}dx}{(ef-dg)^4} - \frac{(e^2g)\int\frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}dx}{(ef-dg)^3} - \frac{(eg)\int\frac{\sqrt{a+bx+cx^2}}{(f+gx)^3}\cdots}{(ef-dg)^2}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{g^2\left(a+\cdots\right.}{3(ef-dg)\left(cf^2\cdots\right.}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf-\cdots}{4(ef-\cdots}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf-\cdots}{4(ef-\cdots}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf-\cdots}{4(ef-\cdots}$$

Mathematica [A] time = 4.16, size = 858, normalized size = 0.92

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

```
[Out] ((48*e^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (12*e*g*(e*f - d*g)
^2*(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-
(b*f) + a*g))*(f + g*x)^2) - (16*g^2*(-(e*f) + d*g)^3*(a + x*(b + c*x))^(3/
2))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^3) + 24*e^2*(((-2*c*d + b*e)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 2*Sqrt[c*d^2
+ e*(-(b*d) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2
+ e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]) + (6*(b^2 - 4*a*c)*e*g*(e*f -
d*g)^2*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) +
 a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (3*g*(2*
c*f - b*g)*(e*f - d*g)^3*((2*Sqrt[a + x*(b + c*x)]*(-2*a*g + 2*c*f*x + b*(f
 - g*x)))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^2) + ((-b^2 + 4*a*c)*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
 + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2)))/(c*f^2 + g*(-(b*f) +
a*g)) - (24*e^2*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x)
)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*f^2 + g
*(-(b*f) + a*g)]))/g + (24*e^3*((2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
 + x*(b + c*x)])]))/(Sqrt[c]*g))/(48*(e*f - d*g)^4)
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 11995, normalized size = 12.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)(gx + f)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{\left(f + g\,x\right)^4\,\left(d + e\,x\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^4*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^4*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)\left(f + gx\right)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**4,x)
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**4), x)
```

3.605 $\int \frac{(f+gx)^3(a+bx+cx^2)^{3/2}}{d+ex} dx$

Optimal. Leaf size=1098

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+\frac{(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}g^2}{60c^2e^2}+\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+a\right.}{}$$

Rubi [A] time = 3.86, antiderivative size = 1098, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1653, 814, 843, 621, 206, 724}

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -((3*(7*b^5*e^5*g^3 - 512*c^5*d^2*(e*f - d*g)^3 + 128*c^4*e*(5*b*d - 4*a*e)
*(e*f - d*g)^3 - 4*b^3*c*e^4*g^2*(9*b*e*f - 3*b*d*g + 8*a*e*g) + 8*b*c^2*e^
3*g*(2*a^2*e^2*g^2 + 6*a*b*e*g*(3*e*f - d*g) + 3*b^2*(3*e^2*f^2 - 3*d*e*f*g
 + d^2*g^2)) - 32*b*c^3*e^2*(2*b*(e*f - d*g)^3 + 3*a*e*g*(3*e^2*f^2 - 3*d*e
*f*g + d^2*g^2))) + 2*c*e*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*
g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (
3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a
*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*x)*Sqrt[a + b*x + c*x^2]
)/(1536*c^4*e^6) + ((7*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(
9*b*e*f - 3*b*d*g + a*e*g) + 24*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)
 + 2*c*e*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3
*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3*e^4)
+ (g^2*(36*c*e*f - 22*c*d*g - 7*b*e*g)*(a + b*x + c*x^2)^(5/2))/(60*c^2*e^2
) + (g^3*(d + e*x)*(a + b*x + c*x^2)^(5/2))/(6*c*e^2) + ((4*c*e*(2*c*d - b*
e)*(8*c*e*(b*d - 2*a*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*
f - 3*b*d*g + a*e*g)) - d*(8*b*c*d - 3*b^2*e - 4*a*c*e)*g*(7*b^2*e^2*g^2 -
4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g
^2))) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*(8*c*e*(2*c*d - b*e
)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g))
- 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 -
4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g
^2))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(3072*c^(9/2
)*e^7) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/
e^7
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} + \int \frac{\left(a+bx+cx^2\right)^{3/2}\left(\frac{1}{2}e\left(12ce^2f^3-d(5bd+2ae)g^3\right)-eg\left(e(6bd-\right.\right.}{}$$

$$= \frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}$$

$$= \frac{\left(7b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(9bef-3bdg+aeg)+24bc^2eg\left(3e^2f\right.\right.}{}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4\right.\right.}{}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4\right.\right.}{}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4\right.\right.}{}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4\right.\right.}{}$$

Mathematica [A] time = 2.38, size = 743, normalized size = 0.68

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (5120*(e*f - d*g)^3*(a + x*(b + c*x))^(3/2) + (1920*e*g*(e*f - d*g)^2*(b +
2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3072*e^2*g^2*(e*f - d*g)*(a + x*(b + c
*x))^(5/2))/c + (2560*e^3*g^2*(f + g*x)*(a + x*(b + c*x))^(5/2))/c + (360*(
b^2 - 4*a*c)*e*g*(e*f - d*g)^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)
] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/
c^(5/2) - (60*e^2*g*(-2*c*f + b*g)*(e*f - d*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[
a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)
^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(7/2) + (e^3*
g*(1792*g*(2*c*f - b*g)*(a + x*(b + c*x))^(5/2) + 5*(24*c^2*f^2 + 7*b^2*g^2
 - 4*c*g*(6*b*f + a*g))*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b
^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c^2 + (9
60*(e*f - d*g)^3*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*
a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*Sqrt[c]*(
e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) - 2*c*e*(-5*b*d
+ 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) +
2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
c*x)])]))))/(c^(3/2)*e^3))/(15360*e^4)
```

IntegrateAlgebraic [F] time = 180.10, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

[Out] $Aborted

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.03, size = 10058, normalized size = 9.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^3\,\left(c\,x^2+b\,x+a\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^{3}\left(a+bx+cx^{2}\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**3*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.606 $\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=662

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(96c^3e^2\left(-a^2e^2g(2ef-dg)-2abe(ef-dg)^2+b^2d(ef-dg)^2\right)+16bc^2e^3\left(3a^2e^2g^2+3\right.}{}$$

Rubi [A] time = 1.55, antiderivative size = 662, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1653, 814, 843, 621, 206, 724}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((3*b^4*e^4*g^2 + 128*c^4*d^2*(e*f - d*g)^2 - 32*c^3*e*(5*b*d - 4*a*e)*(e*f
 - d*g)^2 - 6*b^2*c*e^3*g*(2*b*e*f - b*d*g + 2*a*e*g) + 8*b*c^2*e^2*(2*b*(e
*f - d*g)^2 + 3*a*e*g*(2*e*f - d*g)) + 2*c*e*((16*c^2*d^2 - 3*b^2*e^2 - 4*c
*e*(2*b*d - 3*a*e))*g*(4*c*e*f - 2*c*d*g - b*e*g) - 8*c*e*(2*c*d - b*e)*(2*
c*e*f^2 - b*d*g^2))*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*e^5) - ((3*b^2*e^2*g
^2 - 16*c^2*(e*f - d*g)^2 - 6*b*c*e*g*(2*e*f - d*g) - 6*c*e*g*(4*c*e*f - 2*
c*d*g - b*e*g)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*e^3) + (g^2*(a + b*x + c
*x^2)^(5/2))/(5*c*e) - ((3*b^5*e^5*g^2 + 256*c^5*d^3*(e*f - d*g)^2 - 384*c^
4*d*e*(b*d - a*e)*(e*f - d*g)^2 - 6*b^3*c*e^4*g*(2*b*e*f - b*d*g + 4*a*e*g)
 + 16*b*c^2*e^3*(3*a^2*e^2*g^2 + b^2*(e*f - d*g)^2 + 3*a*b*e*g*(2*e*f - d*g
)) + 96*c^3*e^2*(b^2*d*(e*f - d*g)^2 - 2*a*b*e*(e*f - d*g)^2 - a^2*e^2*g*(2
*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*
c^(7/2)*e^6) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^2*ArcTanh[(b*d -
2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^
2])])/e^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
```

```
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce} + \frac{\int \frac{\left(\frac{5}{2}e\left(2cef^2-bdg^2\right)+\frac{5}{2}eg(4cef-2cdg-beg)x\right)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{5ce^2}$$

$$= -\frac{\left(3b^2e^2g^2-16c^2(ef-dg)^2-6bceg(2ef-dg)-6ceg(4cef-2cdg-beg)x\right)\left(\right.}{48c^2e^3}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-\right.}{}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-\right.}{}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-\right.}{}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-\right.}{}$$

Mathematica [A] time = 1.25, size = 536, normalized size = 0.81

$$\frac{\frac{90eg(b^2-4ac)(ef-dg)\left((b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{5/2}}+\frac{15e^2g(2cf-bg)\left(\frac{3(b^2-4ac)\left((b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{5/2}}+\frac{16(b+2cx)(a+x(b+cx))^{3/2}}{c}\right)}{c}+\frac{240(ef-dg)^2\left(-\left((2cd-be)\left(4ce(3ae-2bd)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)-2\sqrt{c}\left(e\sqrt{a+x(b+cx)}\left(-2ce(4ae-5bd+bex)-b^2e^2+4c^2d(ex-2d)\right)+8c\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)\right)\right)}{c^{3/2}e^3}+1280(a+x(b+cx))^{3/2}(ef-dg)^2+\frac{480eg(b+2cx)(a+x(b+cx))^{3/2}(ef-dg)}{c}+\frac{768e^2g^2(a+x(b+cx))^{5/2}}{c}}{3840e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (1280*(e*f - d*g)^2*(a + x*(b + c*x))^(3/2) + (480*e*g*(e*f - d*g)*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2))/c + (768*e^2*g^2*(a + x*(b + c*x))^(5/2))/c +
(90*(b^2 - 4*a*c)*e*g*(e*f - d*g)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2) + (15*e^2*g*(2*c*f - b*g)*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/
2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2
)))/c + (240*(e*f - d*g)^2*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-
2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2
*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) - 2*c*
e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[
(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
 + x*(b + c*x)])])))/(c^(3/2)*e^3))/(3840*e^3)
```

IntegrateAlgebraic [B] time = 85.15, size = 27845, normalized size = 42.06

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 6860, normalized size = 10.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^2\,\left(c\,x^2+b\,x+a\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^2\left(a+bx+cx^2\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**2*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.607 $\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=441

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2(-d)(ef-dg)\right)-8b^2ce^3(3aeg-bdg+bef)+192c^3\right.}{128c^{5/2}e^5}$$

Rubi [A] time = 0.85, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2(-d)(ef-dg)\right)-8b^2ce^3(3aeg-bdg+bef)+192c^3de(bd-ae)(ef-dg)+3b^4e^4g-128c^4d^3(ef-dg)\right)}{128c^{5/2}e^5}-\frac{\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3aeg-2bdg+2bef)+3b^2e^2g+16c^2d(ef-dg)\right)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(3aeg-2bdg+2bef)+3b^3e^3g-64c^3d^2(ef-dg)\right)}{64c^2e^4}+\frac{(ef-dg)\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5}+\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -((3*b^3*e^3*g - 64*c^3*d^2*(e*f - d*g) + 16*c^2*e*(5*b*d - 4*a*e)*(e*f - d
*g) - 4*b*c*e^2*(2*b*e*f - 2*b*d*g + 3*a*e*g) + 2*c*e*(3*b^2*e^2*g + 16*c^2
*d*(e*f - d*g) - 4*c*e*(2*b*e*f - 2*b*d*g + 3*a*e*g))*x)*Sqrt[a + b*x + c*x
^2])/(64*c^2*e^4) + ((8*c*e*f - 8*c*d*g + 3*b*e*g + 6*c*e*g*x)*(a + b*x + c
*x^2)^(3/2))/(24*c*e^2) + ((3*b^4*e^4*g - 128*c^4*d^3*(e*f - d*g) + 192*c^3
*d*e*(b*d - a*e)*(e*f - d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c
^2*e^2*(a^2*e^2*g - b^2*d*(e*f - d*g) + 2*a*b*e*(e*f - d*g)))*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*e^5) + ((c*d^2 - b*
d*e + a*e^2)^(3/2)*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{(8cef-8cdg+3beg+6cegx)\left(a+bx+cx^2\right)^{3/2}}{24ce^2} - \int \frac{\left(\frac{1}{2}\left(8ce(bd-2ae)f+4acdeg-2bd\left(4c\right.\right.\right.}{}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg\right.}{}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg\right.}{}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg\right.}{}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg\right.}{}$$

Mathematica [A] time = 1.13, size = 420, normalized size = 0.95

$$\frac{\frac{3\left(\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2d(dg-ef)\right)-8b^2ce^3(3aeg-bdg+bef)-192c^3de(bd-ae)(dg-ef)+3b^4e^4g+128c^4d^3(dg-ef)\right)+2\sqrt{c}e\sqrt{a+x(b+cx)}\left(8c^2e(ae(-8dg+8ef+3egx)+2b(ex-5d)(ef-dg))+2bce^2(6aeg+b(-4dg+4ef-3egx))-3b^3e^3g-32c^3d(ex-2d)(ef-dg)\right)+128c^{5/2}(dg-ef)\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)\right)}{16c^{3/2}e^3}+(a+x(b+cx))^{3/2}(3beg+c(-8dg+8ef+6egx))}{24ce^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*(3*b*e*g + c*(8*e*f - 8*d*g + 6*e*g*x)) + (3*(2*Sq
rt[c]*e*Sqrt[a + x*(b + c*x)]*(-3*b^3*e^3*g - 32*c^3*d*(e*f - d*g)*(-2*d +
e*x) + 2*b*c*e^2*(6*a*e*g + b*(4*e*f - 4*d*g - 3*e*g*x)) + 8*c^2*e*(2*b*(e*
f - d*g)*(-5*d + e*x) + a*e*(8*e*f - 8*d*g + 3*e*g*x))) + (3*b^4*e^4*g + 12
8*c^4*d^3*(-(e*f) + d*g) - 192*c^3*d*e*(b*d - a*e)*(-(e*f) + d*g) - 8*b^2*c
*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c^2*e^2*(a^2*e^2*g + 2*a*b*e*(e*f - d*g
) + b^2*d*(-(e*f) + d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c
*x)])] + 128*c^(5/2)*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*(-(e*f) + d*g)*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt
[a + x*(b + c*x)])]))/(16*c^(3/2)*e^3))/(24*c*e^2)
```

IntegrateAlgebraic [A] time = 3.47, size = 687, normalized size = 1.56

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(192*c^3*d^2*e*f - 240*b*c^2*d*e^2*f + 24*b^2*c*e^3*
f + 256*a*c^2*e^3*f - 192*c^3*d^3*g + 240*b*c^2*d^2*e*g - 24*b^2*c*d*e^2*g
- 256*a*c^2*d*e^2*g - 9*b^3*e^3*g + 60*a*b*c*e^3*g - 96*c^3*d*e^2*f*x + 112
*b*c^2*e^3*f*x + 96*c^3*d^2*e*g*x - 112*b*c^2*d*e^2*g*x + 6*b^2*c*e^3*g*x +
 120*a*c^2*e^3*g*x + 64*c^3*e^3*f*x^2 - 64*c^3*d*e^2*g*x^2 + 72*b*c^2*e^3*g
*x^2 + 48*c^3*e^3*g*x^3))/(192*c^2*e^4) - (2*(-(c*d^2*e*Sqrt[-(c*d^2) + b*d
*e - a*e^2]*f) + b*d*e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*f - a*e^3*Sqrt[-(c*
d^2) + b*d*e - a*e^2]*f + c*d^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*g - b*d^2*e*
Sqrt[-(c*d^2) + b*d*e - a*e^2]*g + a*d*e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*g
)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x - e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2)
+ b*d*e - a*e^2]])/e^5 + ((128*c^4*d^3*e*f - 192*b*c^3*d^2*e^2*f + 48*b^2*c
^2*d*e^3*f + 192*a*c^3*d*e^3*f + 8*b^3*c*e^4*f - 96*a*b*c^2*e^4*f - 128*c^4
*d^4*g + 192*b*c^3*d^3*e*g - 48*b^2*c^2*d^2*e^2*g - 192*a*c^3*d^2*e^2*g - 8
*b^3*c*d*e^3*g + 96*a*b*c^2*d*e^3*g - 3*b^4*e^4*g + 24*a*b^2*c*e^4*g - 48*a
^2*c^2*e^4*g)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(128*c^(5/2
)*e^5)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 4188, normalized size = 9.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*f+1
/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a*f+1/
4/e*g*(c*x^2+b*x+a)^(3/2)*x-1/3/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e
^2-b*d*e+c*d^2)/e^2)^(3/2)*d*g-1/e^4*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/
2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/
2)*d^3*f-1/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*a*d*g-1/16/e/c^(3/2)*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e
)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*b^3*f-5/4/e^2*(
(x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d*f+1/4/
e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*f-1
/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e
+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d
/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a^2*f+1/8/e*g/c*(c*x^2+b*x+a
)^(3/2)*b+3/8/e*g*(c*x^2+b*x+a)^(1/2)*x*a-3/64/e*g/c^2*(c*x^2+b*x+a)^(1/2)*
b^3+3/8/e*g/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128/e
*g/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4-1/e^4*((x+d/e)^2
```

```
*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d^3*g+1/e^3*((x+d
/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d^2*f+1/e^5*
ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^4*g+1/8/e/c*((x+d/e)^2*c+(b*e-2*
c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f+5/4/e^3*((x+d/e)^2*c+(b
*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d^2*g+1/16/e^2/c^(3/2)
*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*b^3*d*g-3/2/e^4*ln(((x+d/e)*c+1/2*(b*e-2*
c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))*c^(1/2)*d^3*b*g-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*
d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b
^2*d^2*f-3/8/e^2*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e
-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d*f-2/e^3/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2
)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*b*d^2*g-2/e^5/((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))/(x+d/e))*b*d^4*c*g+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*l
n(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/(x+d/e))*b*d^3*c*f+2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*
(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d
/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*b*
d*f+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^
2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*
d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*c*d^3*g-2/e^3/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/
e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*c*d^2*f-3/4/e^2/c^(1/2)*ln(((x+d/e
)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))*a*b*d*g+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b
*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+
d/e))*b^2*d^3*g+3/2/e^3*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2
*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*a*g-3/
2/e^2*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a*f-1/e^5/((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(x+d/e))*c^2*d^4*f+1/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/(x+d/e))*c^2*d^5*g-1/2/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*x*c*d*f+3/4/e/c^(1/2)*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e
)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)*a*b*f-3/16/e*g/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a+
3/16/e*g/c*(c*x^2+b*x+a)^(1/2)*b*a-3/32/e*g/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/2
/e^3*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d
/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*b*f+3/8/e^3*ln(((x+d/e)*c
+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d^2*g-1/8/e^2/c*((x+d/e)^2*c+(b*e-2*c*d)*(x
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d*g+1/2/e^3*((x+d/e)^2*c+(b*e-2*
c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c*d^2*g-1/4/e^2*((x+d/e)^2*
c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*d*g+1/e^2/((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e
^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a^2*d*g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for mo
re details)Is b*e-2*c*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f + g\,x)\,(c\,x^2 + b\,x + a)^{3/2}}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f + gx)\,(a + bx + cx^2)^{\frac{3}{2}}}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.608 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=252

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\text{t}}{16c^{3/2}e^4}$$

Rubi [A] time = 0.35, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {734, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^4}+\frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4}+\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e^3) + (a + b*x + c*x^2)^(3/2)/(3*e) - ((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/e^4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(a+bx+cx^2\right)^{3/2}}{3e} - \frac{\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{\left(a+bx+cx^2\right)^{3/2}}{3e}$$

Mathematica [A] time = 0.41, size = 236, normalized size = 0.94

$$\frac{2\sqrt{c}\left(e\sqrt{a+x(b+cx)}\left(2ce(16ae-15bd+7bex)+3b^2e^2+4c^2\left(6d^2-3dex+2e^2x^2\right)\right)-24c\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)\right)-3(2cd-be)\left(4ce(3ae-2bd)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{48c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (-3*(2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(e*Sqrt[a + x*(b +
 c*x)]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x) + 4*c^2*(6*d^2 - 3*d
*e*x + 2*e^2*x^2)) - 24*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d)
+ 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b
+ c*x)])]))/(48*c^(3/2)*e^4)
```

IntegrateAlgebraic [A] time = 0.00, size = 278, normalized size = 1.10

$$\frac{\sqrt{a+bx+cx^2}\left(32ace^2+3b^2e^2-30bcde+14bce^2x+24c^2d^2-12c^2dex+8c^2e^2x^2\right)}{24ce^3}+\frac{\left(-12abce^3+24ac^2de^2+b^3e^3+6b^2cde^2-24bc^2d^2e+16c^3d^3\right)\log\left(-2c^{3/2}\sqrt{a+bx+cx^2}+bc+2c^2x\right)}{16c^{3/2}e^4}+\frac{2\sqrt{-ae^2+bde-cd^2}\left(ae^2-bde+cd^2\right)\tan^{-1}\left(\frac{-e\sqrt{a+bx+cx^2}+\sqrt{c}d+\sqrt{c}ex}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(24*c^2*d^2 - 30*b*c*d*e + 3*b^2*e^2 + 32*a*c*e^2 -
12*c^2*d*e*x + 14*b*c*e^2*x + 8*c^2*e^2*x^2))/(24*c*e^3) + (2*Sqrt[-(c*d^2)
 + b*d*e - a*e^2]*(c*d^2 - b*d*e + a*e^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x -
 e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^4 + ((16*c^3*d
^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + 24*a*c^2*d*e^2 + b^3*e^3 - 12*a*b*c*e
^3)*Log[b*c + 2*c^2*x - 2*c^(3/2)*Sqrt[a + b*x + c*x^2]])/(16*c^(3/2)*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1946, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/4
/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b-1/
2/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c
*d+1/8/e/c*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*b^2-5/4/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*b*d+3/4/e/c^(1/2)*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*
c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*b-3/2/e^2*ln(((x+
d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a-1/16/e/c^(3/2)*ln(((x+d/e)*c+1/2*(b*e
-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))*b^3-3/8/e^2*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*
c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d+1/e*(
(x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a+1/e^3*((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d^2+3/2/e
^3*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e
)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*b-1/e^4*ln(((x+d/e)*c+1/2*(
b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))*c^(3/2)*d^3-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*
d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a
^2+2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2
-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d
)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*b*d-2/e^3/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a*c*d^2-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(
```

```
1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/(x+d/e))*b^2*d^2+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c
*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(
(x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*
b*d^3*c-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(
a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-
2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c^2*d^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                               +c*d^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^{2}\right)^{\frac{3}{2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

3.609 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)} dx$

Optimal. Leaf size=491

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+bg^2(-4aeg+bdg+3bef)+8c^2ef^3\right)}{8\sqrt{c}\,eg^3(ef-dg)}+\frac{\sqrt{a+bx+cx^2}\,(ae^2 \ldots}{e^2(ef-\ldots}$$

Rubi [A] time = 0.84, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {895, 734, 843, 621, 206, 724, 814}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+bg^2(-4aeg+bdg+3bef)+8c^2ef^3\right)}{8\sqrt{c}\,eg^3(ef-dg)}+\frac{\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}{e^2(ef-dg)}+\frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3(ef-dg)}-\frac{(2cd-be)\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^3(ef-dg)}-\frac{\sqrt{a+bx+cx^2}\left(-g(-4aeg-bdg+5bef)-2cgx(ef-dg)+4cef^2\right)}{4eg^2(ef-dg)}-\frac{\left(ag^2-bfg+cf^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g^3(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2])/(e^2*(e*f - d*g)) - ((4*c*e
*f^2 - g*(5*b*e*f - b*d*g - 4*a*e*g) - 2*c*g*(e*f - d*g)*x)*Sqrt[a + b*x +
c*x^2])/(4*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*e^3*(e*f - d
*g)) + ((8*c^2*e*f^3 + b*g^2*(3*b*e*f + b*d*g - 4*a*e*g) - 4*c*g*(3*b*e*f^2
 - a*g*(3*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(8*Sqrt[c]*e*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[
(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
 + c*x^2])])/(e^3*(e*f - d*g)) - ((c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g^3*(e*f - d*g))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\sqrt{a+bx+cx^2}}{f+gx}\,dx}{e(ef-dg)} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{e(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{\left(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x\right)}{4eg^2(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{\left(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x\right)}{4eg^2(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{\left(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x\right)}{4eg^2(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{\left(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x\right)}{4eg^2(ef-dg)}$$

Mathematica [A] time = 1.07, size = 323, normalized size = 0.66

$$\frac{\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-12ceg(-aeg+bdg+bef)+3b^2e^2g^2+8c^2\left(d^2g^2+defg+e^2f^2\right)\right)}{\sqrt{c}}+\frac{2\left(-4g^3\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+eg\sqrt{a+x(b+cx)}(ef-dg)(5beg+c(-4dg-4ef+2egx))+4e^3\left(g(ag-bf)+cf^2\right)^{3/2}\tanh^{-1}\left(\frac{2ag-bf+bgx-2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)\right)}{ef-dg}}{8e^3g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (((3*b^2*e^2*g^2 - 12*c*e*g*(b*e*f + b*d*g - a*e*g) + 8*c^2*(e^2*f^2 + d*e*
f*g + d^2*g^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqr
t[c] + (2*(e*g*(e*f - d*g)*Sqrt[a + x*(b + c*x)]*(5*b*e*g + c*(-4*e*f - 4*d
*g + 2*e*g*x)) - 4*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*g^3*ArcTanh[(-(b*d) + 2
*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c
*x)])] + 4*e^3*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a*g - 2
*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))
/(e*f - d*g))/(8*e^3*g^3)
```

IntegrateAlgebraic [B] time = 78.21, size = 2557, normalized size = 5.21

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-4*a*b^2*e*f - 4*a*b^2*d*g + 5*b^3*e*g*x^2) + c*Sqr
t[a + b*x + c*x^2]*(-16*a^2*e*f - 16*a^2*d*g - 16*a*b*e*f*x - 16*a*b*d*g*x
+ 16*a*b*e*g*x^2 + 20*b^2*e*g*x^3) + Sqrt[c]*(16*a^2*b*e*f + 16*a^2*b*d*g +
 12*a*b^2*e*f*x + 12*a*b^2*d*g*x - 2*b^3*e*f*x^2 - 2*b^3*d*g*x^2 - 19*a*b^2
*e*g*x^2 - 14*b^3*e*g*x^3) + c^2*Sqrt[a + b*x + c*x^2]*(-32*a*e*f*x^2 - 32*
a*d*g*x^2 - 48*b*e*f*x^3 - 48*b*d*g*x^3 + 8*a*e*g*x^3 + 20*b*e*g*x^4) + c^(
3/2)*(16*a^2*e*f*x + 16*a^2*d*g*x + 40*a*b*e*f*x^2 + 40*a*b*d*g*x^2 + 4*a^2
*e*g*x^2 + 16*b^2*e*f*x^3 + 16*b^2*d*g*x^3 - 24*a*b*e*g*x^3 - 27*b^2*e*g*x^
4) + c^3*Sqrt[a + b*x + c*x^2]*(-64*e*f*x^4 - 64*d*g*x^4 + 24*e*g*x^5) + c^
(5/2)*(64*a*e*f*x^3 + 64*a*d*g*x^3 + 80*b*e*f*x^4 + 80*b*d*g*x^4 - 20*a*e*g
*x^4 - 32*b*e*g*x^5) + c^(7/2)*(64*e*f*x^5 + 64*d*g*x^5 - 24*e*g*x^6))/(8*b
^2*e^2*g^2*x^2 + 64*c^2*e^2*g^2*x^4 + 8*c*e^2*g^2*x^2*(4*a + 8*b*x) - 32*b*
Sqrt[c]*e^2*g^2*x^2*Sqrt[a + b*x + c*x^2] - 64*c^(3/2)*e^2*g^2*x^3*Sqrt[a +
 b*x + c*x^2]) - (2*c^2*d^4*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^
2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x + c*x^2
])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(e^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*
f - d*g)) + (4*b*c*d^3*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] +
(Sqrt[c]*e*x)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x + c*x^2])/Sq
rt[-(c*d^2) + b*d*e - a*e^2]])/(e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d
*g)) - (4*a*c*d^2*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt
[c]*e*x)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x + c*x^2])/Sqrt[-(
c*d^2) + b*d*e - a*e^2]])/(e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)) +
(-1/2*(a*d^2*Sqrt[a + b*x + c*x^2])/(e^2*(e*f - d*g)) + (a*f^2*Sqrt[a + b*x
 + c*x^2])/(2*g^2*(e*f - d*g)) - (5*b*d*x^2*Sqrt[a + b*x + c*x^2])/(8*e*(e*
f - d*g)) + (5*b*f*x^2*Sqrt[a + b*x + c*x^2])/(8*g*(e*f - d*g)) - (4*a*b*d*
x^2*ArcTan[(-(Sqrt[c]*d) - Sqrt[c]*e*x + e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*
d^2) + b*d*e - a*e^2]])/(Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)) + (2*b
^2*d^2*x^2*ArcTan[(-(Sqrt[c]*d) - Sqrt[c]*e*x + e*Sqrt[a + b*x + c*x^2])/Sq
rt[-(c*d^2) + b*d*e - a*e^2]])/(e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g
)) + (2*a^2*e*x^2*ArcTan[(-(Sqrt[c]*d) - Sqrt[c]*e*x + e*Sqrt[a + b*x + c*x
^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f
- d*g)) + (4*a*b*f*x^2*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x
+ c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/((e*f - d*g)*Sqrt[-(c*f^2) + b*f
*g - a*g^2]) - (2*b^2*f^2*x^2*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a
 + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/(g*(e*f - d*g)*Sqrt[-(c*f
^2) + b*f*g - a*g^2]) - (2*a^2*g*x^2*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g
*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/((e*f - d*g)*Sqrt[
-(c*f^2) + b*f*g - a*g^2]))/x^2 + (2*c^2*f^4*ArcTan[(Sqrt[c]*f)/Sqrt[-(c*f^
2) + b*f*g - a*g^2] + (Sqrt[c]*g*x)/Sqrt[-(c*f^2) + b*f*g - a*g^2] - (g*Sqr
t[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/(g^3*(e*f - d*g)*Sqrt[
```

```
-(c*f^2) + b*f*g - a*g^2]) - (4*b*c*f^3*ArcTan[(Sqrt[c]*f)/Sqrt[-(c*f^2) +
b*f*g - a*g^2] + (Sqrt[c]*g*x)/Sqrt[-(c*f^2) + b*f*g - a*g^2] - (g*Sqrt[a +
 b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/(g^2*(e*f - d*g)*Sqrt[-(c*f
^2) + b*f*g - a*g^2]) + (4*a*c*f^2*ArcTan[(Sqrt[c]*f)/Sqrt[-(c*f^2) + b*f*g
 - a*g^2] + (Sqrt[c]*g*x)/Sqrt[-(c*f^2) + b*f*g - a*g^2] - (g*Sqrt[a + b*x
+ c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/(g*(e*f - d*g)*Sqrt[-(c*f^2) + b
*f*g - a*g^2]) - (c^(3/2)*f^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^
2]])/(e*g^3) - (c^(3/2)*d*f*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
])/(e^2*g^2) - (c^(3/2)*d^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
])/(e^3*g) - (3*b^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(8*Sq
rt[c]*e*g) + ((a*Sqrt[c]*f)/(2*e*g^2) + (a*Sqrt[c]*d)/(2*e^2*g) - (a*Sqrt[c
]*x)/(8*e*g) - (3*b*Sqrt[c]*x^2)/(4*e*g) - (c^(3/2)*x^3)/(8*e*g) + (3*b*Sqr
t[c]*f*x*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2*e*g^2) + (3*b
*Sqrt[c]*d*x*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2*e^2*g) -
(3*a*Sqrt[c]*x*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2*e*g))/x
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 4226, normalized size = 8.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] -1/4/(d*g-e*f)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*x*b-1/8/(d*g-e*f)/c*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*b^2+1/16/(d*g-e*f)/c^(3/2)*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e
)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)*b^3+1/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e
+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*a^2+1/4/(d*g-
e*f)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*b+
1/8/(d*g-e*f)/c*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)
^(1/2)*b^2-1/16/(d*g-e*f)/c^(3/2)*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)/c^(1/2)+
((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*b^3-1/(d
*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-
b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)
*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a^2-1/3/(d*g-e*f)*((x+d
/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/3/(d*g-e*f)*
((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/(d*g-e*
```

```
f)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b
*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*
(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b^2*d^2+1/(d*g-e*f)/e^4/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*
d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*c^2*d^4+1/2/(d*g-e*f)/e*((x+d/e
)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c*d+3/2/(d*g-e
*f)/e*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a+3/8/(d*g-e*f)/e*ln(((x+d
/e)*c+1/2*(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d-3/2/(d*g-e*f)/e^2*ln(((x+d/e)*c+1/2*
(b*e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))*c^(1/2)*d^2*b-1/2/(d*g-e*f)/g*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/
g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c*f-3/2/(d*g-e*f)/g*ln(((x+f/g)*c+1/2
*(b*g-2*c*f)/g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2))*c^(1/2)*f*a-3/8/(d*g-e*f)/g*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g
)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)
)/c^(1/2)*b^2*f-1/(d*g-e*f)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*a+1/(d*g-e*f)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-
b*f*g+c*f^2)/g^2)^(1/2)*a+2/(d*g-e*f)/g^3/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*l
n(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g
^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)
)/(x+f/g))*b*f^3*c-2/(d*g-e*f)/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2
*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)
)*a*b*d-2/(d*g-e*f)/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+
d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)
^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*b*d^3*c
-2/(d*g-e*f)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+
2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b
*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a*c*f^2+3/2/(d
*g-e*f)/g^2*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*
f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(1/2)*f^2*b-1/(d*g-e*f)/g^2/
((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*
f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)
/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b^2*f^2-1/(d*g-e*f)/g^4/((a*g^2
-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^
2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c^2*f^4-5/4/(d*g-e*f)/g*((x+f/g)^2*c+(
b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b*f+3/4/(d*g-e*f)/c^(1/
2)*ln(((x+f/g)*c+1/2*(b*g-2*c*f)/g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g
)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*a*b+2/(d*g-e*f)/g/((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*
f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2))/(x+f/g))*a*b*f+2/(d*g-e*f)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(x+d/e))*a*c*d^2+1/(d*g-e*f)/g^2*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2)*c*f^2-1/(d*g-e*f)/g^3*ln(((x+f/g)*c+1/2*(b*g-2
*c*f)/g)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2))*c^(3/2)*f^3+5/4/(d*g-e*f)/e*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d-3/4/(d*g-e*f)/c^(1/2)*ln(((x+d/e)*c+1/2*(b*
e-2*c*d)/e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2))*a*b-1/(d*g-e*f)/e^2*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*c*d^2+1/(d*g-e*f)/e^3*ln(((x+d/e)*c+1/2*(b*e-2*c*d)/
e)/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))*c^(3/2)*d^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^{2}\right)^{\frac{3}{2}}}{\left(d+ex\right)\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)), x)
```

3.610 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx$

Optimal. Leaf size=787

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce(ef-dg)^2}-\frac{e\sqrt{a+bx+cx^2}\left(-2cg(5bf-4ag)+b^2g^2-2\right.}{8cg^2(ef-dg)^2}$$

Rubi [A] time = 1.38, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {960, 734, 814, 843, 621, 206, 724, 732}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e*(e*f - d*g)^2) + (3*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)) - (e*(8*c^2*f^2 + b^2*g^2 - 2*c*g*(
5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c*g^2*(e*
f - d*g)^2) + (a + b*x + c*x^2)^(3/2)/((e*f - d*g)*(f + g*x)) - ((2*c*d - b
*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^2*(e*f - d*g)^2) + (e*(2*c*f -
 b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e*f - d*g)^2) - (3*(8*c^2
*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/(e^2*(e*f - d*g)^2) + (3*(2*c*f - b*g)*Sqrt[c*f^2
 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b
*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(e*f - d*g)) - (e*(c*f^2 - b*
f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^3*(e*f - d*g)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
```

```
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx=\int\left(\frac{e^2\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(d+ex)}-\frac{g\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)^2}-\frac{eg\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(f+gx)}\right)dx$$

$$=\frac{e^2\int\frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}dx}{(ef-dg)^2}-\frac{(eg)\int\frac{\left(a+bx+cx^2\right)^{3/2}}{f+gx}dx}{(ef-dg)^2}-\frac{g\int\frac{\left(a+bx+cx^2\right)^{3/2}}{(f+gx)^2}dx}{ef-dg}$$

$$=\frac{\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)}-\frac{e\int\frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}dx}{2(ef-dg)^2}+\frac{e\int\frac{(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{f+gx}d}{2(ef-dg)^2}$$

$$=\frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2}+\frac{3(4cf-3bg-2cg}{4g^2(ef-}$$

$$=\frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2}+\frac{3(4cf-3bg-2cg}{4g^2(ef-}$$

$$=\frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2}+\frac{3(4cf-3bg-2cg}{4g^2(ef-}$$

$$=\frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2}+\frac{3(4cf-3bg-2cg}{4g^2(ef-}$$

Mathematica [A] time = 1.40, size = 357, normalized size = 0.45

$$\frac{-2g^3(f+gx)\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+e\left(2g\sqrt{a+x(b+cx)}(dg-ef)(eg(bf-ag)+cdg(f+gx)-cef(2f+gx))-e(f+gx)\sqrt{g(ag-bf)+cf^2}(g(-2aeg+3bdg-bef)+2cf(2ef-3dg))\tanh^{-1}\left(\frac{2ag-bf+bgx-2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)\right)-\sqrt{c}(f+gx)(ef-dg)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)(-3beg+2cdg+4cef)}{2e^2g^3(f+gx)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] (-(Sqrt[c]*(e*f - d*g)^2*(4*c*e*f + 2*c*d*g - 3*b*e*g)*(f + g*x)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*(c*d^2 + e*(-(b*d) + a*e)
)^(3/2)*g^3*(f + g*x)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*
d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])] + e*(2*g*(-(e*f) + d*g)*Sqr
t[a + x*(b + c*x)]*(e*g*(b*f - a*g) + c*d*g*(f + g*x) - c*e*f*(2*f + g*x))
- e*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*(2*c*f*(2*e*f - 3*d*g) + g*(-(b*e*f) + 3
*b*d*g - 2*a*e*g))*(f + g*x)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*
Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(2*e^2*g^3*(e*f -
d*g)^2*(f + g*x))
```

IntegrateAlgebraic [B] time = 149.46, size = 3744, normalized size = 4.76

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-32*b^2*e*f^4*g + 32*a*b*e*f^3*g^2 - 48*b^2*e*f^3*g
^2*x + 48*a*b*e*f^2*g^3*x - 24*b^2*e*f^2*g^3*x^2 + 24*a*b*e*f*g^4*x^2 - 4*b
^2*e*f*g^4*x^3 + 4*a*b*e*g^5*x^3) + c*Sqrt[a + b*x + c*x^2]*(112*b*e*f^5 -
16*b*d*f^4*g - 64*a*e*f^4*g + 184*b*e*f^4*g*x - 40*b*d*f^3*g^2*x - 96*a*e*f
^3*g^2*x + 108*b*e*f^3*g^2*x^2 - 36*b*d*f^2*g^3*x^2 - 48*a*e*f^2*g^3*x^2 +
26*b*e*f^2*g^3*x^3 - 14*b*d*f*g^4*x^3 - 8*a*e*f*g^4*x^3 + 2*b*e*f*g^4*x^4 -
 2*b*d*g^5*x^4) + Sqrt[c]*(-24*b^2*e*f^5 - 8*b^2*d*f^4*g + 96*a*b*e*f^4*g -
```

64*a^2*e*f^3*g^2 + 4*b^2*e*f^4*g*x - 20*b^2*d*f^3*g^2*x + 112*a*b*e*f^3*g^2*x - 96*a^2*e*f^2*g^3*x + 42*b^2*e*f^3*g^2*x^2 - 18*b^2*d*f^2*g^3*x^2 + 24*a*b*e*f^2*g^3*x^2 - 48*a^2*e*f*g^4*x^2 + 27*b^2*e*f^2*g^3*x^3 - 7*b^2*d*f*g^4*x^3 - 12*a*b*e*f*g^4*x^3 - 8*a^2*e*g^5*x^3 + 5*b^2*e*f*g^4*x^4 - b^2*d*g^5*x^4 - 4*a*b*e*g^5*x^4) + c^2*Sqrt[a + b*x + c*x^2]*(64*e*f^5*x - 32*d*f^4*g*x + 128*e*f^4*g*x^2 - 80*d*f^3*g^2*x^2 + 96*e*f^3*g^2*x^3 - 72*d*f^2*g^3*x^3 + 28*e*f^2*g^3*x^4 - 28*d*f*g^4*x^4 + 4*e*f*g^4*x^5 - 4*d*g^5*x^5) + c^(3/2)*(-128*a*e*f^5 + 64*a*d*f^4*g - 144*b*e*f^5*x + 32*b*d*f^4*g*x - 192*a*e*f^4*g*x + 160*a*d*f^3*g^2*x - 248*b*e*f^4*g*x^2 + 80*b*d*f^3*g^2*x^2 - 96*a*e*f^3*g^2*x^2 + 144*a*d*f^2*g^3*x^2 - 156*b*e*f^3*g^2*x^3 + 72*b*d*f^2*g^3*x^3 - 16*a*e*f^2*g^3*x^3 + 56*a*d*f*g^4*x^3 - 40*b*e*f^2*g^3*x^4 + 28*b*d*f*g^4*x^4 + 8*a*d*g^5*x^4 - 4*b*e*f*g^4*x^5 + 4*b*d*g^5*x^5) + c^(5/2)*(-64*e*f^5*x^2 + 32*d*f^4*g*x^2 - 128*e*f^4*g*x^3 + 80*d*f^3*g^2*x^3 - 96*e*f^3*g^2*x^4 + 72*d*f^2*g^3*x^4 - 28*e*f^2*g^3*x^5 + 28*d*f*g^4*x^5 - 4*e*f*g^4*x^6 + 4*d*g^5*x^6))/(4*b*e*g^2*(e*f - d*g)*(f + g*x)*(2*f + g*x)^3 + 8*c*e*g^2*(e*f - d*g)*x*(f + g*x)*(2*f + g*x)^3 - 8*Sqrt[c]*e*g^2*(e*f - d*g)*(f + g*x)*(2*f + g*x)^3*Sqrt[a + b*x + c*x^2]) + ((4*a*c*d^2)/(Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^2) - (4*b*c*d^3)/(e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^2))*ArcTan[(-(Sqrt[c]*d) - Sqrt[c]*e*x + e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]] - (2*c^2*d^4*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^2) - (2*(b*d - a*e)^2*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^2) + ((-2*b*c*e*f^3)/(g^2*(e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) + (4*b*c*d*f^2)/(g*(e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) - (2*a*c*e*f^2)/(g*(e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]))*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]] + ((-8*a*c*d*f)/((e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) - (6*b*c*e*f^3)/(g^2*(e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) + (8*b*c*d*f^2)/(g*(e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) + (6*a*c*e*f^2)/(g*(e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]))*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]] + ((-4*b^2*d*f)/((e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) + (2*b^2*e*f^2)/(g*(e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) + (4*a*b*d*g)/((e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) - (2*a^2*e*g)/((e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]))*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]] + ((a*b)/((e*f - d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]) - (b^2*f)/(g*(e*f - d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]))*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]] + ((-2*b*c*f^2)/(g^2*(-(e*f) + d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]) + (2*a*c*f)/(g*(-(e*f) + d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]))*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]] - (4*c^2*e*f^4*ArcTan[(Sqrt[c]*f)/Sqrt[-(c*f^2) + b*f*g - a*g^2] + (Sqrt[c]*g*x)/Sqrt[-(c*f^2) + b*f*g - a*g^2] - (g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/(g^3*(e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) + (6*c^2*d*f^3*ArcTan[(Sqrt[c]*f)/Sqrt[-(c*f^2) + b*f*g - a*g^2] + (Sqrt[c]*g*x)/Sqrt[-(c*f^2) + b*f*g - a*g^2] - (g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/(g^2*(e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) + (b*c*f^2*ArcTan[(Sqrt[c]*f)/Sqrt[-(c*f^2) + b*f*g - a*g^2] + (Sqrt[c]*g*x)/Sqrt[-(c*f^2) + b*f*g - a*g^2] - (g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/(g^2*(-(e*f) + d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]) - (3*b*Sqrt[c]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2*e*g^2) + ((-8*c^(3/2)*f^5)/(g^3*(-(e*f) + d*g)) - (4*c^(3/2)*f^4*x)/(g^2*(-(e*f) + d*g)) - (4*c^(3/2)*d*f^3*x)/(e*g*(-(e*f) + d*g)) - (6*c^(3/2)*d*f^2*x^2)/(e*(-(e*f) + d*g)) + (2*c^(3/2)*f^3*x^2)/(g*(-(e*f) + d*g)) + (3*c^(3/2)*f^2*x^3)/(-(e*f) + d*g) - (3*c^(3/2)*d*f*g*x^3)/(e*(-(e*f) + d*g)) + (c^(3/2)*f*g*x^4)/(2*(-(e*f) + d*g)) - (c^(3/2)*

```
d*g^2*x^4)/(2*e*(-(e*f) + d*g)) - (16*c^(3/2)*f^5*Log[b + 2*c*x - 2*Sqrt[c]
*Sqrt[a + b*x + c*x^2]])/(g^3*(-(e*f) + d*g)) + (8*c^(3/2)*d*f^4*Log[b + 2*
c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(e*g^2*(-(e*f) + d*g)) + (8*c^(3/2)
*d^2*f^3*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(e^2*g*(-(e*f) +
 d*g)) + (12*c^(3/2)*d^2*f^2*x*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x
^2]])/(e^2*(-(e*f) + d*g)) - (24*c^(3/2)*f^4*x*Log[b + 2*c*x - 2*Sqrt[c]*Sq
rt[a + b*x + c*x^2]])/(g^2*(-(e*f) + d*g)) + (12*c^(3/2)*d*f^3*x*Log[b + 2*
c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(e*g*(-(e*f) + d*g)) + (6*c^(3/2)*d
*f^2*x^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(e*(-(e*f) + d*g
)) - (12*c^(3/2)*f^3*x^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/
(g*(-(e*f) + d*g)) + (6*c^(3/2)*d^2*f*g*x^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[
a + b*x + c*x^2]])/(e^2*(-(e*f) + d*g)) - (2*c^(3/2)*f^2*x^3*Log[b + 2*c*x
- 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(-(e*f) + d*g) + (c^(3/2)*d*f*g*x^3*Log
[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(e*(-(e*f) + d*g)) + (c^(3/2
)*d^2*g^2*x^3*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(e^2*(-(e*f
) + d*g)))/(2*f + g*x)^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 7959, normalized size = 10.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{(ex+d)\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{\left(f+g\,x\right)^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^2*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^2*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}}{(d+ex)\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)**2), x)
```

3.611 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx$

Optimal. Leaf size=1066

$$\frac{(2cf-bg)\left(8c^2f^2-b^2g^2-4cg(2bf-3ag)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{16c^{3/2}g^3(ef-dg)^3}-\frac{\left(cf^2-bgf+ag^2\right)^{3/2}\tanh^{-1}\left(\frac{bf-2ag+(}{2\sqrt{cf^2-bgf+ag}}\right.}{g^3(ef-dg)^3}$$

Rubi [A] time = 1.71, antiderivative size = 1066, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {960, 734, 814, 843, 621, 206, 724, 732, 812}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*(e*f - d*g)^3) + (3*e*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)^2) - (3*(4*c*f - b*g + 2*c*g*x)*Sqrt
[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)*(f + g*x)) - (e^2*(8*c^2*f^2 + b^2*g^
2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(
8*c*g^2*(e*f - d*g)^3) + (a + b*x + c*x^2)^(3/2)/(2*(e*f - d*g)*(f + g*x)^2
) + (e*(a + b*x + c*x^2)^(3/2))/((e*f - d*g)^2*(f + g*x)) - ((2*c*d - b*e)*
(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e*(e*f - d*g)^3) + (3*Sqrt[c]*(2*c*f
 - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(e*f
 - d*g)) + (e^2*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e
*f - d*g)^3) - (3*e*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)^2)
+ ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)^3) + (
3*e*(2*c*f - b*g)*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(
e*f - d*g)^2) - (e^2*(c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^
3*(e*f - d*g)^3) - (3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(
b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x
+ c*x^2])])/(8*g^3*(e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \int \left(\frac{e^3\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^3(d+ex)} - \frac{g\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)^3} - \frac{eg\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(f+gx)^2} - \frac{e^2g\left(a+bx+c\right.}{(ef-dg)^3(f+}\right.$$

$$= \frac{e^3\int\frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{(ef-dg)^3} - \frac{\left(e^2g\right)\int\frac{\left(a+bx+cx^2\right)^{3/2}}{f+gx}\,dx}{(ef-dg)^3} - \frac{(eg)\int\frac{\left(a+bx+cx^2\right)^{3/2}}{(f+gx)^2}\,dx}{(ef-dg)^2} - \frac{g\int\frac{\left(a+bx+cx^2\right)^{3/2}}{(f+gx)^3}}{ef-dg}$$

$$= \frac{\left(a+bx+cx^2\right)^{3/2}}{2(ef-dg)(f+gx)^2} + \frac{e\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(f+gx)} - \frac{e^2\int\frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2(ef-dg)^3} + \frac{e^2\int\frac{(}{}}{}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cg}{4g^2(ef-}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cg}{4g^2(ef-}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cg}{4g^2(ef-}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cg}{4g^2(ef-}$$

Mathematica [A] time = 3.27, size = 1036, normalized size = 0.97

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((2*(a + x*(b + c*x))^(3/2))/((e*f - d*g)*(f + g*x)^2) + (4*e*(a + x*(b + c
*x))^(3/2))/((e*f - d*g)^2*(f + g*x)) + (-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e
^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b +
 c*x)])]) - 2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d
+ e*x) - 2*c*e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(
3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) +
 a*e)]*Sqrt[a + x*(b + c*x)])]))/(4*c^(3/2)*e*(e*f - d*g)^3) - (3*e*((8*c^2
*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])] + 2*Sqrt[c]*(g*(-4*c*f + 3*b*g + 2*c*g*x)*Sqrt[a + x*(b
+ c*x)] + 2*(2*c*f - b*g)*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTanh[(-(b*f) +
2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b +
c*x)])])))/(2*Sqrt[c]*g^3*(e*f - d*g)^2) + (3*(((-2*c*f + b*g)*(a + x*(b +
c*x))^(3/2))/(f + g*x) - (Sqrt[a + x*(b + c*x)]*(b^2*g^2 + 2*c^2*f*(2*f - g
*x) + c*g*(-5*b*f + 2*a*g + b*g*x)))/g^2 + (4*Sqrt[c]*(2*c*f - b*g)*(c*f^2
+ g*(-(b*f) + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
+ (8*c^2*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*Sqrt[c*f^2 + g*(-(b*f) + a*g
)]*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a
```

```
*g)]*Sqrt[a + x*(b + c*x)])])/(2*g^3)))/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a
*g))) - (e^2*((2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 + 4*c*g*(-2*b*f + 3*a*g))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(g*Sqrt[
a + x*(b + c*x)]*(-(b^2*g^2) + 4*c^2*f*(-2*f + g*x) - 2*c*g*(-5*b*f + 4*a*g
 + b*g*x)) + 8*c*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a*g -
 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]
)))/(4*c^(3/2)*g^3*(-(e*f) + d*g)^3))/4
```

IntegrateAlgebraic [F] time = 180.19, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.6Error: Bad Argument Type
```

maple [B] time = 0.02, size = 15927, normalized size = 14.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(ex + d)\left(gx + f\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2 + b\,x + a\right)^{3/2}}{\left(f + g\,x\right)^3\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^3*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^3*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{3}{2}}}{\left(d + ex\right)\left(f + gx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**3,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)**3), x)
```

3.612 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=886

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)\left(cd^2-bed+ae^2\right)^{5/2}}{e^5(ef-dg)}+\frac{\left(cx^2+bx+a\right)^{3/2}\left(cd^2-bed+ae^2\right)}{3e^2(ef-dg)}-\frac{(2cd-be)\left(8c^2d^2-\right.}{}$$

Rubi [A] time = 1.80, antiderivative size = 886, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {895, 734, 814, 843, 621, 206, 724}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c
*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*e^4*(e*f - d*g)) - ((64*c^3
*e*f^4 - 16*c^2*e*f^2*g*(9*b*f - 8*a*g) - b^2*g^3*(5*b*e*f + 3*b*d*g - 8*a*
e*g) + 4*c*g^2*(22*b^2*e*f^2 + 16*a^2*e*g^2 - 3*a*b*g*(13*e*f - d*g)) - 2*c
*g*(16*c^2*e*f^3 + b*g^2*(5*b*e*f + 3*b*d*g - 8*a*e*g) - 4*c*g*(6*b*e*f^2 -
 a*g*(7*e*f - 3*d*g)))*x)*Sqrt[a + b*x + c*x^2])/(64*c*e*g^4*(e*f - d*g)) +
 ((c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2))/(3*e^2*(e*f - d*g)) - ((
8*c*e*f^2 - g*(11*b*e*f - 3*b*d*g - 8*a*e*g) - 6*c*g*(e*f - d*g)*x)*(a + b*
x + c*x^2)^(3/2))/(24*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e +
a*e^2)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^5*(e*f - d*g)) + ((128*c^4*
e*f^5 - 320*c^3*e*f^3*g*(b*f - a*g) - b^3*g^4*(5*b*e*f + 3*b*d*g - 8*a*e*g)
 + 48*c^2*g^2*(5*b^2*e*f^3 - 10*a*b*e*f^2*g + a^2*g^2*(5*e*f - d*g)) - 8*b*
c*g^3*(5*b^2*e*f^2 + 12*a^2*e*g^2 - 3*a*b*g*(5*e*f + d*g)))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(3/2)*e*g^5*(e*f - d*g)) +
((c*d^2 - b*d*e + a*e^2)^(5/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^5*(e*f - d*g)) - ((c
*f^2 - b*f*g + a*g^2)^(5/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt
[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^5*(e*f - d*g))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\left(a+bx+cx^2\right)^{3/2}}{f+gx}\,dx}{e(ef-dg)} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{e(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}}{3e^2(ef-dg)} - \frac{\left(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-d\right.}{24eg^2(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)} -$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)} -$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)} -$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)} -$$

Mathematica [A] time = 2.54, size = 647, normalized size = 0.73

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (3*(5*b^4*e^4*g^4*(-(e*f) + d*g) - 40*b^2*c*e^3*g^3*(e*f - d*g)*(b*e*f + b*
d*g - 3*a*e*g) + 320*c^3*e*g*(-(b*e^4*f^4) + a*e^4*f^3*g + b*d^4*g^4 - a*d^
3*e*g^4) + 128*c^4*(e^5*f^5 - d^5*g^5) + 240*c^2*e^2*g^2*(e*f - d*g)*(a^2*e
^2*g^2 - 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(-(e*g*(-(e*f)
 + d*g)*Sqrt[a + x*(b + c*x)]*(15*b^3*e^3*g^3 + 2*b*c*e^2*g^2*(278*a*e*g +
b*(-132*e*f - 132*d*g + 59*e*g*x)) - 16*c^3*(12*d^3*g^3 - 6*d^2*e*g^2*(-2*f
 + g*x) + 2*d*e^2*g*(6*f^2 - 3*f*g*x + 2*g^2*x^2) + e^3*(12*f^3 - 6*f^2*g*x
 + 4*f*g^2*x^2 - 3*g^3*x^3)) + 8*c^2*e*g*(a*e*g*(-56*e*f - 56*d*g + 27*e*g*
x) + b*(54*d^2*g^2 + 2*d*e*g*(27*f - 13*g*x) + e^2*(54*f^2 - 26*f*g*x + 17*
g^2*x^2))))) - 192*c*(c*d^2 + e*(-(b*d) + a*e))^(5/2)*g^5*ArcTanh[(-(b*d) +
 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
 c*x)])] + 192*c*e^5*(c*f^2 + g*(-(b*f) + a*g))^(5/2)*ArcTanh[(-(b*f) + 2*a
*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x
)])]))/(384*c^(3/2)*e^5*g^5*(e*f - d*g))
```

IntegrateAlgebraic [F] time = 180.14, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 9052, normalized size = 10.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for more
 details)Is d*g-e*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{5/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(5/2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(5/2)/((f + g*x)*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

3.613 $\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=431

$$-\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2eg\left(aeg(4ef-dg)+b\left(d^2g^2-4defg+6e^2f^2\right)\right)-6bce^2g^2(2aeg-bdg+4bef)+\cdots\right)}{16c^{7/2}e^4}$$

Rubi [A] time = 1.37, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1653, 843, 621, 206, 724}

$$\frac{g^2\sqrt{a+bx+cx^2}\left(-4ceg(4aeg-7bdg+18bef)+15b^2e^2g^2+4c^2\left(11d^2g^2-36defg+36e^2f^2\right)\right)}{24c^3e^3}-\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2eg\left(aeg(4ef-dg)+b\left(d^2g^2-4defg+6e^2f^2\right)\right)-6bce^2g^2(2aeg-bdg+4bef)+5b^3e^3g^3-16c^3\left(4d^2efg^2-d^3g^3-6de^2f^2g+4e^3f^3\right)\right)}{16c^{7/2}e^4}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}(-5beg-14cdg+24cef)}{12c^2e^3}+\frac{(ef-dg)^4\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4\sqrt{ae^2-bde+cd^2}}+\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*(15*b^2*e^2*g^2 - 4*c*e*g*(18*b*e*f - 7*b*d*g + 4*a*e*g) + 4*c^2*(36*e
^2*f^2 - 36*d*e*f*g + 11*d^2*g^2))*Sqrt[a + b*x + c*x^2])/(24*c^3*e^3) + (g
^3*(24*c*e*f - 14*c*d*g - 5*b*e*g)*(d + e*x)*Sqrt[a + b*x + c*x^2])/(12*c^2
*e^3) + (g^4*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*e^3) - (g*(5*b^3*e^3*g
^3 - 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^
2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*
e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(16*c^(7/2)*e^4) + ((e*f - d*g)^4*ArcTanh[(b*d - 2*a*e + (2*c
*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^4*S
qrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \frac{\int \frac{\frac{1}{2}e\left(6ce^3f^4-d^2(bd+4ae)g^4\right)-\frac{1}{2}eg\left(de(7bd+8ae)g^3-c\left(24e^3f^3-2d^3g^3\right)\right.}{(d\ldots}}{}$$

$$= \frac{g^3(24cef-14cdg-5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \ldots$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{\ldots}}{24c^3e^3}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{\ldots}}{24c^3e^3}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{\ldots}}{24c^3e^3}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{\ldots}}{24c^3e^3}$$

Mathematica [A] time = 0.89, size = 553, normalized size = 1.28

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((48*e*g^2*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)])/c + (24*e^2*g^2*(e*f - d*g)
*(f + g*x)*Sqrt[a + x*(b + c*x)])/c + (16*e^3*g^2*(f + g*x)^2*Sqrt[a + x*(b
 + c*x)])/c + (24*e*g*(2*c*f - b*g)*(e*f - d*g)^2*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (48*g*(e*f - d*g)^3*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (6*e^2*g*(e*f - d*g)*(6
*Sqrt[c]*g*(2*c*f - b*g)*Sqrt[a + x*(b + c*x)] + (8*c^2*f^2 + 3*b^2*g^2 - 4
*c*g*(2*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
)/c^(5/2) + (e^3*g*((2*g*Sqrt[a + x*(b + c*x)]*(15*b^2*g^2 + 4*c^2*f*(16*f
+ 5*g*x) - 2*c*g*(27*b*f + 8*a*g + 5*b*g*x)))/c^2 + (3*(2*c*f - b*g)*(8*c^2
*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f + 3*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])])/c^(5/2)))/c + (48*(e*f - d*g)^4*ArcTanh[(-2*a*e + 2*
c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)
])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(48*e^4)
```

IntegrateAlgebraic [A] time = 2.71, size = 509, normalized size = 1.18

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(144*c^2*e^2*f^2*g^2 - 96*c^2*d*e*f*g^3 - 72*b*c*e^2
*f*g^3 + 24*c^2*d^2*g^4 + 18*b*c*d*e*g^4 + 15*b^2*e^2*g^4 - 16*a*c*e^2*g^4
+ 48*c^2*e^2*f*g^3*x - 12*c^2*d*e*g^4*x - 10*b*c*e^2*g^4*x + 8*c^2*e^2*g^4*
x^2))/(24*c^3*e^3) + (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e^4*f^4 - 4*d*e^3*f
^3*g + 6*d^2*e^2*f^2*g^2 - 4*d^3*e*f*g^3 + d^4*g^4)*ArcTan[(Sqrt[c]*d + Sqr
t[c]*e*x - e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(e^4*(
c*d^2 - b*d*e + a*e^2)) + ((-64*c^3*e^3*f^3*g + 96*c^3*d*e^2*f^2*g^2 + 48*b
*c^2*e^3*f^2*g^2 - 64*c^3*d^2*e*f*g^3 - 32*b*c^2*d*e^2*f*g^3 - 24*b^2*c*e^3
*f*g^3 + 32*a*c^2*e^3*f*g^3 + 16*c^3*d^3*g^4 + 8*b*c^2*d^2*e*g^4 + 6*b^2*c*
d*e^2*g^4 - 8*a*c^2*d*e^2*g^4 + 5*b^3*e^3*g^4 - 12*a*b*c*e^3*g^4)*Log[b + 2
*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(16*c^(7/2)*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.03, size = 1597, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d
*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*f^4+2*g^3/e^2*b/c^(3/2)*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f+2*g^3/e*x/c*(c*x^2+b*x+a)^(1/
2)*f+3/4*g^4/e^2*b/c^2*(c*x^2+b*x+a)^(1/2)*d-3*g^3/e*b/c^2*(c*x^2+b*x+a)^(1
/2)*f-3/8*g^4/e^2*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d
+3/2*g^3/e*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f+1/2*g^
4/e^2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d-2*g^3/e*a/c^(
3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f-4*g^3/e^2/c*(c*x^2+b*x+a
)^(1/2)*d*f-1/2*g^4/e^3*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*d^2-3*g^2/e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f^2+4*
g^3/e^3*d^2*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-6*g^2/e^2
*d*f^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+4/e^4/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^3*f*d^3-6/e^3/((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x+d/e))*d^2*f^2*g^2+4/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b
```

```
*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+
d/e))*d*f^3*g-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)
/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c
+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^4*d^4+1/3
*g^4/e*x^2/c*(c*x^2+b*x+a)^(1/2)-5/12*g^4/e*b/c^2*x*(c*x^2+b*x+a)^(1/2)+3/4
*g^4/e*b/c^(5/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*g^4/e^2*
x/c*(c*x^2+b*x+a)^(1/2)*d+5/8*g^4/e*b^2/c^3*(c*x^2+b*x+a)^(1/2)-5/16*g^4/e*
b^3/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/3*g^4/e/c^2*a*(c*
x^2+b*x+a)^(1/2)+g^4/e^3/c*(c*x^2+b*x+a)^(1/2)*d^2+6*g^2/e/c*(c*x^2+b*x+a)^
(1/2)*f^2-g^4/e^4*d^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+4
*g/e*f^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^4}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + g x\right)^{4}}{\left(d + e x\right) \sqrt{a + b x + c x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**4/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.614 $\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=270

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)}{8c^{5/2}e^3}+\frac{3g^2\sqrt{a+bx+}}{}$$

Rubi [A] time = 0.71, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1653, 843, 621, 206, 724}

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)}{8c^{5/2}e^3}+\frac{3g^2\sqrt{a+bx+cx^2}(-beg-2cdg+4cef)}{4c^2e^2}+\frac{(ef-dg)^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3\sqrt{ae^2-bde+cd^2}}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (3*g^2*(4*c*e*f - 2*c*d*g - b*e*g)*Sqrt[a + b*x + c*x^2])/(4*c^2*e^2) + (g^
3*(d + e*x)*Sqrt[a + b*x + c*x^2])/(2*c*e^2) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*
(3*b*e*f - b*d*g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*e^3) + ((e*f -
 d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(e^3*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
```

```
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{\int \frac{\frac{1}{2}e\left(4ce^2f^3-d(bd+2ae)g^3\right)-eg\left(e(2bd+ae)g^2-c\left(6e^2f^2-d^2g^2\right)\right)x+\frac{3}{2}e^2g \cdots}{(d+ex)\sqrt{a+bx+cx^2}}}{2ce^3}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{\int \frac{\frac{1}{4}e^3\left(8c^2e^2f^{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{(ef-dg)^3 \cdots}{\cdots}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} - \frac{\left(2(ef-dg)\cdots\right.}{\cdots}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{g\left(3b^2e^2g^2 \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.38, size = 358, normalized size = 1.33

$$\frac{\frac{e^2g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}+\frac{4eg(2cf-bg)(ef-dg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}+\frac{6e^2g^2\sqrt{a+x(b+cx)}(2cf-bg)}{c^2}+\frac{8(ef-dg)^3\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}+\frac{8eg^2\sqrt{a+x(b+cx)}(ef-dg)}{c}+\frac{8g(ef-dg)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}+\frac{4e^2g^2(f+gx)\sqrt{a+x(b+cx)}}{c}}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((6*e^2*g^2*(2*c*f - b*g)*Sqrt[a + x*(b + c*x)])/c^2 + (8*e*g^2*(e*f - d*g)
*Sqrt[a + x*(b + c*x)])/c + (4*e^2*g^2*(f + g*x)*Sqrt[a + x*(b + c*x)])/c +
 (4*e*g*(2*c*f - b*g)*(e*f - d*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])])/c^(3/2) + (8*g*(e*f - d*g)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (e^2*g*(8*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(2
*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2
) + (8*(e*f - d*g)^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2
 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)
])/(8*e^3)
```

IntegrateAlgebraic [A] time = 1.41, size = 308, normalized size = 1.14

$$\frac{\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)\left(4ace^2g^3-3b^2e^2g^3-4bcdeg^3+12bce^2fg^2-8c^2d^2g^3+24c^2defg^2-24c^2e^2f^2g\right)}{8c^{5/2}e^3}+\frac{\sqrt{a+bx+cx^2}\left(-3beg^3-4cdg^3+12cefg^2+2ceg^3x\right)}{4c^2e^2}-\frac{2\left(d^3g^3-3d^2efg^2+3de^2f^2g-e^3f^3\right)\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(\frac{-e\sqrt{a+bx+cx^2}+\sqrt{c}d+\sqrt{c}ex}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^3\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((12*c*e*f*g^2 - 4*c*d*g^3 - 3*b*e*g^3 + 2*c*e*g^3*x)*Sqrt[a + b*x + c*x^2]
)/(4*c^2*e^2) - (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e^3*f^3) + 3*d*e^2*f^2
*g - 3*d^2*e*f*g^2 + d^3*g^3)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x - e*Sqrt[a +
b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(e^3*(c*d^2 - b*d*e + a*e^2)
) + ((-24*c^2*e^2*f^2*g + 24*c^2*d*e*f*g^2 + 12*b*c*e^2*f*g^2 - 8*c^2*d^2*g
```

```
^3 - 4*b*c*d*e*g^3 - 3*b^2*e^2*g^3 + 4*a*c*e^2*g^3)*Log[b + 2*c*x - 2*Sqrt[
c]*Sqrt[a + b*x + c*x^2]])/(8*c^(5/2)*e^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1007, normalized size = 3.73

$$\frac{d^3g^3\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2ae^2-2bde+2cd^2}{e^2}+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\,e^4}-\frac{3d^2fg^2\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2ae^2-2bde+2cd^2}{e^2}+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\,e^3}+\frac{3df^2g\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2ae^2-2bde+2cd^2}{e^2}+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\,e^2}-\frac{f^3\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2ae^2-2bde+2cd^2}{e^2}+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\,e}-\frac{ag^3\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{3}{2}}e}+\frac{3b^2g^3\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{5}{2}}e}+\frac{bdg^3\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{3}{2}}e^2}-\frac{3bfg^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{3}{2}}e}+\frac{d^2g^3\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{\sqrt{c}\,e^3}-\frac{3dfg^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{\sqrt{c}\,e^2}+\frac{3f^2g\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{\sqrt{c}\,e}+\frac{\sqrt{cx^2+bx+a}\,g^3x}{2ce}-\frac{3\sqrt{cx^2+bx+a}\,bg^3}{4c^2e}-\frac{\sqrt{cx^2+bx+a}\,dg^3}{ce^2}+\frac{3\sqrt{cx^2+bx+a}\,fg^2}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*g^3/e*x/c*(c*x^2+b*x+a)^(1/2)-3/4*g^3/e*b/c^2*(c*x^2+b*x+a)^(1/2)+3/8*g
^3/e*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*g^3/e*a/c^
(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-g^3/e^2/c*(c*x^2+b*x+a)^(
1/2)*d+3*g^2/e/c*(c*x^2+b*x+a)^(1/2)*f+1/2*g^3/e^2*b/c^(3/2)*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d-3/2*g^2/e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*f+g^3/e^3*d^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))/c^(1/2)-3*g^2/e^2*d*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/
2)+3*g/e*f^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/e^4/((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)
/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^3*d^3-3/e^3/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(x+d/e))*d^2*f*g^2+3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
(b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(
x+d/e))*d*f^2*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)
/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c
+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*f^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
```

```
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c        *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((f + g*x)**3/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.615 $\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=176

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

Rubi [A] time = 0.30, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1653, 843, 621, 206, 724}

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(c*e) + (g*(4*c*e*f - 2*c*d*g - b*e*g)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*e^2) + ((e*f - d
*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x + c*x^2])])/(e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
```

```
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= \frac{g^2\sqrt{a+bx+cx^2}}{ce}+\frac{\int \frac{\frac{1}{2}e\left(2cef^2-bdg^2\right)+\frac{1}{2}eg(4cef-2cdg-beg)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{ce^2}\\
&= \frac{g^2\sqrt{a+bx+cx^2}}{ce}+\frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^2}+\frac{(g(4cef-2cdg-beg))\int -\cdots}{2ce^2}\\
&= \frac{g^2\sqrt{a+bx+cx^2}}{ce}-\frac{\left(2(ef-dg)^2\right)\operatorname{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\cdots\right.}{e^2}\\
&= \frac{g^2\sqrt{a+bx+cx^2}}{ce}+\frac{g(4cef-2cdg-beg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\,\mathrm{ta}\cdots}{e^2}\end{aligned}$$

Mathematica [A] time = 0.48, size = 170, normalized size = 0.97

$$\frac{-\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)(beg+2cdg-4cef)}{c^{3/2}}+\frac{2(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}+\frac{2eg^2\sqrt{a+x(b+cx)}}{c}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*e*g^2*Sqrt[a + x*(b + c*x)])/c - (g*(-4*c*e*f + 2*c*d*g + b*e*g)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (2*(e*f - d*g)^
2*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)
]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(2*e^2)
```

IntegrateAlgebraic [A] time = 0.77, size = 210, normalized size = 1.19

$$\frac{\log\left(-2c^{3/2}\sqrt{a+bx+cx^2}+bc+2c^2x\right)\left(beg^2+2cdg^2-4cefg\right)}{2c^{3/2}e^2}+\frac{2\left(d^2g^2-2defg+e^2f^2\right)\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(\frac{-e\sqrt{a+bx+cx^2}+\sqrt{c}d+\sqrt{c}ex}{\sqrt{-ae^2+bde-cd^2}}\right)}{e^2\left(ae^2-bde+cd^2\right)}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(c*e) + (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e^2*
f^2 - 2*d*e*f*g + d^2*g^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x - e*Sqrt[a + b*x
 + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(e^2*(c*d^2 - b*d*e + a*e^2)) +
 ((-4*c*e*f*g + 2*c*d*g^2 + b*e*g^2)*Log[b*c + 2*c^2*x - 2*c^(3/2)*Sqrt[a +
 b*x + c*x^2]])/(2*c^(3/2)*e^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 613, normalized size = 3.48

$$-\frac{d^{2} g^{2} \ln \left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^{2}-2bde+2c d^{2}}{e^{2}}+2\sqrt{\frac{a e^{2}-bde+c d^{2}}{e^{2}}}\, \sqrt{\left(x+\frac{d}{e}\right)^{2} c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^{2}-bde+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^{2}-bde+c d^{2}}{e^{2}}}\, e^{3}}+\frac{2dfg \ln \left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^{2}-2bde+2c d^{2}}{e^{2}}+2\sqrt{\frac{a e^{2}-bde+c d^{2}}{e^{2}}}\, \sqrt{\left(x+\frac{d}{e}\right)^{2} c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^{2}-bde+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^{2}-bde+c d^{2}}{e^{2}}}\, e^{2}}-\frac{f^{2} \ln \left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^{2}-2bde+2c d^{2}}{e^{2}}+2\sqrt{\frac{a e^{2}-bde+c d^{2}}{e^{2}}}\, \sqrt{\left(x+\frac{d}{e}\right)^{2} c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^{2}-bde+c d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a e^{2}-bde+c d^{2}}{e^{2}}}\, e}-\frac{b g^{2} \ln \left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{2 c^{\frac{3}{2}} e}-\frac{d g^{2} \ln \left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{\sqrt{c}\, e^{2}}+\frac{2fg \ln \left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{\sqrt{c}\, e}+\frac{\sqrt{c x^{2}+bx+a}\, g^{2}}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] g^2*(c*x^2+b*x+a)^(1/2)/c/e-1/2*g^2/e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))-g^2/e^2*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1
/2)+2*g/e*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/e^3/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/
e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d^2*g^2+2/e^2/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(x+d/e))*d*f*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*
c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(f+g\,x\right)}^2}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx\right)^{2}}{\left(d + ex\right) \sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**2/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.616 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=131

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,e}$$

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {843, 621, 206, 724}

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) + ((
e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e}$$

$$= \frac{(2g)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e} - \frac{(2(ef-dg))\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,\right.}{e}$$

$$= \frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,e} + \frac{(ef-dg)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] time = 0.15, size = 126, normalized size = 0.96

$$\frac{\frac{(dg-ef)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}} + \frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + ((-(e
*f) + d*g)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(
b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/e
```

IntegrateAlgebraic [A] time = 0.53, size = 152, normalized size = 1.16

$$-\frac{2(dg-ef)\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(\frac{-e\sqrt{a+bx+cx^2}+\sqrt{c}\,d+\sqrt{c}\,ex}{\sqrt{-ae^2+bde-cd^2}}\right)}{e\left(ae^2-bde+cd^2\right)} - \frac{g\log\left(-2\sqrt{c}\,e\sqrt{a+bx+cx^2}+be+2cex\right)}{\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(Sqrt[c]*d + Sqrt[
c]*e*x - e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(e*(c*d^
2 - b*d*e + a*e^2)) - (g*Log[b*e + 2*c*e*x - 2*Sqrt[c]*e*Sqrt[a + b*x + c*x
^2]])/(Sqrt[c]*e)
```

fricas [B] time = 31.64, size = 1071, normalized size = 8.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c)*g*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - sqrt(c*d^2 - b*d*e + a
*e^2)*(c*e*f - c*d*g)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c
^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)
*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a
*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c^2*d^2*e - b
*c*d*e^2 + a*c*e^3), 1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c)*g*log(-8*c^2*x^2
- 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*
sqrt(-c*d^2 + b*d*e - a*e^2)*(c*e*f - c*d*g)*arctan(-1/2*sqrt(-c*d^2 + b*d*
e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 -
```

```
 a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e
 + a*b*e^2)*x)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(2*(c*d^2 - b*d*e
+ a*e^2)*sqrt(-c)*g*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*(c*e*f - c*d*g)*log((
8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d
 - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*
e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -((c*d
^2 - b*d*e + a*e^2)*sqrt(-c)*g*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - sqrt(-c*d^2 + b*d*e - a*e^2)*(c*e*f -
c*d*g)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d
- 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*
e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)))/(c^2*d^2*e - b*c*d*e^
2 + a*c*e^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 349, normalized size = 2.66

$$\frac{dg\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\;e^2}-\frac{f\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\;e}+\frac{g\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/e*g*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/e^2/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(x+d/e))*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))      /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f+g\,x}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx}{(d + ex)\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.617 $\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x + c*x^2])]/Sqrt[c*d^2 - b*d*e + a*e^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 0.99

$$-\frac{\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*
e)]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d^2 + e*(-(b*d) + a*e)])
```

IntegrateAlgebraic [A] time = 0.00, size = 138, normalized size = 1.75

$$\frac{2\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+bx+cx^2}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2+bde-cd^2}}\right)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e
- a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x +
 c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(c*d^2 - b*d*e + a*e^2)
```

fricas [B] time = 0.55, size = 343, normalized size = 4.34

$$\left[\frac{\log\left(\frac{8\,abde-8\,a^2e^2-\left(b^2+4\,ac\right)d^2-\left(8\,c^2d^2-8\,bcde+\left(b^2+4\,ac\right)e^2\right)x^2-4\,\sqrt{cd^2-bde+ae^2}\,\sqrt{cx^2+bx+a}\,(bd-2\,ae+(2\,cd-be)x)-2\left(4\,bcd^2+4\,abe^2-\left(3\,b^2+4\,ac\right)de\right)x}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{cd^2-bde+ae^2}},\ \frac{\sqrt{-cd^2+bde-ae^2}\,\arctan\left(-\frac{\sqrt{-cd^2+bde-ae^2}\,\sqrt{cx^2+bx+a}\,(bd-2\,ae+(2\,cd-be)x)}{2\left(acd^2-abde+a^2e^2+\left(c^2d^2-bcde+ace^2\right)x^2+\left(bcd^2-b^2de+abe^2\right)x\right)}\right)}{cd^2-bde+ae^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*
e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x
 + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2))/sqrt(c*d^2 - b*d*e + a*e^2), sqr
t(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x
^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2
+ (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x))/(c*
d^2 - b*d*e + a*e^2)]
```

giac [A] time = 0.23, size = 72, normalized size = 0.91

$$\frac{2\,\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 +
 b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

maple [B] time = 0.01, size = 157, normalized size = 1.99

$$-\frac{\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d
*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                            +c*d^2    positive,
negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.618 $\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{e\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}}-\frac{g\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

Rubi [A] time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {960, 724, 206}

$$\frac{e\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}}-\frac{g\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)) - (g*ArcT
anh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a +
 b*x + c*x^2])])/((e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx &= \int \left(\frac{e}{(ef-dg)(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)\sqrt{a+bx+cx^2}}\right. \\
&= \frac{e\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{ef-dg} \\
&= -\frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{ef-dg} + \frac{(2g)\,\mathrm{Subst}\left(\right.}{} \\
&= \frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}\,(ef-dg)} - \frac{g\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\sqrt{cf^2-bfg+ag^2}}
\end{aligned}
$$

Mathematica [A] time = 0.25, size = 169, normalized size = 0.93

$$
\frac{\frac{g\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\sqrt{g(ag-bf)+cf^2}} - \frac{e\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}}{dg-ef}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-((e*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) +
a*e)]*Sqrt[a + x*(b + c*x)])]])/Sqrt[c*d^2 + e*(-(b*d) + a*e)]) + (g*ArcTanh
[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
+ x*(b + c*x)])]])/Sqrt[c*f^2 + g*(-(b*f) + a*g)])/(-(e*f) + d*g)
```

IntegrateAlgebraic [A] time = 0.87, size = 299, normalized size = 1.64

$$
-\frac{2e\sqrt{-ae^2+bde-cd^2}\,\tan^{-1}\left(-\frac{e\sqrt{a+bx+cx^2}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2+bde-cd^2}}\right)}{(dg-ef)\left(ae^2-bde+cd^2\right)} - \frac{2g\sqrt{-ag^2+bfg-cf^2}\,\tan^{-1}\left(-\frac{g\sqrt{a+bx+cx^2}}{\sqrt{-ag^2+bfg-cf^2}}+\frac{\sqrt{c}\,gx}{\sqrt{-ag^2+bfg-cf^2}}+\frac{\sqrt{c}\,f}{\sqrt{-ag^2+bfg-cf^2}}\right)}{(ef-dg)\left(ag^2-bfg+cf^2\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d
*e - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*
x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/((c*d^2 - b*d*e + a*e^2)*(-(e*
f) + d*g)) - (2*g*Sqrt[-(c*f^2) + b*f*g - a*g^2]*ArcTan[(Sqrt[c]*f)/Sqrt[-(
c*f^2) + b*f*g - a*g^2] + (Sqrt[c]*g*x)/Sqrt[-(c*f^2) + b*f*g - a*g^2] - (g
*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/((e*f - d*g)*(c*f^
2 - b*f*g + a*g^2))
```

fricas [B] time = 123.75, size = 1952, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c*f^2 - b*f*g + a*g^2)*g*log((8*a*b*f*g
 - 8*a^2*g^2 - (b^2 + 4*a*c)*f^2 - (8*c^2*f^2 - 8*b*c*f*g + (b^2 + 4*a*c)*g
^2)*x^2 - 4*sqrt(c*f^2 - b*f*g + a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a*g
+ (2*c*f - b*g)*x) - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*g)*x)/(g^
2*x^2 + 2*f*g*x + f^2)) + (c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(c*d^2 - b*d*e
```

```
+ a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*
c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 +
 b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b
^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/((c^2*d^2*e - b*c*d*e^2 + a
*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f^2*g + (b*c*d^3 + a^
2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*g^3),
-1/2*(2*(c*d^2 - b*d*e + a*e^2)*sqrt(-c*f^2 + b*f*g - a*g^2)*g*arctan(-1/2*
sqrt(-c*f^2 + b*f*g - a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a*g + (2*c*f -
b*g)*x)/(a*c*f^2 - a*b*f*g + a^2*g^2 + (c^2*f^2 - b*c*f*g + a*c*g^2)*x^2 +
(b*c*f^2 - b^2*f*g + a*b*g^2)*x)) + (c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(c*d^
2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*
d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sq
rt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*
e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/((c^2*d^2*e - b*c
*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f^2*g + (b*
c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e + a^2*d*e
^2)*g^3), -1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c*f^2 - b*f*g + a*g^2)*g*log((
8*a*b*f*g - 8*a^2*g^2 - (b^2 + 4*a*c)*f^2 - (8*c^2*f^2 - 8*b*c*f*g + (b^2 +
 4*a*c)*g^2)*x^2 - 4*sqrt(c*f^2 - b*f*g + a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f
 - 2*a*g + (2*c*f - b*g)*x) - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*
g)*x)/(g^2*x^2 + 2*f*g*x + f^2)) - 2*(c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(-c*
d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 +
b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^
2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)))/((c^2*d
^2*e - b*c*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f
^2*g + (b*c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e
 + a^2*d*e^2)*g^3), -((c*d^2 - b*d*e + a*e^2)*sqrt(-c*f^2 + b*f*g - a*g^2)*
g*arctan(-1/2*sqrt(-c*f^2 + b*f*g - a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a
*g + (2*c*f - b*g)*x)/(a*c*f^2 - a*b*f*g + a^2*g^2 + (c^2*f^2 - b*c*f*g + a
*c*g^2)*x^2 + (b*c*f^2 - b^2*f*g + a*b*g^2)*x)) - (c*e*f^2 - b*e*f*g + a*e*
g^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*
sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e +
a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)
*x)))/((c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 -
a*c)*d*e^2)*f^2*g + (b*c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^
3 - a*b*d^2*e + a^2*d*e^2)*g^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage2
```

maple [A] time = 0.02, size = 327, normalized size = 1.80

$$\frac{\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^2-2bde+2c d^2}{e^2}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(dg-ef\right)\sqrt{\frac{a e^2-bde+c d^2}{e^2}}}-\frac{\ln\left(\frac{\frac{\left(bg-2cf\right)\left(x+\frac{f}{g}\right)}{g}+\frac{2a g^2-2bfg+2c f^2}{g^2}+2\sqrt{\frac{a g^2-bfg+c f^2}{g^2}}\,\sqrt{\left(x+\frac{f}{g}\right)^2 c+\frac{\left(bg-2cf\right)\left(x+\frac{f}{g}\right)}{g}+\frac{a g^2-bfg+c f^2}{g^2}}}{x+\frac{f}{g}}\right)}{\left(dg-ef\right)\sqrt{\frac{a g^2-bfg+c f^2}{g^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a
*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2
*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+1/(d*g-e*f)/((a*e^
```

```
2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e
^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(f + g\,x\right)\,\left(d + e\,x\right)\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)\left(f + gx\right)\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

3.619 $\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=340

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}-\frac{eg\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}-\cdots$$

Rubi [A] time = 0.39, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {960, 724, 206, 730}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}-\frac{eg\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}-\frac{g(2cf-bg)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x))
 + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2) -
(g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*
f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2
)^(3/2)) - (e*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f
*g + a*g^2]*Sqrt[a + b*x + c*x^2])]])/((e*f - d*g)^2*Sqrt[c*f^2 - b*f*g + a*
g^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx &= \int \left(\frac{e^2}{(ef-dg)^2(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)^2\sqrt{a+bx+\cdots}} \right. \\
&= \frac{e^2 \int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{1}{(f+gx)^2\sqrt{a+bx\cdots}}}{ef-dg\cdots} \\
&= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} - \frac{\left(2e^2\right)\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-\cdots}\right.}{(ef-\cdots} \\
&= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)\cdots}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+b\cdots}}\right.}{\sqrt{cd^2-bde+ae^2}\,(ef-d\cdots} \\
&= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)\cdots}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+b\cdots}}\right.}{\sqrt{cd^2-bde+ae^2}\,(ef-d\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.95, size = 256, normalized size = 0.75

$$
-\frac{-\frac{2e^2\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}+\frac{2g^2\sqrt{a+x(b+cx)}\,(dg-ef)}{(f+gx)\left(g(ag-bf)+cf^2\right)}+\frac{g(g(2aeg+bdg-3bef)+2cf(2ef-dg))\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{3/2}}}{2(ef-dg)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((2*g^2*(-(e*f) + d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*
g))*(f + g*x)) - (2*e^2*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*
d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a
*e)] + (g*(2*c*f*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(-
2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x
*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2))/(e*f - d*g)^2
```

IntegrateAlgebraic [B] time = 21.82, size = 2266, normalized size = 6.66

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-((b*Sqrt[c]*f*g^2*x^2)/(e*f - d*g)) + (2*a*Sqrt[c]*g^3*x^2)/(e*f - d*g) -
 (2*c^(3/2)*f*g^2*x^3)/(e*f - d*g) + (b*Sqrt[c]*g^3*x^3)/(e*f - d*g) + (2*c
*f*g^2*x^2*Sqrt[a + b*x + c*x^2])/(e*f - d*g) - (b*g^3*x^2*Sqrt[a + b*x + c
*x^2])/(e*f - d*g))/((f + g*x)*(b*f^3 - 2*a*f^2*g + (b*f^2*g - 2*a*f*g^2)*x
 + (-(c*f^2*g) + b*f*g^2 - a*g^3)*x^2)*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x
+ c*x^2])) - (2*e^2*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sq
rt[c]*e*x)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x + c*x^2])/Sqrt[
-(c*d^2) + b*d*e - a*e^2]])/(Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^2)
+ (-((b*Sqrt[c]*f^3*g)/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2))) + (2*a*Sqrt[c
]*f^2*g^2)/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)) - (b*Sqrt[c]*f^2*g^2*x)/((
e*f - d*g)*(c*f^2 - b*f*g + a*g^2)) + (2*a*Sqrt[c]*f*g^3*x)/((e*f - d*g)*(c
*f^2 - b*f*g + a*g^2)) - (b*f^2*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*
```

```
f^2 - b*f*g + a*g^2)) + (2*a*f*g^3*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f
^2 - b*f*g + a*g^2)) + (b^2*f^3*g^2*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*
Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/((e*f - d*g)*Sqrt[-
(c*f^2) + b*f*g - a*g^2]*(c*f^2 - b*f*g + a*g^2)) - (4*a*b*f^2*g^3*ArcTan[(
-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g
 - a*g^2]])/((e*f - d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]*(c*f^2 - b*f*g + a*
g^2)) + (4*a^2*f*g^4*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x +
c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/((e*f - d*g)*Sqrt[-(c*f^2) + b*f*g
 - a*g^2]*(c*f^2 - b*f*g + a*g^2)) + (b^2*f^2*g^3*x*ArcTan[(-(Sqrt[c]*f) -
Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/((e
*f - d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]*(c*f^2 - b*f*g + a*g^2)) - (4*a*b*
f*g^4*x*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[
-(c*f^2) + b*f*g - a*g^2]])/((e*f - d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]*(c*
f^2 - b*f*g + a*g^2)) + (4*a^2*g^5*x*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g
*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/((e*f - d*g)*Sqrt[
-(c*f^2) + b*f*g - a*g^2]*(c*f^2 - b*f*g + a*g^2)) - (b*c*f^2*g^3*x^2*ArcTa
n[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*
f*g - a*g^2]])/((e*f - d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]*(c*f^2 - b*f*g +
 a*g^2)) + (b^2*f*g^4*x^2*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b
*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/((e*f - d*g)*Sqrt[-(c*f^2) +
b*f*g - a*g^2]*(c*f^2 - b*f*g + a*g^2)) + (2*a*c*f*g^4*x^2*ArcTan[(-(Sqrt[c
]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2
]])/((e*f - d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]*(c*f^2 - b*f*g + a*g^2)) -
(3*a*b*g^5*x^2*ArcTan[(-(Sqrt[c]*f) - Sqrt[c]*g*x + g*Sqrt[a + b*x + c*x^2]
)/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/((e*f - d*g)*Sqrt[-(c*f^2) + b*f*g - a*g
^2]*(c*f^2 - b*f*g + a*g^2)) + (2*a^2*g^6*x^2*ArcTan[(-(Sqrt[c]*f) - Sqrt[c
]*g*x + g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*g^2]])/(f*(e*f -
 d*g)*Sqrt[-(c*f^2) + b*f*g - a*g^2]*(c*f^2 - b*f*g + a*g^2)))/(-(b*f^3) +
2*a*f^2*g + (-(b*f^2*g) + 2*a*f*g^2)*x + (c*f^2*g - b*f*g^2 + a*g^3)*x^2) +
 (4*e*g*ArcTan[(Sqrt[c]*f)/Sqrt[-(c*f^2) + b*f*g - a*g^2] + (Sqrt[c]*g*x)/S
qrt[-(c*f^2) + b*f*g - a*g^2] - (g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b
*f*g - a*g^2]])/((e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2]) - (2*d*g^2*A
rcTan[(Sqrt[c]*f)/Sqrt[-(c*f^2) + b*f*g - a*g^2] + (Sqrt[c]*g*x)/Sqrt[-(c*f
^2) + b*f*g - a*g^2] - (g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2) + b*f*g - a*
g^2]])/(f*(e*f - d*g)^2*Sqrt[-(c*f^2) + b*f*g - a*g^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 788, normalized size = 2.32

$$\frac{b g \ln\left(\frac{\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{2a g^2-2bfg+2c f^2}{g^2}+2\sqrt{\frac{a g^2-bfg+c f^2}{g^2}}\sqrt{\left(x+\frac{f}{g}\right)^2 c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a g^2-bfg+c f^2}{g^2}}}{x+\frac{f}{g}}\right)}{2\left(dg-ef\right)\left(a g^2-bfg+c f^2\right)\sqrt{\frac{a g^2-bfg+c f^2}{g^2}}}-\frac{c f \ln\left(\frac{\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{2a g^2-2bfg+2c f^2}{g^2}+2\sqrt{\frac{a g^2-bfg+c f^2}{g^2}}\sqrt{\left(x+\frac{f}{g}\right)^2 c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a g^2-bfg+c f^2}{g^2}}}{x+\frac{f}{g}}\right)}{\left(dg-ef\right)\left(a g^2-bfg+c f^2\right)\sqrt{\frac{a g^2-bfg+c f^2}{g^2}}}-\frac{e \ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2a e^2-2bde+2c d^2}{e^2}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(dg-ef\right)^2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}}+\frac{e \ln\left(\frac{\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{2a g^2-2bfg+2c f^2}{g^2}+2\sqrt{\frac{a g^2-bfg+c f^2}{g^2}}\sqrt{\left(x+\frac{f}{g}\right)^2 c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a g^2-bfg+c f^2}{g^2}}}{x+\frac{f}{g}}\right)}{\left(dg-ef\right)^2\sqrt{\frac{a g^2-bfg+c f^2}{g^2}}}-\frac{\sqrt{\left(x+\frac{f}{g}\right)^2 c+\frac{(bg-2cf)\left(x+\frac{f}{g}\right)}{g}+\frac{a g^2-bfg+c f^2}{g^2}}\, g}{\left(dg-ef\right)\left(a g^2-bfg+c f^2\right)\left(x+\frac{f}{g}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g
+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2
-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^
2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b-1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*
g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)
/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(
a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f+e/(d*g-e*f)^2/((a*g^2-b*f*g+c*f
^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^
2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c
*f^2)/g^2)^(1/2))/(x+f/g))-e/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)\left(gx + f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f + g\,x\right)^2\,\left(d + e\,x\right)\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)\left(f + gx\right)^2\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)**2*sqrt(a + b*x + c*x**2)), x)
```

3.620 $\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=587

$$-\frac{g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}}-\frac{e^2g}{\quad}$$

Rubi [A] time = 0.81, antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {960, 724, 206, 744, 806, 730}

$$-\frac{g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}}-\frac{e^2g\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^3\sqrt{ag^2-bfg+cf^2}}+\frac{eg^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2\left(ag^2-bfg+cf^2\right)}+\frac{3g^2\sqrt{a+bx+cx^2}(2cf-bg)}{4(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}+\frac{g^2\sqrt{a+bx+cx^2}}{2(f+gx)^2(ef-dg)\left(ag^2-bfg+cf^2\right)}-\frac{eg(2cf-bg)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x
)^2) + (3*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 -
b*f*g + a*g^2)^2*(f + g*x)) + (e*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)*(f + g*x)) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b
*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2
- b*d*e + a*e^2]*(e*f - d*g)^3) - (e*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g +
 (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(
2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (e^2*g*ArcTanh[(b*f - 2*a*
g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]
)/((e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (g*(8*c^2*f^2 + 3*b^2*g^2 -
 4*c*g*(2*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2
 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{e^3}{(ef-dg)^3(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)^3\sqrt{a+bx+\cdots}}\right.$$

$$= \frac{e^3 \int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^3} - \frac{\left(e^2 g\right)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{1}{(f+gx)^2\sqrt{\cdots}}}{(ef-\cdots}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\cdots\right.}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+b\cdots}}{4(ef-dg)\left(cf^2-bfg+a\cdots\right.}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+b\cdots}}{4(ef-dg)\left(cf^2-bfg+a\cdots\right.}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+b\cdots}}{4(ef-dg)\left(cf^2-bfg+a\cdots\right.}$$

Mathematica [A] time = 2.43, size = 549, normalized size = 0.94

$$\frac{g(ef-dg)^2\left(\frac{6g\sqrt{a+x(b+cx)}(2cf-bg)}{(f+gx)\left(g(ag-bf)+cf^2\right)^2}-\frac{\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{5/2}}\right)+\frac{8e^3\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}+\frac{8eg^2\sqrt{a+x(b+cx)}(ef-dg)}{(f+gx)\left(g(ag-bf)+cf^2\right)}+\frac{4g^2\sqrt{a+x(b+cx)}(ef-dg)^2}{(f+gx)^2\left(g(ag-bf)+cf^2\right)}+\frac{4eg(bg-2cf)(ef-dg)\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{3/2}}-\frac{8e^2g\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\sqrt{g(ag-bf)+cf^2}}}{8(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*g^2*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f
 + g*x)^2) + (8*e*g^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f
) + a*g))*(f + g*x)) + (8*e^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*S
qrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*
d) + a*e)] + (4*e*g*(-2*c*f + b*g)*(e*f - d*g)*ArcTanh[(-2*a*g + 2*c*f*x +
b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*
f^2 + g*(-(b*f) + a*g))^(3/2) - (8*e^2*g*ArcTanh[(-2*a*g + 2*c*f*x + b*(f -
 g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*f^
2 + g*(-(b*f) + a*g)] + g*(e*f - d*g)^2*((6*g*(2*c*f - b*g)*Sqrt[a + x*(b +
 c*x)])/((c*f^2 + g*(-(b*f) + a*g))^2*(f + g*x)) - ((8*c^2*f^2 + 3*b^2*g^2
- 4*c*g*(2*b*f + a*g))*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f
^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^
(5/2)))/(8*(e*f - d*g)^3)
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 3.26, size = 2256, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c^2*d^2*f^2*g^3 - 8*b*c*d^2*f*g^4 + 3*b^2*d^2*g^5 - 4*a*c*d^2*g^5 -
24*c^2*d*f^3*g^2*e + 28*b*c*d*f^2*g^3*e - 10*b^2*d*f*g^4*e + 4*a*b*d*g^5*e
+ 24*c^2*f^4*g*e^2 - 36*b*c*f^3*g^2*e^2 + 15*b^2*f^2*g^3*e^2 + 20*a*c*f^2*g
^3*e^2 - 20*a*b*f*g^4*e^2 + 8*a^2*g^5*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c^2*d^3*f^4*g^3
 - 2*b*c*d^3*f^3*g^4 + b^2*d^3*f^2*g^5 + 2*a*c*d^3*f^2*g^5 - 2*a*b*d^3*f*g^
6 + a^2*d^3*g^7 - 3*c^2*d^2*f^5*g^2*e + 6*b*c*d^2*f^4*g^3*e - 3*b^2*d^2*f^3
*g^4*e - 6*a*c*d^2*f^3*g^4*e + 6*a*b*d^2*f^2*g^5*e - 3*a^2*d^2*f*g^6*e + 3*
c^2*d*f^6*g*e^2 - 6*b*c*d*f^5*g^2*e^2 + 3*b^2*d*f^4*g^3*e^2 + 6*a*c*d*f^4*g
^3*e^2 - 6*a*b*d*f^3*g^4*e^2 + 3*a^2*d*f^2*g^5*e^2 - c^2*f^7*e^3 + 2*b*c*f^
6*g*e^3 - b^2*f^5*g^2*e^3 - 2*a*c*f^5*g^2*e^3 + 2*a*b*f^4*g^3*e^3 - a^2*f^3
*g^4*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) + 2*arctan(((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^3/((d^3*g^3 - 3
*d^2*f*g^2*e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/4
*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g^3 - 8*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*b*c*d*f*g^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b^2*d*g^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*g^5 - 16*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f^3*g^2*e + 20*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b*c*f^2*g^3*e - 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f
*g^4*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^4*e + 4*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a*b*g^5*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x +
```

```
a))^2*c^(5/2)*d*f^3*g^2 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2
)*d*f^2*g^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^4 -
 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^4 - 40*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f^4*g*e + 44*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*b*c^(3/2)*f^3*g^2*e - 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
b^2*sqrt(c)*f^2*g^3*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f
^2*g^3*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^4*e + 8*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*g^5*e + 24*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g^2 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*b^2*c*d*f^2*g^3 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^
3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*f*g^4 + 28*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b*c*d*f*g^4 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
*b^2*d*g^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*g^5 - 40*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f^4*g*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*b^2*c*f^3*g^2*e + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3
*g^2*e - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f^2*g^3*e - 72*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*g^3*e + 13*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a*b^2*f*g^4*e + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^4
*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*g^5*e + 6*b^2*c^(3/2)*d*f^
3*g^2 - 3*b^3*sqrt(c)*d*f^2*g^3 - 20*a*b*c^(3/2)*d*f^2*g^3 + 11*a*b^2*sqrt(
c)*d*f*g^4 + 12*a^2*c^(3/2)*d*f*g^4 - 8*a^2*b*sqrt(c)*d*g^5 - 10*b^2*c^(3/2
)*f^4*g*e + 7*b^3*sqrt(c)*f^3*g^2*e + 32*a*b*c^(3/2)*f^3*g^2*e - 27*a*b^2*s
qrt(c)*f^2*g^3*e - 20*a^2*c^(3/2)*f^2*g^3*e + 28*a^2*b*sqrt(c)*f*g^4*e - 8*
a^3*sqrt(c)*g^5*e)/((c^2*d^2*f^4*g^2 - 2*b*c*d^2*f^3*g^3 + b^2*d^2*f^2*g^4
+ 2*a*c*d^2*f^2*g^4 - 2*a*b*d^2*f*g^5 + a^2*d^2*g^6 - 2*c^2*d*f^5*g*e + 4*b
*c*d*f^4*g^2*e - 2*b^2*d*f^3*g^3*e - 4*a*c*d*f^3*g^3*e + 4*a*b*d*f^2*g^4*e
- 2*a^2*d*f*g^5*e + c^2*f^6*e^2 - 2*b*c*f^5*g*e^2 + b^2*f^4*g^2*e^2 + 2*a*c
*f^4*g^2*e^2 - 2*a*b*f^3*g^3*e^2 + a^2*f^2*g^4*e^2)*((sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*f + b*f -
 a*g)^2)
```

maple [B] time = 0.02, size = 1817, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)^2*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/
g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+3/4*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2
/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*
b-3/2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)*(x
+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*c*f-3/8*g^2/(d*g-e*f)/(a*g^2-b*f*g+c
*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-
b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)
*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b^2+3/2*g/(d*g-e*f)/(a*
g^2-b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/
g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+
(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b*c*f-3/2/(d
*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*
f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((
x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c
^2*f^2+1/2/(d*g-e*f)*c/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*
ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2
))/(x+f/g))+g*e/(d*g-e*f)^2/(a*g^2-b*f*g+c*f^2)/(x+f/g)*((x+f/g)^2*c+(b*g-2
*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/2*g*e/(d*g-e*f)^2/(a*g^2-b
*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*
g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*
c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b+e/(d*g-e*f)^2/(a*
```

```
g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+
2*(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b
*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f-e^2/(d*g-e
*f)^3/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*
f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(
x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+e^2/(d*g-e*f)^3/((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+
2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\,(ex + d)\left(gx + f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f + g\,x\right)^3\,\left(d + e\,x\right)\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex\right)\left(f + gx\right)^{3}\sqrt{a + bx + cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)**3*sqrt(a + b*x + c*x**2)), x)
```

3.621 $\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=496

$$-\frac{2\left(-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg\right.\right.\right.}{}$$

Rubi [A] time = 1.20, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1646, 1653, 843, 621, 206, 724}

$$-\frac{2\left(x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab(ef-dg)+4b^2df\right)+b^3g^4(bd-ae)+c^3f^2(4ag(2ef-3dg)-bf(4dg+ef))+2c^4df^4\right)-b^2\left(a^2eg^4+4acdfg^3+c^2ef^4\right)+bc\left(a^2g^3(4ef-3dg)+2acf^2g(3dg+2ef)+c^2df^4\right)+2ac\left(a^2eg^4-2acfg^2(3ef-2dg)+c^2f^3(ef-4dg)\right)+ab^3dg^4\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{g^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-3beg-2cdg+8cef)}{2c^{5/2}e^2}+\frac{g^4\sqrt{a+bx+cx^2}}{c^2e}+\frac{(ef-dg)^4\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(a*b^3*d*g^4 - b^2*(c^2*e*f^4 + 4*a*c*d*f*g^3 + a^2*e*g^4) + 2*a*c*(a^2
*e*g^4 + c^2*f^3*(e*f - 4*d*g) - 2*a*c*f*g^2*(3*e*f - 2*d*g)) + b*c*(c^2*d*
f^4 + a^2*g^3*(4*e*f - 3*d*g) + 2*a*c*f^2*g*(2*e*f + 3*d*g)) + (2*c^4*d*f^4
 + b^3*(b*d - a*e)*g^4 - b*c*g^3*(4*b^2*d*f - 3*a^2*e*g - 4*a*b*(e*f - d*g)
) + 2*c^2*g^2*(3*b^2*d*f^2 - 3*a*b*f*(e*f - 2*d*g) - a^2*g*(4*e*f - d*g)) +
 c^3*f^2*(4*a*g*(2*e*f - 3*d*g) - b*f*(e*f + 4*d*g)))*x))/(c^2*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^4*Sqrt[a + b*x + c*x^
2])/(c^2*e) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*e^2) + ((e*f - d*g)^4*ArcTanh[(b*d
 - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c
*x^2])])/(e^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2a\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2a\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2a\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2a\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2a\right.}{}$$

Mathematica [A] time = 2.46, size = 587, normalized size = 1.18

$$\frac{-\frac{2e\left(b^2\left(3a^2e^2g^4+acg^3\left(d^2g+4de(2f+3gx)+e^2x(gx-8f)\right)+c^2\left(d^2g^4x^2-12def^2g^2x+2e^2f^4\right)\right)-2bc\left(a^2eg^3(-5dg+4ef+5egx)+2acg\left(d^2g^3x+deg\left(3f^2+6fgx-g^2x^2\right)+e^2f^2(2f-3gx)\right)+c^2ef^3(d(f-4gx)-efx)\right)-4c\left(2a^3e^2g^4+a^2cg^2\left(d^2g^2+deg(4f+gx)+e^2\left(-6f^2-4fgx+g^2x^2\right)\right)+ac^2\left(d^2g^4x^2-2def^2g(2f+3gx)+e^2f^3(f+4gx)\right)+c^3def^4x\right)+b^3g^3(3aeg(ex-d)+cdx(dg+8ef-egx))-3b^4deg^4x\right)}{c^2\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)}+\frac{g^3\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)(-3beg-2cdg+8cef)}{c^{5/2}}+\frac{2(ef-dg)^4\log(d+ex)}{\left(e(ae-bd)+cd^2\right)^{3/2}}-\frac{2(ef-dg)^4\log\left(2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}+2ae-bd+bex-2cdx\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*e*(-3*b^4*d*e*g^4*x + b^3*g^3*(3*a*e*g*(-d + e*x) + c*d*x*(8*e*f + d*g
 - e*g*x)) + b^2*(3*a^2*e^2*g^4 + c^2*(2*e^2*f^4 - 12*d*e*f^2*g^2*x + d^2*g
```

```
^4*x^2) + a*c*g^3*(d^2*g + e^2*x*(-8*f + g*x) + 4*d*e*(2*f + 3*g*x))) - 2*b
*c*(a^2*e*g^3*(4*e*f - 5*d*g + 5*e*g*x) + c^2*e*f^3*(-(e*f*x) + d*(f - 4*g*
x)) + 2*a*c*g*(d^2*g^3*x + e^2*f^2*(2*f - 3*g*x) + d*e*g*(3*f^2 + 6*f*g*x -
 g^2*x^2))) - 4*c*(2*a^3*e^2*g^4 + c^3*d*e*f^4*x + a*c^2*(d^2*g^4*x^2 - 2*d
*e*f^2*g*(2*f + 3*g*x) + e^2*f^3*(f + 4*g*x)) + a^2*c*g^2*(d^2*g^2 + d*e*g*
(4*f + g*x) + e^2*(-6*f^2 - 4*f*g*x + g^2*x^2)))))/(c^2*(b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (2*(e*f - d*g)^4*Log[d + e*x
])/(c*d^2 + e*(-(b*d) + a*e))^(3/2) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*Lo
g[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2) - (2*(e*f - d*g)^4*
Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqr
t[a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/(2*e^2)
```

IntegrateAlgebraic [B] time = 21.86, size = 5425, normalized size = 10.94

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 4453, normalized size = 8.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*f^4+6/e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^2*f^2*g^2-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^
2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f^
4-1/e^3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)
*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+
d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^4
*d^4-2*g^4/e^4*d^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+4/(a*e^2-b*d*e+c*d^2)/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*
d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d*f^3*g+4*g^3/e^2/c/(c*x^2+b*x+
a)^(1/2)*d*f+3/2*g^4/e*b/c^2*x/(c*x^2+b*x+a)^(1/2)-3/4*g^4/e*b^4/c^3/(4*a*c
```

-b^2)/(c*x^2+b*x+a)^(1/2)+8*g/e*f^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+g^4/e
^2*x/c/(c*x^2+b*x+a)^(1/2)*d-4*g^3/e*x/c/(c*x^2+b*x+a)^(1/2)*f-1/2*g^4/e^2*
b/c^2/(c*x^2+b*x+a)^(1/2)*d+2*g^3/e*b/c^2/(c*x^2+b*x+a)^(1/2)*f-4/e^2/(a*e^
2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*g^3*f*d^3-e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((
b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x
+d/e))*f^4+1/e^3/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*g^4*d^4-4/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e
-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f^3*g+g^4/e*x^2/c/(c*x^2
+b*x+a)^(1/2)-3/4*g^4/e*b^2/c^3/(c*x^2+b*x+a)^(1/2)-3/2*g^4/e*b/c^(5/2)*ln(
(c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*g^4/e/c^2*a/(c*x^2+b*x+a)^(1/2)-
g^4/e^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d+4*g^3/e/c^(3/
2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f-g^4/e^3/c/(c*x^2+b*x+a)^(1
/2)*d^2-6*g^2/e/c/(c*x^2+b*x+a)^(1/2)*f^2+12/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c
-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c
*d^3*f^2*g^2-2/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)
*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*g^4*d^4-16/e^3/(a*e^2-b*d*e
+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*x*c^2*d^4*g^3*f+8/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d*f^3*g-8/e/(a*e^
2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*b*c*d^2*f^3*g+24/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+
d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^3*f^2
*g^2-16/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2*f^3*g-8/e^3/(a*e^2-b*d*e+c*d^2)/
(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*b*c*d^4*g^3*f-6/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*
d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^2*f^2*g^2+2/e^4/(a*e^2-b*
d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*b*c*d^5*g^4-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+
(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f^4+4/e^4/(a*e^2
-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*x*c^2*d^5*g^4+4/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/
e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*g^3*f*d^3+4
*g^4/e/c*a*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-g^4/e^2*b^2/c/(4*a*c-b^2)/(c
*x^2+b*x+a)^(1/2)*x*d+4*g^3/e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f+8*g
^3/e^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f+4*g^3/e^2*b^2/c/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*d*f-24*g^2/e^2*d*f^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*
x+16*g^3/e^3*d^2*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+8/e^2/(a*e^2-b*d*e+c
*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*x*b*c*g^3*f*d^3-12/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+
(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d^2*f^2*g^2+8*g^
3/e^3*d^2*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-12*g^2/e^2*d*f^2/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*b-3/2*g^4/e*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+2
*g^4/e/c^2*a*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+16*g/e*f^3/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)*c*x-4*g^4/e^4*d^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x-g^4/
e^3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d^2-6*g^2/e*b^2/c/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)*f^2+2*g^3/e*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f-2*g^
4/e^3*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d^2-12*g^2/e*b/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*f^2-1/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e
-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*g^4*d^4+4/e^2/(a*e^2-b
*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*
e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*
c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^3*f*d^3-6/e/(a*e^
2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*
(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e
-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d^2*f^2*g^2-1/2*
g^4/e^2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d+4/(a*e^2-b*d*e+c*d^2)/(4*

```
a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
b^2*d*f^3*g+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d
/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d*f^4+4/(a*e^2-b*d*e+c*d^2)/(4*a*c
-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c
^2*d*f^4
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^4}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + g x\right)^{4}}{\left(d + e x\right) \left(a + b x + c x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)**4/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.622 $\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=357

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\left(ae^2\right.}$$

Rubi [A] time = 0.53, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1646, 843, 621, 206, 724}

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\left(a^2eg^3+3acfg(dg+ef)+c^2df^3\right)+b^2\left(adg^3+cef^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{g^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}e}+\frac{(ef-dg)^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*(c*e*f^3 + a*d*g^3) - 2*a*c*(c*f^2*(e*f - 3*d*g) - a*g^2*(3*e*f - d
*g)) - b*(c^2*d*f^3 + a^2*e*g^3 + 3*a*c*f*g*(e*f + d*g)) - (2*c^3*d*f^3 - b
^2*(b*d - a*e)*g^3 + c*g^2*(3*b^2*d*f - 3*a*b*e*f + 3*a*b*d*g - 2*a^2*e*g)
+ c^2*f*(6*a*g*(e*f - d*g) - b*f*(e*f + 3*d*g)))*x))/(c*(b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^3*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*e) + ((e*f - d*g)^3*ArcTanh[(b*d - 2
*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2
])])/(e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
```

```
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2e\right.\right.}{}$$

$$= \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2e\right.\right.}{}$$

$$= \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2e\right.\right.}{}$$

$$= \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2e\right.\right.}{}$$

Mathematica [A] time = 1.04, size = 373, normalized size = 1.04

$$\frac{2\left(b\left(a^2eg^3+3acg(dg(f+gx)+ef(f-gx))+c^2f^2(d(f-3gx)-efx)\right)+2c\left(a^2g^2(dg-e(3f+gx))+acf(ef(f+3gx)-3dg(f+gx))+c^2df^3x\right)+b^2\left(ag^3(ex-d)+c\left(3dfg^2x-ef^3\right)\right)-b^3dg^3x\right)}{c\left(4ac-b^2\right)\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)}+\frac{g^3\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}e}+\frac{(ef-dg)^3\log(d+ex)}{e\left(e(ae-bd)+cd^2\right)^{3/2}}+\frac{(dg-ef)^3\log\left(2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}+2ae-bd+bex-2cdx\right)}{e\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-(b^3*d*g^3*x) + b^2*(a*g^3*(-d + e*x) + c*(-(e*f^3) + 3*d*f*g^2*x)) +
b*(a^2*e*g^3 + c^2*f^2*(-(e*f*x) + d*(f - 3*g*x)) + 3*a*c*g*(e*f*(f - g*x)
+ d*g*(f + g*x))) + 2*c*(c^2*d*f^3*x + a^2*g^2*(d*g - e*(3*f + g*x)) + a*c*
f*(-3*d*g*(f + g*x) + e*f*(f + 3*g*x)))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b
*d) + a*e))*Sqrt[a + x*(b + c*x)]) + ((e*f - d*g)^3*Log[d + e*x])/(e*(c*d^2
 + e*(-(b*d) + a*e))^(3/2)) + (g^3*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)]])/(c^(3/2)*e) + ((-(e*f) + d*g)^3*Log[-(b*d) + 2*a*e - 2*c*d*x + b*
e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)]])/(e*(c*d^2 +
e*(-(b*d) + a*e))^(3/2))
```

IntegrateAlgebraic [A] time = 6.47, size = 437, normalized size = 1.22

$$-\frac{2\left(-a^2beg^3-2a^2cdg^3+6a^2cefg^2+2a^2ceg^3x+ab^2dg^3-ab^2eg^3x-3abcdfg^2-3abcdg^3x-3abcef^2g+3abcefg^2x+6ac^2df^2g+6ac^2dfg^2x-2ac^2ef^3-6ac^2ef^2gx+b^3dg^3x-3b^2cdfg^2x+b^2cef^3-bc^2df^3+3bc^2df^2gx+bc^2ef^3x-2c^3df^3x\right)}{c\left(4ac-b^2\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{g^3\log\left(-2c^{3/2}e\sqrt{a+bx+cx^2}+bce+2c^2ex\right)}{c^{3/2}e}-\frac{2\left(d^3g^3-3d^2efg^2+3de^2f^2g-e^3f^3\right)\tan^{-1}\left(\frac{-e\sqrt{a+bx+cx^2}+\sqrt{c}d+\sqrt{c}ex}{\sqrt{-ae^2+bde-cd^2}}\right)}{e\left(-ae^2+bde-cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(-(b*c^2*d*f^3) + b^2*c*e*f^3 - 2*a*c^2*e*f^3 + 6*a*c^2*d*f^2*g - 3*a*b
*c*e*f^2*g - 3*a*b*c*d*f*g^2 + 6*a^2*c*e*f*g^2 + a*b^2*d*g^3 - 2*a^2*c*d*g^
3 - a^2*b*e*g^3 - 2*c^3*d*f^3*x + b*c^2*e*f^3*x + 3*b*c^2*d*f^2*g*x - 6*a*c
^2*e*f^2*g*x - 3*b^2*c*d*f*g^2*x + 6*a*c^2*d*f*g^2*x + 3*a*b*c*e*f*g^2*x +
```

```
b^3*d*g^3*x - 3*a*b*c*d*g^3*x - a*b^2*e*g^3*x + 2*a^2*c*e*g^3*x))/(c*(-b^2
+ 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) - (2*(-(e^3*f^3) +
3*d*e^2*f^2*g - 3*d^2*e*f*g^2 + d^3*g^3)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x -
e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(e*(-(c*d^2) + b*
d*e - a*e^2)^(3/2)) - (g^3*Log[b*c*e + 2*c^2*e*x - 2*c^(3/2)*e*Sqrt[a + b*x
 + c*x^2]])/(c^(3/2)*e)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 3127, normalized size = 8.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*f^3+g^3/e/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+
6/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^3*f*g^2+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2
)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d*f
^3-6*g^2/e^2*d*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+12*g/e*f^2/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2)*c*x+1/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*g^3*d^3+3/(a*e^2-b*
d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*b^2*d*f^2*g-3/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+
(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^2*f*g^2-2/e^3/(a
*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*b*c*d^4*g^3-12*g^2/e^2*d*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)*c*x-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/
e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f^3-4/e^3/(a*e^2-b*d*e+c*d^2)/(4*
a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
x*c^2*d^4*g^3+2/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d
)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*g^3*d^3-6/e/(a*e^2-b*d*e+c
*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*x*b*c*d^2*f*g^2-6/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(
b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^2*f^2*g-3*g^2/e*b
^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f+4*g^3/e^3*d^2/(4*a*c-b^2)/(c*x^2+b*x
+a)^(1/2)*c*x+1/2*g^3/e*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+2*g^3/e^3*d
^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+6*g/e*f^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
/2)*b+3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)
*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+
```

```
d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d*f
^2*g-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f^3+1/e^2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*g^3*d^3+g^3/e^2/c/(c*x^2+b*x+a)^(1/2)*d-3*
g^2/e/c/(c*x^2+b*x+a)^(1/2)*f-1/e^2/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2
*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*g^3*d^3+g^3/e*b^2/c/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+2*g^3/e^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d-6
*g^2/e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f+g^3/e^2*b^2/c/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)*d+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d
)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d*f^3-3/e/(a*e^2-b*d*e+c*d
^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*
e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d^2*f*g^2+3/e/(a*e^2-b*d*e+
c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^
2*f*g^2-3/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*d*f^2*g-e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x+d/e))*f^3-g^3/e*x/c/(c*x^2+b*x+a)^(1/2)+1/2*g^3/e*b/c^2/(c*x^
2+b*x+a)^(1/2)+6/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(
x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d*f^2*g+12/e^2/(a*e^2-b*d*e+c
*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*x*c^2*d^3*f*g^2-12/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+
(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2*f^2*g
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^3}{\left(d+e\,x\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx\right)^{3}}{\left(d + ex\right) \left(a + bx + cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)**3/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.623 $\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=240

$$\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 0.31, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1646, 12, 724, 206}

$$\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-2dg)\right)+b^2ef^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*e*f^2 + 2*a*(a*e*g^2 - c*f*(e*f - 2*d*g)) - b*(c*d*f^2 + a*g*(2*e*f
 + d*g)) - (2*c^2*d*f^2 + b*(b*d - a*e)*g^2 + c*(2*a*g*(2*e*f - d*g) - b*f*
(e*f + 2*d*g)))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c
*x^2]) + ((e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d
^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+}}$$

Mathematica [A] time = 0.63, size = 265, normalized size = 1.10

$$\frac{2\left(-2a^2eg^2+abg(dg+2ef-egx)-2acdg(2f+gx)+2acef(f+2gx)+b^2\left(dg^2x-ef^2\right)+bcf(d(f-2gx)-efx)+2c^2df^2x\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)}+\frac{(ef-dg)^2\log(d+ex)}{\left(e(ae-bd)+cd^2\right)^{3/2}}-\frac{(ef-dg)^2\log\left(2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}+2ae-bd+bex-2cdx\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-2*a^2*e*g^2 + 2*c^2*d*f^2*x - 2*a*c*d*g*(2*f + g*x) + 2*a*c*e*f*(f + 2
*g*x) + a*b*g*(2*e*f + d*g - e*g*x) + b^2*(-(e*f^2) + d*g^2*x) + b*c*f*(-(e
*f*x) + d*(f - 2*g*x))))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a +
 x*(b + c*x)]) + ((e*f - d*g)^2*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^(3
/2) - ((e*f - d*g)^2*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 +
e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2)
```

IntegrateAlgebraic [A] time = 1.36, size = 323, normalized size = 1.35

$$\frac{2\left(-2a^2eg^2+abdg^2+2abefg-abeg^2x-4acdfg-2acdg^2x+2acef^2+4acefgx+b^2dg^2x-b^2ef^2+bcdf^2-2bcdfgx-bcef^2x+2c^2df^2x\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-ae^2+bde-cd^2\right)}+\frac{2\left(e^2f^2\sqrt{-ae^2+bde-cd^2}-2defg\sqrt{-ae^2+bde-cd^2}+d^2g^2\sqrt{-ae^2+bde-cd^2}\right)\tan^{-1}\left(\frac{-e\sqrt{a+bx+cx^2}+\sqrt{c}d+\sqrt{c}ex}{\sqrt{-ae^2+bde-cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b*c*d*f^2 - b^2*e*f^2 + 2*a*c*e*f^2 - 4*a*c*d*f*g + 2*a*b*e*f*g + a*b*d
*g^2 - 2*a^2*e*g^2 + 2*c^2*d*f^2*x - b*c*e*f^2*x - 2*b*c*d*f*g*x + 4*a*c*e*
f*g*x + b^2*d*g^2*x - 2*a*c*d*g^2*x - a*b*e*g^2*x))/((b^2 - 4*a*c)*(-(c*d^2
) + b*d*e - a*e^2)*Sqrt[a + b*x + c*x^2]) + (2*(e^2*Sqrt[-(c*d^2) + b*d*e -
 a*e^2]*f^2 - 2*d*e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*f*g + d^2*Sqrt[-(c*d^2)
+ b*d*e - a*e^2]*g^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x - e*Sqrt[a + b*x + c*
x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(c*d^2 - b*d*e + a*e^2)^2
```

fricas [B] time = 7.79, size = 2023, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 - 4
*a^2*c)*d^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*g
+ (b^2*c - 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4*a*
b*c)*d*e*f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log(
(8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2
```

```
+ 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*
d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d
*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 4*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e
 + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f^2 - 2*(2*a*c^2*d^3 - 3*a*
b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*f*g + (a*b*c*d^3 + 3*a^2*b
*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e)*g^2 + ((2*c^3*d^3 - 3*b*c^2*d
^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f^2 - 2*(b*c^2*d^3 + 3*a*b*c*d*
e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*f*g - (a^2*b*e^3 - (b^2*c - 2*
a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*g^2)*x)*sqrt(c*
x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^
3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d
*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*
a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^
3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^
2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x), (((a
*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 - 4*a^2*c)*d
^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*g + (b^2*c
- 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4*a*b*c)*d*e*
f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d -
b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 +
(b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e
 + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f^2 - 2*(2*a*c^2*d^3 - 3*a*
b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*f*g + (a*b*c*d^3 + 3*a^2*b
*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e)*g^2 + ((2*c^3*d^3 - 3*b*c^2*d
^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f^2 - 2*(b*c^2*d^3 + 3*a*b*c*d*
e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*f*g - (a^2*b*e^3 - (b^2*c - 2*
a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*g^2)*x)*sqrt(c*
x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^
3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d
*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*
a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^
3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^
2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)]
```

giac [B] time = 0.35, size = 757, normalized size = 3.15

$$-\frac{2\left(\frac{\left(2c^3d^3f^2-2bc^2d^3fg+b^2cd^3g^2-2ac^2d^3g^2-3bc^2d^2f^2e+2b^2cd^2fge+4ac^2d^2fge-b^3d^2g^2e+abcd^2g^2e+b^2cdf^2e^2+2ac^2df^2e^2-6abcdfge^2+2ab^2dg^2e^2-2a^2cdg^2e^2-abcf^2e^3+4a^2cfge^3-a^2bg^2e^3\right)x}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3f^2-4ac^2d^3fg+abcd^3g^2-2b^2cd^2f^2e+2ac^2d^2f^2e+6abcd^2fge-ab^2d^2g^2e-2a^2cd^2g^2e+b^3df^2e^2-abcdf^2e^2-2ab^2dfge^2-4a^2cdfge^2+3a^2bdg^2e^2-ab^2f^2e^3+2a^2cf^2e^3+2a^2bfge^3-2a^3g^2e^3}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}\right)}{\sqrt{cx^2+bx+a}}+\frac{2\left(d^2g^2-2dfge+f^2e^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3*f^2 - 2*b*c^2*d^3*f*g + b^2*c*d^3*g^2 - 2*a*c^2*d^3*g^2 - 3*
b*c^2*d^2*f^2*e + 2*b^2*c*d^2*f*g*e + 4*a*c^2*d^2*f*g*e - b^3*d^2*g^2*e + a
*b*c*d^2*g^2*e + b^2*c*d*f^2*e^2 + 2*a*c^2*d*f^2*e^2 - 6*a*b*c*d*f*g*e^2 +
2*a*b^2*d*g^2*e^2 - 2*a^2*c*d*g^2*e^2 - a*b*c*f^2*e^3 + 4*a^2*c*f*g*e^3 - a
^2*b*g^2*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*
e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8
*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f^2 - 4*a*c^2*d^3*
f*g + a*b*c*d^3*g^2 - 2*b^2*c*d^2*f^2*e + 2*a*c^2*d^2*f^2*e + 6*a*b*c*d^2*f
*g*e - a*b^2*d^2*g^2*e - 2*a^2*c*d^2*g^2*e + b^3*d*f^2*e^2 - a*b*c*d*f^2*e^
2 - 2*a*b^2*d*f*g*e^2 - 4*a^2*c*d*f*g*e^2 + 3*a^2*b*d*g^2*e^2 - a*b^2*f^2*e
^3 + 2*a^2*c*f^2*e^3 + 2*a^2*b*f*g*e^3 - 2*a^3*g^2*e^3)/(b^2*c^2*d^4 - 4*a*
c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2
 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^
3*c*e^4))/sqrt(c*x^2 + b*x + a) + 2*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*arctan(
-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e -
a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
```

maple [B] time = 0.02, size = 2123, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/
e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2
*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d*f*g-1/e
/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e
)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*
c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d^2*g^2-e/
(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*b^2*f^2+e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*
d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*f^2-g^2/e/c/(c*x^2+b*x+a)^(1/2)
+1/e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*d^2*g^2+4/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+
(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^3*g^2+2/e^2/(a
*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*b*c*d^3*g^2-2/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*
c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f*g-e/(a*e^2-b*d*e+c*d^2)/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d
^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/
e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*f^2-4/e/(a*e^2-b*d*e+c*d^2)/(4*a
*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b
*c*d^2*f*g-2/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+
d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d^2*g^2+4/(a*e^2-b*d*e+c*d^2)/(
4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*x*b*c*d*f*g+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d*f*g+2/(a*e^2-b*d*e+c*d^2)/(4*a
*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b
*c*d*f^2-4*g^2/e^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+8*g/e*f/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)*c*x+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b
*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d*f^2-1/e/(a*e^2-b
*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*b^2*d^2*g^2-2*g^2/e^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+4*
g/e*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-2*g^2/e*b/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)*x-g^2/e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-8/e/(a*e^2-b*d*e+c*d^2
)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*x*c^2*d^2*f*g-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*
c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^2}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral((f + g*x)**2/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.624 $\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{e(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 0.13, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {822, 12, 724, 206}

$$\frac{e(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e)f+bcdf\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d*f - b^2*e*f + 2*a*c*e*f - 2*a*c*d*g + a*b*e*g + c*(2*c*d*f + 2*a
*e*g - b*(e*f + d*g))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b
*x + c*x^2]) + (e*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^
(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \cdots$$

$$= -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \cdots$$

$$= -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \cdots$$

$$= -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \cdots$$

Mathematica [A] time = 0.16, size = 183, normalized size = 0.98

$$\frac{2b(aeg+cd(f-gx)-cefx)+4c(-adg+ae(f+gx)+cdfx)-2b^2ef}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)}+\frac{e(dg-ef)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*b^2*e*f + 2*b*(a*e*g - c*e*f*x + c*d*(f - g*x)) + 4*c*(-(a*d*g) + c*d*f
*x + a*e*(f + g*x)))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + x*(
b + c*x)]) + (e*(-(e*f) + d*g)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(
2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d
) + a*e))^(3/2)
```

IntegrateAlgebraic [A] time = 0.99, size = 233, normalized size = 1.25

$$-\frac{2\left(-abeg+2acdg-2acef-2acegx+b^2ef-bcdf+bcdgx+bcefx-2c^2dfx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-ae^2+bde-cd^2\right)}-\frac{2\left(deg\sqrt{-ae^2+bde-cd^2}-e^2f\sqrt{-ae^2+bde-cd^2}\right)\tan^{-1}\left(\frac{-e\sqrt{a+bx+cx^2}+\sqrt{c}d+\sqrt{c}ex}{\sqrt{-ae^2+bde-cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(-(b*c*d*f) + b^2*e*f - 2*a*c*e*f + 2*a*c*d*g - a*b*e*g - 2*c^2*d*f*x +
 b*c*e*f*x + b*c*d*g*x - 2*a*c*e*g*x))/((b^2 - 4*a*c)*(-(c*d^2) + b*d*e - a
*e^2)*Sqrt[a + b*x + c*x^2]) - (2*(-(e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*f)
+ d*e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*g)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x - e
*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(c*d^2 - b*d*e + a
*e^2)^2
```

fricas [B] time = 5.00, size = 1663, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g + ((b^2*c - 4*a*c
^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c)*e^2*f - (b^3 -
4*a*b*c)*d*e*g)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 -
 (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sq
```

```
rt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e
)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*
x + d^2)) + 4*sqrt(c*x^2 + b*x + a)*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e +
 (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f - (2*a*c^2*d^3 - 3*a*b*c*d^
2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*g + ((2*c^3*d^3 - 3*b*c^2*d^2*e
- a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f - (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a
^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*g)*x))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2
*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
- 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a
*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^
3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)
*x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
 - 4*a^3*b*c)*e^4)*x), (((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g
+ ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c
)*e^2*f - (b^3 - 4*a*b*c)*d*e*g)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/
2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d
- b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2
+ (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*sqrt(c*x^2 + b*x + a)*((b*c^2*d^3 -
 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f -
 (2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*g + ((
2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f - (b*c^2
*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*g)*x))/((a*b^
2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b
^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^
4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*
c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a
^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*
a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a
^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)]
```

giac [B] time = 0.32, size = 568, normalized size = 3.04

$$-\frac{2\left(\frac{\left(2c^3d^3f-bc^2d^3g-3bc^2d^2fe+b^2cd^2ge+2ac^2d^2ge+b^2cdfe^2+2ac^2dfe^2-3abcdge^2-abcfe^3+2a^2cge^3\right)x}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3f-2ac^2d^3g-2b^2cd^2fe+2ac^2d^2fe+3abcd^2ge+b^3dfe^2-abcdfe^2-ab^2dge^2-2a^2cdge^2-ab^2fe^3+2a^2cfe^3+a^2bge^3}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}\right)}{\sqrt{cx^2+bx+a}}-\frac{2\left(dge-fe^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3*f - b*c^2*d^3*g - 3*b*c^2*d^2*f*e + b^2*c*d^2*g*e + 2*a*c^2*
d^2*g*e + b^2*c*d*f*e^2 + 2*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 - a*b*c*f*e^3 +
 2*a^2*c*g*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^
3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 +
 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f - 2*a*c^2*d^3*
g - 2*b^2*c*d^2*f*e + 2*a*c^2*d^2*f*e + 3*a*b*c*d^2*g*e + b^3*d*f*e^2 - a*b
*c*d*f*e^2 - a*b^2*d*g*e^2 - 2*a^2*c*d*g*e^2 - a*b^2*f*e^3 + 2*a^2*c*f*e^3
+ a^2*b*g*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e
 + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*
a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^2 + b*x + a) - 2*(d*g*
e - f*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt
(-c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*
e^2))
```

maple [B] time = 0.01, size = 1261, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2/e*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/(a*e^2-b*d*e+c*d^2)/((x+d
/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*g+e/(a*e^2-b
*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*f+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d*g-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)
/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f-
4/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2*g+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d*f+1
/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*b^2*d*g-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^
2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f-2/e/(a*e^2-b
*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*b*c*d^2*g+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*
e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d*f+1/(a*e^2-b*d*e+c*
d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d
*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x
+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*d*g-e/(a*e^2-b*d*e+c*d^2)/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*
d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)
/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `assum
e?` for more details)Is (b/e-(2*c*d)/e^2)^2    -(4*c       *((-(b*d)/e)
    +(c*d^2)/e^2+a))     /e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f+g\,x}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + g x}{\left(d + e x\right) \left(a + b x + c x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.625 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=155

$$\frac{e^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Rubi [A] time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {740, 12, 724, 206}

$$\frac{e^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 -
b*d*e + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int -\frac{\left(b^2-4ac\right)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2} \cdots\right.}{cd^2-bde+\cdots}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+\cdots}}\right.}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.26, size = 162, normalized size = 1.05

$$\frac{\frac{e^2\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}} + \frac{4c(ae+cdx)-2b^2e+2bc(d-ex)}{\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)}}{4ac-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*b^2*e + 4*c*(a*e + c*d*x) + 2*b*c*(d - e*x))/((c*d^2 + e*(-(b*d) + a*e
))*Sqrt[a + x*(b + c*x)]) + ((b^2 - 4*a*c)*e^2*ArcTanh[(-(b*d) + 2*a*e - 2*
c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(
c*d^2 + e*(-(b*d) + a*e))^(3/2))/(-b^2 + 4*a*c)
```

IntegrateAlgebraic [A] time = 0.00, size = 215, normalized size = 1.39

$$\frac{2e^2\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+bx+cx^2}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,ex}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}\,d}{\sqrt{-ae^2+bde-cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^2} - \frac{2\left(-2ace+b^2e-bcd+bcex-2c^2dx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-ae^2+bde-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(-(b*c*d) + b^2*e - 2*a*c*e - 2*c^2*d*x + b*c*e*x))/((b^2 - 4*a*c)*(-(c
*d^2) + b*d*e - a*e^2)*Sqrt[a + b*x + c*x^2]) + (2*e^2*Sqrt[-(c*d^2) + b*d*
e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x
)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2)
+ b*d*e - a*e^2]])/(c*d^2 - b*d*e + a*e^2)^2
```

fricas [B] time = 0.91, size = 1349, normalized size = 8.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a^2*c
)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*
c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b
*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*
```

```
b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) -
4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a
^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^
2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 -
 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*
(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2
*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c
^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*
a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*
e^4)*x), (((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a
^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a
*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*
d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*
b*e^2)*x)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 -
 (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c +
2*a*c^2)*d*e^2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*
(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 -
 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*
c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3
)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*
x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2
*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
- 4*a^3*b*c)*e^4)*x)]
```

giac [B] time = 0.32, size = 447, normalized size = 2.88

$$-\frac{2\left(\frac{\left(2c^3d^3-3bc^2d^2e+b^2cde^2+2ac^2de^2-abce^3\right)x}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3-2b^2cd^2e+2ac^2d^2e+b^3de^2-abcde^2-ab^2e^3+2a^2ce^3}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}\right)}{\sqrt{cx^2+bx+a}}+\frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)e^2}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x
/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2
 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3
+ a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e +
 b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*
d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8
*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*
e^4))/sqrt(c*x^2 + b*x + a) + 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^2/((c*d^2 - b*d*e + a*e^2)
*sqrt(-c*d^2 + b*d*e - a*e^2))
```

maple [B] time = 0.01, size = 603, normalized size = 3.89

$$-\frac{2bcex}{\left(ae^2-bde+cd^2\right)\left(4ac-b^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}+\frac{4c^2dx}{\left(ae^2-bde+cd^2\right)\left(4ac-b^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}-\frac{b^2e}{\left(ae^2-bde+cd^2\right)\left(4ac-b^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}+\frac{2bcd}{\left(ae^2-bde+cd^2\right)\left(4ac-b^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}-\frac{e\ln\left(\frac{\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{2ae^2-2bde+2cd^2}{e^2}+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}{x+\frac{d}{e}}\right)}{\left(ae^2-bde+cd^2\right)\sqrt{\frac{ae^2-bde+cd^2}{e^2}}}+\frac{e}{\left(ae^2-bde+cd^2\right)\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(
x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-
b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^
2*d-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)
^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d-e/(a*e^2-b*
d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e
```

```
^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c
*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                               +c*d^2    positive,
negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.626 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=352

$$-\frac{2e\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ef-dg)\left(ae^2-bde+cd^2\right)}+\frac{2g\left(2acg+b^2(-g)+cx(2cf-bg)+bcf\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

Rubi [A] time = 0.44, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {960, 740, 12, 724, 206}

$$-\frac{2e\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ef-dg)\left(ae^2-bde+cd^2\right)}+\frac{2g\left(2acg+b^2(-g)+cx(2cf-bg)+bcf\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(ef-dg)\left(ag^2-bfg+cf^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{g^3\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g +
 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)*Sqrt[a + b*x + c*x^2]) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x
)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e +
 a*e^2)^(3/2)*(e*f - d*g)) - (g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(
2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \left(\frac{e}{(ef-dg)(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)\left(a+bx+\right.}\right.$$

$$= \frac{e\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx}{ef-dg}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(b\right.}{\left(b^2-4ac\right)(e}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(b\right.}{\left(b^2-4ac\right)(e}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(b\right.}{\left(b^2-4ac\right)(e}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(b\right.}{\left(b^2-4ac\right)(e}$$

Mathematica [A]   time = 1.22, size = 317, normalized size = 0.90

$$\frac{-\frac{2e\left(-2c(ae+cdx)+b^2e+bc(ex-d)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)}+\frac{2g\left(-2c(ag+cfx)+b^2g+bc(gx-f)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(g(bf-ag)-cf^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}-\frac{g^3\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{3/2}}}{ef-dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*e*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d^
2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (2*g*(b^2*g - 2*c*(a*g + c*f*x
) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(
b + c*x)]) + (e^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 +
e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e))^(3/2)
 - (g^3*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f)
+ a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2))/(e*f - d
*g)
```

IntegrateAlgebraic [A]   time = 5.38, size = 463, normalized size = 1.32

$$-\frac{2\left(-3abceg+2ac^2dg+2ac^2ef-2ac^2egx+b^3eg-b^2cdg-b^2cef+b^2cegx+bc^2df-bc^2dgx-bc^2efx+2c^3dfx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(-ae^2+bde-cd^2\right)\left(-ag^2+bfg-cf^2\right)}-\frac{2e^3\sqrt{-ae^2+bde-cd^2}\tan^{-1}\left(-\frac{e\sqrt{a+bx+cx^2}}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}ex}{\sqrt{-ae^2+bde-cd^2}}+\frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{(dg-ef)\left(ae^2-bde+cd^2\right)^2}-\frac{2g^3\sqrt{-ag^2+bfg-cf^2}\tan^{-1}\left(-\frac{g\sqrt{a+bx+cx^2}}{\sqrt{-ag^2+bfg-cf^2}}+\frac{\sqrt{c}gx}{\sqrt{-ag^2+bfg-cf^2}}+\frac{\sqrt{c}f}{\sqrt{-ag^2+bfg-cf^2}}\right)}{(ef-dg)\left(ag^2-bfg+cf^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c^2*d*f - b^2*c*e*f + 2*a*c^2*e*f - b^2*c*d*g + 2*a*c^2*d*g + b^3*e*
g - 3*a*b*c*e*g + 2*c^3*d*f*x - b*c^2*e*f*x - b*c^2*d*g*x + b^2*c*e*g*x - 2
*a*c^2*e*g*x))/((b^2 - 4*a*c)*(-(c*d^2) + b*d*e - a*e^2)*(-(c*f^2) + b*f*g
- a*g^2)*Sqrt[a + b*x + c*x^2]) - (2*e^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Arc
Tan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x)/Sqrt[-(c*d^2
) + b*d*e - a*e^2] - (e*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*d^2) + b*d*e - a*e^
2]])/((c*d^2 - b*d*e + a*e^2)^2*(-(e*f) + d*g)) - (2*g^3*Sqrt[-(c*f^2) + b*
f*g - a*g^2]*ArcTan[(Sqrt[c]*f)/Sqrt[-(c*f^2) + b*f*g - a*g^2] + (Sqrt[c]*g
*x)/Sqrt[-(c*f^2) + b*f*g - a*g^2] - (g*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*f^2
) + b*f*g - a*g^2]])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 1343, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)-2/(d*g-e*f)*g^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/
((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*b*c+4/(
d*g-e*f)*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)
/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c^2*f-1/(d*g-e*f)*g^2/(a*g^2-b*f*g+c*f^
2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^
(1/2)*b^2+2/(d*g-e*f)*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2
*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b*c*f-1/(d*g-e*f)/(a*g^2-b*f
*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(
a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-
2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))-1/(d*g-e*f)/(a*e^
2-b*d*e+c*d^2)*e^2/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)+2/(d*g-e*f)*e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e
-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c-4/(d*g-e*f)*e/(a*e^2
-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*x*c^2*d+1/(d*g-e*f)*e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2-2/(d*g-
e*f)*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d+1/(d*g-e*f)/(a*e^2-b*d*e+c*d^2)*e^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2
```

```
)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(ex+d)\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f+g\,x\right)\,\left(d+e\,x\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(f+gx\right)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.627 $\int \frac{1}{(d+ex)(f+gx)^2\left(a+bx+cx^2\right)^{3/2}} dx$

Optimal. Leaf size=642

$$\frac{g^2\sqrt{a+bx+cx^2}\left(-4cg(2ag+bf)+3b^2g^2+4c^2f^2\right)}{\left(b^2-4ac\right)(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}-\frac{2e^2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(ef-dg)^2\left(ae^2-bde+cd^2\right)}+\cdots$$

Rubi [A] time = 0.91, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {960, 740, 12, 724, 206, 806}

$$\frac{g^2\sqrt{a+bx+cx^2}\left(-4cg(2ag+bf)+3b^2g^2+4c^2f^2\right)}{\left(b^2-4ac\right)(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}-\frac{2e^2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(ef-dg)^2\left(ae^2-bde+cd^2\right)}+\frac{2eg\left(2acg+b^2(-g)+cx(2cf-bg)+bcf\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(ef-dg)^2\left(ag^2-bfg+cf^2\right)}+\frac{2g\left(2acg+b^2(-g)+cx(2cf-bg)+bcf\right)}{\left(b^2-4ac\right)(f+gx)\sqrt{a+bx+cx^2}(ef-dg)\left(ag^2-bfg+cf^2\right)}+\frac{e^4\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{eg^3\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\left(ag^2-bfg+cf^2\right)^{3/2}}-\frac{3g^3(2cf-bg)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b
^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 -
b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c*(
2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*
x)*Sqrt[a + b*x + c*x^2]) + (g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*
g))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)) + (e^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^(3/2)*
(e*f - d*g)^2) - (3*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*
x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(
c*f^2 - b*f*g + a*g^2)^(5/2)) - (e*g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
```

```
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \left(\frac{e^2}{(ef-dg)^2(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)^2\left(a+bx\right.}\right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{}{(f+gx)^2(a+}}{ef-}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{2eg}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{2eg}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{2eg}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{2eg}{\left(b^2-4ac\right)}$$

Mathematica [A] time = 5.08, size = 623, normalized size = 0.97

$$\frac{g^2\left(\frac{3g(bg-2cf)\tanh^{-1}\left(\frac{2ag-bf+bgx-2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{5/2}}-\frac{2\sqrt{a+x(b+cx)}\left(-4cg(2ag+bf)+3b^2g^2+4c^2f^2\right)}{\left(b^2-4ac\right)(f+gx)\left(g(ag-bf)+cf^2\right)^2}\right)}{2(dg-ef)}-\frac{2e^2\left(-2c(ae+cdx)+b^2e+bc(ex-d)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}(ef-dg)^2\left(e(bd-ae)-cd^2\right)}+\frac{2eg\left(-2c(ag+cfx)+b^2g+bc(gx-f)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}(ef-dg)^2\left(g(bf-ag)-cf^2\right)}-\frac{2g\left(-2c(ag+cfx)+b^2g+bc(gx-f)\right)}{\left(b^2-4ac\right)(f+gx)\sqrt{a+x(b+cx)}(dg-ef)\left(g(bf-ag)-cf^2\right)}+\frac{e^4\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{(ef-dg)^2\left(e(ae-bd)+cd^2\right)^{3/2}}-\frac{eg^3\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{(ef-dg)^2\left(g(ag-bf)+cf^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^2*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d
^2) + e*(b*d - a*e))*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g -
```

```
 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*f^2
) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g + c*f*x)
 + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(b*f - a*g)
)*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (g^2*((-2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c
*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 + g*(-(b*f)
+ a*g))^2*(f + g*x)) + (3*g*(-2*c*f + b*g)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*
x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^
2 + g*(-(b*f) + a*g))^(5/2)))/(2*(-(e*f) + d*g)) + (e^4*ArcTanh[(-2*a*e + 2
*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x
)])])/((c*d^2 + e*(-(b*d) + a*e))^(3/2)*(e*f - d*g)^2) - (e*g^3*ArcTanh[(-2
*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*
(b + c*x)])])/((e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g))^(3/2))
```

IntegrateAlgebraic [F] time = 180.07, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.03, size = 2807, normalized size = 4.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x)
[Out] 1/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/(d*g-e*f)^2*e/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^
2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2-b*f*g+c*f^2)/g
^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*
g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))-1/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)
/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*b^2+3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b
*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b^3+3*g^2/(d*g-e*f)/(a*g
^2-b*f*g+c*f^2)^2/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^
2)^(1/2)*c*f-1/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*ln(((b*e-2*c*d)*(x+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
```

```
^2)^(1/2))/(x+d/e))-g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)/((x+f/g)^2*c+(b
*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+3*g^3/(d*g-e*f)/(a*g^2-b
*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c
*f^2)/g^2)^(1/2)*x*b^2*c+12*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/(
(x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c^3*f^2-
6*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*
(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b^2*c*f+6*g/(d*g-e*f)/(a*g^2-b*f*g
+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2)*b*c^2*f^2-2/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((
x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c+4/(d*
g-e*f)^2*e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/
e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d+2/(d*g-e*f)^2*e^2/(a*e^2-b*d*e+
c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)*(x+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*b*c*d-12*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)
^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*b*c^2*f-2/(d*g-
e*f)^2*e*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)
/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b*c*f+2/(d*g-e*f)^2*e*g^2/(a*g^2-b*f*g+c*
f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2)*x*b*c-3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((x+f/g)^2*c+(b*g-2*c
*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b-1/(d*g-e*f)^2*e/(a*g^2-b*f*g
+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/
2)-4/(d*g-e*f)^2*e*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*
f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c^2*f-3*g^2/(d*g-e*f)/(a*g^2-
b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*
(a*g^2-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g
-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f-8*g/(d*g-e*f
)*c^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x-4*g/(d*g-e*f)*c/(a*g^2-b*f*g+c*f^2)/(4*a*c-b
^2)/((x+f/g)^2*c+(b*g-2*c*f)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b+1/(
d*g-e*f)^2*e*g^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)*(
x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b^2+3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+
c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln(((b*g-2*c*f)*(x+f/g)/g+2*(a*g^2
-b*f*g+c*f^2)/g^2+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f
)*(x+f/g)/g+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(ex+d)\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(f+g\,x\right)}^2\,\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex\right)\left(f+gx\right)^{2}\left(a+bx+cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)**2*(a + b*x + c*x**2)**(3/2)), x)
```

3.628 $\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=1064

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)e^5}{\left(cd^2-bed+ae^2\right)^{3/2}(ef-dg)^3}-\frac{2\left(-eb^2+cdb+2ace+c(2cd-be)x\right)e^3}{\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)(ef-dg)^3\sqrt{cx^2+bx+a}}-\frac{g^3\tanh^{-1}\left(\frac{bf-2ag}{2\sqrt{cf^2-bgf+}}\right.}{(ef-dg)^3\left(cf^2-\right.}$$

Rubi [A] time = 1.90, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {960, 740, 12, 724, 206, 834, 806}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^3*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2]) + (2*e^2*g*(b*c*f -
 b^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^3*(c*f^2
- b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c
*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f +
g*x)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b^2*g + 2*a*c*g + c*(2*c*f
- b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)*S
qrt[a + b*x + c*x^2]) + (g^2*(8*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f + 3*a*g)
)*Sqrt[a + b*x + c*x^2])/(2*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)^2) + (e*g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*g))*Sq
rt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^2
*(f + g*x)) + (g^2*(2*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 1
3*a*g))*Sqrt[a + b*x + c*x^2])/(4*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^3*(f + g*x)) + (e^5*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqr
t[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^
(3/2)*(e*f - d*g)^3) - (3*e*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(5/2)) - (e^2*g^3*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e
*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (3*g^3*(16*c^2*f^2 + 5*b^2*g^2
 - 4*c*g*(4*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f
^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \left(\frac{e^3}{(ef-dg)^3(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)^3\left(a+bx\right.}\right.$$

$$= \frac{e^3\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^3} - \frac{\left(e^2g\right)\int \frac{1}{(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{\ }{(f+g}}{(e}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2}{\left(b^2-4ac\right)}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2}{\left(b^2-4ac\right)}$$

Mathematica [A] time = 5.74, size = 1013, normalized size = 0.95

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^3*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d
^2) + e*(b*d - a*e))*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)]) - (2*e^2*g*(b^2*g
 - 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)^3*(-(
c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g + c
*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(b*f -
 a*g))*(f + g*x)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g - 2*c*(a*g + c*f*
x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*f^2) + g*(b*f - a*g
))*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (e*g^2*((2*(4*c^2*f^2 + 3*b^2*g^2 - 4
*c*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 + g*(-(b*f
) + a*g))^2*(f + g*x)) + (3*g*(2*c*f - b*g)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f
*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f
^2 + g*(-(b*f) + a*g))^(5/2)))/(2*(e*f - d*g)^2) - (g^2*((4*(8*c^2*f^2 + 5*
b^2*g^2 - 4*c*g*(2*b*f + 3*a*g))*Sqrt[a + x*(b + c*x)])/(f + g*x)^2 + (2*(2
*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 13*a*g))*Sqrt[a + x*(b
 + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)) + (3*(b^2 - 4*a*c)*g*(16*c
^2*f^2 + 5*b^2*g^2 - 4*c*g*(4*b*f + a*g))*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x
 + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2
 + g*(-(b*f) + a*g))^(3/2)))/(8*(b^2 - 4*a*c)*(-(e*f) + d*g)*(c*f^2 + g*(-(
b*f) + a*g))^2) - (e^5*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/((c*d^2 + e*(-(b*d) + a*e))
^(3/2)*(-(e*f) + d*g)^3) - (e^2*g^3*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x)
```

```
)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/((e*f - d*g)^3
*(c*f^2 + g*(-(b*f) + a*g))^(3/2))
```

IntegrateAlgebraic [F] time = 180.08, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [B] time = 21.91, size = 14731, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
[Out] -2*((2*c^9*d^3*f^9 - 9*b*c^8*d^3*f^8*g + 18*b^2*c^7*d^3*f^7*g^2 - 21*b^3*c^
6*d^3*f^6*g^3 + 15*b^4*c^5*d^3*f^5*g^4 + 6*a*b^2*c^6*d^3*f^5*g^4 - 12*a^2*c
^7*d^3*f^5*g^4 - 6*b^5*c^4*d^3*f^4*g^5 - 15*a*b^3*c^5*d^3*f^4*g^5 + 30*a^2*
b*c^6*d^3*f^4*g^5 + b^6*c^3*d^3*f^3*g^6 + 12*a*b^4*c^4*d^3*f^3*g^6 - 18*a^2
*b^2*c^5*d^3*f^3*g^6 - 16*a^3*c^6*d^3*f^3*g^6 - 3*a*b^5*c^3*d^3*f^2*g^7 - 3
*a^2*b^3*c^4*d^3*f^2*g^7 + 24*a^3*b*c^5*d^3*f^2*g^7 + 3*a^2*b^4*c^3*d^3*f*g
^8 - 6*a^3*b^2*c^4*d^3*f*g^8 - 6*a^4*c^5*d^3*f*g^8 - a^3*b^3*c^3*d^3*g^9 +
3*a^4*b*c^4*d^3*g^9 - 3*b*c^8*d^2*f^9*e + 15*b^2*c^7*d^2*f^8*g*e - 6*a*c^8*
d^2*f^8*g*e - 33*b^3*c^6*d^2*f^7*g^2*e + 24*a*b*c^7*d^2*f^7*g^2*e + 41*b^4*
c^5*d^2*f^6*g^3*e - 34*a*b^2*c^6*d^2*f^6*g^3*e - 16*a^2*c^7*d^2*f^6*g^3*e -
 30*b^5*c^4*d^2*f^5*g^4*e + 9*a*b^3*c^5*d^2*f^5*g^4*e + 66*a^2*b*c^6*d^2*f^
5*g^4*e + 12*b^6*c^3*d^2*f^4*g^5*e + 24*a*b^4*c^4*d^2*f^4*g^5*e - 96*a^2*b^
2*c^5*d^2*f^4*g^5*e - 12*a^3*c^6*d^2*f^4*g^5*e - 2*b^7*c^2*d^2*f^3*g^6*e -
23*a*b^5*c^3*d^2*f^3*g^6*e + 49*a^2*b^3*c^4*d^2*f^3*g^6*e + 48*a^3*b*c^5*d^
2*f^3*g^6*e + 6*a*b^6*c^2*d^2*f^2*g^7*e + 3*a^2*b^4*c^3*d^2*f^2*g^7*e - 54*
a^3*b^2*c^4*d^2*f^2*g^7*e - 6*a^2*b^5*c^2*d^2*f*g^8*e + 15*a^3*b^3*c^3*d^2*
f*g^8*e + 9*a^4*b*c^4*d^2*f*g^8*e + 2*a^3*b^4*c^2*d^2*g^9*e - 7*a^4*b^2*c^3
*d^2*g^9*e + 2*a^5*c^4*d^2*g^9*e + b^2*c^7*d*f^9*e^2 + 2*a*c^8*d*f^9*e^2 -
6*b^3*c^6*d*f^8*g*e^2 - 3*a*b*c^7*d*f^8*g*e^2 + 15*b^4*c^5*d*f^7*g^2*e^2 -
6*a*b^2*c^6*d*f^7*g^2*e^2 - 20*b^5*c^4*d*f^6*g^3*e^2 + 13*a*b^3*c^5*d*f^6*g
^3*e^2 + 16*a^2*b*c^6*d*f^6*g^3*e^2 + 15*b^6*c^3*d*f^5*g^4*e^2 - 48*a^2*b^2
*c^5*d*f^5*g^4*e^2 - 12*a^3*c^6*d*f^5*g^4*e^2 - 6*b^7*c^2*d*f^4*g^5*e^2 - 1
5*a*b^5*c^3*d*f^4*g^5*e^2 + 51*a^2*b^3*c^4*d*f^4*g^5*e^2 + 42*a^3*b*c^5*d*f
^4*g^5*e^2 + b^8*c*d*f^3*g^6*e^2 + 12*a*b^6*c^2*d*f^3*g^6*e^2 - 19*a^2*b^4*
c^3*d*f^3*g^6*e^2 - 50*a^3*b^2*c^4*d*f^3*g^6*e^2 - 16*a^4*c^5*d*f^3*g^6*e^2
 - 3*a*b^7*c*d*f^2*g^7*e^2 - 3*a^2*b^5*c^2*d*f^2*g^7*e^2 + 27*a^3*b^3*c^3*d
*f^2*g^7*e^2 + 24*a^4*b*c^4*d*f^2*g^7*e^2 + 3*a^2*b^6*c*d*f*g^8*e^2 - 6*a^3
*b^4*c^2*d*f*g^8*e^2 - 9*a^4*b^2*c^3*d*f*g^8*e^2 - 6*a^5*c^4*d*f*g^8*e^2 -
a^3*b^5*c*d*g^9*e^2 + 3*a^4*b^3*c^2*d*g^9*e^2 + a^5*b*c^3*d*g^9*e^2 - a*b*c
^7*f^9*e^3 + 6*a*b^2*c^6*f^8*g*e^3 - 6*a^2*c^7*f^8*g*e^3 - 15*a*b^3*c^5*f^7
*g^2*e^3 + 24*a^2*b*c^6*f^7*g^2*e^3 + 20*a*b^4*c^4*f^6*g^3*e^3 - 34*a^2*b^2
*c^5*f^6*g^3*e^3 - 16*a^3*c^6*f^6*g^3*e^3 - 15*a*b^5*c^3*f^5*g^4*e^3 + 15*a
```

^2*b^3*c^4*f^5*g^4*e^3 + 54*a^3*b*c^5*f^5*g^4*e^3 + 6*a*b^6*c^2*f^4*g^5*e^3 + 9*a^2*b^4*c^3*f^4*g^5*e^3 - 66*a^3*b^2*c^4*f^4*g^5*e^3 - 12*a^4*c^5*f^4*g^5*e^3 - a*b^7*c*f^3*g^6*e^3 - 11*a^2*b^5*c^2*f^3*g^6*e^3 + 31*a^3*b^3*c^3*f^3*g^6*e^3 + 32*a^4*b*c^4*f^3*g^6*e^3 + 3*a^2*b^6*c*f^2*g^7*e^3 - 30*a^4*b^2*c^3*f^2*g^7*e^3 - 3*a^3*b^5*c*f*g^8*e^3 + 9*a^4*b^3*c^2*f*g^8*e^3 + 3*a^5*b*c^3*f*g^8*e^3 + a^4*b^4*c*g^9*e^3 - 4*a^5*b^2*c^2*g^9*e^3 + 2*a^6*c^3*g^9*e^3)*x/(b^2*c^8*d^4*f^12 - 4*a*c^9*d^4*f^12 - 6*b^3*c^7*d^4*f^11*g + 24*a*b*c^8*d^4*f^11*g + 15*b^4*c^6*d^4*f^10*g^2 - 54*a*b^2*c^7*d^4*f^10*g^2 - 24*a^2*c^8*d^4*f^10*g^2 - 20*b^5*c^5*d^4*f^9*g^3 + 50*a*b^3*c^6*d^4*f^9*g^3 + 120*a^2*b*c^7*d^4*f^9*g^3 + 15*b^6*c^4*d^4*f^8*g^4 - 225*a^2*b^2*c^6*d^4*f^8*g^4 - 60*a^3*c^7*d^4*f^8*g^4 - 6*b^7*c^3*d^4*f^7*g^5 - 36*a*b^5*c^4*d^4*f^7*g^5 + 180*a^2*b^3*c^5*d^4*f^7*g^5 + 240*a^3*b*c^6*d^4*f^7*g^5 + b^8*c^2*d^4*f^6*g^6 + 26*a*b^6*c^3*d^4*f^6*g^6 - 30*a^2*b^4*c^4*d^4*f^6*g^6 - 340*a^3*b^2*c^5*d^4*f^6*g^6 - 80*a^4*c^6*d^4*f^6*g^6 - 6*a*b^7*c^2*d^4*f^5*g^7 - 36*a^2*b^5*c^3*d^4*f^5*g^7 + 180*a^3*b^3*c^4*d^4*f^5*g^7 + 240*a^4*b*c^5*d^4*f^5*g^7 + 15*a^2*b^6*c^2*d^4*f^4*g^8 - 225*a^4*b^2*c^4*d^4*f^4*g^8 - 60*a^5*c^5*d^4*f^4*g^8 - 20*a^3*b^5*c^2*d^4*f^3*g^9 + 50*a^4*b^3*c^3*d^4*f^3*g^9 + 120*a^5*b*c^4*d^4*f^3*g^9 + 15*a^4*b^4*c^2*d^4*f^2*g^10 - 54*a^5*b^2*c^3*d^4*f^2*g^10 - 24*a^6*c^4*d^4*f^2*g^10 - 6*a^5*b^3*c^2*d^4*f*g^11 + 24*a^6*b*c^3*d^4*f*g^11 + a^6*b^2*c^2*d^4*g^12 - 4*a^7*c^3*d^4*g^12 - 2*b^3*c^7*d^3*f^12*e + 8*a*b*c^8*d^3*f^12*e + 12*b^4*c^6*d^3*f^11*g*e - 48*a*b^2*c^7*d^3*f^11*g*e - 30*b^5*c^5*d^3*f^10*g^2*e + 108*a*b^3*c^6*d^3*f^10*g^2*e + 48*a^2*b*c^7*d^3*f^10*g^2*e + 40*b^6*c^4*d^3*f^9*g^3*e - 100*a*b^4*c^5*d^3*f^9*g^3*e - 240*a^2*b^2*c^6*d^3*f^9*g^3*e - 30*b^7*c^3*d^3*f^8*g^4*e + 450*a^2*b^3*c^5*d^3*f^8*g^4*e + 120*a^3*b*c^6*d^3*f^8*g^4*e + 12*b^8*c^2*d^3*f^7*g^5*e + 72*a*b^6*c^3*d^3*f^7*g^5*e - 360*a^2*b^4*c^4*d^3*f^7*g^5*e - 480*a^3*b^2*c^5*d^3*f^7*g^5*e - 2*b^9*c*d^3*f^6*g^6*e - 52*a*b^7*c^2*d^3*f^6*g^6*e + 60*a^2*b^5*c^3*d^3*f^6*g^6*e + 680*a^3*b^3*c^4*d^3*f^6*g^6*e + 160*a^4*b*c^5*d^3*f^6*g^6*e + 12*a*b^8*c*d^3*f^5*g^7*e + 72*a^2*b^6*c^2*d^3*f^5*g^7*e - 360*a^3*b^4*c^3*d^3*f^5*g^7*e - 480*a^4*b^2*c^4*d^3*f^5*g^7*e - 30*a^2*b^7*c*d^3*f^4*g^8*e + 450*a^4*b^3*c^3*d^3*f^4*g^8*e + 120*a^5*b*c^4*d^3*f^4*g^8*e + 40*a^3*b^6*c*d^3*f^3*g^9*e - 100*a^4*b^4*c^2*d^3*f^3*g^9*e - 240*a^5*b^2*c^3*d^3*f^3*g^9*e - 30*a^4*b^5*c*d^3*f^2*g^10*e + 108*a^5*b^3*c^2*d^3*f^2*g^10*e + 48*a^6*b*c^3*d^3*f^2*g^10*e + 12*a^5*b^4*c*d^3*f*g^11*e - 48*a^6*b^2*c^2*d^3*f*g^11*e - 2*a^6*b^3*c*d^3*g^12*e + 8*a^7*b*c^2*d^3*g^12*e + b^4*c^6*d^2*f^12*e^2 - 2*a*b^2*c^7*d^2*f^12*e^2 - 8*a^2*c^8*d^2*f^12*e^2 - 6*b^5*c^5*d^2*f^11*g*e^2 + 12*a*b^3*c^6*d^2*f^11*g*e^2 + 48*a^2*b*c^7*d^2*f^11*g*e^2 + 15*b^6*c^4*d^2*f^10*g^2*e^2 - 24*a*b^4*c^5*d^2*f^10*g^2*e^2 - 132*a^2*b^2*c^6*d^2*f^10*g^2*e^2 - 48*a^3*c^7*d^2*f^10*g^2*e^2 - 20*b^7*c^3*d^2*f^9*g^3*e^2 + 10*a*b^5*c^4*d^2*f^9*g^3*e^2 + 220*a^2*b^3*c^5*d^2*f^9*g^3*e^2 + 240*a^3*b*c^6*d^2*f^9*g^3*e^2 + 15*b^8*c^2*d^2*f^8*g^4*e^2 + 30*a*b^6*c^3*d^2*f^8*g^4*e^2 - 225*a^2*b^4*c^4*d^2*f^8*g^4*e^2 - 510*a^3*b^2*c^5*d^2*f^8*g^4*e^2 - 120*a^4*c^6*d^2*f^8*g^4*e^2 - 6*b^9*c*d^2*f^7*g^5*e^2 - 48*a*b^7*c^2*d^2*f^7*g^5*e^2 + 108*a^2*b^5*c^3*d^2*f^7*g^5*e^2 + 600*a^3*b^3*c^4*d^2*f^7*g^5*e^2 + 480*a^4*b*c^5*d^2*f^7*g^5*e^2 + b^10*d^2*f^6*g^6*e^2 + 28*a*b^8*c*d^2*f^6*g^6*e^2 + 22*a^2*b^6*c^2*d^2*f^6*g^6*e^2 - 400*a^3*b^4*c^3*d^2*f^6*g^6*e^2 - 760*a^4*b^2*c^4*d^2*f^6*g^6*e^2 - 160*a^5*c^5*d^2*f^6*g^6*e^2 - 6*a*b^9*d^2*f^5*g^7*e^2 - 48*a^2*b^7*c*d^2*f^5*g^7*e^2 + 108*a^3*b^5*c^2*d^2*f^5*g^7*e^2 + 600*a^4*b^3*c^3*d^2*f^5*g^7*e^2 + 480*a^5*b*c^4*d^2*f^5*g^7*e^2 + 15*a^2*b^8*d^2*f^4*g^8*e^2 + 30*a^3*b^6*c*d^2*f^4*g^8*e^2 - 225*a^4*b^4*c^2*d^2*f^4*g^8*e^2 - 510*a^5*b^2*c^3*d^2*f^4*g^8*e^2 - 120*a^6*c^4*d^2*f^4*g^8*e^2 - 20*a^3*b^7*d^2*f^3*g^9*e^2 + 10*a^4*b^5*c*d^2*f^3*g^9*e^2 + 220*a^5*b^3*c^2*d^2*f^3*g^9*e^2 + 240*a^6*b*c^3*d^2*f^3*g^9*e^2 + 15*a^4*b^6*d^2*f^2*g^10*e^2 - 24*a^5*b^4*c*d^2*f^2*g^10*e^2 - 132*a^6*b^2*c^2*d^2*f^2*g^10*e^2 - 48*a^7*c^3*d^2*f^2*g^10*e^2 - 6*a^5*b^5*d^2*f*g^11*e^2 + 12*a^6*b^3*c*d^2*f*g^11*e^2 + 48*a^7*b*c^2*d^2*f*g^11*e^2 + a^6*b^4*d^2*g^12*e^2 - 2*a^7*b^2*c*d^2*g^12*e^2 - 8*a^8*c^2*d^2*g^12*e^2 - 2*a*b^3*c^6*d*f^12*e^3 + 8*a^2*b*c^7*d*f^12*e^3 + 12*a*b^4*c^5*d*f^11*g*e^3 - 48*a^2*b^2*c^6*d*f^11*g*e^3 - 30*a*b^5*c^4*d*f^10*g^2*e^3 + 108*a^2*b

^3*c^5*d*f^10*g^2*e^3 + 48*a^3*b*c^6*d*f^10*g^2*e^3 + 40*a*b^6*c^3*d*f^9*g^
3*e^3 - 100*a^2*b^4*c^4*d*f^9*g^3*e^3 - 240*a^3*b^2*c^5*d*f^9*g^3*e^3 - 30*
a*b^7*c^2*d*f^8*g^4*e^3 + 450*a^3*b^3*c^4*d*f^8*g^4*e^3 + 120*a^4*b*c^5*d*f
^8*g^4*e^3 + 12*a*b^8*c*d*f^7*g^5*e^3 + 72*a^2*b^6*c^2*d*f^7*g^5*e^3 - 360*
a^3*b^4*c^3*d*f^7*g^5*e^3 - 480*a^4*b^2*c^4*d*f^7*g^5*e^3 - 2*a*b^9*d*f^6*g
^6*e^3 - 52*a^2*b^7*c*d*f^6*g^6*e^3 + 60*a^3*b^5*c^2*d*f^6*g^6*e^3 + 680*a^
4*b^3*c^3*d*f^6*g^6*e^3 + 160*a^5*b*c^4*d*f^6*g^6*e^3 + 12*a^2*b^8*d*f^5*g^
7*e^3 + 72*a^3*b^6*c*d*f^5*g^7*e^3 - 360*a^4*b^4*c^2*d*f^5*g^7*e^3 - 480*a^
5*b^2*c^3*d*f^5*g^7*e^3 - 30*a^3*b^7*d*f^4*g^8*e^3 + 450*a^5*b^3*c^2*d*f^4*
g^8*e^3 + 120*a^6*b*c^3*d*f^4*g^8*e^3 + 40*a^4*b^6*d*f^3*g^9*e^3 - 100*a^5*
b^4*c*d*f^3*g^9*e^3 - 240*a^6*b^2*c^2*d*f^3*g^9*e^3 - 30*a^5*b^5*d*f^2*g^10
*e^3 + 108*a^6*b^3*c*d*f^2*g^10*e^3 + 48*a^7*b*c^2*d*f^2*g^10*e^3 + 12*a^6*
b^4*d*f*g^11*e^3 - 48*a^7*b^2*c*d*f*g^11*e^3 - 2*a^7*b^3*d*g^12*e^3 + 8*a^8
*b*c*d*g^12*e^3 + a^2*b^2*c^6*f^12*e^4 - 4*a^3*c^7*f^12*e^4 - 6*a^2*b^3*c^5
*f^11*g*e^4 + 24*a^3*b*c^6*f^11*g*e^4 + 15*a^2*b^4*c^4*f^10*g^2*e^4 - 54*a^
3*b^2*c^5*f^10*g^2*e^4 - 24*a^4*c^6*f^10*g^2*e^4 - 20*a^2*b^5*c^3*f^9*g^3*e
^4 + 50*a^3*b^3*c^4*f^9*g^3*e^4 + 120*a^4*b*c^5*f^9*g^3*e^4 + 15*a^2*b^6*c^
2*f^8*g^4*e^4 - 225*a^4*b^2*c^4*f^8*g^4*e^4 - 60*a^5*c^5*f^8*g^4*e^4 - 6*a^
2*b^7*c*f^7*g^5*e^4 - 36*a^3*b^5*c^2*f^7*g^5*e^4 + 180*a^4*b^3*c^3*f^7*g^5*
e^4 + 240*a^5*b*c^4*f^7*g^5*e^4 + a^2*b^8*f^6*g^6*e^4 + 26*a^3*b^6*c*f^6*g^
6*e^4 - 30*a^4*b^4*c^2*f^6*g^6*e^4 - 340*a^5*b^2*c^3*f^6*g^6*e^4 - 80*a^6*c
^4*f^6*g^6*e^4 - 6*a^3*b^7*f^5*g^7*e^4 - 36*a^4*b^5*c*f^5*g^7*e^4 + 180*a^5
*b^3*c^2*f^5*g^7*e^4 + 240*a^6*b*c^3*f^5*g^7*e^4 + 15*a^4*b^6*f^4*g^8*e^4 -
 225*a^6*b^2*c^2*f^4*g^8*e^4 - 60*a^7*c^3*f^4*g^8*e^4 - 20*a^5*b^5*f^3*g^9*
e^4 + 50*a^6*b^3*c*f^3*g^9*e^4 + 120*a^7*b*c^2*f^3*g^9*e^4 + 15*a^6*b^4*f^2
*g^10*e^4 - 54*a^7*b^2*c*f^2*g^10*e^4 - 24*a^8*c^2*f^2*g^10*e^4 - 6*a^7*b^3
*f*g^11*e^4 + 24*a^8*b*c*f*g^11*e^4 + a^8*b^2*g^12*e^4 - 4*a^9*c*g^12*e^4)
+ (b*c^8*d^3*f^9 - 6*b^2*c^7*d^3*f^8*g + 6*a*c^8*d^3*f^8*g + 15*b^3*c^6*d^3
*f^7*g^2 - 24*a*b*c^7*d^3*f^7*g^2 - 20*b^4*c^5*d^3*f^6*g^3 + 34*a*b^2*c^6*d
^3*f^6*g^3 + 16*a^2*c^7*d^3*f^6*g^3 + 15*b^5*c^4*d^3*f^5*g^4 - 15*a*b^3*c^5
*d^3*f^5*g^4 - 54*a^2*b*c^6*d^3*f^5*g^4 - 6*b^6*c^3*d^3*f^4*g^5 - 9*a*b^4*c
^4*d^3*f^4*g^5 + 66*a^2*b^2*c^5*d^3*f^4*g^5 + 12*a^3*c^6*d^3*f^4*g^5 + b^7*
c^2*d^3*f^3*g^6 + 11*a*b^5*c^3*d^3*f^3*g^6 - 31*a^2*b^3*c^4*d^3*f^3*g^6 - 3
2*a^3*b*c^5*d^3*f^3*g^6 - 3*a*b^6*c^2*d^3*f^2*g^7 + 30*a^3*b^2*c^4*d^3*f^2*
g^7 + 3*a^2*b^5*c^2*d^3*f*g^8 - 9*a^3*b^3*c^3*d^3*f*g^8 - 3*a^4*b*c^4*d^3*f
*g^8 - a^3*b^4*c^2*d^3*g^9 + 4*a^4*b^2*c^3*d^3*g^9 - 2*a^5*c^4*d^3*g^9 - 2*
b^2*c^7*d^2*f^9*e + 2*a*c^8*d^2*f^9*e + 12*b^3*c^6*d^2*f^8*g*e - 21*a*b*c^7
*d^2*f^8*g*e - 30*b^4*c^5*d^2*f^7*g^2*e + 66*a*b^2*c^6*d^2*f^7*g^2*e + 40*b
^5*c^4*d^2*f^6*g^3*e - 89*a*b^3*c^5*d^2*f^6*g^3*e - 32*a^2*b*c^6*d^2*f^6*g^
3*e - 30*b^6*c^3*d^2*f^5*g^4*e + 45*a*b^4*c^4*d^2*f^5*g^4*e + 114*a^2*b^2*c
^5*d^2*f^5*g^4*e - 12*a^3*c^6*d^2*f^5*g^4*e + 12*b^7*c^2*d^2*f^4*g^5*e + 12
*a*b^5*c^3*d^2*f^4*g^5*e - 147*a^2*b^3*c^4*d^2*f^4*g^5*e + 6*a^3*b*c^5*d^2*
f^4*g^5*e - 2*b^8*c*d^2*f^3*g^6*e - 21*a*b^6*c^2*d^2*f^3*g^6*e + 74*a^2*b^4
*c^3*d^2*f^3*g^6*e + 46*a^3*b^2*c^4*d^2*f^3*g^6*e - 16*a^4*c^5*d^2*f^3*g^6*
e + 6*a*b^7*c*d^2*f^2*g^7*e - 3*a^2*b^5*c^2*d^2*f^2*g^7*e - 63*a^3*b^3*c^3*
d^2*f^2*g^7*e + 24*a^4*b*c^4*d^2*f^2*g^7*e - 6*a^2*b^6*c*d^2*f*g^8*e + 21*a
^3*b^4*c^2*d^2*f*g^8*e - 6*a^5*c^4*d^2*f*g^8*e + 2*a^3*b^5*c*d^2*g^9*e - 9*
a^4*b^3*c^2*d^2*g^9*e + 7*a^5*b*c^3*d^2*g^9*e + b^3*c^6*d*f^9*e^2 - a*b*c^7
*d*f^9*e^2 - 6*b^4*c^5*d*f^8*g*e^2 + 9*a*b^2*c^6*d*f^8*g*e^2 + 6*a^2*c^7*d*
f^8*g*e^2 + 15*b^5*c^4*d*f^7*g^2*e^2 - 27*a*b^3*c^5*d*f^7*g^2*e^2 - 24*a^2*
b*c^6*d*f^7*g^2*e^2 - 20*b^6*c^3*d*f^6*g^3*e^2 + 35*a*b^4*c^4*d*f^6*g^3*e^2
 + 50*a^2*b^2*c^5*d*f^6*g^3*e^2 + 16*a^3*c^6*d*f^6*g^3*e^2 + 15*b^7*c^2*d*f
^5*g^4*e^2 - 15*a*b^5*c^3*d*f^5*g^4*e^2 - 75*a^2*b^3*c^4*d*f^5*g^4*e^2 - 42
*a^3*b*c^5*d*f^5*g^4*e^2 - 6*b^8*c*d*f^4*g^5*e^2 - 9*a*b^6*c^2*d*f^4*g^5*e^
2 + 72*a^2*b^4*c^3*d*f^4*g^5*e^2 + 48*a^3*b^2*c^4*d*f^4*g^5*e^2 + 12*a^4*c^
5*d*f^4*g^5*e^2 + b^9*d*f^3*g^6*e^2 + 11*a*b^7*c*d*f^3*g^6*e^2 - 32*a^2*b^5
*c^2*d*f^3*g^6*e^2 - 45*a^3*b^3*c^3*d*f^3*g^6*e^2 - 16*a^4*b*c^4*d*f^3*g^6*
e^2 - 3*a*b^8*d*f^2*g^7*e^2 + 33*a^3*b^4*c^2*d*f^2*g^7*e^2 + 6*a^4*b^2*c^3*
d*f^2*g^7*e^2 + 3*a^2*b^7*d*f*g^8*e^2 - 9*a^3*b^5*c*d*f*g^8*e^2 - 6*a^4*b^3

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*c^2*d*f*g^8*e^2 + 3*a^5*b*c^3*d*f*g^8*e^2 - a^3*b^6*d*g^9*e^2 + 4*a^4*b^4*
c*d*g^9*e^2 - a^5*b^2*c^2*d*g^9*e^2 - 2*a^6*c^3*d*g^9*e^2 - a*b^2*c^6*f^9*e
^3 + 2*a^2*c^7*f^9*e^3 + 6*a*b^3*c^5*f^8*g*e^3 - 15*a^2*b*c^6*f^8*g*e^3 - 1
5*a*b^4*c^4*f^7*g^2*e^3 + 42*a^2*b^2*c^5*f^7*g^2*e^3 + 20*a*b^5*c^3*f^6*g^3
*e^3 - 55*a^2*b^3*c^4*f^6*g^3*e^3 - 16*a^3*b*c^5*f^6*g^3*e^3 - 15*a*b^6*c^2
*f^5*g^4*e^3 + 30*a^2*b^4*c^3*f^5*g^4*e^3 + 60*a^3*b^2*c^4*f^5*g^4*e^3 - 12
*a^4*c^5*f^5*g^4*e^3 + 6*a*b^7*c*f^4*g^5*e^3 + 3*a^2*b^5*c^2*f^4*g^5*e^3 -
81*a^3*b^3*c^3*f^4*g^5*e^3 + 18*a^4*b*c^4*f^4*g^5*e^3 - a*b^8*f^3*g^6*e^3 -
 10*a^2*b^6*c*f^3*g^6*e^3 + 43*a^3*b^4*c^2*f^3*g^6*e^3 + 14*a^4*b^2*c^3*f^3
*g^6*e^3 - 16*a^5*c^4*f^3*g^6*e^3 + 3*a^2*b^7*f^2*g^7*e^3 - 3*a^3*b^5*c*f^2
*g^7*e^3 - 33*a^4*b^3*c^2*f^2*g^7*e^3 + 24*a^5*b*c^3*f^2*g^7*e^3 - 3*a^3*b^
6*f*g^8*e^3 + 12*a^4*b^4*c*f*g^8*e^3 - 3*a^5*b^2*c^2*f*g^8*e^3 - 6*a^6*c^3*
f*g^8*e^3 + a^4*b^5*g^9*e^3 - 5*a^5*b^3*c*g^9*e^3 + 5*a^6*b*c^2*g^9*e^3)/(b
^2*c^8*d^4*f^12 - 4*a*c^9*d^4*f^12 - 6*b^3*c^7*d^4*f^11*g + 24*a*b*c^8*d^4*
f^11*g + 15*b^4*c^6*d^4*f^10*g^2 - 54*a*b^2*c^7*d^4*f^10*g^2 - 24*a^2*c^8*d
^4*f^10*g^2 - 20*b^5*c^5*d^4*f^9*g^3 + 50*a*b^3*c^6*d^4*f^9*g^3 + 120*a^2*b
*c^7*d^4*f^9*g^3 + 15*b^6*c^4*d^4*f^8*g^4 - 225*a^2*b^2*c^6*d^4*f^8*g^4 - 6
0*a^3*c^7*d^4*f^8*g^4 - 6*b^7*c^3*d^4*f^7*g^5 - 36*a*b^5*c^4*d^4*f^7*g^5 +
180*a^2*b^3*c^5*d^4*f^7*g^5 + 240*a^3*b*c^6*d^4*f^7*g^5 + b^8*c^2*d^4*f^6*g
^6 + 26*a*b^6*c^3*d^4*f^6*g^6 - 30*a^2*b^4*c^4*d^4*f^6*g^6 - 340*a^3*b^2*c^
5*d^4*f^6*g^6 - 80*a^4*c^6*d^4*f^6*g^6 - 6*a*b^7*c^2*d^4*f^5*g^7 - 36*a^2*b
^5*c^3*d^4*f^5*g^7 + 180*a^3*b^3*c^4*d^4*f^5*g^7 + 240*a^4*b*c^5*d^4*f^5*g^
7 + 15*a^2*b^6*c^2*d^4*f^4*g^8 - 225*a^4*b^2*c^4*d^4*f^4*g^8 - 60*a^5*c^5*d
^4*f^4*g^8 - 20*a^3*b^5*c^2*d^4*f^3*g^9 + 50*a^4*b^3*c^3*d^4*f^3*g^9 + 120*
a^5*b*c^4*d^4*f^3*g^9 + 15*a^4*b^4*c^2*d^4*f^2*g^10 - 54*a^5*b^2*c^3*d^4*f^
2*g^10 - 24*a^6*c^4*d^4*f^2*g^10 - 6*a^5*b^3*c^2*d^4*f*g^11 + 24*a^6*b*c^3*
d^4*f*g^11 + a^6*b^2*c^2*d^4*g^12 - 4*a^7*c^3*d^4*g^12 - 2*b^3*c^7*d^3*f^12
*e + 8*a*b*c^8*d^3*f^12*e + 12*b^4*c^6*d^3*f^11*g*e - 48*a*b^2*c^7*d^3*f^11
*g*e - 30*b^5*c^5*d^3*f^10*g^2*e + 108*a*b^3*c^6*d^3*f^10*g^2*e + 48*a^2*b*
c^7*d^3*f^10*g^2*e + 40*b^6*c^4*d^3*f^9*g^3*e - 100*a*b^4*c^5*d^3*f^9*g^3*e
 - 240*a^2*b^2*c^6*d^3*f^9*g^3*e - 30*b^7*c^3*d^3*f^8*g^4*e + 450*a^2*b^3*c
^5*d^3*f^8*g^4*e + 120*a^3*b*c^6*d^3*f^8*g^4*e + 12*b^8*c^2*d^3*f^7*g^5*e +
 72*a*b^6*c^3*d^3*f^7*g^5*e - 360*a^2*b^4*c^4*d^3*f^7*g^5*e - 480*a^3*b^2*c
^5*d^3*f^7*g^5*e - 2*b^9*c*d^3*f^6*g^6*e - 52*a*b^7*c^2*d^3*f^6*g^6*e + 60*
a^2*b^5*c^3*d^3*f^6*g^6*e + 680*a^3*b^3*c^4*d^3*f^6*g^6*e + 160*a^4*b*c^5*d
^3*f^6*g^6*e + 12*a*b^8*c*d^3*f^5*g^7*e + 72*a^2*b^6*c^2*d^3*f^5*g^7*e - 36
0*a^3*b^4*c^3*d^3*f^5*g^7*e - 480*a^4*b^2*c^4*d^3*f^5*g^7*e - 30*a^2*b^7*c*
d^3*f^4*g^8*e + 450*a^4*b^3*c^3*d^3*f^4*g^8*e + 120*a^5*b*c^4*d^3*f^4*g^8*e
 + 40*a^3*b^6*c*d^3*f^3*g^9*e - 100*a^4*b^4*c^2*d^3*f^3*g^9*e - 240*a^5*b^2
*c^3*d^3*f^3*g^9*e - 30*a^4*b^5*c*d^3*f^2*g^10*e + 108*a^5*b^3*c^2*d^3*f^2*
g^10*e + 48*a^6*b*c^3*d^3*f^2*g^10*e + 12*a^5*b^4*c*d^3*f*g^11*e - 48*a^6*b
^2*c^2*d^3*f*g^11*e - 2*a^6*b^3*c*d^3*g^12*e + 8*a^7*b*c^2*d^3*g^12*e + b^4
*c^6*d^2*f^12*e^2 - 2*a*b^2*c^7*d^2*f^12*e^2 - 8*a^2*c^8*d^2*f^12*e^2 - 6*b
^5*c^5*d^2*f^11*g*e^2 + 12*a*b^3*c^6*d^2*f^11*g*e^2 + 48*a^2*b*c^7*d^2*f^11
*g*e^2 + 15*b^6*c^4*d^2*f^10*g^2*e^2 - 24*a*b^4*c^5*d^2*f^10*g^2*e^2 - 132*
a^2*b^2*c^6*d^2*f^10*g^2*e^2 - 48*a^3*c^7*d^2*f^10*g^2*e^2 - 20*b^7*c^3*d^2
*f^9*g^3*e^2 + 10*a*b^5*c^4*d^2*f^9*g^3*e^2 + 220*a^2*b^3*c^5*d^2*f^9*g^3*e
^2 + 240*a^3*b*c^6*d^2*f^9*g^3*e^2 + 15*b^8*c^2*d^2*f^8*g^4*e^2 + 30*a*b^6*
c^3*d^2*f^8*g^4*e^2 - 225*a^2*b^4*c^4*d^2*f^8*g^4*e^2 - 510*a^3*b^2*c^5*d^2
*f^8*g^4*e^2 - 120*a^4*c^6*d^2*f^8*g^4*e^2 - 6*b^9*c*d^2*f^7*g^5*e^2 - 48*a
*b^7*c^2*d^2*f^7*g^5*e^2 + 108*a^2*b^5*c^3*d^2*f^7*g^5*e^2 + 600*a^3*b^3*c^
4*d^2*f^7*g^5*e^2 + 480*a^4*b*c^5*d^2*f^7*g^5*e^2 + b^10*d^2*f^6*g^6*e^2 +
28*a*b^8*c*d^2*f^6*g^6*e^2 + 22*a^2*b^6*c^2*d^2*f^6*g^6*e^2 - 400*a^3*b^4*c
^3*d^2*f^6*g^6*e^2 - 760*a^4*b^2*c^4*d^2*f^6*g^6*e^2 - 160*a^5*c^5*d^2*f^6*
g^6*e^2 - 6*a*b^9*d^2*f^5*g^7*e^2 - 48*a^2*b^7*c*d^2*f^5*g^7*e^2 + 108*a^3*
b^5*c^2*d^2*f^5*g^7*e^2 + 600*a^4*b^3*c^3*d^2*f^5*g^7*e^2 + 480*a^5*b*c^4*d
^2*f^5*g^7*e^2 + 15*a^2*b^8*d^2*f^4*g^8*e^2 + 30*a^3*b^6*c*d^2*f^4*g^8*e^2
- 225*a^4*b^4*c^2*d^2*f^4*g^8*e^2 - 510*a^5*b^2*c^3*d^2*f^4*g^8*e^2 - 120*a
^6*c^4*d^2*f^4*g^8*e^2 - 20*a^3*b^7*d^2*f^3*g^9*e^2 + 10*a^4*b^5*c*d^2*f^3*
```

g^9*e^2 + 220*a^5*b^3*c^2*d^2*f^3*g^9*e^2 + 240*a^6*b*c^3*d^2*f^3*g^9*e^2 + 15*a^4*b^6*d^2*f^2*g^10*e^2 - 24*a^5*b^4*c*d^2*f^2*g^10*e^2 - 132*a^6*b^2*c^2*d^2*f^2*g^10*e^2 - 48*a^7*c^3*d^2*f^2*g^10*e^2 - 6*a^5*b^5*d^2*f*g^11*e^2 + 12*a^6*b^3*c*d^2*f*g^11*e^2 + 48*a^7*b*c^2*d^2*f*g^11*e^2 + a^6*b^4*d^2*g^12*e^2 - 2*a^7*b^2*c*d^2*g^12*e^2 - 8*a^8*c^2*d^2*g^12*e^2 - 2*a*b^3*c^6*d*f^12*e^3 + 8*a^2*b*c^7*d*f^12*e^3 + 12*a*b^4*c^5*d*f^11*g*e^3 - 48*a^2*b^2*c^6*d*f^11*g*e^3 - 30*a*b^5*c^4*d*f^10*g^2*e^3 + 108*a^2*b^3*c^5*d*f^10*g^2*e^3 + 48*a^3*b*c^6*d*f^10*g^2*e^3 + 40*a*b^6*c^3*d*f^9*g^3*e^3 - 100*a^2*b^4*c^4*d*f^9*g^3*e^3 - 240*a^3*b^2*c^5*d*f^9*g^3*e^3 - 30*a*b^7*c^2*d*f^8*g^4*e^3 + 450*a^3*b^3*c^4*d*f^8*g^4*e^3 + 120*a^4*b*c^5*d*f^8*g^4*e^3 + 12*a*b^8*c*d*f^7*g^5*e^3 + 72*a^2*b^6*c^2*d*f^7*g^5*e^3 - 360*a^3*b^4*c^3*d*f^7*g^5*e^3 - 480*a^4*b^2*c^4*d*f^7*g^5*e^3 - 2*a*b^9*d*f^6*g^6*e^3 - 52*a^2*b^7*c*d*f^6*g^6*e^3 + 60*a^3*b^5*c^2*d*f^6*g^6*e^3 + 680*a^4*b^3*c^3*d*f^6*g^6*e^3 + 160*a^5*b*c^4*d*f^6*g^6*e^3 + 12*a^2*b^8*d*f^5*g^7*e^3 + 72*a^3*b^6*c*d*f^5*g^7*e^3 - 360*a^4*b^4*c^2*d*f^5*g^7*e^3 - 480*a^5*b^2*c^3*d*f^5*g^7*e^3 - 30*a^3*b^7*d*f^4*g^8*e^3 + 450*a^5*b^3*c^2*d*f^4*g^8*e^3 + 120*a^6*b*c^3*d*f^4*g^8*e^3 + 40*a^4*b^6*d*f^3*g^9*e^3 - 100*a^5*b^4*c*d*f^3*g^9*e^3 - 240*a^6*b^2*c^2*d*f^3*g^9*e^3 - 30*a^5*b^5*d*f^2*g^10*e^3 + 108*a^6*b^3*c*d*f^2*g^10*e^3 + 48*a^7*b*c^2*d*f^2*g^10*e^3 + 12*a^6*b^4*d*f*g^11*e^3 - 48*a^7*b^2*c*d*f*g^11*e^3 - 2*a^7*b^3*d*g^12*e^3 + 8*a^8*b*c*d*g^12*e^3 + a^2*b^2*c^6*f^12*e^4 - 4*a^3*c^7*f^12*e^4 - 6*a^2*b^3*c^5*f^11*g*e^4 + 24*a^3*b*c^6*f^11*g*e^4 + 15*a^2*b^4*c^4*f^10*g^2*e^4 - 54*a^3*b^2*c^5*f^10*g^2*e^4 - 24*a^4*c^6*f^10*g^2*e^4 - 20*a^2*b^5*c^3*f^9*g^3*e^4 + 50*a^3*b^3*c^4*f^9*g^3*e^4 + 120*a^4*b*c^5*f^9*g^3*e^4 + 15*a^2*b^6*c^2*f^8*g^4*e^4 - 225*a^4*b^2*c^4*f^8*g^4*e^4 - 60*a^5*c^5*f^8*g^4*e^4 - 6*a^2*b^7*c*f^7*g^5*e^4 - 36*a^3*b^5*c^2*f^7*g^5*e^4 + 180*a^4*b^3*c^3*f^7*g^5*e^4 + 240*a^5*b*c^4*f^7*g^5*e^4 + a^2*b^8*f^6*g^6*e^4 + 26*a^3*b^6*c*f^6*g^6*e^4 - 30*a^4*b^4*c^2*f^6*g^6*e^4 - 340*a^5*b^2*c^3*f^6*g^6*e^4 - 80*a^6*c^4*f^6*g^6*e^4 - 6*a^3*b^7*f^5*g^7*e^4 - 36*a^4*b^5*c*f^5*g^7*e^4 + 180*a^5*b^3*c^2*f^5*g^7*e^4 + 240*a^6*b*c^3*f^5*g^7*e^4 + 15*a^4*b^6*f^4*g^8*e^4 - 225*a^6*b^2*c^2*f^4*g^8*e^4 - 60*a^7*c^3*f^4*g^8*e^4 - 20*a^5*b^5*f^3*g^9*e^4 + 50*a^6*b^3*c*f^3*g^9*e^4 + 120*a^7*b*c^2*f^3*g^9*e^4 + 15*a^6*b^4*f^2*g^10*e^4 - 54*a^7*b^2*c*f^2*g^10*e^4 - 24*a^8*c^2*f^2*g^10*e^4 - 6*a^7*b^3*f*g^11*e^4 + 24*a^8*b*c*f*g^11*e^4 + a^8*b^2*g^12*e^4 - 4*a^9*c*g^12*e^4))/sqrt(c*x^2 + b*x + a) + 1/4*(48*c^2*d^2*f^2*g^5 - 48*b*c*d^2*f*g^6 + 15*b^2*d^2*g^7 - 12*a*c*d^2*g^7 - 120*c^2*d*f^3*g^4*e + 132*b*c*d*f^2*g^5*e - 42*b^2*d*f*g^6*e + 12*a*b*d*g^7*e + 80*c^2*f^4*g^3*e^2 - 100*b*c*f^3*g^4*e^2 + 35*b^2*f^2*g^5*e^2 + 28*a*c*f^2*g^5*e^2 - 28*a*b*f*g^6*e^2 + 8*a^2*g^7*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c^3*d^3*f^6*g^3 - 3*b*c^2*d^3*f^5*g^4 + 3*b^2*c*d^3*f^4*g^5 + 3*a*c^2*d^3*f^4*g^5 - b^3*d^3*f^3*g^6 - 6*a*b*c*d^3*f^3*g^6 + 3*a*b^2*d^3*f^2*g^7 + 3*a^2*c*d^3*f^2*g^7 - 3*a^2*b*d^3*f*g^8 + a^3*d^3*g^9 - 3*c^3*d^2*f^7*g^2*e + 9*b*c^2*d^2*f^6*g^3*e - 9*b^2*c*d^2*f^5*g^4*e - 9*a*c^2*d^2*f^5*g^4*e + 3*b^3*d^2*f^4*g^5*e + 18*a*b*c*d^2*f^4*g^5*e - 9*a*b^2*d^2*f^3*g^6*e - 9*a^2*c*d^2*f^3*g^6*e + 9*a^2*b*d^2*f^2*g^7*e - 3*a^3*d^2*f*g^8*e + 3*c^3*d*f^8*g*e^2 - 9*b*c^2*d*f^7*g^2*e^2 + 9*b^2*c*d*f^6*g^3*e^2 + 9*a*c^2*d*f^6*g^3*e^2 - 3*b^3*d*f^5*g^4*e^2 - 18*a*b*c*d*f^5*g^4*e^2 + 9*a*b^2*d*f^4*g^5*e^2 + 9*a^2*c*d*f^4*g^5*e^2 - 9*a^2*b*d*f^3*g^6*e^2 + 3*a^3*d*f^2*g^7*e^2 - c^3*f^9*e^3 + 3*b*c^2*f^8*g*e^3 - 3*b^2*c*f^7*g^2*e^3 - 3*a*c^2*f^7*g^2*e^3 + b^3*f^6*g^3*e^3 + 6*a*b*c*f^6*g^3*e^3 - 3*a*b^2*f^5*g^4*e^3 - 3*a^2*c*f^5*g^4*e^3 + 3*a^2*b*f^4*g^5*e^3 - a^3*f^3*g^6*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) - 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^5/((c*d^5*g^3 - 3*c*d^4*f*g^2*e - b*d^4*g^3*e + 3*c*d^3*f^2*g*e^2 + 3*b*d^3*f*g^2*e^2 + a*d^3*g^3*e^2 - c*d^2*f^3*e^3 - 3*b*d^2*f^2*g*e^3 - 3*a*d^2*f*g^2*e^3 + b*d*f^3*e^4 + 3*a*d*f^2*g*e^4 - a*f^3*e^5)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/4*(24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*f*g^6 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*g^7 - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c

```
^2*f^3*g^4*e + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f^2*g^5*e - 11*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^6*e - 4*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*a*c*f*g^6*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*
g^7*e + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*f^3*g^4 - 48*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*f^2*g^5 + 13*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^6 - 28*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*a*c^(3/2)*d*f*g^6 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*
sqrt(c)*d*g^7 - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f^4*g^3*e
+ 68*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f^3*g^4*e - 17*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f^2*g^5*e + 20*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*c^(3/2)*f^2*g^5*e - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*a*b*sqrt(c)*f*g^6*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*
sqrt(c)*g^7*e + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g^4 - 44
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g^5 - 88*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*c^2*d*f^2*g^5 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^3*d*f*g^6 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*f*g^6 - 9*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^2*c*d*g^7 - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f^4*g^3
*e + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f^3*g^4*e + 112*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3*g^4*e - 13*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*b^3*f^2*g^5*e - 104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*
g^5*e + 17*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^6*e + 28*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^6*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a^2*b*g^7*e + 14*b^2*c^(3/2)*d*f^3*g^4 - 7*b^3*sqrt(c)*d*f^2*g^5 - 44
*a*b*c^(3/2)*d*f^2*g^5 + 23*a*b^2*sqrt(c)*d*f*g^6 + 28*a^2*c^(3/2)*d*f*g^6
- 16*a^2*b*sqrt(c)*d*g^7 - 18*b^2*c^(3/2)*f^4*g^3*e + 11*b^3*sqrt(c)*f^3*g^
4*e + 56*a*b*c^(3/2)*f^3*g^4*e - 39*a*b^2*sqrt(c)*f^2*g^5*e - 36*a^2*c^(3/2
)*f^2*g^5*e + 36*a^2*b*sqrt(c)*f*g^6*e - 8*a^3*sqrt(c)*g^7*e)/((c^3*d^2*f^6
*g^2 - 3*b*c^2*d^2*f^5*g^3 + 3*b^2*c*d^2*f^4*g^4 + 3*a*c^2*d^2*f^4*g^4 - b^
3*d^2*f^3*g^5 - 6*a*b*c*d^2*f^3*g^5 + 3*a*b^2*d^2*f^2*g^6 + 3*a^2*c*d^2*f^2
*g^6 - 3*a^2*b*d^2*f*g^7 + a^3*d^2*g^8 - 2*c^3*d*f^7*g*e + 6*b*c^2*d*f^6*g^
2*e - 6*b^2*c*d*f^5*g^3*e - 6*a*c^2*d*f^5*g^3*e + 2*b^3*d*f^4*g^4*e + 12*a*
b*c*d*f^4*g^4*e - 6*a*b^2*d*f^3*g^5*e - 6*a^2*c*d*f^3*g^5*e + 6*a^2*b*d*f^2
*g^6*e - 2*a^3*d*f*g^7*e + c^3*f^8*e^2 - 3*b*c^2*f^7*g*e^2 + 3*b^2*c*f^6*g^
2*e^2 + 3*a*c^2*f^6*g^2*e^2 - b^3*f^5*g^3*e^2 - 6*a*b*c*f^5*g^3*e^2 + 3*a*b
^2*f^4*g^4*e^2 + 3*a^2*c*f^4*g^4*e^2 - 3*a^2*b*f^3*g^5*e^2 + a^3*f^2*g^6*e^
2)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*sqrt(c)*f + b*f - a*g)^2)
```

maple [B] time = 0.04, size = 5459, normalized size = 5.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}(ex + d)\left(gx + f\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(f+g\,x\right)^3\,\left(d+e\,x\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(f+gx\right)^3\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)**3*(a + b*x + c*x**2)**(3/2)), x)
```

3.629 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=220

$$\frac{(d+ex)^{m+3}\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^5(m+3)}+\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^5(m+1)}-\frac{(ef-$$

Rubi [A] time = 0.22, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {947}

$$\frac{(d+ex)^{m+3}\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^5(m+3)}+\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^5(m+1)}-\frac{(ef-dg)(d+ex)^{m+2}(2cd(ef-2dg)-e(2aeg-3bdg+bef))}{e^5(m+2)}+\frac{g(d+ex)^{m+4}(beg-4cdg+2cef)}{e^5(m+4)}+\frac{cg^2(d+ex)^{m+5}}{e^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (
(e*f - d*g)*(2*c*d*(e*f - 2*d*g) - e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*x)
^(2 + m))/(e^5*(2 + m)) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 -
6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^(3 + m))/(e^5*(3 + m)) + (g*(2*c*e*f - 4*
c*d*g + b*e*g)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c*g^2*(d + e*x)^(5 + m))
/(e^5*(5 + m))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^m}{e^4}+\frac{(ef-dg)(-2cd(ef-2dg)}{}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^{1+m}}{e^5(1+m)}-\frac{(ef-dg)(2cd(ef-2dg)-e}{e^5(}$$

Mathematica [A] time = 0.27, size = 198, normalized size = 0.90

$$\frac{(d+ex)^{m+1}\left(\frac{(d+ex)^2\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{m+3}+\frac{(ef-dg)^2\left(e(ae-bd)+cd^2\right)}{m+1}+\frac{(d+ex)(ef-dg)(e(2aeg-3bdg+bef)+2cd(2dg-ef))}{m+2}+\frac{g(d+ex)^3(beg-4cdg+2cef)}{m+4}+\frac{cg^2(d+ex)^4}{m+5}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*((((c*d^2 + e*(-(b*d) + a*e))*(e*f - d*g)^2)/(1 + m) + ((
e*f - d*g)*(2*c*d*(-(e*f) + 2*d*g) + e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*
x))/(2 + m) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 - 6*d*e*f*g +
6*d^2*g^2))*(d + e*x)^2)/(3 + m) + (g*(2*c*e*f - 4*c*d*g + b*e*g)*(d + e*x)
^3)/(4 + m) + (c*g^2*(d + e*x)^4)/(5 + m)))/e^5
```

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2), x]
```

fricas [B] time = 0.66, size = 1381, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*e^4*f^2*m^4 + (c*e^5*g^2*m^4 + 10*c*e^5*g^2*m^3 + 35*c*e^5*g^2*m^2 + 5
0*c*e^5*g^2*m + 24*c*e^5*g^2)*x^5 + (60*c*e^5*f*g + 30*b*e^5*g^2 + (2*c*e^5
*f*g + (c*d*e^4 + b*e^5)*g^2)*m^4 + (22*c*e^5*f*g + (6*c*d*e^4 + 11*b*e^5)*
g^2)*m^3 + (82*c*e^5*f*g + (11*c*d*e^4 + 41*b*e^5)*g^2)*m^2 + (122*c*e^5*f*
g + (6*c*d*e^4 + 61*b*e^5)*g^2)*m)*x^4 - (2*a*d^2*e^3*f*g + (b*d^2*e^3 - 14
*a*d*e^4)*f^2)*m^3 + (40*c*e^5*f^2 + 80*b*e^5*f*g + 40*a*e^5*g^2 + (c*e^5*f
^2 + 2*(c*d*e^4 + b*e^5)*f*g + (b*d*e^4 + a*e^5)*g^2)*m^4 + 4*(3*c*e^5*f^2
+ 2*(2*c*d*e^4 + 3*b*e^5)*f*g - (c*d^2*e^3 - 2*b*d*e^4 - 3*a*e^5)*g^2)*m^3
+ (49*c*e^5*f^2 + 2*(17*c*d*e^4 + 49*b*e^5)*f*g - (12*c*d^2*e^3 - 17*b*d*e^
4 - 49*a*e^5)*g^2)*m^2 + 2*(39*c*e^5*f^2 + 2*(5*c*d*e^4 + 39*b*e^5)*f*g - (
4*c*d^2*e^3 - 5*b*d*e^4 - 39*a*e^5)*g^2)*m)*x^3 + 20*(2*c*d^3*e^2 - 3*b*d^2
*e^3 + 6*a*d*e^4)*f^2 - 20*(3*c*d^4*e - 4*b*d^3*e^2 + 6*a*d^2*e^3)*f*g + 2*
(12*c*d^5 - 15*b*d^4*e + 20*a*d^3*e^2)*g^2 + (2*a*d^3*e^2*g^2 + (2*c*d^3*e^
2 - 12*b*d^2*e^3 + 71*a*d*e^4)*f^2 + 4*(b*d^3*e^2 - 6*a*d^2*e^3)*f*g)*m^2 +
 (60*b*e^5*f^2 + 120*a*e^5*f*g + (a*d*e^4*g^2 + (c*d*e^4 + b*e^5)*f^2 + 2*(
b*d*e^4 + a*e^5)*f*g)*m^4 + ((10*c*d*e^4 + 13*b*e^5)*f^2 - 2*(3*c*d^2*e^3 -
 10*b*d*e^4 - 13*a*e^5)*f*g - (3*b*d^2*e^3 - 10*a*d*e^4)*g^2)*m^3 + ((29*c*
d*e^4 + 59*b*e^5)*f^2 - 2*(18*c*d^2*e^3 - 29*b*d*e^4 - 59*a*e^5)*f*g + (12*
c*d^3*e^2 - 18*b*d^2*e^3 + 29*a*d*e^4)*g^2)*m^2 + ((20*c*d*e^4 + 107*b*e^5)
*f^2 - 2*(15*c*d^2*e^3 - 20*b*d*e^4 - 107*a*e^5)*f*g + (12*c*d^3*e^2 - 15*b
*d^2*e^3 + 20*a*d*e^4)*g^2)*m)*x^2 + ((18*c*d^3*e^2 - 47*b*d^2*e^3 + 154*a*
d*e^4)*f^2 - 2*(6*c*d^4*e - 18*b*d^3*e^2 + 47*a*d^2*e^3)*f*g - 6*(b*d^4*e -
 3*a*d^3*e^2)*g^2)*m + (120*a*e^5*f^2 + (2*a*d*e^4*f*g + (b*d*e^4 + a*e^5)*
f^2)*m^4 - 2*(a*d^2*e^3*g^2 + (c*d^2*e^3 - 6*b*d*e^4 - 7*a*e^5)*f^2 + 2*(b*
d^2*e^3 - 6*a*d*e^4)*f*g)*m^3 - ((18*c*d^2*e^3 - 47*b*d*e^4 - 71*a*e^5)*f^2
 - 2*(6*c*d^3*e^2 - 18*b*d^2*e^3 + 47*a*d*e^4)*f*g - 6*(b*d^3*e^2 - 3*a*d^2
*e^3)*g^2)*m^2 - 2*((20*c*d^2*e^3 - 30*b*d*e^4 - 77*a*e^5)*f^2 - 10*(3*c*d^
3*e^2 - 4*b*d^2*e^3 + 6*a*d*e^4)*f*g + (12*c*d^4*e - 15*b*d^3*e^2 + 20*a*d^
2*e^3)*g^2)*m)*x)*(e*x + d)^m/(e^5*m^5 + 15*e^5*m^4 + 85*e^5*m^3 + 225*e^5*
m^2 + 274*e^5*m + 120*e^5)
```

giac [B] time = 0.27, size = 2740, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*g^2*m^4*x^5*e^5 + (x*e + d)^m*c*d*g^2*m^4*x^4*e^4 + 2*(x*e +
 d)^m*c*f*g*m^4*x^4*e^5 + (x*e + d)^m*b*g^2*m^4*x^4*e^5 + 10*(x*e + d)^m*c*
g^2*m^3*x^5*e^5 + 2*(x*e + d)^m*c*d*f*g*m^4*x^3*e^4 + (x*e + d)^m*b*d*g^2*m
^4*x^3*e^4 + 6*(x*e + d)^m*c*d*g^2*m^3*x^4*e^4 - 4*(x*e + d)^m*c*d^2*g^2*m^
3*x^3*e^3 + (x*e + d)^m*c*f^2*m^4*x^3*e^5 + 2*(x*e + d)^m*b*f*g*m^4*x^3*e^5
 + (x*e + d)^m*a*g^2*m^4*x^3*e^5 + 22*(x*e + d)^m*c*f*g*m^3*x^4*e^5 + 11*(x
*e + d)^m*b*g^2*m^3*x^4*e^5 + 35*(x*e + d)^m*c*g^2*m^2*x^5*e^5 + (x*e + d)^
m*c*d*f^2*m^4*x^2*e^4 + 2*(x*e + d)^m*b*d*f*g*m^4*x^2*e^4 + (x*e + d)^m*a*d
*g^2*m^4*x^2*e^4 + 16*(x*e + d)^m*c*d*f*g*m^3*x^3*e^4 + 8*(x*e + d)^m*b*d*g
^2*m^3*x^3*e^4 + 11*(x*e + d)^m*c*d*g^2*m^2*x^4*e^4 - 6*(x*e + d)^m*c*d^2*f
*g*m^3*x^2*e^3 - 3*(x*e + d)^m*b*d^2*g^2*m^3*x^2*e^3 - 12*(x*e + d)^m*c*d^2
```

```
*g^2*m^2*x^3*e^3 + 12*(x*e + d)^m*c*d^3*g^2*m^2*x^2*e^2 + (x*e + d)^m*b*f^2
*m^4*x^2*e^5 + 2*(x*e + d)^m*a*f*g*m^4*x^2*e^5 + 12*(x*e + d)^m*c*f^2*m^3*x
^3*e^5 + 24*(x*e + d)^m*b*f*g*m^3*x^3*e^5 + 12*(x*e + d)^m*a*g^2*m^3*x^3*e^
5 + 82*(x*e + d)^m*c*f*g*m^2*x^4*e^5 + 41*(x*e + d)^m*b*g^2*m^2*x^4*e^5 + 5
0*(x*e + d)^m*c*g^2*m*x^5*e^5 + (x*e + d)^m*b*d*f^2*m^4*x*e^4 + 2*(x*e + d)
^m*a*d*f*g*m^4*x*e^4 + 10*(x*e + d)^m*c*d*f^2*m^3*x^2*e^4 + 20*(x*e + d)^m*
b*d*f*g*m^3*x^2*e^4 + 10*(x*e + d)^m*a*d*g^2*m^3*x^2*e^4 + 34*(x*e + d)^m*c
*d*f*g*m^2*x^3*e^4 + 17*(x*e + d)^m*b*d*g^2*m^2*x^3*e^4 + 6*(x*e + d)^m*c*d
*g^2*m*x^4*e^4 - 2*(x*e + d)^m*c*d^2*f^2*m^3*x*e^3 - 4*(x*e + d)^m*b*d^2*f*
g*m^3*x*e^3 - 2*(x*e + d)^m*a*d^2*g^2*m^3*x*e^3 - 36*(x*e + d)^m*c*d^2*f*g*
m^2*x^2*e^3 - 18*(x*e + d)^m*b*d^2*g^2*m^2*x^2*e^3 - 8*(x*e + d)^m*c*d^2*g^
2*m*x^3*e^3 + 12*(x*e + d)^m*c*d^3*f*g*m^2*x*e^2 + 6*(x*e + d)^m*b*d^3*g^2*
m^2*x*e^2 + 12*(x*e + d)^m*c*d^3*g^2*m*x^2*e^2 - 24*(x*e + d)^m*c*d^4*g^2*m
*x*e + (x*e + d)^m*a*f^2*m^4*x*e^5 + 13*(x*e + d)^m*b*f^2*m^3*x^2*e^5 + 26*
(x*e + d)^m*a*f*g*m^3*x^2*e^5 + 49*(x*e + d)^m*c*f^2*m^2*x^3*e^5 + 98*(x*e
+ d)^m*b*f*g*m^2*x^3*e^5 + 49*(x*e + d)^m*a*g^2*m^2*x^3*e^5 + 122*(x*e + d)
^m*c*f*g*m*x^4*e^5 + 61*(x*e + d)^m*b*g^2*m*x^4*e^5 + 24*(x*e + d)^m*c*g^2*
x^5*e^5 + (x*e + d)^m*a*d*f^2*m^4*e^4 + 12*(x*e + d)^m*b*d*f^2*m^3*x*e^4 +
24*(x*e + d)^m*a*d*f*g*m^3*x*e^4 + 29*(x*e + d)^m*c*d*f^2*m^2*x^2*e^4 + 58*
(x*e + d)^m*b*d*f*g*m^2*x^2*e^4 + 29*(x*e + d)^m*a*d*g^2*m^2*x^2*e^4 + 20*(
x*e + d)^m*c*d*f*g*m*x^3*e^4 + 10*(x*e + d)^m*b*d*g^2*m*x^3*e^4 - (x*e + d)
^m*b*d^2*f^2*m^3*e^3 - 2*(x*e + d)^m*a*d^2*f*g*m^3*e^3 - 18*(x*e + d)^m*c*d
^2*f^2*m^2*x*e^3 - 36*(x*e + d)^m*b*d^2*f*g*m^2*x*e^3 - 18*(x*e + d)^m*a*d^
2*g^2*m^2*x*e^3 - 30*(x*e + d)^m*c*d^2*f*g*m*x^2*e^3 - 15*(x*e + d)^m*b*d^2
*g^2*m*x^2*e^3 + 2*(x*e + d)^m*c*d^3*f^2*m^2*e^2 + 4*(x*e + d)^m*b*d^3*f*g*
m^2*e^2 + 2*(x*e + d)^m*a*d^3*g^2*m^2*e^2 + 60*(x*e + d)^m*c*d^3*f*g*m*x*e^
2 + 30*(x*e + d)^m*b*d^3*g^2*m*x*e^2 - 12*(x*e + d)^m*c*d^4*f*g*m*e - 6*(x*
e + d)^m*b*d^4*g^2*m*e + 24*(x*e + d)^m*c*d^5*g^2 + 14*(x*e + d)^m*a*f^2*m^
3*x*e^5 + 59*(x*e + d)^m*b*f^2*m^2*x^2*e^5 + 118*(x*e + d)^m*a*f*g*m^2*x^2*
e^5 + 78*(x*e + d)^m*c*f^2*m*x^3*e^5 + 156*(x*e + d)^m*b*f*g*m*x^3*e^5 + 78
*(x*e + d)^m*a*g^2*m*x^3*e^5 + 60*(x*e + d)^m*c*f*g*x^4*e^5 + 30*(x*e + d)^
m*b*g^2*x^4*e^5 + 14*(x*e + d)^m*a*d*f^2*m^3*e^4 + 47*(x*e + d)^m*b*d*f^2*m
^2*x*e^4 + 94*(x*e + d)^m*a*d*f*g*m^2*x*e^4 + 20*(x*e + d)^m*c*d*f^2*m*x^2*
e^4 + 40*(x*e + d)^m*b*d*f*g*m*x^2*e^4 + 20*(x*e + d)^m*a*d*g^2*m*x^2*e^4 -
 12*(x*e + d)^m*b*d^2*f^2*m^2*e^3 - 24*(x*e + d)^m*a*d^2*f*g*m^2*e^3 - 40*(
x*e + d)^m*c*d^2*f^2*m*x*e^3 - 80*(x*e + d)^m*b*d^2*f*g*m*x*e^3 - 40*(x*e +
 d)^m*a*d^2*g^2*m*x*e^3 + 18*(x*e + d)^m*c*d^3*f^2*m*e^2 + 36*(x*e + d)^m*b
*d^3*f*g*m*e^2 + 18*(x*e + d)^m*a*d^3*g^2*m*e^2 - 60*(x*e + d)^m*c*d^4*f*g*
e - 30*(x*e + d)^m*b*d^4*g^2*e + 71*(x*e + d)^m*a*f^2*m^2*x*e^5 + 107*(x*e
+ d)^m*b*f^2*m*x^2*e^5 + 214*(x*e + d)^m*a*f*g*m*x^2*e^5 + 40*(x*e + d)^m*c
*f^2*x^3*e^5 + 80*(x*e + d)^m*b*f*g*x^3*e^5 + 40*(x*e + d)^m*a*g^2*x^3*e^5
+ 71*(x*e + d)^m*a*d*f^2*m^2*e^4 + 60*(x*e + d)^m*b*d*f^2*m*x*e^4 + 120*(x*
e + d)^m*a*d*f*g*m*x*e^4 - 47*(x*e + d)^m*b*d^2*f^2*m*e^3 - 94*(x*e + d)^m*
a*d^2*f*g*m*e^3 + 40*(x*e + d)^m*c*d^3*f^2*e^2 + 80*(x*e + d)^m*b*d^3*f*g*e
^2 + 40*(x*e + d)^m*a*d^3*g^2*e^2 + 154*(x*e + d)^m*a*f^2*m*x*e^5 + 60*(x*e
 + d)^m*b*f^2*x^2*e^5 + 120*(x*e + d)^m*a*f*g*x^2*e^5 + 154*(x*e + d)^m*a*d
*f^2*m*e^4 - 60*(x*e + d)^m*b*d^2*f^2*e^3 - 120*(x*e + d)^m*a*d^2*f*g*e^3 +
 120*(x*e + d)^m*a*f^2*x*e^5 + 120*(x*e + d)^m*a*d*f^2*e^4)/(m^5*e^5 + 15*m
^4*e^5 + 85*m^3*e^5 + 225*m^2*e^5 + 274*m*e^5 + 120*e^5)
```

maple [B] time = 0.02, size = 1347, normalized size = 6.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x)
```

```
[Out] (e*x+d)^(1+m)*(c*e^4*g^2*m^4*x^4+b*e^4*g^2*m^4*x^3+2*c*e^4*f*g*m^4*x^3+10*c
*e^4*g^2*m^3*x^4+a*e^4*g^2*m^4*x^2+2*b*e^4*f*g*m^4*x^2+11*b*e^4*g^2*m^3*x^3
-4*c*d*e^3*g^2*m^3*x^3+c*e^4*f^2*m^4*x^2+22*c*e^4*f*g*m^3*x^3+35*c*e^4*g^2*
```

```
m^2*x^4+2*a*e^4*f*g*m^4*x+12*a*e^4*g^2*m^3*x^2-3*b*d*e^3*g^2*m^3*x^2+b*e^4*
f^2*m^4*x+24*b*e^4*f*g*m^3*x^2+41*b*e^4*g^2*m^2*x^3-6*c*d*e^3*f*g*m^3*x^2-2
4*c*d*e^3*g^2*m^2*x^3+12*c*e^4*f^2*m^3*x^2+82*c*e^4*f*g*m^2*x^3+50*c*e^4*g^
2*m*x^4-2*a*d*e^3*g^2*m^3*x+a*e^4*f^2*m^4+26*a*e^4*f*g*m^3*x+49*a*e^4*g^2*m
^2*x^2-4*b*d*e^3*f*g*m^3*x-24*b*d*e^3*g^2*m^2*x^2+13*b*e^4*f^2*m^3*x+98*b*e
^4*f*g*m^2*x^2+61*b*e^4*g^2*m*x^3+12*c*d^2*e^2*g^2*m^2*x^2-2*c*d*e^3*f^2*m^
3*x-48*c*d*e^3*f*g*m^2*x^2-44*c*d*e^3*g^2*m*x^3+49*c*e^4*f^2*m^2*x^2+122*c*
e^4*f*g*m*x^3+24*c*e^4*g^2*x^4-2*a*d*e^3*f*g*m^3-20*a*d*e^3*g^2*m^2*x+14*a*
e^4*f^2*m^3+118*a*e^4*f*g*m^2*x+78*a*e^4*g^2*m*x^2+6*b*d^2*e^2*g^2*m^2*x-b*
d*e^3*f^2*m^3-40*b*d*e^3*f*g*m^2*x-51*b*d*e^3*g^2*m*x^2+59*b*e^4*f^2*m^2*x+
156*b*e^4*f*g*m*x^2+30*b*e^4*g^2*x^3+12*c*d^2*e^2*f*g*m^2*x+36*c*d^2*e^2*g^
2*m*x^2-20*c*d*e^3*f^2*m^2*x-102*c*d*e^3*f*g*m*x^2-24*c*d*e^3*g^2*x^3+78*c*
e^4*f^2*m*x^2+60*c*e^4*f*g*x^3+2*a*d^2*e^2*g^2*m^2-24*a*d*e^3*f*g*m^2-58*a*
d*e^3*g^2*m*x+71*a*e^4*f^2*m^2+214*a*e^4*f*g*m*x+40*a*e^4*g^2*x^2+4*b*d^2*e
^2*f*g*m^2+36*b*d^2*e^2*g^2*m*x-12*b*d*e^3*f^2*m^2-116*b*d*e^3*f*g*m*x-30*b
*d*e^3*g^2*x^2+107*b*e^4*f^2*m*x+80*b*e^4*f*g*x^2-24*c*d^3*e*g^2*m*x+2*c*d^
2*e^2*f^2*m^2+72*c*d^2*e^2*f*g*m*x+24*c*d^2*e^2*g^2*x^2-58*c*d*e^3*f^2*m*x-
60*c*d*e^3*f*g*x^2+40*c*e^4*f^2*x^2+18*a*d^2*e^2*g^2*m-94*a*d*e^3*f*g*m-40*
a*d*e^3*g^2*x+154*a*e^4*f^2*m+120*a*e^4*f*g*x-6*b*d^3*e*g^2*m+36*b*d^2*e^2*
f*g*m+30*b*d^2*e^2*g^2*x-47*b*d*e^3*f^2*m-80*b*d*e^3*f*g*x+60*b*e^4*f^2*x-1
2*c*d^3*e*f*g*m-24*c*d^3*e*g^2*x+18*c*d^2*e^2*f^2*m+60*c*d^2*e^2*f*g*x-40*c
*d*e^3*f^2*x+40*a*d^2*e^2*g^2-120*a*d*e^3*f*g+120*a*e^4*f^2-30*b*d^3*e*g^2+
80*b*d^2*e^2*f*g-60*b*d*e^3*f^2+24*c*d^4*g^2-60*c*d^3*e*f*g+40*c*d^2*e^2*f^
2)/e^5/(m^5+15*m^4+85*m^3+225*m^2+274*m+120)
```

maxima [B] time = 0.58, size = 684, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b*f^2/((m^2 + 3*m + 2)*e^2) +
 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*f*g/((m^2 + 3*m + 2)*e^2
) + (e*x + d)^(m + 1)*a*f^2/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 +
 m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*f^2/((m^3 + 6*m^2 + 11*m
 + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x
 + 2*d^3)*(e*x + d)^m*b*f*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^2 + 3*m +
2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*g^2/(
(m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3
+ 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(
e*x + d)^m*c*f*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^3 + 6*m^2
+ 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x
^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b*g^2/((m^4 + 10*m^3 + 35*m^2 + 50*m
+ 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 +
11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*
d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c*g^2/((m^5 + 15*m^4 + 85*
m^3 + 225*m^2 + 274*m + 120)*e^5)
```

mupad [B] time = 3.94, size = 1354, normalized size = 6.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2),x)
```

```
[Out] ((d + e*x)^m*(24*c*d^5*g^2 + 40*a*d^3*e^2*g^2 - 60*b*d^2*e^3*f^2 + 40*c*d^3
*e^2*f^2 + 120*a*d*e^4*f^2 - 30*b*d^4*e*g^2 - 120*a*d^2*e^3*f*g + 80*b*d^3*
e^2*f*g + 154*a*d*e^4*f^2*m - 6*b*d^4*e*g^2*m + 71*a*d*e^4*f^2*m^2 + 14*a*d
```

```
*e^4*f^2*m^3 + a*d*e^4*f^2*m^4 + 18*a*d^3*e^2*g^2*m - 47*b*d^2*e^3*f^2*m +
18*c*d^3*e^2*f^2*m - 60*c*d^4*e*f*g + 2*a*d^3*e^2*g^2*m^2 - 12*b*d^2*e^3*f^
2*m^2 - b*d^2*e^3*f^2*m^3 + 2*c*d^3*e^2*f^2*m^2 - 12*c*d^4*e*f*g*m - 94*a*d
^2*e^3*f*g*m + 36*b*d^3*e^2*f*g*m - 24*a*d^2*e^3*f*g*m^2 - 2*a*d^2*e^3*f*g*
m^3 + 4*b*d^3*e^2*f*g*m^2))/(e^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 +
 120)) + (x*(d + e*x)^m*(120*a*e^5*f^2 + 71*a*e^5*f^2*m^2 + 14*a*e^5*f^2*m^
3 + a*e^5*f^2*m^4 + 154*a*e^5*f^2*m + 60*b*d*e^4*f^2*m - 24*c*d^4*e*g^2*m -
 40*a*d^2*e^3*g^2*m + 47*b*d*e^4*f^2*m^2 + 12*b*d*e^4*f^2*m^3 + b*d*e^4*f^2
*m^4 + 30*b*d^3*e^2*g^2*m - 40*c*d^2*e^3*f^2*m - 18*a*d^2*e^3*g^2*m^2 - 2*a
*d^2*e^3*g^2*m^3 + 6*b*d^3*e^2*g^2*m^2 - 18*c*d^2*e^3*f^2*m^2 - 2*c*d^2*e^3
*f^2*m^3 + 120*a*d*e^4*f*g*m + 94*a*d*e^4*f*g*m^2 + 24*a*d*e^4*f*g*m^3 + 2*
a*d*e^4*f*g*m^4 - 80*b*d^2*e^3*f*g*m + 60*c*d^3*e^2*f*g*m - 36*b*d^2*e^3*f*
g*m^2 - 4*b*d^2*e^3*f*g*m^3 + 12*c*d^3*e^2*f*g*m^2))/(e^5*(274*m + 225*m^2
+ 85*m^3 + 15*m^4 + m^5 + 120)) + (c*g^2*x^5*(d + e*x)^m*(50*m + 35*m^2 + 1
0*m^3 + m^4 + 24))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120) + (x^2*(
m + 1)*(d + e*x)^m*(60*b*e^3*f^2 + 12*b*e^3*f^2*m^2 + b*e^3*f^2*m^3 + 120*a
*e^3*f*g + 47*b*e^3*f^2*m + 12*c*d^3*g^2*m + 20*a*d*e^2*g^2*m - 15*b*d^2*e*
g^2*m + 20*c*d*e^2*f^2*m + 24*a*e^3*f*g*m^2 + 2*a*e^3*f*g*m^3 + 9*a*d*e^2*g
^2*m^2 + a*d*e^2*g^2*m^3 - 3*b*d^2*e*g^2*m^2 + 9*c*d*e^2*f^2*m^2 + c*d*e^2*
f^2*m^3 + 94*a*e^3*f*g*m + 40*b*d*e^2*f*g*m - 30*c*d^2*e*f*g*m + 18*b*d*e^2
*f*g*m^2 + 2*b*d*e^2*f*g*m^3 - 6*c*d^2*e*f*g*m^2))/(e^3*(274*m + 225*m^2 +
85*m^3 + 15*m^4 + m^5 + 120)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(20*a*e^2*
g^2 + 20*c*e^2*f^2 + a*e^2*g^2*m^2 + c*e^2*f^2*m^2 + 40*b*e^2*f*g + 9*a*e^2
*g^2*m - 4*c*d^2*g^2*m + 9*c*e^2*f^2*m + b*d*e*g^2*m^2 + 2*b*e^2*f*g*m^2 +
5*b*d*e*g^2*m + 18*b*e^2*f*g*m + 2*c*d*e*f*g*m^2 + 10*c*d*e*f*g*m))/(e^2*(2
74*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (g*x^4*(d + e*x)^m*(11*m +
 6*m^2 + m^3 + 6)*(5*b*e*g + 10*c*e*f + b*e*g*m + c*d*g*m + 2*c*e*f*m))/(e*
(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))
```

sympy [A] time = 14.70, size = 15757, normalized size = 71.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*f**2*x + a*f*g*x**2 + a*g**2*x**3/3 + b*f**2*x**2/2 + 2*
b*f*g*x**3/3 + b*g**2*x**4/4 + c*f**2*x**3/3 + c*f*g*x**4/2 + c*g**2*x**5/5
), Eq(e, 0)), (-a*d**2*e**2*g**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e
**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*a*d*e**3*f*g/(12*d**4*e**5 +
48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 4*a*d
*e**3*g**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8
*x**3 + 12*e**9*x**4) - 3*a*e**4*f**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d
**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 8*a*e**4*f*g*x/(12*d**4*e*
*5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) -
6*a*e**4*g**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*
d*e**8*x**3 + 12*e**9*x**4) - 3*b*d**3*e*g**2/(12*d**4*e**5 + 48*d**3*e**6*
x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*b*d**2*e**2*f*g/
(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e*
*9*x**4) - 12*b*d**2*e**2*g**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e
**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - b*d*e**3*f**2/(12*d**4*e**5 + 4
8*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 8*b*d*
e**3*f*g*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x
**3 + 12*e**9*x**4) - 18*b*d*e**3*g**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x
+ 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 4*b*e**4*f**2*x/(12*
d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
**4) - 12*b*e**4*f*g*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**
2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 12*b*e**4*g**2*x**3/(12*d**4*e**5 + 48
*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*c*d*
*4*g**2*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 4
```

```
8*d*e**8*x**3 + 12*e**9*x**4) + 25*c*d**4*g**2/(12*d**4*e**5 + 48*d**3*e**6
*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*c*d**3*e*f*g/(1
2*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9
*x**4) + 48*c*d**3*e*g**2*x*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 7
2*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 88*c*d**3*e*g**2*x/(12*
d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
**4) - c*d**2*e**2*f**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
+ 48*d*e**8*x**3 + 12*e**9*x**4) - 24*c*d**2*e**2*f*g*x/(12*d**4*e**5 + 48*
d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 72*c*d**
2*e**2*g**2*x**2*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7
*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 108*c*d**2*e**2*g**2*x**2/(12*d**4
*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4)
 - 4*c*d*e**3*f**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 4
8*d*e**8*x**3 + 12*e**9*x**4) - 36*c*d*e**3*f*g*x**2/(12*d**4*e**5 + 48*d**
3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 48*c*d*e**3
*g**2*x**3*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
+ 48*d*e**8*x**3 + 12*e**9*x**4) + 48*c*d*e**3*g**2*x**3/(12*d**4*e**5 + 48
*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*c*e**
4*f**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*
x**3 + 12*e**9*x**4) - 24*c*e**4*f*g*x**3/(12*d**4*e**5 + 48*d**3*e**6*x +
72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*c*e**4*g**2*x**4*lo
g(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x
**3 + 12*e**9*x**4), Eq(m, -5)), (-2*a*d**2*e**2*g**2/(6*d**3*e**5 + 18*d**
2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*a*d*e**3*f*g/(6*d**3*e**5 + 18
*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*d*e**3*g**2*x/(6*d**3*e*
*5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*a*e**4*f**2/(6*d**3
*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*e**4*f*g*x/(6*
d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*e**4*g**2*
x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 6*b*d*
*3*e*g**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e
**8*x**3) + 11*b*d**3*e*g**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2
 + 6*e**8*x**3) - 4*b*d**2*e**2*f*g/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e*
*7*x**2 + 6*e**8*x**3) + 18*b*d**2*e**2*g**2*x*log(d/e + x)/(6*d**3*e**5 +
18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 27*b*d**2*e**2*g**2*x/(6*d
**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - b*d*e**3*f**2/(
6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 12*b*d*e**3*
f*g*x/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 18*b*
d*e**3*g**2*x**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**
2 + 6*e**8*x**3) + 18*b*d*e**3*g**2*x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18
*d*e**7*x**2 + 6*e**8*x**3) - 3*b*e**4*f**2*x/(6*d**3*e**5 + 18*d**2*e**6*x
 + 18*d*e**7*x**2 + 6*e**8*x**3) - 12*b*e**4*f*g*x**2/(6*d**3*e**5 + 18*d**
2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 6*b*e**4*g**2*x**3*log(d/e + x)/
(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 24*c*d**4*g
**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x*
*3) - 44*c*d**4*g**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**
8*x**3) + 12*c*d**3*e*f*g*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d
*e**7*x**2 + 6*e**8*x**3) + 22*c*d**3*e*f*g/(6*d**3*e**5 + 18*d**2*e**6*x +
 18*d*e**7*x**2 + 6*e**8*x**3) - 72*c*d**3*e*g**2*x*log(d/e + x)/(6*d**3*e*
*5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 108*c*d**3*e*g**2*x/(
6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*c*d**2*e**
2*f**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 36*c
*d**2*e**2*f*g*x*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**
2 + 6*e**8*x**3) + 54*c*d**2*e**2*f*g*x/(6*d**3*e**5 + 18*d**2*e**6*x + 18*
d*e**7*x**2 + 6*e**8*x**3) - 72*c*d**2*e**2*g**2*x**2*log(d/e + x)/(6*d**3*
e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 72*c*d**2*e**2*g**2
*x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*c*d
*e**3*f**2*x/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3)
+ 36*c*d*e**3*f*g*x**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e*
*7*x**2 + 6*e**8*x**3) + 36*c*d*e**3*f*g*x**2/(6*d**3*e**5 + 18*d**2*e**6*x
```

```
 + 18*d*e**7*x**2 + 6*e**8*x**3) - 24*c*d*e**3*g**2*x**3*log(d/e + x)/(6*d*
*3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*c*e**4*f**2*x*
*2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 12*c*e**
4*f*g*x**3*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*
e**8*x**3) + 6*c*e**4*g**2*x**4/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x
**2 + 6*e**8*x**3), Eq(m, -4)), (2*a*d**2*e**2*g**2*log(d/e + x)/(2*d**2*e*
*5 + 4*d*e**6*x + 2*e**7*x**2) + 3*a*d**2*e**2*g**2/(2*d**2*e**5 + 4*d*e**6
*x + 2*e**7*x**2) - 2*a*d*e**3*f*g/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 + 4*a*d*e**3*g**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
+ 4*a*d*e**3*g**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - a*e**4*f**2/
(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*a*e**4*f*g*x/(2*d**2*e**5 + 4*
d*e**6*x + 2*e**7*x**2) + 2*a*e**4*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*
d*e**6*x + 2*e**7*x**2) - 6*b*d**3*e*g**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e
**6*x + 2*e**7*x**2) - 9*b*d**3*e*g**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x
**2) + 4*b*d**2*e**2*f*g*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x*
*2) + 6*b*d**2*e**2*f*g/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*b*d**
2*e**2*g**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*b*
d**2*e**2*g**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - b*d*e**3*f**2/(
2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*b*d*e**3*f*g*x*log(d/e + x)/(2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*b*d*e**3*f*g*x/(2*d**2*e**5 + 4*d
*e**6*x + 2*e**7*x**2) - 6*b*d*e**3*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4
*d*e**6*x + 2*e**7*x**2) - 2*b*e**4*f**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e*
*7*x**2) + 4*b*e**4*f*g*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**
7*x**2) + 2*b*e**4*g**2*x**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*
c*d**4*g**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 18*c*d*
*4*g**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*c*d**3*e*f*g*log(d/e
+ x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 18*c*d**3*e*f*g/(2*d**2*e**
5 + 4*d*e**6*x + 2*e**7*x**2) + 24*c*d**3*e*g**2*x*log(d/e + x)/(2*d**2*e**
5 + 4*d*e**6*x + 2*e**7*x**2) + 24*c*d**3*e*g**2*x/(2*d**2*e**5 + 4*d*e**6*
x + 2*e**7*x**2) + 2*c*d**2*e**2*f**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*
x + 2*e**7*x**2) + 3*c*d**2*e**2*f**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x*
*2) - 24*c*d**2*e**2*f*g*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*
x**2) - 24*c*d**2*e**2*f*g*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*
c*d**2*e**2*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 + 4*c*d*e**3*f**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
+ 4*c*d*e**3*f**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*c*d*e**3*
f*g*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*c*d*e**3
*g**2*x**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*c*e**4*f**2*x**2*lo
g(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4*c*e**4*f*g*x**3/(2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + c*e**4*g**2*x**4/(2*d**2*e**5 + 4*d
*e**6*x + 2*e**7*x**2), Eq(m, -3)), (-12*a*d**2*e**2*g**2*log(d/e + x)/(6*d
*e**5 + 6*e**6*x) - 12*a*d**2*e**2*g**2/(6*d*e**5 + 6*e**6*x) + 12*a*d*e**3
*f*g*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 12*a*d*e**3*f*g/(6*d*e**5 + 6*e**
6*x) - 12*a*d*e**3*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 6*a*e**4*f**
2/(6*d*e**5 + 6*e**6*x) + 12*a*e**4*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x
) + 6*a*e**4*g**2*x**2/(6*d*e**5 + 6*e**6*x) + 18*b*d**3*e*g**2*log(d/e + x
)/(6*d*e**5 + 6*e**6*x) + 18*b*d**3*e*g**2/(6*d*e**5 + 6*e**6*x) - 24*b*d**
2*e**2*f*g*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 24*b*d**2*e**2*f*g/(6*d*e**
5 + 6*e**6*x) + 18*b*d**2*e**2*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) +
6*b*d*e**3*f**2*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 6*b*d*e**3*f**2/(6*d*e
**5 + 6*e**6*x) - 24*b*d*e**3*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 9*
b*d*e**3*g**2*x**2/(6*d*e**5 + 6*e**6*x) + 6*b*e**4*f**2*x*log(d/e + x)/(6*
d*e**5 + 6*e**6*x) + 12*b*e**4*f*g*x**2/(6*d*e**5 + 6*e**6*x) + 3*b*e**4*g*
*2*x**3/(6*d*e**5 + 6*e**6*x) - 24*c*d**4*g**2*log(d/e + x)/(6*d*e**5 + 6*e
**6*x) - 24*c*d**4*g**2/(6*d*e**5 + 6*e**6*x) + 36*c*d**3*e*f*g*log(d/e + x
)/(6*d*e**5 + 6*e**6*x) + 36*c*d**3*e*f*g/(6*d*e**5 + 6*e**6*x) - 24*c*d**3
*e*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 12*c*d**2*e**2*f**2*log(d/e
+ x)/(6*d*e**5 + 6*e**6*x) - 12*c*d**2*e**2*f**2/(6*d*e**5 + 6*e**6*x) + 36
*c*d**2*e**2*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 12*c*d**2*e**2*g**2
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*x**2/(6*d*e**5 + 6*e**6*x) - 12*c*d*e**3*f**2*x*log(d/e + x)/(6*d*e**5 + 6
*e**6*x) - 18*c*d*e**3*f*g*x**2/(6*d*e**5 + 6*e**6*x) - 4*c*d*e**3*g**2*x**
3/(6*d*e**5 + 6*e**6*x) + 6*c*e**4*f**2*x**2/(6*d*e**5 + 6*e**6*x) + 6*c*e*
*4*f*g*x**3/(6*d*e**5 + 6*e**6*x) + 2*c*e**4*g**2*x**4/(6*d*e**5 + 6*e**6*x
), Eq(m, -2)), (a*d**2*g**2*log(d/e + x)/e**3 - 2*a*d*f*g*log(d/e + x)/e**2
 - a*d*g**2*x/e**2 + a*f**2*log(d/e + x)/e + 2*a*f*g*x/e + a*g**2*x**2/(2*e
) - b*d**3*g**2*log(d/e + x)/e**4 + 2*b*d**2*f*g*log(d/e + x)/e**3 + b*d**2
*g**2*x/e**3 - b*d*f**2*log(d/e + x)/e**2 - 2*b*d*f*g*x/e**2 - b*d*g**2*x**
2/(2*e**2) + b*f**2*x/e + b*f*g*x**2/e + b*g**2*x**3/(3*e) + c*d**4*g**2*lo
g(d/e + x)/e**5 - 2*c*d**3*f*g*log(d/e + x)/e**4 - c*d**3*g**2*x/e**4 + c*d
**2*f**2*log(d/e + x)/e**3 + 2*c*d**2*f*g*x/e**3 + c*d**2*g**2*x**2/(2*e**3
) - c*d*f**2*x/e**2 - c*d*f*g*x**2/e**2 - c*d*g**2*x**3/(3*e**2) + c*f**2*x
**2/(2*e) + 2*c*f*g*x**3/(3*e) + c*g**2*x**4/(4*e), Eq(m, -1)), (2*a*d**3*e
**2*g**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e
**5*m**2 + 274*e**5*m + 120*e**5) + 18*a*d**3*e**2*g**2*m*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + 40*a*d**3*e**2*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 2*a*d**2*e**3*f*g*m**3*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) - 24*a*d**2*e**3*f*g*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 94*a*d**2*e*
*3*f*g*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m
**2 + 274*e**5*m + 120*e**5) - 120*a*d**2*e**3*f*g*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 2*
a*d**2*e**3*g**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 18*a*d**2*e**3*g**2*m**2*x*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) - 40*a*d**2*e**3*g**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + a*d*e**4*
f**2*m**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 14*a*d*e**4*f**2*m**3*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 71*a*d*e**4*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 154*a*d*e**4*f**2*m*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 120*a*d*e**4*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e
**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*a*d*e**4*f*g*m**4*x*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 24*a*d*e**4*f*g*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 94*a*d*e*
*4*f*g*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e
**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*d*e**4*f*g*m*x*(d + e*x)**m/(e**5
*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5
) + a*d*e**4*g**2*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**
5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10*a*d*e**4*g**2*m**3*x**
2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 2
74*e**5*m + 120*e**5) + 29*a*d*e**4*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20
*a*d*e**4*g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + a*e**5*f**2*m**4*x*(d + e*x)**m
/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 12
0*e**5) + 14*a*e**5*f**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 71*a*e**5*f**2*m**2*x
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 154*a*e**5*f**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**
5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*e**5
*f**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + 2*a*e**5*f*g*m**4*x**2*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
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26*a*e**5*f*g*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 118*a*e**5*f*g*m**2*x**2*(d +
 e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**
5*m + 120*e**5) + 214*a*e**5*f*g*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m
**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*e**5*f*
g*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**
2 + 274*e**5*m + 120*e**5) + a*e**5*g**2*m**4*x**3*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12
*a*e**5*g**2*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 49*a*e**5*g**2*m**2*x**3*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 78*a*e**5*g**2*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m*
*4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*a*e**5*g**2
*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
 + 274*e**5*m + 120*e**5) - 6*b*d**4*e*g**2*m*(d + e*x)**m/(e**5*m**5 + 15*
e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 30*b*d*
*4*e*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 4*b*d**3*e**2*f*g*m**2*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
+ 36*b*d**3*e**2*f*g*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 80*b*d**3*e**2*f*g*(d + e*x)**
m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 1
20*e**5) + 6*b*d**3*e**2*g**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 30*b*d**3*e**2*g
**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) - b*d**2*e**3*f**2*m**3*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 1
2*b*d**2*e**3*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 47*b*d**2*e**3*f**2*m*(d + e*
x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
 + 120*e**5) - 60*b*d**2*e**3*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 4*b*d**2*e**3*f*g*
m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**
2 + 274*e**5*m + 120*e**5) - 36*b*d**2*e**3*f*g*m**2*x*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 80*b*d**2*e**3*f*g*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 3*b*d**2*e**3*g**2*m**3*x**2
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) - 18*b*d**2*e**3*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) -
15*b*d**2*e**3*g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b*d*e**4*f**2*m**4*x*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 12*b*d*e**4*f**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 47*b*d*e**4*
f**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) + 60*b*d*e**4*f**2*m*x*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
+ 2*b*d*e**4*f*g*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*b*d*e**4*f*g*m**3*x**2*
(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274
*e**5*m + 120*e**5) + 58*b*d*e**4*f*g*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 1
5*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*b*
d*e**4*f*g*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 2
25*e**5*m**2 + 274*e**5*m + 120*e**5) + b*d*e**4*g**2*m**4*x**3*(d + e*x)**
m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 1
20*e**5) + 8*b*d*e**4*g**2*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 17*b*d*e**4*g**2
*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5
```

```
*m**2 + 274*e**5*m + 120*e**5) + 10*b*d*e**4*g**2*m*x**3*(d + e*x)**m/(e**5
*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5
) + b*e**5*f**2*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*
m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 13*b*e**5*f**2*m**3*x**2*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) + 59*b*e**5*f**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 107*b*e*
*5*f**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) + 60*b*e**5*f**2*x**2*(d + e*x)**m/(e**5
*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5
) + 2*b*e**5*f*g*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 24*b*e**5*f*g*m**3*x**3*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) + 98*b*e**5*f*g*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e*
*5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 156*b*e**
5*f*g*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e*
*5*m**2 + 274*e**5*m + 120*e**5) + 80*b*e**5*f*g*x**3*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 b*e**5*g**2*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*b*e**5*g**2*m**3*x**4*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 41*b*e**5*g**2*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5
*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 61*b*e**5*g
**2*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5
*m**2 + 274*e**5*m + 120*e**5) + 30*b*e**5*g**2*x**4*(d + e*x)**m/(e**5*m**
5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
24*c*d**5*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) - 12*c*d**4*e*f*g*m*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 60*c*d**4*e*f*g*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 2
25*e**5*m**2 + 274*e**5*m + 120*e**5) - 24*c*d**4*e*g**2*m*x*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 2*c*d**3*e**2*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 18*c*d**3*e**2*f**2*m
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 40*c*d**3*e**2*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**
5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*d**3*
e**2*f*g*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225
*e**5*m**2 + 274*e**5*m + 120*e**5) + 60*c*d**3*e**2*f*g*m*x*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 12*c*d**3*e**2*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**
4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*d**3*e**2*
g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) - 2*c*d**2*e**3*f**2*m**3*x*(d + e*x)**m/(e
**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e
**5) - 18*c*d**2*e**3*f**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 40*c*d**2*e**3*f**2
*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) - 6*c*d**2*e**3*f*g*m**3*x**2*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 36*c*d**2*e**3*f*g*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*
e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 30*c*d**2*e**3*f*g*m*x
**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
 274*e**5*m + 120*e**5) - 4*c*d**2*e**3*g**2*m**3*x**3*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
- 12*c*d**2*e**3*g**2*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 8*c*d**2*e**3*g**2*m*
x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) + c*d*e**4*f**2*m**4*x**2*(d + e*x)**m/(e**5*m**5
```

```
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10
*c*d*e**4*f**2*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 29*c*d*e**4*f**2*m**2*x**2*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 20*c*d*e**4*f**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*c*d*e*
*4*f*g*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 22
5*e**5*m**2 + 274*e**5*m + 120*e**5) + 16*c*d*e**4*f*g*m**3*x**3*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) + 34*c*d*e**4*f*g*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**
4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*c*d*e**4*f*g
*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + c*d*e**4*g**2*m**4*x**4*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 6*c*d*e**4*g**2*m**3*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*c*d*e**4*g**2*m**2*x**4
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 6*c*d*e**4*g**2*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*
e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + c*e**5*
f**2*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*e**5*f**2*m**3*x**3*(d + e*x)**m/
(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120
*e**5) + 49*c*e**5*f**2*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 78*c*e**5*f**2*m*x*
*3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) + 40*c*e**5*f**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*c*e**5
*f*g*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) + 22*c*e**5*f*g*m**3*x**4*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 82*c*e**5*f*g*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 122*c*e**5*f*g*m*x**4
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) + 60*c*e**5*f*g*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5
*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + c*e**5*g**2
*m**4*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5
*m**2 + 274*e**5*m + 120*e**5) + 10*c*e**5*g**2*m**3*x**5*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + 35*c*e**5*g**2*m**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e
**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 50*c*e**5*g**2*m*x**5*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 24*c*e**5*g**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5), True))
```

3.630 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=144

$$\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^4(m+1)}-\frac{(d+ex)^{m+2}(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^4(m+2)}+\frac{(d+ex)^{m+3}(beg-}{e^4(m+}$$

Rubi [A] time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {771}

$$\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^4(m+1)}-\frac{(d+ex)^{m+2}(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^4(m+2)}+\frac{(d+ex)^{m+3}(beg-3cdg+cef)}{e^4(m+3)}+\frac{cg(d+ex)^{m+4}}{e^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)^(1 + m))/(e^4*(1 + m)) - ((c
*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^4*(
2 + m)) + ((c*e*f - 3*c*d*g + b*e*g)*(d + e*x)^(3 + m))/(e^4*(3 + m)) + (c*
g*(d + e*x)^(4 + m))/(e^4*(4 + m))
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^m}{e^3}+\frac{(-cd(2ef-3dg)+e(bef}{}\right.\\ &= \frac{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^{1+m}}{e^4(1+m)}-\frac{(cd(2ef-3dg)-e(bef-2}{e^4(2+}\end{aligned}$$

Mathematica [A] time = 0.33, size = 180, normalized size = 1.25

$$\frac{(d+ex)^{m+1}\left(\frac{(d+ex)\left(ce(2aeg(m+3)+bdg(m-2)+bef(m+4))-b^2e^2g(m+2)+2c^2d(3dg-ef(m+4))\right)}{e^2(m+2)}-\frac{\left(e(ae-bd)+cd^2\right)(beg(m+1)+6cdg-2cef(m+4))}{e^2(m+1)}+(a+x(b+cx))(beg+c(-3dg+ef(m+4)+eg(m+3)x))\right)}{ce^2(m+3)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(-(((c*d^2 + e*(-(b*d) + a*e))*(6*c*d*g + b*e*g*(1 + m)
- 2*c*e*f*(4 + m)))/(e^2*(1 + m))) + ((-(b^2*e^2*g*(2 + m)) + 2*c^2*d*(3*d*
g - e*f*(4 + m)) + c*e*(b*d*g*(-2 + m) + 2*a*e*g*(3 + m) + b*e*f*(4 + m)))*
(d + e*x))/(e^2*(2 + m)) + (a + x*(b + c*x))*(b*e*g + c*(-3*d*g + e*f*(4 +
m) + e*g*(3 + m)*x))))/(c*e^2*(3 + m)*(4 + m))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2), x]
```

fricas [B] time = 0.43, size = 613, normalized size = 4.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*e^3*f*m^3 + (c*e^4*g*m^3 + 6*c*e^4*g*m^2 + 11*c*e^4*g*m + 6*c*e^4*g)*x
^4 + (8*c*e^4*f + 8*b*e^4*g + (c*e^4*f + (c*d*e^3 + b*e^4)*g)*m^3 + (7*c*e^
4*f + (3*c*d*e^3 + 7*b*e^4)*g)*m^2 + 2*(7*c*e^4*f + (c*d*e^3 + 7*b*e^4)*g)*
m)*x^3 - (a*d^2*e^2*g + (b*d^2*e^2 - 9*a*d*e^3)*f)*m^2 + (12*b*e^4*f + 12*a
*e^4*g + ((c*d*e^3 + b*e^4)*f + (b*d*e^3 + a*e^4)*g)*m^3 + ((5*c*d*e^3 + 8*
b*e^4)*f - (3*c*d^2*e^2 - 5*b*d*e^3 - 8*a*e^4)*g)*m^2 + ((4*c*d*e^3 + 19*b*
e^4)*f - (3*c*d^2*e^2 - 4*b*d*e^3 - 19*a*e^4)*g)*m)*x^2 + 4*(2*c*d^3*e - 3*
b*d^2*e^2 + 6*a*d*e^3)*f - 2*(3*c*d^4 - 4*b*d^3*e + 6*a*d^2*e^2)*g + ((2*c*
d^3*e - 7*b*d^2*e^2 + 26*a*d*e^3)*f + (2*b*d^3*e - 7*a*d^2*e^2)*g)*m + (24*
a*e^4*f + (a*d*e^3*g + (b*d*e^3 + a*e^4)*f)*m^3 - ((2*c*d^2*e^2 - 7*b*d*e^3
 - 9*a*e^4)*f + (2*b*d^2*e^2 - 7*a*d*e^3)*g)*m^2 - 2*((4*c*d^2*e^2 - 6*b*d*
e^3 - 13*a*e^4)*f - (3*c*d^3*e - 4*b*d^2*e^2 + 6*a*d*e^3)*g)*m)*x)*(e*x + d
)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*m + 24*e^4)
```

giac [B] time = 0.20, size = 1162, normalized size = 8.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*g*m^3*x^4*e^4 + (x*e + d)^m*c*d*g*m^3*x^3*e^3 + (x*e + d)^m*
c*f*m^3*x^3*e^4 + (x*e + d)^m*b*g*m^3*x^3*e^4 + 6*(x*e + d)^m*c*g*m^2*x^4*e
^4 + (x*e + d)^m*c*d*f*m^3*x^2*e^3 + (x*e + d)^m*b*d*g*m^3*x^2*e^3 + 3*(x*e
 + d)^m*c*d*g*m^2*x^3*e^3 - 3*(x*e + d)^m*c*d^2*g*m^2*x^2*e^2 + (x*e + d)^m
*b*f*m^3*x^2*e^4 + (x*e + d)^m*a*g*m^3*x^2*e^4 + 7*(x*e + d)^m*c*f*m^2*x^3*
e^4 + 7*(x*e + d)^m*b*g*m^2*x^3*e^4 + 11*(x*e + d)^m*c*g*m*x^4*e^4 + (x*e +
 d)^m*b*d*f*m^3*x*e^3 + (x*e + d)^m*a*d*g*m^3*x*e^3 + 5*(x*e + d)^m*c*d*f*m
^2*x^2*e^3 + 5*(x*e + d)^m*b*d*g*m^2*x^2*e^3 + 2*(x*e + d)^m*c*d*g*m*x^3*e^
3 - 2*(x*e + d)^m*c*d^2*f*m^2*x*e^2 - 2*(x*e + d)^m*b*d^2*g*m^2*x*e^2 - 3*(
x*e + d)^m*c*d^2*g*m*x^2*e^2 + 6*(x*e + d)^m*c*d^3*g*m*x*e + (x*e + d)^m*a*
f*m^3*x*e^4 + 8*(x*e + d)^m*b*f*m^2*x^2*e^4 + 8*(x*e + d)^m*a*g*m^2*x^2*e^4
 + 14*(x*e + d)^m*c*f*m*x^3*e^4 + 14*(x*e + d)^m*b*g*m*x^3*e^4 + 6*(x*e + d
)^m*c*g*x^4*e^4 + (x*e + d)^m*a*d*f*m^3*e^3 + 7*(x*e + d)^m*b*d*f*m^2*x*e^3
 + 7*(x*e + d)^m*a*d*g*m^2*x*e^3 + 4*(x*e + d)^m*c*d*f*m*x^2*e^3 + 4*(x*e +
 d)^m*b*d*g*m*x^2*e^3 - (x*e + d)^m*b*d^2*f*m^2*e^2 - (x*e + d)^m*a*d^2*g*m
^2*e^2 - 8*(x*e + d)^m*c*d^2*f*m*x*e^2 - 8*(x*e + d)^m*b*d^2*g*m*x*e^2 + 2*
(x*e + d)^m*c*d^3*f*m*e + 2*(x*e + d)^m*b*d^3*g*m*e - 6*(x*e + d)^m*c*d^4*g
 + 9*(x*e + d)^m*a*f*m^2*x*e^4 + 19*(x*e + d)^m*b*f*m*x^2*e^4 + 19*(x*e + d
)^m*a*g*m*x^2*e^4 + 8*(x*e + d)^m*c*f*x^3*e^4 + 8*(x*e + d)^m*b*g*x^3*e^4 +
 9*(x*e + d)^m*a*d*f*m^2*e^3 + 12*(x*e + d)^m*b*d*f*m*x*e^3 + 12*(x*e + d)^
m*a*d*g*m*x*e^3 - 7*(x*e + d)^m*b*d^2*f*m*e^2 - 7*(x*e + d)^m*a*d^2*g*m*e^2
 + 8*(x*e + d)^m*c*d^3*f*e + 8*(x*e + d)^m*b*d^3*g*e + 26*(x*e + d)^m*a*f*m
*x*e^4 + 12*(x*e + d)^m*b*f*x^2*e^4 + 12*(x*e + d)^m*a*g*x^2*e^4 + 26*(x*e
+ d)^m*a*d*f*m*e^3 - 12*(x*e + d)^m*b*d^2*f*e^2 - 12*(x*e + d)^m*a*d^2*g*e^
2 + 24*(x*e + d)^m*a*f*x*e^4 + 24*(x*e + d)^m*a*d*f*e^3)/(m^4*e^4 + 10*m^3*
e^4 + 35*m^2*e^4 + 50*m*e^4 + 24*e^4)
```

maple [B] time = 0.01, size = 503, normalized size = 3.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x)
```

```
[Out] -(e*x+d)^(m+1)*(-c*e^3*g*m^3*x^3-b*e^3*g*m^3*x^2-c*e^3*f*m^3*x^2-6*c*e^3*g*
m^2*x^3-a*e^3*g*m^3*x-b*e^3*f*m^3*x-7*b*e^3*g*m^2*x^2+3*c*d*e^2*g*m^2*x^2-7
*c*e^3*f*m^2*x^2-11*c*e^3*g*m*x^3-a*e^3*f*m^3-8*a*e^3*g*m^2*x+2*b*d*e^2*g*m
^2*x-8*b*e^3*f*m^2*x-14*b*e^3*g*m*x^2+2*c*d*e^2*f*m^2*x+9*c*d*e^2*g*m*x^2-1
4*c*e^3*f*m*x^2-6*c*e^3*g*x^3+a*d*e^2*g*m^2-9*a*e^3*f*m^2-19*a*e^3*g*m*x+b*
d*e^2*f*m^2+10*b*d*e^2*g*m*x-19*b*e^3*f*m*x-8*b*e^3*g*x^2-6*c*d^2*e*g*m*x+1
0*c*d*e^2*f*m*x+6*c*d*e^2*g*x^2-8*c*e^3*f*x^2+7*a*d*e^2*g*m-26*a*e^3*f*m-12
*a*e^3*g*x-2*b*d^2*e*g*m+7*b*d*e^2*f*m+8*b*d*e^2*g*x-12*b*e^3*f*x-2*c*d^2*e
*f*m-6*c*d^2*e*g*x+8*c*d*e^2*f*x+12*a*d*e^2*g-24*a*e^3*f-8*b*d^2*e*g+12*b*d
*e^2*f+6*c*d^3*g-8*c*d^2*e*f)/e^4/(m^4+10*m^3+35*m^2+50*m+24)
```

maxima [**B**] time = 0.52, size = 352, normalized size = 2.44

$$\frac{\left(e^{2}(m+1)x^{2}+demx-d^{2}\right)(ex+d)^{m}bf}{\left(m^{2}+3m+2\right)e^{2}}+\frac{\left(e^{2}(m+1)x^{2}+demx-d^{2}\right)(ex+d)^{m}ag}{\left(m^{2}+3m+2\right)e^{2}}+\frac{(ex+d)^{m+1}af}{e(m+1)}+\frac{\left(\left(m^{2}+3m+2\right)e^{3}x^{3}+\left(m^{2}+m\right)de^{2}x^{2}-2d^{2}emx+2d^{3}\right)(ex+d)^{m}cf}{\left(m^{3}+6m^{2}+11m+6\right)e^{3}}+\frac{\left(\left(m^{2}+3m+2\right)e^{3}x^{3}+\left(m^{2}+m\right)de^{2}x^{2}-2d^{2}emx+2d^{3}\right)(ex+d)^{m}bg}{\left(m^{3}+6m^{2}+11m+6\right)e^{3}}+\frac{\left(\left(m^{3}+6m^{2}+11m+6\right)e^{4}x^{4}+\left(m^{3}+3m^{2}+2m\right)de^{3}x^{3}-3\left(m^{2}+m\right)d^{2}e^{2}x^{2}+6d^{3}emx-6d^{4}\right)(ex+d)^{m}cg}{\left(m^{4}+10m^{3}+35m^{2}+50m+24\right)e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b*f/((m^2 + 3*m + 2)*e^2) + (
e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*g/((m^2 + 3*m + 2)*e^2) + (e
*x + d)^(m + 1)*a*f/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^
2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*f/((m^3 + 6*m^2 + 11*m + 6)*e^3)
 + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e
*x + d)^m*b*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^3 + 6*m^2 + 11*m + 6)*e^
4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m
*x - 6*d^4)*(e*x + d)^m*c*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4)
```

mupad [**B**] time = 3.59, size = 602, normalized size = 4.18

$$\frac{(d+ex)^{m}\left(24ade^{3}f-6cd^{4}g+8bd^{3}eg+8cd^{3}ef-12ad^{2}e^{2}g-12bd^{2}e^{2}f+9ade^{3}fm^{2}+ade^{3}fm^{3}-7ad^{2}e^{2}gm-7bd^{2}e^{2}fm-ad^{2}e^{2}gm^{2}-bd^{2}e^{2}fm^{2}+26ade^{3}fm+2bd^{3}egm+2cd^{3}efm\right)}{e^{4}\left(m^{4}+10m^{3}+35m^{2}+50m+24\right)}+\frac{x(d+ex)^{m}\left(24ae^{4}f+26ae^{4}fm+9ae^{4}fm^{2}+ae^{4}fm^{3}+7ade^{3}gm^{2}+7bde^{3}fm^{2}+ade^{3}gm^{3}+bde^{3}fm^{3}-8bd^{2}e^{2}gm-8cd^{2}e^{2}fm-2bd^{2}e^{2}gm^{2}-2cd^{2}e^{2}fm^{2}+12ade^{3}gm+12bde^{3}fm+6cd^{3}egm\right)}{e^{4}\left(m^{4}+10m^{3}+35m^{2}+50m+24\right)}+\frac{x^{2}(m+1)(d+ex)^{m}\left(12ae^{2}g+12be^{2}f+7ae^{2}gm+7be^{2}fm-3cd^{2}gm+ae^{2}gm^{2}+be^{2}fm^{2}+4bdegm+4cdefm+bdegm^{2}+cdefm^{2}\right)}{e^{2}\left(m^{4}+10m^{3}+35m^{2}+50m+24\right)}+\frac{cgx^{4}(d+ex)^{m}\left(m^{3}+6m^{2}+11m+6\right)}{m^{4}+10m^{3}+35m^{2}+50m+24}+\frac{x^{3}(d+ex)^{m}\left(m^{2}+3m+2\right)\left(4beg+4cef+begm+cdgm+cefm\right)}{e\left(m^{4}+10m^{3}+35m^{2}+50m+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2),x)
```

```
[Out] ((d + e*x)^m*(24*a*d*e^3*f - 6*c*d^4*g + 8*b*d^3*e*g + 8*c*d^3*e*f - 12*a*d
^2*e^2*g - 12*b*d^2*e^2*f + 9*a*d*e^3*f*m^2 + a*d*e^3*f*m^3 - 7*a*d^2*e^2*g
*m - 7*b*d^2*e^2*f*m - a*d^2*e^2*g*m^2 - b*d^2*e^2*f*m^2 + 26*a*d*e^3*f*m +
 2*b*d^3*e*g*m + 2*c*d^3*e*f*m))/(e^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
+ (x*(d + e*x)^m*(24*a*e^4*f + 26*a*e^4*f*m + 9*a*e^4*f*m^2 + a*e^4*f*m^3 +
 7*a*d*e^3*g*m^2 + 7*b*d*e^3*f*m^2 + a*d*e^3*g*m^3 + b*d*e^3*f*m^3 - 8*b*d^
2*e^2*g*m - 8*c*d^2*e^2*f*m - 2*b*d^2*e^2*g*m^2 - 2*c*d^2*e^2*f*m^2 + 12*a*
d*e^3*g*m + 12*b*d*e^3*f*m + 6*c*d^3*e*g*m))/(e^4*(50*m + 35*m^2 + 10*m^3 +
 m^4 + 24)) + (x^2*(m + 1)*(d + e*x)^m*(12*a*e^2*g + 12*b*e^2*f + 7*a*e^2*g
*m + 7*b*e^2*f*m - 3*c*d^2*g*m + a*e^2*g*m^2 + b*e^2*f*m^2 + 4*b*d*e*g*m +
4*c*d*e*f*m + b*d*e*g*m^2 + c*d*e*f*m^2))/(e^2*(50*m + 35*m^2 + 10*m^3 + m^
4 + 24)) + (c*g*x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 +
10*m^3 + m^4 + 24) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(4*b*e*g + 4*c*e*f +
b*e*g*m + c*d*g*m + c*e*f*m))/(e*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

sympy [**A**] time = 5.86, size = 5930, normalized size = 41.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*f*x + a*g*x**2/2 + b*f*x**2/2 + b*g*x**3/3 + c*f*x**3/3
+ c*g*x**4/4), Eq(e, 0)), (-a*d*e**2*g/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d
*e**6*x**2 + 6*e**7*x**3) - 2*a*e**3*f/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d
*e**6*x**2 + 6*e**7*x**3) - 3*a*e**3*g*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18
*d*e**6*x**2 + 6*e**7*x**3) - 2*b*d**2*e*g/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) - b*d*e**2*f/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*d*e**2*g*x/(6*d**3*e**4 + 18*d**2*e**5*
x + 18*d*e**6*x**2 + 6*e**7*x**3) - 3*b*e**3*f*x/(6*d**3*e**4 + 18*d**2*e**
5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*e**3*g*x**2/(6*d**3*e**4 + 18*d**
2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*d**3*g*log(d/e + x)/(6*d**3*
e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 11*c*d**3*g/(6*d**3
*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 2*c*d**2*e*f/(6*d*
*3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d**2*e*g*x*
log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3)
+ 27*c*d**2*e*g*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x
**3) - 6*c*d*e**2*f*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e*
*7*x**3) + 18*c*d*e**2*g*x**2*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d*e**2*g*x**2/(6*d**3*e**4 + 18*d**2*e
**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*c*e**3*f*x**2/(6*d**3*e**4 + 18*d
**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*e**3*g*x**3*log(d/e + x)/(
6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3), Eq(m, -4)), (
-a*d*e**2*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - a*e**3*f/(2*d**2*e**
4 + 4*d*e**5*x + 2*e**6*x**2) - 2*a*e**3*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*
e**6*x**2) + 2*b*d**2*e*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x
**2) + 3*b*d**2*e*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - b*d*e**2*f/(
2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*b*d*e**2*g*x*log(d/e + x)/(2*d*
*2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*b*d*e**2*g*x/(2*d**2*e**4 + 4*d*e**
5*x + 2*e**6*x**2) - 2*b*e**3*f*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2)
+ 2*b*e**3*g*x**2*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6
*c*d**3*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 9*c*d**3*
g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*d**2*e*f*log(d/e + x)/(2*d
**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 3*c*d**2*e*f/(2*d**2*e**4 + 4*d*e**5
*x + 2*e**6*x**2) - 12*c*d**2*e*g*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x
+ 2*e**6*x**2) - 12*c*d**2*e*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) +
 4*c*d*e**2*f*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*c
*d*e**2*f*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c*d*e**2*g*x**2*lo
g(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*e**3*f*x**2*log(d
/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*e**3*g*x**3/(2*d**2*
e**4 + 4*d*e**5*x + 2*e**6*x**2), Eq(m, -3)), (2*a*d*e**2*g*log(d/e + x)/(2
*d*e**4 + 2*e**5*x) + 2*a*d*e**2*g/(2*d*e**4 + 2*e**5*x) - 2*a*e**3*f/(2*d*
e**4 + 2*e**5*x) + 2*a*e**3*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*b*d*
*2*e*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*b*d**2*e*g/(2*d*e**4 + 2*e**5
*x) + 2*b*d*e**2*f*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*d*e**2*f/(2*d*e
**4 + 2*e**5*x) - 4*b*d*e**2*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*e
**3*f*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*e**3*g*x**2/(2*d*e**4 + 2*
e**5*x) + 6*c*d**3*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 6*c*d**3*g/(2*d*e
**4 + 2*e**5*x) - 4*c*d**2*e*f*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*c*d**
2*e*f/(2*d*e**4 + 2*e**5*x) + 6*c*d**2*e*g*x*log(d/e + x)/(2*d*e**4 + 2*e**
5*x) - 4*c*d*e**2*f*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 3*c*d*e**2*g*x**
2/(2*d*e**4 + 2*e**5*x) + 2*c*e**3*f*x**2/(2*d*e**4 + 2*e**5*x) + c*e**3*g*
x**3/(2*d*e**4 + 2*e**5*x), Eq(m, -2)), (-a*d*g*log(d/e + x)/e**2 + a*f*log
(d/e + x)/e + a*g*x/e + b*d**2*g*log(d/e + x)/e**3 - b*d*f*log(d/e + x)/e**
2 - b*d*g*x/e**2 + b*f*x/e + b*g*x**2/(2*e) - c*d**3*g*log(d/e + x)/e**4 +
c*d**2*f*log(d/e + x)/e**3 + c*d**2*g*x/e**3 - c*d*f*x/e**2 - c*d*g*x**2/(2
*e**2) + c*f*x**2/(2*e) + c*g*x**3/(3*e), Eq(m, -1)), (-a*d**2*e**2*g*m**2*
(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4
) - 7*a*d**2*e**2*g*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2
 + 50*e**4*m + 24*e**4) - 12*a*d**2*e**2*g*(d + e*x)**m/(e**4*m**4 + 10*e**
4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*d*e**3*f*m**3*(d + e*x)**m
```

```
/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 9*a*d*e*
*3*f*m**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m
 + 24*e**4) + 26*a*d*e**3*f*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e
**4*m**2 + 50*e**4*m + 24*e**4) + 24*a*d*e**3*f*(d + e*x)**m/(e**4*m**4 + 1
0*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*d*e**3*g*m**3*x*(d +
e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 7
*a*d*e**3*g*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 +
50*e**4*m + 24*e**4) + 12*a*d*e**3*g*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*
m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*e**4*f*m**3*x*(d + e*x)**m/(
e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 9*a*e**4*f
*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
 24*e**4) + 26*a*e**4*f*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**
4*m**2 + 50*e**4*m + 24*e**4) + 24*a*e**4*f*x*(d + e*x)**m/(e**4*m**4 + 10*
e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*e**4*g*m**3*x**2*(d + e
*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*
a*e**4*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 +
50*e**4*m + 24*e**4) + 19*a*e**4*g*m*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4
*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*a*e**4*g*x**2*(d + e*x)**m
/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 2*b*d**3
*e*g*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
24*e**4) + 8*b*d**3*e*g*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m*
*2 + 50*e**4*m + 24*e**4) - b*d**2*e**2*f*m**2*(d + e*x)**m/(e**4*m**4 + 10
*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 7*b*d**2*e**2*f*m*(d + e
*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 12
*b*d**2*e**2*f*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e
**4*m + 24*e**4) - 2*b*d**2*e**2*g*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4
*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 8*b*d**2*e**2*g*m*x*(d + e*x)
**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*d*e
**3*f*m**3*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**
4*m + 24*e**4) + 7*b*d*e**3*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3
 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*b*d*e**3*f*m*x*(d + e*x)**m/(e*
*4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*d*e**3*g*m
**3*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m
+ 24*e**4) + 5*b*d*e**3*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3
+ 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 4*b*d*e**3*g*m*x**2*(d + e*x)**m/(e
**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*e**4*f*m*
*3*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
 24*e**4) + 8*b*e**4*f*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 3
5*e**4*m**2 + 50*e**4*m + 24*e**4) + 19*b*e**4*f*m*x**2*(d + e*x)**m/(e**4*
m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*b*e**4*f*x**
2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e*
*4) + b*e**4*g*m**3*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m
**2 + 50*e**4*m + 24*e**4) + 7*b*e**4*g*m**2*x**3*(d + e*x)**m/(e**4*m**4 +
 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 14*b*e**4*g*m*x**3*(d
 + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
+ 8*b*e**4*g*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 5
0*e**4*m + 24*e**4) - 6*c*d**4*g*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 3
5*e**4*m**2 + 50*e**4*m + 24*e**4) + 2*c*d**3*e*f*m*(d + e*x)**m/(e**4*m**4
 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*c*d**3*e*f*(d + e
*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 6*
c*d**3*e*g*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e
**4*m + 24*e**4) - 2*c*d**2*e**2*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4
*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 8*c*d**2*e**2*f*m*x*(d + e*x)
**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 3*c*d
**2*e**2*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2
+ 50*e**4*m + 24*e**4) - 3*c*d**2*e**2*g*m*x**2*(d + e*x)**m/(e**4*m**4 + 1
0*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*d*e**3*f*m**3*x**2*(d
 + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
+ 5*c*d*e**3*f*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m
```

```
**2 + 50*e**4*m + 24*e**4) + 4*c*d*e**3*f*m*x**2*(d + e*x)**m/(e**4*m**4 +
10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*d*e**3*g*m**3*x**3*(
d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
 + 3*c*d*e**3*g*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*
m**2 + 50*e**4*m + 24*e**4) + 2*c*d*e**3*g*m*x**3*(d + e*x)**m/(e**4*m**4 +
 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*e**4*f*m**3*x**3*(d
 + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
+ 7*c*e**4*f*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**
2 + 50*e**4*m + 24*e**4) + 14*c*e**4*f*m*x**3*(d + e*x)**m/(e**4*m**4 + 10*
e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*c*e**4*f*x**3*(d + e*x)
**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*e**
4*g*m**3*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e*
*4*m + 24*e**4) + 6*c*e**4*g*m**2*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m*
*3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 11*c*e**4*g*m*x**4*(d + e*x)**m/
(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 6*c*e**4*
g*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m +
24*e**4), True))
```

3.631 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=525

$$\frac{(d+ex)^{m+3}\left(e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(6d^2g^2-6defg+e^2f^2\right)\right)+2ce\left(ae\left(6d^2g^2-6defg+e^2f^2\right)-\right.\right.}{e^7(m+3)}$$

Rubi [A] time = 0.61, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {947}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^7*(1 + m)) -
 (2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2
*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((c^2*d^2*(6*e^2*f^2 -
20*d*e*f*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b
^2*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(a*e*(e^2*f^2 - 6*d*e*f*g + 6
*d^2*g^2) - b*d*(3*e^2*f^2 - 12*d*e*f*g + 10*d^2*g^2)))*(d + e*x)^(3 + m))/
(e^7*(3 + m)) + (2*(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 -
5*d*e*f*g + 5*d^2*g^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*
g + 10*d^2*g^2)))*(d + e*x)^(4 + m))/(e^7*(4 + m)) + ((b^2*e^2*g^2 + 2*c*e*
g*(2*b*e*f - 5*b*d*g + a*e*g) + c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d
 + e*x)^(5 + m))/(e^7*(5 + m)) + (2*c*g*(c*e*f - 3*c*d*g + b*e*g)*(d + e*x)
^(6 + m))/(e^7*(6 + m)) + (c^2*g^2*(d + e*x)^(7 + m))/(e^7*(7 + m))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)^m}{e^6} + \frac{2\left(cd^2-bde+ae^2\right)(\ldots}{\ldots}\right.$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)^{1+m}}{e^7(1+m)} - \frac{2\left(cd^2-bde+ae^2\right)(ef\ldots}{\ldots}$$

Mathematica [A] time = 0.77, size = 492, normalized size = 0.94

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(((c*d^2 + e*(-(b*d) + a*e))^2*(e*f - d*g)^2)/(1 + m) -
(2*(c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)*(c*d*(-2*e*f + 3*d*g) + e*(b*e
```

```
*f - 2*b*d*g + a*e*g))*(d + e*x))/(2 + m) + ((c^2*d^2*(6*e^2*f^2 - 20*d*e*f
*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b^2*(e^2*
f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(b*d*(-3*e^2*f^2 + 12*d*e*f*g - 10*d^
2*g^2) + a*e*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)))*(d + e*x)^2)/(3 + m) + (2*
(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 - 5*d*e*f*g + 5*d^2*g
^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*g + 10*d^2*g^2)))*(
d + e*x)^3)/(4 + m) + ((b^2*e^2*g^2 + 2*c*e*g*(2*b*e*f - 5*b*d*g + a*e*g) +
 c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d + e*x)^4)/(5 + m) + (2*c*g*(c*
e*f - 3*c*d*g + b*e*g)*(d + e*x)^5)/(6 + m) + (c^2*g^2*(d + e*x)^6)/(7 + m)
))/e^7
```

IntegrateAlgebraic [F] time = 0.63, size = 0, normalized size = 0.00

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2, x]
```

fricas [B] time = 0.53, size = 4747, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^6*f^2*m^6 + (c^2*e^7*g^2*m^6 + 21*c^2*e^7*g^2*m^5 + 175*c^2*e^7*g^
2*m^4 + 735*c^2*e^7*g^2*m^3 + 1624*c^2*e^7*g^2*m^2 + 1764*c^2*e^7*g^2*m + 7
20*c^2*e^7*g^2)*x^7 + (1680*c^2*e^7*f*g + 1680*b*c*e^7*g^2 + (2*c^2*e^7*f*g
 + (c^2*d*e^6 + 2*b*c*e^7)*g^2)*m^6 + (44*c^2*e^7*f*g + (15*c^2*d*e^6 + 44*
b*c*e^7)*g^2)*m^5 + 5*(76*c^2*e^7*f*g + (17*c^2*d*e^6 + 76*b*c*e^7)*g^2)*m^
4 + 5*(328*c^2*e^7*f*g + (45*c^2*d*e^6 + 328*b*c*e^7)*g^2)*m^3 + 2*(1849*c^
2*e^7*f*g + (137*c^2*d*e^6 + 1849*b*c*e^7)*g^2)*m^2 + 4*(1019*c^2*e^7*f*g +
 (30*c^2*d*e^6 + 1019*b*c*e^7)*g^2)*m)*x^6 - (2*a^2*d^2*e^5*f*g + (2*a*b*d^
2*e^5 - 27*a^2*d*e^6)*f^2)*m^5 + (1008*c^2*e^7*f^2 + 4032*b*c*e^7*f*g + 100
8*(b^2 + 2*a*c)*e^7*g^2 + (c^2*e^7*f^2 + 2*(c^2*d*e^6 + 2*b*c*e^7)*f*g + (2
*b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*g^2)*m^6 + (23*c^2*e^7*f^2 + 2*(17*c^2*d*e^
6 + 46*b*c*e^7)*f*g - (6*c^2*d^2*e^5 - 34*b*c*d*e^6 - 23*(b^2 + 2*a*c)*e^7)
*g^2)*m^5 + 3*(69*c^2*e^7*f^2 + 2*(35*c^2*d*e^6 + 138*b*c*e^7)*f*g - (20*c^
2*d^2*e^5 - 70*b*c*d*e^6 - 69*(b^2 + 2*a*c)*e^7)*g^2)*m^4 + 5*(185*c^2*e^7*
f^2 + 2*(59*c^2*d*e^6 + 370*b*c*e^7)*f*g - (42*c^2*d^2*e^5 - 118*b*c*d*e^6
- 185*(b^2 + 2*a*c)*e^7)*g^2)*m^3 + 4*(536*c^2*e^7*f^2 + (187*c^2*d*e^6 + 2
144*b*c*e^7)*f*g - (75*c^2*d^2*e^5 - 187*b*c*d*e^6 - 536*(b^2 + 2*a*c)*e^7)
*g^2)*m^2 + 12*(201*c^2*e^7*f^2 + 4*(7*c^2*d*e^6 + 201*b*c*e^7)*f*g - (12*c
^2*d^2*e^5 - 28*b*c*d*e^6 - 201*(b^2 + 2*a*c)*e^7)*g^2)*m)*x^5 + (2*a^2*d^3
*e^4*g^2 - (50*a*b*d^2*e^5 - 295*a^2*d*e^6 - 2*(b^2 + 2*a*c)*d^3*e^4)*f^2 +
 2*(4*a*b*d^3*e^4 - 25*a^2*d^2*e^5)*f*g)*m^4 + (2520*b*c*e^7*f^2 + 2520*a*b
*e^7*g^2 + 2520*(b^2 + 2*a*c)*e^7*f*g + ((c^2*d*e^6 + 2*b*c*e^7)*f^2 + 2*(2
*b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*f*g + (2*a*b*e^7 + (b^2 + 2*a*c)*d*e^6)*g^2
)*m^6 + ((19*c^2*d*e^6 + 48*b*c*e^7)*f^2 - 2*(5*c^2*d^2*e^5 - 38*b*c*d*e^6
- 24*(b^2 + 2*a*c)*e^7)*f*g - (10*b*c*d^2*e^5 - 48*a*b*e^7 - 19*(b^2 + 2*a*
c)*d*e^6)*g^2)*m^5 + ((131*c^2*d*e^6 + 452*b*c*e^7)*f^2 - 2*(65*c^2*d^2*e^5
 - 262*b*c*d*e^6 - 226*(b^2 + 2*a*c)*e^7)*f*g + (30*c^2*d^3*e^4 - 130*b*c*d
^2*e^5 + 452*a*b*e^7 + 131*(b^2 + 2*a*c)*d*e^6)*g^2)*m^4 + ((401*c^2*d*e^6
+ 2112*b*c*e^7)*f^2 - 2*(265*c^2*d^2*e^5 - 802*b*c*d*e^6 - 1056*(b^2 + 2*a*
c)*e^7)*f*g + (180*c^2*d^3*e^4 - 530*b*c*d^2*e^5 + 2112*a*b*e^7 + 401*(b^2
+ 2*a*c)*d*e^6)*g^2)*m^3 + 10*((54*c^2*d*e^6 + 509*b*c*e^7)*f^2 - (83*c^2*d
^2*e^5 - 216*b*c*d*e^6 - 509*(b^2 + 2*a*c)*e^7)*f*g + (33*c^2*d^3*e^4 - 83*
b*c*d^2*e^5 + 509*a*b*e^7 + 54*(b^2 + 2*a*c)*d*e^6)*g^2)*m^2 + 12*(3*(7*c^2
```

```
*d*e^6 + 164*b*c*e^7)*f^2 - (35*c^2*d^2*e^5 - 84*b*c*d*e^6 - 492*(b^2 + 2*a
*c)*e^7)*f*g + (15*c^2*d^3*e^4 - 35*b*c*d^2*e^5 + 492*a*b*e^7 + 21*(b^2 + 2
*a*c)*d*e^6)*g^2)*m)*x^4 - ((12*b*c*d^4*e^3 + 490*a*b*d^2*e^5 - 1665*a^2*d*
e^6 - 44*(b^2 + 2*a*c)*d^3*e^4)*f^2 - 2*(88*a*b*d^3*e^4 - 245*a^2*d^2*e^5 -
 6*(b^2 + 2*a*c)*d^4*e^3)*f*g + 4*(3*a*b*d^4*e^3 - 11*a^2*d^3*e^4)*g^2)*m^3
 + (6720*a*b*e^7*f*g + 1680*a^2*e^7*g^2 + 1680*(b^2 + 2*a*c)*e^7*f^2 + ((2*
b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*f^2 + 2*(2*a*b*e^7 + (b^2 + 2*a*c)*d*e^6)*f*
g + (2*a*b*d*e^6 + a^2*e^7)*g^2)*m^6 - ((4*c^2*d^2*e^5 - 42*b*c*d*e^6 - 25*
(b^2 + 2*a*c)*e^7)*f^2 + 2*(8*b*c*d^2*e^5 - 50*a*b*e^7 - 21*(b^2 + 2*a*c)*d
*e^6)*f*g - (42*a*b*d*e^6 + 25*a^2*e^7 - 4*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^5
- ((64*c^2*d^2*e^5 - 326*b*c*d*e^6 - 247*(b^2 + 2*a*c)*e^7)*f^2 - 2*(20*c^2
*d^3*e^4 - 128*b*c*d^2*e^5 + 494*a*b*e^7 + 163*(b^2 + 2*a*c)*d*e^6)*f*g - (
40*b*c*d^3*e^4 + 326*a*b*d*e^6 + 247*a^2*e^7 - 64*(b^2 + 2*a*c)*d^2*e^5)*g^
2)*m^4 - ((332*c^2*d^2*e^5 - 1134*b*c*d*e^6 - 1219*(b^2 + 2*a*c)*e^7)*f^2 -
 2*(200*c^2*d^3*e^4 - 664*b*c*d^2*e^5 + 2438*a*b*e^7 + 567*(b^2 + 2*a*c)*d*
e^6)*f*g + (120*c^2*d^4*e^3 - 400*b*c*d^3*e^4 - 1134*a*b*d*e^6 - 1219*a^2*e
^7 + 332*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^3 - 8*((76*c^2*d^2*e^5 - 211*b*c*d*e
^6 - 389*(b^2 + 2*a*c)*e^7)*f^2 - (115*c^2*d^3*e^4 - 304*b*c*d^2*e^5 + 1556
*a*b*e^7 + 211*(b^2 + 2*a*c)*d*e^6)*f*g + (45*c^2*d^4*e^3 - 115*b*c*d^3*e^4
 - 211*a*b*d*e^6 - 389*a^2*e^7 + 76*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^2 - 4*((8
4*c^2*d^2*e^5 - 210*b*c*d*e^6 - 949*(b^2 + 2*a*c)*e^7)*f^2 - 2*(70*c^2*d^3*
e^4 - 168*b*c*d^2*e^5 + 1898*a*b*e^7 + 105*(b^2 + 2*a*c)*d*e^6)*f*g + (60*c
^2*d^4*e^3 - 140*b*c*d^3*e^4 - 210*a*b*d*e^6 - 949*a^2*e^7 + 84*(b^2 + 2*a*
c)*d^2*e^5)*g^2)*m)*x^3 + 168*(6*c^2*d^5*e^2 - 15*b*c*d^4*e^3 - 30*a*b*d^2*
e^5 + 30*a^2*d*e^6 + 10*(b^2 + 2*a*c)*d^3*e^4)*f^2 - 168*(10*c^2*d^6*e - 24
*b*c*d^5*e^2 - 40*a*b*d^3*e^4 + 30*a^2*d^2*e^5 + 15*(b^2 + 2*a*c)*d^4*e^3)*
f*g + 24*(30*c^2*d^7 - 70*b*c*d^6*e - 105*a*b*d^4*e^3 + 70*a^2*d^3*e^4 + 42
*(b^2 + 2*a*c)*d^5*e^2)*g^2 + 2*((12*c^2*d^5*e^2 - 108*b*c*d^4*e^3 - 1175*a
*b*d^2*e^5 + 2552*a^2*d*e^6 + 179*(b^2 + 2*a*c)*d^3*e^4)*f^2 + (48*b*c*d^5*
e^2 + 716*a*b*d^3*e^4 - 1175*a^2*d^2*e^5 - 108*(b^2 + 2*a*c)*d^4*e^3)*f*g -
 (108*a*b*d^4*e^3 - 179*a^2*d^3*e^4 - 12*(b^2 + 2*a*c)*d^5*e^2)*g^2)*m^2 +
(5040*a*b*e^7*f^2 + 5040*a^2*e^7*f*g + (a^2*d*e^6*g^2 + (2*a*b*e^7 + (b^2 +
 2*a*c)*d*e^6)*f^2 + 2*(2*a*b*d*e^6 + a^2*e^7)*f*g)*m^6 - ((6*b*c*d^2*e^5 -
 52*a*b*e^7 - 23*(b^2 + 2*a*c)*d*e^6)*f^2 - 2*(46*a*b*d*e^6 + 26*a^2*e^7 -
3*(b^2 + 2*a*c)*d^2*e^5)*f*g + (6*a*b*d^2*e^5 - 23*a^2*d*e^6)*g^2)*m^5 + 3*
((4*c^2*d^3*e^4 - 38*b*c*d^2*e^5 + 180*a*b*e^7 + 67*(b^2 + 2*a*c)*d*e^6)*f^
2 + 2*(8*b*c*d^3*e^4 + 134*a*b*d*e^6 + 90*a^2*e^7 - 19*(b^2 + 2*a*c)*d^2*e^
5)*f*g - (38*a*b*d^2*e^5 - 67*a^2*d*e^6 - 4*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^4
 + ((168*c^2*d^3*e^4 - 750*b*c*d^2*e^5 + 2840*a*b*e^7 + 817*(b^2 + 2*a*c)*d
*e^6)*f^2 - 2*(60*c^2*d^4*e^3 - 336*b*c*d^3*e^4 - 1634*a*b*d*e^6 - 1420*a^2
*e^7 + 375*(b^2 + 2*a*c)*d^2*e^5)*f*g - (120*b*c*d^4*e^3 + 750*a*b*d^2*e^5
- 817*a^2*d*e^6 - 168*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^3 + 2*((330*c^2*d^3*e^4
 - 951*b*c*d^2*e^5 + 3929*a*b*e^7 + 739*(b^2 + 2*a*c)*d*e^6)*f^2 - (480*c^2
*d^4*e^3 - 1320*b*c*d^3*e^4 - 2956*a*b*d*e^6 - 3929*a^2*e^7 + 951*(b^2 + 2*
a*c)*d^2*e^5)*f*g + (180*c^2*d^5*e^2 - 480*b*c*d^4*e^3 - 951*a*b*d^2*e^5 +
739*a^2*d*e^6 + 330*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^2 + 12*((42*c^2*d^3*e^4 -
 105*b*c*d^2*e^5 + 879*a*b*e^7 + 70*(b^2 + 2*a*c)*d*e^6)*f^2 - (70*c^2*d^4*
e^3 - 168*b*c*d^3*e^4 - 280*a*b*d*e^6 - 879*a^2*e^7 + 105*(b^2 + 2*a*c)*d^2
*e^5)*f*g + (30*c^2*d^5*e^2 - 70*b*c*d^4*e^3 - 105*a*b*d^2*e^5 + 70*a^2*d*e
^6 + 42*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m)*x^2 + 4*((78*c^2*d^5*e^2 - 321*b*c*d
^4*e^3 - 1377*a*b*d^2*e^5 + 2007*a^2*d*e^6 + 319*(b^2 + 2*a*c)*d^3*e^4)*f^2
 - (60*c^2*d^6*e - 312*b*c*d^5*e^2 - 1276*a*b*d^3*e^4 + 1377*a^2*d^2*e^5 +
321*(b^2 + 2*a*c)*d^4*e^3)*f*g - (60*b*c*d^6*e + 321*a*b*d^4*e^3 - 319*a^2*
d^3*e^4 - 78*(b^2 + 2*a*c)*d^5*e^2)*g^2)*m + (5040*a^2*e^7*f^2 + (2*a^2*d*e
^6*f*g + (2*a*b*d*e^6 + a^2*e^7)*f^2)*m^6 - (2*a^2*d^2*e^5*g^2 - (50*a*b*d*
e^6 + 27*a^2*e^7 - 2*(b^2 + 2*a*c)*d^2*e^5)*f^2 + 2*(4*a*b*d^2*e^5 - 25*a^2
*d*e^6)*f*g)*m^5 + ((12*b*c*d^3*e^4 + 490*a*b*d*e^6 + 295*a^2*e^7 - 44*(b^2
 + 2*a*c)*d^2*e^5)*f^2 - 2*(88*a*b*d^2*e^5 - 245*a^2*d*e^6 - 6*(b^2 + 2*a*c
)*d^3*e^4)*f*g + 4*(3*a*b*d^3*e^4 - 11*a^2*d^2*e^5)*g^2)*m^4 - ((24*c^2*d^4
```

```
*e^3 - 216*b*c*d^3*e^4 - 2350*a*b*d*e^6 - 1665*a^2*e^7 + 358*(b^2 + 2*a*c)*
d^2*e^5)*f^2 + 2*(48*b*c*d^4*e^3 + 716*a*b*d^2*e^5 - 1175*a^2*d*e^6 - 108*(
b^2 + 2*a*c)*d^3*e^4)*f*g - 2*(108*a*b*d^3*e^4 - 179*a^2*d^2*e^5 - 12*(b^2
+ 2*a*c)*d^4*e^3)*g^2)*m^3 - 4*((78*c^2*d^4*e^3 - 321*b*c*d^3*e^4 - 1377*a*
b*d*e^6 - 1276*a^2*e^7 + 319*(b^2 + 2*a*c)*d^2*e^5)*f^2 - (60*c^2*d^5*e^2 -
 312*b*c*d^4*e^3 - 1276*a*b*d^2*e^5 + 1377*a^2*d*e^6 + 321*(b^2 + 2*a*c)*d^
3*e^4)*f*g - (60*b*c*d^5*e^2 + 321*a*b*d^3*e^4 - 319*a^2*d^2*e^5 - 78*(b^2
+ 2*a*c)*d^4*e^3)*g^2)*m^2 - 12*((84*c^2*d^4*e^3 - 210*b*c*d^3*e^4 - 420*a*
b*d*e^6 - 669*a^2*e^7 + 140*(b^2 + 2*a*c)*d^2*e^5)*f^2 - 14*(10*c^2*d^5*e^2
 - 24*b*c*d^4*e^3 - 40*a*b*d^2*e^5 + 30*a^2*d*e^6 + 15*(b^2 + 2*a*c)*d^3*e^
4)*f*g + 2*(30*c^2*d^6*e - 70*b*c*d^5*e^2 - 105*a*b*d^3*e^4 + 70*a^2*d^2*e^
5 + 42*(b^2 + 2*a*c)*d^4*e^3)*g^2)*m)*x)*(e*x + d)^m/(e^7*m^7 + 28*e^7*m^6
+ 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 13132*e^7*m^2 + 13068*e^7*m +
 5040*e^7)
```

giac [B] time = 0.55, size = 10489, normalized size = 19.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*g^2*m^6*x^7*e^7 + (x*e + d)^m*c^2*d*g^2*m^6*x^6*e^6 + 2*(x
*e + d)^m*c^2*f*g*m^6*x^6*e^7 + 2*(x*e + d)^m*b*c*g^2*m^6*x^6*e^7 + 21*(x*e
 + d)^m*c^2*g^2*m^5*x^7*e^7 + 2*(x*e + d)^m*c^2*d*f*g*m^6*x^5*e^6 + 2*(x*e
+ d)^m*b*c*d*g^2*m^6*x^5*e^6 + 15*(x*e + d)^m*c^2*d*g^2*m^5*x^6*e^6 - 6*(x*
e + d)^m*c^2*d^2*g^2*m^5*x^5*e^5 + (x*e + d)^m*c^2*f^2*m^6*x^5*e^7 + 4*(x*e
 + d)^m*b*c*f*g*m^6*x^5*e^7 + (x*e + d)^m*b^2*g^2*m^6*x^5*e^7 + 2*(x*e + d)
^m*a*c*g^2*m^6*x^5*e^7 + 44*(x*e + d)^m*c^2*f*g*m^5*x^6*e^7 + 44*(x*e + d)^
m*b*c*g^2*m^5*x^6*e^7 + 175*(x*e + d)^m*c^2*g^2*m^4*x^7*e^7 + (x*e + d)^m*c
^2*d*f^2*m^6*x^4*e^6 + 4*(x*e + d)^m*b*c*d*f*g*m^6*x^4*e^6 + (x*e + d)^m*b^
2*d*g^2*m^6*x^4*e^6 + 2*(x*e + d)^m*a*c*d*g^2*m^6*x^4*e^6 + 34*(x*e + d)^m*
c^2*d*f*g*m^5*x^5*e^6 + 34*(x*e + d)^m*b*c*d*g^2*m^5*x^5*e^6 + 85*(x*e + d)
^m*c^2*d*g^2*m^4*x^6*e^6 - 10*(x*e + d)^m*c^2*d^2*f*g*m^5*x^4*e^5 - 10*(x*e
 + d)^m*b*c*d^2*g^2*m^5*x^4*e^5 - 60*(x*e + d)^m*c^2*d^2*g^2*m^4*x^5*e^5 +
30*(x*e + d)^m*c^2*d^3*g^2*m^4*x^4*e^4 + 2*(x*e + d)^m*b*c*f^2*m^6*x^4*e^7
+ 2*(x*e + d)^m*b^2*f*g*m^6*x^4*e^7 + 4*(x*e + d)^m*a*c*f*g*m^6*x^4*e^7 + 2
*(x*e + d)^m*a*b*g^2*m^6*x^4*e^7 + 23*(x*e + d)^m*c^2*f^2*m^5*x^5*e^7 + 92*
(x*e + d)^m*b*c*f*g*m^5*x^5*e^7 + 23*(x*e + d)^m*b^2*g^2*m^5*x^5*e^7 + 46*(
x*e + d)^m*a*c*g^2*m^5*x^5*e^7 + 380*(x*e + d)^m*c^2*f*g*m^4*x^6*e^7 + 380*
(x*e + d)^m*b*c*g^2*m^4*x^6*e^7 + 735*(x*e + d)^m*c^2*g^2*m^3*x^7*e^7 + 2*(
x*e + d)^m*b*c*d*f^2*m^6*x^3*e^6 + 2*(x*e + d)^m*b^2*d*f*g*m^6*x^3*e^6 + 4*
(x*e + d)^m*a*c*d*f*g*m^6*x^3*e^6 + 2*(x*e + d)^m*a*b*d*g^2*m^6*x^3*e^6 + 1
9*(x*e + d)^m*c^2*d*f^2*m^5*x^4*e^6 + 76*(x*e + d)^m*b*c*d*f*g*m^5*x^4*e^6
+ 19*(x*e + d)^m*b^2*d*g^2*m^5*x^4*e^6 + 38*(x*e + d)^m*a*c*d*g^2*m^5*x^4*e
^6 + 210*(x*e + d)^m*c^2*d*f*g*m^4*x^5*e^6 + 210*(x*e + d)^m*b*c*d*g^2*m^4*
x^5*e^6 + 225*(x*e + d)^m*c^2*d*g^2*m^3*x^6*e^6 - 4*(x*e + d)^m*c^2*d^2*f^2
*m^5*x^3*e^5 - 16*(x*e + d)^m*b*c*d^2*f*g*m^5*x^3*e^5 - 4*(x*e + d)^m*b^2*d
^2*g^2*m^5*x^3*e^5 - 8*(x*e + d)^m*a*c*d^2*g^2*m^5*x^3*e^5 - 130*(x*e + d)^
m*c^2*d^2*f*g*m^4*x^4*e^5 - 130*(x*e + d)^m*b*c*d^2*g^2*m^4*x^4*e^5 - 210*(
x*e + d)^m*c^2*d^2*g^2*m^3*x^5*e^5 + 40*(x*e + d)^m*c^2*d^3*f*g*m^4*x^3*e^4
 + 40*(x*e + d)^m*b*c*d^3*g^2*m^4*x^3*e^4 + 180*(x*e + d)^m*c^2*d^3*g^2*m^3
*x^4*e^4 - 120*(x*e + d)^m*c^2*d^4*g^2*m^3*x^3*e^3 + (x*e + d)^m*b^2*f^2*m^
6*x^3*e^7 + 2*(x*e + d)^m*a*c*f^2*m^6*x^3*e^7 + 4*(x*e + d)^m*a*b*f*g*m^6*x
^3*e^7 + (x*e + d)^m*a^2*g^2*m^6*x^3*e^7 + 48*(x*e + d)^m*b*c*f^2*m^5*x^4*e
^7 + 48*(x*e + d)^m*b^2*f*g*m^5*x^4*e^7 + 96*(x*e + d)^m*a*c*f*g*m^5*x^4*e^
7 + 48*(x*e + d)^m*a*b*g^2*m^5*x^4*e^7 + 207*(x*e + d)^m*c^2*f^2*m^4*x^5*e^
7 + 828*(x*e + d)^m*b*c*f*g*m^4*x^5*e^7 + 207*(x*e + d)^m*b^2*g^2*m^4*x^5*e
^7 + 414*(x*e + d)^m*a*c*g^2*m^4*x^5*e^7 + 1640*(x*e + d)^m*c^2*f*g*m^3*x^6
*e^7 + 1640*(x*e + d)^m*b*c*g^2*m^3*x^6*e^7 + 1624*(x*e + d)^m*c^2*g^2*m^2*
```

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x^7*e^7 + (x*e + d)^m*b^2*d*f^2*m^6*x^2*e^6 + 2*(x*e + d)^m*a*c*d*f^2*m^6*x
^2*e^6 + 4*(x*e + d)^m*a*b*d*f*g*m^6*x^2*e^6 + (x*e + d)^m*a^2*d*g^2*m^6*x^
2*e^6 + 42*(x*e + d)^m*b*c*d*f^2*m^5*x^3*e^6 + 42*(x*e + d)^m*b^2*d*f*g*m^5
*x^3*e^6 + 84*(x*e + d)^m*a*c*d*f*g*m^5*x^3*e^6 + 42*(x*e + d)^m*a*b*d*g^2*
m^5*x^3*e^6 + 131*(x*e + d)^m*c^2*d*f^2*m^4*x^4*e^6 + 524*(x*e + d)^m*b*c*d
*f*g*m^4*x^4*e^6 + 131*(x*e + d)^m*b^2*d*g^2*m^4*x^4*e^6 + 262*(x*e + d)^m*
a*c*d*g^2*m^4*x^4*e^6 + 590*(x*e + d)^m*c^2*d*f*g*m^3*x^5*e^6 + 590*(x*e +
d)^m*b*c*d*g^2*m^3*x^5*e^6 + 274*(x*e + d)^m*c^2*d*g^2*m^2*x^6*e^6 - 6*(x*e
 + d)^m*b*c*d^2*f^2*m^5*x^2*e^5 - 6*(x*e + d)^m*b^2*d^2*f*g*m^5*x^2*e^5 - 1
2*(x*e + d)^m*a*c*d^2*f*g*m^5*x^2*e^5 - 6*(x*e + d)^m*a*b*d^2*g^2*m^5*x^2*e
^5 - 64*(x*e + d)^m*c^2*d^2*f^2*m^4*x^3*e^5 - 256*(x*e + d)^m*b*c*d^2*f*g*m
^4*x^3*e^5 - 64*(x*e + d)^m*b^2*d^2*g^2*m^4*x^3*e^5 - 128*(x*e + d)^m*a*c*d
^2*g^2*m^4*x^3*e^5 - 530*(x*e + d)^m*c^2*d^2*f*g*m^3*x^4*e^5 - 530*(x*e + d
)^m*b*c*d^2*g^2*m^3*x^4*e^5 - 300*(x*e + d)^m*c^2*d^2*g^2*m^2*x^5*e^5 + 12*
(x*e + d)^m*c^2*d^3*f^2*m^4*x^2*e^4 + 48*(x*e + d)^m*b*c*d^3*f*g*m^4*x^2*e^
4 + 12*(x*e + d)^m*b^2*d^3*g^2*m^4*x^2*e^4 + 24*(x*e + d)^m*a*c*d^3*g^2*m^4
*x^2*e^4 + 400*(x*e + d)^m*c^2*d^3*f*g*m^3*x^3*e^4 + 400*(x*e + d)^m*b*c*d^
3*g^2*m^3*x^3*e^4 + 330*(x*e + d)^m*c^2*d^3*g^2*m^2*x^4*e^4 - 120*(x*e + d)
^m*c^2*d^4*f*g*m^3*x^2*e^3 - 120*(x*e + d)^m*b*c*d^4*g^2*m^3*x^2*e^3 - 360*
(x*e + d)^m*c^2*d^4*g^2*m^2*x^3*e^3 + 360*(x*e + d)^m*c^2*d^5*g^2*m^2*x^2*e
^2 + 2*(x*e + d)^m*a*b*f^2*m^6*x^2*e^7 + 2*(x*e + d)^m*a^2*f*g*m^6*x^2*e^7
+ 25*(x*e + d)^m*b^2*f^2*m^5*x^3*e^7 + 50*(x*e + d)^m*a*c*f^2*m^5*x^3*e^7 +
 100*(x*e + d)^m*a*b*f*g*m^5*x^3*e^7 + 25*(x*e + d)^m*a^2*g^2*m^5*x^3*e^7 +
 452*(x*e + d)^m*b*c*f^2*m^4*x^4*e^7 + 452*(x*e + d)^m*b^2*f*g*m^4*x^4*e^7
+ 904*(x*e + d)^m*a*c*f*g*m^4*x^4*e^7 + 452*(x*e + d)^m*a*b*g^2*m^4*x^4*e^7
 + 925*(x*e + d)^m*c^2*f^2*m^3*x^5*e^7 + 3700*(x*e + d)^m*b*c*f*g*m^3*x^5*e
^7 + 925*(x*e + d)^m*b^2*g^2*m^3*x^5*e^7 + 1850*(x*e + d)^m*a*c*g^2*m^3*x^5
*e^7 + 3698*(x*e + d)^m*c^2*f*g*m^2*x^6*e^7 + 3698*(x*e + d)^m*b*c*g^2*m^2*
x^6*e^7 + 1764*(x*e + d)^m*c^2*g^2*m*x^7*e^7 + 2*(x*e + d)^m*a*b*d*f^2*m^6*
x*e^6 + 2*(x*e + d)^m*a^2*d*f*g*m^6*x*e^6 + 23*(x*e + d)^m*b^2*d*f^2*m^5*x^
2*e^6 + 46*(x*e + d)^m*a*c*d*f^2*m^5*x^2*e^6 + 92*(x*e + d)^m*a*b*d*f*g*m^5
*x^2*e^6 + 23*(x*e + d)^m*a^2*d*g^2*m^5*x^2*e^6 + 326*(x*e + d)^m*b*c*d*f^2
*m^4*x^3*e^6 + 326*(x*e + d)^m*b^2*d*f*g*m^4*x^3*e^6 + 652*(x*e + d)^m*a*c*
d*f*g*m^4*x^3*e^6 + 326*(x*e + d)^m*a*b*d*g^2*m^4*x^3*e^6 + 401*(x*e + d)^m
*c^2*d*f^2*m^3*x^4*e^6 + 1604*(x*e + d)^m*b*c*d*f*g*m^3*x^4*e^6 + 401*(x*e
+ d)^m*b^2*d*g^2*m^3*x^4*e^6 + 802*(x*e + d)^m*a*c*d*g^2*m^3*x^4*e^6 + 748*
(x*e + d)^m*c^2*d*f*g*m^2*x^5*e^6 + 748*(x*e + d)^m*b*c*d*g^2*m^2*x^5*e^6 +
 120*(x*e + d)^m*c^2*d*g^2*m*x^6*e^6 - 2*(x*e + d)^m*b^2*d^2*f^2*m^5*x*e^5
- 4*(x*e + d)^m*a*c*d^2*f^2*m^5*x*e^5 - 8*(x*e + d)^m*a*b*d^2*f*g*m^5*x*e^5
 - 2*(x*e + d)^m*a^2*d^2*g^2*m^5*x*e^5 - 114*(x*e + d)^m*b*c*d^2*f^2*m^4*x^
2*e^5 - 114*(x*e + d)^m*b^2*d^2*f*g*m^4*x^2*e^5 - 228*(x*e + d)^m*a*c*d^2*f
*g*m^4*x^2*e^5 - 114*(x*e + d)^m*a*b*d^2*g^2*m^4*x^2*e^5 - 332*(x*e + d)^m*
c^2*d^2*f^2*m^3*x^3*e^5 - 1328*(x*e + d)^m*b*c*d^2*f*g*m^3*x^3*e^5 - 332*(x
*e + d)^m*b^2*d^2*g^2*m^3*x^3*e^5 - 664*(x*e + d)^m*a*c*d^2*g^2*m^3*x^3*e^5
 - 830*(x*e + d)^m*c^2*d^2*f*g*m^2*x^4*e^5 - 830*(x*e + d)^m*b*c*d^2*g^2*m^
2*x^4*e^5 - 144*(x*e + d)^m*c^2*d^2*g^2*m*x^5*e^5 + 12*(x*e + d)^m*b*c*d^3*
f^2*m^4*x*e^4 + 12*(x*e + d)^m*b^2*d^3*f*g*m^4*x*e^4 + 24*(x*e + d)^m*a*c*d
^3*f*g*m^4*x*e^4 + 12*(x*e + d)^m*a*b*d^3*g^2*m^4*x*e^4 + 168*(x*e + d)^m*c
^2*d^3*f^2*m^3*x^2*e^4 + 672*(x*e + d)^m*b*c*d^3*f*g*m^3*x^2*e^4 + 168*(x*e
 + d)^m*b^2*d^3*g^2*m^3*x^2*e^4 + 336*(x*e + d)^m*a*c*d^3*g^2*m^3*x^2*e^4 +
 920*(x*e + d)^m*c^2*d^3*f*g*m^2*x^3*e^4 + 920*(x*e + d)^m*b*c*d^3*g^2*m^2*
x^3*e^4 + 180*(x*e + d)^m*c^2*d^3*g^2*m*x^4*e^4 - 24*(x*e + d)^m*c^2*d^4*f^
2*m^3*x*e^3 - 96*(x*e + d)^m*b*c*d^4*f*g*m^3*x*e^3 - 24*(x*e + d)^m*b^2*d^4
*g^2*m^3*x*e^3 - 48*(x*e + d)^m*a*c*d^4*g^2*m^3*x*e^3 - 960*(x*e + d)^m*c^2
*d^4*f*g*m^2*x^2*e^3 - 960*(x*e + d)^m*b*c*d^4*g^2*m^2*x^2*e^3 - 240*(x*e +
 d)^m*c^2*d^4*g^2*m*x^3*e^3 + 240*(x*e + d)^m*c^2*d^5*f*g*m^2*x*e^2 + 240*(
x*e + d)^m*b*c*d^5*g^2*m^2*x*e^2 + 360*(x*e + d)^m*c^2*d^5*g^2*m*x^2*e^2 -
720*(x*e + d)^m*c^2*d^6*g^2*m*x*e + (x*e + d)^m*a^2*f^2*m^6*x*e^7 + 52*(x*e
 + d)^m*a*b*f^2*m^5*x^2*e^7 + 52*(x*e + d)^m*a^2*f*g*m^5*x^2*e^7 + 247*(x*e
```

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 + d)^m*b^2*f^2*m^4*x^3*e^7 + 494*(x*e + d)^m*a*c*f^2*m^4*x^3*e^7 + 988*(x*
e + d)^m*a*b*f*g*m^4*x^3*e^7 + 247*(x*e + d)^m*a^2*g^2*m^4*x^3*e^7 + 2112*(
x*e + d)^m*b*c*f^2*m^3*x^4*e^7 + 2112*(x*e + d)^m*b^2*f*g*m^3*x^4*e^7 + 422
4*(x*e + d)^m*a*c*f*g*m^3*x^4*e^7 + 2112*(x*e + d)^m*a*b*g^2*m^3*x^4*e^7 +
2144*(x*e + d)^m*c^2*f^2*m^2*x^5*e^7 + 8576*(x*e + d)^m*b*c*f*g*m^2*x^5*e^7
 + 2144*(x*e + d)^m*b^2*g^2*m^2*x^5*e^7 + 4288*(x*e + d)^m*a*c*g^2*m^2*x^5*
e^7 + 4076*(x*e + d)^m*c^2*f*g*m*x^6*e^7 + 4076*(x*e + d)^m*b*c*g^2*m*x^6*e
^7 + 720*(x*e + d)^m*c^2*g^2*x^7*e^7 + (x*e + d)^m*a^2*d*f^2*m^6*e^6 + 50*(
x*e + d)^m*a*b*d*f^2*m^5*x*e^6 + 50*(x*e + d)^m*a^2*d*f*g*m^5*x*e^6 + 201*(
x*e + d)^m*b^2*d*f^2*m^4*x^2*e^6 + 402*(x*e + d)^m*a*c*d*f^2*m^4*x^2*e^6 +
804*(x*e + d)^m*a*b*d*f*g*m^4*x^2*e^6 + 201*(x*e + d)^m*a^2*d*g^2*m^4*x^2*e
^6 + 1134*(x*e + d)^m*b*c*d*f^2*m^3*x^3*e^6 + 1134*(x*e + d)^m*b^2*d*f*g*m^
3*x^3*e^6 + 2268*(x*e + d)^m*a*c*d*f*g*m^3*x^3*e^6 + 1134*(x*e + d)^m*a*b*d
*g^2*m^3*x^3*e^6 + 540*(x*e + d)^m*c^2*d*f^2*m^2*x^4*e^6 + 2160*(x*e + d)^m
*b*c*d*f*g*m^2*x^4*e^6 + 540*(x*e + d)^m*b^2*d*g^2*m^2*x^4*e^6 + 1080*(x*e
+ d)^m*a*c*d*g^2*m^2*x^4*e^6 + 336*(x*e + d)^m*c^2*d*f*g*m*x^5*e^6 + 336*(x
*e + d)^m*b*c*d*g^2*m*x^5*e^6 - 2*(x*e + d)^m*a*b*d^2*f^2*m^5*e^5 - 2*(x*e
+ d)^m*a^2*d^2*f*g*m^5*e^5 - 44*(x*e + d)^m*b^2*d^2*f^2*m^4*x*e^5 - 88*(x*e
 + d)^m*a*c*d^2*f^2*m^4*x*e^5 - 176*(x*e + d)^m*a*b*d^2*f*g*m^4*x*e^5 - 44*
(x*e + d)^m*a^2*d^2*g^2*m^4*x*e^5 - 750*(x*e + d)^m*b*c*d^2*f^2*m^3*x^2*e^5
 - 750*(x*e + d)^m*b^2*d^2*f*g*m^3*x^2*e^5 - 1500*(x*e + d)^m*a*c*d^2*f*g*m
^3*x^2*e^5 - 750*(x*e + d)^m*a*b*d^2*g^2*m^3*x^2*e^5 - 608*(x*e + d)^m*c^2*
d^2*f^2*m^2*x^3*e^5 - 2432*(x*e + d)^m*b*c*d^2*f*g*m^2*x^3*e^5 - 608*(x*e +
 d)^m*b^2*d^2*g^2*m^2*x^3*e^5 - 1216*(x*e + d)^m*a*c*d^2*g^2*m^2*x^3*e^5 -
420*(x*e + d)^m*c^2*d^2*f*g*m*x^4*e^5 - 420*(x*e + d)^m*b*c*d^2*g^2*m*x^4*e
^5 + 2*(x*e + d)^m*b^2*d^3*f^2*m^4*e^4 + 4*(x*e + d)^m*a*c*d^3*f^2*m^4*e^4
+ 8*(x*e + d)^m*a*b*d^3*f*g*m^4*e^4 + 2*(x*e + d)^m*a^2*d^3*g^2*m^4*e^4 + 2
16*(x*e + d)^m*b*c*d^3*f^2*m^3*x*e^4 + 216*(x*e + d)^m*b^2*d^3*f*g*m^3*x*e^
4 + 432*(x*e + d)^m*a*c*d^3*f*g*m^3*x*e^4 + 216*(x*e + d)^m*a*b*d^3*g^2*m^3
*x*e^4 + 660*(x*e + d)^m*c^2*d^3*f^2*m^2*x^2*e^4 + 2640*(x*e + d)^m*b*c*d^3
*f*g*m^2*x^2*e^4 + 660*(x*e + d)^m*b^2*d^3*g^2*m^2*x^2*e^4 + 1320*(x*e + d)
^m*a*c*d^3*g^2*m^2*x^2*e^4 + 560*(x*e + d)^m*c^2*d^3*f*g*m*x^3*e^4 + 560*(x
*e + d)^m*b*c*d^3*g^2*m*x^3*e^4 - 12*(x*e + d)^m*b*c*d^4*f^2*m^3*e^3 - 12*(
x*e + d)^m*b^2*d^4*f*g*m^3*e^3 - 24*(x*e + d)^m*a*c*d^4*f*g*m^3*e^3 - 12*(x
*e + d)^m*a*b*d^4*g^2*m^3*e^3 - 312*(x*e + d)^m*c^2*d^4*f^2*m^2*x*e^3 - 124
8*(x*e + d)^m*b*c*d^4*f*g*m^2*x*e^3 - 312*(x*e + d)^m*b^2*d^4*g^2*m^2*x*e^3
 - 624*(x*e + d)^m*a*c*d^4*g^2*m^2*x*e^3 - 840*(x*e + d)^m*c^2*d^4*f*g*m*x^
2*e^3 - 840*(x*e + d)^m*b*c*d^4*g^2*m*x^2*e^3 + 24*(x*e + d)^m*c^2*d^5*f^2*
m^2*e^2 + 96*(x*e + d)^m*b*c*d^5*f*g*m^2*e^2 + 24*(x*e + d)^m*b^2*d^5*g^2*m
^2*e^2 + 48*(x*e + d)^m*a*c*d^5*g^2*m^2*e^2 + 1680*(x*e + d)^m*c^2*d^5*f*g*
m*x*e^2 + 1680*(x*e + d)^m*b*c*d^5*g^2*m*x*e^2 - 240*(x*e + d)^m*c^2*d^6*f*
g*m*e - 240*(x*e + d)^m*b*c*d^6*g^2*m*e + 720*(x*e + d)^m*c^2*d^7*g^2 + 27*
(x*e + d)^m*a^2*f^2*m^5*x*e^7 + 540*(x*e + d)^m*a*b*f^2*m^4*x^2*e^7 + 540*(
x*e + d)^m*a^2*f*g*m^4*x^2*e^7 + 1219*(x*e + d)^m*b^2*f^2*m^3*x^3*e^7 + 243
8*(x*e + d)^m*a*c*f^2*m^3*x^3*e^7 + 4876*(x*e + d)^m*a*b*f*g*m^3*x^3*e^7 +
1219*(x*e + d)^m*a^2*g^2*m^3*x^3*e^7 + 5090*(x*e + d)^m*b*c*f^2*m^2*x^4*e^7
 + 5090*(x*e + d)^m*b^2*f*g*m^2*x^4*e^7 + 10180*(x*e + d)^m*a*c*f*g*m^2*x^4
*e^7 + 5090*(x*e + d)^m*a*b*g^2*m^2*x^4*e^7 + 2412*(x*e + d)^m*c^2*f^2*m*x^
5*e^7 + 9648*(x*e + d)^m*b*c*f*g*m*x^5*e^7 + 2412*(x*e + d)^m*b^2*g^2*m*x^5
*e^7 + 4824*(x*e + d)^m*a*c*g^2*m*x^5*e^7 + 1680*(x*e + d)^m*c^2*f*g*x^6*e^
7 + 1680*(x*e + d)^m*b*c*g^2*x^6*e^7 + 27*(x*e + d)^m*a^2*d*f^2*m^5*e^6 + 4
90*(x*e + d)^m*a*b*d*f^2*m^4*x*e^6 + 490*(x*e + d)^m*a^2*d*f*g*m^4*x*e^6 +
817*(x*e + d)^m*b^2*d*f^2*m^3*x^2*e^6 + 1634*(x*e + d)^m*a*c*d*f^2*m^3*x^2*
e^6 + 3268*(x*e + d)^m*a*b*d*f*g*m^3*x^2*e^6 + 817*(x*e + d)^m*a^2*d*g^2*m^
3*x^2*e^6 + 1688*(x*e + d)^m*b*c*d*f^2*m^2*x^3*e^6 + 1688*(x*e + d)^m*b^2*d
*f*g*m^2*x^3*e^6 + 3376*(x*e + d)^m*a*c*d*f*g*m^2*x^3*e^6 + 1688*(x*e + d)^
m*a*b*d*g^2*m^2*x^3*e^6 + 252*(x*e + d)^m*c^2*d*f^2*m*x^4*e^6 + 1008*(x*e +
 d)^m*b*c*d*f*g*m*x^4*e^6 + 252*(x*e + d)^m*b^2*d*g^2*m*x^4*e^6 + 504*(x*e
+ d)^m*a*c*d*g^2*m*x^4*e^6 - 50*(x*e + d)^m*a*b*d^2*f^2*m^4*e^5 - 50*(x*e +
```

d)^m*a^2*d^2*f*g*m^4*e^5 - 358*(x*e + d)^m*b^2*d^2*f^2*m^3*x*e^5 - 716*(x*e + d)^m*a*c*d^2*f^2*m^3*x*e^5 - 1432*(x*e + d)^m*a*b*d^2*f*g*m^3*x*e^5 - 358*(x*e + d)^m*a^2*d^2*g^2*m^3*x*e^5 - 1902*(x*e + d)^m*b*c*d^2*f^2*m^2*x^2*e^5 - 1902*(x*e + d)^m*b^2*d^2*f*g*m^2*x^2*e^5 - 3804*(x*e + d)^m*a*c*d^2*f*g*m^2*x^2*e^5 - 1902*(x*e + d)^m*a*b*d^2*g^2*m^2*x^2*e^5 - 336*(x*e + d)^m*c^2*d^2*f^2*m*x^3*e^5 - 1344*(x*e + d)^m*b*c*d^2*f*g*m*x^3*e^5 - 336*(x*e + d)^m*b^2*d^2*g^2*m*x^3*e^5 - 672*(x*e + d)^m*a*c*d^2*g^2*m*x^3*e^5 + 44*(x*e + d)^m*b^2*d^3*f^2*m^3*e^4 + 88*(x*e + d)^m*a*c*d^3*f^2*m^3*e^4 + 176*(x*e + d)^m*a*b*d^3*f*g*m^3*e^4 + 44*(x*e + d)^m*a^2*d^3*g^2*m^3*e^4 + 1284*(x*e + d)^m*b*c*d^3*f^2*m^2*x*e^4 + 1284*(x*e + d)^m*b^2*d^3*f*g*m^2*x*e^4 + 2568*(x*e + d)^m*a*c*d^3*f*g*m^2*x*e^4 + 1284*(x*e + d)^m*a*b*d^3*g^2*m^2*x*e^4 + 504*(x*e + d)^m*c^2*d^3*f^2*m*x^2*e^4 + 2016*(x*e + d)^m*b*c*d^3*f*g*m*x^2*e^4 + 504*(x*e + d)^m*b^2*d^3*g^2*m*x^2*e^4 + 1008*(x*e + d)^m*a*c*d^3*g^2*m*x^2*e^4 - 216*(x*e + d)^m*b*c*d^4*f^2*m^2*e^3 - 216*(x*e + d)^m*b^2*d^4*f*g*m^2*e^3 - 432*(x*e + d)^m*a*c*d^4*f*g*m^2*e^3 - 216*(x*e + d)^m*a*b*d^4*g^2*m^2*e^3 - 1008*(x*e + d)^m*c^2*d^4*f^2*m*x*e^3 - 4032*(x*e + d)^m*b*c*d^4*f*g*m*x*e^3 - 1008*(x*e + d)^m*b^2*d^4*g^2*m*x*e^3 - 2016*(x*e + d)^m*a*c*d^4*g^2*m*x*e^3 + 312*(x*e + d)^m*c^2*d^5*f^2*m*e^2 + 1248*(x*e + d)^m*b*c*d^5*f*g*m*e^2 + 312*(x*e + d)^m*b^2*d^5*g^2*m*e^2 + 624*(x*e + d)^m*a*c*d^5*g^2*m*e^2 - 1680*(x*e + d)^m*c^2*d^6*f*g*e - 1680*(x*e + d)^m*b*c*d^6*g^2*e + 295*(x*e + d)^m*a^2*f^2*m^4*x*e^7 + 2840*(x*e + d)^m*a*b*f^2*m^3*x^2*e^7 + 2840*(x*e + d)^m*a^2*f*g*m^3*x^2*e^7 + 3112*(x*e + d)^m*b^2*f^2*m^2*x^3*e^7 + 6224*(x*e + d)^m*a*c*f^2*m^2*x^3*e^7 + 12448*(x*e + d)^m*a*b*f*g*m^2*x^3*e^7 + 3112*(x*e + d)^m*a^2*g^2*m^2*x^3*e^7 + 5904*(x*e + d)^m*b*c*f^2*m*x^4*e^7 + 5904*(x*e + d)^m*b^2*f*g*m*x^4*e^7 + 11808*(x*e + d)^m*a*c*f*g*m*x^4*e^7 + 5904*(x*e + d)^m*a*b*g^2*m*x^4*e^7 + 1008*(x*e + d)^m*c^2*f^2*x^5*e^7 + 4032*(x*e + d)^m*b*c*f*g*x^5*e^7 + 1008*(x*e + d)^m*b^2*g^2*x^5*e^7 + 2016*(x*e + d)^m*a*c*g^2*x^5*e^7 + 295*(x*e + d)^m*a^2*d*f^2*m^4*e^6 + 2350*(x*e + d)^m*a*b*d*f^2*m^3*x*e^6 + 2350*(x*e + d)^m*a^2*d*f*g*m^3*x*e^6 + 1478*(x*e + d)^m*b^2*d*f^2*m^2*x^2*e^6 + 2956*(x*e + d)^m*a*c*d*f^2*m^2*x^2*e^6 + 5912*(x*e + d)^m*a*b*d*f*g*m^2*x^2*e^6 + 1478*(x*e + d)^m*a^2*d*g^2*m^2*x^2*e^6 + 840*(x*e + d)^m*b*c*d*f^2*m*x^3*e^6 + 840*(x*e + d)^m*b^2*d*f*g*m*x^3*e^6 + 1680*(x*e + d)^m*a*c*d*f*g*m*x^3*e^6 + 840*(x*e + d)^m*a*b*d*g^2*m*x^3*e^6 - 490*(x*e + d)^m*a*b*d^2*f^2*m^3*e^5 - 490*(x*e + d)^m*a^2*d^2*f*g*m^3*e^5 - 1276*(x*e + d)^m*b^2*d^2*f^2*m^2*x*e^5 - 2552*(x*e + d)^m*a*c*d^2*f^2*m^2*x*e^5 - 5104*(x*e + d)^m*a*b*d^2*f*g*m^2*x*e^5 - 1276*(x*e + d)^m*a^2*d^2*g^2*m^2*x*e^5 - 1260*(x*e + d)^m*b*c*d^2*f^2*m*x^2*e^5 - 1260*(x*e + d)^m*b^2*d^2*f*g*m*x^2*e^5 - 2520*(x*e + d)^m*a*c*d^2*f*g*m*x^2*e^5 - 1260*(x*e + d)^m*a*b*d^2*g^2*m*x^2*e^5 + 358*(x*e + d)^m*b^2*d^3*f^2*m^2*e^4 + 716*(x*e + d)^m*a*c*d^3*f^2*m^2*e^4 + 1432*(x*e + d)^m*a*b*d^3*f*g*m^2*e^4 + 358*(x*e + d)^m*a^2*d^3*g^2*m^2*e^4 + 2520*(x*e + d)^m*b*c*d^3*f^2*m*x*e^4 + 2520*(x*e + d)^m*b^2*d^3*f*g*m*x*e^4 + 5040*(x*e + d)^m*a*c*d^3*f*g*m*x*e^4 + 2520*(x*e + d)^m*a*b*d^3*g^2*m*x*e^4 - 1284*(x*e + d)^m*b*c*d^4*f^2*m*e^3 - 1284*(x*e + d)^m*b^2*d^4*f*g*m*e^3 - 2568*(x*e + d)^m*a*c*d^4*f*g*m*e^3 - 1284*(x*e + d)^m*a*b*d^4*g^2*m*e^3 + 1008*(x*e + d)^m*c^2*d^5*f^2*e^2 + 4032*(x*e + d)^m*b*c*d^5*f*g*e^2 + 1008*(x*e + d)^m*b^2*d^5*g^2*e^2 + 2016*(x*e + d)^m*a*c*d^5*g^2*e^2 + 1665*(x*e + d)^m*a^2*f^2*m^3*x*e^7 + 7858*(x*e + d)^m*a*b*f^2*m^2*x^2*e^7 + 7858*(x*e + d)^m*a^2*f*g*m^2*x^2*e^7 + 3796*(x*e + d)^m*b^2*f^2*m*x^3*e^7 + 7592*(x*e + d)^m*a*c*f^2*m*x^3*e^7 + 15184*(x*e + d)^m*a*b*f*g*m*x^3*e^7 + 3796*(x*e + d)^m*a^2*g^2*m*x^3*e^7 + 2520*(x*e + d)^m*b*c*f^2*x^4*e^7 + 2520*(x*e + d)^m*b^2*f*g*x^4*e^7 + 5040*(x*e + d)^m*a*c*f*g*x^4*e^7 + 2520*(x*e + d)^m*a*b*g^2*x^4*e^7 + 1665*(x*e + d)^m*a^2*d*f^2*m^3*e^6 + 5508*(x*e + d)^m*a*b*d*f^2*m^2*x*e^6 + 5508*(x*e + d)^m*a^2*d*f*g*m^2*x*e^6 + 840*(x*e + d)^m*b^2*d*f^2*m*x^2*e^6 + 1680*(x*e + d)^m*a*c*d*f^2*m*x^2*e^6 + 3360*(x*e + d)^m*a*b*d*f*g*m*x^2*e^6 + 840*(x*e + d)^m*a^2*d*g^2*m*x^2*e^6 - 2350*(x*e + d)^m*a*b*d^2*f^2*m^2*e^5 - 2350*(x*e + d)^m*a^2*d^2*f*g*m^2*e^5 - 1680*(x*e + d)^m*b^2*d^2*f^2*m*x*e^5 - 3360*(x*e + d)^m*a*c*d^2*f^2*m*x*e^5 - 6720*(x*e + d)^m*a*b*d^2*f*g*m*x*e^5 - 1680*(x*e + d)^m*a^2*d^2*g^2*m*x*e^5 + 1276*(x*e

```
+ d)^m*b^2*d^3*f^2*m*e^4 + 2552*(x*e + d)^m*a*c*d^3*f^2*m*e^4 + 5104*(x*e +
 d)^m*a*b*d^3*f*g*m*e^4 + 1276*(x*e + d)^m*a^2*d^3*g^2*m*e^4 - 2520*(x*e +
d)^m*b*c*d^4*f^2*e^3 - 2520*(x*e + d)^m*b^2*d^4*f*g*e^3 - 5040*(x*e + d)^m*
a*c*d^4*f*g*e^3 - 2520*(x*e + d)^m*a*b*d^4*g^2*e^3 + 5104*(x*e + d)^m*a^2*f
^2*m^2*x*e^7 + 10548*(x*e + d)^m*a*b*f^2*m*x^2*e^7 + 10548*(x*e + d)^m*a^2*
f*g*m*x^2*e^7 + 1680*(x*e + d)^m*b^2*f^2*x^3*e^7 + 3360*(x*e + d)^m*a*c*f^2
*x^3*e^7 + 6720*(x*e + d)^m*a*b*f*g*x^3*e^7 + 1680*(x*e + d)^m*a^2*g^2*x^3*
e^7 + 5104*(x*e + d)^m*a^2*d*f^2*m^2*e^6 + 5040*(x*e + d)^m*a*b*d*f^2*m*x*e
^6 + 5040*(x*e + d)^m*a^2*d*f*g*m*x*e^6 - 5508*(x*e + d)^m*a*b*d^2*f^2*m*e^
5 - 5508*(x*e + d)^m*a^2*d^2*f*g*m*e^5 + 1680*(x*e + d)^m*b^2*d^3*f^2*e^4 +
 3360*(x*e + d)^m*a*c*d^3*f^2*e^4 + 6720*(x*e + d)^m*a*b*d^3*f*g*e^4 + 1680
*(x*e + d)^m*a^2*d^3*g^2*e^4 + 8028*(x*e + d)^m*a^2*f^2*m*x*e^7 + 5040*(x*e
 + d)^m*a*b*f^2*x^2*e^7 + 5040*(x*e + d)^m*a^2*f*g*x^2*e^7 + 8028*(x*e + d)
^m*a^2*d*f^2*m*e^6 - 5040*(x*e + d)^m*a*b*d^2*f^2*e^5 - 5040*(x*e + d)^m*a^
2*d^2*f*g*e^5 + 5040*(x*e + d)^m*a^2*f^2*x*e^7 + 5040*(x*e + d)^m*a^2*d*f^2
*e^6)/(m^7*e^7 + 28*m^6*e^7 + 322*m^5*e^7 + 1960*m^4*e^7 + 6769*m^3*e^7 + 1
3132*m^2*e^7 + 13068*m*e^7 + 5040*e^7)
```

maple [B] time = 0.04, size = 5890, normalized size = 11.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.79, size = 2034, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*b*f^2/((m^2 + 3*m + 2)*e^
2) + 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*f*g/((m^2 + 3*m +
2)*e^2) + (e*x + d)^(m + 1)*a^2*f^2/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3
+ (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*b^2*f^2/((m^3 + 6*
m^2 + 11*m + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2
*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*c*f^2/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 4*
((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x
+ d)^m*a*b*f*g/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^2 + 3*m + 2)*e^3*x^3 +
(m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a^2*g^2/((m^3 + 6*m^
2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2
*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*
b*c*f^2/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 2*((m^3 + 6*m^2 + 11*m
+ 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*
d^3*e*m*x - 6*d^4)*(e*x + d)^m*b^2*f*g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
*e^4) + 4*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3
 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*c*f*g/((m^4
 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4
+ (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6
*d^4)*(e*x + d)^m*a*b*g^2/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4
 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^
4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d
^4*e*m*x + 24*d^5)*(e*x + d)^m*c^2*f^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 +
274*m + 120)*e^5) + 4*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 +
 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(
m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*b*c*f*g/((m^5 + 1
5*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + ((m^4 + 10*m^3 + 35*m^2 + 50
```

```
*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2
+ 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x
 + d)^m*b^2*g^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 2*(
(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*
d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 -
24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*a*c*g^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^
2 + 274*m + 120)*e^5) + 2*((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*
e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^
3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^
2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^m*c^2*f*g/((m^6 + 2
1*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + 2*((m^5 + 15*m^
4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m
^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3
 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 12
0*d^6)*(e*x + d)^m*b*c*g^2/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 +
1764*m + 720)*e^6) + ((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m
 + 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6
*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*
m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360
*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m*c^2*g^2/((m^7
 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7
)
```

mupad [B] time = 5.38, size = 4871, normalized size = 9.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^2,x)
```

```
[Out] ((d + e*x)^m*(720*c^2*d^7*g^2 + 5040*a^2*d*e^6*f^2 + 1680*a^2*d^3*e^4*g^2 +
 1680*b^2*d^3*e^4*f^2 + 1008*b^2*d^5*e^2*g^2 + 1008*c^2*d^5*e^2*f^2 - 1680*
b*c*d^6*e*g^2 - 1680*c^2*d^6*e*f*g + 358*a^2*d^3*e^4*g^2*m^2 + 358*b^2*d^3*
e^4*f^2*m^2 + 44*a^2*d^3*e^4*g^2*m^3 + 44*b^2*d^3*e^4*f^2*m^3 + 2*a^2*d^3*e
^4*g^2*m^4 + 2*b^2*d^3*e^4*f^2*m^4 + 24*b^2*d^5*e^2*g^2*m^2 + 24*c^2*d^5*e^
2*f^2*m^2 - 5040*a*b*d^2*e^5*f^2 - 2520*a*b*d^4*e^3*g^2 + 3360*a*c*d^3*e^4*
f^2 + 2016*a*c*d^5*e^2*g^2 - 2520*b*c*d^4*e^3*f^2 - 5040*a^2*d^2*e^5*f*g -
2520*b^2*d^4*e^3*f*g + 8028*a^2*d*e^6*f^2*m + 5104*a^2*d*e^6*f^2*m^2 + 1665
*a^2*d*e^6*f^2*m^3 + 295*a^2*d*e^6*f^2*m^4 + 27*a^2*d*e^6*f^2*m^5 + a^2*d*e
^6*f^2*m^6 + 1276*a^2*d^3*e^4*g^2*m + 1276*b^2*d^3*e^4*f^2*m + 312*b^2*d^5*
e^2*g^2*m + 312*c^2*d^5*e^2*f^2*m - 2350*a*b*d^2*e^5*f^2*m^2 - 490*a*b*d^2*
e^5*f^2*m^3 - 50*a*b*d^2*e^5*f^2*m^4 - 2*a*b*d^2*e^5*f^2*m^5 - 216*a*b*d^4*
e^3*g^2*m^2 + 716*a*c*d^3*e^4*f^2*m^2 - 12*a*b*d^4*e^3*g^2*m^3 + 88*a*c*d^3
*e^4*f^2*m^3 + 4*a*c*d^3*e^4*f^2*m^4 + 48*a*c*d^5*e^2*g^2*m^2 - 216*b*c*d^4
*e^3*f^2*m^2 - 12*b*c*d^4*e^3*f^2*m^3 - 2350*a^2*d^2*e^5*f*g*m^2 - 490*a^2*
d^2*e^5*f*g*m^3 - 50*a^2*d^2*e^5*f*g*m^4 - 2*a^2*d^2*e^5*f*g*m^5 - 216*b^2*
d^4*e^3*f*g*m^2 - 12*b^2*d^4*e^3*f*g*m^3 + 6720*a*b*d^3*e^4*f*g - 5040*a*c*
d^4*e^3*f*g + 4032*b*c*d^5*e^2*f*g - 240*b*c*d^6*e*g^2*m - 240*c^2*d^6*e*f*
g*m - 5508*a*b*d^2*e^5*f^2*m - 1284*a*b*d^4*e^3*g^2*m + 2552*a*c*d^3*e^4*f^
2*m + 624*a*c*d^5*e^2*g^2*m - 1284*b*c*d^4*e^3*f^2*m - 5508*a^2*d^2*e^5*f*g
*m - 1284*b^2*d^4*e^3*f*g*m + 1432*a*b*d^3*e^4*f*g*m^2 + 176*a*b*d^3*e^4*f*
g*m^3 + 8*a*b*d^3*e^4*f*g*m^4 - 432*a*c*d^4*e^3*f*g*m^2 - 24*a*c*d^4*e^3*f*
g*m^3 + 96*b*c*d^5*e^2*f*g*m^2 + 5104*a*b*d^3*e^4*f*g*m - 2568*a*c*d^4*e^3*
f*g*m + 1248*b*c*d^5*e^2*f*g*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 196
0*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x*(d + e*x)^m*(5040*a^2*e^7*f^2
+ 8028*a^2*e^7*f^2*m + 5104*a^2*e^7*f^2*m^2 + 1665*a^2*e^7*f^2*m^3 + 295*a^
2*e^7*f^2*m^4 + 27*a^2*e^7*f^2*m^5 + a^2*e^7*f^2*m^6 - 1276*a^2*d^2*e^5*g^2
*m^2 - 1276*b^2*d^2*e^5*f^2*m^2 - 358*a^2*d^2*e^5*g^2*m^3 - 358*b^2*d^2*e^5
*f^2*m^3 - 44*a^2*d^2*e^5*g^2*m^4 - 44*b^2*d^2*e^5*f^2*m^4 - 2*a^2*d^2*e^5*
g^2*m^5 - 2*b^2*d^2*e^5*f^2*m^5 - 312*b^2*d^4*e^3*g^2*m^2 - 312*c^2*d^4*e^3
```

*f^2*m^2 - 24*b^2*d^4*e^3*g^2*m^3 - 24*c^2*d^4*e^3*f^2*m^3 - 720*c^2*d^6*e*g^2*m - 1680*a^2*d^2*e^5*g^2*m - 1680*b^2*d^2*e^5*f^2*m - 1008*b^2*d^4*e^3*g^2*m - 1008*c^2*d^4*e^3*f^2*m + 1284*a*b*d^3*e^4*g^2*m^2 - 2552*a*c*d^2*e^5*f^2*m^2 + 216*a*b*d^3*e^4*g^2*m^3 - 716*a*c*d^2*e^5*f^2*m^3 + 12*a*b*d^3*e^4*g^2*m^4 - 88*a*c*d^2*e^5*f^2*m^4 - 4*a*c*d^2*e^5*f^2*m^5 - 624*a*c*d^4*e^3*g^2*m^2 + 1284*b*c*d^3*e^4*f^2*m^2 - 48*a*c*d^4*e^3*g^2*m^3 + 216*b*c*d^3*e^4*f^2*m^3 + 12*b*c*d^3*e^4*f^2*m^4 + 240*b*c*d^5*e^2*g^2*m^2 + 1284*b^2*d^3*e^4*f*g*m^2 + 216*b^2*d^3*e^4*f*g*m^3 + 12*b^2*d^3*e^4*f*g*m^4 + 240*c^2*d^5*e^2*f*g*m^2 + 5040*a*b*d*e^6*f^2*m + 5040*a^2*d*e^6*f*g*m + 5508*a*b*d*e^6*f^2*m^2 + 2350*a*b*d*e^6*f^2*m^3 + 490*a*b*d*e^6*f^2*m^4 + 50*a*b*d*e^6*f^2*m^5 + 2*a*b*d*e^6*f^2*m^6 + 2520*a*b*d^3*e^4*g^2*m - 3360*a*c*d^2*e^5*f^2*m - 2016*a*c*d^4*e^3*g^2*m + 2520*b*c*d^3*e^4*f^2*m + 1680*b*c*d^5*e^2*g^2*m + 5508*a^2*d*e^6*f*g*m^2 + 2350*a^2*d*e^6*f*g*m^3 + 490*a^2*d*e^6*f*g*m^4 + 50*a^2*d*e^6*f*g*m^5 + 2*a^2*d*e^6*f*g*m^6 + 2520*b^2*d^3*e^4*f*g*m + 1680*c^2*d^5*e^2*f*g*m - 5104*a*b*d^2*e^5*f*g*m^2 - 1432*a*b*d^2*e^5*f*g*m^3 - 176*a*b*d^2*e^5*f*g*m^4 - 8*a*b*d^2*e^5*f*g*m^5 + 2568*a*c*d^3*e^4*f*g*m^2 + 432*a*c*d^3*e^4*f*g*m^3 + 24*a*c*d^3*e^4*f*g*m^4 - 1248*b*c*d^4*e^3*f*g*m^2 - 96*b*c*d^4*e^3*f*g*m^3 - 6720*a*b*d^2*e^5*f*g*m + 5040*a*c*d^3*e^4*f*g*m - 4032*b*c*d^4*e^3*f*g*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(840*a^2*e^4*g^2 + 840*b^2*e^4*f^2 + 638*a^2*e^4*g^2*m + 638*b^2*e^4*f^2*m - 120*c^2*d^4*g^2*m + 179*a^2*e^4*g^2*m^2 + 179*b^2*e^4*f^2*m^2 + 22*a^2*e^4*g^2*m^3 + 22*b^2*e^4*f^2*m^3 + a^2*e^4*g^2*m^4 + b^2*e^4*f^2*m^4 + 1680*a*c*e^4*f^2 + 1276*a*c*e^4*f^2*m - 52*b^2*d^2*e^2*g^2*m^2 - 52*c^2*d^2*e^2*f^2*m^2 - 4*b^2*d^2*e^2*g^2*m^3 - 4*c^2*d^2*e^2*f^2*m^3 + 358*a*c*e^4*f^2*m^2 + 44*a*c*e^4*f^2*m^3 + 2*a*c*e^4*f^2*m^4 + 3360*a*b*e^4*f*g - 168*b^2*d^2*e^2*g^2*m - 168*c^2*d^2*e^2*f^2*m + 2552*a*b*e^4*f*g*m - 104*a*c*d^2*e^2*g^2*m^2 - 8*a*c*d^2*e^2*g^2*m^3 + 420*a*b*d*e^3*g^2*m + 420*b*c*d*e^3*f^2*m + 280*b*c*d^3*e*g^2*m + 716*a*b*e^4*f*g*m^2 + 88*a*b*e^4*f*g*m^3 + 4*a*b*e^4*f*g*m^4 + 420*b^2*d*e^3*f*g*m + 280*c^2*d^3*e*f*g*m + 214*a*b*d*e^3*g^2*m^2 + 36*a*b*d*e^3*g^2*m^3 + 2*a*b*d*e^3*g^2*m^4 - 336*a*c*d^2*e^2*g^2*m + 214*b*c*d*e^3*f^2*m^2 + 36*b*c*d*e^3*f^2*m^3 + 2*b*c*d*e^3*f^2*m^4 + 40*b*c*d^3*e*g^2*m^2 + 214*b^2*d*e^3*f*g*m^2 + 36*b^2*d*e^3*f*g*m^3 + 2*b^2*d*e^3*f*g*m^4 + 40*c^2*d^3*e*f*g*m^2 - 208*b*c*d^2*e^2*f*g*m^2 - 16*b*c*d^2*e^2*f*g*m^3 + 840*a*c*d*e^3*f*g*m + 428*a*c*d*e^3*f*g*m^2 + 72*a*c*d*e^3*f*g*m^3 + 4*a*c*d*e^3*f*g*m^4 - 672*b*c*d^2*e^2*f*g*m))/(e^4*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(42*b^2*e^2*g^2 + 42*c^2*e^2*f^2 + 13*b^2*e^2*g^2*m - 6*c^2*d^2*g^2*m + 13*c^2*e^2*f^2*m + b^2*e^2*g^2*m^2 + c^2*e^2*f^2*m^2 + 84*a*c*e^2*g^2 + 26*a*c*e^2*g^2*m + 2*a*c*e^2*g^2*m^2 + 168*b*c*e^2*f*g + 14*b*c*d*e*g^2*m + 52*b*c*e^2*f*g*m + 14*c^2*d*e*f*g*m + 2*b*c*d*e*g^2*m^2 + 4*b*c*e^2*f*g*m^2 + 2*c^2*d*e*f*g*m^2))/(e^2*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^2*(m + 1)*(d + e*x)^m*(360*c^2*d^5*g^2*m + 5040*a*b*e^5*f^2 + 5040*a^2*e^5*f*g + 5508*a*b*e^5*f^2*m + 5508*a^2*e^5*f*g*m + 156*b^2*d^3*e^2*g^2*m^2 + 156*c^2*d^3*e^2*f^2*m^2 + 12*b^2*d^3*e^2*g^2*m^3 + 12*c^2*d^3*e^2*f^2*m^3 + 2350*a*b*e^5*f^2*m^2 + 490*a*b*e^5*f^2*m^3 + 50*a*b*e^5*f^2*m^4 + 2*a*b*e^5*f^2*m^5 + 840*a^2*d*e^4*g^2*m + 840*b^2*d*e^4*f^2*m + 2350*a^2*e^5*f*g*m^2 + 490*a^2*e^5*f*g*m^3 + 50*a^2*e^5*f*g*m^4 + 2*a^2*e^5*f*g*m^5 + 638*a^2*d*e^4*g^2*m^2 + 638*b^2*d*e^4*f^2*m^2 + 179*a^2*d*e^4*g^2*m^3 + 179*b^2*d*e^4*f^2*m^3 + 22*a^2*d*e^4*g^2*m^4 + 22*b^2*d*e^4*f^2*m^4 + a^2*d*e^4*g^2*m^5 + b^2*d*e^4*f^2*m^5 + 504*b^2*d^3*e^2*g^2*m + 504*c^2*d^3*e^2*f^2*m - 642*a*b*d^2*e^3*g^2*m^2 - 108*a*b*d^2*e^3*g^2*m^3 - 6*a*b*d^2*e^3*g^2*m^4 + 312*a*c*d^3*e^2*g^2*m^2 - 642*b*c*d^2*e^3*f^2*m^2 + 24*a*c*d^3*e^2*g^2*m^3 - 108*b*c*d^2*e^3*f^2*m^3 - 6*b*c*d^2*e^3*f^2*m^4 - 642*b^2*d^2*e^3*f*g*m^2 - 108*b^2*d^2*e^3*f*g*m^3 - 6*b^2*d^2*e^3*f*g*m^4 + 1680*a*c*d*e^4*f^2*m - 840*b*c*d^4*e*g^2*m - 840*c^2*d^4*e*f*g*m - 1260*a*b*d^2*e^3*g^2*m + 1276*a*c*d*e^4*f^2*m^2 + 358*a*c*d*e^4*f^2*m^3 + 44*a*c*d*e^4*f^2*m^4 + 2*a*c*d*e^4*f^2*m^5 + 1008*a*c*d^3*e^2*g^2*m - 1260*b*c*d^2*e^3*f^2*m - 120*b*c*d^4*e*g^

```
2*m^2 - 1260*b^2*d^2*e^3*f*g*m - 120*c^2*d^4*e*f*g*m^2 - 1284*a*c*d^2*e^3*f
*g*m^2 - 216*a*c*d^2*e^3*f*g*m^3 - 12*a*c*d^2*e^3*f*g*m^4 + 624*b*c*d^3*e^2
*f*g*m^2 + 48*b*c*d^3*e^2*f*g*m^3 + 3360*a*b*d*e^4*f*g*m + 2552*a*b*d*e^4*f
*g*m^2 + 716*a*b*d*e^4*f*g*m^3 + 88*a*b*d*e^4*f*g*m^4 + 4*a*b*d*e^4*f*g*m^5
 - 2520*a*c*d^2*e^3*f*g*m + 2016*b*c*d^3*e^2*f*g*m))/(e^5*(13068*m + 13132*
m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (c^2*g^2*x^7*
(d + e*x)^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(
13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)
+ (x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(30*c^2*d^3*g^2*m + 420*a*b*e^3
*g^2 + 420*b*c*e^3*f^2 + 420*b^2*e^3*f*g + 214*a*b*e^3*g^2*m + 214*b*c*e^3*
f^2*m + 214*b^2*e^3*f*g*m + 36*a*b*e^3*g^2*m^2 + 2*a*b*e^3*g^2*m^3 + 36*b*c
*e^3*f^2*m^2 + 2*b*c*e^3*f^2*m^3 + 42*b^2*d*e^2*g^2*m + 42*c^2*d*e^2*f^2*m
+ 36*b^2*e^3*f*g*m^2 + 2*b^2*e^3*f*g*m^3 + 840*a*c*e^3*f*g + 13*b^2*d*e^2*g
^2*m^2 + 13*c^2*d*e^2*f^2*m^2 + b^2*d*e^2*g^2*m^3 + c^2*d*e^2*f^2*m^3 + 428
*a*c*e^3*f*g*m + 84*a*c*d*e^2*g^2*m - 70*b*c*d^2*e*g^2*m + 72*a*c*e^3*f*g*m
^2 + 4*a*c*e^3*f*g*m^3 - 70*c^2*d^2*e*f*g*m + 26*a*c*d*e^2*g^2*m^2 + 2*a*c*
d*e^2*g^2*m^3 - 10*b*c*d^2*e*g^2*m^2 - 10*c^2*d^2*e*f*g*m^2 + 168*b*c*d*e^2
*f*g*m + 52*b*c*d*e^2*f*g*m^2 + 4*b*c*d*e^2*f*g*m^3))/(e^3*(13068*m + 13132
*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (c*g*x^6*(d
+ e*x)^m*(14*b*e*g + 14*c*e*f + 2*b*e*g*m + c*d*g*m + 2*c*e*f*m)*(274*m + 2
25*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(e*(13068*m + 13132*m^2 + 6769*m^3 +
 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**2,x)
[Out] Timed out
```

3.632 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx$

Optimal. Leaf size=311

$$\frac{(d+ex)^{m+4}\left(2ce(aeg-4bdg+bef)+b^2e^2g-2c^2d(2ef-5dg)\right)}{e^6(m+4)}+\frac{(d+ex)^{m+3}\left(2ce(ae(ef-3dg)-3bd(ef-2dg))\right.}{e^6(m}$$

Rubi [A] time = 0.39, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {771}

$$\frac{(d+ex)^{m+4}\left(2ce(aeg-4bdg+bef)+b^2e^2g-2c^2d(2ef-5dg)\right)}{e^6(m+4)}+\frac{(d+ex)^{m+3}\left(2ce(ae(ef-3dg)-3bd(ef-2dg))+be^2(2aeg-3bdg+bef)+2c^2d^2(3ef-5dg)\right)}{e^6(m+3)}+\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)^2}{e^6(m+1)}-\frac{(d+ex)^{m+2}\left(ae^2-bde+cd^2\right)(cd(4ef-5dg)-e(aeg-3bdg+2bef))}{e^6(m+2)}+\frac{c(d+ex)^{m+5}(2beg-5cdg+cef)}{e^6(m+5)}+\frac{c^2g(d+ex)^{m+6}}{e^6(m+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)^(1 + m))/(e^6*(1 + m)) - (
(c*d^2 - b*d*e + a*e^2)*(c*d*(4*e*f - 5*d*g) - e*(2*b*e*f - 3*b*d*g + a*e*g
))*(d + e*x)^(2 + m))/(e^6*(2 + m)) + ((2*c^2*d^2*(3*e*f - 5*d*g) + b*e^2*(
b*e*f - 3*b*d*g + 2*a*e*g) + 2*c*e*(a*e*(e*f - 3*d*g) - 3*b*d*(e*f - 2*d*g)
))*(d + e*x)^(3 + m))/(e^6*(3 + m)) + ((b^2*e^2*g - 2*c^2*d*(2*e*f - 5*d*g)
 + 2*c*e*(b*e*f - 4*b*d*g + a*e*g))*(d + e*x)^(4 + m))/(e^6*(4 + m)) + (c*(
c*e*f - 5*c*d*g + 2*b*e*g)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (c^2*g*(d + e
*x)^(6 + m))/(e^6*(6 + m))
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)^m}{e^5}+\frac{\left(cd^2-bde+ae^2\right)(-cd(4e}{}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)^{1+m}}{e^6(1+m)}-\frac{\left(cd^2-bde+ae^2\right)(cd(4ef-}{}$$

Mathematica [B] time = 1.52, size = 655, normalized size = 2.11

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((a + x*(b + c*x))^2*(2*b*e*g + c*(-5*d*g + e*f*(6 + m)
+ e*g*(5 + m)*x)) + (2*(((c*d^2 + e*(-(b*d) + a*e))*(b^3*e^3*g*(3 + 4*m + m
^2) + 12*c^3*d^2*(-5*d*g + e*f*(6 + m)) - b*c*e^2*(1 + m)*(b*d*g*(-6 + m) +
 b*e*f*(6 + m) + 2*a*e*g*(9 + 2*m)) + 2*c^2*e*(-3*b*d*(d*g*(-9 + m) + 2*e*f
*(6 + m)) + 2*a*e*(d*g*(-15 + m + m^2) + e*f*(24 + 10*m + m^2)))))/(e^2*(1
+ m)) + ((b^4*e^4*g*(6 + 5*m + m^2) + 12*c^4*d^3*(5*d*g - e*f*(6 + m)) - b^
2*c*e^3*(2 + m)*(b*e*f*(6 + m) + b*d*g*(-3 + 2*m) + a*e*g*(21 + 5*m)) + 2*c
```

```
^3*d*e*(3*b*d*(d*g*(-14 + m) + 3*e*f*(6 + m)) - 2*a*e*(d*g*(-30 - 4*m + m^2
) + e*f*(42 + 19*m + 2*m^2))) + c^2*e^2*(4*a^2*e^2*g*(15 + 8*m + m^2) + b^2
*d*(d*g*(6 - 13*m + m^2) + 2*e*f*(-6 + 5*m + m^2)) + 2*a*b*e*(e*f*(42 + 19*
m + 2*m^2) + d*g*(-18 + 11*m + 4*m^2))))*(d + e*x))/(e^2*(2 + m)) - (c*e*(4
 + m)*(b*d*(-5*c*d + 2*b*e)*g - 2*a*c*d*e*g*m + a*b*e^2*g*(1 + m) + c*e*(b*
d - 2*a*e)*f*(6 + m)) - (3*c*d - b*e)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g
+ e*f*(6 + m)) - c*e*(b*d*g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m))) +
c*e*(3 + m)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g + e*f*(6 + m)) - c*e*(b*d*
g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m)))*x)*(a + x*(b + c*x))))/(c*e^
2*(3 + m)*(4 + m))))/(c*e^2*(5 + m)*(6 + m))
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (d+ex)^m (f+gx) \left(a+bx+cx^2\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2, x]
```

fricas [B] time = 0.45, size = 2368, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^5*f*m^5 + (c^2*e^6*g*m^5 + 15*c^2*e^6*g*m^4 + 85*c^2*e^6*g*m^3 + 2
25*c^2*e^6*g*m^2 + 274*c^2*e^6*g*m + 120*c^2*e^6*g)*x^6 + (144*c^2*e^6*f +
288*b*c*e^6*g + (c^2*e^6*f + (c^2*d*e^5 + 2*b*c*e^6)*g)*m^5 + 2*(8*c^2*e^6*
f + (5*c^2*d*e^5 + 16*b*c*e^6)*g)*m^4 + 5*(19*c^2*e^6*f + (7*c^2*d*e^5 + 38
*b*c*e^6)*g)*m^3 + 10*(26*c^2*e^6*f + (5*c^2*d*e^5 + 52*b*c*e^6)*g)*m^2 + 1
2*(27*c^2*e^6*f + 2*(c^2*d*e^5 + 27*b*c*e^6)*g)*m)*x^5 - (a^2*d^2*e^4*g + 2
*(a*b*d^2*e^4 - 10*a^2*d*e^5)*f)*m^4 + (360*b*c*e^6*f + 180*(b^2 + 2*a*c)*e
^6*g + ((c^2*d*e^5 + 2*b*c*e^6)*f + (2*b*c*d*e^5 + (b^2 + 2*a*c)*e^6)*g)*m^
5 + (2*(6*c^2*d*e^5 + 17*b*c*e^6)*f - (5*c^2*d^2*e^4 - 24*b*c*d*e^5 - 17*(b
^2 + 2*a*c)*e^6)*g)*m^4 + ((47*c^2*d*e^5 + 214*b*c*e^6)*f - (30*c^2*d^2*e^4
 - 94*b*c*d*e^5 - 107*(b^2 + 2*a*c)*e^6)*g)*m^3 + (2*(36*c^2*d*e^5 + 307*b*
c*e^6)*f - (55*c^2*d^2*e^4 - 144*b*c*d*e^5 - 307*(b^2 + 2*a*c)*e^6)*g)*m^2
+ 6*(6*(c^2*d*e^5 + 22*b*c*e^6)*f - (5*c^2*d^2*e^4 - 12*b*c*d*e^5 - 66*(b^2
 + 2*a*c)*e^6)*g)*m)*x^4 - ((36*a*b*d^2*e^4 - 155*a^2*d*e^5 - 2*(b^2 + 2*a*
c)*d^3*e^3)*f - 2*(2*a*b*d^3*e^3 - 9*a^2*d^2*e^4)*g)*m^3 + (480*a*b*e^6*g +
 240*(b^2 + 2*a*c)*e^6*f + ((2*b*c*d*e^5 + (b^2 + 2*a*c)*e^6)*f + (2*a*b*e^
6 + (b^2 + 2*a*c)*d*e^5)*g)*m^5 - 2*((2*c^2*d^2*e^4 - 14*b*c*d*e^5 - 9*(b^2
 + 2*a*c)*e^6)*f + (4*b*c*d^2*e^4 - 18*a*b*e^6 - 7*(b^2 + 2*a*c)*d*e^5)*g)*
m^4 - ((36*c^2*d^2*e^4 - 130*b*c*d*e^5 - 121*(b^2 + 2*a*c)*e^6)*f - (20*c^2
*d^3*e^3 - 72*b*c*d^2*e^4 + 242*a*b*e^6 + 65*(b^2 + 2*a*c)*d*e^5)*g)*m^3 -
4*((20*c^2*d^2*e^4 - 56*b*c*d*e^5 - 93*(b^2 + 2*a*c)*e^6)*f - (15*c^2*d^3*e
^3 - 40*b*c*d^2*e^4 + 186*a*b*e^6 + 28*(b^2 + 2*a*c)*d*e^5)*g)*m^2 - 4*((12
*c^2*d^2*e^4 - 30*b*c*d*e^5 - 127*(b^2 + 2*a*c)*e^6)*f - (10*c^2*d^3*e^3 -
24*b*c*d^2*e^4 + 254*a*b*e^6 + 15*(b^2 + 2*a*c)*d*e^5)*g)*m)*x^3 - (2*(6*b*
c*d^4*e^2 + 119*a*b*d^2*e^4 - 290*a^2*d*e^5 - 15*(b^2 + 2*a*c)*d^3*e^3)*f -
 (60*a*b*d^3*e^3 - 119*a^2*d^2*e^4 - 6*(b^2 + 2*a*c)*d^4*e^2)*g)*m^2 + (720
*a*b*e^6*f + 360*a^2*e^6*g + ((2*a*b*e^6 + (b^2 + 2*a*c)*d*e^5)*f + (2*a*b*
d*e^5 + a^2*e^6)*g)*m^5 - (2*(3*b*c*d^2*e^4 - 19*a*b*e^6 - 8*(b^2 + 2*a*c)*
d*e^5)*f - (32*a*b*d*e^5 + 19*a^2*e^6 - 3*(b^2 + 2*a*c)*d^2*e^4)*g)*m^4 + (
(12*c^2*d^3*e^3 - 72*b*c*d^2*e^4 + 274*a*b*e^6 + 89*(b^2 + 2*a*c)*d*e^5)*f
+ (24*b*c*d^3*e^3 + 178*a*b*d*e^5 + 137*a^2*e^6 - 36*(b^2 + 2*a*c)*d^2*e^4)
*g)*m^3 + (2*(42*c^2*d^3*e^3 - 123*b*c*d^2*e^4 + 461*a*b*e^6 + 97*(b^2 + 2*
a*c)*d*e^5)*f - (60*c^2*d^4*e^2 - 168*b*c*d^3*e^3 - 388*a*b*d*e^5 - 461*a^2
```

```
*e^6 + 123*(b^2 + 2*a*c)*d^2*e^4)*g)*m^2 + 6*(2*(6*c^2*d^3*e^3 - 15*b*c*d^2
*e^4 + 117*a*b*e^6 + 10*(b^2 + 2*a*c)*d*e^5)*f - (10*c^2*d^4*e^2 - 24*b*c*d
^3*e^3 - 40*a*b*d*e^5 - 117*a^2*e^6 + 15*(b^2 + 2*a*c)*d^2*e^4)*g)*m)*x^2 +
 24*(6*c^2*d^5*e - 15*b*c*d^4*e^2 - 30*a*b*d^2*e^4 + 30*a^2*d*e^5 + 10*(b^2
 + 2*a*c)*d^3*e^3)*f - 12*(10*c^2*d^6 - 24*b*c*d^5*e - 40*a*b*d^3*e^3 + 30*
a^2*d^2*e^4 + 15*(b^2 + 2*a*c)*d^4*e^2)*g + 2*(2*(6*c^2*d^5*e - 33*b*c*d^4*
e^2 - 171*a*b*d^2*e^4 + 261*a^2*d*e^5 + 37*(b^2 + 2*a*c)*d^3*e^3)*f + (24*b
*c*d^5*e + 148*a*b*d^3*e^3 - 171*a^2*d^2*e^4 - 33*(b^2 + 2*a*c)*d^4*e^2)*g)
*m + (720*a^2*e^6*f + (a^2*d*e^5*g + (2*a*b*d*e^5 + a^2*e^6)*f)*m^5 + 2*((1
8*a*b*d*e^5 + 10*a^2*e^6 - (b^2 + 2*a*c)*d^2*e^4)*f - (2*a*b*d^2*e^4 - 9*a^
2*d*e^5)*g)*m^4 + ((12*b*c*d^3*e^3 + 238*a*b*d*e^5 + 155*a^2*e^6 - 30*(b^2
+ 2*a*c)*d^2*e^4)*f - (60*a*b*d^2*e^4 - 119*a^2*d*e^5 - 6*(b^2 + 2*a*c)*d^3
*e^3)*g)*m^3 - 2*(2*(6*c^2*d^4*e^2 - 33*b*c*d^3*e^3 - 171*a*b*d*e^5 - 145*a
^2*e^6 + 37*(b^2 + 2*a*c)*d^2*e^4)*f + (24*b*c*d^4*e^2 + 148*a*b*d^2*e^4 -
171*a^2*d*e^5 - 33*(b^2 + 2*a*c)*d^3*e^3)*g)*m^2 - 12*((12*c^2*d^4*e^2 - 30
*b*c*d^3*e^3 - 60*a*b*d*e^5 - 87*a^2*e^6 + 20*(b^2 + 2*a*c)*d^2*e^4)*f - (1
0*c^2*d^5*e - 24*b*c*d^4*e^2 - 40*a*b*d^2*e^4 + 30*a^2*d*e^5 + 15*(b^2 + 2*
a*c)*d^3*e^3)*g)*m)*x)*(e*x + d)^m/(e^6*m^6 + 21*e^6*m^5 + 175*e^6*m^4 + 73
5*e^6*m^3 + 1624*e^6*m^2 + 1764*e^6*m + 720*e^6)
```

giac [B] time = 0.35, size = 4940, normalized size = 15.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*g*m^5*x^6*e^6 + (x*e + d)^m*c^2*d*g*m^5*x^5*e^5 + (x*e + d
)^m*c^2*f*m^5*x^5*e^6 + 2*(x*e + d)^m*b*c*g*m^5*x^5*e^6 + 15*(x*e + d)^m*c^
2*g*m^4*x^6*e^6 + (x*e + d)^m*c^2*d*f*m^5*x^4*e^5 + 2*(x*e + d)^m*b*c*d*g*m
^5*x^4*e^5 + 10*(x*e + d)^m*c^2*d*g*m^4*x^5*e^5 - 5*(x*e + d)^m*c^2*d^2*g*m
^4*x^4*e^4 + 2*(x*e + d)^m*b*c*f*m^5*x^4*e^6 + (x*e + d)^m*b^2*g*m^5*x^4*e^
6 + 2*(x*e + d)^m*a*c*g*m^5*x^4*e^6 + 16*(x*e + d)^m*c^2*f*m^4*x^5*e^6 + 32
*(x*e + d)^m*b*c*g*m^4*x^5*e^6 + 85*(x*e + d)^m*c^2*g*m^3*x^6*e^6 + 2*(x*e
+ d)^m*b*c*d*f*m^5*x^3*e^5 + (x*e + d)^m*b^2*d*g*m^5*x^3*e^5 + 2*(x*e + d)^
m*a*c*d*g*m^5*x^3*e^5 + 12*(x*e + d)^m*c^2*d*f*m^4*x^4*e^5 + 24*(x*e + d)^m
*b*c*d*g*m^4*x^4*e^5 + 35*(x*e + d)^m*c^2*d*g*m^3*x^5*e^5 - 4*(x*e + d)^m*c
^2*d^2*f*m^4*x^3*e^4 - 8*(x*e + d)^m*b*c*d^2*g*m^4*x^3*e^4 - 30*(x*e + d)^m
*c^2*d^2*g*m^3*x^4*e^4 + 20*(x*e + d)^m*c^2*d^3*g*m^3*x^3*e^3 + (x*e + d)^m
*b^2*f*m^5*x^3*e^6 + 2*(x*e + d)^m*a*c*f*m^5*x^3*e^6 + 2*(x*e + d)^m*a*b*g*
m^5*x^3*e^6 + 34*(x*e + d)^m*b*c*f*m^4*x^4*e^6 + 17*(x*e + d)^m*b^2*g*m^4*x
^4*e^6 + 34*(x*e + d)^m*a*c*g*m^4*x^4*e^6 + 95*(x*e + d)^m*c^2*f*m^3*x^5*e^
6 + 190*(x*e + d)^m*b*c*g*m^3*x^5*e^6 + 225*(x*e + d)^m*c^2*g*m^2*x^6*e^6 +
 (x*e + d)^m*b^2*d*f*m^5*x^2*e^5 + 2*(x*e + d)^m*a*c*d*f*m^5*x^2*e^5 + 2*(x
*e + d)^m*a*b*d*g*m^5*x^2*e^5 + 28*(x*e + d)^m*b*c*d*f*m^4*x^3*e^5 + 14*(x*
e + d)^m*b^2*d*g*m^4*x^3*e^5 + 28*(x*e + d)^m*a*c*d*g*m^4*x^3*e^5 + 47*(x*e
 + d)^m*c^2*d*f*m^3*x^4*e^5 + 94*(x*e + d)^m*b*c*d*g*m^3*x^4*e^5 + 50*(x*e
+ d)^m*c^2*d*g*m^2*x^5*e^5 - 6*(x*e + d)^m*b*c*d^2*f*m^4*x^2*e^4 - 3*(x*e +
 d)^m*b^2*d^2*g*m^4*x^2*e^4 - 6*(x*e + d)^m*a*c*d^2*g*m^4*x^2*e^4 - 36*(x*e
 + d)^m*c^2*d^2*f*m^3*x^3*e^4 - 72*(x*e + d)^m*b*c*d^2*g*m^3*x^3*e^4 - 55*(
x*e + d)^m*c^2*d^2*g*m^2*x^4*e^4 + 12*(x*e + d)^m*c^2*d^3*f*m^3*x^2*e^3 + 2
4*(x*e + d)^m*b*c*d^3*g*m^3*x^2*e^3 + 60*(x*e + d)^m*c^2*d^3*g*m^2*x^3*e^3
- 60*(x*e + d)^m*c^2*d^4*g*m^2*x^2*e^2 + 2*(x*e + d)^m*a*b*f*m^5*x^2*e^6 +
(x*e + d)^m*a^2*g*m^5*x^2*e^6 + 18*(x*e + d)^m*b^2*f*m^4*x^3*e^6 + 36*(x*e
+ d)^m*a*c*f*m^4*x^3*e^6 + 36*(x*e + d)^m*a*b*g*m^4*x^3*e^6 + 214*(x*e + d)
^m*b*c*f*m^3*x^4*e^6 + 107*(x*e + d)^m*b^2*g*m^3*x^4*e^6 + 214*(x*e + d)^m*
a*c*g*m^3*x^4*e^6 + 260*(x*e + d)^m*c^2*f*m^2*x^5*e^6 + 520*(x*e + d)^m*b*c
*g*m^2*x^5*e^6 + 274*(x*e + d)^m*c^2*g*m*x^6*e^6 + 2*(x*e + d)^m*a*b*d*f*m^
5*x*e^5 + (x*e + d)^m*a^2*d*g*m^5*x*e^5 + 16*(x*e + d)^m*b^2*d*f*m^4*x^2*e^
5 + 32*(x*e + d)^m*a*c*d*f*m^4*x^2*e^5 + 32*(x*e + d)^m*a*b*d*g*m^4*x^2*e^5
```

+ 130*(x*e + d)^m*b*c*d*f*m^3*x^3*e^5 + 65*(x*e + d)^m*b^2*d*g*m^3*x^3*e^5
+ 130*(x*e + d)^m*a*c*d*g*m^3*x^3*e^5 + 72*(x*e + d)^m*c^2*d*f*m^2*x^4*e^5
+ 144*(x*e + d)^m*b*c*d*g*m^2*x^4*e^5 + 24*(x*e + d)^m*c^2*d*g*m*x^5*e^5 -
2*(x*e + d)^m*b^2*d^2*f*m^4*x*e^4 - 4*(x*e + d)^m*a*c*d^2*f*m^4*x*e^4 - 4*
(x*e + d)^m*a*b*d^2*g*m^4*x*e^4 - 72*(x*e + d)^m*b*c*d^2*f*m^3*x^2*e^4 - 36
*(x*e + d)^m*b^2*d^2*g*m^3*x^2*e^4 - 72*(x*e + d)^m*a*c*d^2*g*m^3*x^2*e^4 -
80*(x*e + d)^m*c^2*d^2*f*m^2*x^3*e^4 - 160*(x*e + d)^m*b*c*d^2*g*m^2*x^3*e
^4 - 30*(x*e + d)^m*c^2*d^2*g*m*x^4*e^4 + 12*(x*e + d)^m*b*c*d^3*f*m^3*x*e^
3 + 6*(x*e + d)^m*b^2*d^3*g*m^3*x*e^3 + 12*(x*e + d)^m*a*c*d^3*g*m^3*x*e^3
+ 84*(x*e + d)^m*c^2*d^3*f*m^2*x^2*e^3 + 168*(x*e + d)^m*b*c*d^3*g*m^2*x^2*
e^3 + 40*(x*e + d)^m*c^2*d^3*g*m*x^3*e^3 - 24*(x*e + d)^m*c^2*d^4*f*m^2*x*e
^2 - 48*(x*e + d)^m*b*c*d^4*g*m^2*x*e^2 - 60*(x*e + d)^m*c^2*d^4*g*m*x^2*e^
2 + 120*(x*e + d)^m*c^2*d^5*g*m*x*e + (x*e + d)^m*a^2*f*m^5*x*e^6 + 38*(x*e
+ d)^m*a*b*f*m^4*x^2*e^6 + 19*(x*e + d)^m*a^2*g*m^4*x^2*e^6 + 121*(x*e + d
)^m*b^2*f*m^3*x^3*e^6 + 242*(x*e + d)^m*a*c*f*m^3*x^3*e^6 + 242*(x*e + d)^m
*a*b*g*m^3*x^3*e^6 + 614*(x*e + d)^m*b*c*f*m^2*x^4*e^6 + 307*(x*e + d)^m*b^
2*g*m^2*x^4*e^6 + 614*(x*e + d)^m*a*c*g*m^2*x^4*e^6 + 324*(x*e + d)^m*c^2*f
*m*x^5*e^6 + 648*(x*e + d)^m*b*c*g*m*x^5*e^6 + 120*(x*e + d)^m*c^2*g*x^6*e^
6 + (x*e + d)^m*a^2*d*f*m^5*e^5 + 36*(x*e + d)^m*a*b*d*f*m^4*x*e^5 + 18*(x*
e + d)^m*a^2*d*g*m^4*x*e^5 + 89*(x*e + d)^m*b^2*d*f*m^3*x^2*e^5 + 178*(x*e
+ d)^m*a*c*d*f*m^3*x^2*e^5 + 178*(x*e + d)^m*a*b*d*g*m^3*x^2*e^5 + 224*(x*e
+ d)^m*b*c*d*f*m^2*x^3*e^5 + 112*(x*e + d)^m*b^2*d*g*m^2*x^3*e^5 + 224*(x*
e + d)^m*a*c*d*g*m^2*x^3*e^5 + 36*(x*e + d)^m*c^2*d*f*m*x^4*e^5 + 72*(x*e +
d)^m*b*c*d*g*m*x^4*e^5 - 2*(x*e + d)^m*a*b*d^2*f*m^4*e^4 - (x*e + d)^m*a^2
*d^2*g*m^4*e^4 - 30*(x*e + d)^m*b^2*d^2*f*m^3*x*e^4 - 60*(x*e + d)^m*a*c*d^
2*f*m^3*x*e^4 - 60*(x*e + d)^m*a*b*d^2*g*m^3*x*e^4 - 246*(x*e + d)^m*b*c*d^
2*f*m^2*x^2*e^4 - 123*(x*e + d)^m*b^2*d^2*g*m^2*x^2*e^4 - 246*(x*e + d)^m*a
*c*d^2*g*m^2*x^2*e^4 - 48*(x*e + d)^m*c^2*d^2*f*m*x^3*e^4 - 96*(x*e + d)^m*
b*c*d^2*g*m*x^3*e^4 + 2*(x*e + d)^m*b^2*d^3*f*m^3*e^3 + 4*(x*e + d)^m*a*c*d
^3*f*m^3*e^3 + 4*(x*e + d)^m*a*b*d^3*g*m^3*e^3 + 132*(x*e + d)^m*b*c*d^3*f*
m^2*x*e^3 + 66*(x*e + d)^m*b^2*d^3*g*m^2*x*e^3 + 132*(x*e + d)^m*a*c*d^3*g*
m^2*x*e^3 + 72*(x*e + d)^m*c^2*d^3*f*m*x^2*e^3 + 144*(x*e + d)^m*b*c*d^3*g*
m*x^2*e^3 - 12*(x*e + d)^m*b*c*d^4*f*m^2*e^2 - 6*(x*e + d)^m*b^2*d^4*g*m^2*
e^2 - 12*(x*e + d)^m*a*c*d^4*g*m^2*e^2 - 144*(x*e + d)^m*c^2*d^4*f*m*x*e^2
- 288*(x*e + d)^m*b*c*d^4*g*m*x*e^2 + 24*(x*e + d)^m*c^2*d^5*f*m*e + 48*(x*
e + d)^m*b*c*d^5*g*m*e - 120*(x*e + d)^m*c^2*d^6*g + 20*(x*e + d)^m*a^2*f*m
^4*x*e^6 + 274*(x*e + d)^m*a*b*f*m^3*x^2*e^6 + 137*(x*e + d)^m*a^2*g*m^3*x^
2*e^6 + 372*(x*e + d)^m*b^2*f*m^2*x^3*e^6 + 744*(x*e + d)^m*a*c*f*m^2*x^3*e
^6 + 744*(x*e + d)^m*a*b*g*m^2*x^3*e^6 + 792*(x*e + d)^m*b*c*f*m*x^4*e^6 +
396*(x*e + d)^m*b^2*g*m*x^4*e^6 + 792*(x*e + d)^m*a*c*g*m*x^4*e^6 + 144*(x*
e + d)^m*c^2*f*x^5*e^6 + 288*(x*e + d)^m*b*c*g*x^5*e^6 + 20*(x*e + d)^m*a^2
*d*f*m^4*e^5 + 238*(x*e + d)^m*a*b*d*f*m^3*x*e^5 + 119*(x*e + d)^m*a^2*d*g*
m^3*x*e^5 + 194*(x*e + d)^m*b^2*d*f*m^2*x^2*e^5 + 388*(x*e + d)^m*a*c*d*f*m
^2*x^2*e^5 + 388*(x*e + d)^m*a*b*d*g*m^2*x^2*e^5 + 120*(x*e + d)^m*b*c*d*f*
m*x^3*e^5 + 60*(x*e + d)^m*b^2*d*g*m*x^3*e^5 + 120*(x*e + d)^m*a*c*d*g*m*x^
3*e^5 - 36*(x*e + d)^m*a*b*d^2*f*m^3*e^4 - 18*(x*e + d)^m*a^2*d^2*g*m^3*e^4
- 148*(x*e + d)^m*b^2*d^2*f*m^2*x*e^4 - 296*(x*e + d)^m*a*c*d^2*f*m^2*x*e^
4 - 296*(x*e + d)^m*a*b*d^2*g*m^2*x*e^4 - 180*(x*e + d)^m*b*c*d^2*f*m*x^2*e
^4 - 90*(x*e + d)^m*b^2*d^2*g*m*x^2*e^4 - 180*(x*e + d)^m*a*c*d^2*g*m*x^2*e
^4 + 30*(x*e + d)^m*b^2*d^3*f*m^2*e^3 + 60*(x*e + d)^m*a*c*d^3*f*m^2*e^3 +
60*(x*e + d)^m*a*b*d^3*g*m^2*e^3 + 360*(x*e + d)^m*b*c*d^3*f*m*x*e^3 + 180*
(x*e + d)^m*b^2*d^3*g*m*x*e^3 + 360*(x*e + d)^m*a*c*d^3*g*m*x*e^3 - 132*(x*
e + d)^m*b*c*d^4*f*m*e^2 - 66*(x*e + d)^m*b^2*d^4*g*m*e^2 - 132*(x*e + d)^m
*a*c*d^4*g*m*e^2 + 144*(x*e + d)^m*c^2*d^5*f*e + 288*(x*e + d)^m*b*c*d^5*g*
e + 155*(x*e + d)^m*a^2*f*m^3*x*e^6 + 922*(x*e + d)^m*a*b*f*m^2*x^2*e^6 + 4
61*(x*e + d)^m*a^2*g*m^2*x^2*e^6 + 508*(x*e + d)^m*b^2*f*m*x^3*e^6 + 1016*(
x*e + d)^m*a*c*f*m*x^3*e^6 + 1016*(x*e + d)^m*a*b*g*m*x^3*e^6 + 360*(x*e +
d)^m*b*c*f*x^4*e^6 + 180*(x*e + d)^m*b^2*g*x^4*e^6 + 360*(x*e + d)^m*a*c*g*
x^4*e^6 + 155*(x*e + d)^m*a^2*d*f*m^3*e^5 + 684*(x*e + d)^m*a*b*d*f*m^2*x*e

```
^5 + 342*(x*e + d)^m*a^2*d*g*m^2*x*e^5 + 120*(x*e + d)^m*b^2*d*f*m*x^2*e^5
+ 240*(x*e + d)^m*a*c*d*f*m*x^2*e^5 + 240*(x*e + d)^m*a*b*d*g*m*x^2*e^5 - 2
38*(x*e + d)^m*a*b*d^2*f*m^2*e^4 - 119*(x*e + d)^m*a^2*d^2*g*m^2*e^4 - 240*
(x*e + d)^m*b^2*d^2*f*m*x*e^4 - 480*(x*e + d)^m*a*c*d^2*f*m*x*e^4 - 480*(x*
e + d)^m*a*b*d^2*g*m*x*e^4 + 148*(x*e + d)^m*b^2*d^3*f*m*e^3 + 296*(x*e + d
)^m*a*c*d^3*f*m*e^3 + 296*(x*e + d)^m*a*b*d^3*g*m*e^3 - 360*(x*e + d)^m*b*c
*d^4*f*e^2 - 180*(x*e + d)^m*b^2*d^4*g*e^2 - 360*(x*e + d)^m*a*c*d^4*g*e^2
+ 580*(x*e + d)^m*a^2*f*m^2*x*e^6 + 1404*(x*e + d)^m*a*b*f*m*x^2*e^6 + 702*
(x*e + d)^m*a^2*g*m*x^2*e^6 + 240*(x*e + d)^m*b^2*f*x^3*e^6 + 480*(x*e + d)
^m*a*c*f*x^3*e^6 + 480*(x*e + d)^m*a*b*g*x^3*e^6 + 580*(x*e + d)^m*a^2*d*f*
m^2*e^5 + 720*(x*e + d)^m*a*b*d*f*m*x*e^5 + 360*(x*e + d)^m*a^2*d*g*m*x*e^5
 - 684*(x*e + d)^m*a*b*d^2*f*m*e^4 - 342*(x*e + d)^m*a^2*d^2*g*m*e^4 + 240*
(x*e + d)^m*b^2*d^3*f*e^3 + 480*(x*e + d)^m*a*c*d^3*f*e^3 + 480*(x*e + d)^m
*a*b*d^3*g*e^3 + 1044*(x*e + d)^m*a^2*f*m*x*e^6 + 720*(x*e + d)^m*a*b*f*x^2
*e^6 + 360*(x*e + d)^m*a^2*g*x^2*e^6 + 1044*(x*e + d)^m*a^2*d*f*m*e^5 - 720
*(x*e + d)^m*a*b*d^2*f*e^4 - 360*(x*e + d)^m*a^2*d^2*g*e^4 + 720*(x*e + d)^
m*a^2*f*x*e^6 + 720*(x*e + d)^m*a^2*d*f*e^5)/(m^6*e^6 + 21*m^5*e^6 + 175*m^
4*e^6 + 735*m^3*e^6 + 1624*m^2*e^6 + 1764*m*e^6 + 720*e^6)
```

maple [B] time = 0.02, size = 2563, normalized size = 8.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x)
```

```
[Out] -(e*x+d)^(m+1)*(-c^2*e^5*g*m^5*x^5-2*b*c*e^5*g*m^5*x^4-c^2*e^5*f*m^5*x^4-15
*c^2*e^5*g*m^4*x^5-2*a*c*e^5*g*m^5*x^3-b^2*e^5*g*m^5*x^3-2*b*c*e^5*f*m^5*x^
3-32*b*c*e^5*g*m^4*x^4+5*c^2*d*e^4*g*m^4*x^4-16*c^2*e^5*f*m^4*x^4-85*c^2*e^
5*g*m^3*x^5-2*a*b*e^5*g*m^5*x^2-2*a*c*e^5*f*m^5*x^2-34*a*c*e^5*g*m^4*x^3-b^
2*e^5*f*m^5*x^2-17*b^2*e^5*g*m^4*x^3+8*b*c*d*e^4*g*m^4*x^3-34*b*c*e^5*f*m^4
*x^3-190*b*c*e^5*g*m^3*x^4+4*c^2*d*e^4*f*m^4*x^3+50*c^2*d*e^4*g*m^3*x^4-95*
c^2*e^5*f*m^3*x^4-225*c^2*e^5*g*m^2*x^5-a^2*e^5*g*m^5*x-2*a*b*e^5*f*m^5*x-3
6*a*b*e^5*g*m^4*x^2+6*a*c*d*e^4*g*m^4*x^2-36*a*c*e^5*f*m^4*x^2-214*a*c*e^5*
g*m^3*x^3+3*b^2*d*e^4*g*m^4*x^2-18*b^2*e^5*f*m^4*x^2-107*b^2*e^5*g*m^3*x^3+
6*b*c*d*e^4*f*m^4*x^2+96*b*c*d*e^4*g*m^3*x^3-214*b*c*e^5*f*m^3*x^3-520*b*c*
e^5*g*m^2*x^4-20*c^2*d^2*e^3*g*m^3*x^3+48*c^2*d*e^4*f*m^3*x^3+175*c^2*d*e^4
*g*m^2*x^4-260*c^2*e^5*f*m^2*x^4-274*c^2*e^5*g*m*x^5-a^2*e^5*f*m^5-19*a^2*e
^5*g*m^4*x+4*a*b*d*e^4*g*m^4*x-38*a*b*e^5*f*m^4*x-242*a*b*e^5*g*m^3*x^2+4*a
*c*d*e^4*f*m^4*x+84*a*c*d*e^4*g*m^3*x^2-242*a*c*e^5*f*m^3*x^2-614*a*c*e^5*g
*m^2*x^3+2*b^2*d*e^4*f*m^4*x+42*b^2*d*e^4*g*m^3*x^2-121*b^2*e^5*f*m^3*x^2-3
07*b^2*e^5*g*m^2*x^3-24*b*c*d^2*e^3*g*m^3*x^2+84*b*c*d*e^4*f*m^3*x^2+376*b*
c*d*e^4*g*m^2*x^3-614*b*c*e^5*f*m^2*x^3-648*b*c*e^5*g*m*x^4-12*c^2*d^2*e^3*
f*m^3*x^2-120*c^2*d^2*e^3*g*m^2*x^3+188*c^2*d*e^4*f*m^2*x^3+250*c^2*d*e^4*g
*m*x^4-324*c^2*e^5*f*m*x^4-120*c^2*e^5*g*x^5+a^2*d*e^4*g*m^4-20*a^2*e^5*f*m
^4-137*a^2*e^5*g*m^3*x+2*a*b*d*e^4*f*m^4+64*a*b*d*e^4*g*m^3*x-274*a*b*e^5*f
*m^3*x-744*a*b*e^5*g*m^2*x^2-12*a*c*d^2*e^3*g*m^3*x+64*a*c*d*e^4*f*m^3*x+39
0*a*c*d*e^4*g*m^2*x^2-744*a*c*e^5*f*m^2*x^2-792*a*c*e^5*g*m*x^3-6*b^2*d^2*e
^3*g*m^3*x+32*b^2*d*e^4*f*m^3*x+195*b^2*d*e^4*g*m^2*x^2-372*b^2*e^5*f*m^2*x
^2-396*b^2*e^5*g*m*x^3-12*b*c*d^2*e^3*f*m^3*x-216*b*c*d^2*e^3*g*m^2*x^2+390
*b*c*d*e^4*f*m^2*x^2+576*b*c*d*e^4*g*m*x^3-792*b*c*e^5*f*m*x^3-288*b*c*e^5*
g*x^4+60*c^2*d^3*e^2*g*m^2*x^2-108*c^2*d^2*e^3*f*m^2*x^2-220*c^2*d^2*e^3*g*
m*x^3+288*c^2*d*e^4*f*m*x^3+120*c^2*d*e^4*g*x^4-144*c^2*e^5*f*x^4+18*a^2*d*
e^4*g*m^3-155*a^2*e^5*f*m^3-461*a^2*e^5*g*m^2*x-4*a*b*d^2*e^3*g*m^3+36*a*b*
d*e^4*f*m^3+356*a*b*d*e^4*g*m^2*x-922*a*b*e^5*f*m^2*x-1016*a*b*e^5*g*m*x^2-
4*a*c*d^2*e^3*f*m^3-144*a*c*d^2*e^3*g*m^2*x+356*a*c*d*e^4*f*m^2*x+672*a*c*d
*e^4*g*m*x^2-1016*a*c*e^5*f*m*x^2-360*a*c*e^5*g*x^3-2*b^2*d^2*e^3*f*m^3-72*
b^2*d^2*e^3*g*m^2*x+178*b^2*d*e^4*f*m^2*x+336*b^2*d*e^4*g*m*x^2-508*b^2*e^5
*f*m*x^2-180*b^2*e^5*g*x^3+48*b*c*d^3*e^2*g*m^2*x-144*b*c*d^2*e^3*f*m^2*x-4
80*b*c*d^2*e^3*g*m*x^2+672*b*c*d*e^4*f*m*x^2+288*b*c*d*e^4*g*x^3-360*b*c*e^
```

```
5*f*x^3+24*c^2*d^3*e^2*f*m^2*x+180*c^2*d^3*e^2*g*m*x^2-240*c^2*d^2*e^3*f*m*
x^2-120*c^2*d^2*e^3*g*x^3+144*c^2*d*e^4*f*x^3+119*a^2*d*e^4*g*m^2-580*a^2*e
^5*f*m^2-702*a^2*e^5*g*m*x-60*a*b*d^2*e^3*g*m^2+238*a*b*d*e^4*f*m^2+776*a*b
*d*e^4*g*m*x-1404*a*b*e^5*f*m*x-480*a*b*e^5*g*x^2+12*a*c*d^3*e^2*g*m^2-60*a
*c*d^2*e^3*f*m^2-492*a*c*d^2*e^3*g*m*x+776*a*c*d*e^4*f*m*x+360*a*c*d*e^4*g*
x^2-480*a*c*e^5*f*x^2+6*b^2*d^3*e^2*g*m^2-30*b^2*d^2*e^3*f*m^2-246*b^2*d^2*
e^3*g*m*x+388*b^2*d*e^4*f*m*x+180*b^2*d*e^4*g*x^2-240*b^2*e^5*f*x^2+12*b*c*
d^3*e^2*f*m^2+336*b*c*d^3*e^2*g*m*x-492*b*c*d^2*e^3*f*m*x-288*b*c*d^2*e^3*g
*x^2+360*b*c*d*e^4*f*x^2-120*c^2*d^4*e*g*m*x+168*c^2*d^3*e^2*f*m*x+120*c^2*
d^3*e^2*g*x^2-144*c^2*d^2*e^3*f*x^2+342*a^2*d*e^4*g*m-1044*a^2*e^5*f*m-360*
a^2*e^5*g*x-296*a*b*d^2*e^3*g*m+684*a*b*d*e^4*f*m+480*a*b*d*e^4*g*x-720*a*b
*e^5*f*x+132*a*c*d^3*e^2*g*m-296*a*c*d^2*e^3*f*m-360*a*c*d^2*e^3*g*x+480*a*
c*d*e^4*f*x+66*b^2*d^3*e^2*g*m-148*b^2*d^2*e^3*f*m-180*b^2*d^2*e^3*g*x+240*
b^2*d*e^4*f*x-48*b*c*d^4*e*g*m+132*b*c*d^3*e^2*f*m+288*b*c*d^3*e^2*g*x-360*
b*c*d^2*e^3*f*x-24*c^2*d^4*e*f*m-120*c^2*d^4*e*g*x+144*c^2*d^3*e^2*f*x+360*
a^2*d*e^4*g-720*a^2*e^5*f-480*a*b*d^2*e^3*g+720*a*b*d*e^4*f+360*a*c*d^3*e^2
*g-480*a*c*d^2*e^3*f+180*b^2*d^3*e^2*g-240*b^2*d^2*e^3*f-288*b*c*d^4*e*g+36
0*b*c*d^3*e^2*f+120*c^2*d^5*g-144*c^2*d^4*e*f)/e^6/(m^6+21*m^5+175*m^4+735*
m^3+1624*m^2+1764*m+720)
```

maxima [B] time = 0.67, size = 1118, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*b*f/((m^2 + 3*m + 2)*e^2)
 + (e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*g/((m^2 + 3*m + 2)*e^2
) + (e*x + d)^(m + 1)*a^2*f/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 +
 m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*b^2*f/((m^3 + 6*m^2 + 11*m
 + 6)*e^3) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x
 + 2*d^3)*(e*x + d)^m*a*c*f/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^2 + 3*m
+ 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*b*g
/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^
3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)
*(e*x + d)^m*b*c*f/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^3 + 6*m^
2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2
*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b^2*g/((m^4 + 10*m^3 + 35*m^2 + 50*
m + 24)*e^4) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*
e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*c*g/
((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m
+ 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2
*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x +
d)^m*c^2*f/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 2*((m^4
+ 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4
*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^
4*e*m*x + 24*d^5)*(e*x + d)^m*b*c*g/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274
*m + 120)*e^5) + ((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 +
 (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^
2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^
4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^m*c^2*g/((m^6 + 21*m^5 + 175
*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6)
```

mupad [B] time = 4.38, size = 2307, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^2,x)
```

```
[Out] ((d + e*x)^m*(240*b^2*d^3*e^3*f - 360*a^2*d^2*e^4*g - 120*c^2*d^6*g - 180*b
^2*d^4*e^2*g + 720*a^2*d*e^5*f + 144*c^2*d^5*e*f - 720*a*b*d^2*e^4*f + 480*
a*b*d^3*e^3*g + 480*a*c*d^3*e^3*f - 360*a*c*d^4*e^2*g - 360*b*c*d^4*e^2*f +
 1044*a^2*d*e^5*f*m + 24*c^2*d^5*e*f*m + 580*a^2*d*e^5*f*m^2 + 155*a^2*d*e^
5*f*m^3 + 20*a^2*d*e^5*f*m^4 + a^2*d*e^5*f*m^5 - 342*a^2*d^2*e^4*g*m + 148*
b^2*d^3*e^3*f*m - 66*b^2*d^4*e^2*g*m + 288*b*c*d^5*e*g - 119*a^2*d^2*e^4*g*
m^2 + 30*b^2*d^3*e^3*f*m^2 - 18*a^2*d^2*e^4*g*m^3 + 2*b^2*d^3*e^3*f*m^3 - a
^2*d^2*e^4*g*m^4 - 6*b^2*d^4*e^2*g*m^2 + 48*b*c*d^5*e*g*m - 684*a*b*d^2*e^4
*f*m + 296*a*b*d^3*e^3*g*m + 296*a*c*d^3*e^3*f*m - 132*a*c*d^4*e^2*g*m - 13
2*b*c*d^4*e^2*f*m - 238*a*b*d^2*e^4*f*m^2 - 36*a*b*d^2*e^4*f*m^3 - 2*a*b*d^
2*e^4*f*m^4 + 60*a*b*d^3*e^3*g*m^2 + 60*a*c*d^3*e^3*f*m^2 + 4*a*b*d^3*e^3*g
*m^3 + 4*a*c*d^3*e^3*f*m^3 - 12*a*c*d^4*e^2*g*m^2 - 12*b*c*d^4*e^2*f*m^2))/
(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (x*(d
+ e*x)^m*(720*a^2*e^6*f + 580*a^2*e^6*f*m^2 + 155*a^2*e^6*f*m^3 + 20*a^2*e^
6*f*m^4 + a^2*e^6*f*m^5 + 1044*a^2*e^6*f*m + 360*a^2*d*e^5*g*m + 120*c^2*d^
5*e*g*m - 240*b^2*d^2*e^4*f*m + 342*a^2*d*e^5*g*m^2 + 119*a^2*d*e^5*g*m^3 +
 18*a^2*d*e^5*g*m^4 + a^2*d*e^5*g*m^5 + 180*b^2*d^3*e^3*g*m - 144*c^2*d^4*e
^2*f*m - 148*b^2*d^2*e^4*f*m^2 - 30*b^2*d^2*e^4*f*m^3 - 2*b^2*d^2*e^4*f*m^4
 + 66*b^2*d^3*e^3*g*m^2 - 24*c^2*d^4*e^2*f*m^2 + 6*b^2*d^3*e^3*g*m^3 + 720*
a*b*d*e^5*f*m + 684*a*b*d*e^5*f*m^2 + 238*a*b*d*e^5*f*m^3 + 36*a*b*d*e^5*f*
m^4 + 2*a*b*d*e^5*f*m^5 - 480*a*b*d^2*e^4*g*m - 480*a*c*d^2*e^4*f*m + 360*a
*c*d^3*e^3*g*m + 360*b*c*d^3*e^3*f*m - 288*b*c*d^4*e^2*g*m - 296*a*b*d^2*e^
4*g*m^2 - 296*a*c*d^2*e^4*f*m^2 - 60*a*b*d^2*e^4*g*m^3 - 60*a*c*d^2*e^4*f*m
^3 - 4*a*b*d^2*e^4*g*m^4 - 4*a*c*d^2*e^4*f*m^4 + 132*a*c*d^3*e^3*g*m^2 + 13
2*b*c*d^3*e^3*f*m^2 + 12*a*c*d^3*e^3*g*m^3 + 12*b*c*d^3*e^3*f*m^3 - 48*b*c*
d^4*e^2*g*m^2))/(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6
+ 720)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(120*b^2*e^3*f + 15*b^2*e^3*f*m^
2 + b^2*e^3*f*m^3 + 240*a*b*e^3*g + 240*a*c*e^3*f + 74*b^2*e^3*f*m + 20*c^2
*d^3*g*m + 30*a*b*e^3*g*m^2 + 30*a*c*e^3*f*m^2 + 2*a*b*e^3*g*m^3 + 2*a*c*e^
3*f*m^3 + 30*b^2*d*e^2*g*m - 24*c^2*d^2*e*f*m + 11*b^2*d*e^2*g*m^2 - 4*c^2*
d^2*e*f*m^2 + b^2*d*e^2*g*m^3 + 148*a*b*e^3*g*m + 148*a*c*e^3*f*m + 60*a*c*
d*e^2*g*m + 60*b*c*d*e^2*f*m - 48*b*c*d^2*e*g*m + 22*a*c*d*e^2*g*m^2 + 22*b
*c*d*e^2*f*m^2 + 2*a*c*d*e^2*g*m^3 + 2*b*c*d*e^2*f*m^3 - 8*b*c*d^2*e*g*m^2)
)/(e^3*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (x^4
*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(30*b^2*e^2*g + b^2*e^2*g*m^2 + 60*a*
c*e^2*g + 60*b*c*e^2*f + 11*b^2*e^2*g*m - 5*c^2*d^2*g*m + 2*a*c*e^2*g*m^2 +
 2*b*c*e^2*f*m^2 + c^2*d*e*f*m^2 + 22*a*c*e^2*g*m + 22*b*c*e^2*f*m + 6*c^2*
d*e*f*m + 2*b*c*d*e*g*m^2 + 12*b*c*d*e*g*m))/(e^2*(1764*m + 1624*m^2 + 735*
m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (c^2*g*x^6*(d + e*x)^m*(274*m + 225*
m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4
+ 21*m^5 + m^6 + 720) + (x^2*(m + 1)*(d + e*x)^m*(360*a^2*e^4*g + 119*a^2*e
^4*g*m^2 + 18*a^2*e^4*g*m^3 + a^2*e^4*g*m^4 + 720*a*b*e^4*f + 342*a^2*e^4*g
*m - 60*c^2*d^4*g*m + 238*a*b*e^4*f*m^2 + 36*a*b*e^4*f*m^3 + 2*a*b*e^4*f*m^
4 + 120*b^2*d*e^3*f*m + 72*c^2*d^3*e*f*m + 74*b^2*d*e^3*f*m^2 + 15*b^2*d*e^
3*f*m^3 + b^2*d*e^3*f*m^4 - 90*b^2*d^2*e^2*g*m + 12*c^2*d^3*e*f*m^2 + 684*a
*b*e^4*f*m - 33*b^2*d^2*e^2*g*m^2 - 3*b^2*d^2*e^2*g*m^3 + 240*a*b*d*e^3*g*m
 + 240*a*c*d*e^3*f*m + 144*b*c*d^3*e*g*m + 148*a*b*d*e^3*g*m^2 + 148*a*c*d*
e^3*f*m^2 + 30*a*b*d*e^3*g*m^3 + 30*a*c*d*e^3*f*m^3 + 2*a*b*d*e^3*g*m^4 + 2
*a*c*d*e^3*f*m^4 - 180*a*c*d^2*e^2*g*m - 180*b*c*d^2*e^2*f*m + 24*b*c*d^3*e
*g*m^2 - 66*a*c*d^2*e^2*g*m^2 - 66*b*c*d^2*e^2*f*m^2 - 6*a*c*d^2*e^2*g*m^3
- 6*b*c*d^2*e^2*f*m^3))/(e^4*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^
5 + m^6 + 720)) + (c*x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(1
2*b*e*g + 6*c*e*f + 2*b*e*g*m + c*d*g*m + c*e*f*m))/(e*(1764*m + 1624*m^2 +
 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```